INTERTEMPORAL CHOICES WITH TEMPORAL PREFERENCES

by

Hyeon Sook Park

B.A./M.A. in Economics, Seoul National University, 1992

M.A. in Economics, The University of Chicago, 1997

M.S. in Mathematics & C.S., Chicago State University, 2001

M.S. in Financial Mathematics, The University of Chicago, 2004

Submitted to the Graduate Faculty of

the Kenneth P. Dietrich Arts and Sciences in partial fulfillment

of the requirements for the degree of

Doctor of Philosophy

University of Pittsburgh

2012

UNIVERSITY OF PITTSBURGH

DIETRICH SCHOOL OF ARTS AND SCIENCES

This dissertation was presented

by

Hyeon Sook Park

It was defended on

May 24, 2012

and approved by

John Duffy, Professor of Economics, University of Pittsburgh

James Feigenbaum, Associate Professor of Economics,Utah State University

Marla Ripoll, Associate Professor of Economics, University of Pittsburgh

David DeJong, Professor of Economics, University of Pittsburgh

Dissertation Director: John Duffy, Professor of Economics, University of Pittsburgh

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Hyeon Sook Park, PhD

University of Pittsburgh, 2012

This dissertation explores the general equilibrium implications of inter-temporal decision-making from a behavioral perspective. The decision makers in my essays have psychology-driven, non-traditional preferences and they either have short term planning horizons, due to bounded rationality (Essay 1), or have present biased preferences (Essay 2) or their utilities depend not only on the periodic consumption but are also dependent upon their expectations about present and future optimal consumption (Essay 3). Finally, they get utilities from the act of caring for others through giving and volunteering (Essay 4). The decision makers who are defined by these preferences are re-optimizing over time if they realize that their past decisions for today are no longer optimal and this is the key mechanism that helps replicate the mean lifecycle consumption data which is known to be hump-shaped over the lifecycle. In the first essay, I prove that there is an income structure that leads to a consumption hump for each time preference. Searching via simulation, I find the best planning horizon that is compatible with matching data for the US economy. In the second essay, I find that the consumption hump is obtained even without the credit constraint if the agent is naive and keeps re-optimizing over time. In a third essay, I demonstrate that reference-dependent preferences can also generate a hump-shaped consumption profile when the agent has age-dependent loss aversion. In the fourth and the final essay, I show how the inclusion of time endowment generates full-blown lifecycle pattern of not only consumption, but also giving, leisure, and volunteer time, which closely follow the data.

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PREFACE

I am deeply grateful to:

My advisor and chair John Duffy for directing me through this dissertation process, for introducing behavioral economics which is the building block of my essays, for the essential feedbacks on each essay and invaluable comments after each talk or presentation, the insightful suggestions that greatly affect this manuscript, and the very detailed helps as to rewrite each sentence of my essays into proper and stylish formats;

My advisor James Feigenbaum for initiating me into the subject of this dissertation, for the priceless guidance and precious conversation through each step of this dissertation, for teaching me the inevitable tools for analysis regarding how to compute and calibrate, for giving me opportunities to meet many others who are working on related topics, for caring for my works ardently even after he left University of Pittsburgh for Utah State University;

My advisor Marla Ripoll for giving me many decisive and insightful comments that challenge the initial direction of the manuscript and lead to essential alteration of the dissertation, for inspiring me to search for new ideas in each essay, for teaching me how to be flexible in idea to be applicable to other directions, and for helping me at seminars with how to write good macroeconomics papers;

My advisor Dave DeJong for accepting to be my committee even if he was about to leave the department and expected to be busy at the position of Vice Provost, for teaching me the very first step of macroeconomics through concepts and intuitions, for showing me of a big picture of macroeconomy and macroeconomics discipline, and for reviewing my works and giving valuable comments;

Daniele Coen-Pirani, Sourav Bhattacharya, Andreas Blume, Lise Vesterlund, Luca Rigotti, Frank Caliendo, Geng Li, Nick Guo, Hui He, Scott Findley, Fang Yang, Richard Evans, Daniel Cooper, Kirk Phillips, Kartik Athreya, Stanley Cho, David Eli, Attila Ambrus, Shantanu Bagchi for their helpful comments and feedbacks; Thomas Rawski for giving valuable guideline regarding how to edit each of essays for better presentation; Paul Zak, Arthur Denzau, Jeremy Thornton, and Sara Helms for their encouragement of the fourth essay; all the participants at the workshops of Macroeconomics Seminar at University of Pittsburgh,

Kent State University, Manhattan College for their detailed comments; all the participants at conferences of Midwest Economic Association Annual Meetings (MEA), Quantitative Society for Pension and Savings (QSPS), Society for Advancement of Behavioral Economics (SABE), Econometric Society North American Summer Meetings (NASMES) and Midwest Macroeconomics Meetings (MMM) for many helpful discussions.

1.0 INTRODUCTION

This dissertation explores the general equilibrium implications of inter-temporal decision-making from a behavioral perspective. The essays in this dissertation study inter-temporal choices in relation to income structure, time discounting, behavioral attitudes, and asset accumulation. The optimal inter-temporal choice requires that current and future changes in utility as implied by current behavior correspond to the decision maker's inter-temporal preferences. The individuals in this dissertation are allowed to have preferences beyond those in standard model. The contribution of this dissertation is to explain the following observations: first, the standard inter-temporal model fails to predict many known features of macroeconomic data, especially the lifecycle consumption data. Second, there is experimental evidence, as well as field data showing that economic agents often violate the assumption of standard models.

In this dissertation, I aim to replicate mean lifecycle consumption data which is known to be hump-shaped over the lifecycle: mean consumption is increasing while the consumer is young, reaching a peak around middle age and then decreasing afterwards. Because this prominent characteristic of the consumption profile is *not* expected from the standard economic theory, it is called a puzzle. Among my essays, I specifically attempt to address how the *consumption puzzle* can be solved by extending the standard economic models into two directions: allowing for bounded rationality in inter-temporal choice; and incorporating psychology into decision-making. The agents in my dissertation essays do not necessarily follow the standard norm of rationality assumption, because they may not be fully rational or their preferences are beyond the definition of standard ones.

According to behavioral economics, individuals may deviate from the assumptions of standard economic models, in terms of preference, beliefs, and decision making. Among these, this dissertation mainly concentrates on the element of preferences: the decision makers in my essays have psychology-driven, non-traditional preferences. I want to make it clear that although the models in my essays may posit non-standard preference for the decision maker, they still assume that the decision maker follows rational beliefs and non-biased decision making. The psychology-driven preferences deviate from the standard ones in three respects: time preferences (time-inconsistency), risk preferences (reference-dependence) and social preferences (concern for others). This dissertation studies four economic models that address each of these topics, answering directly the questions above. They are inter-temporal models in which (i) decision makers have short term planning horizons, due to *bounded rationality,* or (ii) decision makers have *present biased preferences* or (iii) the decision maker's utility depends not only on the periodic consumption but is also dependent upon his expectations about present and future optimal consumption (*belief dependent preferences*). Additionally, models in which (iv) decision makers get utility from the act of caring for others through giving and volunteering (*social preferences*).

In the first two essays, I explore the possibility that individuals behave myopically rather than rationally. Specifically, I suppose that they possess either time-inconsistent tastes for immediate gratification or they optimize period-by-period without looking far ahead. In the first essay, the boundedly rational agents have less than full planning horizons and because of this, their consumption profiles are more closely tied with income stream when they move their planning positions forwards. In the second essay, agents have generalized hyperbolic discount factors so that their short-run discount factors are lower than long-run discount factors. Thus their initial consumption should be higher at the price of next period consumption. In the third essay, I examine the plausibility that individual's optimization is affected by comparison to a reference status of utility. If their gain utilities to the reference are strong enough to overcome the more pondering loss utilities, then they may deviate from the standard consumption behavior. In fact, these individuals are rational deviators for their personal well-beings. Finally, in the fourth essay, I demonstrate that unlike the standard assumption of self-interested decision makers, it may be plausible to assume that individuals care for (or are concerned about) others in many ways.

All of the economic models in these essays are based on multi-period overlapping generations models, which have been proved to be a useful analytic tool for inter-temporal economic models. Decision makers who are defined by these preferences are *re-optimizing* over time if they realize that their past decision for today is no longer optimal due to (i) imperfect foresight (Essay 1), (ii) negating their earlier resolution (Essay 2: naive agent), (iii) changing taste (Essay 3: time-varying loss aversion). These are the key elements that allow the decision making under those preferences to generate different path than the standard one. This is because, in standard theory with perfect-foresight and full-rationality, the re-optimization does *not* generate choice sets deviating from the determined path of initial optimization. In this sense, agents' preferences are *temporal*. This deviating outcome is important in macroeconomic dynamics. To address and explain the consumption puzzle, for example, models with standard preferences usually require outside mechanisms or certain frictions unless the model has uncertainty. Those mechanisms include borrowing constraints, mortality risk, choice between consumption and leisure, and consumer durables. However, the models under these specifications generally perform incompetent to the models in my essays: for example, the hump in consumption-leisure model is easily disappearing when the model is added with social security, while it is not in my models.

Although the fourth essay is somewhat different from these first three essays, it is still in the behavioral approach to inter-temporal choice, as decision makers have warm glow motivations (Essay 4) for giving a share of their resources to care for others over the lifecycle. The key feature of this essay is that the model explicitly assumes the endowment of *time* as the main resource agents could exploit and its allocation is incorporated into macroeconomics. The decision makers of the model face trade-offs between private and public choice over the lifecycle because they get utility not only from private consumption of good and time, such as consumption and leisure, but also from public choice behavior such as charitable giving and volunteering. This essay addresses how the private provision of public goods can be made through life, which has not been studied rigorously in macroeconomic model. In this essay, I demonstrate a representative lifecycle pattern of consumption, leisure, charitable giving, and volunteering in US data. The main mechanism generating the hump-shaped lifecycle outcomes is the choice between consumption of goods and of time. Like the choice between consumption and leisure, this mechanism generates a non–monotonic consumption and giving profile. In the essay, the decision makers are subject to mortality risks and this adds another channel reinforcing the non-monotonicity.

2.0 BOUNDED RATIONALITY AND LIFECYCLE CONSUMPTION

This essay explores the general equilibrium characteristics of a lifecycle model with a short term planning horizon, in particular whether or not the model can produce a hump-shaped lifecycle consumption profile similar to the data. Using analytic solution, this essay shows that an increasing income profile, together with exogenously imposed retirement is sufficient to induce a hump for a simple model. Then with no other mechanism that can account for a hump, than the short horizon, the model produces a consumption hump with a location and magnitude consistent with data in a well-calibrated general equilibrium. Unlike partial equilibrium where matching the data is trivial given any parameter set, in general equilibrium, there exists stylized relationship among the parameters for the model to be calibrated with standard macroeconomic targets. Moreover, the macroeconomic predictions of the model are essentially independent of intertemporal elasticity of substitution. Finally, this essay demonstrates that the model with a planning horizon of around 20 years provides a best fit to the salient features of the consumption data, which fact is well supported by behavioral evidence found in surveys on retirement planning.

Keywords: bounded rationality, time inconsistency, short-term planning, lifecycle model, general equilibrium, consumption hump.

2.1 INTRODUCTION

The well-known feature of lifecycle consumption data is that a mean consumption is increasing while the consumer is young, reaching peak around middle age and then decreasing afterwards. This prominent characteristic of the consumption profile is known as the consumption hump.[1] The consumption hump has been the central interest among many researchers who study lifecycle consumptions because this property is not expected from the standard economic theory. The standard lifecycle model, in which the agent is fully rational, predicts a monotone consumption over time if the model does not assume any friction like borrowing constraint or any uncertainty like stochastic income processes. This prediction has a strong theoretic implication in lifecycle model because the monotonicity is achieved regardless of the functional specification for the periodic utility of the model, as long as it satisfies strict concavity.[2] The optimal consumption is expected to be monotonically increasing, decreasing, or staying constant over the agent's life. Therefore, there is no way to break the monotonicity and yield the consumption hump in a standard model.[3]

The discrepancy between the model's prediction and the data has not gone unnoticed.[4] Recently, among researchers who explore the consumption hump, there has been growing interest in explaining the consumer's optimization behavior based on bounded rationality, deviating from the traditional assumption of full rationality but without breaking the general optimization rule. By *Bounded Rationality*, it is meant in a way that the agents experience limits in formulating and solving complex problems. It also means that they experience limits in processing information: receiving, storing, retrieving and transmitting information.[5] Therefore bounded rationality implies that the agent's decision making based on rationality may be incomplete if either the agent does not have full information regarding all the options he could consider or there exists cost, physical or mental, related with decision making. This incompleteness of decision making in bounded rationality models is thought to add other possibilities to solve the lifecycle consumption puzzle.

In this paper, I find that a bounded rationality model in which decision making is adapted to changes in economic environment such as information on new income, can solve the dis-

[1] The earliest one shown in literature is from Thurow (1969)[103].

[2] When the periodic utility is strictly increasing and strictly concave, the Euler equation of the standard model tells that $u'(c_t) = \beta R u'(c_{t+1})$. Because marginal utility is strictly decreasing, it must be satisfied that $c_t < c_{t+1}$ or $c_t > c_{t+1}$ or $c_t = c_{t+1}$, whenever $\beta R > 1$ or $\beta R < 1$ or $\beta R = 1$. Examples with CRRA are in the Appendix.

[3] It is often called "consumption puzzle."

[4] Details are described in the next section.

[5] The term is attributable to Herbert Simon. The description here is from the citation in Williamson, O., (1988)[106], "The Economics of Organization: The Transaction Cost Approach."

crepancy between the lifecycle consumption prediction and its data, by generating a consumption hump in a well-calibrated general equilibrium. Specifically, I find that in a short term planning horizon model, where consumers optimize period by period, by maximizing their utilities only over a subset of entire life,[6] the general equilibrium result can be consistent with the known characteristics of the lifecycle consumption data. In fact, the quantitative result of simulation exercise shows that the model with planning horizon of around 20 years provides a best fit to the consumption profile from US data. I demonstrate that my result in the calibrated general equilibrium is robust to alternative set of parameters within reasonable range.

The implication of general equilibrium result is important because in partial equilibrium, matching the data is trivial given any parameter set. One can fit the data simply by changing the consumer parameters[7] for any target variable like interest rate. In general equilibrium, however, the interest rate and thus the ratio[8] of interest rate to wage rate is no longer a free value. Because both labor market and bond market must clear simultaneously, the equilibrium condition of one market directly affects the other: the excess demand for bonds in an economy should equate the economy wide capital stock; and the aggregate labor supply should be equated to the aggregate labor demand at the same time. Therefore, many partial equilibrium results may not be supported in general equilibrium. Moreover, because of the interdependence of the macroeconomic variables, calibration result may be interpreted more properly in general equilibrium. In this sense, my finding with general equilibrium model is encouraging for the view that bounded rationality can be an alternative way to solve the lifecycle consumption puzzle.

I set up the model with assumption that the agent is not fully rational so that he foresees only to a degree. The agent thus plans for only part of his life and the planning horizon is shorter than the usual lifecycle length. For example, a consumer who is 25 years old may plan consumption or saving schedule only for 5, 10, or 15 years instead of full term of 55 years. It is inferred that the shorter the planning horizon, the greater the degree of bounded rationality. Because the agent does not foresee perfectly, he needs to re-optimize as further

[6] This is related with *time inconsistent preferences* as is discussed in Related Literature.

[7] With CRRA, the consumer parameters are risk aversion coefficient and discount factor.

[8] The ratio of returns to each factor of production.

information or new resolution regarding future plan reveals over time. One may imagine that the agent wants to reset his consumption plan as new information about future income is available over time.[9] In the model I posit that each time the agent re-optimizes for a fixed number of planning horizon.[10] By the mechanism of re-optimization, it is shown that the actual consumption is a series of the initial consumption of each planned path, being an envelope. Moreover, the realized consumption is more closely tied with income stream through this rebalancing. The standard lifecycle model with perfect foresight is a special case of this, in which the planning horizon is exactly equal to the lifecycle length.[11]

The *partial equilibrium* model by Caliendo and Aadland (2007a)[20], aims to explain behavioral phenomena noted in several surveys regarding retirement plan among workers.[12] They use a continuous time control model to explain why a good portion of people do not prepare for the retirement adequately. Because the workers are not far-sighted, they don't think seriously until the retirement is eminent and in view. The authors show, along with the workers' saving behavior, that the model produces a consumption hump in partial equilibrium without resorting to any other assumption than the short term planning horizon.

However, the work of Caliendo and Aadland (2007a)[20] is based on the consumer behavior in partial equilibrium and does not answer the following questions: First, can the short term planning model still produce a consumption hump in a well-calibrated general equilibrium? This question is important because in partial equilibrium, matching the data is trivial given any set of consumer parameters like degree of risk aversion or discount factor. Second, if the answer is "Yes," then which planning horizon does best match consumption data in general equilibrium?[13] The answer to this question is important in providing useful policy guideline for economic issues related with retirement and social security. Third, can all three standard macroeconomic targets, i.e. the interest rate, the capital-output ratio and the consumption-output ratio, be calibrated with reasonable set of parameters for this

[9] Although this may be the simplest logic behind re-optimization, bounded rationality model admits other psychological reasoning as well.

[10] Why would the planning horizon be fixed at a specific number of periods and would not be changing may be another question to ask. I study this in the last section, *Modification*.

[11] The *hand to mouth* consumer is another special case where the planning horizon is zero.

[12] They register many evidences from Retirement Confidence Survey, Health and Retirement Study and Survey of Consumer Finances.

[13] Caliendo and Aadland (2007a)[20] set arbitrarily the planning horizon at certain value.

model? Fourth, is there any stylized relationship among parameters or any restriction on the set of parameters for the short term planning general equilibrium to be compatible with data?

I provide a general equilibrium framework that can answer those questions under quite general classes of parameter values. Like Caliendo and Aadland, I do not assume any other modification than the short term planning horizon to obtain the hump. Using discrete time horizon I first derive a closed form solution for a simple model and propose conditions to generate consumption hump for any choice of time preference. Then using full model I assess how well a calibrated, general equilibrium short term planning model can account for the consumption hump. I consider a $(T+1)$-period overlapping generations general equilibrium model where there are $(T+1)$-types of identical cohorts, who consume, save (or dissave) and supply labor for production every period in a stationary economy. The equilibrium condition specifies that consumption loans cancel out in aggregate so that the excess demand for bonds should be equal to the capital stock.

To calibrate the model, I propose three standard macroeconomic targets mentioned above. The model has four scalar parameters and one planning horizon parameter. The risk aversion coefficient and the discount factor are from consumer side and capital share of production function and depreciation rate are from production side. Among the five parameters, the four scalar parameters are jointly set to match the target, taking the planning horizon as given, to see how closely the general equilibrium model replicates the data as the planning horizon changes. Then I look for the best planning horizon for the consumption profile of the model to fit the mean consumption profile estimated by Gourinchas and Parker (2002)[57].

2.1.1 Main Findings

The short term planning model produces consumption hump, with a reasonable size and location of consumption peak, in a well-calibrated general equilibrium model. I first set the consumer's time preference or the discount factor to be free to have any value. Then for any choice of risk aversion parameter between zero to three, the general equilibrium model produces a consumption hump for each selection of the planning horizon from 5 years to 26 years. If the model limits the discount factor not to exceed one, then the equilibrium condition finds a maximum value of risk aversion coefficient. This value becomes bigger as the planning horizon gets longer. Thus the shorter the planning horizon is, the smaller the acceptable range of the parameter is. I find that the discount factor needs to be increased if the risk aversion parameter is increased and the degree of this increment exaggerates as the planning horizon gets shorter. This implies that in a model with very short horizon, the equilibrium discount factor tends to be greater than one if the risk aversion coefficient is relatively high.[14] However with the optimal choice of planning horizon that fits the data best in general equilibrium, the discount factor is low enough to generate a reasonable value of risk aversion parameter.[15]

Next I find that the short term planning models with identical planning horizons produce essentially the same macro variables independent of risk aversion parameter.[16] This implies that given a planning horizon risk aversion parameter and discount factor are not identified from each other once the model is set to the target variables. Consequently, there are many combinations of the two parameters admissible for any calibration.

Regarding the best parameter value of the planning horizon: the model with planning horizon of 18 years fits best the consumption data in terms of minimum deviation, while by the age of consumption peak and by the ratio between the peaked consumption and the initial one, the model with planning horizon of 20 years and 22 years fits better respectively. Moreover, the rank of these fits is remarkably robust to any alternative selection of the risk aversion parameter between zero to three.

Finally, there are some stylized facts with respect to the two consumer parameters. First, the discount factor becomes larger as risk aversion parameter gets larger for all choices of the planning horizon. Second, for a fixed risk aversion parameter, the discount factor decreases as the planning horizon increases. Third, the shorter the planning horizon is, the faster the necessary increment of the discount factor for an adjustment to higher value of risk aversion

[14] The very *short term planning horizon* may itself imply that the agent tends to evaluate highly the near future.

[15] Or the inverse of the elasticity of substitution.

[16] Feigenbaum (2008c)[47] finds a similar result in his baseline model of precautionary saving.

parameter. Fourth, if the discount factor is set to be less than one, then there exists a highest value of the risk aversion parameter for any choice of planning horizon. Finally, from the result of fourth fact, it follows that the longer the horizon, the wider the range of risk aversion parameter available to the parameter set.

2.1.2 Related Literature

There are many related papers both in the bounded rationality framework and the standard full rationality one. Regarding bounded rationality, *hyperbolic discounting*[17] by Laibson (1997)[69] should be addressed. The model is based on the evidence that people tend to value the immediate utilities differently from future ones. That is, delayed choices are devalued heavily relative to immediate ones, especially in terms of discounting. Unlike the standard exponential discounting, the hyperbolic discounting on which his model is formulated captures this sort of time inconsistency. Laibson posits the dynamically inconsistent agent of a certain time as unique self and sets a T-period consumption and saving problem into T-period dynamic game among the T-type selves facing asset constraints. He finds a unique subgame perfect equilibrium (SPE) strategy under certain assumptions. The empirical implication of his work is that the existence of the commitment device like illiquid asset plays a role that can produce a consumption profile which tracks the income flows. That is, consumption and income can co-move. The short term planning approach by Caliendo and Aadland (2007a)[20] belongs to this kind of time inconsistent preference as well.

Although related with, but in a different perspective of the time inconsistency is *procrastination*. As argued in O'Donoghue and Rabin (1999)[84], the self of an agent may be divided into the naive and the sophisticated who have beliefs about future selves. Only the sophisticated agent has correct beliefs and does not procrastinate. This model implies that when there are many errors in retirement planning for the majority of the economy because of the present biased preferences, policies with cautious paternalism may help solve this problem. Thus, a policy such as tax incentives designed to increase savings may increase the cost of procrastination of saving and can boost the saving. Also infrequent transaction

[17] *Quasi-hyperbolic discounting,* precisely. Also $\beta - \delta$ model or present-biased preference.

dates incurs large costs for procrastinators, while small costs for time consistent selves.

Another line of bounded rationality is the deviation from full information,[18] known as *inattentiveness*. Based on information friction, this approach explicitly assumes cost related with information and due to this cost the planning would take place infrequently. Reis (2006)[92] proposes a partial equilibrium model with inattentive consumer who incurs costs of acquiring, absorbing and processing information in forming expectation and making decision. His result shows that in partial equilibrium with fixed interest rates, inattentive consumers face more uncertainty and save more for precautionary reasons.[19]

Regarding the cost itself, there may be other costs than the information. *Implementation cost* would be one. Caliendo and Huang (2007b)[21] show seven stylized macroeconomic features with just the existence of implementation cost of saving. Other than these, habit formations (Fehrer, 2000[39]), dual-self (Levine and Fudenberg, 2006[73]), rational inattentiveness (Luo, 2005[77]), overconfidence (Caliendo & Huang, 2008[22]), are the ones considered for bounded rationality, as well as near-rationality (Caballero, 1995[19]).

Regarding the lifecycle consumption profile implied by the standard assumption on fully rational agent, several literatures should be mentioned. Borrowing constraint, mortality risk, consumption and leisure substitutability, income uncertainty and precautionary savings are the main issues because these can induce a consumption hump. First, the relative importance of precautionary savings related with the borrowing frictions of the model is well studied in general equilibrium model by Feigenbaum (2008c[47]).

He shows that, along with the consumption hump, in a general equilibrium lifecycle model with an exogenous borrowing constraint, observable macroeconomic variables are insensitive to simultaneous changes in the discount factor and risk aversion coefficient that preserve the equilibrium interest rate. This calibration implication has a common element with my work, in that the two parameters are not identified from each other. His work also demonstrates that the unobservable fraction of aggregate saving due to precautionary motives increases with consumers' risk aversion and the effect of the parameter on observable macro variables

[18] See Sims (2003)[95] and Moscarini (2004)[83].

[19] One can easily show that the inattentiveness induces a consumption hump in a lifecycle model through similar mechanism as precautionary saving in standard rational agent model, although Reis does not explicitly work on this topic.

differs depending on the assumption about borrowing constraint in each proposed model.

If I turn to the other literatures that induce a hump, I address the followings: (1) family size effect (Attanasio et al.,1999[12]), (2) borrowing constraint (Deaton,1991[32]), (3) mortality risk (Feigenbaum, 2008a[45]; Hansen & İmrohoroğlu, 2006[60]), (4) choice between consumption and leisure (Heckman, 1974[61]; Bullard & Feigenbaum, 2007[16]), (5) income uncertainty and precautionary saving (Carroll, 1997[24]; Gourinchas & Parker, 2002[57]; Aiyagari, 1994[1]; Feigenbaum, 2008b[46]), (6) consumer durables (Fernández-Villaverde & Krueger, 2010[49]).

A couple of remarks need to be mentioned. Fernández-Villaverde & Krueger (2010)[49] show how the interaction between durable and nondurable consumption may work to explain the hump in a model where durable goods serve as collateral for loans. Bullard and Feigenbaum's (2007)[16] calibration work with choice between consumption and leisure as well as Heckman's (1974)[61] model produce similar result each other. Mortality risk (Feigenbaum, 2008a[45]; Hansen & İmrohoroğlu, 2006[60]) needs also to be noticed as explanation for the hump.

2.1.3 Chapter Organization

In the following section, I present analytic solutions for a simple model of short term planning horizon. Based on this I prove that the model produces a consumption hump under very reasonable condition. In the section, I present a full model and describe the baseline short term planning general equilibrium. First the partial equilibrium and then the overlapping generations general equilibrium is derived. In fourth section, quantitative analysis follows. Targeting US data, calibration method and results are addressed. It discusses my findings about the short term planning model and reports several sensitivity analyses. Thereafter, two modifications to the model are presented and the chapter concludes.

2.2 A SIMPLE MODEL

Consider a boundedly rational agent who lives four[20] periods, $t = 0, 1, 2, 3$, but maximizes his periodic utility only for two periods, $\tau = \{current,\ next\}$. Assume that the agent has a nonnegative income stream of $\{y_0, y_1, y_2, y_3\}$[21] and his utility is specified by a CRRA utility form, i.e. $u(c)$[22] $= \frac{c^{1-\gamma}}{1-\gamma}$ where γ denotes the degree of risk aversion or the inverse of intertemporal elasticity of substitution. The agent discounts future utility by β. Also assume that there is no borrowing constraint and the agent can borrow or lend freely at the market interest rate R. Then the optimization problem of the boundedly rational agent at $t = 0$ is

$$U_0(c_0, c_1) = Max_{\{c_0,\ c_1\}} \frac{c_0^{1-\gamma}}{1-\gamma} + \beta\frac{c_1^{1-\gamma}}{1-\gamma} \tag{2.1}$$

subject to

$$c_0 + b_1 = y_0$$
$$c_1 = y_1 + Rb_1$$

where b_1 is the bond holdings for the next period. Solving the maximization problem yields the optimal consumption profile for $\tau = \{0, 1\}$, which is

$$c_0 = \frac{y_0 + \frac{y_1}{R}}{1 + \frac{1}{\phi}}$$

$$c_1 = \frac{R}{\phi}\left(\frac{y_0 + \frac{y_1}{R}}{1 + \frac{1}{\phi}}\right)$$

where[23] $\frac{1}{\phi} = \frac{(\beta R)^{1/\gamma}}{R}$. Thus he consumes $c_0 = \frac{y_0+\frac{y_1}{R}}{1+\frac{1}{\phi}}$ at $t = 0$ and *intends* to consume $c_1 = \frac{R}{\phi}\left(\frac{y_0+\frac{y_1}{R}}{1+\frac{1}{\phi}}\right)$ next period. At $t = 1$, however, the agent now foresees his future income

[20] Four is the smallest period to generate a consumption hump for a short term planner.

[21] The assumption of two-period utility maximization implies that each time the agent foresees or considers income of those only up to one period ahead.

[22] CRRA utility is defined by $u(c) = \ln(c)$ when $\gamma = 1$.

[23] $\phi = (\beta R)^{-1/\gamma}R$. This parameter is originally introduced by Feigenbaum (2005)[43]. A value of ϕ contains a combined effect from three parameters, i.e. β, R, and γ.

for the next period $t = 2$ and realizes that the consumption he planned for the period is no longer optimal. Thus, instead of consuming $c_1 = \frac{R}{\phi}\left(\frac{y_0+\frac{y_1}{R}}{1+\frac{1}{\phi}}\right)$ he wants to adjust his consumption according to this future income realization. Therefore at $t = 1$, the boundedly rational agent solves again the following maximization problem:

$$U_1(c_1, c_2) = Max_{\{c_1,\, c_2\}} \ \frac{c_1^{1-\gamma}}{1-\gamma} + \beta \frac{c_2^{1-\gamma}}{1-\gamma} \tag{2.2}$$

subject to

$$c_1 + b_2 = y$$
$$c_2 = y_2 + Rb$$

where $y_1^* = y_1 + Rb_1$.[24] The solution to this problem is the optimal consumption profile for $\tau = \{1, 2\}$, starting from $t = 1$. Notice that the optimal consumption from this maximization is different from what the agent calculated from last period. To emphasis the difference between the new consumption and the one that was planned from last period, it is better to call the *new* consumption c_1^1 by denoting the adjustment for the new consumption plan starting from $t = 1$. Likewise, c_2^1 is the planned consumption for the next period, i.e. $t = 2$, calculated from $t = 1$. Thus,

$$c_1^1 = \frac{y_1^* + \frac{y_2}{R}}{1 + \frac{1}{\phi}}$$

$$c_2^1 = \frac{R}{\phi}\left(\frac{y_1^* + \frac{y_2}{R}}{1 + \frac{1}{\phi}}\right).$$

It is noticeable that the adjusted consumption c_1^1 is different from the planned one c_1^0 in that the new consumption is a function of future income realization y_2. Similarly, the optimal consumption profile for $\tau = \{2, 3\}$, starting from $t = 2$, is

$$c_2^2 = \frac{y_2^* + \frac{y_3}{R}}{1 + \frac{1}{\phi}}$$

$$c_3^2 = \frac{R}{\phi}\left(\frac{y_2^* + \frac{y_3}{R}}{1 + \frac{1}{\phi}}\right).$$

[24] In fact, this term corresponds to the 'cash on hand' by Deaton (1990)[32] but, here it is indexed by $t = 1$.

All together, the adjusted consumption profile for the entire four friends, $t = 0, 1, 2, 3$ is,

$$\{c_0, c_1^1, c_2^2, c_3^2\} = \left\{ \frac{y_0 + \frac{y_1}{R}}{1 + \frac{1}{\phi}},\ \ \frac{y_1^* + \frac{y_2}{R}}{1 + \frac{1}{\phi}},\ \ \frac{y_2^* + \frac{y_3}{R}}{1 + \frac{1}{\phi}},\ \frac{R}{\phi}\left(\frac{y_2^* + \frac{y_3}{R}}{1 + \frac{1}{\phi}} \right) \right\}.$$

The last term in the bracket follows from the fact that the agent no longer needs to adjust his consumption in the last period because no more income process is to be realized. Solving recursively for bond demand of each period and substituting into $y_t^* = y_t + Rb_t$ for $t = 0, 1, 2$ pins down the entire consumption profile. The bond demand is

$$b_1 = \frac{\frac{1}{\phi}y_0 - \frac{1}{R}y_1}{1 + \frac{1}{\phi}}$$

$$b_2 = \frac{\left(\frac{R}{\phi}\right)^2 y_0 + \left(\frac{1}{\phi}\right)^2 Ry_1 - \left(\frac{1}{\phi} + 1\right) y_2}{R(1 + \frac{1}{\phi})^2}.$$

Then the realized consumption $\{c_0, c_1, c_2, c_3\} \equiv \{c_0^0, c_1^1, c_2^2, c_3^2\}$ is obtained from the above consumption equations by substituting the bond demands. The consumption is[25]

$$c_0 = \frac{(Ry_0 + y_1)}{R\left(1 + \frac{1}{\phi}\right)}$$

$$c_1 = \frac{\frac{R}{\phi}(Ry_0 + y_1) + (1 + \frac{1}{\phi})y_2}{R\left(1 + \frac{1}{\phi}\right)^2}$$

$$c_2 = \frac{\left(\frac{R}{\phi}\right)^2 (Ry_0 + y_1) + \frac{R}{\phi}(1 + \frac{1}{\phi})y_2 + \left(1 + \frac{1}{\phi}\right)^2 y_3}{R\left(1 + \frac{1}{\phi}\right)^3}$$

$$c_3 = \frac{R}{\phi}\left(\frac{\left(\frac{R}{\phi}\right)^2 (Ry_0 + y_1) + \frac{R}{\phi}(1 + \frac{1}{\phi})y_2 + \left(1 + \frac{1}{\phi}\right)^2 y_3}{R\left(1 + \frac{1}{\phi}\right)^3} \right).$$

It is worthwhile to see how the consumption is related with income. Rewriting the above to get

[25] One may find how the realized consumption is related to the intended one. For example, $c_1^1 = \frac{\frac{R}{\phi}(Ry_0+y_1)+\left(1+\frac{1}{\phi}\right)y_2}{R\left(1+\frac{1}{\phi}\right)^2} = \frac{\phi}{1+\phi}\left(c_1^0 + \frac{y_2}{R}\right).$

$$\{c_0,\, c_1,\, c_2,\, c_3\}^{Bounded} = \{c_0,\, \left(\tfrac{R}{1+\phi}\right) c_0 + \left(\tfrac{\phi}{1+\phi}\right) \tfrac{y_2}{R},\; \left(\tfrac{R}{1+\phi}\right) c_1 + \left(\tfrac{\phi}{1+\phi}\right) \tfrac{y3}{R},\; \left(\tfrac{R}{\phi}\right) c_2\}$$

where $c_0 = \frac{Ry_0+y_1}{R\left(1+\frac{1}{\phi}\right)}$.

Notice that consumption at each period, except for the initial and the last, is directly related with income of the next period, together with previous consumption. In fact, this is the core property of the bounded rationality model: *re-optimization ties the consumption more closely to income*. To characterize further the consumption property of the boundedly rational agent, first look for the standard lifecycle consumption profile of a fully rational agent who is looking forward up to T. The standard optimization predicts that the marginal utility of consumption between any two periods conforms to the Euler rule and thus the consumption profile over all periods exhibits monotonic movement. By monotonicity the consumption profile of the rational agent is increasing, decreasing or stay constant over the entire life time. Thus the fully rational agent's consumption profile with $T = 3$ is characterized by

$$\begin{aligned} \{c_0,\, c_1,\, c_2,\, c_3\}^{Rational} &= \{c_0, c_0, c_0, c_0\} && \text{if } \beta R = 1 \\ \{c_0,\, c_1,\, c_2,\, c_3\}^{Rational} &= \{c_0, \left(\tfrac{R}{\phi}\right)^1 c_0, \left(\tfrac{R}{\phi}\right)^2 c_0, \left(\tfrac{R}{\phi}\right)^3 c_0\} && \text{if } \beta R \neq 1 \end{aligned}$$

where $c_0 = \dfrac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2} + \frac{y_3}{R^3}}{1 + \frac{1}{\phi^1} + \frac{1}{\phi^2} + \frac{1}{\phi^3}}$.

It is important to notice that the monotonicity of these consumption profiles is preserved across any choice of income processes. By comparing the consumption of the boundedly rational agent with this monotone profile, it is easily seen that for the boundedly rational agent, this monotonicity does not hold any longer except at the last stage.[26] The breakage of monotonicity can be obtained for a boundedly rational consumer even with a shorter life time period and a proof with three-period model is presented at the Appendix. Another property to consider is whether the consumption profile of the boundedly rational agent could generate a consumption hump.[27] To derive analytically the conditions for a hump, I

[26] This comes from the assumption that the boundedly rational agent plans for two periods. If he plans for three periods, then the monotonicity holds for the last two periods.

[27] Precautionary motivation is often proposed to explain this property under uncertainty. With bounded rationality, uncertainty is not necessary for a hump.

want to define first the consumption hump for any T-period consumption profile in a strong sense:[28]

Definition *A consumption hump for T-period model is a consumption profile* $\{c_t\}_{t=0}^{T}$ *that satisfies*

i) There is a consumption peak at time $t \in (0, T)$.

ii) Consumption is monotonically increasing up to t.

iii) Consumption is monotonically decreasing beyond t.

For example, if there are four periods $t = 0, 1, 2, 3$, the hump condition requires either $\{c_0 < c_1 < c_2 > c_3\}$ or $\{c_0 < c_1 > c_2 > c_3\}$ depending on whether the peak occurs at c_1 or c_2. Can the hump be obtained with the four-period model solved above? To explore this, assume that the gross interest rate is greater than zero, i.e. $1 + r = R > 0$ and the net interest rate satisfies $\phi > r$.[29] Furthermore assume that $\gamma > 0$ to ensure CRRA.

Proposition 1. *If the boundedly rational agent's income stream* $\{y_0, y_1, y_2, y_3\}$ *satisfies that* $y_2 > \frac{\left(1-\frac{R}{\phi}+\frac{1}{\phi}\right)}{1+\frac{1}{\phi}}(Ry_0 + y_1)$,[30] *then the optimal consumption is increasing initially, i.e.* $c_0 < c_1$, *regardless of the choice of time preference of the agent. For the agent with* $\beta > 1/R$, *the increasing property is achieved with a weaker condition of* $y_2 > \frac{\frac{1}{\phi}}{1+\frac{1}{\phi}}(Ry_0 + y_1)$.

Proof. First, consider the case of $\beta = 1/R$. Because $\frac{R}{\phi} = 1$, the condition reduces to $y_2 + \frac{y_2}{R} > y_0 + \frac{y_1}{R}$ and the consumption of $t = 0$ and $t = 1$ is $c_0 = \frac{Ry_0+y_1}{R+1}$ and $c_1 = \frac{(Ry_0+y_1)+\left(y_2+\frac{y_2}{R}\right)}{(R+1)\left(1+\frac{1}{R}\right)}$. Thus if the condition is satisfied, then $(y_2 + \frac{y_2}{R}) + (Ry_0 + y_1) > (y_0 + \frac{y_1}{R}) + (Ry_0 + y_1)$, RHS of which equals to $(1 + \frac{1}{R})(Ry_0 + y_1)$. Therefore, $c_0 < c_1$. Second, consider the case of $\beta > 1/R$. Because $\frac{R}{\phi} > 1$, it is satisfied that $y_2 + \frac{y_2}{\phi} > \frac{1}{\phi}(Ry_0 + y_1) > \left(1 - \frac{R}{\phi} + \frac{1}{\phi}\right)(Ry_0 + y_1)$. Therefore, $c_0 < c_1$. Third, consider the case of $\beta < 1/R$. Because $\frac{R}{\phi} < 1$, it is satisfied that either $y_2 + \frac{y_2}{\phi} > \left(1 - \frac{R}{\phi} + \frac{1}{\phi}\right)(Ry_0 + y_1) > \frac{1}{\phi}(Ry_0 + y_1)$ or $\left(1 - \frac{R}{\phi} + \frac{1}{\phi}\right)(Ry_0 + y_1) > y_2 + \frac{y_2}{\phi} > \frac{1}{\phi}(Ry_0 + y_1)$. It is clear that only the first one induces $c_0 < c_1$. Therefore, $c_0 < c_1$ regardless of β. □

[28] In a weak sense, it allows to have any wiggle over the horizon with several local peaks.

[29] These assumptions are just for computational purpose. In fact, these assumptions are not restrictive at all and easily satisfied in general.

[30] The meaning of the inequality condition is explained following the proof.

To see the meaning of the condition, rewrite the condition to get

$$y_2 + \tfrac{y_2}{\phi} > \left(1 - \tfrac{R}{\phi} + \tfrac{1}{\phi}\right)(Ry_0 + y_1).$$

If $\beta = 1/R$, then the condition yields $y_2 + \frac{y_2}{R} > y_0 + \frac{y_1}{R}$.[31] In line of roughly hump-shaped income data, this inequality implies: if y_2 is greater than y_0 and if either y_2 is as good as y_1 or, in case y_2 is big enough relative to y_0, if not too small compared to y_1, then the increasing property of the consumption at early stage of life is obtained, i.e. $c_0 < c_1$. Notice that this condition is always satisfied with increasing income profile up to $t = 2$. But this condition may not be satisfied if y_2 is much smaller than y_1.

Now turn to the other two cases. If $\beta < 1/R$ then because $R/\phi < 1$, the condition says that $y_2 + \frac{y_2}{\phi} > \left(1 - \frac{R}{\phi} + \frac{1}{\phi}\right)(Ry_0 + y_1) > \frac{1}{\phi}(Ry_0 + y_1) > y_0 + \frac{y_1}{\phi}$. This inequality implies that, given γ and R, the increasing property of consumption requires higher income level of y_2 than the one of the case with $\beta = 1/R$. Conversely, if $\beta > 1/R$, then the condition implies that given γ and R, the increasing property of consumption can be achieved with lower y_2. Intuitively, when $\beta > 1/R$, the initial saving is greater and this helps maintain consumption to grow even with lower income level at later periods. Therefore, given any combination of γ and R, the desired level of y_2 for the increasing consumption profile, has the ordering of $y_2^{\beta>1/R} < y_2^{\beta=1/R} < y_2^{\beta<1/R}$, while keeping both y_0 and y_1 the same for all three cases.

Proposition 2. *If the boundedly rational agent's income stream $\{y_0, y_1, y_2, y_3\}$ satisfies that there is at least one non-zero income except for the last period and the last income is sufficiently small, i.e. $y_T = y_3 = \varepsilon$, then $c_1 > c_2$, regardless of the choice of time preference of the agent. The sufficient condition of the last income is $y_T = y_3 = 0$.*

Proof. From the consumption profile obtained above, the condition for decreasing, i.e. $c_1 > c_2$ is $\left(1 - \frac{R}{\phi} + \frac{1}{\phi}\right)\left(\frac{R}{\phi}(Ry_0 + y_1) + \left(1 + \frac{1}{\phi}\right)y_2\right) > \left(1 + \frac{1}{\phi}\right)^2 y_3$. When $\beta = 1/R$, the inequality condition reduces to $\frac{1}{R}\left(Ry_0 + y_1 + \left(1 + \frac{1}{R}\right)y_2\right) > \left(1 + \frac{1}{R}\right)^2 y_3$. Therefore if the last period income y_3 is sufficiently small so that $y_3 = \varepsilon < (1 + \frac{1}{R})^{-2}\left(y_0 + \frac{1}{R}y_1 + \frac{1}{R}(1 + \frac{1}{R})y_2\right)$, and at least one income among $\{y_0, y_1, y_2\}$ is non-zero, then $c_1 > c_2$. Because with non-negative income stream it is always true that $y_0 + \frac{1}{R}y_1 + \frac{1}{R}(1 + \frac{1}{R})y_2 > 0$, thus $\varepsilon = 0$ is the sufficient

[31] Notice that there is no longer ϕ in the expression, implying γ does not play any role for this inequality.

condition for $c_1 > c_2$. If $\beta < 1/R$, it is true that $r < R < \phi$ because $r < \phi$. Likewise, if $\beta > 1/R$, it is true that $r < \phi < R$. In either case, it is satisfied that $\left(1 - \frac{R}{\phi} + \frac{1}{\phi}\right) > 0$. Therefore, if y_3 is sufficiently small, then $c_1 > c_2$ for all choices of β. The sufficient condition is $y_3 = 0 < \left(1 + \frac{1}{\phi}\right)^{-2} \left(1 - \frac{R}{\phi} + \frac{1}{\phi}\right) \left(\frac{R}{\phi}(Ry_0 + y_1) + \left(1 + \frac{1}{\phi}\right) y_2\right)$. □

It is easily seen that if the last period income is zero such that $y_T = y_3 = 0$, or very small[32] like $y_3 = \varepsilon$, then the inequality holds for general classes of parameter values. Therefore, if retirement is exogenously imposed, then this condition is seldom violated. Finally, combining both conditions yields the consumption hump.

Proposition 3. *If the boundedly rational agent's income stream $\{y_0, y_1, y_2, y_3\}$ satisfies both (A): $y_2 > \frac{\left(1 - \frac{R}{\phi} + \frac{1}{\phi}\right)}{1 + \frac{1}{\phi}}(Ry_0 + y_1)$ and (B): $y_3 = 0$, then the consumption profile of the agent produces a hump regardless of the choice of time preference of the agent.*

Proof. For the agent with $\beta \leqslant 1/R$, (A) is a necessary and sufficient condition for $c_0 < c_1$ and (B) is a sufficient condition for $c_1 > c_2$. Similarly, for the agent with $\beta > 1/R$, (A) is a sufficient condition for $c_0 < c_1$ and (B) is also a sufficient condition for $c_1 > c_2$. Therefore, combining both conditions yields $c_0 < c_1 > c_2$ regardless of time preferences. Thus the consumption hump is achieved for $\{c_0, c_1, c_2\}$.[33] □

Example *Suppose $\{y_0, y_1, y_2, y_3\} = \{1, 3, 2, 0\}$.[34] Then the consumption profile of a boundedly rational agent who is forward looking only for two periods and has $1/R$ for his time preference, is characterized by*

$\{c_0,\ c_1,\ c_2,\ c_3\}^{Bounded} = \left\{\frac{3+R}{1+R},\ \frac{5+R+\frac{2}{R}}{(1+R)(1+\frac{1}{R})},\ \frac{5+R+\frac{2}{R}}{(1+R)(1+\frac{1}{R})^2},\ \frac{5+R+\frac{2}{R}}{(1+R)(1+\frac{1}{R})^2}\right\}$. *Using standard annual interest rate $R_{ann} = 1.035$ and per length $\tau = 15$* year, *this yields* $\{c_0,\ c_1,\ c_2,\ c_3\}^{Bounded} = \{1.748,\ 1.842,\ 1.153,\ 1.153\}$. *Clearly, $c_0 < c_1 > c_2$ and a hump is achieved.*

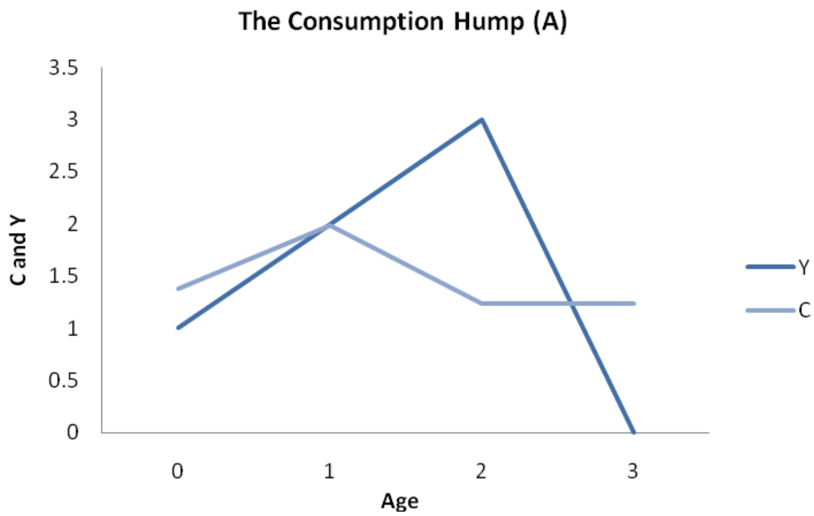
The following two graphs Figure 1 and 2 explain these propositions. The second graph (B) replicates the example above and the first graph (A) is the obtained with $\{y_0, y_1, y_2, y_3\} =$

[32] The common application for the case of zero income in the last period is retirement and the case of small income is social security.

[33] The last consumption c_3 follows the standard path starting from c_2, increasing, constant, or decreasing depending on $\beta >=< 1/R$ and may not be the main interest of the analysis.

[34] This income stream is the case where y_2 is greater than y_0 and y_2 is not too small relative to y_1. But with increasing income profile $\{1, 2, 3, 0\}$, the hump is always achieved with $\beta = 1/R$.

Figure 1: The consumption hump in a simple model. The income stream is $\{y_0, y_1, y_2, y_3\} = \{1, 2, 3, 0\}$.

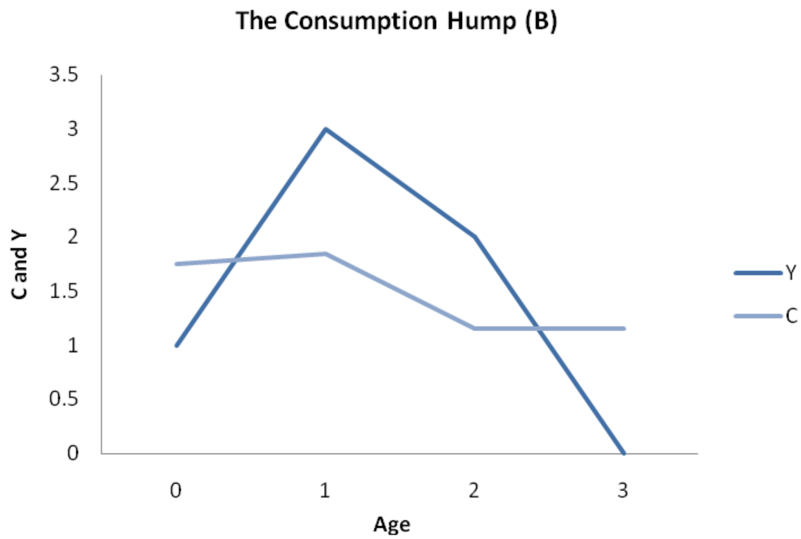


$\{1, 2, 3, 0\}$. The consumption of the last period is residual and because $\beta = 1/R$ it is equal to the consumption of previous period. That is $c_3 = c_2$. By the both figures, it may be noticed that the consumption peak comes no later than the income peak, which fact is related with the length of planning horizon relative to the lifecycle horizon in the short term planning model. In fact, the consumption and income data (Section 4) show that the age of consumption peak comes slightly earlier than that of income peak. In this four period model, the overall period is too small to show this detailed characteristic. Also one cannot miss the observation that the initial consumption is higher than the initial income, which is also supported by the data. I revisit these properties in the quantitative analysis with full model.

2.3 A LIFECYCLE MODEL WITH SHORT TERM PLANNING

In this section, the full model with short-term planning horizon is presented, first in partial equilibrium and then in general equilibrium by including technology in an overlapping

Figure 2: The consumption hump in a simple model. The income stream is $\{y_0, y_1, y_2, y_3\} = \{1, 3, 2, 0\}$.



generations economy.

2.3.1 Consumer

2.3.1.1 Environment Time is discrete and denoted by τ. At each time, a generation of identical cohorts is born. The population is constant over time and each agent who is indexed by age t, lives for $T+1$ periods in a $(T+1)-$ period overlapping generations economy. During working periods agents are endowed with one unit of labor productivity, measured in efficiency units, which is supplied inelastically. There is no borrowing constraint, so that agents can borrow and lend freely under market determined interest rate R. There is no government, and there exists a single good which can be either consumed or saved, in which case it is called capital. There is no uncertainty in this model. However, the agents are not fully rational to foresee perfectly all the way up to their life time T. Instead, they care and plan only up to S, which is shorter than T. Finally, the retirement occurs exogenously at $t = T_w+1$ where $T_w < T$.

2.3.1.2 Consumer Optimization Let us define the consumption notation first. A consumer who is in age t with a planning horizon S has a consumption denoted by $c_s^S(t)$. The subscript s represents consumption time. The age t also represents planning time in the model.[35] For convenience, planning horizon is suppressed whenever the notation is unambiguous. Then the representative consumer who plans for S+1 periods at each planning time, maximizes for $t = 0, 1, ..., T$,

$$U(t) = \sum_{s=t}^{t+S} \beta^{s-t} \frac{c_s^{1-\gamma}(t)}{1-\gamma} \tag{2.3}$$

subject to

$$c_s(t) + b_{s+1}(t) = we_s + Rb_s(t)$$
$$b_0(0) = 0,\ b_t(t) = given,\ \ b_{t+S+1}(t) = 0$$[36]

where $c_s(t)$ is consumption planned at t for time s and $b_{s+1}(t)$ is bond demand purchased at s for the next period, indexed by planning time t. The consumer has a stream of productivity profile over life time so that he supplies e_s efficiency units of labor at s into production and earns labor income of we_s each time, where w is the market determined real wage rate which is assumed to be stationary over time. Because the consumer can save or borrow freely under market determined interest rate, he will earn financial income of $Rb_s(t)$ or incur financial cost of $Rb_s(t)$ if he carries bond to the next period. One thing to notice is that the model does not restrain β less than one.[37] This implies that a consumer with short term planning horizon may have higher evaluation on future consumption than current one.

Solving the consumer optimization problem implies that for $s = t, ..., t$+S–1,

$$c_s^{-\gamma}(t) = (R\beta)c_{s+1}^{-\gamma}(t)$$

or

$$c_{s+1}(t) = (R\beta)^{1/\gamma}c_s(t)$$

[35] The physical age is $t + 25$ if consumers start working when they are 25 years old.

[36] This implies that the agent plans to have no debt or saving by the end of a planning term $[t,\, t+S]$.

[37] Because the model assumes finite time horizon, the usual restriction on β for infinite time horizon is not necessary.

Let $c_t(t)$ be the optimal initial consumption starting from the planning date (or age) t. Using budget constraint, one can pin down $c_t(t)$ which is

$$c_t(t) = \frac{\sum_{s=t}^{t+S} \frac{we_s}{R^{s-t}} + Rb_t(t)}{\sum_{s=t}^{S} \left[\frac{(R\beta)^{1/\gamma}}{R}\right]^{s-t}}$$

where $b_t(t)$ is the initial bond holding at each planning date t and $b_0(0) = 0$. Let us define the total wealth at any time τ over the planning horizon $[t,\ t+S]$, as the sum of human wealth and financial wealth:

$$W_\tau(t) = h_\tau(t) + Rb_\tau(t)$$

where

$$h_\tau(t) = \sum_{s=t}^{t+S} \frac{we_s}{R^{s-\tau}}.$$

Then this produces a compact form for the initial consumption at each planning time t, i.e. $c_t(t)$, which is

$$c_t(t) = \frac{W_t(t)}{\sum_{s=t}^{t+S} \left[\frac{1}{\phi}\right]^{s-t}}$$

where $\phi^{-1} = (R\beta)^{1/\gamma} R^{-1}$. Therefore, for any τ over the planning horizon $[t,\ t+S]$, it is satisfied that

$$c_\tau(t) = \frac{W_\tau(t)}{\sum_{s=\tau}^{t+S} \left[\frac{1}{\phi}\right]^{s-\tau}}.$$

2.3.1.3 Planned and Realized Consumption Assume that the consumer has a productivity schedule, over the working periods, which shows an inverse U shape. This agrees with common sense as well as lifecycle income data. If a consumer at age or time t perfectly foresees only S periods forward, then he maximizes his utility from time t to time $t+S$ following any income stream available for this planning horizon. The solution to this maximization procedure is a vector of consumption schedule for a planning horizon S starting from t,

$$C^P(t) = \{c_t(t), c_{t+1}(t), c_{t+2}(t), ..., c_{t+S}(t)\}.$$

Let us call this *planned consumption* determined at t for the subsequent S periods. Initially the consumer intends to follow the planned consumption stream and consumes $c_t(t)$ at t. However, at t+1 he realizes that the planned consumption for t+1, i.e. $c_{t+1}(t)$ is no longer optimal because a new income of time $(t$+1$)$+S is in view.[38] He has to re-plan to incorporate this and solves again for the consumption stream starting from t+1. He keeps doing this as long as there comes new income in view: as long as there is a drift between the planned consumption and optimal consumption with new planning horizon.

Because each time the consumer follows a planned path only at the initial time of the planning horizon, the actual consumption will be an envelope of the entire planned consumption path over whole life. The realized consumption from age t is,

$$C^R(t) = \{c_t(t), c_{t+1}(t{+}1), c_{t+2}(t{+}2), ..., c_{t+S}(T\text{-}S)\}.$$

Therefore the lifecycle consumption profile of the representative consumer with any planning horizon S is

$$\{c_t\}_{t=0}^{T} = \{c_0^S(0), c_1^S(1), c_2^S(2), ..., c_T^S(T)\}$$[39]

where the consumption is indexed by a planning time, a consumption time, and a planning horizon. Likewise, the profile of lifecycle asset demand is

$$\{b_{t+1}\}_{t=0}^{T-1} = \{b_1^S(0),\ b_2^S(1),\ \ \ldots\ ,\ b_T^S(T-1)\}$$

and

$$b_0^S(0) = 0.$$

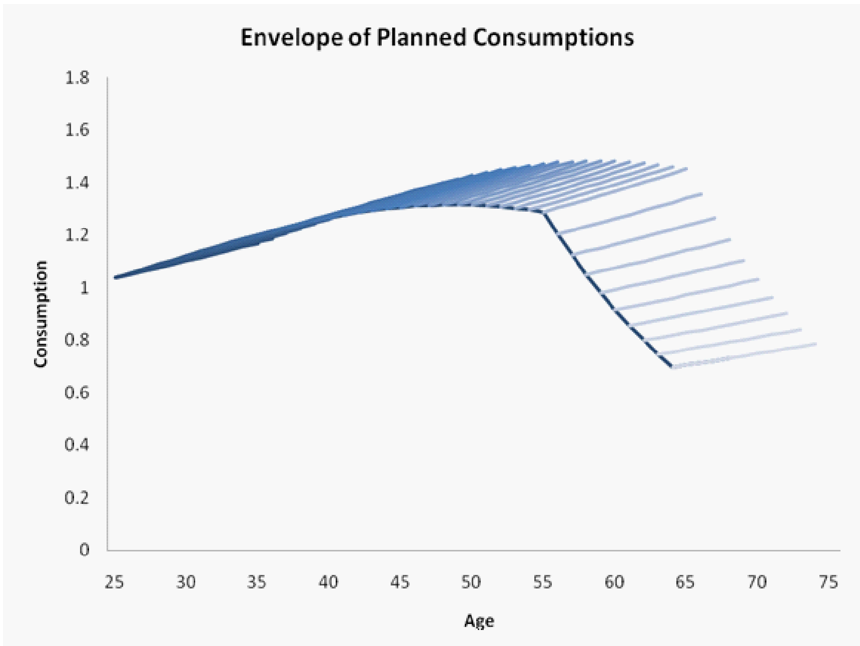
Figure 3 shows a planned and realized consumption profile. The realized consumption is the envelope of all the planned consumption series.[40]

[38] Remember that the agent foresees perfectly S periods forward all the time. At time t+1, the S^{th} income is the one of time t+1+S.

[39] The consumer follows standard consumption path for the residual periods in the last phase of life. Therefore, the consumption beyond time $T-S$, is the same as the planned path.

[40] This result is obtained with S=10, γ=3, R=1.045, and β=0.98.

Figure 3: Planned and Realized Consumption Profile for the model with S=10.



2.3.2 Technology and General Equilibrium

To explore a general equilibrium model, let us add production technology to the economy. Assume that there is a continuum (infinite number) of identical perfectly competitive firms. Specifically, this model introduces the following Cobb-Douglas production function for the representative firm:

$$F(K, N) = K^{\alpha} N^{1-\alpha}$$

The marginal productivity is given by

$$F_K = \alpha \left(\frac{K}{N} \right)^{\alpha - 1}$$

$$F_N = (1-\alpha) \left(\frac{K}{N} \right)^{\alpha}$$

Let us define a competitive equilibrium for the model.

Definition *A competitive equilibrium in this economy is an allocation* $\{c_t\}_{t=0}^{T}$*, a set of bond demands* $\{b_{t+1}\}_{t=0}^{T}$*, an interest rate* R*, and a wage rate* w *such that given* R *and* w*,* *the followings are satisfied:*

i) $\{c_t\}_{t=0}^{T}$ *and* $\{b_{t+1}\}_{t=0}^{T-1}$ *solve the consumer's problem.*

ii) Factors are paid out their marginal productivity:

$w = F_N$ *and* $R - 1 = F_K - \delta$

iii) Labor market and bond market clear:

$K = \sum_{t=0}^{T} b_t$ *and* $N = \sum_{t=0}^{T} e_t$

The market clearing condition in the last line specifies that consumption loans cancel out in the aggregate so that the excess demand for bonds should be equal to the capital stock. Also the aggregate labor supply that sums up over all cohorts should be equal to the aggregate labor demand. By the equilibrium condition ii), it is obvious to have

$$w = (1-\alpha)\left(\frac{K}{N}\right)^{\alpha}$$

$$R - 1 = \alpha\left(\frac{K}{N}\right)^{\alpha-1} - \delta$$

Rewrite the last equation to get

$$\left(\frac{K}{N}\right)^{\alpha-1} = \frac{R-1+\delta}{\alpha}.$$

The capital-to-labor demand ratio is written as a function of the interest rate and other parameters. Rearrange the capital as a function of the interest rate to get

$$K(R) = N\left(\frac{R-1+\delta}{\alpha}\right)^{\frac{1}{\alpha-1}}.$$

By the equilibrium condition iii), the market equilibrium condition to determine R is[41]

$$\sum_{t=0}^{T} b_t(R) = \left(\frac{R-1+\delta}{\alpha}\right)^{\frac{1}{\alpha-1}} \sum_{t=0}^{T} e_t.$$

Once the equilibrium interest rate R is obtained, the wage rate w is determined by

$$w(R) = (1-\alpha)\left(\frac{R-1+\delta}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}.$$

[41] At equilibrium, $K_D(R) = K(R) = K_S(R)$ and $K_S(R) = \sum_{t=0}^{T} b_t(R)$. Also $N_D = N = N_S$ and $N_S = \sum_{t=0}^{T} e_t$.

2.4 QUANTITATIVE ANALYSIS

The goal of quantitative analysis is to assess how well a calibrated, general equilibrium model of short term planning can account for the stylized facts regarding lifecycle consumption data. The following subsections answer the four questions raised in the introduction with respect to the featured consumption hump. One methodological issue in the analysis is how to calibrate the under-identified model parameters[42] given the lesser number of target variables from US data. The *Calibration* discusses this in detail. For simulation exercise, I utilize the mean profile of lifecycle consumption and income estimated by Gourinchas and Parker (2002)[57]. Because the equilibrium series of consumption, income, bond demand, and labor supply in the model of overlapping generation economy can be interpreted as economy wide cohort averages, the mean profiles should work well for the model.

Finally, in order to quantify the lifecycle model, I set a period of the model to a year. The agents are born to be 25 years old. The economy is stationary and there is no population growth. Let the agents live for sure from $t = 0$ to $T = 55$. This corresponds to physical life from 25 years old to 80 years old. The agents work to 65 years old and because there is no other income sources than the earnings from labor, their income is zero after retirement.[43] Then the agents live on savings from working years.

2.4.1 Targets in US Data

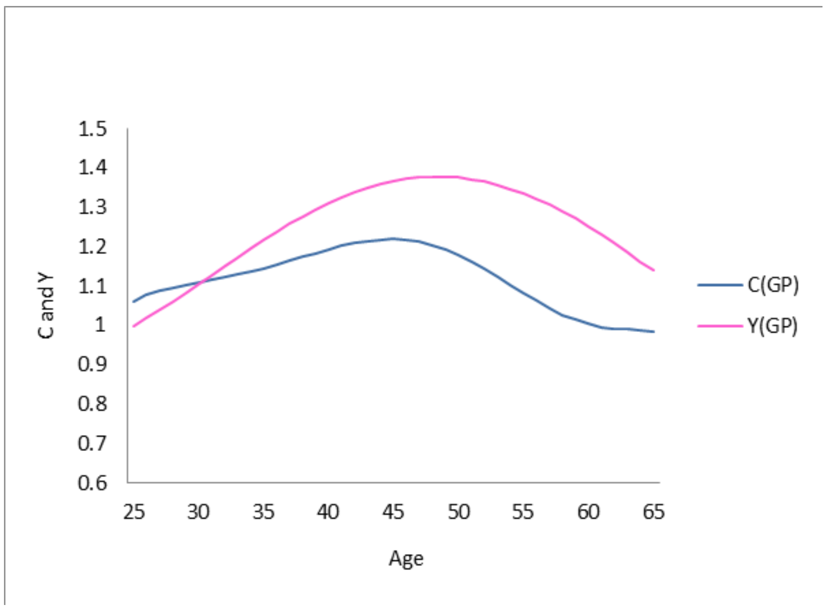
I propose three targets in standard macroeconomic variables representing US data. They are interest rate (R), capital-output ratio (K/Y), and consumption-output ratio (C/Y). Following Rios-Rull (1996)[93], I set 2.94[44] as a target value for capital-output ratio and 0.748 for the target ratio of consumption to output. The third macroeconomic target is the real interest rate, which is determined by the equilibrium condition of the model. Following

[42] There are five model parameters but three target variables.

[43] Thus $T_w = 40$ and $e_t = 0$ for $t > 40$.

[44] Other authors like Feigenbaum (2008c)[47], suggest *2.5* for this ratio. When I simulate the model with this target ratio, I find that the main result of the model is not altered in terms of the best planning horizon. The alternative target ratio produces very similar consumption profile for each γ, but with slightly lower β than the baseline model.

Figure 4: The consumption and productivity profile from US data estimated by Gourinchas and Parker (2002)[57].



McGrattan and Prescott (2000)[81], I set the rate at 3.5%.[45]

Regarding lifecycle consumption data that the model aims to accomplish, I use the mean consumption of Gourinchas and Parker's estimation mentioned above. Feigenbaum (2008a)[45] interpolates their estimation into septic polynomial function of age:

$$\begin{aligned} c_t^{GP} &= 1.062588 + 0.015871t - 0.00184t^2 + 0.000109t^3 + 0.00000413t^4 - 0.00000056t^5 \\ &\quad +0.0000000163t^6 - 0.0000000001475t^7 \end{aligned}$$

According to this profile, the age of consumption peak is 45 years old and the ratio of peak consumption to initial consumption is 1.1476. These values, as well as mean squared error between the data and the model, are used to assess different consumption profiles that models with different planning horizons produce. Also, regarding the income schedule, Feigenbaum (2008a)[45] suggests a quadratic fit for the US data. Since the labor is supplied inelastically in the model, income is proportional to productivity. Therefore, Feigenbaum's quadratic fit to the income data of Gourinchas and Parker (2002)[57] can serve for the productivity profile as well. The profile is

[45] Similarly, Gourinchas and Parker (2002)[57] estimate the rate 3.44%.

$$e_t = 1 + 0.0181t + 0.000817t^2 - 0.000051t^3 + 0.000000536t^4$$

According to this income stream, the peak occurs around 48 years old. Figure 4 shows the consumption and income (productivity) profile from US data. With inelastic labor supply, the mean income schedule serves as productivity profile of the representative consumer. From the figure, it is clear that both lifecycle consumption and income streams are hump-shaped, but the age of consumption peak comes slightly earlier than the age of income peak.[46]

2.4.2 Calibration

The model has four standard parameters: the risk aversion coefficient γ and the discount factor β are from consumer optimization and the capital share α and the depreciation rate δ are from technology. But with the short term planning model of bounded rationality, there is one more parameter to consider because the model assumes a shorter than full term planning horizon. It is the planning horizon parameter, which is denoted by S in the model. Therefore, the model has five parameters and they are $\{\gamma, \beta, \alpha, \delta, S\}$. Unlike other parameters, S allows only a small set of distinct integers for its value, from one to fifty five, at most.[47] Therefore, instead of calibrating all of these five parameters together, I set the planning horizon as given and have the other four parameters jointly set to match the target. Then I explore how well the short term planning general equilibrium model replicates the data as the planning horizon changes. In other words, the planning horizon itself is evaluated in terms of best model fitted to the consumption data.

Among the four scalar parameters $\{\gamma, \beta, \alpha, \delta\}$, I find that the model is well calibrated jointly with a set of three parameters, given the three target values of interest rate (3.5%) and capital-output ratio (2.94), as well as consumption-output ratio (0.748). Those three parameters are: ϕ, α, and δ. Among the three, ϕ represents a collective value of the two consumer parameters $\{\gamma, \beta\}$. The joint set $\{\phi(\gamma, \beta), \alpha, \delta\}$ that minimizes the deviation from the targets provides unique value of ϕ for each planning horizon, along with the other two

[46] This fact is also found in Thurow's consumption hump in Appendix. This implies that the two profiles are closely connected, as I emphasize through this paper.

[47] The planning horizon is year based: the maximum length $S = 55$ corresponds to a plan from 25 years old to 80 years old.

production parameters. For example, with planning horizon $S = 18$, those three calibrated values are: $\phi = 0.986$, $\alpha = 0.289004$ and $\delta = 0.063001$. And with an alternative target ratio of $K/Y = 2.5$, these values are $\phi = 0.981$, $\alpha = 0.3116$ and $\delta = 0.0896$. Unlike ϕ, the two production parameters are found to be independent of the planning horizon S. This leaves the model with two free parameters, γ and β. Therefore, for a choice of γ, β is set from ϕ, matching the target for each planning horizon. The result section shows how the prediction of each of the twenty two[48] models of different planning horizon varies in equilibrium according to a joint choice of γ and β. Table [1] summaries the description of parameters and targets.

Table 1: Parameters and Targets

Variable	Description	Target
γ	Risk Aversion	Free
β	Discount Factor	Free
α	Capital Share	Free
δ	Depreciation Rate	Free
S	Planning Horizon	Given
R	Interest Rate	3.5 %
K/Y	Capital-Output	2.94
C/Y	Consumption-Output	0.748

Another issue is to find the most plausible planning horizon that best describes the stylized fact of consumption data. For this purpose, I employ both quantitative and qualitative comparison. For the quantitative fit, I calculate the model's deviation from the consumption profile implied by the data using mean squared error (MSE) and the least absolute deviation (LAD). For the qualitative fit, I compute the consumption ratio: the ratio of the consumption at its maximum to the consumption at the starting age. This is a common measure to quantify the consumption hump, proposed by Bullard and Feigenbaum (2008a)[45] and Hansen and İmrohoroğlu (2006)[60]. Another measure for the qualitative fit is the age of the

[48] I analyze the model of all the planning horizons from $S = 5$ to $S = 26$.

consumption peak. In the next subsection, I register the result of the consumption ratio and the consumption peak age for each planning horizon.

2.4.3 Results

Simulation exercise reveals that the short term planning model does produce a consumption hump, with right size and location of consumption peak, in a well-calibrated general equilibrium. The location of peak falls within 40 to 54 years old for all choices of planning horizon from 5 to 26 and the location is reversely related with the length of planning horizon. By this, it is inferred that there is a planning horizon that generates the same peak location as the one from the data. Likewise, there is a planning horizon that produces the closest ratio of peak consumption to initial consumption as the one of the data.

Next are the stylized facts among the major parameters that the model needs to hold to be supported in the calibrated general equilibrium. It turns out that β, γ and S are closely related in the model so that β needs to be elevated for all of the planning horizons when γ is high, although the degree of increment is not equal to each other but different by the planning horizon. Therefore, one can imagine a three dimensional relationship among the three parameters $\{\gamma, \beta, S\}$.

