THREE ESSAYS ON ADAPTIVE LEARNING IN MONETARY ECONOMICS

by

Suleyman Cem Karaman

Master of Arts in Economics, Bilkent University 2000

Bachelor of Science in Mathematics Education, Middle East Technical University 1997

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DEPARTMENT OF ECONOMICS

This dissertation was presented

by

Suleyman Cem Karaman

It was defended on

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and approved by

Prof. John Duffy, Department of Economics

Prof. James Feigenbaum, Department of Economics

Prof. Esther Gal-Or, Katz Graduate School of Business

Dissertation Advisors: Prof. John Duffy, Department of Economics,

Prof. David DeJong, Department of Economics

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Suleyman Cem Karaman, PhD

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Adaptive learning is important in dynamic models since it is a process that shows the improvement in the understanding of the agents of the model. Whenever there is a dynamic environment, there is a room for improvement through learning. In this thesis I analyze the adaptive learning of the agents in different setups. In my first paper I show that adaptive learning does not eliminate the multiplicity of stationary equilibria in the Diamond overlapping generations model with money and productive capital; both dynamically efficient and inefficient equilibria are found to be stable under adaptive learning. In my second paper I show that the two agents of a natural-rate model, with different beliefs, learn the economy which leads to convergence or endogenous fluctuations of the inflation rate under different conditions. And in my last paper I show that a central bank with an extraneous instrument, "cheap talk" announcements, can influence the private sector to achieve better outcomes than could be obtained by manipulating the nominal interest rate alone with full knowledge of private sector expectation formation and in anything less than full knowledge, the private sector learns to discount announcements.

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1.0 INTRODUCTION

This thesis is in three parts. In the first part we examine the question of the stability of equilibria under adaptive learning in Diamond's (1965) overlapping-generations model with productive capital and money. In particular, we are interested in whether dynamically inefficient equilibria, which are possible in this model, are stable under adaptive learning. This model has one more asset, capital, than the model considered by Lucas (1986), Marcet and Sargent (1989) and others. Lucas (1986) showed that if agents used a simple adaptive learning rule, they would converge upon the unique monetary equilibrium of a two-period pure exchange OLG model with money as the single outside asset. We show that adaptive learning does not eliminate the multiplicity of stationary equilibria in the Diamond overlapping generations model with money and productive capital; both dynamically efficient and inefficient equilibria are found to be stable under adaptive learning.

In the second part we start with a model of Cho, Williams and Sargent (2002). They consider a natural rate model in which the central bank has imperfect control over inflation and is uncertain of the actual laws of motion of the economy. They show that if the central bank uses a misspecified approximating model to determine inflation there can be endogenous cycling (escape dynamics) between the time-consistent Nash equilibrium outcome and the optimal Ramsey outcome of Kydland and Prescott (1977). They obtain these escape dynamics assuming the central bank and the private sector have the same information and beliefs about the economy. In this paper we assume these two actors have different beliefs about the structure of the economy. The central bank and the private sector learn the economy with their own models separately. If the private sector learns the economy with a fully specified model instead of having rational expectations, escapes disappear and the economy converges to the Nash outcome. With a reverse robustness check we find that escapes can reappear if the private sector uses a misspecified model and the central bank uses a fully specified model. Thus escapes can arise in a model where the central bank is better informed than the private sector. Moreover under certain conditions the difference in beliefs in a two-sided learning model allows the central bank to exploit the expectations of the private sector to achieve an inflation rate lower than the Nash equilibrium outcome level of inflation.

In the last part, using a New Keynesian model, we show that a central bank with an extraneous instrument, "cheap talk" announcements, can influence the private sector to achieve better outcomes than could be obtained by manipulating the nominal interest rate alone. Announcements are effective only if the central bank has full knowledge of how private sector expectations are formed, in which case the central bank can achieve lower inflation and higher output. Otherwise the private sector learns to discount announcements, and we observe convergence to the Nash equilibrium levels of inflation and output.

2.0 LEARNING AND DYNAMIC INEFFICIENCY

2.1 INTRODUCTION

Lucas (1986) suggested that adaptive learning might be useful as an equilibrium selection device in a simple, two period overlapping generations model with money as the single outside asset. He showed that if agents used a simple adaptive learning rule á la Bray (1982), they would converge upon the unique monetary equilibrium of the model. Marcet and Sargent (1989) extended this finding to an environment where a long-lived government financed a fixed deficit by printing money (seigniorage) and where agents learned according to a recursive least squares learning process. The environment they consider gives rise to a Laffer curve and the possibility of two stationary monetary equilibria. They show that the low inflation stationary equilibria is stable and the high inflation equilibrium is unstable under the recursive least squares updating scheme. This work has been interpreted as supporting the notion that low inflation, *monetary* equilibria are attractors under adaptive learning processes in overlapping generations models which are known to admit multiple equilibria.

More recently, Lettau and Van Zandt (2001) and Adam et al. (2006) have shown in the seigniorage inflation overlapping generations monetary model that the high inflation steady state (Lettau and Van Zandt (2001)) or stationary paths near that steady state (Adam et al. (2006)) may be stable under adaptive learning dynamics under certain restrictive timing assumptions, e.g., if agents have contemporary observations of endogenous variables in the information sets they use to form future expectations. These findings cast some doubt on Lucas's suggestion that adaptive learning dynamics might provide a means of selecting between the low and high inflation stationary equilibria of the model as it appears that under certain conditions both equilibria might be learnable. On the other hand, as Marcet and

Sargent (1989) pointed out, the high inflation steady state of the seigniorage model has the counterfactual implication that an increase in the money growth rate is associated with a reduction in the steady state inflation rate.

In all of this prior work involving the stability of monetary equilibria in overlapping generations economies, the models examined leave out alternative means of intertemporal savings, in particular, productive capital. It is of interest to reconsider whether monetary equilibria remain stable under adaptive learning processes when capital is also present, and that is the aim of this paper.

An overlapping generations model with both capital and government liabilities was first proposed by Diamond (1965). Here we consider the stability of the equilibria in the Diamond model under adaptive learning behavior by agents. The version of the Diamond model we consider has fiat money in place of government debt (as in Diamond's original formulation) as the sole outside asset so to maintain comparability with the prior literature on learning. It is well known (see, e.g. Azariadis (1993)) that this model admits three stationary equilibria: an autarkic equilibrium, a nontrivial nonmonetary equilibrium where capital is the only source of savings – the inside money equilibrium – and an "outside money" equilibrium where fiat money and productive capital coexist and pay the same rate of return. The latter equilibrium is only possible if the inside money equilibrium is dynamically inefficient. Under the benchmark assumption of perfect foresight, the autarkic equilibrium is a "source", the inside money equilibrium is a "sink" and the outside money equilibrium is a "saddle". It may seem implausible that a perfect foresight steady state equilibrium with the saddle property can be learned by adaptive agents. However, Packalén (2000) Evans and Honkapohja (2001) have shown that the perfect foresight saddle path of the Ramsey–Cass-Koopmans optimal growth model is indeed locally learnable under standard assumptions about preferences and technology and so it is not so implausible to consider whether individuals are capable of learning such equilibria. Evans and Honkapohja (2001) have shown that the inside money equilibria of a "scalar" Diamond model – one without any outside asset – is learnable by adaptive agents, but the question of whether the outside money equilibrium of the Diamond model is learnable has not, to our knowledge, been previously addressed.

