

HUMAN STRATEGIES IN THE CONTROL OF TIME CRITICAL UNSTABLE SYSTEMS

by

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University of Pittsburgh, 2010

The purpose of this study is to investigate the human manual control strategy when balancing an inverted pendulum under time critical constraints. The strategy was assessed through the quantification and evaluation of human response while performing tasks that require fast reaction from the human operator. The results show that as the task becomes more difficult due to increased time delay or shortened pendulum length, the human operator adopts a more discrete-type strategy. Additionally, dissimilarities between control of a short pendulum and a delayed pendulum are identified and discussed. Finally, the discrete-control mechanism is interpreted by relating the observed human responses to human-performance models. These results can be applied to systems requiring human interaction, such as teleoperation, which could be designed to maximize human response.

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1.0 INTRODUCTION

Evaluation of human manual control when performing a difficult task is important for understanding human movement behavior as well as for identifying human limitations. This study investigates the manual control of an inverted pendulum, which is an unstable system. Experimental results have shown that when the control task is difficult and demands a fast response, such as in the case of a short length pendulum or when time delay affects the task, the human operator adopts a discrete-type control strategy (i.e. the human response is intermittent rather than continuous). The objective of this study is to identify and analyze the continuity/intermittency characteristics of human manual control. Classifying human control strategies is valuable for characterizing and quantifying human performance, and can aid in the design of systems that take maximum advantage of the human control movement.

1.1 BACKGROUND

Research into human control started in the late 1940's with the pioneering work of Tustin [1], who attempted to improve the servomechanism response of gun turrets by replacing the human controller with a transfer function. In the next two decades, seminal research was conducted on the evaluation of human performance, and on the development of correspondent theoretic models of the human operator [2]-[8]. Birmingham and Taylor [2] used evidence that human operators

achieve better results when they perform simple control tasks to suggest solutions for improving the man-machine control system by reducing the contribution of the human operator to a simple amplifier. Simpler control tasks imply less complex mental computation by the human operator. The idea that complex mental computation required by the task directly affects the performance of the human operator will also be discussed in this study in chapter 4.0 . McRuer [3] followed the direction of Tustin, focusing on the identification of a more general transfer function representation of the human operator that would reproduce the advantages of human control. Jex [4] and Smith [5] evaluated the performance of the human operator when performing tasks at the limit of stability. The results were then related to the values of the parameters of the transfer function representation of the human operator, and the boundaries of these values were graphically displayed and interpreted. Meanwhile, Kleinman [6] concentrated on representing the human operator as an optimal controller. A historical overview of the progress made in manual control research is provided by Pew [7], who states that most of the models developed in the 1950's and 1960's are still used today, and that no significant innovation has been done since then. Despite great technological progress in the past decades, which has made possible the implementation of complex human operator models, there are still important tasks that require direct human intervention. Characteristics of human control such as adaptation, learning capabilities, and decision making skills require the human operator to continue to perform certain manual tasks that are too complicated to be accomplished by machines.

The control of inherently unstable systems such as unstable aircraft, booster rockets, and the inverted pendulum has attracted significant attention and research efforts [8]-[12] due to its challenging properties. Young and Meiry [8] discussed manual control of high-order unstable systems, and noted that human subjects tended to adopt a discrete, or bang-bang strategy in this

case. They identified the switching lines in the error trajectory, and suggested a model for the human operator that consists of a proportional-derivative controller and a three-state relay with time delay. This model is also investigated in our study in section 4.2.1. A study by Loram [9] validates the intermittent control strategy when balancing an inverted pendulum with small moment of inertia. Stepan [10] emphasized the importance of the inverted pendulum application in studying human postural control. In his study, the human controller was modeled as a delayed proportional-derivative term, and the stability conditions were evaluated. The conclusion of this work is that both position signals and velocity signals are needed to stabilize an inverted pendulum, and that the human operator is capable of sensing both types of signals with the help of the vestibular system. Research about the control of unstable systems was also conducted by Cabrera and Milton [11], [12], who suggested several methods for identifying discrete control.

Human interaction with computers in a closed loop system is usually separated by a communication link. Thus, both the command signals issued by the human operator, and the feedback signals back to the human operator are affected by time delay, and performance is influenced. Research efforts were limited on evaluating the effect of time delay on simple manual tasks such as reaching and tracking. For example, MacKenzie and Ware [13] conducted a study on predicting the required movement time for accomplishing reaching tasks affected by time delay using Fitts's law. Similarly, Beamish et al. [14] used a servomechanism to investigate the best possible speed-accuracy trade-off when time delay was present.

While this past research outlines important progress in the field, further research is needed, particularly in evaluating human control of unstable systems under time delay constraints.

1.2 MOTIVATION

Teleoperation is an attractive field due to important potential benefits, including handling objects or performing services in locations that are either hostile or impossible to reach for humans. Sheridan [15] identifies some of the current applications of manual control in teleoperation. He recognizes the importance of teleoperation in operating vehicles and systems in outer space. An example is provided by the remote manipulator system (RMS) which is controlled by a human operator with the help of two three-axis joysticks in order to move heavy loads outside a space shuttle. By 1980, remote operated vehicles (ROV) were widely used for underwater operations. Such robots play an important role in the offshore oil and gas industry, including inspection of underwater welds, monitoring pipelines, or placing anodes. Marine biologists rely on remote vehicles for investigating the undersea fauna. Telerobotics is also used in military operations to provide extended vision for soldiers in highly dangerous locations, navigate over mine fields, and observe enemy operations.

An important emerging field centered on human manual control is telesurgery [16], [17]. The use of teleoperation in surgical procedures has many promising benefits because it allows physicians to provide medical expertise without traveling to the location of the patient. Additionally, the surgeon can rely on telerobotics to reach places not accessible by the human hands. Our investigation in this study can be of importance in the field of telesurgery, where the human operator performs a task that requires high precision over a network channel which induces significant amount of time delay.

Some of the above mentioned systems are unstable [8]-[12], which poses a greater challenge for the human operator. Our study is focused on balancing an inverted pendulum, which is an inherently unstable system.

2.0 METHODS

An experiment was conducted to evaluate the strategies of the human operators when controlling an unstable system such as the inverted pendulum. In this chapter the dynamics of the inverted pendulum are introduced, and the experiment setup is described.

2.1 INVERTED PENDULUM SYSTEM

The human-controlled inverted pendulum system [18] considered in the study is shown in Figure 2.1. The dynamics of the inverted pendulum are described by the following differential equation

$$\frac{mL^2}{3} \frac{d^2\theta}{dt^2} + \frac{mL}{2} \cos\theta \frac{d^2x}{dt^2} = \frac{mgL}{2} \sin\theta \quad (1)$$

where m is the mass of the pendulum, L is the length of the pendulum, θ is the angle of the pendulum relative to the vertical position, x is the displacement of the bottom tip of the pendulum, and g is the gravitational acceleration (please refer to the Appendix for a complete derivation of the equation).

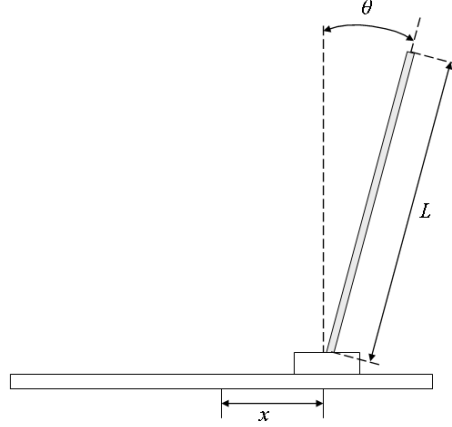


Figure 2.1 The inverted pendulum system adapted from [18].

By approximating the nonlinear terms in (1) such that $\cos \theta \approx 1$, and $\sin \theta \approx \theta$ for small values of the angle θ , the inverted pendulum transfer function is simplified to the form

$$H(s) = \frac{\Theta(s)}{X(s)} = \frac{-\frac{\alpha}{g}s^2}{s^2 - \alpha} = \frac{-\frac{1}{g}s^2}{(Ts + 1)(Ts - 1)} \quad (2)$$

where $\alpha = 3g/2L$, s is the Laplace variable, and $T = 1/\sqrt{\alpha} = \sqrt{2L/3g}$ is the time constant of the inverted pendulum system which varies with the length of the pendulum L .

The above obtained linear time-invariant system has as input the displacement x , and as output the angle of the pendulum relative to the vertical position θ . The poles of the transfer function are $s_{1,2} = \pm\sqrt{\alpha}$. Hence, the system is unstable due to the real positive pole. In order to simulate the inverted pendulum dynamics on the computer, the discrete transfer function with a sample time of $T_s = 50 \text{ ms}$ was obtained from (2), and written in state space form

$$\begin{cases} x[k+1] = \begin{bmatrix} 1 & \alpha T_s \\ T_s & 1 \end{bmatrix} x[k] + \begin{bmatrix} 0 \\ T_s \end{bmatrix} u[k] \\ y[k] = \begin{bmatrix} 0 & -\frac{\alpha}{g} \end{bmatrix} x[k] - \frac{\alpha}{g} u[k] \end{cases} \quad (3)$$

where $x[k]^T = [\dot{\theta}[k], \theta[k]]$.