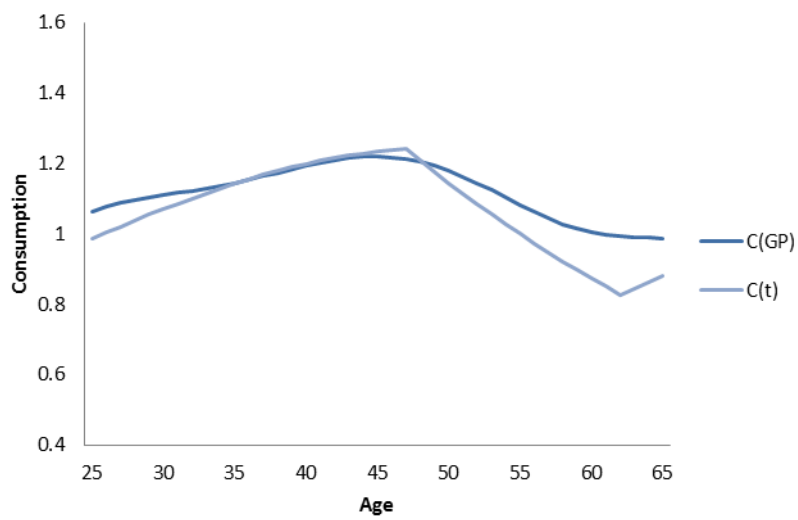
Regarding the best model parameter S I summarize the result by: *about 20 years of planning horizon.*[49] This is because when the planning horizon is around 18 to 22 years, the model fits very well to the consumption data by all means generally considered. Finally, the robustness report confirms the positive contribution of the short term planning general equilibrium model.

2.4.3.1 Simulation Initially let us set the planning horizon at 18. This number is chosen because the general equilibrium consumption profile from 18 years of planning horizon best fits the consumption data in terms of minimum deviation from the objective. I first consider the case where the β belongs to $(0, 1]$.[50] Figure 5 and 6 show the optimal lifecycle

[49] In *Conclusion*, I mention summary remarks of the US retirement survey that strongly supports this result.

[50] Again, in short term planning model, it is natural to allow β to be greater than one because the agent can be very patient for a short period of time and may evaluate the near future more highly than the current.

Figure 5: Optimal Consumption Profile C(t) for the model with S=18, γ=1, β=0.986 and R=1.035. C(GP) represents the data from Gourinchas and Parker (2002)[57].



consumption and the corresponding asset demand from the model of 18 years of planning horizon. In the model the risk aversion parameter is set to 1. Figure 5 includes C(GP), the mean consumption profile of Gourinchas and Parker (2002)[57], for comparison.

In the figure the simulated consumption C(t) shows a hump-shaped profile,[51] together with an increasing part at the tail. This is the residual,[52] mentioned in Section 2, which comes from the assumption that agents follow the planned path once they reach the final stage of planning where their end term T is in view.[53] Figure 7 shows a series of consumption profiles that follow gradual increase of the planning horizon from 10 years to 26 years.[54] In order to compare them together, all of the profiles are generated by the models of the same

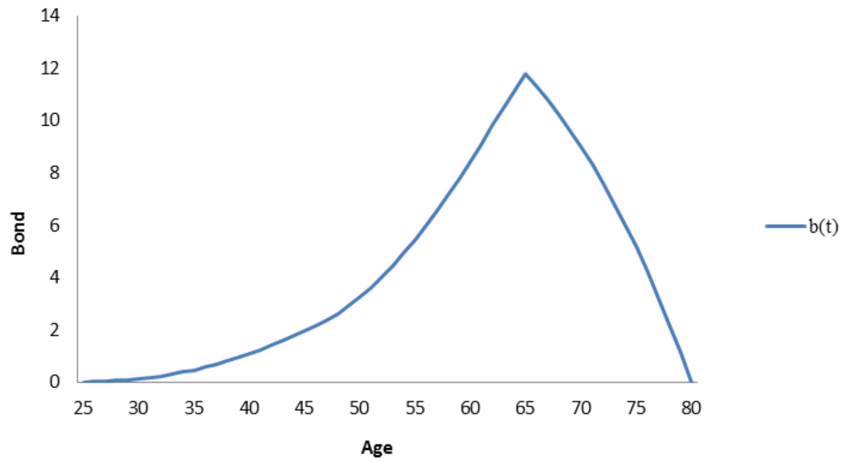
[51] Except for the tail area, the hump is observed by the strong sense of the term, which is formaly defined later.

[52] Because $\beta R > 1$ in this figure, the consumption is increasing during the residual years.

[53] If mortality risk (together with bequest) is introduced to the model, then the tail part would be smoother. But as described in the introduction, the main objective of the paper is to analyze the short term planning model to induce a hump without any other mechanism that might account for it. Mortality risk is one of such factors.

[54] A planning horizon beyond this may not be interesting because the consumption becomes closer to the standard lifecycle model as the planning horizon increases.

Figure 6: Optimal Bond Demand corresponding to the consumption profile for the model with S=18, γ=1, β=0.986 and R=1.035.



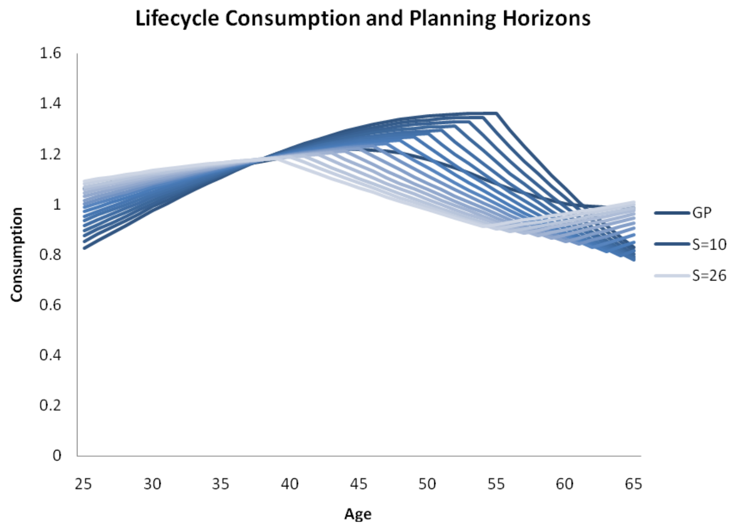
value of $\gamma = 0.5$. This is because 0.5 belongs to the small range of γ that can produce general equilibrium with acceptable β range for the entire planning horizon from 10 to 55. In case of high γ, equilibrium requires very high β if the planning horizon is relatively short. In fact, among the few discrete values of $\gamma = \{0, \ 0.5, \ 1, \ 1.5, \ 2, \ 2.5, \ 3, \ 3.5\}$, $\gamma = 0.5$ is the maximum for the model to generate a consumption profile that agrees with the targets without invoking $\beta > 1$, for all of the planning horizons from 10 to 55. At $\beta = 1$, the general equilibrium is obtained with $\gamma = 0.5967$ for planning horizon 10. Notice that $S = 10$ is the shortest planning horizon in this simulation. The next subsection shows that β becomes lower as planning horizon gets longer. Thus, it follows that when γ is higher than 0.5967, the discount factor exceeds one. Therefore it may be good to set $\gamma = 0.5$ to compare all of the planning horizons.[55] Needless to say, a planning horizon admits many different combinations of β and γ set to match the targets.[56]

If β is allowed to exceed one, then for any choice of γ it is possible to obtain optimal lifecycle consumption profile for every planning horizon. This is because β adjusts accord-

[55] It is also supported by Gourinchas and Parker (2002)[57], in which they estimate $\gamma = 0.5$.

[56] This comes from the fact that the parameters are under-identified: in the model a composition of the two parameters is calibrated, but not the β or γ, separately.

Figure 7: Optimal Consumption Profiles for the models with S from 10 to 26 and $\gamma = 0.5$. The darkest is the shortest horizon and the brightest is the longest.



ingly to any change of risk aversion parameter, keeping the model in equilibrium. From this, I want to highlight two interesting facts with respect to lifecycle consumption. First, given a value of risk aversion parameter, as the planning horizon gets longer, the agents tend to consume more initially.[57] Also the consumption peak arrives earlier but with lower level of it.[58] This means that the lifecycle consumption profile looks more flatten when the planning horizon is longer. This agrees with the common notion that as agents foresee farther, they tend to plan for a longer period. Thus it is likely that a smoother consumption is made.

Notice that the analysis about initial consumption and consumption peak are related with the attribute of the consumption-income data I describe in Section 2. If I revisit them: (A) the initial consumption is higher than the initial income and (B) the age of consumption peak comes slightly earlier than that of income peak. If I compare the model's behavior with the data regarding the two terms, I find the following: for all models with horizon beyond $S = 18$,[59] the initial consumption is higher than the initial income. For all models with horizon below $S = 23$, the age of consumption peak comes earlier than that of income

[57] This is because agents are aware of the income increase in later years.
[58] Again, this is because the agents are aware of the zero income in later years after retirement.
[59] This result is obtained with K/Y=2.94. When I use K/Y=2.5, the horizon is S=17.

peak.[60] Therefore the planning horizons between the two inequalities satisfy both properties.

Another issue to discuss is the limiting behavior of the model. What would happen to the general equilibrium if the planning horizon gets longer and become the full lifecycle length? Given a target value of interest rate $R = 1.035$, the value of $R\beta$ gets smaller as the planning horizon increases because for a fixed γ, β becomes smaller with longer horizon. This implies that $R\beta$ may converge to a value near one so that in the extreme, one gets very smooth consumption profile like the one as the standard theory predicts. To see this, I compute the value of $R\beta$ for the full horizon model, i.e. $S = T = 55$. It turns out that $R\beta = 0.9956$ when $\gamma = 2$ and $R\beta = 0.9989$ when $\gamma = 0.5$. Both yield a very smoothing, but slightly decreasing consumption profile because $R\beta < 1$. This implies that in the well-calibrated general equilibrium, the standard lifecycle consumption profile with $\gamma \leqslant 2$ is not a constant one if the full length is 55 years, although it is nearly flat and monotonic.[61] This suggests that if the model assumes a life span longer than 55, then there is a planning horizon, slightly higher than 55, that yields $R\beta = 1$ near $\gamma = 2$, and another one that yields $R\beta = 1$ near $\gamma = 0.5$. Both should generate a completely flat consumption profile over the entire life.

2.4.3.2 Beta and Gamma Following the description in *Calibration*, three-dimensional graph of the parameters $\{\gamma, \beta, S\}$ can be observed in Figure 8. In the graph, a value of β is determined by a joint position of γ and S. Table [2] shows the optimal β values for different γ over the major horizon grids. Figure 6 extends the result to all values of 22 planning horizons from $S = 5$ to $S = 26$.

Figure 9 demonstrates the following facts. First, β is inversely related with planning horizon S for all values of γ and it is more so with higher γ. Second, when γ is relatively low, β does not change a lot over planning horizon. Third, with longer planning horizon, β values are relatively stable for all choices of γ. These facts help understand why high values of β do not bother with the analysis in general equilibrium. Because both β and γ belong to the reasonable range of parameter values around the optimal planning horizon, i.e. $S = 20$,

[60] If I specify the three year difference between consumption peak (45 years old) and the income peak (48 years old), then S=20 fits exactly to this.

[61] This implies that unless γ is very high, completely flat consumption cannot be expected with $S = T = 55$ years.

this result may justify the claim that the general equilibrium model of short term planning would be a good approach to explain the data. Figure 9 shows the evolution of β for each planning horizon over all risk aversion parameters from 0.5 to 3.

If β is restricted to be less than one, then for some planning horizons not every value of designated γ is available. Table [3] shows result about this fact. From Figure 10 and Table [3], it is easy to notice the following summary facts. First, β is positively related with γ all choices of the planning horizon. Second, β is inversely related with planning horizon for any γ. Third, for any planning horizon longer than 10, γ is good enough to be over 0.5 even with the restriction of $\beta < 1$. Fourth, the longer the horizon, the wider the range of γ admissible for $\beta < 1$.

The logic behind smaller range of γ with shorter planning horizon is this: when the planning horizon is very short, it is less likely to accumulate enough wealth that is transformed into capital, clearing the bond market. Therefore, to compensate this shortage and achieve an equilibrium it is necessary to let the consumer have very low risk aversion or, in other words, very high intertemporal substitution.[62] This implies that the agent should yield enough saving to be matched with the necessary capital in an equilibrium. Table [4] shows the highest values of γ for a couple of major grids in planning horizon.

2.4.3.3 The Best Planning Horizon I look for the best planning horizon that replicates the US consumption data most closely in terms of three measures described earlier. I analyze all the planning horizons from 5 to 26. First, regarding the quantitative fit, I find that both methods of MSE and LAD yield the same result for the best horizon in terms of smallest deviation. The planning horizon of 18 years fits best the consumption data by both methods and the errors are: $MSE = 0.005045$ and $LAD = 0.05308$. Although the two quantitative measures do not produce identical ranking over all the planning horizons, they do produce unambiguously the same ranking near the planning horizon of minimum deviation ($S = 18$) for all values of γ between zero to three. One thing to mention is that the graph of MSE is not symmetric from its lowest value.[63] Rather it shows faster improvement from

[62] With CRRA, risk aversion is equal to the inverse of intertemporal substitution.

[63] With different scale the LAD graph has a similar shape as MSE. Thus the description here also applies to LAD.

the left hand side (horizons shorter than 18), implying the same deviation from the lowest S engenders bigger error for shorter horizon. Thus $S = 23$ fits better than $S = 13$ although both deviate 5 periods from $S = 18$. Any planning horizon less than 10 is not considered to be a good fit in terms of MSE or LAD. Figure 11 summarizes this. The minimum is achieved at the planning horizon of 18 and the MSE value is about 0.005.

Second, for the age of consumption peak, $S = 20$ fits better than any other planning horizon, for all values of risk aversion coefficient, to the data where the peak age is 45. Also the peak age increases constantly (proportionally) as the planning horizon becomes shorter but only up to the point of $S = 11$, beyond which it stalls at 54 and stays the same. Third, the data displays that the ratio of mean consumption at the peak of the hump to the one at the beginning is 1.1476. In term of this measure, $S = 22$ fits best. One remarkable feature is that the consumption ratio decrease as the planning horizon gets longer, suggesting a smoother consumption profile with longer horizon. Figure 12 and Figure 13 summarize these results.

2.4.3.4 Robustness In this section, I want to check the sensitivity of the model to alternative calibrations of the baseline parameters $\{\phi(\gamma, \beta),\ \alpha,\ \delta\}$. Notice that β is going to adjust accordingly to specific γ for a predetermined planning horizon S. Therefore S is not considered to be a parameter for this analysis. For the three targets of macroeconomic variables, i.e. $r = 3.5\%$, $K/Y = 2.94$, $C/Y = 0.748$, I report a couple of sensitivity check around the best planning horizons found above. Table [5] shows two sets of sensitivity report, one with baseline $\gamma = 2$ and the other one with $\gamma = 0.5$. The reports for the planning horizons $S = 15$ and $S = 20$ are based on $\gamma = 0.5$ and the ones for the planning horizons $S = 18$ and $S = 22$ are based on $\gamma = 2$. The second column, *Model* of the table shows different alternative calibration to the baseline model, keeping other parameters intact. Thus, for example, the first group of the table represents the alternative calibration to the baseline model $\{S = 15,\ \gamma = 0.5,\ \beta = 0.979,\ \alpha = 0.289,\ \delta = 0.063\}$, by changing only one parameter, except S, and keeping the others constant.

2.5 MODIFICATIONS

The main model discussed so far assumes two things. First, the planning horizon does not change during the agent's entire life. Also there is no heterogeneity in the model. Because the agents are identical in the economy, this implies that there is only one planning horizon which is stationary over time. Second, the agent is able to adjust his plan at any point during his life time.

The first assumption is based on the premise that agent would not learn from the past and does not improve his planning skills for the future. In other words, the agent's limited ability to foresee the future is persistent through his entire life. Thus the optimization process stays unvarying in an economy as well. This may not be a good assumption if one accepts the argument that human cognitive ability suggests learning through past experiences. If this is the case, then a modification to the baseline model on the consumer optimization is necessary and this is the topic for the section. I consider two modifications: one is the gradual learning until the end of the consumer's life, while the other one is the gradual learning only up to specific horizon S.

The second assumption implies that each agent may not incur any cost related with the planning itself so that he can plan frequently for the future whenever he wants. Thus it follows that the better his welfare would be, the sooner he re-plans if he finds the planned consumption already set up is no longer optimal. If, however, there exists any cost to the frequent planning, like time and energy, then this assumption would be at question. For this reason, a modified version of the model may be considered. Frequent planning is no more an optimal strategy and the planning occurs infrequently. One way to construct this is to assume that the planning occurs only after the previous planning term ends. Another variation may be a model of endogenous planning horizon with planning cost.[65]

[65] Regarding planning cost, there are related works by Reis (2006)[92] and by Caliendo and Huang (2007b)[21]. Reis focuses on endogenous planning horizon for an infrequent planner due to information cost that depends on the unplanned periods. Caliendo and Huang focus on the role of cost related with saving implementation to explain many phenomena on consumption-saving puzzle. Here I focus on how this modification help us understand the prediction of lifecycle consumption of the short term planning model.

2.5.1 Learning

The boundedly rational agent learns from past experience and plans better as his experience accumulates. Thus, for example, each year he plans for one or more additional years to his previous planning horizon. But he is still assumed to plan frequently, year based, without incurring any cost related with planning. Assume that the planning horizon of each time is just one more year than the previous one and the initial planning horizon is zero: the consumer at $t = 10$ (age=35) is able to plan for 10 years and at age 36 for 11 years, etc. Then the representative agent plans for $age + 1$ periods each time and maximizes the following for $t = 0, 1, ..., T$,

$$U(t) = \sum_{s=t}^{t+S_t} \beta^{s-t} \frac{c_s^{1-\gamma}(t)}{1-\gamma}$$

subject to

$$c_s(t) + b_{s+1}(t) = we_s + Rb_s(t)$$
$$b_0(0) = 0,\ b_t(t) = given,\ \ b_{t+S_t+1}(t) = 0$$
$$S_{t+1} = S_t + 1,\ S_0 = 0.$$

The optimal consumption at age t is obtained by

$$c_t(t) = \frac{\sum_{s=t}^{t+S_t} \frac{we_s}{R^{s-t}} + Rb_t(t)}{\sum_{s=t}^{t+S_t} \left[\frac{(R\beta)^{1/\gamma}}{R}\right]^{s-t}}$$

Figure 14 shows the result in general equilibrium when the agent keeps learning through his life.[66] Planning horizon expands each year through consumer's life time. In this graph the age of consumption peak comes earlier than the data. This implies that as the agent learns more from past and plans further, he is more likely to prepare for his retirement early. Consider another case where the agent's learning ability grows only up to S years and then it stalls after that. The summary result for this case is in the next graph. From Figure 15, it is also clear that learning for longer periods give the agent more chances of consumption smoothing. The consumption peak age is closest to the data when the length of the gradual learning is about 20.[67]

[66] As in the baseline model, once the agent is able to plan for up to the end of life, he follows the planned path for the residual years.

[67] In fact, $L = 18$ is the best by this criterion in learning model.

2.5.2 Cost and Infrequent Planning

Let us modify the baseline model into the one in which no adjustment is allowed between any two planning dates. This modification would be justified if the planning horizon is relatively short. For example, if the planning horizon is two to five years, then the total cost to frequent planning would be more than the cost related with the sub-optimal consumption between those periods. However, if the planning horizon is relatively long, then re-planning would be better than following the original profile even though it may incur some cost related with the planning activity. Assume the agent chooses planning dates $t_1, t_2, \ldots, t_n$ from his life time. At each planning date t_i and only at this date, he incurs a planning cost X which does not depend on the duration of non-planned periods.[68]

Assume the agent plans for $S(i)$ periods that may differ by each planning date t_i. He plans only on planning date t_i when the previous planning term ends and the new planning term starts. Therefore, he solves the following maximization problem at t_i, $i = 0, 1, 2, \ldots, n$, for $S(i) + 1$ non-planning periods,

$$U(t_i) = \sum_{s=t_i}^{t_i+S(i)} \beta^{s\text{-}t_i} \frac{c_s^{1-\gamma}}{1-\gamma}$$

subject to

$$\begin{aligned} c_s + b_{s+1} &= we_s + Rb_s, & if \quad s \neq t_i \\ c_s + b_{s+1} &= we_s + Rb_s - X, & if \quad s = t_i \\ b_0 = 0,\ \ b_{t_i} &= given,\ \ b_{t_i+S(i)+1} = 0. \end{aligned}$$

Thus, over the entire life the agent maximizes

$$U = \sum_{i=0}^{n} \sum_{s=t_i}^{t_i+S(i)} \beta^{s\text{-}t_i} \frac{c_s^{1-\gamma}}{1-\gamma}$$

[68] For the case where the planning cost depends on duration of non-planned periods can be modeled by

$$U = \sum_{i=0}^{n} \sum_{s=t_i}^{t_i+S(i)} \beta^{s\text{-}t_i} \frac{c_s^{1-\gamma}}{1-\gamma}$$

subject to

$$\begin{gathered} c_s + b_{s+1} = we_s + Rb_s, \quad if \quad s \neq t_i \\ c_s + b_{s+1} = we_s + Rb_s - X(S(i-1)), \quad if \quad s = t_i \\ b_0 = 0,\ \ b_{t_i} = given,\ \ b_{t_i+S(i)+1} = 0, \quad t_0 = 0,\ \ t_n + S(n) = T, \end{gathered}$$

where $S(i)$ is the planning horizon (duration) set at planning date t_i.

subject to

$$
\begin{aligned}
c_s + b_{s+1} &= we_s + Rb_s, \qquad && if \quad s \neq t_i \\
c_s + b_{s+1} &= we_s + Rb_s - X, \qquad && if \quad s = t_i
\end{aligned}
$$

$$b_0 = 0, \;\; b_{t_i} = given, \;\; b_{t_i+S(i)+1} = 0,$$

$$t_0 = 0, \, t_n + S(n) = T.$$

The optimal consumption at each t_i, $i = 0, 1, 2, ...n$, is obtained by

$$c_{t_i} = \frac{\sum_{s=t_i}^{t_i+S(i)} \frac{we_s}{R^{s-t_i}} + Rb_{t_i} - X_s}{\sum_{s=t_i}^{t_i+S(i)} \left[\frac{(R\beta)^{1/\gamma}}{R}\right]^{s-t_i}}$$

where $X_s = 0 \;\; if \;\; s \neq t_i$ and $X_s = X \;\; if \;\; s = t_i$.

Remember that between two planning dates, the agent follows the planned path that is monotonic. The result is shown in Figure 16. In this figure the model is simulated with the assumption that the planning cost is not monetary cost, but mental or time cost related with the planning. Therefore planning occurs infrequently but there is no actual financial cost with this example. The agent plans for 5 years which is constant over the entire life[69] and the planning occurs infrequently only on the planning dates when the previous planning term ends.

2.6 CONCLUSION

The prominent feature of lifecycle consumption data known as consumption hump has been central issue in lifecycle consumption literature because standard lifecycle theory cannot produce one with such property. The standard theory implies that consumption is not directly affected by income fluctuation: it is the average of income that matters, not the individual ups and downs. Therefore, a hump-shaped lifecycle consumption cannot be expected in a model of standard assumption, even though the income, the resources that the consumption relies on, is roughly hump-shaped over life.

[69] In this example, the planning horizon is fixed for all i. Thus, $S(i) = S = 5$.

In this paper I study a bounded rationality model that is based on time inconsistent preference and that solves the discrepancy between the monotonic prediction and the hump-shaped data of lifecycle consumption. The standard lifecycle models usually require extra assumption about the structure of model beyond preferences to reconcile the discrepancy. Motivated by this, I explore a model of novel framework that deviates from the main core of the traditional theory: the *full rationality* assumption. The rational agent is now allowed to be *boundedly rational* and maximizes his known utility over certain period of time, which is shorter than full life span. By this way I find that it is possible to construct a model in which the optimal consumption profile clearly links to income stream. Moreover, from the fact that existing bounded rationality literatures that deal with the issue analyze only the partial equilibrium features, I highlight the general equilibrium approach through this paper and characterize a model of boundedly rational agent under $(T+1)$-period overlapping generations general equilibrium with production.

With analytic solution, I first prove that the short term planning model produces the consumption hump that is closely related with income profile. Then, with no other assumption than the short term planning horizon, I show that the model produces consumption hump for all of the planning horizons I consider. Among them I find that the planning horizons of 18, 20, and 22 years generate the consumption profile that resembles the data most closely in a well-calibrated general equilibrium. That is, the general equilibrium result is consistent with the known characteristics of the lifecycle consumption profile. I also find the exhaustive inner relationship among the main parameters (time preference, risk aversion, and the planning horizon) that is necessary to be supported in the calibrated general equilibrium. Through sensitivity check, I show that my general equilibrium result is quite robust to alternative choice of parameters. Finally, I introduce two modifications of the model incorporating (1) learning and (2) infrequent planning due to cost. These modifications help understand how the model of short horizon works to induce a consumption hump.

The simulation exercise demonstrates that the general equilibrium model with planning horizon of about 20 years provides a best fit to the salient features of the lifecycle consumption data. One remarkable thing is that this result is very consistent with people's actual behavior reported in many survey literatures, specifically the survey on retirement planning

(Retirement Confidence Survey, 2007). The findings from the survey can be summarized as the following: the average US worker tends to start to save for retirement around age 45, planning to retire at 65, with an expectation of being retired for 20 years. By this statement, it is expected that the representative consumer who has a planning horizon of 20 years and an exogenous retirement age of 65, does not realize that he needs to accumulate assets for the retirement until 45 years old, at which age he could see clearly there is no more income after 20 years and has to set up an retirement account, by dramatically reducing his consumption he has enjoyed so far. This fact exactly coincides with the model's prediction if the planning horizon of 20 years is used, which this paper proves the optimal planning horizon for the model.

Figure 8: Optimal Parameter Surface from the parameters $\{\gamma, \beta, S\}$

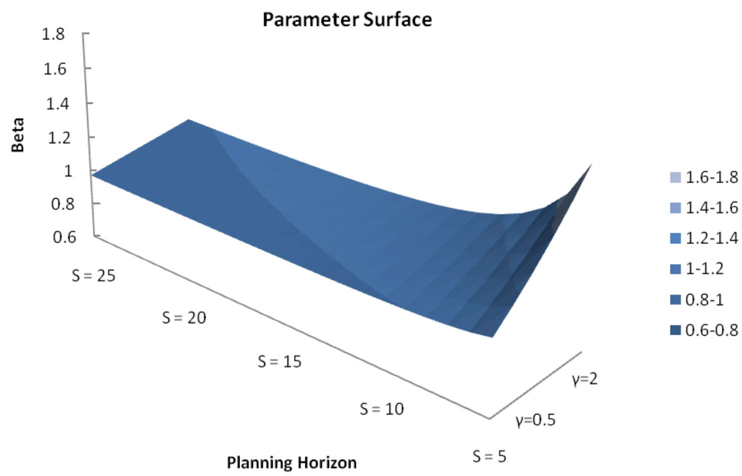


Table 2: Optimal Beta Values over Gamma and Planning Horizon

γ	$S = 5$	$S = 10$	$S = 15$	$S = 20$	$S = 25$
0.5	1.0580	0.9944	0.9800	0.9742	0.9710
1	1.1586	1.0235	0.9940	0.9822	0.9759
1.5	1.2688	1.0517	1.0081	0.9903	0.9809
2	1.3894	1.0818	1.0225	0.9985	0.9858
2.5	1.5215	1.1128	1.0371	1.0068	0.9908
3	1.6661	1.1447	1.0519	1.0151	0.9957

Figure 9: Optimal Beta Values for the models with S from 5 to 26.

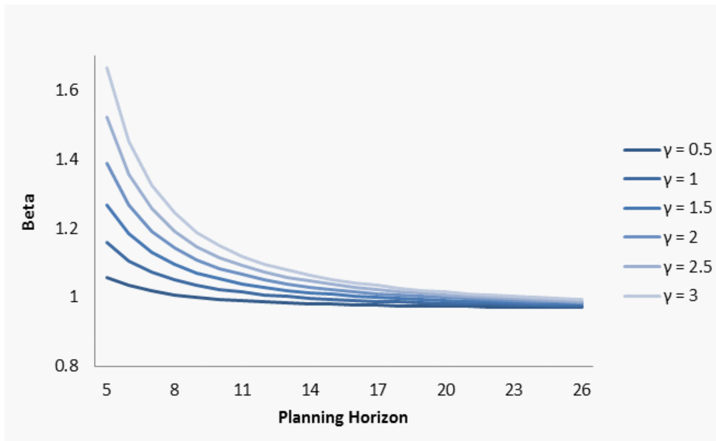


Figure 10: Optimal Beta Values for the models with risk aversion parameters from 0.5 to 3.

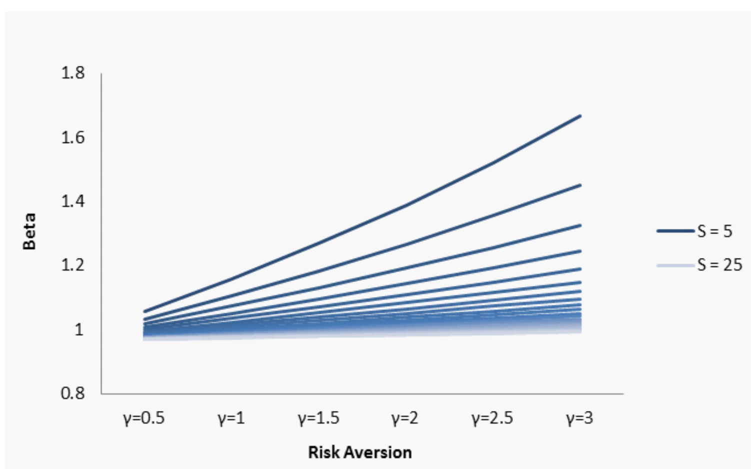


Table 3: Optimal Beta Values over Planning Horizon

γ	$S = 5$	$S = 10$	$S = 15$	$S = 20$	$S = 25$
0.01	0.9679	0.9667	0.9665	0.9664	0.9663
0.05	0.9759	0.9689	0.9676	0.9670	0.9667
0.1	0.9839	0.9718	0.9689	0.9678	0.9672
0.15	0.9929	0.9746	0.9703	0.9686	0.9676
0.2		0.9774	0.9717	0.9694	0.9681
0.5		0.9944	0.9800	0.9742	0.9710
1			0.9940	0.9822	0.9759
1.2			0.9996	0.9855	0.9779
2				0.9985	0.9858
2.5				0.9994	0.9908
3					0.9957
3.42					0.9999

Table 4: The Highest Value of Gamma Less than One

Planning Horizon (S)	Risk Aversion (γ)
5	(0, 0.189401]
10	(0, 0.596736]
15	(0, 1.214161]
20	(0, 2.010267]
25	(0, 3.424499]

*The second number in the brackets is the value when $\beta = 1$

Figure 11: The Best Fitting Planning Horizon: minimum deviation from GP.

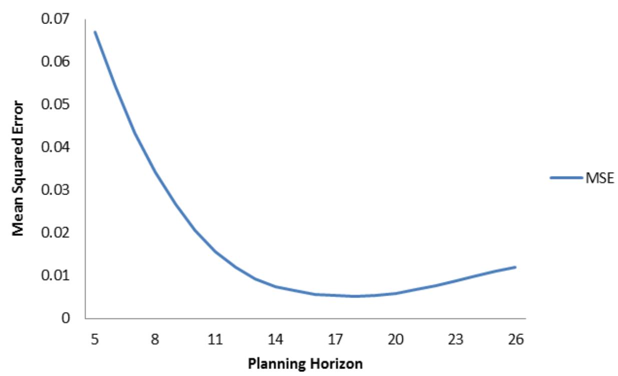


Figure 12: The Best Fitting Planning Horizon based on the age of maximum consumption: S=20 fits best by this criterion (45 years).

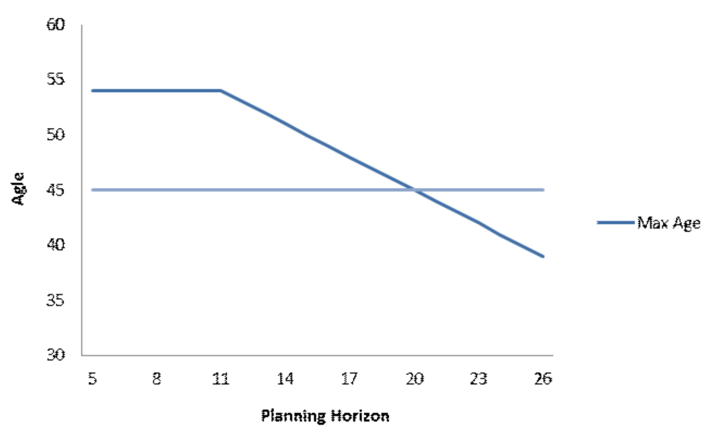


Figure 13: The Best Fitting Planning Horizon based on the ratio of maximum consumption to initial consumption: S=22 fits best by this criterion.

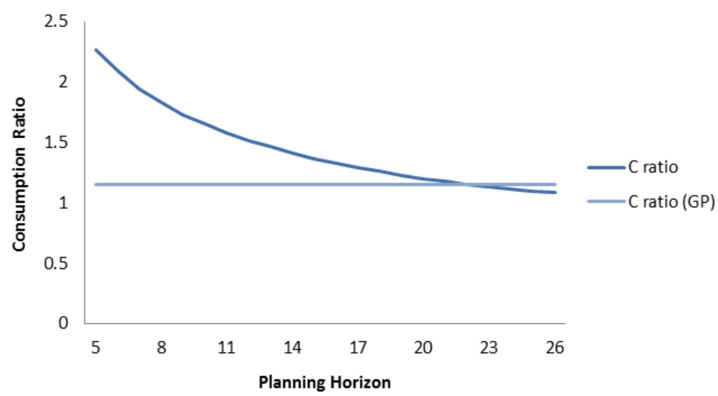


Table 5: Sensitivity to Alternative Calibration

S	Model	r	C/Y	K/Y	Age[64]	C_m/C_o	MSE
Baseline	γ=0.5, β=0.979	3.50%	0.748	2.94	50	1.369	0.0063
S=15	$\gamma = 0.40$	3.60%	0.753	2.91	50	1.365	0.0061
	$\alpha = 0.30$	3.93%	0.747	2.92	51	1.380	0.0054
	$\delta = 0.07$	3.80%	0.751	2.77	50	1.351	0.0058
	$\beta = 0.98$	3.61%	0.732	3.02	50	1.395	0.0060
Baseline	γ=2, β=1.006	3.50%	0.748	2.94	47	1.259	0.0051
S=18	$\gamma = 1.90$	3.39%	0.743	2.97	47	1.262	0.0054
	$\alpha = 0.33$	4.43%	0.712	3.07	47	1.328	0.0019
	$\delta = 0.07$	3.20%	0.746	2.83	47	1.239	0.0071
	$\beta = 1.00$	3.83%	0.763	2.84	47	1.250	0.0041
Baseline	γ=0.5, β=0.974	3.50%	0.748	2.94	45	1.202	0.0058
S=20	$\gamma = 0.60$	3.64%	0.754	2.90	45	1.199	0.0053
	$\alpha = 0.30$	3.58%	0.730	3.01	45	1.210	0.0048
	$\delta = 0.08$	3.34%	0.747	2.66	45	1.169	0.0083
	$\beta = 0.98$	3.08%	0.707	3.17	45	1.232	0.0072
Baseline	γ=2, β=0.993	3.50%	0.748	2.94	43	1.139	0.0074
S=22	$\gamma = 2.10$	3.58%	0.752	2.92	43	1.153	0.0071
	$\alpha = 0.30$	3.73%	0.737	2.98	43	1.168	0.0058
	$\delta = 0.05$	4.09%	0.755	3.18	43	1.189	0.0039
	$\beta = 0.98$	4.32%	0.784	2.71	43	1.139	0.0049

Figure 14: The gradual learning result in general equilibrium: $\gamma = 0.5$.

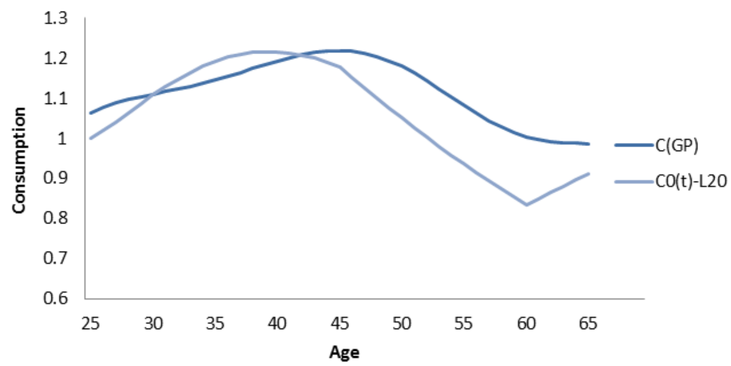


Figure 15: Planning horizon expands each year only up to S periods and stalls: $\gamma = 0.5$, $R = 1.035$.

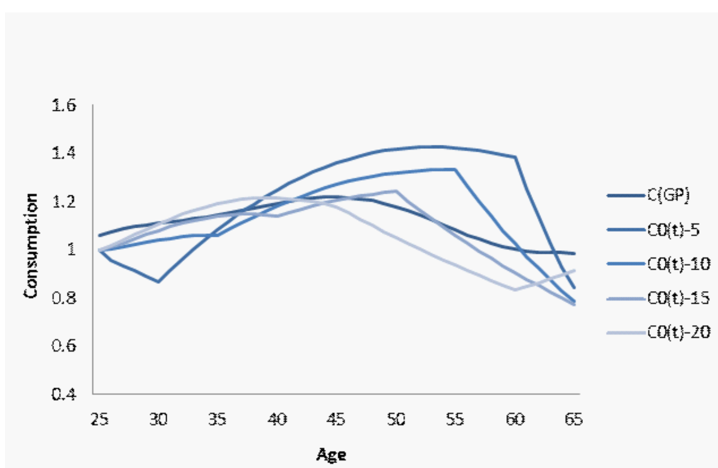
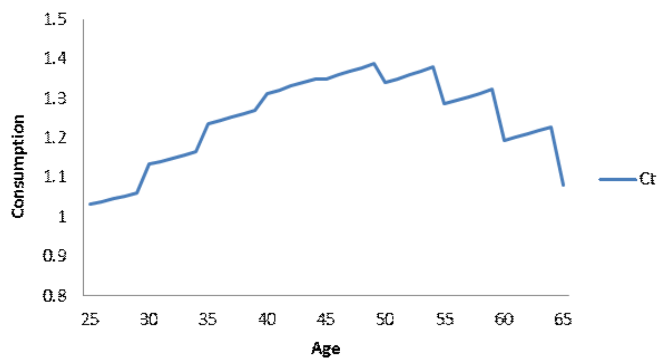


Figure 16: Infrequent planning due to cost: $S = 5$, $\gamma = 2$, $\beta = 0.98$, $R = 1.035$.



3.0 PRESENT BIASED PREFERENCE AND CONSTRAINED CONSUMER

A consumer who has present-biased preference is more likely to accumulate debts because of his time-inconsistent taste for immediate gratification. From this observation, I examine how the consumers with such preference react to credit constraints and explore the general equilibrium characteristics of an economy of these myopic consumers. With analytic solutions that define life time consumption of the agents who re-optimize, negating earlier resolution, both with and without the credit constraint, I show the possibility of consumption and income co-movement without resorting to any other constraint. For a constrained consumer who has a generalized discounting function for immediate gratification my model produces the prominent consumption hump in an empirically plausible, calibrated general equilibrium. Beyond the baseline analysis, I also introduce a pay-as-you-go social security system, as well as mortality risks and bequests to the model, making it even more empirically plausible and keep finding that the model predicts many of salient lifecycle features of consumption data. Finally, the simulation exercise demonstrates that the values of the discount factors estimated from field experiments, are consistent with the model's prediction.

Keywords: present-biased preference, time inconsistency, borrowing constraint, lifecycle model, general equilibrium, consumption hump.

3.1 INTRODUCTION

Traditionally, economic theory has posited that individuals are infinitely far-sighted, fully rational, expected utility maximizers with time-consistent preferences. Increasingly, economic researchers are questioning these assumptions, as the models that rely on them often yield predictions that are not corroborated in the available data (DellaVigna, 2009[33]). In this paper, I explore the possibility that individuals behave *myopically* rather than rationally, demonstrating that this tendency among individuals can be supported in macro level analysis. Specifically, I suppose that they possess time-inconsistent tastes for immediate gratification.[1] Myopic decision makers may not have full self-restraint to be patient for delayed gratification and they are more likely to show greater desire for immediate satisfaction.

Individuals discount the utility of a later reward with many reasons and the discounting usually increases with the length of delay. They discount a delayed reward simply because they are impatient. But individuals also discount a later reward because they don't like the uncertain outcome of any future event.[2] In fact, as argued by Frederick, Loewenstein and O'Donoghue (2002)[52], time discounting admits any motivation for caring less about future consequence and the driving forces include impulsivity, uncertainty, and the possibility of changing tastes. Because the time preference conveys a composite[3] of all different motivations driving their choice behavior, myopic consumer may be represented by a time preference parameter that describes such behavior.

In discounted utility theory, the utility of a myopic consumer is represented by *Present-Biased Preference*,[4] where the valuations fall very rapidly for small delay periods, but then fall more slowly for longer delay periods. This time-inconsistency contrasts with the standard exponential discounting, where valuations fall by a constant factor regardless of the length of delay. Since Laibson (1997)[69] and O'Donoghue and Rabin (1999)[84], by their formalized works about the consumption and saving decision for a myopic consumer, the model of present-biased preferences has been widely confirmed in diverse field studies. Selected field evidences include: Laibson, Repetto, and Tobacman (2009)[71], Skiba and Tobacman (2008)[96], Ashraf, Karlan and Wesley (2006)[11], Thaler and Benartzi (2004)[102]; Choi

[1] Experiments on intertemporal choice are summarized in Loewenstein and Prelec (1992)[76]. See also Frederick, Loewenstein, and O'Donoghue (2002)[52].

[2] Regarding the relationship between uncertainty and hyperbolic discounting, see Thomas Epper, Helga Fehr-Duda, and Adrian Bruhin (2010)[36].

[3] For the experimental analysis on the combined effect of time and uncertainty, see Andersen et al. (2008)[2].

[4] It is also called "Strotzian," or "Hyperbolic Discounting," or "(β, δ)-model," by many other researchers.

et al. (2004)[29], Cronqvist and Thaler (2004)[31]; Ariely and Wertenbroch (2002)[9].[5] The core theoretic framework of these works is based on the intertemporal pattern of discounting: the *short-run* discount factors are lower than *long-run* discount factors.

The Quasi-hyperbolic discounting model[6] by Laibson (1997)[69] is based on the this evidence that per-period discount rate changes over time. With dynamically inconsistent preferences, Laibson finds a unique sub-game perfect equilibrium strategy for a dynamic game in which time-indexed selves who face choice between liquid and illiquid assets, solve a T-period consumption problem. To control future selves, in his model, it is necessary for the long-run self to choose more illiquid assets. The empirical implication of Laibson's work and a related work by Angeletos et al. (2001)[7], is that the existence of the commitment device like illiquid asset or borrowing constraint helps induce the occurrence that consumption may co-move with income flows,[7] an empirically important characteristic of consumption data. Their findings, however, are based on the assumption of asset constraint that induces such property and do not explain the consumption property from the present-biased preference itself. In fact, self-controlled individuals would not show the consumption and income co-movement if they are not equipped with such commitment device.[8] Therefore, the research question to ask is: under what condition does consumption behavior co-move with income profile for an agent with present-biased preference? My paper directly answers this.

The myopic agents in my model possess time-inconsistent tastes for immediate gratification and because of this, they continually *re-optimize* at each date in time as they realize that their past decision for the present period is no longer optimal. They keep negating their earlier resolution from *consume more now* taste. By this mechanism, I find that the model of naive agents with perpetual re-optimization can demonstrate the co-movement property without resorting to any other assumption like the asset constraint. Moreover, my model answers the following important questions in macroeconomic analysis: first, what would be the actual lifecycle consumption profile for a consumer who has a present-biased preference in partial equilibrium as well as in general equilibrium? The next question is regarding

[5] The subjects of these fields include liquid and illiquid savings, 401(k), retirement plan, homeworks and deadlines, and credit cards.

[6] This is a discrete version of Hyperbolic discounting model.

[7] In other words, if income flow is hump-shaped, then it is possible to have such consumption flow.

[8] Section 3 analyzes this for a sophisticated or self-controlled agent.

the consumption hump, an important element describing consumption data. Although the co-movement of income and consumption is implied in several works mentioned above, its details are not specified. Therefore, the second question is: can the consumption profile from the present-biased preferences model match the consumption data, producing the featured consumption hump in general equilibrium?

I provide a general equilibrium framework that can answer these questions for quite general classes of parameter values. I show that the standard macroeconomic targets, the interest rate, capital-output ratio and consumption-output ratio, are calibrated with reasonable set of parameters with this model. Moreover, I answer the questions from very realistic model economy, by including a pay-as-you-go social security system, as well as mortality risks and bequests. I first present closed form solutions to a simple model of present-biased preferences, with and without borrowing constraint. Based on this, I derive analytic condition for the consumption hump.[9] This result demonstrates that the hump is obtained even with myopic consumers if the time preference of the agent remains in a specific range.

An important issue for the consumers with present-biased preferences is self-awareness: whether or not the agent is aware of his self-control problem of future behavior. Unlike the *naïve* agent who is not aware of the self-control problem, the *sophisticated* agent is clearly aware of this and prepares certain commitment device to limit his future behavior of overconsumption. O'Donoghue and Rabin (1999)[84] demonstrate how awareness of self-control problem can successfully moderate the behavioral consequences of those agents with present-biased preferences. Based on this notion, I provide an analytic solution to the optimization problem of the sophisticated consumer. For three-period model, I derive the optimal amount of a commitment device (saving plan) that can limit the overconsumption of the agent by controlling his future behavior and giving him the desired result. Then I compare the three consumption profiles of the model with and without self-control problem.[10] When the consumer successfully controls his future behavior by a commitment device, his consumption behavior is very similar to the one of a consumer who does not have the self-control prob-

[9] Remember that *precautionary saving* is often proposed to explain the consumption hump under uncertainty. Here the hump is obtained in a deterministic world.

[10] The three models are: (1) the model without self control problem, (2) the model for a naive consumer, (3) the model for a sophisticated consumer.

lem. But in the full model of general equilibrium I do not assume that there is a conflict between the short term and long term selves: I posit that each time, because of his taste for immediate gratification, the agent denies his previous resolution without hesitation and re-optimizes his consumption profile for the remaining periods following his current taste.

In a recent field study, succeeding many other theoretic and empirical studies, Meier and Sprenger (2009)[82] show that present-biased individuals are found to be more likely to borrow and borrow more than dynamically consistent individuals. In their field experiments, they find that present-biased individuals are around 15 percentage points more likely than dynamically consistent individuals to have any credit card debt. Moreover, they find that, conditional on borrowing, present-biased individuals borrow around 25 percent more than dynamically consistent individuals. From this observation, I examine how the consumers with present-biased preferences react to credit constraints and explore the general equilibrium characteristics of an economy that is populated by these myopic consumers. The contribution of my work is to assess the reality, generality, and tractability[11] of the assumption of the present-biased preferences for a lifecycle model economy in general equilibrium. I show that reasonable parameterization of the model generates a hump-shaped consumption profile among other empirically plausible features.

Using discrete time horizon, I first show how to incorporate agents with myopic or time-inconsistent preferences into otherwise standard models of intertemporal decision making. I then show how models with myopic decision makers can explain some puzzling features of inter-temporal consumption decisions. In particular, the model I develop can explain why the pattern of household consumption over a lifetime is 'hump' shaped: mean consumption is increasing while the consumer is young, reaching a peak around middle age and then decreasing afterwards. By contrast, standard models with far-sighted rational agents with time-consistent preferences imply that consumption over lifetime should be monotonically increasing, decreasing or constant, but not hump-shaped. In the paper, I demonstrate that the model solves the discrepancy between the lifecycle prediction and its data, by generating a consumption hump under reasonable parametrization, in a well-calibrated general equilibrium model. I find that in my model of constrained myopic consumers, where their

[11] Stigler (1965)[99] suggests this criterion to assess any economic theory.

preferences are specified by either a quasi-hyperbolic discounting or a generalized double exponential discounting, the general equilibrium result can be consistent with the known characteristics of the lifecycle consumption data. Finally, through simulation I look for the best parameter values for the consumption profile produced by the model to fit the mean consumption profile of the US consumption data estimated by Gourinchas and Parker (2002)[57].

3.1.1 Main Findings

I find the model with present-biased preference helps solve the consumption puzzle, i.e. the discrepancy between the hump-shaped consumption data and theoretic prediction of its profile denoted by monotonicity, far from the hump. In particular, the model with a constrained myopic consumer produces a consumption hump, with similar size and location of consumption peak, not only in partial equilibrium, but also in a well-calibrated general equilibrium under reasonable parameterization. In calibrated general equilibrium, I find that the model also predicts many of the salient lifecycle features of consumption data, even in more plausible environment of social security, bequest and mortality risks.

In the baseline model with double exponential discounting, I find that for all of the values of risk aversion parameter that I consider, the model generates a consumption hump, but with a consumption peak age earlier than the data. In term of quantitative fit to the data, MSE as the criterion, it is shown that the model with risk aversion value of 0.5 induces the best fit to the data. By the location of the consumption peak age, the model with around $2 \sim 3$ gives better fit. Also, by the peak to initial consumption ratio, models with lower risk aversion give better fits. When I set the risk aversion parameter at 2, I find that each aggregator $\omega = \{0.3, 0.5, 0.7\}$[12]has a relative strength in explaining the data, according to each different criterion. When I perform simulation exercises to find the optimal values of the discounting factors, I find that the resulting consumption profiles do not change significantly by the choice of alternative β. This may be explained by the fact that in a calibrated general

[12]The parameter ω represents the relative weight between the two exponential discount factors: $U(t) = \omega \sum_{\tau=t}^{T} \beta^{\tau-t} \frac{c_{\tau}^{1-\gamma}(t)}{1-\gamma} + (1-\omega) \sum_{\tau=t}^{T} \delta^{\tau-t} \frac{c_{\tau}^{1-\gamma}(t)}{1-\gamma}$

equilibrium, the other discounting factor, which is δ in the model, jointly adjusts to the macroeconomic targets.

Beyond these findings, I also find the following relationships among the parameters in the general equilibrium: first, δ is inversely related with risk aversion parameter γ, for all values of β and ω. Moreover, given any γ, the δ is inversely related with β, but positively related with ω. Second, when γ is relatively low, the δ may have chances of exceeding one so that it is more likely to violate the assumption of present biased preference as β gets lower and ω gets higher.

When the model is extended to include mortality risks and social security, I find the followings. First, like in baseline model, when the value of γ is greater than one, the model tends to conform to the featured consumption peak, but at somewhat earlier age than the data.[13] In term of quantitative fit to the data, the model with higher risk aversion values like $\gamma = \{2, 2.5, 3\}$, induces a smaller deviation from the data. With these values, the location of the consumption peak age is also close to the data. Also, by the peak consumption to initial consumption ratio, $\gamma = 1.5$ gives the best fit, followed by the group of $\gamma = \{2, 2.5, 3\}$. Therefore, when both mortality risks and social security are introduced, the model with γ around 2 to 3 gives overall good fit to the data .

Second, when the risk aversion parameter is set to the baseline value of 2, the consumption peak age fits better to the data with lower aggregator, i.e. $\omega = \{0.3\}$, but by other criterion, $\omega = \{0.5\}$ is better. Third, like in baseline model, I find that consumption stream does not change a lot by the choices of alternative β. Finally, I compare the four different model specifications and find that the model with both mortality risks and social security produces a best result in terms of quantitative fit and the peak to initial consumption ratio. But by the consumption peak age, the model without either mortality risks or social security fits better. In this case, it is found that the consumption peak age is slightly earlier than the data.

[13] The peak age is also earlier than the age of the baseline model.

3.1.2 Literature Review

There are many related papers both in the non-standard models of economic agents with behavioral assumptions and in standard models of far-sighted and fully rational agents. Regarding behavioral models, time-inconsistent preferences must be addressed. The "Golden Eggs" model by Laibson (1997)[69] is the one, as is discussed in the introduction. Caliendo and Aadland (2007a)[20]'s short term planning approach belongs to this time inconsistent preferences, as well.

Related, but not a direct logical consequence of time inconsistency is the *procrastination*. As O'Donoghue and Rabin (1999)[84] argue, the self is divided into naive and sophisticated ones who have beliefs about future selves. The sophisticated one has correct beliefs and does not procrastinate, while the naive one does. The policy implication of this approach is that in the presence of many errors in retirement planning by the majority of the economy with the present-biased preferences, cautious paternalism may help solve this problem. For example, tax incentives designed to increase savings may also increase the cost of delay in saving. Likewise less frequent transaction dates incurs small cost for time consistent selves, but large cost for procrastinators, and thus help curb the procrastinators from such behavior.

A concept that is related to, but distinct from, the present-biased preferences and that has not been much addressed in the behavioral economics literature is the *shorter than usual lifetime planning horizon* due to bounded rationality of decision maker. *Bounded Rationality* may be defined to be a limitation of human rationality arising from bounds on available information, the cognitive limitations of their minds, and the finite amount of time they have to make decisions. Therefore bounded rationality implies that the economic agent's decision making based on rationality may be incomplete if either the agent does not have full information regarding all the options he could consider or there exists costs, physical or mental, to decision making. Caliendo and Aadland (2007a)[20] and Park (2011)[86] model bounded rationality as short term planning horizon where the boundedly rational consumer maximizes the discounted utility from consumption over a shorter interval of time than his known lifetime. It is shown that this modification of the standard model can generate a hump-shaped consumption profile in general equilibrium whereas the standard model with full planning horizon cannot. Another line of bounded rationality is the deviation from full information, due to information cost (Reis, 2006[92]) or implementation cost (Caliendo and Huang, 2007b[21]). Other than these there are a couple of others that may be considered for bounded rationality. They are rational inattentiveness (Luo, 2005[77]), dual-self (Levine and Fudenberg, 2006[73]), and overconfidence (Caliendo and Huang, 2008[22]).

Regarding the lifecycle consumption profile implied by the standard assumption on fully rational agent, several papers should be mentioned. Borrowing constraint, mortality risk, consumption and leisure substitutability, income uncertainty and precautionary savings are the main topics among them. First, the relative importance of precautionary savings related with the borrowing frictions in the model is well studied in Feigenbaum's (2008c)[47] general equilibrium model. He shows that, along with the consumption hump, in a general equilibrium lifecycle model with an exogenous borrowing constraint, observable macroeconomic variables are insensitive to simultaneous changes in the discount factor and risk aversion that preserve the equilibrium interest rate. This calibration implication has a common element with this paper although my work posits a different preference specification than his. Feigenbaum also shows that the unobservable fraction of aggregate saving due to precautionary motives increases with consumers' risk aversion and the effect of the parameter on observable macro variables depend on the assumptions about borrowing constraints in the model.

Regarding other mechanisms to show a hump, the most successful ones are: Family Size Effect (Attanasio et. al., 1999[12]), Borrowing Constraint (Deaton,1991[32]), Mortality Risk (Feigenbaum, 2008a[45]; Hansen and Imrohoroglu, 2006[60]), Choice Between Consumption and Leisure (Heckman, 1974[61]; Bullard and Feigenbaum, 2007[61]), Income Uncertainty and Precautionary Saving (Carroll, 1997[24], 2009[25]; Aiyagari, 1994[1], Feigenbaum, 2008b[46]), Consumer Durables (Fernandez-Villaverde and Krueger, 2010[49]).

3.1.3 Chapter Organization

This chapter is organized as follows. In section 2, I present closed form solutions for a simple model, with and without borrowing constraint. Using these results, I derive the analytic conditions for the consumption hump. The next step is to analyze the sophisticated agent who is aware of his self control problem. Then in the next section, I present the full model and describe the baseline general equilibrium for the model of present-biased preferences. First the partial equilibrium and then the general equilibrium in an overlapping generations economy are derived under the condition of borrowing constraint. In the fourth section, quantitative analysis follows. Target variables with US data, calibration method and the results are addressed. The result section discusses the main questions addressed in the model. It also reports sensitivity check. Thereafter, the chapter concludes with the emphasis on the implication of the general equilibrium, as well as of the employment of the non-standard preferences to complement the standard ones.

3.2 ANALYTIC SOLUTION TO A SIMPLE MODEL

First, let us introduce a simple version of the present-biased preference. The well-known quasi-hyperbolic discounting model is a discrete-time approximation to hyperbolic discount function.[14] By the quasi-hyperbolic discounting, an individual's intertemporal choice at time t is represented as follows:

$$U(u_t, u_{t+1}, ..., u_T) = u_t + \beta \sum_{\tau=t+1}^{T} \delta^{\tau-t} u_\tau \tag{3.1}$$

where $\beta, \delta \leq 1$. If $\beta = 1$, this preference corresponds to the case of standard exponential discounting. However, for $\beta < 1$ this representation captures the time-inconsistent taste for immediate gratification. The result is a declining per period rate of time discounting and a violation of time-consistency.[15] Notice that the quasi-hyperbolic discount function can be written as a series of β and δ:

$$\{\beta, \delta\} = \left\{ \begin{array}{ll} 1 & \text{if } \ \tau = t \\ \beta\delta^{\tau-t} & \text{if } \ \tau \geqslant t+1 \end{array} \right\}.$$

[14] Hyperbolic discounting is described by a discount factor $\frac{1}{1+\lambda t}$.

[15] Strictly speaking, there are two changes from the standard discounting: (1) it makes the per-period discount rate change over time. (2) it bases discounting on relativistic time rather than absolute time. For other detailed charcteristics, see Rasmusen (2008)[90].

Consider a present biased agent who maximizes consumption utility following his taste for immediate gratification, which is represented by the quasi-hyperbolic discounting with $\beta, \delta < 1$. Assume that the agent has a nonnegative income stream of $\{y_0, y_1, ..., y_T\}$ and his periodic utility is specified by

$$u(c) = \left\{ \begin{array}{ll} \frac{c^{1-\gamma}}{1-\gamma} & \text{if } \ \gamma \neq 1 \\ \ln(c) & \text{if } \ \gamma = 1 \end{array} \right\}.$$[16]

The agent discounts future utility by the sequence of $\{1, \beta\delta^1, \beta\delta^2, ..., \beta\delta^T\}$. For simplicity, assume for now that there is no borrowing constraint and the agent can borrow or lend freely at the market interest rate $1 + r = R$. Furthermore, let us consider a simplest case that the agent lives three periods so that $T = 2$. Although the main analysis through this paper focuses on the lifecycle model with $T \geqslant 55$, this simple model is easy to tract, yet rich enough to demonstrate the analytic property of the intertemporal optimization with the present-biased preference. The optimization problem of the three period model for the agent at $t = 0$ is,

$$U_0(c_0, c_1, c_2) = Max \ \frac{c_0^{1-\gamma}}{1-\gamma} + \beta\delta^1 \frac{c_1^{1-\gamma}}{1-\gamma} + \beta\delta^2 \frac{c_2^{1-\gamma}}{1-\gamma} \tag{3.2}$$

subject to

$$\begin{aligned} c_0 + b_1 &= y_0 \\ c_1 + b_2 &= y_1 + Rb_1 \\ c_2 \qquad &= y_2 + Rb_2 \end{aligned}$$

where b_1 and b_2 are savings (or borrowings if negative) for the subsequent two periods. Solving the maximization problem yields the optimal consumption profile for $t = \{0, 1, 2\}$, which is

$$c_0 = \frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1 + \beta^{1/\gamma}[\frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2]}$$

$$c_1 = \beta^{1/\gamma} \left(\frac{R}{\phi}\right) \left(\frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1 + \beta^{1/\gamma}[\frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2]} \right)$$

[16] As discussed in Laibson (1998)[70] and Geraats (2006)[55], the elasticity of intertemporal substitution is lower than $1/\gamma$.

$$c_2 = \beta^{1/\gamma}\left(\frac{R}{\phi}\right)^2\left(\frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1 + \beta^{1/\gamma}[\frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2]}\right)$$

where $\phi^{-1} = R^{-1}(\delta R)^{1/\gamma}$.[17] Thus he consumes c_0 at $t = 0$ and he *intends* to consume c_1 at the next period. This consumption plan implies that because $\beta < 1$, unless $\frac{R}{\phi} > 1$, it is always satisfied that $c_0 > c_1 \geqslant c_2$ *regardless of* income stream. The tendency of overconsumption at early stage may be curbed if the interest rate is high enough to reverse the inequalities. Notice that the consumption difference between the first two periods is greater in absolute value than that of next two periods, characterizing the consumption outcome from (β, δ)-preferences.[18]

At $t = 1$, however, the present biased agent realizes that the consumption he planned for the period is not fulfilling his taste of immediate gratification. In fact, he wants to consume more now than what he is supposed to have. This desire can be realized at the cost of future consumption c_2. Thus, instead of consuming c_1 obtained above, the agent re-optimizes the current and future consumption according to his *consume-more-now* taste. He solves a revised maximization problem for the remaining periods such as:

$$U_1(c_1, c_2) = Max\ \frac{c_1^{1-\gamma}}{1-\gamma} + \beta\delta^1\frac{c_2^{1-\gamma}}{1-\gamma} \tag{3.3}$$

subject to

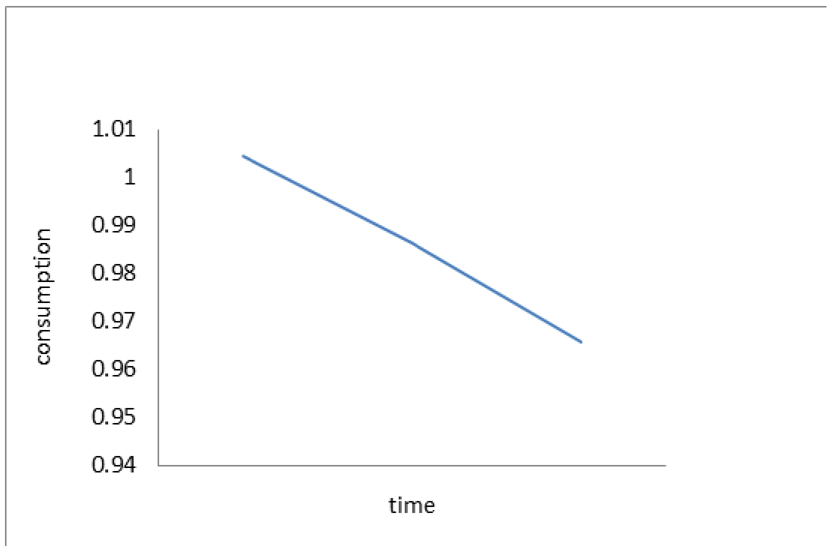
$$\begin{aligned} c_1 + b_2 &= y_1 + Rb_1 \\ c_2 &= y_2 + Rb_2 \end{aligned}$$

The solution to this problem is the optimal consumption profile for $t = \{1, 2\}$. Let the consumption be c_1^1 and c_2^1, emphasizing the difference between the new consumption solved at $t = 1$ and the one planned from $t = 0$. They are

[17] This measure is related with the limit of marginal propensity to consume. For details, see Feigenbaum (2005)[43].

[18] It is worth to notice that except the first period consumption, all others are monotonically increasing or decreasing by the rate of $\frac{R}{\phi}$. This is true for any $T-$period optimzation of the model.

Figure 17: Consumption with Q-Hyperbolic Discounting in Three-Period Model: $R = 1.035$, $\gamma = 2$, $\beta = 0.5$, $\delta = 0.97$. The income is 1 for all three periods.



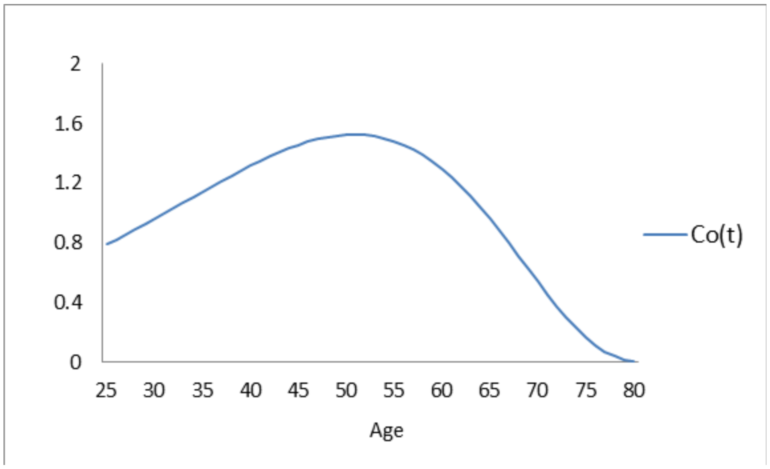
$$c_1^1 = \frac{y_1 + Rb_1 + \frac{y_2}{R^1}}{1 + \beta^{1/\gamma}\frac{1}{\phi}}$$
$$c_2^1 = \beta^{1/\gamma}\left(\frac{R}{\phi}\right)\left(\frac{y_1 + Rb_1 + \frac{y_2}{R^1}}{1 + \beta^{1/\gamma}\frac{1}{\phi}}\right).$$

The rebalanced consumption c_1^1 is different from the planned one c_1^0. The new consumption is a function of *current* cash on hand $y_1 + Rb_1$. All together, the revised consumption profile for the entire three periods, $t = 0, 1, 2$, is

$$\{c_0^0, c_1^1, c_2^1\} =$$
$$\left\{\frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1 + \beta^{1/\gamma}[\frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2]}, \frac{y_1 + Rb_1 + \frac{y_2}{R^1}}{1 + \beta^{1/\gamma}\frac{1}{\phi}}, \frac{\beta^{1/\gamma}R}{\phi}\left(\frac{y_1 + Rb_1 + \frac{y_2}{R^1}}{1 + \beta^{1/\gamma}\frac{1}{\phi}}\right)\right\}$$

The last term in the bracket follows from the fact that the agent no longer needs to renew his consumption at the last period because no more cash on hand is realized. That is $c_2^2 = c_2^1$. Solving for $b_1 = y_0 - c_0$ pins down the consumption profile $\{c_0^0, c_1^1, c_2^1\}$ for the model. Figure 17 shows the resulting consumption with the simple model. In general, for any $t = 0, 1, ..., T,$ solving recursively for bond demand of each period returns the

Figure 18: Lifecycle Consumption with Q-Hyperbolic Discounting in Partial Equilibrium: $R = 1.045$, $\gamma = 0.5$, $\beta = 0.5$, $\delta = 0.98$, $T + 25 = 80$. The income follows GP.



entire consumption profile. The consumption $\{c_0^0, c_1^1, ..., c_T^T\}$ is obtained by substituting the renewed cash on hand $y_t + Rb_t$ into consumption equations. Figure 18 and 19 show the lifecycle consumption profiles simulated from the US income data for the present biased agent with $T + 25 = 80$.

From these graphs, it is easy to notice that the consumption profile may or may not be hump-shaped depending on the parameter values. To study further this issue, I want to define here the consumption hump as follows:[19]

Definition *A consumption hump for T-period model is a consumption profile* $\{c_t\}_{t=0}^T$ *that satisfies*[20]

i) There is a consumption peak at time $t \in (0, T)$.

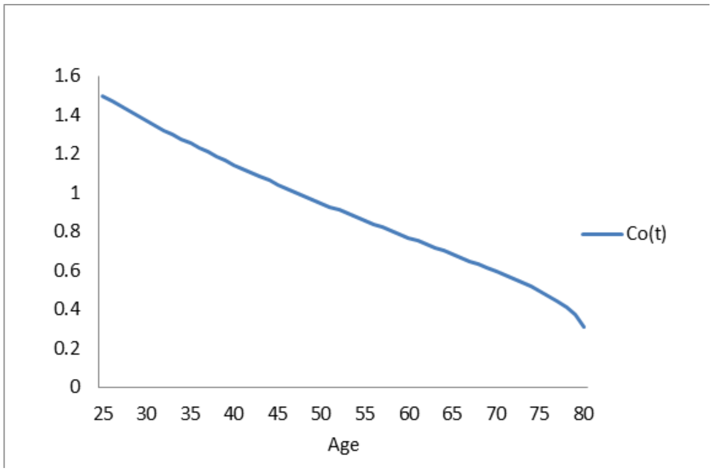
ii) Consumption is monotonically increasing up to t.

iii) Consumption is monotonically decreasing beyond t.

[19] This definition is in a strong sense. In a weak sense, it may allow having any wiggle over the horizon and there can be many local peaks.

[20] For example, if there are three periods, $t = 0, 1, 2$, then the hump condition requires that $\{c_0 < c_1 > c_2\}$ implying the peak occurs at c_1.

Figure 19: Lifecycle Consumption with Q-Hyperbolic Discounting in Partial Equilibrium: $R = 1.035$, $\gamma = 2$, $\beta = 0.7$, $\delta = 0.95$, $T + 25 = 80$. The income follows GP.



Let us go back to the consumption profile of the simple model. The closed form solution to the problem for $t = \{0, 1, 2\}$ is

$$c_0^0 = \frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1 + \beta^{1/\gamma}[\frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2]}$$

$$c_1^1 = \frac{\left(Ry_0 + y_1 + \frac{y_2}{R}\right)\beta^{1/\gamma}[\frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2]}{\left(1 + \beta^{1/\gamma}\frac{1}{\phi}\right) 1 + \beta^{1/\gamma}[\frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2]}$$

$$c_2^2 = \frac{\left(R^2 y_0 + Ry_1 + y_2\right)\beta^{2/\gamma}[\left(\frac{1}{\phi}\right)^2 + \left(\frac{1}{\phi}\right)^3]}{\left(1 + \beta^{1/\gamma}\frac{1}{\phi}\right) 1 + \beta^{1/\gamma}[\frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2]}$$

and the bond demand is

$$b_1 = \frac{\beta^{1/\gamma}[\frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2]y_0 - \left(\frac{y_1}{R^1} + \frac{y_2}{R^2}\right)}{1 + \beta^{1/\gamma}[\frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2]}$$

$$b_2 = \left(\left(\frac{R}{\phi}\beta^{1/\gamma}\left(y_1 + y_0 R\right) - y_2\right)[\frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2] - \frac{y_2}{\phi}\right)\beta^{1/\gamma} - y_2.$$

To explore the consumption property, first compare this with the one of a rational agent who has a standard exponential discounting, i.e. the case where $\beta = 1$. Then

$$\{c_0, c_1, c_2\}^{standard} =$$

$$\left\{ \frac{y_0+\frac{y_1}{R^1}+\frac{y_2}{R^2}}{1+\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2}, \frac{\frac{1}{\phi}\left(Ry_0+y_1+\frac{y_2}{R}\right)}{1+\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2}, \frac{\left(\frac{1}{\phi}\right)^2\left(R^2y_0+Ry_1+y_2\right)}{1+\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2} \right\}$$

Proposition 4. *Given a fixed interest rate R, for any fixed γ value, $0 < \gamma < \infty$, the agent with the present biased preference consumes more initially than the standard, rational agent for all values of time preference parameter δ. That is, $c_0^{present-biased} > c_0^{standard}$ for each δ and γ.*

Proof. This is a direct application of the definition of the present-biased preference, requiring $\beta < 1$. Fix the value of R. Then for all values $0 < \gamma < \infty$, and for all values of $\beta < 1$ and $\delta < 1$, it is satisfied that

$$c_0^{present-biased} = \frac{y_0+\frac{y_1}{R^1}+\frac{y_2}{R^2}}{1+\beta^{1/\gamma}[\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2]} > \frac{y_0+\frac{y_1}{R^1}+\frac{y_2}{R^2}}{1+\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2} = c_0^{standard}. \qquad \square$$

Now, assume that the agent is constrained in asset markets and the agent can only save but not borrow. Then the optimal consumption plan of the unconstrained agent in the previous section may not be feasible and it should be modified. To explore the consumption profile for the agent under the constraint, consider the following three cases regarding binding situation over $\{c_0^0, c_1^1, c_2^2\}$:

- All of the three consumption points $\{c_0^0, c_1^1, c_2^2\}$ are not binding: [NNN]
- $\{c_0^0\}$ is binding, but $\{c_1^1, c_2^2\}$ are not binding: [BNN]
- $\{c_0^0, c_1^1\}$ are binding, but $\{c_2^2\}$ is not binding: [BBN]

Case 1: **[NNN]** When consumption is not binding at all during the entire life, the resulting consumption profile is exactly equal to the one $\{c_0^0, c_1^1, c_2^1\}$ of the unconstrained model above. That is,

$$\{c_0^0, c_1^1, c_2^2\}^{NNN} = \left\{ c_0^0, \quad \left(\frac{\beta^{1/\gamma}[R+\frac{R}{\phi}]}{\beta^{1/\gamma}+\phi}\right) c_0^0, \quad \left(\frac{\beta^{2/\gamma}\frac{R}{\phi}[R+\frac{R}{\phi}]}{\beta^{1/\gamma}+\phi}\right) c_0^0 \right\},$$

where

$$c_0^0 = \frac{y_0+\frac{y_1}{R^1}+\frac{y_2}{R^2}}{1+\beta^{1/\gamma}[\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2]}.$$

If the consumption increases initially and decreases afterwards, i.e. $c_0^0 < c_1^1$ and $c_1^1 > c_2^2$, then the profile yields a hump defined above. Therefore, assuming $0 < \gamma < \infty$ and $R > 0$, I have the following proposition.