This question is important for several reasons. First, the Diamond model with an outside asset is a standard workhorse model in monetary theory. If the monetary equilibrium of this model is unlearnable, it would call into question a large body of work in monetary theory that makes use of this equilibrium. Second, as noted earlier, an implication of prior work in the learning literature is that monetary equilibria are learnable, nonmonetary equilibria are not learnable and hyperinflationary equilibria may be learnable under certain conditions. It is important to examine whether this conclusion is robust to the inclusion of an additional asset by which individuals can save intertemporally, namely capital. Third, this model has an equilibrium that is *dynamically inefficient* – the nontrivial equilibrium without outside money. In this equilibrium, the capital stock is too high; all agents can be made better off by lowering the capital stock to the golden rule level. It is of independent interest to know whether such dynamically inefficient equilibria are learnable or not; if not then the possibility of dynamic inefficiency, which is typically illustrated using the Diamond model, may be taken less seriously. Finally, this work adds to the learning literature by considering learning in another multivariate system which differs from the Ramsey–Cass–Koopmans framework examined by Evans and Honkapohja (2001).

The structure of the paper is as follows: In the next section, we consider the case where capital is the only means of storage between periods. In Section 3, capital and money both can be used as means of storage. In case Section 4 a more general case where consumption is possible in both of the periods of the model. The last section, Section 5, is the conclusion.

2.2 THE CASE WITH CAPITAL AND NO MONEY

2.2.1 The model

Consider a two-period, overlapping generations environment in discrete time. Following the learning literature's examination of such an environment, we assume that there is no technical progress or labor supply growth. At every date $t = 1, 2, ...$ a single representative agent is born. This agent works when young and consumes only when old. Each young agent inelastically supplies his unit labor endowment in exchange for the competitive market wage, w_t.

The young agent must decide how much to save in the form of capital. Savings at time t equal next period's capital stock. Output, Y of the single, perishable consumption good is produced using capital and labor according to a Cobb-Douglas production technology $Y = K^{\alpha}L^{1-\alpha}$, where K is the aggregate capital stock, L is aggregate labor input, and $\alpha \in (0,1)$ is capital's share of output. We will work with the intensive version of the production technology where output per capita is $y = f(k) = k^{\alpha}$, and k denotes capital per worker. Under perfect competition, factors are paid their marginal products, so that net return on capital is $r_t = f'(k_t) - \delta$ and the wage paid per unit of labor is $w_t = f(k_t) - k_t f'(k_t)$.

The representative agent born at time t seeks to maximize:

$$\max_{\{c_{t+1},n_t\}} U(c_{t+1}, n_t) = u\left(c_{t+1}\right) - v\left(n_t\right)$$

subject to:

$$\begin{aligned} k_{t+1} &\leq n_t w_t \\ c_{t+1} &\leq R^e_{t+1} k_{t+1} \end{aligned}$$

Utility from consumption c, u(·), is assumed to be concave and to satisfy the Inada conditions. Agents experience disutility from working which is captured by assuming that v(·) is a convex function. In this paper we use the functional forms, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and $v(n) = \frac{n^{1+\varepsilon}}{1+\epsilon}$ which satisfy all of these properties. The young agent's intertemporal decision is whether to work less today or to consume more tomorrow. Let n_t denote labor demand. In equilibrium labor demand equals labor supply, $n_t = L_t$. We also use R^e_{t+1} to denote expected return gross return on investment in capital.

The maximization problem can be stated as:

$$\max_{n_t} E_t \left\{ u(R^e_{t+1} n_t w_t) \right\} - v(n_t)$$

The first order conditions give

$$u'(R_{t+1} n_t w_t) R^e_{t+1} (w_t + n_t \frac{\partial w_t}{\partial n_t}) = v'(n_t)$$

Using the functional forms $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and $v(n) = \frac{n^{1+\varepsilon}}{1+\epsilon}$ we can rewrite the first order condition as

$$n_t = (1-\alpha)^{\frac{1}{\sigma+\varepsilon}} (R^e_{t+1})^{\frac{1-\sigma}{\sigma+\varepsilon}} w_t^{\frac{1-\sigma}{\sigma+\varepsilon}}$$

Using the market clearing condition, $k_{t+1} = n_t w_t$, together with the fact that factors are paid their marginal products, $w_t = (1-\alpha)k_t^\alpha$, we arrive at a single equation characterizing equilibrium dynamics in the model without money:

$$k_{t+1} = (1-\alpha)^{\frac{2+\varepsilon}{\sigma+\varepsilon}} \left(R^e_{t+1}\right)^{\frac{1-\sigma}{\sigma+\varepsilon}} k_t^{\alpha\frac{1+\varepsilon}{\sigma+\varepsilon}} \tag{2.1}$$

Notice that one equilibrium is the trivial steady state equilibrium where $k_{t+1} = k_t = 0$ for all t. The other non-trivial interior steady state equilibrium can be found using the the definition of R^e_{t+1} in 2.1 and solving the the following nonlinear equation:

$$\overline{k}^{in} = (1-\alpha)^{\frac{1}{\sigma+\varepsilon}} (\alpha(\overline{k}^{in})^{\alpha-1} + 1 - \delta)^{\frac{1-\sigma}{\sigma+\varepsilon}} [(1-\alpha)(\overline{k}^{in})^{\alpha}]^{\frac{1+\varepsilon}{\sigma+\varepsilon}}$$

We label this steady state capital stock k^{in} as it corresponds to the interior steady state for the capital to labor ratio in the model without outside money. Note that in the special case of full depreciation, $\delta = 1$, we can get an explicit expression for $\overline{k}^{in}$:

$$\overline{k}^{in} = \left[(1-\alpha)^{2+\varepsilon}\alpha^{1-\sigma}\right]^{\frac{1}{1-2\alpha+\alpha\sigma-\alpha\varepsilon+\varepsilon}}$$

In order to study stability under adaptive learning dynamics, we will need to linearize (2.1) with respect to R^e_{t+1} and k_t, around the steady states under consideration. Linearization gives:

$$k_{t+1} = c^{in} + \beta_r^{in} r^e_{t+1} + \beta_k^{in} k_t, \tag{2.2}$$

where

$$\begin{aligned}
c^{in} &= (1-\alpha)^{\frac{2+\varepsilon}{\sigma+\varepsilon}} (\alpha(\overline{k}^{in})^{\alpha-1} + 1 - \delta)^{\frac{1-\sigma}{\sigma+\varepsilon}} (\overline{k}^{in})^{\alpha\frac{1+\varepsilon}{\sigma+\varepsilon}}, \\
\beta_r^{in} &= (1-\alpha)^{\frac{2+\varepsilon}{\sigma+\varepsilon}} \frac{1-\sigma}{\sigma+\varepsilon} (\alpha(\overline{k}^{in})^{\alpha-1} + 1 - \delta)^{\frac{1-\sigma}{\sigma+\varepsilon}-1} (\overline{k}^{in})^{\alpha\frac{1+\varepsilon}{\sigma+\varepsilon}}, \\
\beta_k^{in} &= (1-\alpha)^{\frac{2+\varepsilon}{\sigma+\varepsilon}} \alpha\frac{1+\varepsilon}{\sigma+\varepsilon} (\alpha(\overline{k}^{in})^{\alpha-1} + 1 - \delta)^{\frac{1-\sigma}{\sigma+\varepsilon}} (\overline{k}^{in})^{\alpha\frac{1+\varepsilon}{\sigma+\varepsilon}-1.}
\end{aligned}$$

