AN EXAMINATION OF THE ROLE OF TECHNOLOGICAL TOOLS IN RELATION TO THE COGNITIVE DEMAND OF MATHEMATICAL TASKS IN SECONDARY CLASSROOMS

by

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This study investigates the role of digital cognitive technologies in supporting students' mathematical thinking while engaging with instructional tasks. Specifically, the study sought to better understand how the use of technology is related to the cognitive demand of tasks. Data were collected in four secondary mathematics classrooms via classroom observations, collection of student work, and post-lesson teacher interviews. Opportunities for high level thinking by students were evaluated using the *Mathematical Tasks Framework* (Stein, Smith, Henningsen, & Silver, 2009). Technology use was evaluated with respect to whether it served to amplify students' thinking by making students' work more efficient or accurate without changing the nature of the task, or whether it was used to reorganize students' thinking by supporting a shift to something different or beyond what the technology was doing for them (Pea, 1985).

Results indicate that the mere inclusion of technology in a task was not related to the cognitive demand during any of the three phases of implementation, as technology was used in both high and low level tasks. However, results suggested an association between the level of cognitive demand of a task and the way that technology was used. In general, when technology was used as an amplifier, it was not related to the thinking requirements of the task, while the use

of technology as a reorganizer was central to the thinking requirements of the task. The decline of tasks set up at high level often corresponded to technology being used as an amplifier and reorganizer during set up, but as only an amplifier during implementation.

Overall, the role of technology in the decline or maintenance of high level thinking during implementation seems to depend more on teachers' classroom practice than any particular issues related to the use of technology. How prepared students were to engage in high level thinking tasks in general, how teachers anticipated students' needs while using technology to engage with the task, and how teachers responded to student questions and difficulties were influential factors in the maintenance or decline of these tasks.

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1 CHAPTER 1: THE RESEARCH PROBLEM

1.1 INTRODUCTION

The inclusion of technology as one of the six principles in the National Council of Teachers of Mathematics's Principles and Standards for School Mathematics (2000) seems curious when one considers that it is listed along side of such enduring issues in mathematics education as equity, teaching, learning, curriculum, and assessment. While research around the use of digital technology for mathematical learning first appeared 30 to 40 years ago (Kaput, 1992), its commonplace presence in secondary mathematics classrooms is still relatively new, as evidenced by the exclusion of a discussion of technology in a series of NCTM documents until 2000 (National Council of Teachers of Mathematics, 1989, 1991, 1995, 2000). This fact makes NCTM's assertion that technology is "essential to the teaching and learning of mathematics" (p. 24) even more remarkable. One interpretation of that statement is that NCTM's understanding of the role of technology in the teaching and learning of mathematics is part of a larger vision for mathematics education that is articulated across all six principles.

Due to the inclusion of technology in the Principles and Standards, and its apparent incompatibility with a more traditional view of mathematics and mathematics education focused on efficiency with computations and procedures automated by many technological tools, it is tempting to believe that the use of technology for instruction is a mark of a reformed practice. Indeed, research has shown that teachers who do not use technology for mathematics instruction often eschew it because of their beliefs about what mathematics is, or because of its perceived irrelevance to the mathematics they teach (Manoucherhri, 1999; Norton, McRobbie, & Cooper, 2000). In addition, researchers have used the inclusion of technology for instruction as an indicator of a reformed mathematics practice (e.g., Mayer, 1998).

However, research has also demonstrated that the converse is not necessarily true. That is, teachers who use technology do not necessarily have a more constructivist view of mathematics or a student-centered practice. Studies have shown that technology is frequently used as an extension of and consistent with teachers' current practice, even if such a practice is fairly traditional (Cuban, Kirkpatrick, & Peck, 2001; Farrell, 1996; Manoucherhri, 1999; Monaghan, 2004; Russell, Bebell, O'Dwyer, & K. O'Connor, 2003). Thus, it seems that the inclusion of technology is no more a sign of a reformed practice than the inclusion of the ideas included in the other principles: teaching, learning, curriculum, or assessment. As with these other elements of mathematics education, it is not the inclusion of technology which makes a teacher's practice reformed, but rather the type of teaching, learning, curriculum, and assessment that it makes possible.

Indeed, in the executive summary of their meta-analysis of research on the impact of handheld graphing devices on student learning, Burrill and her colleagues state plainly:

> A core finding from the research is that the type and extent of gains in student learning of mathematics with handheld graphing technology are a function, not simply of the presence of handheld graphing technology, but of how the technology is used in the teaching of mathematics. (Burrill et al., 2002, p. iii)

Thus, the mere presence of technology in a mathematics classroom does not thereby induce learning by students, or even necessarily cause teachers to alter their instruction. Three of the teachers in Monaghan's (2004) study who incorporated technology into their practice used the same type of "closed" tasks as they had prior to using technology, with technological procedures replacing mathematical procedures. In considering how the roles that teachers played during technology and non-technology lessons differed, Farrell (1996) noted that "teachers did continue to exhibit some of the characteristics of direct, teacher-led instruction during most of the observed segments" (p. 46). Cuban, Kirkpatrick, and Peck (2001) found that most teachers who did incorporate technology into their instruction maintained a fairly teacher-centered practice while doing so. Manouchehri (1999) summarizes the results of a survey of 181 mathematics teachers with regard to their use of technology for instruction as follows: "there is evidence of the lack of use of computers at both middle and high school levels in ways other than drill and practice" (p. 37). Monaghan (2004) recounts arguments made by researchers (Schwartz, 1989; Heid, Sheets, and Matras, 1990; Hudson and Borba, 1999; Zbiek, 2002) and organizations (The Mathematical Association, 1992) which claim that the use of technology for instruction will cause a transformation of classroom pedagogy. While Monaghan (2004) and Farrell (1996) found differences between lessons taught by teachers with and without the use of technology, some of which indicated a more student-centered, exploratory approach to mathematics instruction, they are careful to point out that such changes are anything but automatic: "In no way is this report meant to imply that the graphing technology causes the reported differences" (Farrell, 1996, p. 51). The National Council of Teachers of Mathematics is quite explicit about this fact in the Technology Principle as well: "Technology is not a panacea. As with any teaching tool, it can be used well or poorly" (2000, p. 25).

A useful analogy may be that of block scheduling for instruction in which classes are 80 to 90 minutes long instead of the traditional 40 to 45 minutes. There is no theoretical reason to believe that doubling the length of an instructional period will result in more or different learning if nothing else about instruction is altered. Teachers may simply teach twice as much material during an instructional period. However, if instruction is altered to take advantage of the longer class period by allowing for more student-centered discovery or explorations, group work on in-depth problems or projects, and presentation and discussion of solutions strategies, what students learn is likely to change as well. The reason for the difference in student learning is not directly associated with a longer class period, but rather the affordances of having more time for instruction supports a different type of instruction. Likewise, the presence of technology in the classroom provides no theoretical reason for student learning to increase or change. Rather, it is what is done with it, and what type of instruction is supported by its use, that has the potential to impact learning.

In the present investigation, technology refers to any digital cognitive technology. Pea defines cognitive technologies as those which "help transcend the limitations of the mind (e.g. attention to goals, short-term memory span) in thinking, learning, and problem-solving activities" (Pea, 1987, p. 91), and may include graphing calculators, dynamic geometry software, interactive whiteboards, computer algebra systems, spreadsheets, and internet applications, but is not necessarily limited to these. A technology-enhanced task is any task in which technology is used as a cognitive technology, whether that use was planned by the teacher or not. The problems that this research seeks to address are both theoretical and practical. The theoretical problem is addressed first.

1.2 THE THEORETICAL PROBLEM

As a theoretical problem, there are few frameworks for examining how the use of technology influences students' mathematical thinking. Many studies have considered how the use of technology has impacted students' learning of algebra, geometry, trigonometry, functions, data analysis, and calculus (Ben-Zvi, 2000; Burrill et al., 2002; Chazan, 1999; Doerr & Pratt, 2008; Glass & Deckert, 2001; Heid, 1997; Heid & Blume, 2008; Hembree & Dessart, 1992; Hollebrands, Laborde, & StraBer, 2008; Jost, 1992; Judson, 1990; Kendal & Stacey, 2001; Mariotti, 2000; O'Callaghan, 1998; Palmiter, 1991; Ruthven, 1990; Schwarz & Hershkowitz, 1999; Zbiek, Heid, Blume, & Dick, 2007), and have demonstrated improved learning of specific concepts or procedures. These studies examine how technology is used to support the learning of particular content, and students' thinking while using these tools is addressed in this context. However, fewer studies have focused on how the opportunity for complex thinking by students using technological tools is supported or hindered in a classroom context (Ben-Zvi, 2000; Doerr & Zangor, 2000; Hoyles & Noss, 1992; Kendal & Stacey, 2001). In terms of NCTM's Principles and Standards for School Mathematics (2000), much of the research conducted on the use of technology for learning has focused on the learning of the Content Standards, but much less on how it supports learning of the Process Standards (Hollebrands, Conner, & R. C. Smith, 2010). The opportunities for students to make connections, use and interpret mathematical representations, engage in problem solving, communicate mathematically, and reason and prove, all characteristic of high level thinking, while not neglected in the above studies, have been backgrounded. While one can engage in such thinking processes only while considering some specific content or problem, few studies have focused on how the thinking demands themselves are influenced by the use of technology. Indeed, the frameworks used in these studies, while not ignoring student thinking, generally relate to the content or specific concept that technology is hypothesized to help students to learn (Schoenfeld, J. P. Smith, & Arcavi, 1993), and are not frameworks for understanding the type of thinking students may do more generally.

An important aspect of the type of thinking afforded by the use of technology is the kind of problem or task that calls for its use. Whether technology is used or not, one way teachers shape students' learning and view of the discipline of mathematics is by the choice of mathematical tasks for instruction (National Council of Teachers of Mathematics, 1991). However, with the introduction of technology comes the need to understand what kinds of tasks utilize the resources provided by the technology to support students' high level thinking: "Some researchers also suggest that the choice of the task in relation to the affordances of the dynamical geometry environment may be critical for the development of the understandings of the students" (Hollebrands et al., 2008, p. 174). Referring to the analogy of block scheduling above, a teacher must understand how the resource of time supports the use of different types of activities than what may be possible in 42 minutes, and how to successfully implement these different types of activities. While a number of studies have examined how the integration of technology might influence curriculum (Ben-Zvi, 2000; Chazan, 1999, 2000; Heid, 1988, 1997; Judson, 1990; O'Callaghan, 1998; Palmiter, 1991; Park & Travers, 1996), few studies have examined how it influences the kinds of tasks that teachers use in relation to students' thinking (Doerr & Zangor, 2000; Hoyles & Noss, 1992; McGraw & Grant, 2005).

The fact that technology can be incorporated into a variety of instructional models points to the need for a framework to describe how the use of technology for instruction may vary. "As educational technology use in and out of the classroom increases, so must our ability to clearly differentiate among the ways teachers can use technology" (Russell et al., 2003, p. 307). Russell et al. identify six categories of use in their study of teachers' use of technology: preparation, email, teacher-directed student use, recording grades, delivery, and special education and accommodation. While these categories are more specific with regard to technology use than earlier frameworks, they are still fairly generic, in part because the teachers in their study taught a variety of subjects across many grade levels (K-12). For example, how is the use of PowerPoint presentations for traditional-style lecture on the properties of triangles distinguished from open-ended explorations of triangles by students using dynamic geometry software which result in conjectures about those properties and the conditions under which they may hold? Currently, there are few ways to describe the differences in how mathematics teachers use technology for instruction.

1.2.1 The *Mathematical Tasks Framework*

The *Mathematical Tasks Framework* (Henningsen & Stein, 1997; Stein & M. S. Smith, 1998; Stein, Grover, & Henningsen, 1996; Stein, M. S. Smith, Henningsen, & Silver, 2009) has been used to describe and differentiate the type of thinking that is called for by a given mathematical task, defined as "a classroom activity, the purpose of which is to focus students' attention on a particular mathematical idea" (Stein et al., 1996, p. 460). This framework distinguishes between low level cognitive demand, including memorization and the use of procedures without any connection to meaning or concepts, and high level cognitive demand, including the use of procedures with connections to meaning or concepts, and *doing mathematics*, of which non-algorithmic thinking is characteristic. The Task Analysis Guide (Stein et al., 2009) describes these four categories in greater detail (APPENDIX A). An important characteristic of this taxonomy is that it is not related to specific mathematical content, but rather characterizes different types of thinking that students may engage in while working on a mathematical task. However, this framework has not been used to investigate and describe the opportunities for thinking specifically in relation to the use of technological tools, which is the aim of this study.

Beyond the distinctions made above with respect to the types of thinking called for by a mathematical task, the *Mathematical Tasks Framework* makes an important connection to classroom practice by recognizing that the thinking requirements of a task may change during its implementation. The task as it appears in curricular materials does not directly influence students' learning by the type of thinking it requires, as those demands may be altered by the teacher when announcing the task to students during instruction, known as the set up phase, and again while students are working on the task, referred to as the implementation phase. This element of the *Mathematical Tasks Framework* makes it especially suitable for describing the impact of using technology on students' thinking in a classroom context.

Stein and Lane (1996) examined student learning in relation to the cognitive demands of mathematical tasks, both during the set up phase and the implementation phase. They developed an instrument which "consists of a set of open-ended tasks that assess students' mathematical problem solving, reasoning, and communication" (p. 65) to measure student learning. They also observed classroom instruction at four schools in order to evaluate the cognitive demands of the mathematical tasks during the phases of set up and implementation, and found a correlation between the cognitive demands of the tasks and student learning as measured by their instrument. That is, students who were more frequently exposed to high level tasks during the set up phase demonstrated greater gains on the measure of student learning, while students who implemented high level tasks at a high level showed the greatest gains in learning. They conclude: "the nature and level of instructional tasks used in the classroom have a substantial impact on student thinking which, in turn, affects student performance and learning" (p. 74).

A critic of these results may claim that the instrument used to measure student learning was biased as it was created to measure precisely what students were hypothesized to learn from engaging with high level tasks. However, this seems to be evidence of the validity of the instrument to measure the type of learning and performance described by the mathematics reform movement (NCTM, 1989; 2000). In this way, the results of this study demonstrate that the *Mathematical Tasks Framework* is a valid way to correlate the type of thinking students do while engaging with instructional tasks to the meaningful learning of mathematics promoted by the mathematics education reform movement. Thus, rather than the mere presence of technology being an indicator of a reform oriented practice, the *Mathematical Tasks Framework* provides a way to gauge whether the way that technology is used for instruction is indeed supporting such a practice.

1.3 THE PROBLEM PRACTICE

As a problem of practice, teacher educators have few resources in terms of rich descriptions of practice that integrate technology in meaningful ways. This relates to the theoretical problem described above, as part of the problem may be that meaningful has not been well-defined. That is, one way to understand meaningful is with regard to the opportunities for high level thinking that are provided by the use of technology. However, such instruction is all too rare, with or without the use of technology (Stigler & Hiebert, 2004). Thus, the problem of mathematics teacher education with regard to the use of technology for instruction is part of a larger problem in mathematics education reform of helping teachers to teach mathematics in ways that are substantially different from the ways that most have learned it, or the prevailing models of instruction common in most secondary mathematics classrooms. Furthermore, using technology for mathematics instruction is not only a case of asking teachers to use unfamiliar tools for instruction, but in many cases a matter of asking them to teach different mathematics made accessible or possible by the use of these tools[1]. The fact that many teachers in Manoucherhri's (1999) study had classroom sets of calculators that they never used suggests that even new teachers who might be considered "digital natives" are likely to not have learned mathematics with digital technology tools.

In reviewing the nascent literature on the use of technology for instruction and learning in 1992, Kaput recognized even then that "major limitations of computer use in the coming decades are likely to be less of a result of technological limitations than a result of limited human imagination and the constraints of old habits and social structure" (p. 515). As recently as 2008, the same point with respect to imagination being a limiting factor in the use of technological tools for mathematics instruction and learning was made (Grandgenett, 2008). In discussing the discouraging results of the study of mathematics teachers' use of technology for instruction, Manoucherhri (1999) concludes that "Little effort is spent on helping teachers conceptualize how technologies can be adopted in their real school settings" (p. 38). Both exemplars of practice which integrate technology, as well as cases that raise specific issues which may relate to planning and implementing technology-enhanced lessons are needed in order to provide practice-based learning opportunities for teachers (D. L. Ball & Cohen, 1999; Putnam & Borko, 2000). In surveying the landscape of research on technology-enhanced mathematics instruction, Galbraith (2006) summarizes the situation as follows:

> So how are current developments in the field of 'technology enhanced' instruction to be assessed? Perhaps it can be agreed that boundless optimism of technophiles, wisely questioned by Fey (1989), has been increasingly challenged through data-based scrutiny.

[1] The ways in which technological tools have been used to reorganize curriculum are discussed in Chapter 2.

> The most compelling data come not from experimental studies, comparing treatment groups with 'controls', as these provide little more than coarse comparisons, leaving untouched insights essential for future progress, and frequently reduce discussion to simplistic input–output relationships. Increasingly, empirical studies have developed case examples detailing in depth what happens when students engage in mathematics learning in the presence of technology. (p. 286-7)

One way to interpret this statement is that more fine-grained descriptions of what students do with technology, and inferences about the type of thinking indicated by such behavior, are needed to inform teachers' understandings and images of how technology can substantially impact students' learning.

While there are a number of studies that investigate students' thinking while using technology to learn specific mathematical concepts, much of this research on student learning with the use of technology has studied novel pedagogical strategies designed and implemented by mathematics education researchers (Ben-Zvi, 2000; Chazan, 1999; Glass & Deckert, 2001; Heid, 1988; Hoyles & Noss, 1992; Judson, 1990; McGraw & Grant, 2005; Palmiter, 1991; Park & Travers, 1996; Ruthven, 1990; Schwartz & Hershkowitz, 1999). Far fewer studies have examined instruction by classroom teachers (Doerr & Zangor, 2000; Farrell, 1996; Heid, Sheets, & Matras, 1990; Kendal & Stacey, 2001), and none of these have explicitly studied it in relation to the thinking opportunities provided by the use of technology. However, this type of research has the potential to directly impact teacher education and professional development by providing insight into the classroom-based factors which may have important implications for students' thinking and learning. This further supports the claim that the *Mathematical Tasks Framework* may be uniquely suited to investigate the use of technology in instructional tasks, as it acknowledges and accounts for changes in the thinking demands of a task during the various phases of implementation. Furthermore, research on how these demands might change during instruction has examined various classroom-based factors associated with such changes (Stein et al., 1996; Henningsen and Stein, 1997). In particular, when tasks are set up at a high level, a number of factors have been identified as being associated with maintaining the cognitive demand during implementation, and another group of factors has been connected to its decline during this phase. These are referred to as classroom-based factors associated with maintenance or decline (Stein et al., 1996, Henningsen and Stein, 1997). Understanding how the use of technology may be related to these factors, or may introduce other factors, could help to inform teacher education and professional development on integrating technological tools for mathematics instruction.

1.4 PURPOSE OF THE STUDY

The purpose of the present study is to examine the use of technology in relation to the types of thinking students engage in while using it in a classroom context, and to describe and begin to characterize these different types of use. The use of the *Mathematical Tasks Framework* to describe students' thinking while using technological tools in classroom contexts, and if and how it may differ from instruction without such tools, is what motivates the research questions. A goal of the present investigation is to contribute to teacher education on the use of technology for secondary mathematics instruction by providing images of effective instruction using technology, as well as being able to describe classroom-based factors which support or hinder such use.

1.5 RESEARCH QUESTIONS

Professional development efforts aimed at improving teachers' ability to select and enact high level cognitive demand tasks have been found to be successful (Boston & M. S. Smith, 2009). However, the implications for the inclusion of technology are not fully understood. Knowing what a high level task is, and how to maintain the demand of a task during implementation, has not been explicitly explored in relation to the use of technology. This points to a need to characterize the use of technology for mathematics instruction in relation to the cognitive demand of the tasks, and motivates the following research questions:

Question 1: How do the cognitive demands of mathematical tasks differ when technology is used as part of the task and when it is not?

a. How is the use of technology associated with the cognitive demand of mathematical tasks as they appear in curricular materials?
b. How is the use of technology associated with the cognitive demand of mathematical tasks as set up by the teacher?
c. How is the use of technology associated with the cognitive demand of mathematical tasks as implemented?

Question 2: How does the role of technology differ in low level and high level cognitive demand tasks? What is the role of technology in each?

a. During Set Up

   What are the features or characteristics of technology use associated with tasks set up at a low level of cognitive demand?

What are the features or characteristics of technology use associated with tasks set up at a high level of cognitive demand?

b. During implementation

What are the features or characteristics of technology use associated with tasks implemented at a low level of cognitive demand?

What are the features or characteristics of technology use associated with tasks implemented at a high level of cognitive demand?

Question 3. How does the use of technology impact the cognitive demand of a task during implementation?

a. How is the use of technology related to factors which have been associated with decline of mathematical tasks set up at a high level of cognitive demand? How is the use of technology related to the decline of mathematical tasks which are set up at a high level of cognitive demand?

b. How is the use of technology related to factors which have been associated with maintenance of mathematical tasks set up at a high level of cognitive demand? How is the use of technology related to the maintenance of mathematical tasks which are set up at a high level of cognitive demand?

The first research question looks at how the use of technology correlates to the tasks enacted by teachers by looking at the entire sample of tasks collected within a classroom and determining how the use of technology within these varies with the cognitive demand of the tasks. The second research question focuses on the sub-sample of tasks collected in a classroom that incorporate the use of technology, and examines how that use varies between high and low level tasks. Finally, the third research examines the sub-sample of tasks set up at a high level, considering how the patterns of maintenance and decline may differ between tasks that use technology and those that don't.

Teaching is a complex process, made more complex when powerful technological tools are incorporated into instruction. Teachers may unwittingly mitigate students' opportunities to engage in high level thinking and reasoning in a number of ways, a fact well documented (Stein et al., 1996; Henningsen and Stein, 1997; Stein and Lane, 1996). What none of these studies have examined, however, is how sustaining or reducing the opportunity for complex thinking and reasoning might be associated with the use of technology. Given the association of students' engagement with cognitively demanding tasks and student achievement, understanding the role of technology within such a task may provide teachers and teacher educators with important principles for its use. By identifying patterns of use associated with different types of mathematical tasks, the present investigation may be able to identify important implications for teachers' use of technology for instruction.

1.6 SIGNIFICANCE

The use of technology by teachers to enhance instruction is not self-evident. Pierson (2001) describes a teacher who she determined to have strong pedagogical content knowledge, but whose teaching was observed to regress when she used technology for instruction due to her inability to integrate it with an otherwise exemplary practice. Pierson also reports that this teacher had a strong belief in the benefits of using technology for instruction which was the basis for her efforts to use it, evidence of the fact that the problem is more than an issue of beliefs.

The question of what teachers need to know to use technology effectively for instruction has recently been given more attention by teacher educators and researchers (AMTE, 2009; Drier, 2001; Garofalo, Drier, S. Harper, Timmerman, & Shockey, 2000; Grandgenett, 2008; Kersaint, 2007; Koehler & Mishra, 2005, 2008; H. S. Lee & Hollebrands, 2008; K. Lee, Suharwoto, Niess, & Sadri, 2006; Mishra & Koehler, 2006; Moursund & Bielefeldt, 1999; Niess, 2005, 2006, 2008; Niess et al., 2009; Pierson, 2001; Richardson, 2009; Suharwoto & K. Lee, 2005; Suharwoto & Niess, 2006; P. H. Wilson, H. S. Lee, & Hollebrands, 2011; P. S. Wilson, 2008). In particular it has been noted that teacher education and professional development that focuses on training teachers to use a particular technology ignores the complexity of the issue: "Underlying this approach is a view of technology that sees it as being a universally applicable skill; unlocking the power and potential of technology can be achieved by acquiring basic competency with hardware and software packages" (Mishra & Koehler, 2006, p. 1031). Such an approach fails to consider how technology is used in practice, i.e., knowing what buttons to push in order to execute a particular command or function is of little use to the teacher that hasn't thought about when or how the use of that command or function will enhance students' understanding or learning.

Just as the framework of pedagogical content knowledge (PCK) (Grossman, 1990; Shulman, 1986, 1987) has been developed as the recognition that knowledge of content alone does not translate into effective instruction and student learning, there is a growing consensus among researchers and teacher educators that knowledge of technologies is necessary but not sufficient for its integration into practice in ways that can positively impact student learning

(Garofalo et al., 2000; Kersaint, 2007; H.S. Lee & Hollebrands, 2008; Mishra & Koehler, 2006; Niess, 2005, 2008; Niess et al., 2009; Pierson, 2001; Richardson, 2009). Rather, the knowledge teachers need is essentially integrated, drawing on teachers' knowledge of technology, pedagogy, and content, which has been termed technological pedagogical content knowledge (TPACK). Mishra and Koehler (2006) propose the model depicted in Figure 1.1 to stress the need to integrate all three types of knowledge in a way that gives rise to a new form of knowledge which is an amalgam of the three.

> TPCK represents a class of knowledge that is central to teachers' work with technology. This knowledge would not typically be held by technologically proficient subject matter experts, or by technologists who know little of the subject or of pedagogy, or by teachers who know little of that subject or about technology. (p. 1029)

The TPACK framework has been taken up and refined within the area of mathematics teacher education, with many researchers and teacher educators using it as a framework for the development of curriculum and standards for teacher education and professional development on teaching mathematics with technology (Grandgenett, 2008; Hollebrands et al., 2008; Kersaint, 2007; H.S. Lee & Hollebrands, 2008; K. Lee et al., 2006; Niess, 2005, 2008; Niess et al., 2009; Richardson, 2009). While the TPACK framework has guided the development of the knowledge that teachers need to teach effectively with technology, rich descriptions of such instruction and how it is "effective" remain sparse, and are generally disconnected from this emerging framework. Thus, the results of the present investigation have the potential to make a contribution to this growing knowledgebase in the area of mathematics education. Ideally, the present study would provide fine-grained descriptions of cases of both non-exemplary and exemplary use which can raise issues for teachers of which they are not currently aware, and from which principles for effective use can be gleaned. Although the present investigation is not intended to focus on or measure teacher knowledge per se, the results of this study have the potential to shed light on important issues related to the use of technology for instructional tasks, and thus make a contribution to an area of teacher education that is developing rapidly.

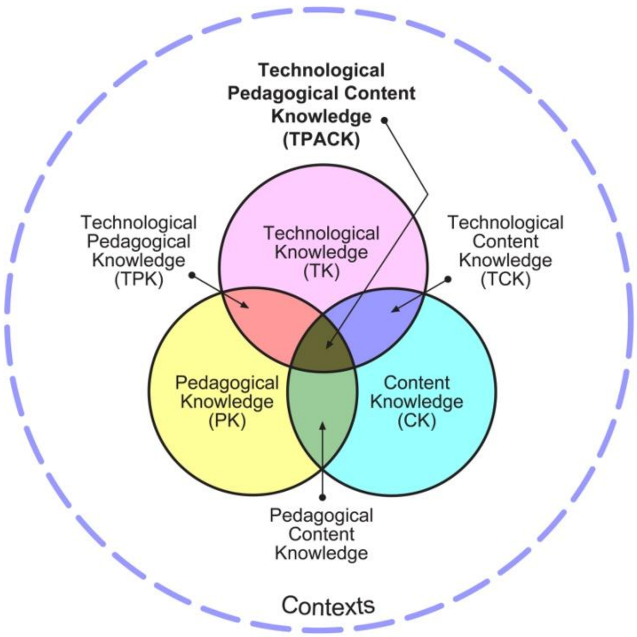


Figure 1.1: A Conceptual Model of TPACK (http://tpack.org/).

In a recent survey, a nationally representative sample of Algebra I teachers reported mixed opinions about the usefulness of computers for instruction. They reported that access to computers for instruction was not an issue, but reported using computers for instruction on average less than once a week (Hoffer, Venkataraman, Hedberg, & Shagle, 2007). For the use of technology to begin to fulfill its potential to enhance student learning, teachers and teacher educators need images of instruction that stretch teachers' imagination about what is possible with technology, and clearly identify important issues that must be taken into consideration from examples that are rooted in teachers' practice. In particular, providing teachers and teacher educators with a description of how the use of technology for instruction has the potential to influence students' thinking as described by the *Mathematical Tasks Framework* is an important anticipated outcome.

1.7 LIMITATIONS

A few methodological issues associated with the data collected limit the strength of the results of this study. The first is that the reliability for coding cognitive demand at four levels (*memorization*, *procedures without connections*, *procedures with connections*, and *doing mathematics*) between the researcher and the field note coder was only 72%, compared with 93% between the researcher and an observer present for the lesson. However, when the coding of cognitive demand was collapsed to the categories of high and low, reliability was 83%. As the research questions only refer to cognitive demand using these two categories, this is the distinction that is used in the analysis and discussion of results.

The low agreement with the field note coder at the four levels of cognitive demand suggest that use of field notes as the primary data source for coding cognitive demand beyond high/low may have limitations. This may not be an issue with using field notes for this purpose in general, but the field notes as constructed in the present study, and the choices made regarding what to observe and how to represent it, may make it a less appropriate data source for capturing those aspects of the lesson needed to inform the evaluation of cognitive demand at a level more specific than high/low. A possible explanation for the difference in agreement between the researcher and the lesson observer, and the researcher and field note coder, is that the researcher generally coded the task from the observation, before or while the field notes were generated, which may have fostered greater agreement with the observer since the coding was based on the same data source, i.e., the observation. The agreement between the field note coder and the researcher was at least 78% for any other dimension of the task that was observed, including factors associated with maintenance and decline (78%), and technology use as an amplifier, reorganizer, both, or neither (86%). These figures support the claim that these field notes were a valid representation of task enactment for many dimensions of classroom instruction of interest in the present study.

Another methodological limitation relates to the evaluation of cognitive demand using the *Mathematical Tasks Framework* in a teacher-centered task. The *Mathematical Tasks Framework* is ideal for evaluating student-centered tasks in which there is a clear introduction by the teacher, students work on the task in pairs or groups, and the teacher concludes the task with a whole class discussion. However, students' thinking is more difficult to assess in a teacher-centered setting, and very often this was how the interactive whiteboard (IWB) was used. For example, when the IWB was used to create novel and interactive representations for demonstrations intended to help students construct meaning for a mathematical procedure, concept, or solution strategy, there is limited data revealing students' thinking, especially if students' participation was limited. When possible, the evaluation of the cognitive demand was made in conjunction with other data sources, such as students' work prior to or following such a teacher-centered demonstration. In general, a lack of high level thinking was inferred from a lack of evidence of high level thinking, and in these tasks a lack of evidence was more common than in others. As a result, the confidence in the results regarding the connection between this use of technology and cognitive demand in these tasks is not as high as in more student-centered tasks.

Although the participating teachers were purposefully selected to provide a sample of tasks capable of answering the research questions, the lack of variation along certain dimensions limits the strength of the results. In particular, more tasks in which technology was not used would have provided a better contrast for the sample of tasks which did use technology. While the tasks that did use technology demonstrate variation with respect to the level of cognitive demand, no comparison can be made with a sample of tasks for which technology was not used as there are simply too few to establish a pattern. For example, the answer to Research Question One based on the data collected was that there is no relationship between the cognitive demand of a task and whether or not technology is used in the task. This result would be stronger if the same variation in cognitive demand was noticed in a larger sample of tasks which did not use technology. More tasks set up and implemented at a high level would also have strengthened the results. While no teacher set up less than 29% of the observed tasks at a high level, only 13% of the entire sample (8 tasks) were implemented at a high level, and six of those tasks were enacted in the same classroom. Furthermore, no tasks were implemented at a high level at two of the four data collection sites. More teachers with a classroom practice supportive of implementing tasks at a high level may have provided greater insight into the role of the teacher in supporting high level thinking with technology. Thus, in terms of the implications for teacher education and professional development, the results of the present study have more to say about what not to do, but are limited to a single teacher in terms of practices which might support students' high level thinking using technology, limiting generalizability. Furthermore, three of the four teachers participating in this study taught geometry, and taught many of the same topics. The small number of teachers and lack of variation in the content being taught limit the potential to glean general principles across multiple teachers and contexts in terms of how to support students' engagement with technology-enhanced tasks at a high level.

1.8 ORGANIZATION OF THE DOCUMENT

In the next chapter, the literatures on mathematical tasks and instructional technology for mathematics instruction are reviewed in relation to the research questions. Research surrounding the construct of mathematical task and the *Mathematical Tasks Framework* are discussed in greater detail. The literature on the use of technology for students' learning of mathematics is broad and varied. Issues identified as relating to teachers' and students' use of technology for instruction and learning that are related to the use of mathematical tasks for instruction are the focus of this review, and hypotheses based on the integration of these literatures in relation to the research questions are discussed.

Chapter 3 describes the research methods that were employed to answer the research questions posed above. Chapter 4 discusses the results of applying the methods to data collected to answer the research questions. Finally, Chapter 5 discusses the results, offering explanations for the results and discussing them in relation to the implications for teacher education and future research.

2 CHAPTER 2: REVIEW OF THE LITERATURE

2.1 INTRODUCTION

There is no shortage of research studies on the use of digital technologies for mathematics learning and instruction. What is unique in the present study is an examination of its use in relation to the mathematical tasks used for instruction. The construct of a mathematical task provides a unit of analysis which captures a meaningful aspect of instruction with clear implications for student learning (Hiebert & Wearne, 1993; Stein & Lane, 1996; Stein et al., 1996; Stigler & Hiebert, 2004; Weiss & Pasley, 2004; Weiss, Pasley, P. S. Smith, Banilower, & Heck, 2003). In particular, the present study considers the use of technological tools in relation to the cognitive demand of instructional tasks and how the demand may change during phases of implementation as described by the *Mathematical Tasks Framework* (Stein et al., 2009).

In the present review, the construct of mathematical task is used to frame the activity of teachers and students within the context of classroom instruction, including the use of technological tools that may be part of that activity. Figure 2.1 is provided as a way to represent this framing, and the interactions between teacher, students, and technological tools around instructional tasks. Further, Figure 2.1 is used to organize the present review. After defining "digital cognitive technologies" there is a brief review of the literature which describes ways in which their use has impacted student learning. The majority of the review discusses, in turn, research which falls into one of the intersections of the three areas in the Venn diagram: teacher and students, students and technology, and teacher and technology.

The purpose of the present study is to use the *Mathematical Tasks Framework* (Stein et al., 2009) to describe classroom activity which would be located at the center of Figure 2.1, i.e., the interactions between teachers, students, and technology as students engage with mathematical tasks.

The construct of a mathematical task provides a lens through which to view classroom activity, but by itself is too general to be used as an analytic framework. Ultimately those aspects of mathematical tasks which mediate student learning are what are important to observe in interactions between the teacher and students, and interactions between students and the tools they use, in this case technology. One dimension of mathematical tasks which has been shown to have important implications for student learning is their cognitive demand (Hiebert & Wearne, 1993; Stein & Lane, 1996). How the cognitive demand of a task may change during implementation is described by the *Mathematical Tasks Framework* (Stein et al., 2009), and thus the research which has provided an empirical basis for it is described.

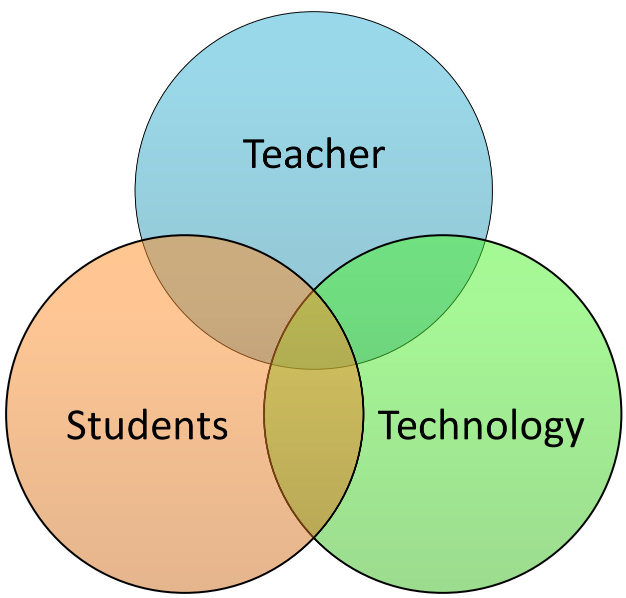


Figure 2.1: Mathematical tasks frame the interactions between the teacher, students, and technology.

The discussion of cognitive demand of instructional tasks and the *Mathematical Tasks Framework* (Stein et al., 2009) encompasses aspects of teacher and student interactions around mathematical tasks used for instruction, seen as the intersection of the teacher and student circles in Figure 2.1. This is followed by a discussion of what lies at the intersection of students and technology in Figure 2.1 – literature regarding issues with students' use of technology for mathematical activity, especially as it is hypothesized to relate to the cognitive demands of the tasks with which they engage. Finally, studies which have focused on teachers' use of technology for instruction are discussed, also with a view toward how the cognitive demand of tasks may be influenced by teachers' use of technological tools. This is depicted in Figure 2.1 as the intersection of the teacher and technology areas.

Two notes of clarification are in order. The first is that the literature that falls into the intersection of students and technology or teachers and technology is not usually discussed in terms of tasks or their cognitive demand. Making the connection between the results of these studies and the Mathematical Task Framework (Stein et al., 2009) in order to make hypotheses for the current study is a primary goal of the present review. Second, the studies identified in this review which discuss students' or teachers' use of technology generally take place in classrooms and not laboratories. As such, it is usually neither possible nor desirable to completely separate teachers and students in the discussion. Rather, the intersections described above in relation to Figure 2.1 are in terms of the focus of the study in question. For example, a number of studies address the issue of students' autonomy in their mathematical work with technological tools, and student behaviors associated with different degrees of autonomy. Because the discussion is focused on student behaviors, this issue is addressed in the section on students' use of technology in spite of the fact that teachers play a role in the amount of freedom that students have to make decisions regarding their use of technological tools for mathematical work. The term technology can encompass an assortment of tools in a variety of contexts. The meaning of instructional technology in the present study is discussed below.

2.2 DIGITAL COGNITIVE TECHNOLOGIES

Digital technologies have become ubiquitous in secondary school settings, but can serve a variety of purposes within mathematics classrooms. Peressini and Knuth (2005) identify five ways that digital technologies can serve as a tool for teachers in the mathematics classroom: as a tool for management, communication, evaluation, motivation, and as a cognitive tool. As a management tool, digital technologies can assist teachers in accomplishing many daily chores involved in teaching and managing a classroom. Using PowerPoint for lectures, the internet as a source of ideas for projects or activities, or software for recording attendance are all examples of using technology as a management tool. As a communication tool, various technologies such as e-mail, online discussion forums, and interactive video all enhance the possibilities for increased communication with administrators, parents, and students, and facilitate collaboration among teachers. As an evaluation tool, teachers may use video, e-mail, discussion forums, and other software packages to evaluate and provide feedback to students. Teachers may also use these technologies to reflect on and evaluate their own practice, as well as to provide and receive feedback from coaches, administrators, or other teachers. As a motivational tool, digital technologies can be engaging for students for a number of reasons: digital technologies are a part of youth culture, its use is novel in most secondary classrooms, they are often interactive, and they provide the potential to explore problems in contexts that may be more interesting or realistic but would otherwise involve tedious manual computations or messy data. Finally, as a cognitive tool, digital technologies can support students' mathematical thinking and activity. It is important to note that these categories of tool use are not necessarily mutually exclusive. For example, a teacher may have students use remote clickers whereby each student is able to enter their response to a series of multiple choice questions remotely. In addition to having the potential to increase student engagement in an exam review (motivational), for example, the teacher is also able to have student responses recorded to a spreadsheet in order to evaluate which questions students struggled with, or which students need help (evaluation). Thus, this is an example of using a digital technology simultaneously as a motivational and evaluation tool.

The interest in the present study is with digital technologies used specifically (but not necessarily exclusively) as cognitive tools, or cognitive technologies (Pea, 1985, 1987). Pea defines cognitive technologies as those which "help transcend the limitations of the mind (e.g. attention to goals, short-term memory span) in thinking, learning, and problem-solving activities" (Pea, 1987, p. 91). That is, unlike other uses of technology as described above, cognitive technologies are tools that support thinking. By mediating human thought they both assist and influence thought and learning. Cognitive technologies are not limited to digital technologies. For example written language and abstract mathematical notation systems are cognitive technologies which help to transcend the limitations of the mind but which are not digital. In this sense both a blackboard and a PowerPoint presentation are cognitive technologies, extending the limited memory capacity of the mind, but neither is a mathematical tool in the sense that they are not specifically designed to allow students to produce or interact with representations of mathematical objects. Examples of digital technologies which constrain users' actions to those which are mathematically meaningful (even if not to the user) include calculators, both scientific and graphing, computers and computer software including dynamic geometry and statistics software, computer microworlds, internet games and applets, computer algebra systems, data collection devices such as Calculator Based Laboratory (CBL) devices, and spreadsheets. Although the studies reviewed here generally investigate the use of digital technologies as a mathematical tool, the present study does not limit the focus to such tools, but rather considers any digital cognitive technology which may serve as a medium for mathematical activity. Thus, the use of PowerPoint for displaying geometric figures, or algebraic equations or graphs, would be an example of a digital cognitive technology under consideration in the current study, while the use of a blackboard for the same purpose would not.

A primary purpose of the present investigation is to better understand the role of technology within mathematics instruction. However, such a focus is justified by its potential to impact student learning. Thus a brief review of research which has demonstrated an empirical connection between the use of technological tools and student learning is presented below.

2.3 TECHNOLOGY AND STUDENT LEARNING

A primary claim about student learning in technology-enhanced environments is that the impact of technology is shaped by many contextual factors which collectively create opportunities for enhanced learning. While technology may provide opportunities for instruction to be structured differently in ways that may not be possible without it, it is not merely the presence of technology in an otherwise unchanged environment that somehow increases learning. The following describes two technologies common in secondary mathematics classrooms, dynamic geometry software and handheld graphing devices, and the conditions under which these have been found to impact students' learning.

Dynamic Geometry Software. Dynamic geometry software (DGS) allows students to create and manipulate mathematical objects graphically, symbolically, or numerically. A key feature of DGS is the ability to "drag" a figure on the screen by clicking on it and moving it with a mouse or touchpad so that it moves or resizes in real time in what appears to be a continuous manner (although in fact it does so in small discrete steps). It is this feature which the descriptor "dynamic" refers to. Another feature of most DGS is their ability to display and coordinate multiple representations. There is an immediate connection between representations in which actions taken on one representation, such as resizing a polygon or moving a line in the plane, are automatically recorded and immediately updated in another representation, such as the numerical side lengths of the polygon or the slope and y-intercept of the equation of a line. Kaput (1992) referred to this connection between representations as a "hot link," and noted that this is a unique affordance of such software environments that has the potential to impact the learning of mathematics in important ways. A third important feature of most DGS is the ability to create a "slider," which allows a given parameter to be directly manipulated. A slider generally appears as a line, which is the representation of the interval of values the parameter can assume, and a button which can be dragged back and forth along the line using a mouse or touchpad to change the current value of the parameter (see Figure 2.2).

As an example of its affordances, a slider can be used to systematically and dynamically vary the slope and y-intercept of a line in order to view the relationship between each parameter in the equation and the graph of a line, as shown in Figure 2.2. The relationship between these representations has been referred to as the Cartesian connection, and research has demonstrated that it can be a difficult connection for students to make, even in the domain of linear functions (Schoenfeld et al.,1993). The series of screenshots depicted in Figure 2.2, Figure 2.3, and Figure 2.4 demonstrate how the use of sliders can allow each parameter to be manipulated individually in order to observe its effect on the graph. In Figure 2.2, the line y = 2x + 2 is given. By changing the slope, m, from the 2 to -1, the equation and graph change to that of y = -x +2, as shown in Figure 2.3. If instead the y-intercept were changed from 2 to -3, the resulting equation y = 2x – 3 and its graph would be displayed, as in Figure 2.4. It is important to note that while this example demonstrates the affordances of the technology, it is not meant to imply that simply providing students with this tool will result in meaningful connections between representations for them. Indeed, how teachers utilize these affordances and their relation to students' thinking is precisely what is under investigation.

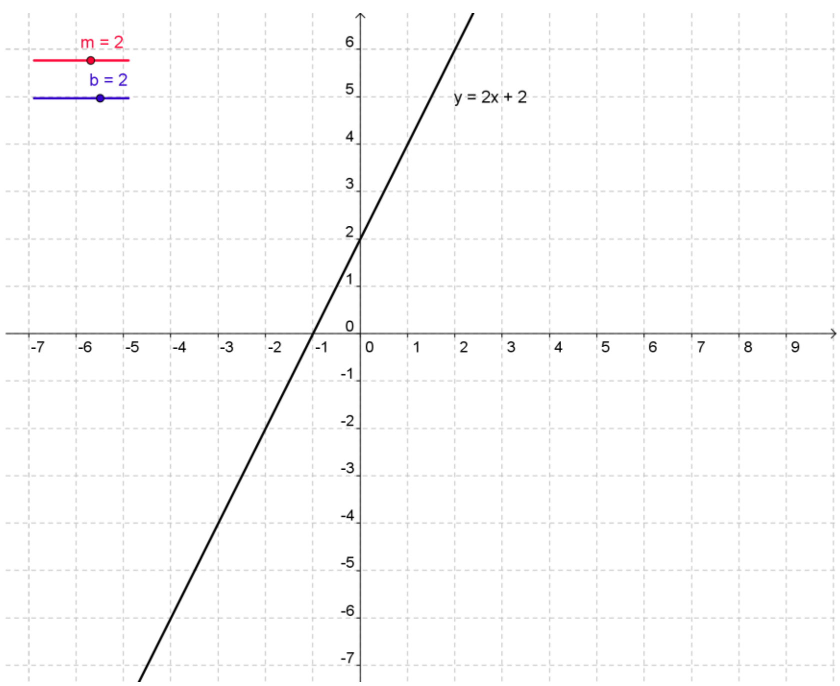


Figure 2.2: Sliders m and b representing the slope and y-intercept, respectively, of a linear function in the DGS GeoGebra.

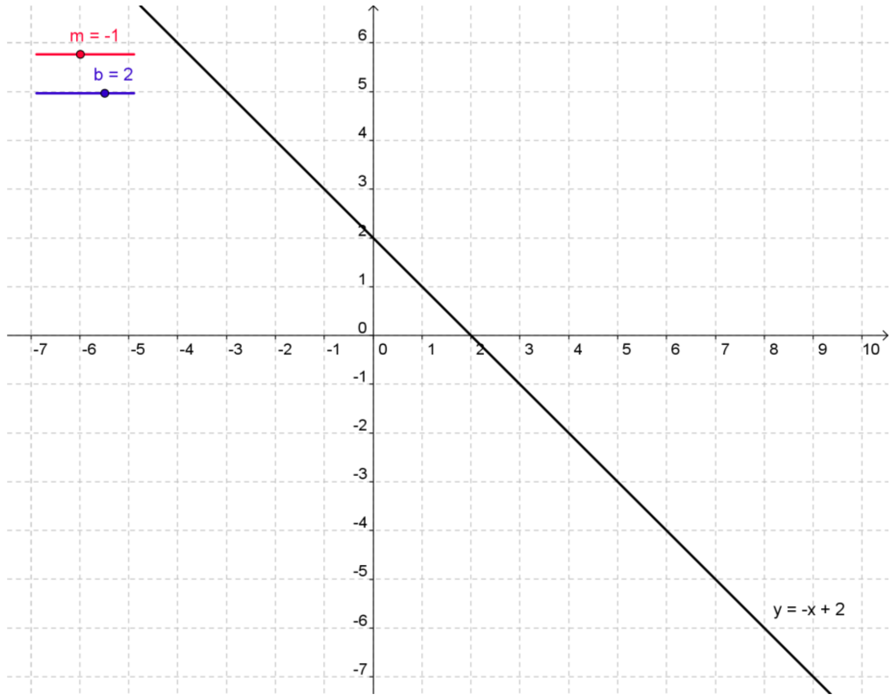


Figure 2.3: Changing m while holding b fixed rotates the line through the point (0,2).

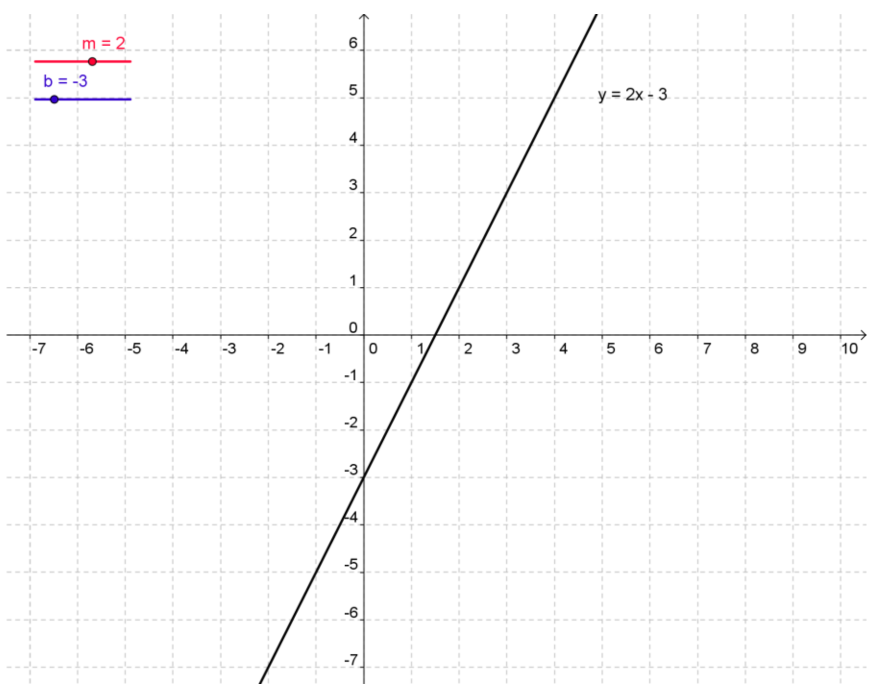


Figure 2.4: Changing b while holding m fixed (from Figure 2.2) moves the line vertically.

The use of DGS has been found to support students' problem solving by providing feedback and allowing for successive refinements of students' solutions: "The capability of the software to incorporate knowledge and to react in a way consistent with theory impacts the student trajectory in the solving process" (Hollebrands et al. 2008, p. 174). In the studies reviewed by Hollebrands and her colleagues, the use of DGS supported students in using multiple, linked representations in problem solving, including geometric diagrams, graphical representations, and symbolic notation. The link between representations that is automated by DGS was found to help students construct meaning for mathematical concepts, such as the idea of curvature (Hollebrands et al., 2008). Dynamic geometry software allows for students to make and test conjectures in ways that would be impossible without it, and a number of studies have demonstrated the diverse and novel ways in which the use of DGS can support students' development of deductive reasoning and proof (Mariotti, 2000; Marrades & A. Gutierrez, 2000; E. Sanchez & Sacristán, 2003).

Hollebrands et al. (2008) warned, "It must be stressed that [learning] results from the conjunction of the use of a DGS, of a careful design of the teaching/learning situation and of the tasks, of the social organization, and of the role of teacher" (p. 186). For example, Glass and Deckert (2001) note that Galindos (1997) found that students might be too willing to accept multiple examples in the form of "dragging" as proof within a dynamic geometry environment. However, Glass and Deckert hypothesize that this may be due to students working on close-ended rather than open-ended tasks: "[S]tudents may view conjectures from close-ended tasks or from given statements as automatically true and therefore may not see a need for formal reasoning" (p. 228). They claim that having students work on open-ended tasks results in the formulation of "shaky conjectures," the truth of which are in question, thus motivating the need for deductive reasoning and proof. This is an example of how the role of the teacher in designing and enacting the mathematical task can have a direct impact on the type of thinking required by the task as implemented in a technology enhanced environment.

Handheld Graphing Devices. Burrill and her colleagues' (Burrill et al., 2002) findings in a meta-analysis of research on handheld graphing technologies in secondary mathematics demonstrate numerous learning benefits by students using these technologies. These handheld devices include computer algebra systems (CAS), which refer specifically to automated symbolic manipulation capabilities, such as the ability to factor or expand an algebraic expression, solve numerical and literal equations, and perform symbolic computations such as finding the limit, derivative, or antiderivative of a function. In short, CAS is to algebra what scientific calculators are to arithmetic. More generally, computer algebra systems refer to any technology which has symbolic manipulation capabilities, usually in addition to graphical and numerical or tabular capabilities. Burrill et al.'s synthesis of research includes both CAS and non-CAS handheld graphing technologies.

With regard to the mathematical knowledge and skills learned by students using handheld graphing technology, numerous benefits were reported across mathematical topics, including algebra, functions, pre-calculus, and calculus (Burrill et al., 2002). Students learning algebra with handheld graphing technology were found to have a better understanding of variables, improved ability to solve problems in "real world" contexts, and an increased understanding and use of graphical representations, while not exhibiting any significant differences in their ability to use procedures effectively. Students using handheld graphing technology for pre-calculus demonstrated a better understanding of properties of function, graphs, and equations than students that did not, and were better able to generate an appropriate symbolic model given a graph. Calculus students using technology for an extended period of time developed a greater number of distinct solution strategies during problem solving. In particular, they developed the ability to use and coordinate both graphical and algorithmic approaches.

In addressing what students gain mathematically when using handheld technologies, Burrill et al. (2002) noted a number of studies in which students' concept of function and ability to use and link multiple representations was more advanced for students who used this technology in their learning than for those who did not. The authors note that the use of technology in these studies was accompanied by particular curriculum or curricular changes. In two studies which investigated the learning effects of using a particular curriculum designed to take advantage of handheld technologies, students using the curricula and technology were more proficient at solving multi-step problems, better at solving contextual problems with the aid of the technology, and more able to deal with problems involving real data than students who did not. Students using CAS calculators in one study demonstrated a more positive attitude toward mathematics and greater self-confidence in their mathematical problem solving. Not surprisingly, students with access to handheld graphing technologies tended to use graphical approaches to problem solving more often and used the technology to get feedback during problem solving. Furthermore, students with access to technology were more active and engaged in problem solving, and were more likely to work together.

These findings are consistent with those reported by Heid (1997) in her synthesis. In it she cites reports by Kieran (1993), Leinhart, Zaslavsky, and Stein (1990), and Dunham and Dick (1994), which all report research that demonstrates that graphing technologies enhance students' learning about graphs and functions. In particular, students in these studies exhibited "higher levels of graphical understanding, better work in interpreting graphs, better work in relating graphs to their symbolic representations, a deeper understanding of functions, and a better understanding of connections among a variety of representations" (Heid, 1997). However, she emphasizes that the overwhelmingly positive results reported in the studies that she summarizes must be interpreted in the context of the overall learning environment of which the graphing technology was a part, and that in most of the studies cited students were engaged in different types of mathematical activities, perhaps as a result of the availability of the graphing technology.

Likewise, Burrill and her colleagues (2002) summarize a key result of their meta-analysis of 43 studies as follows:

> A core finding from the research is that the type and extent of gains in student learning of mathematics with handheld graphing technology are a function, not simply of the presence of handheld graphing technology, but of how the technology is used in the teaching of mathematics. (p. i)

For example, Schwarz and Hershkowitz (1999) reported a more robust concept of mathematical functions and their representations by a class of students that used multirepresentational software in their learning about functions than students who did not. However, the use of technology was only one part of the learning environment that constituted the experimental condition for this group of students. In addition to the use of computer tools, these students used a different curriculum, were asked to make decisions about which representations and medium they worked with and how to make use of them, worked frequently in collaborative groups, were asked to report their solution processes, and frequently participated in whole class discussions about their work orchestrated by the teacher. While the context of the learning environment has important implications for the use of technology for learning, Schwarz and Hershkowitz report that the practices of "reporting, criticizing, and reflecting on one's own and others' solution processes" were more common when students worked with technology "to reduce the burden of symbol manipulation and pencil-and-paper graphing" (1999, p. 386). This is an example of the synergistic relationship between the use of technology and the learning environment in which it is used; each has the potential to support the other in ways that bear upon students' learning of important mathematics. This example provides empirical support for the claim that it is not the presence of technology per se which enhances students' learning, but how the learning environment is structured to take advantage of its affordances.

There exists an abundance of literature documenting improved student learning of mathematics with the use of technology. The point of the small sampling of literature reviewed here is to demonstrate that when students exhibit learning gains in conjunction with technology use, a variety of classroom based factors are present which are likely to be important to how and what students learn. One such factor that has been identified as correlating to student learning is the type of mathematical tasks that students work on as part of classroom instruction. Stigler and Hiebert (2004) in their analysis of the 1999 TIMSS Video Study distinguish two type of problems students might work on during instruction: using procedures which involve "basic computational skills and procedures" (p. 14), and making connections, defined as "rich mathematical problems that focus on concepts and connections among mathematical ideas" (p. 14). They note that the ways in which teachers and students work on problems during a lesson, and especially the way that making connections type problems get implemented, was a common characteristic observed in the classrooms of high achieving countries, while the use of technology varied across these countries. Given the connections made in the literature between mathematical tasks and student learning, the construct of mathematical tasks may be a useful conceptual lens through which to observe the use of technology for instruction. In particular, the types of thinking that students engage in while working on tasks influences the type of learning they do (Hiebert & Wearne, 1993; Stein & Lane, 1996). The next section describes the meaningful learning of mathematics as envisioned by the mathematics education reform movement, and how high level cognitive demand tasks have been shown to foster such learning.

2.4 MATHEMATICS EDUCATION REFORM AND STUDENT LEARNING

The reform movement in mathematics education is not just an idea or plan to improve the teaching and learning of mathematics in the United States. Rather, at its heart it is an assertion about what mathematics is and what it means to do, learn, and teach it. In its Principles and Standards for School Mathematics, the National Council of Teachers of Mathematics states that "Standards are descriptions of what mathematics instruction should enable students to know and do – statements of what is valued for school mathematics education" (2000, p. 7). In addition to describing the content that students should learn, certain habits of mind (Cuoco, Goldenberg, & Mark, 1996; National Council of Teachers of Mathematics, 2009) are enumerated and developed as being essential to the doing of mathematics. These Process Standards, problem solving, reasoning and proof, connections, communication, and representation (National Council of Teachers of Mathematics, 2000), are based on the belief that students should make sense out of the content that they are learning, and that the act of reasoning and sense making is itself an important outcome of mathematics education. Indeed, it is claimed that trying to predict the content that students entering school today will need to know when they graduate is "risky business," and thus the processes in which students engage and the habits of mind , or reasoning habits (Cuoco et al., 1996; National Council of Teachers of Mathematics, 2009) that they develop may in fact be just as important to the purposes of learning mathematics (if not more) than the subject matter that students learn.

Thus, the reform movement in mathematics education is not just about exploring and promoting instructional techniques that will help students learn more or better. It is about the type of learning in which students engage, including authentic and collaborative problem solving, mathematical discussions and communication, the use of novel and standard representations and connections among them, explaining, justifying, challenging and refuting results, and conceptual understanding. These goals for student learning are not in lieu of the ability to use and apply formulas and procedures, but rather provide a meaningful context for it by having students connect them to concepts and prior knowledge. In short, the goal is for students to "learn mathematics with understanding, actively building new knowledge from experience and prior knowledge," in contrast to learning mathematics without understanding, which "has been a persistent problem since at least the 1930s" (National Council of Teachers of Mathematics, 2000, p. 20).

Another important outcome of the mathematics education envisioned by reformers is the view of the discipline of mathematics that is developed over the course of a student's K-12 education. The cumulative effect of traditional instruction and learning can leave students with a view of mathematics as a set, static collection of arbitrary and disconnected rules, formulas, and procedures which they must learn well enough to pass exams but which have no connection to each other or to their lives outside of school. By contrast, reformers' views of mathematics emphasize the dynamic and creative nature of mathematical thinking, where students frame problems and make decisions about how to approach them, generate representations and information needed to solve them, and interpret and check the reasonableness of their results (Romberg, 1994; Schoenfeld, 1992, 1994). This view stresses the active construction of mathematical knowledge as a sense making endeavor relevant to students' lives: "Such a focus on reasoning and sense making will produce citizens who make informed and reasoned decisions, including quantitatively sophisticated choices…It will also produce workers who can satisfy the increased mathematical needs in professional areas…" (National Council of Teachers of Mathematics, 2009, p. 3).

A key factor in students' learning and views of mathematics is the kind of tasks they engage in, and the opportunities that those tasks provide to think in cognitively complex ways:

> Tasks also convey messages about what mathematics is and what *doing mathematics* entails. Tasks that require students to reason and to communicate mathematically are more likely to promote their ability to solve problems and make connections. Such tasks can illuminate mathematics as an intriguing and worthwhile domain of inquiry. (National Council of Teachers of Mathematics, 1991, p. 24)

Thus, the tasks that students engage in do more than mediate their learning by providing opportunities to think in certain ways; they shape students' perceptions of the discipline of mathematics. Students' views of mathematics can in turn influence their thinking and learning by creating expectations for their mathematical behavior.

In the next section evidence from empirical research is presented which demonstrates that the work students engage in, and the opportunities for mathematical thinking contained therein influence what they learn. This is followed by a description of a conceptual framework for understanding how mathematical tasks mediate student learning, and research which has both utilized and developed this framework is discussed.

2.5 THE *MATHEMATICAL TASKS FRAMEWORK*

Prior to discussing the framework developed by Stein and her colleagues (Stein et al., 1996; Stein & Lane, 1996; Henningsen & Stein, 1997; Stein & Smith, 1998; Stein et al., 2009), research connecting mathematical tasks and student learning, either explicitly or implicitly, is discussed more generally. These studies point to the idea that the tasks students engage with have important implications for learning, that certain types of tasks are more effective than others at engendering the kind of learning valued by the mathematics education reform, and that teachers play an important role in that process by the way that they shape and support students' opportunities to engage in these tasks. The studies discussed in this section fall into the areas depicted in Figure 2.5, studying the behaviors of students and teachers in the context of classroom instruction through the lens of mathematical tasks.

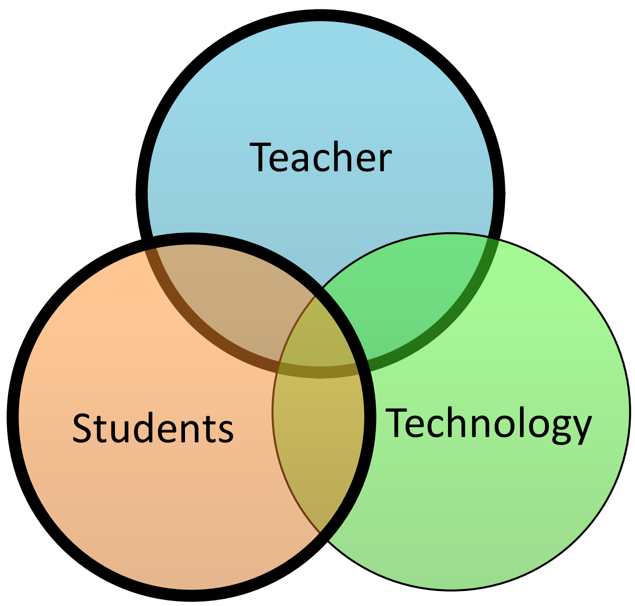


Figure 2.5: The present section reviews literature on the interactions of teachers and students within mathematical tasks.

2.5.1 Mathematical Tasks and Student Learning

In their discussion of the 1999 TIMSS Video Study, Stigler and Hiebert (2004) note that a feature of instruction common to the classrooms observed in high achieving countries was the way in which teachers and students engaged in problems during instruction. In particular, they classified problems as using procedures or making connections, and found that all six countries in their analysis (including the U.S.) had some mix of both types of problems, and that there existed some variation across countries with regard to the proportion of each. However, while most high achieving countries were observed to implement making connections problems faithfully, 100% of the making connections problems observed in U.S. classrooms degenerated during implementation to using procedures problems. Stigler and Hiebert conclude, "U.S. 8th graders spend most of their time in mathematics classrooms practicing procedures. They rarely spend time engaged in the serious study of mathematical concepts" (p. 14). This is one possible explanation for U.S. students' lower achievement on the TIMSS examination than students in comparison countries. The point is that the type of thinking that students have the opportunity to engage in during classroom instruction is likely to have important consequences for what they learn.

These findings are confirmed by research on instructional quality conducted by the Horizon group (Weiss & Pasley, 2004; Weiss et al., 2003). They studied 364 mathematics and science lessons, and classified only 15% of those lessons as being of high quality in terms of rigor and excellence. Features of instruction that were used to rate its quality included the opportunities for students to engage with course content (versus being passive recipients), effective questioning by teachers which goes beyond fill-in-the-blank or yes-or-no questions, and the opportunities that students had to make sense of the content that they were taught. These features all relate to the opportunities students have to think and reason in complex ways, and the manner in which those opportunities are supported by teachers.

Boaler (1998) "was particularly interested to discover whether different forms of teaching would create different forms of knowledge" (p. 42). She identified two schools in the U.K., Amber Hill and Phoenix Park, with almost identical student demographics but at two extremes regarding their learning environment. The mathematical tasks that students engaged with were an important aspect of that environment. At Amber Hill students generally worked on closed problems individually from a traditional textbook following a brief lecture by the teacher. At Phoenix Park students worked on open-ended projects in groups, while the teacher served as a resource for students to consult if needed. While Amber Hill students completed many problems from their text each class period, Phoenix Park students' work on projects of their choosing was self-paced, and they often spent two to three weeks on a single project.

In assessing the mathematical knowledge of students at each of these schools, both an applied problem and a brief set of closed questions were devised by the researcher. Students at Phoenix Park significantly outperformed Amber Hill students on the applied task, but there was no significant difference in the performance of students at each school on the closed written exam. As part of the applied task, students were also administered a brief written test a few weeks before engaging in the task which addressed the mathematics needed to solve the task. A high proportion of students at Amber Hill demonstrated the required mathematical knowledge on this test, but could not use that knowledge to solve the applied task. Phoenix Park students, on the other hand, performed comparably on both, indicating that they could use their mathematical knowledge in a variety of contexts. This is further attested to by Phoenix Park students' performance on a traditional, standardized examination, which was significantly better than those at Amber Hill, in spite of the fact that the exam consisted of the types of questions with which students at Amber Hill were very familiar.

In interviews with students after this exam, Boaler (1998) found that the slightest difference in the wording or problem presentation from that which Amber Hill students were accustomed caused them to falter during the exam, while Phoenix Park students viewed the questions as problems they needed to think about and "work things out." Indeed, these interviews demonstrate that the views of these two groups of students with regard to what it means to do and learn mathematics was influenced in large part by the tasks they worked on, and had a strong impact on their problem solving ability. Boaler concludes that for students at Amber Hill, "their textbook learning has encouraged them to develop an inert, procedural knowledge that was of limited use to them" (p. 56), while at Phoenix Park, "the students had the belief that mathematics involved active and flexible thought…[and] had developed an ability to adapt and change methods to fit new situations" (p. 57). While the mathematical tasks that students completed were just one part of the learning environment at these schools, students' performance on various assessments and remarks during interviews clearly indicate that these tasks impacted their learning in significant ways.

Whereas Boaler (1998) considered how different types of teaching may impact students with very similar characteristics, Boaler and Staples (2008) studied students at three neighboring high schools which differed along important dimensions with regard to the demographics of their students. Students at Railside School were more racially and ethnically diverse and of a lower SES than the two neighboring schools, Hilltop and Greendale. In addition, the manner in which mathematics instruction was structured was similar for Hilltop and Greendale, but quite different at Railside, which had the most reform-oriented mathematics instruction. Mathematical tasks were again an important element of those differences. While students at Hilltop and Greendale were observed to participate in traditional lecture and practice mathematics lessons, “at Railside School the teachers posed longer, conceptual problems and combined student presentations with teacher questioning” (p. 619). Students at Railside were grouped heterogeneously, often worked on problems in groups, on average spent more than twice as much time working on a problem than students at Hilltop or Greendale, and were lectured to by teachers, on average, less than one-fifth of the time than their peers at the two neighboring schools. The differences in these instructional approaches had important implications for student learning at the three schools in the study.

Students at Railside were achieving well below their peers on content-aligned tests and open-ended project assessments when entering high school (Boaler & Staples, 2008). After one year of instruction the achievement of students at Railside had caught up to the other two schools on their performance on these assessments, and after two years they significantly outperformed the other two schools. The cognitive demands placed on students and maintained by teachers during problem solving at Railside was an important difference between Railside and the other two schools: “Importantly the support that teachers gave to students did not serve to reduce the cognitive demand of the work, even when students were showing signs of frustration” (p. 635). Other studies have identified the issues of maintaining a sustained press for explanation and justification, and not lowering the cognitive demand of the task by constraining the problem space or taking over the difficult aspects of the task as being crucial to students’ engagement at a high level during implementation (Stein et al., 1996; Henningsen and Stein, 1997).

