ESSAYS IN LOCAL PUBLIC FINANCE

by

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The first essay, "Producers and Predators in a Multiple Community Setting" investigates how different ways of organizing the provision of local policing services in a multi-community setting affect the level of criminal activity, the spatial distribution of the population, the cost of policing, and overall productivity across all communities. Our analysis shows that if individual local governments are boundedly rational, in the sense that they do not anticipate the effects of their own defense activity on the equilibrium predator/producer ratio and distribution of producer activity, then competition among local governments never achieves a first-best outcome and sometimes yields a lower consumption per capita in equilibrium than would be achieved if there were no local governments and each agent who chose to be a producer also chose his own level of defense. The second essay, "Discriminatory Taxation in a Model of Local Community Competition," analyzes tax competition for new economic resources among local communities within the context of a dynamic, overlapping generations model. We show that in a simple model of discriminatory tax competition, allowing communities to compete for new entrants via the use of entry bonuses and entry taxes does not produce a 'race to the bottom,' does not reduce overall efficiency, and can prevent the economy from getting stuck in an inefficient allocation of resources across communities. The third essay, "A Note on the Effects of Tax Increment Financing on the Path of Land Development," shows that TIFs introduce distortions in the early

use of property even as they reduce tax distortions on later use of property. The net effect of a TIF on the dynamic efficiency of land use depends on the magnitude of the TIF subsidy.

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1.0 INTRODUCTION

The essays that comprise the thesis deal with different aspects of local public finance. Two of the essays apply general equilibrium analysis to the competition among local communities. The third essay studies the nature of a second best policy for taxation of land within a dynamic framework. The first essay, "Producers and Predators in a Multiple Community Setting," (published in the Berkeley Electronic Journals in Topics in Economic Analysis and Policy 2004) analyzes an extension of a model of production and predation due to Grossman (1998) to a multiple community setting. In a multiple community setting, defense expenditures in any one community have the property of a local public good. Such expenditures produce effects on other communities. These effects include changes in the distribution of population among communities, the redistribution of predatory efforts over communities, and an induced change in the predator/producer ratio in the economy as a whole. The question we address is whether the level of defense chosen by local governments so as to maximize the per capita consumption of their own producers, given defense levels elsewhere, always produces a second-best outcome. Our analysis shows that if the number of communities is fixed, fully rational local government decision-making leads to the same level of defense activity and equilibrium per capita consumption as would be chosen by a central planner. However, if individual local governments are boundedly rational, in the sense that they do not anticipate the effects of their own defense activity on the equilibrium predator/producer ratio and distribution of producer activity, then competition among local governments never achieves a first-best outcome. Furthermore, the equilibrium associated with competition among boundedly rational local governments can sometimes yield a lower consumption per capita in equilibrium than would be achieved if there were no local governments and each agent who chose to be a producer also chose his/her own level of defense.

The second essay, "Discriminatory Taxation in a Model of Local Community Competition," analyzes tax competition for new economic resources among independent local communities within the context of a dynamic, overlapping generations model. Agents live for two periods. In the first period, they are mobile and must choose a community in which to reside and to locate their resources. In the second period, an agent is a member of a community chosen in the first period of life and the agent's resources must be used within that particular community. In this model, each community must finance a local public good. A community chooses a fiscal policy each period to maximize the after-tax per capita income of its existing, old residents, taking as given the tax policy of other communities. Existing residents benefit from attracting new residents and their resources whose output can be taxed to help finance the local public good. This benefit is limited by the fact that as the size of the community grows output per capita tends to diminish. Potential entrants are forward looking. They recognize that the discounted present value of expected lifetime after-tax income of entry into a community depends upon the present size of its existing population, its tax policy when they first enter that community, and its expected size and related tax policy of their chosen community in the second period of their lives, when they and their resources have become immobile. In the steady state all communities are of the same size and adopt the same tax policy. The equilibrium tax policy provides an 'entry bonus' to newcomers. The equilibrium bonus is determined by the parameters of the production function and the magnitude of the fixed cost associated with the local public good. The bonus is finite, so that there is not a "race to the bottom." Furthermore, the bonus entails no welfare loss.

The third essay, "A Note on the Effects of Tax Increment Financing on the Path of Land Development" shows that TIF reduces the distortionary effect of property taxes on TIF qualifying investments, but increases the distortionary effect of property taxation on earlier, nonqualifying, investments. The net effect of a TIF on the dynamic efficiency of land use depends on the magnitude of the TIF subsidy.

2.0 PRODUCERS AND PREDATORS IN A MULTIPLE COMMUNITY SETTING¹

2.1 INTRODUCTION

Many good arguments support the establishment of private property rights. But private property rights have to be protected. This is true, regardless of whether or not there exists any formal government to enforce these rights. Protection of property is costly. Furthermore, when an individual decides to protect his/her own property from exploitation by others, this has effects on the security of other property owners. Costly protection of the property of some may not only force others to incur similar protection costs, it can also serve as a deterrent that will reduce the volume of predatory activity in general.

Grossman (1998) presents a general equilibrium model that captures these externalities.² In this model individuals choose to be either producers or predators. An individual's decision to be a predator or producer depends upon his productivity in either role. Producers expend resources to defend what they produce. The more resources that producers as a group devote to defense the less consumable output they produce, but the greater the share of that output they successfully keep from predators. In equilibrium predators and producers have the same expected consumption per capita. Therefore, the equilibrium ratio of predators to producers is related to the equilibrium average defense level among producers.³

¹ A version of this chapter has been published in the BEPress. Han, Shinkyoo and Ochs, Jack (2004) "Producers and Predators in a Multiple Community Setting," *Topics in Economic Analysis & Policy*: Vol. 4 : Iss. 1, Article 11. Available at: http://www.bepress.com/bejeap/topics/vol4/iss1/art11

² Grossman's model is itself a variant of a model first presented by Grossman and Kim (1995).

³ The assumption that an individual chooses whether to become a producer or a predator based upon that individual's expected utility in one 'occupation' relative to the other is central to the 'economic approach' to the analysis of crime that stems from the seminal paper by Gary Becker (1968). There has been a voluminous empirical literature based on a partial equilibrium model of the demand and supply of criminal activity that has been devoted to measuring the magnitude of the response of criminal activity to factors that may affect the returns to criminal

Grossman considers two alternative institutions for organizing defense. In one institution, which we call atomistic competition, each individual producer chooses his own level of defense. In the second institution, which we call global government, a common level of defense is chosen by a single decision-maker whose decision is binding on all producers. In atomistic competition, any one producer cannot affect the probability of being attacked. That probability is determined by the average defense level among the community of producers. Nevertheless, individual producers do have an incentive to invest in defense because individual defense efforts affect the likelihood that a producer will be able to successfully fend off an attack if he is attacked. Of course, the average level of defense in a community of producers does depend upon the level of defense of each producer. Therefore, under atomistic competition, each individual producer's defense decision does generate an externality. Not surprisingly, Grossman can show that if a single decision-maker can choose the common level of defense, binding on all producers, then the level of defense that maximizes consumption per capita is greater than the equilibrium level of defense that emerges under atomistic competition. Indeed, in his model, the common level of defense that maximizes per capita consumption is one at which the return from being a producer exceeds the return from predatory activity, even after the tax necessary to support the common defense.

Individual defense and global defense of property represent polar extreme forms of organization. It is often the case that responsibility of protection of property in a given region is assigned to a local government for that region.⁴ A local government may be concerned with the per capita consumption of its own citizens, but not with that of people in other regions. Unlike an individual citizen's protection activities, a local government's decision with respect to the average level of protection to have in its region will have an effect on the probability that a predatory attack strike its own citizens. Changing the average level of defense in one region will not only have a deterrence effect locally, but will also shift the locus of predatory activity elsewhere. In what follows, we modify the Grossman model in a way that allows for this type of competition among locales.

activity relative to legal activity. For a discussion of that literature and references to other surveys, see Ehrlich (1996).

⁴ A group of property owners in a condominium association may, for our purposes, be considered a local government.

This modification requires us to introduce a raison d'etre of multiple communities, which we do via a productivity index. The productivity index relates the productivity of a producer in a given community to the size of that community. This introduces another channel through which decisions in any one community affect producers in other communities.

This extension of the model allows us to compare three different forms of organization of defense: atomistic competition in which producers make their own decisions with regard to defense; local government competition in which each local government chooses a common level of defense for all producers within its jurisdictions; central planning in which a single authority chooses the level of defense for all producers regardless of jurisdiction. In what follows, we compare the symmetric Nash equilibria associated with atomistic competition and competition among local governments. We find that when the number of communities is fixed, competition among *fully rational* local governments (i.e., governments that fully anticipate how agents will adapt to any policy they adopt) leads to the equilibrium level of defense and consumption per capita that is identical to the common level of defense and consumption per capita that is identical to the common level of defense and consumption per capita that would be chosen by a central planner. However, if local governments are *boundedly rational*, in the sense that they do not fully anticipate how agents will respond to any policy change they adopt, then the equilibrium level of consumption per capita is never as high as that achieved by central planning and, under certain conditions, may be even lower than the equilibrium level of consumption.

2.1.1 Related literature

In the Grossman model, an agent's resources are inalienable. Each agent must decide whether to use those resources in a productive or an appropriative activity. It is the producer's output that must be defended from appropriation. Skaperdas (1992) considers another model in which each agent has inalienable resources that can be used in productive or appropriative activities. Skaperdas analyzes a two-player game in which each agent decides how much of an inalienable resource to make available as an input into a production process whose output depends upon the inputs of both agents. The remainder of each agent's resource is devoted to an appropriative activity. The appropriative efforts of both players are inputs into a conflict function that determines the respective probabilities of each agent claiming the output of the production

process. This setup is not well suited to exploring how different institutions for organizing defensive efforts affect the equilibrium level of output.

Skogh and Stuart (1982) consider how the introduction of a social system of discovery and punishment of predators would affect the equilibrium allocation of effort over production, protection, and predation. Like Grossman, we abstract from any collective effort at the discovery and punishment of predators.

A productive resource may itself be alienable. Hirshleifer (1995) considers a model in which there is a productive resource whose ownership is not well-defined. All agents must fight to control this resource. Therefore, in his model there is no distinction made between predatory and protection activities. Meza and Gould (1992) provide a model of enclosures, where ownership of land is well-defined but the owner must decide whether or not to incur an 'enclosure cost' and deny free access to his/her property. Such enclosures by one landowner can force other landowners also to incur these costs in order to prevent a spillover of individuals who seek free access to unenclosed land. As they demonstrate, this can lead to an equilibrium in which all property is enclosed even though the enclosure costs make this less efficient than a system in which no property is enclosed. In this model, workers choose to work for wages or to work on unenclosed land. There is no predatory activity, as such.

2.2 BASIC STRUCTURE

The basic structure of the environment within which all three forms of organization operate is as follows. The total population is of unit mass. Each agent has one unit of an inalienable resource. Each agent chooses to be either a producer or a predator. An agent who chooses to be a predator devotes all of this resource to predation. The number of locations at which production can take place is n. An agent who chooses to be a producer is located in one of these n locations. We are interested only in symmetric equilibria. Therefore, we assume an equal number of producers in every location. A producer j in location i chooses a fraction, d_{ij} , of his resource to devote to defense. The remainder is devoted to production.

The productivity of a producer's production activity in community *i* depends upon a productivity index, y_i , where y_i depends on a technology parameter, *k*, and on the fraction, π_i , of all producers who reside in community *i*. The form of the index function is $y_i(\pi_i,k) = \pi_i(2k-\pi_i)$. Thus, positive agglomeration effects of community size over some range are more than offset by the negative congestion effects of community size over another range. The output of producer *j* living in community *i* is then $(1-d_{ij})y_i$.