Since factors are paid their marginal product, $r_{t+1} = \alpha k_{t+1}^{\alpha-1} - \delta$. Linearizing this equation around the steady state gives:

$$r_{t+1} = d^{in} k_{t+1}, \tag{2.3}$$

where

$$d^{in} = \alpha\,(\alpha - 1)\,(\overline{k}^{in})^{\alpha-2}.$$

2.2.2 Adaptive Learning

We focus on the case of the interior rational expectations steady state where $k = k^{in}$ as the trivial case is not of economic interest. We now relax the assumption that agents possess rational expectations and assume as in Evans and Honkapohja (2001, section 4.5) that agents form not-necessariliy rational expectations about the value of r_{t+1}^e in the linearized system (2.2). Their expectations, together with the value of the capital stock will determine the value of next period's capital stock.

2.2.2.1 How do agents learn? We suppose that agents form forecasts of the value of r_{t+1} by applying a least squares regression to past data. By contrast, Evans and Honkapohja (2001) used a simpler, deterministic decreasing gain gradient learning rule in their analysis.

Agents' forecasts interact with the actual law of motion (2.3) to determine a new capital stock k_{t+1} each period. Thus, a new observation is added to the historical data set each period and agents use this to update the coefficients of their forecasting model.

We suppose that agents forecast r_{t+1} using the perceived law of motion:

$$r_{t+1} = a_t + b_t k_t + \epsilon_t. \tag{2.4}$$

where ϵ is a white noise term. This rule may be rationalized as follows: Equation (2.3) combined with (2.3) imply that $r_{t+1} = a + bk_t + cr_{t+1}^e$. So the rational expectations solution will be of the form given by the perceived law of motion (2.4). Hence, this forecast model nests the rational expectations solution as a special case and there is some hope agents can learn the REE. If the coefficients a_t and b_t converge to the rational expectations solution, then we say that the rational expectations solution is learnable, or stable under adaptive learning; otherwise we say it is unstable or unlearnable.

For analytical results we rely on the criterion of expectational instability, as developed in Evans and Honkapohja (2001). Consider a class of perceived laws of motion, specified by a finite dimensional parameter $\theta = (a, b)$. Suppose that agents use a given perceived law of motion to formulate their forecasts of variables of interest. Inserting these forecast rules into the structural equations defining the true economic model we can obtain the actual law of motion implied by the perceived law of motion. If the actual law of motion lies in the same space as the perceived law of motion, though with possibly different parameters, then we obtain a mapping $T(\theta)$ from the perceived to the actual laws of motion. Rational expectations solutions $\overline{\theta}$ correspond to fixed points of $T(\theta)$. A given rational expectations solution $\overline{\theta}$ is said to be E-stable if the differential equation

$$\frac{d\theta}{d\tau} = T(\theta) - \theta$$

is locally asymptotically stable at $\overline{\theta}$. Marcet and Sargent (1989) and Evans and Honkapohja (2001) show how satisfaction of this condition will under certain regularity conditions characterize the stability of the dynamics of the stochastic recursive least squares learning algorithm.

Using the perceived law of motion, the expected value of r_{t+1} will be $a + bk_t$. Pugging this value into the linearized equation (2.2) gives the actual law of motion (ALM) for capital:

$$k_{t+1} = c^{in} + \beta_r^{in} a + \left(\beta_k^{in} + \beta_r^{in} b\right) k_t. \tag{2.5}$$

Combining (2.5) with (2.3) gives the actual law of motion for interest rates:

$$r_{t+1} = d^{in}(c^{in} + \beta_r^{in} a) + d^{in} \left(\beta_k^{in} + \beta_r^{in} b\right) k_t \tag{2.6}$$

The mapping from agents' PLM (2.4) to the ALM (2.6) is given by the T-map:

$$T \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} d^{in}(c^{in} + \beta_r^{in} a) \\ d^{in}(\beta_k^{in} + \beta_r^{in} b) \end{pmatrix}$$

The unique rational expectations equilibrium for this model is the unique fixed point of the T-map which is:

$$\frac{d}{d\tau}\begin{pmatrix} a \\ b \end{pmatrix} = T\begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} a \\ b \end{pmatrix}$$

where τ denotes "notional" time. It is said that the rational expectations equilibrium is expectationally stable, or E-stable, if the rational expectations equilibrium is locally asymptotically stable under the above equation.

$$\frac{da}{d\tau} = d^{in}c^{in} + \left(d^{in}\beta_r^{in} - 1\right)a$$

$$\frac{db}{d\tau} = d^{in}\beta_k^{in} + \left(d^{in}\beta_r^{in} - 1\right)b$$

The rational expectations equilibrium is E-stable if and only if $d^{in}\beta_r^{in} < 1$.

Proposition 1. *Suppose that $\alpha \in (0,1)$, $\delta = 1$ and $\sigma > 1$. Then $d^{in}\beta_r^{in} < 1$ and the unique non-trivial steady state of the economy where capital investment is the only means of intertemporal savings is expectationally stable.*

The proof of Proposition 1 is provided in the appendix.

2.2.2.2 Numerical Analysis Assuming less than full depreciation we need to use numerical methods, as in that case, it is not possible to find a closed form solution for the steady state value of capital, $\overline{k}^{in}$. Nevertheless, we can show that in all instances examined, the interior steady state exists and is unique.

Specifically, we conducted a simulation exercise where we change all model parameters within an empirically plausible range. Table 1 gives the parameter ranges we used. For each parameter value we used a step-size of 0.001

For all parameter values given in Table 1 the value of $d^{in}\beta_r^{in}$ is less than 1 which provides numerical confirmation that the nonmonetary equilibrium is learnable for empirically plausible cases.

Parameter	Lower Bound	Upper Bound
δ	0.1	0.8
α	0.1	0.5
σ	1.001	3.001
ε	1.001	3.001

Table 1: Parameter Values for the Non-Monetary Model

2.3 THE CASE WITH CAPITAL AND MONEY

2.3.1 The Model

Consider next, the same model, but now allow money as another mean of intertemporal savings. The growth rate of money is assumed to be exogenously set and equal to μ, i.e.,

$$m_t = (1+\mu)m_{t-1}$$

This implies endogenous determination of real government consumption, $g_t = \frac{\mu}{(1+\mu)}m_t$ per period. As our focus is on monetary equilibria and less on fiscal policy, we assume that government consumption leaves the economy.