In both of Boaler’s studies (1998; Boaler & Staples, 2008) she was able to identify unique schools, i.e., Phoenix Park and Railside, which had adopted novel instructional practices that had clear positive consequences for student learning. While the tasks that students worked on in these schools were an important element of the learning environment, both studies identify other school-level factors which were likely to have influenced the observed learning outcomes. This raises the question of the importance of tasks in relation to other elements of the learning environment. A partial answer to that question may be given by Huntley and her colleagues (Huntley, Rasmussen, Villarubi, Sangtong, & Fey, 2000), who studied the effect on student learning of implementing two different types of curricula at multiple school sites. The Core-Plus Mathematics Project (CPMP) curriculum was developed to align with the principles of mathematical learning described above as valued by the mathematics education reform and embodied by the Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics, 1989). Their study was intended to determine how students' learning of algebra differed between students who used the CPMP curriculum and those who used a variety of more traditional curricula. CPMP is an integrated problem-based curriculum, with seven units covering algebra and functions over the first three years of the curriculum, while the comparison group students completed the traditional Algebra I – Geometry – Algebra II sequence. A total of approximately 300 students in each condition (CPMP vs. traditional) at six different sites were administered three assessments. Part 1 of the assessment consisted of an individual multi-step real-life context problem in which graphing calculators were allowed; Part 2 included a small number of short, closed problems which emphasized by-hand symbol manipulation in a purely mathematical context without the aid of a graphing calculator; and Part 3 was a single open-ended problem set in a real-life context that students worked on with a partner and with access to a graphing calculator. The CPMP students significantly outperformed the control group on Parts 1 and 3 of the assessment at five of the six sites. Conversely, the achievement of students using more traditional curricula was significantly better than the CPMP students on Part 2 of the assessment, including all six of the individual sites. In general, the CPMP students were better at representing problem situations and interpreting models in terms of a situation, and translating between representations. Not surprisingly, these are the types of activities that students encountered in the CPMP curriculum, using open ended multi-day tasks, group work, and graphing calculators. Conversely, the control group spent two years learning and practicing manual procedures for manipulating expressions and solving equations in purely mathematical contexts.

It is important to not overstate the significance of the results of this study. Given that it was not a strict experimental design, many factors which may influence student learning were not controlled for. It is likely that schools or districts which chose to implement the CPMP curriculum had some important characteristics in common. Furthermore, a curriculum is more than just a collection of tasks, and implementation of both the CPMP and traditional curricula are likely to have varied across schools. Nonetheless, this study demonstrates that the kinds of activities that students participate in impact what they know and can do, and have implications for their learning that goes beyond classroom or school level factors.

The studies discussed above suggest that the types of thinking afforded by different tasks influence student learning. Cognitive demand is an important dimension of mathematical tasks which helps to explain how they mediate student learning, and, as Stigler and Hiebert (2004) and Boaler and Staples (2008) note, how students actually engage with tasks is an important element of that mediation. A framework for assessing the cognitive demands of a mathematical task, how it may change during implementation, and classroom factors which influence that process are described below, and research associated with it is discussed.

2.5.2 Cognitive Demand of Mathematical Tasks

In order to connect the discussion of the use of technology with the use of mathematical tasks for instruction, the frameworks for understanding how mathematical tasks mediate students' learning are described below. The idea of the cognitive demand of mathematical tasks and the *Mathematical Tasks Framework* (Stein et al., 2009) have been developed as frameworks to help describe, quantify, and analyze the type of thinking that students engage in during instruction, and if and how the thinking required changes during implementation. Stein and her colleagues define a mathematical task as "a classroom activity, the purpose of which is to focus students' attention on a particular mathematical idea" (Stein et al., 1996, p. 460). Given this broad definition, it is clear that mathematical tasks are a part of every mathematics classroom. Furthermore, tasks have the potential to influence students' learning by the opportunities for mathematical thinking that they afford.

The mathematical tasks in which students engage have been found to correlate with students' learning (Hiebert & Wearne, 1993; Stein & Lane, 1996). In particular, the ability to think and reason at high levels, problem solve, make conceptual connections, use representations, notice patterns, and make conjectures, has been associated with the use of instructional mathematical tasks which require a high level of cognitive demand (Stein & Lane, 1996). The cognitive demand of a mathematical task refers to the kinds of cognitive processes required of students who engage with it. High level cognitive demand task are those which require students to think and reason in complex or non-algorithmic ways characteristic of *doing mathematics*, or tasks that focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas, referred to as *procedures with connections* (Stein et al., 2009).

An example of a *doing mathematics* task which utilizes technology is the construction of a square within a dynamic geometry software (DGS) using Euclid's postulates. This activity provides students with the opportunity to think about the mathematical properties of a square, and to use technological tools to construct a figure with such properties. Scher (2005) notes that while attempting to construct a square using DGS, students might create a drawing, in which they simply draw an object which "looks like" a square. This would be analogous to using a ruler and protractor in a pencil and paper environment to ensure that the figure has four equal sides and four right angles. However, a ruler and protractor are not tools which reflect Euclid's postulates, which is why these constructions are also referred to as compass and straightedge constructions. The affordances of a DGS provide a way for students to monitor their progress by "dragging" their construction to determine if it maintains the defining properties of a square. Thus, a student who had "drawn" a square as described above could dynamically drag their figure while measuring the side lengths and angles to see that the figure does not retains its "squareness," as in Figure 2.6.

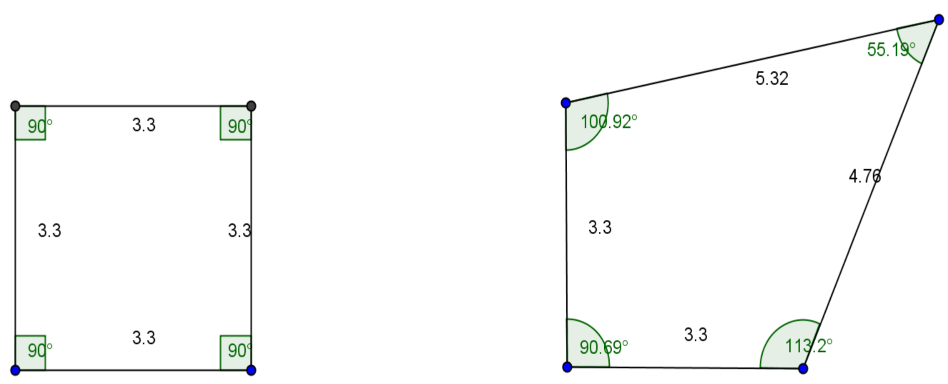


Figure 2.6: A "drawing" looks like a square, but "dragging" reveals that it does not have any properties built in.

Scher (2005) notes that while attempting to construct a square, students may also create an object that is underconstrained, or overconstrained. Quadrilaterals that are underconstrained may possess some of the properties of a square, such as four right angles (a rectangle), or two pairs of parallel sides, but do not possess all the properties proper to a square. A rhombus or trapezoid is another example of a quadrilateral which is underconstrained. A square would be considered overconstrained if it possessed the defining properties of a square but could not be resized. Such a figure is a square with a fixed side length, a particular square. Since having a certain side length is not part of the definition of a square, the figure is considered overconstrained. The most challenging aspect of this task is thinking about how to build these properties into the figure so that the figure retains them while being resized, rotated, or translated in the plane, and thus is appropriately constrained. This may lead students to consider both the minimal properties needed to determine a square, as well as equivalent conditions. Indeed, Scher demonstrates that while the classifications of drawing, underconstrained, overconstrained, and appropriately constrained are helpful in evaluating students' work, the open-ended nature of the task allows for multiple strategies by students with some familiarity with the technology, many of which do not fall neatly into one of these categories.

An important part of the choice to have students make this construction using a DGS is that construction of a square in a static medium, for example using paper, pencil, straightedge and compass, may actually obscure the mathematical significance of this activity for a student. The differences between drawing, underconstrained, overconstrained, and appropriately constrained may not be apparent since all look the same and cannot be manipulated to reveal the properties that are built into the figure. As Kaput (1992) notes, "One very important aspect of mathematical thinking is the abstraction of invariance. But, of course, to recognize invariance –

to see what stays the same – one must have variation" (p. 525). The freedom to make decisions about how to use the technology to accomplish the task, the ability to use it to monitor their own work, and the possibility of multiple solution strategies are key aspects of this task which make it a *doing mathematics* task.

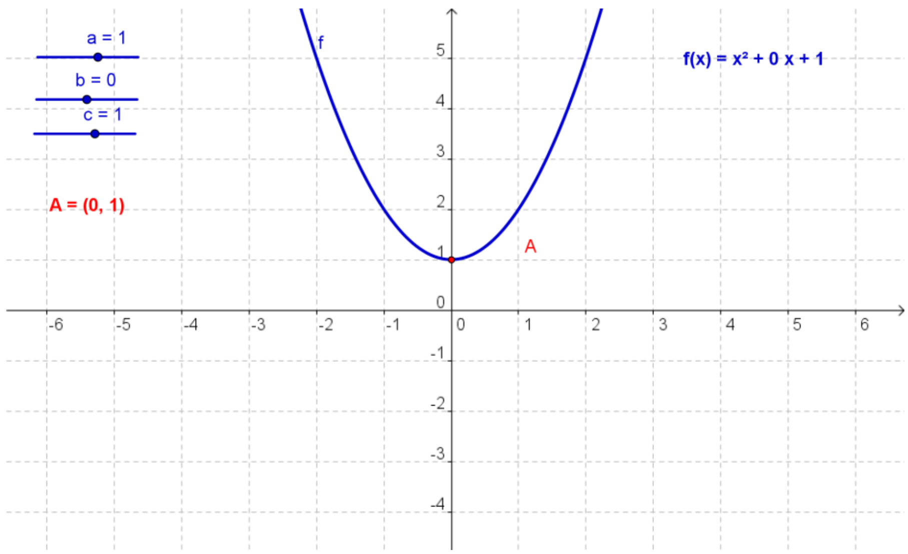


Figure 2.7: The parabola representing a quadratic function with parameters a, b, and c controlled by "sliders".

An example of a *procedures with connections* (Stein et al., 2009) task using technology is given in Figure 2.7, where the sliders for a, b, and c correspond to the parameters of the quadratic function . Students can vary these parameters individually to see how each affects the graph of the function in general, and in particular the trajectory of the vertex. The vertex is constructed such that a visible "trace" is left behind on the graph as it moves. Students are led to notice that manipulating b while leaving a and c fixed traces out a parabola. Students are then instructed to fix the sliders for parameters a and c such that a = 1 and c = 1, adjust the slider for parameter b, and find the equation of the shape traced out by the vertex, which is the dotted parabola facing downwards in Figure 2.8. There are a number of ways students may do this, including guess and check, identifying points on this graphs and solving a system of equations, or by using the symmetry of the graph, e.g., the y-intercept is (0, 1) and f(-1) = f(1) = 0. Students are able to check their work by using the DGS to graph the function they've produced to determine if it coincides with the parabola traced out by the vertex (note the thin black curve through the dotted parabola in Figure 2.9).

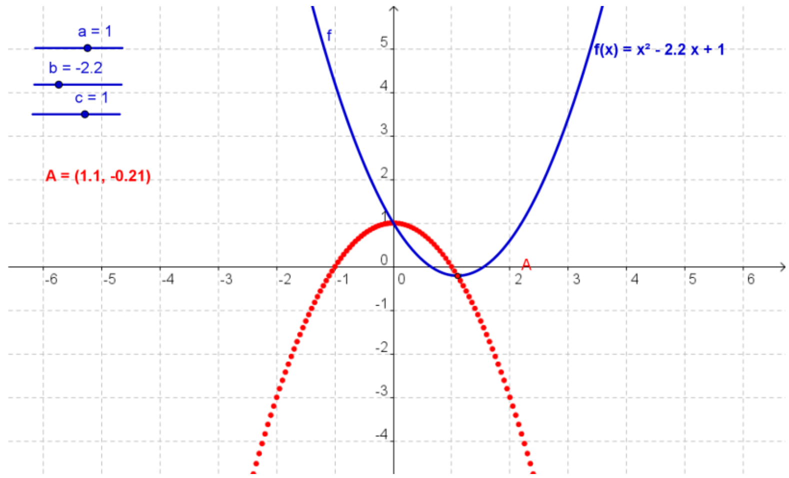


Figure 2.8: The shape traced out by the vertex of the parabola representing f(x) as the parameter b is changed.

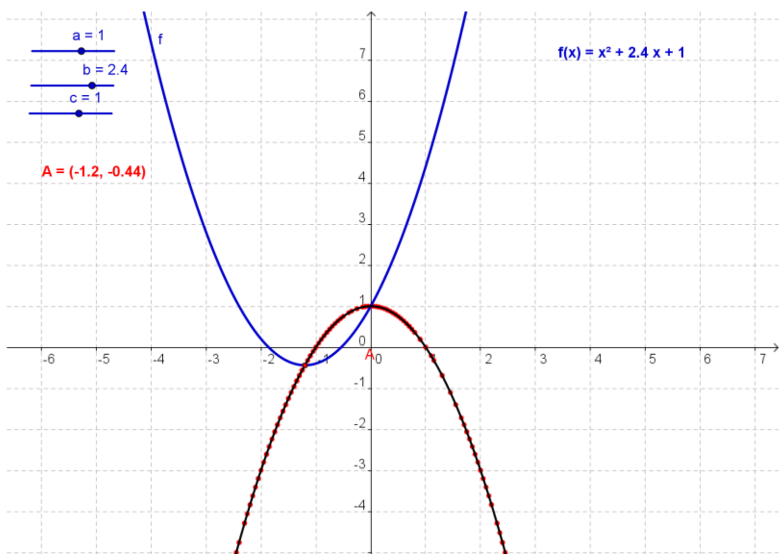


Figure 2.9: Students can check their work by graphing the function they think is traced out in Figure 2.8.

Students are asked to repeat the process by fixing a = -1 and c = -2, and changing b, and then to generalize their findings by determining the equation of the shape traced out by the vertex when adjusting b for any value of a and c. The characteristics of this task which classify it as *procedures with connections* are that students are given procedures, albeit technical rather than mathematical, which constrain their actions and specify precisely what they are to investigate and how, and scaffolds their generalization. However, students are required to make connections between representations, and in particular this task calls for students to describe graphs symbolically and generalize their findings.

Tasks which require only a low level of cognitive demand for their successful completion include those which involve memorization, or the use of procedures without any connection to meaning or concepts, i.e. *procedures without connections* (Stein et al., 2009). An example of a *memorization* task which makes use of technology is the use of an applet for a speed quiz of multiplication facts. A single multiplication problem is displayed in which both factors are natural numbers less than or equal to twelve, and students are to type in their answer in a blank box after an equal sign. Students press a button for feedback, which displays a message that the answer is correct or otherwise gives the correct answer. Finally, students click on a button for another problem. The quiz is timed for one minute, and students take the quiz repeatedly, recording the number of problems they answer correctly in a minute, and reporting to the teacher the largest number of problems they got right over all their attempts. The applet makes no connections to alternative representations of multiplication, and requires only recall of facts.

An example of a *procedures without connections* (Stein et al., 2009) task which makes use of technology involves students solving proportions of the form — − while using a calculator for arithmetic computations, namely multiplication and division. This task would be considered *procedures without connections* if students had already been taught a procedure for solving these types of problems, were not required to make connections to other representations, including a non-mathematical context which might be mathematically modeled by the proportion, and were not required to set up the proportion[2]. This example highlights the role of prior knowledge in assessing the cognitive demand of the task, especially when distinguishing between *doing mathematics* and *procedures without connections* tasks. The same task might be considered *doing mathematics* if students had received no prior instruction about how to solve these types of problems, but were asked to "invent" strategies based on their prior knowledge and generalize their strategies. A description of the critical features of all four types of tasks is given in APPENDIX A, The Task Analysis Guide (Stein et al., 2009).

2.5.3 The *Mathematical Tasks Framework*

Researchers have noted that the cognitive demand of instructional tasks as found in curricular materials may change as they are implemented. There are two phases in which this can occur, and thus impact student learning: set-up and implementation[3] (Stein et al., 2009). This process by which instructional tasks that appear in curricular materials pass through these phases of implementation to impact student learning is called the *Mathematical Tasks Framework*, and is depicted in Figure 2.10. The task set-up phase is the task as it is announced to students in class initiating their work on it. The set up may be quite elaborate, including detailed explanations, rubrics, and exemplars, grouping and accountability strategies, and expectations for how students

[2] Setting up the proportion would not guarantee that the task would then be considered procedures with connections as students might blindly follow a procedure to do this as well.

[3] Some researchers have recently suggested that there may exist an additional phase of implementation after "tasks as implemented by students" which they have termed the "wrap up" phase, during which the task as implemented is discussed in a whole class setting (Otten, 2010) However, research connecting this phase of implementation to student learning has yet to be conducted and thus this modified *Mathematical Tasks Framework* is not employed in the current study.

are to report their work; or the set up may be as simple as handing out a worksheet. A primary purpose of the set up phase is for the teacher to communicate his expectations to the students about how they are to engage with the task. During set up the cognitive demand is measured in terms of the potential best case scenario for the type of thinking students could engage in while working on the task as announced by the teacher.

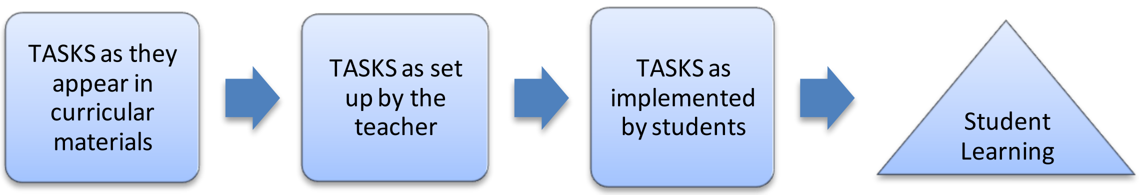


Figure 2.10: The Mathematical Tasks Framework (Stein et al., 2009).

For example, students might be instructed to use Excel to find a rectangular prism with the same volume as a cylindrical pop can which minimizes surface area. Students take measurements of the can and 12-pack box dimensions for two brands of pop, and proceed to compute the surface area and volume of one of the cans using a scientific calculator. The task is set in the context of trying to find a shape for a pop can that minimizes the cost of materials, including sheet metal for the can (surface area) and cardboard for the box (surface area). Students work in groups of two or three with one laptop and an Excel spreadsheet with columns for height, length, width, surface area, and volume. The expectations are that students would use a guess and check strategy to determine dimensions of a rectangular prism which has the same volume as the can but with the smallest surface area[4]. Students are expected to create a formula

[4] Considered over all possible rectangular prisms. In fact, it is not possible to create a rectangular prism with the same volume and smaller surface area than a cylinder.

in Excel for the surface area and volume columns in terms of the height, length, and width cells, and then to vary those three dimensions using a successive guess and check strategy while keeping the volume close to the volume of the cylindrical can, and notice what happens to the surface area as they change these dimensions. The purpose of using Excel was to support the guess and check strategy by automating the surface area and volume computations and providing students with feedback about the effect of their guesses. Ideally, students would use this feedback to notice that as the three dimensions get closer to the same value, i.e., as the rectangular prism gets closer to being a cube, the surface area is minimized.

The set up of the task would include the teacher's description for what students are to do, as outlined above, as well as the resources that students are given to work on the task. For example, are they given a handout or worksheet which outlines what they are to do and on which they are to document their work? Are they given a spreadsheet with the correct formulas already inputted, or is this the responsibility of the students? Furthermore, part of the set up is communicating expectations about the product that students are to create. For example, is each group expected to make a conjecture about the dimensions of the rectangular prism with the smallest surface area, i.e., that it is a cube? Or are they simply to report the dimensions of the rectangular prism which they found to have the smallest surface area? These are important elements of the set up of the task which have the potential to put students in the right cognitive space to engage productively with the task.

Due to a number of classroom-based factors, however, the type of thinking that students actually do may or may not realize the potential of the task, a key finding of the 1999 TIMSS Video Study (Stigler & Hiebert, 2004). Thus, the cognitive demand of the task as implemented may differ from that of the task as set up. This can have important implications for student learning. Indeed, it is quite common for tasks that are set up at a high level of cognitive demand to decline to a low level during implementation, while very rarely, if ever, does the cognitive demand of a low level task increase during implementation (Boston & M.S. Smith, 2009; Henningsen & Stein, 1997; Stein et al., 2009).

For example, in Doerr and Zangor's (2000) qualitative case study of an experienced teacher using graphing calculators with her precalculus classes, the teacher regularly allowed students to utilize the graphing calculator to model problem situations, with the goal of having students understand how their mathematical model represented the situation. However, a few of her students regularly bypassed this goal by using regression and curve fitting functions of the calculator based on numerical data, resulting in models that they could not explain or interpret in terms of the problem situation. As the cognitive demand framework is a classroom level indicator, i.e., it assesses the type of thinking in which a majority of students were engaged for a majority of the task, a majority of students would need to engage in such behavior for this task to be classified as *procedures without connections* during implementation. Nonetheless, it demonstrates how students might engage with a task during implementation at a low level even if the task was set up at a high level.

Stein and Lane (1996) studied the connection between the types of tasks that students encounter during instruction and the type of mathematical learning that they exhibit. They studied instruction and learning of students at four schools participating in the QUASAR project (Quantitative Understanding: Amplifying Student Achievement and Reasoning), using schools as the unit of analysis. They found that the cognitive demand of mathematical tasks are important since the types of thinking that students do during instruction seem to influence students' learning to engage in important mathematical behaviors which involve complex thinking.

In evaluating the types of tasks that were set up and implemented at the four schools, data were collected about the tasks that teachers used, with special attention paid to the features of the tasks and the type of thinking required by the tasks as set up by the teacher, and the type of thinking that students actually engaged in while working on the task. Task features include whether or not the task was capable of being solved in multiple ways, entailed the use of multiple representations, and required mathematical explanations. They collected task artifacts and observed and videotaped lessons in order to make these determinations. To assess students' learning, an instrument was created to measure students' abilities to think, reason, problem solve, and communicate mathematically on open ended tasks. They found the greatest gains in students' learning on the assessment at School A, moderate and comparable gains at Schools B and C, and the smallest gains at School D. These results reflected the degree to which teachers and students at each of these schools were observed to set up and implement high level cognitive demand tasks with the task features described above. That is, of the four schools in their study, teachers at School A were observed to both set up the most tasks at a high level of cognitive demand and maintain a requirement for high level thinking during implementation, and to use tasks which included multiple solution strategies, representations, and explanations. The instructional tasks used at schools B and C had similar high level characteristics during set up as those used at School A, but the demands of the tasks degenerated to low levels more frequently than at School A. The sample of tasks used at School D was an outlier in terms of their cognitive demand, the majority of which were set up and implemented at a low level. Furthermore, when tasks at School D were observed to be set up at a high level, the thinking requirement almost always decreased to a low level during implementation.

Another important finding of this study is that students at schools B and C still experienced greater learning gains than students at School D in spite of the fact that the level of thinking was frequently found to decrease to a low level during implementation. Stein and Lane (1996) conclude:

> [I]t appears in this study that exposure to instructional tasks that are set up to be challenging but that are implemented at lower cognitive levels confers greater benefit to students than does exposure to tasks that emphasize lower levels of thinking from the start. (p. 74)

Perhaps due to the implicit message that tasks carry about what mathematics is and what it means to do it, tasks set up at a high level may influence students' beliefs in ways that may affect their problem solving, even if they fail to implement such tasks at a high level. While school level factors may account for some of the differences in learning gains reported by this study, these results nonetheless represent empirical evidence of the connection between the types of tasks in which students engage and the type of mathematical learning that they do. That is, students learn what they have the opportunity to learn, and high level tasks with certain features seem to be effective at promoting the mathematical behaviors described by the Process Standards (National Council of Teachers of Mathematics, 2000), allowing students to achieve conceptual understanding of important content, and gain the ability to use that knowledge in a variety of situations.

These results are crucial to the design of the present study as they point to an aspect of classroom instruction which correlates to student learning, and therefore an important lens through which to examine the use of technology for mathematics instruction. That is, different types of tasks require different types of thinking, which in turn lead to different types of learning.

While faithful implementation of high level tasks has important implications for student learning (Boaler & Staples, 2008; Stein & Lane, 1996; Stigler & Hiebert, 2004), exposure to high level tasks which degenerate during implementation still seems to be more valuable than not being exposed to them at all. These results lead to the hypothesis that the ways in which technological tools are used to support high and low level thinking requirements may vary as well. Exploring how the role and characteristics of the use of technology differs in high and low level tasks is a primary purpose of the present study. The present study takes as a starting point the importance of mathematical tasks for student learning, and seeks to investigate the use of technology therein. Measuring the effect of the use of technology on student learning is not a purpose of this study.

2.5.4 Classroom-based factors associated with decline and maintenance of task demand

Stein and her colleagues (Henningsen & Stein, 1997; Stein et al., 1996) found that certain classroom factors were associated with the maintenance of the cognitive demand of a high level task during implementation, while other factors were present when students' thinking degenerated to a low level. These factors are related to the ways in which access to the challenging aspects of a task with high level cognitive demands is provided to students without removing those requirements.

An important dynamic that emerged in the analysis of the implementation of instructional tasks set up at a high level was that some tasks declined in cognitive demand during implementation, but not to *memorization* or *procedures without connections*. Two other possibilities emerged when analyzing students' thinking during implementation. In one case, students' activity within the task became random or aimless, resulting in a lack of progress and a failure to make relevant connections. This type of activity was termed unsystematic exploration

(Henningsen and Stein, 1997; Stein et al., 1996), and is unique to the implementation phase of the *Mathematical Tasks Framework*. That is, while the seeds of such activity may be planted during the set up phase, unsystematic exploration specifically refers to how students implement a task. In the optimization example described above, the fact that students had no way to determine if indeed they had found the rectangular prism with smallest surface area, or even if such a thing existed or was unique, seemed to lead some students to engage in unsystematic exploration, randomly plugging in numbers to their Excel spreadsheet. Others made more systematic progress and observed that the surface area decreased as the three dimensions of height, width, and length seemed to be getting closer to the same value. Unsystematic exploration is considered a decline in cognitive demand despite the fact that students' behaviors do not become procedural or simply a matter of recall, and has been found to be most often associated with tasks that are set up as *doing mathematics* (Henningsen & Stein, 1997; Stein et al., 1996).

A second way in which the cognitive demand during implementation was observed to decline to something besides *procedures without connections* or *memorization* is that students' activity was non-mathematical in nature (Stein et al., 2009). This includes the possibility of students being off task as well as focusing on non-mathematical aspects of the task. For example, students might be talking about nothing to do with mathematics or the task at hand, or may be texting on their phones or sleeping. On the other hand, students may be engaged with some aspect of the task which is not necessarily mathematical in nature, such as creating a display or summary of results on poster board to present to the class or making a computer generated representation of data or results. While there are inherently mathematical elements to these activities, if the students' focus is more on the aesthetical aspects then the task may be considered to have no mathematical activity during implementation.

The factors most commonly associated with the decline and maintenance of the cognitive demand of mathematical tasks during implementation are summarized in APPENDIX C. It is important to note that these factors are not mutually exclusive. Once a determination had been made with regard to whether or not the cognitive demand of a task deemed to be high level during set up had declined or been maintained during implementation, coders chose as many factors as seemed relevant to this outcome from the list in APPENDIX C. That is, if the demand of the task had been judged to decline during implementation, coders chose factors from the list on the left hand side of **Error! Reference source not found.**; if it was maintained, factors from the right hand side of APPENDIX C were coded. An interesting pattern that emerged was that, on average, more factors were present when the task demand was maintained than when it declined (Stein et al., 1996). Of the tasks that were set up and implemented at a high level, an average of four of these factors were present in each task, while for tasks that declined during implementation, an average of only 2.5 factors were present per task (Stein et al, 1996). Thus, it seems that a difficulty in maintaining the cognitive demand of a task during implementation can be likened to following a recipe in the following sense: most of the ingredients (factors) must be included for it to turn out well, while missing one or two key ingredients is sufficient for its failure. These factors may be especially important in relation to the use of technology by students while working on a task or problem, as there may be many, often unanticipated ways in which the use of technological tools can support or hinder students' thinking.

Referring again to the optimization problem example described above, the factors associated with the decline of this task might be described in terms of classroom management issues, the inappropriateness of the task for this group of students, and a lack of accountability. By putting students in groups of two or three with a single laptop, almost two-thirds of the students did not have a laptop at their disposal. The feedback provided by the spreadsheet with respect to the dimensions that were being inputted, whether or not these dimensions produced the same volume as the cylindrical can, and whether or not students produced a rectangular prism with a smaller surface area than any previous guesses, was essential to the high level thinking opportunities afforded by this task. Most students who did not have direct access to a laptop doodled on their paper, texted on their cell phones, or simply sat and stared off into space. This is perhaps counterintuitive in the sense that one might expect students sharing a laptop to work better as a group than if each had their own. Thus, it may be that this task was inappropriate for this group of students in the sense that they were unaccustomed to working in groups, a factor independent of the number of laptops in each group. The lack of accountability or direction for students working in groups may have contributed to this phenomenon as well. Specific expectations, including outcomes and accountability, are needed in order to ensure students' implementation of the task in ways that align with the teacher's goal(s) for the task. Furthermore, the task may have been inappropriate for these students in the sense that students had no way to know how many iterations of guess and check were sufficient. Asking students to make a conjecture about the dimensions of the rectangular prism with the smallest surface area, for example, may have supported students' engagement by alerting them to the fact that they should notice a pattern emerging, and motivating them to continue with the task until they detected one.

It is important to note that factors associated with high level engagement were also present in this task, such as providing students with a way to monitor their own progress through the feedback provided by the spreadsheet, and giving students an appropriate amount of time to explore[5]. However, these factors would not be coded since the task was judged to have declined during implementation, thus factors associated with decline are considered to be the most influential.

2.6 ISSUES RELATED TO STUDENTS' USE OF TECHNOLOGY

The *Mathematical Tasks Framework* provides a frame for evaluating student opportunity for thinking mathematically, and the teacher's role in shaping it. In the present study it also serves as a conceptual framework for understanding teacher and student interactions with tasks that utilize technology. There are many dimensions along which to look at teachers' and students' use of technology for teaching and learning mathematics. The present investigation considers the use of technological tools by teachers and students in terms of their potential influence on the cognitive demand of the tasks during the various phases of enactment. The presence of technology as a cognitive tool for mathematical activity is hypothesized to play a role in the cognitive demand of mathematical tasks during implementation due to the ways in which it supports and constrains students' mathematical thinking. Research on students' use of technology for mathematical thinking and learning has identified student behaviors which support this idea, and allow for specific hypotheses to be formulated.

This section presents a review of the literature on issues identified as being associated with students' use of technology for the learning of mathematics that may be related to the cognitive demand of the tasks with which they engage. The studies reviewed in this section

[5] Too much time to explore has been associated with unsystematic exploration (Henningsen & Stein, 1997), which was noticed in this task as well. However, only a minority of students engaged in such behaviors; for the majority of students, the amount of time seemed appropriate.

generally fall into areas highlighted in Figure 2.11. As discussed above, however, since most of these studies take place in the context of classrooms, teachers' influences are present and often shape students' interactions with the technology, but are not the focus of the study.

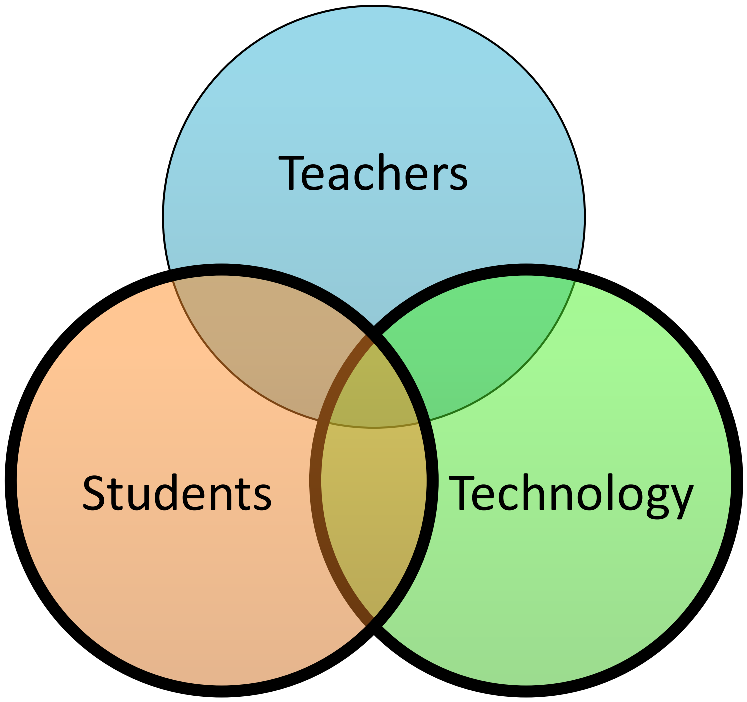


Figure 2.11: The present section reviews literature on the interactions of student and technology within mathematical tasks.

2.6.1 Student Autonomy

One issue related to students' work in technologically enhanced environments that has been investigated in the literature is the issue of students' autonomy while working on tasks. McGraw and Grant (2005) distinguish between what they call Type 1 and Type 2 lessons. In Type 1 lessons students follow a set of specific directions which determine both what students will investigate and how they are to investigate it. Student activity is constrained by the structure of the lesson in such a way that students' explorations will lead to the same conclusions. While students have the opportunity to notice patterns and make conjectures in Type 1 lessons, there is little room for decision making – students are generally "funneled" toward the same conjectures. Although not essential to their conception of a Type 1 lesson, teachers can extend students' thinking from what was noticed to why they think it occurs. An example of a Type 1 lesson given by McGraw and Grant is that of giving students explicit instructions about how to create a parallelogram in a dynamic geometry environment, having them use the software to take specific measurements of side lengths and angle measures, dragging the figure dynamically, and then recording what they notice about these measurements and making conjectures about the properties of parallelograms. While this Type 1 lesson might scaffold students' use of the technology, especially students unfamiliar with it, the authors note that these types of lessons have a tendency to shift students' attention away from the mathematics they were intended to investigate to carefully following directions. Type 2 lessons, by contrast, are much more open-ended, prompting students to make decisions about what is to be investigated and how. For example, the above lesson could be restructured to allow students to manipulate a quadrilateral (not necessarily a parallelogram) and make conjectures about its properties, and explore which of those conjectured properties might hold for other quadrilaterals and why. The key difference in this lesson is that students make the decisions about what they will investigate and how, e.g., which quadrilaterals they will explore and which measurements will help them to do so. By allowing students to make these types of decisions for themselves, students are likely to make different conjectures and come to different conclusions, creating a need to communicate their reasoning to each other and the teacher and thus creating the potential for a rich classroom discussion. While the authors believe that Type 1 lessons can be productive, they note the advantages of using Type 2 lessons: "[A]llowing students to make some decisions about what to do and how to do it helped create richer learning experiences for [students] and also helped create more manageable classroom environments for us" (p. 316).

Researchers elsewhere have made a distinction between exploratory and expressive activity (Zbiek et al., 2007), drawing on and generalizing the distinction made between exploratory and expressive mathematical models (Bliss & Ogborn, 1989; Doerr & Pratt, 2008). Exploratory and expressive activity seems to generally align with the Type1 and Type 2 lessons, respectively, as described above. An exploratory model is one created by someone else (generally an expert) that students engage with in order to investigate a problem, while an expressive model is one that students build themselves. Zbiek and her colleagues extend this distinction to encompass other types of activity besides mathematical modeling that students may engage in while working with technological tools: "[W]hen students are given a procedure to carry out, they are engaging in exploratory activity; however, when students decide which procedures to use they are engaging in expressive activity" (p. 1181). They note that both exploratory and expressive activity can take place under the broader heading of discovery learning, and that each refers to two ends of a continuum with respect to student autonomy within such a learning environment, articulated in their ability to make decisions about what to investigate and how to use technological tools in doing so. In general, if a process or procedure for investigating a specific idea is given or suggested, the activity is considered exploratory; if students need to make decisions about what to investigate and how, the activity is considered expressive.

The issue of autonomy may have important implications for understanding students' thinking: "[D]ifferent insights into learning result from observing what occurs when one does what one is directed to do with a tool as opposed to when one initiates what is to be done with the tool" (Zbiek et al., 2007, p. 1181). As exploratory activity might be generally associated with *procedures with connections* tasks, and expressive activity more like *doing mathematics* tasks, the implications of engaging with each type of activity may not only provide different insights to learning, but different learning.

While expressive activity may embody more completely the principles of reform oriented use of technology as described by Heid (1997), implementing such activity poses pedagogical problems as well. In particular, when students are given the type of freedom characteristic of expressive activity, they may unintentionally (or intentionally) avoid the mathematical goal of the activity they are engaged in.

> Because many cognitive tools offer to students such a wide variety of approaches to solving problems, students might not encounter, in the course of their explorations of a problem with the tool, the particular mathematical ideas that were identified as goals by their teacher or by the developers of the curriculum materials. (Zbiek et al., 2007, p. 1182).

Hoyles and Noss (1992) refer to this tension between giving students the freedom to make decisions about when and how to use technology for exploration and the desire for them to encounter certain mathematical ideas as the "play paradox." They provide an example from their own work with students using computers to investigate the concepts of ratio and proportion using programs created using Logo. They gave students a picture of a pentagon in the shape of a house with side lengths labeled, and ask students to use the computer to produce a "house" bigger than the given house but in proportion to it. While the possibility of students adding a fixed number to the length of each side of the house was anticipated, the researchers, serving as the teachers of these students, expected that the results of such an action, i.e., a house which was not a closed figure, would prompt students to move toward a multiplicative strategy for solving the problem. Instead, some students added the necessary amount to the short side to close the figure, interpreting the goal of the problem to be to produce a larger house that "looks like" the original house. While students had previously engaged in an activity and had a class discussion about what it means for two figures to be "in proportion" to each other, some students nonetheless used the tools at their disposal to accomplish their interpretation of the goal of the activity.

Another example given by Dugdale (2008) is related to her development of a computer game called Green Globs, in which students are given points, i.e. Green Globs, in the Cartesian plane, and asked to find a function which passes through as many of the points as possible. Students are able to use an automatic graphing technology in order to check their work and make adjustments accordingly. However, some students sidestepped the goal of the activity, namely to think about the graphs of various functions and how to transform them, by graphing a function such as which would cover a large proportion of the graphing area and thus ensure that it hits every "glob". While students used mathematical reasoning in addressing the goal of the game, their strategy avoided the specific mathematical ideas that students were intended to engage with while playing the game.

Another case of this type of behavior is reported by Doerr and Zangor (2000). The teacher regularly allowed students to utilize the graphing calculator to model problem situations, with the goal of having students understand how their mathematical model represented the situation. Some students regularly bypassed the connections they were intended to make while working on the task by using regression and curve fitting functions of the calculator based on numerical data, resulting in models that they could not explain or interpret in terms of the problem situation. Thus, the use of technology in this case provided students with a strategy which allowed them to avoid the mathematical behaviors that they were intended to engage in during these types of tasks.

Hoyles and Noss (1992) resolve the play paradox by structuring their interventions to encourage students to reflect on the task with which they are engaged, and in particular with the mathematical goals of the task. "Our strategy for achieving this was not by direct questioning but rather by suggesting an activity during which we anticipated the issues would have to be confronted" (p. 44). For example, the teacher in Doerr and Zangor's (2000) study asked students who had used numerical procedures on the calculator to explain the meaning of their answer in terms of the context of the problem statement. Hoyles and Noss distinguish between the practices of the computer environment in which students worked to achieve certain goals with a degree of autonomy, and that of formal mathematics. As students are unfamiliar with the mathematics that the teacher intends for them to encounter, they often fail to make the connection between their work on the computer and the mathematical ideas embedded in the activity. While the computer supports goal-oriented behavior based on prior knowledge, it also provides strategies which allow for the goal to be accomplished in unanticipated ways. The challenge for the teacher is to suggest activities with technology that will provide the type of feedback needed for students to engage with the mathematical goals of the task. Another pedagogical strategy for helping students engage with the intended mathematics is small group and whole class discussions in which students have the opportunity to consider other solution strategies employed by classmates which get at the intended mathematical goal of the task.

The play paradox seems most likely to become an issue when students are given more freedom to make decisions about when and how to use technology to work on an open ended task. Within the cognitive demand framework, the play paradox would likely be associated with *doing mathematics* tasks due to their open-ended nature and the requirement for non-algorithmic thinking. In particular, the issues related to the play paradox seem related to the student behavior reported by Henningsen and Stein (1997) in which high level cognitive demand tasks degenerated into unsystematic exploration in which "students explored around the edges of significant mathematical ideas but failed to make systematic and sustained progress in developing mathematical strategies or understandings" (p. 532). While students' activity with the computer might very well be structured, and may utilize mathematical ideas in accomplishing the goal of the task, what the episodes reported in connection with the play paradox have in common with the description of unsystematic exploration is a lack of sustained progress in developing the intended mathematical understandings while engaging in the task. Furthermore, appropriate scaffolding of the task, a factor found to be associated with the implementation of mathematical tasks at a high level, is clearly related to the resolution of the play paradox suggested by Hoyles and Noss (1992). The need to guide students toward the mathematical ideas without taking over the mathematical reasoning for the student is a tension the dynamics of which may become more complex when technological tools are present. In the studies of Stein et al. (1996; Stein & Lane, 1996; Henningsen & Stein, 1997), taking over the difficult aspects of a high level task was the most common way in which teachers were found to lower the demand of a task during implementation (Stein et al., 1996). Given the unique challenges associated with using technology, it is hypothesized that this behavior on the part of teachers may be even more prevalent in a technology enhanced task.

Another response to the play paradox is the use of more Type 1 (exploratory) lessons. The pressure that teachers feel to have students master content in a timely manner, whether perceived or real, may prompt teachers to resolve the play paradox by reducing the amount of autonomy students have when working with technological tools. However, a major criticism of Type 1 (exploratory) tasks by McGraw and Grant (2005) was that they create a situation in which students are more focused on using the technology correctly than on the mathematics they are intended to encounter.

> We observed students spending large portions of their class time attempting to understand and follow our carefully written directions rather than thinking deeply about the mathematics they were investigating. We were equally frustrated by the amount of time we spent during class telling students what to do and how to do it when our hope had been that the technology would help students become the investigators. (p. 308)

Thus, scripting lessons with teacher-produced worksheets has the potential to turn a high level task into a *procedures without connections* task, with the difference being that the procedures are technical rather than mathematical. This may be especially true if the focus of the task becomes using the technology to complete a worksheet, i.e., shifting the focus to correct or complete answers. With respect to student autonomy in using technology tools for exploration it seems as if teachers and students are walking a "high level" tightrope: given too much freedom, students may "fall off" in the direction of unsystematic exploration, and given too much direction students may "fall off" into *procedures without connections*. An interesting (and open) question is whether, and how, a teacher can assist students with using the technology, in either type of activity, without taking over the challenging mathematical aspects of the task.

2.6.2 Mathematical Fidelity

Another construct which influences students' thinking and learning with technological tools has been called mathematical fidelity (Dick, 2008; Zbiek et al., 2007). This refers to the degree to which technological tools faithfully represent mathematical objects and actions. Any concrete representation of abstract mathematical objects will necessarily have limitations and will be a sort of approximation of the objects they are intended to represent. “It is precisely because mathematical objects are not in the tangible world that the differentiation between a mathematical object and its representation is at the heart of the learning process” (Guin & Trouche, 1999, p. 207). Technological tools are no different. Software developers have to make choices about how to represent mathematical objects, choices which are constrained by the numerical precision (number of decimal places) and graphical accuracy (number of pixels) of the hardware within which the software will be used. How students make sense of technological output, feedback, and syntax has important implications for their thinking and learning.

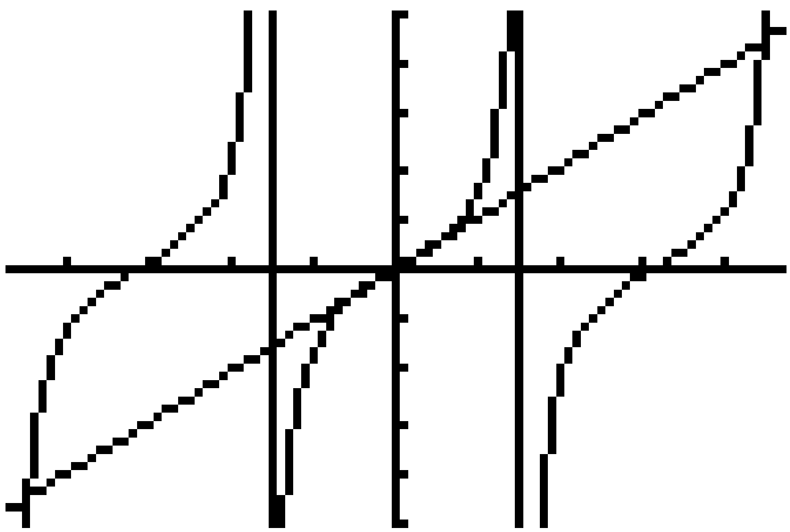


Figure 2.12: The graph of y=x and y=tanx on the same screen.

Guin and Trouche (1999) describe a number of examples of how the graphs produced by graphing calculators can affect students’ thinking and reasoning. When confronted by the graph in Figure 2.12 of the functions and , most students believed that there were only a finite number of solutions to the equation , namely those which are displayed on the graph produced by the calculator, reasoning that beyond a certain point in the domain, the function y = x has no intersections with the graph of . Other students included intersections of y = x with the asymptotes as solutions because they were part of the graph, and other students believed that there were an infinite number of solutions near the origin because the graphs of the two functions seem to coincide in that area of the graph.

In another case reported by Guin and Trouche (1999), two sets of 50 students, one with calculators and one without, were asked to determine . All students not using a calculator drew on their knowledge of the natural logarithm function to answer correctly, while only five students who used a calculator answered correctly. Those students who did not answer correctly inferred from the periodic nature of the graph, which did not appear to be strongly increasing in the graphing window, that the function did not have a limit. These are two different examples of the limitations of the ability of technology to accurately represent mathematical objects. In the first case, the graph is distorted, while in the second case the end behavior of the function is difficult to capture using technology, both graphically and numerically. The inability of technology to faithfully represent these mathematical objects can lead students with fragile mathematical knowledge to second guess themselves. While ideally students will attempt to make sense of their mathematical work, Guin and Trouche report that students' inability to reconcile the information given to them by their calculators with the prior mathematical knowledge became a stumbling block to understanding. For example, they report that when asked to evaluate one student inferred a new "theorem." Originally the student believed that the limit tended to infinity because the coefficient of the highest term of the polynomial function is positive, but modified that response to account for the graphical and numerical data obtained from the calculator, which appeared to be strongly decreasing based on his calculator explorations. Ultimately he decided that if the coefficient of the second highest term is much larger than that of the highest term, then it will determine the end behavior of the function instead. Thus, the mathematical fidelity of technological tools, or lack thereof, exposes issues of mathematical knowledge and authority. Students who lack confidence in their mathematical knowledge or reasoning are likely to interpret confusing or unexpected feedback from technological tools by doubting themselves rather than the tool. Other researchers (Doerr & Zangor, 2000; Schwartz & Hershkowitz, 1999), however, have identified ways in which the issue of mathematical fidelity can be used to deepen students' mathematical understanding.

In Doerr and Zangor's (2000) qualitative case study of a precalculus teacher with considerable experience using graphing calculators for instruction, the teacher was aware of the limitations of graphing calculators in faithfully representing mathematical objects, and believed that these limits provided important opportunities to deepen her students' mathematical knowledge and to help them become more judicious users of technology. The authors report an episode in which students were investigating an exponential decay situation using candies with the letter "m" on one side. Students shake a container holding the candies and remove all those candies which have the "m" facing upward. The class worked together to model the situation using the function . Students understood that due to the discrete nature of the experiment they were engaged in that they would eventually have no candies left. However, they also understood that one can divide by two infinitely without getting zero, so the function which modeled the situation should never reach zero. While investigating the function numerically students realized that for large values of x, the calculator gave a value of zero for the function. A class discussion followed in which the limitations of the calculator were discussed and students concluded that the calculator does not always “tell the truth.” “The students began to see the calculator as a tool that should be checked based on their own understanding of mathematical results” (p. 150).

Schwarz and Hershkowitz (1999) citing Schwartz and Dreyfus (1995), refer to these “partial embodiments of function representations” (p. 369) that are characteristic of multirepresentational software, including graphing calculators, as representatives. For example, two graphs of the same function using different scales, or two tables of function values using different values of the independent variable, are considered different representatives of the same function. Schwarz and Hershkowitz report that students using multirepresentational software in conjunction with a reform oriented curriculum and learning environment developed a more robust conception of function, including the ability to coordinate representatives and recognize representatives of the same function. Thus, the ability to interpret output and representations generated by technological tools requires a certain amount, or perhaps type, of knowledge of the mathematical objects under investigation, and the tools used for investigating these objects. For example, Pierce and Stacey (2002) have described in some detail the algebraic knowledge students need in order to make productive use of a computer algebra system.

These examples raise the question of how students’ mathematical knowledge mediates their use of technological tools, an issue rooted in the limitations of the technology to represent mathematical objects faithfully or according to classroom conventions. Conversely, the use of technological tools mediates students’ mathematical thinking and learning by the mathematics that it makes accessible (or not) via the representations and operations that it affords, and the limitations of those. As Hiebert et al. (1997) point out, students must construct meaning for and with tools, and they do so best when given the opportunity to use the tool. Regardless of the degree of autonomy that students have in using technology for thinking about and learning mathematics, they will inevitably encounter the limits of technology to provide an accurate representation of the mathematics they are engaging.

How students interpret output, at times unexpected or misleading, and "representatives" generated by technological tools, and whether and how the teacher deals with these issues may have consequences for the cognitive demand of the mathematical tasks during implementation. In particular, a factor associated with the decline of a high level cognitive demand task into unsystematic exploration is what Henningsen and Stein (1997) call "inappropriateness of the task for the particular group of students" (p. 537). A reason identified for a task being inappropriate is a lack of prior knowledge. In the case of mathematical fidelity, both a lack of knowledge of the technology, including technological functions and syntax, as well as the mathematical knowledge needed to use the technology productively, make sense of the output, and recognize limitations, could contribute to the inappropriateness of the task when technology is involved. How the teacher deals with these issues during implementation are important to note in relation to the cognitive demand of the task.

2.6.3 Instrumental Genesis

Instrumental genesis refers to the process by which an artifact, such as a calculator, becomes a tool, or instrument, for students' mathematical thinking and learning. While much of this research is developed at a level of detail which goes beyond the scope of the current study, the general idea is useful in understanding student and teacher roles and behavior in a technology enhanced environment, and may be related to how a task is enacted. Instrumental genesis is seen to have two parts: instrumentalisation, in which the student begins to make sense of the artifact as a tool for thinking, and instrumentation, in which the student constructs mathematical meaning with the tool (Guin & Trouche, 1999; Zbiek et al., 2007). For example, Burrill et al. (2002) noted in their meta-analysis of research on handheld graphing devices that numerous studies demonstrated that students who had used this technology during instruction were more likely to use graphical approaches in problem solving. This is an example of instrumentation, as the mathematical meaning that these students had constructed was shaped by the affordances of the technological tools that they used while learning, namely the availability of graphs and graphical strategies. However, in order to use the tool effectively during problem solving, these students would have had to learn what the affordances of the technology are and how they relate to the mathematics at hand. That is, pressing a sequence of buttons must be connected to mathematical actions taken on mathematical objects, such as locating an intersection of the graphs of two functions. Thus, the process of instrumental genesis is a complex dialectic in which mathematical knowledge shapes students' attitude toward and use of technology, and the use of technology shapes their mathematical understanding, as the influence of learner and artifact goes both directions. Indeed, Hoyles, Noss, and Kent (2004) claim that the type of mathematical knowledge constructed with technological tools is different than that constructed without it. From a socio-cultural perspective, technology is an important part of the environment which influences what is learned, and affects the thinking of students accustomed to its use even when it is not present.

Researchers believe that teachers play a crucial role in the instrumental genesis of their students (Guin and Trouche, 1999; Hoyles et al., 2004), i.e. the process of making the technological tools instruments for mathematical thinking and learning. "A key challenge, then, for the integration of technology into classrooms and curricula is to understand and to devise ways to foster the process of instrumental genesis" (Hoyles et al., 2004, p. 314). Studying the process of instrumental genesis and how it is fostered by teachers is beyond the scope of the present investigation. However, a teacher must take into account the relationship her students have with technology, i.e., where they may be in a trajectory of instrumental genesis, when enacting mathematical tasks which make use of technology. Failure to do so would be equivalent to planning the task without considering the necessary prior mathematical knowledge needed to engage with the task.

Guin and Trouche (1999) describe the various work methods that students used while solving problems, described in terms of the types of reasoning that they engaged in and the tools that they made use of. The relevant aspect of this line of research for the present study is the fact that students' experience with using technology, and its interaction with their mathematical knowledge, shapes their use of technological (and other) tools when engaging with mathematical tasks. While this research focuses on students' behaviors, the questions it raises are in relation to how teachers deal with the variety of behaviors that are likely to be present in a technology enhanced task. A teacher will likely consider the prior technical and mathematical knowledge needed to engage with the task, but how aware is she of the variety of ways that students may work on a task with technology? How do teachers set up and implement tasks which utilize technology given these factors? Do they curtail students' autonomy and restrict the task to avoid a variety of work methods? Do they allow enough time for students using various work methods to produce a solution? Given the variety of potential solution strategies, do they shift the focus during implementation to getting the correct answer, or do they press students for justification and explanation? In particular, do students work at the task in ways that are unanticipated by the teacher, and if so, how does the teacher deal with these unanticipated solutions? While these are issues related to students' use of technology in the context of mathematical problem solving, in the context of instructional tasks the teacher becomes an important factor in shaping students' opportunities for high level thinking. In the next section research related to teachers' use of technology for instruction is discussed.

2.7 FACTORS ASSOCIATED WITH TEACHERS' USE OF TECHNOLOGY FOR INSTRUCTION

As the discussion of instrumental genesis and students' work methods above demonstrates, discussions of students' and teachers' use of technology for learning and instruction are often two sides of the same coin. Patterns or issues identified as being associated with students' use of technology raise questions about how teachers may structure instruction to account for these. Likewise, considering issues that research on teachers' use of technology for instruction has identified is likely to have implications for students' use. While this section focuses on the teachers' use of technology for instruction, as depicted in Figure 2.13, it is also discussed in relation to the potential implications for the cognitive demand of mathematical tasks for students and the factors that have been found to be associated with it.

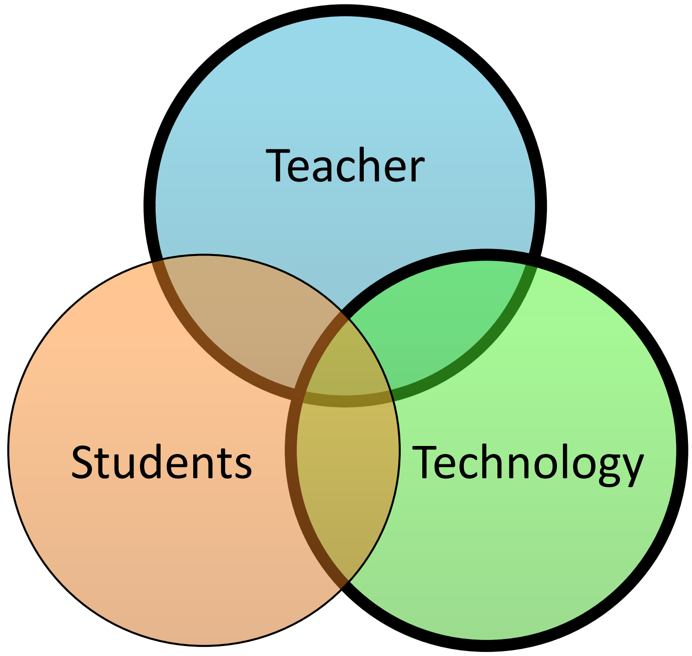


Figure 2.13: The present section reviews literature on the interactions of teachers and technology within mathematical tasks.

2.7.1 Amplifier and Reorganizer Use of Cognitive Technologies

Cognitive technologies, as defined by Pea (1985; 1987), are discussed above; a further distinction that Pea makes within cognitive technologies is between its use as an amplifier or a reorganizer of mental activity (1985; 1987). When technology is used as an amplifier, it performs more efficiently tedious processes that might be done by hand, like computations or the generation of standard representations. In this use of technology, what students do or think about is not changed, but can be done with significantly less time and effort. The use of PowerPoint to display representations of mathematical objects is an example of using a cognitive technology as an amplifier as the representations could just as well be displayed on a blackboard. The use of PowerPoint may allow for more accurate, digitally generated representations which can be embedded prior to its use for instruction, allowing for greater efficiency. However, if the use of a digital technology in this example does not change what students are thinking about with respect to the purpose of presenting these representations during instruction, then such use is precisely what is meant by amplifier.

As a reorganizer, technology has the power to affect students' thinking by providing novel representations which make salient some aspect of a concept which is difficult to make explicit without it, or to allow for students' cognitive focus to shift by offloading tedious or time consuming computations to technological tools or providing feedback to students that they would otherwise not have access to. For example, in answering questions about the relationship between smoking and lung cancer from a data set, there are a number of digital technologies that can quickly produce a scatterplot, an equation and a graph of the line of best fit, and the correlation coefficient. By using technological tools to generate these computations and representations, students are able to focus on interpretation within the context of the larger problem posed, i.e., the relationship between smoking and lung cancer. It also allows students to view these computations and representations within a larger problem solving context, so that they can focus on why they are generating them, rather than how (Greeno & Hall, 1997), i.e., on using representations rather than on producing them. In these ways the technology reorganizes the mathematical activity and thinking of students. The focus is no longer on computation, but rather on choosing and interpreting representations of data; the *what* of the mathematical activity has changed.

It is important to note that while a cognitive technology is regarded as such due to the affordances of the technology, its classification as an amplifier or reorganizer is based on how the technology is used. As the use of a tool depends on the user and not on the tool, the same cognitive technology may be used as either an amplifier or a reorganizer. In the example described above, for instance, if the focus of instruction is not shifted to interpretation and problem solving, then the use of technology simply to generate mathematical representations such as a scatterplot and the equation and graph of the line of best fit, or to compute the coefficient of correlation, would be an example of an amplifier use of technology. The potential of the use of technology to shift students' thinking is insufficient to consider it a reorganizer under the present interpretation of this construct. The fact that using technology can shift students' mathematical activity to representation, interpretation, and communication does not mean it is being used as a reorganizer unless that affordance is explicitly utilized in some way by the teacher. With reference to the above example, if some aspect of instruction, e.g., a worksheet or class discussion, pushes students to make decisions about which representations would be most suited to answering certain questions, or asked students to use the representations generated with technology to make interpretations about the relationship between the quantities, then such use of technology would be considered a reorganizer, even if students did not successfully achieve the goals of the lesson[6].

As suggested by this example, a hypothesis of the present study is that a given technology can be, and often is, used as both an amplifier and reorganizer. Its ability to automate what would otherwise be tedious and time consuming procedures and computations can be leveraged to shift students' thinking away from the procedures and computations per se, to the underlying concepts or some other concept or reasoning process. For example, Shwarz and Hershkowitz (1999) noticed an increase in communication, reflection, and reasoning about solution process when by-hand procedures were offloaded to technology.

[6] The difference between the teacher's intentions and students' work is an important one that is captured by the *Mathematical Tasks Framework*.

The idea of using cognitive technologies as an amplifier or a reorganizer has been used to describe and organize the literature on the use of technology (Heid, 1997; Zbiek et al. 2007), as well as by researchers in describing uses of technology (Ben-Zvi, 2000; H. S. Lee & Hollebrands, 2008; Pea, 1985, 1987; Zbiek et al., 2007). Indeed, the use of technology as a reorganizer is often an important factor in the studies which have identified it as enhancing student learning, even if its use was not framed as such. In general, however, it has been used at the grain size of curriculum, to describe how the structure or content of a course was changed to emphasize concepts or ideas different from the status quo. Although the amplifier and reorganizer distinction is not cited explicitly, it is precisely what the National Council of Teachers of Mathematics refers to in the Technology Principle:

> Most of the arithmetic and algebra procedures long viewed as the heart of the school mathematics curriculum can now be performed with handheld calculators. Thus, more attention can be given to understanding the number concepts and the modeling procedures used in solving problems. (2000, p. 20)

That is, the presence of technology allows for the K-12 curriculum to be reorganized, giving priority to a different set of skills and abilities than what has traditionally been the goal of mathematics education. The present study seeks to apply this distinction at the grain size of a mathematical task, and thus how the use of technology allows for the sequence of topics in a particular course to be reorganized is not a primary concern. However, a discussion of these studies can provide greater insight into this distinction, and is related to how individual tasks are reorganized within the curriculum.

In her review of the literature, Heid (1997) discusses the role of technology in reorganizing the K-12 mathematics curriculum, and uses this distinction to help organize the studies included in her review: "When technology is used as a reorganizer in the mathematical curriculum, it alters mathematics content in a far more fundamental way than it would alter content in other less quantitative disciplines" (p. 7). She emphasizes that the availability of technology should provide the impetus for a re-examination of the current content of the mathematics curriculum: "Some mathematical concepts come into less demand with technology, and others come into more demand" (Heid, 1997, p. 44).

Reorganizing Curriculum. An example of such a transformation of the content of a course is given by Heid (1988). She served as the teacher and researcher for a university applied calculus course in which she changed both the sequence of instruction with regard to procedures and concepts, as well as the amount of time spent on each. By using the equivalent of a computer algebra system (CAS), instruction was altered to develop the mathematical concepts of the course, such as the limit, derivative, and integral of a function by allowing the CAS to perform these procedures before students had learned how to do them by hand. The CAS class of 39 students learned these concepts in the context of problem solving, and used the concept to understand problem situations and to interpret computer generated results in the context of the problem. Thus, the use of CAS allowed for concepts and problem solving to be taught prior to the mastery of by-hand procedures. Students were able to focus on analyzing problem situations and make decisions about which procedure would be relevant to the task at hand rather than focusing on the correct execution of the procedure.

The experimental group spent 12 weeks on concepts and problem solving, followed by three weeks of instruction on manual techniques of differentiation and integration, while a comparison class of 100 students which did not use technology spent 15 weeks on the traditional curriculum of pencil-and-paper techniques and applications. In a common course final exam which emphasized the correct execution of manual procedures, no significant difference was detected in the performance of the two groups. However, on conceptual questions developed by the researcher to measure students' understanding of course concepts, CAS students significantly outperformed the comparison class. In particular, they demonstrated a greater ability to apply and interpret problem situations in terms of those concepts, and use them to solve problems or interpret representations.

During interviews conducted by the researcher, students in the CAS sections of the course cited three ways in which the use of the computer allowed them to shift, or reorganize, their thinking: "[T]he computer relieved them of some of the manipulative aspects of calculus work…, gave them confidence in the results on which they based their reasoning…, and helped them focus attention on more global aspects of problem solving" (Heid, 1988, p. 22). While the difference between the understandings of the two groups of students in this study may be interpreted as merely providing evidence that students learn what they have the opportunity to learn, the point is that the use of technology for the CAS sections was a key to restructuring the course in such a way that those students had the opportunity to learn something different than what is traditionally learned in such a course. Furthermore, these results have been replicated under similar conditions (Judson, 1990; Palmiter, 1991; Park & Travers, 1996), providing further evidence of the efficacy of this pedagogical transformation.

Similar restructurings of the curriculum have been reported in the area of algebra (Chazan, 1999, 2000; Heid et al., 1990; Huntley et al., 2000; O'Callaghan, 1998). Chazan (1999; 2000) describes his experience of teaching beginning algebra in two different contexts. Although not an empirical study, it is an important example of how the use of technology enables a fundamental change in the conceptualization of a course's subject matter. The first experience of teaching algebra that he describes used a very traditional algebra curriculum, and involved having students learn a litany of procedures for manipulating symbols:

> I was concerned that my teaching was focused on a long list of techniques, that students depended on me and on the text to tell them right from wrong and exercised little independent judgment, and that students did not understand what the course was all about. (Chazan, 1999, p. 124)

The concept of mathematical function as the relationship between variable quantities became the object of study for his second experience of teaching algebra, an approach rooted in the conceptualization of x as variable rather than as an unknown number. He describes the role of technology in making this shift: "[T]he capacity of graphing calculators and computers to carry out many calculations rapidly supports the transition from examination of single cases towards the examination of groups of cases at once" (p. 123). Although Chazan does not use this terminology, this quote illustrates an example of technology being used as an amplifier in order to reorganize students' understanding of algebra. The concept of x as variable on which the functions-based approach to algebra depends is made possible by the number and speed at which examples can be generated using technology.

Another way in which the use of technology reorganized Chazan's algebra course was to make functions and their representations the objects of study from the beginning of the course, while postponing instruction on methods for solving equations. The affordances of graphing technologies were instrumental in resequencing the course: "a second strand of work asked students to use technological tools to deepen their understandings of representations of functions by examining changes to those representations" (Chazan, 1999, p. 128). A specific example of this is having students solve quadratic equations of the form . In the traditional algebra course Chazan admits that he would never have asked students to do a problem for which he had not already given them an algorithm and a number of examples of how to use it. Such a description is very similar to what would be considered a *procedures without connections* task in the Task Analysis Guide. He was unaccustomed to stating a problem in terms of the properties of its solution as his students had few resources with which to engage with such a problem. However, the functions based approach and the use of technology "gave them resources which they could use to solve the problems even before being taught standard methods. Standard methods could then be introduced to students as ways of solving problems which they already understood" (p. 126). In this approach, "solving an equation" is restated as "finding shared inputs which will generate equal outputs" for two functions, and , with technology providing graphical information which could be used to identify the number of solutions and approximate them graphically or numerically. More importantly, the entire problem is conceived of differently in this approach, in terms of functions as relationships between quantities, an idea developed based on students' experiences at the beginning of the course. It is also interesting to note how the use of technology in this example has the potential to transform a *procedures without connections* task to a *procedures with connections* or even *doing mathematics* task.

While the use of technology to restructure the curricular content of courses provides important examples of the use of technology as a reorganizer, these examples are based on researchers' experiences as both the teachers and designers of these courses. Most classroom teachers have neither the resources nor the autonomy to make the kinds of sweeping changes described above, and research bears this out. "Despite the opportunities offered by technology for teachers to change their teaching practice, researchers report that teachers generally use handheld graphing technology as an extension of the way in which they have always taught" (Burrill et al., 2002, p. iv), i.e., as an amplifier of their practice.

In a survey of 181 middle and high school mathematics teachers in the state of Missouri, Manoucherhri (1999) reports that 97 out of 116 high school teachers felt that there was not room or time in their curriculum for exploratory activities using technology, and more than half of the middle school teachers felt that the use of technology had no relevance for curriculum that they taught. Of the teachers reporting the use of technology, thirty middle school teachers and 51 high school teachers used computers for drill and practice activities by individual students. This is a clear example of an amplifier use of technology in the sense that what students are learning has not changed, but rather students are able to practice the same techniques more efficiently using the computer. Manoucherhri summarizes the findings of the study as follows: "the paramount message highlighted throughout the range of data reported in this study is that teachers are not convinced of the usefulness of computers in their instruction, and of the potential of computers to enhance the curriculum they teach" (p. 37).

Reorganizing Tasks. At least part of the issue for teachers may be that, unlike researchers and curriculum developers, using technology as a reorganizer at the grain size of curriculum may not be practical. A claim of the current study is that the amplifier-reorganizer distinction may be a useful way to describe the use of technology within a mathematical task, a unit of instruction much more accessible to classroom teachers.

Hoyles and Noss (1992) refer to the pedagogical strategies made available by the use of technology as strategic apertures:

> [W]e contend that the computer opens a range of alternatives, strategic apertures through which children can gain access to approaches and solutions which are simply unavailable

> with pencil-and-paper…[which] can be exploited in order to increase the possibility of specific learning outcomes. (p. 43)

They give a simple but poignant example using nothing more than the computer's ability to perform multiplication and display a series of results. In order to confront middle school students' conception that "multiplication makes bigger," allow them to discover that multiplication by numbers between 0 and 1 are relevant exceptions, and to distinguish these numbers from those less than 0, the researchers (also serving as the teachers) created the Target Game. A group of two students are given the number 13 and asked to produce a number as close as possible to the target number 100 using only the arithmetic operation of multiplication, and taking turns in determining the multiplicative factor.

The mathematical goals of this task could be addressed by telling students that multiplication by numbers between 0 and 1 produce a smaller product than the original factor, and demonstrating with a few examples. However, they articulate another goal for this activity that such an approach could not address, namely that of "extending the range of pupils' situated abstractions – the ways in which people make mathematical sense of the results of their actions" (p. 41). In this task students were able to take specific mathematical actions and obtain immediate feedback from the computer, reflect on that feedback and discuss it with their partner, make conjectures about results, especially reasons for unexpected output, and adjust their thinking accordingly. This version of the task has characteristics of a *doing mathematics* task, specifically that it "requires students to explore and to understand the nature of mathematical concepts, processes, or relationships" and "demands self-monitoring or self-regulation of one's own cognitive processes." By contrast, the "tell and show" approach would likely be considered a *procedures without connections* task, depending on students' opportunity to make connections to meaning. This example demonstrates how a single task can be reorganized by the use of technology, and leads to the hypothesis that such reorganization may allow teachers to create tasks with a higher cognitive demand than might otherwise be possible.