2.2 EXPERIMENT DESCRIPTION

In order to observe the human control strategies when balancing an inverted pendulum, an experiment with seven subjects was conducted. A planar inverted-pendulum system was considered in this study [Figure 2.1]. Instead of physical implementation of the inverted pendulum, a real-time simulation in Matlab/Simulink was used following the idea of Bodson [18] [Figure 2.2]. The subjects balanced the inverted pendulum using a Logitech ATK3 joystick connected to a computer whose screen provided visual feedback to the subjects. Artificial delays were introduced in the simulation to emulate transport delays in teleoperation.

The subjects were asked to try their best to maintain a long pendulum of 20 m in the upright position under zero delay and under a delay of 150, 300, and 500 ms. The subjects were also asked to balance a short pendulum of 5 m without time delay. Note that the pendulum length considered for the computer simulation should not be regarded as corresponding to the pendulum-balancing task in the real environment because in the experiment the human operator did not experience the force feedback from the inverted pendulum. Moreover, in the computer simulation the movement of the joystick corresponded to the bottom tip displacement of the pendulum which was proportional to the pendulum length. However, using the simulated pendulum system, the difficulty of the stabilization task was still closely related to the length of the pendulum. When the pendulum was long enough and under zero time delay, the human operator experienced no difficulty in control, but when the pendulum was sufficiently short (i.e. 5 m), the human operator was challenged.

For each considered scenario, five successful trials were recorded in which the pendulum was balanced without falling. Each trial was 45 seconds long, and the trials were tested in a random order with a 15 second break between successive trials.



Figure 2.2 The inverted pendulum computer simulation [18].

The quantitative measures of the human performance that were considered in our study are the following:

- Velocity of hand movement
- Magnitude of the pendulum's angular sway
- Frequency of the pendulum's angular sway (i.e. number of times the pendulum crossed the upright position in one second)
- Reaction time of the human operator to the angle deviation of the pendulum
- Pendulum angle when the correction movement started, and when the correction movement stopped.

3.0 RESULTS

It was observed that, as the balancing task became more difficult, the subjects adjusted their strategy in order to keep the pendulum in the stable upright position. When balancing the long-length pendulum without time delay, the subjects experienced no difficulties, and they exhibited a more continuous type of control. However, as the time delay increased, the human operators exhibited more apparent discrete-type, or bang-bang-like actions in their control. A similar type of control was identified when balancing the short-length pendulum.

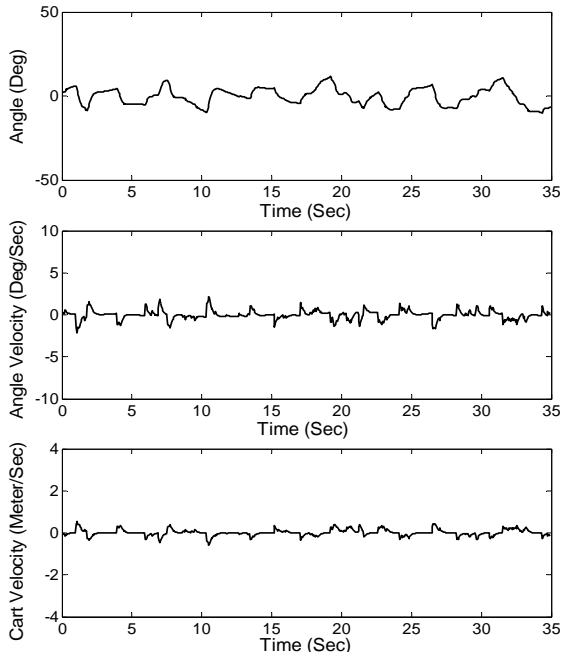
3.1 BALANCING THE INVERTED PENDULUM WITH TIME DELAY

The trajectory of the pendulum angle, the time derivative of the pendulum angle, and the velocity of the movement of a representative subject (Subject 4) are shown in Figure 3.1 for comparison between manual control strategies with different time delays. Although a recorded trial was 45 seconds long, the figures do not include the first 10 seconds of the trial, when the subjects tended to make use of the initial upright position of the pendulum. The subjects waited during this time period for the pendulum to start falling to one side before triggering any corrective movement. In order to perceive the change of strategy of the human operator as the time delay increased, the case when no time delay affected the task [Figure 3.1 (a)] is shown first. The human operator strategy in this case yields characteristics of more continuous control. An important

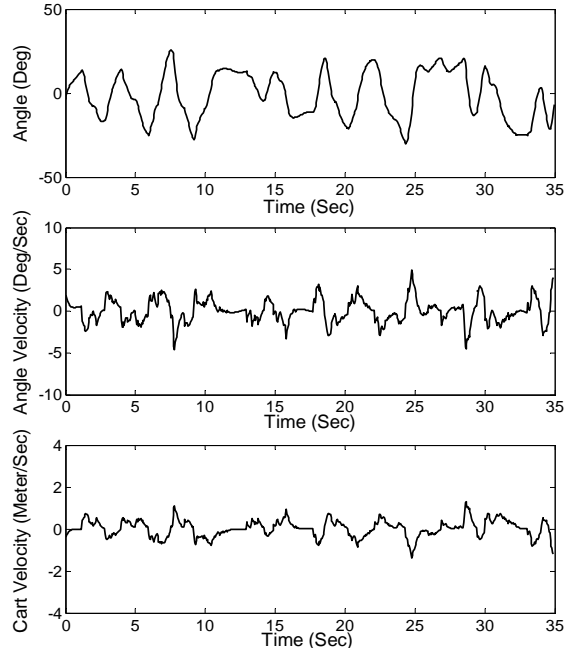
characteristic of this type of control is that movements are not time critical (i.e. the pendulum system has a large time constant), which allows the human operator enough time to initiate the balancing movement, and then to make corresponding corrections if needed. This behavior can be noticed in Figure 3.1 (a) in the velocity profile of the movement between 18 and 23 seconds or between 27 and 31 seconds, where the subject executes 3 - 4 consecutive tiny movements in the same direction to correct the displacement of the pendulum from the upright position. The small magnitudes of the angle velocity and the movement velocity reveal that a smoother hand movement trajectory was exhibited when no time delay was present, compared to the behavior affected by time delay.

As the amount of time delay increased, the performance of the human operator was affected, and a change in the control strategy could be recognized. Figure 3.1 (b)-(d) show the adopted human operator control approach when time delay was 150 ms, 300 ms, and 500 ms, respectively. As the time delay was increased, spike-form profiles of the movement velocity became more apparent. This indicates that the human operator tended to make more sudden moves. The magnitude of these peaks increased with time delay, such that when the time delay was 500 ms, one corrective movement was enough to cause the pendulum to pass the stable upright position to the opposite side.

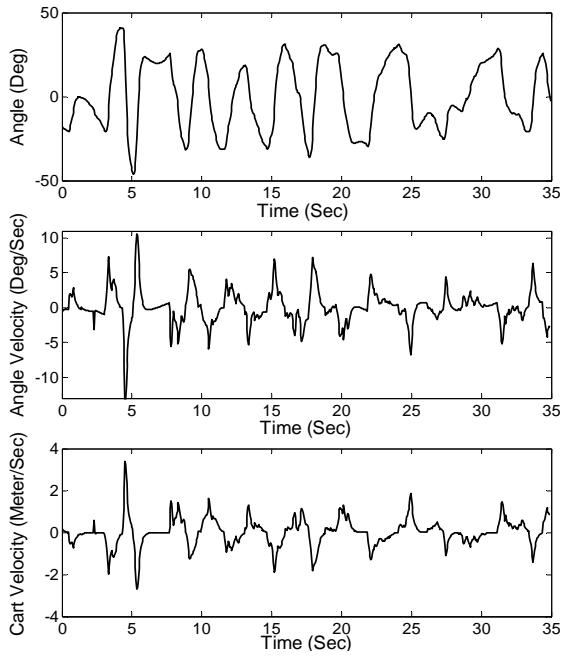
The average trend of the movement velocity, angular sway, angular sway rate and the reaction time of the human operator will be investigated in the next sections.



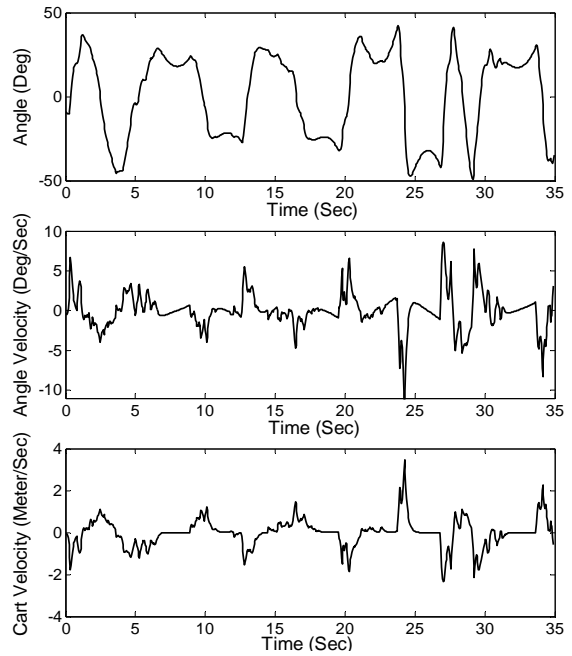
(a)



(b)



(c)



(d)

Figure 3.1 Inverted pendulum control with (a) no delay, (b) 150ms, (c) 300ms, and (d) 500ms time delay.