Agents can now choose to hold their savings in both money and capital. The possibility of arbitrage requires that return on capital and return on money are same. We will assume this condition throughout the learning process. Savings can be thought of as mutual fund investing in two assets which yields a unique rate of return for investors. The R^e_{t+1} in the model represents this return. The equality of returns on money and capital will be used in finding the steady states of the economy.

$$\max U(c_{t+1}, n_t) = u(c_{t+1}) - v(n_t)$$

subject to:

$$\begin{aligned} m_t + k_{t+1} &\leq n_t w_t \\ c_{t+1} &\leq R^e_{t+1}(m_t + k_{t+1}) \end{aligned}$$

Simplifying the budget constraints gives $c_{t+1} \leq R^e_{t+1} n_t w_t$. The maximization problem thus becomes:

$$E_t u(R^e_{t+1} n_t w_t) - v(n_t)$$

The first order conditions give:

$$u'(R^e_{t+1} n_t w_t) R^e_{t+1} (w_t + n_t \frac{\partial w_t}{\partial n_t}) = v'(n_t)$$

Using the functions $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and $v(n) = \frac{n^{1+\varepsilon}}{1+\epsilon}$ we get:

$$n_t^{\varepsilon} = (R^e_{t+1} n_t w_t)^{-\sigma} R^e_{t+1} w_t (1-\alpha)$$

From this equation we get:

$$n_t = (1-\alpha)^{\frac{1}{\sigma+\varepsilon}} (R^e_{t+1})^{\frac{1-\sigma}{\sigma+\varepsilon}} w_t^{\frac{1-\sigma}{\sigma+\varepsilon}}$$

Using the above first order conditions we can derive the following equilibrium conditions. First the budget constraint implies that

$$\begin{aligned} k_{t+1} &= n_t w_t - m_t \\ &= (1-\alpha)^{\frac{2+\varepsilon}{\sigma+\varepsilon}} \left(R^e_{t+1}\right)^{\frac{1-\sigma}{\sigma+\varepsilon}} k_t^{\alpha \frac{1+\varepsilon}{\sigma+\varepsilon}} - m_t \\ &= (1-\alpha)^{\frac{1}{\sigma+\varepsilon}} (R^e_{t+1})^{\frac{1-\sigma}{\sigma+\varepsilon}} [(1-\alpha) k_t^{\alpha}]^{\frac{1+\varepsilon}{\sigma+\varepsilon}} - m_t \end{aligned} \tag{2.7}$$

Second, the absence of arbitrage opportunities, $E(R^k_{t+1}) = E(R^m_{t+1})$ implies that:

$$\begin{aligned} \frac{1}{1+\mu} \frac{m_{t+1}}{m_t} &= \alpha k_{t+1}^{\alpha-1} + 1 - \delta \\ \text{or} m_{t+1} &= (1+\mu) m_t R^e_{t+1} \end{aligned} \tag{2.8}$$

We can use these equilibrium conditions to derive steady state values for k and m in the case where both assets coexist: Using (2.8) we have:

$$\overline{k}^{out} = \left[\frac{1}{\alpha}(\frac{1}{1+\mu} - 1 + \delta)\right]^{\frac{1}{\alpha-1}}$$

and using (2.7) we have:

$$\bar{m} = (1-\alpha)^{\frac{1}{\sigma+\varepsilon}} (\alpha(\overline{k}^{out})^{\alpha-1} + 1 - \delta)^{\frac{1-\sigma}{\sigma+\varepsilon}} [(1-\alpha)(\overline{k}^{out})^{\alpha}]^{\frac{1+\varepsilon}{\sigma+\varepsilon}}] - (\overline{k}^{out})$$

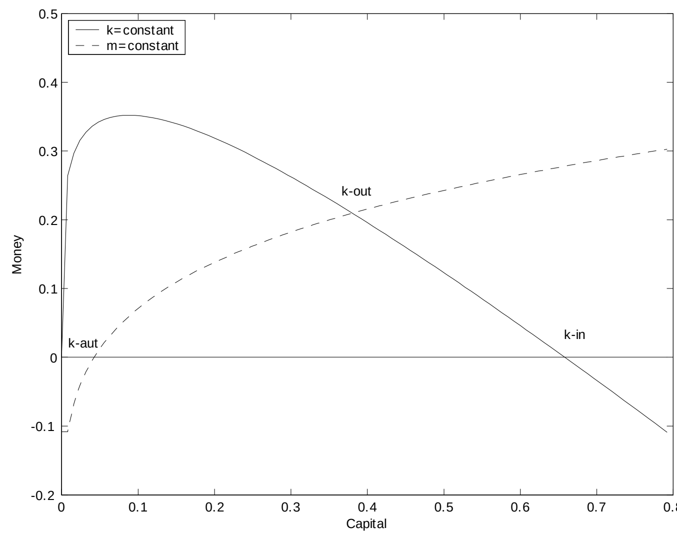

Figure 1: Illustration of Phase Diagram for the Planar Model with Capital and Money

A phase diagram that illustrates the possible steady state values for money and capital can be developed by plotting equations (2.7)-(2.8) Figure 1 provides an illustration.

This model with productive capital and money as means of savings has three rational expectations equilibria. $(k, m) = (0, 0)$, $(k, m) = (k^{in}, 0)$, $(k, m) = (k^{out}, m^{out})$. In Figure 1, the autarkic equilibrium is labeled "k-out", the nontrivial nonmonetary equilibrium where capital is the only source of savings is labeled as "k-in" and the "outside money" equilibrium where fiat money and productive capital coexist and pay the same rate of return is labeled "k-out". In this section we will consider only the latter two equilibria which are the ones of greatest interest.

Under rational expectations the outside money equilibrium (if it exists) is a saddle path and the inside money equilibrium is a sink. If the return on money is more than the return on capital in the case where capital is the only medium of exchange, (that is, if the economy is dynamically inefficient) then a monetary equilibrium exists. The condition for dynamic inefficiency can be written as:

$$\frac{1}{1+\mu} > f'(k^{in}) + 1 - \delta \tag{2.9}$$

The left hand side is the gross steady state return on real money balances.[1] The right hand side is the gross steady state return on capital when capital is the only mean of savings. This condition states that when the steady state return on money is greater than the steady state return on capital in the environment where there is no money, that money can serve as an additional store of value. Otherwise money will not be valued by agents.

2.3.2 The existence of dynamically inefficient equilibrium in Diamond's overlapping generations model:

Unlike the Ramsey-Cass-Koopmans infinitely lived agent model, it is possible for competitive equilibria to be dynamically inefficient in Diamond's model. The capital stock of the Diamond model may exceed the golden-rule level, so that a permanent increase in consumption is possible. If individuals in the market economy want to consume in the old age, their only choice is to hold capital, even if its rate of return is low. But a planner can divide the resources available for consumption between the young and old in any manner. If this change is required for every generation, a planner makes every generation better off. In our model, instead of a planner, money is introduced as a mean to decrease the capital stock to its golden rule level and eliminate the dynamic inefficiency.