Doerr and Zangor's (2000) description of the use of the graphing calculator as a transformational tool seems to align with the idea of a reorganizer. Specifically, they cite the use of the graphing calculator "whereby tedious computational tasks were transformed into interpretive tasks" (p. 152-3), and emphasize the teacher's role in this process. This teacher required students to interpret the results of data analysis and modeling in terms of the context of the problem. Offloading computations such as the calculation of regression equations from numerical data, does not guarantee that students will attend to the interpretive aspects of their work. "By her focusing the students' attention on the interpretation of the results, rather than on the actual computation, the students attended to making sense of the result and validating it in the context of the problem situation" (p. 153). That is, the presence of technology does not transform the task, but rather the teacher transforms the task, using technology to do so.

In his discussion of using technological tools in the learning of data analysis, Ben-Zvi (2000) uses the amplifier-reorganizer distinction to discuss the ways in which students' activity and thinking may be reorganized by the use of these tools. In particular, he claims that the use of technological tools can reorganize students' work by shifting their activity to a higher cognitive level, changing the objects of the activity, focusing the activity on transforming and analyzing representations, supporting the situated cognition mode of thinking and problem solving, accessing statistical conceptions by the use of graphics, and constructing meaning of conceptions by the use of representative ambiguity (pp. 140-143). While Ben-Zvi describes the ways that students' activity and thinking can be altered by the use of technological tools in greater detail than most authors, what all these descriptions have in common, in keeping with its original description by Pea (1985, 1987), is a focus on the potential of the technology to influence students' thinking. That is, the focus is on the technology and its affordances, not on what students do with it and how it affects their thinking. The present study seeks to extend the use of the amplifier-reorganizer distinction to describe if and how technology may actually reorganize students' thinking while working on a mathematical task. The difference between the potential of technology to reorganize students' activity and whether or not it actually does so aligns with the distinction made between the set up and implementation phases of enacting a mathematical task, and may be a useful way to correlate technology use to students' thinking via the *Mathematical Tasks Framework*.

The teacher's role in using technology as a reorganizer within a given task relates to the cognitive demand of the task in a number of ways. As the examples described above demonstrate, a primary effect of using technology to change the nature of the task may be that it raises the cognitive demand of the task. This seems especially true in the case of transforming *procedures without connections* tasks to *procedures with connections*, or even *doing mathematics* as described in the example from Hoyles and Noss (1992). However, in making this transformation it seems especially crucial that teachers not shift the emphasis back to getting correct results, a factor found to be associated with lowering the cognitive demand of the task during implementation (Stein et al., 1996; Henningsen & Stein, 1997). For example, it is not hard to imagine a teacher who would like students to discover a procedure or explore the conceptual basis for it, but when perceived time restrictions force her to choose, places a higher priority on making sure that students know and can use the procedures. Given the teacher's role in transforming a task by using technology as a reorganizer, there may be a greater need for teachers to hold students accountable for high level processes and products, a factor associated with maintaining a high level of thinking during implementation, as the teacher in Doerr and Zangor's (2000) study did.

2.7.2 Teachers' mediation of student use

As with many aspects of students' mathematics education, teachers mediate students' use of technological tools as well. Students' attitudes and use of technology for thinking and learning in mathematics are heavily influenced by those of their teacher, beginning with whether or not students are allowed to use it all. In recent survey of a nationally representative sample of Algebra I teachers, over 60% reported using graphing calculators less than once a week, including 33% who reported never using them (Hoffer et al., 2007), although part of the problem for these teachers was related to their availability. However, over 75% of these teachers reported using computers less than once a week, including 43% who never used them, despite reporting that access was not a problem. Kastberg and Leatham (2005) point out that access to calculators is mediated by teachers even for students who have their own as teachers may disallow its use in their class, noting that these decisions are most often associated with teachers' beliefs about what mathematics is and what it means to do it.

Kastberg and Leatham (2005) note that in a series of studies focusing on how students' performance varies with access (Harskamp, Suhre, and van Streun, 1998; 2000; van Streun, Harskamp, and Suhre, 2000), students with access to graphing calculators for one year performed better, produced a wider variety of solution strategies, and attempted more problems on a posttest than students with access for one unit of instruction and students with no access at all. It is important to note that in most of the studies discussed in this review which report enhanced student understanding in conjunction with the use of the technological tools (Ben-Zvi, 2000; Doerr & Zangor, 2000; Farrell, 1996; Guin & Trouche, 1999; Hoyles & Noss, 1992; Ruthven, 1990; Schwarz & Hershkowitz, 1999), students were given constant and continuous access to these tools. As Hiebert et al. (1997) point out with tools in general, the best way for students to construct meaning for the tool, and mathematical meaning with it, is to allow them to use it. Thus, an important question with regard to the use of technology within mathematical tasks is the degree of students' access to these tools, and the freedom to make decision about how to use them. Given the previous discussion of instrumental genesis, the use of technological tools may not support high level engagement with a mathematical task if students are unfamiliar with the tools that they are being asked to use within the task. This suggests that access and autonomy with regard to the use of technology may have important implications for the cognitive demand of mathematical tasks.

For example, how do teachers and students negotiate the use of technology for *procedures with connections* tasks? These types of tasks are characterized by the use of procedures in order to make connections to meaning between related concepts, or between procedures and concepts. The procedures used to make these connections may be technical, and may be specified by the teacher, as in exploratory activity as described above. In this case the use of technology is initiated by the teacher, what students are to do with it is clearly indicated, and generally students should all arrive at the same conclusion. The challenge may be for students to move beyond button pushing to reflecting on their work and making the connections which are the goal of the task. The difficulty may lie specifically in moving students from a passive frame of mind in which they follow directions and record results, to one in which they are active constructors of their own mathematical knowledge, requiring agency and authority in their mathematical work which is not called for during a large portion of the task.

A specific way that teachers mediate students' use of technology is through privileging. Kendal and Stacey (2001) use this term to describe how the teacher may shape students' views of mathematics and the role of technology in thinking and learning about mathematics. Borrowing from Wertsch (1990), they define privileging as "a construct to describe an individual teacher's way of teaching and includes decisions about what is taught and how it is taught" (p. 145). In order to examine the impact of privileging on student learning, they studied two teachers' instruction on a unit of differential calculus using CAS enabled graphing calculators with 11th grade girls. The two teachers had worked together to redesign a 20 lesson unit on differentiation, sharing ideas, lesson notes, and worksheets. By having the teachers teach not only from a common curriculum, but one which they had created, the study was able to examine how different emphases and classroom norms manifested during implementation impacted students' learning. Kendal and Stacey identify three areas in which a teacher's privileging may have strong implications for student learning: "teaching approach, emphasis given to different representations of differentiation, and use of technology" (p. 146).

Of the two teachers in their study, they characterize Teacher A as having a teaching approach which was content-focused with an emphasis on performance, in which he privileged the knowledge and application of rules and procedures using a lecture-based teaching style. He admittedly preferred the symbolic representation, and stressed its connection to the numerical representation of the derivative during his instruction. Teacher B's teaching approach was classified as content-focused with an emphasis on conceptual understanding, as he used a more student-centered approach and encouraged the construction of meaning and understanding of ideas by students. Teacher B also favored the symbolic representation, but stressed its connection more to the graphical representation of the derivative more than Teacher A.

Contrary to the findings of Kastberg and Leatham (2005) in their review of the literature on the use of graphing calculators by mathematics teachers, Teacher A, although more focused on the performance of symbolic procedures, allowed students free use of the CAS calculator for such procedures. That is, the belief that the ability to execute procedures efficiently by hand is an important mathematical skill for students to learn is rarely associated with free access to technology which can perform such procedures. Furthermore, although Teacher B stressed conceptual understanding and student construction of mathematical knowledge, he limited the use of the CAS calculator in favor of mastering by-hand procedures, using technology more during instruction in order to help students connect procedures with concepts. However, the authors point out that these findings with respect to teachers' privileging are consistent with those of Jost (1992), who observed that "teachers who used calculators for procedures viewed learning as listening, and those who used calculators for learning used student-centered teacher styles" (Kendal and Stacey, 2001, p. 156).

While the overall performance of the two classes on a measure developed by the authors was similar, each class exhibited competency on different aspects of the concept of the derivative. The instrument used measured students' abilities to formulate problems in terms of a derivative, and to interpret a derivative in terms of natural language or the context of a problem situation. It was designed to test these competencies both within and across the three standard representations, i.e., graphical, numerical, and symbolic. Overall, Class A outperformed Class B in formulation type problems, and problems which involved the numerical and graphical representations, while Class B did better on interpretation items, and those involving the symbolic representation. Although information about teaching methods or implementation were not available in Huntley et al.'s (2000) study, these results are similar in that they are evidence that students learn what they have the opportunity to learn. What Kendal and Stacey's (2001) results demonstrate is that what students have the opportunity to learn is shaped in important ways by the teacher, and not just the curriculum, as in Huntley et al.'s study.

The significance of these results for the present study is twofold. First, it points to the fact that student learning with technology is perhaps not as strongly correlated to student access as the studies cited above might suggest (Ben-Zvi, 2000; Doerr & Zangor, 2000; Farrell, 1996; Guin & Trouche, 1999; Heid, 1997; Hoyles & Noss, 1992; Ruthven, 1990; Schwarz & Hershkowitz, 1999). Rather, the teaching approach, with its innate privileging, may be a more influential factor than the degree of access that students have. Teacher A allowed free and constant access to the CAS calculator, especially its automated symbolic manipulation capabilities, but ultimately Teacher B's students performed better on symbolic items, despite the fact that he carefully controlled its use by students. Thus, the question of access in relation to students' high level engagement with tasks may be more open than the studies reviewed above suggest.

Second, using the calculator primarily for pedagogical purposes, as Teacher B did, may be an effective way to help students connect procedures to conceptual meaning and understanding. Teacher B's students' ability to interpret their symbolic work with the derivative seems to support this. This suggests that using technology in *procedures with connections* tasks is a promising approach to using technology to help students engage in cognitively demanding thinking. However, few details of how such instruction was enacted by this teacher are available. Given the issues raised above with regard to students' exploration degenerating to button pushing and the recording of results (Doerr & Zangor, 2000; McGraw & Grant, 2005; Zbiek et al., 2007), one is left to wonder how this teacher negotiated the procedural aspects of the task in order to help students make meaningful connections. A goal of the present study is to describe this process in detail in order to identify the ways in which tasks which involve the use of technology help or hinder students in making meaningful connections.

2.7.3 Judicious use of technology

While teachers mediate students' access to technological tools, given some degree of access, they also mediate students' use of it. The idea of judicious use of technology for mathematical activity by students considers the teachers' role in mediating students' attitudes toward and relationship with technological tools. The discussion of judicious use can be viewed as part of the teacher side of instrumental genesis. That is, instrumental genesis is likely to involve more than judicious use, but this aspect of it addresses important issues related to the distribution of mathematical authority between students and technological tools.

Doerr and Zangor (2000) conducted a qualitative investigation of an experienced mathematics teacher in an effort to understand how the teacher's practice mediated students' use of graphing calculators in a precalculus course. The teacher who was the focus of the study had 20 years of teaching experience and was extremely familiar with the use of the graphing calculator. Two sections of precalculus students were observed for three units of instruction (linear, exponential, and trigonometric functions), for a total of 21 weeks. The students had constant access to graphing calculators in and out of class, worked on modeling activities in which they were asked to interpret and represent data, and made generalizations based on these. Using field notes from classroom observations, audio taped small group discussions, videotaped whole class discussions, and interviews with the teacher, the researchers identified five roles that graphing calculator fulfilled in students' work. It served as a tool for computation, data collection and analysis, visualizing, checking results, and as a transformational tool as described above.

An important contribution of this study is the connection between the teacher's practice and the various roles that the calculator played in students' work. For example, the teacher's consistent requirement that students explain the output of their calculator had "become part of the mathematical norms established by the teacher, whereby the accepted truth or falsehood of a statement had to be supported by mathematical reasoning or justification, not by an appeal to any authority ascribed to the calculator" (p. 152). This practice was identified in connection with students' development of the use of the graphing calculator as a computational tool. A specific example is that of having a student explain why the graphing calculator gave an error message when the student tried to use it to calculate the logarithm of -1. The explanation given by the student included an explicit mention of the domain of the logarithm function. This is a concrete example of how a teacher mediates her students' instrumental genesis (Guin & Trouche, 1999; Hoyles et al., 2004) by helping them to connect their use of it to their mathematical knowledge. In this way students are able to construct mathematical meaning for and with the tool, a process clearly mediated by the teacher in this example, and one that echoes a factor associated with high level of cognitive demand, namely the "sustained press for justifications, explanations, and/or meaning through teacher questioning, comments, and/or feedback" (Stein et al., 2009, p. 16).

In this way the teacher did not allow the calculator to become a source of mathematical authority in the classroom, but rather a tool the results of which needed to be checked based on students' mathematical knowledge and reasoning. This is especially important in light of concerns about the role of graphing calculators as a source of mathematical authority for students (Williams, 1993; M. R. Wilson & Krapfl, 1994). Students are unlikely to engage in complex thinking or problem solving characteristic of high level tasks if they do not view themselves as having the ability to discern mathematical meaning. An uncritical stance toward the use of technological tools may be both a symptom and a cause of such an attitude. The teacher in Doerr and Zangor's (2000) study is a concrete example of how the teacher mediates the relationship between her students and the tools in a way that advances the mathematical authority of her students. This is hypothesized to be a key factor in promoting high level engagement in technology enhanced tasks, and noticing if and how teachers address this issue during instruction will be important to note.

These classroom practices, and others described by Doerr and Zangor (2000), indicate how this particular teacher shaped her students' views of the technological tools at their disposal by instilling in them the idea that the graphing calculator is a tool that can provide information and assist in problem solving, but that cannot do the thinking for them. This was evident in the vignettes that were described in which students did not appeal to the graphing calculator in justifying a conjecture or making a mathematical argument, but to their own mathematical reasoning. "Because the calculator told me so" was not accepted as an argument in this classroom; students were required to look for mathematical reasons to explain the information or representations that their calculators produced. They were to use their calculators to do mathematics, and were discouraged from believing that the calculator could do the mathematics for them.

Ball and Stacey (2005) also discuss the issue of judicious use of technology by students. They describe judicious use as a thoughtful and purposeful use of technology in which students pause to think about the problem they are working on before reaching for it, and consider the affordances of technology in relation to other techniques and sources of information at their disposal. They note that students' pencil-and-paper skills must be fairly strong in order to use technology judiciously, contrasting this with "'fishing' or 'zapping' behavior shown by some students who press buttons seemingly at random with just a vague hope that something might work" (p. 4). This type of behavior is also characteristic of the random work method described by Guin and Trouche (1999), who note that it is enabled by the technology. This is not to say that students cannot work mindlessly in a pencil-and-paper environment, but rather that little thought and effort is needed to randomly mash buttons on a calculator or computer. In fact, this behavior may be one way that a task set up at a high level could degenerate into *procedures without connections* during implementation that is introduced by the presence of technology.

In Ball and Stacey's (2005) case study of a single teacher and her students, the teacher notes that her weaker students are more prone to this type of behavior, and that it requires conscious effort on her part to try to get students to think before they pick up their calculator. Ball and Stacey cite Pierce (2002) with reference to the possibility of teaching students to use technology judiciously for mathematical activity, who claims that while some students seem to persist in using technology in an uncritical and unthinking manner, most can be taught to use it judiciously. This issue seems likely to be a crucial factor in maintaining the cognitive demand of a high level task during implementation, as students may avoid the thinking required by the task in favor of "fishing" for a correct answer. Whether students engage in this type of behavior, and even whether the task allows this possibility, and how a teacher deals with it may be important considerations in examining the interaction between students' use of technology and the thinking required by mathematical tasks.

Ball and Stacey (2005) identify four habits connected to teaching students to use technology for their mathematical work judiciously: promoting careful decision making about the use of technology, integrating technology with the curriculum, strategically restricting the use of the technology at times, and promoting the use of "algebraic insight" (Pierce & Stacey, 2002) for monitoring their work. To promote decision-making about the use of technology, teachers must allow for multiple solution strategies in both technological and pencil-and-paper mediums, and take the time to discuss the affordances and constraints of each. Furthermore, there must be a certain amount of freedom given to students at times in order to exercise their judgment based on these experiences. Integrating technology with the curriculum involves teachers making use of mathematical tasks for which the use of technology has the potential to give students access to mathematical ideas that would be difficult to gain without it, i.e., using technology as a reorganizer. The requirement to strategically restrict the use of technology at times helps students to develop the mathematical knowledge and reasoning, and pencil-and-paper skills that are needed to use technology productively. If instrumental genesis involves the bi-directional influence of mathematical knowledge and use of technology, this strategy ensures that the mathematical knowledge side of the equation has an opportunity to develop independently of the use of technology. Finally, the idea of "algebraic insight" described more fully elsewhere (Pierce & Stacey, 2002), is the algebraic equivalent of number sense, and involves what they call algebraic expectation, which is the ability to anticipate the effects of certain mathematical actions across representations. This is important in being able to critically evaluate the output of technological tools. The point of elaborating these strategies for developing judicious use among students is that research has shown that this is a teachable habit of mind, and has identified important teacher moves in instilling it in students. While there are likely many ways to develop this habit among students, whether or not teachers make an explicit attempt to do so may impact how their students engage in mathematical tasks during implementation.

2.8 HYPOTHESES FOR THE PRESENT STUDY

The research reviewed above has been organized by considering the construct of mathematical tasks in relation to the intersection of areas in Figure 2.1, i.e., teachers and students, students and technology, and teachers and technology. In the present section, explicit hypotheses are made about the interaction of all three, i.e., the center of Figure 2.1, by connecting the research reviewed above to the research questions posed in the present investigation.

2.8.1 Hypotheses about Research Question One

Question 1: How do the cognitive demands of mathematical tasks differ when technology is used as part of the task and when it isn't?

a. How is the use of technology associated with the cognitive demand of mathematical tasks as they appear in curricular materials?
b. How is the use of technology associated with the cognitive demand of mathematical tasks as set up by the teacher?
c. How is the use of technology associated with the cognitive demand of mathematical tasks as implemented?

The hypothesis is that teachers will use technology to plan high level tasks, but will often fail to maintain the cognitive demand during implementation, for two reasons: (1) teachers have been shown to have difficulty maintaining the cognitive demands of high level tasks during implementation (Boston & M. S. Smith, 2009; Henningsen & Stein, 1997; Stein et al., 1996), and (2) even teachers who are able to implement tasks at a high level without technology might struggle to do so while using technology due to their students' and/or their own unfamiliarity with the technology, especially in relation to the task goals. In the first case, teachers have not developed a practice which can support the implementation of high level tasks, and introducing technology does not change that. This is in agreement with Monaghan (2004) who challenged the idea that introducing technology into the mathematics classroom forces teachers to adapt their practice to its presence in ways that may raise the level of thinking by students. I hypothesize that it is more likely that teachers will adapt the use of the technology to their practice than the other way around.

The second case above highlights the construct of Technological Pedagogical Content Knowledge (TPACK) (Koehler & Mishra, 2008; Mishra & Koehler, 2006) in the sense that a teacher with strong pedagogical content knowledge (Shulman, 1986, 1987) may be able to enact high level tasks faithfully, but does not thereby understand how to do so when technology is involved. Teachers may not be able to implement lessons with the same degree of skill, understanding, and anticipation when using an unfamiliar tool. That is, their understanding of the role of technology in supporting students' thinking is not integrated with an otherwise exceptional practice. One of the teachers in Pierson's (2001) study was an example of a teacher with strong pedagogical content knowledge whose practice was observed to regress when she incorporated technology. Beliefs are not the issue for these teachers, as their beliefs in the usefulness of technology for learning are borne out in their willingness to use it for instruction. However, they lack a deep understanding of how the affordances of the technology can be leveraged to achieve certain mathematical goals, an issue perhaps related to their lack of familiarity with the technology.

Guin and Trouche (1999) and others have described the importance of the process of instrumentation through which students must progress in order for an artifact to become an instrument of mathematical thought. I hypothesize that teachers must go through a similar process in which an artifact becomes an instrument for mathematics instruction. Whether teachers maintain the cognitive demand during implementation or not, these cases are of particular interest as they represent occasions in which the decline or maintenance of the cognitive demand of the task goes beyond explanation by the teachers' practice. That is, much of teachers' use of technology has been explained by their practice. However, these instances provide the opportunity to investigate what teachers need to know and be able to do while incorporating technology into instruction that goes beyond their practice without it. Of particular interest will be teachers who frequently implement high level tasks without the use of technology, but struggle to do so while using it, as these cases highlight the difference between pedagogical content knowledge (Shulman, 1986, 1987) and technological pedagogical content knowledge (Koehler & Mishra, 2008; Mishra & Koehler, 2006).

2.8.2 Hypotheses about Research Question Two

Question 2: How does the role of technology differ in low level and high level cognitive demand tasks? What is the role of technology in each?

a. During Set Up

   What are the features or characteristics of technology use associated with tasks set up at a low level of cognitive demand?

What are the features or characteristics of technology use associated with tasks set up at a high level of cognitive demand?

b. During implementation

What are the features or characteristics of technology use associated with tasks implemented at a low level of cognitive demand?

What are the features or characteristics of technology use associated with tasks implemented at a high level of cognitive demand?

It is hypothesized that in general technology will be used to transform the learning of content into an activity in which students are asked to explore and investigate regularities in the behavior of mathematical objects, and to make conjectures about them. These types of activities would be considered exploratory (vs. expressive), as students are told what to investigate and how to investigate it and are asked to reflect on their work in order to make connections. In terms of the cognitive demand, most of the tasks for which technology is central to the task will be *procedures with connections* during set up. This is partly due to the fact that *doing mathematics* tasks are more rare than *procedures with connections* (Boston & M. S. Smith, 2009; Stein et al., 1996), and that teachers often replace curricular materials with worksheets when using technology for instruction (Monaghan, 2004). However, technology has been shown to provide students with solution strategies which are unanticipated by the teacher and which skirt the mathematical goals of the lesson (Hoyles & Noss, 1992). Furthermore, it may provide students with strategies which not only miss the mathematical goal of the task, but which do not lead to a solution, i.e., unsystematic exploration. Even when students are given explicit instructions about what to do, the nature of exploring mathematics is that one doesn't know what to expect or what one might find. That is, the teacher wants students to grapple with certain mathematical ideas, but by the nature of the activity, the students are unaware of the ideas about which they should be thinking and reasoning about. Furthermore, in the case where exploring a mathematical concept or procedure is not a regular part of the classroom practice, students may be further hindered from engaging in productive exploration due to their lack of familiarity in engaging in such type of tasks, i.e., the habits of mind (Cuoco et al., 1996) needed to do so.

Related to this hypothesis is another regarding the use of technology by teachers for instruction. Although not explicit in the research questions, the metaphors of amplifier and reorganizer (Pea, 1985, 1987) may be useful in answering questions about how technology gets used for instruction. In particular, the use of technology in the context of low level tasks will generally align with using it as an amplifier, while using it to reorganize students' cognitive focus will be more commonly associated with high level tasks.

The use of technology as a reorganizer is hypothesized to almost always be based on first using it as an amplifier. Computations, the generation of representations, or the measurement of objects using technology are all examples of tasks which students would generally be able to do by hand, and thus would be considered an amplification of students' mental processes, allowing them to do more efficiently and accurately what they could do without it. However, in order to change students' cognitive focus during a task, it will almost always be the case that these types of tasks will need to be offloaded to the technology in order to give them the opportunity to focus on something else. Thus, a hypothesis is that instead of seeing the use of technology as dichotomous, i.e., amplifier or reorganizer, it may be more appropriate to discuss the use of technology for doing mathematics as only an amplifier, or as also a reorganizer.

2.8.3 Hypotheses about Research Question Three

Question 3: How does the use of technology impact the cognitive demand of a task during implementation?

a. How is the use of technology related to factors which have been associated with decline of mathematical tasks set up at a high level of cognitive demand?

b. How is the use of technology related to factors which have been associated with maintenance of mathematical tasks set up at a high level of cognitive demand?

It is hypothesized that the use of technology is unlikely to introduce a factor associated with the maintenance of cognitive demand during implementation that is distinct from those already identified. This is based on the fact that these factors are sufficiently broad that the use of technology would likely fall under one of the factors already identified. For example, the use of technology can provide a scaffold for students' thinking and allow them to monitor their own progress, by giving them access to easily generated information, and the ability to check their work or see the results of certain mathematical actions (Zbiek et al., 2007). Such a scaffold may allow teachers to redirect students back to this source of information and feedback if pressed for additional information or help, as turning challenges into non-problems was identified as the most common factor present in tasks which declined during implementation (Stein et al., 1996). Furthermore, the presence of some scaffolding of student thinking was a factor in 58% of the tasks observed to stay at a high level in general during implementation, and in 73% of tasks implemented as *doing mathematics* in particular (Henningsen & Stein, 1997; Stein et al., 1996). The fact that a lack of scaffolding may create a situation in which teachers are more likely to be pressed by students for additional information is not introduced by the use of technology. However, what is not known is exactly how the use of technology may provide a scaffold for students in the context of engaging in a high level task, and how the teacher supports or undermines its potential to do so. How does a teacher use technology to scaffold students' engagement? Does she redirect students back to the technology when pressed for information or assistance? The role of the technology in the task as set up, how students' use of it is structured, anticipated solution strategies which use technology, and how the teacher responds to unexpected student use or technology output are some issues that are hypothesized to influence the cognitive demand of the task during implementation. The present study seeks to identify and describe these issues in the context of classroom instruction, and determine how they may relate to factors associated with the maintenance of a high level of demand during implementation. While the use of technology may not introduce new factors associated with the maintenance of cognitive demand during implementation, it may be possible to provide a more detailed description of how this happens in a classroom context.

On the other hand, because of the numerous ways in which the demand of high level tasks can be lowered during implementation, it is hypothesized that the use of technology as part of such tasks may introduce new factors associated with this phenomenon. For example, when using technological tools, there is the possibility that students' attention will shift from the mathematics to the technology. As Heid (1997) points out, the extent of this tendency may be related to the transparency of the tool, but it may also be an indicator of students' familiarity with the tool and how teachers address this issue. Students unfamiliar with a given technology cannot be expected to use it with any degree of sophistication, and attempts to have them do so may result in students who are more focused on using the technology correctly than the mathematical ideas that are to be explored with it. A common response of teachers to this situation is to create instructional materials that describe the use of the technology for the task in minute detail in order to ensure that students are able to use it without problems or frustration (Monaghan, 2004). However, such detailed instructions on the use of technology have the potential to shift students' attention from the mathematics to pressing the right buttons. This general phenomenon has been noted by Gutierrez (1995), termed "metadidactical gliding" by Brousseau, and has the potential to lower the cognitive demand of a high level task during implementation if the teacher is not able to salvage the mathematical goal of the lesson by shifting students' focus back to the mathematics at some point during the task. At least part of this issue with the use of technology may fall under the factor of "inappropriateness of the task" for the particular group of students for which it was designed or adapted. This factor includes an assortment of possibilities, including "low levels of motivation, lack of prior knowledge, and a lack of suitably specific task expectations" (Henningsen & Stein, 1997, p. 537). A lack of prior knowledge of the technology may be a contributing factor in this phenomenon, but whether it captures all instances of it is an open question. Are there cases in which the mathematical task may be appropriate for students, but that the technology serves as a distracter from the mathematics and thus lowers the demand of the task? More generally, what role does the use of technology play when the cognitive demand of the task is lowered during implementation?

2.9 CONCLUSION

The research reviewed here is an attempt to integrate two literatures: research on students' thinking while engaging in mathematical tasks and how it may change during the phases of implementation, i.e, the *Mathematical Tasks Framework*, and research on the use of technological tools for instruction and learning. While not exhaustive, the purpose is to demonstrate the potential of applying the *Mathematical Tasks Framework* to students' use of technology to understand how its use as designed and enacted by the teacher impacts and shapes the students' thinking while working on technology enhanced tasks. Given the ubiquity of these tools in secondary classrooms, and the importance of students' thinking to the type of learning they do, it is imperative to understand how these tools shape that thinking in a classroom context, and the teacher's role in that process. Indeed, studying the use of technology through the lens of the *Mathematical Tasks Framework* is what motivates the research questions posed in the present investigation. On one hand, the *Mathematical Tasks Framework* has not been used to study technology use before, and conversely, the use of technology for instruction and learning has generally not been studied in terms of the opportunities for high level thinking that it may provide.

The hypotheses above are meant to further demonstrate what this integration might yield in terms of the research questions. However, it is important to note that the design of the current study is not that of hypothesis testing. While the use of a deductive framework will be used to understand the classroom interactions of teachers, students, and technology around mathematical tasks, a major emphasis of the present investigation is to explore and describe how technology mediates students' thinking, and how teachers influence that process, a level of detail which can inform teacher education. The research methods that will be used to accomplish these goals are described in the next chapter.

3 CHAPTER 3: METHODS

3.1 INTRODUCTION

The use of technology for mathematics instruction has been studied in many contexts using a variety of methods. What is novel in the present study is the investigation of the use of technology through the theoretical lens of the *Mathematical Tasks Framework* (Stein et al., 2009). Through this lens, the role of technology was examined in relation to the cognitive demands of mathematical tasks, how it may change during the various phases of implementation, and classroom-based factors which may be associated with this dynamic. In this sense the present study is an exploratory study aimed at identifying and describing the relationship between technology and the cognitive demand of tasks as it plays out in classrooms. For these reasons, the answers to the research questions posed in this study are most suited to qualitative methods, a strength of which is to describe processes in context and provide rich illustrations (Denzin & Lincoln, 2003). The purpose of this study is to describe how the use of technology is related to and may differ between the kinds of tasks within which it is situated. In particular, the present study investigates how the use of technology supports or undermines the type of thinking called for by a particular task. Obtaining a qualitative, fine-grained description of the role of technology in this process is anticipated to be an important contribution of the present investigation.

The research methods described in this section were developed and refined through a series of pilot studies. In the fall of 2009, one teacher was studied for eleven observations over a two month period as part of a course on observational research methods. These observations assisted in the development of the purpose of the study and the research questions, and provided practical experience in collecting data in a classroom context, resulting in a better understanding of what can be captured by a single researcher in this context, and the types of questions that could be answered using this data. After specific research questions and methods were developed, three small pilot studies were conducted in April 2010 ranging from one to three days of observations at three of the data collection sites. These small studies were helpful in refining the research methods proposed, and indicated that a longer, sustained pilot study was warranted. This was conducted during June 2010, during which twelve lessons were observed over a four week period. As the methods used and refined in that pilot study are very similar to those described below, the data collected during that pilot study are part of the data corpus for the present investigation as well.

3.2 STUDY DESIGN

The present study follows a mixed methods design in which a deductive framework was imposed on the data in order to understand the use of technology for instruction through the theoretical lens of the *Mathematical Tasks Framework*, as well as a more inductive qualitative analysis to reveal in detail how the use of technology influences the thinking students do while engaging with instructional tasks. The cognitive demands of mathematical tasks as present in curricular materials, and during set up and implementation, and the factors associated with maintenance and decline of the thinking required by tasks during implementation, were used to code the data using the *Mathematical Tasks Framework*. Coding contrasts were used to quantify, summarize and compare the features of tasks within and across four different classrooms.

The qualitative aspect of this study is based on lesson observation field notes and transcribed interviews with the teachers after lessons. The purpose of collecting this data is to gain a better understanding of how the use of technology for instruction influences students' thinking at a level of detail that can inform and guide teacher education. Thus, while the quantitative element of the study was useful in identifying general patterns of use of technology for instruction in relation to the cognitive demand of mathematical tasks within classrooms, the qualitative component investigates those patterns in depth so as to describe how they are manifested in classroom instruction.

A methodological challenge of the present study was that the *Mathematical Tasks Framework* (Stein et al., 2009) is a classroom level indicator, i.e., the level of thinking exhibited by the majority of students for a majority of the task. However, the research questions regarding the use of technology require a fine-grained description of its use by the teacher and/or individual students. Thus, data was needed that can be analyzed at both grain sizes. Lesson observation provides the flexibility to move from student to student and to linger to observe specific actions a student performs with the technology, and to record them in a way that can be reproduced for qualitative analysis. Lesson observation field notes also allowed for data to be collected at a larger grain size, noticing what the teacher was doing at the board, how she directed students' work, and in what types of activity the class of students as a whole was engaged. The flexibility of this method is crucial in collecting data at more than one grain size, so that both information about classroom cognitive demand and individual students' behaviors using technology were observed and recorded.

3.3 DATA SOURCES

The unit of analysis for the present study is a mathematical task nested within a mathematics classroom. A mathematical task is defined “as a classroom activity, the purpose of which is to focus students’ attention on a particular mathematical idea…[and] is not classified as a different or new task unless the underlying mathematical idea toward which the activity is oriented changes” (Stein et al., 1996, p. 460). Data were collected from each of four classrooms each at a different school. A summary of the data sources, their purpose, and frequency of collection for the present investigation is given in Figure 3.1.

Data Source	**Focus of Data Collection**					**Frequency**
	Curricular Materials	Set up	Implementation	Classroom-based factors	Technology Use	
Initial teacher interview		X	X	X	X	Once
Lesson observation field notes		X	X	X	X	Each observation
Task artifacts	X	X				Each observation
Student work			X			Each observation
Technology Use Checklist	X				X	Tasks using technology
Post-lesson teacher interview		X	X	X	X	Each observation
Analytic memos		X	X	X	X	Irregular

Figure 3.1: Summary of data sources.

3.3.1 Initial Orienting Interview

As the set up and implementation of mathematical tasks, and factors associated with maintenance and decline of tasks set up at a high level are strongly influenced by the learning environment in which they are enacted, information about the school, the course, the teacher, and the students was collected in the context of an interview with the teacher at each site, prior to the beginning of data collection, when possible. Given individual teachers' time constraints, this information was gathered over a few interviews in some cases. This information was important in interpreting what was observed, and in formulating hypotheses about the results in the discussion section of the dissertation. In particular, the information elicited during this interview shed light on factors which may influence the kinds of tasks that the teacher selects, her perception of the opportunity to enact high level tasks, the way she sets up and implements tasks, and her view of the role of technology for mathematics instruction and learning. This information provided the researcher with a way to interpret classroom instruction, make connections and formulate hypotheses about contextual factors and observed tasks, and inquire about these with the teacher during post-lesson interviews. The results of this interview were used to inform the site descriptions provided in Section 4.1, and also provided the necessary context for the conclusions proposed in Chapter 5. The protocol for this interview is provided as APPENDIX D.

3.3.2 Lesson Observation Field Notes

Lesson observation field notes were the primary data source in evaluating the cognitive demand of a task during set up and implementation, the factors associated with maintenance or decline of tasks set up at a high level, and the ways in which students' thinking is influenced or supported by the use of technology. Lesson observation field notes have been used in previous studies, in conjunction with the collection of task artifacts and students' work, to assess cognitive demand during set up and implementation (Boston & M. S. Smith, 2009; Boston & Wolf, 2006), and field notes from the pilot study demonstrated that they could be used reliably to make these evaluations.

Field notes were developed based on written jottings (Emerson, Fretz, & Shaw, 1995) recorded during observations, with particular attention paid to the set up and implementation of tasks, and the use of technology by the teacher and students (reliability coding results are reported in separate section below). Field notes were constructed from these jottings immediately following an observation, by combining the written jottings with head notes, and the recollection of events recorded in jottings (Emerson et al., 1995; Graue & Walsh, 1998). That is, jottings are a record of the sequence of events, observations of the activity of the teacher and individual and groups of students, and utterances and exchanges between the teacher and individual students, recorded in real time as a form of shorthand which elicits the memory of the specific events that were observed, which are then constructed in greater detail in the field notes. Although jottings from lesson observations are intended to be a description of events with minimal interpretation, a layer of interpretation was already present in deciding what to focus on when taking jottings during lesson observations. Furthermore, not everything recorded in written jottings is used to construct field notes. For example, written jottings may include a record of a teacher demonstrating three or four examples of similar problems to explain how to use a procedure, but the field notes may only include the first one or two examples in detail, and a note that two or three more examples were explained at the board in a similar fashion. Moreover, in order to compare tasks within and across classrooms, the main instructional task for a given lesson was identified by the largest proportion of instructional time devoted to it[7]. The process of constructing field notes benefitted from retrospection so that only those aspects of the task which are relevant to the cognitive demand during set up and implementation, and the use of technology by students are reconstructed in the field notes. Yet, details recorded in the jottings which were not originally used when constructing the field note were available for reference if needed.

Observation Focus by Phase. The lesson observation field notes are a central data source in the present study, and represent a first level of interpretation in the sense that the observer makes choices about what to focus on and record in the written jottings during observations. In order to be explicit about these choices, the focus of lesson observations is discussed in greater detail in the present section, especially in terms of how the focus changes with the phases of implementation.

The task set up phase is defined as "the task that is announced by the teacher" (Stein et al. 1996, p. 460). Set up includes both written and oral directions given by the teacher when announcing the task. These directions are an indication of the teacher's expectations for how students will engage with the task. These may be explicit or implicit, including how long students may have to work on the task, if students will work independently or with a partner or group, what students will be held accountable for both in terms of processes and products, and how it may relate to previous work and/or subsequent activities planned for that day or another class. As set up is done by the teacher, the teacher's announcement of the task, examples given, connections made by the teacher to prior knowledge or previous tasks before having students work on the task, and if and how she answered questions from students prior to their beginning the task were the focus of the observation at this point.

[7] If two (or more) tasks take an equal amount of time, both will be considered main instructional tasks.

The task implementation phase is defined as "the manner in which students actually work on the task" (Stein et al. 1996, p. 460). The focus of the observation at this phase shifted to students' behaviors while working on a mathematical task, the interactions between students, between students and the teacher, and between students and the technology. In particular, the questions students asked and how the teacher responded, the questions the teacher asked and how students replied, conversations between students, questions students asked of one another, and what students were writing or doing with technological tools were noted during the implementation phase.

Observing Students' Use of Technology. Doerr and Zangor (2000) describe some of the difficulties in observing students' use of graphing calculators during classroom instruction: "there were occasions where it was simply not clear what the students had done with their calculators" (p. 148). While there are limits to what can be observed while students are working on a task with technology, there is still much that can be captured about students' use of technology.

Students' work on PCs, laptops, or an interactive whiteboard is much more apparent than work on a calculator, and these were the primary technologies employed during instruction at these sites. Calculators were used only occasionally, and when it was not possible to observe exactly what students did with the calculator, it was possible to infer it from the task that they were working on. Whole class discussions, when they occurred, were potentially revealing of the strategies that individual or groups of students used while engaging in a task if the teacher asked for that information. The researcher discreetly roamed the classroom while observing in order to notice what the majority of students were doing with the technology, while simultaneously collecting detailed descriptions of what individual students were doing. These observations focused on what students were constructing or manipulating with the technology, what questions or difficulties arose during its use, and conversations between students or between students and the teacher that took place in relation to the use of the technology.

Generally the first page of the field note is a summary of the lesson, as interpreted by the observer. Topic refers to the mathematical content of the lesson, while pedagogical arrangement describes how instruction was organized or structured, such as direction instruction, individual seat work, or small group investigation, for example. The outline of class is a brief chronological outline of what students did and when; it is arranged by activity and not necessarily by task[8]. The purpose is the observer's interpretation of the mathematical goal of the lesson, i.e., what students were to learn as a result of the lesson. Technology is a brief description of if and how technology was used during the lesson that day, and discussion with teacher after class is a one or two sentence summary of what the post-lesson interview was about. The purpose of this summary was to serve as a reference when reviewing field notes during analysis.

The field notes generally follow a chronological order from the beginning of class to the end, with some minor adjustments made in order to describe events coherently that may have occurred simultaneously or overlapped in time. In addition to being used for coding cognitive demand and technology use, the lesson observation field notes were the primary data source for qualitative analysis, as described below. Excerpts from the field notes are used throughout Chapter 4 in order to exemplify results or provide evidence for claims.

[8] Recall that a task can span a number of activities if the same mathematical idea is the focus of these activities.

3.3.3 Task Artifacts

In addition to field notes, artifacts from the main task of the lesson, determined by the greatest amount of instructional time spent on it, were collected from the teacher either before or after instruction. This included warm-up problems, homework assignments (if a large proportion of class time is spent reviewing homework), PowerPoint slides from a lecture, software files created before or during instruction by the teacher, example problems, in-class activities, and handouts. A copy of the task as it appears in curricular materials prior to any modification by the teacher was also collected in order to evaluate the cognitive demand at this initial phase of implementation using the Task Analysis Guide. Furthermore, a "blank" copy of the artifact as it appears prior to students' beginning to work on it was collected and, in conjunction with the teacher's announcement of the task, was used in the evaluation of the cognitive demand of the task during the set up phase.

3.3.4 Student Work

An important indicator of students' engagement with a task, and thus cognitive demand of the task during implementation, is the record of work they produce (Boston, 2006; Boston & M. S. Smith, 2009; Boston & Wolf, 2006). As much as possible, student work was collected for the main instructional task each day, including software files if the work for the day was primarily on the computer. The purpose of collecting student work is to evaluate the thinking and reasoning students engaged in while working on the task. Student work was evaluated according to the type of thinking the majority of students seemed to be engaged in while working on the task, and was used to triangulate the evaluation of the cognitive demand made from the lesson observation and field using the Task Analysis Guide. As a means of triangulation, student work provided insights into students' thinking as captured by the written work that might not be apparent during lesson observations. Although the cognitive demand of the task during set up and implementation was made using the lesson observation, field notes, and tasks artifacts, student work was evaluated for consistency with that judgment, and provided further details to support it.

3.3.5 Technology Use Checklist

The Technology Use Checklist (APPENDIX E) was completed from the lesson observation for all main instructional tasks which made use of technology. The purpose of this instrument is to document the context in which technology was used which may be associated with the cognitive demand of the task within which it is used. Particularly noted are grouping strategies and the amount of freedom that students were given while using technological tools. The categories for the Technology Use Checklist are primarily drawn from the distinctions gleaned from studies reviewed in Chapter 2. Many of the distinctions are straightforward and dichotomous, such as whether or not technology is used, and whether or not students work in groups while using it. There are also a number of issues related to students' autonomy while using technological tools, including who directly manipulates the technology, whether or not the use of technological tools was initiated by the teacher or students, the amount of autonomy that students have with regard to their use of the technology while working on the task, and whether or not technology is used for exploratory or expressive activity (Doerr & Pratt, 2008; Zbiek et al., 2007).

Finally, whether or not technology is used as an amplifier, a reorganizer, both, or neither (Pea, 1985, 1987) was coded. As an amplifier, technology allows for more efficient execution of by-hand procedures, and as a reorganizer it has the potential to change the cognitive focus of the task, for example, by giving students access to mathematical concepts, representations, or behaviors that might otherwise be difficult or impossible. Technology can act as both an amplifier and a reorganizer when the purpose for offloading tedious or time consuming computations or constructions is for the express purpose of having students think about some aspect of the task that makes use of those results. For example, students might construct a triangle in a dynamic geometry software environment, and rotate and resize it by dragging to create numerous examples in order to investigate the relationship between the lengths of the sides of a triangle, i.e., the Triangle Inequality Theorem. The process of drawing numerous triangles is offloaded to the program so that students can focus on making observation, generalizations, and conjectures, and thus the use of the technology as an amplifier provides the basis for changing the focus of students' activity. Finally, if students are unable or unwilling to do with technology that which they could do by hand, such as creating and dragging triangles, then they will be unable to use it as a reorganizer as well. In such a case the code "neither" was used.

How the teacher plans and intends students to use technology, and what purpose it serves in the task, may not be the same as how students actually use it. In order to capture if and when such a difference existed, the use of technology as an amplifier, reorganizer, both, or neither was coded separately during both the set up and implementation phases. However, the coding of the use of technology was not intended to be a classroom level indicator of what students do with it. That is, technological tools may act as a reorganizer for some students' but not others, and in such a case the use of technology was coded as a reorganizer. The reason for using a different unit of analysis for the coding of technology use than for the coding of cognitive demand was so that the use of technology would not merely be redundant with the cognitive demand of the task.

For example, if the reorganizer use of technology simply became synonymous with high level cognitive demand, subtle distinctions with regard to how technology acted as a reorganizer might be lost.

Most of the distinctions made in the Technology Use Checklist are low inference decisions, and therefore reliability for these dimensions was unnecessary. However, the use of technology as an amplifier, reorganizer, both, or neither is a distinction which requires a degree of interpretation, and thus was double coded for reliability, and results are reported below.

3.3.6 Post-lesson interview

In many studies of students' use of technology in a classroom setting, the researchers were also the classroom instructors (e.g., Ben-Zvi, 2000; Chazan, 1999; Heid, 1988; Hoyles & Noss, 1992; McGraw & Grant, 2005; Schwarz & Hershkowitz, 1999). Much of the data collected in those studies was based on the researchers' knowledge of students' actions and thinking during their interactions with them as a teacher. As teachers have access to students' thinking via their interactions with students that may not be available to an observer, informal interviews were conducted after most lesson observations that allowed for a discussion of what the teacher noticed students doing or thinking about during the task. In addition, these informal interviews allowed for the opportunity to discuss teachers' expectations for students' engagement with a task, whether or not students met those expectations, the prior knowledge that students were expected to draw upon when engaging with the task, how a particular lesson is situated in the instructional unit or relates to previous or upcoming lessons, and teachers' beliefs regarding the purpose of using technology in a particular task, or in general.

The use of technology as an amplifier is defined by whether or not students could do by hand what they are using the technology to do. As this determination is relative to students' prior knowledge, this was inquired about as part of the post-lesson protocol. As a reorganizer, technology is used to change students' cognitive focus from whatever the technology is doing to some other goal. This includes the case where students are unable to perform by hand the procedures that the technology is being used for. For example, students may use a graphing calculator to find the equation of a line of best fit for a set of data, with the goal of the task being to have students understand how the equation models the context from which the data is taken. By contrast, technology can also be used as a reorganizer when students can perform the procedures by hand, but doing so would limit their ability to focus on the goal of the task. For example, students may use a graphing calculator to generate the graphs of five lines with different slopes and the same y-intercept in order to explore the connection between the equation of a line and its graph. For students who do not understand the meaning of slope, graphing these equations by hand will likely prevent them from achieving the goal of the task. Thus, a teacher's goal for a task, and the purpose of using technology, were also inquired about in the post-lesson protocol to assist in making this assessment.

The post-lesson interview also provides a venue for discussing and validating issues that may be identified by the researcher during data collection. In this way, the post-lesson interview serves as a form of member check (Elliott, Fischer, & Rennie, 1999). Regular post-lesson discussions with the teacher allowed for related pieces of information to be connected by the teacher, and to gain the teacher's perspective on themes or hypotheses that were identified the analytic memos. Insights into teachers' beliefs were an important factor in developing hypotheses and explanations for the results of the study as discussed in Chapter 5. Conversations with the teacher were conducted after each observed lesson, and were transcribed in a separate section at the end of the field note for a given observation. Occasionally the teacher would have another obligation which prevented an interview on a given day. Overall, ten interviews were conducted at Sites One and Two, 15 interviews at Site Three, and 17 interviews at Site Four. A protocol for the post-lesson interview is included in APPENDIX F.

3.3.7 Analytic Memos

A preliminary qualitative and integrative analysis began during data collection in the form of memoing or analytic memos (Emerson et al., 1995; Graue & Walsh, 1998; Strauss, 1987). These are records of the researcher's thoughts about how observations and interviews might be related to each other or to theory, and the possible implications of these. Analytic memos began to be written during data collection, and constituted a first layer of analysis in which the researcher reflected on what was being observed, made connections, and developed hypotheses. Memos were crucial to guiding future data collection as themes came into focus. Although there was a need to remain open to alternative interpretations and conflicting evidence, as well as other patterns or themes, memos helped to make the researcher more aware of certain behaviors or phenomena in the classroom. Thus, while a tension existed between being open and being focused during observations, in a context as complex as a classroom memos helped the researcher focus his attention along productive dimensions.

Memos also served as the beginning of qualitative analysis by allowing for ongoing identification of patterns or themes which might be systematically investigated. Indeed, during the data analysis phase, analytic memos served an important purpose by allowing the researcher to step back from fine-grained data analysis and document thoughts, connections, themes, or patterns that may exist across tasks or sites. Hence, while memoing began during data collection, it continued during the data analysis phase as a way to reflect on what was being noticed and to make connections between coding categories, observations, or data collection sites. Memos were the first opportunity to record thoughts about how the use of technology, the teacher's practice, and students' thinking were related. Ultimately, ideas or themes identified in analytic memos were connected and reviewed, and provided the basis for the discussion in Chapter 5. A sample analytic memo is included as APPENDIX G.

3.4 SAMPLE

3.4.1 Description of Sample

The unit of analysis for the present investigation is a mathematical task nested within a classroom. The kinds of tasks that were needed to answer the research questions posed required a purposeful selection of the four classrooms within which this sample was collected, which is described in the following. A primary purpose of the present study is to provide a fine-grained description of how the use of technology supports student thinking at a high level or may be associated with its decline. In order to determine the role that technology plays in this process, a significant proportion of tasks needed to be set up at a high level. Furthermore, it was important to observe tasks set up at a high level in which technology is not used in order to determine if there is a similar pattern of maintenance or decline between tasks that use technology and those that do not. Given the importance of these features of the sample to be collected, it was important to observe teachers who both use high level cognitive demand tasks with their students, and who use technology for instruction on a regular basis.

Among teachers' reasons for not using technology, a lack of knowledge and experience, and teachers' beliefs about the usefulness of technology for mathematics instruction have been cited (Manoucherhri, 1999; Norton et al., 2000). The choice of teachers who believe in and are using technology for instruction allows for an investigation into other factors which may be obstacles to the effective use of technology for mathematics instruction, such as curriculum, pacing, and assessment, as well as the beliefs and decisions of the teachers themselves.

Three of the four teachers who participated were identified through their association with a teacher preparation program which utilizes the *Mathematical Tasks Framework* (Stein et al., 2009) as part of the curriculum, and the fourth was identified by a contact familiar with this program. These teachers were hypothesized to have a greater capacity to identify, set up, and implement high level cognitive demand tasks than those who had not been exposed to this framework (Boston & Smith, 2009). With respect to technology use, teachers were recruited who believe in and have experience with using technology for instruction, identified either through the researcher's knowledge of them as former students in a course designed to prepare teachers to use technology for instruction, or through recruitment which identified technology use as an important focus of the study. Finally, participating teachers were observed for one to three lessons prior to data collection to help ensure that these features were present in their instruction.

3.4.2 Observational Period

The sample of mathematical tasks was collected in the classrooms of four teachers each teaching one instructional unit[9]. An instructional unit is taken to be any conceptually coherent set of lessons, such as a chapter from a textbook, and was identified by participating teachers. Teachers were asked to identify potential instructional units for observation from a single class (e.g., fourth period Geometry) which have a mix of high and low level tasks, as well as a mix of tasks which utilize technology and those that do not. There are three reasons that the sample of tasks were collected across a unit of instruction: the variation in cognitive demand and technology use that is anticipated to be present, the need to understand students' prior knowledge in assessing cognitive demand, and the necessity of understanding how a series of tasks is related.

It was important to collect a sample of tasks from each classroom in which the presence of technology in the observed lessons and the cognitive demands of the tasks vary in order to answer the research questions. The pilot studies and discussions with teachers indicated that an instructional unit will contain such variation, as within a unit students are often required to understand concepts, perform procedures proficiently, and memorize particular formulas or facts. Data collection at each site was scheduled so as to avoid overlap with other sites due to the time intensive nature of the data collection methods.

In order to accurately assess the cognitive demand of a task, one must be aware of students' prior knowledge. This is especially true when deciding whether a given task is a *procedures with connections* or a *doing mathematics* task, as the difference could simply be whether or not a procedure for solving the task had already been taught to students. For example, students in one class were observed to work on a page in their text titled "Inventing

[9] Including the data collected during the June pilot study.

Rules" in which they were asked to invent a method for solving proportions. However, during a previous lesson students had been taught a procedure for solving proportions, and this assignment was given as a way to practice the procedure. Without this information this task may have erroneously been coded as *doing mathematics* instead of *procedures without connections*. While not all of students' prior knowledge was able to be observed, discussions with the teacher will include inquiring into students' prior knowledge and the teacher's expectations for students' work on a task. However, observing a unit of instruction reduces the reliance on the teacher for making this assessment.

It was also important to know how lessons were related, or how teachers may build on lessons which incorporate technology. For example, in one classroom students engaged in a lesson using Geometer's Sketchpad (GSP) in which they had to determine the position of line segment connecting two sides of a triangle so that a smaller similar triangle would be formed within it. The teacher wanted students to notice that the line segment must be parallel to the third side of the triangle. A week later as students were independently preparing for an exam using a variety of worksheets from the past week, the teacher noticed that most students were still struggling with finding the missing side length in a triangle with a similar triangle inscribed within it. She discussed this difficulty in the context of a post-lesson discussion, which allowed for this difficulty to be connected back to the GSP activity which she used to introduce this idea, and to inquire with her about the relationship between the two, and the purpose of the GSP activity in this context. This was also the first opportunity to notice a disconnect between the kinds of tasks that students engaged in using GSP, and what they are held accountable for on worksheets and assessments. In short, observing a unit of instruction allows for the role of technology within a task to be analyzed from a broader context than a single task.

3.5 DATA COLLECTION

Data collection began in June of 2010 and concluded by the end of March 2011, with a total of 63 tasks observed at the four data collection sites. Figure 3.2 summarizes the number of tasks collected at each, and the data collection period.

	Site One	Site Two	Site Three	Site Four
Data collection period	June 7 – 30	September 27 – October 14	November 1 – December 3	January 25 – March 3
Number of tasks collected	12	17	17	17

Figure 3.2: The data collection period and number of tasks collected at each site.

Figure 3.3 depicts the data collection and generation process for a single lesson observation. Written jottings were recorded during the lesson, a post-lesson interview with the teacher was audio recorded after the lesson, and artifacts from the day's task, including student work, were collected when available. Generally, the construction of field notes occurred immediately after leaving the site, using the jottings and head notes taken during the observation. Coding of the main instructional task with respect to cognitive demand during set up and implementation, the factors associated with maintenance and decline (when applicable), and the use of technology via the Technology Use Checklist were completed in conjunction with field note generation. Analytic memos were generated as a way to document and reflect on the significance of new insights or connections within the data, across tasks or sites, or between the data and theory. As such, analytic memos were generated on an ongoing and irregular basis throughout the data collection process.

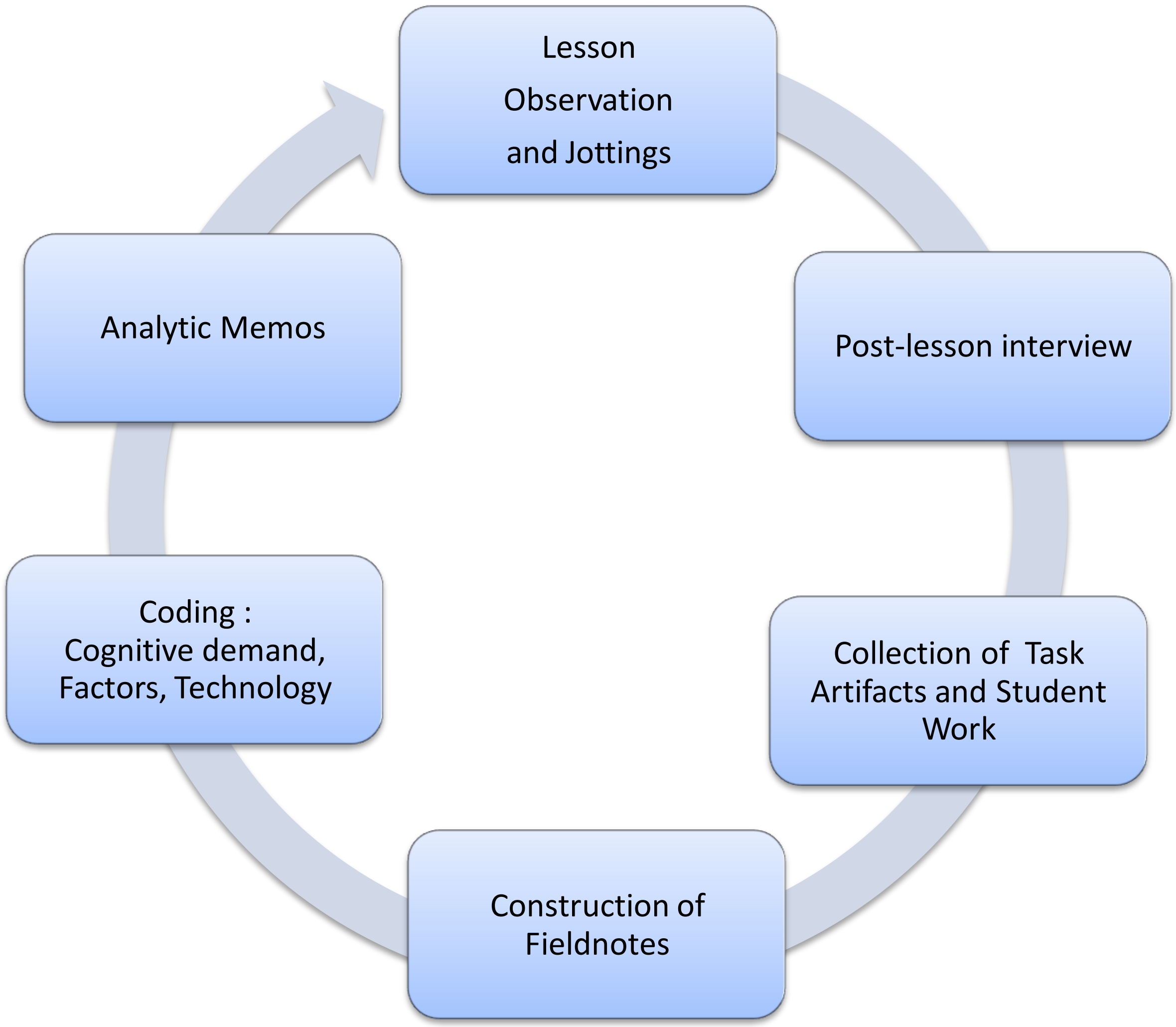


Figure 3.3: A depiction of the data collection process for a single observation.

One way to conceive of the process depicted in Figure 3.3 is that the data collection activities moved progressively from observation toward analysis. That is, the researcher's stance toward the phenomenon under investigation moves from one of an observer very close to the data, to an increasingly reflective stance, and comparisons and contrasts are made between tasks or classrooms. The patterns or themes which surface during the more analytic part of the process then help to guide subsequent observations and qualitative analysis.

3.6 DATA MANAGEMENT

Each task was imported into NVivo8 Qualitative Data Analysis software. That is, the field note text, and scanned task artifacts and student work were imported, and coded according to the site at which they were collected. Each site was created as a case in NVivo with certain properties corresponding to classroom factors such as students' academic level, class size, curriculum, teacher's degree and experience, instructional period length, school size, and course subject. An example is given in Figure 3.4.

Casebook

	A : Academic level	B : Class size	C : Curriculum	D : Degree	E : Instructional period length	F : School size	G : Subject	H : Teacher experience
1 : Cases\Site 1	Advanced	10-15	Traditional	PY	Regular	<500	Geometry	3
2 : Cases\Site 2	Regular	21-25	Traditional	MEd	Block	1000-1500	Geometry	3
3 : Cases\Site 3	Regular	16-20	Reformed	MAT	Regular	500-1000	Other	3
4 : Cases\Site 4	Regular	21-25	Reformed	MAT	Block	500-1000	Algebra	2

Figure 3.4: Each site created as a case with given attributes.

The reason for using NVivo for data analysis is that this is a qualitative study, and NVivo makes the analysis of qualitative data both more efficient and more productive. NVivo allows the researcher to employ a hierarchal node structure, so that sections of text can be coded, and all instances of a given code can be collected into a single document with references to the lesson observation field note in which it appears. The ability of this software to code text and images, and store those codes for later retrieval, is an example of how it can make qualitative data collection and analysis more efficient[10].

A way in which NVivo helped to make analysis more productive is by its ability to execute queries regarding the relationships and intersections between coding categories. For example, after coding all tasks as being set up at a low or high level during set up, as being implemented at a low or a high level, and whether or not the task included the use of technology

[10] One could say that NVivo is being used as an amplifier in this case.

or not, a query was run which returned all tasks which included the use of technology during the decline of a task during implementation that was set up at a high level, as shown in Figure 3.5.

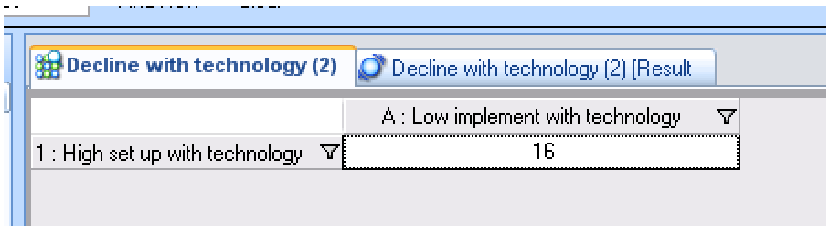


Figure 3.5: Frequency of the intersection of codes: tasks set up a high level, tasks implemented at a low, tasks that use technology.

Furthermore, clicking on the cell automatically displays a summary of all such tasks, as shown in Figure 3.6. Results can also be displayed as text for qualitative analysis, as in Figure 3.7. Queries such as this were used to execute coding contrasts, as described below, that were relevant for answering a given research question.

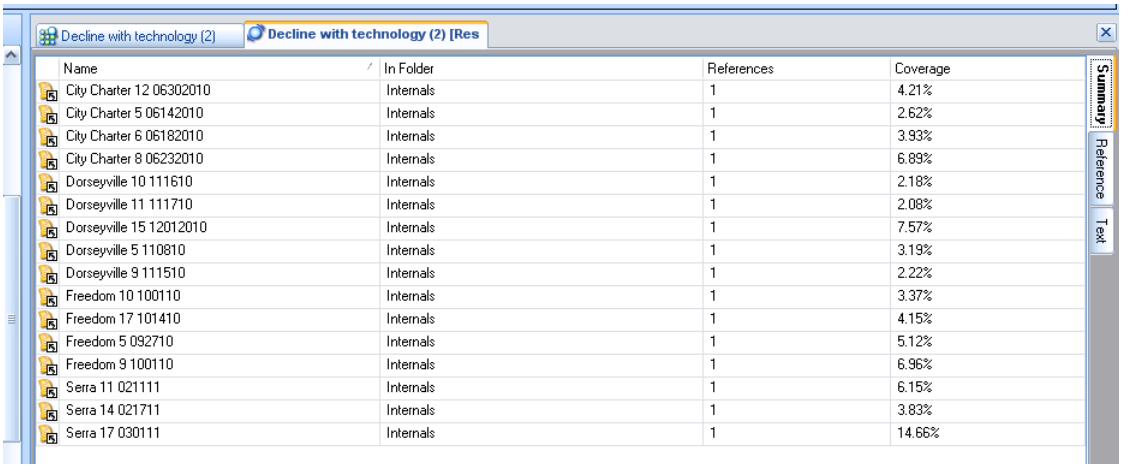


Figure 3.6: Clicking on the cell in Figure 3.5 returns a summary of all tasks that declined during implementation and used technology.

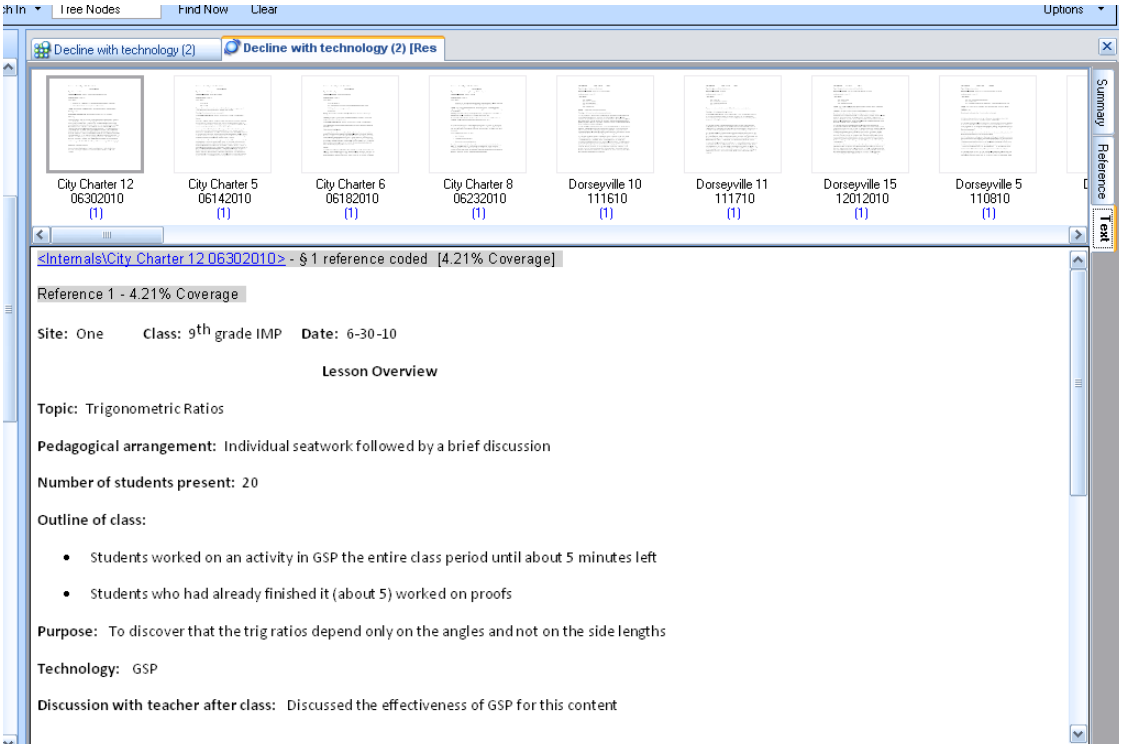


Figure 3.7: Coding query results can be displayed as text in NVivo.

Another important way that NVivo was used was to facilitate qualitative coding of the field notes and interview transcripts according to themes or ideas identified in the analytic memos. For example, the theoretical idea of instrumental genesis was identified during data collection as a potentially useful way to describe and explain the results. "Instrumental genesis" was created as a node in NVivo, and during qualitative analysis all specific instances that were considered to be associated with students' instrumental genesis were coded to this node so that all instances could be collected into a single document. Thus, what was an idea that emerged during data collection was rigorously and systematically investigated, including the identification of disconfirming evidence, and was related to other elements of the deductive framework, such as the factors associated with maintenance or decline of cognitive demand during implementation.

3.7 DATA CODING

3.7.1 Cognitive Demand

The Task Analysis Guide (Stein et al., 2009) provided as APPENDIX A was used to code tasks with respect to the cognitive demand in curricular materials and during the set up and implementation phases. In particular, tasks were coded as involving *memorization*, *procedures without connections*, *procedures with connections*, or *doing mathematics* within curricular materials, and during set up and implementation (see Figure 3.8).

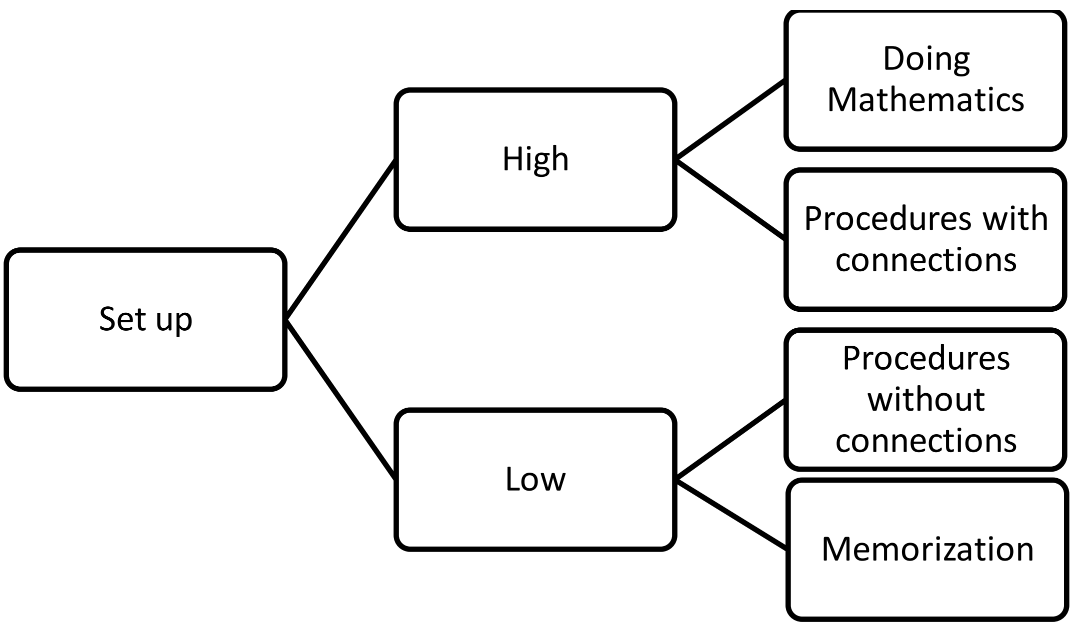


Figure 3.8: Coding for cognitive demand during set up.

For the purposes of the present study, curricular materials included task artifacts such as copies of PowerPoint slides, worksheets, or activity handouts which include tasks that students engaged with, and these were traced back to a commercially published curriculum, textbook, or other resources when possible. These teachers generally used their textbook and support materials as a scope and sequence guide, having created a sequence of lectures and activities that have some overlap with the topics in their text. However, this was not a process that took place during the data collection period. Part of the criteria for selecting a class to observe was that it was not the first time the teacher was teaching the observed course, and thus the process of creating the set of materials that was collected took place prior to data collection.