3.1.1 Movement velocity

The hand movement velocity exhibited by all the seven subjects and the average movement velocity are shown in Figure 3.2. Most subjects followed a similar trend, with movements becoming faster with increasing time delay. Thus, the human operator tried to compensate for the effect of delay by making faster and more sudden hand movements. Subject 2 slowed down the control movements when the time delay was 150 ms, and Subjects 3, 5 and 7 slowed down their movement when the time delay was 300 ms. However, Subject 1 did not seem to follow the average tendency and can be identified as an exception for which we will try to find an explanation later in the study.

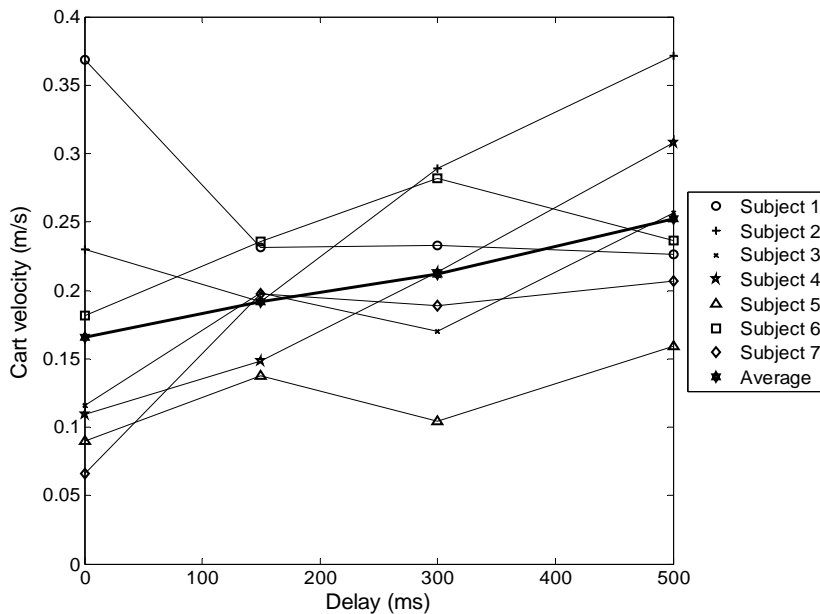


Figure 3.2 Movement velocity relative to time delay.

3.1.2 Magnitude of angular sway

A consequence of speeding up the movements as the time delay increased was the change in magnitude of the pendulum sway around the vertical position. It is apparent from Figure 3.3 that, on average, the magnitude of the angular sways also increased with time delay. The tendency of increased pendulum sway with time delay is clearly observed when the 500 ms time delay affected the performance. In this situation the average magnitude of oscillations was approximately 50% greater than that under no time delay control. This result is consistent with the observations from Figure 3.1 where the maximum magnitude of the angular sway when no time delay was present was about 10 degrees. As the task was increasingly affected by time delay, the maximum magnitude of the pendulum sway reached up to 40 degrees.

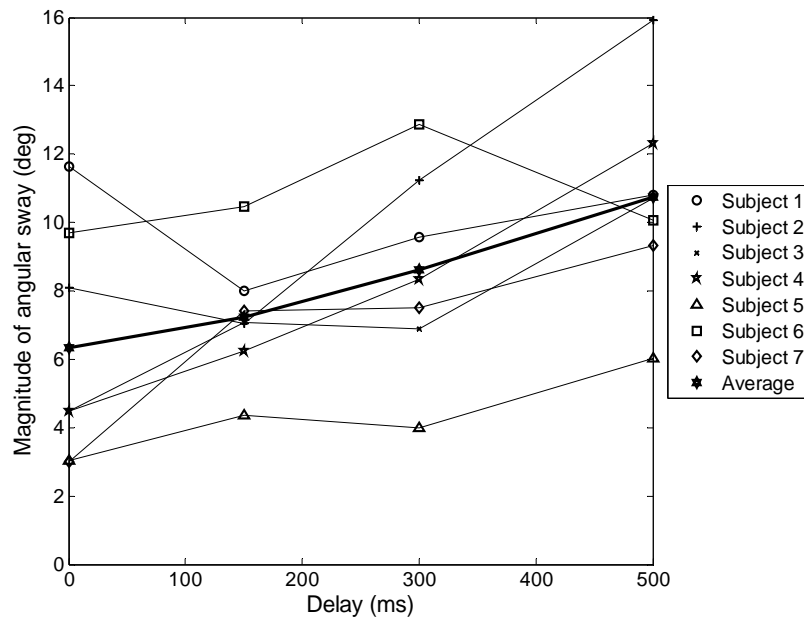


Figure 3.3 Magnitude of the angular sway increased with time delay.

Individually, each subject exhibited in general an increased magnitude of pendulum sway as the time delay increased. The performance of Subject 4 is consistent with the average tendency of the magnitude of the pendulum sway as time delay increased. Subjects 1 and 2 have a slight decrease in the pendulum sway at 150 ms time delay, and Subjects 3, 5 and 7 experienced the same tendency at 300 ms. The magnitude of the pendulum sway of Subject 6 increased when time delay was 150 ms and 300 ms, but when time delay was 500 ms, it was comparable to the case when no time delay affected the task. It is important to note how these results are in concordance with the individual performances of the subjects presented in the previous section.

3.1.3 Frequency of angular sway

Both the magnitude of the angular sway and the frequency of oscillations of the pendulum around the upright position are affected by the amount of time delay. Figure 3.4 shows the frequency of the angular sway over the different time delays of all the subjects averaged over all trials. For Subjects 1 and 5 a decrease in the frequency of the angular sway with each time delay was obvious, but Subjects 2, 3, 4 and 7 exhibited only a general trend of decrease in the frequency of oscillation. However, Subject 6 proved once again to be an exception to the average trend, as the frequency of the angular sway increased slightly with time delay. The average frequency of oscillation, which is shown with the bold line in Figure 3.4, changed inversely with the amount of time delay. This result is expected as we observed that the magnitude of angular sway increased with time delay. Thus, the observed manual strategy of balancing an inverted pendulum under time delay resembled less frequent movements of larger magnitude as the delay increased.

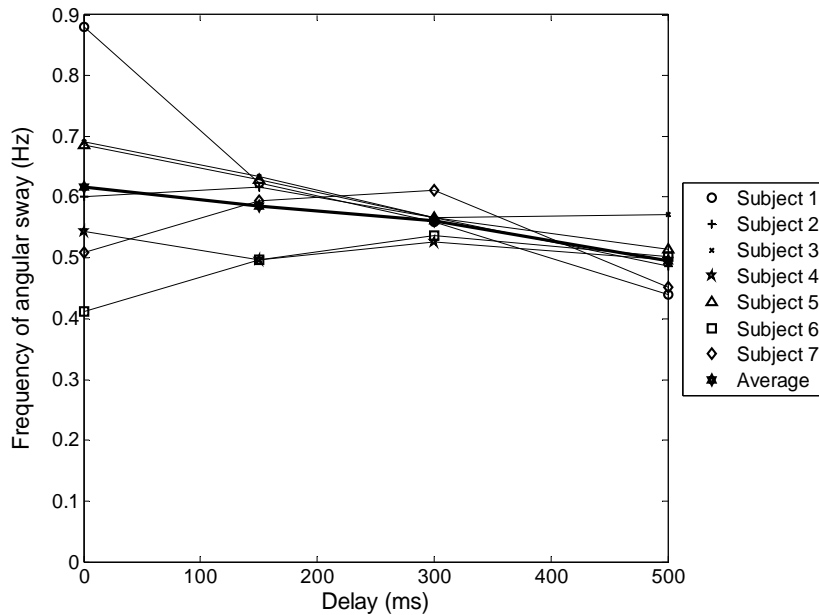


Figure 3.4 Frequency of angular sway relative to time delay.

3.1.4 Reaction time

The human operator reaction time is defined in this study as the time interval between two consecutive corrective movements (Figure 3.5). Figure 3.6 presents the average reaction time over all trials and over all subjects as the subjects experienced different time delays. The average reaction time was observed to decrease when time delay was 150 ms, but seemed to follow an increasing trend as time delay increased to 300 ms and 500 ms. The large value for the reaction time when no time delay was present is caused by the performance of the Subjects 1 and 6 which can be considered exceptions. The increasing trend of the reaction time with the amount of time delay supports the argument that movements become less frequent as time delay increases.

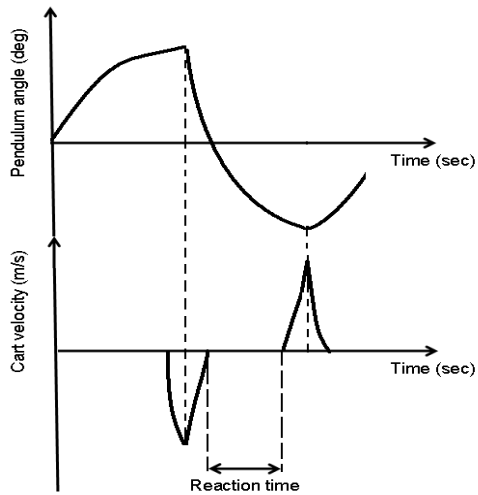


Figure 3.5 Reaction time of human operator to the angle deviation.