Predators are free to roam over all locations. The fraction of total predatory effort that is directed against community *i*, $G_i(\mathbf{d}) \ge 0$, is a function of the vector of average defense level across communities $\mathbf{d} = (d_i, d_{-i})$ such that $\sum_{i=1}^n G_i = 1$. The fraction of all agents who are predators is γ . Therefore, the expected number of predators attacking community *i* is { $\gamma G_i(\mathbf{d})$ }. Predators are aware of the average level of defense in any community, but do not know the level of defense chosen by any particular producer. Therefore, having chosen a community to attack, a predator strikes one producer, chosen at random from the producers in that community. Consequently, the probability of producer *j* in community *i* being attacked is $\wp_{ij}(\mathbf{d}, \pi_i) = \{\gamma G_i(\mathbf{d})/\pi_i\}$. Given any positive probability of being attacked, the probability that a predator's attack on a producer will be successful in transferring what that producer has produced to the predator depends upon that producer's level of protection d_{ij} . This probability is determined by the ratio d_{ij}/t , where *t* is a predation efficiency parameter.

A successful attack transfers the output of the producer who is attacked to the predator. Therefore, the expected consumption, C_{ij} , of producer *j* in community *i* depends upon his own defense effort d_{ij} , the average defense effort in his community d_i , the average defense level in other communities d_{-i} , and the productivity index in his community y_i :

$$C_{ij} = \left(\wp_{ij} (\mathbf{d}, \pi_i) \frac{d_{ij}}{t} + \left(1 - \wp_{ij} (\mathbf{d}, \pi_i) \right) \right) (1 - d_{ij}) y_i.$$

$$\tag{1}$$

For future reference, the notation is summed up as follows:

- γ fraction of population who are predators
- $(1-\gamma)$ fraction of population who are producers

- *p* ratio of predators to producers
- π_i number of producers in community *i*
- y_i $\pi_i(2k-\pi_i)$ = productivity index of community *i*
- d_{ij} defense effort of producer *j* in community *i*
- d_i average defense level in community *i*
- d_{-i} average defense level in the communities except *i*
- G_i fraction of predatory effort of a predator directed against community *i*
- \wp_{ij} probability of producer *j* in community *i* being attacked
- *n* number of local communities
- t predation efficiency parameter (0 < t < 1)
- k production technology parameter (0 < k < 1)
- C_{ij} expected consumption of producer *j* in community *i*
- D_i expected consumption of a predator in community *i*

2.3 ATOMISTIC EQUILIBRIUM

Atomistic competition involves no collective decision-making with regard to protection activity. In this individual non-cooperative competition, a producer's protection activity affects his expected consumption through two channels. First, the level of defense chosen by each producer, when aggregated, determines the average defense in his community. Since each individual is of measure zero, however, the defense level chosen by a producer has an imperceptible effect on the average level of defense. Accordingly, this indirect impact of any individual's choice on the probability of being attacked is infinitesimal.⁵ Second, the protection activity of a producer directly affects the probability that a predator's attack on the producer will be successful in transferring what that producer has produced to the predator, conditional on being attacked. This

⁵ Note that, like Grossman (1998), we assume that the probability of a producer being attacked depends only on the average level of defense in his community, and is independent of his own defense level. If individual producers were allowed to influence the probability of being attacked, given a continuum of producers, the equilibrium level of protection activities would absorb all the resources and there would be no production for a given predator/producer ratio.

provides a motivation for an individual producer to invest in protection of his own property, given any positive probability of being attacked.

We are interested in the symmetric equilibrium in which identical agents employ the same strategy. Imposing the symmetric equilibrium condition, we simply drop the subscripts in the expected consumption for producer *j* since, in equilibrium, this is the same for any producer in any community. Also, notice that symmetry implies that in equilibrium the fraction of total predatory effort must be the same across communities, i.e. $G_i = G_{-i} = 1/n$. Symmetry further means that in equilibrium every community has the same number of producers, $\pi = (1-\gamma)/n$. Therefore, since predators strike at random in a community, the probability of a producer in community *i* being attacked, denoted as \wp , is $\{\gamma(1/n)(1/\pi)\}$. Since $\pi = (1-\gamma)/n$ and $p = \gamma/(1-\gamma)$, $\wp = p$. Then the expected consumption of producer *j* is

$$C_{j} = \left(p\frac{d}{t} + (1-p)\right)(1-d)y.$$
(2)

Producers must determine the fraction of resources to devote to defense to maximize their expected consumption. Since each producer is of measure zero, in making this choice any producer must *take the predator/producer ratio*, *p*, *as given*. Imposing the equilibrium condition that each producer has the same defense level, we derive the equilibrium level of defense, d(p;t), by solving the first-order condition to the producer maximization problem,

$$\underset{d}{Max} \quad C_{j} = \left(p \frac{d}{t} + (1-p) \right) (1-d) y \quad \text{s. t.} \qquad p \ge 0, \quad 0 \le d \le t < 1.^{6}$$

The first-order conditions for the solution to this problem imply the equilibrium relationship between the defense level, d, the equilibrium predator/producer ratio, p, and the effectiveness of predation parameter, t.

⁶ For $p \ge 0$, the expected consumption per predator must be nonnegative. This requires $d \le t$, where *t* is a measure of effectiveness of predation.

$$d(p;t) = \frac{p-t+pt}{2p}.$$
(3)

The optimal level of defense given p is decreasing in the predation efficiency parameter, t, because an increase in t reduces the marginal benefit of defense to a producer who faces any given probability of being attacked. For any given average level of defense in the community, an increase in the predator/producer ratio, p, increases the probability of any producer being attacked. This increases the marginal benefit of defense expenditure to an individual producer and accounts for a positive relationship between d(p;t) and p. Because an individual cannot affect the average level of defense in his own community or the global predator/producer ratio, p, no individual has an incentive to react as the number of communities, n, changes, if such a change does not affect p.

To find equilibrium values of both *d* and *p* in terms of the parameters of the model, we exploit a second equilibrium condition: in equilibrium an individual must be indifferent between being a producer and a predator. This implies that in equilibrium the expected consumption of an agent is the same regardless of his chosen role. As described above, the equilibrium probability of any particular producer being attacked is *p*, and conditional on that producer being attacked the expected consumption of the predator who makes that attack is (1-d/t)(1-d)y. Since $(1-\gamma)$ individuals are producers, the expected consumption of all predators is $\{p(1-d/t)(1-d)y\}(1-\gamma)$.

 D_i denotes the expected consumption of a predator in community *i*, which in equilibrium is the same regardless of the community he attacks. Let this common equilibrium level of expected consumption per predator be *D*. Therefore, with γ predators, the expected consumption per predator is

$$D = p\left(1 - \frac{d}{t}\right)(1 - d)y\frac{1 - \gamma}{\gamma} = \left(1 - \frac{d}{t}\right)(1 - d)y, \qquad (4)$$

where the second equality is due to the predator/producer ratio $p = \gamma / (1-\gamma)$. Similarly, in equilibrium individual producers' expected consumption must be the same across all communities. Let this common equilibrium level of expected consumption per producer be *C*. In

equilibrium an individual is indifferent between being a producer and a predator. Therefore, in equilibrium D = C, or:

$$D = \left(1 - \frac{d}{t}\right)(1 - d)y = \left(p\frac{d}{t} + (1 - p)\right)(1 - d)y = C.$$

This implies

$$d(p;t) = \frac{pt}{1+p}.$$
(5)

Solving (3) and (5) simultaneously, we derive the atomistic equilibrium defense, d(t), and predator/producer ratio, p(t).

$$d(t) = \frac{t(-1+\sqrt{1-4t^2+4t})}{1-2t+\sqrt{1-4t^2+4t}}, \qquad p(t) = \frac{1-\sqrt{1-4t^2+4t}}{2(t-1)}.$$

In the atomistic equilibrium, both the predator/producer ratio and defense are an increasing function of the efficiency of predation, t, but are independent of the parameters n and k for the following reason. The productivity index, y, affects the equilibrium consumption per capita, but the equilibrium values of p and d simply insure that all agents have the same expected consumption regardless of the level of that consumption. Since the equilibrium values of p and d are independent of y, they are independent of the parameters n and k of the productivity index, y.

Substituting the equilibrium defense and predator/producer ratio into the expected per capita consumption function, we derive the atomistic equilibrium consumption as a function of the parameters, (n,t,k), as shown in figure 2.1 for fixed k. Note that the equilibrium consumption is decreasing in t. It is also inverse U-shaped in n, which reflects the fact that the productivity of a producer is subject to positive agglomeration effects over some ranges of community size and

negative congestion effects over another ranges, for a fixed value of the production parameter, k.⁷



Figure 2.1: Atomistic equilibrium consumption C(n,t), given k = 0.222

2.4 GLOBAL EQUILIBRIUM

We now consider the case with a central government that establishes a common defense expenditure for all communities. The objective of the central government is to choose a defense policy that maximizes consumption per producer, given that individuals are free to choose whether to be producers or predators. An individual's choice of role depends upon the expected consumption of a producer relative to the expected consumption of a predator. For n fixed, we formalize the central government's problem as follows:

⁷ The parameter values (n,t,k) used in the numerical analysis are chosen so as to allow for a clear comparison of equilibria in the subsequent sections. Extreme parameter values expand the range of degenerate cases such as the negative equilibrium level of defense or consumption. For the equilibrium consumption with different values of k, see figure A.1 in Appendix B.

$$\begin{aligned} & M_{ax} \quad C = \left(p \, \frac{d}{t} + (1 - p) \right) (1 - d) y \\ & \text{s. t.} \qquad y = \pi (2k - \pi), \quad \pi = \frac{1 - \gamma}{n}, \quad 0 \le p(d; t) = \frac{d}{t - d} \le 1, \quad 0 \le d \le t < 1, \end{aligned}$$

where p(d;t) is the condition that must be satisfied for individuals to be indifferent between being producers and predators. Figure 2.2 represents the graph of the solution to this problem, d(n,t), for a fixed value of the production parameter, k.