In order to assess the E-stability of the stationary equilibria of the model it is necessary to linearize equations (2.7)-(2.8). This gives:

$$k_{t+1} = c_k^{out} + \beta_r^{out} r_{t+1}^e + \beta_k^{out} k_t - m_t$$

$$m_{t+1} = c_m^{out} + m_t + \eta \left(r_{t+1}^e \right)$$

[1] The gross return on real money balances is simply the inverse of the expected inflation factor: $\frac{p_t}{p_{t+1}} = \frac{M_{t+1}/p_{t+1}}{(1+\mu)M_t/p_t} = \frac{1}{1+\mu}\frac{m_{t+1}}{m_t}$.

where

$$
\begin{aligned}
c_k^{out} &= (1-\alpha)^{\frac{2+\varepsilon}{\sigma+\varepsilon}} (\overline{R})^{\frac{1-\sigma}{\sigma+\varepsilon}} (\overline{k}^{out})^{\alpha\frac{1+\varepsilon}{\sigma+\varepsilon}} - \overline{m}, \\
\beta_r^{out} &= (1-\alpha)^{\frac{2+\varepsilon}{\sigma+\varepsilon}} \frac{1-\sigma}{\sigma+\varepsilon} (\alpha(\overline{k}^{out})^{\alpha-1} + 1 - \delta)^{\frac{1-\sigma}{\sigma+\varepsilon}-1} (\overline{k}^{out})^{\alpha\frac{1+\varepsilon}{\sigma+\varepsilon}}, \\
\beta_k^{out} &= (1-\alpha)^{\frac{2+\varepsilon}{\sigma+\varepsilon}} \alpha \frac{1+\varepsilon}{\sigma+\varepsilon} (\alpha(\overline{k}^{out})^{\alpha-1} + 1 - \delta)^{\frac{1-\sigma}{\sigma+\varepsilon}} (\overline{k}^{out})^{\alpha\frac{1+\varepsilon}{\sigma+\varepsilon}-1}, \\
c_m^{out} &= (1+\mu)\overline{m}(\alpha(\overline{k}^{out})^{\alpha-1} + 1 - \delta), \\
\eta &= 1+\mu)\bar{m}.
\end{aligned}
$$

2.3.3 Expectational Stability

We will use the first linearized equations to analyze the expectational stability

$$k_{t+1} = c_k^{out} + \beta_r^{out} r_{t+1}^e + \beta_k^{out} k_t - m_t \tag{2.10}$$

$$m_{t+1} = c_m^{out} + m_t + \eta r_{t+1}^e \tag{2.11}$$

Substitute the lagged value of (2.11) into (2.10) to get

$$k_{t+1} = c_k^{out} + \beta_r^{out} r_{t+1}^e + \beta_k^{out} k_t - c_m^{out} - m_{t-1} - \eta r_t$$

using $r_t = d^{out} k_t$

$$k_{t+1} = c_k^{out} - c_m^{out} + \beta_r^{out} r_{t+1}^e + \left(\beta_k^{out} - \eta d^{out}\right) k_t - m_{t-1}$$

We can write the perceived law of motion equation (PLM) as

$$r_{t+1} = a + bk_t + cm_{t-1} + \varepsilon_t$$

The expected value of r_{t+1} will be $a + bk_t + cm_{t-1}$. Pugging this value into the linearized equation gives the actual law of motion (ALM) equation which is:

$$k_{t+1} = c_k^{out} - c_m^{out} + \beta_r^{out} a + \left(\beta_k^{out} - \eta d^{out} + \beta_r^{out} b\right) k_t + (\beta_r^{out} c - 1) m_{t-1}$$

Since factors are paid their marginal product, $r_{t+1} = \alpha k_{t+1}^{\alpha-1} - \delta$. Linearizing this equation around the steady state gives

$$r_{t+1} = \alpha (\alpha - 1) (\overline{k}^{out})^{\alpha-2} k_{t+1}$$

Or shortly,

$$r_{t+1} = d^{out} k_{t+1}$$

where $d^{out} = \alpha (\alpha - 1) (\overline{k}^{out})^{\alpha-2}$ Using this equality in the actual law of motion gives

$$r_{t+1} = d^{out} \left(c_k^{out} - c_m^{out} + \beta_r^{out} a \right) + d^{out} \left(\beta_k^{out} - \eta d^{out} + \beta_r^{out} b \right) k_t + d^{out} \left(\beta_r^{out} c - 1 \right) m_{t-1}$$

Thus, the mapping from the PLM to ALM is given by the T-map:

$$T \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} d^{out} \left(c_k^{out} - c_m^{out} + \beta_r^{out} a \right) \\ d^{out} \left(\beta_k^{out} - \eta d^{out} + \beta_r^{out} b \right) \\ d^{out} (\beta_r^{out} c - 1) \end{pmatrix}$$

The unique rational expectations equilibrium for this model is the unique fixed point of the T-map which is:

$$\frac{d}{d\tau} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = T \begin{pmatrix} a \\ b \\ c \end{pmatrix} - \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

where τ denotes "notional" time. It is said that the rational expectations equilibrium is expectationally stable, or E-stable, if the rational expectations equilibrium is locally asymptotically stable under the above equation.

$$\frac{da}{d\tau} = d^{out} \left(c_k^{out} - c_m^{out} \right) + \left(d^{out} \beta_r^{out} - 1 \right) a$$

$$\frac{db}{d\tau} = d^{out} \left(\beta_k^{out} - \eta d^{out} \right) + \left(d^{out} \beta_r^{out} - 1 \right) b$$

$$\frac{dc}{d\tau} = -d^{out} + \left(d^{out} \beta_r^{out} - 1 \right) c$$

The rational expectations equilibrium is E-stable if and only if $d^{out} \beta_r^{out} < 1$. Although there is an explicit expression for the steady state value of capital, the value of $d^{out} \beta_r^{out}$ is dependent on many parameters which makes it impossible to find an analytic solution. We therefore conducted numerical analysis to check the plausibility of the condition that $d^{out}\beta_r^{out} < 1$ for a more plausible parameterization of the model. Specifically, we considered the same grid of parameter values used for the model without money and provided earlier in Table 1. In addition, to those parameters, we now also vary the parameter μ from 0 to 1.0 with step-size 0.1. The case of $\mu = 0$ represents a constant money stock, while values of $\mu > 0$ imply a growing supply of money. We focus only on cases where the equilibrium with both money and capital exists, i.e., the condition for dynamic inefficiency (2.9) is satisfied.

Of all parameter combinations satisfying (2.9), we find that $d^{out}\beta_r^{out}$ is less than 1 in 36968 cases out of 38777 cases when δ is between 0.1 and 0.8. When $\delta = 1$, $d^{out}\beta_r^{out}$ is less than 1 in 13300 cases out of 17187 cases. When we look at the cases where the system is not stable we observe that δ is always equal or greater than 0.6 and α is either 0.1 or 0.2. Even though we do not observe a clear pattern for the parameter values, our numerical analysis suggests that higher levels of depreciation and lower levels of capital share may lead to instability.

Thus for empirically plausible versions of the model, the dynamically inefficient equilibrium where capital and money coexist as means of intertemporal savings is learnable, for most of the time, by agents. As the equilibrium where only capital serves as a store of value is also learnable, we conclude that the E-stability principle (adaptive learning dynamics) do not enable us to select from among the nontrivial equilibria of the Diamond overlapping generations model as both equilibria can be learned by agents who do not initially possess rational expectations.

2.4 A MORE GENERAL CASE

2.4.1 The Model

Now we will consider the case where consumption in both periods of life is possible. In the first period, agents will make an additional choice between youthful consumption and savings. The setup of this model is the same as the previous one except for the extra choice of consumption in the first period.