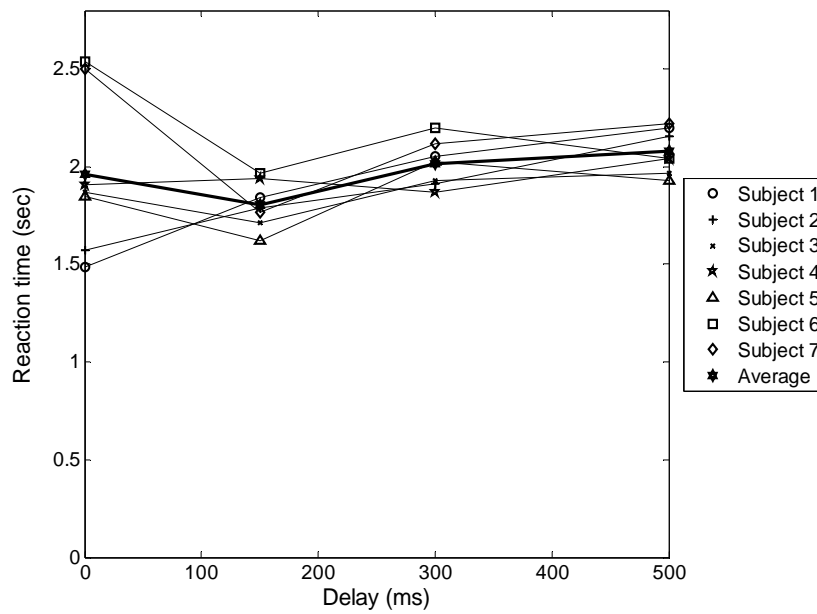


Figure 3.6 Reaction time between consecutive movements.

3.1.5 Precision of corrective movements

The angle deviations from the upright position were recorded when the corrective movement started (marked as Γ), and then when it ended (referred as γ). Figure 3.7 captures the normalized

distributions of Γ and γ averaged over all subjects and all trials. The widths of the distribution of Γ and γ seem to increase with time delay. This result was expected since we already observed in section 3.1.2 that the magnitude of the angular sway increased with delay.

The reduction of the difference between the widths of the distribution Γ and the distribution γ , was considered an index of how the accuracy of the corrective movements changed with time delay. Table 3-I provides the standard deviations of each distribution Γ and γ relative to the various time delays. The reduction of the difference of the standard deviation from the distribution Γ and the distribution γ decreased from 15%, when no time delay was present, to 10% when the time delay was 500 ms. The result supports the argument that the human operator loses the precision of his or her corrective movements as time delay increases.

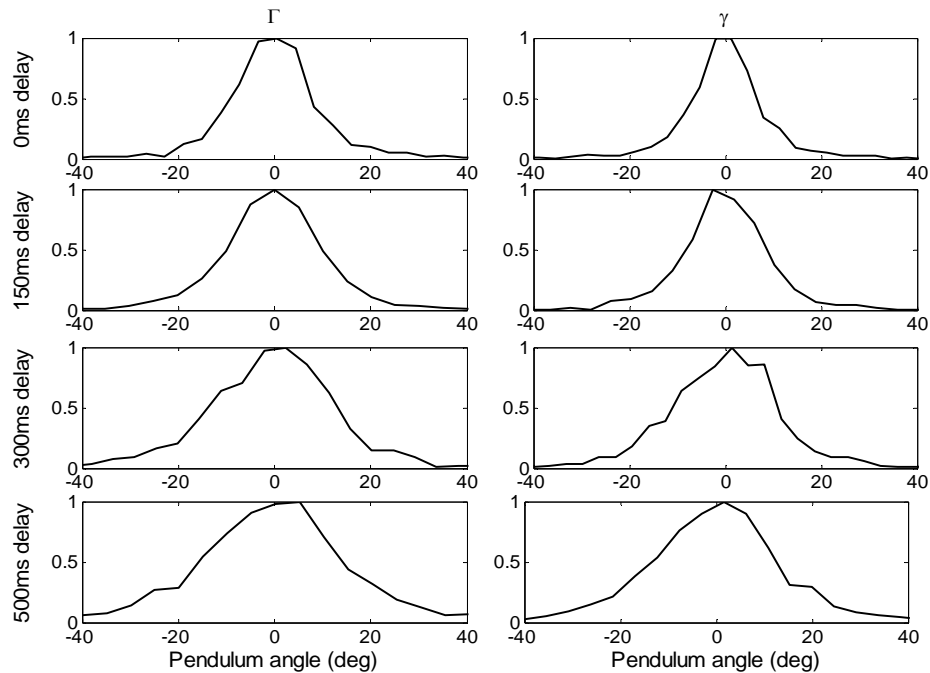


Figure 3.7 Distribution of the pendulum angle when movement starts Γ , and when movement ends γ .

Table 3-I Standard deviations of the distribution of Γ and γ relative to time delay

	<i>0 ms delay</i>	<i>150 ms delay</i>	<i>300 ms delay</i>	<i>500 ms delay</i>
Stv(Γ)	0.21	0.22	0.24	0.31
Stv(γ)	0.17	0.18	0.21	0.25
$\frac{\text{Stv}(\Gamma)-\text{Stv}(\gamma)}{\text{Stv}(\Gamma)} * 100$	15.02%	14.48%	13.84%	10.55%

3.2 BALANCING A SHORT-LENGTH PENDULUM

We also conducted an experiment to observe the human strategy when a short pendulum was balanced for the purpose of comparing the results with those of the time delay control. A simulated pendulum of length 5 m was determined empirically to be short enough to challenge the human operator. Similarities and differences between the two strategies will be examined.

The trajectory profile of the pendulum angle, velocity of the pendulum angle, and velocity of the movement of a representative subject (the same as in Figure 3.1) are illustrated in Figure 3.8. It is apparent that the profile of the movement velocity exhibits a very similar spike-form pattern (a characteristic of discrete control) as the one in Figure 3.1 (d). The magnitude of these spikes is much smaller than that of the spikes which represent controlling the pendulum under time delay [refer to Figure 3.1 (b)-(d)], but comparable with the magnitude of the movement velocity when a long pendulum without time delay was balanced. Due to the short length of the pendulum, each of the peaks observed in the movement velocity profile corresponded to a sway of the pendulum on the other side relative to the upright position. Therefore, the frequency of the angular sway is expected to be higher for the shorter pendulum

than for the long pendulum. Moreover, the frequency of the spikes in the velocity profile seemed greater when a short pendulum was balanced. Thus, when balancing a short pendulum, the human operator appeared to make sudden movements similar to those made when balancing a delayed pendulum, but the movements were quicker and of smaller magnitude. This result will be investigated for validation in the next sections.

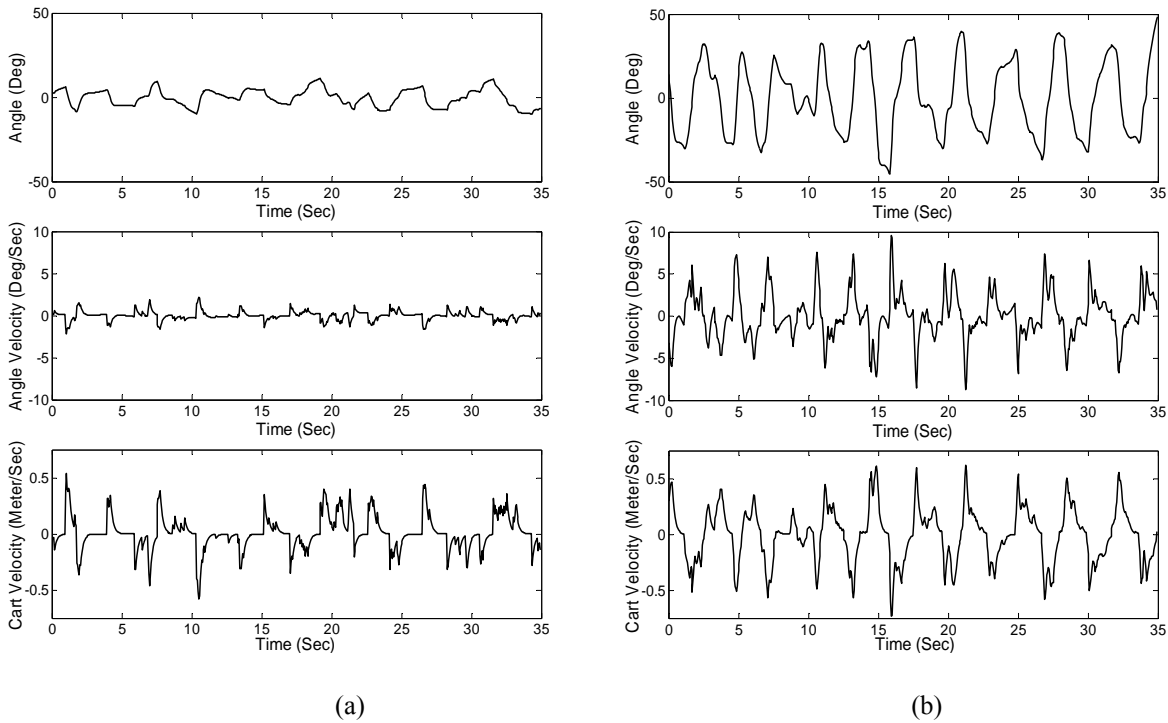


Figure 3.8. Inverted pendulum control of a long beam (a), and a short beam (b).