Figure 2.2: Global equilibrium d(n,t), given k = 0.222

Since agents adjust to any defense level chosen by the central planner so as to equalize the per capita consumption of producers and predators, the net effect of the central planner's decision on per capita consumption of producers depends upon how it influences the productivity index, y. This index depends upon both the ratio of predators to producers, p, and on the number of communities, n. The marginal effect of p on y depends upon n. When n is small (or, equivalently, when π is large), an increase in p increases y as it reduces the congestion effect. But when n is large, an increase in p reduces y since it reduces the agglomeration effect. Therefore, because the equilibrium value of p is positively related to the defense level, the optimal level of defense is lower, the larger is the number of communities. This is reflected in figure 2.2. The relationship between *d* and *n* that is induced by the indifference condition relating *p* to *d* implies that for *n* sufficiently large the optimal level of defense is 0. That is, a sufficiently large number of communities means that, *if* all agents were producers and no resources were devoted to defense, then no agent would be able to secure a higher consumption by switching all of his effort from production to predation. With the parameters used in the illustration above, this occurs when $n \ge 4$. An equilibrium with no defense and no predation is unstable, however, unless a switch from producer to predator by a measurable set of agents can be deterred by a threat of increasing defense from zero to a level that would reduce the consumption of predators relative to producers. Therefore, we assume that when the equilibrium levels of defense and predation are 0, the central government's policy can be described as:

$$\sigma(p) = [d = 0 \text{ if } p = 0, d = min \{d \mid C > D\} \text{ if } p > 0].$$
(6)

2.5 COMMUNITY EQUILIBRIUM

2.5.1 Fully rational community equilibrium

Community defense may be a local government responsibility, rather than a central government responsibility. In this event, the level of defense chosen by any one community will have an impact not only on (*i*) the global predator/producer ratio but also upon (*ii*) the distribution of predatory activity across communities, and (*iii*) the distribution of producers across communities. In this section, we characterize the symmetric equilibrium that emerges when each community takes into account the way its own policy affects the decisions of individuals with regard to (*i*) whether to produce or predate, (*ii*) where to live, conditional on being a producer, and (*iiii*) where to attack, conditional on being a predator. We characterize the game as a two-stage game. In the first stage, communities choose their defense levels. In the second stage, individuals choose their roles, with producers choosing where to live and predators choosing where to attack. Allowing

individuals to move after every community has set its policy allows each community to account for how its policy choice affects patterns of production and predation.⁸

A sequentially rational equilibrium requires that following any policy vector chosen by local governments in the first stage, the strategies chosen by individuals in the second stage form an equilibrium. So we begin with the second stage, given $\mathbf{d}=(d_i,d_{-i})$. Fix a global predator/producer ratio, p. Then the fraction of population who are predators, γ , and the fraction of population who are producers, $(1-\gamma)$, become

$$\gamma = \frac{p}{1+p}, \qquad \qquad 1-\gamma = \frac{1}{1+p}.$$

Then the expected consumption per producer in community *i* can be expressed as follows:

$$C_{i} = \left(\frac{p}{(1+p)}\frac{G_{i}}{\pi_{i}}\frac{d_{i}}{t} + \left(1 - \frac{p}{(1+p)}\frac{G_{i}}{\pi_{i}}\right)\right)(1-d_{i})\pi_{i}(2k-\pi_{i}).$$
(7)

And the expected per predator consumption in community *i* is

$$D_{i} = \wp_{i} \left(1 - \frac{d_{i}}{t} \right) (1 - d_{i}) \pi_{i} (2k - \pi_{i}) \frac{\pi_{i}}{\gamma G_{i}} = \left(1 - \frac{d_{i}}{t} \right) (1 - d_{i}) \pi_{i} (2k - \pi_{i}).$$
(8)

For the per capita consumption in a community other than *i*, we assume that all communities except community *i* follow a symmetric level of defense, d_{-i} , such that the probability of any particular community other than *i* being attacked by a predator is $G_{-i} = (1-G_i)/(n-1)$. We also assume that all producers except those in community *i* are uniformly distributed over (n-1) number of communities such that $\pi_{-i} = ((1-\gamma)-\pi_i)/(n-1)$. Then the expected per producer consumption in a community other than *i*, becomes

⁸ By contrast, if communities and individuals had to move simultaneously, then each community would have to act as if it had no influence on the predator/producer ratio, on the distribution of producers over communities, or on the likelihood of being attacked. In effect, in a one-shot simultaneous move game, a community would act as though it had no more influence on the behaviors of individuals than does any individual producer within that community.

$$C_{-i} = \left(\frac{p}{(1+p)}\frac{G_{-i}}{\pi_{-i}}\frac{d_{-i}}{t} + \left(1 - \frac{p}{(1+p)}\frac{G_{-i}}{\pi_{-i}}\right)\right)(1-d_{-i})\pi_{-i}(2k - \pi_{-i}).$$
(9)

Similarly, the expected per predator consumption in a community other than *i* is

$$D_{-i} = \left(1 - \frac{d_{-i}}{t}\right) (1 - d_{-i}) \pi_{-i} (2k - \pi_{-i}).$$
(10)

Given the policy vector **d**, the optimal behavior of individuals in the second stage requires that individuals must be indifferent about where they choose to live and what role to assume. This yields three indifference conditions, $C_i = D_i = C_{-i} = D_{-i}$. For any given community policy vector **d** and the parameters (*n*,*t*,*k*), these conditions determine the symmetric equilibrium *p*, **G**, and distribution of producers over communities.⁹

Figures 2.3, 2.4, 2.5, and 2.6 below show the properties of the symmetric equilibrium in the second stage as a function of the policy vector chosen by the communities and particular values for the parameters, *n*, *t*, and *k*. These reflect the equilibrium indifference conditions, $C_i = D_i = C_{-i} = D_{-i}$. For the given parameter values when the policy vector is $(d_i, d_{-i}) = (0, 0.30809)$, notice that 20% of the population is predators but none of the predators attack community *i*. This is because more than 50% of the producers live in community *i*. In this community, production is taking place in the region where congestion dominates agglomeration while in the other community agglomeration relatively outweighs congestion in the productive and represent a better 'target' for predators. As community *i* raises its defense level above zero, the tax drives some producers in its community into predation. Furthermore, this increased productivity in community *i* as well. Therefore, the overall predator/producer ratio increases as d_i increases.

⁹ In the interests of space, the explicit derivation of the equilibrium strategy in the second stage is omitted and is available upon request.



Figure 2.3: $\pi_i(d_i)$, given $(d_{-i}, n, t, k) = (0.30809, 2, 0.9, 0.222)$



Figure 2.4: $G_i(d_i)$, given $(d_{-i}, n, t, k) = (0.30809, 2, 0.9, 0.222)$



Figure 2.5: $p(d_i)$, given $(d_{-i}, n, t, k) = (0.30809, 2, 0.9, 0.222)$



Figure 2.6: $C_i(d_i) = D_i(d_i) = C_{-i}(d_i) = D_{-i}(d_i)$, given $(d_{-i}, n, t, k) = (0.30809, 2, 0.9, 0.222)$

Given the strategy chosen by individuals in the second stage, the problem facing community *i* in the first stage is

$$\begin{split} & \underset{d_i}{Max} \quad C_i = \left(\frac{p}{(1+p)} \frac{G_i}{\pi_i} \frac{d_i}{t} + \left(1 - \frac{p}{(1+p)} \frac{G_i}{\pi_i}\right)\right) (1 - d_i) \pi_i (2k - \pi_i) \\ & \text{s. t.} \qquad p = p(d_i, d_{-i}) , \quad G_i = G_i(d_i, d_{-i}) , \quad \pi_i = \pi_i(d_i, d_{-i}) , \\ & p \ge 0 , \qquad 0 \le d_i \le t < 1, \quad \sum_{i=1}^n G_i = 1, \end{split}$$

where $p(d_i, d_{-i})$, $G_i(d_i, d_{-i})$, and $\pi_i(d_i, d_{-i})$ are the equilibrium second-stage responses of individuals to the policy vector **d**.

Differentiating the per capita consumption with respect to d_i and substituting the equilibrium condition $d_{-i} = d_i$ into the first-order condition yields the fixed-point at which all communities have the same defense level. The fixed-point is the symmetric equilibrium level of defense of local communities (see Appendix A for a full description of equilibrium).

In the second stage equilibrium, agents adjust to any policy vector so as to satisfy the indifference conditions. This implies that if community *i* can raise the per capita consumption of its producers by changing its defense level, it will induce a change in the behavior of agents that raises the consumption of all agents, not simply those who are producers in community *i*.

2.5.1.1 The fully rational community equilibrium is socially optimal.

The responses made in the second stage to any variation in the policy vector chosen in the first stage are independent of the process that produced that variation. Therefore, the response of agents to a change in di is the same whether this change is made by the local government in community i or by a central planner. Therefore, a central planner who wishes to choose a policy vector to maximize the per capita consumption of producers is subject to the same constraints created by the action of agents in the second stage game as is any local government. If we rewrite the central planner's objective as

$$M_{\mathbf{d}} Z = \frac{1}{n} \sum_{j=1}^{n} C_j(\mathbf{d}),$$

where $C_j(\mathbf{d})$ incorporates the equilibrium second-stage responses, then the first-order conditions are

$$\frac{\partial Z}{\partial d_i} = \frac{1}{n} \sum_{j=1}^n \frac{\partial C_j}{\partial d_i} = 0, \qquad \forall i.$$

The nature of the second stage response equalizes the consumption of all agents, regardless of whether an agent is a producer or a predator, regardless of where a producer produces, and regardless of where a predator chooses to attack. Therefore, the following two conditions hold:

$$\frac{\partial C_{j}}{\partial d_{i}} = \frac{\partial C_{i}}{\partial d_{i}}, \quad \forall j , \qquad \qquad \frac{\partial C_{i}}{\partial d_{i}} = 0 \Leftrightarrow \frac{\partial Z}{\partial d_{i}} = 0.$$

A local government has no interest in the consumption of any agent who is not a producer in its own jurisdiction. Nevertheless, a fully rational local government recognizes that a change in its own policy can only benefit its own producers if that change induces a response by agents that leads to an increase in the consumption per capita of all agents in the economy. This is reflected in the best response function of a fully rational local government. Consequently, given the defense policy of all other local governments, the local government in community *i* changes its own defense level in exactly the same way as a central planner would do so. This is what produces the identity between the equilibrium achieved among fully rational local governments and the optimal defense chosen by a central planner.

2.5.2 Boundedly rational community equilibrium

We now assume that local governments are boundedly rational in the sense that they take as given the global predator/producer ratio, p, and the distribution of producer activity, (π_i, π_{-i}) . But each community is assumed to anticipate that predators distribute their attacks across communities in a way that will maximize a predator's expected consumption. By changing its own level of defense, therefore, a community believes it can affect the probability that a predator will attack one of its producers. We continue to assume that producers are distributed uniformly over communities such that $\pi_i = \pi_{-i} = (1-\gamma)/n$, conditional on p. Assuming that all other communities except community i follow a common level of defense, d_{-i} , the anticipated behavior of a predator is characterized as the solution to the following problem:

$$M_{G} ax \quad D = \sum_{i=1}^{n} G_{i} \left(1 - \frac{d_{i}}{t} \right) (1 - d_{i}) y \qquad \text{s. t.} \qquad \sum_{i=1}^{n} G_{i} = 1, \quad G_{i} \ge 0, \quad \forall i .$$

Or, by the symmetry assumption for the communities other than *i*,

$$\underset{G_{i}}{\operatorname{arg\,max}} \quad D_{i} + D_{-i} = G_{i} \left(1 - \frac{d_{i}}{t} \right) (1 - d_{i}) y + \frac{1 - G_{i}}{n - 1} \left(1 - \frac{d_{-i}}{t} \right) (1 - d_{-i}) y .$$
(11)

Differentiating (11) with respect to G_i , we can derive the critical level of defense of community *i* from the first-order condition. Let this critical level of defense be $\hat{d}_i(d_{-i};n,t)$. Then the rule for distribution of predatory efforts of a predator is derived as follows:

$$G_i = 1$$
 if $0 \le d_i < \hat{d}_i(d_{-i}; n, t)$, $G_i = 0$ if $d_i > \hat{d}_i(d_{-i}; n, t)$.

Community *i* must, therefore, decide whether it is better to set its level of defense at the critical level, and be free of predators, or below the critical level. Let $C_{ia}(d_{-i},p;n,t,k)$ denote the per capita consumption of its producers when community *i* sets its defense at $\hat{d}_i(d_{-i};n,t)$. Alternatively, given that it chooses a level of defense below the critical level, its optimal choice satisfies the solution to

$$\underset{d_i}{Max} C_i = \left(pn \frac{d_i}{t} + (1 - pn) \right) (1 - d_i) y \qquad \text{s. t.} \qquad 0 \le p \le \frac{1}{n}, \qquad 0 \le d_i \le t < 1.$$

This implies

$$d_i(p;n,t) = \frac{pn-t+pnt}{2\,pn} \,.$$

Let $C_{ib}(p;n,t,k)$ denote the per capita consumption when community *i* chooses the optimal level of defense, $d_i(p;n,t)$, conditional on $d_i < \hat{d}_i(d_{-i};n,t)$. Therefore, its best choice is the *max* $\{C_{ia}(d_{-i},p;n,t,k), C_{ib}(p;n,t,k)\}$. The symmetric Nash equilibrium requires that the symmetric level of defense elsewhere, d_{-i} , must be chosen so as to make community *i* indifferent between $C_{ia}(d_{-i},p;n,t,k)$ and $C_{ib}(p;n,t,k)$. Using this indifference condition, we derive the fixed-point level of defense, d(p;n,t), as shown in figure 2.7 for fixed *t*.