Proposition 5. *With the present biased preference, i.e. $\beta < 1$, and $\delta < 1$, the consumption hump is obtained regardless of income stream if it is satisfied that $\frac{R}{R-1+(\delta R)^{1/\gamma}} < (\beta\delta R)^{1/\gamma} < 1$.*[21] *Therefore, given R, δ and $\gamma \in (0,\infty)$, the bound for β is given by $\frac{1}{\delta R} < \beta < \left(\frac{R}{(\delta R)^{1/\gamma}\{R-1+(\delta R)^{1/\gamma}\}}\right)^{\gamma}$.*

Proof. Suppose that consumption is not increasing initially. Then it should be satisfied that $c_0^0 \geqslant c_1^1$, which means $\left(1+\beta^{1/\gamma}\frac{(\delta R)^{1/\gamma}}{R}\right) \geqslant R\beta^{1/\gamma}\left(\frac{(\delta R)^{1/\gamma}}{R}+\left(\frac{(\delta R)^{1/\gamma}}{R}\right)^2\right)$. This inequality implies $R \geqslant (R\text{-}1+(\delta R)^{1/\gamma})(\beta\delta R)^{1/\gamma}$ which contradicts $\frac{R}{R-1+(\delta R)^{1/\gamma}} < (\beta\delta R)^{1/\gamma}$. Suppose secondly that consumption is not decreasing at later periods. Then it must be true that $c_1^1 \leqslant c_2^1$, which means that $1 \leqslant R\beta^{1/\gamma}\frac{(\delta R)^{1/\gamma}}{R}$. This again contradicts $1 > (\beta\delta R)^{1/\gamma}$. Therefore, $c_0^0 < c_1^1 > c_2^2$. □

The condition can be rewritten directly from the consumption function as:

$$1 + \beta^{1/\gamma}\frac{1}{\phi} < \frac{R}{\phi}\left(\beta^{1/\gamma} + \beta^{1/\gamma}\frac{1}{\phi}\right) < 1 + \frac{R}{\phi}\beta^{1/\gamma}\frac{1}{\phi}$$

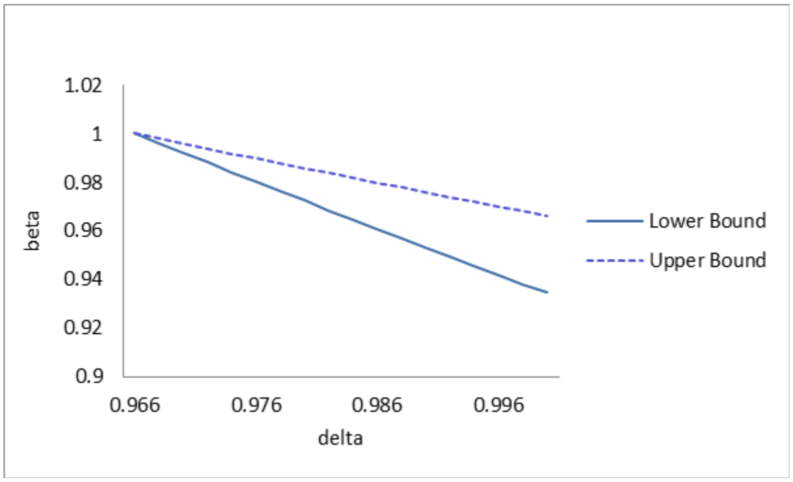
It is easy to notice that the inequalities are not satisfied when $\frac{R}{\phi} \leqslant 1$ because $\beta^{1/\gamma} < 1$. However, when $\frac{R}{\phi} > 1$, the condition is satisfied for certain parameter values that induces $1-\varepsilon < \frac{R}{\phi}\beta^{1/\gamma} < 1$. Intuitively, when the interest rate is high enough relative to agent's time preference, even the present biased agent is willing to curb his overconsumption for future benefit.[22] The key implication of the proposition is that there exists a range of parameter values that can induce a consumption hump regardless of income stream. Moreover, the hump is achieved without any asset constraint. This is important because in most of the literature that deals with present biased preferences, a sort of borrowing constraint plays a crucial role to induce a consumption hump.[23] In my model the hump is achieved without

[21] Here I assume that the interest rate satisfies $R + (\delta R)^{1/\gamma} > 1$ for technical purpose. Unless the real interest rate is deep in negative value, this condition is always satisfied.

[22] Remember that this does not mean he violates the tenet of quasi-hyperbolic discounting or consume more now taste.

[23] As described in *Introduction*, the possibility of consumption hump in Laibson (1997)[69] comes from the fact that there exist illiquid assets (Golden eggs), in his model, which play an equivalent economic role as a borrowing constraint. Unlike Laibson or others, my model demonstrates the property without resorting to

Figure 20: Lower and Upper Bound for the beta over various δ when $R = 1.035$ and $\gamma = 0.5$.



those constraints, but under certain parameter values when the agent negates his earlier resolution and re-optimizes following his *consume more now* taste. Figure 20 shows the upper and the lower bounds for β over various δ in the simple model when $R_{ann} = 1.035$ and $\gamma = 0.5$, together with per length of 20 years.

Case 2: **[BNN]** If the borrowing constraint is binding at $t = 0$, the agent cannot achieve his optimal consumption c_0^0 because it is not feasible. Instead, his consumption is limited by the current wealth or cash on hand, which is $y_0 + Rb_0 = y_0$. Thus, the feasible consumption for the first period is y_0. At $t = 1$, because he saved none (in fact, borrowed none) last period, the agent solves a revised maximization problem for the rest of the periods, with current resource (cash on hand) and the expected future income afterwards. Because he inherits nothing from last period, the current financial wealth is $y_1 + Rb_1 = y_1$ and his future income is $\frac{y_2}{R}$ when discounted. Thus, for $t = 0, 1, 2,$ the consumption profile is

$$\{c_0^0, c_1^1, c_2^2\}^{BNN} = \left\{ y_0, \ \frac{y_1 + \frac{y_2}{R^1}}{1 + \beta^{1/\gamma}\frac{1}{\phi}}, \ \beta^{1/\gamma}\frac{R}{\phi}\left(\frac{y_1 + \frac{y_2}{R^1}}{1 + \beta^{1/\gamma}\frac{1}{\phi}}\right) \right\}.$$

To explore further the consumption property with BNN, consider two agents who face the same interest rate but differ either in their income or taste. Between the two agents, it is the one who has a lower initial income if both have the same time preference; and the one

any asset constraint.

who has a stronger taste if both have the same income stream, that may face a borrowing constraint. By this it is inferred that a constrained agent is more likely to have either a lower income or a stronger taste unless the interest rate is quite low.[24] Therefore, unlike Case 1 where the income profile does not matter to induce a hump-shaped consumption profile, in Case 2, the structure of income stream matters. Thus I propose conditions for the hump under constraint like the following:

Proposition 6. *When a borrowing constraint is binding initially, the consumption hump is obtained if i) the income stream satisfies* $y_0 + \frac{y_0(\beta\delta R)^{1/\gamma}}{R} < y_1 + \frac{y_2}{R}$ *and ii)* $(\beta\delta R)^{1/\gamma} < 1$.

Proof. Suppose that the consumption is not a hump. Then it should be satisfied that either $c_0^0 \geqslant c_1^1$ or $c_1^1 \leqslant c_2^2$. The former implies $y_0(1 + \beta^{1/\gamma}\frac{1}{\phi}) \geqslant y_1 + \frac{y_2}{R^1}$, which contradicts the condition $y_0 + \frac{y_0(\beta\delta R)^{1/\gamma}}{R} < y_1 + \frac{y_2}{R}$. The latter, on the other hand, implies $1 \leqslant R\beta^{1/\gamma}\frac{1}{\phi}$, which contradicts $(\beta\delta R)^{1/\gamma} < 1$. Therefore, the consumption is a hump. □

This first condition implies that a lowest initial income is a key factor for the hump when the interest rate is sufficiently low.[25] Notice that if the income keeps increasing over time, $y_0 < y_1 < y_2$, then the inequality condition is always satisfied when the second condition cooperates. If, however, the initial income is not the lowest, then it is necessary to have a relatively high income for y_1 to satisfy the condition. The second condition implies that the interest rate should be low relative to the time preference of the agent so that the consumption of later stage is decreasing. Finally,

Case 3: **[BBN]** If the borrowing constraint is binding at the first and the second periods, no part of his initial plan can be realized. His consumption is limited by the cash on hand each time. The consumption for the first period is y_0, the same as [BNN]. But the consumption for the second period is also y_1 because his planned consumption[26] for $t = 1$ is not feasible, either. At the final period, $t = 2$, because he has no financial income from earlier periods, he should live on his current income. Thus, $\{c_0^0, c_1^1, c_2^2\}^{BBN} = \{y_0, y_1, y_2\}$.

[24] When interest rate is sufficiently low there is little incentive to save and even the patient consumer may want to borrow.

[25] If the interest rate is low, it is more likely for the agent to borrow when his income is not sufficient.

[26] The planned consumption is $\dfrac{y_1 + \frac{y_2}{R^1}}{1 + \beta^{1/\gamma}\frac{1}{\phi}}$.

Then the hump is obtained if his income has a hump-shaped profile, i.e. $y_0 < y_1 > y_2$. In this case, the consumption and the income co-move over life time.

3.3 SELF AWARENESS

In this section I analyze the case in which the agent is not naïve but sophisticated enough to be aware of his future behavior of overconsumption. Consider again an agent with present-biased preference who lives three periods, $t = 0, 1, 2$, and maximizes his periodic utility over the three periods given a known income stream $\{y_0, y_1, y_2\}$. Although the agent has a time-inconsistent preference and discounts the future utility by the sequence of $\{1, \beta\delta^1, \beta\delta^2\}$, the sophisticated agent is *aware of his inconsistency.* Because there is no borrowing constraint, the present-biased agent is able to consume as much as he wants at earlier periods, so long as it satisfies the three-period lifecycle wealth constraint.

The sophisticated agent wants to solve this problem by controlling his future behavior. Because, as shown in previous section, overconsumption occurs on the prey of final stage consumption, the agent wants to control his behavior of the second period to guarantee a comfortable life for the final period.[27] At $t = 0$, the agent considers a saving plan in which he deposits certain amount in the first period. Let the saving be s. By this saving contract, he is not allowed to withdraw the saving till the maturity, which is $t = 2$. Assume for simplicity that the interest rate for this saving follows the same market interest rate $1 + r = R$ by which the bond rate is made.[28]

To analyze the self control problem and get the sub-game perfect equilibrium, it is necessary to solve the maximization problem by backward induction. I want to solve for the saving amount s that the initial self needs to have to control his future behavior. Let each self at three different times be self (0), self (1), and self (2). At the final stage $t = 2$, because there is no more period to consider, it is clear that self (2) consumes everything available to

[27] The sophisticated agent can control both the first and the second period overconsumption. This is sometimes called the behavior of *long-run self,* and analyzed with $\beta = 1$. In this case the outcome goes back to the standard exponential one.

Here, I focus on more interesting case where the agent does not give up his initial preference but wants to control his next-stage self.

[28] The model assumes the agent utilizes bond market between periods to borrow or lend as before.

him regardless of his taste:

$$c_2 \geqslant R^2 s$$

$R^2 s$ is the minimum amount self (2) can get when there is nothing left from earlier periods except for the forced saving amount. In fact, the last consumption c_2 is the residual from the consumption behavior by self (1) from his optimization at $t = 1$. Therefore, the key solution needed for the analysis is the one by self (1). The optimization problem of the agent who has a taste of *consume more now* at $t = 1$ is

$$U_1(c_1|c_2) = Max\ \frac{c_1^{1-\gamma}}{1-\gamma} + \beta\delta^1 \frac{c_2^{1-\gamma}}{1-\gamma} \tag{3.4}$$

subject to

$$\begin{aligned} c_1 + b_2 &= y_1 + Rb_1 \\ c_2 \qquad &= y_2 + Rb_2 + R^2 s \\ c_2 &\geqslant R^2 s \end{aligned}$$

where $b_1 = [y_0 - s] - c_0$ is the bond holding by the initial self, representing self (1) inherits financial wealth, positive or negative, from the maximization at $t = 0$. Remember that s is not available at $t = 1$ and the maximum amount he can borrow should not go beyond the present value of the income at $t = 2$. Thus it leaves self (2) at least $R^2 s$. The maximum borrowing is

$$-\tfrac{y_2}{R} \leqslant b_2$$

Solving the problem gives optimal solution to the self (1) at $t = 1$. If the optimal consumption $\{c_1^1, c_2^1\}$ does not violate the minimum allowance constraint $c_2 \geqslant R^2 s$, then the solution is not different from the one in the previous section.[29] Upon this result there will be no need of commitment device like the saving plan by self (0). However, if the constraint is binding, then the solution is $c_1^1 = y_1 + Rb_1 + \frac{y_2}{R}$

$$c_2^1 = R^2 s$$

[29] This is because the model assumes the same interest rate for both bond demand and the saving plan.

which requires for self (0) to occupy the commitment device of the saving plan. Thus letting $\{c_1, c_2\} = \{c_1^1, c_2^1\}$, self (0) solves

$$U_0(c_0|c_1, c_2) = Max\ \frac{c_0^{1-\gamma}}{1-\gamma} + \beta\delta^1 \frac{c_1^{1-\gamma}}{1-\gamma} + \beta\delta^2 \frac{c_2^{1-\gamma}}{1-\gamma} \tag{3.5}$$

subject to

$$c_0 + b_1 = y_0 - s$$

Following the Appendix for maximization procedure, the saving is obtained

$$s = R^{-2} (R\delta)^{1/\gamma} \left(y_1 + Rb_1 + \tfrac{y_2}{R}\right)$$

From the first order condition $y_0 - s - b_1 = \left(\beta\delta^2 R^2\right)^{-1/\gamma} R^2 s$, substituting b_1 into the saving equation returns

$$s = R^{-2} (R\delta)^{1/\gamma} \left(y_1 + R(y_0 - [1 + R^2 \left(\beta\delta^2 R^2\right)^{-1/\gamma}]s) + \tfrac{y_2}{R}\right)$$

Rearranging to get the necessary saving amount and the bond holding as follows:

$$s = \frac{\frac{\beta^{1/\gamma}}{\phi^2}\left(y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}\right)}{1 + \beta^{1/\gamma}[\frac{1}{R\phi} + \left(\frac{1}{\phi}\right)^2]}$$

$$b_1 = \frac{\frac{\beta^{1/\gamma}}{R\phi} y_0 - \left(\frac{\beta^{1/\gamma}}{\phi^2} + 1\right)\left(\frac{y_1}{R^1} + \frac{y_2}{R^2}\right)}{1 + \beta^{1/\gamma}[\frac{1}{R\phi} + \left(\frac{1}{\phi}\right)^2]}$$

Thus, the optimal consumption at $t = 0$ is

$$c_0 = \frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1 + \beta^{1/\gamma}[\frac{1}{R\phi} + \left(\frac{1}{\phi}\right)^2]},$$

because $c_0 = y_0 - s - b_1$. By the value of b_1 and s, the consumption points of the subsequent periods are

$$c_1 = y_1 + Rb_1 + \tfrac{y_2}{R} = \frac{\frac{\beta^{1/\gamma}}{\phi}\left(y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}\right)}{1 + \beta^{1/\gamma}[\frac{1}{R\phi} + \left(\frac{1}{\phi}\right)^2]}$$

$$c_2 = R^2 s = R^2 \frac{\frac{\beta^{1/\gamma}}{\phi^2}\left(y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}\right)}{1 + \beta^{1/\gamma}[\frac{1}{R\phi} + \left(\frac{1}{\phi}\right)^2]}$$

All together, the complete consumption profile is

$$(A) : \{c_0, c_1, c_2\}^{Controlled} =$$

$$\left\{ \frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1+\beta^{1/\gamma}[\frac{1}{R\phi} + \left(\frac{1}{\phi}\right)^2]}, \frac{\frac{\beta^{1/\gamma}}{\phi}\left(y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}\right)}{1+\beta^{1/\gamma}[\frac{1}{R\phi} + \left(\frac{1}{\phi}\right)^2]}, \frac{\frac{\beta^{1/\gamma}R^2}{\phi^2}\left(y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}\right)}{1+\beta^{1/\gamma}[\frac{1}{R\phi} + \left(\frac{1}{\phi}\right)^2]} \right\}$$

The controlled consumption of the sophisticated agent tells us that the last consumption c_2 is large enough that it can be higher than the second c_1 even in the case of $\frac{R}{\phi} < 1$. Let us compare this controlled consumption (A) with the consumption profile made by present biased self (0) who is without the self control problem (B)[30] and the consumption profile of the present biased naïveté obtained in previous subsection (C). Thus,

$$(B) : \{c_0, c_1, c_2\}^{No\ Self-Control} =$$

$$\left\{ \frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1+\beta^{1/\gamma}[\frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2]}, \frac{\frac{\beta^{1/\gamma}R}{\phi}\left(y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}\right)}{1+\beta^{1/\gamma}[\frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2]}, \frac{\frac{\beta^{1/\gamma}R^2}{\phi^2}\left(y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}\right)}{1+\beta^{1/\gamma}[\frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2]} \right\}$$

and

$$(C) : \{c_0, c_1, c_2\}^{Uncontrolled} =$$

$$\left\{ \frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1+\beta^{1/\gamma}[\frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2]}, \quad \frac{\frac{\beta^{1/\gamma}R}{\phi}\left(1+\frac{1}{\phi}\right)}{\left(1+\beta^{1/\gamma}\frac{1}{\phi}\right)}(c_0), \quad \frac{\frac{\beta^{2/\gamma}R^2}{\phi^2}\left(1+\frac{1}{\phi}\right)}{\left(1+\beta^{1/\gamma}\frac{1}{\phi}\right)}(c_0) \right\}$$

It is easy to see that for a special case of $R = 1$, the initial consumptions by all three methods are equal to each other. In fact, when $R = 1$, the controlled consumption profile (A) is identical to the one without self control problem (B). That is, $\{c_0, c_1, c_2\}^{Controlled} = \{c_0, c_1, c_2\}^{No\ Self-Control}$. But with usual interest rate of $R > 1$, it is clear that

$$\{c_0\}^{Controlled} > \{c_0\}^{No\ Self-Control} = \{c_0\}^{Uncontrolled}$$

And regarding the last period consumption, it is also true that

$$\{c_2\}^{Controlled} > \{c_2\}^{No\ Self-Control} > \{c_2\}^{Uncontrolled}$$

Figure 21 and Figure 22 show this result. Once the agent controls his future selves, the result is not much different from the one without self control problem.

[30] In other words, the consumption plan by the initial self is honored by subsequent selves.

Figure 21: Three consumption profiles with and without self control problem. With self control problem, agent is either naive (Uncontrolled) or sophisticated (Controlled): $R = 1.035$, $\gamma = 0.5$, $\beta = 0.9$, $\delta = 0.95$.

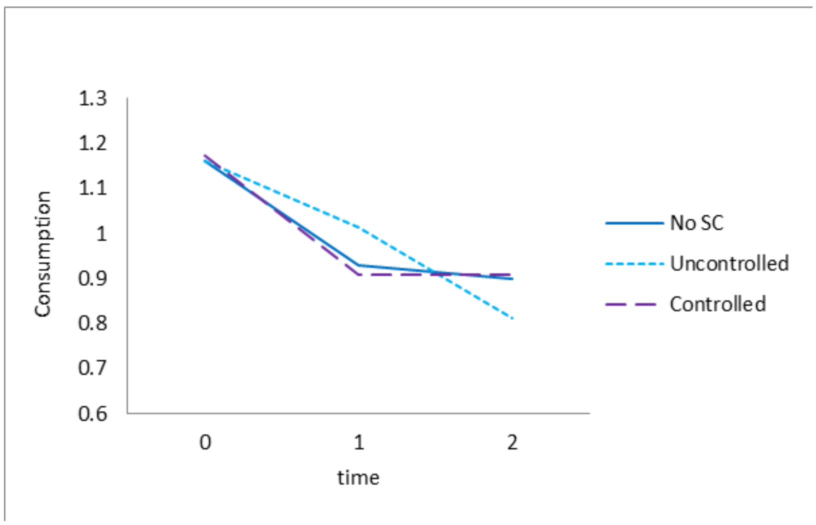
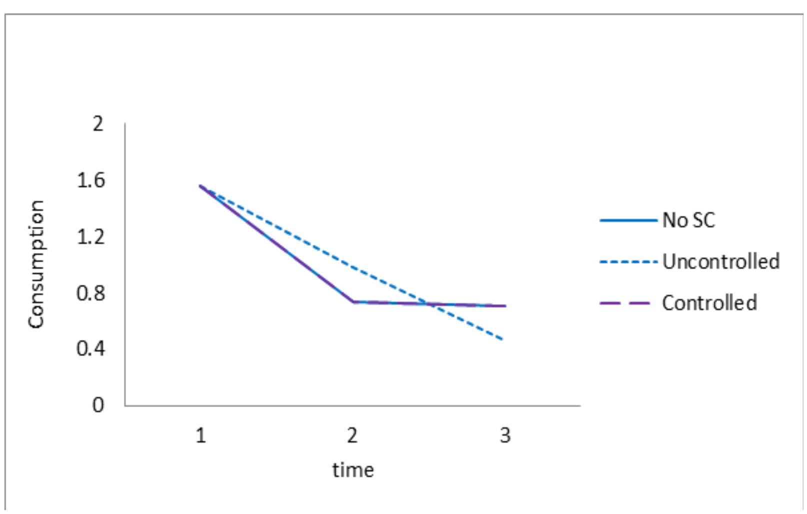


Figure 22: Three consumption profiles with and without self control problem. When R=1, the self controlled consumption coincides with the one where there is no self control problem: $R = 1$, $\gamma = 0.5$, $\beta = 0.7$, $\delta = 0.98$.



3.4 A LIFECYCLE MODEL WITH PRESENT BIASED PREFERENCES

In this section, a general equilibrium model with a constrained consumer who has present-biased preference is presented. Consumer's time preference is represented by the usual quasi-hyperbolic discounting. But a more generalized discounting form can also be used. Here I analyze both cases. I consider a $(T+1)$-*period* overlapping generations general equilibrium where there are $(T+1)$ types of identical cohorts, who consume, save or dissave, and supply labor for production every period. The consumer's optimization results in a series of rebalanced consumption and bond demand, net of which meets with capital demand in a stationary competitive equilibrium.

3.4.1 Consumer

3.4.1.1 Environment Time is discrete. At each time a generation of identical cohorts is born. Each agent who is indexed by age t, lives for T periods in a $(T+1)$ - period overlapping generations economy. There is no population growth. During working years, agents are endowed with one unit of labor productivity, measured in efficiency units, which is supplied inelastically. There exists a single good that can be either consumed or saved, in which case it is called capital. Each agent has present-biased preference represented by either Q-hyperbolic discounting or generalized discounting defined later. Although the agent is able to foresee perfectly his future labor productivity or the income stream, all the way up to his life time T, he is not rational enough to control his future taste in advance. Therefore, there is no self control problem in this model and the agent optimizes each time following his myopic taste given his income stream and cash on hand. Moreover, there is a borrowing constraint which the agent cannot anticipate in advance.[31] Thus under market determined interest rate, the agent can earn the financial income if he ever saves. Finally, for this section, assume that there is no government running social security and there is no bequest. No bequest assumption comes from the premise that agents live for sure up to T without any mortality risk. These assumptions are relaxed in next section where we could see what

[31] This implies that the agent can not internalize the borrowing constraint into the his optimization process until the actual consumption time and there is no anticipatory saving for this.

happen to the equilibrium consumption in more realistic environment.

3.4.1.2 Constrained Consumer Optimization A consumer who is in the age t and lives for $T-t+1$ periods has a consumption denoted by $c_s(t)$. The subscript s represents the consumption time. The age also represents the planning time in the model.[32] The myopic consumer at age t, who has a quasi-hyperbolic discounting function, discounts his future consumption at the rate of $\beta\delta^{\tau-t}$ for $\tau = t+1, t+2, .., T$. Then the representative consumer who is credit constrained in asset market, plans for $T-t+1$ periods at each planning time and maximizes for $t = 0, 1, ..., T$,

$$U(t) = \frac{c_t^{1-\gamma}(t)}{1-\gamma} + \beta \sum_{\tau=t+1}^{T} \delta^{\tau-t} \frac{c_\tau^{1-\gamma}(t)}{1-\gamma} \tag{3.6}$$

subject to

$$c_\tau(t) + b_{\tau+1}(t) = we_\tau + Rb_\tau(t)$$
$$b_0(0) = 0,\ b_t(t) = given,\ \ b_{T+1}(t) = 0$$
$$c_\tau(t) \geqslant 0$$
$$b_{\tau+1}(t) \geqslant 0^{33}$$
$$\text{for } \tau = t, ..., T.$$

$c_\tau(t)$ is consumption planned at t for time τ and $b_{\tau+1}(t)$ is bond demand purchased at τ for the next period, indexed by planning time t. The consumer has a stream of productivity over life time so that he supplies e_τ efficiency units of labor at age τ to production and earns labor income of we_τ for $\tau = t, ..., T$, where w is the market determined real wage rate which is assumed to be stationary over time. Likewise, under market determined interest rate, the agent earns $Rb_\tau(t)$ if he carries bond to the next period.

[32] The physical age is $t + 25$ if the consumer starts working at 25 years old.

[33] The borrowing constraint can be modified to accomodate the situation where borrowing is limited by the extent of a portion of current wealth level: $-b_{\tau+1}(t) < -ax(t)$. I find that the result is not much different from the model I propose here.

Before solving the constrained consumer optimization problem, let us first look for the optimal condition for the unconstrained consumer's problem. The first order condition says that, for $\tau = t, ..., T$,

$$c_\tau^{-\gamma}(t) = (R\beta\delta)c_{\tau+1}^{-\gamma}(t), \quad if\ \tau = t$$

and

$$c_\tau^{-\gamma}(t) = (R\delta)c_{\tau+1}^{-\gamma}(t), \quad \text{if} \ \ \tau = t{+}1, ..., T$$

If the borrowing constraint is binding at τ, these equalities are not satisfied. Let $c_t(t)$ be the optimized 'initial' consumption starting from the planning date or age t. Using the budget constraint, one can pin down $c_t(t)$, which is,

$$c_t(t) = \frac{\sum_{s=t}^{T} \frac{we_s}{R^s} + Rb_t(t)}{1 + \beta^{1/\gamma} \sum_{s=t+1}^{T} \left[\frac{(R\delta)^{1/\gamma}}{R}\right]^{s-t}}$$

where $b_t(t)$ is the initial bond holding each planning date and it is assumed that $b_0(0) = 0$. Let us define the total wealth at any time τ over the planning horizon $[t,\, T]$, as the sum of human wealth and financial wealth, i.e.

$$W_\tau(t) = \sum_{s=\tau}^{T} \frac{we_s}{R^{s-\tau}} + Rb_\tau(t),$$

then at $\tau \in [t,\, T]$, it is satisfied that

$$c_\tau(t) = \frac{W_\tau(t)}{1 + \beta^{1/\gamma} \sum_{s=\tau+1}^{T} \left[\frac{1}{\phi}\right]^{s-\tau}},$$

where $\phi^{-1} = (R\delta)^{1/\gamma}R^{-1}$.

Now that the consumer is constrained in asset market, whenever the constraint is binding, the consumer cannot follow the optimal rule, not fulfilling his taste of immediate gratification. Instead, his consumption is curbed by the available cash in his hand. Thus given $\chi_t = we_t + Rb_t$, the feasible consumption under the borrowing constraint is,

$$\begin{aligned} c_t^*(t) &= c_t(t) \quad if \quad c_t(t) \leq \chi_t \\ c_t^*(t) &= \chi_t \qquad if \quad c_t(t) > \chi_t. \end{aligned}$$

In this subsection, I want to explore the model with more general discount factor than the simple quasi-hyperbolic discounting. Consider

$$U(t) = \sum_{\tau=0}^{T-t} f_\beta(\tau) \frac{c_\tau^{1-\gamma}(t)}{1-\gamma} + \sum_{\tau=0}^{T-t} f_\delta(\tau) \frac{c_\tau^{1-\gamma}(t)}{1-\gamma} \tag{3.7}$$

By this preference, the agent discounts future consumption using combined discount factors of $f_\beta(\tau)$ and $f_\delta(\tau)$. One possibility for the form is a linear combination of two exponential discount factors: $\{\omega\beta^\tau + (1-\omega)\delta^\tau\}$.[34] Some argue that the double exponential discounting has a better fit to the lab data than the Q-hyperbolic. Then the representative consumer who is credit constrained maximizes for $t = 0, 1, ..., T$,

$$U(t) = \omega \sum_{\tau=t}^{T} \beta^{\tau-t} \frac{c_\tau^{1-\gamma}(t)}{1-\gamma} + (1-\omega) \sum_{\tau=t}^{T} \delta^{\tau-t} \frac{c_\tau^{1-\gamma}(t)}{1-\gamma} \tag{3.8}$$

subject to

$$\begin{gathered} c_\tau(t) + b_{\tau+1}(t) = we_\tau + Rb_\tau(t) \\ b_0(0) = 0,\ b_t(t) = given,\ b_{T+1}(t) = 0 \\ c_\tau(t) \geqslant 0 \\ b_{\tau+1}(t) \geqslant 0 \\ \text{for } \tau = t, ..., T. \end{gathered}$$

The aggregator ω shows a relative strength between the two discount parameters. Solving the consumer optimization problem implies that when the borrowing constraint is not binding, the inter-temporal optimal rule yields that, for $\tau = t, ..., T$,

$$[\omega\beta^\tau + (1-\omega)\delta^\tau] c_\tau^{-\gamma}(t) = R[\omega\beta^{\tau+1} + (1-\omega)\delta^{\tau+1}] c_{\tau+1}^{-\gamma}(t)$$

and the feasible consumption under the borrowing constraint is,

[34] As shown in the subsequent sections, this choice of preference representation has a better fit to the lifecycle consumption data than the simple quasi-hyperbolic discounting.

$$c_t^*(t) = c_t(t) \quad if \quad c_t(t) \leq \chi_t$$
$$c_t^*(t) = \chi_t \quad if \quad c_t(t) > \chi_t$$

where χ_t is cash on hand at t and the initial consumption at each period is

$$c_t(t) = \frac{\sum_{\tau=t}^{T} \frac{we_\tau}{R^{\tau-t}} + Rb_t(t)}{\sum_{\tau=t}^{T} \frac{R^{(\tau-t)/\gamma}[\omega\beta^{\tau-t}+(1-\omega)\delta^{\tau-t}]^{1/\gamma}}{R^{\tau-t}}}.$$

3.4.1.3 Planned and Realized Consumption in Partial Equilibrium The consumer's optimization at t leads to a vector of consumption schedule for the $T-t+1$ periods starting from t,

$$C^P(t) = \{c_t(t), c_{t+1}(t), c_{t+2}(t), ..., c_T(t)\}.$$

Initially the consumer intends to follow the planned optimal consumption stream and consumes $c_t(t)$ at t and $c_{t+1}(t)$ at $t+1$. However at $t+1$, the myopic consumer realizes that the planned consumption from t for $t+1$, i.e. $c_{t+1}(t)$ is now no longer desirable because he wants to consume more due to his taste for immediate gratification. He has to rebalance the consumption plan to incorporate this, rather than follow the planned consumption. He keeps doing this as if he were reborn each time to optimize for the remaining periods of his life.

Because each time he follows the planned path only at the initial time of a planning horizon, the actual consumption is the envelope of all the planned consumption path over the life time. That is, the realized consumption is,

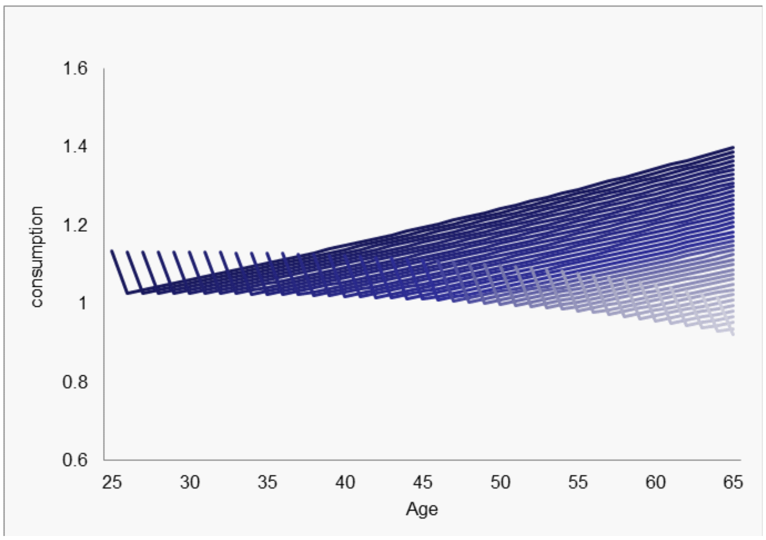
$$C^R(t) = \{c_t(t), c_{t+1}(t+1), c_{t+2}(t+2), ..., c_T(T)\}.$$

Therefore the lifecycle consumption profile of the representative consumer with present biased preference is

$$\{c_t\}_{t=0}^T = \{\ c_0(0),\ c_1(1),\ c_2(2),\ ...\ ,\ c_T(T)\ \}$$

where each consumption is indexed by a planning time and a consumption time. Likewise, the lifecycle asset demand profile is

Figure 23: Planned and Realized Consumption Profile for the model with $\gamma = 1$, $\beta = 0.9$, $\delta = 0.97$, $R = 1.035$. The realized consumption is the envelope of the planned consumption series.



$$\{b_{t+1}\}_{t=0}^{T} = \{\ b_1(0),\ b_2(1),\ \dots\ ,\ b_{T+1}(T)\ \}$$

and

$$b_0(0) = 0,\ b_{T+1}(T) = 0\ .$$

Figure 23 shows the planned and the realized consumption profile of model with Quasi-hyperbolic. The realized consumption is the envelope of the planned consumption series in partial equilibrium.

3.4.2 Technology and General Equilibrium

Adding production technology to the economy completes the model. Assume there are a continuum(infinite number) of identical perfectly competitive firms who operate the following Cobb-Douglas production function:

$$F(K, N) = K^{\alpha} N^{1-\alpha}$$

The marginal productivity is given by

$$F_K = \alpha \left(\frac{K}{N}\right)^{\alpha-1}$$
$$F_N = (1-\alpha)\left(\frac{K}{N}\right)^{\alpha}$$

Now define a competitive equilibrium for the model.

Definition *A competitive equilibrium in this economy is an allocation $\{c_t\}_{t=0}^T$, a set of bond demands $\{b_{t+1}\}_{t=0}^T$, an interest rate R and wage rate w such that given R and w, the followings are satisfied:*

i) $\{c_t\}_{t=0}^T$ and $\{b_{t+1}\}_{t=0}^T$ solve consumer's problem.

ii) Factors are paid out their marginal productivity:

$w = F_N$ *and* $R - 1 = F_K - d$.

iii) Labor market and bond market clear:

$K = \sum_{t=0}^T b_t$ and $N = \sum_{t=0}^T e_t$

The last market clearing condition specifies that consumption loans cancels out in the aggregate so that the excess demand for bonds should be equal to the capital stock. Also the aggregate labor supply that sums up over the labor supply of each cohort should be equal to the aggregate labor demand. Therefore, by the above equilibrium condition ii),

$$w = (1-\alpha)\left(\frac{K}{N}\right)^{\alpha}$$
$$R - 1 = \alpha\left(\frac{K}{N}\right)^{\alpha-1} - d$$

Rewrite the last equation to get

$$\left(\frac{K}{N}\right)^{\alpha-1} = \frac{R-1+d}{\alpha}.$$

Thus the capital-to-labor demand ratio is written as a function of the interest rate and other parameters. Similarly, rewrite the capital stock as a function of the interest rate to get

$$K(R) = N\left(\frac{R-1+d}{\alpha}\right)^{\frac{1}{\alpha-1}}$$

By the equilibrium condition iii), the market equilibrium condition to determine R is

$$\sum_{t=0}^{T} b_t(R) = \left(\frac{R-1+d}{\alpha}\right)^{\frac{1}{\alpha-1}} \sum_{t=0}^{T} e_t.$$

Once it is solved for an equilibrium interest R, then the real wage w is determined by

$$w(R) = (1-\alpha)\left(\frac{R-1+d}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}$$

3.4.3 Extended General Equilibrium Model with Mortality Risk, Bequest and Social Security

In this section, I want to extend the model by including social security, another source of the retirement assets and analyze the effect of social security, together with mortality risk and bequests in the general equilibrium model. Consider the model economy in which agents have present-biased preference and live to a maximum age of T. The agents survive until age $s \geqslant t$, with a probability Q_t, which is assumed to be generation (or cohort)-independent. Then the agents of age t maximize the expected utility:

- if the agent follows a quasi-hyperbolic discounting:

$$U(t) = Q_t \frac{c_t^{1-\gamma}(t)}{1-\gamma} + \beta \sum_{\tau=t+1}^{T} \delta^{\tau-t} Q_\tau \frac{c_\tau^{1-\gamma}(t)}{1-\gamma}$$

- if the agent follows a double exponential discounting:

$$U(t) = \omega \sum_{\tau=t}^{T} \beta^{\tau-t} Q_\tau \frac{c_\tau^{1-\gamma}(t)}{1-\gamma} + (1-\omega) \sum_{\tau=t}^{T} \delta^{\tau-t} Q_\tau \frac{c_\tau^{1-\gamma}(t)}{1-\gamma}$$

Also let us assume that agents who do not survive up to t leave their assets to the other remaining agents in the form of bequest B_t. The bequests are spread uniformly over the surviving population. Now I introduce government in the model and assume the government implements *pay as you go* ($PAYG$) Social Security system financed by a payroll tax of s.[35] Also the government is assumed to be in balanced budget each time. The agent receives the social security benefit S_t from retirement time T_{RET}, where T_{RET} is determined exogenously by government policy. Therefore, an agent starts to receive social security benefit if he

[35] The notation should not be confused with the saving amount in previous section, where I discuss about the self control problem.

survives beyond this. Thus the consumer's problem is now to maximize for $t = 0, 1, ..., T$,

$$U(t) = \omega \sum_{\tau=0}^{T-t} \beta^{\tau} Q_{\tau} \frac{c_{\tau}^{1-\gamma}(t)}{1-\gamma} + (1-\omega) \sum_{\tau=0}^{T-t} \delta^{\tau} Q_{\tau} \frac{c_{\tau}^{1-\gamma}(t)}{1-\gamma} \tag{3.9}$$

subject to

$$\begin{gathered} c_{\tau}(t) + b_{\tau+1}(t) = (1-s)\, we_{\tau} + Rb_{\tau}(t) + B_{\tau} + S_{\tau} 1\,(t \geqslant T_{RET}) \\ b_0(0) = 0, \;\; b_t(t) = given, \;\; b_{T+1}(t) = 0 \\ c_{\tau}(t) \geqslant 0 \\ b_{\tau+1}(t) \geqslant 0 \end{gathered}$$

where $1\,(t \geqslant T_{RET})$ shows the condition under which the social security is included to the income of the agent.

The difference now is the extra mechanism, than the personal saving, that transfers income from the young to the old. This transfer will reduce the demand for saving by the young so that the interest rate will be increased if there is no distortional effect between consumption and leisure. Because there is no leisure in the periodic utility in the model, there is no relative price change from tax between the consumption good and leisure, i.e. no substitution effect arises in intra-temporal optimization.

To accommodate these modification to the model, I want to adjust our definition of general equilibrium. So the competitive equilibrium condition in the model is modified to

$$K(R) = N \left(\frac{R-1+d}{\alpha} \right)^{\frac{1}{\alpha-1}}$$

and

$$\textstyle\sum_{t=0}^{T} Q_t b_t\,(R) = N \left(\frac{R-1+d}{\alpha} \right)^{\frac{1}{\alpha-1}} \sum_{t=0}^{T} Q_t e_t .$$

By the second equation, the market interest rate R is determined.

The size of Bequest is determined by

$$\textstyle\sum_{t=0}^{T} B_{\tau} Q_t = \sum_{t=0}^{T} (Q_t - Q_{t+1})\, Rb_{t+1}.$$

Lastly, the size of social security is obtained by

$$s\sum_{t=0}^{Tw-1} Q_t we_t = \sum_{t=Tw}^{T} Q_t S_\tau.$$

where s is the payroll tax rate, which is set by the government. If $S_\tau = S^*$ for $Tw \leq t \leq T$, then

$$s\sum_{t=0}^{Tw-1} Q_t we_t = S^* \sum_{t=Tw}^{T} Q_t S_\tau.$$

3.5 QUANTITATIVE ANALYSIS

The goal of the quantitative analysis is to assess how well a calibrated, general equilibrium model of the constrained consumer with present biased preference can account for stylized facts regarding consumption hump. Following many other works, I propose three standard macroeconomic variables to be targeted for the US data. The targets are interest rate, capital-output ratio and consumption output ratio. But the model has six scalar parameters,[36] of which three parameters are related to the discounting factor[37], one parameter to the risk aversion, and the remaining two parameters to the production[38]. In case of quasi-hyperbolic discounting function, the model has only two time-preference parameters. Among those time-preference parameters, I first set the other parameter(s), except for the δ, at certain values, and let all other three parameters of (δ, α, d) jointly set to match the target variables given those parameter values. Then, the preference parameter(s) itself is calibrated for the consumption profile produced by the model to fit the mean consumption profile of Gourinchas and Parker's (2002)[57] estimation. Because the equilibrium profiles of consumption, income, bond demand, and labor supply in the overlapping generation model can be interpreted as economy wide cohort averages, these averages can be used for comparison to US data. To asses the best matching planning horizon, I use a couple of standard measures, which are MSE, Consumption Peak Age and Peak Consumption to Initial Consumption Ratio.

[36] These are β, δ, ω, γ, α, d in the model.
[37] These are β, δ, ω in the model.
[38] These are α, d in the model.

3.5.1 Targets in US Data

Three macroeconomic targets are proposed for the analysis and they are interest rate (R), capital-output ratio (K/Y) and consumption output ratio (C/Y). Following Rios-Rull (1996)[93], I first set 2.94 as a target value for capital-output ratio and 0.748 for the target ratio of consumption to output. The third macroeconomic target is the real interest rate, which is independently determined in the lifecycle framework. Following McGrattan and Prescott (2000)[81], I set the target real interest rate at 3.5%.[39]

In the context of lifecycle profile of consumption, the mean consumption profile in Gourinchas and Parker's (2002)[57] estimation and its septic polynomial fit by Feigenbaum (2008a)[45] is adopted.

$$\begin{aligned} c_t^{GP} = {} & 1.062588 + 0.015871t - 0.00184t^2 + 0.000109t^3 + 0.00000413t^4 \\ & -0.00000056t^5 + 0.0000000163t^6 - 0.0000000001475t^7 \end{aligned}$$

According to this fit, the age of lifecycle mean consumption peak is 45 years old and the ratio of peak consumption to initial consumption is 1.1476. These values, as well as mean squared errors for the fit, are used to compare the consumption profiles along the different planning horizons. Also, regarding the income schedule, Feigenbaum (2008a)[45] suggests a quadratic fit for the US data. Since the labor is supplied inelastically in the model, income is proportional to productivity. Therefore, Feigenbaum's quadratic fit to the income data of Gourinchas and Parker can serve for the productivity profile as well. The profile is

$$e_t = 1 + 0.0181t + 0.000817t^2 - 0.000051t^3 + 0.000000536t^4.$$

3.5.2 Calibration

The standard lifecycle model of general equilibrium has four parameters, two from consumer's optimization and the other two from technology; the risk aversion coefficient γ and the discount factor δ are from consumer side and the capital share α in production function and the depreciation rate d are from production side. But in the present biased preference

[39] Similary, Gourinchas and Parker (2002)[57] estimate 3.44% for the rate .

model, I should consider two more parameters, i.e. β, ω (or just one more parameter for quasi-hyperbolic discounter; β), because the model assumes three parameters for the discount factor. Following the recent experimental result for β ,from Laibson et al., in quasi-hyperbolic discounting model, I set $\beta = 0.7$ initially and explore more results with $\beta = \{0.3, \ \ 0.5, \ \ 0.7, \ 0.9, \ 1\}$. For the double-exponential model, I set $\{\beta = 0.7, \omega = 0.5\}$ initially and find more results with $\{\beta \text{x} \omega\} = \{0.3, \ \ 0.5, \ \ 0.7, \ \ 0.9, \ 1\}\text{x}\{0.3, \ \ 0.5, \ \ 0.7\}$. Therefore, instead of calibrating all of these six(or five) parameters together, I set the β and ω as given and explore how well the general equilibrium model fits to the data as the parameters change.

Among the remaining four scalar parameters $\{\gamma, \delta, \alpha, d\}$, I first find that the model is well calibrated with a function of the combined consumer parameters $\psi(\gamma, \delta : \beta, \omega)$ and the other two parameters of production side, i.e. the capital share α and depreciation rate d, given the three target values. The optimal parameter function $\psi(\gamma, \delta : \beta, \omega)$ that minimizes the deviation from the target values has different values for each (β, ω) with two production parameters. This leaves the model with four free parameters δ, β, ω and γ for double-exponential model and three free parameters β, δ and γ for quasi-hyperbolic model. Therefore, for a choice of γ and (ω, β), δ is set to match the target for each combination of consumer parameters. The subsequent result section shows how the predictions of each model of the different consumer parameters vary in equilibrium with a joint choice of γ, ω, β and δ. Table [6] summaries the description of parameters and targets for double-exponential model. Notice that there is no ω of the table for the quasi-hyperbolic model.

Another issue for consideration is to find the discount factor(s) that best describes the stylized fact of consumption data profile. For this purpose, I use both the quantitative and qualitative comparison. For the quantitative fit, I calculate the model's deviation from the consumption profile implied by the data using mean squared error (MSE).

For the qualitative fit, I compute the consumption ratio; the ratio of the consumption at the maximum to the consumption at the starting age. This is a common measure for the size of the consumption hump, proposed by Bullard and Feigenbaum (2007)[16] and Hansen and Imrohoroglu (2006)[60]. Another measure for the qualitative fit is the age of the consumption peak. In the next subsection, I register the comparison of the consumption ratio and the consumption peak age, indexed by different consumer parameters for general

Table 6: Parameters and Targets with Double Exponential

Variable	Description	Target
γ	Risk Aversion	Free
ω	Aggregation Parameter	Free
β, δ	Discount Factor	Free
α	Capital Share	Free
d	Depreciation Rate	Free
R	Interest Rate	3.5 %
K/Y	Capital-Output	2.94
C/Y	Consumption-Output	0.748

set of parameters.

Finally, for the computation of lifecycle profile, the model assumes the agents can live up to maximum 100 years old with mortality risk. This corresponds to the model age from $t = 0$ to $T = 75$. Also the model assumes the agent retires after completion of 41 years of work to $T_w = 40$, i.e. he continues to work up to 65 years old and then retires. During the working years, workers are paying payroll tax of $s = 0.0765$.[40] When retired, the retirees get social security benefits of S each year as long as they survive. There is no other income sources than the earnings from labor and social security, so the income profile sets $e_t = S$ for $t > 40$ and the agents live on both social security benefits and savings from working years.

3.5.3 Results

In the following subsections, I report the main results into four model categories: General Equilibrium with and without Mortality Risk, General Equilibrium with and without Social Security. The simulation exercises reveal that the model with constrained myopic consumer does produce a consumption hump, with similar size and location of consumption peak, in partial equilibrium, and in a well-calibrated general equilibrium within reasonable parameter

[40] This choice is from current US payroll tax. If self-employed included, the overall tax rate is about 10%.

ranges. In partial equilibrium, the featured consumption hump is easily obtained with most sets of parameters. With a certain rage of parameters, the hump can be obtained even without the borrowing constraint or mortality risk.

In calibrated general equilibrium, borrowing constraint and mortality risks help the model conform to the hump-shaped data. The location of consumption peak falls within 30 to 54 years old for most of consumer parameters in general equilibrium. Details are below.

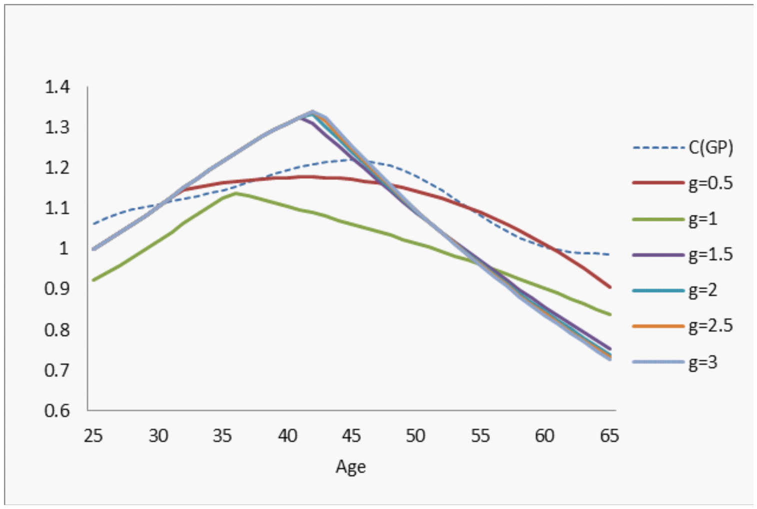
3.5.3.1 The Result for the Baseline Model Initially let us set the baseline parameters of the double discounting model like the followings: The risk aversion $\gamma = 2$ and the discounting parameters $\beta = 0.7,\ \ \omega = 0.5$. The value of $\beta = 0.7$ is chosen following recent experimental literatures. Notice that the simulation sets only β and the other discounting parameter δ is not preset. Then given any choice of risk aversion parameter γ, the discount factor δ is set to match the macroeconomic targets. With these baseline model parameters, I first show how the consumption profiles are varying following the risk aversion parameter γ in the baseline model without mortality risks or social security assumptions. The γ values I analyze are, $\gamma = \{0.5,\ 1,\ 1.5,\ 2,\ 2.5,\ 3\}$. I want to compare the different model parameters based on the following criterion which are i) quantitative fit, ii) location of the consumption of peak age and iii) the value of peak to initial consumption ratio.

Figure 24 shows that when the value of γ is greater than 1, the model tends to conform a consumption peak, but at earlier age than the data. In term of quantitative fit to the data, MSE is used and by this criterion, it is shown that $\gamma = 0.5$ induces the smallest MSE, which is 0.005212. But with respect to the location of the consumption peak age, i.e. 45 years old in the data, $\gamma = \{2,\ 2.5,\ 3\}$ gives closer fit and the peak age is 42. Also, by the value of peak to initial consumption ratio,[41] lower γ gives better fit.

Second, given the risk aversion value at $\gamma = \{2\}$ I show how the consumption profiles are moving under different aggregators of the two discount factors. That is, I check the model for $\omega = \{0.3, 0.5, 0.7\}$. Figure 25 summarizes the result. Consumption peak age looks closer to the data with lower aggregator, i.e., $\omega = \{0.3\}$, but by other criterion, $\omega = \{0.7\}$ is better.

[41] In GP, this value is 1.1476.

Figure 24: Consumption and Gamma in General Equilibrium Model.



Finally I look for the best beta by setting $\gamma = 2$ and $\omega = 0.5$ for all different models. Given each β, I let δ adjust accordingly to match the three targets in the general equilibrium setting. The γ values I analyze are, $\beta = \{0.3, 0.5, 0.7\}$. Figure 26 summarizes the result. From this simulation exercise I find that in calibrated general equilibrium, β would not play a significant role to give a different conjecture to the consumption stream. This is a difference from partial equilibrium result, where I do not find this sort of result. This is because, in the general equilibrium the other discounting parameter is now adjusting accordingly to match the macroeconomic targets, while in partial equilibrium, there is no possibility of this.

Table [7] shows the optimal δ values for different γ over the predetermined discount parameters of $\{\beta, \omega\}$ in the baseline, which is $\{\beta = 0.7, \omega = 0.5\}$. Thus, for example, the δ value of 0.98661 in the first cell is the optimal discount value when $\{\gamma = 0.5, \beta = 0.5, \omega = 0.5\}$ in the calibrated general equilibrium. Likewise, 0.94773 in the last cell is the optimal one when $\{\gamma = 3, \beta = 0.7, \omega = 0.7\}$ Notice that when $\{\gamma = 0.5, \beta = 0.7, \omega = 0.7\}$, δ exceeds one, which violates present biased preference assumption, implying the consumer cares for his future more than now after the initial heavy discount.

The table demonstrates the following facts: First, δ is inversely related with γ for all values of β and ω. And given any γ, the δ is inversely related with β, but positively related

Figure 25: Consumption and Omega in General Equilibrium Model.

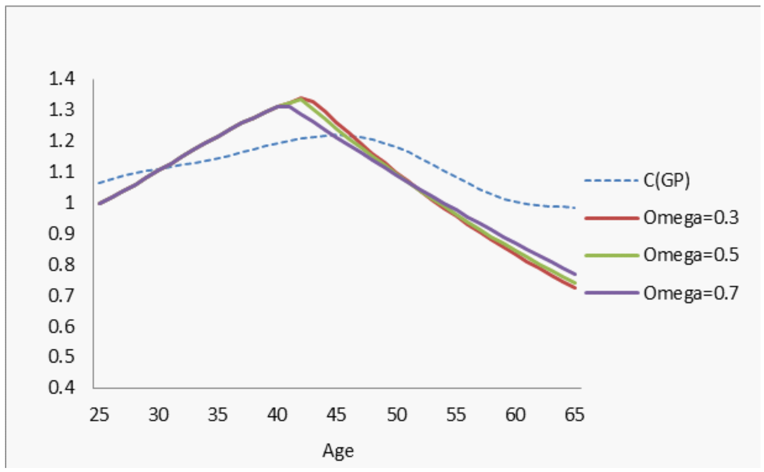


Figure 26: Consumption and Beta in General Equilibrium Model.

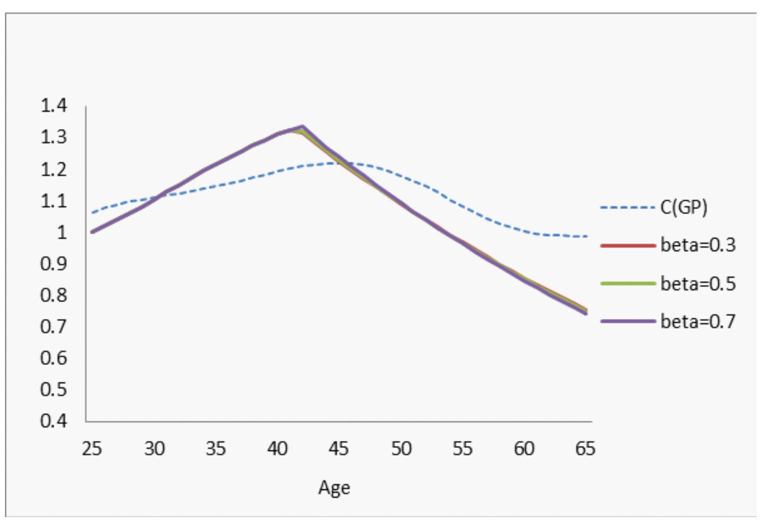


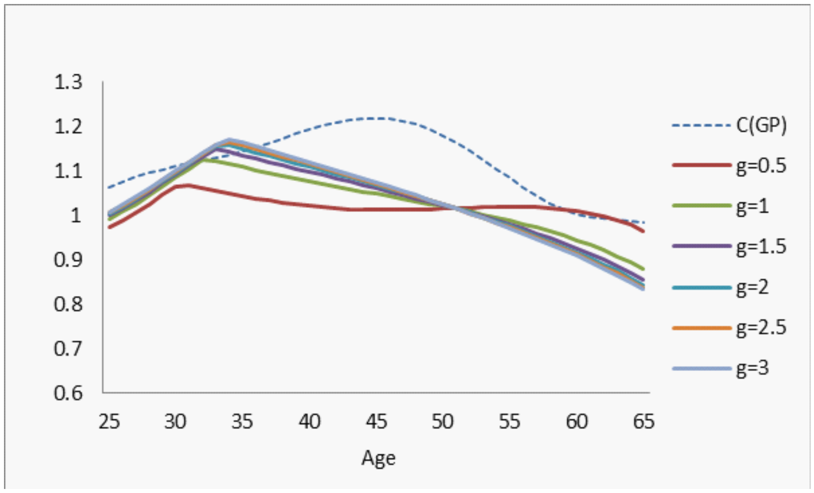
Table 7: Optimal Beta Values over Discounting Factor Parameters

γ	$\beta = 0.5$	$\beta = 0.7$	$\omega = 0.3$	$\omega = 0.5$	$\omega = 0.7$
0.5	0.98661	0.98549	0.97164	0.98549	*1.00045*[42]
1	0.97485	0.98217	0.95871	0.98217	0.99773
1.5	0.96648	0.96292	0.94637	0.96292	0.98621
2	0.95352	0.94936	0.93121	0.94936	0.97501
2.5	0.94123	0.93557	0.91634	0.93557	0.96283
3	0.92694	0.92175	0.90394	0.92175	0.94773

with ω. Second, when γ is relatively low, the δ may have a chance of exceeding one so that violating of present biased preference as β gets lower and ω gets higher. These facts help understand how the value of δ from lab experiments may be obtained: for example, Laibson, Repetto, and Tobacman (2009)[71] estimate annual time preference parameters ($\beta = 0.7$, $\delta = 0.96$) on lifecycle accumulation data. Thus $\delta = 0.96$ is from the following combination of parameters in the model, $\{(\beta = 0.5, \gamma = 1.5, \omega = 0.5)$, $(\beta = 0.7, \gamma = 1.5, \omega = 0.5)$, $(\beta = 0.7, \gamma = 0.5, \omega = 0.3)$, $(\beta = 0.7, \gamma = 1.5, \omega = 0.5)$, $(\beta = 0.7, \gamma = 2.5, \omega = 0.7)\}$.

3.5.3.2 Simulation Result for the Extended Model I set the same baseline parameters and criterion as in the previous subsection and have the following result for my extended model where it has both mortality risks and social security. First, like in baseline model, I find that when the value of γ is greater than one, the model tends to conform to a consumption peak, but at somewhat earlier age than the data, and than the baseline model, as well. In terms of quantitative fit to the data, the model with $\gamma = \{2, 2.5, 3\}$ induces smaller MSEs, which are about 0.016. Like in baseline model, for models with these parameters, the location of the consumption peak age is close to the GP. Also, by the value of peak to initial consumption ratio, $\gamma = 1.5$ gives best fit, followed by the model with higher gammas, i.e. $\gamma = \{2, 2.5, 3\}$. In sum, $\gamma = \{2, 2.5, 3\}$ gives better fit to the data when both mortality risks and social security are introduced. Figure 27 demonstrates the result.

Figure 27: Consumption and Gamma in Extended General Equilibrium Model.



Second, given the risk aversion value at $\gamma = \{2\}$ I show how the consumption profiles are moving under different aggregators of the two different discount factors. That is, I check the model with $\omega = \{0.1, 0.3, 0.5, 0.7\}$. Figure 28 summarizes the result. Consumption peak age looks better when simulated with lower omega $\omega = \{0.3\}$, but by other criterion, the middle one $\omega = \{0.5\}$ is better. Third, like in the baseline model, I find that consumption does not change significantly by the many different choices of β. Figure 29 summarizes the result.

Finally, I compare the four different model specifications and show the result. By quantitative fit, the model with both mortality risks and social security produces best result. By the consumption of peak age, the model without either mortality risks or social security fits best and the age is 42. Also by the value of peak to initial consumption ratio, the model with both mortality risks and social security gives a closest fit to the data. From this, it may be inferred that inclusion of both mechanisms improve overall shape of the model. Figure 30 shows the comparison among the models. Remember that all of these results are from model with double discounting function. If Q-hyperbolic discounting function is used, then the result is in the Figure 31. From both figures, it may be demonstrated that overall the models with generalized discounting fit better than the ones with Q-hyperbolic discounting.

Figure 28: Consumption and Omega in Extended General Equilibrium Model.

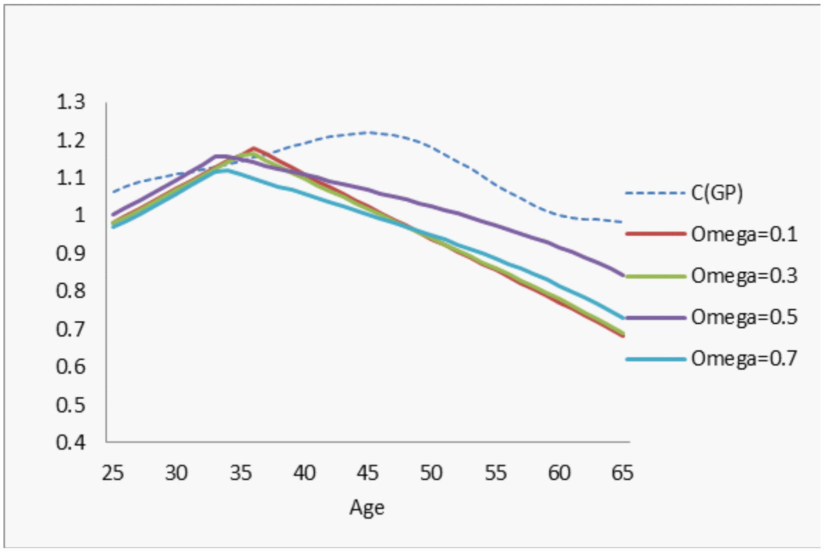


Figure 29: Consumption and Beta in Extended General Equilibrium Model.

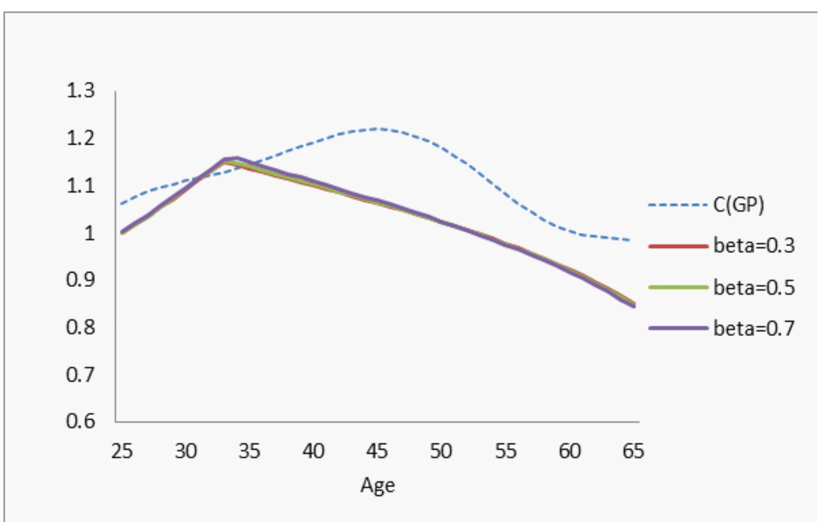


Figure 30: Comparison of the models regarding inclusion of mortality risks (M) and social security(S): N implies No (Not included) and Y implies Yes (Included) of the specification.

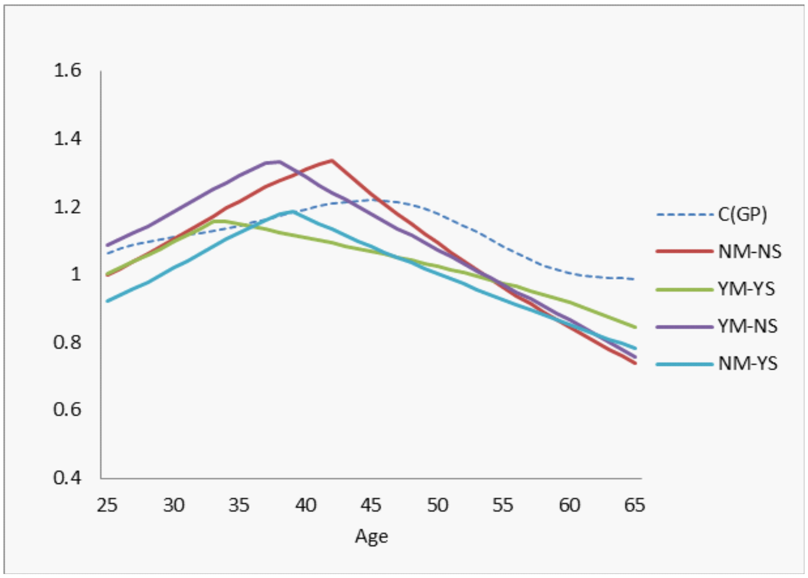
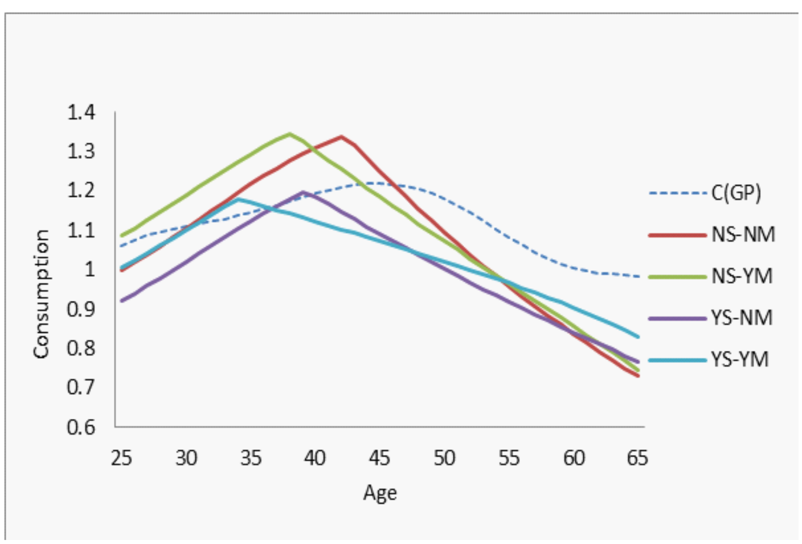


Figure 31: Result with Q-Hyperbolic Discounting. Comparison of the models regarding inclusion of mortality risks (M) and social security(S): N implies No (Not included), Y implies Yes (Included) of the specification.



3.5.3.3 Robustness Following the discussion in above subsections, I first set the baseline parameters to $\{\gamma = 0.5,\ \beta = 0.7,\ \omega = 0.5,\ \alpha = 0.289,\ d = 0.063\}$ and then set them to different γ and ω. Notice that δ is going to adjust accordingly to these parameters for the given discounting parameter set $\{\beta, \omega\}$. However, ω is not considered here to be a parameter for sensitivity check. For the three targets of macroeconomic variables, i.e. $r = 3.5\%$, $K/Y = 2.94$, $C/Y = 0.748$, I report a couple of sensitivity check around the best fitting calibrations.

Table [8] shows two sets of sensitivity check, one with baseline $\gamma = 0.5$ and the other one with $\gamma = 2$. The results for the $\omega = 0.5$ and $\omega = 0.3$ are based on $\gamma = 0.5$ and $\gamma = 2$. The second column 'Model' of the table shows different alternative calibration to the baseline model, keeping other parameters intact. Thus, for example, the first group of the table represents the alternative calibration to the baseline model $\{\alpha = 0.289,\ d = 0.063,$ $\gamma = 0.5,\ \beta = 0.7, \omega = 0.5\}$, by changing only one parameter, except for ω, and keeping others constant.

3.6 CONCLUSION

In this essay, I consider consumers with present-biased preferences who are also credit constrained in asset markets and I explore the general equilibrium characteristics of an economy that is populated by these myopic consumers. I show that reasonable parameterization of my model generates a hump-shaped consumption profile among the other empirically plausible features. I first derive analytic solution for a simple model. Then using full model I assess how well my model can account for the consumption hump, in a calibrated general equilibrium.

I find that in my model of myopic consumers, where their preferences are specified by either a quasi-hyperbolic discounting function or a generalized double exponential function, the general equilibrium result can be consistent with the known characteristics of the lifecycle consumption data. In fact, the quantitative result from simulation exercises shows that the model supports well the general equilibrium with discounting factors of $\{\beta = 0.7,\ \delta = 0.96\}$, coinciding with the field data on the time preference. In the paper I document the set of parameters and their stylized facts for the general equilibrium model to match with the data.

Many observations on behaviors with 'myopia,' defined by 'short-sightedness, a narrow view and a lack of foresight' in decision making, have interested many researchers who study behavioral assumptions in economics modeling. Myopic decision maker, who has time-inconsistent taste for immediate gratification or who optimizes period-by-period without looking forward to the end, has become an important focus area, particularly for inter-temporal dynamics. The merit of these non-traditional assumptions is not only to increase the explanatory power of economics by providing its models with more realistic assumptions about human behavior, but also to explain a variety of puzzling market outcomes. The contribution of my work is to assess the reality, generality, and tractability of the assumption of present-biased preferences in a realistic model economy, by predicting the well known lifecycle features in a well-calibrated general equilibrium. My findings in general equilibrium result are encouraging for the view that analysis with behavioral assumptions may be an alternative way to get insights into the lifecycle consumption dynamics.