The question arises as to whether or not written materials that were written or created by the teacher are considered "curricular materials," or only materials that appear in a commercially published textbook or teacher support materials. The answer to this question has direct bearing upon the answer to Research Question 1a. For example, Ms. Lowe created a GeoGebra activity with a handout to guide students through the task. The mathematical topic, perpendicular bisectors of a triangle, and their intersection, the circumcenter, is covered in the Holt Geometry textbook which was the text she used for this course. If the handout created by Ms. Lowe is considered the curricular materials, then the curricular materials include the use of technology, but if the section of the text that covers perpendicular bisectors and the circumcenter is considered the curricular materials, then it does not include the use of technology.

Ultimately, the notion of mathematical task was used to resolve the issue. That is, a mathematical task is something that students engage with or "do" in some way. The section of the Holt Geometry text described above did not ask students to "do" anything; it was merely a description of theorems and proofs, and applications in example problems, and thus was not a "task" per se. Teachers may include the statement of some of these example problems in a PowerPoint lecture with the expectation that students solve them as part of lecture, and at this point it would be considered an instructional task because students were being asked to do something. In this case the PowerPoint slides would be considered the curricular materials. In general, the mathematical task as it appears in curricular materials was traced back to the written materials that asked students to do something. In some cases, such as the Interactive Mathematics Project or Connected Mathematics Project curricula, the task appeared in the textbook, and in these cases this is what was coded.

During the set up phase the cognitive demand was measured in terms of the potential best case scenario for the type of thinking students would engage in while working on the task as announced by the teacher. Field notes from classroom observations and task artifacts were analyzed for the set up of the main instructional task of a lesson in order to evaluate the cognitive demand of the task during the set up phase. As the set up of a task includes expectations for how students are to engage in the task, including what resources they have available while working on the task, the use of technology during set up was coded either if it was actually used during the set up of the task, or if its use during the implementation phase was suggested, required, or implied during the set up phase, including expectations for how it would be used. This is contrasted from tasks in which the use of technology was not suggested, required, or implied during the set up phase, which were coded as not using technology during the set up phase.

The Task Analysis Guide was used to code the task during set up and implementation (Stein et al., 2009) based on the thinking expected of or engaged in by the majority of students for a majority of the task, as represented by the lesson observation field notes and task artifacts, and triangulated by student work (when available). First, the task as set up was assessed with regard to whether it was low level or high level. For example, if students were being asked only to recall previous knowledge, such as identifying angle relations, or practice a previously learned procedure, such as solving equations by isolating the variable, then the task was most likely considered low level, *memorization* or *procedures without connections*, respectively. If students were asked to discover a formula or procedure that they did not already know or make connections between representations, or were asked to engage in open-ended problem solving, then the task was most likely high level, *procedures with connections* or *doing mathematics*, respectively. During implementation, the focus was on students' engagement with the task. For example, while general procedures may be followed, were students making some cognitive effort while working on the task, or were they asking a classmate or the teacher for help before making any effort? Students' conversations with the teacher or classmates were revealing of whether student were trying to make the required conceptual connections, or whether they were looking for a shortcut or the answer. Less attention was paid to the correctness of a student's answer than the effort they were making to understand the connections intended by the task. For example, were students using multiple representations, and focused on interpreting one representation in terms of another? A detailed example of how students' activity while engaging with a task was interpreted using the Task Analysis Guide is provided as APPENDIX B.

Two additional codes, unsystematic exploration and no mathematical activity, were used in evaluating the cognitive demand during implementation. Unsystematic exploration is defined as a manner of implementing a task "in which students explored around the edges of significant mathematical ideas but failed to make systematic and sustained progress in developing mathematical strategies or understandings" (Henningsen & Stein, 1997, p. 532), while no mathematical activity refers a situation in which students are off task or engaged with non-mathematical aspects of the task (Stein et al., 2009). As Figure 3.9 demonstrates, the coding of cognitive demand of the task during implementation followed the evaluation of the same during set up. Thus, the first rectangle in Figure 3.9 is one of the rectangles on the far right of Figure

3.8 This is important to note as whether or not the cognitive demand of the task is reduced or upheld during implementation depends on how the task was set up.

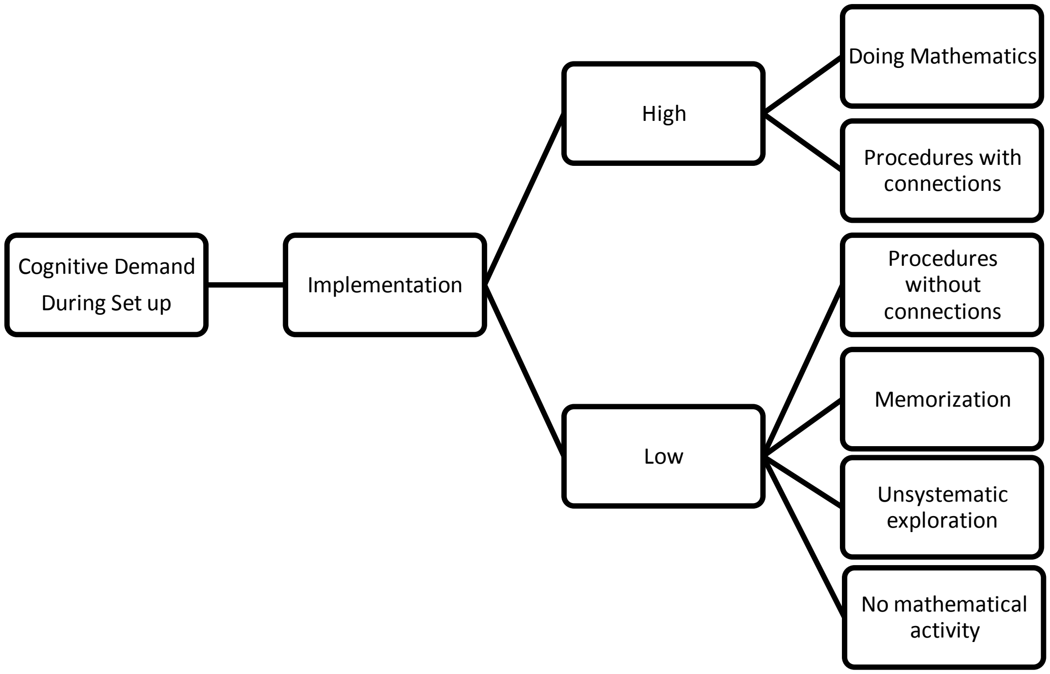


Figure 3.9: Coding for cognitive demand implementation.

3.7.2 Classroom-Based Factors Associated with Maintenance or Decline

In addition to coding the type of thinking that students engaged in, tasks set up at a high level were coded with regard to the classroom-based factors associated with the maintenance or decline of the cognitive demand (Henningsen & Stein, 1997; Stein et al., 1996), as depicted in Figure 3.10 (the list of factors can be found in APPENDIX C). Whether or not the cognitive demand was sustained or reduced was first determined for each task set up at a high level. If the cognitive demand was maintained, the task was coded with as many factors as applied from a list of factors associated with maintenance; likewise, for tasks that declined during implementation, factors associated with decline were coded. This procedure was used for this purpose in previous studies which sought to determine the classroom-based factors associated with maintenance or decline of cognitive demand during implementation (Henningsen & Stein, 1997; Stein et al., 1996).

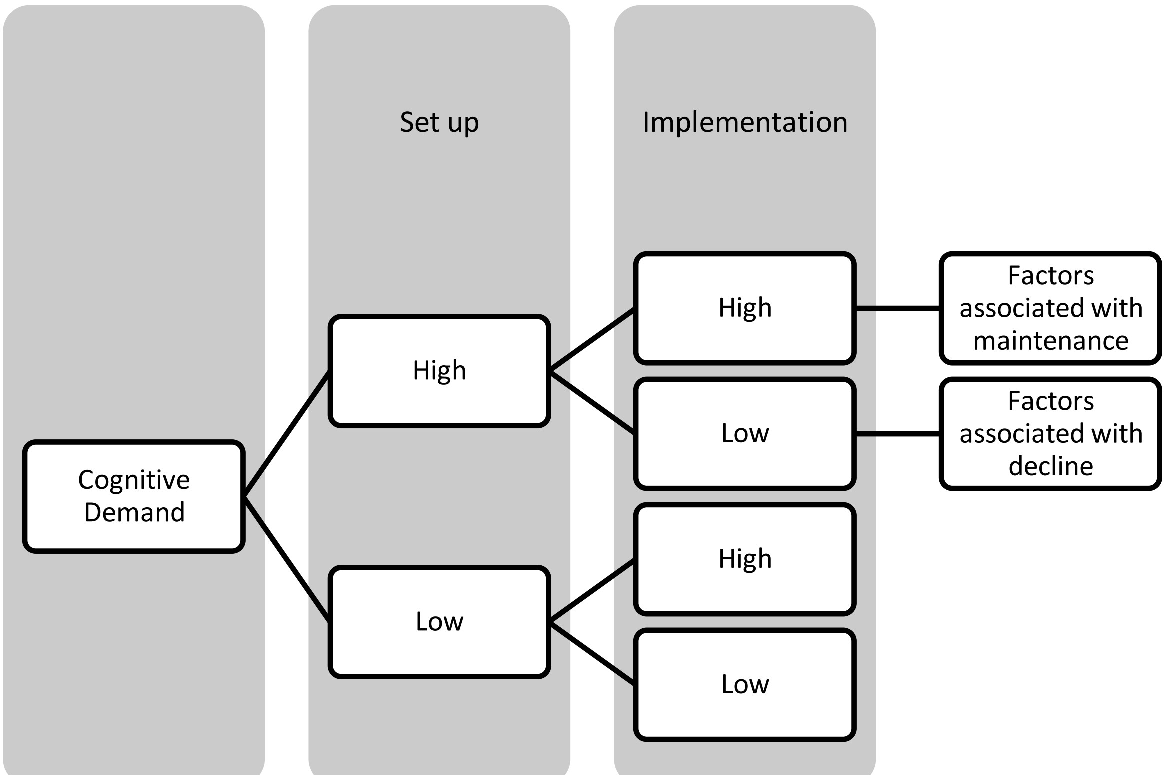


Figure 3.10: Coding of classroom-based factors associated with maintenance or decline.

3.7.3 Technology Use

Each task that made use of technology by the teacher or students was coded using the Technology Use Checklist (APPENDIX E). Furthermore, each use of technology was coded with respect to how the technology was used. For example, if within the same task the teacher used the IWB for a PowerPoint lecture with embedded practice problems, and students used calculators while working on the practice problems, then each of use of technology was coded separately using the Technology Use Checklist.

Each main instructional task which made use of technology was coded in NVivo using the same categories as those which appear in the Technology Use Checklist. The purpose of coding these tasks in NVivo was to be able to systematically examine the use of technology in relation to the cognitive demands of the task during set up and implementation, and in the case of tasks set up at a high level, factors associated with maintenance or decline. For example, after identifying all tasks set up at a high level which declined during implementation that made use of technology, a further query was used to identify which tasks made use of technology as an amplifier and which made use of technology as a reorganizer.

Codes were used for qualitative analysis that were not part of the deductive framework as well. One example is given above with regard to the idea of instrumental genesis. Another example is that of the teacher not using technology for the whole class discussion of a task which incorporated the use of technology by students, which was referred to as "mode-switching." In systematically reviewing lesson observation field notes, it was verified that this teacher switched mediums during the whole class discussion, using meter sticks or dry erase markers to create angles, for example, in three out of the four lessons for which technology played a central role in the task. Furthermore, analysis of the field notes revealed that in subsequent discussions of the mathematical idea which was the goal of the task, the teacher referred to the non-technology demonstration ("that's like the meter stick example") rather than anything students had done using the technology. Such codes often emerged as themes or patterns in the analytic memos, and were subsequently used to analyze the data qualitatively.

3.7.4 Reliability

To ensure the validity and reliability of the field notes in capturing those dimensions of classroom instruction and students' thinking necessary to make evaluations of cognitive demand during set up and implementation, and factors associated with maintenance or decline, and how technology was used during set up and implementation, two reliability coders were employed. With the exception of Site One, a coder accompanied the researcher to lesson observations, and both the observer and researcher coded these dimensions directly from the observation prior to the generation of field notes. After field notes were constructed, a second coder coded the task from the field notes. The roles of these coders were exchanged on a regular basis, with each coding approximately the same number of tasks from observations and from field notes.

These two coders were trained separately. The first reliability coder, who was the only coder for Site One, was already familiar with the Task Analysis Guide, the *Mathematical Tasks Framework*, and the factors associated with maintenance and decline (Stein et al., 2009). The distinction between the use of technology as an amplifier and a reorganizer was described and discussed with the researcher. In order to ensure that these frameworks and definitions were being interpreted and applied consistently, both the researcher and the reliability coder coded three lessons, with the reliability coder using the lesson observation field notes. Discrepancies were discussed and resolved after the coding of each lesson. The second field note coder was less familiar with the *Mathematical Tasks Framework*. After reading descriptions of the Task Analysis Guide, the *Mathematical Tasks Framework*, and the factors associated with maintenance and decline (Stein et al., 2009), sample tasks provided in this resource were coded and discussed with the researcher prior to coding the same three lessons from the field notes as the first reliability coder. After discussing and resolving discrepancies after the coding of each lesson, a fourth lesson was coded in order to achieve a higher proportion of agreement.

Table 3.1: The number and percent of tasks coded for reliability in each classroom.

	Site One	Site Two	Site Three	Site Four	Total
Number (percent) of tasks coded by an observer	0 (0%)	6 (35%)	4 (24%)	5 (29%)	15 (24%)
Number (percent) of tasks coded from the field note	4 (33%)	6 (35%)	7 (41%)	5 (29%)	22 (35%)

Including Site One, 24% of the 63 observed tasks were coded by a second observer of the lesson (besides the researcher); the percent of tasks observed by a second observer at each site is given in Table 3.1. Approximately 35% of the observed tasks were coded for reliability from the field notes, with the percentage at each site given in Table 3.1. There are two reasons for the discrepancy between the number of tasks coded for reliability by an observer, and the number coded by a field note coder. The first is that Site One was coded for reliability only from the field notes. The second is that three additional tasks were coded from Site Three from the field notes in attempt to increase the agreement between the researcher and the field note coder with respect to coding the cognitive demand at four levels (*memorization*, *procedures without connections*, *procedures with connections*, *doing mathematics*) during the three phases of implementation. Site Three was chosen for additional coding as originally it had had the smallest percentage of tasks coded for reliability.

Each task was coded with respect to cognitive demand during the three phases of implementation: curricular materials, set up, and implementation. The Task Analysis Guide (Stein et al., 2009) was used to code the cognitive demand as *memorization*, *procedures without connections*, *procedures with connections*, or *doing mathematics*, and the percent agreement with the lesson observer and field note coder is given in Table 3.2. As noted in the limitations section, the agreement with the field note coder was much lower than with the lesson observer. The agreement with the field note coder was much higher (83%) with regard to whether a given task was high or low level during the three phases of implementation, indicating that many of the discrepancies were distinctions between whether a task was a *memorization* or *procedures without connections* task, or whether it was a *procedures with connections* or *doing mathematics* task. Discrepancies between low and high level tasks were almost always with regard to whether a given task was *procedures with connections* or *procedures without connections*. All discrepancies with both coders were resolved and the consensus code was assigned to the task.

Table 3.2: Percent agreement in coding of cognitive demand.

	Observer	Field note coder
Coding of cognitive demand (*memorization*, procedures without connections, procedures with connections, doing mathematics)	93%	72%
Coding of cognitive demand (high, low)	98%	83%

Each task observed by a second observer set up at a high level of cognitive demand was coded by the primary investigator and both reliability coders using the list of classroom-based factors associated with the decline of cognitive demand during implementation given in APPENDIX C. Agreement was around 80% with both the lesson observer and the field note coder, as shown in Table 3.3. The general nature of the discrepancies was that each of the coders coded the same behavior associated with decline, but interpreted them differently with regard to the list of factors. For example, what one coder called insufficient time the other called reducing the complexity of the task, as the teacher did not give students time to explore or investigate the concepts under consideration, but while roaming the classroom and working with individual students she reduced the task to a series of short questions and answers in order to ensure their progress on the task. These discrepancies provided the opportunity for the researcher and coders to clarify the type of behavior each factor refers to.

Table 3.3: Percent agreement in coding of factors associated with decline or maintenance.

	Observer	Field note coder
Coding of classroom-based factors associated with decline or maintenance	80%	78%

For each task that was coded for reliability which included the use of technology, each use of technology was coded with respect to how the technology was used along the dimensions identified in the Technology Use Checklist. Most of these dimensions are low inference judgments, such as who manipulates the technology, and how students are grouped while technology is being used. Thus, the only dimensions which for which agreement was computed was the use of technology as an amplifier, reorganizer, both, or neither during the set up and implementation phases. The agreement was fairly high and exactly the same with the lesson observer and the field note coder, as shown in Table 3.4.

Table 3.4: Percent agreement in coding of technology use.

	Observer	Field note coder
Coding of the use of technology (amplifier, reorganizer, both, neither)	86%	86%

3.8 ANALYSIS

3.8.1 Quantitative Coding Contrasts

This section describes how data coded using the deductive framework were analyzed. As all of the research questions investigate the use of technology in relation to the thinking demands of the tasks in which they are situated, contrasts using the data coding described above were used to examine quantitative patterns in the data. These contrasts provided preliminary answers to some of the research questions by revealing patterns of use which may warrant closer examination via qualitative analysis. The contrasts that were used are discussed below in relation to the research questions that they are intended to answer. Although cognitive demand was coded at four levels during data collection (*memorization*, *procedures without connections*, *procedures with connections*, *doing mathematics*), these were collapsed to two categories (high and low) for analysis.

Research Question One. How do the cognitive demands of mathematical tasks differ when technology is used as part of the task and when it is not?

a. How is the use of technology associated with the cognitive demand of mathematical tasks as they appear in curricular materials?
b. How is the use of technology associated with the cognitive demand of mathematical tasks as set up by the teacher?
c. How is the use of technology associated with the cognitive demand of mathematical tasks as implemented?

In order to answer Research Question One, a contrast was used that considered the cognitive demand of the task (high or low) and whether or not technology was used as part of the task (yes or no), resulting in a frequency distribution table. These frequencies were computed separately for the curricular materials, set up, and implementation phases of the task at each site in order to answer each subquestion of Research Question One. Table 3.5, Table 3.6, and Table 3.7 demonstrate these contrasts.

Table 3.5: Cognitive Demand as it appears in curricular materials vs. Technology Use.

Curricular Materials	Technology		
		Yes	No
Cognitive Demand	High		
	Low		

Table 3.6: Cognitive Demand during set up vs. Technology Use.

Set Up	Technology		
		Yes	No
Cognitive Demand	High		
	Low		

Table 3.7: Cognitive Demand during implementation vs. Technology Use.

Implemented	Technology		
		Yes	No
Cognitive Demand	High		
	Low		

These contrasts allowed for patterns to be observed between technology use and the kinds of tasks that are used in each classroom. The tables were reviewed both for patterns within each classroom, as well as comparisons across classrooms. Given the lack independence of tasks observed at a single site, and the important differences in numerous contextual factors between sites, it was inappropriate to group all the tasks across sites into a single sample for this analysis. Although the proportion of tasks using or not using technology within high and low level tasks is reported for the entire sample, it is important to interpret these proportions in the context of the individual classrooms as aggregating the data has the potential to mask importance differences in the way technology was used at each site. For example, one teacher was observed to set up tasks at a high level only when technology was used. Another teacher set up more tasks at a low level using technology, thus combining the tasks observed at these two sites might give the false impression of a fairly even distribution of high and low level tasks which make use of technology during set up. In order to answer the research questions, similarities and differences between sites were identified in the coding contrasts and investigated qualitatively.

Research Question Two. How does the role of technology differ in low level and high level cognitive demand tasks? What is the role of technology in each?

a. During Set Up

   What are the features or characteristics of technology use associated with tasks set up at a low level of cognitive demand?

   What are the features or characteristics of technology use associated with tasks set up at a high level of cognitive demand?

b. During implementation

   What are the features or characteristics of technology use associated with tasks implemented at a low level of cognitive demand?

   What are the features or characteristics of technology use associated with tasks implemented at a high level of cognitive demand?

Table 3.8 displays the primary contrast that was conducted in answering Research Question Two, and was conducted for the set up and implementation phases of task enactment. The results of these contrasts were instrumental in providing a basis for qualitative analysis by grouping tasks along the dimensions of cognitive demand and technology. For example, by grouping all low levels tasks that used technology as an amplifier, this subsample of tasks was analyzed with respect to the role that technology played in these tasks, including if there was a general pattern, whether there were any exceptions to that pattern, and what the nature of those exceptions were.

Table 3.8: Association of technology use and the cognitive demand of the task during set up.

Technology Use during Set Up									
		Site One		Site Two		Site Three		Site Four	
	Cognitive Demand	High	Low	High	Low	High	Low	High	Low
Technology Use	Amplifier								
	Reorganizer								
	Both								
	Neither								

Another contrast that was conducted examined the associations between technology use and the type of technology used. The contrast displayed in Table 3.9 was used to investigate how the use of technology as an amplifier during set up was related to the type of technology. This contrast was executed in order to better understand how the technological tools available were used by these teachers and students.

Table 3.9: Amplifier use of technology during the set up of tasks at a low level.

	Site One	Site Two	Site Three	Site Four
IWB				
Calculator				
DGS				

Qualitative analysis of tasks in relation to Research Question Two was also necessary in order to identify dimensions of technology use in relation to the type of thinking students do while engaging with a task. This included gaining a deeper understanding of how technology acts as an amplifier or reorganizer in these tasks by identifying specific qualitative examples, as well as identifying other roles that technology might play with regard to cognitive demand that is not captured by the amplifier and reorganizer distinction.

Research Question Three. How does the use of technology impact the cognitive demand of a task during implementation?

a. How is the use of technology related to factors which have been associated with decline of mathematical tasks set up at a high level of cognitive demand?
b. How is the use of technology related to factors which have been associated with maintenance of mathematical tasks set up at a high level of cognitive demand?

Analysis for Research Question Three was limited to tasks that were set up at a high level. For each classroom, coding results were used to identify tasks which were implemented at a high level and those which declined to a low level during implementation, both with and without the use of technology. The first step in this analysis was a coding contrast used to examine patterns of maintenance or decline within and across classrooms. The results of coding were summarized in Table 3.10 in order to examine this pattern.

Table 3.10: The association of technology and student engagement in tasks set up at a high level.

	Declined	Maintained	Total
Site One			
Site Two			
Site Three			
Site Four			
Total			

In order to investigate the factors that were associated with the decline during implementation of tasks set up at a high level, the percent of tasks which declined during implementation that had a particular factor present were compiled by factor and by teacher, as in Figure 3.11. As only one task was set up at a high level which did not make use of technology during the set up or implementation phases, no comparable summary was made for the decline of tasks which did not use technology.

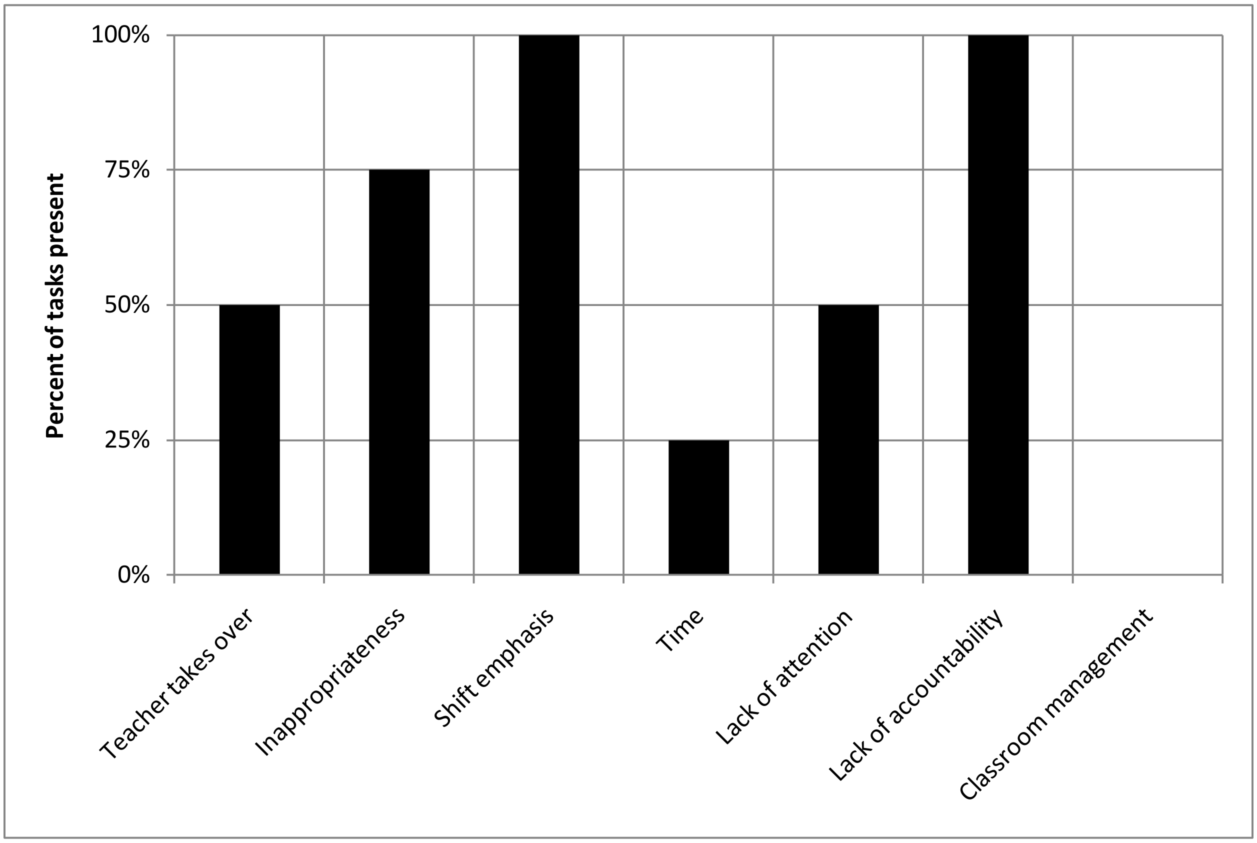


Figure 3.11: Factors associated with decline of tasks set up at a high level using technology.

Likewise, the results of coding of factors associated with maintenance were compiled in order to examine the patterns within and across classrooms as shown in Figure 3.12. This analysis was also limited to tasks which used technology as only one task was implemented at a high level that did not use technology during set up or implementation Once these factors were summarized and compiled, qualitative analysis focused on those factors associated with maintenance or decline which were most closely related to the use of technology, as this is the focus of Research Question Three.

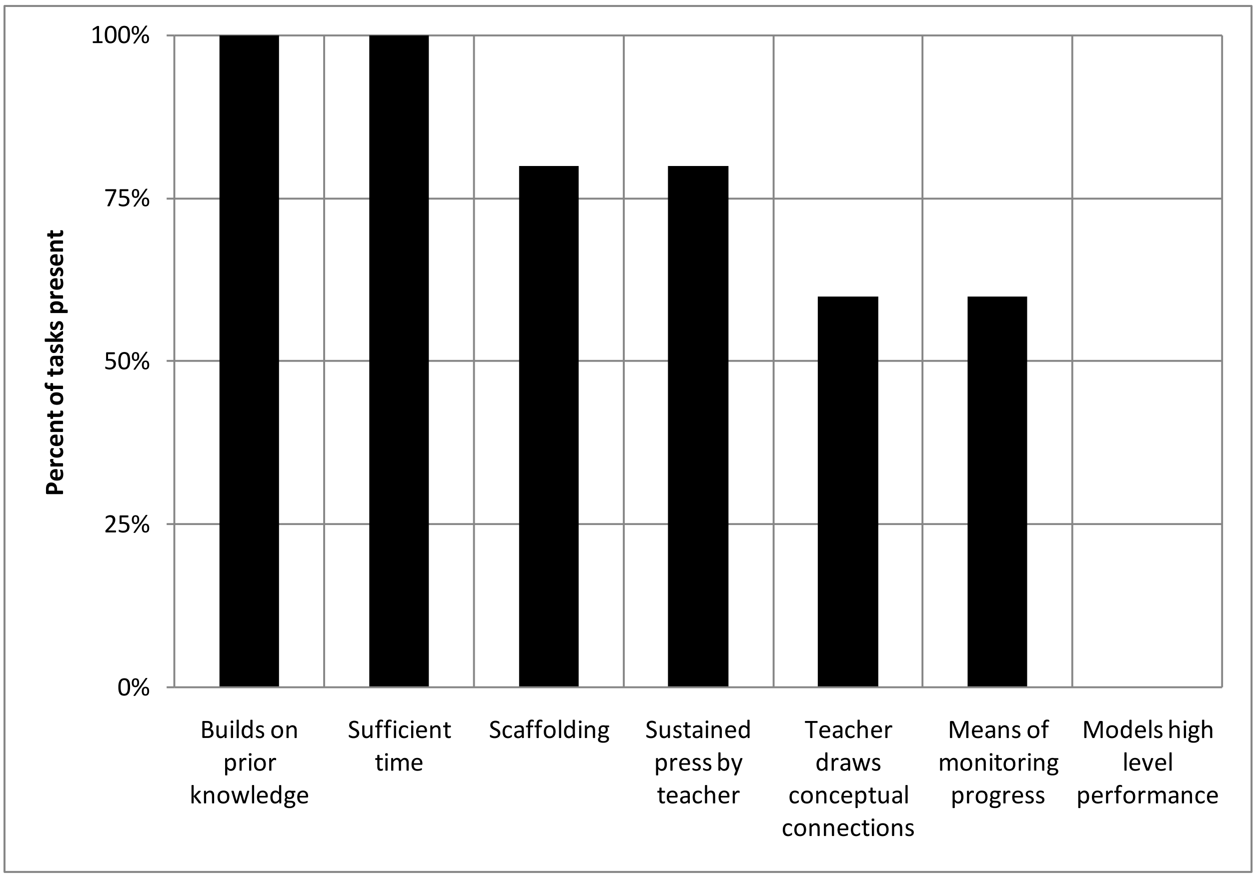


Figure 3.12: Factors associated with maintenance of tasks set up at a high level using technology.

Qualitative analysis of the factors associated with decline focused on those factors that were most related to issues related to students' use of technology, versus factors that were more related to how the teacher reacted to these issues. For example, the teacher shifting the emphasis to getting the correct answer, or taking over the demanding aspects of the task for the students, while not uncommon when students engaged with a high level task using technology, are more about the teacher's response to an issue related to students' use of the technology. Thus, the primary criterion used in selecting factors to focus on for qualitative analysis was, "which factors seem unique to the technological context of the task?," as these were considered to have the greatest to potential to make a new contribution to what is known about classroom-based factors related to students' high level engagement. This is not to say that these factors were the most influential in the decline of these tasks, but rather they involved students doing something with the technology that was unexpected, or was related to their engagement with the task as they used technology. In considering which factors that were associated with maintenance to focus on for qualitative analysis, the question was less about the factors and more about the teacher, as only one teacher implemented any tasks at a high level which used technology as a reorganizer. Thus, the focus of qualitative analysis was on those factors that were most instrumental to the high level demands of the task during implementation. A fuller description of how these factors were related to students' use of technology is given in the analysis of these factors in Section 4.4.

3.8.2 Qualitative Analysis

The purpose of the qualitative data analysis was to analyze in detail the processes by which the use of technological tools for instruction influenced the cognitive demand of tasks, and to provide rich examples of how this happened in the context of classroom instruction. While the results of the quantitative analysis provided patterns of association related to the research questions, qualitative analysis allowed for some explanation of these patterns, and description and generalization across examples of the similarities and differences of tasks that are part of this pattern. Furthermore, qualitative analysis provided greater insight into potential causal connections between the use of technology for instruction and the cognitive demands of the tasks, as opposed to mere associations identified by the quantitative analysis.

The qualitative analysis was an iterative and non-linear process, not beginning or ending at any definite points in the data collection and analysis cycle. This analysis was both deductive and inductive. The deductive element consisted of investigating the results of the quantitative analysis which utilized the deductive coding framework to provide further insight and explanations for those results. A separate, inductive analysis examined the data apart from the coding framework, in light of the purpose of the study and the research questions posed, to identify themes, patterns, and explanations relating the use of technology to the cognitive demands of the tasks within which it situated.

Qualitative data analysis began during data collection via analytic memos as described above, a process which itself was iterative and non-linear, spanning data collection and analysis. Field notes were coded using the deductive framework provided by the *Mathematical Tasks Framework* and the Technology Use Checklist, with coding contrasts executed to identify patterns in the data. The analytic memos constituted the beginning of the inductive analysis by identifying themes or ideas during data collection that may be relevant to answering the research questions or explaining the results. In this way, the analytic memos and field note coding bridged data collection and analysis, providing deductive patterns and inductive themes to be investigated more in depth qualitatively.

Codes for the conceptual categories identified in the analytic memos were created via tree nodes within NVivo, and field notes, including post-lesson interviews, analyzed for instances of them. Coding was more general at first, with codes being refined or re-defined in order to better describe the data. Figure 3.13 displays a screenshot from NVivo. A portion of the hierarchical node structure in NVivo that is the result of the inductive coding process appears in the pane on the left, with the text of one of the field notes on the right. To code a text passage, it is highlighted using the mouse and "dragged and dropped" into the appropriate node on the left. A given passage can be coded at as many nodes as are relevant. When coding is complete, all instances of a code can be collected into a single document, and passages of text that have been coded with more than one code can be identified using a query if such an intersection is deemed to be potentially important.

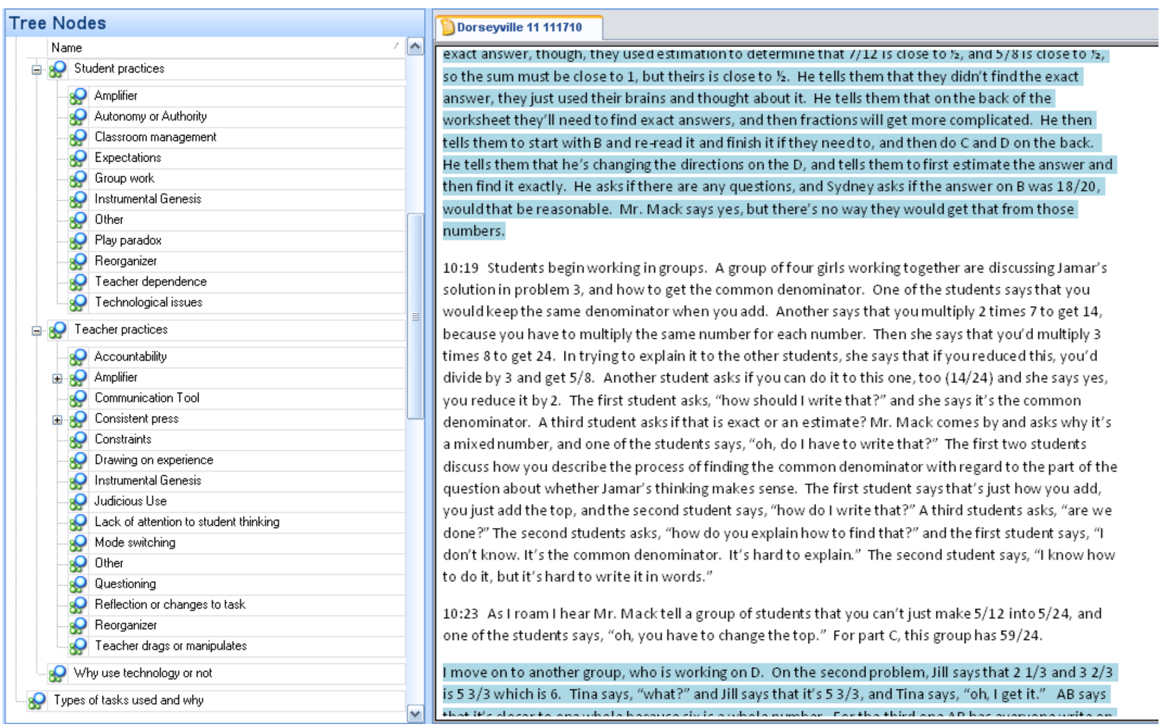


Figure 3.13: Qualitative coding of field notes in NVivo.

Only a portion of the node tree is displayed in Figure 3.13, and this portion was selected to demonstrate the types of codes that were created during the data collection and analysis process. This node tree began with nodes that had been identified during the data collection process, either from the deductive framework or through the analytic memos. For example, amplifier and reorganizer were part of the original node tree taken from the coding framework. While tasks had been coded as using technology as an amplifier, reorganizer, both, or neither, the purpose of the qualitative analysis was to identify specific examples in the field notes of technology being used in this way by students or the teacher. These examples were referred to later in consider the role of technology in the set up and implementation of high and low level tasks (Research Question Two).

Examples of inductive codes that were identified in the analytic memos that were created as part of the original node tree were the ideas of instrumental genesis, mode-switching, and accountability. These refer to how students construct meaning for and with tools, using a different medium for the discussion of a task than students used while working on the task, and how students were held accountable for their work using technology, respectively. Each of these ideas seemed relevant to answering the research questions, although it was not yet clear what their significance was. By making these ideas part of node tree, instances of these ideas in the field notes were able to be coded qualitatively, collected, and examined across tasks and sites, and refined. Throughout this process, the question of how these instances may be related to the research questions and the theoretical framework were considered.

As the inductive qualitative analysis was a "bottom- up" process in the sense that one starts with the data and creates nodes as patterns and ideas emerge, it was necessary to group and restructure the node tree throughout this process. As Figure 3.13 shows, two conceptual categories that were used to group individual codes were "student practices" and "teacher practices." Many of the individual codes fell into one of these two categories, but others did not, such as "why use technology or not," which dealt more with teachers' beliefs than their practices, and was primarily used when coding post-lesson interviews with the teacher.

During the coding process, codes were created for behaviors or trends that were related to the purpose of the study, even if how that code may or may not provide an answer to a specific research question. The researcher made an explicit attempt during the inductive qualitative analysis to place a priority on attending to the data in terms of the broad ideas of technology use and mathematical thinking and behaviors rather than on making the data fit into the deductive framework. In many cases, interesting examples or insights did not relate directly to the current study, but may be pursued in a subsequent analysis or study. NVivo allowed for such instances to be captured, stored, quantified, and related to other cases or elements of the framework in order to determine their relevance and significance. Ultimately, most the codes created during the inductive qualitative analysis were not explicitly used in the results or discussion sections of the study, but helped to develop and inform the thinking of the researcher with respect to the research questions and results.

Another methodological tool utilized during the inductive field note coding was the ability to annotate field notes in NVivo by highlighting a section of text and making a comment about its meaning or significance. For example, in Figure 3.13, part of the text is highlighted in blue, indicating that an annotation has been made referring to that particular passage of text. Annotations were generally made whenever the researcher had an insight into the significance of a passage of text at a grain size that was more specific than the codes in the node tree. For example, in the passage depicted in Figure 3.13, a portion of text was coded as "decline," indicating that something specific happened at that point which was considered to lower the cognitive demand of the task during implementation. In addition, the following annotation was made: "In lieu of thinking about the situation, Mr. Mack encourages students to look for key words. There is no evidence that most students understood the answer to this question." This captures and preserves the specific reason that this passage was coded as “decline.” In this example, the annotation related to how the passage was coded, but this was not always the case. At other times annotations were not necessarily related to or a more specific explanation for how a passage was coded. In general, annotations were used to make notes about specific passages of text from the field notes or interviews that were not general enough to be given their own code, and to document insights into the significance of a given excerpt.

While annotations were used to document thoughts or insights at a grain size finer than the codes in the node tree, analytic memos provided the opportunity to document insights at a larger grain size than a code in the node tree. In this way the purpose of the analytic memos shifted slightly from data collection to data analysis. During data collection, analytic memos were used to step back from the data being collected in order to make hypotheses about the significance of what is being observed. During analysis and coding, analytic memos were used to step back from the data to make connections among isolated codes or ideas, either within the node tree, or across sites. They were used both to distill insights from the coding process, and to raise awareness of how codes might be related. Below is an example of a memo with regard to fostering instrumental genesis. Up to this point in the analysis process the idea of instrumental genesis was salient as a code, but less thought had been given to how to foster it, or whether there was any evidence or examples in the data that might provide insight into the answer to this question.

> There is a contrast between Ms. Jones, who took a very "hands-off" approach to students' use of technology, and Ms. Lowe who was constantly looking over students' shoulders, ensuring that students were constructing their figures correctly and dragging them, and asking them questions about "what does it mean"

> Having students construct figures versus manipulate figures may be an important factor in fostering instrumental genesis for two reasons: 1) it gives students more freedom, 2) it allows them to become familiar with the tool. Developing meaning for a tool by using its affordances to accomplish a goal goes much farther in developing instrumental genesis than simply having a student manipulate a figure that someone else made. In a sense, this isn't a tool at all. It may be important to have students use the tool in context. However, this may not be enough, as both Ms. Jones and Ms. Lowe did this, but only the latter successfully. It's interesting that Ms. Jones and Ms. Young struggle to get students to follow directions, while Ms. Lowe wants students to go beyond following directions (see Serra 6). (Analytic memo, 4/30/11)

While this memo was created on the date indicated, other insights were added to it during the analysis process, and thus it was not written all at once. In other cases and memo was created on a given topic that was not added to after the original entry. This process highlights the inductive process in which it is not possible to know which insights will ultimately be most significant or relevant to answering the research questions or explaining the results.

The purpose of the present section has been to provide insight into the process by which the results of the quantitative analysis were used to guide the qualitative analysis, as well as how the inductive component of the qualitative analysis was conducted. Ultimately, the results of the qualitative coding and analysis process described above were used to formulate answers to the research questions, provide examples to support claims, counterexamples and exceptions to general trends, and explanations for the results. The process of formulating qualitative answers to the research question was facilitated by returning to the research questions and quantitative coding results with a strong familiarity with the qualitative data. Given the volume and depth of the qualitative data that was collected, returning to the research questions was a crucial step in the process of gleaning specific answers and examples from the qualitative data. Those results are discussed in detail in the following chapter.

4 CHAPTER 4: RESULTS

In this chapter, the results of the study are discussed in response to each research question. The first research question investigates how the inclusion of technology within a task is associated with cognitive demand of the task by examining the results of the coding of these dimensions at each phase of implementation defined by the *Mathematical Tasks Framework* (Stein et al., 2009). In Research Question Two, the role of technology is investigated during the set up and implementation phases in tasks making use of it. Results for Research Question One and Two are discussed by phase of implementation. In general, each of these sections begins with the results of coding of cognitive demand and technology use at each of the phases of implementation, and trends and exceptions in these results are discussed qualitatively. Finally, Research Question Three addresses the factors associated with maintenance and decline of tasks set up at a high level, and investigates how the use of technology may be related to each.

4.1 SITE DESCRIPTIONS

The description of the results of the study begins with a summary of the learning environment at each of the data collection sites. Although specific features of the classroom learning environment will be discussed in greater detail as part of the results for Research Questions Two and Three, the purpose of this more general description is to provide the reader with a context for the data reported here. For example, many of the results for Research Question One are summarized in tables describing the cognitive demands of the tasks enacted by teachers at the various phases of implementation. Providing a general description of these sites at the outset allows the reader to understand the context in which these data were collected. In addition, a brief summary of the tasks observed, whether and what kind of technology was used, and the results of the coding of the observed tasks enacted by each of the teachers in the study is included as APPENDIX H, APPENDIX I, APPENDIX J, and APPENDIX K. Each site description provides information about the teacher and their experience, the students, the curriculum, the available technology, structural features such as the period length and classroom arrangement, and unique aspects of the site.

Site One. Ms. Jones is a third year teacher of ninth grade integrated mathematics at an urban charter high school. The class that was observed for the present study consisted of 28 ninth-grade students, with a support teacher (referred to as a "paraprofessional") who was present for seven of the twelve observed tasks. The school setting was unique for a number of reasons. The school did not track based on ability and, according to the teacher, the class contained an unusually wide range of students in terms of prior knowledge and achievement background. Differentiated instruction was a term used frequently by Ms. Jones in discussing how she dealt with this perceived variation in background knowledge and ability, and often resulted in her allotting portions of class time for students to work individually at their own pace on a set of assignments for a given unit, with some optional enrichment activities for more advanced students. Often two or more different activities were being implemented simultaneously by the students. In terms of coding cognitive demand, the criterion used was that of evaluating what the majority of students were doing the majority of time. Nonetheless, this was a more difficult determination to make in this setting, for at times no clear majority of the class worked on the same task within the same period. Another unique element of the structure of the school was that a teacher moves with students through each grade, a factor which occasionally influenced the instructional decisions that this teacher made, given that she would be teaching most of these students again the following year.

The class met for 65 minutes each day, from 12:25 to 1:30pm. The fact that Ms. Jones had a planning period after this class was the primary reason for selecting it for observation. The unit observed involved the mathematical ideas of similarity and proportion, the Triangle Inequality Theorem, setting up and solving proportions related to similar triangles, the relationship between angles formed by parallel lines cut by a transversal, and an introduction to trigonometric ratios (sine, cosine, and tangent). The curriculum that was used for mathematics at the school was the Interactive Mathematics Program (IMP), although for the unit observed it served primarily as a reference. The sequence of topics was not followed, and many of the sections were either skipped or replaced with worksheets. Ms. Jones noted that the problem with IMP is that students "didn't get enough reps," and she appears to have addressed this perceived deficiency by supplementing the curriculum with worksheets.

The room was arranged with students seated at tables in pairs facing the front of the room, with the teacher's desk at the front left corner of the room facing the students, and whiteboard at the front of the room. Students had free access to scientific calculators at their tables, and each had a school-issued laptop for their individual use. The primary instructional technology that students used in this class, besides their calculators, was Geometer's Sketchpad (GSP), a dynamic geometry software program published by Key Curriculum Press. Ms. Jones was the only teacher participating in the study who did not have an IWB.

Site Two. Ms. Young is a third year teacher teaching Geometry and Algebra at a suburban high school. The class observed for the present study was an 11th grade regular inclusion Geometry class of ten students, three girls and seven boys, with a special education support teacher who was present for five of the nine observations. Six students in the class had an IEP, although not all of the IEPs were academic in nature. This class was chosen because Ms. Young anticipated using technology for the unit, and because this class met during the last period of the day, allowing for a post-lesson interview on a regular basis. In addition, Ms. Young was working with another researcher as part of a different project to plan lessons for her Honors Geometry classes, making these classes less ideal for observation, as the purpose of the present study was not to study the effect of an intervention.

The unit observed included topics related to angle relations such as supplementary, complementary, vertical, and adjacent, angles formed by a transversal, such as alternate interior, alternate exterior, corresponding, and same side interior, classifying triangles as acute, right, or obtuse, and polygons, including the theorem for the sum of the interior angles of a convex polygon. This school was on an alternating block schedule, meeting for about 80 minutes every other day, from approximately 1:05 to 2:28pm. This posed a unique methodological problem in that each class generally consisted of more than one main task. That is, Ms. Young generally treated the 80 minute period as back to back classes of varying lengths, i.e. not always as two 40 minute periods. The decision was made to code the main two tasks, defined by duration, for a given lesson, unless there was only one. Task demarcation was determined by a change in the mathematical focus, or when there was a clear closure to one instructional task and introduction/set up of a new task.

Students were seated at desks in pairs in a semi-circle facing the corner of the room where a free standing interactive whiteboard (IWB) was placed. The teacher's desk was at the opposite corner of the room, but she usually used a student desk facing the students placed off-center to the right of the IWB. Students had free access to a set of scientific calculators at the rear of the room, and each had a school issued laptop for their individual use. Besides calculators, the students in this class used both Geometer's Sketchpad and GeoGebra on their laptops to complete instructional tasks.

Site Three. Mr. Mack is a third year 6th grade teacher at a suburban middle school who teaches four sections of mathematics and one section of reading. Although a 6th grade teacher, he is certified to teach secondary (7-12) mathematics as well, and thus his educational background and pedagogical training is similar to the other teachers who participated in the study. The class that was observed included 20 6th grade students, including six boys and 14 girls, and was chosen primarily due to the planning period which followed the class. Although some 6th graders were accelerated into pre-algebra, most were enrolled in regular 6th grade math. Thus, Mr. Mack's class had a fairly broad range of students in terms of background and ability. Classes were about 50 minutes long, from approximately 10:05 to 10:55am. Methodologically, this class was fairly straightforward to observe since all students generally worked simultaneously on a single task during the class period.

A unique element to this data collection site was that two units of instruction were observed instead of one. The first unit that was observed was comparatively short, consisting of only eight tasks (compared with twelve tasks observed in Ms. Jones' class and seventeen in Ms. Young's). In addition to being a short unit, the topics and curriculum used were quite different from the subsequent unit. Order of operations, solving one step equations, and number properties such as the commutative and associative properties of addition and multiplication were taught from a more traditional curriculum (Scott-Foresman). According to the teacher, these topics were added to the scope and sequence for the 6th grade curriculum because the mathematics teachers at the school had decided that the curriculum that was generally used (Connected Mathematics Project) did not cover these topics sufficiently to prepare students for the state standardized exam. The second unit of instruction included fractions, i.e., estimating, comparing, and adding and subtracting fractions and mixed numbers. In addition to being a conceptually richer topic using a more reformed curriculum (CMP), the teacher had explained that his use of technology in this unit was more creative as well. Thus, the primary reason for observing a second unit of instruction was the potential for the contrast it might provide with the first in the terms of the types of tasks and the use of technology. As hypothesized, there was a stark contrast between the cognitive demand of the tasks in this unit and the first unit, as well as differences in the way that technology was used. Furthermore, the nine additional tasks observed results in a total of seventeen tasks, which was the same number observed in Ms. Young's class, and ultimately in Ms. Lowe's class as well.

Mr. Mack's students were seated in groups of three or four for instruction, with their desks pushed together as a group table. The whiteboard at the front of the room has an IWB projector attached to it, and this was the primary technology used in this classroom. The teacher's desk was located at the front of the room to the left of the whiteboard, but many of the tools that he used for instruction (such as his laptop and other hardware, a document projector, and a podium) were located to the right of the whiteboard near the entrance to the classroom. Although a set of calculators was located at the back of the room, students did not have free access to them and were rarely given permission to use them, using them once during the seventeen observed tasks. In addition, Mr. Mack had access to a cart of student laptops which he did not use with his math students during the two units of instruction observed.

Site Four. Ms. Lowe is a third year teacher at small Catholic high school. She teaches two sections of honors geometry, two sections of regular geometry, one section of basic geometry, and one section of fundamentals of algebra. One of the sections of honors geometry was observed because, as she put it, "my chapter 5 in my honors class has such a nice mix of technology/non-technology activities." Another reason for choosing this class is that this was her third year teaching it, whereas, for example, she had not taught the fundamentals of algebra class before. This section of honors geometry was chosen because it was the last period of the day, allowing for a post-lesson interview. The class was 39 minutes long, from 1:49 to 2:28 each day, although many observed classes were shorter than this due to numerous weather related two-hour delays.

The unit observed included topics related to triangles, such as perpendicular and angle bisectors, medians, and altitudes, and points of concurrency related to them such as circumcenter, incenter, centroid, and orthocenter. Other topics included triangle midsegments and the Triangle Inequality Theorem. The text she used for the class was Holt Geometry, but most students didn't have a copy of the text as Ms. Lowe received new textbooks half-way through the year. She generally gave students worksheets from the Holt Geometry curricular materials, and included portions of the text in her PowerPoint lectures. The observed class consisted of 16 students (six boys and ten girls), who were seated in three rows of side-by-side desks facing an IWB secured to the wall at the front of the room. Ms. Lowe's desk was behind the students at the far corner of the room, but she also had a small station at the front of the room to the right of the IWB with books and materials and a laptop connected to the IWB projector. In addition to the IWB, students all had free access to graphing calculators which they kept with them for their individual use.

One unique element of this site was that Ms. Lowe took the class to a computer lab for five of the 17 observed lessons to engage in a student-centered task using the dynamic geometry software program GeoGebra. Although she did have a cart of laptops available, her opinion was that the laptops took too long to start up and get connected to the internet, and that the computers in the lab were much quicker in that regard.

4.2 ASSOCIATIONS OF COGNITIVE DEMAND WITH THE USE OF TECHNOLOGY WITHIN TASKS

Results related to Research Question One are discussed below, with each section addressing associations of technology with cognitive demand during each phase of implementation. The curricular materials phase was interpreted to be the task as it appeared prior to set up, usually in some written form. This included activity handouts, worksheets, PowerPoint slides used for instruction, or the statement of a homework problem(s), if the task consisted of reviewing homework. The task did not always appear in a commercially published curriculum, although when the task could be traced back to such materials this was taken to be the task as it appears in curricular materials, and any modifications to the task by the teacher were considered to be part of the set up. The set up phase is the task as announced to students, which generally includes the teacher's expectations for how students are to work on the task, both explicit and implied. Set up includes expectations about what students are to do, what knowledge they might draw upon, if they work on the task alone, with a partner, or with a group, what resources they have available, such as manipulatives or technology, and what students will be held accountable for from their work on the task. The implementation phase is what students actually do while working on the task, and includes discussion and task conclusion when this occurred. Task conclusion is not considered a separate phase as has been proposed elsewhere (Otten, 2010).

For Research Question One, the question is whether or not the use of technology is related to the cognitive demand during these phases of implementation. As this research question does not address how technology is used, but only whether or not it is used, it is important to note that the following tables do not imply causal relationships, but rather associations between cognitive demand and the inclusion of technology in the task. How the use of technology contributed to the cognitive demand will be addressed in Research Questions Two and Three.

4.2.1 Curricular Materials Phase: Associations of the Use of Technology with Cognitive Demand

Table 4.1 summarizes the tasks as they appeared in curricular materials for each teacher, including the cognitive demand of the task and whether or not technology was proposed as part of the task.

Table 4.1: Technology use in relation to the cognitive demand in curricular materials.

Cognitive Demand within Curricular Materials										
Teacher		Ms. Jones		Ms. Young		Mr. Mack		Ms. Lowe		
	Technology	Yes	No	Yes	No	Yes	No	Yes	No	Total
Cognitive Demand	High	3	5	4	1	0	6	6	4	29
	Low	0	4	7	4	8	3	1	5	32
	Total	3	9	11	5	8	9	7	9	
		12		16*		17		16*		61

* Not all tasks that were set up and implemented appeared in curricular materials

Individual Patterns. The number of tasks which used technology or not, and whether the cognitive demand was high low at the curricular materials phase, is summarized in Table 4.2, Table 4.3, Table 4.4, Table 4.5, for Ms. Jones, Ms. Young, Mr. Mack, and Ms. Lowe, respectively.

Table 4.2: Technology use in relation to the cognitive demand in curricular materials for Ms. Jones.

Ms. Jones		Technology		
		Yes	No	Total
Cognitive Demand	High	3	5	8
	Low	0	4	4
	Total	3	9	12

Table 4.3: Technology use in relation to the cognitive demand in curricular materials for Ms. Young.

Ms. Young		Technology		
		Yes	No	Total
Cognitive Demand	High	4	1	5
	Low	7	4	11
	Total	11	5	16

Table 4.4: Technology use in relation to the cognitive demand in curricular materials for Mr. Mack.

Mr. Mack		Technology		
		Yes	No	Total
Cognitive Demand	High	0	6	6
	Low	8	3	11
	Total	8	9	17

Table 4.5: Technology use in relation to the cognitive demand in curricular materials for Ms. Lowe.

Ms. Lowe		Technology		
		Yes	No	Total
Cognitive Demand	High	6	4	10
	Low	1	5	6
	Total	7	9	16

For Ms. Jones, a majority of the observed tasks did not include the use of technology at the phase of curricular materials, but the three that did were considered high level tasks. Ms. Young seems to favor curricular materials that include the use of technology, but uses it more for low level tasks than high level tasks. At the level of curricular materials, the only high level tasks that Mr. Mack used did not include the use of technology, while the majority of low levels tasks did, although across cognitive demand there is a fairly even distribution of tasks which use technology and those that do not. Ms. Lowe is the only teacher to use tasks with both of the following properties: more high level tasks use technology than do not, and the use of technology is associated with more high level tasks than low, with a fairly even distribution of technology and non-technology tasks.

Comparisons and contrasts across sites. Not reflected in Table 4.1 is the fact that there was very little support for the use of technology for instruction within the commercially published curricula that these teachers used for instruction. Of the 61 tasks analyzed, only one that appeared in a commercially published curriculum included support for the use of technology. Furthermore, none of the high level tasks selected by Ms. Jones[11] and Mr. Mack as they appeared in their curricula (Interactive Mathematics Project and Connected Mathematics Project, respectively) included the use of technology. Although most of the tasks analyzed did

[11] The three high level tasks selected by Ms. Jones that used technology did not appear in IMP, but in a curriculum supplement published by Key Curriculum Press, as described below.

not come directly from commercially published curricula, the fact that these tasks were created by these teachers and included the use of technology in some way reflects the lack of support provided by the commercially published texts that they used. That is, these teachers modified from their textbook or support materials or created their own curricular materials in order to utilize technology.

The exception to this lack of support for using technology in commercially published curricular materials is a curriculum supplement published by Key Curriculum Press that contains activities that can be used in conjunction with the GSP software. Both Ms. Jones and Ms. Young used tasks that appeared in this resource. These activities are written so that teachers can simply make copies for the students to follow while using the program. At the curricular materials phase, most of these activities were considered high level, as they give students step-by-step instructions for creating and manipulating a geometric object in order to explore properties and make generalizations about its behavior. For Ms. Jones, two of the three tasks that used technology came from this supplement, and in Ms. Young's case, two of the four high level tasks that used technology also originated from this source, although she adapted one of them to use GeoGebra instead, a dynamic geometry software program similar to GSP but freely available for download on the internet. By contrast, five of the six high level tasks which called for the use of technology used by Ms. Lowe involved activities that she created in conjunction with the use of GeoGebra. As this software is not commercially published, no such coherent collection of activities exists which can be purchased and used by teachers[12]. Thus, Ms. Lowe drew on her knowledge of the program and her knowledge of her students to create activity handouts to guide them through the tasks.

[12] Many activities have been developed by users of GeoGebra, but these are an eclectic collection which do not usually have accompanying handouts.

Mr. Mack was the only teacher who did not use a dynamic geometry software program with his students. The five high level tasks that he used appeared in the CMP curriculum without the use of technology. Although his students did not use dynamic geometry software, Mr. Mack created interactive files that can be dragged or manipulated on the IWB. Thus, many of the affordances of DGS which allow the user to create, drag, and manipulate object interactively were present in Mr. Mack's use of the IWB.

Although the IWB was the primary technology used in Mr. Mack's classroom, he did have a cart of laptops available for use with his students, but which he never used during the two units of instruction observed in his classroom. When queried about this, he said that he uses them a couple of times a year, but does not use them more due to time and reliability issues. He said that the computers are older and slow, and it takes too long for students to log on. He also notes that the reliability of the network is a factor:

> There are a couple of lessons where I use it, and I think it's too important to not have it, but not every day, because it's, just like you plan this great activity and then it's, "oh, my internet doesn't work. Oh, I can't log in." Well, that kind of blows it up. I've never really had that happen, I just know it's happened to enough teachers. (Interview, 12/2/10)

In spite of Mr. Mack's involvement in the decision-making and implementation of technology at the district and school level, or perhaps because of it, he seemed to perceive the constraints involved with using technology in a student-centered way as not worth the affordances that such an approach may provide. His choice to use the IWB almost daily for instruction had a strong influence on the curricular materials that he created and used with his students.

Overall patterns and summary. Across sites, there was a fairly even distribution of high and low level tasks, with 29 high level tasks and 32 low level tasks. Within the high level tasks, 13 of the 29 (45%) tasks utilized technology, while 16 of the 32 (50%) low level tasks included the use of technology. Similarly, 29 of the 61 tasks (48%) included the use of technology in some way. Thus, while the distribution of tasks across all sites is fairly even, there exists variation in the association of technology use and cognitive demand within these teachers. In the curriculum materials phase, Ms. Jones tended to favor high level tasks which did not utilize technology; Ms. Young and Mr. Mack chose, or created, more low level task which do include the use of technology than any other combination of cognitive demand and technology; and Ms. Lowe was the most inclined to select high level tasks which did include the use of technology in some way. While these teachers' preferences are in some way revealed by the curricular materials they chose, an important distinction made by the *Mathematical Tasks Framework* is that student learning from these materials is mediated by the teacher during two additional phases of implementation. The second subquestion of Research Question One examines the association between the cognitive demand and the use of technology during the set up phase.

4.2.2 Set Up Phase: Associations of the Use of Technology with Cognitive Demand

For the purposes of the present study, "set up" is interpreted to mean how the task was presented to students, including expectations for what students should do, what products they will be responsible for, how students would be grouped while working on the task, what tools or resources they should or shouldn't use, and if and how students would be held accountable for their work on the task. An important distinction regarding technology use during set up is that whether and how it was used refers either to its actual use during the set up of the task, or that its use during the implementation phase was suggested, required, or implied, including expectations for how it would be used. This is contrasted from tasks in which the use of technology was not suggested, required, or implied during the set up phase. Table 4.6 summarizes the cognitive demand of the tasks set up by these teachers, and whether or not the task involved the use of technology.

Table 4.6: Technology use in relation to the cognitive demand during set up.

Technology Use During Set Up										
		Ms. Jones		Ms. Young		Mr. Mack		Ms. Lowe		
	Technology	Yes	No	Yes	No	Yes	No	Yes	No	Total
Cognitive Demand	High	4	0	4	1	6	1	9	1	26
	Low	0	8	9	3	9	1	7	0	37
	Total	4	8	13	4	15	2	16	1	
		12		17		17		17		63

Overall patterns. Overall, the use of technology was associated with both low and high level cognitive demand tasks during the set up phase across all data collection sites. Of the 26 tasks set up at a high level, only three did not utilize technology, which suggests a strong association between the use of technology and setting up high cognitive demand tasks. However, this pattern may say more about these teachers' preference for using technology than the relationship between technology use and cognitive demand. Indeed, only 15 out of 63 total tasks (24%) did not use technology during set up, and of those 15, eight were set up by Ms. Jones. Furthermore, 25 of the 37 low level tasks (68%) were also set up using technology. This makes Ms. Jones, who did not use technology to set up any low level tasks, a clear outlier in this regard. Excluding Ms. Jones, 86% (25 of 29) of the low level tasks were set up using technology. In general, the other three teachers included technology as part of the set up for almost all of the observed tasks. Out of the 48 tasks which did include technology as part of the set up, 25 were set up to use technology at a low level, while 23 were set up at a high level, which demonstrates a fairly even distribution in this regard. Thus, the only clear pattern across sites is that, with the exception of Ms. Jones, these teachers had a strong preference for using technology during the set up phase. However, considering the results across all sites together masks important differences in how the individual teachers set up tasks in their classrooms.

Individual patterns. The cognitive demand during set up and whether or not technology was used or implied during set up by Ms. Jones, Ms. Young, Mr. Mack, and Ms. Lowe are shown in Table 4.7, Table 4.8, Table 4.9, and Table 4.10, respectively.

Table 4.7: Technology use in relation to the cognitive demand during set up for Ms. Jones.

Ms. Jones		Technology		
		Yes	No	Total
Cognitive Demand	High	4	0	4
	Low	0	8	8
	Total	4	8	12

Table 4.8: Technology use in relation to the cognitive demand during set up for Ms. Young.

Ms. Young		Technology		
		Yes	No	Total
Cognitive Demand	High	4	1	5
	Low	9	3	12
	Total	13	4	17

Table 4.9: Technology use in relation to the cognitive demand during set up for Ms. Lowe.

Ms. Lowe		Technology		
		Yes	No	Total
Cognitive Demand	High	9	1	10
	Low	7	0	7
	Total	16	1	17

Table 4.10: Technology use in relation to the cognitive demand during set up for Mr. Mack.

Mr. Mack		Technology		
		Yes	No	Total
Cognitive Demand	High	6	1	7
	Low	9	1	10
	Total	15	2	17

Although Ms. Jones selected mostly high level tasks during the curricular materials phase, or simply used a curriculum (Interactive Mathematics Project) which contained many such tasks, the majority of which did not use technology, two-thirds of the twelve observed tasks were set up at a low level. As an example of how this occurred, Ms. Jones chose to use an activity from the Interactive Mathematics Project curriculum called "Inventing Rules" in which students are asked to invent strategies for solving proportions arising from similar triangles. As no strategy for solving proportions was given or assumed in this task as it appears in the curricular materials, it was coded as high level. However, prior to having students engage with the task, Ms. Jones taught students a procedure for "clearing the denominator" to solve proportions, emphasizing that students were not to use cross multiplication to solve these. Thus, the task was set up as a way to practice a procedure that students had just been taught. The fact that the task was set up at a low level is further evidenced by the mantra that she taught students for dividing by a fraction: "Ours is not to reason why, take the inverse and multiply."

An important pattern in the results for Ms. Jones is that there is a strong association of the use of technology with setting up high level tasks. The only high level tasks that she set up included the use of technology, and the only tasks that included the use of technology were set up at a high level. This pattern suggests that Ms. Jones viewed the affordances of technology as suited to promoting high level thinking by her students. All four of these tasks followed a similar structure in that they were guided explorations of mathematical objects and properties using GSP.

The pattern of cognitive demand and technology use for Ms. Young at the set up phase is almost exactly the same as it was during the curricular materials phase, favoring low level tasks which utilized technology. In general these consisted of a PowerPoint lecture with problems embedded for students to solve during the lecture in order to have them practice the procedure they were being taught, or to have them use terminology they had just been taught to identify different types of angles in a figure.

The results for Mr. Mack indicate a strong preference for the use of technology during set up, with 15 of 17 tasks using technology or implying its use. Mr. Mack's inclusion of technology was often associated with setting up tasks at a high level that had appeared in the written curriculum he used. Mr. Mack often cut and pasted portions of an electronic version of his text into a SMART notebook file (like an interactive PowerPoint) to set these tasks up. However, his modification of these tasks usually involved constraining the open-ended nature of these tasks as they appeared in the curricular materials. For example, a task from CMP requires students to develop their own strategies and reasoning for estimating fractions and fraction sums using benchmarks, and for determining under what circumstances an overestimate or underestimate is most appropriate. Mr. Mack cut and pasted sample problems from an electronic copy of the text into a SMART notebook file, and discussed a number of examples with the class before having them work on the task in groups. While the strategy he used in the examples with the class made important connections between numerical representations and an interactive number line representation made possible by the technology, the task no longer had the open-ended characteristic that it had had in the curricular materials.

Ms. Lowe also demonstrated a strong preference for using technology during the set up of tasks, setting up all but one of the seventeen observed tasks using technology or preparing students to. Ms. Lowe set up 10 of the 17 observed tasks at a high level, the most of any of the four teachers. Furthermore, she set up nine of these ten high level tasks with technology, which was also more than any other teacher in the study. More detail about the nature of these tasks is discussed below in connection with her use of the dynamic geometry software (DGS) GeoGebra.

Comparisons and contrasts across sites. With the exception of Ms. Jones, a notable trend in the set up of tasks across all levels of cognitive demand is that a large majority (76%) of these tasks were set up using technology. This is in contrast to the tasks as they appeared in curricular materials, which exhibited a much more even distribution, with 48% of all tasks using technology. In the case of Mr. Mack, as noted above, most of this shift from non-technology to technology tasks from curricular materials to set up corresponded with taking tasks as they appeared in the CMP curriculum without technology, and setting them up as a task using the IWB. In the case of Ms. Lowe, five tasks that did not include the use of technology in the written materials made use of the IWB during set up. In four out of the five cases, these tasks were coded as high level both in the curricular materials and during the set up phase. These tasks generally involved the use of the IWB for presentation and discussion of problems in which students applied known theorems or procedures to solve a problem.

The converse of the pattern of using the IWB for setting up low level tasks by three of the teachers is confirmed by Ms. Jones, who did not have an IWB. This seems to be the primary factor in the distinct pattern exhibited in her classroom in which, of the twelve tasks observed, the only tasks which were set up at a high level were those which included technology as part of the set up by the teacher. Conversely, every task in which technology was part of the set up was set up at high level. In all four cases, these tasks involved the use of a GSP as part of a high level student-centered task. The absence of an IWB in Ms. Jones' classroom, as well as the absence of low level tasks set up which utilize technology, raises the question of the association between the type of technology being used and cognitive demand of the task.

Table 4.11: Types of technology used in relation to cognitive demand during set up.

Technology Use During Set Up									
		Ms. Jones		Ms. Young		Mr. Mack		Ms. Lowe	
	Cognitive Demand	High	Low	High	Low	High	Low	High	Low
Technology	Interactive Whiteboard	0	0	0	11	6	9	1	7
	Calculators	0	0	0	1	0	0	0	0
	Dynamic Geometry Software	4	0	5	0	0	0	6	2

Patterns of cognitive demand with technology type. Table 4.11 displays the types of technology used by each teacher and the associated cognitive demand of the task. Note that some tasks are double counted in this table when two forms of technology were used simultaneously, such as using GeoGebra on the IWB for a demonstration. The data in this table demonstrate that for these teachers, the IWB was most commonly used in tasks set up at a low level.