Figure 2.7: Fixed-point d(p;n), given t = 0.9

As n increases, the fraction of producers who are located in any one community decreases. When one small community increases its defense level, the induced increase in predatory activity summed over all other communities is smaller than when a large community increases its defense level. Consequently, the induced response of other communities to an increase in the defense level of community i is smaller, the smaller is community i (or, equivalently, the larger is n). Because the conditional responsiveness of other communities to an increase in defense by community i depends on the percentage of population in i, as n increases and this percentage decreases, the equilibrium level of defense increases.

Solving simultaneously the indifference condition (5) and the symmetric equilibrium condition $C_{ia}(d_{-i},p;n,t,k) = C_{ib}(p;n,t,k)$, we obtain the symmetric equilibrium level of defense, d(n,t), and predator/producer ratio, p(n,t).

2.6 A COMPARISON OF EQUILIBRIA

Figures 2.8 and 2.9 compare the equilibrium defense levels and consumption levels under atomistic competition and boundedly rational community competition.



Figure 2.8: Atomistic equilibrium d(n,t) (top) and boundedly rational community equilibrium d(n,t) (bottom)



Figure 2.9: Atomistic equilibrium C(n,t) (bottom with large *n*) and boundedly rational community equilibrium C(n,t) (top with large *n*), given k = 0.222

Observe that the boundedly rational community equilibrium level of defense is decreasing in the number of communities, and it is lower than the atomistic equilibrium level of defense that is independent of the number of communities. The reason is that an increase in any one community's defense level reduces p, and, ceteris paribus, reductions in p reduce the optimal level of defense for any community. Since an individual is of measure zero, no individual's level

of defense relative to that of other individuals can influence p and, therefore, cannot influence any other individual's best response function through this channel. However, a community is not of measure zero, so it does influence other community's reactions to its own level of defense through two channels: (1) the relative defense level effect, which induces a positive reaction and (2) the average defense level effect, an effect on p that induces a reduction in defense by others. It is this second effect that accounts for the lower equilibrium level of defense in the community equilibrium than in the atomistic equilibrium.

A lower equilibrium defense level and predator/producer ratio achieved by switching from atomistic competition to community competition does not imply that equilibrium consumption levels will always be increased by such a switch. While the producers in a community gain by paying lower taxes and by being subject to a lower probability of attack, fewer predators mean that each community will have a larger fraction of the global population engaged in production within that community. The productivity of a producer depends nonlinearly on the fraction of the global population that is producing in the same location. Therefore, it is possible that when the number of communities is small, the decline in productivity induced by the congestion created by a larger population of producers more than offsets the consumption savings associated with the lower tax rate and the lower probability of being attacked. Figure 2.9 shows that this is, in fact, true when k = 0.222, $t \le 0.9$, n < 4, and communities do not anticipate how p will change as they change their own defense levels.¹⁰

The assumption that boundedly rational communities do not anticipate the change in p that their choice of d induces is crucial. If, as in fully rational community model, this effect is fully anticipated, then a community's best response function will reflect the effect of its own choice of defense on the size of its producer population and, therefore, on the productivity of its producers. When increases in defense spending per producer induce a reduction in the population of producers that raises output per producer by more (less) than the increased sum of tax per producer and expected loss to predation per producer, communities that fully anticipate this effect will select a higher (lower) defense level than would be chosen by boundedly rational communities. This is reflected in figure 2.10, where for a sufficiently small number of communities, defense expenditure per capita is higher in the fully rational community

¹⁰ For a comparison of equilibria with different values of k, see figure A1 in Appendix B.

equilibrium than in the boundedly rational community equilibrium. For a larger number of communities, the opposite is true. Consequently, as is shown in figure 2.11, for all values of n the consumption level in the competitive community equilibrium when communities fully anticipate this effect is always higher than that which is achieved in either the atomistic equilibrium or the community competitive equilibrium without anticipation of this effect.



Figure 2.10: Atomistic d(n,t) (top), boundedly rational community d(n,t) (bottom), and fully rational community d(n,t) (penetrating), given k = 0.222



Figure 2.11: Fully rational community C(n,t) (top), boundedly rational community C(n,t) (middle with large *n*), and atomistic C(n,t) (bottom with large *n*), given k = 0.222

2.7 CONCLUSION

In this paper, we have analyzed the extension of a model of production and predation due to Grossman (1998) to a multiple community setting. In a multiple community setting, defense expenditures in any one community have the property of a local public good within that community. We assume that predators are free to roam. We also assume, as in Grossman, that individuals can choose whether to be producers or predators. Therefore, defense against predation in any one locale generates a series of effects on other communities. These effects include changes in the distribution of population among communities, the redistribution of predatory effort over communities, and an induced change in the predator/producer ratio raises the percentage of output consumed by producers. However, a reduction in the predator/producer ratio raises the number of producers in each locale. This increase in the number of producers can either raise, or lower productivity through scale effects. These scale effects depend upon both the number of communities and properties of the production function.

In equilibrium all individuals have the same level of consumption regardless of whether they are producers or predators and regardless of where any producer lives. The question we address in this paper is whether the level of defense chosen by local governments so as to maximize the per capita consumption of their own producers, given defense levels elsewhere, always produces a second-best outcome. Our analysis shows that the answer to this question is a qualified "No." Under some parametric conditions, unless local governments anticipate the effects of their defense decisions on the global predator/producer ratio and its consequences for productivity, the equilibrium level of per capita consumption is actually higher when each individual producer must determine how much effort to devote to defense than when each community decides how much its own producers will devote to defense. In particular, with a small number of large local communities and with a high level of effectiveness of defense, competition among local governments who do not take into account the effects of their actions on the global predator/producer ratio may produce a lower equilibrium level of per capita consumption than would occur if all defense decisions were made at the individual level. However, if communities are fully rational and anticipate the equilibrating behavior of agents, then the community equilibrium is the first best.

In our model, all agents are identical. Therefore, there is no inherent reason for communities to be different in any respect in equilibrium. By concentrating on this special case, we are able to highlight the important difference between the equilibrium outcomes that emerge when local governments are fully rational and when they are not. Of course, we observe that communities in the real world are not identical. To capture this aspect of reality we would have to consider agents who differ either by productivity, or by preferences, or both. More productive agents have more to lose to a predator than do less productive agents. Therefore, they would presumably be willing to devote a larger fraction of their endowments to defense than would less productive types. How this difference in willingness to pay for defense would be reflected in equilibrium generated by competing local governments, and indeed whether an equilibrium exists, might well depend upon assumptions that are made with regard to how a government determines how much each individual in its jurisdiction is required to pay for their own defense.

3.0 DISCRIMINATORY TAXATION IN A MODEL OF LOCAL COMMUNITY COMPETITION

3.1 INTRODUCTION

Communities compete to attract and retain economic resources.¹¹ This competition typically involves a community adopting a fiscal policy that discriminates in favor of economic resources that are not currently located in that community. Such discrimination in favor of 'newcomers,' or 'outsiders,' tends to be a permanent feature of local fiscal policies. Since resources that locate in a particular community tend to remain in that same community for an extended period of time, current 'outsiders' can anticipate being subject to future tax treatment as an 'insider' and have to pay for the favorable tax treatment extended to future 'newcomers.' Consequently, it is not obvious why the local favorable tax treatment of 'outsiders' should have any effect on the location decisions of 'outsiders.' But, if the location decisions of 'outsiders' cannot be expected to be influenced by such incentives, then why have they become a permanent feature of the tax systems of local governments? The objective of this paper is to explain why local fiscal competition leads to such discriminatory tax policies and to analyze the consequences of this type of tax discrimination.

There are controversies in the voluminous literature of tax competition as to whether competition among localities for new investments leads to an outcome which is worse than would occur when there is no competition at all.¹² According to a negative view of tax competition, communities try to outbid one another in order to attract new investments the

¹¹ For an empirical study of strategic tax competition, see Brueckner and Saavedra (2001).

¹² See, for example, Wilson and Wildasin (2004) and Wilson (1999) for a comprehensive survey of tax competition literature.

amount of which is fixed in the economy as a whole, leading themselves to a "race to the bottom." The results of our analysis do not support this negative view of tax competition. Specifically, we will show that there is a natural limit to the degree of discrimination in local tax policies between old investments and new investments. Furthermore, this discrimination does not lead to an equilibrium that is worse for anybody than would occur if competition were not allowed.

Our results share some features in common with other papers that have addressed differential tax treatment of economic resources based on mobility. Wilson (1985) presents a model where a large number of small regions engage in tax competition for mobile capital which is both used as a complementary input to the labor supplied by immobile residents and taxed to finance local public expenditures. He was able to show that the optimal property taxation chosen by local communities so as to maximize their own residents' utility calls for a lower tax on the capital that is used to produce a nationally traded good, which is exchanged for capital and whose demand is infinitely price elastic, than the tax on the capital that is used to produce a non-tradable local good. In contrast, we treat the mobile resources as technically identical to the immobile resources, i.e. perfect substitutes. Accordingly, in our model a local government's policy affects the *size* of its community, rather than the mix of economic factors.

The boundedness in the degree of tax competition is investigated in Wilson (1986). Based on the same model as in Wilson (1985), Wilson studies the optimal public policy from a local region's viewpoint, i.e. the tax rates on mobile capital and capital-labor ratio in public production. His analysis emphasizes that the condition for tax competition to take place depends upon the production technologies for public goods, factor substitutability, and elasticity of substitution between private goods. The equilibrium tax policy in our model also shows explicitly the intensity of tax competition as a function of certain parameters in closed form.

In an economy in which agents' productivity is exposed to risks, factor mobility can have significant implications for income redistribution. Wildasin (1995) utilizes a simple model that captures the risk shifting of mobile factors to immobile factors, which results in a greater expected mean income for immobile factors than would occur in the absence of variable factor's spatial arbitrage. The patterns and consequences of risk shifting are similar to our results, although the process that generates such results is different. In his model, the mobility of agents and the size of immobile agents are exogenously given and there is no incentive for local

governments to engage in tax competition. By contrast, in our model all agents are mobile when "young" and immobile when "old" and the size of a community is endogenously determined as a result of strategic interactions among communities who compete for mobile factors.

Wildasin and Wilson (1998) consider the spatial diversification of income risks through cross-ownership of shares in firms in a tax competition setting. In their model, the first-best outcome of risk-pooling through full diversification is achieved in the absence of tax competition. This is so because local governments engage in confiscatory taxation on immobile factors owned by non-residents. However, when local governments must levy uniform property taxes on both variable and fixed factors, the mobility of one factor alleviates the confiscatory taxation on the other immobile factor, given a reasonable substitutability between these factors. Yet, in their model, the fact that non-residents might anticipate such confiscatory taxation of communities on immobile assets is not considered. By contrast, the anticipatory behavior of nonresidents before they commit their resources to a particular community is an essential element of the model we propose below. Somewhat surprisingly, agents' rational expectations, combined with the competitive pressure facing communities, greatly simplify the derivation of equilibrium tax policies. Furthermore, in our analysis tax competition does not introduce distortions.