Table 8: Sensitivity to Alternative Calibration

	Model	r	C/Y	K/Y	Age[43]	C_m/C_o
ω=0.5	γ=0.5, β=0.7, δ=0.98549	3.50%	0.748	2.94	41	1.171
	$\gamma = 0.49$	3.51%	0.749	2.93	42	1.182
	$\beta = 0.73$	3.45%	0.746	2.95	40	1.179
	$\delta = 0.986$	3.39%	0.743	2.98	40	1.174
ω=0.5	γ=2, β=0.7, δ=0.94936	3.50%	0.748	2.94	42	1.335
	$\gamma = 2.1$	3.23%	0.735	3.02	41	1.325
	$\beta = 0.69$	3.53%	0.749	2.93	42	1.337
	$\delta = 0.940$	4.43%	0.788	2.68	44	1.354
ω=0.3	γ=0.5, β=0.7, δ=0.97164	3.50%	0.748	2.94	40	1.299
	$\gamma = 0.53$	3.42%	0.744	2.96	40	1.301
	$\beta = 0.75$	3.43%	0.745	2.96	40	1.305
	$\delta = 0.973$	3.35%	0.745	2.98	39	1.294
ω=0.3	γ=2, β=0.7, δ=0.93121	3.50%	0.748	2.94	42	1.338
	$\gamma = 1.9$	3.75%	0.759	2.87	43	1.349
	$\beta = 0.72$	3.46%	0.746	2.95	42	1.338
	$\delta = 0.940$	2.78%	0.713	3.19	41	1.315

4.0 A GE MODEL OF DECISION MAKERS WITH BELIEF DEPENDENT PREFERENCES

In this essay, I explore macroeconomic dynamics for a decision maker whose preference depends not only on his actual consumption but also on comparisons to his beliefs about optimal consumption. The standard decision maker is loss averse with respect to this belief-dependent reference point. When loss aversion is low, the decision maker can deviate from standard lifecycle consumption behavior. This deviation can help explain some puzzling features of inter-temporal consumption data in general equilibrium. When the decision maker has age-related time-varying degrees of loss aversion and rebalances his consistent consumption through adjusted beliefs, the model generates hump-shaped consumption profile that closely tracks the data in a well-calibrated general equilibrium.

Keywords: reference dependent preference, loss aversion, gain-loss utility, belief updating, lifecycle consumption

4.1 INTRODUCTION

People tend to react differently to the same situations (events) when they have different expectations. It may often be observed that individual utility from an outcome that has met one's expectation is quite different from an outcome that has not.[1] As argued in Kőszegi and Rabin (2006[64], 2007[65]), individuals may not evaluate utilities in absolute level of outcomes but in gains or losses of the outcomes relative to their reference expectations. Ref-

[1] People see unknown risks differently from the risks that are common (Matthey, 2005[79]), because the common risk is anticipated while the unknown risk may turn out to be a surprise.

erence dependence of utility is widely confirmed in lab experiments[2]: it helps understand why standard economic theory cannot explain such findings as the failure of the independence axiom in expected utility.[3] It also explains a variety of field data (*Labor supply*: Farber (2008)[38], Fehr and Goette (2007)[40]; *Housing market*: Genesove and Mayer (2001)[54]; *Finance*: Barberis, Huang and Santos (2001)[14], Karlsson, Loewenstein and Seppi (2005)[63]; *Insurance*: Sydnor (2006)[100]). Inspired by this, I demonstrate that an individual's *loss aversion* with respect to the reference expectation about optimal consumption can solve the well-known consumption puzzle,[4] by providing richer macroeconomic dynamics. The innovative contribution of my work is to build the model around reference dependent *utility,* based on "prospect theory," instead of reference dependent *consumption,* the latter being well-established in macroeconomics but has not yet explained the consumption puzzle.[5] Moreover, the inclusion of the reference status is made through a forward-looking mechanism. Loss aversion plays a key role to induce consumption dynamics that solve the puzzle.

The choice of expectation for the reference point follows from the recent development in reference-dependent preferences models. A key issue in these models is what would be the reference point, because as Pesendorfer (2006)[87] expressed, the reference point can be anything and may be selected arbitrarily by researchers.[6] Providing one way to solve this problem, Kőszegi and Rabin (2006[64], 2007[65]) propose a model of reference-dependent preferences where the reference point is the individual's rational expectation formed in the recent past about outcomes. Kőszegi and Rabin provide a solution concept, by which the reference point is determined *endogenously* as a function of the decision maker's beliefs on the available strategies combined with his planned action for each strategy. In another work,

[2] The original work is Prospect Theory (1979)[62] by Kahneman and Tversky. The key feature of the theory is (1) reference dependence, (2) gain-loss utility with more weights on loss (*loss aversion*) (3) diminishing sensitivity.

[3] Rabin (2000)[88] demonstrates that reference dependence is an important factor in explaining people's attitude toward risk.

[4] The consumption data is known to be a hump-shaped, which is not obtained from standard economic theory because the theory predicts monotonic consumption.

[5] There have been long-standing literatures of external habit and recently, of internal habit formation. These models are based on 'adaptive' evolution of reference points and thus are *backward looking*. Examples of this type of reference dependence include: Caroll, Overland, and Weil (2000)[26]; Christiano, Eichenbaum, and Evans (2005)[30]; Chetty and Szeidl (2009)[28].

[6] Many times the reference point is assumed to be the current status, such as current consumption, position, or endowment.

Kőszegi and Rabin (2009)[66] propose a dynamic reference-dependent model by which they specifically construct that the decision maker should meet the rational consistency condition: given any expectation generated by a dynamic strategy, the strategy maximizes the decision maker's utility each period, under the condition that the continuation strategies must be consistent with rationality.

Based on this solution concept, I develop an intertemporal choice model with reference-dependent preference and examine its macroeconomic implication. Given any information about the income stream, the decision maker forms beliefs on what should be the optimal consumption over time if he follows the standard practice[7] of utility maximization.[8] The optimal consumption here may be compared to the solution concept provided by a financial planner when he gives advice regarding consumption and saving plan to a client who is in a specific wealth position with respect to lifetime income and assets/liabilities. The solution serves as a guideline to which the client should refer in deciding his consumption over his lifetime. This is the reference status that I postulate for my model. With this ex ante optimal plan on hand, the reference-dependent decision maker decides whether he should follow this rule or not.

A reference-dependent decision maker derives utility from comparison to the reference status: it may be a gain to the reference point or a loss to it. A loss is assumed be more important to decision maker than a gain of the same size.[9] In two-period intertemporal choice of consumption and saving, decision maker feels a "contemporaneous gain" utility if he consumes more now than the suggested ex ante optimal solution. As a result, his consumption is lowered next period and this yields a "prospective loss" utility relative to the reference point. If the contemporaneous gain utility is greater than the prospective loss utility, then he chooses not to follow the ex ante optimal rule but to deviate for more consumption now. Likewise, if the prospective loss utility is greater than the contemporaneous gain utility, then he sticks to the standard consumption rule. The decision maker's greater concern for the loss (high loss aversion) deters him from over-consuming.

This analysis can be applied the other way around: the decision maker may have a

[7] A rational, utility maximizing agent posited in the standard economic theory.

[8] A rational, utility maximizing agent posited in the standard economic theory.

[9] The property of "loss aversion" is a standard assumption in models with reference dependent preferences.

"contemporaneous loss" utility if he consumes less and saves more than the ex ante solution. As a result, his consumption is elevated next period and this gives him a "prospective gain" utility. If his contemporaneous loss feeling is greater than the prospective gain one, then he does not reduce his consumption but follow the suggested consumption plan. If, however, his prospective gain utility is more than the contemporaneous loss utility, then he is willing to reduce his consumption and save more now. In this case, the high loss aversion of decision maker deters him from under-consuming. By both stories, it is clear that when the decision maker's loss aversion is high, the agent does not deviate from the ex ante optimal solution. But if the decision maker cares more about the gain due to low loss aversion, then he deviates from standard rules for his personal well-being. The deviation is possible in either direction of more or less than the ex ante optimal consumption.

In the model, I assume that consumer's utility has two preference components following Kőszegi and Rabin.[10] One is the usual consumption utility (absolute level) and the other one is the reference dependent utility (contrast level) and these two payoffs interact each other through consumer optimization. The total utility is maximized by choosing the best combination of current and future consumption points according to his *intended* consumption plan.[11] If a decision maker has high loss aversion, then the intended consumption is not realized and he stays with the standard consumption. If a decision maker has low loss aversion and wants to deviate, then an alternative consumption can be chosen to fulfill his intention and the consumption must satisfy the consistency condition: the alternative consumption must be optimal in that it maximizes his ex post utility based on his intention.[12]

There are two main issues when constructing a macroeconomic model of intertemporal decision makers with the reference-dependent preferences. One is how to construct the multi-period reference points from overlapping layers of belief formation. The other is what is a proper macroeconomic model that conveys the idea described above. This is because the framework on which the model relies is a descriptive one and it admits many different formats for a model. To tackle the first issue, I first restrict the dimension of commodity

[10] Classical prospect theory posits only comparison utility (gain-loss). Krähmer and Stone (2010)[67] also proposes a model with two components; the comparison and the intrinsic characteristics of utility.

[11] In two-period model, it is described by $\{c_0 > c_0^*, c_1 < c_1^*\}$ for an over-consumer and $\{c_0 < c_0^*, c_1 > c_1^*\}$ for a natural born saver, where $\{c_0^*, c_1^*\}$ is the reference point.

[12] It should be the solution to a maximization problem of his intended consumption plan.

space to be one[13] so that the agent of an economy has only consumption space from which both consumption utility and gain-loss utility are derived.[14] For the second issue, I focus mainly on the consistent consumption plans of two typical types of decision makers, assuming that the agent is either an over-consumer or a natural born saver, but not both, if his loss aversion is low. This implies that my model does not consider the case where decision maker reverses his natural type of spending, such as from an over-consumer to a saver, although he may change the degree of loss aversion over time within the same type of spending.

Then I construct intertemporal models of decision making, first in two periods and later up to T periods, searching the consistent consumption strategy for any intended plan. Unlike in two-period model, in three or more-period models a decision maker can have several alternative intended plans. For example, a deviator may want to consume more than the ex ante optimal amount for the first two consecutive periods and as a result he has to accept a very low consumption amount for the last period. Alternatively, he may want to consume more only in the first period. The two intended plans yield two different outcomes and I assume that the agent should select the most desirable plan between them.[15] Therefore, I propose the best consumption plan that a deviator employs because it gives him the highest utility among all the available consistent consumption plans. In fact, it turns out that a plan that keeps over- or under-consumption up to the middle point of the planning horizon gives the highest utility for a deviator.[16] This property implies that it may be necessary to follow the consumption smoothing rule even among the deviators.

Based on the these preliminary works, I analyze the model of dynamic decision makers for cases where the decision maker may or may not change his mind over time. When the agent's preference (loss aversion, in particular) stays constant, then because his expectation is met, there are no gain-loss utilities in the subsequent periods. However, if the agent's preference does change, then depending on how the new reference points are formed, there are different types of gain-loss utilities following his intended plan in subsequent periods.

[13] Many microeconomic applications assume two-dimensional commodity space where the agent trades between the two goods.

[14] This restriction does not limit the application of the model to real economy, as consumption at two different times is considered as two different goods.

[15] This corresponds to the "preferred personal equilibrium" in Kőszegi and Rabin's models.

[16] Section 2 explains this in detail with utility comparison.

I provide optimality condition, as well as closed form solutions for each of the alternative paths that the decision maker may choose.

Next is the main topic: I propose a self-corrective "Sub-period Perfect Reference Point (SPRP)" in the dynamic model and explore the consumption dynamics when the decision maker changes his mind over time due to time-varying degrees of loss aversion. The decision maker rebalances his consumption based on the adjusted belief regarding the ex ante optimal consumption through the SPRP for his remaining lifetime. Because the SPRP is constructed based on current asset and liability position, whenever decision maker changes his mind, he solves a new maximization problem relative to this updated reference point. The resulting consumption profile reflects his current financial situation. Since the decision maker rebalances the consistent consumption as he keeps adjusting his belief, the actual consumption profile turns out to be the envelope of each planned path. I demonstrate that age-related loss aversion that varies over time can produce a consumption hump similar to the data under plausible parameter values.

Modeling belief dependent preference in general equilibrium is as follows. First, I develop a basic lifecycle model of representative agent who may deviate from the standard consumption rules. Then I study the consumption dynamics of those who have time-varying degrees of loss aversion. A hump-shaped lifecycle consumption profile for a representative saver and for an over-consumer are provided. If heterogeneity is introduced, a hybrid model improves the overall fit to the consumption data. Finally, I perform macroeconomic computation to get optimal parameter values of general equilibrium in an overlapping generations economy. The objective of this computation is to see whether the consumption profiles generated by the model can be consistent with many salient features of the consumption characteristics in a well-calibrated general equilibrium.

4.1.1 Other Related Literatures

Because there are few works in macroeconomics that are directly related to this paper, I refer instead to microeconomics literatures for conceptual liaison to reference-dependent preference.[17] Matthey (2005)[79] provides a model of reference dependence where the risk itself is included in the reference states. When the risk is included, it changes the evaluation of contrast utility and this is named *risk inclusion* effect, compared to endowment effect or attachment effect in the usual model. Recent work by Krähmer and Stone (2010)[67] discusses uncertainty aversion based on fear of regret. Unlike Kőszegi and Rabin (2006)[64], their model posits that the reference point depends on the agent's ex post beliefs about what he should have done ex ante if he has the wisdom of hindsight and thus in their model the loss aversion arises endogenously. Selecting a compound lottery reveals information, and this alters the ex post assessment of what the best choice would have been. In fact, they seek for psychological foundation of the uncertainty aversion and in line of this, a similar work by Halevy and Felkamp (2005)[59] may be considered. About narrow framing (evaluate decisions separately), another element of reference-dependence preferences, there is a work by Baberis, Ming, Huang and Thaler (2006)[15], as well as Rabin and Weizsäcker (2009)[89].

If I turn to the models of non-traditional preferences for macroeconomic dynamics, there are "Hyperbolic discounting" model by Laibson (1997)[69] and O'Donoghue and Rabin (1999)[84]. Hyperbolic discounting model, or Present-biased preference, formalizes the evidence that people tend to value the immediate utilities differently from distant future ones and thus the preference induces time inconsistent optimization. The empirical implication of these works is that the existence of the commitment device like illiquid asset (Laibson) or cost (O'Donoghue and Rabin) play a role that can produce a consumption profile which tracks the income flows. The short term planning approach by Caliendo and Aadland (2007a)[20] belongs to this time inconsistent preferences too.

Regarding the featured consumption hump which has been a central issue in lifecycle data, several literatures should be mentioned. Borrowing constraint (Deaton, 1991[32]; Feigenbaum, 2009[48]), mortality risk (Feigenbaum, 2008a[45]; Hansen and Imrohoroglu, 2006[60]), consumption and leisure substitutability (Heckman, 1974[61]; Bullard and Feigenbaum, 2007[16]), income uncertainty and precautionary savings (Carroll, 2009[25]; Feigenbaum, 2008b[46]) are the main causes that can produce a consumption hump with standard

[17] Many experimental studies with prospect theory or reference-dependent models are not considered in this review.

preference. Other mechanisms to show a hump are household size effect (Attanasio et al., 1999[12]) and consumer durables (Fernandez-Villaverde and Krueger, 2010[49]). They show how the interaction between durable and nondurable consumption may work to explain the hump in a model where durable goods serve as collateral for loans. Bullard and Feigenbaum's (2007)[16] calibration work with choice between consumption and leisure as well as Heckman's (1974)[61] model produce the similar result.

4.1.2 Modeling Reference-Dependent Preference

I first introduce a couple of short definitions about the reference dependent preference presented by Kőszegi and Rabin (KR hereafter) and induce a functional form to build my model. The reference dependent utility is defined by,

$$U(c|r) = m(c) + \mu(c|r)$$

where $m(c)$ is classical consumption utility which is increasing, concave or quasi-concave, and differentiable. $\mu(c|r)$ is additively separable gain-loss utility related to deviation from reference point r. It is specified by

$$\mu(c|r) = \textstyle\sum_k \mu_k(c_k - r_k|r)$$ [18]

where k is the dimension of the state space. If it is stochastic, it is called the Reference-Dependent Von Neumann-Morgenstern Preference, implying for each r, decision maker maximizes the modified expected utility[19] $\sum_c P(c)u(c|r)$.

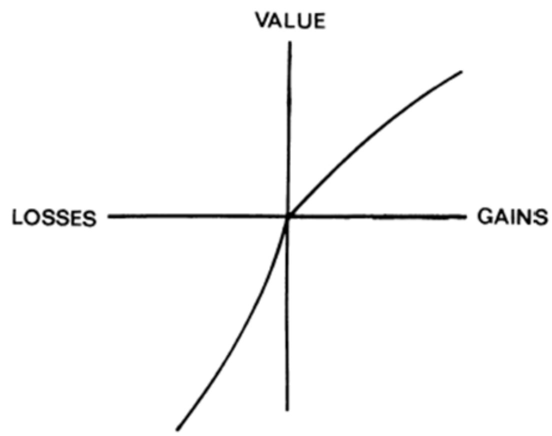
For all k, the gain-loss utility $\mu_k(x|r)$ satisfies the following:

- A0 Continuous, differentiable except at $x = 0$: $\mu_k(0) = 0$
- A1 Strictly increasing
- A2 If $y > z > 0$, then $\mu_k(y) + \mu_k(-y) < \mu_k(z) + \mu_k(-z)$
- A3 $\mu_k''(x) < 0$ for $x > 0$ and $\mu_k''(x) > 0$ for $x < 0$
- A4 $lim_{x \to 0+} \frac{\mu'(-x)}{\mu'(x)} \equiv \lambda_k > 1$

[18]Below the gain-loss utility is specified by $\sum_k \mu(m_k(c_k) - m_k(r_k))$ using consumption utility $m(c)$.

[19]This is one of the points that KR model is different from Prospect Theory by Kahneman and Tversky (1979). In prospect theory, a subjective weight fucntion is used to evaluate a lottery.

Figure 32: Value Function



A2 implies that losses get more weight than the same size of gains. By A3, it is assumed that the utility function is concave in positive valuation and convex in negative valuation,[20] implying diminishing sensitivity. Furthermore, it is assumed μ_k to be proportional to marginal consumption utility $\frac{\partial m}{\partial c_k}|_{c=r}$. All of these properties of the gain-loss utility can be summarized by a hypothetical value function, the shape of which is like the following Figure 32. One may find that the two points in opposite direction of same distance from the origin give two different values, higher absolute value for loss than for gain, due to the steeper value function for loss.

Assume that there is only one dimensional state space so that $k = 1$. Then the reference dependent utility of the decision maker who has a functional form of CRRA for the consumption utility is,

$$u(c, z) = \frac{c^{1-\gamma}}{1-\gamma} + \eta\mu(z)$$

where $\mu(z)$ is a gain-loss utility related to deviation from a reference point and η is the relative weight of gain-loss utility to consumption utility.[21] The gain-loss utility $\mu(z)$ is defined by

[20] This explains why the often used functional form of $\frac{(c_t-c^*)^{1-\gamma}}{1-\gamma}$ is *not* proper for gain-loss utility. The main difference between *reference dependence of consumption* and *reference dependence of utility* arises here.

[21] If η is very small, then the gain-loss effect is negligible. Many researchers simply set $\eta = 1$.

$$\mu(z) = \left\{ \begin{array}{cc} z & \text{if } z \geqslant 0 \\ \lambda z & \text{if } z < 0 \end{array} \right\}$$ [22]

in which

z : Deviation in outcome from a reference point, i.e. $z = u(c) - u(r)$

$\lambda > 1$: Coefficient of loss aversion

Then the total utility in a *static* model is specified by the following, assuming the decision maker has a reference point $u(r) = u(c^*) = \frac{c^{*1-\gamma}}{1-\gamma}$,

$$u(c|c^*) = \left\{ \begin{array}{lll} \frac{c^{1-\gamma}}{1-\gamma} + \eta\left(\frac{c^{1-\gamma}}{1-\gamma} - \frac{c^{*1-\gamma}}{1-\gamma}\right) & \text{if} & \frac{c^{1-\gamma}}{1-\gamma} - \frac{c^{*1-\gamma}}{1-\gamma} \geqslant 0 \\ \frac{c^{1-\gamma}}{1-\gamma} + \eta\lambda\left(\frac{c^{1-\gamma}}{1-\gamma} - \frac{c^{*1-\gamma}}{1-\gamma}\right) & \text{if} & \frac{c^{1-\gamma}}{1-\gamma} - \frac{c^{*1-\gamma}}{1-\gamma} < 0 \end{array} \right\}$$

In a *dynamic* model of intertemporal decision making, it is necessary to modify the gain-loss utility to incorporate decision maker's *psychological weight* on the gain-loss utilities over time.[23] And it is natural to posit that the weighting is likely to decay as its effect fades. To simplify the model for multi-period analysis, I assume that the initial strength of the concern for loss relative to gain is $\omega > 0$ and that the decay follows standard time discounting.[24]

4.2 A SIMPLE MODEL

4.2.1 Two-Periods

To explore the implication of the decision making with reference dependent preference (RDP hereafter), let us first construct a simple model of consumption-saving in a standard way and compare it with the RDP model. Consider a decision maker who lives two periods ($t = 0, 1$) and discounts the future utility by β. He has a nonnegative income stream $\{y_0, y_1\}$ and his utility is specified by a CRRA functional form, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, where γ[25] denotes the degree

[22] Following KR, I posit a *linear* gain-loss function to get closed form solutions. Other candidates for the gain-loss functions may be power function and exponential function.

[23] KR propose, for $\tau = t, ..., T$, $u_t = m(c_t) + \sum_{\tau=t}^{T} \varphi_{t,\tau} n(F_{t,\tau} \mid F_{t-1,\tau})$ where $F_{t-1,\tau}$ represents fixed beliefs, inherited from last period and $F_{t,\tau}$ represents new beliefs that decision maker forms. $\varphi_{\tau,\tau} \geqslant \varphi_{\tau-1,\tau} \geqslant ... \geqslant \varphi_{0,\tau} \geqslant 0$ are weights on gain-loss utilities.

[24] This is because the psychological discounting may or may not follow the usual time discounting. When $\omega = 0$, there is no gain-loss utility and this returns to the standard model.

[25] CRRA utility is defined by $u(c) = \ln(c)$ when $\gamma = 1$.

of risk aversion.[26] Assume that there is no borrowing constraint and the agent can borrow or lend freely at the market interest rate $1 + r = R$. In the standard model of two-period consumption and saving decision, the agent chooses his optimal consumption $\{c_0, c_1\}$, by maximizing life-time utility

$$u(c_0, c_1) = \frac{c_0^{1-\gamma}}{1-\gamma} + \beta \frac{c_1^{1-\gamma}}{1-\gamma}$$

subject to

$$c_0 + b_1 = y_0$$
$$c_1 = y_1 + Rb_1$$

where b_1 is bond holding at $t = 0$ for the next period. Solving the maximization problem yields the optimal consumption profile of the standard decision maker for two periods. The two-period consumption is

$$c_0^* = \frac{y_0 + y_1 R^{-1}}{1 + \phi^{-1}}$$
$$c_1^* = \frac{R}{\phi}\left(\frac{y_0 + y_1 R^{-1}}{1 + \phi^{-1}}\right)$$

where $\phi^{-1} = (\beta R)^{1/\gamma} R^{-1}$.[27] In the standard model, it is clear that this ex ante solution is optimal ex post, as well.[28] Let us call this solution $\{c_0^*, c_1^*\}$ the "optimal consumption plan," solved from the maximization problem of the standard decision maker given any income stream.

Now turn to the model of reference dependent preference.[29] To build a simple RDP version of the consumption-saving model, define ω the initial psychological weight on loss relative to gain utility. Then at the beginning of the first period $t = 0$, the decision maker forms belief regarding the optimal consumption, as described in the introduction. Because the optimal consumption plan above gives the decision maker ex ante maximum utility, this outcome serves as the reference point to the agent. The main question here about RDP

[26] Or the inverse of intertemporal elasticity of substitution.

[27] $\phi = (\beta R)^{-1/\gamma} R$. This notation is introduced by Feigenbaum (2005)[43]. A value of ϕ contains a combined effect from three parameters: β, R, and γ.

[28] This is easy to show: if otherwise, then there exists always at least one consumption point that can increase the decision maker's utility.

[29] I assume that both RDP agent and standard agent are rational decision maker.

decision maker is, "Would the decision maker choose the consumption which maximizes his ex ante utility among all the strategies available to him?" That is to say: would the ex ante solution be optimal ex post for the decision maker with belief-dependent preference? To answer this question, consider the following stories.

First, if the decision maker intends to consume more than the optimal plan at the first period $t = 0$, and as a result he ends up with consuming less than the optimal at $t = 1$, i.e. $\{c_0 > c_0^*,\ c_1 < c_1^*\}$, then the situation of this decision maker at the beginning of the first period is described by the following maximization

$$u(c_0, c_1 | c_0^*, c_1^*) =$$

$$\tfrac{c_0^{1-\gamma}}{1-\gamma} + \eta\left[\left(\tfrac{c_0^{1-\gamma}}{1-\gamma} - \tfrac{c_0^{*1-\gamma}}{1-\gamma}\right) + \beta\omega\lambda\left(\tfrac{c_1^{1-\gamma}}{1-\gamma} - \tfrac{c_1^{*1-\gamma}}{1-\gamma}\right)\right] + \beta\tfrac{c_1^{1-\gamma}}{1-\gamma}$$

subject to

$$c_0 + b_1 = y_0$$
$$c_1 = y_1 + Rb_1$$

The total utility comes from consumption utility for each period, which is represented by $u(c_0)$ and $\beta u(c_1)$, and the gain-loss utility for each period. If the decision maker consumes more than the reference point at the first period, he has a contemporaneous gain utility for that period, which is $[u(c_0) - u(c_0^*)]$ when $\eta = 1$.[30] As a result, his consumption is lowered next period and this yields a prospective loss utility relative to the reference point. This is described by $\beta\omega\lambda[u(c_1) - u(c_1^*)]$.[31] Notice that there is no gain-loss utility in the second period ($t = 1$). Because the decision maker is clearly aware of the consumption of the first period, he forms new expectation, based on his past behavior, that determines his next reference point.[32]

Is consuming more than the ex ante optimal amount at the first period is worthwhile to this decision maker? To answer this, construct the following derivative to see if the deviation is profitable. Using the life-time resource constraint $c_1 = y_1 + R(y_0 - c_0)$

[30] η : the weight of gain-loss utility relative to consumption utility

[31] $\lambda > 1$: coefficient of loss aversion. $\omega > 0$: initial strength of the concern for loss relative to gain. β : usual time discounting. Thus $\beta\omega\lambda$ represents the "prospective" loss relative to contemporaneous gain.

[32] The rational decision maker does not forget where he is from or what his path was.

$$\frac{du}{dc_0} = (1+\eta)c_0^{-\gamma} - \beta R(1+\eta\omega\lambda)[y_0 + R(y_0 - c_0)]^{-\gamma}$$

If the derivative is evaluated at the optimal consumption $c_0^* = \dfrac{y_0 + y_1 R^{-1}}{1+\phi^{-1}}$, then

$$\frac{du}{dc_0^*} = \eta(1-\omega\lambda)\left(\frac{y_0 + y_1 R^{-1}}{1+\phi^{-1}}\right)^{-\gamma}$$
$$= \eta(1-\omega\lambda)c_0^{*-\gamma}\left\{\begin{array}{lll} \leqslant 0 & if & \omega\lambda \geqslant 1 \\ > 0 & if & \omega\lambda < 1 \end{array}\right\}.$$

The condition implies that there is no incentive to deviate from the optimal consumption level c_0^* if $\omega\lambda \geqslant 1$, because the deviation does not increase the utility ex post. The ex post optimal consumption of this RDP decision maker is the same as the one of standard decision maker. In other words, the standard agents are those who care more about the prospective loss utility than the contemporaneous gain utility. This is what KR noticed: prospective loss from lowering future consumption tends to act as internal commitment device. This implies that RDP agent would not over-consume even though he does not have any external commitment device like illiquid asset.[33]

However, if $\omega\lambda < 1$ then it may be better for the decision maker to choose another strategy than the ex ante optimal.[34] This comes from the fact that deviation from ex ante solution does increase the utility ex post. Thus, to the decision maker whose concern about loss is not very high, new optimality condition should apply and the condition is

$$(1+\eta)c_0^{-\gamma} = R\beta(1+\eta\omega\lambda)c_1^{-\gamma}$$

The consistent consumption path for the intended plan $\{c_0 > c_0^*,\, c_1 < c_1^*\}$ is[35]

$$\left\{\begin{array}{c} c_0 = \dfrac{y_0 + y_1 R^{-1}}{1+\phi^{-1}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}} \\ c_1 = \frac{R}{\phi}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}\left(\dfrac{y_0 + y_1 R^{-1}}{1+\phi^{-1}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}}\right) \end{array}\right\}$$

It is easy to see $c_0 > c_0^*$ and $c_1 < c_1^*$ because

[33] The golden eggs in Laibson (1997)[69] is an external commitment device for over-consumers.

[34] Remember that $\omega > 0$, thus $0 < \omega\lambda < 1$.

[35] This is a closed form solution to KR's conjecture.

$$c_0 = \frac{y_0 + y_1 R^{-1}}{1 + \phi^{-1}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}} > \frac{y_0 + y_1 R^{-1}}{1 + \phi^{-1}} = c_0^*$$

and

$$c_1 = \frac{R}{\phi}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}\left(\frac{y_0 + y_1 R^{-1}}{1 + \phi^{-1}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}}\right) < \frac{R}{\phi}\left(\frac{y_0 + y_1 R^{-1}}{1 + \phi^{-1}}\right) = c_1^*$$

When decision maker cares more about contemporaneous gain utility than about prospective loss utility, his present consumption can be higher than the usual consumption of the standard model solved earlier. This implies that RDP model yields the similar result as in the model with present biased preference or hyperbolic discounting that is based on time inconsistent preferences.[36]

However, the reference dependent preference model can be illuminated in another way.[37] If the decision maker intends to *consume less than the optimal plan* in the first period[38] and as a result he ends up with consuming more than the optimal amount in the next period, i.e. $\{c_0 < c_0^*,\, c_1 > c_1^*\}$, then the situation for the decision maker is described by

$$u(c_0, c_1 |\, c_0^*, c_1^*) =$$

$$\frac{c_0^{1-\gamma}}{1-\gamma} + \eta\left[\omega\lambda\left(\frac{c_0^{1-\gamma}}{1-\gamma} - \frac{c_0^{*1-\gamma}}{1-\gamma}\right) + \beta\left(\frac{c_1^{1-\gamma}}{1-\gamma} - \frac{c_1^{*1-\gamma}}{1-\gamma}\right)\right] + \beta\frac{c_1^{1-\gamma}}{1-\gamma}$$

subject to

$$c_0 + b_1 = y_0$$
$$c_1 = y_1 + Rb_1$$

Because the decision maker consumes less than the optimal in the first period, he has contemporaneous loss feeling (utility) relative to the reference consumption behavior. This loss utility is $\omega\lambda[u(c_0) - u(c_0^*)]$. As a result, his consumption is elevated next period and this gives him prospective gain feeling, which is $\beta[u(c_1) - u(c_1^*)]$. Like in the over-consumer's maximization procedure, because the expectation is met, there is no gain-loss utility in the second period. The solution to this problem is summarized by

[36] In fact, this result comes from the Hyperbolic discounting-like component in gain-loss utility.
[37] Few researchers, if any, have noticed this property.
[38] This may be the case for a miser (natural born saver) who likes to save than consume.

$$\left\{\begin{array}{c} c_0 = \dfrac{y_0 + y_1 R^{-1}}{1+\phi^{-1}} = c_0^* \\ c_1 = \ \frac{R}{\phi}\left(\dfrac{y_0 + y_1 R^{-1}}{1+\phi^{-1}}\right) = c_1^* \end{array}\right\} \text{ if } \omega\lambda \geqslant 1$$

and

$$\left\{\begin{array}{c} c_0 = \dfrac{y_0 + y_1 R^{-1}}{1+\phi^{-1}\left(\frac{1+\eta}{1+\eta\omega\lambda}\right)^{1/\gamma}} \\ c_1 = \frac{R}{\phi}\left(\frac{1+\eta}{1+\eta\omega\lambda}\right)^{1/\gamma}\left(\dfrac{y_0 + y_1 R^{-1}}{1+\phi^{-1}\left(\frac{1+\eta}{1+\eta\omega\lambda}\right)^{1/\gamma}}\right) \end{array}\right\} \text{ if } \omega\lambda < 1$$

Notice that unlike the previous example of over-consumer, the optimization of miser, i.e. who has $\omega\lambda < 1$ for contemporaneous loss, yields $c_0 < c_0^*$ and $c_1 > c_1^*$. When the decision maker cares more about prospective gain utility than about contemporaneous loss utility, he would deviate, from standard prediction, for less consumption (more saving) in the current period. This explanation justifies the behavior of natural born saver. The savers save because their expected gain feelings in the future outweigh the current loss feeling. Likewise, the standard agents are those who care more about the contemporaneous loss utility than the prospective gain utility.[39] These two exercises show that the two-period RDP model induces both ways of deviation: consuming either more or less than the level which is considered optimal based on his belief formed at the beginning of the planning period.

4.2.2 Beyond Two-Periods

Constructing an intertemporal model of belief dependent preference for three and more periods requires deeper understanding of how the decision maker forms expectation regarding the reference status because there are potentially many reference points to consider.[40] It is also necessary to figure out what type of strategy gives the decision maker a highest utility among all the alternative consumption plans because there are more alternative plans with longer periods. In this section, I focus on the second issue and study the model with three periods in detail and get implication of the model with longer periods.

[39] Combining with the previous example of over-consumer, the standard agents are said to be those who care more about loss for any period.

[40] This is analyzed in detail in the next section of Dynamic Model.

4.2.2.1 Three-Periods Following the two-period model, I first explore if the belief dependent decision maker (DM: hereafter) selects the consumption bundle that can maximize his ex ante utility among all the choices available to him. I assume the same environment as in the two-period model and the riskless bond can be sold or bought at riskless interest rate R. First of all, notice that given any income stream of three periods $\{y_0, y_1, y_2\}$, the ex ante utility maximizer for the three-period consumption $\{c_0,\, c_1,\, c_2\}$ is

$$\{c_0^*,\, c_1^*,\, c_2^*\} = \left\{ \frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1 + \frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2},\, \frac{R}{\phi} \left(\frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1 + \frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2} \right),\, \left(\frac{R}{\phi}\right)^2 \left(\frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1 + \frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2} \right) \right\}$$

Then choosing $\{c_0^*, c_1^*, c_2^*\}$ maximizes both consumption utility and ex ante expected gain-loss utility (which is zero), so this is the ex ante optimal strategy. Is this strategy consistent with the agent who has belief dependent preference? To see this, let us construct the following consumption plan. At the beginning of the first period, DM plans a consumption profile for the three periods $\{c_0,\, c_1,\, c_2\}$ that satisfies his intention of over-consumption for the first two periods: $\{c_0 > c_0^*,\, c_1 > c_1^*,\, c_2 < c_2^*\}$. Then the strategic situation of the DM at the beginning of $t = 0$ is

$$u(c_0, c_1, c_2 | c_0^*, c_1^*, c_2^*) =$$

$$\frac{c_0^{1-\gamma}}{1-\gamma} + \eta \left[\left(\frac{c_0^{1-\gamma}}{1-\gamma} - \frac{c_0^{*1-\gamma}}{1-\gamma} \right) + \beta \left(\frac{c_1^{1-\gamma}}{1-\gamma} - \frac{c_1^{*1-\gamma}}{1-\gamma} \right) + \beta^2 \omega \lambda \left(\frac{c_2^{1-\gamma}}{1-\gamma} - \frac{c_2^{*1-\gamma}}{1-\gamma} \right) \right] + \beta \frac{c_1^{1-\gamma}}{1-\gamma} + \beta^2 \frac{c_2^{1-\gamma}}{1-\gamma}$$

subject to

$$\begin{aligned} c_0 + b_1 &= y_0 \\ c_1 + b_2 &= y_1 + R b_1 \\ c_2 \qquad &= y_2 + R b_2 \end{aligned}$$

The total utility comes from consumption utility for each period, which is represented by $u(c_0)$, $\beta u(c_1)$ and $\beta^2 u(c_2)$, and the gain-loss utility related with the consumption choice each period. Because the DM consumes more than his reference point at the first two periods, he has the contemporaneous gain utility for the first period, which is $[u(c_0) - u(c_0^*)]$ and the prospective gain utility $\beta[u(c_1) - u(c_1^*)]$ for the second period, when $\eta = 1$. Likewise, the DM feels a prospective loss for the third period because of his expected consumption which is lower than the reference point. This is described by $\beta^2\omega\lambda[u(c_2) - u(c_2^*)]$ if $\eta = 1$. Notice that there will be no gain-loss utility in the second and the third period for this DM who plans the consumption strategy at the beginning of the first period.[41] This is because DM forms adapted belief on reference points for the subsequent periods following the plan. DM knows the expectation is met those periods, if he follows the intended consumption plan. Let us see again if the ex ante optimal consumption is consistent with this plan by evaluating the derivative at the reference points. The total utility of the original problem may be re-written by

$$\begin{gathered}\tfrac{c_0^{1-\gamma}}{1-\gamma} + \eta\left(\tfrac{c_0^{1-\gamma}}{1-\gamma} - \tfrac{c_0^{*1-\gamma}}{1-\gamma}\right) + \beta\left\{\tfrac{c_1^{1-\gamma}}{1-\gamma} + \eta\left(\tfrac{c_1^{1-\gamma}}{1-\gamma} - \tfrac{c_1^{*1-\gamma}}{1-\gamma}\right)\right\} \\ + \beta^2\left\{\tfrac{c_2^{1-\gamma}}{1-\gamma} + \omega\eta\lambda\left(\tfrac{c_2^{1-\gamma}}{1-\gamma} - \tfrac{c_2^{*1-\gamma}}{1-\gamma}\right)\right\}\end{gathered}$$

To examine the consistency condition, consider the following derivatives to see if the deviation is profitable. Because $c_2 = [y_2 + Ry_1 + R^2(y_0 - c_0) - Rc_1]$, the derivatives with respect to the other two variables (c_0, c_1) are

$$\tfrac{du}{dc_0} = (1+\eta)c_0^{-\gamma} - \beta^2R^2(1+\eta\omega\lambda)[y_2 + Ry_1 + R^2(y_0 - c_0) - Rc_1]^{-\gamma}$$

and

$$\tfrac{du}{dc_1} = \beta[(1+\eta)c_1^{-\gamma} - \beta R(1+\eta\omega\lambda)[y_2 + Ry_1 + R^2(y_0 - c_0) - Rc_1]^{-\gamma}$$

If these are evaluated at the ex ante optimal consumption $\{c_0^*, c_1^*, c_2^*\}$, then

$$\begin{aligned}\tfrac{du}{dc_0^*} &= (1+\eta)c_0^{*-\gamma} - \beta^2R^2(1+\eta\omega\lambda)[y_2 + Ry_1 + R^2(y_0 - c_0^*) - Rc_1^*]^{-\gamma} \\ &= \eta(1-\omega\lambda)\left(\frac{y_0 + y_1R^{-1} + y_2R^{-2}}{1+\phi^{-1}+\phi^{-2}}\right)^{-\gamma} \\ &= \eta(1-\omega\lambda)c_0^{*-\gamma}\left\{\begin{array}{ll}\leqslant 0 & if \;\; \omega\lambda \geqslant 1 \\ > 0 & if \;\; \omega\lambda < 1\end{array}\right\}\end{aligned}$$

$$\begin{aligned}\tfrac{du}{dc_1^*} &= \beta(1+\eta)c_1^{*-\gamma} - \beta^2R(1+\eta\omega\lambda)[y_2 + Ry_1 + R^2(y_0 - c_0^*) - Rc_1^*]^{-\gamma} \\ &= \eta(1-\omega\lambda)R^{-1}\left(\frac{y_0 + y_1R^{-1} + y_2R^{-2}}{1+\phi^{-1}+\phi^{-2}}\right)^{-\gamma} \\ &= \eta(1-\omega\lambda)R^{-1}c_0^{*-\gamma}\left\{\begin{array}{ll}\leqslant 0 & if \;\; \omega\lambda \geqslant 1 \\ > 0 & if \;\; \omega\lambda < 1\end{array}\right\}\end{aligned}$$

[41] Following many researchers who work on multi-selves model, this agent may be called self (0).

The first lines of each evaluation of the derivatives follow from the fact that because $c_2(c_0^*, c_1^*) = c_2^*$, the resource constraint satisfies $c_2^* = [y_2 + Ry_1 + R^2(y_0 - c_0^*) - Rc_1^*]$. If $\omega\lambda \geqslant 1$, there is no incentive to deviate from the ex ante optimal consumption levels, c_0^* and c_1^*, because the deviation does not increase the utility ex post. This implies that consuming more than the optimal for the first two periods would not pay off for the agents who concerns a lot about the loss feeling of the final period due to the over-consumption of earlier periods.

However this does not apply to the agent who has lower weight on loss and cares more about the gains. If $\omega\lambda < 1$ then it is always better for the agent to choose another strategy than the ex ante solution because deviation increases his total utility ex post. Those people who cares more about the gain feeling because of their low weights on the prospective pain from loss, would deviate from the ex ante solution for an alternative path following new optimality condition.

Proposition 7. *When the DM has a high weight ($\omega\lambda \geqslant 1$) on prospective loss, he would not deviate from the optimal consumption plan. If, however, his weight on loss is low, then the consistent consumption plan for DM who intends to consume $\{c_0 > c_0^*,\ c_1 > c_1^*,\ c_2 < c_2^*\}$ is*

$$c_0 = \frac{y_0 + y_1 R^{-1} + y_2 R^{-2}}{1 + \phi^{-1} + \phi^{-2}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}}$$

$$c_1 = \tfrac{R}{\phi}\left(\frac{y_0 + y_1 R^{-1} + y_2 R^{-2}}{1 + \phi^{-1} + \phi^{-2}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}}\right)$$

$$c_2 = \left(\tfrac{R}{\phi}\right)^2\left(\tfrac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}\left(\frac{y_0 + y_1 R^{-1} + y_2 R^{-2}}{1 + \phi^{-1} + \phi^{-2}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}}\right).$$

Again it is easy to see $c_0 > c_0^*$ and $c_1 > c_1^*$, but $c_2 < c_2^*$, because $\frac{1+\eta\omega\lambda}{1+\eta} < 1$ when $\omega\lambda < 1$. When DM cares more about gain utilities within near future than about remote loss utilities, his earlier consumption can be higher than the usual one suggested by the standard model. It is also a possible scenario that the DM with a low weight may intend to overconsume only the first period, but not the second period: DM intends to consume $\{c_0 > c_0^*,\ c_1 < c_1^*,\ c_2 < c_2^*\}$. Then the consistent consumption plan for this DM is

$$\{c_0,\ c_1,\ c_2\} = \left\{c_0,\quad c_0\tfrac{R}{\phi}\left(\tfrac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma},\quad c_0\left(\tfrac{R}{\phi}\right)^2\left(\tfrac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}\right\},$$

$$c_0 = \frac{y_0 + y_1 R^{-1} + y_2 R^{-2}}{1 + \phi^{-1}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma} + \phi^{-2}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}}.$$

Likewise, if DM with $\omega\lambda < 1$ intends a consumption strategy, in which he clings to one of the optimal consumption such as $c_1 = c_1^*$, then the consistent consumption for the plan $\{c_0 > c_0^*,\, c_1 = c_1^*,\, c_2 < c_2^*\}$ is

$$\{c_0,\, c_1,\, c_2\} = \left\{c_0, \quad c_1^*, \quad c_0\left(\frac{R}{\phi}\right)^2\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}\right\},$$
$$c_0 = \frac{y_0 + y_1 R^{-1} + y_2 R^{-2} - \frac{1}{\phi}\left(\frac{y_0 + y_1 R^{-1} + y_2 R^{-2}}{1+\phi^{-1}+\phi^{-2}}\right)}{1 + \phi^{-2}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}}.$$

Because the second period consumption is fixed at the ex ante optimal c_1^*, it is necessary for the consumption levels of the other two (the first and the third period) to be adjusted to fulfill DM's intention. It is interesting to notice that when the DM sticks to the intended plan for the subsequent periods, the ex post life time total utility[42] from the three scenarios has the following ranking: $\{c_0 > c_0^*,\, c_1 < c_1^*,\, c_2 < c_2^*\} \succeq \{c_0 > c_0^*,\, c_1 > c_1^*,\, c_2 < c_2^*\} \succeq \{c_0 > c_0^*,\, c_1 = c_1^*,\, c_2 < c_2^*\}$. In the next section, I describe this in detail, together with other intended plans with longer periods.

4.2.2.2 Utility Comparison

In this section I want to compare the overall utility from alternative consumption plans and determine the ranking of the total ex post utility among the plans.[43] The key assumption for the comparison here is that DM follows the intended plan for the subsequent periods without changing his mind over time.[44] In a three period life time plan, the agent wants to maximize his utility based on the following comparison:

$$\begin{aligned}
&\frac{c_0^{1-\gamma}}{1-\gamma} + \eta\left(\frac{c_0^{1-\gamma}}{1-\gamma} - \frac{c_0^{*1-\gamma}}{1-\gamma}\right)\mathbf{1}(c_0 > c_0^*) + \omega\eta\lambda\left(\frac{c_0^{1-\gamma}}{1-\gamma} - \frac{c_0^{*1-\gamma}}{1-\gamma}\right)\mathbf{1}(c_0 < c_0^*)\\
&+ \beta\{\frac{c_1^{1-\gamma}}{1-\gamma} + \eta\left(\frac{c_1^{1-\gamma}}{1-\gamma} - \frac{c_1^{*1-\gamma}}{1-\gamma}\right)\mathbf{1}(c_1 > c_1^*) + \omega\eta\lambda\left(\frac{c_1^{1-\gamma}}{1-\gamma} - \frac{c_1^{*1-\gamma}}{1-\gamma}\right)\mathbf{1}(c_1 < c_1^*)\}\\
&+ \beta^2\{\frac{c_2^{1-\gamma}}{1-\gamma} + \eta\left(\frac{c_2^{1-\gamma}}{1-\gamma} - \frac{c_2^{*1-\gamma}}{1-\gamma}\right)\mathbf{1}(c_2 > c_2^*) + \omega\eta\lambda\left(\frac{c_2^{1-\gamma}}{1-\gamma} - \frac{c_2^{*1-\gamma}}{1-\gamma}\right)\mathbf{1}(c_2 < c_2^*)\}
\end{aligned}$$

DM has ex ante optimal consumption plan on hand as his reference point and his utility comes from both consumption utility and gain-loss utility relative to the reference point.

[42] Total utility means consumption utility and gain-loss utility altogether over three periods.

[43] This analysis is necessary to get insight about the best strategy for the longer period model that I consider.

[44] The *changing mind* is studied in the next section.

From the above, it is easy to find that there are five possible consumption plans if he is not an initial saver.[45] In the Appendix B, there are solutions regarding the consistent consumption strategies for these five alternatives. From those solutions I have the following utility ranking in Table [9]. Although the table shows the result from a specific income stream $\{y_0 = 1, y_1 = 1, y_2 = 1\}$ and a set of parameter values $\{R = 1.05, \beta = 0.98, \gamma = 0.9, \omega\lambda = 0.8\}$, this ranking is robust with almost all other parameter values given any alternative income profile.[46]

Table 9: Three-period Plan and Utility Rank

Plan	$u(c)$	$u(G/L)$	*Total U*	*Rank*
c^*: $c_0 = c_0^*$, $c_1 = c_1^*$, $c_2 = c_2^*$	29.4461	0	29.4461	4
A: $c_0 > c_0^*$, $c_1 > c_1^*$, $c_2 < c_2^*$	29.4421	0.0115	29.4536	2
B: $c_0 > c_0^*$, $c_1 < c_1^*$, $c_2 < c_2^*$	29.4419	0.0119	**29.4538**	1
C: $c_0 = c_0^*$, $c_1 > c_1^*$, $c_2 < c_2^*$	29.3483	−0.0766	29.2717	5
D: $c_0 > c_0^*$, $c_1 = c_1^*$, $c_2 < c_2^*$	**29.4430**	0.0088	29.4518	3
E: $c_0 > c_0^*$, $c_1 < c_1^*$, $c_2 = c_2^*$	29.3418	−0.0823	29.2595	6

It is interesting that the best plan in terms of total utility does not provide a highest consumption utility. In fact, the highest consumption utility, except for the ex ante optimal solution, comes from the plan D which gives DM the smoothest consumption bundle among all the deviation strategies. This implies that DM gets the highest consumption utility if he follows the optimal plan but because of his gain-loss feeling, this is suboptimal to his preferred choice. Notice also that plan C and plan E have negative gain-loss utilities, implying these choices are not fulfilling his taste at all, as well as giving DM the lowest consumption utility. Once DM intends to deviate, clinging to any part of the original plan would not help for the new plan and this may explain the result of the table.

[45] In other words, the assumption here is that DM is an initial over-consumer.

[46] The plan of alternating consumption, $\{c_0 > c_0^*, c_1 < c_1^*, c_2 > c_2^*\}$, is excluded in this comparison because the plan not only gives the lowest total utility but also it does not clearly characterize the type of an over-consumer.

Among the above five, it may be proper to compare analytically those plans that do not have any fixed consumption (c^*) because it is clear from the table that those profiles are suboptimal to the strategies of free choice. Thus let us compare between the two plans, {A: $c_0 > c_0^*$, $c_1 > c_1^*$, $c_2 < c_2^*$} and {B: $c_0 > c_0^*$, $c_1 < c_1^*$, $c_2 < c_2^*$ }. From the work in earlier chapters, it is obvious that if $\omega\lambda < 1$, then deviation is justified. The deviation could happen in a way of more consumption only at the first period (B) or more consumption both of the two periods (A). When DM deviates for more consumption in earlier periods, he should accept the penalty of low consumption in later periods. Thus DM may consider whether he consumes more for both periods and takes the burden of deep loss at the last period or smooth the burden, by over-consuming only in the first period. Then the question is: which one is better for a deviator? Although the table shows direct answer to this question, it may also be intuitive. Assume $R\beta = 1$. Then because $(R\beta)^{1/\gamma} = 1$, it follows that $c_0 = c_1$ in the plan A. Thus if $\omega\lambda < 1$, then $c_0 = c_1 > c_2$. If $\omega\lambda \geqslant 1$, then no deviation occurs and $c_0 = c_1 = c_2$. In plan B, because $c_1 = c_2$, it follows that $c_0 > c_1 = c_2$ if $\omega\lambda < 1$. From the solutions in Appendix, it is easy to see that $c_0^B > c_0^A$ and $c_2^B > c_2^A$ for the deviator and thus plan B is more likely to give higher utility. The utility difference between the two plan is $(1+\eta)[u(c_0^B) - u(c_0^A)] + \beta[(1+\eta\omega\lambda)u(c_1^B) - (1+\eta)u(c_1^A) - (1-\omega\lambda)u(c_1^*)] + \beta^2(1+\eta\omega\lambda)[u(c_2^B) - u(c_2^A)]$.

How about four-period model? In the Appendix, the alternative consistent consumptions are derived for DM with $\omega\lambda < 1$. The resulting utility order is:[47] $\{c_0 > c_0^*,\ c_1 > c_1^*,\ c_2 < c_2^*,\ c_3 < c_3^*\} \succ \{c_0 > c_0^*,\ c_1 < c_1^*,\ c_2 < c_2^*,\ c_3 < c_3^*\} \succ \{c_0 > c_0^*,\ c_1 > c_1^*,\ c_2 > c_2^*,\ c_3 < c_3^*\} \succ \{c_0^*, c_1^*, c_2^*, c_3^*\}$. This ranking is preserved for a general class of parameter values. With five-period model, I have the result in Table [10] for $\{\beta = 0.95,\ R = 1.035,\ \gamma = 0.9,\ \eta = 1,\ \omega\lambda = 0.4\}$.

If I describe each plan using the notation of $\{1\}$ for $c_t > c_t^*$ and $\{0\}$ for $c_t < c_t^*$, the order is described in this fashion: $\{1,1,0,0,0\} \succ \{1,1,1,0,0\} \succ \{1,0,0,0,0\} \succ \{1,1,1,1,0\} \succ \{(Standard)\}$. This order is preserved for most values of parameters.[48] As shown in the tables, I find that for general class of parameters, the overall utility culminates ex post when

[47] The plan with fixed consumptions are excluded by the same argument.

[48] I have not found any violation for most of parameter values.

Table 10: Five-period Plan and Utility Rank

Plan	$u(c)$	$u(G/L)$	*Total U*	R
$c_0 > c_0^*$, $c_1 > c_1^*$, $c_2 > c_2^*$, $c_3 > c_3^*$, $c_4 < c_4^*$	46.6852	0.1258	46.8110	4
$c_0 > c_0^*$, $c_1 > c_1^*$, $c_2 > c_2^*$, $c_3 < c_3^*$, $c_4 < c_4^*$	46.6562	0.2024	46.8586	2
$c_0 > c_0^*$, $c_1 > c_1^*$, $c_2 < c_2^*$, $c_3 < c_3^*$, $c_4 < c_4^*$	46.6491	0.2178	46.8669	1
$c_0 > c_0^*$, $c_1 < c_1^*$, $c_2 < c_2^*$, $c_3 < c_3^*$, $c_4 < c_4^*$	46.6707	0.1570	46.8277	3

DM keeps the over-consumption behavior up to the half of the whole periods. So if DM has a consumption plan for S periods, shifting from high consumption ($c > c^*$) to low consumption ($c < c^*$) at the middle point of the planning horizon, which is $S/2$, gives him the highest ex post utility.[49] Although I register the utility comparison only for the over-consumer here, this rule also applies to the case of a saver, who gets the highest utility when he keeps the under-consumption behavior up to the middle point of the planning horizon.

4.2.2.3 Consumption Hump in Simple Model Besides the utility ranking, there is one more interesting topic that many researchers are looking for. It is known that the standard lifecycle theory cannot explain why the pattern of household consumption over life-time is "hump" shaped: mean consumption is increasing while the consumer is young, reaching a peak around middle age and then decreasing afterwards. The consumption hump is a well-known feature of lifecycle consumption data. But this property is not obtained in standard model with far-sighted rational agents with time-consistent preference because the model predicts that consumption over lifecycle should be monotonically increasing, decreasing or constant, but not hump-shaped.[50]

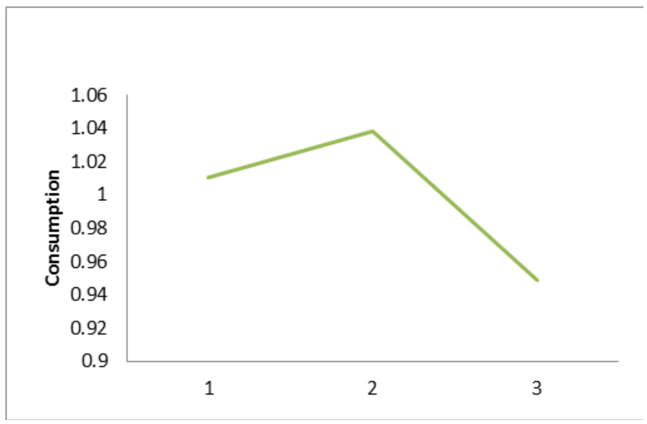
Thus the question is: can the model of belief dependent preference produce a hump over lifecycle when deviation is desirable, i.e. $\omega\lambda < 1$?[51] Although I deal with this issue in depth

[49] For example, if $S = 4$, then $\{1, 1, 0, 0\}$ is the best plan. If S is odd number, then over-consumption up to one period less than the middle point gives the best result.

[50] This follows from the Euler equation in standard model: $u'(c_t) = R\beta u'(c_{t+1})$.

[51] When $\omega\lambda \geqslant 1$, the model returns to the standard one and there is no room for a hump.

Figure 33: The consumption is obtained with flat income stream, for a plan of over-consumption for the first two periods. The gain-loss parameters are $\eta = 1$, $\lambda = 2$, $\omega = 0.4$. The other parameters are $R = 1.035$, $\beta = 0.99$ and $\gamma = 0.9$.



in later sections, introducing formal definition of the consumption hump, I want to show here an exemplary result that demonstrates such property. With $\omega\lambda < 1$, it is possible to have a hump with three, four, or five-period model as is seen in the following figure.[52] What would be the mechanism for a hump here? When $\omega\lambda < 1$, this consumer is more like a myopic (over-consumer) or a miser (saver). So he wants to consume more or save more than the standard monotonic prediction. If this intention combines with time preference and the market interest rate, then there is room for a consumption hump.

Although the hump-shaped consumption in 33 is obtained from an over-consumer's behavior, the model of belief dependent preference can produce another hump with other consumption plans as well. I want to discuss more on this in detail since this is one of the most important features of reference dependent utility. So far I focus on the over-consumers or the savers who are characterized by initial over- or under-consumption and who are not changing their natural types of consumption behavior over time.[53] However, it is possible to imagine that a consumer may have both characteristic over the lifecycle. Thus let us assume that there is such agent who plans for three periods $t = 0, 1, 2$. He intends to consume less

[52] Those specifications that may produce a hump would not give the agent the highest welfare ex post in general.

[53] Through the paper in later sections I assume this for simplicity as described in the introduction.

than the ex ante optimal in the first period, more than the optimal in the second period and finally less than the optimal in the last period: $\{c_0 < c_0^*,\, c_1 > c_1^*,\, c_2 < c_2^*\}$. When deviation is desirable ($\omega\lambda < 1$) for the DM, the consistent consumption is

$$\{c_0,\, c_1,\, c_2\} = \left\{c_0,\quad c_0\frac{R}{\phi}\left(\frac{1+\eta}{1+\eta\omega\lambda}\right)^{1/\gamma},\quad c_0\left(\frac{R}{\phi}\right)^2\right\},$$

$$c_0 = \frac{y_0 + y_1R^{-1} + y_2R^{-2}}{1 + \phi^{-1}\left(\frac{1+\eta}{1+\eta\omega\lambda}\right)^{1/\gamma} + \phi^{-2}}.$$

It is straightforward that $c_0 < c_0^*$, and $c_2 < c_2^*$, but $c_1 > c_1^*$ because $\omega\lambda < 1$. I want to demonstrate that this consumption profile produces the consumption hump even in the simplest environment such as $\beta = 1$ and $R = 1$.[54] The RDP consumption under this condition is

$$\left\{\frac{y_0 + y_1 + y_2}{1 + \left(\frac{1+\eta}{1+\eta\omega\lambda}\right)^{1/\gamma} + 1},\quad \frac{\left(\frac{1+\eta}{1+\eta\omega\lambda}\right)^{1/\gamma}(y_0 + y_1 + y_2)}{1 + \left(\frac{1+\eta}{1+\eta\omega\lambda}\right)^{1/\gamma} + 1},\quad \frac{y_0 + y_1 + y_2}{1 + \left(\frac{1+\eta}{1+\eta\omega\lambda}\right)^{1/\gamma} + 1}\right\}$$

Because $\left(\frac{1+\eta}{1+\eta\omega\lambda}\right)^{1/\gamma} > 1$ when $\omega\lambda < 1$ for all values of the risk aversion parameter $\gamma > 0$, it is clear that $c_0 < c_1 > c_2$. *The hump is obtained with RDP agent.* This result is important because this profile is not dependent on any assumption about either income, time preference, or the magnitude of interest rate. Only the loss aversion matters with this specific consumption plan to produce a hump. In later sections, I assume the DMs have fixed intrinsic types of consumption behavior and derive conditions for the hump.

4.2.3 A Simple Lifecycle Model

If DM lives for T life-time periods, and if subsequent selves do not change the preference, then the lifecycle maximization problem of RDP agent is

$$u_0(c|c^*) =$$

$$\textstyle\sum_{t=0}^{T} \beta^t \left\{\frac{c_t^{1-\gamma}}{1-\gamma} + \eta I_t\left(\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{c_t^{*1-\gamma}}{1-\gamma}\right) + \eta\omega\lambda(1 - I_t)\left(\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{c_t^{*1-\gamma}}{1-\gamma}\right)\right\}$$

[54] It id easy to notice that when $\beta = 1$ and $R = 1$, the consumption profile of the standard model is $\{c_0, c_1, c_2\}^{Standard} = \left\{\frac{y_0 + y_1 + y_2}{3},\ \frac{y_0 + y_1 + y_2}{3},\ \frac{y_0 + y_1 + y_2}{3}\right\}$.

subject to

$$c_t + b_{t+1} = y_t + Rb_t$$
$$b_0 = 0,\, b_{T+1} = 0$$

For example, if the DM has a plan of $\{\ c_0 > c_0^*,\ c_1 > c_1^*, \ldots.\ c_{T-1} > c_{T-1}^*,\ c_T < c_T^*\ \}$, then this plan is described by $I_t = 1\ \forall t = 0, \ldots T-1$, except for $I_T = 0$. It is also a standard exercise that DM would not deviate from c^* when his loss feeling is grave at any period of life time. Likewise, if $\omega\lambda < 1$, it is always profitable for DM to deviate from the ax ante optimal consumption path c^*. In case of deviation, it must satisfy the following optimality condition for all t :

$$c_t^{-\gamma} + \eta I_t c_t^{-\gamma} + \eta\omega\lambda(1-I_t)c_t^{-\gamma} = R\beta[c_{t+1}^{-\gamma} + \eta I_{t+1} c_{t+1}^{-\gamma} + \eta\omega\lambda(1-I_{t+1})c_{t+1}^{-\gamma}]$$

Combining with the resource constraint gives the consistent consumption profile for each intended plan. However, which one gives the highest welfare over life time is not very deterministic although it depends on the consumer parameters and the interest rate, as well as the composition of the consumption plan itself. If the income is deterministic, the discrete index for the gain-loss utility gives two-phase monotonic lifecycle consumption profile with a kink. The following two figures show this.

In Figure 34, DM has a plan of over-consumption up to retirement, i.e. $I_t = 1$ for all t, up to $t = 65$. It is interesting to see that the over-consumer enjoys just a tiny extra consumption than the standard and as a result he has to accept substantial decrease in later consumption after retirement. The Figure 35 shows the opposite. This figure is from the saver's consistent consumption and the saver enjoys high consumption in retirement years at a small cost of consumption reduction during working years.

As shown in these examples, this model produces a graph with kink because of the discrete index.[55] The kink arises at the point when the DM shifts his plan from over-consumption to under-consumption or vice versa. If income is deterministic, what would be the I_t that is not discrete? As an exercise with this simple model, I propose the following non-discrete index as a function of income level:

[55] In the next section, I introduce a dynamic model where the discrete index gives a smooth and hump-shaped consumption profile.

Figure 34: Two-phase consumption path for an over-consumer: $R = 1.035$, $\beta = 0.98$, $\gamma = 0.5$, $\omega\lambda = 0.8$.

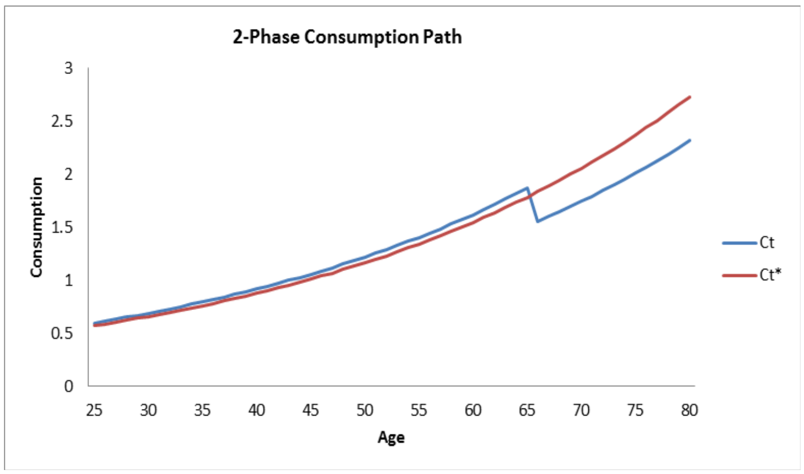


Figure 35: Two-phase consumption path for a saver: R=1.035, $\beta = 0.98$, $\gamma = 0.5$, $\omega\lambda = 0.8$.

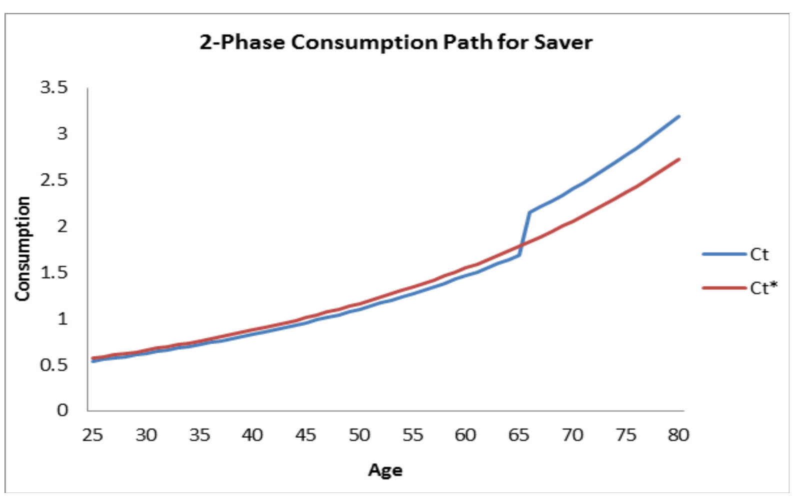
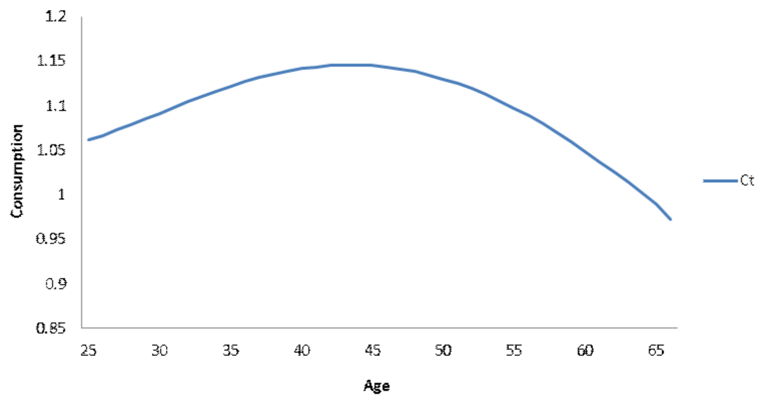


Figure 36: Exemplary lifecycle consumption profile in a simple model ($R = 1.035$, $\beta = 0.95$, $\gamma = 0.5$, $\omega\lambda = 0.8$).



$$I_t \equiv \frac{y_t - y_{\min}}{y_{\max} - y_t}$$

This simple specification gives continuous function between 0 and 1. Thus each index gives the ratio of the consumer's willingness to consume compared to the reference points. At individual levels, this implies the likelihood of the overconsumption increases with higher income. At an aggregate level, this implies that the portion of the population who consumes $c_t > c^*$ is directly related with the relative income position. The next Figure 36 shows the featured consumption hump over life.

4.3 DYNAMIC REFERENCE-DEPENDENT MODEL

4.3.1 Dynamic Decision Makers

As mentioned in three-period model, there are no gain-loss utilities in the second and third period, because the decision maker thinks he would follow his plan in the subsequent periods and this forms a new expectation that determines his reference point on those periods. If the DM of the next period would *not* follow the rule he has intended in the previous period, what would happen to the optimality? To analyze this, let us call the agent of each period in terms of decision time such as DM(0), DM(1), and DM(2).

4.3.1.1 Reference Points in the Dynamic Model The analysis of the first scenario in the three period model (section 2.2.1) is for the DM(0) ($\omega_0\lambda < 1$) who is at period zero and who realizes a great pleasure for consuming more both the first and the second period, but also who knows he will feel sorry for not consuming as much at the third period. Thus DM(0) expects that he would consume on the second period the planned amount he set, which is more than the ex ante optimal ($c_1 > c_1^*$), and he would not have any gain or loss feeling for the period because his expectation of high consumption is met with such consumption. Likewise, DM(0) expects that the third period he would consume the planned amount which is lower than the ex ante optimal ($c_2 < c_2^*$) and because of his already lowered reference point, he expects he would not feel any loss regarding the low consumption.

At $t = 1$, however, the new DM(1), instead of following the whatever planned path for the period, may change his mind and intend to plan a new strategy for the remaining periods. Then the initial plan by DM(0) may not be consistent with the intention of subsequent DMs. So far it is found that if the preference of DM stays the same, deviating in the subsequent periods is not profitable. Because the DM can have a different taste here, it is possible to have different result than before. To construct an optimization problem for DM(1), it must be clarified what the reference point for the DM(1) is? Would the reference point be the consumption utility that the DM(0) sets for the period to fulfill his taste, which is $u(c_1)$? Or would it be the optimal consumption utility solved from the original problem for the period, $u(c_1^*)$?

To see this, assume first that DM(0) has the usual high weight on loss ($\omega_0\lambda \geqslant 1$). Then $c_1 = c_1^*$ and there is no difference between the two reference points regardless of the weights of subsequent DMs. Thus deviating from the planned path is not consistent choice for the DM(1) if he has also a high loss aversion ($\omega_1\lambda \geqslant 1$). But if DM(1) has a low weight ($\omega_1\lambda < 1$) and wants to consume more, [56] then the consistent choice for the DM(1) is a new consumption solved by himself $c_1(1)$ which should be bigger than c_1^*. In summary, with the initial high

[56] Remember that an agent with ($\omega_1\lambda < 1$) can be a saver, too. Through out this analysis, I assume DM(t) is an over-consumer, unless otherwise specified.

weight $\omega_0\lambda \geqslant 1$, the two reference points are the same and the consistent consumption profiles are derived from optimality condition to the intended plan by each subsequent DM. The bracket in the new consumption denotes the choice by DM(1)

Proposition 8. *When DM(0) has a preference (taste) of $\omega_0\lambda \geqslant 1$ and made a plan of $\{c_0^*,\ c_1^*,\ c_2^*\}$, if DM(1) has a preference of $\omega_1\lambda \geqslant 1$, then it does not pay off for him to deviate from DM(0)'s plan.*

Proposition 9. *When DM(0) has a preference (taste) of $\omega_0\lambda \geqslant 1$ and made a plan of $\{c_0^*,\ c_1^*,\ c_2^*\}$, if DM(1) has a preference of $\omega_1\lambda < 1$, then it pays off for him to deviate from DM(0)'s plan and the consistent consumption for DM(1) is* $c_1(1) = \frac{\left(1+\frac{1}{\phi}\right)\frac{R}{\phi}\left(y_0+\frac{y_1}{R}+\frac{y_2}{R^2}\right)}{\left[1+\frac{1}{\phi}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}\right]\left(1+\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2\right)}$

$= \left(\frac{1+\frac{1}{\phi}}{1+\frac{1}{\phi}\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}}\right) c_1^*.$ *Thus* $c_1(1) > c_1^*$.

Assume secondly that the initial DM(0)'s weight for loss is low ($\omega_0\lambda < 1$) and he wants to deviate with a plan of $\{c_0 > c_0^*,\, c_1 > c_1^*,\, c_2 < c_2^*\}$. Then the ex ante optimal consumption for $t = 1$ is no more optimal and the planned consistent consumption by DM(0) for the period is not equal to c_1^*. Thus the two specifications of the reference point are different: $u(c_1) \neq u(c_1^*)$. To analyze further, assume that the consistent consumption choice by DM(0) for $t = 1$ is c_1'. Then, with respect to the loss utility, the second definition of reference point measures the utility loss of the new plan which deviates from the original plan: $u(c_1(1)) - u(c_1^*)$, while by the first one, the loss feeling arises from not fulfilling the adjusted plan recently revised by DM(0): $u(c_1(1)) - u(c_1')$. Although in the next section I define a proper reference point that can be applied to any subsequent period in dynamic model, I want to explore here the implication of dynamic decision making based on the notion of both reference points.[57]

First, let the reference point be defined by the second notion $u(c_1^*)$. Consider an optimization problem of DM(1), at the beginning of $t = 1$, who has cash on hand[58] and who is supposed to consume c_1' which is more than the ex ante optimal initially set for the period. This consumer may or may not want to keep the high consumption for this period. DM(1)

[57] For the reference point, the proposal by Kőszegi & Rabin (2009)[66] vaguely suggests "the most recent past expectation," which is more likely to indicate the latter. But I find the first one is more appealing in this application.