What these data do not show, however, is how it was used. That is, not captured in this table is whether or not the use of the IWB contributed to or supported the low level demand of the task, or whether it was merely the medium for displaying what would otherwise be a low level task without the IWB. Indeed, the type of technology used may have only a secondary association with the cognitive demand of the task with which it is used. The primary association may be with the way that technology is used, and what Table 4.11 really reflects is the perceived affordances of these types of technologies by these teachers. These questions will be investigated in greater depth in Research Question Two.

Table 4.11 also makes clear that these teachers rarely, if ever, used a calculator, scientific or graphic, to set up tasks. For three of the teachers, this may primarily be due to the fact that they taught Geometry, and the units observed required little in the way of graphing or computations. For Mr. Mack, the use of scientific calculators was rare, as his students did not have free access to them and he rarely invited them to use one.

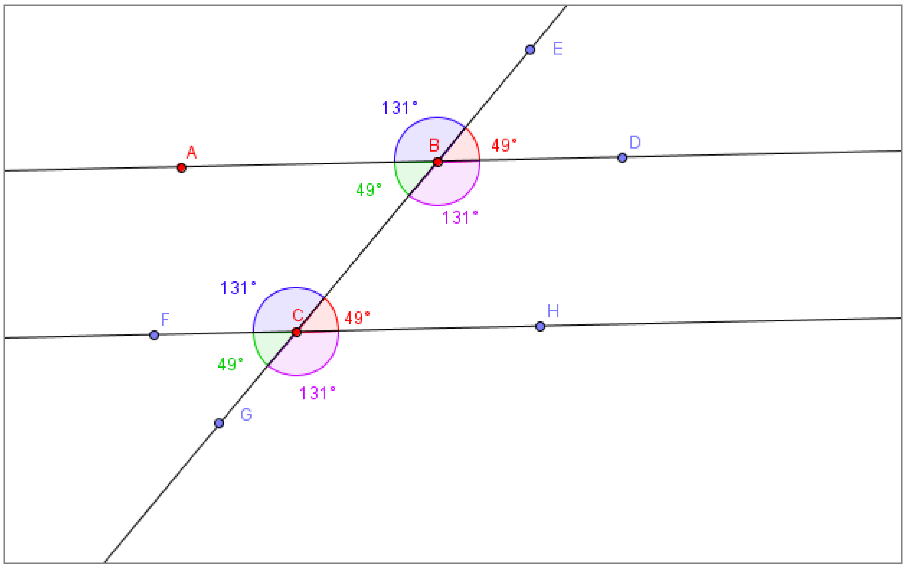
Another notable pattern in Table 4.11 is that of the three teachers who used DGS, it was used in almost all of the observed tasks (15 out of 17) to set up high level tasks. A way in which Ms. Jones' set-up of these tasks was unique was that in two of the four tasks that made use of technology while setting up high level tasks, Ms. Jones gave students a copy of an activity worksheet that she copied from a GSP activity curriculum. In both cases the set up consisted of little more than handing out the worksheet and telling students to complete it. By contrast, Ms. Young and Ms. Lowe generally created their own activity and handout to guide students through the task, and included a brief discussion of their expectations for the task before students began work on it.

A unique aspect to Ms. Young's use of dynamic geometry software was that she was the only teacher to use prefabricated applets to support students' exploration of mathematical objects and their properties. Such a task differs from the way in which Ms. Jones and Ms. Lowe used dynamic geometry software in that students were not required to construct the figure that they would manipulate during the exploration. That is, they did not start with a blank GSP or GeoGebra file and follow a set of directions to use the tools provided in these environments to construct a figure, for example, parallel lines cut by a transversal. Rather, the teacher creates the figure of parallel lines cut by a transversal and publishes it to the web, or uses an applet already published to the web by someone else. This generally results in a figure that can be manipulated with a mouse or touchpad, but contains no tools or commands, thereby constraining students' possible actions in this environment. The teacher can then post a link to the applet that students simply click on and manipulate. Important affordances of this approach are that students do not need to know how to use the software program, it prevents students from constructing erroneous figures, and it saves the time involved in having students create the figure for themselves. Ms. Jones' students' work using GSP to investigate the same concept underscores these affordances, as many of her students pieced together line segments which appeared to be parallel lines cut by a transversal, and thus completely deformed when dragged. As the figures that they created did not possess any of the properties of the object they were intended to investigate, their exploration of these properties was futile.

By contrast, Ms. Young found a file that had been published to the GeoGebra wiki (http://www.geogebra.org/en/wiki/index.php/English) and created a handout to guide students through an investigation which made use of the applet, including what they were to do with the file and what she wanted them to make observations or generalizations about. A screenshot of the file that students used for this task is given in Figure 4.1. Although there are questions included under the figure, Ms. Young created a two-page handout to accompany and guide students' work on this task. Of the four high level tasks that Ms. Young set up that used technology, two were of this nature, while the other two required students to construct the figures themselves before manipulating them as part of the task. Although Ms. Young supported her students' engagement in construction tasks differently than Ms. Jones did, her students also struggled with many of the same issues of using the software to create mathematically accurate constructions.

Properties of Parallel Lines

Lines AB and FC are parallel. Line BC is a transversal of the two parallel lines AB and FC.

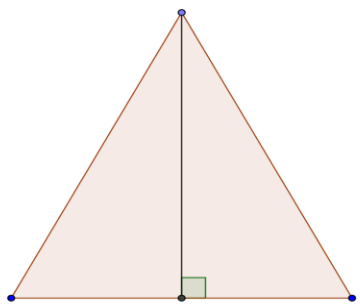


Use points A, B, and C to change the angle values. When a transversal intersects two parallel lines, what angle relationships are formed? Make as many observations as you can.

Barbara Perez, 10/31/06, Created with GeoGebra

Figure 4.1: Parallel lines cut by a transversal used by Ms. Young's students.

Ms. Lowe set up by far the most high level tasks of any of the four teachers, a total of ten, nine of which utilized technology in some way. The most common way that she set up these tasks was to create an activity in which students were to construct a mathematical object and explore its properties using GeoGebra. These tasks always included a worksheet that she created to guide students through the technological procedures needed to construct the object, and prompt them to make observations and conjectures about the properties of the object. An important part of the set up by Ms. Lowe, which was usually less than five minutes, was a few key questions to help students access the prior knowledge that they either needed or that she wanted them to connect to in the activity. For example, the following field note excerpt is typical of how Ms. Lowe set up these kinds of tasks:

> Ms. Lowe calls the students' attention to the whiteboard, reminding them that they've looked at perpendicular bisectors and angle bisectors in GeoGebra, and today they're going to look at something different, called an altitude. She tells them that they've worked with them before, and explains that they are also perpendicular to the side of the triangle, but they're different from perpendicular bisectors because instead of passing through the midpoint of the side, it passes through the opposite vertex. She draws a figure on the board:
>
> 
>
> She points out that it's "this dimension" referring to the altitude in the figure above. She asks them when they remember using that before, and a student says when finding the area of a triangle. Ms. Lowe says yes, drawing a smaller triangle next to the figure above, labeling the altitude h, writing the formula for the area of a triangle on the whiteboard, and telling students that it's h.
>
> She tells students that the GeoGebra activity is short, and that on the back of the worksheet there is a problem that she wants them to do. She tells them that they need to

find the new point analytically, and that this is the type of problem that they'll need to know how to do. (Field note, 2/1/11)

Five of the nine tasks that she set up at a high level that used technology followed this general process for creating and setting up the activity.

Table 4.11 also shows that Ms. Lowe set up two tasks at a low level using GeoGebra. However, the way in which the task was set up in these cases differed from the process described above. Both of these were cases in which she used GeoGebra on the IWB to demonstrate the behavior of some mathematical object. For example, GeoGebra and the IWB were used by Ms. Lowe to review the points of concurrency and their properties based on the student-centered investigations that her students had already completed. As such, it was primarily used to prompt students to recall the results of explorations they had already conducted. Thus, one important difference between the way in which these tasks were set up involved who would be directly manipulating the technology during the implementation of the task. Tasks set up at a high level usually, but not always, indicated that students would use the technology during implementation, while in the tasks that were set up at a low level, it was clear that the teacher would be using it while students watched.

Summary. The tasks as set up by these teachers reveal a strong preference for the use of technology for enacting instructional tasks, with the exception of Ms. Jones. This pattern is unremarkable, since these teachers were chosen to participate in the study due to their belief and interest in using technology for instruction. The adaptation of numerous tasks, from curricular materials to set up, to include the use of technology provides empirical evidence of this characteristic in these teachers. The pervasive use of the IWB for instruction by three of the teachers, however, may tend to obscure what was a very clear pattern in Ms. Jones' classroom of only using technology to set up high level tasks and only setting up high level task with technology. However, research has demonstrated that the implementation phase has the strongest connection to student learning (Stein & Lane, 1996), and thus the pattern of cognitive demand and technology use during implementation is examined in the next section.

4.2.3 Implementation Phase: Associations of the Use of Technology with Cognitive Demand

"Implementation" is interpreted as the portion of the task in which students do or produce something that requires some type of thinking, as described by the task analysis guide (Stein, Smith, Henningsen, & Silver, 2009). This includes both the portion of the task in which students work individually or in groups, as well as a whole class discussion of that work, if there was one. Table 4.12 summarizes the tasks implemented in each of the four classrooms, including the cognitive demand and whether or not technology was used.

Table 4.12: Technology use in relation to the cognitive demand during implementation.

Technology Use During Implementation										
		Ms. Jones		Ms. Young		Mr. Mack		Ms. Lowe		
	Technology	Yes	No	Yes	No	Yes	No	Yes	No	Total
Cognitive Demand	High	0	0	0	0	2	0	5	1	8
	Low	8	4	16	1	14	1	11	0	55
	Total	8	4	16	1	16	1	16	1	
		12		17		17		17		63

In some cases technology was used during implementation although it was not used or expected to be used during the set up phase. For example, Ms. Lowe set up four tasks using technology and eight tasks without it, but during implementation these numbers were reversed, i.e., eight used technology and four did not. This was due to students using calculators during implementation although their use was not suggested, required, or implied during the set up phase of the task.

Overall patterns. The most direct answer to Research Question 1c is that the use of technology was generally associated with low level cognitive demand tasks during the implementation phase across all data collection sites. Specifically, 49 of the 63 total tasks (78%) were implemented at a low level using technology. However, as very few tasks did not make use of technology, it is difficult to gauge the influence of technology on this trend as these teachers may have implemented a similar proportion of tasks at a low level without using technology. Thus, the use of technology during the implementation of low level tasks may say more about these teachers than the impact of technology.

Considering the cognitive demand of all tasks which used technology, out of the 56 tasks that used technology during implementation, only seven (12.5%) were implemented at a high level, and two of the four teachers implemented no tasks at a high level with technology. This further suggests an association between the use of technology and low level tasks, although it is important to note that no causal relationship is implied by this association. Even by restricting the sample of tasks to those which include technology, it is not possible to determine why this association exists from these results. Research Question Two addresses how technology is used, and its role in high and low level tasks, and thus is potentially more revealing of causal connections that may be embedded in this association.

Comparisons and contrasts across sites. Research has shown that it is difficult for teachers to maintain the cognitive demand during implementation of tasks set up at a high level (Stein et al., 1996; Henningsen & Stein, 1997), so the prevalence of tasks implemented at a low level does not come as a great surprise. It is interesting to note, however, that the two teachers

(Mr. Mack and Ms. Lowe) who were successful at implementing at least some tasks at a high level were both graduates of the same teacher preparation program which emphasizes the use of the *Mathematical Tasks Framework* in selecting and implementing instructional tasks, providing further evidence of the potential for teacher education to influence teachers' ability to plan and implement high level tasks with their students (Boston & M.S. Smith, 2009).

Table 4.13, Table 4.14, Table 4.15, and Table 4.16 summarize the cognitive demand of tasks during implementation, and whether or not technology was used, for Ms. Jones, Ms. Young, Mr. Mack, and Ms. Lowe respectively.

Table 4.13: Technology use in relation to the cognitive demand during implementation for Ms. Jones.

Ms. Jones		Technology		
		Yes	No	Total
Cognitive Demand	High	0	0	0
	Low	8	4	12
	Total	8	4	12

Table 4.14: Technology use in relation to the cognitive demand during implementation for Ms. Young.

Ms. Young		Technology		
		Yes	No	Total
Cognitive Demand	High	0	0	0
	Low	16	1	17
	Total	16	1	17

Table 4.15: Technology use in relation to the cognitive demand during implementation for Mr. Mack.

Mr. Mack		Technology		
		Yes	No	Total
Cognitive Demand	High	2	0	2
	Low	14	1	15
	Total	16	1	12

Table 4.16: Technology use in relation to the cognitive demand during implementation for Ms. Lowe.

Ms. Lowe		Technology		
		Yes	No	Total
Cognitive Demand	High	5	1	6
	Low	11	0	11
	Total	16	1	17

Ms. Jones and Ms. Young did not implement a single task at high level, and Mr. Mack only implemented two tasks out of seventeen at a high level. However, since almost all of the tasks that these teachers implemented involved the use technology, the significance of this result in relation to the use of technology cannot be determined from these results. That is, it is not possible to know from these data whether the use of technology contributed to the implementation of these tasks at a low level, or whether it was simply used within a task that would have been low level even without the use of technology.

Ms. Lowe was a clear outlier as she implemented more than a third of the tasks observed in her classroom at a high level. The five tasks that Ms. Lowe implemented at a high level with technology all followed a similar pattern in terms of the curricular materials being created by Ms. Lowe and that the students were in the computer lab for these tasks. The role of technology in these tasks, and specific factors related to the maintenance of the high level thinking demands will be discussed in detail in response to Research Questions Two and Three. However, in terms of using technology, there is no difference between the proportion of tasks which Ms. Lowe implemented that used technology (16 out of 17) and the proportion of tasks that Ms. Young and Mr. Mack implemented which used technology. This provides further evidence that, with respect to cognitive demand during implementation, how technology is used is more important than if it is used. The fact that Ms. Lowe used technology to implement five tasks at high level, while the rest of the teachers used technology in tasks almost all of which were implemented at a low level, underscores the fact that the use of technology in a task is not directly related to the cognitive demand merely by its presence.

In terms of the association of the cognitive demand with technology, the fact that only 1/9 of the tasks were implemented without technology makes any pattern difficult to discern at this grain size. Of the seven tasks that did not use technology, only one was implemented at a high level (14.3%) while 7 out of 56 tasks which used technology were implemented at a high level (12.5%). Thus, while the proportion of tasks implemented at a high level is slightly higher without using technology, the small number of tasks which did not use technology, and the fact that the proportion of high level implementation with or without technology is similar, suggest a lack of association between cognitive demand and technology use during implementation.

Summary. Three out of four of these teachers implemented very few tasks at a high level, a result which is consistent with previous research (Boston & M. S. Smith, 2009; Henningsen & Stein, 1997; Stein et al., 1996). Furthermore, the use of technology in some way was present in a large majority of these tasks, but the absence of a comparable group of tasks that did not use technology makes it difficult to discern the role of technology in the general pattern of low level implementation noted in these results. What can be concluded, however, is that technology did not play a significant role in maintaining the cognitive demand during implementation for most of these teachers, as this rarely occurred. The outlier in this regard is Ms. Lowe, who implemented more tasks at a high level than the other three teachers combined. This fact begs the question of how Ms. Lowe used technology in these tasks, and what role it played in supporting high level implementation. Conversely, how were the teachers who implemented the vast majority of the observed tasks at a low level using technology, and what role, if any, did it play in contributing to the decline of tasks set up at a high level? For example, Mr. Mack and Ms. Young used the IWB extensively while enacting instructional tasks, many of which were implemented at a low level. Was the IWB used as a medium for enacting what would otherwise be a low level task, or did its use play an active role in contributing to low level set up or implementation?

These questions highlight the limit of the analysis associated with Research Question One. As numerous studies have demonstrated (e.g., Burrill et al., 2000, Hollebrands, Laborde, & StraBer, 2008), the presence or absence of technology does not influence student learning, but rather how it is used. Thus, Research Question Two examines how technology is used in relation to the cognitive demand of tasks during set up and implementation, and in particular what role it plays in supporting the implementation of high or low level tasks.

4.3 COGNITIVE DEMAND AND THE ROLE OF TECHNOLOGY

Research Question Two investigates the role of technology in low and high level cognitive demand tasks, and how that role may differ, with the goal of characterizing the use of technology in each. That is, does the intended or enacted use of technology contribute to or support the cognitive demand of the task, and if so, how? If not, how is the way in which technology is used related to the cognitive demand? Thus, the analysis for this research question is limited to the sub-sample of tasks which utilize technology in some way during set up or implementation of classroom tasks.

The results for Research Question Two will be discussed by phase. Beginning with the set up phase, the role of technology intended by teachers when setting up low and high level tasks will be summarized and discussed; this will be followed by the same for the implementation phase. If and how the role of technology changed from set up to implementation, and how it is relates to the decline or maintenance of tasks set up at a high level is the focus of Research Question Three.

4.3.1 Cognitive Demand and the Role of Technology during the Set Up Phase

Table 4.17 summarizes the use of technology in relation to cognitive demand during the set up phase. One way to characterize the use of technology within mathematics instruction is as an amplifier or a reorganizer of students' thinking. As an amplifier, technology has the potential to make certain actions more efficient or accurate without changing the focus of students' thinking.

Table 4.17: The intended use of technology during set up in relation to cognitive demand.

Technology Use During Set Up										
		Ms. Jones		Ms. Young		Mr. Mack		Ms. Lowe		
	Cognitive Demand	High	Low	High	Low	High	Low	High	Low	Total
Technology Use	Amplifier	0	0	0	8	5	9	2	7	31
	Both	4	0	4	0	2	0	6	0	16
	Total	4	0	4	8	7	9	8	7	
		4		12		16		15		47

Students are essentially doing or thinking about the same concepts or procedures that they would be if they were not using the technology. Indeed, whether or not a task could be implemented without the use of technology was a criterion used in determining if a particular case of technology use was classified as an amplifier or reorganizer. If the purpose of using technology was to allow students to shift the focus of their thinking to something different or beyond what the technology was doing for the students, then that particular use of technology was coded as a reorganizer. The use of technology as an amplifier or a reorganizer was coded during both the set up and implementation phases, that is, how it was intended to be used, and how it was actually used by students while engaging with the task.

In practice, each instance of technology use was coded as an amplifier, reorganizer, both, or neither. By offloading tedious or time consuming tasks to technology for the express purpose of having students focus on some other mathematical concept, procedure, or practice, technology may be used as both an amplifier and a reorganizer, and this is what the “both” code refers to. As the use of technology as a reorganizer was always associated with its use as an amplifier during the set up phase, no tasks were coded as using technology as a reorganizer only, and therefore that code is not included in Table 4.17. Furthermore, the “neither” code referred to those situations in which students’ inability to use technology as an amplifier prevented the intended shift in focus, and thus it did not act as a reorganizer either. During the set up phase, there were no instances of this code, and thus it is not included in Table 4.17.

In the following, the summary of this coding within and across sites is discussed, followed by a discussion of precisely how technology acted as an amplifier or reorganizer in specific cases.

The role of technology in tasks set up at low level. The discussion of the role of technology in relation to tasks set up at a low level begins with a discussion of the patterns noted across sites as captured in Table 4.17. Patterns within individual teachers, including detailed descriptions of the way in which individual teachers used technology in setting up tasks are discussed both to exemplify these patterns and to describe exceptions, attempting to get inside the numbers in Table 4.17 and describe exactly how the use of technology is related to the set up of low level tasks.

The most notable pattern in Table 4.17 is that within tasks set up at a low level, the use of technology was always intended as an amplifier. A summary of the type of technology used in such tasks is given in Table 4.18. As Table 4.18 indicates, Ms. Jones did not use technology during the set up of low level tasks, and therefore she is not included in Table 4.18. Note that the set up of tasks that require the use of dynamic geometry software (DGS) by students is absent from Table 4.18, as all of these tasks were set up at a high level. As the first row of Table 4.18 indicates, tasks in which the interactive whiteboard was used for a class lecture and practice problems comprise the large majority of these tasks. In all 24 tasks set up at a low level with technology by these teachers, the interactive whiteboard was used either by itself or in conjunction with calculators or DGS. It was used to display lecture notes and/or practice problems, to project a worksheet while discussing problems or solutions, and to display problems for an exam review game. In many cases, teachers took electronic copies of the textbook and copied and pasted them into PowerPoint presentations, and used this to create handouts for students. In some cases, teachers used it in conjunction with DGS in order to provide a dynamic demonstration or example.

Table 4.18: Amplifier use of technology during the set up of tasks at a low level.

	Ms. Young	Mr. Mack	Ms. Lowe	Total
IWB	6	8	5	19
Calculator & IWB	1	1	0	2
DGS on the IWB	1	0	2	3
Total	8	9	7	24

What all of these examples have in common is that the interactive whiteboard is used to display text and images that everyone in the classroom can see. This is an affordance that is shared by chalkboards and whiteboards as well, and thus is a perfect example of using technology as an amplifier. What is represented, and whether and how that representation makes use of other affordances of the IWB is at the discretion of the teacher. However, these teachers primarily used it in the set up of low level tasks as a medium for the display of content to be memorized, procedures to be learned, or problems for which the recall of facts or the execution of procedures was called for. When used merely as a medium, the use of the IWB as an amplifier has no connection to the cognitive demand of the task, or at least nothing beyond what using a chalkboard would provide. For example, Mr. Mack set up a high level task in which the IWB was only used for the statement of the problem that students worked on. Clearly having a shared space for representing the problem statement or students' thinking, and to which teacher and students can refer while working on the task, supports students' engagement in the task. But if this is the only role that the use of the interactive whiteboard is serving in these tasks, it has no impact on the cognitive demand of the task beyond what would be provided in a non-digital environment.

Another way in which technology was used as an amplifier was the use of calculators for computations implied or suggested by the teacher during the set up of a task, although this was rare. During set up, Ms. Jones and Ms. Lowe never suggested the use of the calculator on a task, and Ms. Young and Mr. Mack did so only once each. For example, while conducting an exam review bingo game, Ms. Young reminded students to get a calculator before the game began, with the calculator being used for computations. During the review, Ms. Young asked the class to use the formula $180*(n-2)$ to find the sum of the interior angles of a 16-gon, and students used the calculator to compute 14 times 180. In general, its use as a computational aid in these types of tasks had no connection to the cognitive demand. These were low level tasks whether the calculator was used or not. Thus, while the use of the calculator as an amplifier during the set up of low level tasks was rare, it is similar to the use of the IWB as an amplifier in such tasks insofar as the use of technology does not affect the cognitive demand.

The role of technology in tasks set up at a high level. The use of technology as both an amplifier and a reorganizer is exclusively associated with high level tasks. While some high level tasks were set up using technology as an amplifier only, when technology was used as both an amplifier and reorganizer, it was always within a task set up at a high level. This pattern suggests that for these teachers the purpose of using technology as a reorganizer is strongly related to engaging students with high level cognitive demand tasks. As the role of technology differs in setting up high level tasks depending on whether or not it is used as both an amplifier and reorganizer, or as an amplifier only, these are discussed separately.

Technology used as both an amplifier and reorganizer in tasks set up at a high level. In general, the use of technology as an amplifier and reorganizer involved offloading tedious or time consuming constructions or measurements to technology in order to allow students' to focus on some other aspect of the task. Examples of how this occurred during instruction are described below. It is important to note that the "both" code in these instances refers to a connection between these two uses of technology. That is, the use of technology as an amplifier is directly connected to an intention to provide students with the opportunity to focus on some other aspect of the task. This is distinguished from a use of technology as an amplifier and as a reorganizer that are unrelated[13].

If a reorganizer use of technology is one which enables students to shift their focus to something other than what the technology is doing for them, then for this group of teachers that shift involved having students engage in high level thinking. The purpose is to clear the way for

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13 Such use of technology did occur during the implementation of a task in Mr. Mack's classroom, and in this case the use of technology was coded as "amplifier" and "reorganizer" but not "both."

students to construct meaning for a mathematical concept or procedure, or to engage in mathematical behavior, such as observing, reasoning, generalizing, and/or conjecturing. With the exception of Mr. Mack, all of these teachers used technology as both an amplifier and reorganizer to set up tasks at a high level using dynamic geometry software (DGS) within a student-centered exploration.

An example of how technology was intended to be used as both an amplifier and reorganizer in order to support high level thinking is Ms. Jones' set up of a task with her students that involved using Geometer's Sketchpad (GSP) to explore the conditions under which a line segment could be drawn connecting two sides of a triangle such that a smaller, similar triangle was created within the larger triangle. She led the class as they worked in GSP on their laptops while her laptop was projected to the front of the room where all students could observe what she was doing while they worked. She began by reminding students of the definition of similarity, i.e., corresponding angles are equal and corresponding sides are proportional, and then led the students in step-by-step instructions on the use of GSP for creating triangles and connecting two sides with a segment, as shown in Figure 4.2.

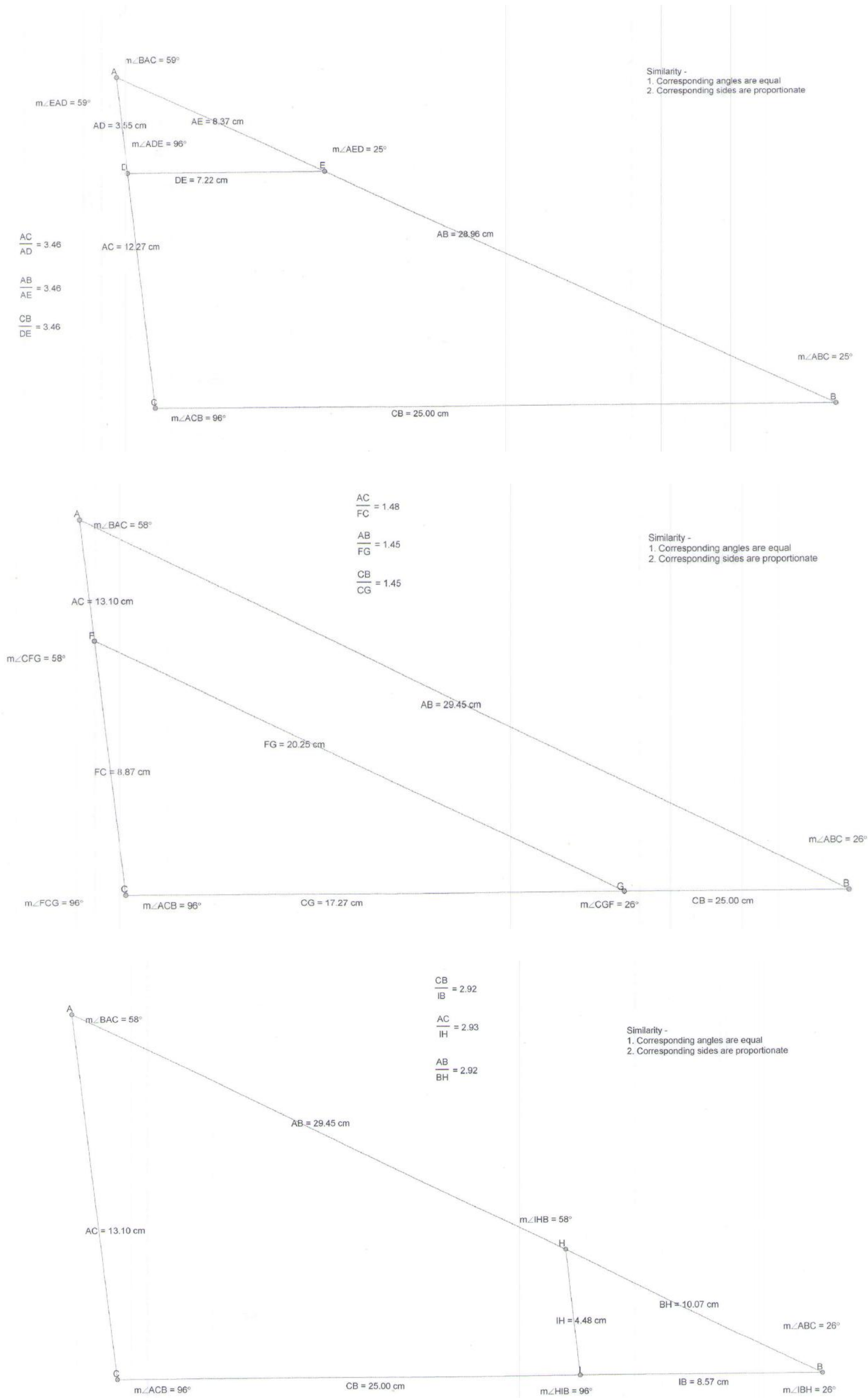


Figure 4.2: Student work in GSP: a similar triangle within a triangle.

In a similar manner, she demonstrated how to measure the angles within each triangle, and the side lengths of each triangle, while students followed along on their laptops. Finally, she showed students how to create dynamic ratios within GSP such that when the object being measured is dragged, the measurements in the ratio change dynamically, and the ratio updates in real time.

The purpose of this task was to draw an arbitrary line segment within a triangle, as in Figure 4.2, and then to move one end of the segment until the inside triangle is similar. Students were led by Ms. Jones in creating three triangles in GSP, connecting different pairs of sides in each, and adjusting the endpoint of the line segment until they had created a similar triangle inside the given triangle. Students were expected to draw on their prior knowledge of similarity and the dynamically updated corresponding angle and side ratio measurements in order to determine the location of the endpoint of the line segment that creates a similar inside triangle. Finally, students were to examine the results of their work across all three cases to make and verify the conjecture that the line segment was parallel to the third side in all three cases.

Drawing, measuring, and computing ratios can be done without the use of a tool like GSP, and thus using GSP for this purpose constitutes its use an amplifier. However, the dynamic and interactive properties of GSP which update the ratios in real time as one end of the segment is moved allowed students to focus on the movement of the line segment and its correspondence with the precise mathematical conditions for similarity rather than on drawing and measuring triangles. In this way, GSP had the potential to act as both an amplifier and a reorganizer of students' thinking on this task.

In the same way that what may be a high level task for one group of students might be low level for another group, whether or not technology has the potential to act as both an amplifier and reorganizer, or merely an amplifier depends on the students, and their background experiences and prior knowledge. For example, Ms. Jones noted that one of her students in a previous class forgot his laptop and successfully completed this task using pencil, paper, and ruler. For this student, the use of technology would only have served as an amplifier since he was able to focus on the goal of the task without the use of technology. However, Ms. Jones emphasized that this was one of her more advanced students, and this approach would not likely have been successful for most of her students. The affordance provided by the technology in helping students to accurately verify the conditions of similarity are necessary in scaffolding the engagement of students who would not know where to begin to draw the line segment by hand.

Another way in which the use of GSP was intended to act as an amplifier and reorganizer in this task is in identifying and verifying the main result: that in order to create a similar triangle within a triangle, the segment connecting to two sides of the outer triangle must be parallel to the third side. Looking across examples is certainly not an affordance of technology, as looking across pencil-and-paper examples would be just as effective in this regard. However, as described above, the construction of these mathematically accurate examples was supported by the use of technology in a way that would be difficult or perhaps impossible for some students. Furthermore, while students may recognize that the line segments which they created appear to be parallel to the third side, GSP provides measurement tools which can assist students in verifying that conjecture.

Figure 4.3 summarizes the tasks that these teachers set up using DGS as both an amplifier and a reorganizer. Although teachers did not all use the same DGS, what Geometer's Sketchpad, GeoGebra, and applets all have in common is the ability to display accurate and precise representations of mathematical objects which can be directly manipulated by the user. In terms of the role that technology played in this group of tasks, these affordances allow student to interact directly with the mathematics, to explore and discover the properties of these objects for themselves.

Providing the opportunity to have students engage directly with the mathematics rather than having that interaction mediated by the teacher was voiced by Ms. Young, Ms. Lowe, and Ms. Jones. Ms. Young said her purpose for using technology is to let students see it for themselves, for them to realize that “there is no math god, he doesn’t just snap his fingers and say ‘all triangles equal 180 because I said so’.” She said that using these “manipulative-type of things” helps her students to see that “I don’t make this stuff up.” She says it lets them see all the different possibilities, and gives them more background, “so they believe me a little more.” She goes on to state that she thinks that her students “may feel more connected” to something that they’ve constructed and investigated themselves. Likewise, Ms. Lowe said that she wants students to be able to investigate things for themselves, to make connections for themselves, and to take control over their learning. Ms. Lowe remarked that she believes that technology “levels the playing field” in terms of her students’ achievement abilities, and allows more students access to the mathematics that is usually only available to her more advanced students. While not as explicit as Ms. Young and Ms. Lowe about having students investigate mathematics for themselves, Ms. Jones claims that a couple of the investigations that she did with technology would have been impossible without it, and that she notices that students are much more engaged and are willing to persist with the exploration longer when using GSP.

Connected to these teachers’ professed purposes for using technology is the potential for having students engage in mathematical behaviors or practices. In almost all these tasks, students were required to construct a figure, measure it in some way, and drag and manipulate it, with the purpose of making an observation or conjecture about the properties of the figure. In particular, the role of dragging in these tasks is a subtle but important one. Once students have constructed their figure, they must begin to make decisions about how to drag it, decisions which are informed by previous dragging, so that their manipulation of these objects becomes more focused and strategic based on previous observations. For example, while investigating the properties of the midsegments of a triangle, i.e. segments that connect the midpoint of each side of a triangle, Neil has the following exchange with Ms. Lowe:

> Neil observes that the midsegment and the segment across from a point don't change when the point is dragged, and tells Ms. Lowe. After Neil shows Ms. Lowe what he's noticing by dragging the triangle, he asks her if that "has anything to do with it" and she says, "I think it does. What's not changing?" Neil replies, "the lengths" and Ms. Lowe says, "what else?" Neil says "the midpoints" and Ms. Lowe again replies, "what else?" and asks him to think in terms of the coordinate plane, and Nick says something about the x-axis, and then says he doesn't know. Ms. Lowe tells him to keep playing with it and walks away. (Field note, 2/16/11)

Although Ms. Lowe supports Neil in his exploration with the questions that she asks, this exchange makes Neil's thinking while dragging explicit in a way that observing a student dragging silently does not. Neil used observations about his previous dragging to guide his continued dragging of the object in a search for mathematically meaningful regularities. Dragging puts students in the position of having control over their mathematical work, providing the opportunity to develop their own mathematical authority by making observations and conjectures, if appropriately supported by the teacher. What that "appropriate support" might

Teacher	Technology	Task Description
Ms. Jones	GSP	Teacher leads the class in using GSP in order to determine how a line segment connecting two sides of a triangle can create a similar triangle within the given triangle
Ms. Jones	GSP	Students use GSP to explore the relationship between the lengths of the sides of a triangle, i.e., the Triangle Inequality Theorem
Ms. Jones	GSP	Students use GSP to individually explore the relationship between the angles formed by parallel lines cut by a transversal and between the angles formed by intersecting lines
Ms. Jones	GSP	Students use GSP to discover that trig ratios (sine, cosine, and tangent) depend only on the angles and not on the side lengths
Ms. Young	GeoGebra applet	Students use dynamic GeoGebra applet to discover angle relationships formed by parallel lines cut by a transversal
Ms. Young	GeoGebra applet	Students use a dynamic GeoGebra applet to discover that the sum of the interior angles of a triangle equal 180
Ms. Young	GeoGebra	Students use GeoGebra to construct a triangle and an exterior angle to discover that the sum of the two remote interior angles is equal to the exterior angle
Ms. Young	GSP	Students use GSP to explore the relationship between the lengths of the sides of a triangle, i.e., the Triangle Inequality Theorem
Ms. Lowe	GeoGebra	Students use GeoGebra to explore the properties of the perpendicular bisector and circumcenter of a triangle
Ms. Lowe	GeoGebra	Students use GeoGebra to explore the properties of the angle bisector and incenter of a triangle.
Ms. Lowe	GeoGebra	Students use GeoGebra to explore properties of altitudes and the orthocenter, and to use their results to solve for the coordinates of the orthocenter of a triangle analytically
Ms. Lowe	GeoGebra	Students use GeoGebra to explore properties of medians and the centroid of a triangle, and to discover the relationship between the median segments
Ms. Lowe	GeoGebra	Students use GeoGebra to explore the properties of the midsegments of a triangle
Ms. Lowe	GeoGebra and IWB	The teacher leads the class in discovering the Euler line in a GeoGebra construction on the IWB

Figure 4.3: Tasks using DGS as both an amplifier and a reorganizer.

consist of will be discussed in terms of the factors associated with the maintenance of high level tasks in response to Research Question Three.

One task set up by Mr. Mack provides an exception to the general pattern of using DGS with students as both an amplifier and reorganizer to set up tasks at a high level. Mr. Mack used the IWB as both an amplifier and reorganizer while setting up a task involving estimating fractions and fraction sums. Mr. Mack created a number line and used the interactive whiteboard pen to estimate a given fraction, drawing a line segment just above or below the number line. Students were to estimate the sum of two fractions using benchmark fractions, e.g., ¼, ½, ¾, etc. For example, for the problem "what is ¼ + 1.2 nearest?" Mr. Mack used the IWB pen as a yellow highlighter[14] to estimate 1.2 as a line segment on the number line, and as a green highlighter to estimate ¼ as a line segment on the number line as in Figure 4.4. He was then able to change the function of the IWB pen to use it like a mouse to "grab" and move objects, and moved the line segment representing ¼ to the end of the line segment representing 1.2, concluding that the sum is closest to 1 ½ as in Figure 4.5. By being able to use the IWB to estimate the length of a line segment as a measurement at one moment, and then to use the IWB to grab these measurements as object and put them end to end to estimate the sum, the IWB allows for a process to be reified as an object almost immediately. That is, the process of measuring results in an object which can be moved about and lined up end to end with the results of other measurements. Facilitating the transition of students thinking from a process, in this case measurement, to an object which can be manipulated is an important step in the development of students' mathematical thinking.

[14] Mr. Mack's slides were re-created in a different color in order to be more visible.

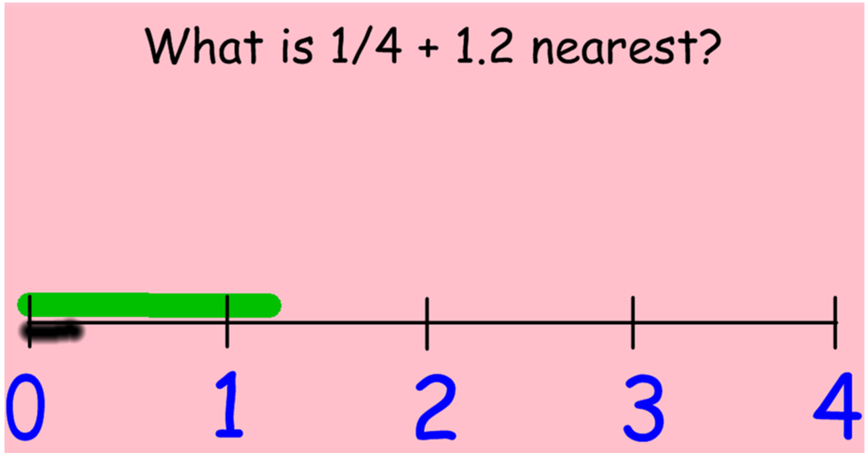


Figure 4.4: Estimating fractions on the number line.

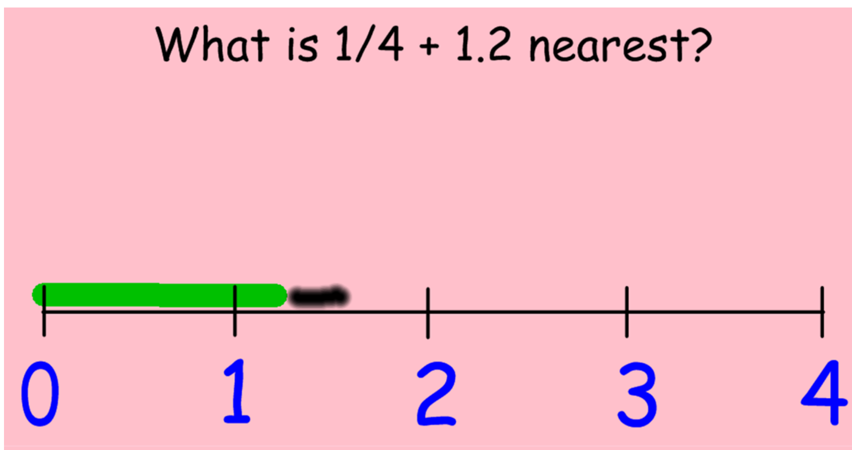


Figure 4.5: Placing measurements end-to-end on the number line to represent addition.

As an amplifier, using a number line is certainly not new, nor is it unique to the use of technology. Indeed, students were asked to draw a number line on their papers as Mr. Mack used the IWB to work some example problems with the students, such as that in Figure 4.4 and Figure 4.5.

The ability to start from 0 and represent two fractions as line segments, and then move them end to end to estimate the sum is an action that cannot be exactly duplicated in a pencil-and-paper environment. However, something which attempts to achieve the same goal could be designed in a pencil-and-paper environment, perhaps with measuring and cutting strips of paper. As an amplifier, then, this use of technology provides a quicker way to execute this procedure, as well as a shared representation of it.

As a reorganizer, the interactive whiteboard has affordances which can provide novel representations capable of supporting conceptual understanding by shifting students' thinking from traditional ways of understanding procedures, such as the addition of fractions, to an alternative representation of the procedure which makes salient the mathematical significance of the procedure in terms of a representation that students may be familiar with, such as a number line. Furthermore, the dynamic and interactive aspects of the representation allow for actions to be represented in a way that would be difficult or impossible to do without technology. In this way, the use of technology has the potential to support students in constructing meaning for the procedure by connecting it to a visual representation.

The use of the IWB as both an amplifier and reorganizer in this example differs from using DGS for student-centered investigations of mathematical objects, but most of those differences relate more to the implementation of the task than the set up. For example, one way in which the use of the IWB for providing a visual representation of a procedure differs from having students use DGS for mathematical investigations is that in the former students do not have access to the file, and cannot manipulate it while working on the task. Based on the reasons given for using technology by Ms. Young, Ms. Lowe, and Ms. Jones, i.e., modifying students' views of the teacher as the sole mathematical authority in the classroom and providing students with more control of their learning, this is a potentially important difference. However, this difference is related to the implementation of the task, and not to the set up. During the set up phase, what these tasks that utilized technology as both an amplifier and a reorganizer had in common is that both provided students access to the task at a high level. However, not all tasks set up at a high level used technology as both an amplifier and a reorganizer, as discussed below.

Technology used an amplifier only in tasks set up at a high level. The exceptions to the general association of reorganizer use of technology, i.e., both amplifier and reorganizer, with high level cognitive demand are six tasks which used technology as an amplifier only to set up a high level task, five by Mr. Mack and one by Ms. Lowe. In general, the interactive whiteboard was used as an amplifier to display the statement or description of a high level task in most of these cases. For example, Mr. Mack displayed the "Spice Problem" from the Connected Mathematics curriculum by cutting and pasting the problem statement from an electronic version of the textbook into a SMART notebook file. The problem was set up as a high level task in which students were to use their background knowledge of fractions and sums in order to invent a method for subtracting mixed numbers when the fractional part of the subtrahend is larger than the fractional part of the minuend. Similar to the use of the interactive whiteboard as an amplifier in tasks set up at a low level, the use of the technology in cases such as this does not relate to the cognitive demand of the task. It is simply a medium for communicating or describing a task which would have the same cognitive demand if it were displayed or communicated using another medium.

There is one notable exception to this use of the interactive whiteboard as merely a medium in setting up high level tasks. Mr. Mack enacted the Land Sections problem from the Connected Mathematics curriculum with his students over two days of instruction. The first day students were given a paper version of the "map" in Figure 4.6 and asked to determine what fraction of a section each person owns (note that there are two sections). The second day of the task involved having students use the results of their work on Day 1 to determine fraction sums, as show in Figure 4.7.

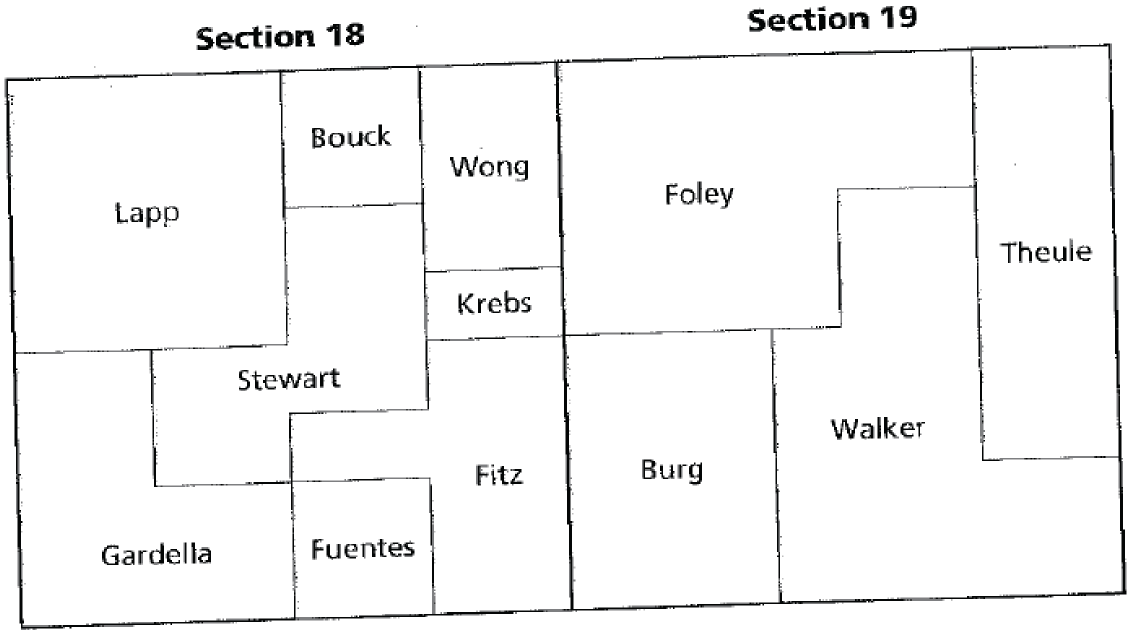


Figure 4.6: The Land Sections problem map.

For example, one question asks, "suppose Fuentes buys Theule's land. What fraction of a section will Fuentes own? Use a number sentence to show your solution." Mr. Mack created an interactive version of the Land Sections map in which each person's section of land can be moved, rotated, or duplicated. This interactive version of the Land Sections problem map was used to discuss students' work from Day 1.

B. Suppose Fuentes buys Theule's land. What fraction of a section will Fuentes own? Write a number sentence to show your solution.

C. **1.** Find a group of owners whose combined land is equal to $1\frac{1}{2}$ sections of land. Write a number sentence to show your solution.

2. Find another group of owners whose combined land is equal to $1\frac{1}{2}$ sections of land.

D. **1.** Bouck and Lapp claim that when their land is combined, the total equals Foley's land. Write a number sentence to show whether this is true.

2. Find two other people whose combined land equals another person's land. Write a number sentence to show your answer.

3. Find three people whose combined land equals another person's land. Write a number sentence to show your answer.

E. How many acres of land does each person own? Explain your reasoning.

F. Lapp and Wong went on a land-buying spree and together bought all the lots of Section 18 that they did not already own. First, Lapp bought the land from Gardella, Fuentes, and Fitz. Then Wong bought the rest.

1. When the buying was completed, what fraction of Section 18 did Lapp own?

2. What fraction of Section 18 did Wong own?

3. Who owned more land? How much more land did he or she own?

Figure 4.7: Day 2 of the Land Sections Task.

After discussing students' solutions for Section 18 at the end of Day 1, Mr. Mack sets up their work on Day 2 by discussing the solutions for Section 19 with the class. An excerpt from the field note of the set up is given below:

> He tells them to look at section 19, and says, "this is why this is tricky." He says that the lines can be deceptive. He draws on the IWB map saying that the line under Foley (drawing it all the way across) is OK, and the line to the left of Theule is OK and the vertical line between Foley and Walker is OK, but the line separating Burg and Walker might have thrown them off. He asks if anyone used section 18, and one of the students says that she used Krebs to figure out Walker. Mr. Mack says that Krebs helps in a couple of ways, and he clones Krebs to fill up Walker, including turning a Krebs piece sideways to fill in the space in Walker directly to the right of Burg, and then counting the number of 1/32 pieces, i.e. Krebs, that it takes to fill Walker, and then counts and says it

> should be 10. [note: “cloning” refers to an affordance of the IWB that allows the user to automatically generate copies of an object. In this example, Mr. Mack creates multiple copies of the Krebs piece and uses them to cover Walker’s area] Mr. Mack then asks if anyone used Bouck or Fuentes, and about half the hands go up. He then clones Bouck to fill Foley, and says that Bouck also fits nicely in Theule. (Field note, 11/19/10)

Although Mr. Mack takes over the discussion of the solutions for Section 19, the purpose of the set up is to ensure that students have the map completed correctly before beginning work on Day 2 of the task. The second day of the task requires that students use the map to answer questions as shown in Figure 4.7. For this reason the set up was not considered to lower the demand of the task.

Mr. Mack uses affordances of the IWB that go beyond a mere medium for displaying problem statements, but this use of technology was also coded as an amplifier for two reasons. First, perhaps by using construction paper, multiple individual pieces of each person’s land could be created and students may have been able to use the same strategy that Mr. Mack did, i.e., filling up one person’s land whose fractional part of section is unknown with another person’s land whose fractional part of a section is known. The second reason that this use of technology was coded as an amplifier is because there did not seem to be any intention to shift students’ thinking to another strategy or way of thinking about the problem. While there are many solutions to the task, the general strategy of using one person’s land to find another’s is embedded in the task, whether technology is used or not.

By utilizing the dynamic and interactive properties of the IWB, Mr. Mack uses this technology in a way that goes beyond a mere medium for display, and supports students’ thinking. In terms of the cognitive demand of the task, Mr. Mack’s purpose in using the IWB this way was to ensure that all groups had determined each person's fractional section correctly so that they could productively engage in the day's task which made use of those results. It also served to support students' understanding of alternative strategies, thereby providing a scaffold for their work on the task for the day. Making the connection between the visual areas and the number sentences students were required to write was viewed to constitute the high level aspect of this task. By providing a way to visualize a general strategy that could be used on the task that would have been difficult for some of the students to visualize without it, this use of technology may have supported some students' access to the task at a high level. Students' work on the first day of the task supports this hypothesis, as described below.

The previous day, students were observed to have discussions about how many times one person's land fit into another's. For example, two boys discussed how many times Bouck fit into Lapp, with one saying that it was four, and the other insisting that it was more than four, either six, or possibly nine. Both used their fingers to estimate the size of Bouck, and moved their fingers separated by that fixed distance through the area representing Lapp's land, and each came up with a different answer. Another group was observed to have ripped a piece of paper into small rectangular shapes that they could move through sections in order to determine how many times one person's land fit into another's. However, these students were unable to rip pieces of paper precisely enough to estimate the land sections accurately. For these students, the interactive version of the Land Sections map provided a way to see how the strategy they struggled to use could be executed more precisely, and also served to provide a way of understanding solutions different from their own. Thus, given these students' prior knowledge and work on this task, the use of technology in this way seemed to support the high level set up of this task. Nonetheless, it did so as an amplifier by providing a way to visualize a strategy that some students may have been able to implement without this scaffold.

As merely a medium for display, the use of technology as an amplifier has no direct relationship with the cognitive demand of the task. The high level aspect of these tasks did not depend in any way on this particular use of technology. However, the Land Sections task provides an example of how the use of technology as an amplifier may facilitate students' access to a task by providing a visualization of a strategy which is not altered by the use of technology.

4.3.2 Cognitive Demand and the Role of Technology during the Implementation Phase

Because classrooms are complex environments with multiple dynamics at play, the opportunity to analyze the role of technology during the set up phase allows for an understanding of its potential to support students' thinking before those dynamics influence students' engagement. The role of technology during the set up phase captures teachers' intentions for using technology as part of a task, which can provide important insight into the role of technology in relation to the cognitive demand of mathematical tasks used for instruction. However, teachers often have difficulty maintaining the cognitive demand of mathematical tasks set up at a high level (Henningsen & Stein, 1997; Stein et al., 1996). Furthermore, research has shown that student learning is most strongly associated with the cognitive demand during the implementation phase (Stein & Lane, 1996). Thus, while understanding the role of technology in relation to the cognitive demand of mathematical tasks during implementation is not a simple task, it is an important one. The focus in this section will be on the role of technology in the tasks as implemented by students, but without discussing specifically why technology played the role that it did. These questions will be addressed in answering Research Question Three, which examines the role of technology in contributing to the decline or maintenance of high level tasks during implementation.

Table 4.19 displays the cognitive demand of tasks and how technology was used by students while implementing the tasks that were set up by the teachers in these classrooms. Two general patterns are worth noting. First, the overwhelming majority of these tasks (87.5%) were implemented at a low level, including all of Ms. Jones' and Ms. Young's observed tasks. Second, the general association of using technology as an amplifier in low level tasks and as both an amplifier and reorganizer in high level tasks persisted during the implementation phase.

Table 4.19: The use of technology during implementation in relation to cognitive demand.

Technology Use During Implementation										
		Ms. Jones		Ms. Young		Mr. Mack		Ms. Lowe		
	Cognitive Demand	High	Low	High	Low	High	Low	High	Low	Total
Technology Use	Amplifier	0	7	0	16	2	13	0	10	47
	Reorganizer	0	0	0	0	0	0	0	0	0
	Both	0	0	0	0	0	2	5	1	8
	Neither	0	1	0	0	0	0	0	0	1
	Total	0	8	0	16	2	14*	5	11	
		8		16		16		16		56

*One task was coded as amplifier and reorganizer separately, but not both.

The role of technology in tasks implemented at a low level. As Table 4.19 indicates, there is a strong association of low level implementation with the use of technology as an amplifier. While the amplifier/reorganizer distinction is useful for describing the use of technology for instruction, it is still a fairly broad distinction. In the following, the use of technology within tasks implemented at a low level will be described in greater detail in order to develop this distinction and articulate its relationship with cognitive demand of instructional tasks during implementation.

A comparison of Table 4.19 with Table 4.17 reveals that while 47 tasks were set up to use technology, 56 utilized technology during implementation. These nine additional tasks involved the use of technology that was not explicitly called for by the teacher or the task during set up. In seven of these tasks students used calculators, and the other two tasks involved the teacher using the interactive whiteboard. In all nine cases, the use of technology during implementation that was not called for during the set up involved using technology as an amplifier in a low level task.All of the tasks in which students initiated the use of a calculator during implementation occurred in Ms. Jones and Ms. Young's classrooms while working on worksheets which, during set up, were coded as being low level. Ms. Jones' and Ms. Young's students had free access to a classroom set of scientific calculators, which they used to complete worksheets such as the one shown in Figure 4.8, in which they were to solve for missing angles in a diagram of parallel lines cut by a transversal. In general, the role of the calculator in these tasks was to aid students with the computations involved in solving these problems. All the steps to solving the problem are exactly the same whether or not a calculator is used, the only difference being that when solving the equation, the calculator is used for computations rather than mental or pencil-and-paper arithmetic. In this sense, the calculator is used as an amplifier, as it does not change the nature of the task or how students think about it. The use of the calculator in these tasks has no influence on the cognitive demand of the task during implementation, as the arithmetic involved in solving these problems was not related to the cognitive demand of the task. These are low level tasks whether one does the arithmetic with a calculator or not.

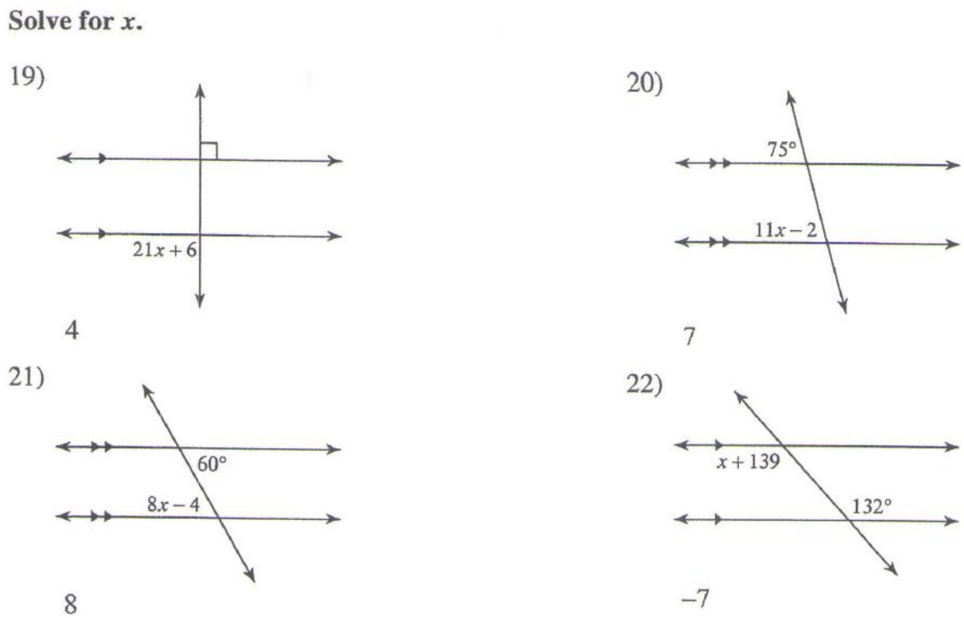


Figure 4.8: A task for which calculators are used for computations while solving equations.

In general, the use of technology as an amplifier during implementation of tasks at a low level included such tasks as using the IWB for a lecture interspersed with practice problems for students to work on from their seats, discussing students' work on a warm-up problem or the previous night's homework, or using a calculator to complete practice worksheets. In all of these cases, it was the types of problems that students were given to work on that was the basis for the coding of the cognitive demand. The use of technology did not contribute to the low cognitive demand of these tasks, but was merely used as a part of a task that would have been low level even if technology had not been used. The fact that there is a strong association of the use of technology as an amplifier with low level cognitive demand tasks seems to say more about the decisions these teachers made in selecting tasks than it does about the use of technology. For example, giving students the "PEMDAS"[15] rule for order of operations and having them practice

[15] **P**arentheses, **E**xponents, **D**ivision and **M**ultiplication, **A**ddition and **S**ubtraction.

it repeatedly on a set of practice problems is a low level task whether done on an IWB or a chalkboard. Thus, the role of technology in these tasks is fairly transparent: it is merely a medium or computational aid for the task that the teacher had chosen to enact with their students in a given lesson. Based on how the task was implemented, and the role that technology played in these episodes, in all of these cases the task would almost certainly have been implemented at a low level even if technology was not used.

There are, however, a few exceptions to using the IWB or calculators as an amplifier during implementation of a task at a low level as described above. Four tasks used DGS in conjunction with the IWB to provide an interactive representation of a mathematical object in order to demonstrate or have students discover a general rule or result. Three of these tasks were coded as using technology as an amplifier only, while the fourth used technology as both an amplifier and a reorganizer. These examples differ from the amplifier use of technology described above in that how the technology was used was directly connected to the low level implementation of these tasks.

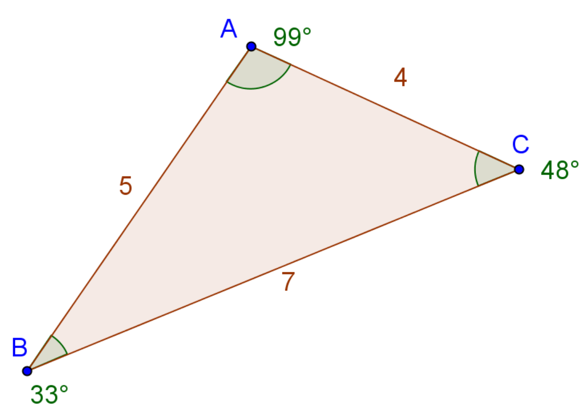
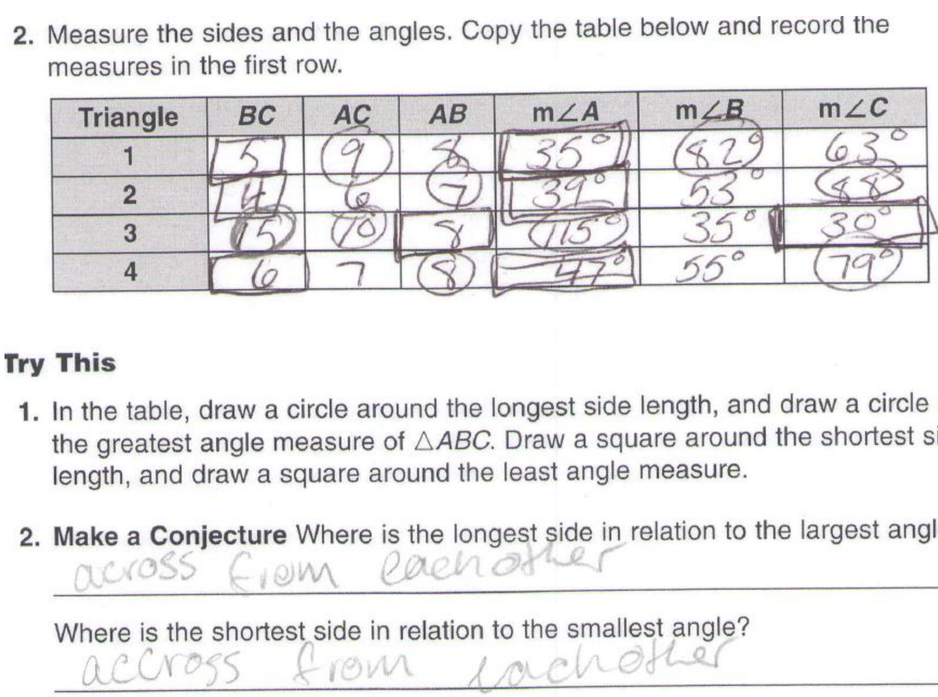


Figure 4.9: A triangle in GeoGebra with side lengths and angle measures displayed.

An example of a task in which DGS was used in conjunction with the IWB as an amplifier only occurred in Ms. Lowe's classroom. To help students "discover" that the longest side in a triangle is always opposite the largest angle, Ms. Lowe created a triangle in GeoGebra which she projected onto the IWB, and displayed the angle measures and side lengths, as in Figure 4.9. Note that she limited the number of decimal places displayed (the default is two decimal places) to make recording the measurements simpler for students. Students remained in their seats and were asked to fill in a table on the worksheet that they were given by Ms. Lowe, as shown in Figure 4.10. Ms. Lowe had students fill in the information for a single triangle, dragged it to create a new triangle, and had students record the measurements for the new triangle. This was repeated for all four triangles. In accordance with the directions, students were to draw a circle around the largest side length and the largest angle, and a square around the shortest side length and the smallest angle, and then make a conjecture about these relationships.


2. Measure the sides and the angles. Copy the table below and record the measures in the first row.

Triangle	BC	AC	AB	m∠A	m∠B	m∠C
1	5	9	8	35°	82°	63°
2	4	6	7	39°	53°	88°
3	15	10	8	115°	35°	30°
4	6	7	8	47°	55°	79°

Try This

1. In the table, draw a circle around the longest side length, and draw a circle at the greatest angle measure of △*ABC*. Draw a square around the shortest side length, and draw a square around the least angle measure.

2. **Make a Conjecture** Where is the longest side in relation to the largest angle?
across from eachother

Where is the shortest side in relation to the smallest angle?
accross from eachother

Figure 4.10: Student work on an investigation of side lengths and angle measures in a triangle.

This particular task was coded as low level during implementation, with the use of technology coded as an amplifier. The use of technology is considered an amplifier, as drawing these triangles by hand is, in fact, how the worksheet directs students to complete the task. The use of GeoGebra on the IWB is a modification to the task as it appears in the curricular materials made by Ms. Lowe during set up. In this instance the use of technology as an amplifier did seem to influence the cognitive demand of the task, although not exclusively. The primary issue with constraining the opportunities for high level thinking associated with this task is the handout, which specifies what students are to do and how to do it to a degree that leaves little room for student thinking. The main issue with the use of technology is that students did not have control over how to use it. However, this is related to the task as it appears in the handout. With such tight constraints on what students were expected to do, it seems to make little difference who manipulates the triangle. However, one can imagine a more open-ended version of this task in which students construct and manipulate their own triangle in GeoGebra, providing the opportunity for students to connect this result to a visual understanding of why it must be true. Such an approach would allow students to make and test their own conjectures, and to make subtle but important decisions about how to investigate this relationship in triangles. All of these decisions were made by the teacher in this task or, more accurately, by the worksheet that the teacher chose to use, and all students simply recorded the same information onto their worksheets. This approach to the task prevents the possibility of generalizing across numerous examples generated by different students during a whole class discussion. Furthermore, students spent most of their time copying the information onto their worksheets before Ms. Lowe changed the triangle again, providing little time for them to connect the visual representation to the relationship they were intended to identify. Instead, students identified the relationship by examining numerical patterns in their table, patterns made more obvious by directing students to draw a circle or square around certain numbers. Thus, the way technology was used in this task constrained students' thinking in such a way as to thwart their opportunity to engage in high level thinking and to make meaningful connections.

Another example from Ms. Lowe's classroom that is similar insofar as GeoGebra was used on the IWB was coded as using technology as both an amplifier and a reorganizer during the implementation of a task at a low level. Ms. Lowe created a triangle (as shown in Figure 4.11) to help students discover the Euler line, i.e., that the orthocenter, circumcenter, and centroid of a triangle all lie on a line, and that in an equilateral triangle, all three of these points of concurrency and the incenter (which does not lie on the Euler line) are concurrent. Although set up at a high level, and with evidence of high level implementation on the part of some of the students, ultimately this task was coded as low level during implementation. One student comes to the IWB to drag the figure while the rest of the class makes observations, but when they do not make the observations that Ms. Lowe wants them to, in particular that three of the points are collinear, she takes over the investigation by asking the class leading questions and constraining the investigation. For example, she tells the student at the IWB to connect the orthocenter and the circumcenter with a line before students make observations about the collinearity of the points of concurrency, and instructs the student at the board to drag the triangle into certain configurations, such as making an obtuse triangle. As only a minority of the students in the class makes observations either before or after this intervention by Ms. Lowe, the cognitive demand of the task is considered to be lowered at this point.

This task draws on students' prior knowledge and experience engaging with student-centered explorations of each of these points of concurrency, and the use of technology was considered both an amplifier and reorganizer during implementation. As an amplifier, it allows for quick and accurate construction and measurement of the triangle and points of concurrency. As a reorganizer, it allows the triangle to be dragged in order to notice that the orthocenter, centroid, and circumcenter lie on a line, to create that line using two of the points, and to drag the triangle to confirm the conjecture by seeing that all three points remain on the line as the triangle changes.

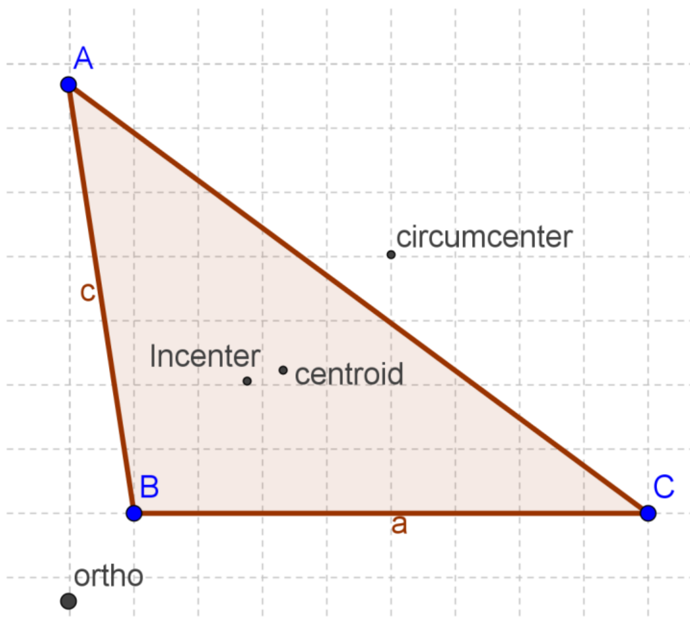


Figure 4.11: A triangle in GeoGebra with all four points of concurrency displayed.

The role of the technology in this episode was to provide an interactive, dynamic, shared representation for a whole class exploration and discussion, and this purpose seems to be fulfilled during the implementation. Students do make observations during the implementation, and some of the manipulations that the students at the IWB performed were not at the suggestion of Ms. Lowe. The representation allowed for students to notice and reason about the relationships between the points of concurrency in a triangle that would have been difficult or impossible to do without the use of GeoGebra and the IWB.

This use of the IWB was not merely to provide a novel representation or interactive demonstration, but for the class to make observations and conjectures. This example differs from the task that Ms. Lowe implemented using GeoGebra on the IWB described above in which students were to notice that the longest side of a triangle is opposite the largest angle as it was much more open-ended, and drew on students' prior knowledge of using GeoGebra to investigate separately each of these points of concurrency. Drawing on those experiences engaged students with the task and making observations in a way that was lacking in the other task. Furthermore, the other task was far too constrained to allow for much thinking on the part of students. Thus, while the affordances of the technologies used in the Euler line task had the potential to engage students in high level thinking, the way that Ms. Lowe used it, or controlled its use, prevented students' high level thinking from fully developing.