The closest model to ours is presented in Wildasin and Wilson (1996) in which workers who live for two periods are free to choose a town to reside and work while young so as to maximize their expected discounted lifetime net income. After the first period of life, they can relocate themselves with a moving cost which is randomly distributed and privately known to each worker in the second period. Appropriately, each local government sets tax policies so as to maximize its land value. Given the moving costs, which give the town monopsony power over the stayer wage, the steady state young wage is greater than that of old stayer wage, resulting in inefficiency borne by workers.

In what follows, we take a slightly different modeling strategy, one that is actually more realistic and greatly simplifies our analysis of the questions we want to address. We will assume an infinite cost of relocation. The objective of a local government in our model is to maximize the welfare of its existing old residents whose resources are embedded in its community. In our analysis, different from the result in Wildasin and Wilson (1996), the "intergenerational income transfer" from the old to the young has a natural bound, the magnitude of which is determined by parameters.

The rest of paper is organized as follows. The model is introduced in the following section. Then we derive the steady state equilibrium tax policy. Conclusion sums up the results.

3.2 THE MODEL

The economy consists of two regions, A and B, in which production takes place. In each period, a unit mass of two-period lived agents are born such that there are two generations of the same size of individuals at any point in time. Let $O_{i,t}$ be the size of old agents and $N_{i,t}$ the size of new agents in community i in period t such that $O_{A,t} + O_{B,t} = N_{A,t} + N_{B,t} = 1$. New agents are endowed with a unit mass of inalienable and durable resources.¹³ In the first period, agents are mobile and must choose a community in which to reside and to locate their resources. In the second period, an agent is a member of a community chosen in the first period of life and the agent's resources must be used within that particular community. The resources of both old and new residents in a community are used to produce an output through a production technology that exhibits diminishing marginal productivity. The price of output is normalized to one, so the total value of output in community i in period t is $(O_{i,t} + N_{i,t})^{\alpha}$, $0 < \alpha < 1$.

Each community must finance a local public good, which requires a fixed amount of expenditures, F.¹⁴ Existing residents benefit from attracting new residents and their resources whose output can be taxed to help finance the local public good. However, this benefit is limited by the fact that as the size of the community grows output per capita tends to diminish. In effect, old-timers will have two tax bills, one for financing the local public good and the other for attracting newcomers. Equivalently, all residents pay the same tax per capita, but upon arrival new residents receive an entry bonus, *b*, which can be either positive or negative. It is this bonus that represents a discriminatory tax system.

¹³ Since economic factors are not differentiated from one another in the present model, we simply regard an agent's resources as *labor* and the income generated by his/her resources as *wage* income.

¹⁴ We assume F is large enough for interior solutions such that both communities have population.

In each period, the game proceeds in two stages. In stage one, each community sets a tax policy so as to maximize the after-tax per capita income of its existing, old residents, taking as given the tax policy of the other community. In stage two, each new agent chooses a community in which to reside and to locate their resources so as to maximize the discounted present value of expected lifetime after-tax income, given the profile of tax policies adopted by communities. Potential entrants are forward looking. They recognize that the expected lifetime net income of entry into a community depends upon (1) the present size of its existing population, (2) its tax policy when they first enter that community, and (3) its expected size and related tax policy of their chosen community in the second period of their lives, when they and their resources have become immobile. In the following period, the old residents die, new residents become old, and new agents are born. And the game proceeds in the same way.

We are interested in the symmetric Nash equilibrium in which identical agents or communities choose the same strategy. Also the equilibrium must be sequentially rational such that each community chooses a tax policy in anticipation of the new agents' response, taking as given the tax policy of the other community. Then the problem facing community A is stated as follows:

$$\max_{\tau_{A,t}} y_{A,t} \equiv (1 - \tau_{A,t}) (O_{A,t} + N_{A,t})^{\alpha} \frac{1}{O_{A,t} + N_{A,t}}$$

s.t.
$$F + b_{A,t} N_{A,t} = \tau_{A,t} (O_{A,t} + N_{A,t})^{\alpha}$$
,

$$N_{A,t} = N_{A,t}(b_{A,t}; b_{B,t}, O_{A,t}, O_{B,t}),$$

where $\tau_{A,t}$ is an income tax rate and $b_{A,t}$ is the size of bonus per newcomer of community A in period t. Similarly, we can define the after-tax per capita income of old residents in community B, $y_{B,t}$, and its objective with constraints. The first constraint reflects the assumption of balanced budget. That is, communities spend their tax revenue on financing the local public good and bonuses to new entrants without surplus or deficit. The second constraint is a function of vectors of bonuses and distribution of old agents, which is derived from the new agents' behavior in stage two. From a new agent's viewpoint, his/her strategy is

Go to community *A* if $\{y_{A,t} + b_{A,t} + \delta y_{A,t+1}\} > \{y_{B,t} + b_{B,t} + \delta y_{B,t+1}\},\$ Go to community *B* if $\{y_{A,t} + b_{A,t} + \delta y_{A,t+1}\} < \{y_{B,t} + b_{B,t} + \delta y_{B,t+1}\},\$ Choose randomly if $\{y_{A,t} + b_{A,t} + \delta y_{A,t+1}\} = \{y_{B,t} + b_{B,t} + \delta y_{B,t+1}\},\$

where δ is a discount factor, $0 < \delta < 1$.¹⁵

Observe that when a community determines the income tax rate, the size of bonus is also determined through the constraints on budget balance and the strategy of possible newcomers. Then we can rewrite a community's problem as choosing the size of bonus paid to each new entrant:

$$\max_{b_{A,t}} y_{A,t} = (O_{A,t} + N_{A,t})^{\alpha - 1} - \frac{F}{O_{A,t} + N_{A,t}} - \frac{b_{A,t} N_{A,t}}{O_{A,t} + N_{A,t}}$$
(1)

s.t.
$$N_{A,t} = N_{A,t}(b_{A,t}; b_{B,t}, O_{A,t}, O_{B,t})$$

3.3 THE STEADY STATE

Proposition 1.

Suppose that there is $k \ge 2$ number of communities. In the steady state, all communities are of the same size and adopt the same tax policy:

¹⁵ Variables $y_{A,t}$, $y_{A,t+1}$, $y_{B,t}$, $y_{B,t+1}$, should be understood in expected terms. For notational simplicity, the notation for expectation is suppressed.

$$O_{i} = N_{i} = \frac{1}{k}, \quad b_{i} = k \left(\left(\alpha - 1 \right) \left(\frac{2}{k} \right)^{\alpha} + F \right).$$
 The steady state annual after-tax per capita income is
$$y_{i} = k \left(\left(\frac{2}{k} \right)^{\alpha} - F \right) - \alpha \left(\frac{2}{k} \right)^{\alpha-1} \text{ for old agents, } y_{i} + b_{i} = \alpha \left(\frac{2}{k} \right)^{\alpha-1} \text{ for new entrants, } i = 1, 2, 3, \dots k.$$

Proof. The first-order condition for the solution, b_i , to community's problem (1), given $b_{j \neq i}$, is

$$\frac{dy_i}{db_i} = \frac{\partial y_i}{\partial N_i} \frac{\partial N_i}{\partial b_i} + \frac{\partial y_i}{\partial b_i} = 0, \quad \text{or} \quad \frac{\partial y_i}{\partial N_i} \frac{\partial N_i}{\partial b_i} = -\frac{\partial y_i}{\partial b_i}.$$
(2)

We know that in the steady state a new entrant must be indifferent to place of entry, i.e. $y_i + b_i + \delta y_i = y_j + b_j + \delta y_j$. In the steady state by symmetry, $y_i = y_j$. So the indifference condition in the steady state is $y_i = y_j + b_j - b_i$. Differentiating the steady state indifference condition with respect to b_i yields

$$\left\{\frac{\partial y_i}{\partial N_i}\frac{\partial N_i}{\partial b_i} + \frac{\partial y_i}{\partial b_i}\right\} = \frac{\partial y_j}{\partial N_i}\frac{\partial N_i}{\partial b_i} - 1.$$

Since the terms in brackets are zero in equilibrium due to community *A*'s optimization, we can rearrange the equation as follows:

$$\frac{\partial N_i}{\partial b_i} = \frac{1}{\frac{\partial y_j}{\partial N_i}} = \frac{-1}{\frac{\partial y_j}{\partial N_j}}.$$
(3)

The second equality is due to $\frac{\partial N_j}{\partial N_i} = \frac{\partial N_i}{\partial N_j} = -1$. Substitute equation (3) into the first-order

condition (2) for community A.

$$\frac{\partial y_i}{\partial N_i} = -\frac{N_i}{O_i + N_i} \frac{\partial y_j}{\partial N_j}.$$
(4)

Evaluation of equation (4) when $O_i = N_i = O_j = N_j = \frac{1}{k}$ yields the following reaction function for community *i*:

$$\left((\alpha - 1) \left(\frac{2}{k}\right)^{\alpha - 1} + F\left(\frac{k}{2}\right) - \frac{1}{2}b_i \right) = -\frac{1}{2} \left((\alpha - 1) \left(\frac{2}{k}\right)^{\alpha - 1} + F\left(\frac{k}{2}\right) - \frac{1}{2}b_j \right), \text{ or}$$
$$b_i = -\frac{1}{2}b_j + 3 \left((\alpha - 1) \left(\frac{2}{k}\right)^{\alpha - 1} + F\left(\frac{k}{2}\right) \right).$$

Similarly, community *j*'s reaction function is $b_j = -\frac{1}{2}b_i + 3\left((\alpha - 1)\left(\frac{2}{k}\right)^{\alpha-1} + F\left(\frac{k}{2}\right)\right)$. Then we impose the symmetry condition $b_i(b_j) = b_j(b_i)$ to find the fixed point. Thus, we have the steady state bonus, $b^* = k\left[(\alpha - 1)\left(\frac{2}{k}\right)^{\alpha} + F\right]$. The steady state annual after-tax per capita income of the old and the young follows immediately. Q.E.D.

In equilibrium, the tax system provides positive entry bonuses to newcomers, even when both communities are identical and could attract the same number of newcomers as they would if there were no bonuses at all.¹⁶ Note that, in stage two, as long as bonuses are the same across communities, newcomers are equally divided. However, competition between communities implies that in equilibrium their tax policy must discriminate in favor of newcomers by paying positive bonuses.

Importantly, the bonus is finite, so that there is not a "race to the bottom," but instead the equilibrium bonus is determined by the parameters of the production function and the magnitude of the fixed cost associated with the local public good.¹⁷ The intuition is straightforward. The

¹⁶ We assume $(\alpha - 1 + F) > 0$.

¹⁷ When k increases, the equilibrium bonus per new agent increases, altering the lifetime distribution of income by raising income when young and reducing income when old. The intuition behind this comparative statics is as follows. As k increases, the size of a community decreases. Consequently, each new worker has a smaller negative impact on average productivity than the positive impact on reducing the cost per old worker of financing the local

larger α is, the smaller is the marginal cost of newcomers to old-timers associated with the declining average productivity. And a large *F* implies a large marginal contribution of newcomers to old-timers through sharing the fixed cost of the local public good. Therefore, an increase in (α , *F*) makes communities more aggressive in attracting newcomers by offering larger bonuses.

Notice that tax competition results in an "intergenerational income transfer" from the old to the young.¹⁸ Would this income transfer have an effect on efficiency? The answer is "No." Although agents receive a higher income when young, in our model there is no opportunity to save at a positive interest rate, which would allow a higher sustainable level of consumption throughout their lifetime.¹⁹ Therefore, if individuals smooth their consumption path over their lifetime, they have the same consumption path as they would have in the absence of competition.