[58] It consists of current labor income and the financial wealth accrued from last period.

may regret his over-consumption of the first period and want to go back to the optimal path by reducing his consumption when he has a reference point of $\{c_0^*,\ c_1^*,\ c_2^*\}$. Thus DM(1) realizes that he would have a prospective loss utility if he ends up with consuming much less than the optimal for the next period as a result of consuming more this period, as well as the first period.[59] The situation for DM(1) who has gain-loss utility by deviating from the original plan c^*, is described by

$$u_1(c_1, c_2 \mid c_1^*, c_2^*) =$$

$$\tfrac{c_1^{1-\gamma}}{1-\gamma} + \eta\left(\tfrac{c_1^{1-\gamma}}{1-\gamma} - \tfrac{c_1^{*1-\gamma}}{1-\gamma}\right) + \beta\left\{\tfrac{c_2^{1-\gamma}}{1-\gamma} + \eta\omega_1\lambda\left(\tfrac{c_2^{1-\gamma}}{1-\gamma} - \tfrac{c_2^{*1-\gamma}}{1-\gamma}\right)\right\}$$

subject to

$$\begin{aligned} c_1 + b_2 &= y_1 + Rb_1 \\ c_2 \quad\; &= y_2 + Rb_2 \end{aligned}$$

Notice that the new utility is indexed by the time of decision making, $t = 1$. Also notice that the cash on hand is defined by $x_1' = y_1 + Rb_1 = y_1 + R(y_0 - c_0')$. Because $c_0' > c_0^*$, it is true that $x_1' < x_1^*$ incorporating the over-consumption at the first period. The preference of DM(1) is represented by $\omega_1\lambda$. When DM(1) consumes more than the ex ante optimal at $t = 1$, he has a contemporaneous gain utility for the period but a prospective loss feeling for the next period. Is it profitable for DM(1) to deviate from the optimal plan? To see this, rewrite $c_2 = [y_2 + Ry_1 + R^2b_1 - Rc_1]$, then the derivative with respect to the consumption by DM(1) is

$$\tfrac{du_1}{dc_1(1)} = (1+\eta)c_1^{-\gamma} - \beta R(1+\eta\omega_1\lambda)[y_2 + Ry_1 + R^2b_1 - Rc_1]^{-\gamma}$$

where

$$b_1 = y_0 - c_0' = \frac{y_0\frac{1}{\phi} + y_0\left(\frac{1}{\phi}\right)^2\left(\frac{1+\eta\omega_0\lambda}{1+\eta}\right)^{1/\gamma} - \frac{y_1}{R^1} - \frac{y_2}{R^2}}{1 + \frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2\left(\frac{1+\eta\omega_0\lambda}{1+\eta}\right)^{1/\gamma}}$$

The bracket in the notation of derivative denotes the choice by DM(1). Remember that the bond demand (or borrowing) is a result from DM(0)'s plan of $\{c_0 > c_0^*, c_1 > c_1^*, c_2 < c_2^*\}$

[59] Notice that in this exercise, DM(0) has $\omega_0\lambda < 1$ and he over consumed at $t = 0$.

from his taste of $\omega_0\lambda < 1$. If the derivative is evaluated at the planned consumption $c_1' = \frac{R}{\phi}c_0'$, then

$$\frac{du_1}{dc_1'} = (1+\eta)\left(\frac{R}{\phi}c_0'\right)^{-\gamma} - \beta R(1+\eta\omega_1\lambda)[y_2 + Ry_1 + R^2 b_1 - R\frac{R}{\phi}c_0']^{-\gamma}$$

$$= \left(\frac{R}{\phi}c_0'\right)^{-\gamma}(1+\eta)\left[1 - \left(\frac{1+\eta\omega_1\lambda}{1+\eta\omega_0\lambda}\right)\right]\left\{\begin{array}{ll} \leqslant 0 & if \;\; \omega_1\lambda \geqslant \omega_0\lambda \\ > 0 & if \;\; \omega_1\lambda < \omega_0\lambda < 1 \end{array}\right\}$$

The first condition says that deviation for any larger amount than the planned c_1' is not profitable if DM(1) cares the future as much as DM(0).[60] The second says that it pays off to consume more than the planned if DM(1) has a lower weight. That is to say, as long as DM(1) has a lower weight than DM(0), deviation from the new plan revised by DM(0), i.e. c_1' for even larger consumption is possible. Because $\omega_0\lambda < 1$, if DM(1) cares a lot about his future ($\omega_1\lambda > 1$), it is always better for him to choose a consumption strategy other than the one set by DM(0). Also, even though DM(1) does not care his future as much ($\omega_1\lambda < 1$), as long as his weight is higher than DM(0), then it is still profitable for him not to follow DM(0)'s plan of overconsumption. Solving the maximization problem yields consistent consumption plan for each case:

$$\begin{array}{ll} c_1'(1) = c_1'(0)\left(\dfrac{1+\frac{1}{\phi}\left(\frac{1+\eta\omega_0\lambda}{1+\eta}\right)^{1/\gamma}}{1+\frac{1}{\phi}\left(\frac{1+\eta\omega_1\lambda}{1+\eta}\right)^{1/\gamma}}\right) < c_1'(0) & \text{if } \omega_0\lambda < 1 \leqslant \omega_1\lambda \\ c_1'(1) = c_1'(0)\left(\dfrac{1+\frac{1}{\phi}\left(\frac{1+\eta\omega_0\lambda}{1+\eta}\right)^{1/\gamma}}{1+\frac{1}{\phi}\left(\frac{1+\eta\omega_1\lambda}{1+\eta}\right)^{1/\gamma}}\right) < c_1'(0) & \text{if } \omega_0\lambda < \omega_1\lambda < 1 \\ c_1'(1) = c_1'(0)\left(\dfrac{1+\frac{1}{\phi}\left(\frac{1+\eta\omega_0\lambda}{1+\eta}\right)^{1/\gamma}}{1+\frac{1}{\phi}\left(\frac{1+\eta\omega_1\lambda}{1+\eta}\right)^{1/\gamma}}\right) > c_1'(0) & \text{if } \omega_1\lambda < \omega_0\lambda < 1 \end{array}$$

The bracket in the consumption notation denotes the choice by DM(t). Now look for the same analysis with an alternative scenario: DM(0) with a taste of $\omega_0\lambda < 1$ has a three-period plan by which he wants to consume more than the predetermined value initially and less for the later two periods, $\{c_0 > c_0^*,\ c_1 < c_1^*,\ c_2 < c_2^*\}$. The derivative evaluated at the consumption point set by DM(0) is

$$\frac{du_1}{dc_1'} = (1-\omega_1\lambda)\eta\left(\frac{R}{\phi}c_0'\right)^{-\gamma}\left(\frac{1+\eta}{1+\eta\omega_0\lambda}\right)\left\{\begin{array}{ll} \leqslant 0 & if \;\; \omega_1\lambda \geqslant 1 \\ > 0 & if \;\; \omega_1\lambda < 1 \end{array}\right\}$$

[60] Although the condition says about just loss aversion, I assume an over-consumer (or a myopic consumer) due to low loss aversion through this analysis. The other case of saver is analyzed later.

Notice that unlike the first scenario, the sign of this derivative does not depend on $\omega_0\lambda$, implying only the attitude of DM(1) matters.

Proposition 10. *When DM(0) has a plan $\{c_0 > c_0^*,\, c_1 < c_1^*,\, c_2 < c_2^*\}$ from his taste of $\omega_0\lambda < 1$, DM(1) should not increase his consumption from the preset value of $c_1' = \frac{R}{\phi}\left(\frac{1+\eta\omega_0\lambda}{1+\eta}\right)^{1/\gamma} c_0'$ if he cares about future ($\omega_1\lambda \geqslant 1$). If however, DM(1) also has a low weight (does not care about the future, or the last period), then it is good for DM(1) to deviate from the plan by consuming more. Thus, $c_1 = c_1'$ if $\omega_1\lambda \geqslant 1$ and $c_1 = \frac{1+\frac{1}{\phi}}{1+\frac{1}{\phi}\left(\frac{1+\eta\omega_1\lambda}{1+\eta}\right)^{1/\gamma}} c_1'$ if $\omega_1\lambda < 1$. It is clear that $c_1 > c_1'$ when $\omega_1\lambda < 1$.*

The proposition implies that when DM(0) is myopic and spends a lot in the first period, not caring about future, DM(1), who wants to correct this, should accept the lower consumption amount set by his precedent self because it is optimal. But this would not apply when DM(1) is also myopic.

Now for the second specification of the reference point: The reference point is the consumption utility from the solution that the DM(0) solves for the period to fulfill his taste of $\omega_0\lambda < 1$. Assume that $\{c'\} \equiv \{c_0', c_1', c_2'\}$ is the solution to the consumption schedule of DM(0) who intends to consume more for the first two periods: $\{c_0 > c_0^*,\, c_1 > c_1^*,\, c_2 < c_2^*\}$. DM(1) may or may not want to keep the high consumption for the second period. Should he regret his over-consumption of last period, can he go back to the *lower* consumption? Because the reference point is now $u(c_1')$, the deviator's solution, DM(1) realizes that he would have a contemporaneous loss utility if he consumes less than this solution, but would have a prospective gain utility for the next period as a result of consuming less this period. Then DM(1) who has gain-loss utility by deviating from the predetermined consumption $\{c'\}$ for an alternative plan $\{c_1 < c_1',\, c_2 > c_2'\}$, maximizes

$$u_1(c_1, c_2 \,|c_1', c_2') =$$

$$\frac{c_1^{1-\gamma}}{1-\gamma} + \eta\omega_1\lambda\left(\frac{c_1^{1-\gamma}}{1-\gamma} - \frac{c_1'^{1-\gamma}}{1-\gamma}\right) + \beta\left\{\frac{c_2^{1-\gamma}}{1-\gamma} + \eta\left(\frac{c_2^{1-\gamma}}{1-\gamma} - \frac{c_2'^{1-\gamma}}{1-\gamma}\right)\right\}$$

subject to

$$\begin{aligned} c_1 + b_2 &= y_1 + Rb_1 \\ c_2 \quad &= y_2 + Rb_2 \end{aligned}$$

This may be called 'reverting saver's optimization problem.' Remember that the consumer at period one inherits the financial wealth, positive or negative, from period zero. To see if it is profitable to deviate from the high consumption that DM(0) set, look for the derivative $w.r.t$ c_1.

$$\frac{du_1}{dc_1} = (1+\eta\omega_1\lambda)c_1^{-\gamma} - \beta R(1+\eta)[y_2 + R(y_1 + Rb_1 - c_1)]^{-\gamma}$$

where

$$b_1 = y_0 - c_0' = \frac{y_0\frac{1}{\phi} + y_0\left(\frac{1}{\phi}\right)^2\left(\frac{1+\eta\omega_0\lambda}{1+\eta}\right)^{1/\gamma} - \frac{y_1}{R^1} - \frac{y_2}{R^2}}{1+\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2\left(\frac{1+\eta\omega_0\lambda}{1+\eta}\right)^{1/\gamma}}$$

Evaluate this at the planned consumption $c_1' = \frac{R}{\phi}c_0'$. Then

$$\frac{du_1}{dc_1'} = \left(\frac{R}{\phi}c_0'\right)^{-\gamma}\left\{(1+\eta\omega_1\lambda) - (1+\eta)\left(\frac{1+\eta}{1+\eta\omega_0\lambda}\right)\right\}$$
$$\left\{\begin{array}{lll} \leqslant 0 & if & \frac{1+\eta\omega_1\lambda}{1+\eta} \leqslant \frac{1+\eta}{1+\eta\omega_0\lambda} \\ > 0 & if & \frac{1+\eta\omega_1\lambda}{1+\eta} > \frac{1+\eta}{1+\eta\omega_0\lambda} \end{array}\right\}$$

The first condition tells that because $\omega_0\lambda < 1$, deviation from c_1' for *smaller* consumption is not profitable whenever DM(1) is myopic, $\omega_1\lambda \leqslant 1$.[61] The condition is also satisfied even if $\omega_1\lambda > 1$ as long as it is not too high. Therefore, deviation for a lower amount is good only if $\omega_1\lambda \gg 1$, which is what the second condition says. This means indirectly[62] that if DM(1) does not have a strong attachment to DM(0)'s plan then he could reduce the consumption for the period. If however, DM(1) cares strongly about the DM(0)'s intention, then he would not deviate from the plan. Only if DM(1) overcomes the strong attachment to DM(0), in vision of remote gain, it is possible for DM(1) to lower his consumption. The new consistent consumption for the saver is

$$c_1''(1) = c_1'(0)\left\{\frac{1+\frac{1}{\phi}\left(\frac{1+\eta\omega_0\lambda}{1+\eta}\right)^{1/\gamma}}{1+\frac{1}{\phi}\left(\frac{1+\eta}{1+\eta\omega_1\lambda}\right)^{1/\gamma}}\right\}$$

Thus

$$c_1''(1) < c_1'(0) \quad \text{if} \quad \omega_1\lambda \gg 1 > \omega_0\lambda$$
$$c_1''(1) \geqslant c_1'(0) \quad \text{if} \quad \omega_1\lambda \leqslant 1$$

[61] There is one technical issue here: to compare the two weights in a consumption plan, the weights should be related to the same reference point. Here this is not satisfied. Because of this, I explain indirectly the loss aversion to the new reference point in terms of the initial reference point.

[62] The implication here follows from DM(1)' intention of reverting the DM(0)'s choice.

It is clear that the new consumption $c_1''(1)$ is smaller than $c_1'(0)$ only if $\omega_1\lambda$ is much greater than one. As an exercise, let us extend these analyses for $T-$period optimization problem where there are $(T+1)$ selves of DM(0), DM(1),..., DM(T).[63] Assume that the initial DM sets a T-period plan and the reference point $\{c_0^*,\ c_1^*, ..., c_T^*\}$ is known to all the subsequent DMs and their consistent consumption plans are revised based on this reference status each time. Assume also that all DMs up to period $\tau - 1$ have high loss aversion, $\omega_0\lambda \geqslant 1, \omega_1\lambda \geqslant 1, ..., \omega_{\tau-1}\lambda \geqslant 1$, and thus $\{c_0^*,\ c_1^*, ..., c_{\tau-1}^*\}$ has been chosen. If at $t = \tau$, DM(τ) has a low loss aversion $\omega_\tau\lambda < 1$, then it pays off for the DM(τ) to choose an alternative consumption path to fulfill his taste and a new plan $\{c_\tau, c_{\tau+1}, .., c_T\}$ starts. The first proposition below tells the equilibrium dynamics when DM(τ) intends more consumption just for the current period and the very next period: $(c_\tau > c_\tau^*, c_{\tau+1} > c_{\tau+1}^*)$.Then I have the following.

Proposition 11. *If there is a deviation by DM(τ), who has a preference of $\omega_\tau\lambda < 1$ and sets an alternative consumption $c_t'(\tau)$ for $t = \tau, \tau+1, ..., T$, then the deviation by the DM($\tau+1$), from $c_{\tau+1}'$ for a larger consumption would not pay off if $\omega_{\tau+1}\lambda \geqslant \omega_\tau\lambda$. His choice is $c_{\tau+1}'$ at most. If however, $\omega_{\tau+1}\lambda < \omega_\tau\lambda < 1$, then it is possible for DM($\tau+1$) to deviate from $c_{\tau+1}'$ for even larger amount, $c_{\tau+1}'' > c_{\tau+1}'$.*

This implies that as long as DM($\tau+1$) has a lower weight than DM(τ), deviation from the plan of DM(τ), i.e. c_1' is profitable. Because $\omega_\tau\lambda < 1$, it is always better for DM($\tau+1$) to choose a consumption strategy which is lower than the intended plan of DM(τ) if DM($\tau+1$) cares more about the future: $(\omega_{\tau+1}\lambda > 1)$. Even though DM($\tau+1$) does not care about his future as much $(\omega_{\tau+1}\lambda < 1)$, as long as his weight is higher than DM(τ), it still pays off for him to abandon DM(τ)'s plan and reduce current consumption. The next proposition describes the alternative dynamics when DM(τ) intends more consumption at the cost of the very next period: $(c_\tau > c_\tau^*,\ c_{\tau+1} < c_{\tau+1}^*)$.

Proposition 12. *If there is a deviation by DM(τ), who has a preference of $\omega_\tau\lambda < 1$ and sets a consumption $c_t'(\tau)$ for $t = \tau, \tau+1, ..., T$, then DM($\tau+1$) would not deviate from $c_{\tau+1}' < c_{\tau+1}^*$ if $\omega_{\tau+1}\lambda \geqslant \omega_\tau\lambda$. If however, $\omega_{\tau+1}\lambda < \omega_\tau\lambda < 1$, then DM($\tau+1$) would deviate from c' for a larger consumption $c_{\tau+1}'' > c_{\tau+1}'$.*

[63] A formal T-period model with dynamic reference points is presented in section 4.

The second proposition says: when DM(τ) over consumes for the period from his taste ($\omega_\tau \lambda < 1$), and lower subsequent consumption as a result, then DM($\tau + 1$) should not increase his consumption from the predetermined value of $c'_{\tau+1}$ if he cares about future more than previous DM ($\omega_{\tau+1}\lambda \geqslant \omega_\tau \lambda$). If however, DM($\tau + 1$) has also a lower weight than DM(τ), then it is good for DM($\tau + 1$) to deviate from the plan by consuming more. Thus, $c_{\tau+1} = c'_{\tau+1}$ if $\omega_{\tau+1}\lambda \geqslant \omega_\tau \lambda$ and $c_{\tau+1} = c''_{\tau+1} > c'_{\tau+1}$ if $\omega_{\tau+1}\lambda < \omega_\tau \lambda < 1$. The proposition implies that when DM(τ) with $\omega_\tau \lambda < 1$ consumes more than the optimal for the current period, not caring about future, DM($\tau + 1$), who wants to correct this, should accept the low consumption amount set by his precedent self because it was optimal. But this would not apply when DM($\tau + 1$) has lower loss aversion than the precedent self.

4.3.1.2 Saver's Dynamic Optimization So far the analysis focuses on the optimization rule of an initial over-consumer ($\omega_0 \lambda < 1$) who intends to deviate for more consumption than the ex ante optimal. In this section, I examine the optimization procedure of an initial saver (miser: $\omega_0 \lambda < 1$), who plans to consume less for the first two periods, $\{c_0 < c_0^*, c_1 < c_1^*, c_2 > c_2^*\}$. It is easy to find the ex ante optimal solution to the initial saver's problem. That is

$$\{c_0,\, c_1,\, c_2\} = \left\{c_0,\ (R\beta)^{1/\gamma} c_0,\ (R\beta)^{2/\gamma} \left(\frac{1+\eta}{1+\eta\omega_0\lambda}\right)^{1/\gamma} c_0\right\}$$

$$c_0 = \frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1 + \frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2 \left(\frac{1+\eta}{1+\eta\omega_0\lambda}\right)^{1/\gamma}}$$

Contrary to DM(0)'s intention, DM(1) may not want to keep the low consumption for $t = 1$. Then can he deviate from the plan of DM(0) and replan to increase consumption? Following the above analysis, let his reference point be the DM(0)'s solution. Because the reference point is now $u(c'_1)$, the saver's solution, DM(1) realizes that he would have a contemporaneous gain utility if he consumes more than this, but would have a prospective loss next period as a result of consuming more this period. Then the utility status for DM(1) when he has gain-loss utility by deviating for $\{c_1 > c'_1, c_2 < c'_2\}$, is described by

$$u_1(c_1, c_2 \mid c'_1, c'_2) =$$

$$\frac{c_1^{1-\gamma}}{1-\gamma} + \eta\left(\frac{c_1^{1-\gamma}}{1-\gamma} - \frac{c_1'^{1-\gamma}}{1-\gamma}\right) + \beta\left\{\frac{c_2^{1-\gamma}}{1-\gamma} + \eta\omega_1\lambda\left(\frac{c_2^{1-\gamma}}{1-\gamma} - \frac{c_2'^{1-\gamma}}{1-\gamma}\right)\right\}$$

This may be called 'reverting plunger's optimization problem.' To see if it is profitable to deviate from the alternative plan of DM(0),

$$\frac{du_1}{dc_1'} = \left(\frac{R}{\phi}c_0'\right)^{-\gamma}\left\{(1+\eta) - (1+\eta\omega_1\lambda)\left(\frac{1+\eta\omega_0\lambda}{1+\eta}\right)\right\}$$
$$\left\{\begin{array}{lll} \leqslant 0 & if & \frac{1+\eta}{1+\eta\omega_1\lambda} \leqslant \frac{1+\eta\omega_0\lambda}{1+\eta} \\ > 0 & if & \frac{1+\eta}{1+\eta\omega_1\lambda} > \frac{1+\eta\omega_0\lambda}{1+\eta} \end{array}\right\}$$

By the second condition, it is clear that the deviation for a larger consumption amount is always profitable if he does not care much about the next period ($\omega_1\lambda < 1$). The condition follows from $\frac{1+\eta}{1+\eta\omega_1\lambda} > 1 > \frac{1+\eta\omega_0\lambda}{1+\eta}$ because $\omega_0\lambda < 1$. This implies that if DM(1)'s attachment to the plan of low consumption is not too strong he can deviate and replan. Now consider the second story: If DM(0) is a saver ($\omega_0\lambda < 1$) and his plan is to save only in the first period, $\{c_0 < c_0^*,\, c_1 > c_1^*,\, c_2 > c_2^*\}$, then the ex ante optimal strategy is

$$\{c_0,\, c_1,\, c_2\} = \left\{c_0,\ (R\beta)^{1/\gamma}\left(\frac{1+\eta}{1+\eta\omega_0\lambda}\right)^{1/\gamma} c_0,\ (R\beta)^{2/\gamma}\left(\frac{1+\eta}{1+\eta\omega_0\lambda}\right)^{1/\gamma} c_0\right\}$$

where

$$c_0 = \frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1 + \frac{1}{\phi}\left(\frac{1+\eta}{1+\eta\omega_0\lambda}\right)^{1/\gamma} + \left(\frac{1}{\phi}\right)^2\left(\frac{1+\eta}{1+\eta\omega_0\lambda}\right)^{1/\gamma}}$$

Instead of enjoying high consumption, if DM(1) desires to save again for $t = 1$, then DM(1)'s utility position for $\{c_1 < c_1',\, c_2 > c_2'\}$ is described by

$$u_1(c_1, c_2 \mid c_1', c_2') =$$

$$\frac{c_1^{1-\gamma}}{1-\gamma} + \eta\omega_1\lambda\left(\frac{c_1^{1-\gamma}}{1-\gamma} - \frac{c_1'^{1-\gamma}}{1-\gamma}\right) + \beta\left\{\frac{c_2^{1-\gamma}}{1-\gamma} + \eta\left(\frac{c_2^{1-\gamma}}{1-\gamma} - \frac{c_2'^{1-\gamma}}{1-\gamma}\right)\right\}$$

The deviation condition is

$$\frac{du_1}{dc_1'} = \left(\frac{R}{\phi}c_0'\left(\frac{1+\eta}{1+\eta\omega_0\lambda}\right)^{1/\gamma}\right)^{-\gamma}\{\eta(\omega_1\lambda - 1)\}$$
$$= -\eta(1-\omega_1\lambda)\left(c_1'\right)^{-\gamma}\left\{\begin{array}{lll} \leqslant 0 & if & \omega_1\lambda \leqslant 1 \\ > 0 & if & \omega_1\lambda > 1 \end{array}\right\}$$

The deviation for *less* consumption is profitable if DM(1) cares more about the future consumption than about current consumption ($\omega_1\lambda > 1$). This implies that if DM(1) has low attachment to the previous plan of high consumption, then he can reduce the consumption again to save more. However, if DM(1) cares strongly about the DM(0)'s intention of high consumption, then he would not deviate from the plan.

4.3.2 The Sub-Period Perfect Reference Point

The main result so far can be summarized as follows: if DM does not change his mind over time and thus the subsequent reference points stay on the same track as the initial conjecture, then deviation for more or less consumption is not profitable at any time when DM has high loss aversion ($\omega\lambda \geqslant 1$). In two-period model this can be explained by simple mechanism that whenever DM tries to consume more today than the optimal, he ends up with giving up this intention because of the high pain from loss related with the low consumption next period.[64] But when $\omega\lambda < 1$, deviation for more consumption is justified because DM cares more about the current pleasure of extra consumption than the concern about less consumption in the next period. The deviation is also possible in the direction of less consumption (save more) now for more consumption next period. In fact, these are the cases where deviation from the standard norm is a better choice: whenever DM tries to consume more (or less), it turns out that this deviation is the the best strategy for the DM in that it gives him highest utility. Thus a deviator is deviating for good. He could be an over-consumer or an under-consumer if his loss aversion is low. But whatever he intends, he should be consistent with his intention.[65]

In dynamic model, where the subsequent reference points are changing over time because DM changes his mind, there is no universal way to describe DM's behavior as is analyzed in section 3.1. Because there can be many reference points to consider at any time of decision making, the consistent consumption profiles are many based on each of the alternative reference points. Here I introduce a novel way to solve this problem and suggest a reference point that can be applied to any period of dynamic model. Assume that there is no uncertainty and the deterministic income stream is known.

Definition *Given any income stream, a Sub-Period Perfect Plan at time t is the optimal solution* $\{c_t^*,\ c_{t+1}^*, ..., c_T^*\}$ *to the maximization problem of DM whose preference follows the standard model* ($\textit{or RDP with } \omega\lambda \geqslant 1$) *and whose preference is not changing over the*

[64] DM is an over-consumer here.

[65] The resulting behavior should be consistent with his plan.

planning horizon of $T-t+1$ periods:

$$c^*(t) \equiv \{c_t^*,\, c_{t+1}^*, ..., c_T^*\} = \arg\max \textstyle\sum_{\tau=t}^{T} \beta^\tau u(c_\tau)$$

subject to

$$c_\tau + b_{\tau+1} = y_\tau + Rb_\tau$$
$$\tau = t,\, t+1, ..., T$$
$$b_t = given$$
$$b_0 = 0$$

At any time t, given any debt or saving inherited from last period, the sub-period perfect plan gives DM a clear notion of what the standard consumption looks like for the remaining period.[66] I postulate this plan serves as the reference point (sub-period perfect reference point: SPRP) to which DM should refer.[67] The merit of this definition is that the *current* asset/liability of DM directly affects the optimal consumption stream for the remaining lifetime so that the plan itself is *self-corrective* over the subsequent periods.[68] SPRP should be a proper reference point in the dynamic model in which DM may change his degree of loss aversion and revise the strategy following his new taste. Whenever DM changes his mind, the reference point is updated and the updating reflects his current financial situation.

Notice that if there is no deviation at time t-1, then the new reference point for the next period is exactly the same as DM(t-1) sets for the period because DM(t) does not have any extra debt or saving than the planned from last period. Likewise, if there is no deviation at all up to time t-1, then the reference point for the next period is exactly the same as the initial DM sets for the period. Because of this, the initial sub-period perfect plan, $c^*(0) = \{c_0^*,\, c_1^*, ..., c_T^*\}$ may be called the super plan. Any plan other than the $c^*(t)$ is called an alternative plan $c'(t) = \{c'_t,\, c'_{t+1}, ..., c'_T\}$.

Proposition 13. *If DM*(t) *has a taste of* $\omega_t\lambda \geqslant 1$, *then there is no deviation from the optimal plan at* t. *If DM*(t) *has* $\omega_t\lambda < 1$, *then it pays off for the DM*(t) *to choose an alternative*

[66] The plan is a sort of financial blueprint that DM with any wealth (positive or negative), may get from a financial avdisor regarding his lifecycle consumption and saving project.

[67] When DM is not deviating, this corresponds to the sub-game perfect equilibrium.

[68] For example, if there was over-consumption by the previous DM, leaving great debt, then the new plan by subsequent DM automatically incorporates this debt. By SPRP, the new reference point has been adjusted accordingly to this debt.

consumption path other than the sub-period perfect plan and $c'(t) \neq c^*(t)$*. If DM*(t) *is an over-consumer, then he consumes more at* t, $c'_t > c^*_t$.

Proposition 14. *Assume that all DMs up to period* τ *have high loss aversion,* $\omega_0\lambda \geqslant 1, \omega_1\lambda \geqslant 1, ..., \omega_{\tau-1}\lambda \geqslant 1$*, and thus* $\{c^*_0,\ c^*_1, ..., c^*_{\tau-1}\}$ *has been chosen based on the reference point of the sub-period perfect plan up to* τ*-1. If at* τ*, DM*(τ) *has low loss aversion* $\omega_\tau\lambda < 1$*, then it pays off for the DM*(τ) *to choose an alternative consumption path to fulfill his taste and a new plan starts, which is* $c'(\tau) = \arg\max \sum_{t=\tau}^{T} \beta^t\{u(c_t) + \eta I_t[u(c_t) - u(c^*_t)] + \eta\omega_\tau\lambda(1 - I_t)[u(c_t) - u(c^*_t)]\}$*,*[69] *where the reference point is the sub-period perfect plan at time* τ*.*

4.4 A GENERAL EQUILIBRIUM WITH DYNAMIC MODEL

4.4.1 A Dynamic Lifecycle Model

In this section, I study a dynamic model in which DM is allowed to change his mind over lifetime. Each time DM decides either to follow the consumption rule determined by earlier selves or to start new plan following his current taste. Once DM decides to change his mind, he re-optimizes for the remaining periods. The reference point where DM's new belief is formed, is now the optimal consumption to the maximization problem over the remaining lifetime, the "Sub-Period Perfect Reference Point," which reflects that DM has inherited a financial wealth from previous consumption. Because DM rebalances his consistent consumption as he re-optimizes following his new belief on the ex ante optimal plan for the remaining periods, the actual consumption profile is the envelope of the initial consumption set by each DM.

4.4.1.1 A Simple Dynamic Model First let's solve a simple model where the DM who lives for four periods,[70] equipped with a dynamic loss aversion parameter ω_t and optimizes for $t = 0, 1, 2, 3$. Because DM is allowed to have different weight each time, it may be useful to denote each agent with time index, DM(t). If, any time t, DM(t) changes his mind, he

[69] Notice that ω_τ is fixed and does not change over time *within* the planning horizon of DM(τ).
[70] To show the consumption hump, it is enough to have three period model.

has to replan for the remaining lifetime following his new taste. And the reference point is the SPRP defined in section 3.2. Assume that the agent may want to deviate from the optimal path for more consumption for the half of the lifetime, lowering his consumption in the subsequent half of the lifetime.[71] At $t = 0$, the agent maximizes his lifetime utility for $t = \{0, 1, 2, 3\}$:

$$u_0(c_0, c_1, c_2, c_3|c^*(0)) =$$

$$\sum_{t=0}^{3} \beta^t \left[\frac{c_t^{1-\gamma}}{1-\gamma} + \eta I_t \left(\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{c_t^{*1-\gamma}}{1-\gamma} \right) + \omega\eta\lambda(1 - I_t) \left(\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{c_t^{*1-\gamma}}{1-\gamma} \right) \right]$$

subject to

$$c_t + b_{t+1} = y_t + Rb_t$$
$$b_0 = 0, \;\; b_4 = 0$$
$$I_t = \{1, 1, 0, 0\}$$

By the analysis in the earlier sections, it is clear that deviation from optimal path $(c_0^*, c_1^*, c_2^*, c_3^*)$ is not profitable if $\lambda\omega_0 \geqslant 1$. But if $\lambda\omega_0 < 1$, then deviation is desirable and his consistent consumption is

$$\{c_0, c_1, c_2, c_3\} = \{c_0, \frac{R}{\phi} c_0, \left(\frac{R}{\phi}\right)^2 \mu_0 c_0, \left(\frac{R}{\phi}\right)^3 \mu_0 c_0\}$$
$$c_0 = \frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2} + \frac{y_3}{R^3}}{1 + \frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2 \mu_0 + \left(\frac{1}{\phi}\right)^3 \mu_0}$$
$$\mu_0 = \left(\frac{1+\eta\omega_0\lambda}{1+\eta}\right)^{1/\gamma}$$

Notice that $\mu_0 < 1$ for the deviators, corresponding to $\lambda\omega_0 < 1$ and $\mu_0 = 1$ for non-deviators. In fact, μ_t is the parameter that captures the main implication of gain-loss utility and this may be called the "relative loss aversion parameter." For DM with $\mu_0 < 1$, it is also obvious that $c_0 > c_0^*$, $c_1 > c_1^*$ but $c_2 < c_2^*$, $c_3 < c_3^*$. Thus he consumes c_0 at the first period and *intends* to consume c_1 in the next period. At $t = 1$, if DM(1) changes his mind and decides to deviate from this plan, then he must re-optimize for the remaining periods following his current taste ω_1 as well as the financial debt from last period.[72] The

[71] This premise comes from the result (utility comparison) in section 2.2.

[72] This is an extra debt beyond the scheduled bond holding. This debt arises from the over-consumption of the first period.

reference point of DM(1) is the optimal solution to the standard maximization problem of the remaining three periods. So at $t = 1$, he re-optimizes his consumption plan following the new reference status for the remaining periods $t = \{1, 2, 3\}$:

$$u_1(c_1, c_2, c_3|c^*(1)) =$$

$$\sum_{t=1}^{3} \beta^t \left[\frac{c_t^{1-\gamma}}{1-\gamma} + \eta I_t \left(\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{c_t^{*1-\gamma}}{1-\gamma}\right) + \eta\omega_1\lambda(1 - I_t)\left(\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{c_t^{*1-\gamma}}{1-\gamma}\right)\right]$$

subject to

$$c_t + b_{t+1} = y_t + Rb_t$$
$$b_1 = y_0 - c_0$$
$$I_t = \{1, 0, 0\}.$$ [73]

Notice that b_1 is given at time $t = 1$ by the consumption behavior at $t = 0$. The solution for a deviator is,

$$\{c_1, c_2, c_3\} = \{c_1, \left(\frac{R}{\phi}\right)\mu_1 c_1, \left(\frac{R}{\phi}\right)^2 \mu_1 c_1\}$$
$$c_1 = \frac{y_1 + \frac{y_2}{R^1} + \frac{y_3}{R^2} + Rb_1}{1 + \frac{1}{\phi}\mu_1 + \left(\frac{1}{\phi}\right)^2 \mu_1}$$
$$\mu_1 = \left(\frac{1+\eta\omega_1\lambda}{1+\eta}\right)^{1/\gamma}.$$

This solution is the consistent consumption profile for $t = \{1,\ 2,\ 3\}$, starting from $t = 1$. Notice that the solution c_1 to the maximization problem of DM(1) is different from what DM(0) calculated last period. To emphasis the difference, it is convenient to call this c_1^1 by denoting the new consistent consumption plan starting from $t = 1$. Likewise, c_2^1 is the consistent consumption for the next period, i.e. $t = 2$, planned at $t = 1$.

Similarly, at $t = 2$ DM(2) replans for the remaining two periods, following his new taste and when $\lambda\omega_2 < 1$, the consistent consumption plan is

$$\{c_2^2, c_3^2\} = \{c_2, \frac{R}{\phi}\mu_2 c_2\}$$
$$c_2 = \frac{y_2 + \frac{y_3}{R^1} + Rb_2}{1 + \frac{1}{\phi}\mu_2}$$

[73] In case of odd number of periods, $I = 1$ for the highest integer less than $(T - t)/2$. This specification follows from the previous result in utility comparison.

$$\mu_2 = \left(\frac{1+\eta\omega_2\lambda}{1+\eta}\right)^{1/\gamma}.$$

All together, following each DM's taste of $\omega_t\lambda$, the consistent consumption profile for the entire four periods is[74]

$$\{c_0^0, c_1^1, c_2^2, c_3^2\} =$$

$$\left\{\frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2} + \frac{y_3}{R^3}}{1 + \frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2 \mu_0 + \left(\frac{1}{\phi}\right)^3 \mu_0},\ \frac{y_1 + \frac{y_2}{R^1} + \frac{y_3}{R^2} + Rb_1}{1 + \frac{1}{\phi}\mu_1 + \left(\frac{1}{\phi}\right)^2 \mu_1},\ \frac{y_2 + \frac{y_3}{R^1} + Rb_2}{1 + \frac{1}{\phi}\mu_2},\ \frac{R}{\phi}\mu_2 c_2^2\right\},$$

$$\mu_t = \left(\frac{1+\eta\omega_t\lambda}{1+\eta}\right)^{1/\gamma}.$$

4.4.1.2 The Consumption Hump In this section, I show that if the taste of DM represented by $\omega_t\lambda$ is changing over time, then it is possible to obtain the featured consumption hump even with a flat (or monotonic) income profile. This is important because the property of hump-shaped consumption profile is not expected at all in standard theory unless there exists extra assumptions like friction or uncertainty.[75] The consumption hump can be obtained with model of time-inconsistent preferences, but even with this, the model usually requires certain properties of income.[76]

To derive analytically the condition for a Hump, I want to define the consumption hump as follows:[77]

Definition *A consumption hump for T-period model is a consumption profile* $\{c_t\}_{t=0}^T$ *that satisfies*

i) There is a consumption peak at time $t \in (0, T)$*.*

ii) Consumption is monotonically increasing up to t*.*

[74] The fourth consumption of the bracket is the residual from the plan of the third period. DM(4) does not have any his own choice.

[75] In the literature review, I include other mechanisms that can induce a hump profile under the standard preference.

[76] For example, a hump-shaped income profile can generate a consumption hump in certain models.

[77] This is a strong sense of the definition. In a weak sense, it may allow having any wiggle over the horizon with many local peaks.

iii) Consumption is monotonically decreasing beyond t.

For example, if there are four periods $t = 0, 1, 2, 3$, the hump condition requires either $\{c_0 < c_1 < c_2 > c_3\}$ or $\{c_0 < c_1 > c_2 > c_3\}$ depending on whether the peak occurs at c_1 or c_2. Can the hump be obtained with the four-period model above? In the previous section, the consistent consumption profile for the entire four periods is

$$\left\{ \frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2} + \frac{y_3}{R^3}}{1 + \frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2 \mu_0 + \left(\frac{1}{\phi}\right)^3 \mu_0},\ \frac{y_1 + \frac{y_2}{R^1} + \frac{y_3}{R^2} + Rb_1}{1 + \frac{1}{\phi}\mu_1 + \left(\frac{1}{\phi}\right)^2 \mu_1},\ \frac{y_2 + \frac{y_3}{R^1} + Rb_2}{1 + \frac{1}{\phi}\mu_2},\ \frac{R}{\phi}\mu_2 c_2^2 \right\},$$

$$\mu_t = \left(\tfrac{1+\eta\omega_t\lambda}{1+\eta}\right)^{1/\gamma}.$$

Solving recursively for bond demand in each period and substituting into the above consumption equations pins down the entire consumption profile. For further analysis, let us define a "human wealth" at time t by $h_t = \sum_{\tau=t}^{3} \frac{y_\tau}{R^{\tau-t}}$. Then the bond demand is

$$b_1 = \frac{y_0\left(\frac{1}{\phi} + \frac{\mu_0}{\phi^2} + \frac{\mu_0}{\phi^3}\right) - \frac{h_1}{R}}{1 + \frac{1}{\phi} + \frac{\mu_0}{\phi^2} + \frac{\mu_0}{\phi^3}}$$

$$b_2 = \frac{\left([y_1 + Ry_0]\left(\frac{\mu_1}{\phi} + \frac{\mu_1}{\phi^2}\right) - \frac{h_2}{R}\right)\left(\frac{1}{\phi} + \frac{\mu_0}{\phi^2} + \frac{\mu_0}{\phi^3}\right) - \frac{h_2}{R}\left(1 + \frac{\mu_1}{\phi} + \frac{\mu_1}{\phi^2}\right)}{\left(1 + \frac{\mu_1}{\phi} + \frac{\mu_1}{\phi^2}\right)\left(1 + \frac{1}{\phi} + \frac{\mu_0}{\phi^2} + \frac{\mu_0}{\phi^3}\right)}$$

Thus the resulting consistent consumption $\{c_0, c_1, c_2, c_3\} \equiv \{c_0^0, c_1^1, c_2^2, c_3^2\}$ is

$$c_0 = \frac{h_0}{1 + \frac{1}{\phi} + \frac{\mu_0}{\phi^2} + \frac{\mu_0}{\phi^3}}$$

$$c_1 = \frac{(h_1 + Ry_0)\left(\frac{1}{\phi} + \frac{\mu_0}{\phi^2} + \frac{\mu_0}{\phi^3}\right)}{\left(1 + \frac{\mu_1}{\phi} + \frac{\mu_1}{\phi^2}\right)\left(1 + \frac{1}{\phi} + \frac{\mu_0}{\phi^2} + \frac{\mu_0}{\phi^3}\right)}$$

$$c_2 = \frac{(h_2 + Ry_1 + R^2 y_0)\left(1 + \frac{\mu_0}{\phi} + \frac{\mu_0}{\phi^2}\right)\left(\frac{\mu_1}{\phi^2} + \frac{\mu_1}{\phi^3}\right)}{\left(1 + \frac{\mu_2}{\phi}\right)\left(1 + \frac{\mu_1}{\phi} + \frac{\mu_1}{\phi^2}\right)\left(1 + \frac{1}{\phi} + \frac{\mu_0}{\phi^2} + \frac{\mu_0}{\phi^3}\right)}$$

$$c_3 = \frac{R}{\phi}\mu_2 c_2$$

Notice that when DM is not deviating, the consumption profile is obtained by letting $\mu_t = 1$. To characterize the consumption property of this simple version of the dynamic reference dependent preference (RDP) model, I want to compare this with the standard lifecycle consumption profile of DM who does not have reference dependent utility. The optimization for standard model predicts that the marginal utility of consumption between any two-periods conforms to the Euler rule and thus the consumption profile over all periods exhibits monotonic movement. By monotonicity the consumption profile is either increasing, decreasing, or stay constant over the entire lifetime. The consumption profile of standard agent is characterized by

$$\{c_0,\, c_1,\, c_2,\, c_3\}^{Standard} = \{c_0, c_0, c_0, c_0\}, \quad \text{if } \; \beta R = 1$$

and

$$\begin{gathered}\{c_0,\, c_1,\, c_2,\, c_3\}^{Standard} = \{c_0, \left(\tfrac{R}{\phi}\right)^1 c_0, \left(\tfrac{R}{\phi}\right)^2 c_0, \left(\tfrac{R}{\phi}\right)^3 c_0\}, \quad \text{if } \; \beta R \neq 1 \\ c_0 = \frac{h_0}{1 + \frac{1}{\phi^1} + \frac{1}{\phi^2} + \frac{1}{\phi^3}} \\ h_0 = y_0 + \tfrac{y_1}{R^1} + \tfrac{y_2}{R^2} + \tfrac{y_3}{R^3}.\end{gathered}$$

The monotonicity of these consumption profiles is preserved across any choice of income processes. By comparing the consumption of the RDP agent with this monotone consumption profile, it may be seen that for the RDP agent, this monotonicity holds for a special case of $\mu_t = 1$. In fact this is the situation where DM does not deviate from the optimal path because of the high loss aversion. Therefore, by letting $\mu_t = 1$ for all $t = 0, 1, 2, 3$, the RDP consumption equations return to

$$\begin{gathered}c_0 = \frac{h_0}{1 + \frac{1}{\phi} + \frac{1}{\phi^2} + \frac{1}{\phi^3}} \\ c_1 = \frac{(h_1 + Ry_0)\,\frac{1}{\phi}}{\left(1 + \frac{1}{\phi} + \frac{1}{\phi^2} + \frac{1}{\phi^3}\right)} = \left(\tfrac{R}{\phi}\right) c_0 \\ c_2 = \frac{(h_2 + Ry_1 + R^2 y_0)\,\frac{1}{\phi}}{\left(1 + \frac{1}{\phi} + \frac{1}{\phi^2} + \frac{1}{\phi^3}\right)} = \left(\tfrac{R}{\phi}\right) c_1 = \left(\tfrac{R}{\phi}\right)^2 c_0 \\ c_3 = \left(\frac{R}{\phi}\right) c_2 = \left(\tfrac{R}{\phi}\right)^3 c_0\end{gathered}$$

This comes from the fact that for any t,

$$h_t = y_t + \frac{h_{t+1}}{R}.$$

Now turn to the main question of whether the consistent consumption profile of the RDP agent can generate a consumption hump. To explore this assume about the gross interest rate that $1+r=R>0$[78] and that the risk aversion parameter $\gamma>0$.[79] For the four-period model, it is shown above that a consumption hump is obtained if the consumption stream follows $\{c_0<c_1<c_2>c_3\}$ or $\{c_0<c_1>c_2>c_3\}$. In either case it is enough to show that $c_0<c_1$ and $c_2>c_3$.

Proposition 15. *Given R and β, if the preference of RDP dynamic agent, $\{\mu_0,\mu_1,\mu_2\}$,*[80] *satisfies that $\left(1+\frac{1}{\phi}\mu_1+\left(\frac{1}{\phi}\right)^2\mu_1\right)<\frac{R}{\phi}\left(1+\frac{1}{\phi}\mu_0+\left(\frac{1}{\phi}\right)^2\mu_0\right)$, then the consistent consumption is increasing initially, i.e. $c_0<c_1$, regardless of the income stream of the agent. The upper bound of μ_1 for a hump is $\mu_0\left(\frac{R}{\phi}\right)+\frac{R-\phi}{1+\phi^{-1}}$.*

It is easily noticed that if DM is not deviating, $\mu_0=\mu_1=\mu_2=1$, the condition returns to the simple condition of $1<\frac{R}{\phi}$, which implies that $\beta R>1$. This is the characterization for any increasing consumption in standard model. Therefore, it can be inferred that when DM deviates, increasing property of consumption may be obtained without the condition that $\beta R>1$. To see clearly the meaning of the condition, first look at the condition with $\beta=1/R$:

$$1+\tfrac{1}{R}\mu_1+\left(\tfrac{1}{R}\right)^2\mu_1<1+\tfrac{1}{R}\mu_0+\left(\tfrac{1}{R}\right)^2\mu_0$$

This condition implies that $\mu_1<\mu_0$. Whenever the loss aversion of DM(1) is less than the one of DM(0),[81] the consumption is increasing. When $\beta R>1$, the restriction on loss aversion parameter can be relaxed due to the time preference. Only if $\beta R<1$, it requires $\mu_1<<\mu_0$ to insure the above property and the upper bound is determined by $\mu_0\left(\frac{R}{\phi}\right)+\frac{R-\phi}{1+\phi^{-1}}$. The next proposition describes the decreasing property of the consumption hump.

Proposition 16. *Given R and β, if the preference of RDP dynamic agent, $\{\mu_0,\mu_1,\mu_2\}$, satisfies that $\left(\dfrac{R}{\phi}\right)\mu_2<1$, then the consistent consumption is decreasing later periods, $c_2>c_3$, regardless of income stream of DM.*[82]

[78] r is the usual net interest rate. This notation should not be confused with the notation for reference (r) introduced in the begining of this paper.

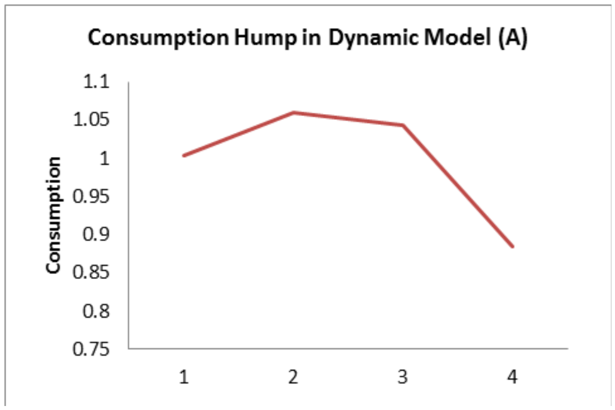
[79] These assumptions are just for the computational purpose. In fact, these assumptions are not restrictive at all and easily satisfied.

[80] μ_t represents the preference of DM: given other parameter values, μ_t is directly (one to one) related to $\omega_t\lambda$.

[81] The loss aversion parameter is ω_t and μ_t is positively related to ω_t.

[82] The preference of DM(3) does not affect this condition because his consumption is residual.

Figure 37: The consumption is obtained with flat income stream, for a dynamic over-consumer. The gain-loss parameters are $\eta = 1$, $\lambda = 2$, $\omega_t = \{0.45, 0.4, 0.35\}$. The other parameters are $R = 1.035$, $\beta = 0.98$ and $\gamma = 0.9$.



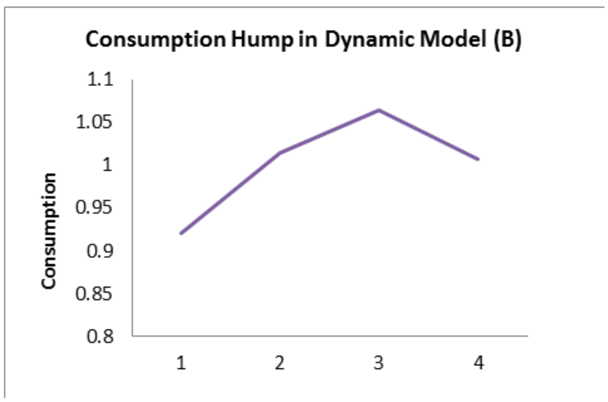
Again with $\beta = 1/R$, the condition says $\mu_2 < 1$. If $\beta \neq 1/R$, then the time preference and the loss aversion jointly operate to yield the decreasing property. This result contrasts to the standard model: when DM is not deviating,[83] the time preference is the sole determinant of the inequality, $\dfrac{R}{\phi} < 1$. However, with RDP model, it is no more a necessary condition. Even though $\beta R > 1$, it is possible to get the result if μ_2 is sufficiently small. Finally combining both conditions yields the consumption hump.

Proposition 17. *Given R and β, if the preference of RDP dynamic agent, $\{\mu_0, \mu_1, \mu_2\}$, satisfies the above two conditions, then the consumption profile of the agent produces a hump.*

Figure 37 shows the consumption hump for a dynamic decision maker who deviates from the optimal plan to over consume. So far I have addressed the condition for a hump with the four-period model of an initial over-consumer. For a three-period model, the hump condition is simpler than these propositions. For an agent with $\{\mu_0, \mu_1\}$, the increasing condition for three periods is $1 + \frac{1}{\phi}\mu_1 < \frac{R}{\phi}\mu_0 \left(1 + \frac{1}{\phi}\right)$ and the decreasing condition is $\left(\frac{R}{\phi}\right)\mu_1 < 1$. The implication of these conditions is straightforward. How about the *saver's hump*? In a four-period simple model, it is obvious that the consumptions are

[83] Remember that when DM is not deviating, the result is the same as the one in the standard model.

Figure 38: The consumption is obtained with flat income stream, for a dynamic saver. The gain-loss parameters are $\eta = 1$, $\lambda = 2$, $\omega_t = \{0.35, 0.45, 0.5\}$. The other parameters are $R = 1.035$, $\beta = 0.94$ and $\gamma = 0.5$.



$$c_0 = \frac{h_0}{1 + \frac{1}{\phi} + \frac{\nu_0}{\phi^2} + \frac{\nu_0}{\phi^3}}$$

$$c_1 = \frac{(h_1 + Ry_0)\left(\frac{1}{\phi} + \frac{\mu_0}{\phi^2} + \frac{\mu_0}{\phi^3}\right)}{\left(1 + \frac{\nu_1}{\phi} + \frac{\nu_1}{\phi^2}\right)\left(1 + \frac{1}{\phi} + \frac{\nu_0}{\phi^2} + \frac{\nu_0}{\phi^3}\right)}$$

$$c_2 = \frac{(h_2 + Ry_1 + R^2 y_0)\left(1 + \frac{\mu_0}{\phi} + \frac{\mu_0}{\phi^2}\right)\left(\frac{\mu_1}{\phi^2} + \frac{\mu_1}{\phi^3}\right)}{\left(1 + \frac{\nu_2}{\phi}\right)\left(1 + \frac{\nu_1}{\phi} + \frac{\nu_1}{\phi^2}\right)\left(1 + \frac{1}{\phi} + \frac{\nu_0}{\phi^2} + \frac{\nu_0}{\phi^3}\right)}$$

$$c_3 = \frac{R}{\phi}\nu_2 c_2$$

where the relative loss aversion parameter is

$$\nu_s = \left(\frac{1+\eta}{1+\eta\omega_s\lambda}\right)^{1/\gamma} \geqslant 1.$$

Therefore, with $\{\nu_0, \nu_1, \nu_2\}$, the condition for increasing part for a hump is

$$\left(1 + \frac{1}{\phi}\nu_1 + \left(\frac{1}{\phi}\right)^2 \nu_1\right) < \frac{R}{\phi}\left(1 + \frac{1}{\phi}\nu_0 + \left(\frac{1}{\phi}\right)^2 \nu_0\right)$$

The condition for decreasing consumption is

$$\left(\frac{R}{\phi}\right)\nu_2 < 1, \text{ together with } \frac{R}{\phi} < 1.^{84}$$

[84] This is necessary because $\nu_s \geqslant 1$.

Although these conditions look to be mirrors to the over-consumer's, its interpretation is the opposite. For example, the increasing condition is satisfied $\nu_1 < \nu_0$ whenever $\frac{R}{\phi} \geqslant 1$. Because ν_s (relative loss aversion of saver) is inversely related to ω_s, the consumption is increasing for the saver when the loss aversion of the subsequent DM is greater than the precedent one. In this case, DM can reduce his saving by consuming more than before. Likewise, the decreasing property is obtained when the saver reduces his intention to save. Figure 38 shows the consumption hump for a dynamic decision maker who deviates from the optimal plan to save more.

4.4.2 An Overlapping Generations General Equilibrium

In this section I propose a general equilibrium based on overlapping generations economy. The consumers are the dynamic decision makers who has low loss aversion and plans to deviate from the standard consumption norm. Moreover the consumer can change the degree of his loss aversion over time. Although each consumer optimizes following his dynamically inconsistent taste, the consumer keeps the rule of "middle-point shift" in any consumption plan because this type of planning gives the consumer highest utility among all the deviation strategies available to him.[85]

4.4.2.1 The RDP Consumer Time is discrete and denoted by τ. At each time, a generation of identical cohorts is born. The population is constant over time and each agent who is indexed by age t, lives for T periods in a $(T+1)$ - period overlapping generations economy. During working periods agents are endowed with one unit of labor productivity, measured in efficiency units, which is supplied inelastically. There is no borrowing constraint and the agent can borrow and lend freely under market determined interest rate. There is no government, and there exists a single good which can be either consumed or saved. Finally, the retirement occurs exogenously at $t = T_w + 1$ where $T_w < T$ and the consumer has no whatsoever income during the retirement years.

The representative RDP consumer initially plans for lifetime $T+1$ periods with his taste

[85] In other words, consumer keeps over- or under-consumption up to the half of the planning periods. In section 2, we have seen why this is desirable.

of ω_0. A deviator from the optimal consumption of the standard lifecycle problem, is assumed to consume more than the ex ante optimal for the first half of his remaining lifetime.[86] At any time if the consumer changes his mind, which is the degree of loss aversion, his subsequent self replans for the remaining lifetime. Then the consumer maximizes for $t = 0, 1, ..., T$,

$$u_t(c_t, c_{t+1}, ..., c_T | c^*) =$$

$$\sum_{\tau=t}^{T} \beta^{\tau-t} \left\{ \frac{c_\tau^{1-\gamma}(t)}{1-\gamma} + \eta I_\tau \left(\frac{c_\tau^{1-\gamma}(t)}{1-\gamma} - \frac{c_\tau^{*1-\gamma}(t)}{1-\gamma} \right) + \eta \omega_\tau(t) \lambda (1 - I_\tau) \left(\frac{c_\tau^{1-\gamma}(t)}{1-\gamma} - \frac{c_\tau^{*1-\gamma}(t)}{1-\gamma} \right) \right\}$$

subject to

$$c_\tau(t) + b_{\tau+1}(t) = we_t + Rb_\tau(t)$$
$$b_0(0) = 0,\ b_t(t) = given,\ \ b_{T+1}(t) = 0$$
$$I_\tau = 0, 1$$

where $c_\tau(t)$ is consumption planned at t for time τ and $b_{\tau+1}(t)$ is bond demand purchased at τ for the next period, indexed by planning time t. The consumer has a lifetime stream of productivity profile so that he supplies e_t efficiency units of labor at t to production and earns labor income of we_t each time, where w is the market determined real wage rate which is assumed to be stationary over time. The weight on loss is represented by $\omega_\tau(t)$, which is assumed to be constant within the planning periods $T - \tau + 1$ of each self. Therefore $\omega_\tau(t) = \omega_t$.

Solving the consumer optimization problem implies the first conjecture that if $\omega_t \lambda \geqslant 1$ $for\ all\ t$, then no deviation occurs and the consistent consumption profile is not different from the optimal solution of the standard model and the model does not produce a consumption hump. If the consumer's weight on loss is low $\omega_t \lambda < 1$, then deviation would occur and there are many scenarios for a consumption plan. The solution to the above maximization at any time t yields a consumption plan for planning horizon $T - t + 1$ starting from t. Initially the consumer intends to follow the planned consistent consumption by consuming $c_t(t)$ at t. At $t+1$, however, the consumer realizes that the planned consumption for $t+1$, which is $c_{t+1}(t)$, is now no more agreeable based on his new loss aversion. He has to replan to incorporate this. Because each time he follows the planned path only at the initial time of a planning

[86] This is the over-consumer's maximization problem. The analysis for the saver can also be considered.

horizon, the actual consumption follows the envelope of all the planned consumption path over the entire periods. Thus the realized lifecycle consumption profile is

$$\{c_t\}_{t=0}^{T} = \{\ c_0(0),\ c_1(1),\ c_2(2),\ ...\ ,\ c_T(T)\ \}.$$

And the realized lifecycle bond demand is

$$\{b_{t+1}\}_{t=0}^{T} = \{\ b_1(0),\ b_2(1),\ b_3(2),\ ...\ ,\ b_T(T-1)\ \}$$
$$b_0 = 0,\ b_{T+1} = 0.$$

These are the actual consumption and bond demand that constitute the partial equilibrium, as well as the general equilibrium I consider in the next subsection. In partial equilibrium, the profile gives a lifecycle consumption for the representative RDP consumer. In general equilibrium, because $(T+1)$ types of cohorts from each age group coexist at any time, the aggregate consumption is obtained by summing up the consumption from all age groups.[87] Likewise, the aggregate bond demand is obtained by summing up the bond demand of each cohort.

The next figures (Figure 39, Figure 40) show a set of realized lifecycle consumption and bond demand of a RDP consumer in partial equilibrium. The result is from a dynamic deviator (over-consumer) who relocates his reference point each time following his new taste (degree of loss aversion: ω_t). For simulation exercises, I propose a simple linear weight function for the loss aversion parameter over age:[88]

$$\omega_t : \quad \left\{ \begin{array}{ll} \omega_t = K - 0.01(t-25) \text{ for} & 25 \leqslant t \leqslant 45 \\ \omega_t = \omega_{t-1} + 0.01 \qquad \text{for} & 45 < t \leqslant 80 \end{array} \right\}$$

where K is a constant between 0 to 0.5.[89] Consumer's loss aversion keeps increasing after the age of 45 until the retirement. This weight function is specified from US retirement survey (Retirement Confidence Survey, 2007) which shows that many people start planning the retirement around age 45. This implies that from this age people are likely to curb their over-consumption behavior. In the figure, K is set to 0.5.

[87] Because I consider stationary equilibrium and the population is conststant over time, the aggregate consumption must be the same over time.

[88] In general equilibrium, the consumer is assumed to live from age 25.

[89] Because I set $\eta = 1, \lambda = 2$ for simulation exercises, the deviator's preference $\omega_t \lambda < 1$ implies $\omega_t < 0.5$.

Figure 39: Loss aversion parameters: $\eta = 1, \lambda = 2$, and $\omega_t = 0.5 - 0.01(t - 25)$ for $t \leqslant 45$ and $\omega_t = \omega_{t-1} + 0.01$ for $45 < t \leqslant 80$. Other parameters: $R = 1.035$, $\gamma = 0.5$, $\beta = 0.976$.

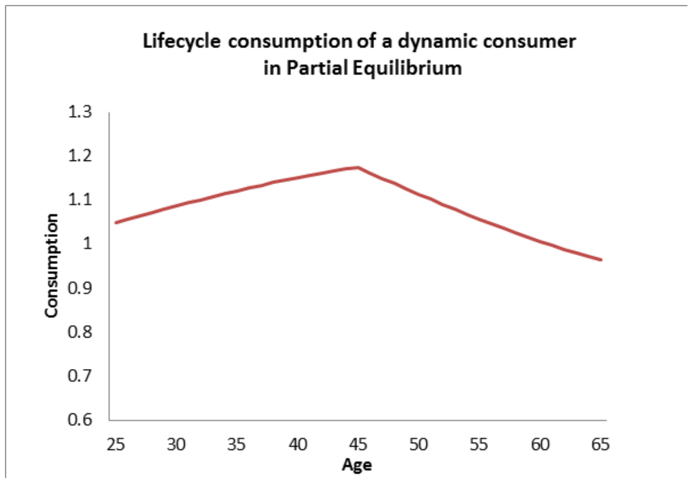
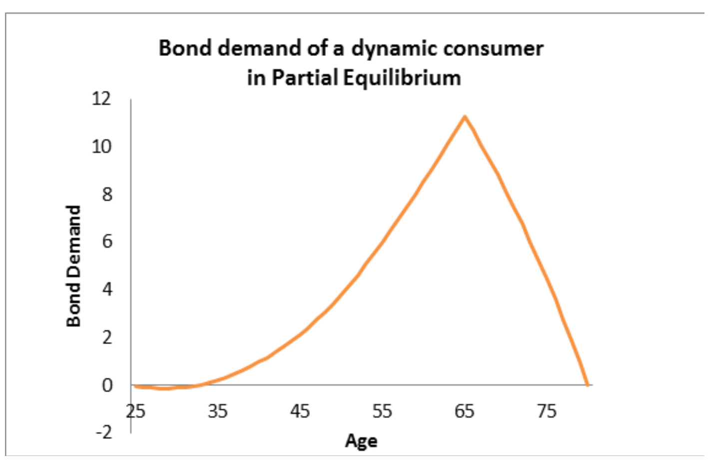


Figure 40: Loss aversion parameters: $\eta = 1, \lambda = 2$, and $\omega_t = 0.5 - 0.01(t - 25)$ for $t \leqslant 45$ and $\omega_t = \omega_{t-1} + 0.01$ for $45 < t \leqslant 80$. Other parameters: $R = 1.035$, $\gamma = 0.5$, $\beta = 0.976$.



In partial equilibrium, changing the parameter values like the interest rate and time preference give different result in realized consumption and bond demand. In general equilibrium, however, this does not apply and not all parameters are independent. I set ω_t and γ to be free. All other variables are jointly set to match the macroeconomic targets of the economy.

4.4.2.2 Production and Calibrated General Equilibrium In general equilibrium, I assume there is a continuum of identical perfectly competitive firms whose production function is specified by $F(K,N) = K^{\alpha}N^{1-\alpha}$ The marginal productivity is given by

$$F_K = \alpha\left(\frac{K}{N}\right)^{\alpha-1} \quad \text{and} \quad F_N = (1-\alpha)\left(\frac{K}{N}\right)^{\alpha}.$$

Then I define the following competitive equilibrium.

Definition *A competitive equilibrium in this economy is an allocation $\{c_t\}_{t=0}^{T}$, a set of bond demands $\{b_{t+1}\}_{t=0}^{T}$, an interest rate R and a wage rate w such that given R and w, the followings are satisfied: (i) $\{c_t\}_{t=0}^{T}$ and $\{b_{t+1}\}_{t=0}^{T}$ solve consumer's problem, (ii) Factors are paid out their marginal productivity, $w = F_N$ and $R-1 = F_K - \delta$, (iii) Labor market and bond market clear, $K = \sum_{t=0}^{T} b_t$ and $N = \sum_{t=0}^{T} e_t$.*

The last market clearing condition specifies that the bond demand among the consumers cancels out in the aggregate and that the excess demand for bonds should be equal to the capital stock. Also the aggregate labor supply that sums up over the labor supply of each cohort should be equal to the aggregate labor demand. Then, by substituting the marginal productivity of capital into the market clearing condition, the equilibrium gross rate R is determined by

$$\textstyle\sum_{t=0}^{T} b_t(R) = \left(\frac{R-1+\delta}{\alpha}\right)^{\frac{1}{\alpha-1}} \textstyle\sum_{t=0}^{T} e_t.$$

Thus,

$$w(R) = (1-\alpha)\left(\frac{R-1+\delta}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}$$

To get a calibrated general equilibrium, three macroeconomic targets are proposed: the interest rate (R), capital-output ratio (K/Y) and consumption output ratio (C/Y). Following Rios-Rull (1996), I set 2.94 as a target value for capital-output ratio and 0.748 for the target ratio of consumption to output. The third macroeconomic target is the real interest rate, which is independently determined in the lifecycle framework. Following McGrattan and Prescott (2000), I set the target of real interest rate at 3.5% ($R = 1.035$). For simulation, the model assumes that agents live for sure to T=65 ($Age = 80$) and when the agents die there will be no bequest left for the next generation. In the context of lifecycle profile of consumption, the mean consumption profile in Gourinchas and Parker's (2002) estimation and its septic polynomial fit by Feigenbaum (2008a) is adopted. According to this estimation, the ratio of peak consumption to initial consumption is 1.1476. For the income schedule, I use Feigenbaum's quadratic fit to the income data of Gourinchas and Parker (2002). The income schedule is[90]

$$e_t = 1 + 0.0181t + 0.000817t^2 - 0.000051t^3 + 0.000000536t^4$$

The next two figures, Figure 41 and Figure 42, show the general equilibrium result. A dynamic model in which consumers deviate for more consumption due to low loss aversion ($\omega_t\lambda < 1$), can reproduce the hump-shaped consumption profile which closely tracts the data in a well-calibrated general equilibrium. In the first figure, three loss aversion profiles are proposed with the same risk aversion parameter (γ), which is 0.5. Using the same weight function ω_t specified above in partial equilibrium, I set $K = 0.5$ for Profile A, $K = 0.4$ for Profile B and $K = 0.3$ for Profile C. Among these, the best quantitative fit to the data in terms of mean squared error is Profile A (mse: 0.0014). Regarding the ratio of peak consumption to initial consumption, Profile A also gives the best match (1.1340) to the data (1.1476). Risk aversion bigger than $\gamma = 0.5$ does not improve the overall fit. For $\gamma = 0.9$, the best profile is $K = 0.4$ in terms of quantitative fit ($mse : 0.0022$) and $K = 0.3$ in terms of the consumption ratio. The second figure summarizes the result.

[90] The labor efficiency replaces the income profile.

Figure 41: GE result with over-consumer. Parameters are $\eta = 1$, $\lambda = 2$, $\gamma = 0.5$.

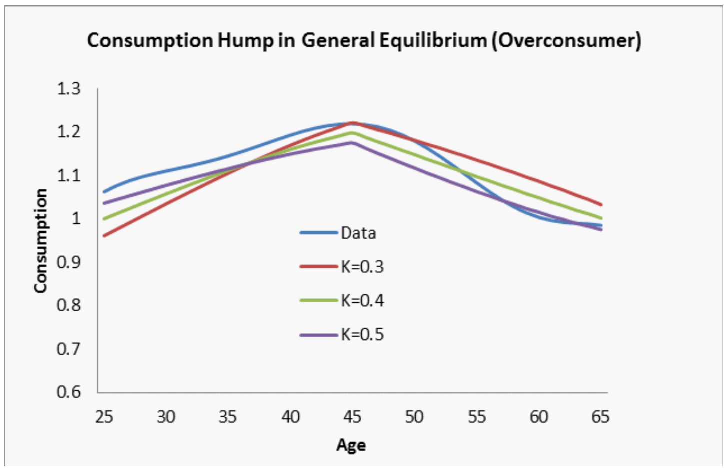
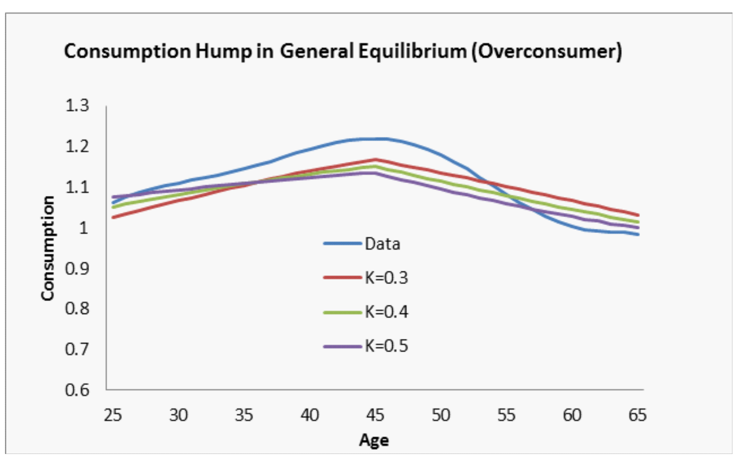


Figure 42: GE result with over-consumer. Parameters are $\eta = 1$, $\lambda = 2$, $\gamma = 0.9$.



4.4.2.3 General Equilibrium with both Savers and Over-consumers If the RDP consumer is not an over-consumer but a saver, then the consumer deviates from the ex ante optimal level for more saving (under-consumption). Assume the agent consumes less than the optimal for the first half of his lifetime. Thus the consumer expects losses for the beginning half of the planning horizon and gains for the other half of it. At any t, if the consumer changes his degree of loss aversion, he re-solves the maximization problem starting from t for the remaining lifetime years. All other economic assumptions remain the same as in the over-consumer's case. For the purpose of simulation exercise, I propose a very simple weight function of the representative saver as follows:

$$\omega_t : \qquad \{\omega_t = K + 0.01(t - 25) \quad \text{for} \;\; 25 \leqslant t \leqslant 80 \;\}$$

where K is a constant term. This specification shows a linearly increasing weight function over the whole life-time. This implies that a saver's propensity to save due to low loss aversion is getting moderate as he gets old. For example, when $K = 0.2$, the saver's loss aversion gets bigger than 0.5 from age 55, implying his consumption behavior converges to the one of the standard agents from this age. Even with this simplest version of the parameter, the model produces a consumption hump in general equilibrium. Figure 43 summarizes the result. In the figure, K is set to 0.2. And the risk aversion parameters (γ) are 0.5 and 0.3. By all of the criterion, profile with $\gamma = 0.5$ fits best (mse : 0.0031). Risk aversion parameter higher than 0.5 does not help the overall fit and the hump-shaped figure is disappearing when γ is very high. Unlike in the the equilibrium of over-consumer, the saver's consumption peak usually comes later in life than the data in this exercise.

If both the savers and over-consumers coexist in the economy, then I have the following result of general equilibrium in figure 11. The weight functions are the same as before with the constant term of $K(saver) = 0.2$ and $K(over\text{-}consumer) = 0.5$. Introducing heterogeneity of consumers improves overall fit and the result is quite robust to alternative parameter values.

Figure 43: GE result with saver. $\eta = 1, \lambda = 2, K = 0.2$.