While there are some differences in these tasks that used DGS in conjunction with the IWB, such a use of DGS seems to neutralize one of its primary affordances, that of being able to interact directly with representations of mathematical objects. Even in the tasks in which the teacher had some students come up individually and manipulate the figure on the IWB themselves, only one person had control of the actions taken on the object in question. As a result, the majority of students were passive observers, not actively engaged in making decisions about what to drag and how. As the opportunities for high level thinking in such an investigation include the ability to make these kinds of decisions, using DGS in conjunction with the IWB may hinder students' high level thinking. On the other hand, the Euler line task seemed to have the potential to engage students in high level thinking if the discussion around this representation had been allowed to fully develop. What is clear, however, is that the role of technology differs in these tasks compared with those implemented at a low level in which technology is only meant to be used as a medium for display. In the latter case, the way in which technology is used seems to have no influence on the cognitive demand, while in the former case at least part of the issue is related to who controls the use of the technology.

The role of technology in tasks implemented at a high level. A total of seven tasks were implemented at a high level across all sites; two in Mr. Mack's classroom, and five in Ms. Lowe's. The tasks implemented in Mr. Mack's classroom both involved the use of the IWB as an amplifier in connection with the Land Sections task from the Connected Mathematics curriculum, and were implemented on consecutive days. All five tasks that Ms. Lowe implemented at a high level included the use of GeoGebra for individual, student-centered investigations in the computer lab, exploring the properties of the four points of concurrency in a triangle (one task for each), and the properties of triangle midsegments. The Land Sections tasks are discussed first, with the goal of gaining a deeper understanding of the role of technology as an amplifier in tasks implemented at a high level, followed by a discussion of the role of technology as a reorganizer in more student-centered explorations using DGS.

In terms of using the IWB as an amplifier within a task implemented at a high level, the claim is that the use of technology as an amplifier has little or no connection to the cognitive demand of the task. Within the Land Sections problem that Mr. Mack enacted with his students, the IWB was used to create an interactive and dynamic version of the Land Sections map, as shown in Figure 4.6 above, that Mr. Mack used for an eight minute discussion of students' solutions to Section 18 on the first day of the task during which students were to determine the fraction of a section each person owns. Students' explanations of their strategies and solution during the whole class discussion provided evidence of high level thinking by the majority of the students during the exploration portion of the task, but did not necessarily result in high level thinking during the discussion. By being able to move the pieces of the Land Sections, Mr. Mack was able to demonstrate visually different solutions that students described, which may have supported students' understanding of strategies different than their own. The whole class, teacher-centered nature of the discussion makes it difficult to assess the degree to which Mr. Mack's demonstration of students' solutions supported high level thinking, but it is possible to assert that the use of technology during the discussion did not lower the demand of the task in any way. However, the basis for coding the task as being implemented at a high level was the students' work in groups.

The second day on this problem was considered a separate task as students were asked to use the results of their work on the first day of determining what fraction of a section each person owned in order to determine collections of land sections that equal other land sections as shown in Figure 4.7. For example, "Find three people whose combined land equals another person's land. Write a number sentence to show your answer." The interactive Land Sections map was used to demonstrate various solution strategies to these problems. However, due to the open-ended nature of the task, not all student solutions could be modeled using the interactive Land Sections map. The following field note excerpt provides an example:

> Then Mr. Mack asks the class if they were able to find three people whose land equals another person's… Irene says Wong plus Theule plus Krebs equals Walker, and Mr. Mack tells the class that it doesn't look like it would fit, but if they used a number sentence, that's more exact, and writes "3/16 + 3/32 + 1/32" [representing Theule, Wong, and Krebs, respectively] and then points out that they can't add it up until they change 3/16 to 6/32, and writes "6/32 + 3/32 + 1/32 = 10/32". Then Mr. Mack tells the class, "so now I know it works." [he now uses the interactive map to show the class this solution

> visually by trying to cover Walker with the Theule, Wong, and Krebs pieces] He puts Theule in the middle [of Walker], and Wong in the lower right corner, and points out that Wong is too big by a Krebs, and that the left side can be filled in by Krebs and the "Krebs" that is left over from the Wong. He tells the class that the diagram is not always the best way because sometimes it's not easy to visualize, and the number sentence is easier. (Field note, 11/19/10)

Although Mr. Mack does use the interactive version of the Land Sections map to explain Irene's solution, students have to do the same mental rearranging of the pieces that would be required if there were no interactive Land Sections map, since the way that Mr. Mack constructed the map doesn't allow for individual sections to be divided and separated. Furthermore, the use of the IWB in this way does not change students' thinking about the problem, in part because they do not manipulate it themselves, but also because the general strategy of determining how one or more people's land fits into another's is not influenced by the use of the IWB in this way. Rather, the use of the IWB seems to support students' visualization of this strategy. These facts suggest that the use of the IWB did not play a crucial role in the implementation of this task at a high level. It did not change the exploration portion of the task at all as it was not used during this part of the task, and while it may have supported students' visualization of alternative strategies during the whole class discussion, the above excerpt demonstrates that students were still required to do some of the mental rearranging of the pieces that would have been required if the IWB was not used. For these reasons, the use of the IWB as an amplifier during the whole class discussion seemed to have little connection to the cognitive demand of these tasks.

The kinds of tasks that Ms. Lowe implemented with her students are described briefly in Figure 4.3. Note that five of the six tasks listed in Figure 4.3 were implemented at a high level, with the Euler line task being the exception. In all of these cases Ms. Lowe created a worksheet to guide students in using GeoGebra individually at their own computer for most of the period in order to investigate the properties of some mathematical object. As an example of this type of task, Ms. Lowe created a worksheet that guided students through using GeoGebra to construct a triangle and the medians[16] of the triangle, to construct the intersection of the medians (the centroid of the triangle), to measure the segments from the vertex to the centriod and from the centroid to the midpoint of the opposite side, and then to record these measurements in a table and look for a relationship (see Figure 4.12).

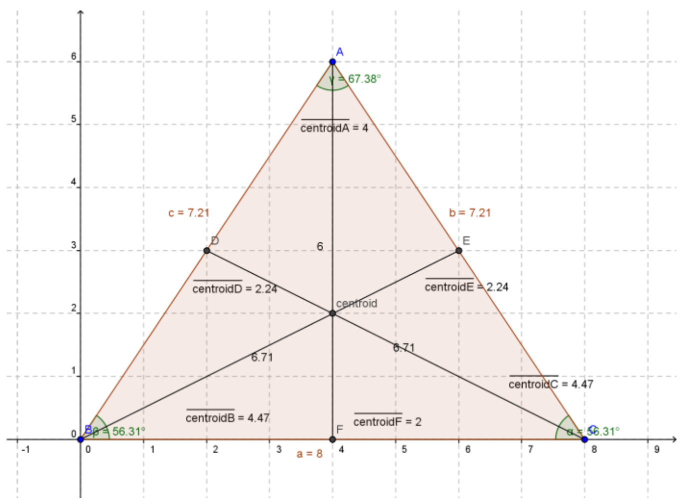


Figure 4.12: A student's construction on the centroid task.

The relationship that they were intended to discover is that the segment from the midpoint to the centroid is 1/3 the length of the median, and the segment from the centroid to the opposite vertex is 2/3 the length of the median. In this case, the opportunity to drag and explore the properties of the medians individually was directly connected to the cognitive demand of the

[16] A segment connecting the midpoint of a side of a triangle to the opposite vertex

task. As an example of the type of thinking that students engaged in while working on the task, the following conversation between two students was observed while working on the task:

> Nick and Brian are dragging their figures and discussing what it is that they're supposed to be noticing. Nick says that there are lots of things to notice, and Brian says, "yeah, but most of them are obvious." Nick replies, "I'm going to make it a right triangle. What would that do?" Nick says to Brian, "it would stay at the center of the triangle, right?" but Brian just stares at his triangle. About 30 seconds later, Brian says to Nick, "look at this" and show him his table, pointing out the 6.17 and the 3.08, and telling him that "this one is almost exactly double that one." Nick says that "you can't make assumptions from one triangle" and they both start dragging their triangles. Nick says that he sees something like that, "but if you stretch it far enough…" Nick says that "one is always half of the other" and Brian says that "the distance from the vertex is always double the distance to the midpoint." Ms. Lowe tells him to "change it, see if you can disprove it" but he replies that he can't disprove it because it's true.

A few minutes later, the following exchange is observed:

> Brian and Nick are still discussing their observation. Nick appears to have started over and created a new triangle, medians, and centroid. Brian is pointing to Nick's screen, and then Nick tells Brian to do it. Brian grabs the mouse and begins to measure the distances from the centroid to the vertex and from the centroid to the midpoint for each median. He says, "that is double that, and that is double that, and that is double that," as he measures each segment. Nick takes over the mouse again and starts dragging the figure and says, "yes, it does stand true." Ms. Lowe comes over and asks them if they can state

their observation in terms of the overall length of the median, because that's how the theorem is stated in the book. (Field note, 2/7/11)

Ms. Lowe's comment at the end of the excerpt refers to having them state their conjecture in terms of the length of the median rather than in terms of the segments.

This excerpt demonstrates both the amplifier and reorganizer use of technology in this task. Students constructed a triangle, the medians of the triangle, and the centroid quickly and precisely. Students measured and labeled on the figure the angle measures, the lengths of the medians, and the lengths of the segments. Most students had completed this part of the task within 10 minutes. While students might be able to construct the centroid of a triangle and use a protractor and ruler to make the same measurements, this might be difficult for most students to do accurately in 10 minutes. However, even if they could, they've only created one triangle. By dragging the triangle, in essence students are creating many triangles and their medians and centroids. But dragging actually does more than just create multiple examples quickly. One can observe, for example, how the centroid moves in response to a vertex being dragged. Thus, one can observe the location of the centroid as the triangle is changed from an acute triangle, to a right triangle, to an obtuse triangle, and back again. This sort of "real-time" motion of one object in relation to another is simply not possible in a pencil-and-paper environment. It is these affordances of the DGS that support its use as a reorganizer by students.

In the above excerpt, for example, students are not focused on making the necessary measurements, but on using them to discern regularities in the behavior of the segments and on understanding what they mean. Nick's statement, "I'm going to make it a right triangle. What would that do? It would stay at the center of the triangle, right?" indicates the open-ended nature of having students directly manipulating the object created within a DGS, that there are many possibilities to choose from in terms of how to drag the object. It also reveals the making and testing of conjectures that is inherent in the development of a more strategic investigation of an object using dragging. These possibilities, and the decisions that students must make in relation to achieving some goal, support students' high level thinking. They must consider the purpose of dragging in terms of an overarching goal, what information would be helpful in achieving that goal, and what sort of dragging might provide that information. Once that move is made, students must assess if the object behaved in the anticipated manner, and if not, why, and what the next move should be in light of this information. By no means does this description constitute a claim that that this process is explicit for the student. Rather, the point is to explicate the type of thinking inherent in such a task, and the role that the use of a DGS plays in that thinking.

The technology acts as a reorganizer by supporting these students' focus on looking for relationships, and making and testing conjectures. Nick's statement that one "can't make assumptions from one triangle," and the subsequent dragging of his triangle indicates that he understands the goal of this task to be a relationship that can be generalized. It also reveals that he understands affordances of dragging the figure in relation to that goal, which is further confirmed after Brian points out the relationship in the median segments while helping him to measure those segments, and Nick drags the triangle to test the claim before agreeing with it. Another important element of this task is the relative absence of Ms. Lowe in the process that Brian and Nick are engaged in. She appears briefly, but only after they've made and tested their conjecture, and only to advance the students' thinking. This is a concrete example of how the use of technology supports independent investigation by students, a goal that Ms. Lowe cites in having students engage in these types of tasks.

Students working informally in pairs was not uncommon during these tasks, but this excerpt is not presented as necessarily representative of the discourse that took place during these tasks. This discourse is revealing of these students' thinking in a way that observing a student silently working alone is not, and this is the reason for its being chosen as an exemplary case. However, the interactions of the students with the technology in the above excerpt are representative of what was observed during these tasks, i.e., making and testing conjectures via dragging.

While the affordances of DGS, and the way those affordances were made use of by these students, support high level thinking, there is nonetheless nothing about the use of a DGS for an exploratory task that causes students to engage in high level thinking. Indeed, numerous tasks were observed in Ms. Jones' and Ms. Young's classrooms in which students implemented such tasks at a low level, using the technology as an amplifier only. The purpose of Research Question Two has been to attempt to identify the role of technology in relation to the cognitive demand of the task during set up and implementation, while not focusing on why a given task was implemented at a high or low level. Many classroom-based factors have the potential to influence whether or not a task set up at a high level was implemented at a high level or not. Research Question Three seeks to identify what those factors are, and especially how technology may have contributed to the decline or maintenance of tasks set up at a high level.

4.4 THE ROLE OF TECHNOLOGY IN THE DECLINE AND MAINTENANCE OF HIGH LEVEL TASKS

Research Question Three investigates the influence of using technology for enacting tasks set up at a high level, especially the role it may play in the decline or maintenance of those tasks during implementation. Research Question Two addressed the role of using technology in setting up tasks, and the role it played in the implementation of those tasks by students. The role of technology was examined only as it pertained to each of these phases of implementation as discrete entities. An important contribution of the *Mathematical Tasks Framework* is the recognition that the cognitive demand of mathematical tasks can change from set up to implementation. Indeed, research has shown that in many cases it declines, and that maintaining the cognitive demand of a task set up at a high level during implementation is difficult for teachers (Boston & M. S. Smith, 2009; Henningsen & Stein, 1997; Stein & Lane, 1996; Stein & M. S. Smith, 1998; Stein et al., 1996, 2009). For this reason it is important to examine the role of technology, and how it may change or influence the cognitive demand from set up to implementation.

Research has also shown that certain classroom based factors are associated with decline and maintenance of the cognitive demand of tasks set up at a high level. In Research Question Three, those factors are identified and discussed, with the aim of providing some explanation of how the technology played the role that it did in decline or maintenance. Thus, for this research question, the sample being analyzed consists of those tasks which were set up at a high level, and which used technology during implementation. Table 4.20 summarizes the number of those tasks from these four sites. The factors associated with the decline of tasks observed in this study, and how technology is related to the decline, will be discussed first, followed by factors associated with maintenance, and the role of technology in those tasks.

Perhaps the most prominent patterns in Table 4.20 is that neither Ms. Jones nor Ms. Young implemented a single task at a high level which involved the use of technology, and that across all sites an overwhelming majority of tasks set up at a high level declined during implementation (72%). However, across all sites only one task set up at a high level did not make use of technology in some way. In conjunction with the discussion of the role of technology in setting up high level tasks in response to Research Question Two, this suggests that without the inclusion of technology, these tasks may not have been set up at a high level to begin with. Ms. Jones, Ms. Young, and Ms. Lowe all used DGS as both an amplifier and reorganizer to set up student-centered tasks (see Figure 4.3). These tasks accounted for over 68% of the tasks that these teachers set up at a high level, including about half (13) of the 25 tasks set up at a high level which used technology across all sites.

Table 4.20: Frequency of tasks set up at a high level using technology, and whether the demand declined or was maintained during implementation.

	Declined	Maintained	Total
Ms. Jones	4	0	4
Ms. Young	5	0	5
Mr. Mack	5	2	7
Ms. Lowe	4	5	9
Total	18	7	25

Ms. Lowe is the exception to the general pattern of decline by these teachers apparent in Table 4.20, maintaining the demand of more tasks set up at a high level than those that declined. In addition, she is the only teacher to implement a task at a high level which did not make use of technology. These results suggest that Ms. Lowe's practice is quite different than the other three teachers who participated in this study. In fact, the results presented up to this point understate the matter. As will be described in the following, Ms. Lowe's practice is unique among this group of teachers, and the strongest evidence of this claim will be presented below in examining the decline and maintenance of tasks set up at a high level in these four classrooms.

4.4.1 The use of technology related to decline

As Table 4.12 demonstrates, overall there was a strong association between the use of technology as an amplifier and the implementation of a task at a low level. In fifteen of the eighteen tasks set up at a high level and implemented at a low level using technology, this was the case. When Ms. Jones and Ms. Young set up and implemented these tasks, the task generally included student-centered tasks which made use of GSP or GeoGebra that was intended to be used as both an amplifier and reorganizer during set up, as shown in Figure 4.3. During low level implementation, the technology was used as only an amplifier for most of the students. An examination of the classroom-based factors associated with the decline of tasks set up at a high level during implementation can provide insight into how this happens.

The factors associated with the decline of tasks during implementation that were set up at a high level (Stein et al., 2009) by these teachers are depicted in Figure 4.13. These tasks incorporated the use of technology at some point during the set up and/or implementation of the task. These factors are given in terms of the percent of tasks that declined when the given factor was present. These are ordered from left to right in terms of their prevalence across sites. For example, as noted in Table 4.20, Ms. Jones and Ms. Lowe each enacted four tasks which declined during implementation, while Ms. Young and Mr. Mack each had five. The "other" category is replaced by "lack of attention to student thinking," as this was the only "other" factor that was coded. A brief overview of how the factors were interpreted in the present study is given below.

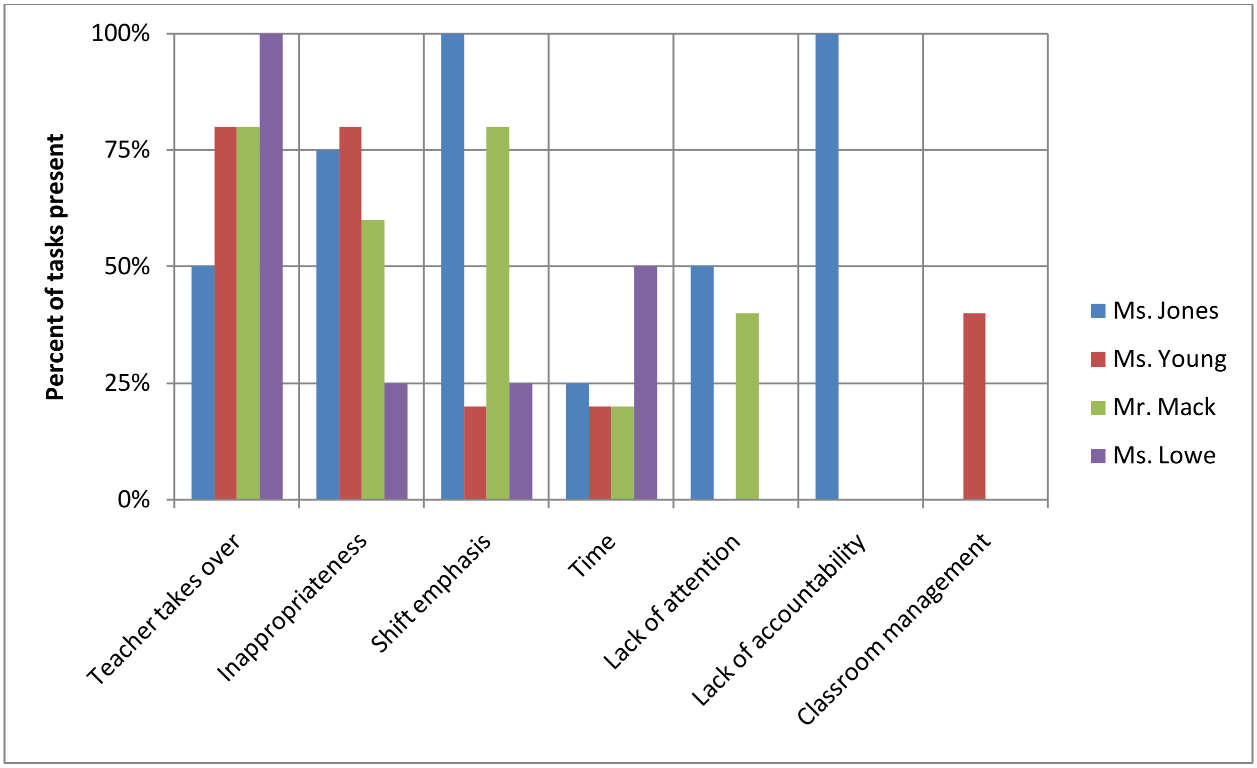


Figure 4.13: Percentage of tasks which includes each factor associated with decline, by teacher.

"Teacher takes over" refers to the category "problematic aspects of the task become routinized" (Stein et al., 2009). In particular, when the teacher takes over the high level thinking aspects of the task, and reduces the task to having students answer a series of low level, short answer questions, this code was assigned. For example, by not allowing students to think about the problem's larger issue of what needs to be done to make progress on the task, and in what order, the cognitive demand of the task is reduced. The teacher carries the burden of reasoning, and simply has students answer a series of low level questions that she has ordered in such a way as to lead to a correct solution.

"Inappropriateness" refers to the inappropriateness of the task for a given group of students. Of the seven factors associated with decline, this code was most related to the use of technology which played an active role in the decline of the task, and will be described in greater detail below. It was also used for other situations, for example when students' prior knowledge made the task inappropriate, or when a task asked students to make generalizations or conjectures from a single example.

"Shifting emphasis" refers to shifting the emphasis of the task from the making conceptual connections or *doing mathematics* to obtaining correct answers, and generally refers to situations in which the teacher ceased pressing students for explanations and simply accepted answers if they were correct, or told students when they were wrong. "Time" refers to having an inappropriate amount of time to work on the task, either too much or too little. In almost all cases in these data it referred to not enough time for students to struggle with the demanding aspects of the task. This code was often associated with the teacher taking over and shifting the emphasis to the correct answer, as a perceived need by the teacher to wrap up the task and draw conclusions by the end of the period all conspired to undermine the high level thinking opportunities. At other times, the teacher may begin a whole class discussion of the task before students have had the time to grapple with the task at a high level.

"Lack of attention" means a lack of attention to students' thinking by teachers, and generally refers to situations in which students' thinking is disregarded to the detriment of their engagement at a high level. For example, students' constructions in a dynamic geometry software environment are mathematically incorrect or inaccurate, preventing productive exploration using the technology, but the teacher never notices as they do not monitor students' work or ask for explanations. Another way in which a teacher may disregard students' thinking is by not considering or exploring alternative solution strategies proposed by students. "Lack of accountability" refers to the idea that the product of students' work on a high level task is not collected by the teacher or shared by the students with the teacher or the rest of the class. This code was exclusive to Ms. Jones ‘classroom, as she did not collect students’ work on high level tasks using GSP, and rarely even referred to their work on GSP tasks in discussing the results. Finally “classroom management” was exclusive to Ms. Youngs’ classroom, and refers to students being off task.

The coding of factors associated with decline did not include the inference that a particular factor contributed to the decline, but only that the factor was present during the task. As these factors have been identified and described in numerous classroom settings without consideration of the use of technology, the present analysis focuses on those factors in which the use of technology was more central to the decline than in others. Many of the factors associated with the decline of the cognitive demand of tasks during implementation are related to how the teacher responds to issues that students experience while working on the task. For example, if students get frustrated and press the teacher for help, she may take over the high level aspects of the task and/or shift the emphasis of the task to finding the correct answer. The qualitative analysis of these factors focused on those issues which were deemed to be directly related to students’ engagement with the task as it related to the use of technology, and not how teachers reacted to these issues. As classrooms are organic and dynamic settings in which these factors are often connected and intertwined, this is an artificial distinction made for the sake of analysis. The two factors that were identified as most related to the use of technology were the inappropriateness of the task and lack of accountability. There is no claim that these were the only factors present, or even that they were most influential in the decline of the task. The goal of the present analysis is to investigate the role of technology in the decline of tasks set up at a high level, as this is the focus of Research Question Three.

Inappropriateness of the task. Inappropriateness of the task included a variety of issues related to students' prior knowledge, the lack of mathematical precision within a given task, or issues associated with the use of technology. Given the variation in the ways that these tasks were considered inappropriate, further refinement resulting in new, distinct factors may be warranted. However, during data collection, the issues described below were coded as inappropriateness of the task, and thus they are discussed as such. This code was used in connection to the use of technology in a total of 10 tasks that declined during implementation, three each for Ms. Jones and Mr. Mack, and four for Ms. Young[17]. Conspicuous for her absence is Ms. Lowe. While four tasks using technology were coded as having declined in her classroom, none were deemed to be related to the inappropriateness of the task as it related to the use of technology. The inappropriateness of the task generally fell into two categories, one having to do with students' use of DGS for student-centered explorations, and the other related to the teacher's use of the IWB. These are discussed separately below.

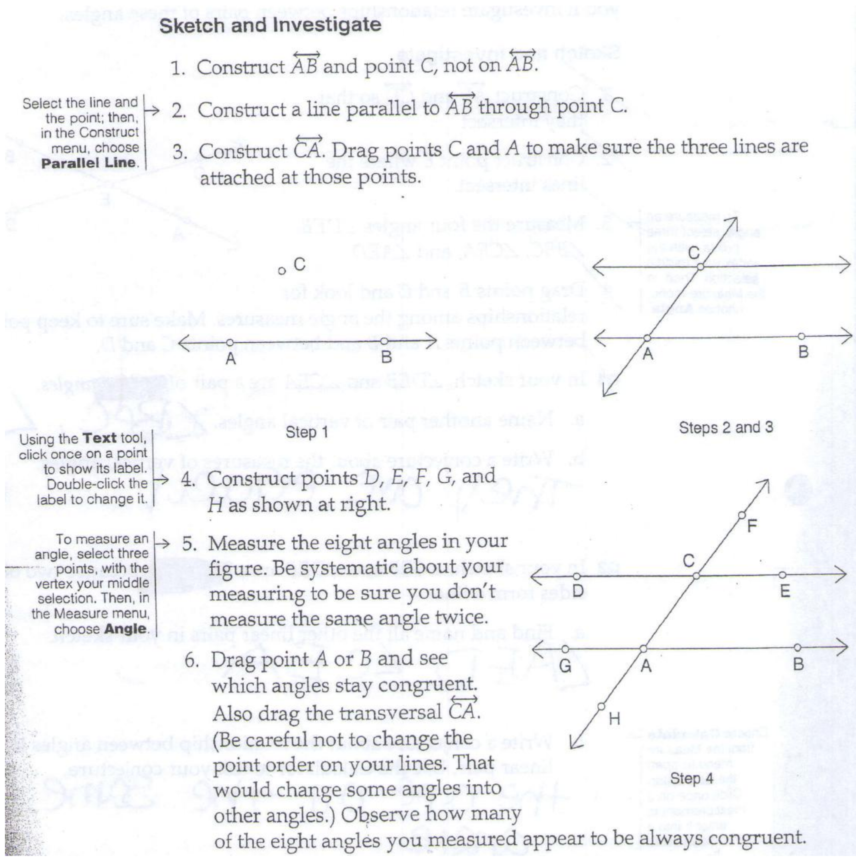
Inappropriateness of the task associated with students' use of technology. The characterization of these issues as being associated with students' use does not imply that this issue was the fault of the students. Likewise, blaming the technology would constitute a failure to recognize that it is merely a tool, and does not cause thinking or learning, or a lack thereof. The issues described below can be characterized as a lack of fit between the tool and the task within which it was to be used. The issue was not the tool per se, but rather that students were asked to do things with the tool that they were unprepared to do, either because they had failed to construct meaning for the tool and thus could not use it appropriately, or because they were asked to engage in mathematical behaviors with the tool for which they had no basis or

[17] The one time this code was used for a task in Ms. Lowe's classroom it was not related to the use of technology, and therefore will not be discussed.

experience. These tasks suggest that teachers may have unrealistic expectations regarding what students will be able to do using technology. For example, if students have never been asked to make a conjecture before, providing them with technological tools will not necessarily result in their ability to do so. While the use of technology can support students' ability to make conjectures by providing numerous examples to analyze as the basis for a conjecture, it does nothing to support students in understanding the importance of examining a variety of examples, what is mathematically meaningful to look for across those examples, how to make a mathematically precise statement as a conjecture, the importance of testing a conjecture or looking for counterexamples, or the difference between a conjecture and a proof. Much could be said regarding these teachers' lack of anticipation of the need for this kind of support, and how they responded to issues related to students' use of technology on these tasks. For example, when some of Ms. Jones' students make an incorrect inference based on their work with GSP, she lowers the cognitive demand of the task by posing a different task to these students which does not require high level thinking. However, the focus of this analysis remains on the role of the technology, and describes teachers' actions and reactions in connection to this, but does not treat them separately or make them an explicit focus of analysis. The role of these teachers' practice in understanding the issues that they and their students experienced while enacting high level tasks with technology is an important idea, and will be discussed in the next chapter as way to explain these results.

The tasks for which the use of technology was problematic for students were implemented in Ms. Jones' and Ms. Young's classrooms, and involved students' being unable to connect the affordances of the technological tools to the requirements of the task. Both an understanding of the affordances of the tool and the mathematical behaviors and products required by the task are necessary for students' productive engagement. A familiar tool may be of little use on an unfamiliar task just as much as an unfamiliar tool on a familiar task. It is the connection between tool and task that these students were unable to make. However, for Ms. Jones' students, the inability to make the connection lay more on the side of the tool, while for Ms. Young's students the task requirements seemed more problematic.

In the task described below that was enacted in Ms. Jones' classroom, students failed to use the technology effectively as an amplifier, which then prevented it from acting as a reorganizer, and thus was coded as "neither." Students were given a worksheet in which they were directed to create parallel lines cut by a transversal as in Figure 4.14. The idea behind the task was to make a conjecture about the relationship between the angles formed by such a figure. Although a primary goal was to determine which angles are congruent and which angles are supplementary, the task is open-ended enough to allow for other observations and conjectures to be made. However, seven students were observed to construct something that looked like parallel lines cut by a transversal, but actually were not. That is, they had something on their computer monitor that looked like Figure 4.15, but when dragged, behaved like Figure 4.16.



Sketch and Investigate

1. Construct $\overleftrightarrow{AB}$ and point *C*, not on $\overleftrightarrow{AB}$.

Select the line and the point; then, in the Construct menu, choose **Parallel Line**.

2. Construct a line parallel to $\overleftrightarrow{AB}$ through point *C*.
3. Construct $\overleftrightarrow{CA}$. Drag points *C* and *A* to make sure the three lines are attached at those points.

Using the **Text** tool, click once on a point to show its label. Double-click the label to change it.

4. Construct points *D*, *E*, *F*, *G*, and *H* as shown at right.

To measure an angle, select three points, with the vertex your middle selection. Then, in the Measure menu, choose **Angle**.

5. Measure the eight angles in your figure. Be systematic about your measuring to be sure you don't measure the same angle twice.
6. Drag point *A* or *B* and see which angles stay congruent. Also drag the transversal $\overleftrightarrow{CA}$. (Be careful not to change the point order on your lines. That would change some angles into other angles.) Observe how many of the eight angles you measured appear to be always congruent.

Figure 4.14: Parallel lines cut by a transversal task worksheet.

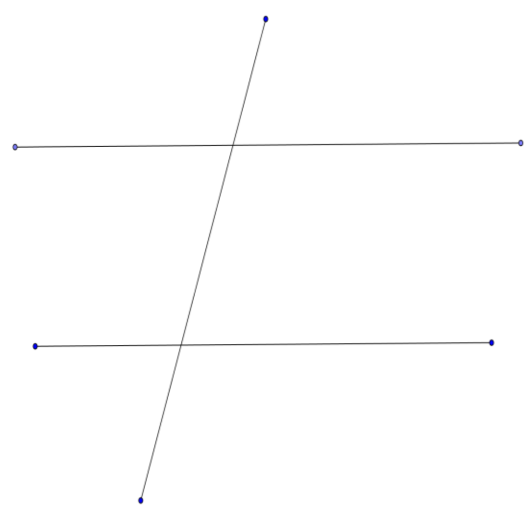


Figure 4.15: Ms. Jones' students created what appear to be parallel lines cut by a transversal.

These students were not observed to make any corrections to their figure after dragging, but simply adjusted it until it looked like Figure 4.15 again, and continued working through the handout. Thus, when these students used GSP to measure the eight angles formed by this figure, none of them were congruent even though the lines looked parallel, as in the student work shown in Figure 4.17.

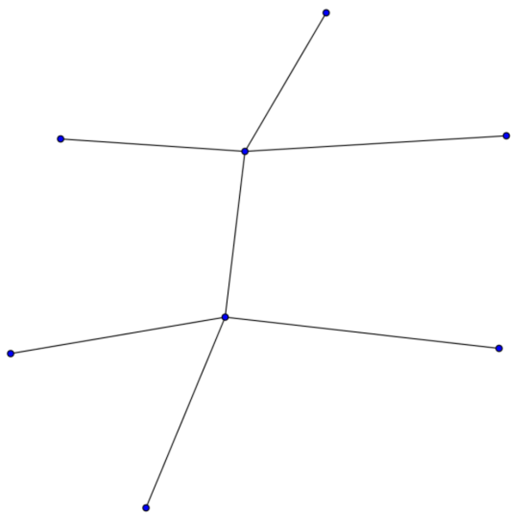


Figure 4.16: When dragged, the figure reveals that the lines are not parallel and the transversal is a collection of segments.

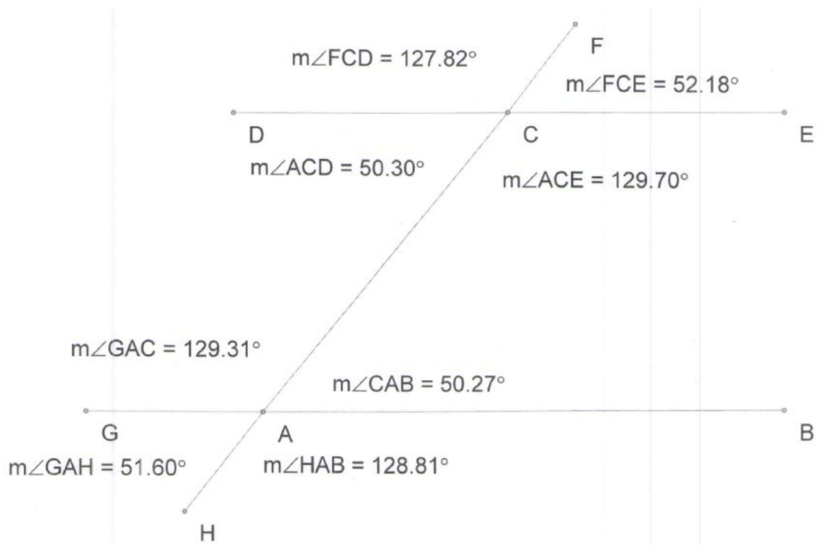


Figure 4.17: Student work in GSP on the parallel lines cut by a transversal task.

The use of technology in this task was coded as neither an amplifier nor a reorganizer. As an amplifier technology allows the user to do more accurately and efficiently what they could do without it; these students did not use GSP as an amplifier, as they did not construct parallel lines cut by a transversal. Further, without the properties built into the construction that were intended to be investigated, any sort of exploration, observation, and generalization of the properties of parallel lines cut by a transversal was futile, and thus the technology failed to act as a reorganizer as well.

This particular task exemplifies the idea that it is not enough to know what buttons to push in order to use a technological tool for mathematical investigation. These students were given directions for how to make this construction in a handout provided by the teacher (see Figure 4.14). In terms of the mathematics, the students seemed to understand what parallel means, as Ms. Jones told students that if their lines were not parallel, "this won't work." She gave no further explanation of what parallel meant, suggesting that she expected that students understood this term. Furthermore, students always adjusted their figures back to something that looked parallel, which also suggests that they knew what parallel meant, although they may have simply been looking at the diagram on the handout that they were following, as in Figure 4.14. The directions in the handout instructed students to create a line, and a point not on the line, and a line parallel to the original line through the point. Furthermore, there are instructions in smaller font in a sidebar: "Select the line and the point; then, in the Construct menu, choose Parallel Line" (see Figure 4.14). So, if students understood what parallel lines are, and the directions for making the figure are straightforward, how does a student fail to properly construct parallel lines cut by a transversal? When informed of these observed cases by the researcher[18], Ms. Jones said that the students did not follow the directions. There is no denying that, but how could students think that they had followed directions?

One explanation is that these students did not understand what parallel means in a dynamic geometry environment. That is, they do not understand that when parallel lines are constructed in a dynamic geometry environment, the parallel quality will be maintained; moving one line will result in the line parallel to it automatically mirroring the same movement in order to maintain the "parallel-ness" of the two lines. These students seem to consider "parallel" to be a contingent rather than necessary property of the lines displayed on their screen. Thus, the lines are parallel when they look parallel. Furthermore, these students are unable to verify that the two lines are indeed parallel, either because they do not understand how to verify this property mathematically, or they do not know how to use the tools in GSP to do this. In addition to a misunderstanding of parallel in a dynamic geometry environment, these students also did not distinguish between a line and a line segment in GSP, as the figure that many of them constructed was a collection of connected line segments.

Ultimately, the issue in this example is that the students lacked the necessary prior knowledge both of the mathematics, and how it is built into the technology. In GSP, there is a definite difference between a line and a line segment, and between lines constructed to be parallel and lines that are made to look parallel. Students' inability to understand those differences, and how they are represented in GSP, prevented them from encountering the mathematics that was the goal of the task. In this way the task was considered inappropriate for this group of students.

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[18] Ms. Jones did not see these cases as she was not circulating the classroom while students worked on the construction.

In the above example, students did not use the technology as an amplifier, which then prevented its use as a reorganizer. In most cases, the decline of the cognitive demand from set up to implementation corresponded with technology being set up to act as both an amplifier and a reorganizer during set up, but used as only an amplifier during implementation. Figure 4.18 depicts a task in which the inappropriateness of the task was due to both the use of the tool and an understanding of the task. Students were to use GSP to discover and make a conjecture about the Triangle Inequality Theorem by constructing a triangle and attempting to manipulate it such that the sum of two sides of the triangle is equal to the third, or less than the third. Students were expected to see that this was impossible, and make a conjecture along those lines.

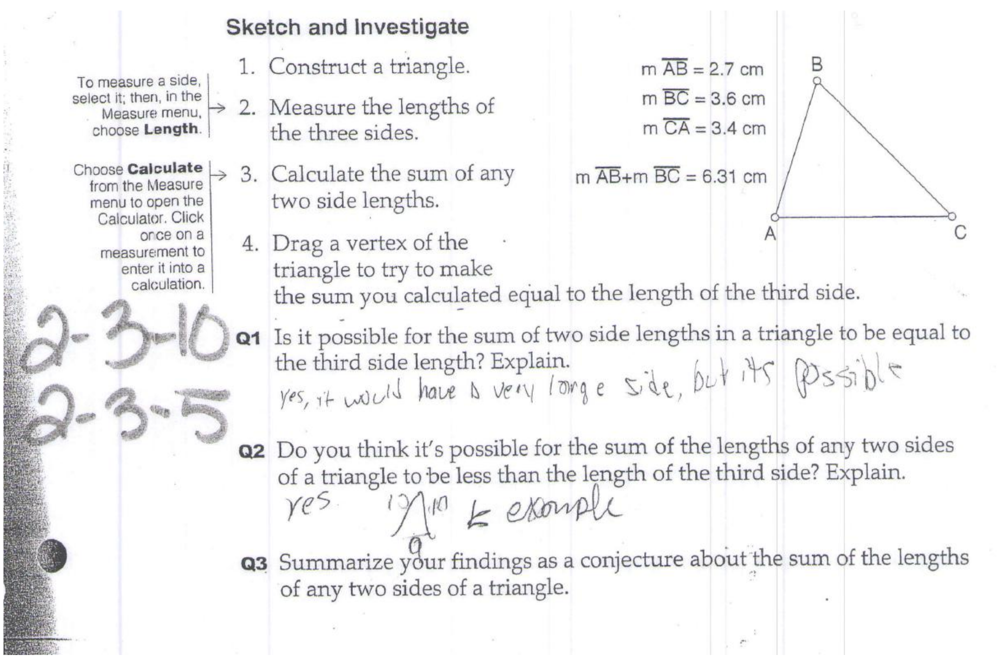


Figure 4.18: Student work on the Triangle Inequality Theorem task in GSP.

Many students were observed to create a triangle but to not drag it, about half the students stated that it was possible to create a triangle where the sum of two sides is equal to the third, or less than the third, or both, and about 70% of the students did not make a conjecture. As

Ms. Jones circulated the classroom and monitored students' progress, she asked students who had answered "yes" to the question, "Is it possible for the sum of two side lengths in a triangle to be equal to the third side length?" to create a triangle with side lengths 2-3-5, as shown in Figure 4.18. Likewise if a student answered "yes" to the question, "Do you think it is possible for the sum of the lengths of any two sides of a triangle to be less than the length of the third side?" she asked them to create a triangle with side lengths 2-3-10. When I inquired with her about the students who were using GSP but answered "yes" to Questions 1 or 2, she said that she felt that she needed to give them specific triangles to look at because they created the triangles themselves, and the sides were in "decimals and were random," and that that may have impeded them from investigating the problem productively. She also mentioned that the students may not know how to add the side lengths in GSP (although it is clearly in the directions), and this was confirmed by observations of some students using calculators. She said that by asking them to create a specific triangle (such as 2-3-5 or 2-3-10) the students did not have to worry about doing calculations.

However, this move by Ms. Jones changes the essence of this task, and removes the high level thinking demands by having students try to construct a single case rather than thinking generally about what is possible in any triangle, and why. Furthermore, the use of technology for this new task that she gives students is superfluous at best, and in some cases gets in the way, as Ms. Jones mentioned that some of these students were able to create a 2-3-5 triangle due to issues of rounding and precision in GSP, so she had them change the precision to see that it does not work. She said that "that's downside of using GSP, but the upside is that you could never do 4.9999 with a ruler, so it's more effective that way." Thus, even the new task needed revision in order for students to use GSP to come to the conclusion that she intended. Constructing a specific triangle, i.e., 2-3-5 or 2-3-10 would likely have been easier for students to accomplish using pencil, paper, and a ruler.

It is difficult to know what students were doing with GSP that resulted in the answers they gave above, as Ms. Jones did not inquire about how they arrived at those answer. However, what is known is that students generally did very little dragging, if any, and most did not have a conjecture, and these two phenomenon are related insofar as the conjecture was intended to be a general statement about the relationship between the side lengths of any triangle, and dragging is the primary affordance by which a user can generate numerous examples very quickly. Ms. Jones mentioned that in an activity like this, when her students get to making a conjecture, "that's where most of them stumble." This is confirmed by the dearth of written conjectures on the student work that was collected. Thus, this task may have been a case in which the inappropriateness of the task was both on the side of the task, in that most students did not make a conjecture, but also on the side of the tool, as students did not use the affordance of dragging in relation to making a conjecture. Being able to connect the demands of making a conjecture to the affordances provided by the tool is the basis for productive engagement with this task. If either is lacking, it is unlikely that students will be able to engage with the task at a high level, and in this sense the task was inappropriate for these students.

Ms. Young's students struggled with some of the same issues of being able to use the technological tools appropriately when asked to construct their own figure for investigation. However, in three of the four tasks for which Ms. Young's students used DGS to explore mathematical objects, Ms. Young's students struggled more with the task requirements than with the use of the tool. For example, Ms. Young's students engaged with the same Triangle Inequality Theorem task using GSP as Ms. Jones' students, except that it included an additional investigation of the relationship between angle measures and side lengths, e.g., the longest side of a triangle is opposite the largest angle. However, Ms. Young's students did not seem to struggle with using the technology as Ms. Jones' students did. When asked how she thought the task went, Ms. Young remarked that the students did very well with GSP, and did not have a lot of technical questions. She said that they had questions "about the questions" but not questions about "how do I do this?" As a specific example of what she means by questions "about the questions," Ms. Young told me that the students "don't know what a conjecture is. We haven't done a lot of that in this class." When a pair of students asked her about this during the task, she said that she told them:

> there is a relationship between the biggest angle and the biggest side. Can you explain it to me? Trying to get her to say that it's like opposite each other, or something along those lines. I thought that maybe giving her a hint like that would help. I don't actually know what they wrote down. I don't think it was think it was anything of substance. (Interview, 10/14/10)

When I ask if she thinks it is the word, or the behavior implied by the word, she replies, "probably not actually knowing what they were doing up here" (referring to the worksheet) and "not saving any of it in their heads, or analyzing it as they went along."

These comments from Ms. Young are consistent with the classroom observations and student work that was collected, and suggest that for her students, using the tools was not so much the problem as understanding what the task was asking for. The same issue arose in the first technology-based exploration that was observed in Ms. Young's classroom, at which time Ms. Young explained to them what a conjecture is and helped some of them to make one. Although it's not possible to know if a better understanding of the task would have resulted in students using the technology in a productive manner, it seems that it would be a prerequisite for such use.

This is another sense in which the inappropriateness of the task for this group of students was related to the use of technology, as students did not seem to understand what the task was asking them to use the technology to do. This is confirmed by the fact that only two students even attempted to make a conjecture on the worksheet. This is not to imply that it was the use of the technology that was the main problem in this task, and that students would have been successful in achieving the goals of the task if technology had not been used. Rather, the point is that students are asked to do something with technology that they have never done before, and are unprepared to do, i.e., looking for patterns and making generalizations and conjectures. If affordances are perceived in relation to a specific goal to be accomplished, then the affordances of the technology that would support making conjectures were lost on these students. Furthermore, as students were not generally expected to engage in high level thinking in this classroom, asking them to do so with technology may have also resulted in a refusal on the part of some students to engage with the task at that level. Nonetheless, asking students to do something with the technology that they never or rarely are expected to do in their mathematics class is likely the root of the problem, and reveals an implicit expectation on the part of Ms. Young that using technology will both motivate and support students' ability to engage in mathematical behaviors that they are unfamiliar with.

This problem was pervasive in the four student-centered technology-enhanced explorations that Ms. Young enacted with her students. Ms. Young avoided many of the issues that Ms. Jones' students had with making mathematically correct or accurate constructions by not having her students make the constructions. Rather, she used prefabricated applets published on the web in which the figure was already constructed, and students' entry to the task began with dragging the figure and making observations and conjectures. For example, Ms. Young had students investigate parallel lines cut by a transversal using an applet as shown in Figure 4.1, and avoided the issues that Ms. Jones' students experienced of having the figure deform when dragged. Below is an excerpt from the field note from the observation of the enactment of the task:

> Ms. Young demonstrates how the points can be dragged on the applet, and that to put the figure back to the beginning they can just refresh the page. She points out that there is a table for them to complete on their worksheet and that there are some directions before the table about what to do. She explains that on the worksheet she gives them a pair of angles with a certain relationship, and asks them to find another one. She explains that they need to make an observation about each type of angle relationship, and explains that this means, "what do you see, notice, what's true?" At this point students whose laptops have started up begin working on the handout. Almost immediately students begin asking each other what they are supposed to be doing. One student is heard to say, "what are we doing?" with the response from another, "I don't know." Mitch[19], while looking at his computer screen says, "corresponding? Where?" and Emily tells him to look at his angle. A couple of students are trying to identify the angles in the applet that are listed on the worksheet, moving their cursor from point to point, or using their finger or a pencil to follow the points in order as given in the angle name on the worksheet. Two students are noticed not moving any of the points on the applet, but simply trying to identify angles and filling in their worksheet. A student is heard to ask Ms. Young what they should write for an observation. He asks if he could write that both angles are 131

[19] All student names are pseudonyms.

> degrees. Ms. Young tells him yes, and she asks "what happens when you move the points around? Are there others that stay the same when moved?" Mitch announces to Ms. Young, "I'm so lost. Are we supposed to move the angles around?" (Fieldnote, 9/29/10)

Students' complete bewilderment as to what they are to do with the applet is evidence of the inappropriateness of the task for this group of students. Although this is the first time these students have used DGS for a student-centered exploration, the issues that students seem to struggle with have more to do with the task than how to use the technology, especially given that Ms. Young demonstrates the dynamic and interactive nature of the applet before students begin the task. Rather, the type of behavior that students are to engage in by investigating and exploring is what these students seem to have trouble with, which is confirmed in my conversation with her. When I asked her about what students seemed to have difficulty with or were asking about while working on the GeoGebra activity, she tells me

> they don't understand the word 'observations,' and neither did my Honors kids, because they had that same worksheet only they didn't have the conjecture fill-in-the-blank. They don't know what to write for observations. They're like "they're both blue" or "one is blue and one is green." These kids were like, "they are both 141 degrees." Which, they were on the right track, but that doesn't help when you move A, and now that angle is 107, so now you're observation is not right. (Interview, 9/29/10)

Her students' lack of mathematically meaningful observations, whether because they did not know how, or were unwilling to, prevents them from using the technology to engage in high level mathematical thinking

Referring to the difficulties that her students have with making observations, Ms. Young states:

> They're not the best observation-makers. I think they don't know what's important." When asked why she thinks that is, she replied I guess maybe they almost think that it's too obvious. Like, "oh yeah, they form a line. Big deal." It isn't clicking that that's what I'm getting at. We're only four and a half weeks into the school year. We haven't done much of this observing, theorem-ing, and stuff in other classes. We've only had Algebra 1 and Algebra 2. So it's kind of a new idea, we don't do much of that in algebra, or at least I don't. 'Look at this picture, and what do you notice?' So, it's different for them. So hopefully we'll get better at making observations, or I'll get better at…the questions I ask. But I feel that these ones are so…not basic, but there isn't much I can say without telling them the answer. There's not a lot of leading that I can do. (Interviewer, 10/1/10)

Ms. Young is clear about the fact that she is asking students to do things with technology that they have very little experience with. In addition to not having much experience with these types of tasks in previous classes, 12 of the 17 observed tasks were set up at a low level. So, although she has made intentional decisions to have students use applets in some of these tasks in order to prevent the use of technology from being an issue, the requirements of the task, and the thinking it calls for, are still an issue for her students.

This was an issue for Ms. Jones' students as well. The only four tasks (of twelve observed) she set up at a high level included the use of GSP for an exploration, but she acknowledges the difficulty in getting her students to use the tools appropriately to engage in high level thinking. She says that many of her students engage in what she calls "button-pushing" when using technology for an investigation, in this case the Triangle Inequality Theorem task described above:

> Ms. Jones: The "button-pushing" is more of the, you know, they're creating it, they're putting answers in, yes's and no's, they don't explain anything, they can't come to a conjecture…or they just followed the steps and measured and then don't carefully read or understand the question, or stop to think about it. I think that they just want to answer it, they have a very "let me just get done" type of mentality, versus, let me look back at this. They're ninth graders, so you're teaching them to go back and look at their notes, you're teaching them to go back and think about the activity, and they want you to tell them the answer. So that's part of the process even just with them being ninth graders, not just with the fact that it's technology. A little bit of both.
>
> MS: do you see that with other worksheets, or more with the technology?
>
> Ms. Jones: I definitely see that more with the technology, for sure, the button-pushing type of, like, thoughtless going through it. I definitely see it more with the technology. They miss the point of the lesson, often, because they can complete the lesson, but they can't analyze, reason, critical thinking-type…
>
> MS: do you think part of that is the type of things you ask them to do with the technology? You ask them to explore, whereas some of the other worksheets are more practice?
>
> Ms. Jones: (pause) Yeah, to some degree, because usually with a worksheet there's a right answer, and it's very concrete, and they are very programmed, for the past 8 years, especially in math class, that it's just about getting the right answer…so when it's

> exploratory it's even worse, because they're missing that. When it's worksheets, they're searching for the right answer. (Interview, 6/18/10)

Ms. Jones acknowledges that she is asking her students to engage in a different type of behavior when using technology than without, resulting in her students using the technology inappropriately, i.e., to just fill in answers on a worksheet. However, her students may simply be fulfilling their perceived role in the mathematics classroom, which is to finish their worksheets. The inclusion of technology in the task is insufficient by itself to communicate to students a different set of expectations regarding their work.

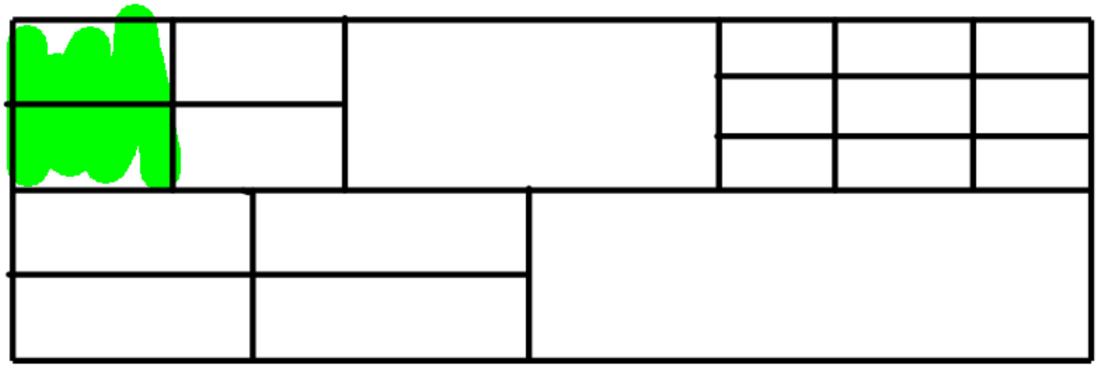
These teachers are enacting different types of tasks using DGS than what their students are normally accustomed to. Since the type of task and the tools are both new to these students, their ability to engage with these tasks at a high level was compromised. Without a clear understanding of the mathematical thinking goals of the tasks, students do not perceive the affordances of the technology in supporting those goals. Conversely, for those students who might understand the goals, a lack of familiarity with the mathematical affordances of the tool could also prevent progress at a high level. They may understand what they need to do, but do not understand how the tool they have been provided with might help. Thus, the inappropriateness of the task in these cases derives from students' inability to connect the affordance of unfamiliar tools to the requirements of novel tasks, including their refusal to engage in mathematical behaviors which they view as outside the scope of their role in the mathematics classroom.

While the use of technology in the tasks described above was problematic, and related to the decline of the cognitive demand, it is important to acknowledge that how these teachers responded to these difficulties may have been a more influential factor in the decline of the tasks than the issues described above. Failing to monitor students' work, changing the task to a low level task in response to students' incorrect answers, or failing to motivate or support students' ability to engage in high level thinking are reactions, or a lack thereof, on the part of the teacher that may have been the fatal flaw during implementation. Greater anticipation of students' needs, or different responses to the issues that students experienced, may have salvaged the high level thinking demands of these tasks for a majority of their students. How these moves, or a lack thereof, on the part of these teachers are associated with their classroom practice is discussed in Chapter 5.

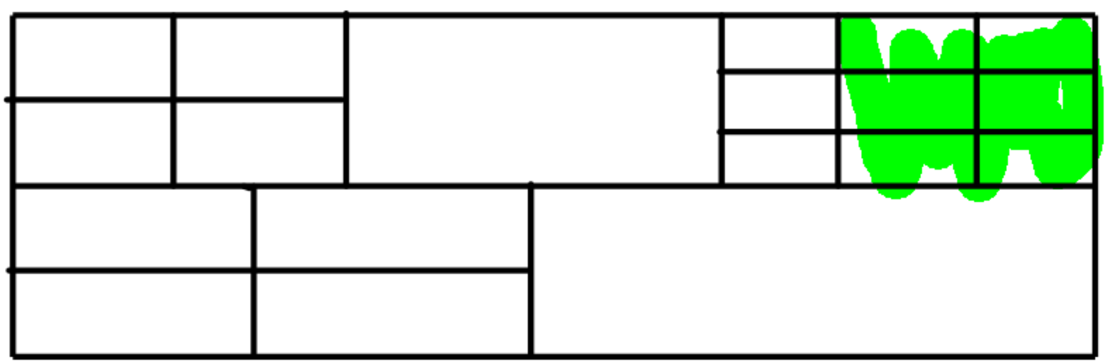
Inappropriateness use associated with teachers' use of the IWB. Mr. Mack's use of the IWB as an amplifier and reorganizer in two tasks which declined during implementation demonstrates another sense in which the task might be considered inappropriate. In these cases the inappropriateness of the task refers to how the technology was used in relation to the task goals. Mr. Mack used the IWB to create dynamic representations of solution strategies to a task, but these representations lacked the precision necessary to resolve mathematical discrepancies between solutions. Furthermore, students' lack of opportunity to manipulate these representations limited its ability to support their mathematical thinking.

For example, to introduce the unit on fractions, Mr. Mack enacted a task in which students were to construct meaning for fractions using an area representation. In this task, students were given a piece of paper with multiple blank copies of this figure on it, and asked to shade certain fractions of the figure. The following is an excerpt from the field note for this task in which students are attempting to shade 1/8 of the figure.

As I roam the class, most students either don't have anything, or don't have anything correct. Mr. Mack asks Brian if he has it, and he says no. He asks Aubrey, and she says, "I think so" and comes to the board and shades the following:



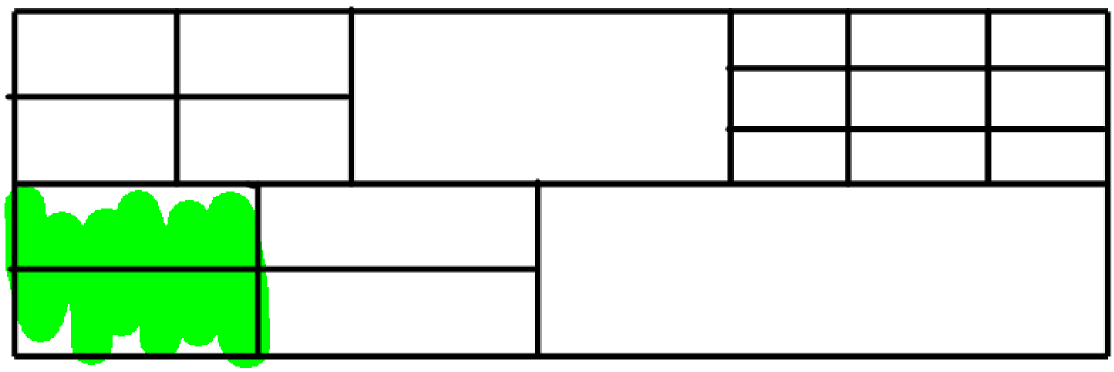
Mr. Mack says that there is an easy way to check. He says, "if this is 1/8, how many times should it fit into the whole thing?" and students say 8. He drags the shading over along the top half, counting how many times the shaded region fits. He says that it fits along the top "five and change" so it doesn't work. He asks AB[20] if she's got it, and she says "Yes!" and comes to the board:



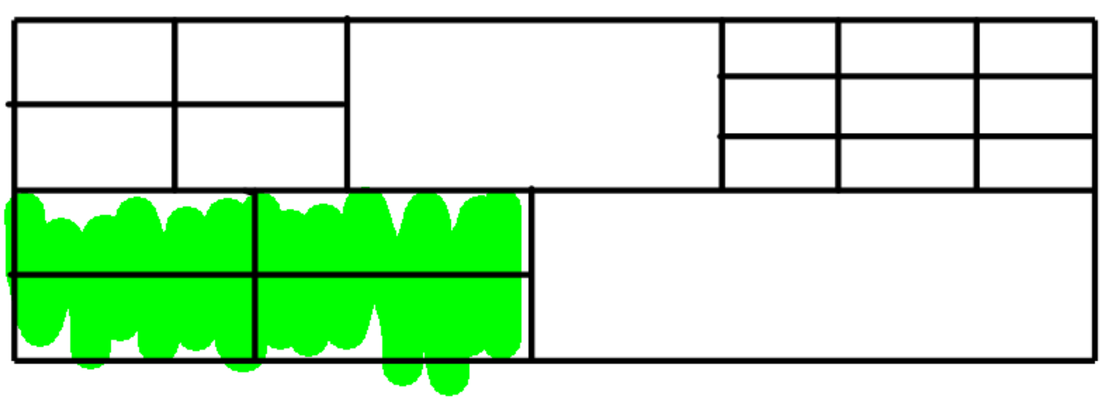
David says, "that's the same thing!" and Mr. Mack tells the class that David is right, but AB protests. Mr. Mack asks who thinks they have it, and 6 hands go up. He asks one

20 AB is the student's initials, and how Mr. Mack referred to her.

student, but she says she doesn't know, and then another student comes up and shades the following:



Mr. Mack asks the class how many people did it this way, and about 6 hands go up. He says, "that's a winner." AB says that's what she had, and Mr. Mack says that the two on the bottom are not the same as the two on top, and demonstrates by moving the shaded area across the bottom and showing that the shaded area fits along the bottom four times, and therefore four more times along the top. He tells the students that another way to think of it is what he calls 'part of a part' thinking. He asks them how much of the region is shaded in the following figure:



And they say 1/4, and one of the students says that ½ of ¼ is 1/8. He asks the class how many students used that type of thinking, and no one raises their hand. He tells them that he didn't tell them this way of doing it before, but that this is the quickest way to do it. He erases the shading and shades 1/8 in the bottom left corner and tells them that they could also see it as ¼ of ½, that they don't need lines, they can just look at it.

One of the students asks why it has to be on the bottom, and says that half the bottom is the same as half of the top. Mr. Mack says that it's not, and shows them that region AB shaded fits into the whole figure 9 times. A couple of students continue to try to argue their point, and Mr. Mack tells them that they'll have to trust him on this, and that it had to be two of these (referring to the rectangles on the bottom left). He says that this is the only answer that he is accepting for this one. (Fieldnote, 11/15/10)

The use of technology in this episode is considered inappropriate as the use of shading with a virtual highlighter for discussing solution strategies is not suited to the accuracy required by the task, as is evident in the excerpt above. Indeed, when a student shades the two rectangles on the top far left, Mr. Mack demonstrates that this cannot be 1/8 by dragging the shaded region across the top of the figure and counting that it fits "five and change" times, meaning that it fit more than five times but not six. In fact, that shaded region is 1/12, and should fit into the top half of the figure exactly six times. While his purpose may have been to simply show that what was shaded could not be 1/8, both the inaccuracy of the shading as well as the estimation inherent in moving the shaded region and approximating the area that it was in prior to moving it result in a crude approximation that is unsuitable for resolving the disagreement that arises. In particular, the strategy used by Mr. Mack is not effective in helping students to see difference between 1/8 and 1/9 in the figure. Due to its inherent imprecision, and therefore its use as a way to approximate the relative size of a given fraction in the figure, a group of students are not convinced that their solution is not correct, and Mr. Mack is unable to convince them using the representation he has created on the IWB. Rather, Mr. Mack must simply assert the correctness of a given answer and ask his students to trust him in order to settle the issue and move on.

When discussing the task afterward, Mr. Mack himself recognizes the shortcomings of the strategy of shading a region and dragging it, stating, "I should have just made individual ones, and that way I could move them around. Probably would have been a more beneficial way to do it, to be perfectly honest…Instead of having to move the highlighted region, I could just move the block." Thus, in addition Mr. Mack's inability to convince students of the difference between 1/8 and 1/9 in figure using the highlighting and dragging strategy, Mr. Mack himself feels that that strategy is flawed. It is not so much the inappropriateness of the task in this case as the inappropriateness of how the technology is used within the task.

In addition to this use of technology failing to support the intended goals of the task, i.e., understanding an area representation of fractions, Mr. Mack was the one who actively manipulated the representation on the IWB, thus the primary role of the technology seems to be to provide a novel representation for fractions. Students did not have access to this representation in order to work on the task themselves. A common theme in the way that Mr. Mack uses affordances of the IWB that go beyond a mere medium for display is that he creates a dynamic representation to demonstrate solutions that students generated without this tool, and that they do not have available to them while working on the task. This is true in the Land Sections task as well, and might be considered "switching modes" in the sense that students work on the task without using technology, but Mr. Mack uses technology to discuss students' solution strategies. Mr. Mack did have access to a cart of laptops that he used with his reading class, but he did not use it for any of the 17 observed tasks with his math class. Our discussion following the fraction area task provides some insight into the reasons for this:

> Mr. Mack: The ones who did it quicker were able to tell me, verbally, it's half of a third, or whatever it might be. So that's why that's a preferred method of thinking, because it just leads to it a little bit quicker.
>
> MS: Is it important for them to visualize in addition to being able to do the 'part of a part' thinking, or is it OK if they go straight to the 'part of a part' thinking?
>
> Mr. Mack: I don't want to say that it's not important to visualize it, you know, because it definitely is, but you can certainly get by without visualizing. Like, I'm not a visual person at all. I don't like visuals. I never was. And so, you could do fine without it. For our purposes, I think it's important that they get the visualization here, because when we do the addition of fractions, I think it's important, but it's not as important as just being able to do it, which sounds bad, but it's kind of the ugly truth. You need to be able to add fractions, and if you can't visualize the adding of fractions it's really not that big of a deal. (Interview, 11/15/10)

Mr. Mack seems to privilege quick and efficient strategies, and views the representations that he creates using the IWB as a scaffold to help students develop a more efficient strategy. This belief is likely a factor in not providing students with the opportunity to interact directly with these dynamic representations. That is, if being able to visualize fractions is optional, then there is no need to have students manipulate these representations directly in a way that would help them construct meaning for them.

In terms of students' thinking, however, it is not clear that seeing a novel representation necessarily results in or encourages high level thinking on its own. Students would likely need to have the opportunity to actively engage with and construct meaning for this novel representation in order for it to result in high level thinking. Thus, while other factors actively contributed to the decline of the cognitive demand during implementation, such as taking over the thinking and shifting the emphasis to the correct answer, the use of technology did not serve to counter this tendency, and in the case noted above, seemed to induce it at certain points. If students had been provided the opportunity to work with the area representation themselves, the factors which contributed to the decline may have been avoided. What all of these tasks had in common during implementation was that students were not able to manipulate the representation directly, and thus were limited in their ability to construct meaning for the representation or the underlying mathematics. In this sense, the use of technology was inappropriate for supporting students' high level thinking.

In summary, there are multiple ways in which a task that utilizes technology may be inappropriate for a given group of students, or the use of technology may be inappropriate in a given task. A task may be inappropriate for students who do not understand the mathematical significance of their action in a DGS environment, as this seems to be prerequisite to using it for high level thinking. On the other hand, students may not be able to understand the requirements of the task that would make use of such tools for high level thinking, or perhaps are unwilling to engage with the task at this level. This issue seems to be related to the classroom culture and expectations regarding students' roles, and is considered inappropriate insofar as the expectations of the task contradict the established classroom norms. While not exclusively an issue related to technology, there was a strong association of such tasks with the use of technology in Ms. Jones' and Ms. Young's classrooms. However, in both these cases, the task was deemed inappropriate for the given group of students.

The use of the IWB may provide novel representations of mathematical objects and actions on those objects, there may be a discrepancy with respect to the precision afforded by the technology and that required by the task. Furthermore, not providing access to these representations to students may limit the potential of the technology to influence students' thinking. "Show and tell" style instruction seems unlikely to engage students in high level thinking regardless of the representation used, and thus such a use of technology seems inappropriate for achieving high level goals.

Given that the factor "inappropriateness of the task" may also refer to issues with the task that have nothing to do with technology, it may be useful to consider the issues described above as new factors associated with decline introduced by the use of technology. Grouping all of these issues under the category of "inappropriateness of the task" may dilute the meaning of this factor as it has been used previously in the literature. However, using the framework as developed, this is how these issues were coded in the present study.

Lack of accountability. The other factor that seems most closely related to the decline of tasks using technology was a lack of accountability for high level processes or products with regard to students' work using technology for tasks set up at a high level. Although this factor was only present in a Ms. Jones' classroom, it has important implications for students' use of technology while engaging with tasks set up at a high level. In all four tasks set up at a high level, Ms. Jones did not collect the computer files or the student worksheet, and usually did not even discuss their work as a class. While a lack of accountability for high level products or processes may be associated with the decline of any task set up at a high level, the important connection of this factor with the use of technology is the difference in accountability when technology was used and when it was not used. As she explains during one of our discussions:

Ms. Jones: I would say easily 75% of the kids are just breezing through the worksheet.

MS: the worksheet or the GSP handout?

Ms. Jones: The worksheet.

MS: OK. And how much of that do you think is –

Ms. Jones: because I don't really check the GSP handouts. I don't read them, I don't – I will force them to go back and look at their answers, but I don't, you know – they think they're doing it for me rather than for their own understanding, and I'm trying to push them to, that's why I don't collect them in any other class, because when I collect them it gives them the impression that I'm checking them, they're doing it for me. When I don't collect them I can reinforce with them that this has nothing to do with me: "this is for you learn from, did you learn from this?" (Interview, 6/23/10)

Ms. Jones seems to want students to take responsibility for their work in GSP, to use the opportunities provided by these tasks to construct their own understandings. However, the fact that she does not take this approach with other classroom activities or assignments may send her students the message that their work using technology is less important or optional.

For example, when Ms. Jones' students engage with the parallel lines cut by a transversal task, she begins the class by putting the diagram depicted in Figure 4.19 on the board, and students are instructed to find the seven missing angles after they've completed the GSP activity and before Ms. Jones will give them the worksheet.

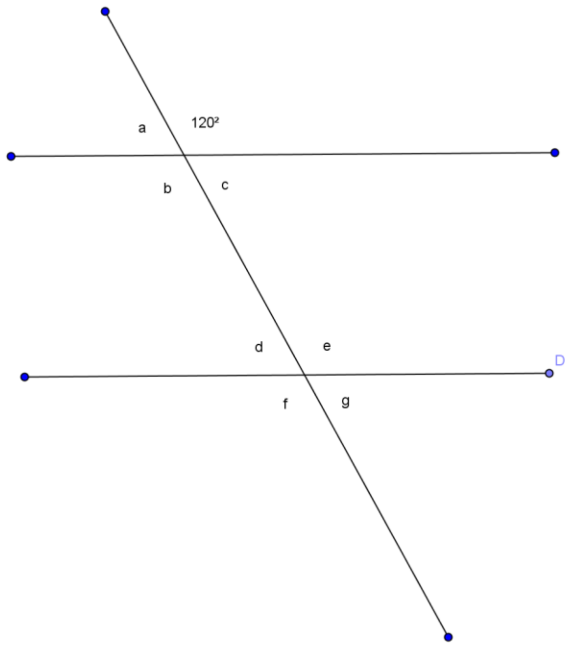


Figure 4.19: The problem that Ms. Jones' students were expected to complete after finishing the parallel line cut by a transversal task in GSP.