3.3.1 Discriminatory taxation is welfare improving.

In the absence of discriminatory taxation, there are three steady state distributions of population: One of the two communities contains entire population. Each community has the same size of old-timers and newcomers. Because of declining marginal productivity, it will not generally be in the interest of old timers who are concentrated in a single community to have all of the newcomers join them. However, without the ability to impose an entry fee, if that community should become populated with all of the newcomers, no newcomer would find it in his/her interest to move away since a newcomer who does so would have to bear the full cost of providing the public good in the other community. With the possibility of charging a negative bonus, or entry fee, a community that currently has entire population of old-timers can control

public good. Therefore, as k increases, old workers in any community have an increased incentive to attempt to attract new workers.

¹⁸ In the absence of competition, the discounted present value of expected lifetime after-tax per capita income would be $(1-F) + \delta(1-F)$, while with competition it is $\alpha + \delta\{(1-F)-(\alpha-1+F)\}$. Thus, the amount of income transfer is $(\alpha-1+F)$.

¹⁹ This kind of income transfer might have an effect on a long-term growth path of an economy, if one were to consider a model in which individuals have opportunities to place savings into productive assets. Since young people receive a bonus, they would have an incentive to save some portion of the bonus in order to smooth their consumption path at a higher sustainable level. However, we abstract from such opportunities of investment.

the number of newcomers who will find it profitable to enter. They can, therefore, prevent the volume of new entrants from exceeding the number that maximizes the after-tax income of the old-timers in that community. Consequently, in the presence of discriminatory taxation, when it is not optimal for everyone to be in a single community the only steady state is the equal division of population. Since a degenerate distribution of population generally provides a lower after-tax income per capita than would be generated by the equilibrium in which population is equally divided, the availability of discriminatory taxation can actually guarantee that the Pareto superior equilibrium is realized. That is, discriminatory taxation serves social purpose.²⁰

3.4 CONCLUSION

In this paper, we have analyzed discriminatory taxation in a model of local community competition within the context of a dynamic, overlapping generations model. A community chooses a tax policy each period to maximize the after-tax per capita income of its existing, old residents, taking as given the tax policy of other communities, anticipating the new agents' response. Existing residents benefit from attracting new residents and their resources whose output can be taxed to help finance the local public good. But this benefit is limited by the fact that as the size of the community grows output per capita diminishes. The tax policy specifies that all residents pay the same tax per capita, but upon arrival new residents receive an entry bonus, which can be either positive or negative. It is this bonus that represents a discriminatory tax system.

The bonus is finite, so that there is not a "race to the bottom." Although the equilibrium tax policy results in an "intergenerational income transfer" from the old to the young, agents have the same consumption path as they would have in the absence of competition. While entry bonuses and fees are motivated by individual community strategic concerns, our analysis does

 $^{^{20}}$ In this model, entry fees and bonuses have the same effect as zoning regulations on minimum housing expenditures in the model of Hamilton (1976). They serve to insure that newcomers take into account the effect of their location decisions on others.

suggest that strategic discriminatory taxation may well be welfare improving, relative to the tax regime that forces communities to adopt the same tax policy. In a world with both local congestion and local public goods they help to internalize externalities.

4.0 A NOTE ON THE EFFECTS OF TAX INCREMENT FINANCING ON THE PATH OF LAND DEVELOPMENT

4.1 INTRODUCTION

Tax increment financing (TIF) is a financing device a local government uses for development. The sponsoring jurisdiction (municipality) makes a commitment to financing a portion of the development of the property. Typically, it issues bonds to finance its share of the development cost. The taxing authority then pays the bonds by dedicating the increment of tax revenue the property generates once developed, relative to its predevelopment assessed value, to paying off these bond obligations. In effect, the private developer receives assets from the taxing authority.

TIF began in California in 1952 as a means of generating local matching funds for federally funded development projects (Huddleston 1982). However, it was not until the mid-1970s that TIF gained its popularity.²¹ There are three principal reasons for adoption of TIF. First, TIF creates a development zone (TIF district) to internalize externalities associated with the development of various parcels in the neighborhood that has become blighted. A blighted neighborhood is one in which it would be profitable to convert *all* of the properties in the neighborhood to another use, but would not be profitable to convert only a portion of the properties. That is, the conversion of properties to more intensive uses require public investments, for example, in site assembly, transportation, and communication, which involve

²¹ By 1970 seven states had authorized TIF (Wyatt 1990). By 1984 twenty-eight states (Greuling 1987), by 1992 forty-four states (Forgey 1993), and by 1997 forty-eight states had passed legislation authorizing TIF (Johnson and Kriz 2001). Johnson and Man (eds. 2001) provide comprehensive studies on TIF, such as working processes, scales of TIF bonds, determinants of TIF adoption, effects on economic development, history, and selected state by state examples.

collective action among developers for that type of investment to take place.²² Therefore, a parcel of land that is left underdeveloped exerts negative externalities on its neighbors by not sharing the development cost and by lowering the value of other developed land in its neighborhood. The capital investment made by the municipality in a TIF district internalizes these externalities.²³

Second, TIF coordinates development investments of multiple taxing jurisdictions. Once development is made in a TIF district by a municipality with subsequent increase in tax base, other taxing jurisdictions would benefit from the development without having made any contributions. The municipality alone would bear the burden of the development cost, while the benefits of development have to be shared by other local taxing bodies. TIF preempts such a free-riding problem by committing multiple taxing jurisdictions to sharing the development cost in proportion to their benefits by not collecting the increment of tax revenue in the TIF district for the duration of TIF.

Third, TIF reduces the economic distortions of property taxation on new development. It is this motivation for adoption of TIF on which we focus. Typically, the pattern of transformation of land use is affected by a property tax, when it is levied on both land and structures.²⁴ We know that a tax on structures tends to discourage the intensity of development and tends to delay the conversion of properties to more intensive uses. TIF can be viewed as a fiscal policy to compensate for these negative effects of property taxes and improve the intertemporal use of the developed land.²⁵

Tax increment financing provides a subsidy to developers who convert qualifying property from less intensive to more intensive use. Arnott and Lewis (1979) provide a dynamic

²² Brueckner (1997) presents a model in which he investigates how different schemes for financing public investments affect the speed of urban spread. TIF can be viewed as a financing scheme for infrastructure in which the city's existing landowners share the cost of required public investments for newly-developed land.

²³ Most states require a "blight finding" to establish a TIF district and to make TIF serve as a revitalization tool (Johnson and Kriz 2001). However, the notion of "blight" has been used in a very expansive fashion by municipalities so as to add "economic development" as one of the allowable purposes, effectively making TIF a regular development tool. See Luce (2003) for a critical review of the use of TIFs as a revenue-capturing device by the suburban affluent cities in St. Louis metropolitan areas.

²⁴ See, for example, Ladd (1998) for summaries on the effects of property taxation on investment decisions.

²⁵ Prior empirical studies have focused on the degree of which TIF has actually influenced the aggregate level of capital investment in communities that have adopted TIF. The central question on the effectiveness of TIF is whether TIF creates investments that otherwise could not take place. The critics of TIF suggest that TIFs simply relocate investments that otherwise would occur outside the TIF district. See Man and Rosentraub (1998) for the positive effects of TIFs on economic activities. For the ineffectiveness of TIFs, see Dye and Merriman (2003).

optimization model of an investor's decision to convert property from one use to another. McFarlane (1999) uses this model to investigate the effects of various types of development fees as well as property taxes on the timing of development, capital intensity, and urban spread. Capozza and Li (1994) also use an optimal stopping time approach to study the effects of property taxes on the pattern of land development when rents are stochastic. These models assume that conversion of property only occurs once. However, unlike the taxes and fees that are analyzed in these models, the magnitude of the subsidy that a developer gets from tax increment financing depends upon the intensity of use of the property in its pre-conversion, non-qualifying use. Therefore, now that TIF is a regular feature of local fiscal policies, it is important to ask how this policy will affect prior, non-qualifying uses of property. In particular, while TIFs reduce the distortionary effects of property taxation on uses that qualify, they introduce distortions on nonqualifying uses. As we will show, the net effect of TIFs on the dynamic path of property transformation depends on the magnitude of the effective subsidy to TIF qualifying uses. We identify what distortions TIF creates and evaluate their consequences, relative to desirable consequences of TIF, for shaping the dynamic path of land transformation from one use to another.

We consider the evolution of use of property to take place in two phases. In the early phase, land is used in a way that does not qualify for tax increment financing. In the second phase, the property is converted to a use that does qualify for TIF. Our analysis shows that unlike a reduction in property tax that induces an increase in capital investment in both phases, a TIF will induce a *reduction* in capital investment in the first phase. The reason for this effect is that by reducing the level of capital investment in the first, non-qualifying stage, a developer secures a lower tax base from which the TIF subsidy is computed and gets a larger subsidy for any given amount of TIF qualifying investment. Consequently, as we show, the net effect of a TIF on the dynamic efficiency of land use depends critically on the size of the TIF, relative to the capital investment that qualifies for the TIF.

The next section introduces the model and studies the path of property conversions to more intensive uses in the absence of taxes and subsidies as a benchmark. Then we will analyze the effects of TIF in the interaction with property taxation on the path of property conversions. The results are summed up in the conclusion.

4.2 THE MODEL

We assume that the community is interested in the value of its land and the tax stream that development of that land can generate. It makes TIF available to developers who wish to convert land from less capital intensive to more capital intensive use. Therefore, we utilize a model in which land has two possible uses, "residential" and "commercial." Only "commercial" use qualifies for a TIF. Further, we assume that in the earliest stages it is more profitable to use the land in a way that does not qualify for TIF. Therefore, the use of the land must be transformed at some point in time from a non-qualifying use to a qualifying use. We analyze how the availability of a TIF affects (1) the time at which the land is developed for a non-qualifying use, (2) the level of investment in the early non-qualifying use, (3) the time at which the use is converted to a qualifying use, and (4) the level of investment made in the qualifying use.

These four decisions made by the land developer will, in turn, determine the discounted present values of the community's land and the tax stream that the use of that land generates. Therefore, our analysis will reveal how the sum of discounted present values of land plus tax revenue is affected by the magnitude of the TIF subsidy.

We extend the Arnott and Lewis model so as to allow for a replacement of structures with conversion costs so that the use of land can be transformed twice. Associated with each possible use is a rental flow per unit of housing/office space created by the capital, *K*, devoted to that use: $r_r(t)$ for residential use and $r_c(t)$ for commercial use at time *t*. Like Arnott and Lewis, we assume that rents before development are zero, and rental rates are expected to grow at a constant rate, $\eta > 0$:

$$r(t) = r(0)e^{\eta t},$$

where r(0) is the rent at time zero such that the residential use of land generates a higher return per unit of capital than the commercial use of land does at time zero: $r_r(0) > r_c(0)$. But the rate of increase of rent per unit of capital from commercial use is greater than that from residential use: $\eta_c > \eta_r$. This reflects the fact that it takes time for a market to evolve in favor of commercial activities.

We further assume that the price of a unit of capital, p, is constant, structures do not depreciate, and the output of housing/office space on a unit of land, Q(K), increases in K at a

diminishing rate: Q'(K) > 0 and Q''(K) < 0. We also assume that it costs a fraction of the development cost to demolish the old (residential) structure, $K_{(r)}$, before a new (commercial) structure, $K_{(c)}$, can be built on the land. That is, the replacement cost is $\{dpK_{(r)}\}$ at the time when the new investment is made, where *d* is a replacement cost parameter: 0 < d < 1. Let *T* denote the timing of property conversions to more intensive uses.