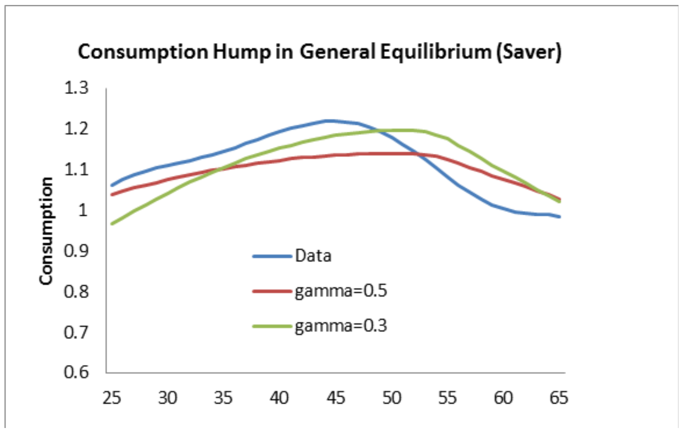
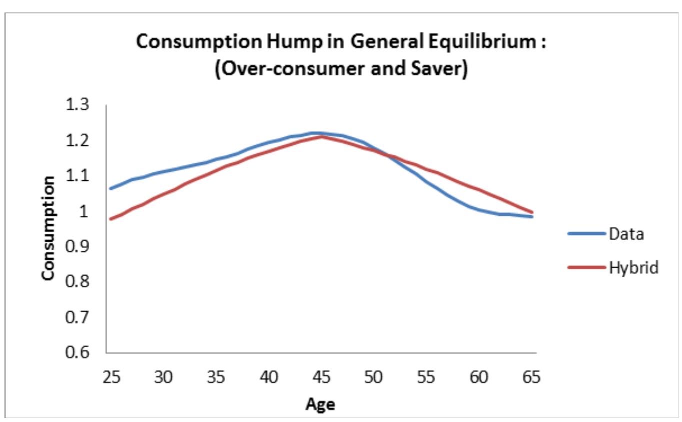


Figure 44: GE result with both saver and over-consumer with ratio of 0.2 for saver. Parameters: $\eta = 1, \lambda = 2$, and saver $(K = 0.2, \gamma = 0.9)$ and over-consumer $(K = 0.5, \gamma = 0.3)$.



4.5 COMPARISON WITH BOUNDED RATIONALITY MODEL

Park (2011)[86] proposes a general equilibrium model of boundedly rational agents who maximize their utilities only over shorter than lifetime planning horizon. The main result is that no mechanism other than the short term planning horizon produces a consumption hump in a well-calibrated general equilibrium. The hump in the model is closely related with hump-shaped income stream.[91] In this section I address the implication of the short term planning model when the consumer has a reference dependent preference and deviate from the standard norm for over-or under-consumption. Specifically I want to focus on the condition of the consumption hump for this modified model and examine how the new condition interacts with the one presented in Park (2011)[86].

4.5.1 A RDP Model of S-Period Updating

The short term planning model assumes that DM foresees only $S < T$ periods and optimizes only S periods. DM updates his belief on the optimal consumption through forward looking as new information arrives and unfolds his future income each year. The reference point where the agent's belief is formed, is now the optimal consumption to the maximization problem over shorter than lifetime planning horizon. Because DM re-optimizes each year by updating his beliefs on the optimal consumption as his income evolves over time, the realized consumption tracks the initial consumption of each planned path for the S periods.

Following the similar steps as in the dynamic model,[92] I derive a closed form solution to a simple model in which DM lives four lifetime periods while he is updating only for two periods through forward looking mechanism. The rebalanced consistent consumption for an over-consumer is

$$\left\{\frac{y_0+\frac{y_1}{R}}{1+\frac{1}{\phi}\mu_0},\quad \frac{y_1{+}Rb_1+\frac{y_2}{R}}{1+\frac{1}{\phi}\mu_1},\quad \frac{y_2{+}Rb_2+\frac{y_3}{R}}{1+\frac{1}{\phi}\mu_2},\ \frac{R}{\phi}\mu_2\left(\frac{y_2{+}Rb_2+\frac{y_3}{R}}{1+\frac{1}{\phi}\mu_2}\right)\right\}.$$

$$\mu_s=\left(\tfrac{1+\eta\omega_s\lambda}{1+\eta}\right)^{1/\gamma}$$

[91] The hump-shaped income stream is the necessary condition for the hump in consumption.

[92] Section 4.1.

The last term in the bracket follows from the fact that the DM at the last period has no choice but consume whatever left for him because no more income is to be realized. Solving recursively for bond demand for each period and substituting into $y_t + Rb_t$ for $t = 0, 1, 2$ pins down the entire consumption profile. The bond demand is

$$b_1 = \frac{y_0\left(\frac{1}{\phi}\mu_0\right) - \frac{y_1}{R}}{1+\left(\frac{1}{\phi}\mu_0\right)}$$

$$b_2 = \frac{R^2 y_0\left(\frac{1}{\phi}\mu_0\right)\left(\frac{1}{\phi}\mu_1\right) + Ry_1\left(\frac{1}{\phi}\mu_0\right)\left(\frac{1}{\phi}\mu_1\right) - y_2\left(\frac{1}{\phi}\mu_0\right)}{R\left(\frac{1}{\phi}\mu_0\right)\left(\frac{1}{\phi}\mu_1\right)}$$

Then the entire consistent consumption $\{c_0,\ c_1,\ c_2,\ c_3\}$ is obtained by substituting the bond demand into the consumption equations. The consumption is[93]

$$c_0 = \frac{Ry_0+y_1}{R\left(1+\frac{1}{\phi}\mu_0\right)}$$

$$c_1 = \frac{\left(R^2y_0+Ry_1\right)\left(\frac{1}{\phi}\mu_0\right)+y_2\left(1+\frac{1}{\phi}\mu_0\right)}{R\left(1+\frac{1}{\phi}\mu_0\right)\left(1+\frac{1}{\phi}\mu_1\right)}$$

$$c_2 = \frac{(Ry_0+y_1)\left(\frac{R}{\phi}\mu_0\right)\left(\frac{R}{\phi}\mu_1\right)+y_2\left(1+\frac{1}{\phi}\mu_0\right)\left(\frac{R}{\phi}\mu_1\right)+y_3\left(1+\frac{1}{\phi}\mu_0\right)\left(1+\frac{1}{\phi}\mu_1\right)}{R\left(1+\frac{1}{\phi}\mu_2\right)\left(1+\frac{1}{\phi}\mu_0\right)\left(1+\frac{1}{\phi}\mu_1\right)}$$

$$c_3 = \left(\frac{R}{\phi}\mu_2\right)\left(\frac{(Ry_0+y_1)\left(\frac{R}{\phi}\mu_0\right)\left(\frac{R}{\phi}\mu_1\right)+y_2\left(1+\frac{1}{\phi}\mu_0\right)\left(\frac{R}{\phi}\mu_1\right)+y_3\left(1+\frac{1}{\phi}\mu_0\right)\left(1+\frac{1}{\phi}\mu_1\right)}{R\left(1+\frac{1}{\phi}\mu_0\right)\left(1+\frac{1}{\phi}\mu_1\right)\left(1+\frac{1}{\phi}\mu_2\right)}\right)$$

It is clear that when the DM is not deviating at any time ($\mu_0 = \mu_1 = \mu_2 = 1$),[94] the consumption profile collapse into a simpler version. This is the case where DM, because his loss aversion is high, does not deviate from the optimal path of S-period maximization with updating. Regarding the consumption hump the model provides the following propositions.[95]

Proposition 18. *Given R and β, if the belief-updating DM's preference $\{\mu_0, \mu_1, \mu_2\}$ and income stream $\{y_0, y_1, y_2, y_3\}$ jointly satisfy that $\left(1 - \frac{R}{\phi}\mu_0 + \frac{1}{\phi}\mu_1\right)(Ry_0+y_1) < y_2(1+\frac{1}{\phi}\mu_0)$, then the consistent consumption is increasing initially, i.e. $c_0 < c_1$.*

To see the meaning of the condition, simplify the condition by setting $\beta = 1/R$,

$$\left(1 - \frac{R}{R}\mu_0 + \frac{1}{R}\mu_1\right)(Ry_0+y_1) < y_2 + \frac{y_2}{R}\mu_0$$

Furthermore, if the DM is not deviating, this returns to

[93] One may find how the realized consumptions are related to the intended ones. Thus, for example, $c_1^1 = \frac{\left(R^2y_0+Ry_1\right)\left(\frac{1}{\phi}\mu_0\right)+y_2\left(1+\frac{1}{\phi}\mu_0\right)}{R\left(1+\frac{1}{\phi}\mu_0\right)\left(1+\frac{1}{\phi}\mu_1\right)} = \frac{\frac{R}{\phi}\mu_0}{\left(1+\frac{1}{\phi}\mu_1\right)}\left(c_1^0 + y_2 + \frac{y_2}{R}\right)$

[94] μ_s is the relative loss aversion parameter.

[95] For computational purpose, I assume that $1+r = R > 0$ and $\phi > r$, as well as the risk aversion parameter $\gamma > 0$.

$$y_0+\frac{y_1}{R} < y_2+\frac{y_2}{R}$$

This inequality implies that if y_2 is greater than y_0 and if either y_2 is as good as y_1 or, in case y_2 is big enough relative to y_0, if not too small compared to y_1, then the increasing property of the consumption at early stage of life is obtained, i.e. $c_0 < c_1$. Notice that this condition is always satisfied with increasing income profile up to $t = 2$. But this condition may not be satisfied if y_2 is much smaller than y_1. Therefore, the proposition implies that with RDP agent, it is possible to have an increasing consumption profile even though the income is not increasing.

Proposition 19. *Given R and β, if the belief-updating DM's preference $\{\mu_0, \mu_1, \mu_2\}$ and income stream $\{y_0, y_1, y_2, y_3\}$ jointly satisfy that, either the relative loss aversion parameter (μ_0, μ_1) or the last income is sufficiently small, i.e.,*

$\left(\frac{1}{\phi}\mu_0\left[R^2 y_0+R y_1+y_2\right]+y_2\right)\left(1+\frac{1}{\phi}\mu_2-\frac{R}{\phi}\mu_1\right) > y_3\left(1+\frac{1}{\phi}\mu_0\right)\left(1+\frac{1}{\phi}\mu_1\right)$, *then the consistent consumption is decreasing, i.e. $c_1 > c_2$. The sufficient condition is $y_2 > 0$ and $y_T = y_3 = 0$.*

Again when $\beta = 1/R$, the condition says

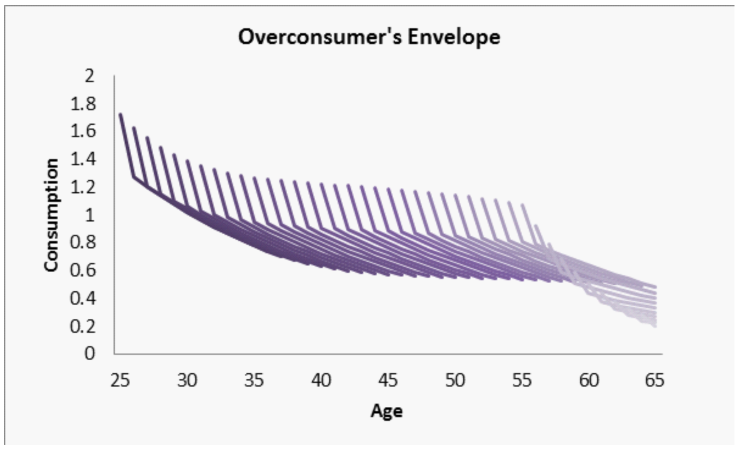
$$\left(\mu_0\left[R y_0+y_1+\frac{y_2}{R}\right]+y_2\right)\left(1+\frac{1}{R}\mu_2-\mu_1\right) > y_3\left(1+\frac{1}{R}\mu_0\right)\left(1+\frac{1}{R}\mu_1\right)$$

If DM is not deviating, then

$$\left(R y_0+y_1+\frac{y_2}{R}+y_2\right)\left(\frac{1}{R}\right) > y_3\left(1+\frac{1}{R}\right)^2$$

It is easily seen that if the DM is not deviating, then income property is the sole determinant of the inequality and that small income at the end of life such as social security is required to get the decreasing property. However, with RDP agent, it is no more a necessary condition. *Even though $y_T = y_3 \neq \varepsilon$, it is possible to get the result if the (μ_0, μ_1) is sufficiently small.* This means that if young DM does not care much about the future, then his consumption of later years can be low even though his income is not tiny in those years. This is because he has to pay off the large amount of borrowing he made when he was young. Finally, combining both conditions yields the consumption hump.

Figure 45: Planned and realized lifecycle consumption for an over-consumer: planning horizon = 10 years, $\omega\lambda = 0.6$, $R = 1.035$, $\beta = 0.96$, $\gamma = 0.9$.



Proposition 20. *If the belief-updating DM's preference* $\{\mu_0, \mu_1, \mu_2\}$ *and income stream* $\{y_0, y_1, y_2, y_3\}$ *jointly satisfy above two conditions, then the consumption profile of the agent produces a hump.*

Example *Suppose* $\{y_0, y_1, y_2, y_3\} = \{1, 1, 1, 1\}$. *Then the consumption profile of a RDP DM who is forward-looking only for two periods and has* $1/R$ *as his time preference, and* $\{\mu_0 = 0.8, \mu_1 = 0.4, \mu_2 = 1\}$ *as his loss aversion parameter is characterized by*

$$\{c_0,\ c_1,\ c_2,\ c_3\}^{Deviating} =$$
$$\left\{\frac{1+R}{0.8+R}, \frac{(0.8+R)+0.4R(R+1)}{(0.8+R)(0.4+R)}, \frac{(0.8+R)(0.4+R)+0.4(0.8+R)R+(0.4)(0.8)RR(1+R)}{(0.8+R)(0.4+R)(1+R)}, c_2\right\}.$$

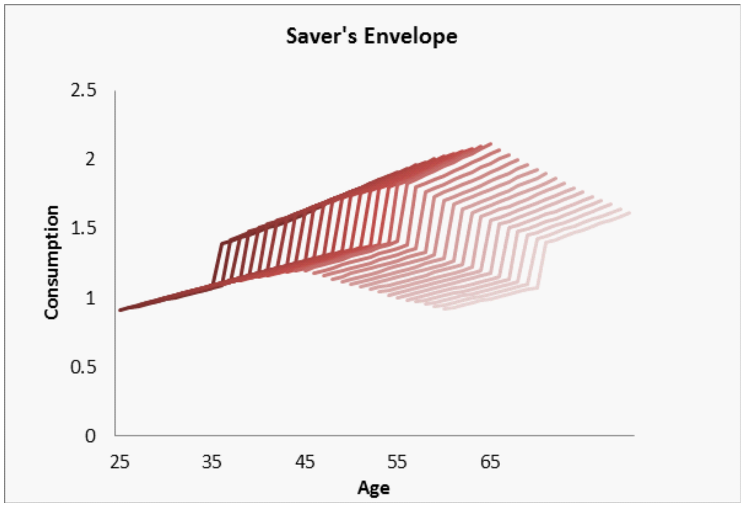
Using the standard interest rate $R_{ann} = 1.035$, *and per length* $\tau = 15$ year, *this yields*

$$\{c_0,\ c_1,\ c_2,\ c_3\}^{Deviating} = \{1.081,\ 1.179,\ \ 0.669,\ \ 0.669\}.$$

Clearly, $c_0 < c_1 > c_2$ *and the hump is achieved because of the preference structure*.

This result contrasts to the example in Park (2011)[86], where the hump is achieved because of the income structure. Figure 45 and Figure 46 show two different types of lifecycle result from this updating model. It may be an interesting question if the consistent consumption profile in Section 2 still preserves the utility ranking in the same model with updating. In fact, the ex post utility ranking is not preserved with the updating for the

Figure 46: Planned and realized lifecycle consumption for a saver: planning horizon = 20 years, $\omega\lambda = 0.6$, $R = 1.035$, $\beta = 0.98$, $\gamma = 0.9$.



consumer with $\omega_t\lambda < 1$. For example, in three period model, the consumption profile of plan A $\{c_0 > c_0^*,\, c_1 < c_1^*,\, c_2 < c_2^*\}$ has a higher utility than the plan B $\{c_0 > c_0^*,\, c_1 > c_1^*,\, c_2 < c_2^*\}$ in the original model of no-updating. But with updating, the second one (Plan B) gives higher utility than the first (Plan A) ex post. This reversion of ranking is explained by the fact that in the updating model, $\{c_0 > c_0^*,\, c_1 > c_1^*,\, c_2 < c_2^*\}$ implies the consumer cannot spend a lot for each of the first two periods because he spreads the total over-consumption amount over the two periods. The opposite happens to $\{c_0 > c_0^*,\, c_1 < c_1^*,\, c_2 < c_2^*\}$ so that the consumer can plunge into one big over-consumption at the first period at the cost of two subsequent under-consumption later. Thus the plan B yields more asset accumulation than Plan A and pays off later.

By the same logic, the next simulation result in partial equilibrium can be justified. This simulation exercise is obtained with $\gamma = 0.9$ and $S = 10$. Remember that any plan for a consumer with $\omega_t\lambda < 1$ can be represented by a binary index $I_t = (0, 1)$. Then I have the following result in Table [11]for an over-consumer.

The table shows that when $\beta R > 1$, that is the market interest rate is high relative to the consumer's time preference, most plans induce a hump. In fact, in the plan of

Table 11: Utility Rank for a Over-consumer

	Initial Plan	Utility Rank	Profile
$\beta R > 1$	$(1, 1, 1, 1, 1, 1, 1, 1, 1, 0)$	1	Hump
	$(1, 1, 1, 1, 1, 0, 0, 0, 0, 0)$	2	Mild Hump
	$(1, 0, 0, 0, 0, 0, 0, 0, 0, 0)$	3	No Hump
$\beta R = 1$	$(1, 1, 1, 1, 1, 1, 1, 1, 1, 0)$	1	Hump
	$(1, 1, 1, 1, 1, 0, 0, 0, 0, 0)$	2	Mild Hump
	$(1, 0, 0, 0, 0, 0, 0, 0, 0, 0)$	3	No Hump
$\beta R << 1$	$(1, 1, 1, 1, 1, 1, 1, 1, 1, 0)$	3	No Hump
	$(1, 1, 1, 1, 1, 0, 0, 0, 0, 0)$	2	No Hump
	$(1, 0, 0, 0, 0, 0, 0, 0, 0, 0)$	1	No Hump

$(1, 1, 1, 1, 1, 1, 1, 1, 1, 0)$, even the consumption profile of an over-consumer can induce a big hump. Because the plan implies that the consumer spreads the resource over long periods, he does not spend a lot in each of the first nine periods. Thus the consumption envelope stays low, yielding more wealth accumulation later. However, as the interest rate is getting lower relative to consumer's time preference, the mechanism that induce a hump becomes weaker. Thus when $\beta R < 1$, the humps are gone for all and the *ranking reverses.*

Turn to the saver's case. In partial equilibrium, it is shown that a plan shifting of plan in about half and half, $(0, 0, 0, 0, 0, 1, 1, 1, 1, 1)$, gives DM the highest ex post utility when there is no updating.[96] But with updating, the plan of $(0, 1, 1, 1, 1, 1, 1, 1, 1, 1)$ gives the highest consumption utility ex post for most of parameter values. Also the plan of $(0, 0, 0, 0, 0, 0, 0, 0, 0, 1)$ gives the lowest consumption utility ex post, as shown in Table [12].

When $\beta R > 1$ all plans produce a hump. And the hump is more apparent with the plan of $(0, 1, 1, 1, 1, 1, 1, 1, 1, 1)$ because the plan implies an initial heavy saving to afford higher consumption for the rest of the nine periods and thus the saving should be high. DM would

[96] Although it depends on parameters.

save in the same manner for the remaining planning periods. Thus in this case the consumer saves a lot at the first period and enjoys high consumption in the subsequent years. With the case of $(0, 0, 0, 0, 0, 0, 0, 0, 0, 1)$, the consumer spreads his pains of saving over long (9) periods and he doesn't need to save much at the first period so that his saving is not much for each time and thus less wealth accumulation occurs. The consumption envelope tracks this implication.

With case of $(0, 0, 0, 0, 0, 0, 0, 0, 0, 1)$, he spreads his pains of saving over long (9) periods and he doesn't need to save much at the first period so that his saving is not much for each time and thus less wealth accumulation occurs. For saver, the hump is obtained with $\beta R = 1$ and even with $\beta R < 1$, although the hump intentness is weaker. However, when $\beta R << 1$[97] the ranking is reversed and the plan of $(0, 0, 0, 0, 0, 0, 0, 0, 0, 1)$ provides best consumption outcome.

Finally, when I apply the same concept of general equilibrium of the previous section, I have the following result: (1) If the preference of representative DM is staying constant over lifetime, i.e. $\omega_t\lambda = \omega\lambda$, (2) If the deviator consumes more (less) for the first half of the S periods and consumes less (more) for the remaining half periods,[98] (3) Then a hybrid of two types of consumers with $\omega\lambda < 1$ can reproduce the hump-shaped consumption data in a well-calibrated general equilibrium. The result in the Figure 47 comes from the hybrid of saver and over-consumer, with a ratio of 0.4 for the saver.

4.6 UNCERTAINTY AND PRECAUTIONARY SAVING

In this section I introduce a RDP model under uncertainty. I specifically study risky choices with an endogenous reference point, under the two schemes of state-independent and state-dependent stochastic references points. The former posits that the decision maker evaluates every possible outcome of a prospect with all possible outcomes of the reference point, while the latter assumes that the decision maker evaluates them only in the same state. Therefore, the decision maker experiences a loss if the outcome of the prospect in a state falls short

[97] $\beta = 0.92$ for $\beta R << 1$, while $\beta = 0.96$ for $\beta R < 1$.

[98] This follows from the previous analysis that the utility culminates when the shift occurs in the middle of the planning horizon.

of the outcome of the reference point in the other states in the state-independent world, while in the state-dependent world, losses are experienced only if they happen in the same state. In this section I derive a two-period general equilibrium result with two agents who are different from each other in their attitudes toward losses.

4.6.1 Stochastic Reference Points

An important question to ask here is what is the reference point in the model of uncertainty. Unlike in deterministic RDP model, where the reference points are the solutions to the dynamic maximization over time for a forward looking agent, with uncertainty, it may be necessary to define the reference point *over different states of the world* for a state contingent agent.[99]

To introduce uncertainty model formally, let us define a couple of notations first. Let $\Omega = \{1, 2, ..., s\}$ be the state-space with finitely many elements. Let $\mathcal{X}$ be the collection of feasible prospects $X : \Omega \to \mathbb{R}$ and $\mathbb{P}[A]$ for the probability that event $A \subset \Omega$ occurs. Following Kőszegi and Rabin (2006[64], 2007[65], 2009[66]), let us define the reference-dependent utility of $X \in \mathcal{X}$ given the reference point $Y \in \mathcal{X}$ as follows:

$$U(X|Y) = \int \int u(x|y) dF_Y(y) dF_X(x),$$

where $F_X(x) = \mathbb{P}[X \leq x]$ and $F_Y(y) = \mathbb{P}[Y \leq y]$ are the probability distribution functions of X and Y. With discrete case,

$$U(X|Y) = \sum_s \sum_{s'} p_s p_{s'} u(X(s)|Y(s'))$$

And $u(x|y)$ is defined by

$$u(x|y) = m(x) + \eta\mu(m(x) - m(y))$$

where m is a continuously differentiable, strictly increasing consumption utility and μ is a gain-loss utility function, with relative importance of η, which satisfies the four properties defined in the chapter introduction. And the expected gain-loss utility over the probability distribution is,

[99] It is possible to defien the reference point over both state and time. Because the main focus here is uncertain outcome, the reference point over different states has priority.

$$G(X|Y) = \int\int \eta\mu(m(x) - m(y))dF_Y(y)dF_X(x).$$

This specification implies DM considers all combinations over every possible outcome of the prospect with all outcomes of the reference point, evaluating all combinations at the product of the two marginal probabilities.[100] The DM is therefore indifferent to the statistical dependence between the prospect X and the reference point Y :

$$U(X|Y) = U(\widetilde{X}|\widetilde{Y})$$

Assume that the DM lives for two periods $t = \{0, 1\}$ and he is subject to face uncertainty at the second period. Thus he receives income $\widetilde{y_1}$ following a known probability distribution of $F(y_1)$ for the second period, but receives certain income y_0 at $t = 0$. Having the consumption utility from different states of the world as his reference points, the DM forms an expectation about the optimal consumption with probability distribution F. Then his utility is described by the reference-dependent expected utility formalized above. Let the maximization problem be

$$max\ u(c_0) + \beta E\left[u(\widetilde{c_1}) \mid u(c_1^r)\right]$$

subject to

$$c_0 + b_1 = y_0$$
$$c_1 = y_1 + Rb_1$$

If there are only two states of the world with respect to second period income (H or L) with probability p, then the DM expects either high consumption or low consumption with the probability and these two consumption outcomes serve as the reference points: $u(c_{1h})$ and $u(c_{1l})$. The gain-loss utility relative to these reference points arise in the second period. When the consumption is high due to high income, DM has a gain feeling relative to the low possible consumption utility that he might have, but zero gain feeling relative to the high consumption utility since the expectation is met. Likewise if the consumption utility is low due to low income, he has a loss feeling relative to the high possible consumption utility, but zero relative to the low one. There is no gain-loss utility in the first period, because there is no uncertainty. Thus the total utility is given by

[100] This implies that the decision maker considers a total of $\{(\#\text{ of state})^2\}$ outcomes in his prospect.

$$u(c|\ p) =$$

$$\frac{c_0^{1-\gamma}}{1-\gamma} + \beta\left(p\frac{c_{1h}^{1-\gamma}}{1-\gamma} + (1-p)\frac{c_{1l}^{1-\gamma}}{1-\gamma}\right)$$
$$+ \beta p\left(p\eta\left[\frac{c_{1h}^{1-\gamma}}{1-\gamma} - \frac{c_{1h}^{1-\gamma}}{1-\gamma}\right] + (1-p)\eta\left[\frac{c_{1h}^{1-\gamma}}{1-\gamma} - \frac{c_{1l}^{1-\gamma}}{1-\gamma}\right]\right)$$
$$+ \beta(1-p)\left(p\eta\omega\lambda\left[\frac{c_{1l}^{1-\gamma}}{1-\gamma} - \frac{c_{1h}^{1-\gamma}}{1-\gamma}\right] + (1-p)\eta\left[\frac{c_{1l}^{1-\gamma}}{1-\gamma} - \frac{c_{1l}^{1-\gamma}}{1-\gamma}\right]\right)$$

Notice that only two terms survive from the four contemporaneous expected gain-loss utility components: there will be one gain utility and one loss utility at $t = 1$. However at $t = 0$, there is no gain-loss utility. Solving the model returns the consistent consumption as well as the bond holding for the DM in partial equilibrium given the interest rate and income distribution. In the next subsection, I derive a closed form solution for a simplified version of the model.

Let's turn to the general equilibrium in which there are two different types of RDP agent. In partial equilibrium, where the interest rate is fixed, the policy variables (consumption and bond demand) are derived given any parameter values. However, in general equilibrium, the bond demand is a function of the equilibrium interest rate and the net bond supply should be equal to zero in equilibrium: b_1+b_2=*0* is the market clearing condition. The candidates for the two different agents of GE are many.[101] Because the main issue of the work with RDP is about the strength of concern about loss, the first step may be to build a general equilibrium with two different types of DMs with respect to their degrees of loss aversion.

4.6.2 A Two-Period Uncertainty Model

With this section, I analyze a simplified 2-period model of RDP under uncertainty that can be solved analytically. The purpose of this step is to demonstrate the implication of gain-loss utility (loss aversion) and disentangle partial equilibrium from general equilibrium. I want to consider an economy with two types of DMs who are receiving the same stochastic income at their second period but different from each other in their attitudes toward loss. Assume that one DM follows standard model ($\omega\lambda > 1$), while the other one follows RDP model ($\omega\lambda < 1$).

[101] DM with i) two different *time preference* (β), ii) two different *risk attitudes* (γ), iii) two different *weights for loss* (ω), iv) two different *wealth or income positions* (y).

First I construct a simple model in which DM faces uncertainty of his income in the second period. Assume that $y_0 = 1$ and $y_{1h} = 1 + \varepsilon$ and $y_{1l} = 1 - \varepsilon$ with probability $p = 1/2$. For simplicity, I assume logalithmic utility function and the gross interest rate is fixed at $1/\beta$ in partial equilibrium. As before DM has consumption utility for both periods[102] and may have gain-loss utilities depending on his references. As described above, the reference point is the outcome of the other state of the world and DM has gain-loss utility for the second period. The situation for the DM who has gain-loss utility due to uncertain outcome, is summarized by

$$Max \ \ln c_0 + \beta \left(\tfrac{1}{2} \ln c_{1h} + \tfrac{1}{2} \ln c_{1l} \right) +$$
$$\beta \tfrac{1}{2} \tfrac{1}{2} \eta \left(\ln c_{1h} - \ln c_{1l} \right) + \beta \tfrac{1}{2} \tfrac{1}{2} \eta \omega \lambda \left(\ln c_{1l} - \ln c_{1h} \right)$$

subject to

$$c_0 + b_1 = y_0$$
$$c_1 = \widetilde{y_1} + R b_1$$

The second term is expected consumption utility and the third and fourth terms are expected contemporaneous gain and loss utilities for the second period. The optimality condition leads to

$$c_0^{-1} = R\beta \{ \tfrac{1}{2} c_{1h}^{-1} + \tfrac{1}{2} c_{1l}^{-1} - \eta(\lambda\omega - 1) \tfrac{1}{2} \tfrac{1}{2} (c_{1h}^{-1} - c_{1l}^{-1}) \}.$$

Except for the third term, this condition coincides with the standard Euler equation. For those who have high loss aversion ($\lambda\omega > 1$), the marginal utility of the second period is lower than the one in the standard model. To equate the marginal utilities over the two periods, DM has to reduce his consumption of the first period.[103] In fact, the last term reflects the DM's intention to save more due to loss aversion. DM may have a gain sense when high outcome realized but a loss feeling when low outcome realized. Because the loss is more painful than the gain is pleasant, his overall utility is negative when the bad outcome happen. Thus he saves more to compensate this loss feelings for bad situation. This explains how the precautionary saving is obtained in RDP model.

[102] The consumption utility for the first period is deterministic but for the second period it is expected utility.

[103] This is because the marginal utility is decreasing.

Before solving this further, let us turn to the other DM who follows the standard model under uncertainty and look for the closed form solution. The two-period maximization problem of the standard model is

$$max\ Eu(c) = \ln c_0 + \beta E \ln c_1$$

Assume for simplicity, $R = 1/\beta = 1$ and the income is given by $y_0 = 1$ and $y_{1h} = 1 + \varepsilon$ and $y_{1l} = 1 - \varepsilon$ with $p = 1/2$. Then the solution to this standard model is

$$c_0 = \tfrac{3}{2} - \tfrac{1}{2}\sqrt{1 + 2\varepsilon^2} < 1 \qquad \text{if } \ \varepsilon > 0,$$

which is decreasing with uncertainty parameter ε. The bond demand is

$$b_1 = 1 - c_0 = -\tfrac{1}{2} + \tfrac{1}{2}\sqrt{1 + 2\varepsilon^2} > 0 \qquad \text{if } \ \varepsilon > 0,$$

which is increasing with uncertainty parameter ε. How about c_1? Because $c_1 = y_1 + Rb_1$,

$$c_1 = \tfrac{1}{2} \pm \varepsilon + \tfrac{1}{2}\sqrt{1 + 2\varepsilon^2}.$$

Thus, it is clear that the expected consumption at the second period is greater than the one with uncertainty $\varepsilon > 0$. When there is uncertainty, DM consumes less at the first period but more at the second period, via increased saving. This property of consumption under uncertainty is well known and the saving is called 'Precautionary Saving.' The precautionary saving may be defined better with 'Cash on Hand,' which is $x = y_0 + Rb_0$. Then the consumption and bond demand over x (cash on hand) are

$$\begin{aligned} c_0 &= \tfrac{3(x+1)/2}{2} - \tfrac{1}{2}\sqrt{(x^2+2x+1)/4 + 2\varepsilon^2} \\ b_1 &= \tfrac{x-3}{4} + \tfrac{1}{2}\sqrt{(x^2+2x+1)/4 + 2\varepsilon^2} \\ c_1 &= \tfrac{1+x}{4} \pm \varepsilon + \tfrac{1}{2}\sqrt{(x^2+2x+1)/4 + 2\varepsilon^2}. \end{aligned}$$

Notice that both directions of the consumption and saving remain the same in ε, while the increase in cash on hand leads to mixed result. However, saving itself increases in both ε and x. Also it is apparent that

$$\begin{aligned} c_0(x; \varepsilon > 0) &< c_0(x; \varepsilon = 0) = \tfrac{1+x}{2} \\ b_1(x; \varepsilon > 0) &> b_1(x; \varepsilon = 0) = \tfrac{x-1}{2} \\ c_1(x; \varepsilon > 0) &> c_1(x; \varepsilon = 0) = \tfrac{1+x}{2}. \end{aligned}$$

These imply that with uncertainty, the expected bond demand is always greater than the one without it.[104] The precautionary saving is defined by $P(x;\varepsilon) = b_1(x;\varepsilon) - b_1(x;\varepsilon = 0)$. Thus,

$$P(x;\varepsilon) = \tfrac{-(x+1)}{4} + \tfrac{1}{2}\sqrt{(x^2+2x+1)/4 + 2\varepsilon^2}$$

The precautionary saving increases as income uncertainty increases. However the effect of cash on hand is mixed. Figure 48 shows the precautionary saving $P(x;\varepsilon)$ (for $x > 0$) as functions of x for ε.Let us go back to the RDP maximization problem. In RDP model, the consumption profiles and savings are affected by the gain-loss parameters and those are[105]

$$\begin{aligned} c_0^{RDP} &= \tfrac{3(1+x)}{4} + \tfrac{\varepsilon\eta(\omega\lambda-1)}{2} - \tfrac{1}{2}K(x,\varepsilon,\eta\omega\lambda) \\ b_1^{RDP} &= \tfrac{x-3}{4} - \tfrac{\varepsilon\eta(\omega\lambda-1)}{2} + \tfrac{1}{2}K(x,\varepsilon,\eta\omega\lambda) \\ c_1^{RDP} &= \tfrac{1+x}{4} \mp \tfrac{\varepsilon\eta(\omega\lambda-1)}{2} + \tfrac{1}{2}K(x,\varepsilon,\eta\omega\lambda) \\ K(x,\varepsilon,\eta\omega\lambda) &= \sqrt{\{3(1+x)/2 + \varepsilon\eta(\omega\lambda-1)\}^2 - 2[1+2x+x^2-\varepsilon^2]}. \end{aligned}$$

The following figures (Figure 49, 50, 51, 52) show c_0, b_1 as functions of x for ε for RDP consumer with three specifications of loss aversion parameter ($\omega\lambda \lesseqgtr 1$). Remember that when $\omega\lambda = 1$, this represents the standard model. And the precautionary saving is

$$P^{RDP}(x;\varepsilon) = -\tfrac{(1+x)}{4} - \tfrac{\varepsilon\eta(\omega\lambda-1)}{2} + \tfrac{1}{2}K(x,\varepsilon,\eta\omega\lambda)$$

Next figures (Figure 53, 54) show $P^{RDP}(x;\varepsilon)$, and P^{RDP}/x (for $x > 0$) as functions of x for ε.

It is easily noticed that if $\lambda\omega = 1$ or $\eta = 0$, this yields the solution to the standard model above. Now let us set $x = 1$ to get the consumption and saving of the RDP DM at partial equilibrium:

$$\begin{aligned} c_0^{RDP} &= \tfrac{3}{2} + \tfrac{\varepsilon\eta(\lambda\omega-1)}{2} - \tfrac{1}{2}\sqrt{1+2\varepsilon^2+6\varepsilon\eta(\lambda\omega-1)+\varepsilon^2\eta^2(\lambda\omega-1)^2} \\ b_1^{RDP} &= -\tfrac{1}{2} - \tfrac{\varepsilon\eta(\lambda\omega-1)}{2} + \tfrac{1}{2}\sqrt{1+2\varepsilon^2+6\varepsilon\eta(\lambda\omega-1)+\varepsilon^2\eta^2(\lambda\omega-1)^2} \\ c_1^{RDP} &= \tfrac{1}{2} \mp \tfrac{\varepsilon\eta(\lambda\omega-1)}{2} + \tfrac{1}{2}\sqrt{1+2\varepsilon^2+6\varepsilon\eta(\lambda\omega-1)+\varepsilon^2\eta^2(\lambda\omega-1)^2}. \end{aligned}$$

[104] Below I compare the standard model with RDP model regarding consumption and bond demand under uncertainty.

[105] Unlike the intertemporal optimzation, the loss aversion arises here with respect to bad outcomes due to uncertainty. Thus high loss aversion $\omega\lambda > 1$ implies people save more than the standard uncertainty model ($\omega\lambda = 1$). Likewise, low loss aversion $\omega\lambda < 1$ implies people save more than the standard uncertainty model.

First look at the bond demand. If $\lambda\omega = 1$ the expected bond demand is $b_1 = \frac{1}{2}[\sqrt{1+2\varepsilon^2} - 1] > 0$ and it is increasing with ε. How about $\lambda\omega > 1$? When $\lambda\omega > 1$, high ε decreases the second term but increases the third term. Because $6\varepsilon\eta(\lambda\omega - 1) > 0$, bond demand increases unambiguously more in RDP model than in the standard model. This implies that given the same uncertainty parameter ε, DM with loss aversion saves more than the DM without this feeling. If however, $\lambda\omega < 1$ (loss-loving: low pain from any loss), the overall effect is ambiguous depending on on the magnitude of ε and z.

Second, look at the first period consumption c_0^{RDP} with respect to ε. If $\lambda\omega = 1$,then high ε decreases current consumption. If $\lambda\omega > 1$, then high ε increases the second term but decreases the third term. By the same argument as in bond demand, current consumption decreases unambiguously more in RDP than in the standard model. If $\lambda\omega < 1$,then high ε induces lower current consumption when $\varepsilon > 6/(1-\lambda\omega)$ but higher c_0 when $\varepsilon > 6/(1-\lambda\omega)$.[106] This suggests that with usual income uncertainty without huge fluctuation, then the effect is positive with $\lambda\omega < 1$. Loss loving people who care less about the pain from loss may not save but consume more now.

4.6.2.1 Market Clearing and General Equilibrium In this section, I consider a general equilibrium model of agents with two different degrees of loss aversion. Although it is possible to construct a general equilibrium of a loss averse agent and a standard agent, $\{\lambda\omega_A > 1$ and $\lambda\omega_B = 1\}$, it is more agreeable to assume $\{\lambda\omega_A > 1$ and $\lambda\omega_B < 1\}$ because in a two-agent economy, one must save while the other must borrow to clear the market. The first specification would not generate this structure. Therefore, the optimality conditions are

$$c_0^{-1} = R\beta\{\tfrac{1}{2}c_{1h}^{-1} + \tfrac{1}{2}c_{1l}^{-1} + \eta(1-\lambda\omega_A)\tfrac{1}{2}\tfrac{1}{2}(c_{1h}^{-1} - c_{1l}^{-1})\}$$
$$c_0^{-1} = R\beta\{\tfrac{1}{2}c_{1h}^{-1} + \tfrac{1}{2}c_{1l}^{-1} + \eta(1-\lambda\omega_B)\tfrac{1}{2}\tfrac{1}{2}(c_{1h}^{-1} - c_{1l}^{-1})\}$$

How can we compare this? Because the gain-loss utility is negative for $\lambda\omega_A > 1$ and positive for $\lambda\omega_B < 1$ it is expected

$$\eta(1-\lambda\omega_B)\tfrac{1}{2}\tfrac{1}{2}(c_{1h}^{-1} - c_{1l}^{-1}) > \eta(1-\lambda\omega_A)\tfrac{1}{2}\tfrac{1}{2}(c_{1h}^{-1} - c_{1l}^{-1})$$

[106] With the assumption of $y = 1$, it is easy to infer that this condition($\varepsilon < 6/(1-\lambda\omega)$) is satisfied more easily than the opposite one. For example, if $\lambda\omega = 0.6$, then the latter requires $\varepsilon > 10$, which is not very realistic, while the former requires only $\varepsilon < 10$.

This implies that $MU(c_{t+1}) > MU(c_t)$ for the DM with $\lambda\omega_B < 1$ and thus he would not save more than DM with $\lambda\omega_A > 1$. Unlike in the partial equilibrium, the bond price or the interest rate is not fixed in general equilibrium and it is determined by the term that the net supply of bond equals to zero. Bond price clears the market at equilibrium. Therefore it is necessary to get the bond demand as a function of interest rate. (But still assume $\beta = 1/R$ for simplicity)

The bond demand of the DM(A) who is loss averse, $\lambda\omega_A > 1$, is

$$b_1^A = \frac{-(1+R)-2\varepsilon\eta(\omega_A\lambda-1)}{2R(R+1)} + \frac{\sqrt{[(2R+1)(1+R)+2\varepsilon\eta(\omega_A\lambda-1)]^2-4(R+1)R[(1+R)^2-\varepsilon^2]}}{2R(R+1)}$$

while bond demand of the DM(B) who is tolerant to loss, $\lambda\omega_B < 1$, is

$$b_1^B = \frac{-(1+R)+2\varepsilon\eta(1-\omega_B\lambda)}{2R(R+1)} + \frac{\sqrt{[(2R+1)(1+R)-2\varepsilon\eta(1-\omega_B\lambda)]^2-4(R+1)R[(1+R)^2-\varepsilon^2]}}{2R(R+1)}.$$

Now with market clearing condition for the economy where only two types of agents exist:

$$\theta b_1^A + (1-\theta)b_1^B = 0.$$

Figure 55 shows the general equilibrium result with two different DMs. The uncertainty specification is $\varepsilon = 0.2$, and θ is adjusted accordingly to set an equilibrium.

4.6.3 State Dependence

In this subsection, I examine the case where the reference points are state-dependent and the reference point may be chosen endogenously due to this dependency. Let us consider the following state-dependent RDP. Assume the same state space as above. Let $\mathcal{Y} \subseteq \mathcal{X}$. For a risky prospect $X \subseteq \mathcal{X}$ and a reference point $Y \subseteq \mathcal{X}$, the state-dependent RDP valuation of X relative to Y is given by

$$V(X|Y) = \int\int u(x|y)d^2 J_{X,Y}(x,y),$$

where $J_{X,Y}(x,y) = \mathbb{P}[X \le x; Y \le y]$ is the joint c.d.f of X and Y. The state-dependent expected utility $V(X|Y)$ evaluates the outcome of prospect X and reference point Y not by the product of the marginal distribution, but by their joint distribution. Through this, it incorporates the statistical dependence between the prospect and the reference point. In a discrete case with S states of the world, this allows DM to compare the outcomes of the prospect with those of the reference point in the same state, but not with those in other states. Thus,

$$V(X|Y) = \sum_s p_s u(X(s)|Y(s))$$

According to this state-dependent utility, the DM does not experience loss feeling from the fact that bad states yield worse outcomes than good states, as in the previous subsection of the state-independent reference points. In fact, DM experiences loss feeling when the utility from selected prospect falls below the reference point in the same state.

If two random variables X and Y are independent, then the joint c.d.f of X and Y is the product of the corresponding marginal distributions functions.

$$J_{X,Y}(x,y) = F_X(x)F_Y(y)$$

In this case, the two specifications of reference-dependent RDP coincide. The expected utilities are,

$$U(X|Y) = \int\int u(x|y)dF_Y(y)dF_X(x) = V(X|Y)$$

However, the two specifications generally differ if the prospect and the reference point are dependent. Compared to the state-dependent utility $V(X|Y)$, the state-independent utility $U(X|Y)$ generally overestimates the true occurrence of gains or losses in the case of positive dependence between X and Y and underestimates it in the case of negative dependence.

In the example of two-period consumption-saving model, the state-dependent RDP is given by

$$Max \ \ln c_0 + \beta\left(\tfrac{1}{2}\ln c_{1h} + \tfrac{1}{2}\ln c_{1l}\right) +$$
$$\beta\tfrac{1}{2}\eta\left(\ln c_{1h} - \ln c_{1l}\right) + \beta\tfrac{1}{2}\eta\omega\lambda\left(\ln c_{1l} - \ln c_{1h}\right)$$

subject to

$$c_0 + b_1 = y_0$$
$$c_1 = \widetilde{y}_1 + Rb_1$$

Notice that the gain or loss utility arises with $p = 1/2$ instead of $p = 1/4$ although it is the same as in the state-independent case that only two terms survive from the four contemporaneous gain-loss utility components. The optimality condition leads to

$$c_0^{-1} = R\beta\{\tfrac{1}{2}c_{1h}^{-1} + \tfrac{1}{2}c_{1l}^{-1} - \eta(\lambda\omega - 1)\tfrac{1}{2}(c_{1h}^{-1} - c_{1l}^{-1})\}.$$

Except for the third term, this condition is equal to the one for state-independent RDP in previous subsection. Compared to state-independent RDP, for those who have high loss aversion ($\lambda\omega > 1$), DM's intention to save more due to loss aversion is slightly weaker with state-dependent RDP. State-dependent consumption profile is

$$\begin{aligned} c_0^{RDP(dep)} &= \tfrac{3x}{2} + \tfrac{\varepsilon\eta(\omega\lambda-1)}{4} - \tfrac{1}{2}K(x,\varepsilon,\eta\omega\lambda) \\ b_1^{RDP(dep)} &= -\tfrac{x}{2} - \tfrac{\varepsilon\eta(\omega\lambda-1)}{4} + \tfrac{1}{2}K(x,\varepsilon,\eta\omega\lambda) \\ c_1^{RDP(dep)} &= \tfrac{2-x}{2} \mp \tfrac{\varepsilon\eta(\omega\lambda-1)}{4} + \tfrac{1}{2}K(x,\varepsilon,\eta\omega\lambda) \\ K(x,\varepsilon,\eta\omega\lambda) &= \sqrt{x^2 + 3x\varepsilon\eta(\omega\lambda-1) + \{\varepsilon\eta(\omega\lambda-1)/2\}^2 + 2\varepsilon^2}. \end{aligned}$$

Likewise, the precautionary saving is obtained by

$$P^{RDP(dep)}(x;\varepsilon) = -\tfrac{x}{2} - \tfrac{\varepsilon\eta(\omega\lambda-1)}{2} + \tfrac{1}{2}K(x,\varepsilon,\eta\omega\lambda).$$

However, it is still true that if $\lambda\omega = 1$ or $\eta = 0$, this yields the solution to the standard model. The consumption and saving for $x = 1$ are

$$\begin{aligned} c_0^{RDP} &= \tfrac{3}{2} + \tfrac{\varepsilon\eta(\omega\lambda-1)}{4} - \tfrac{1}{2}K(x,\varepsilon,\eta\omega\lambda) \\ b_1^{RDP} &= -\tfrac{1}{2} - \tfrac{\varepsilon\eta(\omega\lambda-1)}{4} + \tfrac{1}{2}K(x,\varepsilon,\eta\omega\lambda) \\ c_1^{RDP} &= \tfrac{1}{2} \mp \tfrac{\varepsilon\eta(\omega\lambda-1)}{4} + \tfrac{1}{2}K(x,\varepsilon,\eta\omega\lambda) \\ K(x,\varepsilon,\eta\omega\lambda) &= \sqrt{1 + 3\varepsilon\eta(\omega\lambda-1) + \{\varepsilon\eta(\omega\lambda-1)/2\}^2 + 2\varepsilon^2}. \end{aligned}$$

Figure 56 displays RDP consumption when $x = 1$ for both state-independent and state-dependent specifications. As seen in the figure, state-independent case gives lower consumption at the first period with stronger precautionary saving motivation.[107] Figure 57 displays RDP bond demands when $x = 1$ for both state-independent and state-dependent specifications. From the figure, it is clear that state-dependent RDP exhibit more moderate saving or over-consumption intensity than one in the state-independent RDP world.

[107] This is the case for $\omega\lambda > 1$. In case of $\omega\lambda < 1$, the opposite is true.

4.7 CONCLUSION

Reference-dependent preferences help explain many phenomena: excessive aversion to small risk, the reluctance to sell a houses at a loss, equity premium puzzle, insurance against small risk and target earnings in labor supply decision. Yet, there are few works, if any, that demonstrate that this type of preferences can help in explaining consumption dynamics in macroeconomics. Based on this motivation, I develop a macroeconomic model of belief dependent preferences for intertemporal decision making and examine its macroeconomic implication when the reference status is the consumer's belief on the optimal consumption for current and future given any information about income stream. To dissolve the key issue of the determination of reference points in these models, I take the solution concept of rational expectation equilibrium by Kőszegi and Rabin.

The main subjects in this essay are those decision makers who have low loss aversion with respect to the standard consumption rule. Through this paper I look for the consistent intertemporal optimization rule for a decision maker when he has a certain degree of loss aversion, or weight on loss relative to gain. Any decision maker who cares future is considered to have a high weight since this agent would not over consume early in life at the cost of low future consumption, because he does not want to have severe loss feeling later. Likewise, any decision maker who cares more of the current pleasure in extra consumption is said to have a low weight since this agent has tolerance of the pain from loss in the future. In any of these cases, the choice strategy of decision maker should meet the rational intertemporal consistency. Any choice of consumption strategy should belong to the consistent plans of the decision maker.

Because decision makers or consumers who are expecting age-dependent average life-cycle income stream should form expectation regarding his future consumption from the income realization each period, the uncertainty in income stream can give greater depth in consumption dynamics. In this context, the model specifies two alternatives regarding the consumer's belief formation from future income stream. The first one is the usual deterministic/or stochastic income stream and its individual realization. And the second one is the non-stochastic income stream but visible only in a certain period ahead but not earlier. Both cases give greater insight on how the news about the distribution of future income would generate multiple layers of consumption dynamics.

The main result of my work is that the reference dependent preferences helps a lot in explaining many noted features of consumption dynamics in lifecycle theories, like the consumption hump and precautionary saving, as well as providing multi-layered understanding of dynamic decision making for the agents who may change his mind over lifetime. Because my model targets a calibrated general equilibrium, the result from my model should closely resemble the actual macro economy. Therefore my findings with reference-dependent preferences are meaningful in that they are consistent with the known macroeconomic characteristics from data.

Table 12: Utility Rank for a Saver

	Initial Plan	Utility Rank	Profile
	$(0,1,1,1,1,1,1,1,1,1)$	1	Hump
$\beta R > 1$	$(0,0,0,0,0,1,1,1,1,1)$	2	Hump
	$(0,0,0,0,0,0,0,0,0,1)$	3	Hump
	$(0,1,1,1,1,1,1,1,1,1)$	1	Hump
$\beta R = 1$	$(0,0,0,0,0,1,1,1,1,1)$	2	Hump
	$(0,0,0,0,0,0,0,0,0,1)$	3	Hump
	$(0,1,1,1,1,1,1,1,1,1)$	1	Hump
$\beta R < 1$	$(0,0,0,0,0,1,1,1,1,1)$	2	Hump
	$(0,0,0,0,0,0,0,0,0,1)$	3	Hump

Figure 47: Realized consumption with hybrid: $\omega\lambda = 0.6$, $\gamma = 0.5$, S=18.

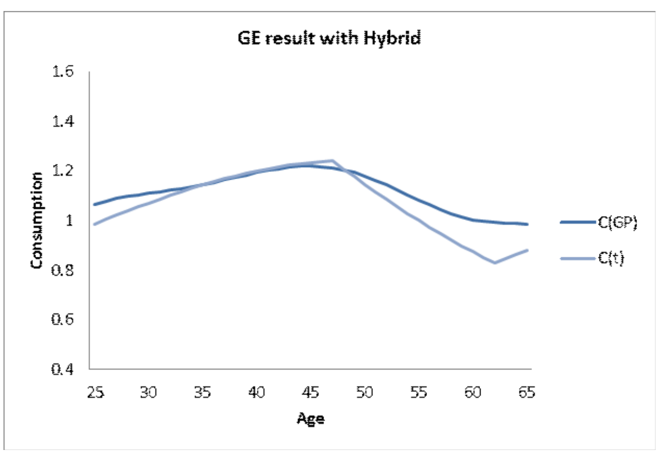


Figure 48: Precautionary Saving in standard model ($\omega\lambda = 1$): darker lines represent the savings when the cash on hand (x) is lower.

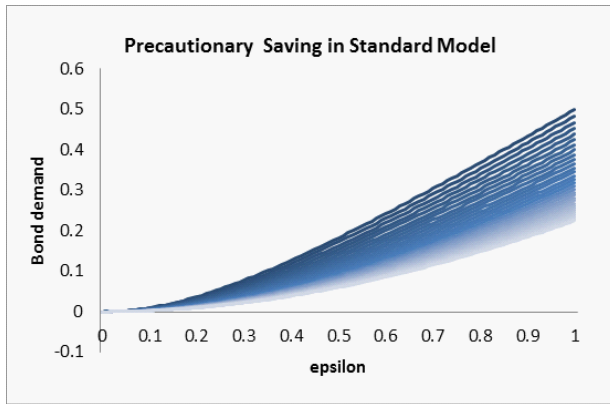


Figure 49: RDP plots of c_0 when $x = 1$. The loss aversion parameters are $\omega\lambda = 0.8$, $\omega\lambda = 1$, and $\omega\lambda = 1.2$.

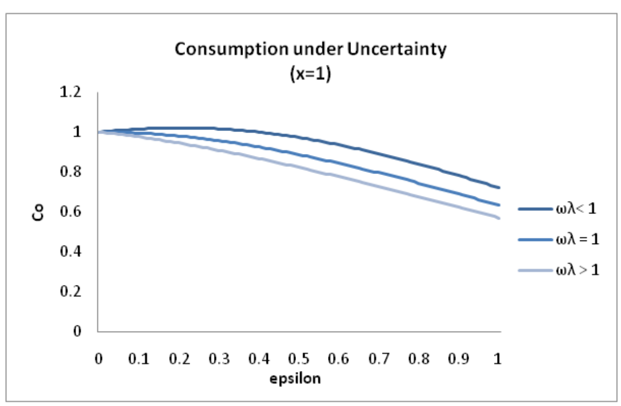


Figure 50: RDP plots of c_0 when $\varepsilon = 0.4$. The loss aversion parameters are $\omega\lambda = 0.8$, $\omega\lambda = 1$, and $\omega\lambda = 1.2$.

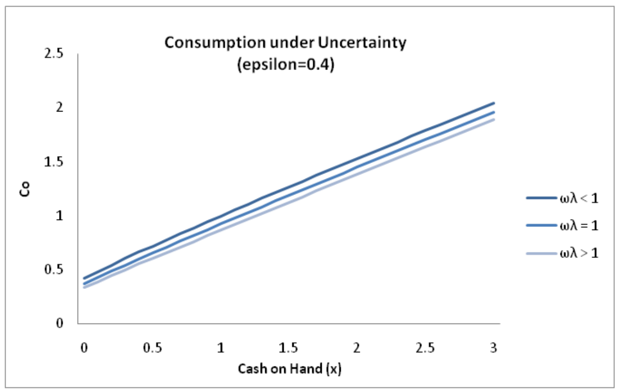


Figure 51: RDP plots of b_1 when $x = 1$. The loss aversion parameters are $\omega\lambda = 0.8$, $\omega\lambda = 1$, and $\omega\lambda = 1.2$. When $\omega\lambda < 1$, the bond demand can be negative.

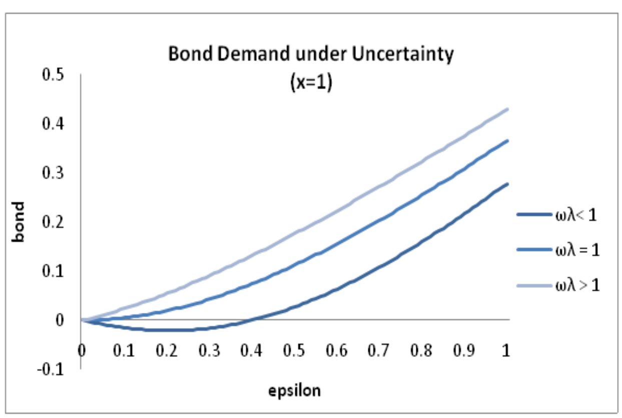


Figure 52: RDP plots of b_1 when $\varepsilon = 0.4$. The loss aversion parameters are $\omega\lambda = 0.8$, $\omega\lambda = 1$, and $\omega\lambda = 1.2$. Each bond demand cuts $b_1 = 0$ at $x \leqslant 1$.

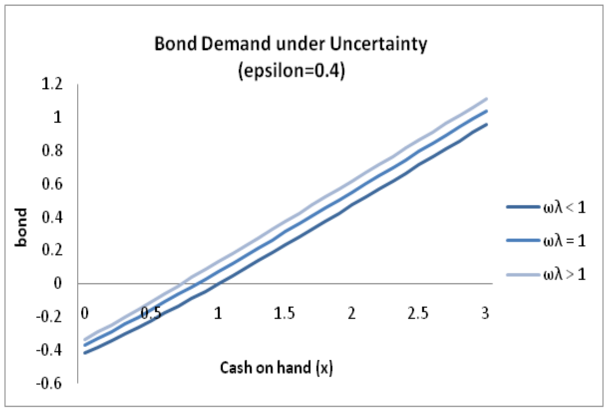


Figure 53: RDP precautionary saving: $\omega\lambda = 0.8$. The darker lines are from lower cash on hand (x).

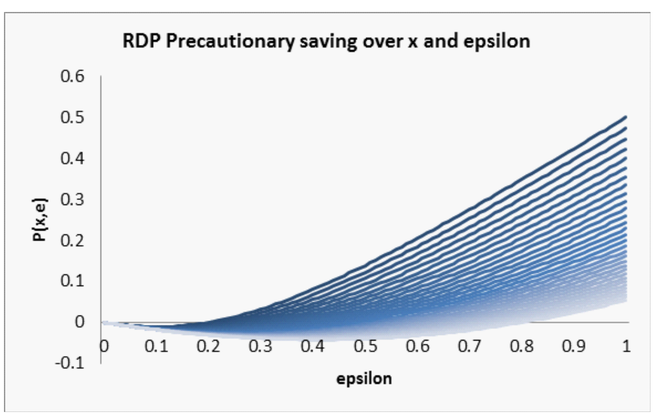


Figure 54: RDP precautionary saving rate: $\omega\lambda = 0.8$. The darker lines are from lower epsilon (ε).

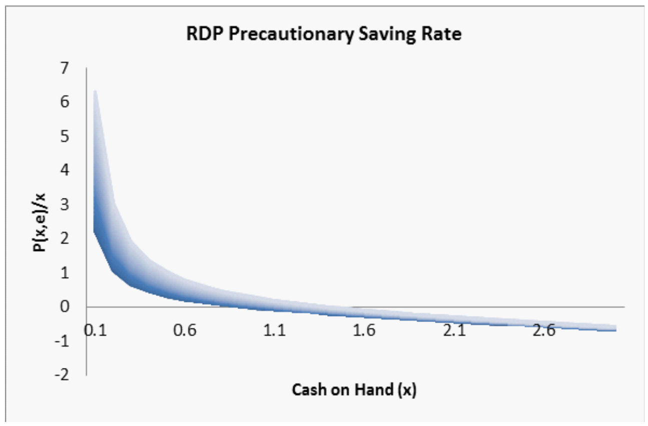


Figure 55: The two agents are represented by $\omega_A\lambda = 2$ and $\omega_B\lambda = 0.2$. The uncertainty is given by $\varepsilon = 0.2$.

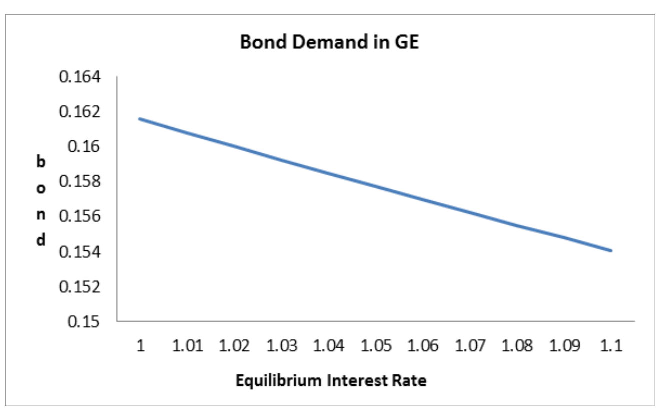


Figure 56: RDP plots of c_0 when $x = 1$ for both state-independent and state-dependent specifications. The loss aversion parameters are $\omega\lambda = 0.8$, $\omega\lambda = 1$, and $\omega\lambda = 1.2$.

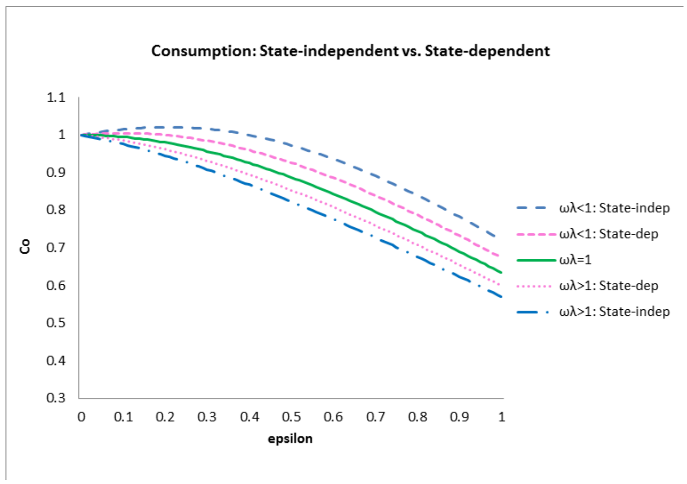
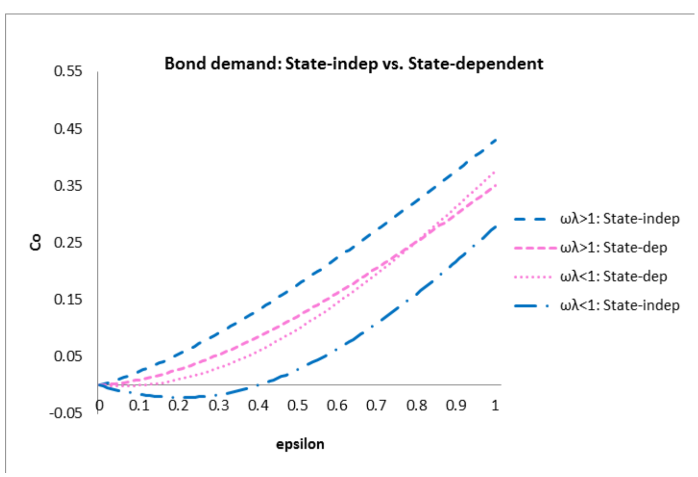


Figure 57: RDP plots of b_1 when $x = 1$ for both state-independent and state-dependent specifications. The loss aversion parameters are $\omega\lambda = 0.8$ and $\omega\lambda = 1.2$.



5.0 A LIFECYCLE MODEL OF CHARITABLE GIVING: A QUANTITATIVE ANALYSIS OF SOCIAL PREFERENCES

I develop a lifecycle model of warm glow for consumers who derive utility from the act of giving both goods and volunteering time and explore the general equilibrium characteristics of an economy that is populated by these pro-social consumers. By separating charitable deduction rate from income tax rate together with occupying non-separable utility between consumption and charitable giving, the model unambiguously determines welfare direction from any change in a tax system, enlightening the role of policy in private provision of public goods. I first derive analytic solutions that define the optimal resource level committed to spend on giving and consumption each period with respect to leisure or volunteer constraint. In the full model where consumers are subject to mortality risks for charitable bequests and choose endogenously their retirement age, I demonstrate that the model features salient facts regarding lifecycle giving/volunteer and consumption/leisure behaviors in an empirically plausible, calibrated overlapping-generations general equilibrium. Reasonable parameterization of my model generates an inverse U-shaped consumption and labor, an inverse J-shaped giving, similar to the data, while it confers highest welfare (CV) for the tax rate of 23% when the rate is equal to charitable deduction rate.

Keywords: social preferences, charitable giving, warm glow, constrained optimization, mortality risks, charitable deduction, general equilibrium.

5.1 INTRODUCTION

5.1.1 Charitable Giving and Warm Glow

The standard economic theory is based on the core assumption that individuals are *self-interested* and affected only by their own well-being.[1] However, as seen in many lab and field experiments (Fehr and Gächter, 2000[41]; Charness and Rabin, 2002[27]), individuals show a great concern for the welfare of others. Fisman, Kariv, and Markovits (2007)[50] show that there exists well-behaved rationalizing preference ordering confirmed by the data for majority of subjects who lean towards a social welfare conception of preferences.[2]

One example of such observation that individuals exhibit *social preferences*[3] is charitable giving. The size and frequency[4] of charitable giving, qualitatively consistent with the experimental findings, suggests that the individual's utility depends also on the payoff or well-being of other people, providing evidence of social preferences in the field. In the US, \$240.9 billion were donated to charities in 2002, representing an approximate 2 % of GDP (Andreoni, 2006)[3]. By recent data the number arises to \$290.89 billion in 2010.[5] Donations of time in the form of volunteering are also substantial. In 2002, 44 % of respondents to a survey report giving time to a charitable organization in the prior year, with volunteers averaging about fifteen hours per month (Andreoni, 2006[3]). This represents nearly 84 million individuals. The volunteer workforce give approximately 15.5 billion hours, representing the equivalent of over 9 million full-time employees at a value of \$239 billion.[6]

Although several demographic characteristics have been found useful in predicting charitable giving, income is by far the most important predictor of giving behavior (McClelland and Brooks, 2004). With data from 1995, Andreoni (2006)[3] shows that giving as a function of income is a U-shaped pattern, implying people in the lowest and highest income groups

[1] The standard model, in its rigid form, assumes purely self-interested decision makers. Decision theorists argue that many lab evidences suggest possible deviations from the standard theory in roughly three ways: nonstandard preferences, nonstandard beliefs, and biased decision making.

[2] Fisman et al. (2007)[50] estimate CES utility functions to demonstrate this.

[3] This is one of the deviations from the standard model with respect to self-interested preferences. Other deviations may be with respect to time preferences and risk preferences.

[4] Every year 90% of Americans give money or time to charity (DellaVigna et al., 2009[34]).

[5] "Giving USA 2011 : The Annual Report on Phylanthropy for the Year 2010."

[6] "Giving and Volunteering in the US," 2001.

give larger proportions of their incomes to charity than individuals in middle-income groups. McClelland and Brooks (2004)[80] who analyze 1997 data,[7] find that this U-shaped pattern persists even when accounting for additional variables associated with income. However, with more recent data of 2001, the pattern is shown to be more like inversely-related with income, implying the right hand side of the U-shaped pattern is disappearing or mitigated. This may come from the rapid income expansion among the high income level groups in later data. But both old and new data sets show that charitable giving generally increases with age except for the very old age.[8]

Despite its size and its close relationship with macroeconomic variables like income and tax widely confirmed in data, charitable giving has not been rigorously studied in macroeconomics. The majority work on charitable giving has been on lab or field studies searching for the motivation of giving or instrumental devices to enhance giving. However, like many other lab findings regarding social preferences, the prediction of giving in the lab usually overpredicts the actual amounts shown in data. That is why a quantitative analysis that correctly predicts the giving behavior with respect to other variables is needed. Moreover, it is shown in data that individual giving varies significantly with age, although the giving pattern over lifecycle is stable across income level. Motivated by this, this paper aims to develop a lifecycle model of charitable giving that incorporates a social preference and study its macroeconomic implications in general equilibrium. Specifically, I study the giving behavior in both goods (money) and time (volunteering) among the economic agents over lifecycle and perform the welfare analysis of charitable deductions, the rate of which is modeled to be separate from income tax rate.

To analyze the giving behavior, which is the main issue related with how to construct the model of a social preference among individuals, it is necessary to understand why people give. Regarding giving of time, it is important to understand what happens to economic agents when they undergo constrained optimization due to their limited time availability. Regardless of income level, all agents are constrained by their physical time endowment. Like borrowing or credit constraint, the time constraint is important in analyzing optimal

[7] Consumer Expenditure Survey.

[8] Details are shown with data in the next subsection.

choices between consumption and leisure (and volunteer) to induce conditions for retirement behavior. I want to address each of these.

First, why do individuals give? Although there are difficulties with field studies due to limited control, the motivation of giving has been well-studied in lab oriented game situations.[9] The motivation may be summarized by (i) intrinsic motivation (altruism, warm glow), (ii) extrinsic motivation (tax breaks, thank-you gestures), (iii) image motivation (social approval, status). Along with these candidates, many researchers argue that while intrinsic social preferences are a leading interpretation for giving, charitable donations may also be motivated by other factors, such as desire for status and social pressure by the fund-raisers (DellaVigna, List, and Malmendier, 2009[34]).

Regarding intrinsic motivation, substantial researchers suggest that individuals are not entirely altruistic when they give. Individuals seem to derive more benefits from the act of giving itself than from the benefits that their givings generate for others. This finding is confirmed by simple explanation: an individual giver is indifferent between giving a dollar to a charity and the charity's receiving the dollar from someone else. His preference is represented by a type of pure altruism,[10] $u(x, G)$ where x is consumption of private good and G is sum of contributions to the public good $G = \sum gi$ and gi is individual contribution by i. Thus the individual increases utility from what others receive. This model of perfect altruism implies that individual donations can be completely *crowded out* by government contributions, which is not supported by the empirical evidence on private charities (Andreoni, 1988[4]). Studies like Steinberg (1989)[98] finds that the crowding out is only partial because individuals get more benefit when they contribute on their own. This suggests an alternative:[11] impure altruism, or warm glow denoted by $u(x, g)$ if it is pure warm glow and $u(x, g, G)$ if partial warm glow.

Under the proposition of *warm glow,* individuals derive internal satisfactions from giving itself, although the nature of these satisfactions is not very clear.[12] By this specification,

[9] For example, Dictator game (one giving to one other) and Public good games (many giving to many other).

[10] According to Becker (1974), the pure altruism may be defined by "a desire to improve the general well-being of recipients."

[11] Andreoni (1990)[6] and others has develped this.

[12] It is hard to know whether givers may care about the objective of the charities or they simply enjoy the act of giving.

givers view the contributions of others as imperfect substitutes of their own ones. That is, they prefer that the contribution for someone else's well-being come from themselves rather than from others. This implies that the crowding out by government provisions in public goods should be incomplete. Likewise, government taxation does not reduce private contributions by the same amount.[13] By Andreoni (1990[6], 2006[3]), the findings in warm glow also apply to giving of time, i.e. volunteering. If individuals were perfectly altruistic, there should be little volunteering observed, which fact is not true as seen by the data. Substantial portion of individuals provide their time to charitable or public activities. Thus, it may be reasonable to think of volunteering as having some independent warm glow component as well.

Recent works, however, are more focusing on the third explanation, an element complementing the first. Signaling (Vesterlund, 2003[105]) or status (Kumru and Vesterlund, 2009[68]) are among those, as well as social information in field study (Frey and Meier, 2004[53]; Shang and Croson, 2009[94]). Status is an example of a private benefit from giving. Likewise, image can be an incentive: individuals concern for what others think as a driver for pro-social behavior. Thus, people may give without any true concern for the receiver's welfare. By these arguments, individual's preference may be represented by a general social norm like $u(x, g, s)$.[14] In this paper, I want to explore macroeconomic modeling from both types of explanation with the choice variables of consumption, giving, leisure and volunteer.

Regarding the second issue, i.e. the time constraint, I want to emphasize that in choice between consumption and leisure, for example, it is important to know if any optimal choice variable is obtained as an interior solution or not. This also applies to the case of choice between charitable goods and charitable time. Because I consider individuals of warm glow who care the four components simultaneously, it is necessary to understand the intrinsic role of constraint in the model. I explain this in detail with analytic works next section.