The problem of the representative agent is:

$$\max u\left(c_t\right)+u\left(c_{t+1}\right)-v\left(n_t\right)$$

subject to:

$$\begin{aligned} m_t+k_{t+1} &\leq n_t w_t-c_t \\ c_{t+1} &\leq R^e_{t+1}(m_t+k_{t+1}) \end{aligned}$$

Simplifying the budget constraints gives $c_{t+1} \leq R^e_{t+1}(n_t w_t - c_t)$, So the maximization problem becomes:

$$u\left(c_t\right)+E_t u(R^e_{t+1}(n_t w_t-c_t))-v(n_t)$$

The first order conditions give:

$$-u'(c_{t+1})R^e_{t+1}+u'(c_t)=0 \tag{2.12}$$

$$u'(c_{t+1})R^e_{t+1}(w_t+n_t\frac{\partial w_t}{\partial n_t})=v'(n_t) \tag{2.13}$$

using $u'(c)=c^{-\sigma}$ and $v'(n)=n^{\varepsilon}$ (2.12) and (2.13) become

$$c^{-\sigma}_{t+1}R^e_{t+1}=c^{-\sigma}_t$$

$$c^{-\sigma}_{t+1}R^e_{t+1}(w_t+n_t\frac{\partial w_t}{\partial n_t})=n^{\varepsilon}_t \tag{2.14}$$

$$c_t=(R^e_{t+1})^{-1/\sigma}c_{t+1}$$

The budget constraint is $c_{t+1}=R_{t+1}(n_t w_t-c_t)$

$$c_{t+1}=R_{t+1}(n_t w_t-(R_{t+1})^{\frac{-1}{\sigma}}c_{t+1})$$

or

$$c_{t+1}=\frac{R_{t+1}n_t w_t}{1+(R_{t+1})^{1-\frac{1}{\sigma}}} \tag{2.15}$$

Substitute (2.15) into the following equation which is equation (2.14)

$$n_t^{\varepsilon} = (1-\alpha) w_t R_{t+1} c_{t+1}^{-\sigma}$$

$$n_t = (1-\alpha)^{\frac{1}{\sigma+\varepsilon}} w_t^{\frac{1-\sigma}{\sigma+\varepsilon}} g(R_{t+1}) \tag{2.16}$$

where $g(R_{t+1}) = R_{t+1}^{\frac{1-\sigma}{\sigma+\varepsilon}} [1 + (R_{t+1})^{1-\frac{1}{\sigma}}]^{\frac{\sigma}{\sigma+\varepsilon}}$. The market clearing condition is

$$k_{t+1} = n_t w_t - c_t - m_t$$

We know that $c_t = [(1-\alpha)w_t]^{\sigma} n_t^{\frac{-\varepsilon}{\sigma}}$. So

$$k_{t+1} = n_t w_t - (1-\alpha)^{\sigma} w_t^{\sigma} n_t^{\frac{-\varepsilon}{\sigma}} - m_t$$

Using (2.16),

$$k_{t+1} = (1-\alpha)^{\frac{2+\varepsilon}{\sigma+\varepsilon}} k_t^{\alpha\frac{1+\varepsilon}{\sigma+\varepsilon}} g(R_{t+1}) - (1-\alpha)^{\sigma - \frac{\varepsilon}{\sigma(\sigma+\varepsilon)} + \sigma - \frac{\varepsilon}{\sigma}\frac{1-\sigma}{\sigma+\varepsilon}} k_t^{\alpha(\sigma - \frac{1-\sigma}{\sigma+\varepsilon}\frac{\varepsilon}{\sigma})} [g(R_{t+1})]^{\frac{-\varepsilon}{\sigma}} - m_t$$

In short,

$$k_{t+1} = A k_t^{z_1} g(R_{t+1}) - B k_t^{z_2} [g(R_{t+1})]^{\frac{-\varepsilon}{\sigma}} - m_t$$

where $A = (1-\alpha)^{\frac{2+\varepsilon}{\sigma+\varepsilon}}$, $z_1 = \alpha\frac{1+\varepsilon}{\sigma+\varepsilon}$, $B = (1-\alpha)^{\sigma - \frac{\varepsilon}{\sigma(\sigma+\varepsilon)} + \sigma - \frac{\varepsilon}{\sigma}\frac{1-\sigma}{\sigma+\varepsilon}}$, $z_2 = \alpha(\sigma - \frac{1-\sigma}{\sigma+\varepsilon}\frac{\varepsilon}{\sigma})$.

Linearization gives the following equation.

$$\begin{aligned} k_{t+1} &= \left\{ A z_1 \overline{k}^{z_1 - 1} g(\overline{R}) - B z_2 \overline{k}^{z_2 - 1} [g(\overline{R})]^{\frac{-\varepsilon}{\sigma}} \right\} k_t ... \\ &... + \left\{ A \overline{k}^{z_1} g(\overline{R}) + B \overline{k}^{z_2} \frac{\varepsilon}{\sigma} [g(\overline{R})]^{\frac{-\varepsilon}{\sigma} - 1} g(\overline{R}) \right\} r_{t+1} - m_t + const \end{aligned}$$

This equation has the same structure as the version presented in the previous section. The expectational stability requirement for this system is for the coefficient on r_{t+1} to be less than 1. We used the same parameter values given in Table 1 to conduct a further numerical analysis. With this model we observed that the inside money equilibrium together with the outside money equilibrium are always stable. Out of 17556 monetary equilibrium where it exits, all of them are stable. This suggests that the results that we found in the previous model are robust to the addition of an intratemporal consumption/savings decision.

2.5 CONCLUSION

Diamond's (1965)overlapping-generations model with productive capital and money is used by many researchers. The question of whether the equilibria of this model are learnable by adaptive agents who do not initially possess rational expectations has not been previously explored. In particular, one might hope to use learning to reduce the set of rational expectations equilibria and in particular, to rule out the possibility of dynamically inefficient equilibria. Our results suggest that stability analysis under adaptive learning does not provide a means for selecting from among the multiple equilibria in this model. In particular, we find that dynamically inefficient equilibria *are* learnable. While the finding that learning does not work as a selection device in this model might be viewed as a negative result, the finding that dynamically inefficient equilibria are learnable might be viewed (positively or negatively!) as rationalizing some kind of government intervention, e.g. fiat money or social security transfer schemes that restore the economy to one of dynamic efficiency.