Then, in the absence of property taxation and tax increment financing, the land developer's problem can be written as follows:

$$\begin{array}{l}
\underset{\{\mathbf{T},\mathbf{K}\}}{\max} \quad L(\mathbf{T},\mathbf{K}) \\
= \int_{T_r}^{T_c} r_r(t) Q(K_r) e^{-it} dt - pK_r e^{-iT_r} + \int_{T_c}^{\infty} r_c(t) Q(K_c) e^{-it} dt - p(dK_r + K_c) e^{-iT_c}, \quad (1)
\end{array}$$

where $\mathbf{T} = (T_r, T_c)$, $\mathbf{K} = (K_r, K_c)$, and *i* is the interest rate. Subscripts represent the type of development: '*r*' for residential use and '*c*' for commercial use of land. For the solution to this dynamic problem, we invoke backward induction and solve first for the optimal timing, T_c , and structural density, K_c , in the second stage, taking the first-stage decisions T_r and K_r as given. The first-order conditions for the second-stage problem are:

$$\frac{\partial L}{\partial K_c} = \left(\frac{r_c(T_c)Q'(K_c)}{i - \eta_c} - p\right)e^{-iT_c} = 0,$$
(2)

$$\frac{\partial L}{\partial T_c} = \left(r_r(T_c)Q(K_r) + ip(dK_r + K_c) - r_c(T_c)Q(K_c)\right)e^{-iT_c} = 0.$$
(3)

Equation (2) implies the optimal scale of capital investment in the commercial structure at which the marginal cost of capital–which is simply the price of a unit of capital, p–equals the marginal benefit of capital which is the increase in rents associated with an additional increase in capital. Equation (3) shows that, at the optimal timing T_c , the marginal cost of delay–which is the rents forgone from making the new conversion of property to a commercial use–equals the marginal benefit of delay which is the rents that continue to accrue to the incumbent residential use of land plus the interest saved by delaying the new conversion additional period.

The first-order conditions for the first-stage problem are obtained by differentiating the objective function (1) with respect to K_r and T_r , after substituting the second-stage solution, $T_c(T_r,K_r)$ and $K_c(T_r,K_r)$, into the objective function. Or, by the envelope theorem,

$$\frac{\partial L}{\partial K_r} = \left(\frac{r_r(T_r)Q'(K_r)}{i - \eta_r} - p\right)e^{-iT_r} - \left(\frac{r_r(T_c)Q'(K_r)}{i - \eta_r} + dp\right)e^{-iT_c} = 0,$$
(4)

$$\frac{\partial L}{\partial T_r} = \left(-r_r(T_r)Q(K_r) + ipK_r\right)e^{-iT_r} = 0.$$
(5)

Table A.2 in the appendix C contains numerical solutions for equations $(2)\sim(5)$ for various parameter values. Table 4.1 provides a summary of the comparative statics.

	i	η_r	η_c	$r_r(0)$	$r_c(0)$	р	d
T_r	+	?	-	-	-	+	+
K_r	-	+	-	+	-	-	+
T_c	-	+	-	+	-	+	+
K _c	—	+	+	+	—	—	+

Table 4.1 Summary of comparative statics

Note: i = interest rate, $\eta =$ growth rate of rent (η_r for residential use and η_c for commercial use of land), r(0) = rent at time zero ($r_r(0)$ for residential use and $r_c(0)$ for commercial use of land), p = price of a unit of capital, d = replacement cost parameter.

4.3 PROPERTY TAXATION AND TIF

Now we introduce property taxation and tax increment financing. Let x_s denote a property tax rate that applies to land and x_k to structures. For a TIF subsidy, note that the sponsoring jurisdiction finances its share of the development cost by issuing bonds. Then it pays the bonds by dedicating the increment of tax revenue the property generates, relative to its predevelopment assessed value, to paying off these bond obligations. In effect, the private developer receives assets from the taxing authority. These assets are essentially "gifts," the magnitude of which is proportional to the increment of the development cost.²⁶ Therefore, the size of the TIF subsidy

²⁶ The legal ownership of public investments belongs to the local government. However, from an economic perspective the public investments, once made, have an effect of transferring assets to the developer, since they become inseparable from the private investments that are made in conjunction with the public investments.

becomes $\{wp(K_c-K_r)\}$, where w is a TIF subsidy rate that determines the portion of the development cost shared by the sponsoring jurisdiction.

Then the objective function facing a private developer can be written as follows:

$$\underset{\{\mathbf{T},\mathbf{K}\}}{Max} P(\mathbf{T},\mathbf{K}) = R(\mathbf{T},\mathbf{K}) - \int_0^\infty x_s P(\mathbf{T},\mathbf{K}) e^{-it} dt = \frac{i}{i+x_s} R(\mathbf{T},\mathbf{K}), \qquad (6)$$

where $P(\mathbf{T},\mathbf{K})$ denotes the discounted present value of land. $R(\mathbf{T},\mathbf{K})$, the discounted present value of land, gross of land tax, is

$$R(\mathbf{T}, \mathbf{K}) = \int_{T_r}^{T_c} r_r(t) Q(K_r) e^{-it} dt - pK_r e^{-iT_r} - x_k \int_{T_r}^{T_c} pK_r e^{-it} dt + \int_{T_c}^{\infty} r_c(t) Q(K_c) e^{-it} dt - p\{dK_r + K_c - w(K_c - K_r)\} e^{-iT_c} - x_k \int_{T_c}^{\infty} pK_c e^{-it} dt.$$
(7)

The same procedure employed to solve the problem in the absence of property taxation and TIF yields the following first-order conditions for this problem:

$$\frac{\partial P}{\partial K_c} = \frac{i}{i+x_s} \left\{ \frac{r_c(T_c)Q'(K_c)}{i-\eta_c} - p\left(1-w+\frac{x_k}{i}\right) \right\} e^{-iT_c} = 0, \tag{8}$$

$$\frac{\partial P}{\partial T_c} = \frac{i}{i+x_s} \left\{ r_r(T_c)Q(K_r) + ip\left((d+w-\frac{x_k}{i})K_r + (1-w+\frac{x_k}{i})K_c\right) - r_c(T_c)Q(K_c) \right\} e^{-iT_c} = 0, \tag{9}$$

$$(9)$$

$$\frac{\partial P}{\partial K_r} = \frac{i}{i+x_s} \left\{ \left(\frac{r_r(T_r)Q'(K_r)}{i-\eta_r} - p\left(1+\frac{x_k}{i}\right) \right) e^{-iT_r} - \left(\frac{r_r(T_c)Q'(K_r)}{i-\eta_r} + p\left(d+w-\frac{x_k}{i}\right) \right) e^{-iT_c} \right\} = 0,$$
(10)

$$\frac{\partial P}{\partial T_r} = \frac{i}{i+x_s} \left\{ -r_r(T_r)Q(K_r) + ipK_r\left(1+\frac{x_k}{i}\right) \right\} e^{-iT_r} = 0.$$
(11)

Equations (8)~(11) correspond to equations (2)~(5), but incorporate the tax rate on structures, x_k , and the effect of TIF, w, on the cost of capital for "commercial" development. Notice that the tax rate on land, x_s , will not be taken into account for the first-order conditions and, therefore, has no effect on the investment decisions. In effect, this land tax is equivalent to a tax on the profit from the 'highest and best' use of the land so that the private developer cannot do better than choosing the same path of development as would choose in the absence of the land tax. By contrast, the tax rate on structures, x_k , and TIF subsidy rate, w, do enter into the first-

order conditions. Their effects on the sequence of property conversions to more intensive uses are summarized in Table 4.2.

	x_s	x_k	W
T_r	0	+	_
K _r	0	-	_
T_c	0	+	_
K _c	0	-	+

Table 4.2 Summary of comparative statics (property taxation and TIF)

Note: x_s = land tax rate; x_k = structure tax rate; w = TIF subsidy rate

The economic intuition for the comparative statics results with respect to a property tax and a TIF subsidy is straightforward. Notice that both the tax on structures and the TIF subsidy work in the similar way as the price of capital works for the first-order conditions. However, unlike a reduction in property tax, which increases capital investment in both development phases, an increase in TIF increases capital investment in the "commercial," TIF qualifying phase, *but decreases capital investment in the "residential," non-qualifying phase*. The reason for its negative effect on the "residential" investment phase is straightforward. The residential capital serves as the base on which the tax is levied during the interval in which "commercial" capital is subject to TIF. The smaller this base, the larger the effective subsidy created by the TIF. Therefore, TIF induces a land developer to sacrifice some value from "residential" use during the "residential" phase in order to secure a larger effective subsidy per dollar invested in the "commercial" phase.

Thus, TIF is not equivalent to a reduction in the property tax on structures. It reduces the distortionary effect of property taxes in the "commercial," TIF qualifying phase, *but increases the distortionary effect of property taxation in the non-qualifying phase*. Therefore, the net effect of a TIF on our welfare measure–denoted total surplus and measured by the sum of the discounted present values of land and net tax revenue–depends on the magnitude of the TIF subsidy, *w*. This is reflected in the numerical results displayed in Table A3 in the appendix. The largest welfare is achieved when there are neither taxes nor subsidies on structures. As that table

shows, when there is a tax on structures, total surplus first increases and then decreases as the TIF subsidy increases. Interestingly, the net of tax value of land, secured by the landowner, will increase as TIF increases, even as the discounted present value of taxes (total surplus – land value) decreases. Therefore, TIFs are always good for landowners, even when they are disadvantageous for tax payers as a group.

4.4 CONCLUSION

In this paper, we focused on the way in which TIF interacts with property taxation to affect the intensity and timing of capital investments. Unlike prior studies, which only analyze the effect of TIF on the intensity of investments that qualify for tax increment financing, we also consider the effect of TIF on earlier non-qualifying investments. We do so because TIF is now a regular feature of local fiscal policy. Therefore, private developers may anticipate the opportunity to secure a TIF at a later point in time, when making investment decisions that do not currently qualify for a TIF subsidy. This anticipated behavior of private developers affects the entire path of land development.

Our analysis highlights the importance of the way by which the incremental tax revenue is computed. The "residential" capital, that does not qualify for TIF, serves as the base from which the TIF subsidy is computed. This creates incentives for the developer to invest on a lower scale in the earlier non-qualifying phase in order to secure a larger effective subsidy per dollar invested in the TIF qualifying phase. As a result, TIF reduces the distortionary effect of property taxes in the "commercial," TIF qualifying phase, but *increases* the distortionary effect of property taxation in the non-qualifying phase. The net effect of a TIF on total surplus—the sum of the discounted present values of land and net tax revenue—depends on the magnitude of the TIF subsidy. In the presence of a property tax on structures, total surplus first increases and then decreases as the TIF subsidy increases. Our results also show that TIFs are always beneficial for landowners, even when they are disadvantageous for tax payers as a group.

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APPENDIX A

Table A.1: Table of Equilibria

	<i>n</i> = 2	<i>n</i> = 3	<i>n</i> = 4	<i>n</i> = 5	<i>n</i> = 6	<i>n</i> = 7	<i>n</i> = 8	<i>n</i> = 9
Atomistic Eqm								
d (defense)	0.40845	0.40845	0.40845	0.40845	0.40845	0.40845	0.40845	0.40845
р								
(predator/producer)	0.83095	0.83095	0.83095	0.83095	0.83095	0.83095	0.83095	0.83095
C (consumption)	0.01508	0.01541	0.01356	0.01181	0.01038	0.00923	0.00829	0.00752
Boundedly Rational								
Community Eqm								
d (defense)	0.10593	0.09983	0.09167	0.08466	0.07885	0.07400	0.06991	0.06640
p								
(predator/producer)	0.13340	0.12476	0.11341	0.10383	0.09602	0.08959	0.08421	0.07965
C (consumption)	0.00099	0.03502	0.04020	0.03949	0.03731	0.03487	0.03251	0.03036
Fully Rational								
Community Eqm**								
d (defense)	0.30809	0.00865	0.00000	0.00000*	0.00000	0.00000	0.00000	0.00000
р								
(predator/producer)	0.52050	0.00971	0.00000	0.00000*	0.00000	0.00000	0.00000	0.00000
C (consumption)	0.01723	0.03691	0.04850	0.04880*	0.04622	0.04302	0.03988	0.03699

NOTE. * Global equilibrium when n is selectable. ** Fully rational community equilibrium is the same as the global equilibrium when a central government planner takes n as fixed.