3.2.1 Movement velocity

The average movement velocity is shown in Figure 3.9, which indicates that the average movement velocity when balancing a short pendulum is similar to the hand movement velocity when balancing a pendulum without time delay. Subjects 1 and 6 are again exceptions. The fact that the average movement velocity when balancing a long pendulum without time delay is close

to the average movement velocity when balancing a short pendulum seems counterintuitive. However, we have to recall that, because the human subjects made use of a joystick to control the pendulum and the pendulum lengths differ in the two situations, the former case resembles a similar spike-form velocity profile, but the spikes are not alternating as in the latter case.

To be noticed that the average hand movement velocity of all subjects was measured to be 0.1 m/s when controlling a short pendulum, which was less than any average movement velocity measured under time delay [refer to Figure 3.2].

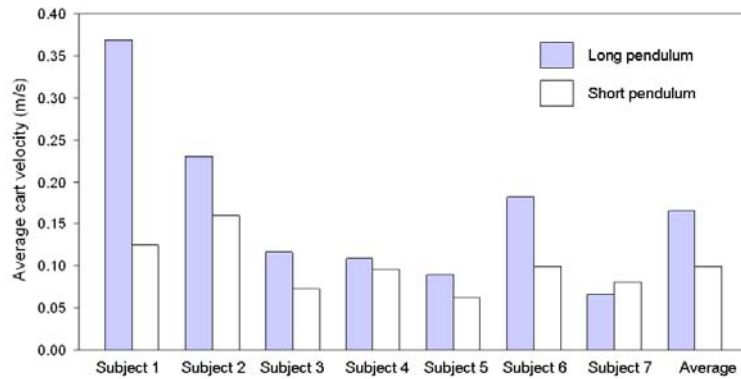


Figure 3.9 Average movement velocity.

3.2.2 Magnitude of angular sway

The averaged absolute value of the magnitude of angular sway over all subjects when controlling a short pendulum is illustrated in Figure 3.10. Disregarding the results of Subjects 1 and 6, the magnitude of the angular sway of all other subjects was larger when controlling a short pendulum than when controlling a long pendulum. Averaged over all subjects, this tendency was clear as the magnitude of the short pendulum sway was 9 deg, and the magnitude of the long

pendulum sway was 6 deg. This result was expected since in the previous section we noticed that the magnitude of the movement velocity when balancing a short pendulum was similar to the magnitude of the movement velocity when balancing a long pendulum. When applying similar force to balance pendulums of different lengths, it makes sense that the short pendulum would expose larger magnitude sway.

When comparing these results with those obtained when controlling the delayed pendulum, we can observe that the magnitude of sway of the short pendulum was smaller than that of the pendulum under 500 ms delay (11 deg), but similar to that of the pendulum under 300 ms time delay (approximately 9 deg).

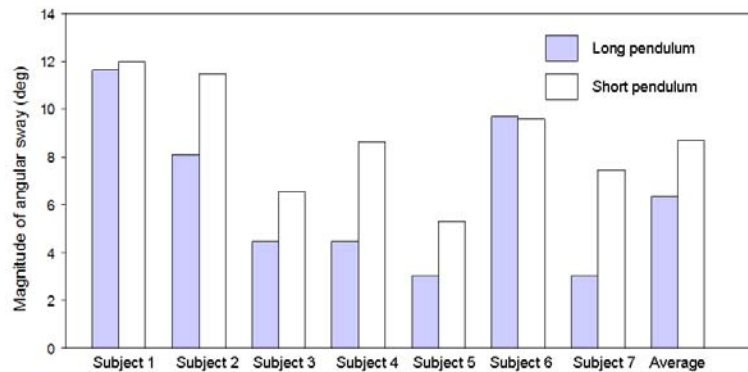


Figure 3.10 Magnitude of angular sway of the long pendulum and the short pendulum.

3.2.3 Frequency of angular sway

The average frequency of crossing the upright position is shown in Figure 3.11 for all subjects. By omitting the performance of Subjects 1 and 6, it is evident that the frequency of the angular sway was higher when controlling a short pendulum (0.8 Hz) than when controlling a long pendulum (0.6 Hz). This result is predictable, as the frequency of occurrence of the “peaks” in

the movement velocity profile of the short pendulum was higher than that of the peaks from the movement velocity of the long pendulum.

The frequency of the angular sway when controlling a short pendulum (0.8 Hz) was visibly higher than the frequency of the angular sway when time delay was involved (from Figure 3.4: 150 ms time delay – 0.6 Hz, 300 ms time delay – 0.55 Hz, 500 ms time delay – 0.5 Hz). This observation confirms the idea that different types of discrete control can be distinguished.

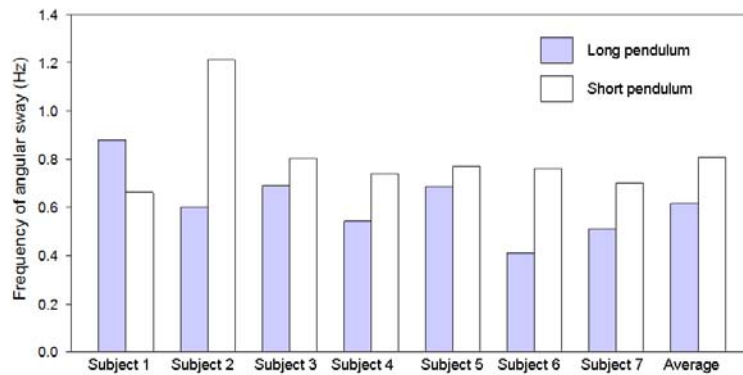


Figure 3.11 Frequency of angular sway of the long pendulum and the short pendulum.

3.2.4 Reaction time

The reaction time of all subjects when controlling a short pendulum and a long pendulum are illustrated in Figure 3.12. The reaction time was noticeably smaller when controlling a short pendulum than when controlling a long pendulum. The averaged reaction time of all subjects was 1.2 seconds when controlling a short pendulum, and 1.9 seconds when controlling a long pendulum. As we have already observed, the reaction time increased as time delay affected the performance. Hence, the human operator was making quicker movements when controlling a

short pendulum, which can be classified as another dissimilarity when compared with the control of the pendulum under time delay constraints.

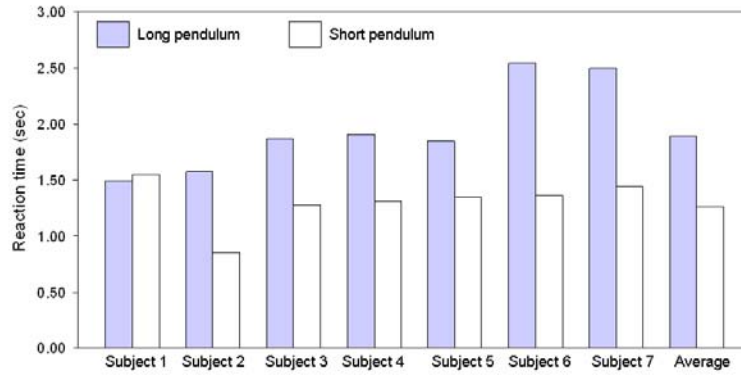


Figure 3.12 Reaction time between consecutive movements.

3.2.5 Precision of corrective movements

The preciseness of the movement was quantified by comparing the distributions of the angle deviation from the upright position recorded when the corrective movement started (marked as Γ), and then when it ended (also referred as γ). Figure 3.13 captured the distribution of Γ and γ averaged for all subjects and all trials. When balancing a short pendulum the distribution of Γ and γ reveal a saddle shape at the 0 degree angle deviation. This feature is the consequence of the refractory period (characteristic of discrete control), as the human operator could not react quickly enough to correct the falling pendulum. In Figure 3.13 it is apparent that the human operator could generate the corrective movement only when the angle deviation was around 15 degrees deviation from the upright position. This angle deviation is expected to increase as the pendulum length gets shorter, until the human operator is not able to keep it stable (in the sense of bounded oscillations). The widths of the distribution of Γ and γ were larger when controlling a

short pendulum as compared to both the control of a long pendulum without time delay, and the control of the pendulum under any time delay.