In our post-lesson interview, she explains:

> Ms. Jones: The reason I put the question on the board, was that it was a very quick way for me to check, "do they get it?" So I don't know if they, when they come up, and they were able to fill that out, and tell me what all those angles in that parallel line set was [sic], I don't know if they got that out of today's activity, or if they already knew it, but they got it. They got it right away. So that was my quick check of "do you have enough understanding to move on to this worksheet?"
>
> MS: So they had to show you the problem on the board in order to get the worksheet?
>
> Ms. Jones: right. So, to me, I'm not sure which thing got it for them. (Interview, 6/23/10)

The fact that filling in the missing angles in the figure is ultimately what students are held accountable for illustrates a general trend for Ms. Jones of only holding students accountable for the content they were to learn through engaging in a GSP activity, but not the thinking processes involved in the task. Ms. Jones is clear in the quote above that how her students gained the understanding needed to complete the worksheet was not important to her. This is reinforced by a remark she made earlier in the interview:

> If they get it, that's fine, I'm not going to push them to do exploration of something they already understand. If they're comfortable and they can take the worksheet and do it on their own, who am I to tell them 'no, go back and do it in Sketchpad.' They already get it. (Interview, 6/23/10)

This statement makes it clear that the ability to complete the worksheet is the ultimate test of whether or not the task was successful in achieving its goal. Most of the worksheets that Ms. Jones gave to students to complete are "practice" worksheets in which the application of previously learned facts or procedures is sufficient for success. These worksheets were generally low level, but these are what students are ultimately held responsible for. Thus, there is little or no accountability for the high level thinking involved in the task.

Ms. Jones' primary concern in all these tasks which utilized GSP is whether or not the students "got it," which here seems to refer to an exclusively content-oriented goal. After another GSP task, the first thing she says in response to how she thought the task went is, "So, to me, I feel very conflicted as to whether they got the point, and I'm guessing that although there's a lot of quiet, and not a lot said, most of them don't get the point of it. " This concern for students "getting it" without any mention of the thinking or process involved in arriving at the conclusion seems to be an important factor in Ms. Jones' accountability strategies. There is little or no concern for how students arrived at the intended conclusion as long as they can complete the worksheet.

Accountability is not limited to having students submit something concrete for the teacher to check or grade. Having students share their thinking and solution strategies can be an effective means of holding students accountable for their work on a task. However, in most cases, Ms. Jones does not discuss what students did or discovered in their work with GSP, and in some cases follows up students' work with non-digital manipulatives. For example, after having students engage with the Triangle Inequality Theorem task using GSP on their laptops as described above (Figure 4.18), Ms. Jones began a whole class discussion as follows:

Ms. Jones: we know that all triangles have how many degrees?

Class: 180

Ms. Jones: no matter how big, the interior angles of a triangle add up to 180. And how many sides do they have? Three. So, we should be able to make a triangle out of any three sides, right?

She coaxes students until she gets a couple to agree, and identifying one student, throws a meter stick, a dry erase marker, and a pen on the floor and tells him to make her a triangle.

After a couple of students make futile attempts, Ms. Jones goes on:

She tells the class that some students said yes to Question 1 and Question 2, and she gave them triangles to make: 2-3-5 and 2-3-10. She asks students how they did with those, and the students say that you can't do it. Ms. Jones says that some came close, and asks one of the students how close she came, and she said two sides were right and one was within .5, to which Ms. Jones says that she was super duper close. She asks a student who she gave 2-3-10, and he says that he did not come close, and Ms. Jones says that that situation is like the meter stick. (Fieldnote, 6/18/10)

Ms Jones does refer to the work of some of the students who were asked to engage with the revised task in GSP, but no one is asked to share their conjecture or what they observed or discovered through their work in GSP. That is, students are not invited to share their thinking, and in fact, this is the only mention of students' work in GSP in a whole class setting that was observed in her classroom. Furthermore, throughout the rest of this lesson in which she develops the criteria for determining whether three segments can form a triangle, she makes reference to the "meter stick" and not students' work in GSP, as she did in the excerpt above.

In another task in which students explored vertical angles using GSP, rather than discussing the investigation that students did in GSP, she begins her follow-up whole class discussion by using two meter sticks crossed over each other to represent the vertical angles, and rotates them to show that no matter how they intersect, the vertical angles are always congruent. While the issue of accountability in these examples is more related to students' use of technology, the use of technology as both an amplifier and reorganizer was instrumental to the high level demands of this task.

What students are held accountable for is an important indicator of a teacher's purpose in enacting a given task, and a powerful message to students regarding what is valued in the classroom. The fact that these students' work on the computer is treated differently than other classroom assignments in this regard may send an unintended message to students that it is less important or worthwhile. Furthermore, a failure to make use of students' work or thinking while using GSP during discussion of the task may undermine students' engagement with future tasks, as they may be given the impression that it is not important. At the very least, it may lead students to a sense that correct conclusions, and not their thinking, are what are valued. Indeed, such an impression seems to be an accurate description of Ms. Jones' expectations.

4.4.2 The use of technology related to maintenance

As Table 4.20 demonstrates, across all sites only seven tasks were implemented at a high level using technology, two by Mr. Mack and five by Ms. Lowe. Although this constitutes only 11% of all observed tasks, the qualitative nature of the data allows for an investigation of how the use of technology supported high level thinking in these tasks.

In terms of the factors associated with the maintenance of high level cognitive demand (Stein et al., 2009), Figure 4.20 depicts the factors present in the tasks implemented by Ms. Lowe and Mr. Mack. "Building on prior knowledge" refers to students clearly using their prior knowledge to provide access to or make progress on the task, or the teacher explicitly activating students' prior knowledge through questioning while students worked on the task. "Sufficient time" indicates that the majority of students arrived at a solution to or completed the task before the task was concluded, or to students not having too much time and getting off task or distracted after completing the task.

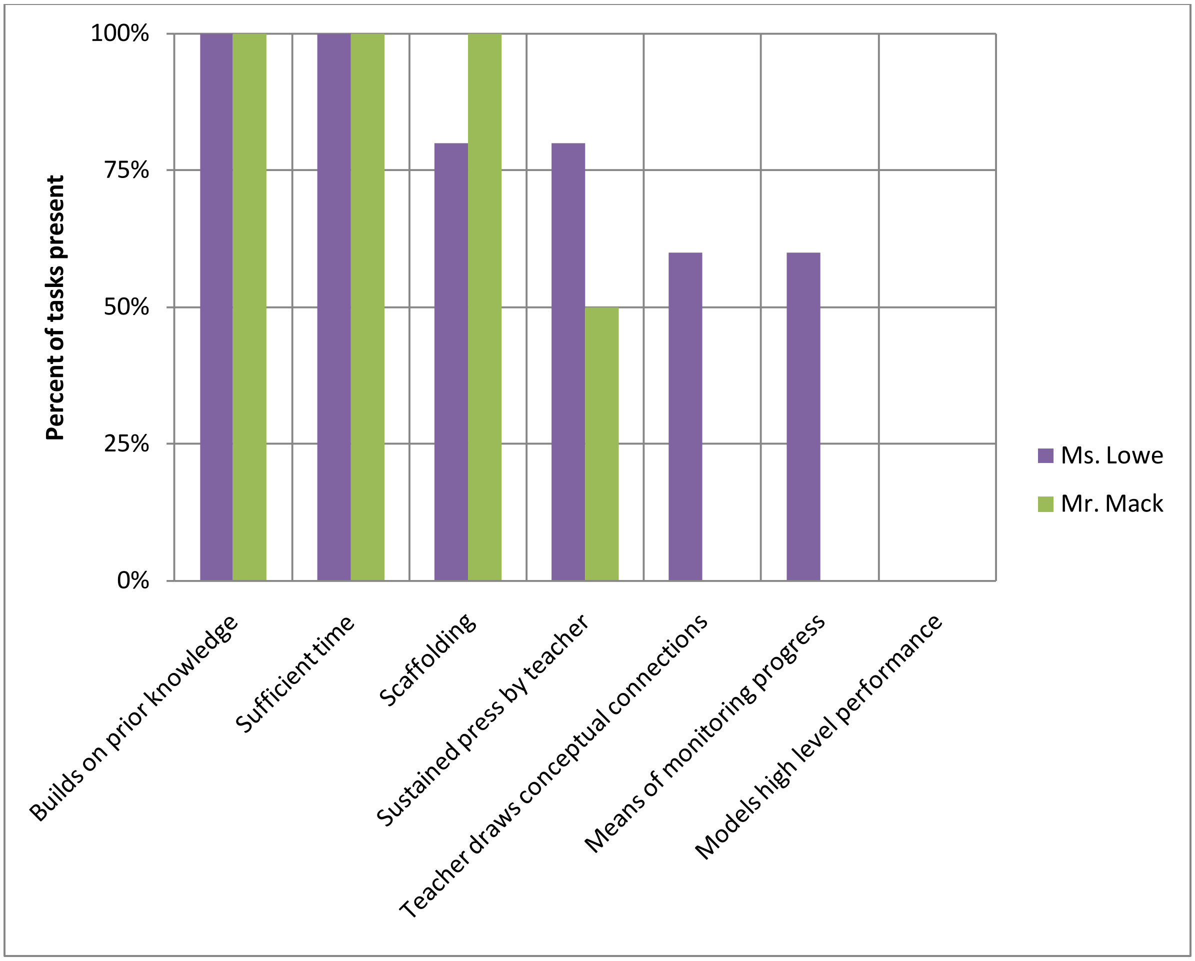


Figure 4.20: Percentage of tasks which includes each factor associated with maintenance, by teacher.

"Scaffolding" refers to the teacher helping students make progress on the task without lowering the demand of the task. This is often done through questioning students in a way that helps them to make connections needed to progress in the task. It may also refer to the teacher noticing a student making an incorrect assumption or inference which would prevent further progress on the task at a high level. For example, the teacher might help students correct the construction of a figure in a DGS which is mathematically incorrect or inaccurate. The key feature in this factor is that the teacher provides guidance needed by students in order to make progress at a high level. "Sustained press by teacher" refers to the teacher questioning students in a way that ensures that students grapple with the high level aspects of the task, or requiring that students explain their thinking or strategies precisely. While this may include questioning that would help students to activate prior knowledge, such exchanges with students needed to do more than ask students to recall or connect to prior knowledge in order to be given this code.

"Teacher draws frequent conceptual connections" generally refers to helping students make connections between their work on a given task and other relevant concepts that they are familiar with or that they will study in the future. As the word "frequent" indicates, these connections needed to be made more than once during the task, and had to consist of more than simply recall of facts or procedures. "Models high level performance" includes a demonstration by the teacher or a student of the type of thinking, behavior, or solution that was expected while engaging with the task. "Means of monitoring progress" is interpreted to means that students have an alternative to the teacher in verifying their progress on the task at a high level. For example, students might drag a figure that they've constructed in order to determine whether or not it is a square. The progress is not necessarily only in relation to the task goals. For example, if the goal of task is to make a conjecture, then students being able to verify their conjecture for themselves, or ensure that the examples they are generating are mathematically accurate and therefore have the potential to support a conjecture, are both ways in which students might monitor their own progress. Two factors present in all of these tasks were enacting tasks that build on students' prior knowledge and allowing sufficient time for students to grapple with the high level demands. Providing scaffolding of students' access to the task, and pressing students for justifications and explanations were also common to both teachers, although not present in all tasks.

Figure 4.21 provides a brief description of the seven tasks which were implemented at a high level using technology. The Land Sections task was discussed in connection with Research Question Two, with the primary result being that the use of technology in that task was not directly related to the cognitive demand of the task during implementation. Building on students' prior knowledge, providing sufficient time for students to work on the task, and scaffolding students' engagement in the task were present in both of the tasks. The use of the interactive version of the Land Sections map during set up was considered to provide a scaffold for students' engagement with the task on the second day. However, its use was not deemed to have a significant impact on students' thinking during implementation of the task. As such, there is no further discussion of these tasks from Mr. Mack's classroom in terms of the role that technology played in the maintenance of the high level demand of the task.

As Ms. Lowe was the only teacher to implement any tasks at a high level for which the use of technology was deemed to be instrumental in the cognitive demand of the task, qualitative analysis focuses on those practices of Ms. Lowe which seemed most instrumental in maintaining the cognitive demand of these tasks. All five tasks that Ms. Lowe implemented at a high level with her students were very similar in structure. Students met in the computer lab instead of their normal classroom, and were given a worksheet which guided them through the activity, and asked them to make observations and conjectures. Ms. Lowe generally set up the task in less than five minutes, and the remainder of the class period was spent in independent exploration while Ms. Lowe circulated the room monitoring students' work and asking and answering questions.

Teacher	Technology	Task Description
Mr. Mack	IWB	The Land Sections Task: the teacher uses an interactive version of the task to explain strategies for determining the fraction of a section each person owns.
Mr. Mack	IWB	The Land Sections Task: the teacher uses the interactive version of the map to set up the task and explain strategies for combining sections, i.e., fraction addition.
Ms. Lowe	GeoGebra	Students use GeoGebra to explore the properties of the perpendicular bisector and circumcenter of a triangle
Ms. Lowe	GeoGebra	Students use GeoGebra to explore the properties of the angle bisector and incenter of a triangle.
Ms. Lowe	GeoGebra	Students use GeoGebra to explore properties of altitudes and the orthocenter, and to use their results to solve for the coordinates of the orthocenter of a triangle
Ms. Lowe	GeoGebra	Students use GeoGebra to explore properties of medians and the centroid of a triangle, and to discover the relationship between the median segments.
Ms. Lowe	GeoGebra	Students use GeoGebra to explore the properties of the midsegments of a triangle.

Figure 4.21: Summary of tasks using technology implemented at a high level across all sites.

Analysis of these five tasks revealed that several practices were particularly crucial in the maintenance of the cognitive demand during implementation of these tasks. These practices are listed below as specific moves that Ms. Lowe made that are more specific than, but in most cases are instance of, one of the factors named in the literature and used to code these tasks, as indicated in parentheses:

- allowing the entire class period for the investigation (sufficient time)
- holding students accountable for their work
- carefully monitoring students' work on the computer (scaffolding)
- asking questions which forced students to interpret their results mathematically (sustained press for explanation, justification, and meaning)
- using technology as a means for students to monitor their own work, often in the context of a sustained press for meaning or explanation

Another issue not limited to a single task was that three of these five tasks were enacted within a span of five days toward the beginning of the unit. This regular use of GeoGebra early in the unit provided students the opportunity to familiarize themselves with the affordances of GeoGebra, and the requirements of these tasks which made use of it.

Sufficient Time. Ms. Lowe set up three of the tasks in two minutes each, and the other two tasks in four minutes each. Her set up generally gave students an idea of the overarching purpose of the exploration and connected it to their prior knowledge. This allowed students almost the entire period to work on the exploration. Students were rarely observed to finish more than five minutes before the end of the period, and the worksheets collected indicated that most students did complete the entire activity. When students did finish early, Ms. Lowe often kept students engaged by extending the investigation for them. For example, at the end of an investigation of medians and their point of concurrency, the centroid, she asks students to explore these ideas for isosceles and equilateral triangles as well, which was not part of the original task. Thus, the amount of time allotted for the task seemed appropriate for this group of students.

Accountability. Related to the amount of time allotted were the expectations for a finished product by Ms. Lowe. For example, without any class discussion of the conclusions that students were to draw, Ms. Lowe asked students to complete a homework assignment based on the results of their own investigation. The following excerpt from the field notes describes how these expectations were communicated during the set up of the task:

> She tells them to listen to her while they're starting up GeoGebra: "here's the deal. What you're doing on GeoGebra is intended to give you enough to do the homework. These activities are replacing me standing in front of the room and telling you, yakking at you." She tells them that they need to really focus on what they're doing, and if they don't understand what they're doing or why, they need to ask her. She tells them that they shouldn't walk out of here not knowing what's going on, and if they do it's their own fault. She tells them that when she gives them the homework, she'll see how much attention they were paying to what they were doing. (Fieldnote, 1/28/11)

This particular accountability strategy employed by Ms. Lowe allowed more time for students to engage in the exploration portion of the task, as class time was not needed for a follow up discussion or lecture. Thus, the amount of time allotted and the accountability for students' work on these tasks are closely related in this case.

Perhaps more importantly, however, this kind of accountability for students' work on the task seems to have important implications for their engagement, a fact confirmed by Ms. Lowe.

This was the first time that she had deviated from the practice of giving a follow-up lecture, and in our post-lesson interview she seemed very pleased both with the focus on the part of the students, and with their ability to apply what they learned from the task to the homework she gave them. She told me that students were beginning to think about how what they were noticing and learning during the investigation might be applied. The novelty of being held accountable may certainly have been a factor in any increased engagement that Ms. Lowe noticed, or perhaps she was simply more aware of their engagement because she anticipated this accountability strategy having this effect. It is not possible to know what these students' engagement with the task would have been if this strategy hadn't been employed. However, the factors that have been identified in the literature were coded based on their presence, not on whether or not they were deemed to contribute to the high level demand of the task during implementation. Although this is not one of the factors previously identified as being associated with high level maintenance, the lack of accountability in Ms. Jones classroom, and its association with decline, resulted in an increased awareness on the part of the researcher of how teachers held students accountable for their work on the computer. If students know that the teacher is going to tell them what they were supposed to gain from engaging with the task, it may provide little motivation for students to engage seriously in the task. It stands to reason that if a lack of accountability is associated with decline, then a clearly communicated and meaningful accountability system may be an important factor in maintenance.

Scaffolding students' work with technology. Scaffolding students' engagement with the task by carefully monitoring students' work on the computer was a crucial factor in Ms. Lowe's implementation of these tasks, and was a consistent part of her practice when implementing these types of tasks. One example occurred during a task using GeoGebra to explore the circumcenter of a triangle. She wanted students to notice that the three perpendicular bisectors[21] of a triangle intersect at one point, the circumcenter, and then to notice that the location of the circumcenter varies depending on whether the triangle is acute, right, or obtuse. Another property that she wanted students to discover was that the circumcenter is equidistant from the three vertices of the triangle, as in Figure 4.22. One way to do this in GeoGebra is simply to measure the distance from the circumcenter to each vertex. Instead, Ms. Lowe had students construct a circle with the circumcenter as the center, and passing through one of the vertices. Since the circumcenter is equidistant from the three vertices, this circle passes through the other two vertices as well. Thus, students were expected to observe that the circumcenter is equidistant from each vertex by noticing that the distance from the circumcenter to each vertex is the radius of the circle, and that this relationship holds when the triangle is dragged. As Figure 4.22 demonstrates, the fact can be less than obvious because the perpendicular bisectors appear on the figure, but not segments from the circumcenter to each vertex. One must reason that because the circle passes through the vertices, and the circumcenter is the center of the circle, then the distance from the circumcenter to each vertex is the radius of the circle, and therefore the distance to each vertex is the same.

[21] A perpendicular bisector of a triangle is a line that intersects a side of the triangle at right angles and divides the side of the triangle it intersects into two equal segments.

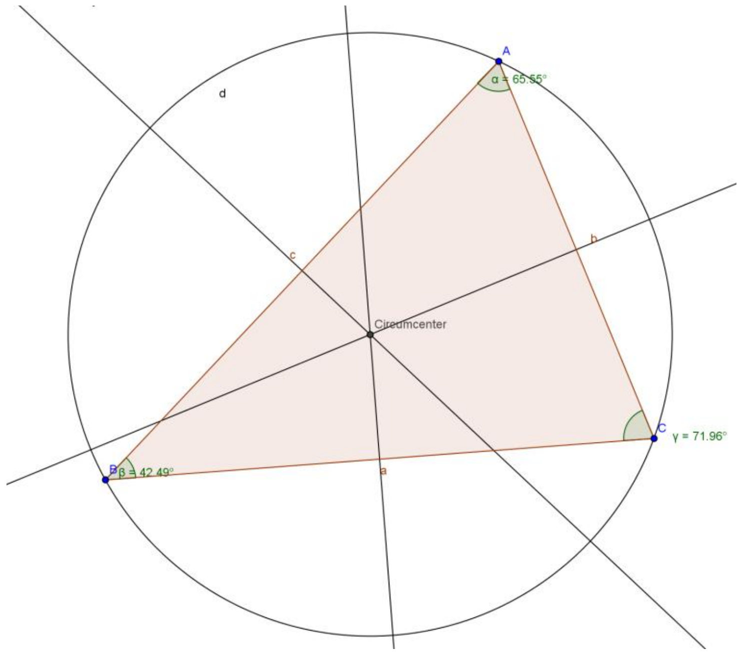


Figure 4.22: Representation of a circle with the circumcenter of a triangle as the center in GeoGebra.

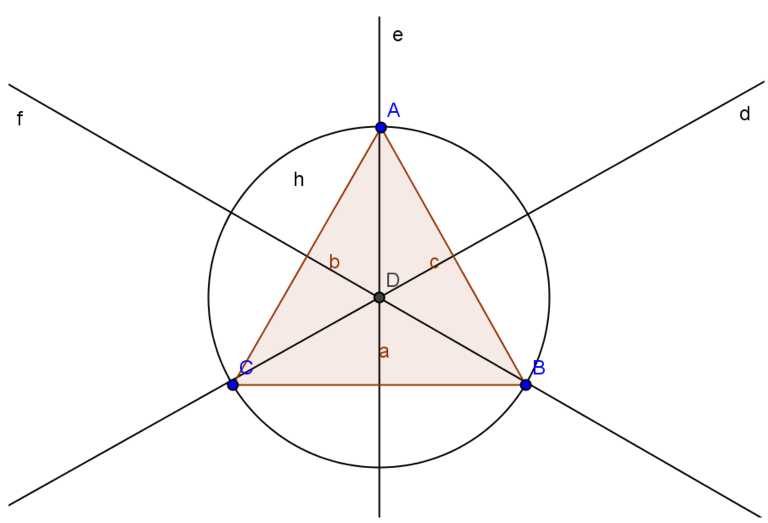
The following field note excerpt from the observation of this task provides an example of how Ms. Lowe monitors students' work as they begin the task:

> I don't see any students dragging their constructions, but rather they have constructed perpendicular bisectors and are writing on their worksheet. Ms. Lowe asks a student what she's noticing, and asks her if she's moved her triangle around. When the student begins to drag her triangle Ms. Lowe asks her, "what's going on?" One student says that the perpendicular bisectors intersect at a single point, and a second student next to her says that her intersection point is not inside the triangle when it's obtuse. Ms. Lowe says, "that's exactly what I want you to see," and the first student asks the second "what? When it's obtuse it's not in the triangle?" as Ms. Lowe walks away. The second student says yes, and demonstrates on her computer while the first student looks on. (Fieldnote, 1/27/11)

This is the first GeoGebra activity of the unit, and her students' experience with the program up to this point has been limited. Ms. Lowe ensures that her students utilize the affordances of the technology by telling them to drag their figure and asking them what they notice. This apparently simple instruction is the beginning of students learning to connect the affordances of the tool to the requirements of the task. Furthermore, the feedback that she provides to students ("that's exactly what I want you to see") helps students to know that they are meeting Ms. Lowe's expectations for the task, and that this is an important observation. Given the plethora of things to notice or observe in this environment, this is a subtle but important move on Ms. Lowe's part to not only ensure that students are in the right cognitive space for the task, but to help students begin to understand what a valid observation is so that they can begin to monitor their own work.

The following field note excerpt from the same task demonstrates how Ms. Lowe supports students' engagement with the task, both in terms of generating constructions that are mathematically accurate, and in holding students accountable for their work on the task.

She comes to another student who has the following on his screen:



> And she remarks, "oh, isn't that nice!" but when she asks him to drag a point it is clear that his circle is drawn over top of the triangle and not connected to it because it comes right off. She tells him to delete his circle, and then instruct him to do #12 on the worksheet which he has skipped over (drawing the three types of triangles, acute, obtuse, and right, and making a conjecture about the location of the circumcenter in each case). To make his circle correctly she tells him to pick one of his vertices, but he doesn't understand (he is a foreign exchange student with some language issues). She helps him to understand what she means by vertex and that he only needs to choose one. When he finishes she has him drag the same point as initially, and says, "there you go. That what I'm after." (Fieldnote, 1/27/11)

This student has made a figure that appears to be what he is supposed to construct, but in fact is not. Ms. Lowe, however, is constantly vigilant, moving from one student to another and asking them to drag their figures, which serves two purposes. First, as noted above, she has students begin to use the affordances of the tool that are suited to making generalizations and conjectures by generating many examples quickly and asking students what they notice. Secondly, as this example demonstrates, she teaches students to assess the correctness of their own construction, and helps them to make corrections when necessary. This type of behavior on Ms. Lowe's part was constant throughout the lesson, but especially during the first 10-15 minutes of the task when students were making their constructions. It is important to note, however, that while she is helping students to test and correct their constructions, she never touches the mouse. The students remain in control of the use of the technology, while Ms. Lowe helps them to correct their constructions, as in the case above.

In another task, students use GeoGebra to construct the incenter of a triangle (the intersection of the angle bisectors of a triangle), to discover that the incenter is always located inside the triangle, and to discover that it is equidistant from the sides of the triangle as in Figure 4.23.

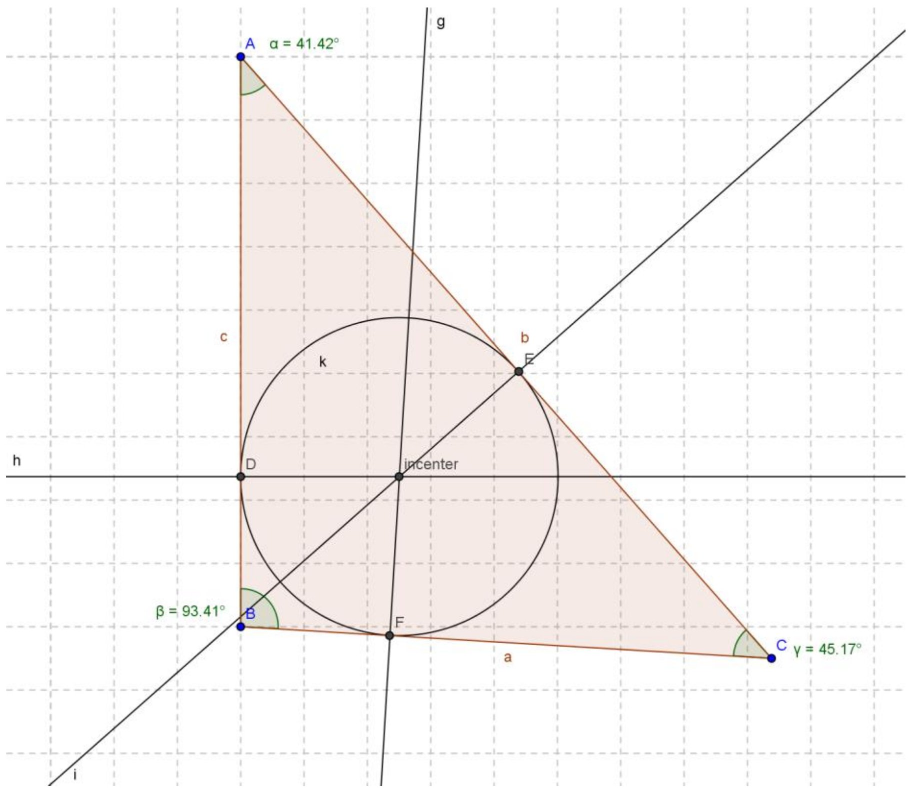


Figure 4.23: A circle inscribed within a triangle with the incenter as the center.

In the following excerpt from our post-lesson interview for this task, Ms. Lowe discusses how she supports students' work with the tool:

MS: So like, when you went, whether they asked for help, or you intervened, what were the issues that you were seeing?

Ms. Lowe: Well I mostly intervened when I saw their constructions not reflecting what they needed to reflect. It seemed like the most common one, at least for activity two, was that they could find and construct the incenter just fine, what they were having trouble

> doing was finding, they were putting the center of the circle on the incenter, and then I think in most cases just picking a point, or a random point that may look like where it's supposed to be…they were creating the picture they thought it was supposed to look like, but it wasn't actually the construction that they were supposed to make…
>
> MS: So before when she did it, and she dragged, the circle didn't move when she dragged the triangle, so the circle stayed the same size.
>
> Ms. Lowe: Exactly. Or if she happened to drag the circle instead of the triangle, then the circle could be gigantic and the triangle could be on the side, and it just wasn't, you know. So in order to not waste their efforts, and to get them where I needed them to be, when I saw that, that's when I would stop, or I'd look at someone's screen and say "move your triangle for me" so I can see that they indeed have the construction.... But that's what I was after. (Interview, 1/28/11)

While much of this conversation relates to the specific issues that Ms. Lowe was noticing in this task, this conversation demonstrates that she is aware of the danger of students using the tools to create figures that looked correct but in fact were not, and that doing so would prevent students from achieving the goals of the task. There is a mathematical difference between a figure that looks like a circle inscribed in a triangle, and a circle that actually is inscribed in a triangle. Using GeoGebra to make these constructions helps to make these differences salient to students, as well as to construct mathematical meaning for the tool. For example, when a student sees that "the circle is stuck to that point," that is part of understanding that difference between a point that looks like an intersection point, and a point that actually is an intersection point. However, understanding these differences can only be fostered if the teacher is aware of this danger and vigilant in monitoring students' work.

Evidence that these exchanges influence students' ability to monitor their own work over time is given by an exchange during the last GeoGebra activity of the unit on properties of midsegments:

> Will tells Ms. Lowe that he realized that his figure is wrong, and asks for another activity worksheet. He explains that when he dragged his figure, he realized that he hadn't created a midpoint because it didn't stay on the segment of the triangle. (Fieldnote, 2/16/11)

This is the fifth (and last) GeoGebra activity in the unit, and although it's been a few weeks since the last one, Will understands the need to drag his figure to test his construction, and how to interpret the results of his dragging. This is also an example of the factor "providing students with the means of monitoring their own work." This student is likely drawing on prior knowledge of the importance and method for checking his construction that he has learned from previous tasks, and using the affordances provided by the tools at his disposal in order to construct a mathematically accurate figure capable of supporting exploration and conjectures. Helping students to understand the mathematical meaning of their work with a technological tool is an important way to deepen students' mathematical understandings, as well as their meaning for the tool.

Sustained press for meaning and explanation. Another important factor in the maintenance of high level tasks in Ms. Lowe's classroom was her insistence that students interpret their observations, that is, that making an observation was not enough to satisfy the task requirements. Ms. Lowe does not assume that as long as students have mathematically accurate and correct constructions, then the mathematical meaning or importance of that construction will be obvious. In particular, she engaged in the following practices while students used GeoGebra:

- she asks questions that require students to think about the mathematical meaning and connections embedded in the task
- she turns students' questions back to them and their construction
- she walks away from a student in order to allow him or her to grapple with cognitively demanding aspects of the task.

An example of how Ms. Lowe requires students to interpret their work mathematically is related to a particular challenge that Ms. Lowe noted on the circumcenter task. She said during the post-lesson interview that students struggled to understand the implication of a circle with the circumcenter as the center and passing through the vertices of the triangle, i.e., that the circumcenter is equidistant from the vertices. Below are collected excerpts from the field notes that demonstrate how Ms. Lowe presses students to make this connection:

> Talking with another student, the student tells Ms. Lowe that all three perpendicular bisectors intersect at a point which is the center of a circle. She tells the student to think about what that means, and to think about the parts of a circle. (Fieldnote, 1/27/11)
>
> She asks another student, "what do you think?" She tells him, "you're seeing what I want you to see. What does it mean?" The student struggles to make a generalization, perhaps unsure of what Ms. Lowe is looking for. She tells him to think about it, and then tells him to think about the parts of a circle, and she walks away. (Fieldnote, 1/27/11)
>
> Ms. Lowe: move the triangle and show me what you're seeing. (student moves her triangle) What's it doing?
>
> Student: it stays on it.

Ms. Lowe: what does that mean? What is the relationship between the circumcenter and the vertices?

Student: it keeps equal distance.

Ms. Lowe: what is? What is the equal distance from the center to the points?

Student: the radius

Ms. Lowe: the radius is what?

Student: the same

Ms. Lowe: so what does that mean?

Student: that the distances are congruent.

Ms. Lowe: write me a theorem. (Fieldnote, 1/27/11)

Although Ms. Lowe's use of the term "theorem" in the last instance is imprecise, these excerpts demonstrate that Ms. Lowe requires students to interpret their observations mathematically. In each of the three excerpts above, Ms. Lowe asks students, "what does that mean?" in response to students' observations. Ms. Lowe does not simply have students make observations, but presses them to interpret those observations mathematically and to make connections to prior knowledge. This kind of questioning is important to the maintenance of the high level demand of the tasks, and builds on the monitoring that she has done. By ensuring that students' constructions are accurate, she puts them into a position to make observations that are mathematically meaningful.

Another practice that Ms. Lowe used to press students for meaning and justification is to turn students' questions back to them. For example, while working on the midsegment triangle task using GeoGebra, Neil has made some observations, but has not discovered all of the properties of midsegment triangles that Ms. Lowe had intended. She pushes him to do more:

> Ms. Lowe looks at Neil's paper and says, "there's a little more." She tells him that he labeled his triangle differently than hers, and she wants to make sure that he's seeing the things that she wants him to see….She tells him to "look at this" referring to DF and AC, which she reminds him don't change when he "bounces" point B. Neil asks, "is that half of the whole?" and she replies, "I don't know. Is it? If you bounce A, what changes?" and Neil replies, "FE?" Ms. Lowe tells him to try it, to move A, and asks what doesn't change… She tells him to look at the measures, and Neil says, "oh, ½ !" …Neil asks, "what does that mean?" and Ms. Lowe replies, "I don't know, what does it mean?" and asks him about the other pairs. Neil says, "this is also ½ of this, and this is ½ of this, and this is ½ of this" referring to the segments and midsegments. (Fieldnote, 2/16/11)

While Ms. Lowe scaffolds his observations by helping him to know where to look, she refuses to confirm them, but rather refers him to his construction. In this way she keeps the onus on him to make and confirm observations and conjectures, which is considered to be the high level aspect of this task. Thus, when answering a question a student asks would lower the cognitive demand, one strategy Ms. Lowe uses is to pose the question back to the student. In addition, this excerpt is another example of "providing students with the means to monitor their own work." In general, when she reflects students' questions back to them, she is encouraging them to use the technology in this way, and to interpret their observations while doing so.

Ms. Lowe also reflected students' questions back to students when they asked a question which extended the exploration in the task. For example, Brian asked many of these types of questions, and invariably Ms. Lowe turned the question back to him. Below is an example in which she has him pursue his own conjecture when he has finished the incenter task:

> Brian asks "if the triangle is an equilateral triangle, will the incenter be the same distance to the sides as the vertices?" Ms. Lowe tells him that that's a great question, and tells him that he has 9 minutes and a tool to investigate it with. (Fieldnote, 1/28/11)

While this was not a task that Ms. Lowe had prepared for students, she does not start handing out the homework to students as they finish the task, but allows students to remain engaged with the task at a high level by extending the task for them or encouraging them to continue the exploration. In fact, Brian stayed after school (the observed class was the last of the school day) for about 30 minutes on a Friday to conduct his investigation, concluding that the distance from the incenter to a vertex of the triangle is twice the distance from the incenter to the side of the triangle, and that in an equilateral triangle the incenter and the circumcenter coincide, as shown in Figure 4.24. This is also an example of "providing students with the means to monitor their own progress" in the sense that Brian has come up with his own conjecture, and Ms. Lowe is referring him to the tools that he has available in order to investigate and confirm it. The practice of reflecting students' questions back to them is often associated with students using the technology to monitor their own progress, and a concrete example of how the use of technology can help to redistribute the mathematical authority in the classroom. The potential for students to use something like GeoGebra to form and verify their own conjectures has important implications for students' mathematical agency and authority.

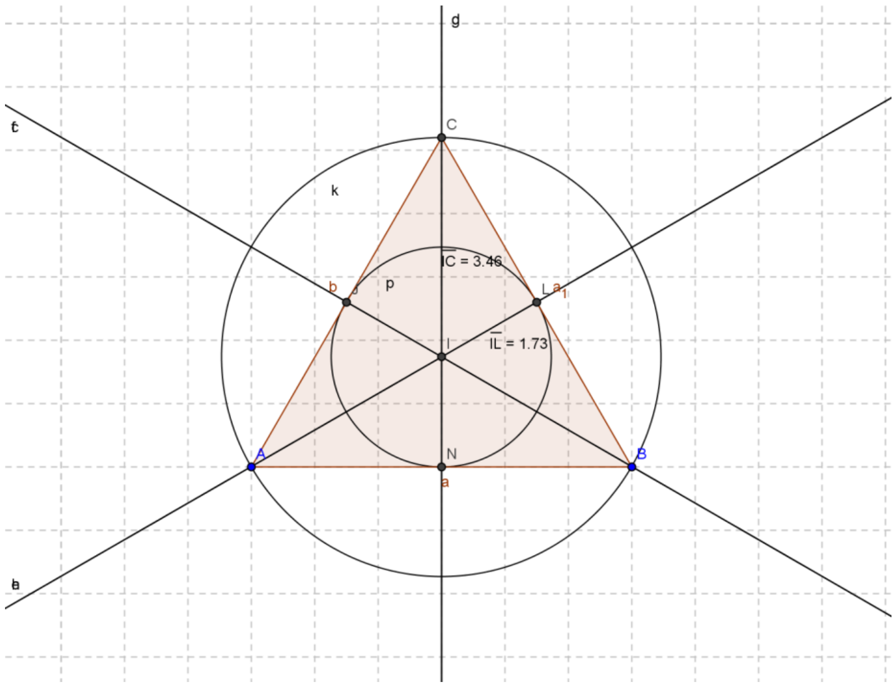


Figure 4.24: The figure Brian constructed in GeoGebra while investigating the location of the incenter in an equilateral triangle.

Another way in which Ms. Lowe sustains the press for meaning and justification is by being willing to walk away from a student before they have come to a conclusion if she feels that they have enough information to make progress on the task. She ensures that students have made the construction accurately and have made relevant observations that can be used to make progress on the task, and then "stirs the pot" by asking students to interpret those observations. One example of this was cited above to illustrate the questioning Ms. Lowe uses to get students to make meaning of their work.

> She asks another student, "what do you think?" She tells him, "you're seeing what I want you to see. What does it mean?" The student struggles to make a generalization, perhaps unsure of what Ms. Lowe is looking for. She tells him to think about it, and then tells him to think about the parts of a circle, and she walks away. (Field note, 1/27/11)

A similar example precedes the exchange above in which Ms. Lowe reflects Neil's questions back to him:

> After Neil shows Ms. Lowe what he's noticing by dragging the triangle, he asks her if that "has anything to do with it" and she says, "I think it does. What's not changing?" Neil replies, "the lengths" and Ms. Lowe says, "what else?" Neil says "the midpoints" and Ms. Lowe again replies, "what else?" and asks him to think in terms of the coordinate plane, and Nick says something about the x-axis, and then says he doesn't know. Ms. Lowe tells him to keep playing with it and walks away. (Field note, 2/16/11)

By walking away, she prevents further discussion or questions from the student which could result in lowering the cognitive demand. She is effectively telling the student, "you don't need to ask more questions, you need to think about what you've observed." Furthermore, it communicates to the student her confidence in their ability to interpret their observations and make conceptual connections for themselves.

The practices associated with sustaining a consistent press for meaning, explanation, and justification are consistent with Ms. Lowe's goals of having students actively construct their own mathematical knowledge, as evidenced by her comment below:

> I'm hoping that they will come to appreciate what they're doing, and feel accomplished like, "You know what, I did this." You know? Here's me, here's them. They walked three-quarters of the way instead of me walking three-quarters of the way. Do you know what I mean? (Interview, 2/1/11)

Ms. Lowe wants her students to have the feeling of accomplishment that comes from constructing their own mathematical knowledge, and believes that she has a role to play in that process as implied by the phrase "they walked three-quarters of the way". That is, she walks one-quarter of the way, which may be a way to describe her active role in scaffolding students' engagement and requiring that they interpret their work on the computer mathematically.

Ms. Lowe certainly views the use of technology as playing an important role in students' mathematical work. However, while it provides a means for supporting students in working independently and exploring mathematics, Ms. Lowe was the only teacher who was successful in facilitating such an experience without the use of technology as well. In order to better understand the role of technology in the factors associated with maintenance, this task is analyzed below by way of comparison.

The task required students to use uncooked sticks of spaghetti and a ruler to investigate the Triangle Inequality Theorem. As shown in Figure 4.25, students were to create three triangles and record the lengths of the sides, and then to create three "non-triangles" and record the lengths of the sides. Ultimately they were to observe that the small plus the medium side is always larger than the larger side in a triangle, and that this is not true for segments of spaghetti which do not form a triangle. Finally, students were to generalize their results for all triangles in the form of a conjecture. Students broke their pieces of spaghetti and made measurements individually, but were required to discuss their observations in groups before making conjectures. Most groups had little difficulty in making the conjecture that a triangle can be formed when the sum of any two sides is longer than the third, and had done so half-way through the class period. However, all the groups either struggled to determine if the sum of two sides could equal the third in a triangle, or incorrectly concluded that it could. The second half of the class was spent wrestling with this question.

The factors associated with maintenance that were coded as being present in this task were scaffolding of students' engagement, consistent press for meaning and explanation, and sufficient time. The following is included as an example of the kinds of interactions that Ms. Lowe had with students and groups:

> Ms. Lowe asks Bruce what he's found, and he says that the small plus the medium sides is greater than the long side. She asks him to explain that, and he draws a triangle, and referring the angles says something about "bigger than these two" and Ms. Lowe says, "not necessarily." She looks at numbers 5, 6, and 7 on the back of sheet and asks him if that's a strict inequality, and he says "yes." She asks him if he's checked with the other students in his group and he says "no." She says to him, "So if it's true for you, that's good enough? You don't need to check the results of the other people in your group to see if it's true in general?" Bruce replies that it's obvious. She tells him to think outside of "that relationship," referring to the "small + medium > large" that he has written for number 5. She tells him to think about all the sides, and says, "what's down here is more" referring to the Triangle Inequality that he has filled out at the bottom of the page. Circling those three inequalities with her finger, she tells him, "explain what you're telling me down here. What does this mean?" He begins to read it to her, and she says "I can read it. Tell me what it means." Bruce says, "when you make a triangle, the sides have to be like this. The two little lines have to be bigger than the longest one. Then you can make some angle." Ms. Lowe turns to Jennifer, who is sitting next to Bruce and has been listening, and asks, "do you understand what he's saying?" She replies, "no" and Bruce says, "me neither." Ms. Lowe has the group members turn toward one another and begin to look at it together. She tells the other students (Jennifer, Grace, and Laura) "he has good ideas. You can help him with the words." (Fieldnote, 2/18/11)

Objective: In this activity, you will compare the sum of the measures of two sides of a triangle to the measure of the third side.

1. Break a piece of spaghetti into three pieces, and use the pieces to form a triangle. Measure each side length to the nearest tenth of a centimeter. In the table below, record the measures of each side of the triangle from smallest to largest; then, find the sum of the measures of the small and medium sides. Repeat this twice, with two other triangles to complete the chart.

Small	Medium	Large	Small + Medium
9	11.5	14	20.9
5 ~~6.2~~	6.2	7	11.2
7.8	10.5	10.7	18.3

2. Break a piece of spaghetti into three pieces so that it is impossible to form a triangle. Measure each side of the non-triangle to the nearest tenth of a centimeter. In the table below, record the measures of each side of the non-triangle from smallest to largest; then, find the sum of the measures of the small and medium sides. Repeat this activity twice, with two other non-triangles to complete the chart.

Small	Medium	Large	Small + Medium
4	6	19.7	10
1.5	3	8	4.5
3.3	4	10.3	7.3

3. Compare the sum of the measures of the small and medium sides to the measure of the large side for each triangle you created. Describe what you notice.

If small + Medium length is ~~shorter~~ longer than Large, you can r triangle.

4. Compare the sum of the measures of the small and medium sides to the measure of the large side for each non-triangle you created. Describe what you notice.

If small + Medium length is shorter than Large length, you can't make triangle.

5. Based on your observations, write a conjecture about the relationship between the sum of the measures of the small and medium sides of a triangle and the measure of the large side of the triangle.

If the measure of the small & medium sides is greater tha the large then you can form a triangle

6. Test your conjecture by using a classmate's measurements. Does your conjecture hold true? If not, revise it. If so, would your conjecture hold true for ***any*** triangle? Explain. Yes, it holds true for everyones.

7. Is it possible to have a triangle such that the sum of the measures of the small and medium sides is equal to the measure of the large side? Explain.

No, because the ends won't touch when it's really obtuse w/ the spaghetti.

8. If the sum of the measures of the small and medium sides of a triangle is greater than the measure of the large side, what can you conclude about the sum of any two sides when compared to the third? Explain.

If the two small and mediums sides are greater than the large one then they will always make a triangle.

9. Write three inequalities that are always true for a triangle with side lengths ***s***, ***m***, and ***l***.

The Triangle Inequality

In a triangle with side lengths *s*, *m*, and *l*,

m + s > l

s + m > m

l + m > s

Figure 4.25: Student worksheet for the Triangle Inequality Theorem task.

In this excerpt, Ms. Lowe scaffolds Bruce's engagement by not allowing him to make a premature generalization, and encourages the group to work together. Her requirement for explanation in this excerpt is explicit; she asks him to explain his answers and tell her what they mean. In other exchanges with students she is observed to reflect students' questions back to them and to ask them a question or tell them to "think about it" and walk away. Thus, in many ways Ms. Lowe's implementation of this task is similar to practices observed while implementing tasks with GeoGebra in the computer lab. However, there is one important difference related to using spaghetti for this investigation. Namely, this manipulative does not provide a means for students to monitor their own work when wrestling with the question of whether or not the sum of two sides can equal the third side in a triangle in the same way that GeoGebra did in the tasks described above.

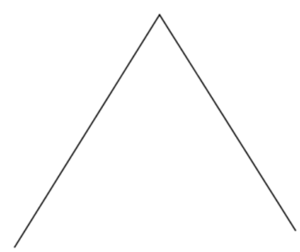
This issue arises when students are attempting to determine if the sum of two sides can equal the third in a triangle. Due to the thickness of the spaghetti and a lack of precision in measuring the pieces, some students believe that they have created triangles in which the sum of two sides equals the third. In fact, Bruce had previously concluded that it was not possible for the sum of two sides to equal the third in a triangle, and after discussing this with his group, he revises his conclusion:

> Ms. Lowe returns to Bruce's group, and he announces that he was wrong. Ms. Lowe asks him who convinced him he was wrong, and he said that he saw them make a triangle. Ms. Lowe asks what the measurements of the sides are, and Jennifer says that the long side is 1 inch, and the other two sides are a ½ inch, and she shows her the triangle made with spaghetti on Grace's desk. Laura asks, "did we do it wrong?" and Ms. Lowe says, "I didn't say that." She asks Grace what she's doing, and Grace says that she's trying to

draw it, but it doesn't work out. Laura says something about a triangle with sides 6, 4, and 2 cm, and then she says, "I need GeoGebra!" (Fieldnote, 2/18/11)

When Ms. Lowe notices that some students believe that they have created triangles in which the sum of two sides equals the third, she begins to ask students to try to draw it on their paper using a ruler. Eventually, Ms. Lowe provides students with the following demonstration:

Ms. Lowe picks up two whole pieces of spaghetti, telling the group that they're the same length so this would be an isosceles triangle, 10 inches and 10 inches, for example.



She tells them that she is going to make it more and more obtuse by spreading them apart where they meet at the top. Grace says that it can't equal the long side, and Ms. Lowe asks her why. She says that in a right triangle, the hypotenuse of the longest side, and Ms. Lowe says, "OK, but what if I keep making it more and more obtuse? What will it make eventually?" and Grace responds, "20 inches, a straight line." (Fieldnote, 2/18/11)

Although she repeats this demonstration for two other groups of students, it was not deemed to lower the cognitive demand, as most students still appeared to struggle with and discuss the conjecture about whether the sum of two sides could equal the third in a triangle.

The point of describing this issue is that students did not have the means to monitor their own progress in this task. Ms. Lowe's continued press after students had come to an incorrect conclusion had the potential to lower the cognitive demand, as some students interpreted this as implying that their conclusion was incorrect. Furthermore, Ms. Lowe eventually feels the need to intervene and provide a demonstration to scaffold students' engagement with the task so that they did not formulate incorrect conjectures. That is, the means that were provided to students led to incorrect conjectures, and Ms. Lowe's attempts to address these required her to take over the investigation to some degree, coming very close to lowering the demand of the task, and may in fact have done so for some of the students. Indeed, the fact that class time ran out may have prevented further decline. In fact, sufficient time was coded precisely because Ms. Lowe did not conclude the investigation before the class period ended, but left the question open and assigned it as homework for students to think about.

Although this is a single example, it demonstrates that many of Ms. Lowe's practices related to factors associated with maintenance were present in a task implemented at a high level which did not make use of technology. Thus, on one hand, these practices may not be exclusively associated with maintaining the cognitive demand of tasks which utilize technology as a reorganizer. On the other hand, the kind of scaffolding that Ms. Lowe provides to students seems related to whether or not technology is used, as most of the scaffolding coded in those tasks specifically related to students' work on the computer, by ensuring that her students' constructions were mathematically accurate and teaching them to do the same. Technology also plays an important role in how Ms. Lowe presses students for meaning and explanations, as she regularly refers students back to their work in GeoGebra, both asking students to use it to verify their conjectures, and asking them to interpret their work mathematically. By providing a means for students to monitor their own work, it also provides an important affordance for Ms. Lowe. This was clear from the Triangle Inequality Theorem task in which she struggled to find a way to help students revise their incorrect conjectures, eventually resorting to providing a demonstration for them.

The practices that Ms. Lowe employed provide further evidence of the claim made above with respect to decline: that specific moves the teacher makes, either in anticipating students' needs while using technology within a task, or in responding to issues or questions, may be more influential in the maintenance of the cognitive demand during implementation than issues directly related to the technology. The specific behaviors that were a part of Ms. Lowe's practice, as well as the way that she used technology in these tasks, seem to account for the differences between the tasks that she maintained and those that declined in Ms. Jones' and Ms. Young's classroom.

Although contrasts between teachers are not addressed by Research Question Three, the preceding description of results regarding the factors associated with maintenance in connection with the use of technology all take place in Ms. Lowe's classroom, providing further evidence of her uniqueness in this sample of teachers. An important issue which likely accounts for many of the differences in the way that technology was used during implementation of tasks set up at a high level is the classroom practice of each of these teachers. In particular, teachers' practice shapes students' instrumental genesis, the process by which an artifact becomes a tool for mathematical thinking. A hypothesis about these results to be discussed in the next chapter is that students' ability to implement tasks at a high level using technology may be directly related to where they are on a trajectory of instrumental genesis. Comparisons and contrasts of the teachers who participated in this study are made in the discussion of the results in the next chapter as part of the explanation of these results.

5 CHAPTER 5: DISCUSSION

In this chapter, the results presented in the previous chapter are discussed in terms of how they can inform both research in mathematics education, and mathematics teacher education and professional development. This chapter begins by situating the results in the context of the research problem that this study was intended to address. This is followed by an explanation and interpretation of the main results of the study, and implications for teacher education and professional development. Finally, the chapter ends with recommendations for further research and concluding remarks.

5.1 IMPORTANCE OF THIS STUDY

The kind of thinking that students have the opportunity to do in classroom tasks has been shown to have important implications for their learning (Boaler, 1998; Henningsen & Stein, 1997; Stein & Lane, 1996; Stein et al., 1996; Stigler & Hiebert, 2004). Students in classrooms where teachers enact tasks which are cognitively demanding have demonstrated a greater capacity for problem solving, reasoning, and mathematical communication than those who engaged with more low level tasks (Stein & Lane, 1996). Furthermore, while students in classrooms in which these tasks are implemented at a high level have displayed the greatest gains on a measure of problem solving and reasoning, students in classrooms in which tasks are set up at a high level benefitted more in terms of student learning than those who were not, even if those tasks frequently declined during implementation. Thus, the mathematical tasks that teachers select and enact with their students have important implications for their students' learning. In particular, choosing and implementing high level tasks is correlated with students' mathematical behaviors such as problem solving, reasoning and proof, using and interpreting representations, making conceptual connections, and mathematical communication and discourse. It is this type of mathematical learning that has been promoted by organizations leading the reform of mathematics education in the U.S., such as the National Council of Teachers of Mathematics (NCTM) and the Association of Mathematics Teacher Educators (AMTE), and is included in the Standards for Mathematical Practice in the Common Core State Standards. If students' K-12 mathematics education is to succeed in preparing them to think mathematically, then they must have such opportunities as they engage with classroom tasks.

These same organizations have created position statements and articulated principles for the use of instructional technology for the teaching and learning of mathematics (Association of Mathematics Teacher Educators, 2006, 2009; National Council of Teachers of Mathematics, 2000, 2008). One of the Common Core's Standards for Mathematical Practices is the ability to "use appropriate tools strategically," including students being "able to use technological tools to explore and deepen their understanding of concepts". Indeed, the use of digital cognitive technologies to support students' mathematical learning, and research on the same, has proliferated over the last decade (Burrill et al., 2002; Heid & Blume, 2008; Zbiek et al., 2007), including research on how teachers use technology for mathematics instruction (cf.,Zbiek & Hollebrands, 2008). This research has considered the use of technology in enhancing students' learning of specific content, and much of this research has shown that how technology is used is more important than if it is used (e.g.,Ben-Zvi, 2000; Burrill et al., 2002; Chazan, 1999; Doerr &

Zangor, 2000; Glass & Deckert, 2001; Hollebrands et al., 2010; Hoyles & Noss, 1992). However, fewer studies have focused on how the use of technology supports students' mathematical thinking more generally (Ben-Zvi, 2000; Doerr & Zangor, 2000; Hollebrands et al., 2010; Hoyles & Noss, 1992; Suh, 2010), and in all but one of these studies the researchers were also the instructors, or planned instruction with the teachers. While such research designs provide access to students' work and thinking and teachers' planning and decisions that are difficult to gain via other designs, they nonetheless limit the generalizability of the results for teacher education in the sense that the teachers in these studies were generally not subject to the same set of constraints as the average classroom teacher. Furthermore, only one of these studies was conducted in a secondary mathematics classroom. Thus, there exists a dearth of research on how classroom teachers use technology to support students' mathematical thinking in a secondary setting.

The overarching purpose of this study was to provide insight into the ways that technology can be used to support students' mathematical thinking. In particular, the present study uses the *Mathematical Tasks Framework* (Stein et al., 2009) to assess the thinking opportunities in tasks which incorporate technological tools, describe the role of technology in supporting the thinking demands of tasks, and to examine the classroom-based factors related to the decline or maintenance of the cognitive demand of those technologically enhanced tasks. This includes understanding the role that technology plays in low and high level tasks, how the teacher shapes that role by the way she sets up the task, and specific moves she makes during implementation that help to maintain the cognitive demand, or allow or contribute to its decline. As little is known about how secondary teachers' use of technology is related to students' thinking, an exploratory design was used to provide a benchmark for future research. That is, the results of this study are expected to provide some insight into fruitful directions that future research may pursue these questions.

5.1.1 Summary of results

Results for Research Question One indicate that the inclusion of technology in a task is not related to the cognitive demand during any of the three phases of implementation, as technology was used in both high and low level tasks. The association between low level tasks and the use of technology during set up and implantation does not seem to be a meaningful or causal connection. A majority of the tasks that three of these teachers set up and implemented were at a low level, and use of technology was ubiquitous in the classroom instruction of all four teachers. These patterns seem to explain most of the association between low level tasks and technology use.

Results for Research Question Two, which inquired about the role of technology in high and low level tasks, suggest a general association of the level of cognitive demand of a task with the way in which technology is used. When technology is used as an amplifier, it generally had little or no influence on the cognitive demand of the task. Both high and low level tasks used technology as an amplifier, and the use of technology in these tasks was not related to the structure or thinking requirements. On the other hand, teachers generally used technology as an amplifier and a reorganizer to set up high level tasks, and in these tasks the use of technology was central to the thinking requirements of the task. The distinction between the amplifier and reorganizer use of technology, and the use of the *Mathematical Tasks Framework* in distinguishing phases of implementation allowed for more detailed and meaningful results. In particular, technology was used simultaneously as both an amplifier and reorganizer to set up high level tasks. That is, by virtue of its use as an amplifier, technology allowed for students' focus to be shifted to higher level thinking, and thus there is an essential connection of these two uses of technology in these situations. In those cases in which the cognitive demand was maintained during implementation, it continued to be used as both an amplifier and reorganizer, whereas it was used as only an amplifier when the task declined. Furthermore, the use of technology as an amplifier was generally associated with low level tasks, both during the set up and implementation phases. However, it was also used as an amplifier in some high level tasks, and qualitative analysis of these tasks revealed that the use of technology as an amplifier generally had no meaningful connection to the thinking demands of the task. More generally, then, the use of technology played an active role in the cognitive demand of the task only when it was used as both an amplifier and a reorganizer. The use of technology as only an amplifier by these teachers seemed to have no implications for the thinking demands of the tasks that they enacted.

With respect to Research Question Three regarding the factors associated with maintenance and decline of tasks set up at a high level and the role of technology, only one teacher was successful in maintaining the cognitive demand of tasks set up at a high level using technology on a fairly consistent basis. Most of the observed issues which seemed to influence the cognitive demand of the tasks during implementation were captured by the classroom-based factors associated with maintenance or decline identified by previous research (Henningsen & Stein, 1997; Stein & Lane, 1996; Stein et al., 1996, 2009). However, the specific manifestations of these factors were generally related to the use of technology in some specific way. For example, the inappropriateness of the task for a given group of students included students' inability to use technological tools appropriately to achieve the high level goals of a task, and scaffolding students' engagement with the task was often related to supporting students' use of technological tools to make progress on the task at a high level. A common factor in the decline of tasks set up at high level with technology was the inappropriateness of the task for a given group of students. This included students having difficulty connecting the affordances of the technological tools to the task requirements, using digitally generated novel representations that were unsuitable for fulfilling the task requirements, and teacher-centered use of the interactive whiteboard. An important factor in maintaining the cognitive demand of tasks set up at a high level with technology was the teacher using technology to provide a means for students to monitor their own work.

Overall, these results show that the use of technology differs within high and low level cognitive demand tasks, and that issues related to the use of technology are associated with the maintenance or decline of tasks set up at a high level. The distinction between the amplifier and reorganizer uses of technology suggests that, in terms of students' mathematical thinking, only the use of technology as a reorganizer, i.e., both an amplifier and reorganizer, is related to such thinking. Furthermore, these results show that while technological tools can support high level thinking, recognizing and using these affordances in this way is neither obvious nor straightforward for teachers or students.

In terms of the implications for teacher education, these results suggest that for teachers to use technology to support their students' high level mathematical thinking, the focus should be on using technology as a reorganizer. However, classrooms are complex environments, and these results further suggest that teacher education and professional development which focus only on teachers' use of technology for mathematics instruction is not likely to be successful unless it addressed within the context of their overall teaching practice. The case for that claim is made in the next section which provides an explanation and interpretation of the results.

5.2 EXPLANATION AND INTERPRETATION OF THE RESULTS

In the following, some explanation and interpretation of these results are offered, and implications for teacher education and professional development associated with each of these conclusions are discussed.

5.2.1 Technology and teachers' practice

These results align with the results of previous research that has shown teachers' use of technology is often integrated into their general practice (Cuban et al., 2001; Farrell, 1996; Manoucherhri, 1999; Monaghan, 2004; Russell et al., 2003), and confirm one of the hypotheses of this study, that teachers are more likely to adapt the use of technology to their practice than the other way around. Although characterizing teachers' practice was not a specific goal of the present study, observing teachers' classroom practice for a duration of 3 – 5 weeks, including the tasks that they selected and enacted with their students, their interactions with students, and interviews with the teachers providing insight into their own thoughts and beliefs about what was observed allows for these results to be interpreted within the context of these teachers' practice. The fine-grained nature of the data collected in the present study allows for a deeper understanding of exactly how a teachers' practice is related to their use of technology for instruction. In the following, each of the four teachers' practice is described and related to the results of the study as a way to explain both the results and how these teachers' practice may have constrained or supported their use of technology in engaging their students in high level thinking.

Ms. Jones' classroom provides the clearest example of using technology as an amplifier of her current practice. Through the post-lesson interviews she made it clear that her primary goal was for her students to learn the content, which primarily consisted of procedures, but that she wanted it to be more student-centered. However, it was not clear why being more student-centered was important to her, or what concrete benefit she expected in terms of student learning. This is connected to the fact that of these four teachers, Ms. Jones seemed the most ambivalent about the use of technology. She did not seem to have clear goals for her use of technology for instruction, and this may have been the root cause of all the tasks that she set up at a high level using technology being implemented at a low level. Given her emphasis on ensuring that students "get the point of it" when using technology, her students may be more focused on producing correct answers than on the opportunities for thinking involved in the task. "As long as it is assumed that content primes over media, the new media will be used to support the old content and will often do this badly since the content was defined for the old media" (Papert, 1996, p. 101). Insofar as the "new media," i.e., GSP, has the potential to support students' mathematical thinking, this quote is an accurate description of Ms. Jones' use of technology for instruction. She seems to have the hope that using technology to learn new content will be more efficient, but does not seem to trust it to accomplish this goal, as she consistently questioned whether students "got it" after they engaged in one of these tasks, and generally followed up these student-centered investigations with lectures which reiterate the main result of the investigation. As she says,

> So technology helps them get through it faster. Does it help them learn any better? I don't know...My experience with that bottom 25% is still that direction instruction helps them more. And I know that does not line up with what we read, but that's still my experience. They don't like to be frustrated. They don't like to be frustrated with the technology, they don't like to be frustrated with, 'what is it you're asking me?' I mean, I'm asking them to come to conclusions and make conjectures, and they want to be told the answer. (Interview, 6/23/10)

However, the routine of following student-centered explorations with a lecture is actually more inefficient than just lecturing to begin with, and there is no evidence of enhanced student learning, at least in terms of the assessments that Ms. Jones used. She seemed to have an intuitive sense that the "new media" supports "old content" badly, and thus reverts to the "old media" to ensure that her content goals are achieved. Ms. Jones does not seem to take advantage of the affordances provided by the "new media" in order to pursue new or different pedagogical goals.

Although Ms. Jones agreed to participate in a study in which the use of instructional technology was under investigation, she was the only teacher participant not hand-picked by the researcher based on beliefs which were expressed in a commitment to its use for instruction. As the quote above demonstrates, she is not fully convinced of this value, and would often offer contradictory statements within the same interview about its role and value for mathematics instruction and learning. Although the present study does not focus on the role of teachers' beliefs with regard to the use of technology for instruction, previous research has examined this topic with mathematics teachers, finding that, in general, teachers' use of technology for instruction is often constrained by and consistent with their views of mathematics (Cuban et al., 2001; Farrell, 1996; Jost, 1992; Kastberg & Leatham, 2005; Kendal & Stacey, 2001; Manoucherhri, 1999; Monaghan, 2004; Norton et al., 2000; Pierce & L. Ball, 2009). The results of the present study suggest the need for further research with regard to how teachers' beliefs influence the opportunities for using technology in the support of high level thinking.

Ms. Young's students not only have little experience with using technology for mathematical tasks, but also have little experience with the types of activities and behaviors that are called for in the tasks she enacts with them which use technology. However, the most serious impediment to her ability to implement tasks at a high level with her students may be the dependence on the teacher that Ms. Young seems to foster with her students. She provided notes for students and told them when and how to fill in the blanks in the notes during her lectures, consistently yielded to students' demands for help, often by asking them a series of low level questions which reduced the demand of the task, gave unsolicited hints to the class while working on a task, conducted reviews immediately prior to both the quiz and the exam that were administered during the unit, and even retrieved calculators from the back of the classroom for students who did not want to get up and get them for themselves. Thus, a potential explanation for the consistent low level implementation in her classroom may be that this dependence on the teacher simply does not position students well to engage in independent exploration, with or without technology. They do not seem to have the agency needed to make mathematically meaningful observations. The dependence on the teacher is likely related to low expectations for her students, and was further expressed in the predominance of low level tasks that she enacted with her students. Thus, in the case of Ms. Young, her students not only seemed to lack an understanding of how to use technology to support high level mathematical thinking, but by asking her students to engage in high level thinking, she violated the didactical contract she had with her students (Brousseau, 1997), resulting in the refusal of some of her students to take these tasks seriously.

Ms. Jones and Ms. Young both seemed to want to use technology to have students engage in different types of tasks than what they were accustomed to, but factors related to their existing practice seemed to prevent these tasks from achieving the intended goals. With over two-thirds of the tasks these teachers used set up at a low level, their students are fairly accustomed to not being required to engage in high level thinking. Not only does this pattern of low level tasks create expectations for students regarding what is required of them while engaging with instructional tasks, it also does little to provide resources to support high level thinking when it is expected. That is, the type of thinking students needed to engage in during independent or collaborative problem solving, mathematical exploration, noticing and observing, making connections between concepts and /or representations, and generalizing and conjecturing can be learned, as research has demonstrated (Boaler, 1998; Boaler & Staples, 2008; Stein & Lane, 1996). However, the teacher must provide opportunities for engaging in high level thinking, model the type of thinking required, press students to explain and justify their thinking, hold students accountable for high level engagement, and, in general, communicate the value of such thinking. In short, teachers must create a classroom culture which supports high level thinking in order to implement high level tasks as such, and these teachers had not done that. The majority of tasks that these two teachers enacted with their students contributed to a classroom culture which values the ability to apply known procedures in a fairly straightforward manner to obtain a single correct answer. The expectations that these tasks created, and type of thinking that students were familiar with did not support high level implementation when these students were presented with such an opportunity.

Mr. Mack's use of the IWB as a teacher-centered technology in what was an otherwise teacher-centered classroom provides further evidence of how a teacher's practice influences their use of technology. At least part of the differences in the tasks and the use of technology that were observed in Mr. Mack's classroom compared to the three other teachers is that he was teaching a different topic (order of operations and fraction arithmetic vs. geometry) to a different group of students (6th graders vs. high school students). In spite of the fact that he had a classroom set of laptops available for use by his students, he never used them for mathematics instruction during a month-long period during which his use of technology for instruction was being observed. While he claimed that this was due to the slow speed and poor reliability of the laptops that were available, careful observation of his daily practice suggests that it may be just as much an issue of control. In many ways he appears to be a technologically savvy version of Mrs. Oublier (Cohen, 1990). That is, his classroom possesses many of the marks of a reform-oriented classroom: the ubiquitous and creative use of technology, students seated and working in groups, students coming to the IWB to explain their solutions, and the use of a standards-based curriculum (Connected Mathematics Project). However, further examination reveals that most of the mathematical activity in his classroom was funneled through the teacher. Although students worked in groups and came to the IWB to share solution strategies, he was clearly the sole authority in this classroom, mathematically and otherwise. For example, when students came to the board to explain their answer, they explained it to him, and he explained it to the class. When students worked in groups, he generally circulated from group to group telling students if their answer was right or wrong by looking at their paper and simply saying "yes" or "no", and settling mathematical disputes among students. During lectures he often pointed out mistakes or errors that students might make when using a given procedure, presumably in order to help them avoid making them. However, when students offered erroneous solutions in a whole class discussion, they were not invited to share their thinking so that the error might be avoided by other students. This curious pattern suggests that if it is a mistake that Mr. Mack did not anticipate, then it is not worth investigating. This habit of carefully monitoring and controlling the mathematical activity in his classroom makes the pervasive use of the IWB by the teacher a natural extension of his classroom practice.

While the classroom practices of Ms. Jones, Ms. Young, and Mr. Mack did not support the implementation of tasks at a high level using technology, Ms. Lowe's did appear to support high-level implementation, based on limited contrasting data. That is, the one task that Ms. Lowe implemented at a high level which did not use technology exhibited many of the same factors associated with the maintenance of high level tasks during implementation. In particular, Ms. Lowe's practice of scaffolding students' engagement with the task at a high a level, insisting that students make meaning of their work and explain it, and providing sufficient time for students to grapple with the high level aspects of the task were present in all the tasks that were implemented at a high level in Ms. Lowe's classroom, whether technology was used or not. While the nature of the scaffolding and what students were required to interpret or explain were different when technology was not used, these general behaviors on the part of Ms. Lowe were instrumental in the high level implementation of these tasks.