Lastly, regarding lifecycle perspective, to solve the intertemporal optimization problem with four simultaneous choice variables, I take three steps of optimization. The first is to

[13] See Andreoni (1988)[4].

[14] Ariely, Bracha, and Meier (AER, 2009)[10], suggest a simpe form of this like the following: $u = (1 - a)log(c + s) + alog(g)$.

induce two selective optimal variables from the four variables via intra-temporal optimization. Next is to analyze the choice between the two composite variables, examining whether the constraint is binding or not. Finally, by defining an optimal expenditure as a value from the second step, the model generates optimal solution over time.

5.1.2 US Giving Reports

There are two major sources of data regarding US charitable givings.[15] The first one is household surveys[16] and the second is samples from tax returns. Charitable givings mainly come from individuals, charitable foundations, corporations, and bequests. While all are significant, by far the dominant source of giving is from individuals. Table [13] shows the 2010 US giving features from both collected and estimated data. By this in 2010 individuals give over 211 billion dollars to charity, or 73% of the total dollars donated. Foundations, which is the second biggest source, is responsible for 14% of all donations. By the source, the combined charitable giving by individuals, bequests, and family foundations amounts to an estimated $254.10 billion or 87% of total.

The major recipient's sectors are: Religion (35%), Education (14%), Foundations (11%), Human services (9%), Health (8%), Public society benefit (8%), Arts-culture (5%), International Affairs (5%), Individuals (2%). Over time, total giving has been on a steady rise, but when measured as a percent of income, giving is much more stable and it is around 2% to income. The average rate between 1970-2000 is 1.9%[18] and between 2000-2005, it is 2.3%. In 2010, by the above source, giving is 2% of GDP. Figure 58 shows the trend of total giving from 1970 to 2010[19] and Figure 59 shows it as percentage of GDP.

Next is the giving statistics with respect to income level of the contributors. The pro-

[15] In US, the contribution is usually made through 1.23 million charitable nonprofit organizations and religious congregations.

[16] For example, "Giving and Volunteering in the United States" is the one that produces a series of biennial national surveys that report trends in giving and charitable behavior. The survey mainly asks about the followings: 1. What are the characteristics of individuals who volunteer? How much time do they give and to whom do they volunteer? 2. What are the characteristics of the households that give to charities? How much do they contribute and to whom? 3. What variables are associated with volunteering and giving behavior?

[18] Author's calculation from annual percentage rate.

[19] From the Figure 1, one may notice a shift around 1997 reflecting a tax reform favorable to charitable giving.

Table 13: Giving Statistics by Contributors

Sources	Billions of Dollars	% of Total
Individual	211.77	73
Foundations	41.00	14
Bequests	22.83	8
Corporations	15.29	5
TOTAL	290.89	100

Source: Giving USA, 2011[17]

portion of households making formal contributions increases as household income increases. Table [14] shows both the average amount and the percentage of income contributed by level of income.

Table [14] explains the following: household income is strongly and positively related with the likelihood of giving. This likelihood ranges from a low of 76.8 percent for households with incomes under \$25,000 to a high of 97.2 percent for households with incomes of \$100,000 or more. Average household contributions also increase significantly with household income. Households with an annual income under \$25,000 contributed \$587, on average, while those with an annual income of \$100,000 or more give \$3,976 on average. Conversely, *average contributions as a percentage of household income is inversely related to household income.*[20] In Figure 60 and Figure 61, I show the relationship between giving contribution and income level.

Table [15] illustrates the amount and percent of giving contributed by age group. From the table, the followings are noticeable. First, the composition of givers are almost equally distributed along the main age groups, i.e. {30-39, 40-49, 50-64}. Second, the average household contribution increases with age, with the exception of the over-65 age group. The proportion of households making formal contributions is significantly lower in the 21 to 29 age group, compared to all other age groups. In summary, giving varies significantly with

[20] Notice that this finding is a deviation from the U-shape pattern of giving explained in the Introduction.

Figure 58: Total Giving from 1970 to 2010 in Millions of Dollars (Inflation-adjusted). Source: Giving USA, 2011.

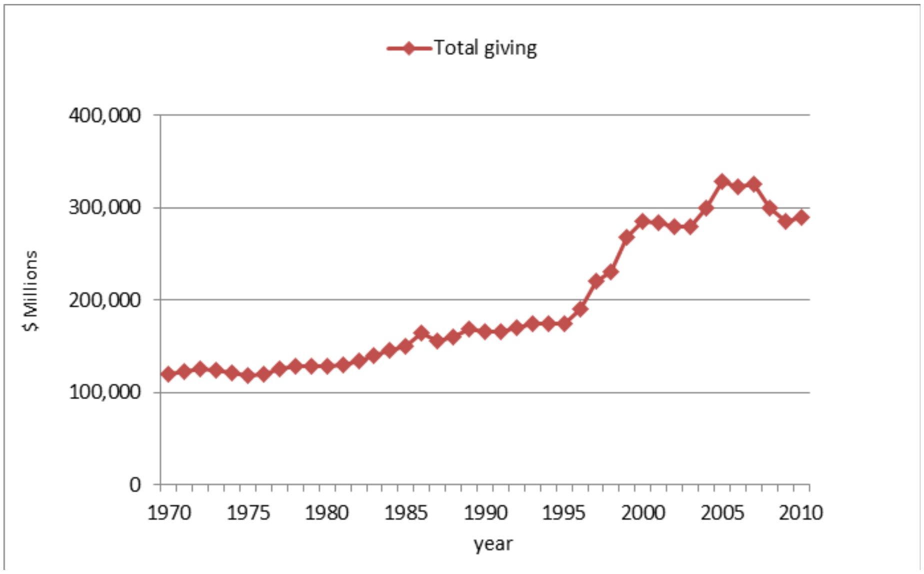


Figure 59: Total Giving as Percentage of GDP from 1970 to 2010. Source: Giving USA, 2011.

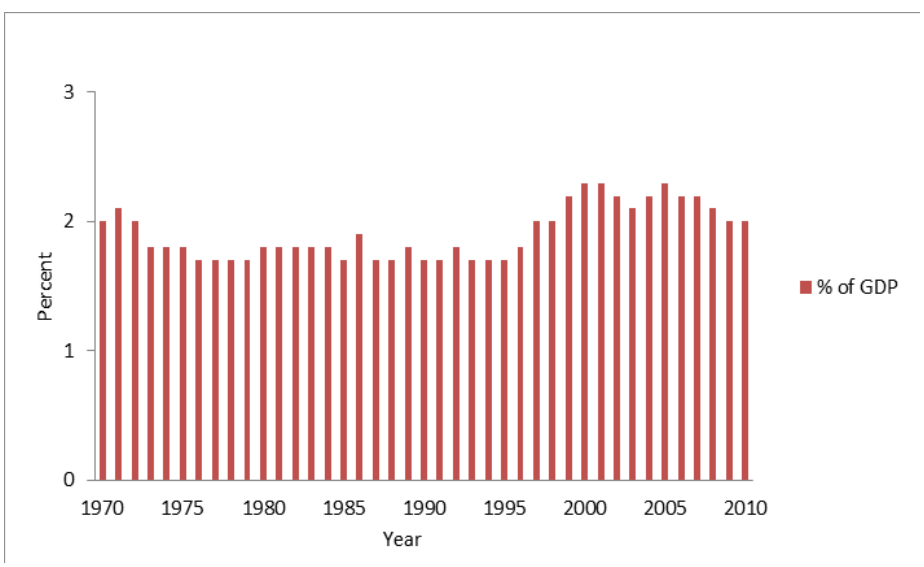


Table 14: Amount and Percent of Giving Contributed by Income Level

Income	Giving Likelihood		Avg. Amount		% Income	
	Givers	All	Givers	All	Givers	All
Under $25,000	76.8%	24.4%	587	439	4.2	3.2
$25,000-$49,999	87.5%	32.0%	1027	891	3.0	2.6
$50,000-$74,999	93.1%	21.4%	1766	1633	3.0	2.8
$75,000-$99,999	96.9%	9.8%	2109	2038	2.7	2.5
$100,000 or more	97.2%	12.4%	3976	3854	2.7	2.6
Total	89.0%	100.0%	1620	1415	3.1	2.7

Source: Giving and Volunteering in the United States, 2001

the age.[21] As people get older, they are typically more likely to give to charity and to give a greater fraction of their incomes. However, the average contribution as a percentage of household income also varies with age: those in the 65 and over age group give significantly more than all other age groups (4.7%), while the 21 to 29 age group give significantly less (2.1%). In summary, older households contribute more as a percentage of household income, compared to their younger counterparts. Figure 62 shows the amount of giving by age groups. From the data, it is easily seen that the pattern of giving over lifecycle is inverse-J shaped, increasing over age with decreasing rate. Also it describes the giving as percent of income is increasing over age. As households get older, they give more portions from their income.

Regarding volunteering time, from Table [16], it is shown that, 34.4 percent of those in the youngest age group volunteered in the past year, compared to the 30 to 39 (48.3%), 40 to 49 (48.4%), and the 50 to 64 (44.9%) age groups. According to this survey, although data is not shown here, it is also found that volunteering results display differences between employed

[21] By the data source, giving also varies significantly with educational attainment of the givers, although this part is not reported here. Those with more education give more often, give more dollars, and generally give a higher fraction of income.

Figure 60: Average Dollar Amounts of Giving Contributed by Income Level.

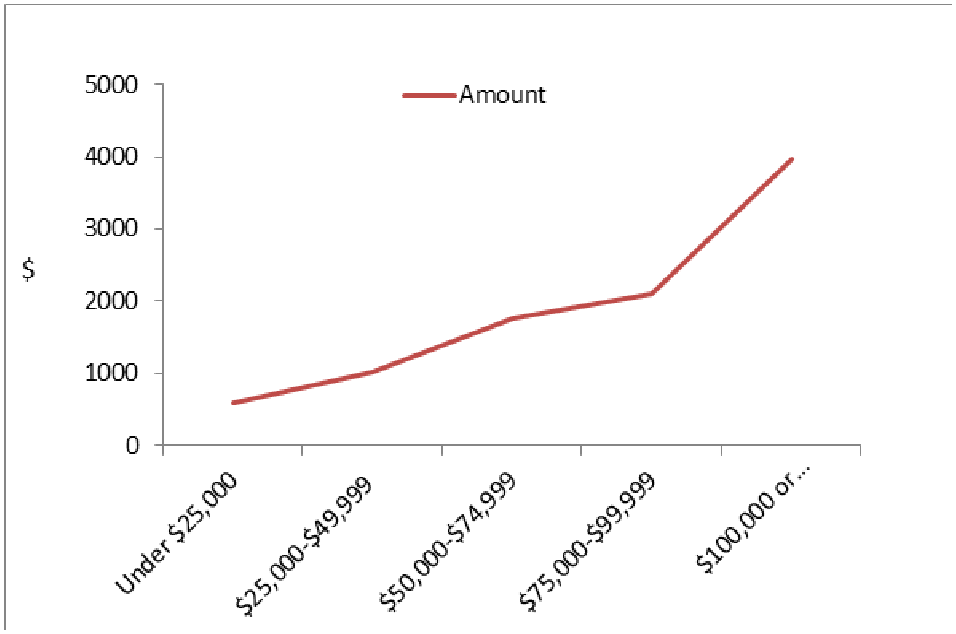
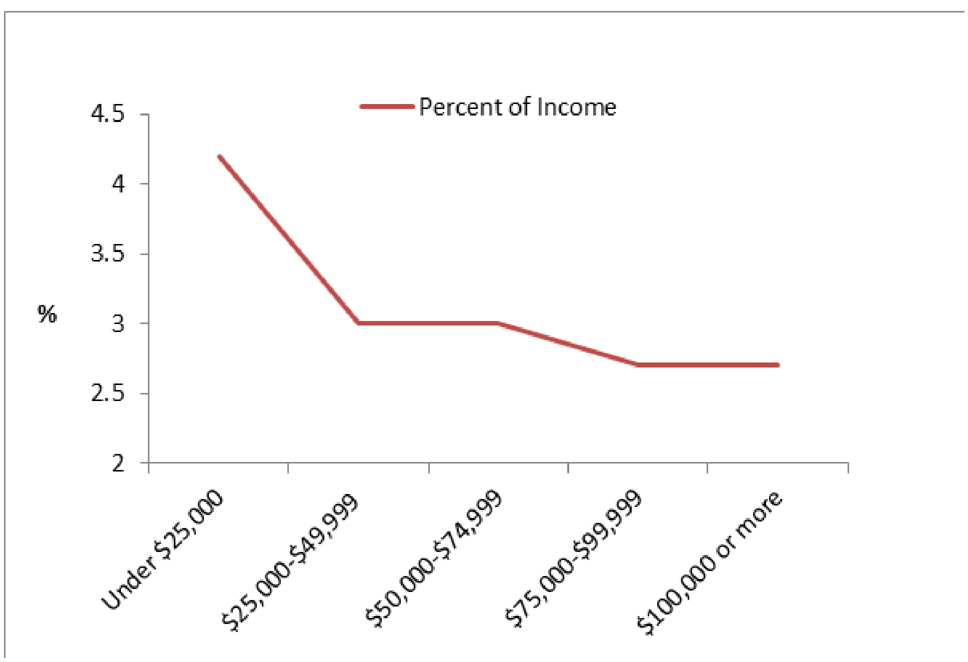


Figure 61: Percent of Income for Giving Contributed by Income Level.



and unemployed respondents. Employed respondents are more likely to have volunteered in the past year, compared to unemployed respondents (46.1% vs. 39.6%). Among past-year volunteers, however, employed respondents volunteer significantly fewer hours in the past month, compared to their unemployed counterparts (13.8 hours vs. 18.2 hours).

5.1.3 Literatures

Although there is no directly linked paper that I can refer to, there are two lines of related literatures. One is the literatures on charitable giving both in lab and field experiments and the other one is literatures related with lifecycle models from macroeconomic perspective. Regarding charitable giving, Andreoni (2006)[3] surveys many earlier lab models of charitable giving and provides evidence for warm glow dominance explained in the beginning of this paper.

Other than the motivation of giving, there are field research works that evaluate the effect of other variables like seed money[22] (List and Lucking-Reiley, 2002[74]), or the identity of the solicitor (Landry et al., 2006[72])[23] as an instrumental effect on giving. DellaVigna et al. (2009)[34] revisit the motivation of giving and perform a field experiment designed to distinguish intrinsic motivation from instrumental device, specifically social pressure from the solicitors. In their model, the social pressure yields negative utility. This implies that individuals donate because the cost of saying *NO* to a solicitor is very high. They confirm a clear role of social pressure in charitable giving, but also approve indirect evidence of intrinsic motivation among givers. Unlike the social pressure due to an unknown solicitor, Rege and Telle (2004)[91] examine the effect of social approval among peer groups in public good games. Frey and Meier (2004)[53] show that social information influences participation rates in fund-raising campaigns in mail fund-raising campaign.

Another consideration related with charitable giving, is the policy issue, especially the effect of tax treatment in charitable giving. From panel data, Auten et al. (2002)[13] estimate the income tax effect on giving and show that persistent income shocks have substantially

[22] Acording to List and Lucking-Reiley (2002)[74], charitable giving increases in the seed money because of the signaling effect from quality of the charity.

[23] Acording to the authors, in door-to-door fund-raising, charitable giving is affected by the type of solicitors.

larger impacts on charitable behavior than transitory shocks. They also show that the price elasticity is greater than income elasticity in giving behavior.[24]

Regarding the second line of macroeconomics literatures, there are two main topics: choice between consumption and leisure (Heckman, 1974[61]; Bullard and Feigenbaum, 2007[16]) and mortality risk (Hansen and Imrohoroglu, 2006[60]; Feigenbaum, 2008c[47]). Bullard and Feigenbaum (2007)[16] examine whether the inclusion of leisure in the utility function can help explain the hump-shaped lifecycle consumption pattern and related phenomena. It is Heckman (1974)[61] who demonstrates that a theoretical possibility exists in a lifecycle context when household productivity follows a hump-shaped lifecycle pattern, as it does in the data. Bullard and Feigenbaum (2007)[16] extend Heckman's partial equilibrium into general equilibrium and verify Heckman's findings in quantitative theoretic general equilibrium analysis.

Hansen and Imrohoroglu (2006)[60] explore the quantitative implications of uncertainty about the length of life and a lack of annuity markets for lifecycle consumption in a general equilibrium model in which markets are otherwise complete. Their model exhibits lifecycle consumption pattern which is consistent with data. In a lifecycle model with mortality risks, Feigenbaum (2008a)[45] shows that the equilibrium parameters of his model is close to the estimated ones in a buffer-stock saving model by Gourinchas and Parker (2002)[57], where borrowing constraints primarily account for the consumption hump. In Feigenbaum (2008a)[45], borrowing is virtually eliminated by the mortality risk and thus mortality supplants borrowing constraint as the explanation for the hump.

Regarding consumption pattern over lifecycle due to other mechanisms: Borrowing Constraint (Deaton,1991[32]), Income Uncertainty and Precautionary Saving (Carroll, 1997[24], 2009[25]; Gourinchas and Parker, 2002[57]; Aiyagari, 1994[1]; Feigenbaum, 2008b[46]), Consumer Durables (Fernandez-Villaverde and Krueger, 2010[49]).

[24] I revisit this issue with model's prediction.

5.1.4 Chapter Organization

The chapter is organized as follows. In the next section, I present closed form solution for a warm glow model, solving three steps of intra-temporal and intertemporal optimization. Using these results, I derive analytic conditions for the endogenous choice of time between leisure (or volunteer time) and labor supply. Then in the subsequent section, I present a full model and describe the baseline general equilibrium for the model of warm glow preferences in an overlapping generations economy. In the fourth section, quantitative analysis follows. Target variables with US data, calibration method and the results are addressed. The result section discusses the main questions addressed in the model. Thereafter, the chapter concludes with the emphasis on the implication of social preferences, as well as of the employment of the non-standard preferences to complement the standard ones.

5.2 A MODEL OF WARM GLOW

This section introduces a tractable version of the warm glow model that a closed form solution can be obtained. The main purpose of this section is to get insight into the structure of optimization procedure for a utility specification of warm glow through analytic solution. Consider an agent with warm glow[25] who gets utility from the act of giving in the form of both goods and time. This agent receives utility from consuming his own consumption goods and leisure, as well. He lives for T-period and each period he has one unit of time endowment, as well as a stream of productivity measured in efficiency units. Each time, the agent can choose freely a portion of time endowment to work for consumption or charitable goods at market-determined real wage rate w per efficiency unit and enjoy the remaining free time either in leisure for himself or volunteering for charitable works. He discounts future period utility with rate of $\beta \in (0, 1)$. Finally, there is no borrowing constraint so that the agent can borrow or save freely at market determined interest rate R which is exogenous.

[25] As explained in the Introduction, the preference of warm glow agent is represented by $u(x, g)$ where x is private consumption and g is private giving. In this case, the warm glow is pure warm glow.

Then the optimization problem of the agent at $t = 0$ is,

$$Max \sum_{t=0}^{T} \beta^t u(x_t, g_t, l_t, v_t) \tag{5.1}$$

subject to

$$x_t + (1 - t_1)g_t + b_{t+1} = (1 - t_2)we_t(1 - l_t - v_t) + (1 - t_3)Rb_t$$
$$x_t \geq 0$$
$$g_t \geq 0$$
$$l_t \geq 0$$
$$v_t \geq 0$$
$$1 \geq l_t + v_t \geq 0$$

where x_t is consumption, g_t is charitable giving, l_t is leisure, and v_t is volunteering time. Also e_t is productivity unit of labor supply and 1 is the normalized endowment of time. There are three tax rates: t_1 is tax rate on charitable deduction, implying $1 - t_1$ is the price of giving, while t_2 and t_3 are the tax rates on labor income and financial income, respectively. Thus $(1 - t_2)we_t$, the opportunity cost of leisure, is the price of volunteering. Given the productivity profile, the agent provides $1 - (l_t + v_t)$ units of time for labor so that he earns $(1 - t_2)we_t(1 - l_t - v_t)$ of labor income each time. Because he can save or borrow freely under market determined interest rate R, he earns financial income of $(1 - t_3)Rb_t$ if he carries the bond to next period. Rewrite the budget constraint to get

$$x_t + (1 - t_1)g_t + (1 - t_2)we_t[l_t + v_t] + b_{t+1} = (1 - t_2)we_t + (1 - t_3)Rb_t \tag{5.2}$$

The RHS of the equation implies all the explicit and implicit resources in monetary value. The LHS consists of two parts: (a) expenditure on goods and time and (b) saving (borrowing) for the next period. It may be convenient to separate (a) from (b). Let M_t be defined for (a) by $M_t \equiv x_t + (1 - t_1)g_t + (1 - t_2)we_t[l_t + v_t]$. Thus, M_t implies the monetary resource necessary for both goods expenditure and time spending for each period. Then the maximization problem is decomposed into two sub-problems:

(i) Intra-temporal optimization

$$V\left(M, we\right) = Max_{x,l,g,v} u\left(x, g, l, v\right) \tag{5.3}$$

subject to

$$x + (1 - t_1)g + (1 - t_2)we[l + v] = M$$
$$x, g, l, v \geq 0$$
$$1 \geq l + v \geq 0$$

(ii) Inter-temporal optimization

$$Max_{M_t, b_{t+1}} \sum_{t=0}^{T} \beta^t V\left(M_t \ , we_t\right) \tag{5.4}$$

subject to

$$M_t + b_{t+1} = (1 - t_2)we_t \ + (1 - t_3)Rb_t$$
$$b_0 = 0$$
$$b_{T+1} = 0$$

By solving the first problem one gets optimal consumption-giving and leisure-volunteer level as a function of total resource level M committed to spend each time, given the price of good and time.[26] The indirect utility function is determined by the four choice functions, from which the agent derives utility. Given this value function, by solving the second problem, the optimal expenditure level and bond demand over time are obtained. From now on, the within-period utility is specified by the following CRRA utility function which satisfies Inada condition:

$$u(x, g, l, v) = \left\{ \begin{array}{ll} \frac{1}{1-\gamma}[c^{\eta}h^{1-\eta}]^{1-\gamma} & \text{if } \ \gamma \neq 1 \\ & \\ \ln \ c^{\eta}h^{1-\eta} & \text{if } \ \gamma = 1 \end{array} \right\} \tag{5.5}$$

where

$$c = x^{\theta}g^{1-\theta}$$

[26] Here it is assumed that the price of consumption goods and price of giving goods are equal to each other, other than the tax deduction on giving.

represents total good consumption and the total hour spending other than labor is

$$h = l^{\sigma} v^{1-\sigma}$$

with $\eta\epsilon(0,1)$, $\theta\epsilon(0,1)$, $\sigma\epsilon(0,1)$ and $\gamma > 0$. Notice that with this specification, the non-negativity constraint of the consumption, giving, leisure, and volunteer would not be binding. The first step to tackle the problem for closed form solution is to reduce the four choice variables into two variables. Thanks to the Cobb-Douglas utility specification, this can be done without difficulty. From the first order condition, it is satisfied that

$$\frac{x}{(1-t_1)\theta} = \frac{g}{(1-\theta)} \tag{5.6}$$

$$\frac{l}{\sigma} = \frac{v}{(1-\sigma)} \tag{5.7}$$

These conditions dictate relative importance of the two arguments in each utility component of goods and time: $c = x^{\theta} g^{1-\theta}$ and $h = l^{\sigma} v^{1-\sigma}$. Once giving (g) and volunteer (v) are written in terms of consumption (x) and leisure (l), or vice versa, it is convenient to rewrite the agent's intra-temporal problem in terms of (x, l) as in the following:

$$Max_{x,l} \frac{D}{1-\gamma} \left(x^{\eta} l^{1-\eta}\right)^{1-\gamma} \tag{5.8}$$

subject to

$$[1 + \tfrac{(1-\theta)}{\theta}]x + (1-t_2)we[1 + \tfrac{(1-\sigma)}{\sigma}]l = M$$
$$\sigma \geq l \geq 0$$

for some constant $D = (A^{\eta} B^{1-\eta})^{1-\gamma}$, where $A = \left(\frac{1-\theta}{(1-t_1)\theta}\right)^{1-\theta}$ and $B = \left(\frac{1-\sigma}{\sigma}\right)^{1-\sigma}$. Notice that the leisure is constrained by a fixed variable σ. Once again the budget constraint can be reduced to simpler form of $\frac{1}{\theta}x + (1-t_2)we(\frac{1}{\sigma})l = M$. Therefore, the maximization problem of four choice variables is reduced to that of two choice variables only. With this preliminary work, the model is analyzed via intra-temporal and inter-temporal optimization.

5.2.1 Solving Intra-temporal optimization

$$V(M, we) = Max_{x,l} \frac{D}{1-\gamma} \left(x^{\eta} l^{1-\eta}\right)^{1-\gamma} \tag{5.9}$$

subject to

$$\frac{1}{\theta}x + (1-t_2)we(\tfrac{1}{\sigma})l = M$$
$$x \geq 0$$
$$\sigma \geq l \geq 0$$

The objective of intra-temporal optimization is to allocate consumption and leisure level, given any expenditure level and wage rate, subject to leisure constraint. The last line implies that when the agent does not work at all, he can enjoy full leisure of σ (and thus, full volunteer of $1-\sigma$) and if he ever works, it should be less than σ. From FOC, when leisure constraint does not bind and an interior solution is obtained, it is satisfied that

$$\frac{x}{\theta\eta} \leq \frac{(1-t_2)wel}{\sigma(1-\eta)} \tag{5.10}$$

and when leisure does bind,

$$\frac{x}{\theta\eta} > \frac{(1-t_2)wel}{\sigma(1-\eta)} \tag{5.11}$$

This shows that if the leisure is not binding, then the market value of consumption and leisure is equalized by their utility shares which are η and $1-\eta$. Likewise, if the leisure is binding to the constraint, then the market value of leisure would be less than that of the consumption, relative to their utility shares. Using the resource constraint, $\frac{1}{\theta}x + (1-t_2)we(\frac{1}{\sigma})l = M$, the optimal consumption and leisure levels are derived for any specific resource level and wage rate. Because it is satisfied $(1-t_2)we < (1-\eta)M$ when leisure binds, the solution is determined by the threshold level $M^* \equiv \frac{(1-t_2)we}{(1-\eta)}$. Thus, depending on whether the leisure is binding or not, the optimal choice sets are:

$$x(M,we) = \begin{Bmatrix} \theta\eta M & if \quad M \leq M^* \\ & \\ \theta[M-(1-t_2)we] & if \quad M > M^* \end{Bmatrix}^{27} \tag{5.12}$$

$$l(M,we) = \begin{Bmatrix} \sigma(1-\eta)\frac{M}{(1-t_2)we} & if \quad M \leq M^* \\ & \\ \sigma & if \quad M > M^* \end{Bmatrix} \tag{5.13}$$

The first line of each term is the optimal consumption and leisure when the constraint is not binding. The second line of each term implies that when the market value of his time is low relative to its optimal value, then the agent would not work at all but spend all his time for leisure and volunteer. From the two choice functions, two facts are noticed: first, the risk aversion parameter γ does not affect any of the optimal choices intra-temporally. Second, as η increases, the threshold resource level M^* increases for a given wage rate. This implies that as the utility share of consumption (and giving) relative to leisure (and volunteer) increases, the agent is more likely to retire later than sooner. Furthermore, because $g = \frac{(1-\theta)}{(1-t_1)\theta}x$ and $v = \frac{(1-\sigma)}{\sigma}l$, it is also true that

$$g(M,we) = \begin{Bmatrix} (1-\theta)\eta\frac{M}{(1-t_1)} & if \quad M \leq M^* \\ & \\ \frac{1-\theta}{1-t_1}[M-(1-t_2)we] & if \quad M > M^* \end{Bmatrix} \tag{5.14}$$

$$v(M,we) = \begin{Bmatrix} (1-\sigma)(1-\eta)\frac{M}{(1-t_2)we} & if \quad M \leq M^* \\ & \\ 1-\sigma & if \quad M > M^* \end{Bmatrix} \tag{5.15}$$

[27] $M > M^*$ implies $(1-\eta)M > we(1-t_2)$.

Figure 63 and 64 show the four choice functions over the domain for certain values of parameters by which binding occurs within the domain range. The parameters are $\{\gamma = 0.5,\ \eta = 0.5,\ \theta = 0.8,\ \sigma = 0.8,\ t_1 = 0.1,\ t_2 = 0.1,\ we = 15\}$. It is easy to notice that consumption and giving are increasing functions of the expenditure, although the increasing rate is not same for all expenditure domain. Likewise, leisure and volunteer are increasing functions of the expenditure up to the point where the agent decides not to work but enjoy full leisure and volunteer.[28] The indirect utility function, i.e. the value function of the optimization is given by

$$V\left(M, we\right) =$$

$$\left\{ \begin{array}{ll} \frac{D}{1-\gamma}\left[(\theta\eta)^{\eta}\left(\frac{\sigma(1-\eta)}{(1-t_2)we}\right)^{1-\eta} M\right]^{1-\gamma} & if \quad M \leq M^* \\ \\ \frac{D}{1-\gamma}\left[\theta^{\eta}[M-(1-t_2)we]^{\eta}\sigma^{1-\eta}\right]^{1-\gamma} & if \quad M > M^* \end{array} \right\} \tag{5.16}$$

Now let us explore several properties of the value function, together with those of consumption, giving, leisure, and volunteer. First, regarding the value function:

Proposition 21. *The value function $V(M, we)$ is continuous in M all over the domain including threshold M^*.*

Proof. See Appendix C-1. □

Second, it is useful to know if the derivative of the value function is continuous. This property is important for further analysis in inter-temporal problem. The derivative of $V\left(M, we\right)$ is obtained by

[28] Beyond the threshold, any increase in resource is fully absorbed into the consumption and giving because the agent has reached full level of leisure and volunteer and does not need to buy extra time for leisure or volunteer.

$$\frac{dV(M,we)}{dM} =$$

$$\left\{ \begin{array}{ll} D\left[(\theta\eta)^{\eta}\left(\frac{\sigma(1-\eta)}{(1-t_2)we}\right)^{1-\eta}\right]^{1-\gamma} M^{-\gamma} & if \quad M \leq M^* \\ D\eta\theta^{\eta(1-\gamma)}\left[M-(1-t_2)we\right]^{\eta(1-\gamma)-1}\sigma^{(1-\eta)(1-\gamma)} & if \quad M > M^* \end{array} \right\} \tag{5.17}$$

Moreover, one can take further step to get its second derivative. Thus,

$$\frac{dV^{\ 2}(M,we)}{dM^{\ 2}} =$$

$$\left\{ \begin{array}{ll} -\gamma D\left[(\theta\eta)^{\eta}\left(\frac{\sigma(1-\eta)}{(1-t_2)we}\right)^{1-\eta}\right]^{1-\gamma} M^{-\gamma-1} & if \ M \leq M^* \\ D\eta(\eta(1\text{-}\gamma)\text{-}1)\theta^{\eta(1-\gamma)}\left[M\text{-}(1\text{-}t_2)we\right]^{\eta(1-\gamma)-2}\sigma^{(1-\eta)(1-\gamma)} & if \ M > M^* \end{array} \right\} \tag{5.18}$$

Proposition 22. *The value function is continuously differentiable all over the domain M including threshold and is strictly concave in M.*

Proof. See Appendix C-2. □

The value function is continuous, continuously differentiable and strictly concave with respect to M. How about the behavior of its derivative $\frac{dV(M,we)}{dM}$? Although $\frac{dV(M,we)}{dM}$ is continuous at the threshold and thus continuous all over the domain, it has two different slopes along the threshold. Thus the agent does not work, i.e. retire endogenously and enjoys full leisure and volunteer time, i.e. $\sigma = l$ if and only if

$$\frac{dV\left(M,we\right)}{dM} \leq D\left(\theta\eta\right)^{\eta(1-\gamma)}\left(\frac{1-\eta}{(1-t_2)we}\right)^{1-\eta+\gamma\eta}\sigma^{(1-\gamma)(1-\eta)}. \tag{5.19}$$

Third, let us check the properties of consumption, giving, leisure and volunteer functions:

Proposition 23. *The optimal consumption, giving, leisure, and volunteer choices are continuous at the threshold.*

Proof. See Appendix C-3. □

However, it is easy to see that none of the policy functions are continuously differentiable everywhere because at the threshold,

$$\lim_{M\uparrow\frac{(1-t_2)we}{1-\eta}} \frac{dx(M,we)}{dM} = \theta\eta \neq \theta = \lim_{M\downarrow\frac{(1-t_2)we}{1-\eta}} \frac{dx(M,we)}{dM}$$

for the consumption and

$$\lim_{M\uparrow\frac{(1-t_2)we}{1-\eta}} \frac{dl(M,we)}{dM} = \frac{\sigma(1-\eta)}{(1-t_2)we} \neq 0 = \lim_{M\downarrow\frac{(1-t_2)we}{1-\eta}} \frac{dl(M,we)}{dM}$$

for the leisure.[29] It is clear to see that the value function is continuous and smooth, and thus differentiable over all area of M. Likewise, the marginal value is continuous, although it is not differentiable at the threshold.

[29] The other two choice variables also are not differentiable because

$$\lim_{M\uparrow\frac{(1-t_2)we}{1-\eta}} \frac{dg(M,we)}{dM} = \frac{(1-\theta)\eta}{(1-t_1)} \neq \frac{(1-\theta)}{(1-t_1)} = \lim_{M\downarrow\frac{(1-t_2)we}{1-\eta}} \frac{dg(M,we)}{dM}$$
$$\lim_{M\uparrow\frac{(1-t_2)we}{1-\eta}} \frac{dv(M,we)}{dM} = \frac{(1-\sigma)(1-\eta)}{(1-t_2)we} \neq 0 = \lim_{M\downarrow\frac{(1-t_2)we}{1-\eta}} \frac{dv(M,we)}{dM}$$

Table 15: Amount and Percent of Giving Contributed by Age Group

Age	Population		Amount (Income)		% of Income	
	Givers	All	Givers	All	Givers	All
21-29	81.4%	16.9%	825 (48,513)	668 (45,705)	2.1	1.7
30-39	88.2%	22.0%	1,466 (63,011)	1,285 (60,099)	2.7	2.4
40-49	90.5%	21.3%	1,827 (67,254)	1,643 (63,712)	2.8	2.5
50-64	90.1%	22.7%	1,912 (60,573)	1,704 (57,636)	3.2	2.8
over 65	88.0%	17.1%	1,718 (38,315)	1,484 (35,958)	4.7	4.1
Total	89.0%	100%	1,620 (56,535)	1,415 (53,432)	3.1	2.7

Source: Giving and Volunteering in the United States, 2001

Figure 62: The Amounts of Giving by Age Groups.

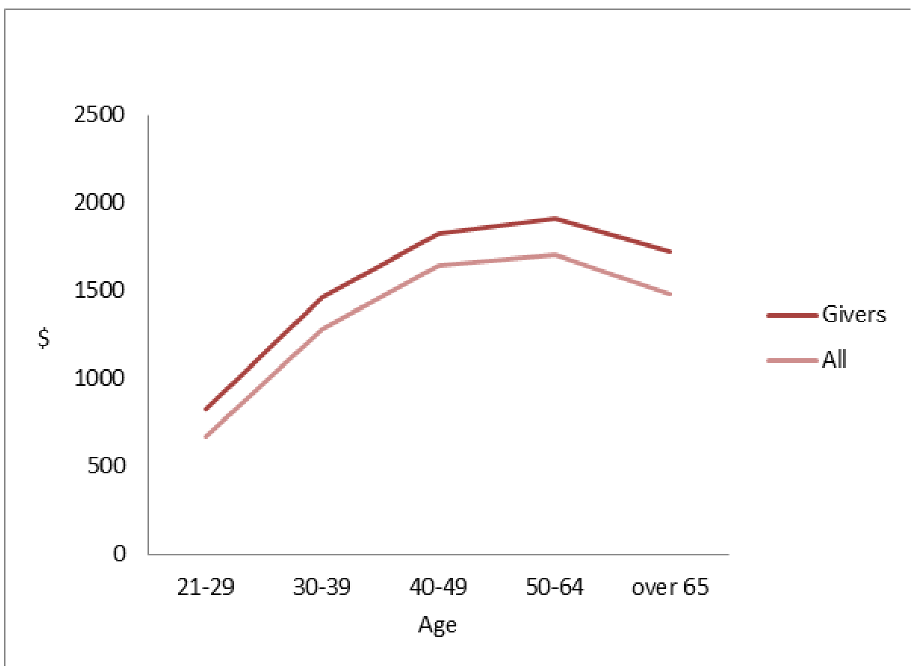


Table 16: Volunteer and Giving among Volunteers by Age Group

Age	Population		Volunteer Hours		Giving % (Income)	
	Volunteers	All	Volunteers	All	Volunteers	All
21-29	34.4%	16.9%	15.5	5.3	2.4% (49,880)	1.7% (45,705)
30-39	48.3%	22.0%	15.1	7.2	3.3% (67,990)	2.4% (60,099)
40-49	48.4%	21.3%	15.5	7.4	3.4% (73,596)	2.5% (63,712)
50-64	44.9%	22.7%	13.7	6.1	3.9% (66,352)	2.8% (57,636)
over 65	40.6%	17.1%	16.2	6.5	5.7% (42,829)	4.1% (35,958)
Total	44.2%	100%	15.1	6.6	3.8% (62,375)	2.7% (53,432)

Source: Giving and Volunteering in the United States, 2001

Figure 63: Optimal Consumption and Giving. Both Consumption and Giving increase as the resource level increases.

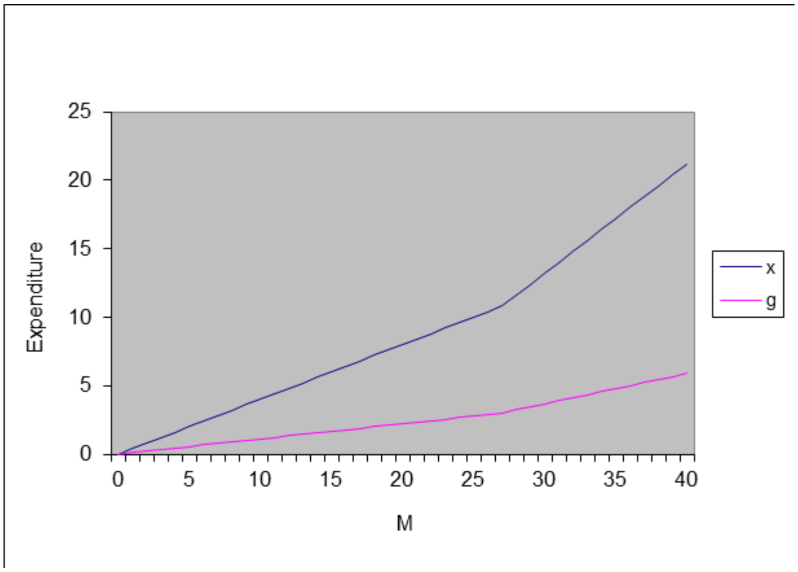
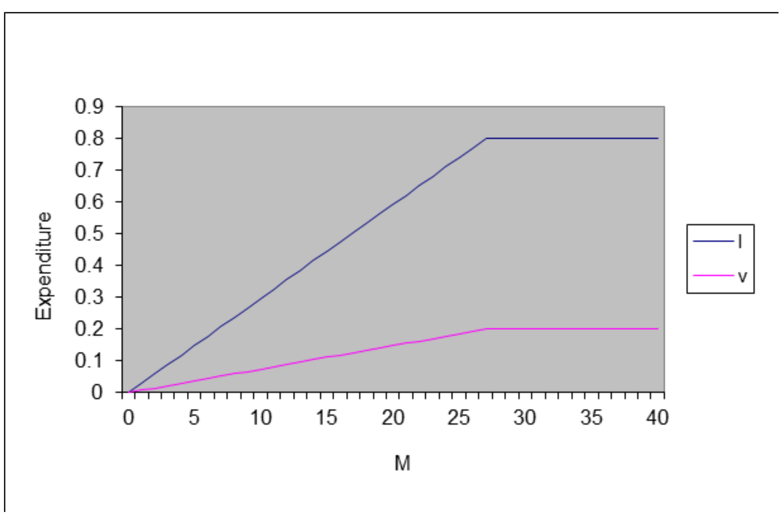


Figure 64: Leisure and Volunteer increase up to the full leisure/volunteer level as the resource level increases.



5.2.2 Solving Inter-temporal optimization

Let the time-indexed resource level be $M_t = \frac{1}{\theta}x_t + \left(\frac{(1-t_2)we_t}{\sigma}\right) l_t$ and let $V(M_t, we_t)$ be the indirect utility function obtained from the analysis in previous section. Then the maximization problem for the agent over time is

$$Max_{M_t, b_{t+1}} \sum_{t=0}^{T} \beta^t V(M_t, we_t) \tag{5.20}$$

subject to

$$M_t + b_{t+1} = (1-t_2)we_t + (1-t_3)Rb_t$$
$$b_0 = 0$$
$$b_{T+1} = 0$$

Solving for the first order condition to get[30]

$$\beta^t V_M(M_t, we_t) = \frac{\lambda}{[(1-t_3)R]^t} \tag{5.21}$$

or

$$V_M(M_t, we_t) = \frac{\lambda}{[(1-t_3)\beta R]^t}. \tag{5.22}$$

where λ is Lagrangian multiplier and $V_M = \frac{dV}{dM}$. Because for a value of we_t, the derivative of value function $V_M(M_t, we_t)$ is strictly decreasing in M_t,[31] there exists an inverse function over the domain. Let $\Psi = V_M^{-1}(M_t, we_t)$ be the inverse function. Then from first order condition above, it is true that

$$M_t = V_M^{-1}\left(\frac{\lambda}{[(1-t_3)\beta R]^t}, we_t\right) = \Psi\left(\frac{\lambda}{[(1-t_3)\beta R]^t}, we_t\right). \tag{5.23}$$

Plugging M_t back into the budget constraint to get

$$\sum_{t=0}^{T} \frac{\Psi\left(\frac{\lambda}{[(1-t_3)\beta R]^t}, we_t\right)}{[(1-t_3)R]^t} = \sum_{t=0}^{T} \frac{(1-t_2)we_t}{[(1-t_3)R]^t}. \tag{5.24}$$

[30] The (T+1) inter-temporal budget constraints are reduced to a single equation budget constraint over life time: $\sum_{t=0}^{T} \frac{(1-t_2)we_t}{[(1-t_3)R]^t} = \sum_{t=0}^{T} \frac{M_t}{[(1-t_3)R]^t}$.

[31] This is proved in Proposition 2 where $V(M, we)$ is strictly concave in M.

Then solving for λ that satisfies this equation, the optimal M_t is obtained by $M_t = \Psi\left(\frac{\lambda^*}{[(1-t_3)\beta R]^t}, we_t\right)$, where λ^* is the value solved from the constraint equation. Once M_t is obtained, the next step is straightforward. In earlier section, the intra-temporal optimization produces the optimal consumption, giving, leisure, and volunteer levels that depend on M_t. Therefore, given the optimal resources schedule over time, the expenditure portion for each of these is obtained by those equations.

5.2.3 Endogenous Retirement

As seen above, the agent chooses freely his optimal working hours, and thus optimal leisure and volunteer hours each period. If the agent optimally chooses full leisure, $l = \sigma$ and thus full volunteer $v = 1 - \sigma$, this implies that he retires endogenously from his life time optimization procedure. In this section, I want to explore the conditions under which the agent chooses either to retire early or postpone entering into the labor market. Assume $t_3 = 0$ for notational simplicity and revisit the equation (22), the optimality condition of the inter-temporal problem above.

$$V_M\left(M_t,\ we_t\right) = \frac{\lambda}{(\beta R)^t}$$

Thus, following the value of V_M in equation (17), rewrite the condition into[32]

$$D\left[(\theta\eta)^\eta\left(\frac{\sigma\left(1-\eta\right)}{(1-t_2)we_t}\right)^{1-\eta}\right]^{1-\gamma} M_t^{-\gamma} = \frac{\lambda}{(\beta R)^t} \tag{5.25}$$

or

$$M_t = \left(\frac{\lambda}{\kappa_t\left(\beta R\right)^t}\right)^{-\frac{1}{\gamma}} \tag{5.26}$$

where $\kappa_t = D\left[(\theta\eta)^\eta\left(\frac{\sigma(1-\eta)}{(1-t_2)we_t}\right)^{1-\eta}\right]^{1-\gamma}$. Combining this with the budget constraint gives the solution for λ which is[33]

$$\lambda^* = \left(\frac{\sum_{t=0}^{T}\frac{(1-t_2)we_t}{R^t}}{\sum_{t=0}^{T}\kappa_t^{\frac{1}{\gamma}}\frac{(\beta R)^{\frac{t}{\gamma}}}{R^t}}\right)^{-\gamma}. \tag{5.27}$$

[32] This is because $V_M\left(M_t, we_t\right) = D\left[(\theta\eta)^\eta\left(\frac{\sigma(1-\eta)}{(1-t_2)we_t}\right)^{1-\eta}\right]^{1-\gamma} M_t^{-\gamma}$

[33] This follows from $\sum_{t=0}^{T}\frac{\left(\frac{\lambda}{\kappa_t(\beta R)^t}\right)^{-\frac{1}{\gamma}}}{R^t} = \sum_{t=0}^{T}\frac{(1-t_2)we_t}{R^t}$.

The Lagrangian multiplier λ^* in the inter-temporal optimization is the key value that determines the marginal value function and thus M_t, which is obtained via inverse image of the marginal value. Therefore the optimal resource schedule is obtained through this term. When the agent is working, the marginal value of one unit of resource spending is greater than its implicit value (threshold) and he is not working, the marginal value is less than the threshold. Thus the optimal resource schedule is represented by the two functional forms of λ^* subject to the marginal value area relative to the threshold. Therefore the optimal resource commitment schedule for *work and partial leisure/volunteer*, $\{\sigma > l > 0,\ 1-\sigma > v > 0\}$ or *no work and full leisure/volunteer*, $\{\sigma = l > 0,\ \ 1-\sigma = v > 0\}$ is:

$$M_t = \left\{ \begin{array}{ll} \left(\frac{\lambda^*}{\kappa_t(\beta R)^t}\right)^{-\frac{1}{\gamma}} & if \quad M_t \leq M_t^* \iff \frac{\lambda^*}{(\beta R)^t} \geq \kappa_t^* \\ \\ \left(\frac{\lambda^*}{\overline{\kappa}_t(\beta R)^t}\right)^{-\frac{1}{\overline{\gamma}}} + (1-t_2)we_t & if \ M_t > M_t^* \iff \frac{\lambda^*}{(\beta R)^t} < \kappa_t^* \end{array} \right\} \tag{5.28}$$

where $\kappa_t^* = \kappa_t \left(\frac{(1-t_2)we_t}{1-\eta}\right)^{-\gamma} = \overline{\kappa}_t \left(\frac{\eta(1-t_2)we_t}{1-\eta}\right)^{-\overline{\gamma}}$ with $\overline{\kappa}_t = D\eta\theta^{\eta(1-\gamma)}\sigma^{(1-\eta)(1-\gamma)}$ and $\overline{\gamma} = 1-\eta\,(1-\gamma)$. Next two figures (Figure 65 and Figure **??**) show the marginal value and optimal resource over time when income has an inverse-U shaped schedule. Both functions are obtained from choice functions solved above when binding occurs. From those, it is clear that the agent keeps working until the marginal value gets lower than the threshold as in Figure 65. The parameters are $\{\gamma = 0.5,\ \eta = 0.5,\ \theta = 0.8, \sigma = 0.8,\ \beta = 0.98,\ t_1 = 0.1,\ t_2 = 0.1,\ w = 15,\ R = 1.045\}$.

Now let us explore some properties for special cases. It may be interesting to look for necessary condition for endogenous choice of leisure or work for specific cases of βR condition. Assume that the agent has a schedule of *constant productivity* $e_t = 1$ over time, thus $we_t = w$.

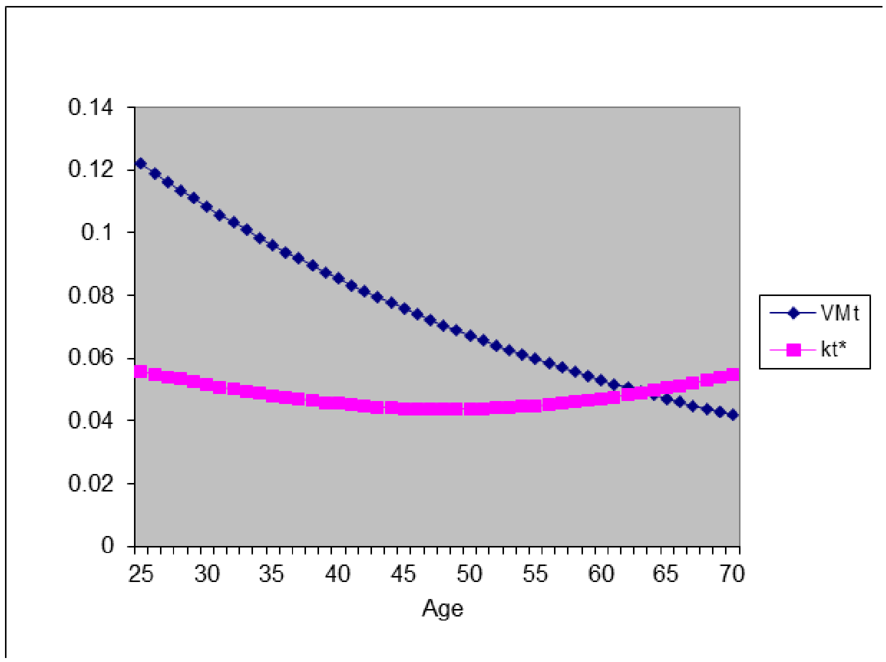
Proposition 24. *It is impossible to have full leisure and full volunteer all the time.*

Proof. See Appendix C-4. □

The proposition implies that the agent has to work in some period of his life time.

Proposition 25. *If $\beta R = 1$, then the agent never gets full leisure nor full volunteer all the time.*

Figure 65: Marginal Value Function and Threshold Function with Non-Fixed Income.



Proof. See Appendix C-5. □

The proposition implies that the agent works all the time without retirement under this specific condition, together with constant productivity.

Proposition 26. *If $\beta R > 1$, then once the agent reaches full leisure and full volunteer stage, then he never reduces them.*

Proof. See Appendix C-6. □

This is because the scheduled resource level keeps increasing over time and never reduces. In other words, if he stops working then he never comes back to work again, i.e. permanent withdrawal from work force or retirement.[34] Therefore, together with $\beta R > 1$, the early retirement (or full leisure and volunteer) condition is

$$V_M \leq D\,(\theta\eta)^{\eta(1-\gamma)} \left(\frac{1-\eta}{(1-t_2)w}\right)^{1-\eta+\gamma\eta} \sigma^{(1-\gamma)(1-\eta)}. \tag{5.29}$$

[34] Remember that this is obtained with the special condition of $R\beta$, together with constant labor income.

Thus an agent retires early and enjoys full leisure and full volunteer at T^* if and only if

$$\frac{\lambda^*}{(\beta R)^{T^*}} \leq \omega^*. \tag{5.30}$$

where $\omega^* = D\left(\theta\eta\right)^{\eta(1-\gamma)}\left(\frac{1-\eta}{(1-t_2)w}\right)^{1-\eta+\gamma\eta}\sigma^{(1-\gamma)(1-\eta)}$. On the other hand, If $\beta R < 1$, then $V_M\left(M_t, w\right) = \frac{\lambda}{(\beta R)^t}$ is increasing and M_t is decreasing over time. So if he ever enjoys full leisure and volunteer without working, then it should be at the beginning of life. Therefore the agent delays entering into the labor force as long as

$$V_M \leq \omega^*. \tag{5.31}$$

Therefore he starts giving up full leisure and full volunteer to start working at t^* if and only if

$$\frac{\lambda}{(\beta R)^{t^*}} \geq \omega^*. \tag{5.32}$$

where ω^* is defined above.

5.3 A LIFECYCLE MODEL OF CHARITABLE GIVING

In this section a lifecycle model of agents with warm glow preference is introduced first in a partial equilibrium, then in a general equilibrium. There are $(T+1)$ types of identical cohorts, who consume goods and time, save or dissave, and supply labor for production every period in an overlapping generations (OLG) economy.

5.3.1 Consumer

5.3.1.1 Environment Time is discrete. At each time a generation of identical cohorts is born. The population is constant. Each agent lives to a maximum age of T in a $(T+1)$ - period overlapping generations economy. Agents are subject to mortality risks and survive until age $s \geqslant t$, with a probability Q_t, which is assumed to be generation (or cohort)-independent. The agents who do not survive up to t leave their assets in the form of bequest B_t to charity B_{1t} and to the remaining offsprings B_{2t}. Thus $B_t = B_{1t} + B_{2t}$. The bequests to offsprings are spread uniformly over the surviving population. Agents with a warm glow preference get utility from both consuming goods and leisure and act of giving goods and time to charity. Over lifecycle agents are endowed with one unit of labor productivity, measured in efficiency units e_t, which is supplied *elastically* at the market determined real wage rate w which is assumed to be stationary over time. There exists a composite good that can be either consumed by agents themselves or given to charity. The remaining portion is saved for next period, in which case it is called capital, each unit of which earns market determined gross return $R = r + 1$.

5.3.1.2 Consumer Optimization Behavior Because a member of each cohort maximizes his expected utility following the survivor function Q_t, the consumer's problem is to maximize his expected utility

$$Max \sum_{t=0}^{T} \beta^t Q_t \frac{1}{1-\gamma} [c_t^{\eta} h_t^{1-\eta}]^{1-\gamma} \tag{5.33}$$

subject to

$$c_t = x_t^{\theta} g_t^{1-\theta}$$

$$h_t = l_t^{\sigma} v_t^{1-\sigma}$$

$$x_t + (1-t_1)g_t + b_{t+1} = (1-t_2)[we_t(1-l_t-v_t) + B_{2t}] + (1-t_3)Rb_t$$

$$1 \geq l_t + v_t \geq 0$$

where t_1, t_2 and t_3 are tax rates on charity, labor income, and financial income respectively. Parameters satisfy $\eta\epsilon(0,1)$, $\theta\epsilon(0,1)$, $\sigma\epsilon(0,1)$ and $\gamma > 0$. Solving the problem follows the same maximization procedure described in section 2. If the total resource is defined as $M_t = \frac{1}{\theta}x_t + \left(\frac{(1-t_2)we_t}{\sigma}\right) l_t$, then the intra-temporal result is same as before. Because of Q_t and B_{2t}, however, the inter-temporal optimization procedure should be modified to

$$Max_{M_t, b_{t+1}} \sum_{t=0}^{T} \beta^t Q_t V(M_t, we_t) \tag{5.34}$$

subject to

$$M_t + b_{t+1} = (1-t_2)[we_t + B_{2t}] + (1-t_3)Rb_t$$

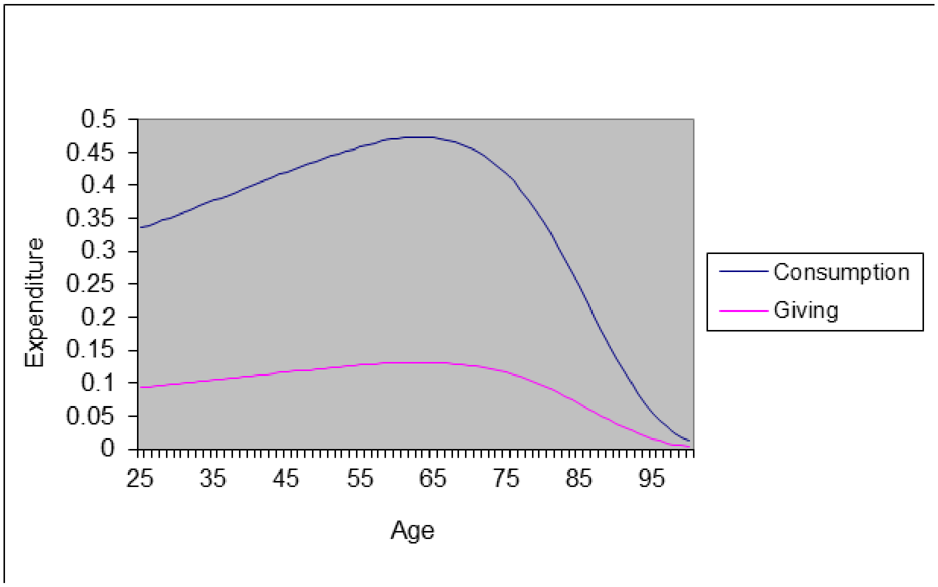
Notice that the first order condition is now $\beta^t Q_t V_M(M_t,\ we_t) = \frac{\lambda}{[R(1-t_3)]^t}$. Similarly, the optimal expenditure schedule conditional on the threshold is modified to

$$M_t{}^* = \left\{ \begin{array}{ll} \left(\frac{\lambda^*}{Q_t \kappa_t((1-t_3)\beta R)^t}\right)^{-\frac{1}{\gamma}} & if \ \ \frac{\lambda^*}{Q_t((1-t_3)\beta R)^t} \geq \kappa_t^* \\ \left(\frac{\lambda^*}{Q_t \overline{\kappa}_t((1-t_3)\beta R)^t}\right)^{-\frac{1}{\gamma}} + (1-t_2)we_t & if \ \ \frac{\lambda^*}{Q_t((1-t_3)\beta R)^t} < \kappa_t^* \end{array} \right\} \tag{5.35}$$

with

$$\lambda^* = \left(\frac{\sum_{t=0}^{T} \frac{Q_t(1-t_2)[we_t + B_{2t}]}{[(1-t_3)R]^t}}{\sum_{t=0}^{T} \kappa_t^{\frac{1}{\gamma}} Q_t^{\frac{1}{\gamma}} \frac{((1-t_3)\beta R)^{\frac{t}{\gamma}}}{[(1-t_3)R]^t}} \right)^{-\gamma}$$

Figure 66: Lifecycle Consumption and Giving for An Individual in Partial Equilibrium.



where κ_t^* and $\overline{\kappa}_t$, as well as $\overline{\gamma}$ are defined in section 2.

Once solved for M_t for each t in partial equilibrium, then the policy variables are determined within each period as shown in Intra-temporal optimization. Figure 66 and 67 show the optimal consumption, giving, leisure, and volunteer over lifecycle for an agent who survives up to T in partial equilibrium with a productivity schedule described later. Figure 68 and 69 demonstrate the expected value of them over life time. Because the model assumes homogeneous cohort other than age, the expected consumption, giving, leisure, and volunteer can be interpreted as economy wide policy variables when its population is normalized. The parameters for all of these figures are $\{\gamma = 0.9,\ \ \eta = 0.4,\ \theta = 0.8,\ \sigma = 0.9,\ \ \beta = 0.978,$ $t_1 = 0.1,\ t_2 = 0.1,\ \ t_3 = 0,\ \ B_2 = 0.05,\ w = 1,\ \ R = 1.035\}$.

5.3.1.3 Extended Preference with Status or Social Pressure As explained in Introduction, recent research works regarding charitable giving in experimental economics are dealing with image motivation, social approval or social status as a motivation for giving. Wether it comes from social pressure by solicitors or from status/image concern, the outcome is the same: increase in giving. Therefore, regardless of its specification, a general

Figure 67: Lifecycle Leisure and Volunteer for An Individual in Partial Equilibrium.

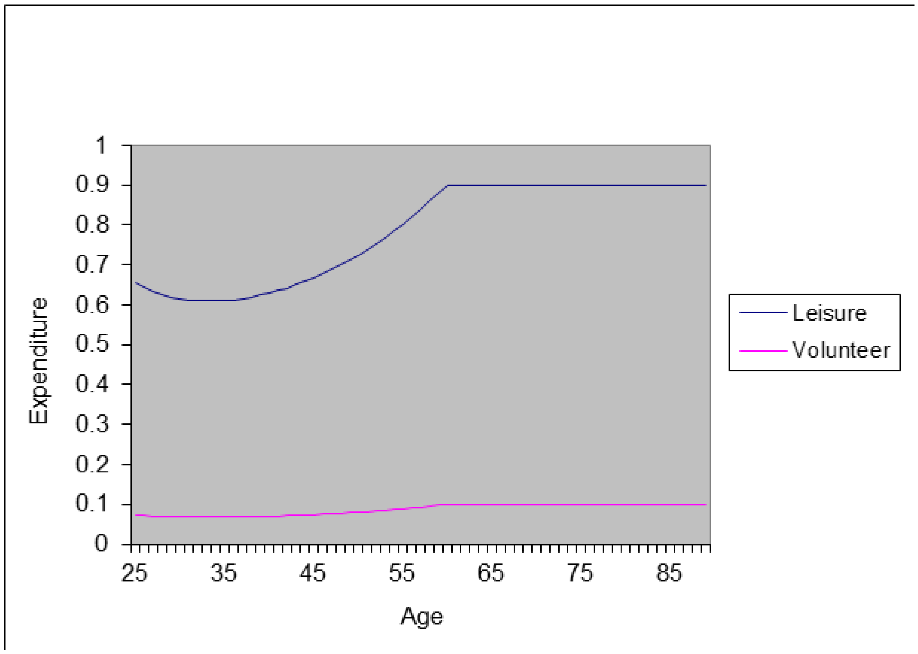


Figure 68: Expected Consumption and Giving in Partial Equilibrium.

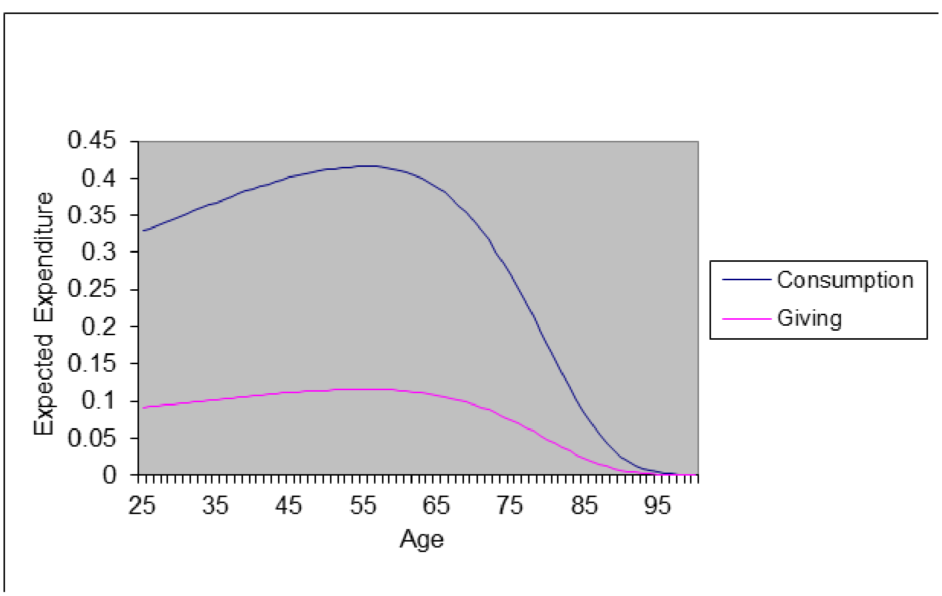
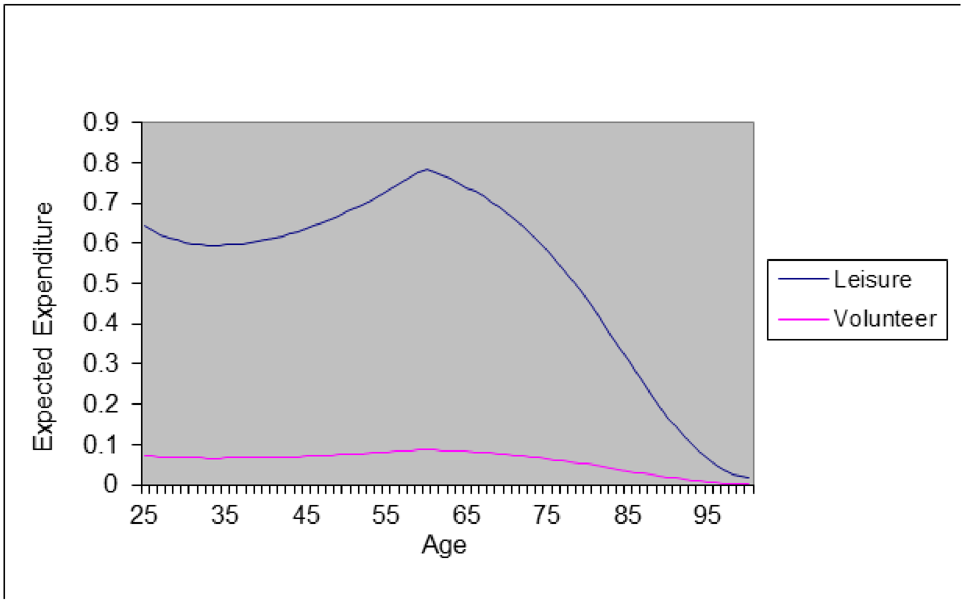


Figure 69: Expected Leisure and Volunteer in Partial Equilibrium.



social preference form $u(x, g, s)$ may be utilized. Here let us introduce a simple version of it in that the agent is indifferent to whether he consumes more private good or gives more to charity, replacing consumption into giving from whatsoever $s-$motivation. If this type of preference is modeled, then the consumer optimization is modified to

$$Max \sum_{t=0}^{T} \beta^t Q_t \frac{1}{1-\gamma}[c_t^{\eta} h_t^{1-\eta}]^{1-\gamma} \tag{5.36}$$

subject to

$$c_t = (x_t + s_t)^{\theta} g_t^{1-\theta}$$
$$h_t = l_t^{\sigma} v_t^{1-\sigma}$$
$$x_t + (1-t_1)(g_t + s_t) + b_{t+1} = (1-t_2)[we_t(1-l_t-v_t) + B_{2t}] + (1-t_3)Rb_t$$
$$1 \geq l_t + v_t \geq 0$$

Now, it is not the MRS between x and g in intra-temporal choice, but $x+s$ and g that is equated to its relative price. Accordingly, the condition for threshold should be related not with x but with $x+s$. Moreover, it is easy to see that $\frac{\partial u}{\partial x \partial s} > 0$ for $0 < \theta < 1$, implying x and s are complementing to each other. However, if the total committed resource is defined accordingly, the intertemporal optimization follows the same steps as before. By simple guessing, one may expect an increasing in total giving, accompanied by reduction of consumption by the agent. This is discussed later.

5.3.2 Technology, Government and General Equilibrium

To explore a general equilibrium model, let us add a production side to the economy and assume there are a continuum (infinite number) of identical perfectly competitive firms who produce the consumption and giving goods. Specifically, this model introduces the following Cobb-Douglas production function for the representative firm:

$$F(K, N) = K^{\alpha} N^{1-\alpha} \tag{5.37}$$

Thus, the firm maximizes its profit $F(K, N) - wN - (r + \delta)K$. The marginal productivity is given by

$$F_K = \alpha \left(\frac{K}{N}\right)^{\alpha-1}$$
$$F_N = (1-\alpha)\left(\frac{K}{N}\right)^{\alpha}$$

Finally, there is a government in this economy, running a balanced budget all the time. The government collects taxes to finance its expenditure. Thus,

$$Gov_t + t_1 \sum_{t=0}^{T} Q_t g_t = t_2 \sum_{t=0}^{T} Q_t [we_t(1 - l_t - v_t) + B_{2t}] + t_3 \sum_{t=0}^{T} Q_t [Rb_t]$$

To close the economy, I assume that the three sectors of consumption, government expenditure and the total giving jointly contribute to the income composition. Let us define a competitive equilibrium for the model with mortality risks and bequest.[35]

Definition *A competitive equilibrium in the economy of warm glow is an allocation* $\{x_t, g_t, l_t, v_t\}_{t=0}^{T}$, *a set of bond demands* $\{b_{t+1}\}_{t=0}^{T}$, *a bequest function* $\{B_t\}_{t=0}^{T}$, *government*

[35] The model economy does not include public sector for equilibrium. The charitable bequest and other variables like volunteer time are measured for calibration purpose in quantitaive analysis.

policy $\{t_1, t_2, t_3\}$*, an interest rate* R *and a wage rate* w *such that given* R*,* w*,* $\{t_1, t_2, t_3\}$*, and* B_t*, the followings are satisfied;*

i) $\{x_t,\ g_t,\ l_t,\ v_t\}_{t=0}^{T}$ *and* $\{b_{t+1}\}_{t=0}^{T}$ *solve the consumer's problem;*

ii) Factors are paid out their marginal productivity;

$$w = F_N \quad \text{and } R - 1 = F_K - \delta$$

iii) Labor market and bond market clear:

$$K_D = \sum_{t=0}^{T} Q_t b_t \text{ and } N_D = \sum_{t=0}^{T} Q_t e_t (1 - l_t - v_t)$$

iv) Bequest B_t satisfies *the balance equation;*

$$\sum_{t=0}^{T} B_t Q_t = \sum_{t=0}^{T} (Q_t - Q_{t+1})\, R b_{t+1}$$

v) Government budget constraint is satisfied.

The market clearing condition in (iii) specifies that consumption loans will cancel out in the aggregate so that the excess demand for bonds should be equal to the capital stock. Also the aggregate labor supply that sums up over the labor supply of each cohort should be equal to the aggregate labor demand. Therefore by the above equilibrium condition ii),

$$w = (1-\alpha)\left(\frac{K}{N}\right)^{\alpha} \tag{5.38}$$

$$R - 1 = \alpha\left(\frac{K}{N}\right)^{\alpha-1} - \delta \tag{5.39}$$

Rewrite the last equation to get

$$\left(\frac{K}{N}\right)^{\alpha-1} = \frac{R-1+\delta}{\alpha}. \tag{5.40}$$

Thus the capital-to-labor demand ratio is written as a function of the interest rate and other parameters. Similarly, rewrite the capital stock as a function of the interest rate to get

$$K(R) = N\left(\frac{R-1+d}{\alpha}\right)^{\frac{1}{\alpha-1}} \tag{5.41}$$

By the equilibrium condition iii), the market equilibrium condition to determine R is

$$\sum_{t=0}^{T} Q_t b_t (R) = N \left(\frac{R - 1 + d}{\alpha} \right)^{\frac{1}{\alpha - 1}} \sum_{t=0}^{T} Q_t e_t (1 - l_t - v_t). \tag{5.42}$$

Once it is solved for an equilibrium interest R, then the real wage w is determined by

$$w(R) = (1 - \alpha) \left(\frac{R - 1 + \delta}{\alpha} \right)^{\frac{\alpha}{\alpha - 1}} \tag{5.43}$$

5.4 QUANTITATIVE ANALYSIS

The objective of the quantitative analysis is to assess how well a calibrated, general equilibrium model of the consumer with warm glow preference can account for stylized facts regarding charitable giving and consumption, as well as leisure and volunteering. In other words, the question I examine is whether a stationary competitive equilibrium of the model can be calibrated to be consistent with characteristics of the macroeconomic data and the giving data. Following many other works, I propose three standard macroeconomic variables to be targeted for the US data. The targets are interest rate, capital-output ratio and consumption output ratio. Regarding charitable goods and time, two targets can be proposed and they are giving to output and volunteer hours to leisure hours (or working hours). But the model has seven scalar parameters, of which five parameters are related to the preference, [36] and the remaining two parameters to the production.[37]

5.4.1 Targets in US Data

Three macroeconomic targets are proposed for the analysis and they are interest rate (R), capital-output ratio (K/Y) and consumption output ratio (C/Y). Following Rios-Rull (1996)[93], let us first set 0.748 for the target ratio of consumption to output. For the target value for capital-output ratio, I set 3 which is close to Rios-Rull (1996)[93].[38] The third macroeconomic target is the real interest rate, which is independently determined in the lifecycle framework. Following McGrattan and Prescott (2000)[81], I set the target real interest rate at 3.5%.[39] The two targets from charitable giving data is the total giving-output ratio (G/Y) and volunteer to work hours (v/N). From giving data over time, the ratio G/Y is found to be stable and I set this for the fourth target which is 2%. Regarding fifth target, Bullard and Feigenbaum (2007)[16] suggest 30 hours of market work per person per week from CPS data. That is labor $N = 120$ hours per month. Because volunteer hours are about 15 hours per month among the volunteers and about half of it among the whole population,

[36] These are β, γ, η, θ, σ in the model.
[37] These are α, δ in the model.
[38] The value in Rios-Rull is 2.94.
[39] Similary, Gourinchas and Parker (2002) estimate 3.44% for the rate .

the working hours to volunteer hours is 16. Thus, I set the $v/N = 0.0625$.[40] Related with this, it is also noticeable that the individuals work about $30/168 = 17.8$ % percent of their available time based on a 24 hour day. These ratios are most directly affected by the parameter η and σ, and these parameters play important roles in the lifecycle consumption and labor/volunteer supply profiles described earlier. Table [17] summarizes the description of targets for the model.