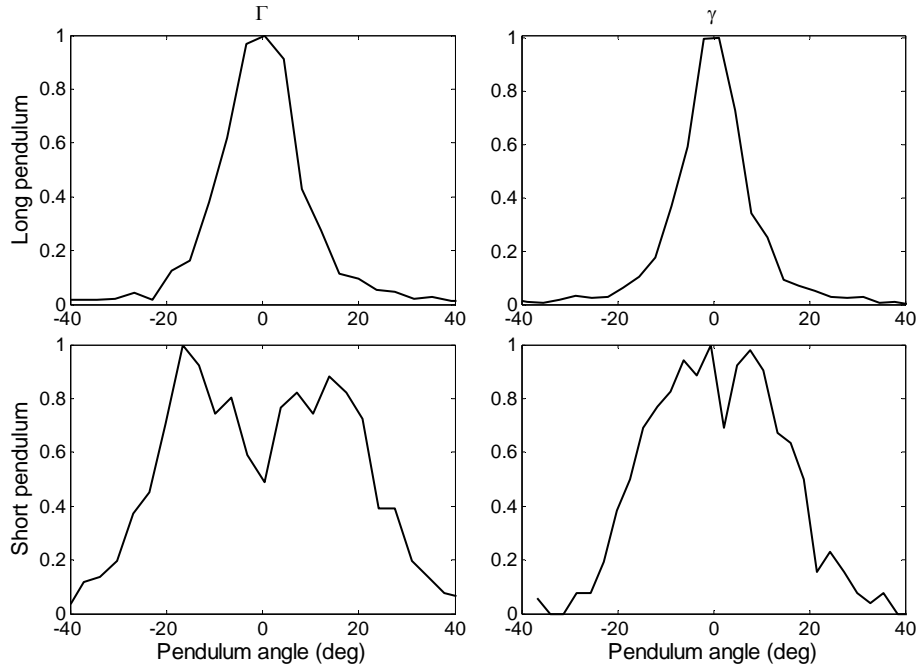


Figure 3.13 Distribution of the pendulum angle when movement starts Γ , and when movement ends γ .

Table 3-II Standard deviations of the distribution of Γ and γ relative to pendulum length.

	<i>Long pendulum</i>	<i>Short pendulum</i>
Stv(Γ)	0.21	0.31
Stv(γ)	0.17	0.23
$\frac{\text{Stv}(\Gamma) - \text{Stv}(\gamma)}{\text{Stv}(\Gamma)} * 100$	15.02%	26.79%

The reduction of the difference between the widths of the distribution Γ and the distribution γ , is shown in Table 3-II. Particularly, the reduction of the difference of the standard deviation between the distribution Γ and γ increased from 15%, as in the case of the long

pendulum, to 26.8% as in the case of the short pendulum. From these results, it appears that the human operator achieves more precise movements when controlling a short pendulum, which is partially true. However, it is important to keep in mind that the standard deviation of Γ was larger when controlling the short pendulum (0.31) than in all the other cases (no delay – 0.21; 150 ms delay – 0.22; 300 ms delay – 0.24; 500 ms delay – 0.28). Also, the standard deviation of γ (0.23) was comparable with the cases when time delay was 300 ms (0.21) and 500 ms (0.25). Thus, the corrective movements of the human operator when balancing a short pendulum, although generated at a larger angle deviation, are similar to the corrective movements generated when time delay affected the movement.

4.0 DISCUSSION

The experimental results show that, when the task of balancing an inverted pendulum becomes more difficult, human control becomes more discrete. The human operators did not experience major difficulties when balancing a sufficiently long pendulum with no delay. The long pendulum with large a time constant allowed the human subjects enough time to prepare the movement before actually performing it. However, when the task became more difficult due to increased time delay or shortened pendulum length, the subjects adopted a discrete, or bang-bang, type of control. This has a certain “open-loop” nature for each stroke or pulse of movement, because the feedback information cannot be evaluated in time in order to generate the best possible performance. Discrete or bang-bang control is known to appear in minimum-time tasks and to exhibit an abrupt change between two states [19]. Another relevant characteristic of discrete control is the occurrence of a refractory period between switching states [11], [12].

In the rest of this section we will compare the discrete-type strategies between the control of the pendulum with time delay and the control of a short pendulum. We will also consider different human-performance models to suggest possible explanations for the discrete control of the human operator. Finally, we will evaluate the conducted experiment.

4.1 DIFFERENT STRATEGIES OF DISCRETE CONTROL

The pendulum-angle and movement-velocity profiles in Figure 3.1 (d) (subject to a time delay of 500 ms) exhibit strong similarities with the corresponding profiles in Figure 3.8 (b), where the human operator controlled a short pendulum. In both cases, the pendulum swayed in a range of ± 40 deg, and the maximum amplitude of the angle velocity was about 5 deg/s. However, there are dissimilarities between the two cases in certain aspects of discrete control.

First, the average movement velocity in balancing the short pendulum was smaller than those observed in control of the long pendulum with time delays, but close to the average movement velocity seen when balancing the long pendulum without time delay. Second, the frequency with which the pendulum crossed the upright position (with or without time delay) was smaller than the frequency observed when balancing the short pendulum without delay. Moreover, the average reaction time when controlling a short pendulum is smaller than the reaction time when controlling any of the long pendulums, and is approximately half the average reaction time when controlling the long pendulum with the maximum considered time delay.

The above differences are characteristics that can help to distinguish between different types of discrete control. When balancing the long pendulum with time delay, the task is difficult because it demands the ability of the human operator to make predictions in order to compensate for the delayed perception of system responses. Here the adopted strategy for discrete control is to create sudden faster but less frequent movements. On the other hand, when the balancing task is difficult due to the shortened length of the pendulum (with reduced time constant), faster reactions of lesser intensity are required to stabilize the pendulum. In this case, the movements are discrete in such a way that more frequent switching is exhibited. Note that Loram [9] reached the same results on the inverted pendulum balancing task when the moment of inertia was very

small (i.e. the moment of inertia is directly proportional with the square of the length of the pendulum).

4.2 HUMAN PERFORMANCE MODELING

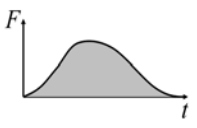
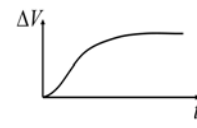
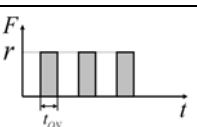
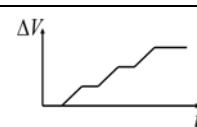
In human manual control, the neural system needs time to plan and execute a movement. This process is constrained not only by the latency in sensory feedback and muscle activation but also by the mental computation required for generating the desired command signal for the muscles. When the task becomes difficult due to the fast dynamics of the system under control (the short pendulum) or to the demanded capability of motion prediction (control under time delay), mental computation is challenged and the discrete-control strategy can be considered a solution to resolve these challenges.

4.2.1 Human operator as a PD controller with three-state relay and time delay

Young and Meiry [8] revealed a direct relationship between the characteristics of discrete control and the required mental computation. They suggested that when the generation of human force involves evaluation of displacements or velocities, the human controller has to mentally compute at least one integration operation [refer to Table 4-I]. The complex integration operation is time consuming. When performing an easy balancing task, the human controller has sufficient time to perform the mental computation and to prepare relatively accurate continuous movements. However, when the system dynamics require fast action, the human controller does not have enough time to perform the complex computation for integration and to implement a smooth

continuous-type control. Rather, the human controller can adopt a discrete-type control to reduce the complexity of the mental computation. A pulse-like force pattern makes the mental-integration process much easier to implement, because the area of the exerted force requires only the computation of the duration of the action, assuming that the magnitude of the pulses is constant [Table 4-I]. These are the characteristics exhibited by the discrete control as can be seen in Figure 3.1 and Figure 3.8.

Table 4-I Required mental computation for evaluation of movement.

Controller	Force	Δ Velocity	Required Mental Computation
Continuous			Full Integration $\Delta V = \int_0^t F(\tau) d\tau$
Discrete			Count pulses $\Delta V = \sum r \cdot t_{ON}$

Adapted from [8]

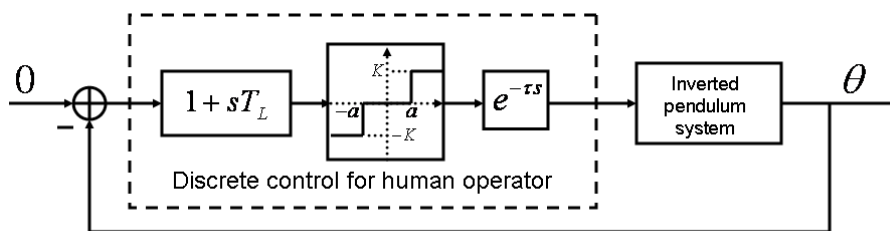


Figure 4.1 Inverted pendulum control model from [8].

Young and Meiry [8] further proposed a human-performance model for manual control as shown in Figure 4.1. The model consists of a proportional-derivative (PD) controller, a three-state relay, and a delay component.

In order to gain insight into how the human operators may internally adjust their strategy according to the task difficulty, we implemented the above model in Simulink/Matlab. The inverted pendulum system used in the simulation was the same discrete state space model representation that was used for the experiments with the human subjects. For the simulation, the discrete corresponding components of Figure 4.1 were implemented with the same sample time of 50 ms seconds as in the experiments.

Table 4-II Parameters of the PD controller with three-state relay and time delay resembling the average behavior of the human operator.

<i>Pendulum length</i>	20 m				5 m
<i>Time delay τ</i>	0 ms	150 ms	300 ms	500 ms	0 ms
<i>a</i>	0.12	0.35	0.53	0.7	0.7
<i>K</i>	9	23	22	18	47
<i>T_L</i>	0.2	0.5	0.8	1.35	0.35

The parameters of the PD controller were adjusted such that the response of the system yielded similar behavior to the average human movement strategy observed in the experiments. The average response of the human operator is characterized by the frequency within which the pendulum crosses the vertical upright position, and by the pendulum's magnitude sway for each scenario. Table 4-II shows the values of all parameters of the considered human operator model. To illustrate intuitive parameter tuning we refer to the following procedure: the value a of the three-state relay (an angle deviation of more than a radians generated a pulse of magnitude K , and an angle deviation of less than $-a$ radians generated a pulse of magnitude $-K$) was set for each scenario to the corresponding angle deviation when the human generated a corrective movement; afterward, the values of K and T_L were generated exhaustively to yield the average

human performance regarding the frequency and the magnitude of the angular sway of the pendulum for each scenario. The value of a is given in radians, as the entire simulation uses this unit of measure for the pendulum angle.