2.6 APPENDIX

2.6.1 Proof of Proposition 1

We assume full depreciation, $\delta = 1$. together with the usual assumptions for α , ε and σ; $\alpha \in (0,1)$; ε, $\sigma > 0$. $d^{in}\beta_r^{in} = \alpha\left(\alpha-1\right)(\overline{k}^{in})^{\alpha-2}(1-\alpha)^{\frac{2+\varepsilon}{\sigma+\varepsilon}}\frac{1-\sigma}{\sigma+\varepsilon}(\alpha(\overline{k}^{in})^{\alpha-1})^{\frac{1-\sigma}{\sigma+\varepsilon}-1}(\overline{k}^{in})^{\alpha\frac{1+\varepsilon}{\sigma+\varepsilon}}$

$d^{in}\beta_r^{in} = \alpha\left(\alpha-1\right)\frac{1-\sigma}{\sigma+\varepsilon}(1-\alpha)^{\frac{2+\varepsilon}{\sigma+\varepsilon}}\alpha^{\frac{1-\sigma}{\sigma+\varepsilon}-1}(\overline{k}^{in})^{\alpha-2+(\alpha-1)\frac{1-\sigma}{\sigma+\varepsilon}-1+\alpha\frac{1+\varepsilon}{\sigma+\varepsilon}}$

Substituting the value of capital, $\overline{k}^{in} = [(1-\alpha)^{2+\varepsilon}\alpha^{1-\sigma}]^{\frac{1}{1-2\alpha+\alpha\sigma-\alpha\varepsilon+\varepsilon}}$ we get

$d^{in}\beta_r^{in} = \alpha\left(\alpha-1\right)\frac{1-\sigma}{\sigma+\varepsilon}(1-\alpha)^{\frac{2+\varepsilon}{\sigma+\varepsilon}}\alpha^{\frac{1-\sigma}{\sigma+\varepsilon}-1}\left((1-\alpha)^{2+\varepsilon}\alpha^{1-\sigma}\right)^{\frac{\alpha-2+(\alpha-1)\frac{1-\sigma}{\sigma+\varepsilon}-1+\alpha\frac{1+\varepsilon}{\sigma+\varepsilon}}{1-2\alpha+\alpha\sigma-\alpha\varepsilon+\varepsilon}}$

$d^{in}\beta_r^{in} = \alpha\left(\alpha-1\right)\frac{1-\sigma}{\sigma+\varepsilon}(1-\alpha)^{\frac{2+\varepsilon}{\sigma+\varepsilon}}\alpha^{\frac{1-\sigma}{\sigma+\varepsilon}-1}\left((1-\alpha)^{\frac{2+\varepsilon}{\sigma+\varepsilon}}\alpha^{\frac{1-\sigma}{\sigma+\varepsilon}}\right)^{-1}$

$d^{in}\beta_r^{in} = \alpha\left(\alpha-1\right)\frac{1-\sigma}{\sigma+\varepsilon}\alpha^{-1} = \left(1-\alpha\right)\frac{\sigma-1}{\sigma+\varepsilon} < 1$

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3.0 TWO-SIDED LEARNING IN A NATURAL RATE MODEL

3.1 INTRODUCTION

Differences in people's perceptions play an important role in economics. Whenever we assume multiple agents, the possibility for disagreement in beliefs opens up the possibility of exploiting these differences. For example, agents with different views about the structure of the economy may derive different decision rules. Or agents may have different beliefs about the commitment technology of the government. In this paper, we study the effect of these differences in a natural rate model where the beliefs of the private sector affect the ability of the central bank to achieve its goals.

Kydland and Prescott (1977) use a natural rate model to argue that, if at each time policymakers select the best action given the current situation, the social objective function will typically not be maximized. Rather, they suggested that, economic performance can be improved by committing ahead of time to policy rules. The current decisions of economic agents depend on their expectations of future policy actions. If agents are rational and have the same information as policy makers, they can infer the actions the government will take. The resulting game dynamics can lead to suboptimal behavior that would not occur if the government sets its future policy independent of what other agents do in the meantime. The optimal policy maximizes the social objective function but it is not consistent due to the rationality of the agents. Specifically, Kydland and Prescott (1977) show that doing what is best given the current situation – i.e. a discretionary or time-consistent policy – results in an excessive level of inflation without any improvement in unemployment.

Sargent (1999) studied the post World War II American inflation under the assumption that policy makers learned to believe in natural unemployment rate hypotheses during this period. He relaxed the rational expectations hypothesis for the policy maker –but not the private sector– and assumed adaptive learning behavior in its place. This model exhibits recurrent escapes from the time-consistent outcome to the optimal outcome of Kydland and Prescott (1977). Cho, Williams and Sargent (2002) showed that the escapes from the time-consistent outcome occur via accidental experimentation induced by the government's adaptive algorithm and its misspecified model.

Assuming the private sector has rational expectations reduces the analysis to a single-agent decision problem. Barro and Gordon (1983) argued that this approach cannot deal with the game-theoretic situation that arises when decisions are made on an ongoing basis. Pursuing this idea, in this paper, neither the central bank nor the private sector know the true model but instead build independent approximating models that incorporate separate beliefs about how the economy works. Thus the model involves a dual-agent decision problem. On one side, the central bank constructs a model with its own beliefs and commitment technology. It derives a policy rule as a function of its current information set. On the other side, the private sector constructs another model with the goal of predicting the policy of the central bank. Its expectation of the central bank's policy will be a function of its own current information set.

In this paper the central bank chooses the rate of price inflation and the private sector determines the rate of wage inflation (or the expected inflation rate) in a dynamic natural rate model. The two players can have different specifications for the laws of motion of the economy. They may also have different beliefs/knowledge about the commitment technology of the central bank. They update their information set every period as new data is generated.

Our results are as follows: 1) When the private sector learns the economy with a correctly specified model rather than having rational expectations, we observe the disappearance of the escapes of Cho, Williams and Sargent (2002) and convergence to the Nash equilibrium. The additional distortion from the learning model of the private sector makes it more difficult for an unusual sequence of shocks to deceive the central bank.

2) In a reverse robustness check we let the private sector have a misspecified approximating model while the central bank has a correctly specified approximating model. We observe escapes but this time the source of the fluctuations is the private sector rather than

the central bank. This establishes that escapes can occur in a more plausible environment where the central bank is better informed than the private sector.

3) We observe that in some scenarios a difference in beliefs between the central bank and the private sector allows the central bank to exploit the private sector and achieve inflation lower than the Nash level. With this result we can explain the efforts of central banks to influence the private sector's expectations through announcements, release of more frequent policy forecasts, and fuller statements explaining interest-rate policy.

The structure of the paper is as follows: In sections 3.2 and 3.3 the model and the learning algorithm are introduced. In section 3.4 we review what happens when the private sector is rational as in Cho, Williams and Sargent (2002). We also show the convergence of the inflation rate to the central bank target when the central bank correctly specifies the economy. In section 3.5 we analyze what happens under different scenarios of two-sided learning. Finally in section 3.6 we talk about possibilities for further research.

3.2 THE MODEL

The model we develop is a general model that encompasses the properties of the model used by Kydland and Prescott (1977) and Cho, Williams and Sargent (2002). The model describes the behavior of a central bank, which imprecisely chooses the rate of price inflation π_t, and the private sector, whose actions imprecisely determine the rate of wage inflation w_t. The private sector sets the rate of wage inflation aiming to set to equal to π_t. Thus w_t can also be viewed as the private sector's expectation of inflation.

The expectational Phillips curve determines the unemployment rate:

$$U_t = U^n - (\pi_t - w_t) + v_{1t}, \tag{3.1}$$

where

$$\pi_t = \mu_t + v_{2t}. \tag{3.2}$$

and

$$w_t = q_t + v_{3t}. \tag{3.3}$$

Here U^n is the constant natural rate of unemployment, U_t is the unemployment rate, μ_t is the central bank determined inflation rate or money growth rate, q_t is the private sector determined rate of wage inflation before its noise, and v_{1t}, v_{2t} and v_{3t} are normally distributed independent noises. The unemployment rate U_t is a convenient proxy for real activity in the economy. The slope of the Phillips curve is taken to be unity for convenience. Using another constant value will not change our results. In this model, surprise inflation lowers the unemployment rate but anticipated inflation does not. Equation (3.2) states that the central bank controls the money supply with some noise just as equation (3.3) states that the private sector determines the rate of wage inflation with some noise. The optimal choices of μ_t and q_t are explained in detail below.