Table 17: Targets

Variables	Description	Target
R	Interest Rate	3.5 %
K/Y	Capital-Output	3
C/Y	Consumption-Output	0.748
G/Y	Giving-Output	2%
v/N	Volunteer-Labor	6.25 %

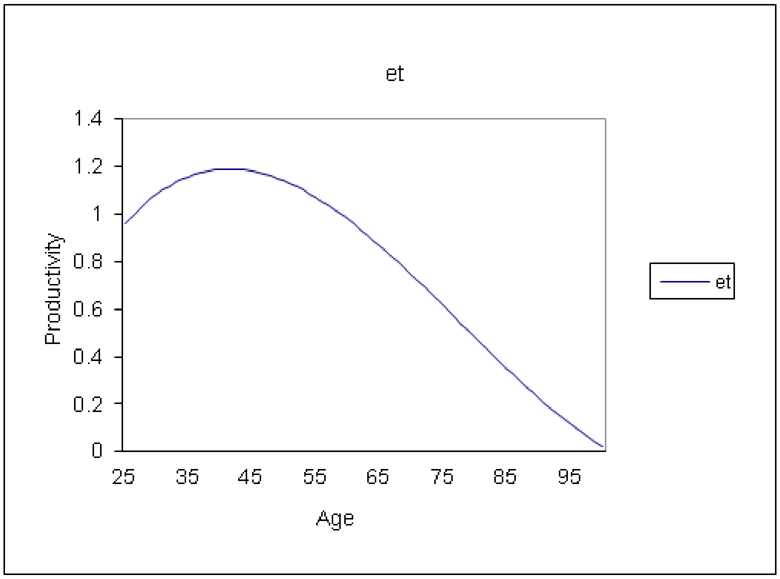
5.4.2 Calibration Method

The model assumes that consumers are born at actual age 25, which corresponds to the model time of zero: $t = 0$. Because the model assumes charitable bequest over lifecycle, it induces a mortality model which is represented by a survivor function. According to actuarial life tables (Arias, 2004), a maximum life span of individuals is 100. Thus, T = 75 is set in the model. I obtain the survivor probabilities from Feigenbaum's (2008a[45]) sextic polynomial function of t, fit to the mortality data in Arias (2004)[8], which is given by the following:

$$Q_t = \exp(-0.01943 - 0.00031t + 0.000006t^2 - 0.00000328t^3 - 0.0000000306t^4 + 0.000000003188t^5 - 0.00000000005199t^6)$$

[40] The labor data is conditional on working population while volunteer data is from all population. The ratio is proposed assuming the difference is not big among the retired population.

Figure 70: Productivity Schedule over Life



Because the household's age-dependent productivity endowment measured in efficiency unit is difficult to observe, I adopt average cross-sectional hourly income rate from the 2001 CPS as a proxy for this.[41] The usual method for linear interpolation to induce average hourly earnings up to 65 and polynomial interpolation beyond that, gives the following productivity schedule over life.

Table [18] summarizes the description of parameters for calibration in the model. Among these parameters, θ and σ are directly related with the last two targets from giving statistics. The economy-wide total giving G is given by $\sum_{t=0}^{75} Q_t(g_t + B_{1t})] + \pi$ where π is corporate contribution to charity and B_{1t} is charitable bequest at each age or contribution from each age group. Following the description of 5% for corporate contribution to overall giving, I set $\pi = 0.05G$. This implies that the corporate contribution to output is 0.1%. Likewise,[43] $B_1 = \sum_{t=0}^{75} Q_t B_{1t} = 0.08G$ and this implies that the charitable bequest to output is 0.16%. Also from volunteer-labor ratio, it is clear that σ is determined via $v/N = 0.0625$. Therefore, among the remaining five parameters I can determine four values given a parameter value.

[41] This is the conditional productivity profile, measured conditional on working agents. It is assumed that the difference between the conditional and unconditional profiles is small.

[43] By the same calcaulation, it is easy to see $g/output = 0.174$.

Table 18: Parameters

Variable	Description	Predetermined
γ	Risk aversion	
η	Goods share	
θ	Consumption share	
σ	Leisure share	
δ	Discount Factor	
α	Capital share	
δ	Depreciation rate	
t_1	Charitable deduction	20%, 15%, 10%
t_2	Income tax rate	15%, 10%
t_3	Capital gain tax rate	0%[42]

Moreover, for the baseline calibration in the following section, I set the bequests from all age groups to be charitable bequests and none for offsprings: that is $B = B_1$ and $B_2 = 0$.

5.4.2.1 Results I parameterize the baseline stationary competitive equilibrium of the model using empirical evidence mentioned above. Because it is very difficult to define properly the empirical evidence regarding η, I allow it as free variable among the five parameters for the calibration, although it is desirable to have it lower than 0.5.[44] Thus, I choose the best discrete $\{\gamma, \eta\}$ combination that makes, along with other targets both α and δ closest to the historically observed values: $\alpha \in [0.25, 0.39]$ and $\delta \in [0.5, 0.9]$. The baseline calibration is shown in Table [19].

These parameters jointly match all the targets while all the value of standard variables $\{\gamma, \beta, \alpha, \delta\}$ are in desirable range. By this calibration, it is found that given η, the value of risk aversion parameter (γ) matters to support proper range of α and δ values. With high γ, other things being equal, the value of $\{\alpha, \delta\}$ runs below its historically observed ones and

[44] This is because Bullard and Feigenbaum (2007)[16], suggest 0.17 or 0.33 for the consumption share to leisure.

Table 19: Summary of Baseline Calibration

$t_1 = 0.20$	γ	β	η	θ	σ	α	δ
$t_2 = 0.15$	0.80	0.970	0.28	0.981	0.988	0.30	0.064
$t_2 = 0.10$	0.85	0.971	0.28	0.981	0.988	0.34	0.079
$t_1 = t_2$	γ	β	η	θ	σ	α	δ
$t_2 = 0.15$	0.81	0.969	0.27	0.980	0.988	0.29	0.060
$t_2 = 0.10$	0.88	0.970	0.30	0.979	0.988	0.31	0.067

vice versa. Also given the risk aversion parameter, increased η returns a higher β.

Based on this, I want to explore how the model features with respect to the lifecycle giving/volunteer and consumption/leisure evidence. In terms of consumption, it is well-known that consumption over a lifecycle exhibits a hump-shaped profile. By Gourinchas and Parker (2002)[57], along the hump, the maximum consumption is obtained at age 45. By the calibrated model, it is found that the maximum consumption is achieved at age 54 conditional on the premise that the agent survives full lifecycle length, while it is around 41 years old when economy-wide average or expected consumption is calculated. Therefore, it is inferred that the model reasonably replicates the consumption data.

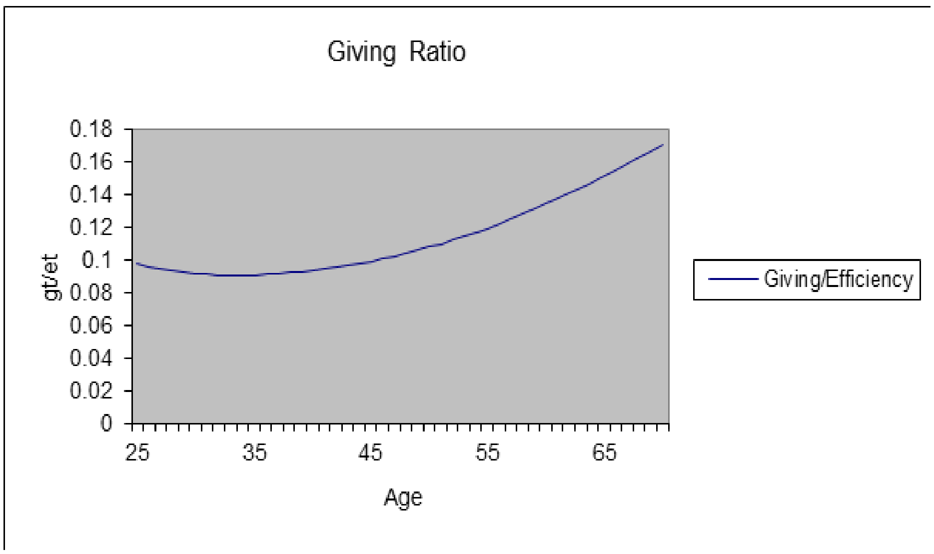
How about giving? Although there is no specification data attainable regarding lifecycle giving, the survey mentioned in the Introduction gives us proper interpretation. By the survey data, charitable giving is increasing with age up to (50-64) age group and reduces beyond that. This fact is true for both measure of *givers only* and *all*. In the calibrated model, the giving increases with age and peaks around 54, which fact conforms to the data.[45]

One more important feature is that giving has an inversely related pattern relative to the income level. The portion of giving out of income is higher for those of low income and the portion of it is lower for with higher income. Although there is no direct quantitative method to measure this from the model, the following giving ratio may explain such property. Because efficiency can be interpreted as income when equilibrium wage is fixed at its

[45] The peak age comes sooner and the giving ratio measured for the amount at peak to initial one looks smaller than the data: this can be corrected by adding social pressure or status in next subsection.

stationary value in the model, the giving to efficiency can proxy giving to income level shown in the data. Because the efficiency is inverse-U shaped over age (low when young and old, and high during the middle age), the U shaped giving ratio implies that giving to income (wage) is inversely related. By this graph, it may be alright to say that the model exhibits such property also. The following Figure 71 summarizes this finding.

Figure 71: Giving to Productivity Ratio



The next topic is time allocation over the lifecycle. It is well-known that the leisure is U-shaped and labor is mildly hump-shaped over age. Figure 72 demonstrates this finding.[46] Moreover, Figure 73 demonstrates the economy-wide population average. Erosa et al. (2010)[37] estimate labor supply over the lifecycle for male workers using heterogeneous agent model with non-linear wages.[47] According to this estimation, the annual hours worked peaks around age 38. The following graph exhibits the comparison of their estimation and the model.

When time endowment is given by (a) 24x7x52=8736 and (b) 24x5x52=6240, the estimated profile lies between the two model profiles with almost same peak age. Volunteer time is generally increasing with age. In the next graph, Figure 75when average volunteering time

[46] Although it is not clearly visible here, the volunteer profile is mildly increasing over age, except at the very young age. See the next figure.

[47] Their estimation is obtained on PSID (1968-1996). See Also Bullard and Feigenbaum (2007)[16].

Figure 72: Individual Lifecycle Time Allocation in General Equilibrium.

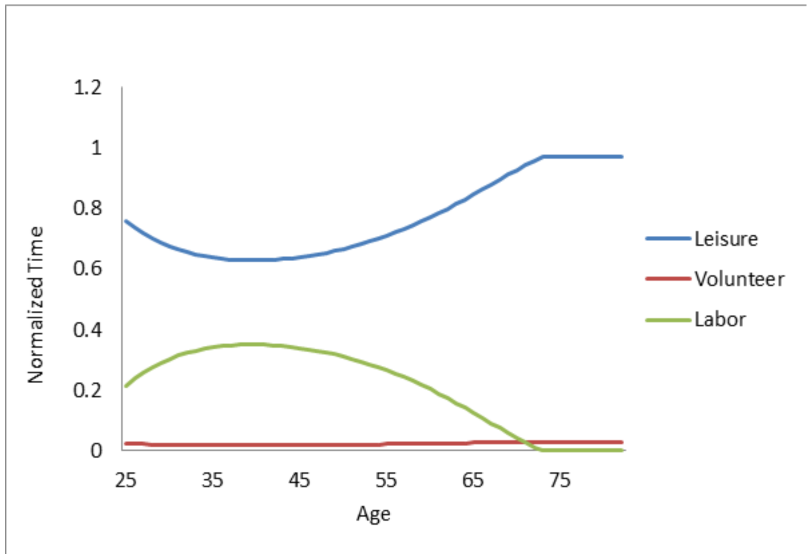


Figure 73: Population Time Allocation over Age in General Equilibrium.

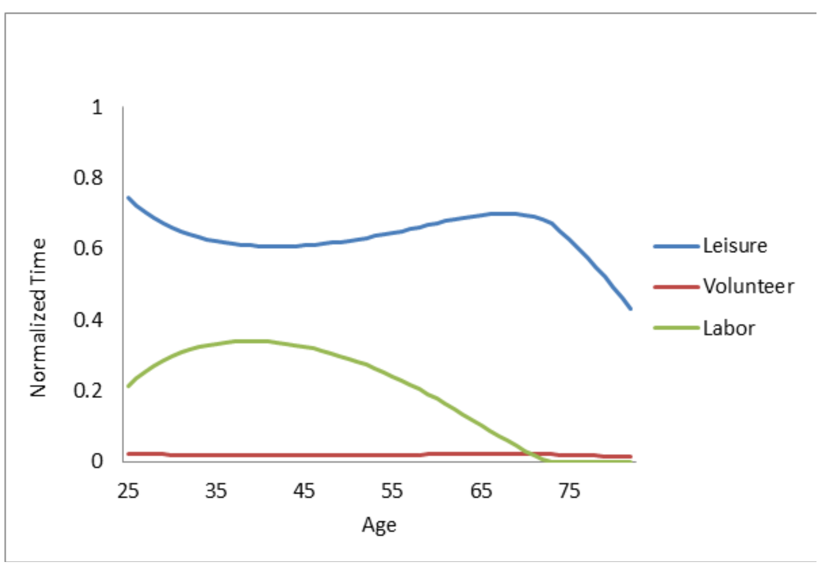
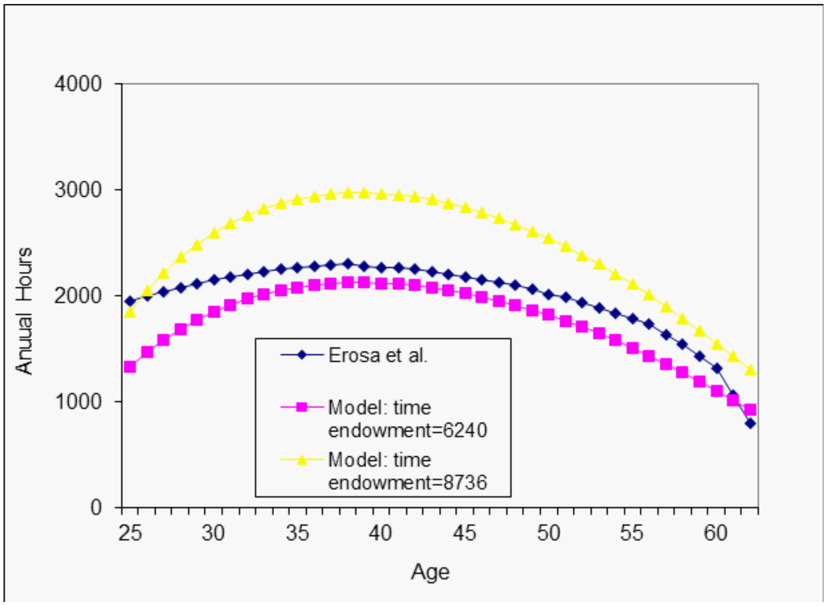


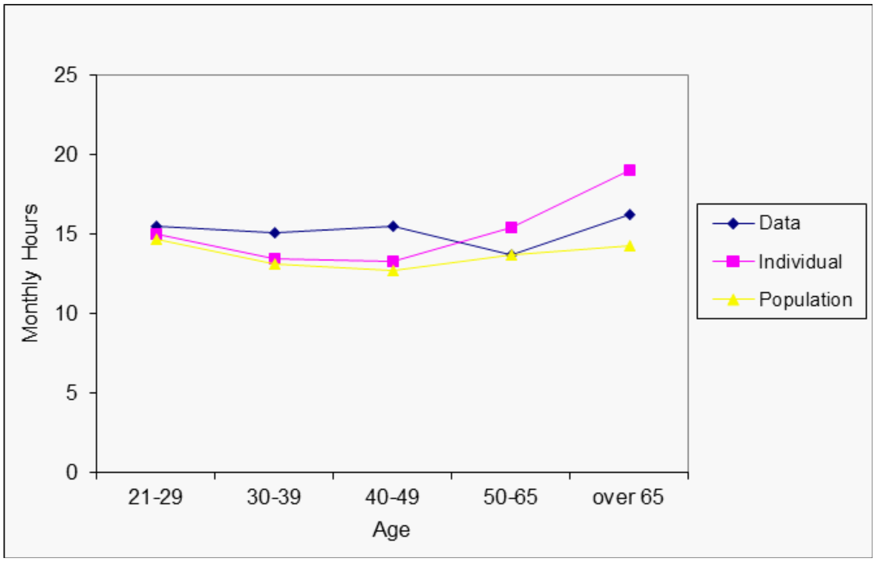
Figure 74: Annual Labor Hours over Age



is calculated over age groups, the model reasonably tracks the survey data. Remember that a volunteer in the survey should be the one surviving up to the age profile in the model. This explains why the data profile stays between the individual and population profile at older age groups.

5.4.2.2 Experiments and Sensitivity In this subsection, I want to explore the general equilibrium result when the consumers exhibit both motivation of warm glow and social norm. As seen in the result section, the peak age in the baseline model comes sooner than the one in the data. Also the giving ratio measured for the peak amount to initial one is smaller than the data. This part of the baseline model can be improved by introducing social pressure or status. I assume $c_t = (x_t + s_t)^{\theta} g_t^{1-\theta}$ as proposed in earlier section following Ariely, Bracha, and Meiercation (2009)[10]. Thus consumers are indifferent to whether they consume more x_t or give more s_t by reducing private consumption x_t. Furthermore, I assume that over age the intensity of s_t is increasing and consumers are willing to replace their private consumption goods into giving as they get older. The following is the giving result when the consumers replace their private consumption into giving by the intensity of $0.00005Age(= t)$.

Figure 75: Data and Model Profile of Volunteer Hours among Volunteers.



In the Figure 76, the peak ages is around 60 and the giving ratio is higher than the baseline. Thus the time of giving peak comes later than the one without this specification. By this transfer, consumption gets lower and its peak comes sooner than the baseline. From this experiment, it may be inferred that the existence of s_t allows more degree of freedom to improve overall fitting, suggesting the complementing effect of the this type of motivation to the intrinsic one. The next graph of Figure 77 summarizes this result by demonstrating its close track to the data.

Some argue that tax rate[48] on charitable giving matters in increasing individual contribution to charity. This argument is important especially in line of tax reform regarding estate tax. The estate tax encourages charitable giving at death by allowing a deduction for charitable bequests. It also encourages giving during life. However, in the baseline model the agent does not plan for the bequest in advance. In the model, the bequest is made upon accidental death. Therefore the tax here does not include estate tax.

As explained in earlier sections, one may argue the consumers with warm glow would not be dramatically affected by the change of tax rates. However it is important to understand

[48] Or the deductible rate on taxable income.

Figure 76: Giving with Social Incentive or Social Pressure.

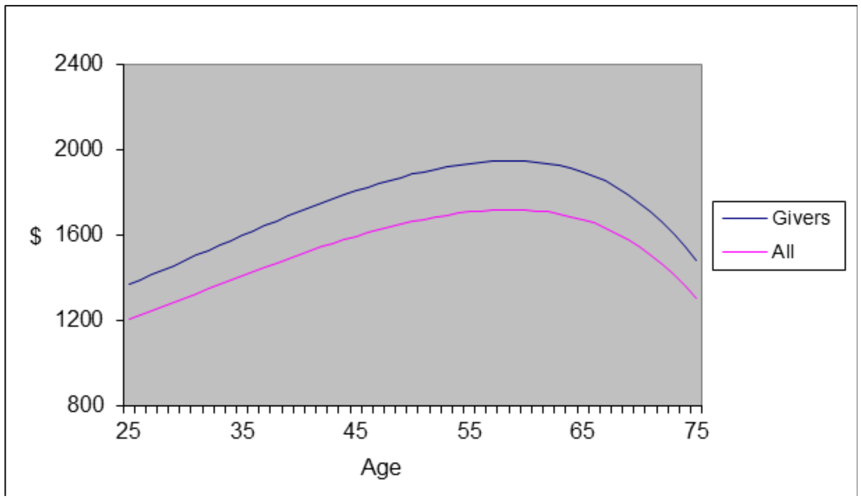
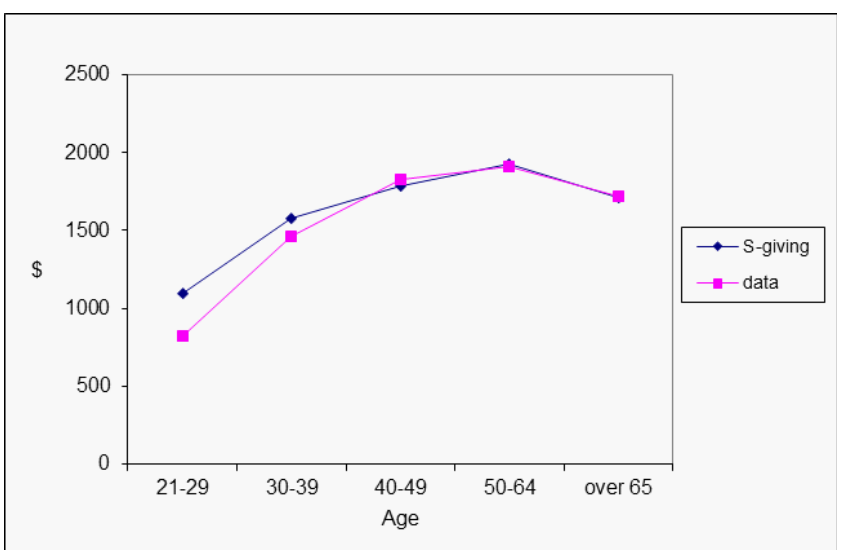


Figure 77: Data and Model of Giving with Social Incentive among Givers.



the giving direction when an increase (decrease) in both of income tax and deduction rate occurs simultaneously. In this subsection, I want to experiment on this topic with the cases when the two tax rates are equal to each other, i.e. $t_1 = t_2$. For the initial income tax rate of 15%, when the deduction rate is increased to 30%, giving is increased by 2% in general equilibrium and 14% in partial equilibrium.[49] Likewise, for the initial income tax rate of 10%, when the deduction is increased to 30%, giving is increased by 1.7% in general equilibrium and 14% in partial equilibrium. Two facts are noticeable. Unlike in partial equilibrium, the general equilibrium result shows that the tax effect toward higher rate on charitable giving is not substantial. This may be direct from the fact that when the tax rate increased, this means a lesser disposable income as well as lower price of charitable giving. Therefore considering the income effect and price effect are working in opposite direction here, the result implies that the price elasticity is greater than the income elasticity. This finding is qualitatively conforming to the empirical result by Auten, Sieg, and Clotfelter (2002)[13] who estimate the price elasticity and income elasticity and show that income elasticity is lower than price elasticity.[50]

Another issue is regarding welfare change due to a tax change. As in the giving, it is also important to determine a welfare direction when there is a simultaneous increase or decrease in both of income tax and deduction rate. Although the welfare can be measured by a change in life time utility, it is more desirable to use compensated variation, which measures the expenditure increment to achieve the same utility level due to price change. The following Figure 78 shows the welfare changes in partial equilibrium where there is a simultaneous increase in income tax (t2) and deduction (t1). By the figure the peak CV arrives at the tax rate of 23%.

Next one in Figure 79 shows the welfare changes in partial equilibrium and general equilibrium when deductible tax rate is increasing while the income tax rate is fixed at baseline rate. The parameter in the partial equilibrium is set at $\{\gamma = 0.8,\ \eta = 0.28,\ \theta = 0.984,\ \sigma = 0.988,\ \beta = 0.97,\ t_2 = 0.15,\ t_3 = 0,\ B_2 = 0.05,\ w = 1,\ R = 1.035\}$.

Following the discussion in calibration, I first set the baseline parameters to $\{\gamma = 0.8,$

[49] The parameters are set to the same values as in the general equilibrium.

[50] Their estimation is -1.26 (-0.40) for permanent (transitory) price elasticity and 0.87 (0.29) for permanent (transitory) income elasticity.

Figure 78: Non-Distorionary Deductible Rate and Welfare Change (CV).

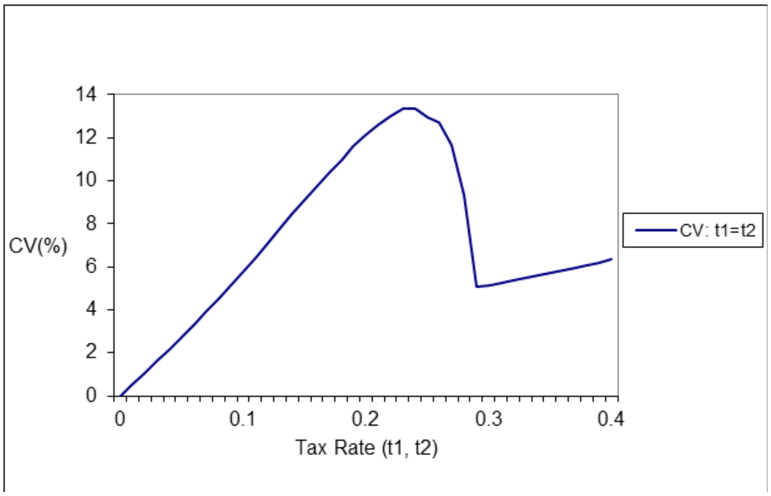
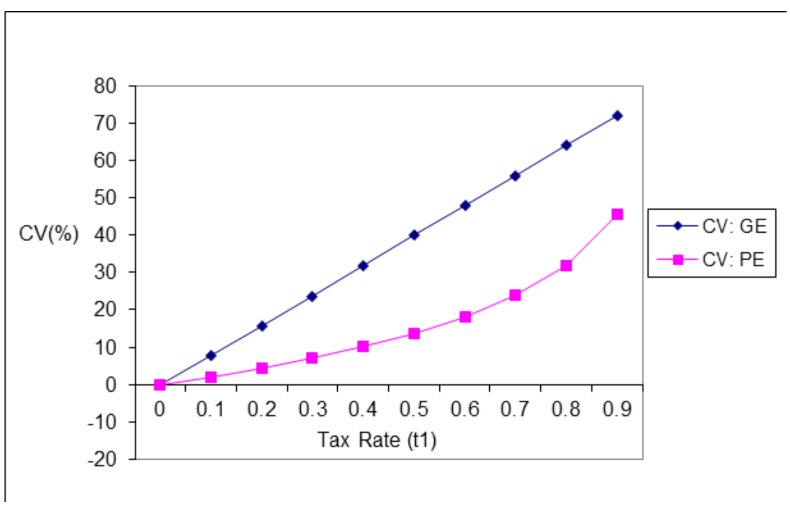


Figure 79: Deductible Rate and Welfare Change in GE relative to PE.



$\eta = 0.28$, $\theta = 0.981$, $\sigma = 0.988$, $\beta = 0.970$, $\alpha = 0.30$, $\delta = 0.064\}$ for the model with $t_2 = 0.15$ and $\{\gamma = 0.85$, $\eta = 0.28$, $\theta = 0.981$, $\sigma = 0.988$, $\beta = 0.971$, $\alpha = 0.34$, $\delta = 0.079\}$ for the model with $t_2 = 0.10$. Notice that β is fixed by initial calibration and is going to adjust accordingly to any parameter set. Thus and are not considered here to be parameters for sensitivity check. For the three targets of macroeconomic variables, i.e. $r = 3.5\%$, $K/Y = 3$, $C/Y = 0.748$, and two targets from giving data, $G/Y = 2\%$, $v/N = 6.25\%$, I report a couple of sensitivity check around the best fitting calibrations. The next result in Table [] shows two sets of sensitivity check, one with $t_2 = 0.15$ and the other one with $t_2 = 0.10$. The second column *Model* of the table shows different alternative calibration to the baseline model, keeping other parameters intact. Thus, for example, the first group of the table represents the alternative calibration to the baseline model with $t_2 = 0.15$ by changing only one parameter and keeping others constant.

Table 20: Sensitivity

	Model	r	C/Y	K/Y	G/Y	v/N
$t_2 = 0.15$	Baseline	3.50%	0.748	3	2.00%	6.25%
	$\eta = 0.290$	2.56%	0.689	3.43	1.98%	6.75%
	$\gamma = 0.801$	4.32%	0.780	2.78	2.06%	7.03%
	$\alpha = 0.330$	4.44%	0.705	2.83	1.92%	6.12%
	$\delta = 0.060$	4.38%	0.751	2.77	2.10%	6.38%
	$\sigma = 0.968$	2.61%	0.721	3.28	1.93%	6.87%
$t_2 = 0.10$	Baseline	3.50%	0.748	3	2.00%	6.95%
	$\eta = 0.245$	4.56%	0.768	2.79	2.05%	6.61%
	$\gamma = 0.899$	3.46%	0.720	3.08	1.95%	5.98%
	$\alpha = 0.300$	3.14%	0.772	2.98	2.06%	6.83%
	$\delta = 0.065$	3.57%	0.748	2.89	2.01%	5.87%
	$\sigma = 0.971$	4.43%	0.759	2.82	2.03%	5.96%

5.5 CONCLUSION

The essay in this chapter studies charitable giving from warm glow motivation and performs quantitative analysis on a social preference in macroeconomic perspective. Shown in many lab studies, social preferences help explain certain phenomena: giving to charities, the response of workers to wage cuts in strikes, the response of giving to gifts in fund-raisers, and the response of effort to non-monetary gifts. However, the models of social preferences that match the laboratory results do not apply to fields but tend to over-predict the data. Motivated by this, this paper aims to develop a quantitative model for a social preference, specifically a model for giving behaviors to charities. The key feature of the giving data is 1) it is positively related with income and 2) individual giving varies significantly with age, although the giving pattern over lifecycle is stable across income level. Encouraged by this fact, in this essay, I construct a lifecycle model for consumers with warm glow preferences who get utility from giving both goods and volunteering time and I explore the general equilibrium characteristics of an economy with warm glow.

I find that the model features salient facts regarding lifecycle giving/volunteer and consumption/leisure behaviors. Like in the data, in my model, charitable giving increases with age except for the very old age and giving in terms of income (productivity) is inversely related. I also show that lifecycle giving follows an inverse J-shaped pattern similar to data while volunteer time is nearly flat till retirement, beyond which it increases with age among surviving population. Moreover, by the scheme of charitable deductions, separated from income tax treatment, together with non-separable utility between consumption and charitable giving, my model unambiguously determines welfare direction from any tax change, enlightening the role of policy in private provision of public goods. The model shows that an increase in deductible tax rate improves the welfare among the consumers faster in general equilibrium than in partial equilibrium. Moreover, it confers highest welfare (compensated variation) for the tax rate of 23% when the rate is not distortionary. This implies that the price elasticity of giving is greater than the income elasticity when tax rate is not very high. I believe this paper is the first serious work that studies a general equilibrium model of social preferences in aggregate level of analysis.

6.0 CONCLUSION

Through this dissertation, I demonstrate that behavioral approaches to inter-temporal choice have many merits in explaining macroeconomic dynamics. One good reason for pursuing this type of approach is that the predictions by standard models are found to be a subset of the predictions of the behavioral models in this dissertation. Moreover, as a by-product, each of my dissertation essays can solve the consumption puzzle introduced in the Introduction, by generating the empirical consumption hump, without any other mechanism than preferences, that may account for the hump.

In the first essay, I prove that there is an *income structure* that leads to a consumption hump for each time preference. That is to say, the hump in consumption is closely related to hump-shaped income, as the data shows, regardless of the time preferences of the consumer. Searching via simulation, I find the best planning horizon that is compatible with matching data for the US economy. I also find the exhaustive inner relationships among the main parameters, i.e., time preference, risk aversion, and the planning horizon, that is necessary to be supported in the calibrated general equilibrium. Finally I show this result is quite robust to alternative choices of parameters.

In the second essay, I find that the consumption hump is obtained even without the credit constraint if the agent is *naive* and keeps re-optimizing over time. In fact, the hump is directly related to his time preference, interacting with the market interest rate, regardless of the income structure. For a constrained agent, I show that the hump-shaped profile for consumption relies not only on preferences, but also on an income condition. Finally when I include social security, bequests, and mortality risks to the baseline model, I find that the results are not conclusive in terms of 'the' best model specification that outperforms the others by several criteria altogether, but the results are consistent in each specification with respect to each criterion.

The first two essays above explore the possibility that individuals behave myopically rather than rationally, each in different way. When I combine the two model specifications into one framework, I find that the combined model is not better in contribution than each model alone: one of the mechanisms tends to lose its key characteristic by adding the other one and vice versa. Moreover, the result is not very appealing or conclusive relative to its many parameters. This implies that "more is better" approach is not applicable to the models of my dissertation.

In a third essay, I demonstrate that reference-dependent preferences *can* also generate a hump-shaped consumption profile. I find that the consumption hump is closely related to loss aversion, by itself or at most combined with time preferences, regardless of the income structure: in a static model when the deviator has a hump-shaped consumption intention and in dynamic model when the deviator has age-dependent loss aversion. To do this I first propose the best consumption scheme for deviators and show how to eliminate the problem of many reference points in dynamic model by introducing the novel concept of 'sub-period perfect reference' points. Moreover, I find the indirect relationship between consumption and income through age and illuminate the existence of savers via this model. Finally, I find that the meaning of *precautionary saving* can be defined in a different way from the usual one with this model of reference dependent preference under uncertainty.

The models in all of the first three essays have a single utility component, i.e. consumption. Regarding the best model specification among the three approaches to explain the featured consumption hump, I can say that the *third one* would be the best one as that specification requires least assumptions on the income structure while its prediction can be chosen to fit even more closely to the data than the other two. However, this claim should be accompanied with the understanding that the result in the third essay is based on the property of the model: a *descriptive* model that may have more parameters to describe the agent's specific behavior.

In the fourth and the final essay, I show how the inclusion of time endowment generates full-blown lifecycle pattern of consumption, giving, leisure, and volunteer time, which closely follow the data. I specifically show that my model reproduces a giving profile that follows an inverse J-shaped pattern over the lifecycle and volunteer time that is nearly flat till retirement, beyond which it increases with age *among volunteers.* I also find that giving in terms of income (productivity) is inversely related. Regarding policy, I find that the price elasticity of giving is greater than the income elasticity when the tax rate is not very high. Finally when the deductible rate is non-distortionary, the highest welfare is obtained with tax rate of 23%.

My findings in general equilibrium are encouraging for the view that analysis with preferences beyond the limit of standard preference can be useful in explaining macroeconomic data and complementing standard economic theories.

APPENDIX A

PROOFS FOR CHAPTER 2

Breakage of monotonicity in three-period model

This part is to show how the bounded rationality model induces non-monotonic consumption profile even with three-period planning horizon when the income is non-monotonic. Assume that a boundedly rational agent lives three periods $t = 0, 1, 2$ but plans only for two periods, $\{current, next\}$.[1] From the consumption functions of Section 2.2, it is easy to see that the consumption schedule for this agent is

$$\{c_0,\, c_1,\, c_2\} = \left\{ \frac{(Ry_0+y_1)}{R\left(1+\frac{1}{\phi}\right)},\, \frac{\frac{R}{\phi}(Ry_0+y_1)+\left(1+\frac{1}{\phi}\right)y_2}{R\left(1+\frac{1}{\phi}\right)^2},\, \frac{R}{\phi}\left(\frac{\frac{R}{\phi}(Ry_0+y_1)+\left(1+\frac{1}{\phi}\right)y_2}{R\left(1+\frac{1}{\phi}\right)^2}\right) \right\}.$$

Assume $y_T = y_2 = 0$. Then the consumption is

$$\{c_0,\, c_1,\, c_2\} = \left\{ \frac{(Ry_0+y_1)}{R\left(1+\frac{1}{\phi}\right)},\, \frac{\frac{1}{\phi}(Ry_0+y_1)}{\left(1+\frac{1}{\phi}\right)^2},\, \frac{R}{\phi}\left(\frac{\frac{R}{\phi}(Ry_0+y_1)}{R\left(1+\frac{1}{\phi}\right)^2}\right) \right\}.$$

And the bond demand is

$$b_1 = \frac{\frac{1}{\phi}y_0 - \frac{1}{R}y_1}{1+\frac{1}{\phi}} = \frac{\frac{1}{\phi}\left[y_0-(\beta R)^{-1/\gamma}y_1\right]}{1+\frac{1}{\phi}} = \frac{\frac{1}{R}\left[(\beta R)^{1/\gamma}y_0-y_1\right]}{1+\frac{1}{\phi}}.$$

Rearrange the consumption to get,

$$\{c_0,\, c_1,\, c_2\} = (Ry_0+y_1)\left\{ \frac{\frac{1}{R}}{1+\frac{1}{\phi}},\quad \frac{\frac{1}{\phi}}{\left(1+\frac{1}{\phi}\right)^2},\quad \frac{\frac{R}{\phi}\frac{1}{\phi}}{\left(1+\frac{1}{\phi}\right)^2} \right\}.$$

It is easy to see that the income stream applies as a scalar and does not affect the consumption ratio. For example, if $\{y_0, y_1, y_2\} = \{x, 2x, 0\}$, then

$$\{c_0,\, c_1,\, c_2\} = (R+2)x\left\{ \frac{\frac{1}{R}}{1+\frac{1}{\phi}},\quad \frac{\frac{1}{\phi}}{\left(1+\frac{1}{\phi}\right)^2},\quad \frac{\frac{R}{\phi}\frac{1}{\phi}}{\left(1+\frac{1}{\phi}\right)^2} \right\}.$$

[1] If the agent plans for only current period, this returns to the *hand to mouth* consumer. In this case consumption and income coincides and monotonicity breaks if income is non-monotonic.

To explore further, consider the three terms in the bracket of the RHS,

$$\left\{\frac{\frac{1}{R}}{1+\frac{1}{\phi}},\quad \frac{\frac{1}{\phi}}{\left(1+\frac{1}{\phi}\right)^2},\quad \frac{\frac{R}{\phi}\frac{1}{\phi}}{\left(1+\frac{1}{\phi}\right)^2}\right\}.$$

To show the breakage of monotonicity, it is enough to look at the case βR=1. Because $\frac{1}{\phi} = \frac{1}{R}$, the expression returns to

$$\left\{\frac{\frac{1}{R}}{1+\frac{1}{R}},\quad \frac{\frac{1}{R}}{\left(1+\frac{1}{R}\right)^2},\quad \frac{\frac{1}{R}}{\left(1+\frac{1}{R}\right)^2}\right\}$$

which implies $c_0 > c_1 = c_2$ for all $R \neq 0$. Therefore, the monotonicity breaks.

APPENDIX B

PROOFS FOR CHAPTER 4

Three-period consumption plans The solutions for the five alternative plans are as follows: Let $\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma} = \mu$.

(1) $c_0 > c_0^*$, $c_1 > c_1^*$, $c_2 < c_2^*$

$$c_0 = \frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1 + \frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2 \mu}$$
$$c_1 = \frac{R}{\phi}\left(\frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1 + \frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2 \mu}\right)$$
$$c_2 = \left(\frac{R}{\phi}\right)^2 \mu \left(\frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1 + \frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2 \mu}\right)$$

(2) $c_0 > c_0^*$, $c_1 = c_1^*$, $c_2 < c_2^*$

$$c_0 = \frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2} - \left(\frac{\frac{1}{\phi}\left(y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}\right)}{1 + \frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2}\right)}{1 + \left(\frac{1}{\phi}\right)^2 \mu}$$
$$c_1 = \frac{R}{\phi}\left(\frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1 + \frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2}\right)$$
$$c_2 = \left(\frac{R}{\phi}\right)^2 \mu \left(\frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2} - \left(\frac{\frac{1}{\phi}\left(y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}\right)}{1 + \frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2}\right)}{1 + \left(\frac{1}{\phi}\right)^2 \mu}\right)$$

(3) $c_0 > c_0^*$, $c_1 < c_1^*$, $c_2 < c_2^*$

$$c_0 = \frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1 + \left(\frac{1}{\phi}\right)\mu + \left(\frac{1}{\phi}\right)^2 \mu}$$
$$c_1 = \frac{R}{\phi}\mu\left(\frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1 + \left(\frac{1}{\phi}\right)\mu + \left(\frac{1}{\phi}\right)^2 \mu}\right)$$
$$c_2 = \left(\frac{R}{\phi}\right)^2 \mu \left(\frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1 + \left(\frac{1}{\phi}\right)\mu + \left(\frac{1}{\phi}\right)^2 \mu}\right)$$

(4) $c_0 = c_0^*$, $c_1 > c_1^*$, $c_2 < c_2^*$

$$c_0 = \frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1+\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2}$$

$$c_1 = \frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2} - \left(\frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1+\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2}\right)}{1+\frac{1}{\phi}\mu}$$

$$c_2 = \left(\frac{R}{\phi}\right)\mu\left(\frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2} - \left(\frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1+\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2}\right)}{1+\frac{1}{\phi}\mu}\right)$$

(5) If $c_0 > c_0^*$, $c_1 < c_1^*$, $c_2 = c_2^*$

$$c_0 = \frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2} - \left(\frac{\left(\frac{R}{\phi}\right)^2\left(y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}\right)}{1+\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2}\right)}{1+\left(\frac{1}{\phi}\right)\mu}$$

$$c_1 = \frac{R}{\phi}\mu\left(\frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2} - \left(\frac{\left(\frac{R}{\phi}\right)^2\left(y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}\right)}{1+\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2}\right)}{1+\left(\frac{1}{\phi}\right)\mu}\right)$$

$$c_2 = \left(\frac{R}{\phi}\right)^2\left(\frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1+\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2}\right)$$

Five-period consumption plans The solutions for the four alternative five-period plans are as follows: Let $w_0 = \frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2} + \frac{y_3}{R^2} + + \frac{y_4}{R^4}}{1+\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2+\left(\frac{1}{\phi}\right)^3+\left(\frac{1}{\phi}\right)^4}$.

(1) $c_0 > c_0^*$, $c_1 > c_1^*$, $c_2 > c_2^*$, $c_3 > c_3^*$, $c_4 < c_4^*$

$$\{c_0,\ c_1,\ c_2,\ c_3\} =$$
$$\{c_0,\ \ \frac{R}{\phi}c_0,\ \left(\frac{R}{\phi}\right)^2 c_0,\ \left(\frac{R}{\phi}\right)^3 c_0,\left(\frac{R}{\phi}\right)^4 \mu c_0\}$$
$$c_0 = (w_0)/\left(1+\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2+\left(\frac{1}{\phi}\right)^3+\left(\frac{1}{\phi}\right)^4\mu\right)$$

(2) $c_0 > c_0^*$, $c_1 > c_1^*$, $c_2 > c_2^*$, $c_3 < c_3^*$, $c_4 < c_4^*$

$$\{c_0,\ c_1,\ c_2,\ c_3\} =$$
$$\{c_0,\ \ \frac{R}{\phi}c_0,\ \left(\frac{R}{\phi}\right)^2 c_0,\ \left(\frac{R}{\phi}\right)^3 \mu c_0,\left(\frac{R}{\phi}\right)^4 \mu c_0\}$$
$$c_0 = (w_0)/\left(1+\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2+\left(\frac{1}{\phi}\right)^3\mu+\left(\frac{1}{\phi}\right)^4\mu\right)$$

(3) $c_0 > c_0^*$, $c_1 > c_1^*$, $c_2 < c_2^*$, $c_3 < c_3^*$, $c_4 < c_4^*$

$$\{c_0,\ c_1,\ c_2,\ c_3\} =$$
$$\{c_0,\ \ \frac{R}{\phi}c_0,\ \left(\frac{R}{\phi}\right)^2 \mu c_0,\ \left(\frac{R}{\phi}\right)^3 \mu c_0,\left(\frac{R}{\phi}\right)^4 \mu c_0\}$$
$$c_0 = (w_0)/\left(1+\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2\mu+\left(\frac{1}{\phi}\right)^3\mu+\left(\frac{1}{\phi}\right)^4\mu\right)$$

(4) $c_0 > c_0^*$, $c_1 < c_1^*$, $c_2 < c_2^*$, $c_3 < c_3^*$, $c_4 < c_4^*$

$$\{c_0,\, c_1,\, c_2,\, c_3\} =$$
$$\{c_0,\ \tfrac{R}{\phi}\mu c_0,\, \left(\tfrac{R}{\phi}\right)^2 \mu c_0,\, \left(\tfrac{R}{\phi}\right)^3 \mu c_0,\, \left(\tfrac{R}{\phi}\right)^4 \mu c_0\}$$
$$c_0 = (w_0)/\left(1 + \tfrac{1}{\phi}\mu + \left(\tfrac{1}{\phi}\right)^2 \mu + \left(\tfrac{1}{\phi}\right)^3 \mu + \left(\tfrac{1}{\phi}\right)^4 \mu\right)$$

Proof of Proposition 7 First, it is straightforward that DM with $\omega\lambda \geqslant 1$ would not deviate because deviation would not increase his utility at all. The maximization problem of DM who has low loss aversion ($\omega\lambda < 1$) and intends to overconsume for the first two periods, $\{c_0 > c_0^*,\, c_1 > c_1^*,\, c_2 < c_2^*\}$, is

$$u(c|c^*) = \tfrac{c_0^{1-\gamma}}{1-\gamma} + \eta\left(\tfrac{c_0^{1-\gamma}}{1-\gamma} - \tfrac{c_0^{*1-\gamma}}{1-\gamma}\right)$$
$$+\ \beta\{\tfrac{c_1^{1-\gamma}}{1-\gamma} + \eta\left(\tfrac{c_1^{1-\gamma}}{1-\gamma} - \tfrac{c_1^{*1-\gamma}}{1-\gamma}\right)\} + \beta^2\{\tfrac{c_2^{1-\gamma}}{1-\gamma} - \omega\eta\lambda\left(\tfrac{c_2^{*1-\gamma}}{1-\gamma} - \tfrac{c_2^{1-\gamma}}{1-\gamma}\right)\}$$

The optimality conditions are

$$\tfrac{(1+\eta)c_0^{-\gamma}}{(1+\eta)c_1^{-\gamma}} = R\beta \quad \text{and} \quad \tfrac{(1+\eta)c_1^{-\gamma}}{(1+\omega\eta\lambda)c_2^{-\gamma}} = R\beta$$

Using budget constraint, this yields

$$\{c_0,\, c_1,\, c_2\} = \{c_0,\ (R\beta)^{1/\gamma}\, c_0,\, (R\beta)^{2/\gamma}\, \mu c_0\}$$
$$\text{where}\ \ c_0 = \left(y_0 + \tfrac{y_1}{R} + \tfrac{y_2}{R^2}\right)/\left(1 + \tfrac{1}{\phi} + \left(\tfrac{1}{\phi}\right)^2 \mu\right).$$

Then the result follows.

Proof of Proposition 8 By the same logic as DM(0) would not deviate, if DM(1) has $\omega_1\lambda \geqslant 1$, he would not deviate from the optimal solution to the maximization problem that starts from t=1, which is $u_1(c|c^*)$. The solution to this problem is

$$c_1 = \frac{y_1 + \frac{y_2}{R} + Rb_1}{1 + \frac{1}{\phi}} \ \text{and}\ \ c_2 = \left(\tfrac{R}{\phi}\right) c_1$$

Because DM(0) consumes c_0^*, leaving $b_1 = y_0 - c_0^*$,

$$b_1 = y_0 - \frac{y_0 + \frac{y_1}{R^1} + \frac{y_2}{R^2}}{1 + \frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2} = \frac{y_0\left(\frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2\right) - \left(\frac{y_1}{R^1} + \frac{y_2}{R^2}\right)}{1 + \frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2},$$

Thus, c_1 is

$$c_1 = \frac{y_1 + \frac{y_2}{R} + Rb_1}{1 + \frac{1}{\phi}} = \frac{\left(Ry_0 + y_1 + \frac{y_2}{R}\right)\left(\frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2\right)}{\left(1 + \frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2\right)\left(1 + \frac{1}{\phi}\right)}$$
$$= \frac{\left(Ry_0 + y_1 + \frac{y_2}{R}\right)\frac{1}{\phi}}{\left(1 + \frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2\right)} = \frac{\left(y_0 + \frac{y_1}{R} + \frac{y_2}{R^2}\right)\frac{R}{\phi}}{\left(1 + \frac{1}{\phi} + \left(\frac{1}{\phi}\right)^2\right)} = c_1^*.$$

Therefore, $c_1 = c_1^*$. His consistent consumption is not different from the one DM(0) planned for the period.

Proof of Proposition 9 When DM(1) has a taste of $\omega_1\lambda < 1$, he would deviate from the optimal solution because deviation increases his utility:

$$\frac{du_1(c|c^*)}{dc_1^*} =$$
$$(1+\eta)\left(\frac{R}{\phi}c_0^*\right)^{-\gamma} - (1+\eta\omega_1\lambda)\beta R[y_2 + R(y_1 + R\left(\frac{y_0\frac{1}{\phi}+y_0\left(\frac{1}{\phi}\right)^2-\frac{y_1}{R}-\frac{y_2}{R^2}}{1+\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2}\right) - \frac{R}{\phi}c_0^*)]^{-\gamma}$$
$$= (1+\eta)\left(\frac{R}{\phi}c_0^*\right)^{-\gamma} - (1+\eta\omega_1\lambda)\beta R\left[\frac{RR}{\phi}c_0^*\left(1+\frac{1}{\phi}\right) - \frac{RR}{\phi}c_0^*\right]^{-\gamma}$$
$$= \left(\frac{R}{\phi}c_0^*\right)^{-\gamma}[\eta(1-\omega_1\lambda)] > 0 \text{ if } \omega_1\lambda < 1.$$

The new consistent consumption is

$$c_1' = \frac{y_1+\frac{y_2}{R}+Rb_1}{1+\frac{1}{\phi}\left(\frac{1+\eta\omega_1\lambda}{1+\eta}\right)^{1/\gamma}} = \frac{\frac{R}{\phi}\left(1+\frac{1}{\phi}\right)\left(y_0+\frac{y_1}{R}+\frac{y_2}{R^2}\right)}{\left[1+\frac{1}{\phi}\left(\frac{1+\eta\omega_1\lambda}{1+\eta}\right)^{1/\gamma}\right]\left(1+\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2\right)}.$$

Therefore, $c_1' > c_1^*$ because

$$c_1' > \frac{\frac{R}{\phi}\left(1+\frac{1}{\phi}\right)\left(y_0+\frac{y_1}{R}+\frac{y_2}{R^2}\right)}{\left[1+\frac{1}{\phi}\right]\left(1+\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2\right)} = \frac{R}{\phi}\left(\frac{y_0+\frac{y_1}{R}+\frac{y_2}{R^2}}{1+\frac{1}{\phi}+\left(\frac{1}{\phi}\right)^2}\right) = c_1^*$$

Proof of Proposition 10 The maximization problem of DM(1) is

$$u_1(c(1) \mid c^*) =$$
$$\frac{c_1^{1-\gamma}}{1-\gamma} + \eta\left(\frac{c_1^{1-\gamma}}{1-\gamma} - \frac{c_1^{*1-\gamma}}{1-\gamma}\right) + \beta\eta\omega_1\lambda\left(\frac{c_2^{1-\gamma}}{1-\gamma} - \frac{c_2^{*1-\gamma}}{1-\gamma}\right) + \beta\frac{c_2^{1-\gamma}}{1-\gamma}$$
$$s.t$$
$$c_2 = y_2 + R(y_1 + Rb_1 - c_1)$$

The optimality condition is $\frac{(1+\eta)c_1^{-\gamma}}{(1+\eta\omega_1\lambda)c_2^{-\gamma}} = R\beta$. Because

$$b_1 = y_0 - c_0' = \frac{y_0\frac{1}{\phi}\left(\frac{1+\eta\omega_0\lambda}{1+\eta}\right)^{1/\gamma}+y_0\left(\frac{1}{\phi}\right)^2\left(\frac{1+\eta\omega_0\lambda}{1+\eta}\right)^{1/\gamma}-\frac{y_1}{R}-\frac{y_2}{R^2}}{1+\frac{1}{\phi}\left(\frac{1+\eta\omega_0\lambda}{1+\eta}\right)^{1/\gamma}+\left(\frac{1}{\phi}\right)^2\left(\frac{1+\eta\omega_0\lambda}{1+\eta}\right)^{1/\gamma}}$$

It is true that

$$c_1'(1) = \frac{y_1+\frac{y_2}{R}+Rb_1}{1+\frac{1}{\phi}\left(\frac{1+\eta\omega_1\lambda}{1+\eta}\right)^{1/\gamma}}$$
$$= \frac{\left(Ry_0+y_1+\frac{y_2}{R}\right)\left(\frac{1}{\phi}\left(\frac{1+\eta\omega_0\lambda}{1+\eta}\right)^{1/\gamma}+\left(\frac{1}{\phi}\right)^2\left(\frac{1+\eta\omega_0\lambda}{1+\eta}\right)^{1/\gamma}\right)}{\left(1+\frac{1}{\phi}\left(\frac{1+\eta\omega_1\lambda}{1+\eta}\right)^{1/\gamma}\right)\left(1+\frac{1}{\phi}\left(\frac{1+\eta\omega_0\lambda}{1+\eta}\right)^{1/\gamma}+\left(\frac{1}{\phi}\right)^2\left(\frac{1+\eta\omega_0\lambda}{1+\eta}\right)^{1/\gamma}\right)}$$
$$= c_0'\frac{R}{\phi}\left(\frac{1+\eta\omega_0\lambda}{1+\eta}\right)^{1/\gamma}\frac{1+\frac{1}{\phi}}{1+\frac{1}{\phi}\left(\frac{1+\eta\omega_1\lambda}{1+\eta}\right)^{1/\gamma}} = c_1'(0)\frac{1+\frac{1}{\phi}}{1+\frac{1}{\phi}\left(\frac{1+\eta\omega_1\lambda}{1+\eta}\right)^{1/\gamma}}$$

Therefore, the consistent consumption is

$$c_1'(1) = c_1'(0)\frac{1+\frac{1}{\phi}}{1+\frac{1}{\phi}\left(\frac{1+\eta\omega_1\lambda}{1+\eta}\right)^{1/\gamma}} = c_1'(0) \quad \text{if } \omega_1\lambda \geqslant 1 \ \text{ and}$$

$$c_1'(1) = c_1'(0)\frac{1+\frac{1}{\phi}}{1+\frac{1}{\phi}\left(\frac{1+\eta\omega_1\lambda}{1+\eta}\right)^{1/\gamma}} > c_1'(0) \quad \text{if } \omega_1\lambda < 1$$

Proof of Proposition 11 Suppose that DM($\tau+1$) has a higher weight than previous DM such that $\omega_{\tau+1}\lambda \geqslant \omega_\tau\lambda$, but consumes $c''_{\tau+1}$ which is larger than c and as a result $c''_{\tau+s} < c^*_{\tau+s}$, $s = 1, 2, ..., T - s - \tau$. Then this consumption is not a consistent consumption profile with respect to his belief on his reference point: because the reference point for the the subsequent periods are $u(c^*_{\tau+s})$, he has loss utility $-[u(c^*_{\tau+s}) - u(c''_{\tau+s})]$, and if his loss aversion is greater than his previous DM, then this loss utility should not be bigger than $-[u(c^*_{\tau+s}) - u(c'_{\tau+s})]$ for each of the subsequent periods. Therefore, $c''_{\tau+s} \leqslant c'_{\tau+s}$. If however, $\omega_{\tau+1}\lambda < \omega_\tau\lambda < 1$, then because his loss aversion is lower than before, $c'_{\tau+1}$ is no more a consistent solution. Thus he deviates.

Proof of Proposition 12 Suppose DM($\tau + 1$) with $\omega_{\tau+1}\lambda \geqslant \omega_\tau\lambda$ consumes $c''_{\tau+1}$ which is larger than $c'_{\tau+1}$ and as a result $c''_{\tau+s} < c'_{\tau+s} < c^*_{\tau+s}$, $s = 1, 2, ..., T - s - \tau$. Then this consumption must be a consistent consumption profile with respect to his belief on his reference point. Because the reference point for the the subsequent periods are $u(c^*_{\tau+s})$, he definitely has loss utility $-[u(c^*_{\tau+s}) - u(c''_{\tau+s})]$ which is bigger than $-[u(c^*_{\tau+s}) - u(c'_{\tau+s})]$ for each of the subsequent periods. Since his loss aversion is greater than his previous DM, $c''_{\tau+s}$ is not the consistent consumption. If however, $\omega_{\tau+1}\lambda < \omega_\tau\lambda < 1$, then because his loss aversion is lower than before, $c'_{\tau+1}$ is no more a consistent solution. He deviates for more.

Proof of Proposition 13 Suppose DM(t) with $\omega_t\lambda \geqslant 1$ deviates for more consumption than the sub-period perfect plan and consumes c'_t which is higher than c^*_t. Then by doing this, DM incurs prospective losses of $-[u(c^*_t) - u(c'_t)]$. Comparing his expected loss utility of zero if he follows the sub-period perfect plan, it is clear that the plan is not fulfilling DM's high loss aversion. Thus he would not deviate for more. Likewise deviation for less produces a current loss of $-[u(c^*_t) - u(c'_t)] < 0$ and thus this can't be an optimal choice to DM either. If however, $\omega_t\lambda < 1$, then deviation is justified because the losses $-[\eta\omega_t\lambda[u(c_t) - u(c^*_t)]]$ are not bigger than the expected gains $\eta[u(c_t) - u(c^*_t)]$ from the alternative consumption he chooses. That is, $\eta[u(c_t) - u(c^*_t)] - \eta\omega_t\lambda[u(c_t) - u(c^*_t)] > 0$.

Proof of Proposition 14 Because the path $\{c^*_0, c^*_1, ..., c^*_{\tau-1}\}$ has been chosen, this path leaves the DM with no extra debts or savings for the period τ than the amount that is necessary for the optimal path for the remaining periods. Let this be the optimal bond demand at $t = \tau$, b^*_τ. Since the sub-period perfect plan at time t is defined by the optimization rule based on the current financial wealth at period τ, this wealth is equal to the optimal bond demand for time τ. Thus, if the new plan starting from period τ, then the reference point must be from the sub-period perfect plan at time τ, which is c_τ^*. The proof of deviation by DM(τ) for more or less consumption follows the same logic as in the previous proposition.

Proof of Proposition 15 Notice that $Rh_0 = h_1 + Ry_0$. The first two consumptions are

$$c_0 = \frac{h_0}{1+\frac{1}{\phi}+\frac{\mu_0}{\phi^2}+\frac{\mu_0}{\phi^3}},\ c_1 = \frac{(h_1+Ry_0)\left(\frac{1}{\phi}+\frac{\mu_0}{\phi^2}+\frac{\mu_0}{\phi^3}\right)}{\left(1+\frac{\mu_1}{\phi}+\frac{\mu_1}{\phi^2}\right)\left(1+\frac{1}{\phi}+\frac{\mu_0}{\phi^2}+\frac{\mu_0}{\phi^3}\right)}$$

Thus,

$$c_1 = c_0\frac{R\left(\frac{1}{\phi}+\frac{\mu_0}{\phi^2}+\frac{\mu_0}{\phi^3}\right)}{\left(1+\frac{\mu_1}{\phi}+\frac{\mu_1}{\phi^2}\right)}$$

which does not depend on income stream (h_t). It is straight forward to get $c_0 < c_1$ if the condition is satisfied. The upper bound of μ_1 is obtained directly from the condition.

Proof of Proposition 16 It is clear that $\dfrac{c_3}{c_2}$ does not depend on h_t, because

$$c_2 = \frac{(h_2+Ry_1+R^2y_0)\left(1+\frac{\mu_0}{\phi}+\frac{\mu_0}{\phi^2}\right)\left(\frac{\mu_1}{\phi^2}+\frac{\mu_1}{\phi^3}\right)}{\left(1+\frac{\mu_2}{\phi}\right)\left(1+\frac{\mu_1}{\phi}+\frac{\mu_1}{\phi^2}\right)\left(1+\frac{1}{\phi}+\frac{\mu_0}{\phi^2}+\frac{\mu_0}{\phi^3}\right)}$$
$$c_3 = \frac{R}{\phi}\mu_2 c_2$$

Therefore, it is straight forward that $c_2 > c_3$, if the condition is satisfied.

Proof of Proposition 17 By the above two propositions, it is clear that combining both conditions yields $c_0 < c_1 > c_2$. Thus the consumption hump is achieved for $\{c_0, c_1, c_2, c_3\}$.

Proof of Proposition 18 First, suppose that $\beta = 1/R$. Because $\frac{R}{\phi} = 1$, the condition reduces to $(1-\mu_0+\mu_1)(Ry_0+y_1) < y_2+\frac{y_2}{R}\mu_0$, and the first two consumptions are $c_0 = \frac{Ry_0+y_1}{R+\mu_0}$ and $c_1 = \frac{(Ry_0+y_1)\mu_0+\left(y_2+\frac{y_2}{R}\mu_0\right)}{(R+\mu_0)\left(1+\frac{1}{R}\mu_1\right)}$. Thus, if the condition is satisfied then $\left(1-\mu_0+\frac{1}{R}\mu_1\right)(Ry_0+y_1) < y_2+\frac{y_2}{R}\mu_0$ or $\left(1+\frac{1}{R}\mu_1\right)(Ry_0+y_1) < y_2+\frac{y_2}{R}\mu_0 + (Ry_0+y_1)\mu_0$. Therefore $c_0 < c_1$. Second, consider the case $\beta < 1/R$. Because $\frac{R}{\phi} < 1$, it is satisfied that $\left(1-\mu_0+\frac{1}{\phi}\mu_1\right)(Ry_0+y_1) < \left(1-\frac{R}{\phi}\mu_0+\frac{1}{\phi}\mu_1\right)(Ry_0+y_1) < y_2(1+\frac{1}{\phi}\mu_0)$. Therefore $c_0 < c_1$. Third, consider the case $\beta > 1/R$. Because $\frac{R}{\phi} > 1$, it is satisfied that either

$\left(1-\frac{R}{\phi}\mu_0+\frac{1}{\phi}\mu_1\right)(Ry_0+y_1) < \frac{1}{\phi}\left(1-\frac{R}{\phi}\mu_0+\frac{1}{\phi}\mu_1\right)(Ry_0+y_1) < y_2(1+\frac{1}{\phi}\mu_0)$ or

$\left(1-\frac{R}{\phi}\mu_0+\frac{1}{\phi}\mu_1\right)(Ry_0+y_1) < y_2(1+\frac{1}{\phi}\mu_0) < \frac{1}{\phi}(Ry_0+y_1)$. In either case, it is true that

$\left(1-\frac{R}{\phi}\mu_0+\frac{1}{\phi}\mu_1\right)(Ry_0+y_1) < y_2(1+\frac{1}{\phi}\mu_0)$. Therefore $c_0 < c_1$.

Proof of Proposition 19 Suppose $\beta = 1/R$. Then the inequality condition reduces to

$\left(\mu_0\left[Ry_0+y_1+\frac{y_2}{R}\right]+y_2\right)\left(1+\frac{1}{R}\mu_2-\mu_1\right)>y_3\left(1+\frac{1}{R}\mu_0\right)\left(1+\frac{1}{R}\mu_1\right)$. If (μ_0,μ_1) is sufficiently small:

$\left(1+\frac{1}{R}\mu_0\right)\left(1+\frac{1}{R}\mu_1\right)<(\,y_3)^{-1}\left(\mu_0\left[Ry_0+y_1+\frac{y_2}{R}\right]+y_2\right)\left(1+\frac{1}{R}\mu_2-\mu_1\right)$ or y_3 is sufficiently small: $y_3<(1+\frac{1}{R}\mu_0)^{-1}(1+\frac{1}{R}\mu_1)^{-1}\left(\mu_0\left[Ry_0+y_1+\frac{y_2}{R}\right]+y_2\right)\left(1+\frac{1}{R}\mu_2-\mu_1\right)$, then $c_1>c_2$. Because with $y_2>0$ it is always true that $\left(\mu_0\left[Ry_0+y_1+\frac{y_2}{R}\right]+y_2\right)\left(1+\frac{1}{R}\mu_2-\mu_1\right)>0$, since $0\leqslant\mu_s\leqslant 1$. Thus $\varepsilon=0$ is the sufficient condition for $c_1>c_2$. Similarly, if either (μ_0,μ_1) or y_3 is sufficiently small, then $c_1>c_2$ for all choices of β.

Proof of Proposition 20 By the above two propositions, it is clear that combining both conditions yields $c_0<c_1>c_2$. Thus the consumption hump is achieved for $\{c_0,c_1,c_2\}$.

APPENDIX C

PROOFS FOR CHAPTER 5

Proof of Proposition 21 From the indirect utility function obtained, it is clear that $V(M, we)$ is continuous in M for both $M > \frac{(1-t_2)we}{1-\eta}$ and $M < \frac{(1-t_2)we}{1-\eta}$. Therefore, to see the continuity, it is necessary to check if it is continuous on the threshold $M^* = \frac{(1-t_2)we}{1-\eta}$ where the binding starts. Thus,

$$\lim_{M\uparrow M^*} V(M, we) = \frac{D}{1-\gamma}\left[\left(\frac{\theta\eta(1-t_2)we}{1-\eta}\right)^{\eta}\sigma^{1-\eta}\right]^{1-\gamma} = \lim_{M\downarrow M^*} V(M, we)$$

Therefore, $V(M, we)$ is continuous all over the domain M.

Proof of Proposition 22 The value function is continuously differentiable over M for both $M > \frac{(1-t_2)we}{1-\eta}$ and $M < \frac{(1-t_2)we}{1-\eta}$. Therefore, to verify the value function continuously is differentiable, it is necessary to check if the derivative is continuous on the threshold where the binding starts. So,

$$\lim_{M\uparrow M^*} \frac{dV(M,we)}{dM} = D\,(\theta\eta)^{\eta(1-\gamma)}\left(\frac{1-\eta}{(1-t_2)we}\right)^{1-\eta+\gamma\eta}\sigma^{(1-\gamma)(1-\eta)} = \lim_{M\downarrow M^*} \frac{dV(M,we)}{dM}$$

Therefore, $V(M, we)$ is continuously differentiable all over the domain M. Because it is differentiable, we can show the strict concavity by demonstrating that its derivative is strictly decreasing. To see this,

$$\frac{dV^{\,2}(M,we)}{dM^{\,2}} = \left\{\begin{array}{ll} -\gamma D\left[(\theta\eta)^{\eta}\left(\frac{\sigma(1-\eta)}{(1-t_2)we}\right)^{1-\eta}\right]^{1-\gamma} M^{-\gamma-1} & if\quad M \le M^* \\ \\ D\eta(\eta(1\text{-}\gamma)\text{-}1)\theta^{\eta(1-\gamma)}\left[M\text{-}(1\text{-}t_2)w\right]^{\eta(1-\gamma)-2}\sigma^{(1-\eta)(1-\gamma)} & if\quad M > M^* \end{array}\right\}$$

With the choice of $\eta\epsilon\,(0,1)$, $\theta\epsilon\,(0,1)$ and $\gamma > 0$, it is true that both terms are negative:

$$-\gamma D\left[(\theta\eta)^{\eta}\quad\left(\frac{\sigma(1-\eta)}{w(1-t_2)}\right)^{1-\eta}\right]^{1-\gamma} M^{-\gamma-1} < 0$$

and

$$D\eta(\eta(1-\gamma)-1)\theta^{\eta(1-\gamma)}\left[M-(1-t_2)w\right]^{\eta(1-\gamma)-2}\sigma^{(1-\eta)(1-\gamma)} < 0$$

Therefore, the value function is continuously differentiable and strictly concave with respect to M.

Proof of Proposition 23 The same logic as above can be applied to prove the proposition. The optimal choice functions are

$$x\left(M,we\right)=\begin{cases}\theta\eta M & if \quad M\leq M^{*}\\ \theta[M-(1-t_{2})we] & if \quad M>M^{*}\end{cases}$$
$$l\left(M,we\right)=\begin{cases}\sigma\left(1-\eta\right)\frac{M}{(1-t_{2})we} & if \quad M\leq M^{*}\\ \sigma & if \quad M>M^{*}\end{cases}$$
$$g\left(M,we\right)=\begin{cases}(1-\theta)\eta\frac{M}{(1-t_{1})} & if \quad M\leq M^{*}\\ \frac{1-\theta}{1-t_{1}}[M-(1-t_{2})we] & if \quad M>M^{*}\end{cases}$$
$$v\left(M,we\right)=\begin{cases}\left(1-\sigma\right)\left(1-\eta\right)\frac{M}{(1-t_{2})we} & if \quad M\leq M^{*}\\ 1-\sigma & if \quad M>M^{*}\end{cases}$$

From the optimal choice functions obtained above, it is clear that

$\lim_{M\uparrow M^{*}}\ x\left(M,we\right)=\frac{\theta\eta(1-t_{2})we}{1-\eta}=\lim_{M\downarrow M^{*}}\ x\left(M,we\right)$
$\lim_{M\uparrow M^{*}}\ g\left(M,we\right)=\frac{(1-\theta)\eta(1-t_{2})we}{(1-t_{2})(1-\eta)}=\lim_{M\downarrow M^{*}}\ g\left(M,we\right)$
$\lim_{M\uparrow M^{*}}\ l\left(M,we\right)=\sigma=\lim_{M\downarrow M^{*}}\ l\left(M,we\right)$
$\lim_{M\uparrow M^{*}}\ v\left(M,we\right)=1-\ \sigma=\lim_{M\downarrow M^{*}}\ v\left(M,we\right)$

Proof of Proposition 24 Notice that the agent has only one source of income which is labor income during the working years. Therefore, if he does not work at all over life time then

$$\sum_{t=0}^{T}\frac{M_t}{R^t}\geq\sum_{t=0}^{T}\frac{(1-t_2)w}{(1-\eta)R^t}>\sum_{t=0}^{T}\frac{(1-t_2)w}{R^t}=\sum_{t=0}^{T}\frac{M_t}{R^t}$$

which is impossible. Therefore, the agent has to work in some period of time.

Proof of Proposition 25 When $\beta R=1$, then $V_M\left(M_t\ ,w\right)=\frac{\lambda}{(\beta R)^t}$ is constant. This implies that M_t is constant over time, so the agent works all the time without retirement.

Proof of Proposition 26 When $\beta R>1$, then $V_M\left(M_t\ ,w\right)=\frac{\lambda}{(\beta R)^t}$ is strictly decreasing over time, which implies M_t is increasing over time so that

$$M_0\leq M_1\leq\cdots\leq M_T$$

Thus, if he stops working then he never comes back to work again because M_t never gets lower.

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