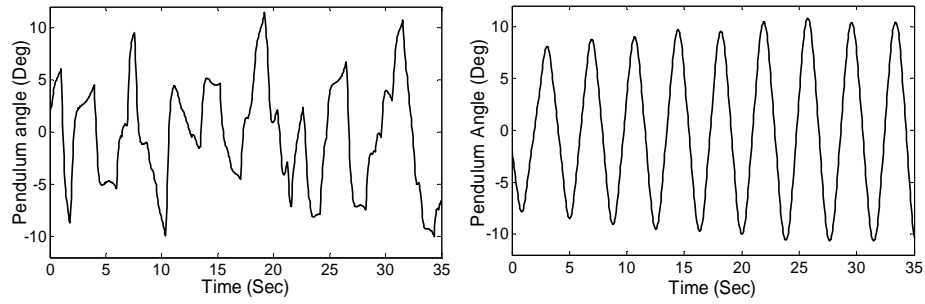
Figure 4.2 and Figure 4.3 illustrate a comparison between the trajectories of the pendulum angle from the experiments (on the same subject as in Figure 3.1) and from simulations using the proposed model for the human operator. A careful inspection of the values for the human operator model showed in Table 4-II reveals a certain tendency. When balancing a delayed pendulum, the parameter T_L that produced the desired behavior of the proposed human operator model increased with time delay, while the value of K decreased as time delay increased. When controlling a short pendulum, the value for T_L was smaller than in the cases where a delayed pendulum was balanced, and was comparable with T_L of the long pendulum without time delay. The value K for the same case was much larger than all the rest. It is important to note that the parameters of the PD controller are consistent with the stability conditions of the delayed inverted pendulum system presented by Stepan [10].

The three-state relay switches its output $-K/0/K$ relative to the magnitude of the input a , which can be regarded as the weighted sum of the angle deviation and the velocity of the angle deviation from the upright position. To achieve similar performance with the human operator, the coefficient of the angle velocity T_L appeared to increase with the time delay. When the time delay was 500 ms, the value of derivative gain T_L exceeded the unity weight coefficient of the pendulum angle. This result demonstrates that as time delay affected the task, the human operator had a tendency to rely increasingly on the changing rate of the error signal (i.e. angle velocity of the pendulum) to generate the control signal (i.e. the corrective balancing movement). This prediction capability is consistent with the intuitive idea that the human operator tends to

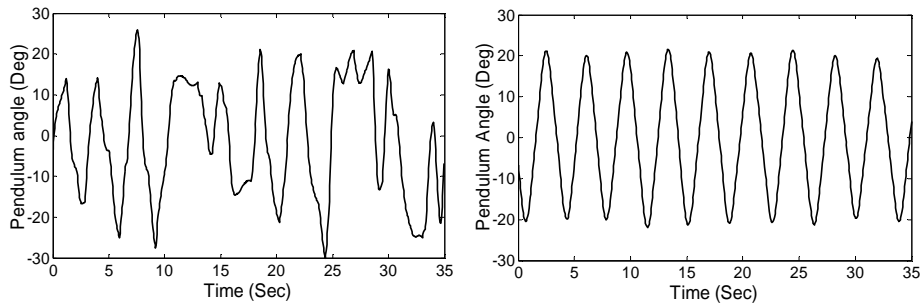
use predictive behavior in order to compensate for latency. Neurological studies identified the cerebellum to serve as a motion predictor in movement control [20].

The parameter K of the output of the three-state relay generates pulses corresponding to the amplitude of the force applied on the pendulum (as mentioned in Table 4-II). As the input to the inverted pendulum system was the displacement of the bottom tip of the pendulum (2), the force pattern had to be integrated twice. The integration operation is linear and, therefore, the magnitude of the force is directly related to the magnitude of the displacement of the pendulum's bottom tip. Because the parameter a is limiting the magnitude of the angular sway, the parameter K directly correlates with the frequency of oscillations of the pendulum around the upright position. To resemble the human operator performance, the value of the parameter K was noted to decrease with time delay. This result is consistent with the observation that the frequency of the angular sway decreases with time delay.

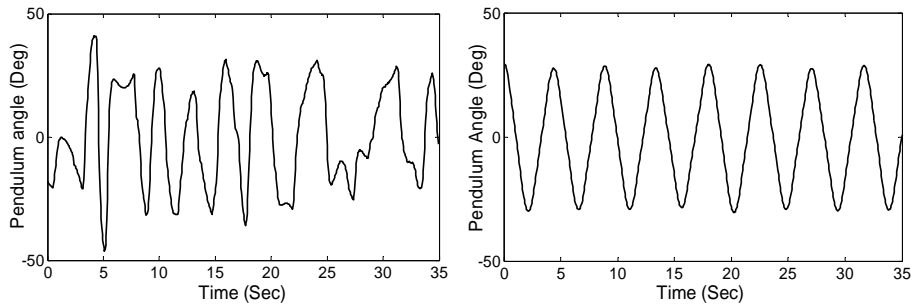
When controlling the short pendulum, the parameters of the human operator model reinforce the idea that the task is difficult due to the need to make quick movements in order to keep the pendulum balanced. The strategy exhibited by the subjects is consistent with the tendency observed in the parameter values of the human operator model. The derivative gain T_L is comparable with the case where a long pendulum was balanced without time delay, invoking limited prediction capabilities in performing the task. Moreover, the parameter K is larger than in any other scenario, which is in concordance with the high frequency of the angular sway observed in the experiments from chapter 3.2.



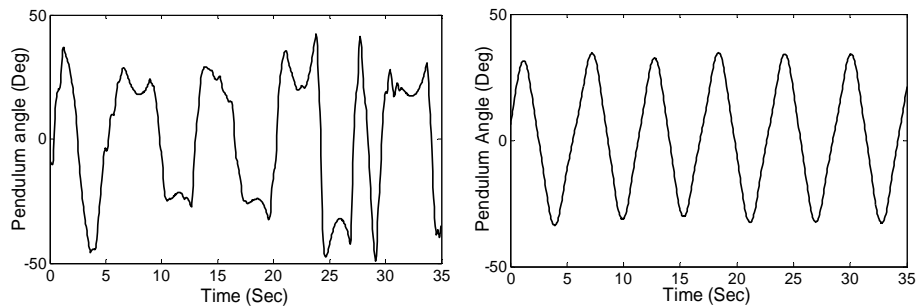
(a)



(b)



(c)



(d)

Figure 4.2 Pendulum angle trajectory of long pendulum control with (a) no delay, (b) 150ms, (c) 300ms, and (d) 500ms time delay, from experiment (left), and simulation (right).

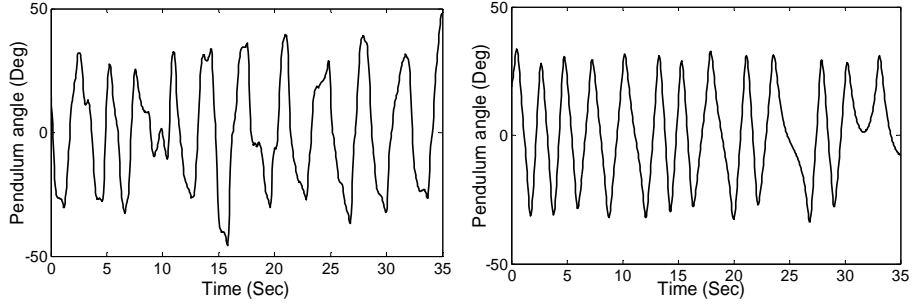


Figure 4.3 Pendulum angle trajectory of short pendulum control with no time delay: experiment (left), and simulation (right).

4.2.2 Human operator as an act-and-wait controller

In addition to the previously discussed model, another interpretation of the human operator's discrete-control strategy can be obtained by representing the human controller as an act-and-wait controller. The act-and-wait controller [21] is a special example of a periodic controller, because feedback is periodically switched on (acting period) and off (waiting period). Thus, the act-and-wait control strategy is similar to bang-bang control, where the waiting period from the act-and-wait controller corresponds to the refractory period in bang-bang control. The controller in this case is a linear mapping of the delayed state variables into command signals:

$$u(t) = Dx(t - \tau) \quad (3)$$

where the command signal u is an m dimensional vector, the state x is an n dimensional vector, the mapping matrix D is an m by n matrix, and τ is the delay of the feedback.