There has been some debate in the literature about the impact of digital cognitive technologies on teachers' practice. Monaghan (2004) recounts arguments made by researchers (Schwartz, 1989; Heid, Sheets, & Matras, 1990; Hudson & Borba, 1999; Zbiek, 2002) and organizations (The Mathematical Association, 1992) which claim that the use of technology for instruction will cause a transformation of classroom pedagogy. Other researchers have found evidence that teachers tend to use technology as an extension of a fairly traditional practice (Cuban et al., 2001; Farrell, 1996; Manoucherhri, 1999; Monaghan, 2004). The results of the current study suggest that in practice, it may be both. With the exception of Mr. Mack, these teachers set up high level tasks with their students which included the use of technology as a reorganizer much more than when they did not use technology, or when they used it as an amplifier only. This is an important result as Stein and Lane (1996) found that on measures of reasoning, problem solving, and communication of solutions, students in classroom where tasks were set up at a high level but declined during implementation outperformed students from classrooms that had not enacted high level tasks during set up or implementation.

On the other hand, the fact that in Ms. Jones' and Ms. Young's classrooms, these high level tasks utilizing technology were situated within a curriculum which emphasizes memorization and the rote use of procedures may have prevented, or least failed to support students engaging in high level thinking processes. By making the distinction between the set up and implementation phases of task enactment, the *Mathematical Tasks Framework* allows for a more nuanced understanding of how secondary mathematics teachers' practice impacts their use of technology for instruction.

5.2.2 Implications for teacher education

The implication of these results for teacher education is that it must address teachers' general practice, and the use of technology within that, rather than attend to their use of technology for instruction separately. As Earle (2002) notes:

> Integrating technology is not about technology – it is primarily about content and effective instructional practices. Technology involves the tools with which we deliver

> content and implement practices in better ways. Its focus must be on curriculum and learning. Integration is defined not by the amount or type of technology used, but by how and why it is used. (p. 8)

In particular, if the goal of engaging students in high level thinking and reasoning is considered an important goal of classroom instruction, then teacher education and professional development must situate the use of technology for instruction within this context. Teacher education and professional development which focuses only on the use of technology in supporting the implementation of high level tasks, and does not address this issue within the broader context of teachers' general practice, may amount to asking teachers to do something completely different with technology than they generally do without it, and may lead to the same results reported in Ms. Jones' and Ms. Young's classrooms. Indeed, Mr. Mack and Ms. Lowe, the two teachers who had any success implementing tasks at a high level using technology, are products of such a teacher preparation program. This fact supports the efficacy of such an approach in comparison with teachers without such a background.

However, given that both Mr. Mack and Ms. Lowe implemented a majority of the tasks observed in their classrooms at a low level, the question remains as to what more is needed to help teachers implement a majority of their instructional tasks at high level. Observation of and interviews with these teachers suggests that their specific school context does more to shape their perceptions of opportunities for enacting high level tasks with their students than their pre-service teacher preparation program. While these teachers may have the necessary preparation to implement tasks at a high level, they perceive the curriculum, including the pacing schedule, preparing students for state-mandated assessments, and their own daily teaching schedule, including the amount of instructional time, number of classes taught, and daily adjustments to the schedule, as factors which constrain their ability to do so. These factors suggest that professional development that works with teachers in their existing school contexts may be needed to help teachers identify and pursue opportunities for using technology in supporting high level thinking with their students.

Professional development which focuses on selecting, setting up, and implementing high level tasks has been demonstrated to be effective (Boston & M.S. Smith, 2009). Given that a major factor in these teachers' inability to implement tasks at a high level was the inexperience of both the teacher and her students in enacting these kinds of tasks, a necessary first step to helping teachers to use technology to support high level thinking may be for them to plan and enact high level tasks with their students on a more regular basis. While there may be certain strategies that are unique to a learning environment which includes technological tools, both implementing high level tasks and integrating technology are complex endeavors for teachers; gaining experience supporting students' high level engagement without technology may help to prepare teachers to do the same when students are using technology as part of the task.

An important open question, however, is whether doing both at the same time may be the most optimal approach. Based on the reflections and insights that teachers shared regarding their instruction during post-lesson interviews, having teachers reflect on the role of technology in supporting high level thinking opportunities within specific tasks, planning and implementing such tasks in their own classrooms, and reflecting on how the use of technology influenced students' thinking on the task may be a promising strategy for helping teachers to use technology effectively to that end.

5.2.3 Amplifier and Reorganizer Uses of Technology

A way in which the results of the present study may contribute to research in mathematics education, and to mathematics teacher education, is by characterizing the use of technology in relation to students' thinking in a way that can differentiate superficial from meaningful use of technology for mathematical instruction and learning. The results for Research Question One provide empirical evidence that the mere inclusion of technology does not have any inherent implications for students' opportunity for high level thinking. However, how technology is used may. The distinction between amplifier and reorganizer was hypothesized to be a way to distinguish superficial from meaningful use with regard to students' thinking, had not been used to describe classroom instruction and learning. In this section, how this distinction was refined and related to the results is described.

Amplifier Use. Another important result is that the use of technology as an amplifier generally had no relationship to the cognitive demand of the task it was used within. This result was unexpected, as one of the hypotheses of the study was that the use of technology as an amplifier would generally be aligned with low level cognitive demand tasks as efficient execution of procedures is not in itself a high level process. That is, as the use of technology as an amplifier does not change the nature of what students are doing. Although there exists an association of low level tasks with amplifier use, qualitative analysis of these tasks reveal that the way the technology was used was not directly related to the low level demand of the task. Indeed, technology was also used as an amplifier in high level tasks, and likewise had no relationship with the cognitive demand of the task in the sense that the use of technology as an amplifier within the task did not support the high level thinking demands, and often was merely used for displaying the description of a task that would have been high level without it. Thus, while these data demonstrate a general association of the use of technology as an amplifier with low level tasks, it did not contribute to the low level cognitive demand. Given the way that the amplifier use of technology is defined, i.e., making some process more accurate or efficient that could be accomplished without it, the use of technology is not directly related to the cognitive demand. Rather, the association revealed in these data seems to be mediated through the teachers, and the affordances they perceive of the technology available to them in relation to low level tasks. Thus, the selection of the task may be the primary factor in the cognitive demand when technology is used as an amplifier.

This result provides empirical evidence that the inclusion of technology in mathematics instruction is not necessarily an indication of a reform-oriented practice, an association that has been made previously in the literature (Mayer, 1999). Such an association is most likely due to the promotion of its use by organizations which are generally aligned with such a view of mathematics instruction and learning (Association of Mathematics Teacher Educators, 2006, 2009; National Council of Teachers of Mathematics, 2000, 2008). However, the use of technology by no means ensures that students will be exposed to instructional tasks which require high level thinking. As the results of this study demonstrate, technology can just as well be used for traditional mathematics instruction. While making the process of teaching more efficient, its use as an amplifier is generally not related to thinking requirements of the task. Indeed, what these results show is that with respect to students' thinking, the real dividing line seems not to be between using technology or not, but between the use of technology as an amplifier or as a reorganizer.

Amplifier use of the IWB. In terms of the implications for teacher education and professional development, the use of the interactive whiteboard for mathematics instruction is an issue raised by these results, as this was the most common use of technology as an amplifier. In the case of the IWB, its use as an amplifier generally involved using it as a medium for display. As a chalkboard or whiteboard can also be used for display, the interactive whiteboard was often no more related to the cognitive demand of the task it displayed than its non-digital counterpart. The use of the IWB often does not influence the essence of the task in any way, and for that reason is most likely the easiest way for teachers to adopt technology for instruction. From this point of view, the popularity of interactive whiteboards makes sense (M. K. O'Connor, 2011; Wood & Ashfield, 2008). If teachers tend to use technology in a way that is consistent with their practice, then the interactive whiteboard most easily lends itself to this sort of adaptation by teachers. However, an open question with regard to the IWB in particular is the degree to which its affordances promote such a use of technology, and perhaps could influence teachers' views of instructional technology as simply a more efficient, accurate, and aesthetically enhanced way of doing what they have always done. Such a view of the role of technology on the part of mathematics teachers could have important implications for their ability to use it effectively as a reorganizer.

The pervasive use of the IWB as an amplifier may be at least in part due to the fact that the affordances of the interactive whiteboard are not specifically mathematical in nature. For example, while reading the field note from a lesson observation in which the teacher is conducting a class discussion or presenting curricular material to students using the IWB, one would not even know that the IWB is being used if it were not noted at the beginning of the field note. Unlike dynamic geometry software, in which tools are defined for creating, manipulating, and measuring mathematical objects, the interactive whiteboard is a more generic technology, a primary affordance of which is the ability to provide a shared and interactive representation.

This is not to say that the interactive whiteboard does not have affordances that could be used as a reorganizer. Indeed, an important affordance of this use of technology is the potential for not having students copying notes from the board into their notebooks by providing copies of the notes and problems to students, and not having to write the material on the board themselves, teachers may be able to discuss the meaning of statements of definitions and theorems, and to pose problems related to these. A shared, dynamic representation allows for a whole class investigation in a way that may be difficult or impossible without it. Thus, there exists the potential for students' attention to be shifted to understanding the meaning of definitions, interpreting them in problem solving contexts, and to investigate mathematics as a group.

However, because the mathematics is not embedded in the technology, connecting these affordances to problems of pedagogy makes greater demands on teachers. Teachers need to learn how to select high level tasks, how to assess the affordances of the IWB in relation to supporting the high level cognitive demands of the task, including whether or not the use of technology is necessary or appropriate for the given task, and then how to implement the task using the technology in a way that supports the high level thinking goals of the task. They must not only understand issues that students struggle with when learning a given mathematical concept or procedure, but know the affordances of the IWB well enough to reason about how they might be used to address those issues and support students' meaningful learning.

For example, Mr. Mack's use of the IWB to provide an interactive number line to support students' estimating fractions and fraction sums connects affordances of the IWB to Mr. Mack's perception of students' learning needs. Although analysis of this task and the way in which technology was used reveals certain shortcomings in its ability to promote high level thinking, this task provides an example of a teacher attempting to connect the affordances of the IWB to promoting conceptual understanding of a procedure. As the use of the IWB in this way requires that teachers have a deep understanding of pedagogical issues surrounding specific concepts and procedures to be taught, and the ability to imagine how the affordances of the technology might be used to address these problems of pedagogy, the use of the IWB in supporting high level thinking poses a significant challenge for professional development and teacher education.

Mr. Mack's use of the IWB suggests that while the IWB may be especially suited to a teacher-centered classroom, such a use of the IWB may frustrate the goal of having students construct meaning for mathematical concepts and procedures. Thus, helping teachers to develop strategies for a more student-centered use of the IWB that could promote mathematical discourse among students is another challenge for teacher education and professional development with regard to using the IWB to support students' high level thinking. As noted above, however, the deeper issue that needs to be addressed is teachers' general practice. Specific techniques for using the IWB to support students' high level thinking may only be useful to a teacher committed to creating a learning environment which supports a student-centered, constructivist approach to mathematics instruction.

Given the widespread availability and popularity of the IWB (M. K. O'Connor, 2011; Wood & Ashfield, 2008), research which explores ways to exploit the affordances of the IWB that could be more student-centered and promote high level mathematical thinking among students is needed. Furthermore, given that the IWB is especially suited to a "show and tell" style of instruction, more research is needed to determine whether or not this is the primary reason for its popularity, at least among mathematics teachers. Creating professional development and teacher education aimed at helping teachers use the IWB in a more student-centered manner capable of influencing students' thinking is a wasted effort if the real issue is teachers' beliefs about mathematics instruction.

Reorganizer Use of Technology. The use of technology as a reorganizer was strongly associated with the set up and implementation of high level tasks. As hypothesized, its use as a reorganizer was in all cases related to its use as an amplifier, in the sense that by offloading the construction, labeling, and measuring of mathematical objects to the technological tools there existed the potential for students to shift the focus of their mental activity to such behaviors such as dragging, observing, generalizing, and making and testing conjectures. In general, teachers used a dynamic geometry software package such as GeoGebra or Geometer's Sketchpad to have students investigate and explore the properties of geometric objects such as triangles.

The use of technology in this way has the potential to put students in the position of having control over their mathematical work, providing the opportunity to develop their own mathematical authority by making their own observations and conjectures. Indeed, an overarching role that technology played during the set up of high level tasks such as these is to support a shift in the locus of mathematical authority in the classroom, a shift that is intentional on the part of at least some of these teachers, based on the reasons that they cite for using it. Providing students with a tool that supports mathematical investigation that is independent of the teacher has the potential to balance the mathematical authority in the classroom.

However, the results of this study demonstrate that not only can such a use of technology result in greater mathematical authority on the part of the students, but also requires it. In particular, Ms. Young's students' inability, or unwillingness, to use GeoGebra as a reorganizer to make mathematically meaningful observations or conjectures may have been due to their dependence on her and the type of low level tasks that they are accustomed to. On the other hand, Ms. Lowe's students seemed to grow in their ability to make, test, and pursue their own conjectures using GeoGebra. This point is also connected to teachers' general practice, which must support the development of students' mathematical authority and agency in the classroom in order for them to gain the necessary traction to use technology as a reorganizer in independent mathematical investigations.

The tasks in which technology was used as a reorganizer in these data are all very similar. Dynamic geometry software was used for mathematical explorations of geometric objects which were considered high level tasks. This is likely due, at least in part, to the fact that these three teachers were teaching geometry, including many of the same topics. This is an important use of technology as a reorganizer, as it allows students to learn new content in the context of opportunities to participate in important mathematical behaviors and high level thinking. In terms of analysis, these similarities allowed for more meaningful comparisons of tasks between these teachers. However, this collection of tasks sheds little light on other ways in which technology might be used as a reorganizer in a secondary mathematics classroom. For example, one could imagine open-ended problem solving supported by technological tools in an algebra class, perhaps using a graphing calculator. Thus, the results of this study as they pertain to the role of technology as a reorganizer are limited in their generalizability. Qualitative analysis of other kinds of tasks which use technology as a reorganizer may reveal other roles that technology may play when used as a reorganizer. However, a hypothesis of this study based on the results is that the use of technology as a reorganizer depends on the development of students' instrumental genesis, as discussed in the next section.

5.2.4 The Role of Instrumental Genesis

In considering the results of the present study, it would be an oversimplification of the matter to attribute students' difficulties in using technology as a reorganizer to merely not knowing how to use the technological tools there were provided with. While knowing how to use the particular technology in question may be a necessary condition for students using it as a reorganizer to support high level thinking, it is by no means sufficient, as many of the tasks discussed on the previous chapter demonstrate. The idea of instrumental genesis provides a way to understand and explain the differences in the ways that these students did or did not use technology as a reorganizer. Indeed, a hypothesis based on the results of this study is that students' ability to use technology as a reorganizer while implementing high level tasks may be directly related to where they are on a trajectory of instrumental genesis.

Evidence of Instrumental Genesis. Instrumental genesis refers to the process by which an artifact, such as a calculator or computer, becomes a tool, or instrument, for students' mathematical thinking and learning, and in a certain sense becomes an extension of his or her thinking. Two components of instrumental genesis have been identified in the literature: instrumentalisation, in which the student begins to make sense of the artifact as a tool for thinking, and instrumentation, in which the student constructs mathematical meaning with the tool (Drijvers & Trouche, 2008; Guin & Trouche, 1999; Zbiek et al., 2007). These two processes are not independent, but rather form a complex dialectic in which meaning constructed for the tool results from using it to construct mathematical meaning, and students' ability to construct mathematical meaning with the tool is supported by the meaning they have constructed for it. A few examples from the data and results discussed in the previous chapter are recalled in order to identify evidence of students' instrumental genesis, and to explain how progress along a trajectory of instrumental genesis can support high level thinking.

In Ms. Lowe's class, one of her students, Brian, asked the following question toward the end of the incenter investigation in GeoGebra:

> Brian asks "if the triangle is an equilateral triangle, will the incenter be the same distance to the sides as the vertices?" Ms. Lowe tells him that that's a great question, and tells him that he has 9 minutes and a tool to investigate it with. (Field note, 1/31/11)

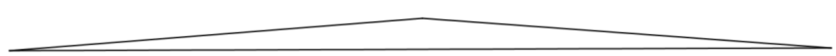
The circumcenter of any triangle is equidistant to its vertices, and the incenter is equidistant to the sides of the triangle. The meaning that Brian has constructed for the incenter and circumcenter is evident in his wondering if there is a relationship between these points of concurrency in special triangles. His use of GeoGebra has resulted in the construction of mathematical meaning that makes this question possible, but also an understanding of how this tool can help to answer the question. That is, the affordances of the tool that were utilized in investigating the incenter and the circumcenter suggest a way for him to investigate this new question that arises from the results of those previous investigations. His resulting construction shown in Figure 4.24 demonstrates that Brian was able to connect the affordances of the technology to the question he had posed. In addition to constructing an equilateral triangle and measuring the distance from the incenter to the vertices and sides of the triangle, he drags the triangle in order to look for a pattern or invariant relationship. He makes the conjecture that the distance from the incenter to a vertex is twice the distance from the incenter to a side of the triangle, and recognizes that due to rounding in the program this relationship may not always appear to be true. This last detail is significant as it demonstrates that the results of his investigation transcend the limitations of the tool used to conduct it. Understanding the limitations of tools in specific contexts is an important element of using them meaningfully.

This episode demonstrates that the meaning that is constructed for and with tools is a complex, back-and-forth process, and suggests that perhaps the best way to foster instrumental genesis is in the context of using tools purposefully. For example, demonstrating where all the functions are in GeoGebra and what they do apart from any purposeful activity which makes use of them may be less effective in assisting students in developing meaning for GeoGebra as a tool to support mathematical thinking than having students use the various functions in the context of conducting an investigation or problem solving. Meaning may best be constructed for the tool when it is being used by a learner to construct mathematical meaning, as instrumentation and instrumentalisation are hypothesized to occur simultaneously. In the episode above, Brian did not know how to create an equilateral triangle in GeoGebra, i.e. a triangle that always remains equilateral no matter how it is dragged. Brian had a need for a specific function, but did not know how GeoGebra might meet that need. The students in Ms. Jones' class who considered the "parallel-ness" of lines to be a contingent rather than necessary property, or at least did not appreciate the difference. Brian understood that what he needed to answer his question was a triangle that remained equilateral no matter how it was dragged or resized, even before he knew how to use GeoGebra to create it. Thus, when he was shown how to create an equilateral triangle, he already understood the mathematical significance of this affordance of GeoGebra.

Another way to describe the process of instrumental genesis is the perception and connection of affordances to a specific goal or activity. In the episode above, Brian needed to create, measure, and drag a figure that could verify his conjecture. He connected affordances of GeoGebra to that mathematical activity and was able to interpret mathematically the result of his work in order to gain deeper mathematical insights, such as the realization that in an equilateral triangle the incenter and the circumcenter are concurrent. This episode is evidence both of the result of instrumental genesis, as well its deepening, and suggests that the best way for students to construct meaning for tools may be in the context of purposeful mathematical activity.

Further evidence of instrumental genesis is given in Ms. Lowe's class investigation of the Triangle Inequality Theorem using uncooked spaghetti. Perhaps ironically, this task did not make use of technology in any way, but for that reason may best portray how the process of instrumental genesis influences students' thinking. The following scenario ensued while students were trying to determine if the sum of two sides of a triangle can equal the third (they had already determined that the sum of two sides cannot be less than the third). They are having difficulty coming to a conclusion because some students have been able to create a triangle in which the sum of two sides equals the third using spaghetti. For example,

> Hannah says that it has to be an isosceles triangle. Ms. Lowe asks Wendy what the measurement of her two shorter pieces are, and Wendy measures them and says 4 and 6.2. Ms. Lowe asks how long her long side is, and Wendy measures and tells her 10.2. Ms. Lowe asks her put them together for her, Wendy asks her if she wants her to make a triangle with them, and Ms. Lowe asks her, "can you?" Wendy replies "yes" and makes the following using her pieces of spaghetti:



However, when asked to do so by Ms. Lowe, some students have not been able to draw it on their paper using a pencil and a ruler, and there is some confusion within groups of students about whether or not this a triangle can be formed if the sum of two sides is equal to the third:

> Ms. Lowe returns to Bruce's group, and he announces that he was wrong. Ms. Lowe asks him who convinced him he was wrong, and he said that he saw them make a triangle. Ms. Lowe asks what the measurements of the sides are, and Jennifer says that the long side is 1 inch, and the other two sides are a ½ inch, and she shows her the triangle made with spaghetti on Grace's desk. Laura asks, "did we do it wrong?" and Ms. Lowe says, "I didn't say that." She asks Grace what she's doing, and Grace says that she's trying to draw it, but it doesn't work out. Laura says something about a triangle with sides 6, 4, and 2 cm, and then she says, "I need GeoGebra!" (Field note, 2/18/11)

This is a clear example of instrumental genesis on the part of Laura insofar as she has a specific problem that she is investigating, and seems to have a clear idea of how the affordances provided by GeoGebra might be used to investigate the problem. Furthermore, her statement implies that she expects that the use of GeoGebra would provide some insight into the problem that the use of spaghetti or pencil and paper is unable to do. Furthermore, class ended with the question of whether or not the sum of two sides could equal the third in a triangle, and when Ms. Lowe created a discussion board for students to post their conjectures for homework, at least one student used GeoGebra to come to the conclusion that such a triangle is impossible. Allowing students to choose the tools that they use to investigate a problem such as the Triangle Inequality Theorem could both reveal and develop students meaning for these tools, and might contribute the maintenance of the high level thinking demands of the task by having students who used different tools explain their findings and reconcile any discrepancies.

A third example of a student who had constructed meaning for the tools in GeoGebra in Ms. Lowe's class was Will, who realized that his construction was incorrect when he dragged it. Will told Ms. Lowe that he realized that his figure is wrong, and asked her for another activity worksheet, explaining that when he dragged his figure, he realized that he had not created a midpoint because it did not stay on the segment of the triangle. Unlike Ms. Jones students who did not understand the difference between two lines that look parallel and two lines that are parallel in GSP, Will understands the mathematical difference between a point that looks like it is the midpoint of a segment and one that is constructed to be. These contrasting examples of Ms. Jones' students and Ms. Lowe's students also provide some insight into the role of dragging in the process of instrumental genesis with a DGS. When Ms. Jones' students dragged their figures and they deformed, they simply adjusted them until they looked parallel again. Will, on the other hand, recognizes that dragging is a way to test a figure and assess whether or not it has been constructed appropriately. Will understands the mathematical significance of creating the midpoint of a segment, that when such an object is constructed it is a property inherent in the figure. Implicit in his statement is the understanding that the midpoint is unique, must lie on the segment, and is fixed for a given segment. The difference in this example and the work of Ms. Jones' students on the parallel lines cut by a transversal task suggests that technology does not provide a means for students to monitor their own progress until they have constructed some meaning for the tool. This is significant insofar as the potential for students to monitor their own work seems to be a particular affordance of technology in terms of maintaining the cognitive demand of a task during implementation. This is a specific example of how the development of instrumental genesis may support high level implementation by students. Indeed, if instrumental genesis can be described as the process by which a learner comes to be able to use a particular tool to think with, then it follows that this process is a necessary condition for using tools to support high level mathematical thinking.

Fostering Instrumental Genesis. An important question for teacher education is how teachers can foster their students' instrumental genesis, and thus create a learning environment conducive to using technological tools in support of high level mathematical thinking. The potential for analyzing students' instrumental genesis is limited using the data collected in this study as specific students and specific behaviors that might inform how this process develops were not targeted. However, by observing the teacher and students in a classroom context, it may be possible to identify promising strategies for fostering students' instrumental genesis by teachers by way of comparison and contrast of the teachers observed in this study.

One way to frame the results of the present study regarding decline in general, and to the inappropriateness of the task for a given group of students in particular, is that difficulty on the side of the tool is related to its use as an amplifier, corresponding to instrumentalisation, or the construction of meaning for the tool. Difficulty on the side of the task may be more related to its use as a reorganizer, corresponding to instrumentation, or the construction of mathematical meaning with or using the tool. As the processes of instrumentalisation and instrumentation are inseparable in practice, the "inappropriateness of the task for a given group of students" described in the previous chapter can be understood to not lie wholly on the side of the tool or the side of the task, as the two are inextricably linked. The difficulties that Ms. Jones' and Ms. Young's students experienced in using technological tools to construct mathematical meaning further suggests the need to foster students' instrumental genesis in the context of their mathematical work. The tasks that Ms. Lowe enacted which used technology as a reorganizer begin to provide some insight into what this might look like. In particular, her scaffolding of students' work with the tool on the amplifier side seems to correspond to developing students' instrumentalisation, and her consistent press for students to make meaning of their work on the reorganizer side may help to foster their instrumentation.

Mr. Mack's classroom provides an example of the claim that for students to construct meaning for a tool, they must use the tool (Hiebert et al., 1997). As Mr. Mack's students rarely had the opportunity to manipulate the IWB directly, their ability to use it meaningfully to conduct investigations or solve problems was limited. Indeed, at no point did Mr. Mack use the IWB in such a manner with his students. The fact that technology was not used as a reorganizer in his classroom supports the hypothesis that the potential of technology to act as a reorganizer may be directly related to where students are in the process of instrumental genesis, and that this process requires that students use the technology. The degree to which the IWB can be used to support students' thinking by providing novel and interactive representations that are manipulated by the teacher or another student is an open and separate question. However it may influence students' mathematical thinking, students' ability to construct meaning for the tool is likely very limited if they do not use it themselves. Thus, an important strategy for promoting students' instrumental genesis is to have them use the tools.

Another strategy which may help students develop meaning for the technological tools that they use is allowing students to work in the program and make their own constructions versus simply manipulating an applet. This was an issue mentioned by Ms. Jones, Ms. Young, and Ms. Lowe. All three commented that they believe that it was more beneficial for students to work in the program directly rather than manipulating an applet in terms of giving students more control. When discussing the difficulty that students had creating 30-60-90 triangles in GSP, Ms. Jones notes that she had considered creating an applet for them to manipulate in order to eliminate those issues, but decided not to because she "felt that it wouldn't be as convincing to them." Ms. Young, who was the only teacher to use applets with her students, remarks in our conversation at the end of the unit that her students may feel more connected to something that they have constructed themselves than just moving a slider with triangles that add to 180, and that this may be more convincing for them because it is something they created.

In general, the range of possible actions in an applet is much more limited, and usually consists of simply dragging an interactive representation. While it may be simpler and more efficient to have students work with applets, especially in light of the issues that Ms. Jones' and Ms. Young's student experienced when attempting to make their own constructions, limiting students' freedom in a DGS environment allows for little growth in students' ability to construct meaning for these tools. Rather than limiting their freedom by using an applet, it seems that a key to fostering students' instrumental genesis is to support their use of the tool, as Ms. Lowe did. Many of the excerpts from Ms. Lowe's classroom in which students discovered important mathematics or pursued their own conjectures would not have been possible if her students had been limited to using applets. However, this process requires a substantial investment of time and energy on the part of the teacher, and consequently a real commitment on her part. She must firmly believe that the payoff of having students able to use such tools meaningfully is worth the investment.

Ms. Lowe admitted that the five tasks that she did in the lab, each of which took an entire class period of instruction, were covered in just a few section of the text. She explained that she believed that the opportunity to explore such rich mathematical topics was worth the extra time that it required. The previous chapter contains numerous examples of how she worked tirelessly in supporting students' use of GeoGebra and required them to interpret and explain their work, and turned them back to their work on the computer when they asked for help. On the other hand, Ms. Jones' lack of accountability for students' work in GSP allowed her students to flounder, and did not support their ability to make use of this tool meaningfully. Her habit of switching modes by using non-digital manipulatives to explain the results of what students had just been investigating with GSP likely further undermined opportunities for students to make meaning for the tool. First, having students explain their work in GSP may have supported their ability to construct meaning for the tool, and serves as a form of accountability that would likely influence their engagement. Thus, not doing so represents missing an important opportunity to foster students' instrumental genesis. Second, not using the same medium for her demonstrations does not allow her the chance to even model the use of this tool, and sends a subtle but clear message that other mediums are more effective for constructing mathematical meaning.

Finally, if students are to use technological tools while engaging with more open-ended tasks involving mathematical investigation and problem solving, these results suggest that they would need to be fairly advanced on a trajectory of instrumental genesis. The tasks which used technology as a reorganizer in this study were generally guided explorations. Students were given specific directions on what to do, and how to use the technology in question to do it. The students' role was to make observations, generalizations, and conjectures. While these tasks often involved high level thinking, they lacked the open-ended nature and multiple solution strategies that would require students to have to have enough familiarity with the tools, and the affordances they offer, to use them independently. The use of technological tools at all may be suggested but not required during a task, thus keeping the solution strategies, including the available tools, open to students' choice. If the goal of instrumental genesis is for students to be able to use tools to support and extend their thinking, then requiring explanations which make use of the tools they used may be an important element of such tasks.

5.3 CONCLUDING REMARKS AND DIRECTIONS FOR FUTURE RESEARCH

The results of this study are important as they contribute to the growing knowledgebase of mathematics technological pedagogical content knowledge (TPACK) (Grandgenett, 2008; Koehler & Mishra, 2005, 2008; Mishra & Koehler, 2006; Niess, 2005, 2006, 2008; Niess et al., 2009; Pierson, 2001). These results help to identify the knowledge that mathematics teachers need in order to teach well using technology. In particular, these results identify issues that teachers must be aware of and pay attention to if they are to use technology to successfully support students' high level mathematical thinking. Teachers must move beyond novel and visually appealing uses of technology as an amplifier, to determine how a particular technology might be used to influence students' thinking as a reorganizer. They must allow students to use the tools for purposeful activity that will provide the opportunity to construct meaning for and with the tool, must scaffold students' use of these tools appropriately, and must require students to interpret their work mathematically using these tools.

TPACK not only relates to teacher knowledge, but also extends to informing teacher education. A primary conclusion in this regard is that teacher education and professional development must address teachers' use of technology within the context of their general practice, as their practice shapes and constrains their use of technology for instruction. Having teachers reflect on their practice and whether and how it supports high level thinking by their students would necessarily entail an examination of the use of technology for that purpose. More consistency across tasks which do and do not use technology could better prepare both the teacher and her students to use technological tools to support high level thinking. Professional development which addresses the selection and enactment of high level tasks has been designed and studied, and shown to be effective (Boston & M. S. Smith, 2009). Future work might include the development and integration of the use of technology into this curriculum, and research which examines its effectiveness.

A case is made above for the importance of students' instrumental genesis in using technological tools to think with, and the hypothesis put forward that students' ability to use technology as a reorganizer is directly related to their location on a trajectory of instrumental genesis. Given the identification of the potential importance of this aspect of students' mathematical thinking, and the charge articulated by the Common Core's Standards for Mathematical Practice that students learn to use appropriate tools strategically, further study of this process is warranted. Observing a single classroom from the beginning of a school year, prior to students' use of technology on a regular basis, and for a longer period of time, perhaps an entire school year, would allow for a better understanding of how this process is developed. Furthermore, such a study should include a specific focus on the types of behaviors that are hypothesized to be associated with students' instrumental genesis. Such a study design could provide data more appropriate for an in-depth analysis of the development of students' instrumental genesis, and the teachers' role in that process.

In order to refine these results, it would be useful to work with a teacher, or teachers, whose teaching practice generally supports high level thinking by her students, but who may struggle to use technology effectively in this way. A teacher like Ms. Lowe might be a good candidate for such a study. While she implemented five tasks at a high level using technology as a reorganizer, she also used technology in a number of tasks which were implemented at a low level. On the other hand, she set up more high level tasks than any other teacher, and was the only teacher who implemented a task at a high level which did not use technology, providing evidence that her practice was more supportive of students' high level thinking in general. The results of such a study could both identify specific ways that the use of technology might support students' high level thinking that go beyond issues related to teachers' general practice, and provide the basis for a curriculum for mathematics teacher professional development by identifying effective means of supporting teachers' development of this aspect of their practice.

The use of technology for mathematics instruction and learning is still an emerging area of research and teacher education, but one that is gaining momentum and is certainly not a passing fad in mathematics education. In order for the use of technology to not become a guiding principle for mathematics instruction, other principles, such as students' mathematical thinking, reasoning, and sense-making must take precedence. Thus, it behooves mathematics education researchers and teacher educators to better understand the role of technology as a tool for achieving worthwhile goals in K-12 mathematics education, and to better prepare teachers to use it as such.

APPENDIX A

TASK ANALYSIS GUIDE (Stein et al., 2009)

Low Level Cognitive Demand Tasks
Memorization Tasks • Involve either producing previously learned facts, rule, formulas, or definitions or committing facts, rule, formulas, or definitions to memory • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure • Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlay the facts, rules, formulas, or definitions being learned or reproduced.
Procedures without Connections Tasks • Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedure being used. • Are focused on producing correct answers rather than developing mathematical understanding. • Require no explanations or explanations that focus solely on describing the procedure that was used.
The next two low level categories only apply to the implementation phase
Unsystematic exploration

• Students explore around the edges of significant mathematical ideas but fail to make systematic or sustained progress in developing mathematical strategies or understandings.	
No Mathematical Activity • This includes students being off task as well as focusing on non-mathematical aspects of the task, such as making an attractive poster to display group work.	
High Level Cognitive Demand Tasks	
Procedures with Connections Tasks • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Student need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.	
Doing Mathematics Tasks • Require complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • Require students to explore and to understand the nature of mathematical concepts, processes, or relationships. • Demand self-monitoring or self-regulation of one's own cognitive processes. • Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • Require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.	

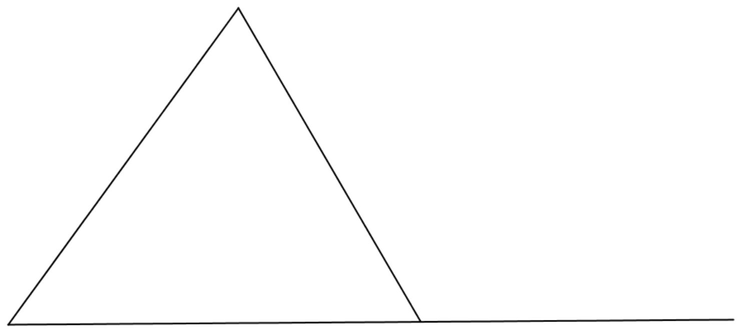
APPENDIX B

ANNOTATED FIELDNOTE USING THE TASK ANALYSIS GUIDE TO EVALUATE COGNITIVE DEMAND

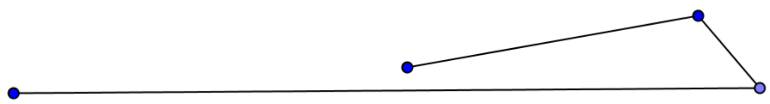
The following descriptors are for the *Procedures with Connections* level of the Task Analysis Guide:

1. Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
2. Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
3. Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning.
4. Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Student need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.

1:50 Ms. Lowe puts students into two groups of four and one group of three. She tells them that she has an activity that she wants them to do, and she wants them to begin by reading. She tells them that she wants them to work independently on the first part of it, and on the rest of it she wants them to discuss what they find. She tells them that they'll need a ruler, and Hannah begins to pass out rulers to everyone.	A strategy or procedure is provided to students that they are to follow to discover the Triangle Inequality Theorem.	2
1:53 Ms. Lowe passes out spaghetti to the students, and she notices that Zack is already answering #5 on the worksheet, and she asks him how he could be answering the questions without doing the investigation. She asks the class what #1 tells them to do, and tells them that the critical thing about the triangles they are to make is that the segments should be end to end, just as they learned that a polygon consists of segments put end to end. She tells them that she doesn't want to see triangles like this:	Zack attempts to go directly to the conclusion, but Ms. Lowe won't let him.	4

		
She tells them that she suggest that they measure in centimeters because it's easier to read than inches. Bruce asks Ms. Lowe if he should make an acute or obtuse triangle, and she tells him the classification doesn't matter. All students seem to be making triangles with spaghetti on their desks and taking measurements with their ruler.		
Ms. Lowe announces to the class that when they're completing the worksheet, they should fill it out as complete as possible. She tells them that she is going to grade it based on how complete it is, and on how thoughtful their responses are. She tells them she doesn't want just "yeah," "no," or "whatever" but Level 5 Geometry answers.	Ms. Lowe communicates expectations for high level engagement.	
11:59 Students are making triangles, measuring, and recording their measurements in the table. Bruce asks about number 2, which asks for an impossible triangle. Ms. Lowe asks him what impossible means, and he says "you can't do it." Ms. Lowe says, "show me," and Bruce says that he doesn't think it's impossible. Ms Lowe tells him to think and asks him if he can put three pieces together so that they don't form a triangle. Shortly after, Elena asks Ms. Lowe how it can be impossible to make a triangle, and she says, "I don't know. Can you break the spaghetti such that it doesn't form a triangle if you put the segments end to end?" Elena says, "End to end? Don't <>" Ms. Lowe replies, "I don't know. That's what I want you to think about." 2:01 Ms. Lowe asks Neil to show her.	Students appear to be making some cognitive effort, and not simply following directions mindlessly.	4

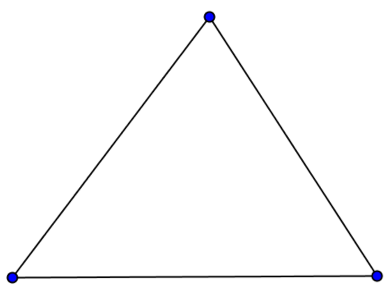
Neil says that there's no vertex. Elena (behind Neil) then says, "I got it!" and Ms. Lowe asks her to show her. Elena has something like this:

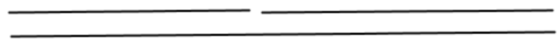
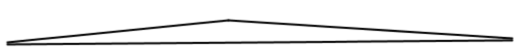


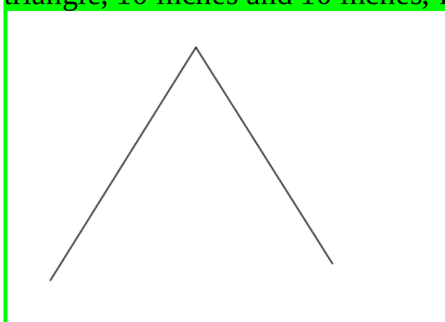
Ms. Lowe asks her what would have to happen in order for this to form a triangle, and Elena says that she'd have cut "that one" and Ms. Lowe asks her where, and Elena points to the long side. Ms. Lowe returns to Bruce, who says he's got it now, and asks what he did. He says that these two lengths must be longer than this.

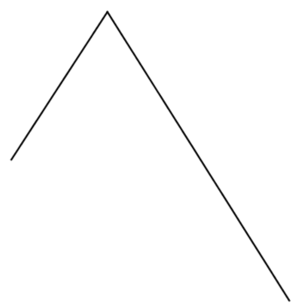
Ms. Lowe asks Grace, and what she'd have to do to make it a triangle, and she says to make it longer or shorter, and Ms. Lowe asks how much, and Grace replies "a lot." Ms. Lowe looks at Laura's paper, the top row of the second table, and asks her if

she's sure that won't make a triangle, and asks if she still has her spaghetti for that, but she doesn't. After Ms. Lowe walks away, Laura erases her first row. Ms. Lowe asks Alice how much longer she'd have to make the short side for it to be a triangle, and she replies, "about twice as long." She asks Will "what's going on here?" and he looks at this paper for a moment, and says, "I don't know." Ms. Lowe tells him to look at both tables. 2:06 Ms. Lowe asks Neil about his group's work and he says that they've made an observation (I don't catch what it is), but that his and Wendy's don't match up. Ms. Lowe tells him that he might examine the number patterns in both tables. Neil says that there are larger numbers, Ms. Lowe asks where, and he says in the top table. He says that the small and medium sides are smaller, and Ms. Lowe asks, "than what?" and Neil says the large side. She asks the group, "what's going on up here?" and Wendy says that the small plus the medium is bigger than the large side. Ms. Lowe asks if that's different between the two tables, and they say "yes." Ms. Lowe asks Bruce what he's found, and he says that the small plus the medium sides is greater than the long side. She asks him to explain that, and he draws a triangle, and referring the angles says something about "bigger than these two" and Ms. Lowe says, "not necessarily." She looks at numbers 5, 6, and 7 on the back of sheet and asks him if that's a strict inequality, and he says "yes." She asks him if he's checked with the other students in his group and he says "no." She says to him, "So if it's true for you, that's good enough? You don't need to check the results of the other people in your group to see if it's true in general?" Bruce replies that it's obvious. She tells him to think outside of "that relationship," referring to the "small + medium > large" that he has written for number 5. She tells him to think about all the sides, and says, "what's down here is more" referring to the Triangle Inequality that he has filled out at the bottom of the page. Circling those three inequalities with her finger, she tells him, "explain what you're telling me down here. What does this mean?" He begins to read it to her, and she says "I can read it. Tell me what it means." Bruce says, "when you make a triangle, the sides have to be like this. The two little lines have to be bigger than the longest one. Then you can make some angle." Ms. Lowe turns to Jennifer, who is sitting next to Bruce and has been listening, and asks, "do you understand what he's saying?" She replies, "no" and Bruce says, "me neither." Ms. Lowe has the group members turn toward one another and begin to look at it together. She tells the other students (Jennifer,	While using a general procedure, students' seem to be developing a deeper geometric understanding of why the Triangle Inequality Theorem is true. Students are required by the task and the teacher to put forth some cognitive effort to complete the task. While Ms. Lowe asks guiding questions, students are still required to make some effort in answering them. Students are required to make connections between representations: geometric, numerical, and general inequalities (algebraic). While guided by the worksheet, students are required to make some cognitive effort in completing the task.	2 4 1, 4 3 4

Grace, and Laura) "he has good ideas. You can help him with the words." 2:13 Ms. Lowe tells Will and Alice to talk to Zack about number 7. She tells the class that they need to get cracking and finish before they leave today. Brian walks into class and Ms. Lowe gives him a ruler and spaghetti and tells him to get started. While Ms. Lowe is talking with Laura, she says that they all have the same relationship, and that if small plus medium is bigger than the large side, then it forms a triangle. Ms. Lowe asks her is the small side plus the medium side are equal to the long side, does it form a triangle, and Bruce (sitting nearby) says "no." She asks the group why, and Laura says, "I don't know." Ms. Lowe replies, "think about it." Laura says, "that would be an equilateral triangle," and Ms. Lowe asks, "would it?" and Laura backtracks and says, "no." Grace says that it would be isosceles. Jennifer says that it would look like:  Ms. Lowe asks her if the small side plus the medium side equals the long side in that triangle, and Jennifer says that it's equilateral, so there is no small or medium side. Ms. Lowe replies, "so if there's no small, medium, and large side, then this whole activity is out the window?" Jennifer says that it would be medium plus medium, and Ms. Lowe asks her if medium plus medium would be bigger than the large side, and Jennifer says "yes." Ms. Lowe tells Bruce to convince his group that he has something to say, and he says that it would look something like this:	Students are grappling the central mathematical ideas by connecting and interpreting the algebraic inequalities to the geometric representation using spaghetti.	3

 Ms. Lowe asks him if that means that there's no triangle, if small plus medium equals long means that there's no triangle. Bruce says, "maybe not," and Jennifer says you could still make a triangle. Grace adds that it would be really obtuse. 2:17 She tells the group, "I want you to think," and tells Bruce, "convince them." Bruce says that if you draw it with a pen, you can't do it. Ms. Lowe asks, "why? Think." As she roams the room, Ms. Lowe announces that she sees lots of "yes's." She comes over to Will, Alice, Elena, and Zack's group and asks, "what's going on here? Summarize for me." Will says that if the small side plus the medium side is less than the long side, then there is no triangle, but if the small side plus the medium side is bigger than the long side, then you can form a triangle. Ms. Lowe asks him, "what if they're equal? Then does it form a triangle?" Zack and Elena say, "yes." Ms. Lowe asks them if they've tried it with the spaghetti, and they say yes, and that it forms an obtuse triangle. Zack shows her use spaghetti with short sides measuring 4.5 cm and long side of 9 cm. Ms. Lowe asks them, "if you used your pencil to draw it, could you draw that hugely obtuse triangle with one side 9cm and the other two 4.5cm?" Then she leaves and moves on to the next group. 2:20 Hannah says that it has to be an isosceles triangle. Ms. Lowe asks Wendy what the measurement of her two shorter pieces are, and Wendy measures them and says 4 and 6.2. Ms. Lowe asks how long her long side is, and Wendy measures and tells her 10.2. Ms. Lowe asks her put them together for her, Wendy asks her if she wants her to make a triangle with them, and Ms. Lowe asks her, "can you?" Wendy replies "yes" and makes the following using her pieces of spaghetti:  Ms. Lowe tells the group to draw it for her, a triangle with side	Ms. Lowe requires students to connect the algebraic inequalities to the geometric representations.	4

lengths 4, 6.2, and 10.2, and to be as accurate as they can.		
2:22 Ms. Lowe returns to Bruce's group, and he announces that he was wrong. Ms. Lowe asks him who convinced him he was wrong, and he said that he saw them make a triangle. Ms. Lowe asks what the measurements of the sides are, and Jennifer says that the long side is 1 inch, and the other two sides are a ½ inch, and she shows her the triangle made with spaghetti on Grace's desk. Laura asks, "did we do it wrong?" and Ms. Lowe says, "I didn't say that." She asks Grace what she's doing, and Grace says that she's trying to draw it, but it doesn't work out. Laura says something about a triangle with sides 6, 4, and 2 cm, and then she says, "I need GeoGebra!" and Ms. Lowe says, "that's a great comment!" Ms. Lowe then asks them, "why must small plus medium be bigger than the large side? What if small plus medium equals the large side?" Laura says that they'd have just two sides, and Jennifer says that it would be 4 – 2 – 2. Ms. Lowe picks up two whole pieces of spaghetti, telling the group that they're the same length so this would be an isosceles triangle, 10 inches and 10 inches, for example.  She tells them that she is going to make it more and more obtuse by spreading them apart where they meet at the top. Grace says that it can't equal the long side, and Ms. Lowe asks her why. She says that in a right triangle, the hypotenuse of the longest side, and Ms. Lowe says, "OK, but what if I keep making it more and more obtuse? What will it make eventually?" and Grace responds, "20 inches, a straight line."	Laura wants to know the answer, but Ms. Lowe won't tell her, but rather requires the students to continue to put forth the cognitive effort required to interpret the inequalities geometrically. Although Ms. Lowe provides a geometric representation of the situation for students, they are still required to interpret its meaning.	4 1
2:26 Ms. Lowe returns to Zack, Elena, Will, and Alice's group, and they tell her that she confused them. She repeats the demonstration above, telling them that she'll start with an isosceles since that's what most of the claim the triangle would have to be if small plus medium equals large. She does the demonstration between this group, and Wendy, Neil, and Hannah's groups so that both can watch. As she spreads the two pieces of spaghetti apart, she asks them what it's approaching, and Hannah says, "a straight line." Ms. Lowe asks how long it would be, and Elena says "20." Ms. Lowe then asks if small	Cognitive effort is still required, as evidenced by students giving incorrect answers and changing them. The demonstration provided by Ms. Lowe has not removed the high level demand of	4

plus medium can equal long, and Elena and Neil both say, “yes.” Zack says “no, because it would be a single line.” Ms. Lowe says “as soon as I hit <>” and Zack says that it would be a line and not a triangle. She asks if in the demonstration that she just did, does the triangle have to be isosceles, and holds up two pieces of spaghetti that aren’t the same length:  And a few students say “no.” She asks them, “what does small plus medium equals long form?” and Hannah says “a line.” 2:28 The bells rings and Ms. Lowe tells them they have an assignment to do over the break, and to look for it on Dashboard.	the task.	

The overall result of the above coding is that this task was coded as a *procedures with connections* task during set up and implementation.

APPENDIX C

CLASSROOM-BASED FACTORS ASSOCIATED WITH MAINTENANCE OR DECLINE (Stein et al., 2009)

Factors Associated with the Decline of High-Level Cognitive Demand Tasks	Factors Associated with the Maintenance of High-Level Cognitive Demand Tasks
• Problematic aspects of the task become routinized (e.g., students press the teacher to reduce the complexity of the task by specifying explicit procedures or steps to perform; the teacher "takes over" the thinking and reasoning and tells students how to do the problem). • The teacher shifts the emphasis from meaning, concepts, or understanding to the correctness or completeness of the answer. • Not enough time is provided to wrestle with the demanding aspects of the task or too much time is allowed and students drift into off-task behavior • Classroom management problems prevent sustained engagement in high-level cognitive activities. • Inappropriateness of task for given group of students (e.g., students do not engage in high-level cognitive activities due to lack of interest, motivation, or prior knowledge need to perform; task expectations not clear enough to put students in the right cognitive space). • Students are not held accountable for high-level products or processes (e.g., although asked to explain their thinking, unclear or incorrect student explanations are accepted; students are given the impression that their work will not "count" toward a grade). • Other:	• Scaffolding of student thinking and reasoning. • Students are provided with means of monitoring their own progress. • Teacher or capable student models high-level performance. • Sustained press for justifications, explanations, and/or meaning through teacher questioning, comments, and/or feedback. • Tasks build on students' prior knowledge. • Teacher draws frequent conceptual connections. • Sufficient time to explore (not too little, not too much). • Other:

APPENDIX D

INITIAL ORIENTING INTERVIEW

About the school:

- How many students? What are the demographics? How many math teachers?
- How are students grouped for instruction (by grade, ability level, etc.)?
- In what ways do you think this school in unique?
- What kind of support do you get from the administration?

About the class:

- What curriculum do you use? In what ways do you think it's effective? How might or do you change it?
- Why types of activities do students generally work on during class?
- What is the unit about that I'm going to observe?
- How many days of instruction are dedicated to it?
- How many times have you taught this unit in the past?
- How long is each class period?
- How many students?
- Tell me about your students.

About the teacher:

- What degree do you have?
- How long have you been teaching?
- How long have you been at this school?
- How does your experience at this school compare to other schools that you've taught at?

About technology:

- What is your background with using technology for instruction?
- How do you generally use it?
- What experience do your students have with using it?
- How do you think it's effective?
- What concerns does it raise for you?

Is there anything you want to add, or questions that you have for me?

APPENDIX E

TECHNOLOGY USE CHECKLIST

Observation: Date: Site:

Briefly describe the use of technology being coded

Who directly manipulates the technology? Teacher ___ Students ____ Both ____

Who initiates the use of the technology? Teacher ____ Student ___

How are students grouped while the technology is used?

Individual ____ Groups ___ Both ___

What type of activity are students engaged in?

Exploratory ____ Expressive ___ Neither ___

How is the technology intended to be used during set up?

Amplifier ___ Reorganizer ___ Both ___ Neither ___

How is the technology actually utilized during implementation?

Amplifier ___ Reorganizer ___ Both ___ Neither ____

APPENDIX F

POST-LESSON INTERVIEW PROTOCOL

- What was the goal of the main instructional task?
- In what ways was that goal achieved? In what ways wasn't it achieved?
- What did most students seem to be doing or thinking about during the task? What kinds of questions were they asking you?
- What did you notice students doing that you didn't anticipate?
- What changes adjustments did you make, or will you make for the next lesson?

If technology was used

- What was the purpose of using technology in this task?
- Could students do by hand what they used the technology for in today's task?

APPENDIX G

SAMPLE ANALYTIC MEMO

6-21-10

Another theme that arose today is the issue of assessment and the use of technology. In particular, students are asked to do different types of activities with technology than those that they are assessed on. This came up in the context of preparing for an exam today, and most students still struggled with being able to solve for a missing side when given a triangle drawn within another triangle. The GSP activity that they did a week ago had them discover the fact that if a line segment connecting two sides of a triangle is parallel to the third side, then the smaller triangle created within the triangle is similar to the larger triangle.

If assessment makes a statement to students about what is important for them to know and be able to do, what is the message here? Some possible explanations for the lack of alignment between the type of activities that students engage in when using technology and what they're held accountable for on assessments: (1) it's more difficult to test the conceptual understanding that students might gain from these activities than it is to test their ability to execute procedures (2) there is a hope on the part of the teacher that these activities will give the procedures a conceptual grounding so that they can know when to apply them, and do so with understanding. However, the connection between the concepts and procedures is not made effectively, and this is where the disconnect occurs, not between technology and assessment. (3) The technology activities are designed to allow students to engage in mathematical behaviors and processes, i.e., reasoning, conjecturing, communicating, that teachers haven't figured out how to assess.

This issue may be particularly interesting to look at in light of technology as an amplifier or reorganizer. It seems that technology provides the means to reorganize tasks and give students access to concepts or behaviors that would be difficult or impossible to get at otherwise. However, there may be a disconnect if assessment isn't also reorganized in these terms. This may be especially prevalent in classrooms in which technology is used sparingly, as separate "enrichment" type activities. That is, if technology is added on to an otherwise traditional practice as a way to spruce up a traditional curriculum or mix in some reform type instruction, it is likely that the mathematical goals of tasks which use technology will not be assessed. Over a period of time, students may begin to see technology lessons or tasks as less important or relevant. If there is a strong alignment of the use of technology with high level tasks, this might also result in an unintended and subtle message that sense making and conceptual understanding aren't important.

APPENDIX H

SITE ONE TASK SUMMARY

Date	Task	Phase	Cognitive Demand	Factors	Technology	Technology Use	Task Description
6-7	1	Curricular	DM		None	None	Lecture and practice worksheet on clearing the denominator to solve proportions
		Set up	PWO		None	None	
		Implementation	PWO		Calculators	Amp	
6-8	2	Curricular	PW		None	None	The teacher leads students in completing a worksheet on setting up and solving proportions to solve for missing sides in similar triangles
		Set up	PWO		None	None	
		Implementation	PWO		Calculators	Amp	
6-9	3	Curricular	PW		None	None	The teacher leads students in determining if two triangles are similar by checking to see if corresponding sides are proportional within each triangle
		Set up	PWO		None	None	
		Implementation	PWO		None	None	
6-10	4	Curricular	PWO		None	None	Students work individually on worksheets to practice setting up and solving proportions to prepare for the next day's quiz
		Set up	PWO		None	None	
		Implementation	PWO		Calculators	Amp	
6-14	5	Curricular	DM	Decline 2,3,6	None	None	Students are led by the teacher in using Geometer's Sketchpad (GSP) in order to determine how a line segment connecting two sides of a triangle can create a similar triangle within the given triangle
		Set up	PW		GSP	Both	
		Implementation	PWO		GSP	Amp	
6-18	6	Curricular	DM	Decline 1,2,5,6,7	None	None	Students use GSP to explore the relationship between the lengths of the sides of a triangle, i.e., the Triangle
		Set up	PW		GSP	Both	

		Implementation	PWO		GSP	Amp	Inequality, followed by a whole class discussion/lecture
6-21	7	Curricular	PWO		None	None	Students work individually on worksheets to practice classifying triangles and solving for missing sides of similar triangles to prepare for the next day's exam
		Set up	PWO		None	None	
		Implementation	PWO		None	None	
6-23	8	Curricular	PW	Decline 2,5,6,7	GSP	Both	Students use GSP to individually explore the relationship between the angles formed by parallel lines cut by a transversal and between the angles formed by intersecting lines
		Set up	PW		GSP	Both	
		Implementation	PWO		GSP	Neither	
6-24	9	Curricular	PW		GSP	Both	Teacher discusses the previous day's GSP activity by summarizing the main points in a lecture, i.e., vertical angles are equal, linear pairs add to 180, etc.
		Set up	Mem		None	None	
		Implementation	Mem		None	None	
6-28	10	Curricular	PWO		None	None	Students work on a worksheet to practice solving for missing angles in diagrams of parallel lines cut by a transversal in order to prepare for a quiz
		Set up	PWO		None	None	
		Implementation	PWO		Calculators	Amp	
6-29	11	Curricular	PWO		None	None	Students practice solving two step equation and solving diagrams of parallel lines cut by a transversal, and make quiz corrections
		Set up	PWO		None	None	
		Implementation	PWO		None	None	
6-30	12	Curricular	PW	Decline 1,2,5,6	GSP	Amp	Students use GSP to discover that trig ratios (sine, cosine, and tangent) depend only on the angles and not on the side lengths
		Set up	PW		GSP	Amp	
		Implementation	PWO		GSP	Amp	

APPENDIX I

SITE TWO TASK SUMMARY

Date	Task	Phase	Cognitive Demand	Factors	Technology	Technology Use	Task Description
9-21	1	Curricular Materials	Memorization		IWB	Amp	Powerpoint lecture on the interactive whiteboard (IWB) Angle Relations: complementary, supplementary, adjacent, vertical, linear pairs
		Set up	Memorization		IWB	Amp	
		Implementation	Memorization		IWB and calculators	Amp	
9-21	2	Curricular Materials	Memorization		None	None	Activote Quiz (remote clickers) covering new and review material
		Set up	Memorization		IWB	Amp	
		Implementation	Memorization		IWB	Amp	
9-23	3	Curricular Materials	Memorization		IWB	Amp	Powerpoint lecture on IWB followed by a student worksheet Angles formed by lines cut by a transversal
		Set up	Memorization		IWB	Amp	
		Implementation	Memorization		IWB &	Amp	
9-23	4	Curricular Materials	n/a		None	None	Angles collage: students work in pairs to find and cut out examples from magazines of each of the types of angles formed by a transversal
		Set up	Memorization		None	None	
		Implementation	Memorization		None	None	
9-27	5	Curricular Materials	PW	Decline 1,2,4,5	GeoGebra	Both	Students use GeoGebra applet to discover angle relationships formed by parallel lines cut by a transversal
		Set up	PW		GeoGebra	Both	
		Implementation	Memorization		GeoGebra	Amp	
9-27	6	Curricular Materials	PWO		IWB	Amp	Powerpoint lecture on the IWB and student worksheet
		Set up	PWO		IWB	Amp	

		Implementation	PWO		IWB &	Amp	Angles relationships formed by parallel lines cut by
9-29	7	Curricular Materials	PWO		IWB	None	IWB is used by teacher and students during the warm up to fill in missing angles on a figure Angles formed by parallel lines cut by a transversal
		Set up	PWO		IWB	Amp	
		Implementation	PWO		IWB	Amp	
9-29	8	Curricular Materials	PWO		None	None	Student worksheet Angles formed by parallel lines cut by a transversal
		Set up	PWO		None	None	
		Implementation	PWO		Calculators	Amp	
10-1	9	Curricular Materials	PW	Decline 1,5	GeoGebra	Both	Students use a dynamic GeoGebra applet to see that the sum of the interior angles of a triangle equal 180
		Set up	PW		GeoGebra	Both	
		Implementation	PWO		GeoGebra	Amp	
10-1	10	Curricular Materials	PW	Decline 1,4,5	GeoGebra	Both	Students use GeoGebra to construct a triangle and an exterior angle to see that the sum of the two remote interior angles is equal to the exterior angle
		Set up	PW		GeoGebra	Both	
		Implementation	PWO		GeoGebra	Amp	
10-5	11	Curricular Materials	PWO		IWB	Amp	Powerpoint lecture on IWB followed by a student worksheet Triangle sum theorem, exterior angle and remoter interior angles relationship
		Set up	PWO		IWB	Amp	
		Implementation	PWO		IWB and calculators	Amp	
10-5	12	Curricular Materials	PW	Decline 1,3	None	None	Sum of the interior angles of a convex polygon worksheet
		Set up	PW		None	None	
		Implementation	PWO		Calculators	Amp	
10-7	13	Curricular Materials	PWO		IWB	Amp	Powerpoint lecture on IWB followed by a student worksheet Interior and exterior angles in polygons
		Set up	PWO		IWB and	Amp	
		Implementation	PWO		Calculators	Amp	
10-12	14	Curricular Materials	Memorization		IWB	Amp	Bingo Exam Review Angle relations
		Set up	Memorization		IWB and	Amp	
		Implementation	Memorization		IWB and	Amp	
10-12	15	Curricular Materials	Memorization		None	None	Practice Test Angle relations
		Set up	Memorization		None	None	
		Implementation	No Math		Calculators	Amp	
10-14	16	Curricular Materials	PWO		None	None	Exam Review on the IWB
		Set up	PWO		None	None	
		Implementation	PWO		IWB	Amp	
10-14	17	Curricular Materials	PW	Decline	GSP	Both	Geometer's Sketchpad exploration

		Set up	PW	5	GSP	Both	Triangle inequality
		Implementation	PWO		GSP	Amp	

APPENDIX J

SITE THREE TASK SUMMARY

Date	Task	Phase	Cognitive Demand	Factors	Technology	Technology Use	Task Description
11-1	1	Curricular Materials	PWO		IWB	Amplifier	Lecture and practice problems on the interactive whiteboard (IWB) introducing and practicing the order of operations (PEMDAS)
		Set up	PWO		IWB	Amplifier	
		Implementation	PWO		IWB	Amplifier	
11-2	2	Curricular Materials	PWO		IWB	Amplifier	Lecture and practice problems on the IWB practicing the order of operations (PEMDAS)
		Set up	PWO		IWB	Amplifier	
		Implementation	PWO		IWB	Amplifier	
11-3	3	Curricular Materials	PWO		IWB	Amplifier	Lecture and practice problems on the IWB identifying and using variables and evaluating expressions
		Set up	PWO		IWB	Amplifier	
		Implementation	PWO		IWB	Amplifier	
11-4	4	Curricular Materials	PWO		None	None	Whole class “speed PEMDAS” competition to review and practice the order of operations
		Set up	PWO		None	None	
		Implementation	PWO		None	None	
11-8	5	Curricular Materials	PWO	Decline 1	IWB	Amplifier	Lecture and practice on the IWB using and solving equations. Teacher uses students’ work to try to help them discover that one step equations can be solved using inverse operations.
		Set up	PW		IWB	Amplifier	
		Implementation	PWO		IWB	Amplifier	
11-9	6	Curricular Materials	PWO		IWB	Amplifier	Lecture and practice on the IWB using inverse operations to solve equations.
		Set up	PWO		IWB	Amplifier	
		Implementation	PWO		IWB	Amplifier	
11-10	7	Curricular Materials	Mem		IWB	Amplifier	Lecture and practice on knowing and using the properties of numbers, such as the commutative and
		Set up	Mem		IWB	Amplifier	

		Implementation	Mem		IWB	Amplifier	associative properties of addition and multiplication, the identity properties, and the distributive property.
11-11	8	Curricular Materials	PWO		None	None	Exam review packet
		Set up	PWO		Calculators	Amplifier	
		Implementation	PWO		Calculators	Amplifier	
11-15	9	Curricular Materials	Doing Math	Decline 1,2,5,7	None	None	Students explore area representations of fractions using a worksheet.
		Set up	PW		IWB	Reorganizer	
		Implementation	PWO		IWB	Reorganizer	
11-16	10	Curricular Materials	PW	Decline 1,2,5	None	None	Estimating fractions and fraction sums: IWB is used to display fractions on a number line and demonstrate how to use benchmark fractions and decimals. Students work on a worksheet estimating fraction
		Set up	PW		IWB	Reorganizer	
		Implementation	PWO		IWB	Reorganizer	
11-17	11	Curricular Materials	Doing Math	Decline 1,2,5	None	None	Students work on a worksheet requiring them to estimate and find exact answers for fraction sums, and to determine whether an overestimate or underestimate is more appropriate for a given situation
		Set up	PW		IWB	Amplifier	
		Implementation	PWO		IWB	Amplifier	
11-18	12	Curricular Materials	Doing Math	Maintain 1,4,5,7	None	None	The Land Sections Problem: using a visual diagram to determine, compare, add, and subtract fractions. Students work in groups.
		Set up	Doing Math		IWB	Amplifier	
		Implementation	Doing Math		IWB	Amplifier	
11-19	13	Curricular Materials	Doing Math	Maintain 1,5,7	None	None	Students work in groups to use the visual diagram from the previous day to write number sentences for adding and subtracting fractions.
		Set up	PW		IWB	Amplifier	
		Implementation	PW		IWB	Amplifier	
11-30	14	Curricular Materials	PWO		IWB	Amplifier	Use the IWB for lecture and practice problems on rules for adding and subtracting fractions with unlike denominators, and making improper fractions into mixed numbers and vice versa
		Set up	PWO		IWB	Amplifier	
		Implementation	PWO		IWB	Amplifier	
12-1	15	Curricular Materials	Doing Math	Decline 2,3	None	None	Developing strategies for the subtraction of mixed numbers: students work in groups on the Spice Problem
		Set up	PW		IWB	Amplifier	
		Implementation	PWO		IWB	Amplifier	
12-2	16	Curricular Materials	PWO		IWB	Amplifier	Use the IWB for lecture and practice problems on rules for subtracting mixed numbers
		Set up	PWO		IWB	Amplifier	
		Implementation	PWO		IWB	Amplifier	

12-3	17	Curricular Materials	PWO		None	None	More practice on subtracting mixed numbers. The IWB is used as a writing space for problems and solutions.
		Set up	PWO		Amplifier	Amplifier	
		Implementation	PWO		Amplifier	Amplifier	

APPENDIX K

SITE FOUR TASK SUMMARY

Date	Task	Phase	Cognitive Demand	Factors	Technology	Technology Use	Task Description
1-25	1	Curricular Materials	DM	Decline 1, 5	None	None	The teacher leads the class in folding paper to construct perpendicular and angle bisectors, and a discussion of the results
		Set up	PW		None	None	
		Implementation	PWO		IWB	Amplifier	
1-26	2	Curricular Materials	PWO		None	None	Uses the interactive whiteboard (IWB) for lecture and examples of using the various forms of an equation of a line, i.e., slope-intercept, point-slope, and standard form.
		Set up	PWO		IWB	Amplifier	
		Implementation	PWO		IWB	Amplifier	
1-27	3	Curricular Materials	PW	Maintain 1,4,5,6,7	GeoGebra	Both	Students use GeoGebra to explore the properties of the perpendicular bisector and circumcenter of a triangle.
		Set up	PW		GeoGebra	Both	
		Implementation	PW		GeoGebra	Both	
1-28	4	Curricular Materials	PW	Maintain 1,4,5,6,7	GeoGebra	Both	Students use GeoGebra to explore the properties of the angle bisector and incenter of a triangle.
		Set up	PW		GeoGebra	Both	
		Implementation	PW		GeoGebra	Both	
1-31	5	Curricular Materials	PWO		None	None	Teacher leads the class in a discussion of the warm-up problem and the homework
		Set up	PWO		IWB	Amplifier	
		Implementation	PWO		IWB	Amplifier	
2-1	6	Curricular Materials	PW	Maintain 1,5,6,7	GeoGebra	Both	Students use GeoGebra to explore properties of altitudes and the orthocenter, and to use their results to solve for the coordinates of the orthocenter of a triangle analytically
		Set up	PW		GeoGebra	Both	
		Implementation	PW		GeoGebra	Both	
2-7	7	Curricular Materials	PW	Maintain 1,4,5,7	GeoGebra	Both	Students use GeoGebra to explore properties of medians and the centroid of a triangle, and to discover the relationship between the median segments
		Set up	PW		GeoGebra	Both	
		Implementation	PW		GeoGebra	Both	
2-8	8	Curricular Materials	PWO		None	None	Students work individually at their desks on two warm-up problems and two worksheets using medians and altitudes
		Set up	PWO		IWB	Amplifier	

		Implementation	PWO		IWB	Amplifier	of triangles
2-9	9	Curricular Materials	PW		None	None	Leads the class in a discussion of three homework problems (8-10 on 5-3 Practice B)
		Set up	PWO		IWB	Amplifier	
		Implementation	PWO		IWB	Amplifier	
2-10	10	Curricular Materials	Mem		IWB & GeoGebra	Amplifier	Uses GeoGebra on the IWB to review the points of concurrency in a triangle and have students complete a summary chart.
		Set up	Mem		IWB & GeoGebra	Amplifier	
		Implementation	Mem		IWB & GeoGebra	Amplifier	
2-11	11	Curricular Materials	PW	Decline 1,2,3	DGS	Both	The teacher leads the class in discovering the Euler line in a GeoGebra construction on the IWB
		Set up	PW		IWB & Geo	Both	
		Implementation	PWO		IWB & Geo	Both	
2-14	12	Curricular Materials	PWO		None	None	The teacher leads the class in a discussion of the homework as an exam review
		Set up	PWO		IWB	Amplifier	
		Implementation	PWO		IWB	Amplifier	
2-16	13	Curricular Materials	PW	Maintain 1,4,5,7	GeoGebra	Both	Students use GeoGebra to explore the properties of the midsegments of a triangle
		Set up	PW		GeoGebra	Both	
		Implementation	PW		GeoGebra	Both	
2-17	14	Curricular Materials	PW	Decline 1,3	None	None	Students work on the following problem, followed by a teacher-led discussion of the solution: The vertices of are X(-1,8), Y(9,2), and Z(3,-4). M and N are the midpoints of XZ and YZ. Show that MN is parallel to XY, and MN = ½ XY.
		Set up	PW		IWB	Amplifier	
		Implementation	PWO		Calculators	Amplifier	
2-18	15	Curricular Materials	PW	Maintain 1,4,7	None	None	Students work through an activity using spaghetti to explore the Triangle Inequality Theorem.
		Set up	PW		None	None	
		Implementation	PW		None	None	
2-28	16	Curricular Materials	PWO		None	None	Teacher uses GeoGebra to create triangles on the IWB for students to fill in a table and discover that the longest side of a triangle is across from the largest angle.
		Set up	PWO		IWB and GeoGebra	Amplifier	
		Implementation	PWO		IWB and GeoGebra	Amplifier	
3-1	17	Curricular Materials	PW	Decline	None	None	Students work on single problem (#11 on Practice C) for

		Set up	PW	1	IWB	Amplifier	the last 15 minutes of class.
		Implementation	PWO		Calculators	Amplifier	

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