APPENDIX B

Expected Consumption in the Atomistic Equilibrium and Boundedly Rational Community Equilibrium in (n,k) parameter space, given t = 0.9



Figure A.1: Atomistic equilibrium C(n,k) (bottom with large *n*) and boundedly rational community equilibrium C(n,k) (top with large *n*), given t = 0.9

APPENDIX C

Table A.2: Numerical Solutions

					Land
	T_r	K_r	T_c	K _c	Value
i = 0.08	0.35	4.70	46.86	73.71	0.41232
i = 0.10	5.15	3.94	40.46	28.80	0.10358
i = 0.12	9.78	3.63	39.71	19.13	0.03235
$\eta_r = 0.028$	5.21	3.39	35.99	22.52	0.08764
$\eta_r = 0.035$	5.15	3.94	40.46	28.80	0.10358
$\eta_r = 0.042$	5.74	4.78	47.44	42.26	0.12660
$\eta_c = 0.048$	5.69	4.21	48.57	27.71	0.07739
$\eta_c = 0.055$	5.15	3.94	40.46	28.80	0.10358
$\eta_c = 0.062$	4.80	3.76	36.46	35.32	0.15652
$r_r(0) = 0.009$	12.09	3.18	35.00	21.32	0.09279
$r_r(0) = 0.012$	5.15	3.94	40.46	28.80	0.10358
$r_r(0) = 0.014$	1.24	4.19	43.22	33.51	0.13607
$r_c(0) = 0.004 r_c(0) = 0.007 r_c(0) = 0.010$	6.07	4.39	57.04	40.96	0.07604
	5.15	3.94	40.46	28.80	0.10358
	4.16	3.38	29.75	22.82	0.14896
p = 0.042	0.39	4.05	38.43	30.65	0.13270
p = 0.050	5.15	3.94	40.46	28.80	0.10358
p = 0.058	9.18	3.83	42.19	27.30	0.08461
d = 0.05	5.14	3.94	39.47	27.26	0.10449
d = 0.30	5.15	3.94	40.46	28.80	0.10358
d = 0.60	5.15	3.95	41.54	30.55	0.10260

i = interest rate, η = growth rate of rent (η_r for residential use and η_c for commercial use of land), r(0) = rent at time zero ($r_r(0)$ for residential use and $r_c(0)$ for commercial use of land), p = price of a unit of capital, d = replacement cost parameter.

When each of the parameters is varied for comparative statics, the other parameters are set as follows: i = 0.1, $\eta_r = 0.035$, $\eta_c = 0.055$, $r_r(0) = 0.012$, $r_c(0) = 0.007$, p = 0.05, d = 0.3. The range of parameter values is selected with the consideration of interior solutions. For the output of housing/office space, the natural logarithmic function is used, Q(K) = lnK, which satisfies the conditions for Q'(K) > 0 and Q''(K) < 0. The sufficient condition for a local maximum is a decreasing output elasticity of capital. With Q(K) = lnK, the output elasticity is Q'(K)K/Q(K) = 1/lnK.

APPENDIX D

Table A.3: Numerical Solutions with Property Taxation and TIF

 x_s = land tax rate; x_k = structure tax rate; w = TIF subsidy rate. Other parameter values are set as follows: i = 0.1, $\eta_r = 0.035$, $\eta_c = 0.055$, $r_r(0) = 0.012$, $r_c(0) = 0.007$, p = 0.05, d = 0.3.

*Net tax revenue is computed as the sum of the discounted present values of tax on land and structures less TIF subsidy.**Total surplus is measured by the sum of the discounted present values of land and net tax revenue.

					Land	Net Tax	Total
	T_r	K_r	T_c	K_c	Value	Revenue*	Surplus ^{**}
$x_{\rm s} = 0.00, \ x_k = 0.00, \ w = 0.00$	5.15	3.94	40.46	28.80	0.10358	0	0.10358
$x_{\rm s} = 0.03, \ x_k = 0.00, \ w = 0.00$	5.15	3.94	40.46	28.80	0.07968	0.02390	0.10358
$x_{\rm s} = 0.03, \ x_k = 0.00, \ w = 0.10$	5.03	3.88	39.39	30.16	0.08149	0.02189	0.10338
$x_{\rm s} = 0.03, \ x_k = 0.00, \ w = 0.20$	4.89	3.81	38.17	31.74	0.08365	0.01895	0.10260
$x_s = 0.03, x_k = 0.02, w = 0.00$	10.09	3.81	41.68	25.66	0.06278	0.03609	0.09886
$x_{\rm s} = 0.03, \ x_k = 0.02, \ w = 0.10$	9.98	3.74	40.75	26.60	0.06417	0.03500	0.09916
$x_{\rm s} = 0.03, \ x_k = 0.02, \ w = 0.20$	9.85	3.67	39.71	27.63	0.06578	0.03345	0.09923
$x_{\rm s} = 0.03, \ x_k = 0.02, \ w = 0.30$	9.70	3.59	38.53	28.78	0.06767	0.03124	0.09891
$x_s = 0.03, x_k = 0.02, w = 0.40$	9.54	3.49	37.17	30.04	0.06993	0.02798	0.09790
$x_s = 0.03, x_k = 0.03, w = 0.00$	12.26	3.74	42.21	24.39	0.05679	0.03805	0.09485
$x_{\rm s} = 0.03, \ x_k = 0.03, \ w = 0.10$	12.15	3.68	41.33	25.18	0.05804	0.03724	0.09527
$x_{\rm s} = 0.03, \ x_k = 0.03, \ w = 0.20$	12.02	3.61	40.36	26.04	0.05946	0.03609	0.09554
$x_s = 0.03, x_k = 0.03, w = 0.30$	11.88	3.53	39.26	26.96	0.06110	0.03445	0.09556
$x_s = 0.03, x_k = 0.03, w = 0.40$	11.73	3.43	38.02	27.97	0.06304	0.03210	0.09514
$x_s = 0.03, x_k = 0.03, w = 0.50$	11.55	3.31	36.57	29.06	0.06536	0.02862	0.09398

BIBLIOGRAPHY

- [1] Gary Becker. "Crime and Punishment: An Economic Approach." *Journal of Political Economy* 76 (2): 169–217, 1968.
- [2] Isaac Ehrlich. "Crime, Punishment, and the Market for Offenses." *Journal of Economic Perspectives* 10 (1): 43–67, 1996.
- [3] Herschel Grossman. "Producers and Predators." *Pacific Economic Review* 3 (3): 169–187, 1998.
- [4] Herschel Grossman and Minseong Kim. "Swords or Plowshares? A Theory of the Security of Claims to Property." *Journal of Political Economy* 103 (6): 1275–1288, 1995.
- [5] Jack Hirshleifer. "Anarchy and Its Breakdown." *Journal of Political Economy* 103 (1): 26–52, 1995.
- [6] David Meza and J.R. Gould. "The Social Efficiency of Private Decisions to Enforce Property Rights." *Journal of Political Economy* 100 (3): 561–580, 1992.
- [7] Stergios Skaperdas. "Cooperation, Conflict, and Power in the Absence of Property Rights." *American Economic Review* 82 (4): 720–739, 1992.
- [8] Goran Skogh and Charles Stuart. "A Contractarian Theory of Property Rights and Crime." *Scandinavian Journal of Economics* 84 (1): 27–40, 1982.

- [9] Jan K. Brueckner and Luz A. Saavedra. "Do Local Governments Engage in Strategic Property–Tax Competition?" *National Tax Journal* 54 No. 2: 203–29, June 2001.
- [10] Bruce W. Hamilton. "Zoning and Property Taxation in a System of Local Governments." *Urban Studies 12*: 205–211, 1975.
- [11] David E. Wildasin and John D. Wilson. "Imperfect Mobility and Local Government Behaviour in an Overlapping-Generations Model." *Journal of Public Economics* 60 No. 2: 177–98, May 1996.
- [12] David E. Wildasin and John D. Wilson. "Risky Local Tax Bases: Risk-Pooling vs. Rent-Capture." *Journal of Public Economics* 69 No. 2: 229–47, June 1998.
- [13] John D. Wilson. "Optimal Property Taxation in the Presence of Interregional Capital Mobility." *Journal of Urban Economics* 18 No. 1: 73–89, July 1985.
- [14] John D. Wilson. "A Theory of Interregional Tax Competition." *Journal of Urban Economics* 19 No. 3: 296–315, May 1986.
- [15] John D. Wilson. "Theories of Tax Competition." *National Tax Journal* 52 No. 2: 269–304, June 1999.
- [16] John D. Wilson and David E. Wildasin. "Capital Tax Competition: Bane or Boon." *Journal of Public Economics* 88 No. 6: 1065–91, June 2004.
- [17] John Anderson. "Tax Increment Financing: Municipal Adoption and Growth." *National Tax Journal* 43 No. 2: 155–63, June 1990.

- [18] Richard Arnott and Frank Lewis. "The Transition of Land to Urban Use." *Journal of Political Economy* 87 No. 1: 161–9, February 1979.
- [19] Jan K. Brueckner. "Infrastructure Financing and Urban Development: The Economics of Impact Fees." *Journal of Public Economics* 66 No. 3: 383–407, December 1997.
- [20] Dennis Capozza and Yuming Li. "The Intensity and Timing of Investment: The Case of Land." *American Economic Review* 84 No. 4: 889–904, September 1994.
- [21] Richard Dye and David Merriman. "The Effect of Tax Increment Financing on Land Use." In *The Property Tax, Land Use and Land Use Regulation*, edited by Dick Netzer, 37–61. Northampton MA: Edward Elgar, 2003.
- [22] Jack Huddleston. "A Comparison of State Tax Increment Financing Laws." *State Government* 55 No. 1: 29–33, 1982.
- [23] Craig Johnson and Kenneth Kriz. "A Review of State Tax Increment Financing Laws." In *Tax Increment Financing and Economic Development: Uses, Structures, and Impact,* edited by Craig Johnson and Joyce Man, 31–56. Albany: State University of New York Press, 2001.
- [24] Craig Johnson and Joyce Man (eds.). *Tax Increment Financing and Economic Development: Uses, Structures, and Impact.* Albany: State University of New York Press, 2001.
- [25] Helen Ladd. "Theoretical Controversies: Land and Property Taxation." In *Local Government Tax and Land Use Policies in the United States: Understanding the Links,* primary author Helen Ladd, 25–40. Northampton MA: Edward Elgar, 1998.
- [26] Thomas Luce. "Reclaiming the Intent: Tax Increment Finance in the Kansas City and St. Louis Metropolitan Areas." A Discussion Paper Prepared for The Brookings Institution Center on Urban and Metropolitan Policy, April 2003.

- [27] Joyce Man and Mark Rosentraub." Tax Increment Financing: Municipal Adoption and Effects on Property Value Growth." *Public Finance Review* 26 No. 6: 523–47, November 1998.
- [28] Alastair McFarlane. "Taxes, Fees, and Urban Development." *Journal of Urban Economics* 46 No. 3: 416–36, November 1999.