The act-and-wait control assumes a constant feedback delay and imposes an additional constraint: the waiting period should be equal to or larger than the time delay of the feedback loop. The main advantage of this control method is that the system can be represented by a finite dimensional monodromy matrix. Additionally, the eigenvalues of this matrix, which are the

poles of the closed-loop system, depend on the values of the control matrix D . Thus, the elements of the matrix D are chosen such that the response of the system is not only stable, but also allows the system to deliver a good performance. Adaptive or optimal control theory methods may be used to determine such a control matrix. As in the previous model, the human operators are assumed to possess capabilities to adjust some internal gains in order to adapt their movement given the constraints of time delay and small time constant of the controlled system. Moreover, the act-and-wait controller not only simplifies pole placement of the closed-loop system but can also stabilize systems that cannot be stabilized by autonomous controllers [21]. This suggests that the act-and-wait control method may be superior compared to other control methods, and may provide an explanation for why the human operator adopts a discrete-type control strategy when the task is constrained by time delay.

Beyond the proposed human operator models, many other models have been suggested in the literature [1]-[5], [10], [21]. They all have in common the idea that adjustments of some parameters have to be performed in order for the task to be controlled. An adaptive adjustment process of such parameters to yield the desired human performance could also give insight into the mechanism involved in obtaining the parameters. Applying control dependent noise in the considered strategy of the human controller would explain variation in human performance on each trial.

4.3 EVALUATION OF THE EXPERIMENT

The average behavior and the individual performances of the subjects in the conducted experiment provide evidence that human subjects adopt specific strategies in manual control. However, certain aspects may have interfered with the obtained results. The subjects did not know ahead of time how difficult the task would be, as the length of the pendulum and the time delay of each scenario were not apparent to them. Therefore, they had to use their intuition to estimate these features after the first few balancing movements. Additionally, the requirement was to successfully balance the pendulum for 45 seconds, and no trial was recorded when the pendulum fell in this time period. Each subject had some failed trials, which relates to the individual capacity of learning the task. As the subjects started to become familiar with the task, the balancing movements sometimes appeared to become slower not only when the time delay increased, but also when the pendulum length was shorter. This is due to the capacity of humans to adapt to the task and continuously improve their performance. This aspect was not accounted for in our experiment, as we would have liked to evaluate human manual performance at its limits. The embodiment of such properties in the design of a human controller has been the aspiration of many researchers in the past decades, and is still an important current research field, and we may regard this direction for future investigation.

As previously mentioned, two of the seven subjects (Subjects 1 and 6) have proven to be exceptions in comparison to the other subjects. On one hand, Subject 1 displayed unusually fast movements, the highest frequency of angular sway, and the smallest reaction time when the pendulum was long and no time delay was included. On the other hand Subject 6 illustrated the lowest frequency of angular sway, and the largest reaction time. Additionally, the two largest sway magnitudes of the pendulum were recorded from these two subjects. These results show

that Subject 1 exhibited characteristics of the behavior expected when balancing a short pendulum. A possible interpretation for this behavior could be the subject's inability to discern between a long pendulum and a short pendulum. Subject 6 was observed to have a slow reaction to the angle deviation of the pendulum. The movements were triggered late such that large angular sway and large reaction times were recorded. Moreover, Subject 6 had the most difficulties with the task, as double the time was needed to accomplish the experiment.

5.0 CONCLUSION

The analysis of the experiment performed in this study shows that human operators tend to adopt a discrete control strategy when the task is difficult due to time delay, or small time constant. When the task of balancing the inverted pendulum became difficult due to time delay constraints, the discrete-type control method was observed, and subjects exhibited less frequent switching but with higher speed. This is slightly different from the task of controlling a shorter pendulum with no delay, where the human operator switched more frequently.

A simple nonlinear model for the human controller was implemented in order to interpret the discrete-control mechanism. The proposed manual model (consisting of a PD controller, a delay component, and a three-state relay) resembled general characteristics of discrete-type control of human operators. The parameters of the PD controller for each scenario were adjusted such that the response of the system yielded a similar response to that observed in the conducted experiments. This gave insight into how the human operator adopts the control strategy relative to the challenging index of the task and the required performance. The derivative gain T_L was increased as time delay increased, implying on one hand that the human operators relied on the rate of change of the pendulum angle when shaping their corrective balancing movement, and on the other hand that the pendulum angle had an attenuating role in determining the movement. This observation confirms the hypothesis that human operators tend to adjust their predictive gains in order to compensate for latency. Another possible explanation for the human strategy of

discrete control can be obtained via a model of act-and-wait control. This theory assumes that the human operator is capable of adjusting the elements of the matrix that maps the state variables into command signals in order to stabilize an unstable system.

The results of our study not only reflect the human performance when dealing with challenging manual control tasks, but also provide a source for investigation of new control theoretic models that can improve or even reproduce human performance. This exploration is of great relevance in the field of teleoperation, where human performance plays a key role.

Further research is required to quantify the information content of the human hand movements in order to provide a qualitative measurement of human performance, and to identify the limits of manual control. Moreover, the investigation of the information rate achieved by the human operator has to be made relative to the amount of time delay and the length of the pendulum.

APPENDIX

INVERTED PENDULUM DYNAMICS

The inverted pendulum dynamics are derived with the Lagrangian method. The inverted pendulum is shown in Figure A.1, and the description of the parameters are given in Table A-I.

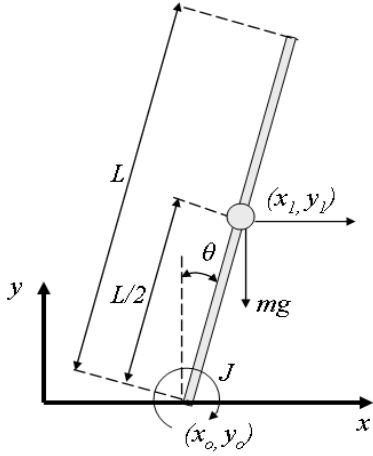


Figure A.1 Inverted pendulum system.

Table A-I Description of the inverted pendulum parameters.

Parameter	Description
(x_o, y_o)	Coordinates of the bottom tip of the pendulum
(x_l, y_l)	Coordinates of the center of gravity of the pendulum
θ	Angle of the pendulum
L	Length of the pendulum
m	Mass of the pendulum
J	Moment of inertia of the pendulum

The Lagrangian is defined as

$$L_g = T - V \quad (\text{A1})$$

where T is the translational and rotational kinetic energy, and V is the potential energy. Both quantities are defined at the center of gravity (x_l, y_l) of the pendulum. The derivation yields

$$\begin{aligned}
T &= \frac{1}{2}m(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}J\dot{\theta}^2 \\
&= \frac{1}{2}m \left\{ \left[\frac{d}{dt} \left(\frac{L}{2} \sin \theta + x_0 \right) \right]^2 + \left[\frac{d}{dt} \left(\frac{L}{2} \cos \theta \right) \right]^2 \right\} + \frac{1}{2}J\dot{\theta}^2 \\
&= \frac{1}{2}m \left\{ \left[\frac{L}{2} \dot{\theta} \cos \theta + \dot{x}_0 \right]^2 + \left[\frac{L}{2} \dot{\theta} \sin \theta \right]^2 \right\} + \frac{1}{2}J\dot{\theta}^2 \\
&= \frac{1}{2}m\dot{x}_0^2 + \frac{1}{2}m\frac{L^2}{4}\dot{\theta}^2 + m\dot{x}_0\dot{\theta}\frac{L}{2}\cos\theta + \frac{1}{2}J\dot{\theta}^2
\end{aligned} \tag{A2}$$

$$V = mg\frac{L}{2}\cos\theta \tag{A3}$$

Due to readability purposes, the notation $\dot{x} = dx/dt$ was used for the first time derivative of x , and $\ddot{x} = d^2x/dt^2$ for the second time derivative of x . Thus, the Lagrangian becomes

$$L_g = \frac{1}{2}m\dot{x}_0^2 + \frac{1}{2}m\frac{L^2}{4}\dot{\theta}^2 + m\dot{x}_0\dot{\theta}\frac{L}{2}\cos\theta + \frac{1}{2}J\dot{\theta}^2 - mg\frac{L}{2}\cos\theta \tag{A4}$$

The equation that describe the dynamics of the inverted pendulum system were obtained by computing the Euler-Lagrange equation

$$\frac{\partial L}{\partial \theta} - \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) = 0 \tag{A5}$$

Which yields
$$\frac{\partial L_g}{\partial \theta} = -m\dot{x}_0\dot{\theta}\frac{L}{2}\sin\theta + mg\frac{L}{2}\sin\theta \tag{A6}$$

$$\frac{\partial L_g}{\partial \dot{\theta}} = m\frac{L^2}{4}\dot{\theta} + m\dot{x}_0\frac{L}{2}\cos\theta + J\dot{\theta} \tag{A7}$$

$$\frac{d}{dt} \left(\frac{\partial L_g}{\partial \dot{\theta}} \right) = m\frac{L^2}{4}\ddot{\theta} + m\ddot{x}_0\frac{L}{2}\cos\theta - m\dot{x}_0\frac{L}{2}\dot{\theta}\sin\theta + J\ddot{\theta} \tag{A8}$$

Substituting (A6) and (A8) into (A5)

$$\left(J + m\frac{L^2}{4} \right) \frac{d^2\theta}{dt^2} + \frac{mL}{2}\cos\theta \frac{d^2x}{dt^2} = \frac{mgL}{2}\sin\theta \tag{A9}$$

After substituting the moment of inertia $J = mL^2/12$ into (A9), and rearranging the terms, the relation (1) from section 2.1 is obtained.

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