The central bank's objective is summarized by the single-period return or payoff function, $Z_{cb,t}$, which depends on that period's values for the unemployment rate and inflation. Following the literature we assume a simple quadratic form:

$$Z_{cb,t} = -E\left\{\frac{1}{2}\pi_t^2 + \frac{b}{2}\left(U_t - (U^n - \alpha)\right)^2\right\} \tag{3.4}$$

The first term in this objective function captures the cost of inflation or, more precisely, penalizes deviations of the inflation rate π_t from the central bank's target of zero. Direct costs of changing prices would be a simple explanation for why inflation is costly. The nonnegative constant b is the weight that the central bank places on achieving its goal for unemployment, relative to its goal for inflation. The second term is the deviation from the targeted unemployment rate, which is α less than the natural unemployment rate, where α is a nonnegative constant. The natural rate of unemployment will tend to exceed the efficient level of unemployment in the presence of unemployment compensation and income taxation. The constant α captures this possibility. The central bank maximizes the single-period objective function (3.4) by choosing an inflation rate μ_t. The constraints on this maximization are explained below. The objective of the private sector is to maximize

$$Z_{ps,t} = -E\left\{\frac{1}{2}(\pi_t - w_t)^2\right\} \tag{3.5}$$

The private sector wants to set the wage inflation as close as possible to the central bank determined inflation rate.

The determination of the unemployment rate can be characterized as a game between the central bank and the private-sector. At period t, the central bank sets the inflation rate, μ_t, with the information set I_{t-1} and the belief set $\mathcal{B}_{cb}$. Private-sector agents set the wage inflation, q_t, with the same information set I_{t-1}, but with their own belief set $\mathcal{B}_{ps}$. We will define the belief sets $\mathcal{B}_{cb}$ and $\mathcal{B}_{ps}$ and the information set I_{t-1} below. We will consider cases where they choose their variables at the same time or sequentially with the private sector going first. The timing of decisions plays an important role and will be explained in detail below. It is also important to note that the belief sets of the agents are not time-dependent.

It should be stressed that in forming inflationary expectations, the private-sector knows that the choice of μ_t will emerge from the central bank's maximization function given in equation (3.4). After the random disturbances $v_t = (v_{1t}, v_{2t}, v_{3t})$ are realized, equations (3.1) - (3.3) determine the unemployment rate.

Information and Belief Sets

The information set I_t includes all the data available up to and including time t. The data consists of all past values of the unemployment rate, inflation rate and wage inflation. The information set, I_t, is available both to the central bank and the private sector. Moreover the central bank and the private sector may have different beliefs about how the economy works. Each will learn the economy separately with its own approximating model based on its beliefs about the structure of the economy. We will talk more about the differences in the approximating models in the next section.

We also allow the two players to have different beliefs about the commitment technology of the central bank. Commitment technology is the ability of the central bank to credibly commit to a policy choice even if the optimal choice might be different in the following periods. Without the commitment technology the central bank makes policy under discretion.

Thus the belief sets are defined as

$$\mathcal{B}_{cb}, \mathcal{B}_{ps} = \{structure\ of\ the\ economy, commitment\ technology\ of\ the\ central\ bank\}$$

Note that the central bank knows correctly and with certainty what its commitment technology, but the private sector may be misinformed about this.

Assuming that the central bank and the private sector have the same belief set means they believe in the same structure of the economy and the private sector knows the commitment technology of the central bank. For this section we assume that the central bank and the private-sector agents have the same belief sets, $\mathcal{B}_{cb} \equiv \mathcal{B}_{ps}$. This assumption makes it possible to assume rational expectations for the private sector. Later in the paper in section 3.5 we will look for the implications of having different belief sets.

Expectation Formation

In the formation of expectations, q_t, private-sector agents consider the central bank's maximization problem, which determines the choice of μ_t. Suppose that, given it's belief set, the private sector perceives this process as described by a strategy function, $F^e_{ps}(I_{t-1} \mid \mathcal{B}_{ps})$. Therefore inflationary expectations are given by

$$q_t = F^e_{ps}(I_{t-1} \mid \mathcal{B}_{ps}) \tag{3.6}$$

We also assume that the central bank understands that q_t is generated from equation (3.6).

Solutions to the Model

Substituting (3.2), (3.3) and (3.6) into (3.1) yields

$$U_t = U^n - (\mu_t - F^e_{ps}(I_{t-1} \mid \mathcal{B}_{ps}) + v_{2t} - v_{3t}) + v_{1t} \tag{3.7}$$

Assuming that the policymaker knows the true model, he selects μ_t that maximizes (3.4) with respect to the constraints, including equation (3.7). There are two possible timing protocols we could use, depending on the central bank's commitment technology. If the central bank cannot commit to a policy, it effectively makes its choice of μ_t after the private sector has embedded its expectations into a particular choice of q_t. Thus the central bank can take q_t as given, and maximize its objective function accordingly. Given its beliefs, the non-committed central bank has a strategy function that depends on its information set and q_t:

$$\mu_t^{nc} = F_{cb}^{nc}(I_{t-1}, q_t \mid \mathcal{B}_{cb})$$

In the second case, the central bank can does commit to a particular policy before the private sector institutionalizes its expectations. In this case, given its beliefs, the committed central bank has a strategy function that depends only on its information set, $\mu_t^c = F_{cb}^c(I_{t-1} \mid \mathcal{B}_{cb})$. In the following two definitions these policies are derived.

Definition 2. *Assume that the central bank is ether unwilling or unable to precommit to a policy and selects its policy choice μ_t after observing the private sector's expectations, q_t, given in (3.6). The solution to the problem*

$$\max_{\mu_t} Z_{cb,t} \text{ subject to } (3.7)$$

is called the Nash outcome since the solution is the best response to private sector expectations. Following the literature we also call this the policy of a non-committed central bank.

The strategy function of the non-committed central bank is

$$\mu_t^{nc} = F_{cb}^{nc}(I_{t-1}, q_t \mid \mathcal{B}_{cb}) = \frac{b}{1+b}\left(F_{ps}^e(I_{t-1} \mid \mathcal{B}_{ps}) + \alpha\right) \tag{3.8}$$

The property $E(v_t \mid I_{t-1}) = 0$ has been used in the computation of the strategy function. A private sector with the same information and belief sets with the central bank, $\mathcal{B}_{ps} \equiv \mathcal{B}_{cb}$, understands the optimization problem of the policymaker. In particular the private sector understands that the actual choice, μ_t^{nc} satisfies equation (3.8). Solving its maximization problem given in (3.5) and using equation (3.8), the private sector calculates $F_{ps}^e(I_{t-1} \mid \mathcal{B}_{ps})$ in equation (3.6). The private sector sets $F_{ps}^e(I_{t-1} \mid \mathcal{B}_{ps}) = \mu_t^{nc}$ which leads to the policy

$$\mu_t^{nc} = b\alpha$$

A non-committed central bank will be tempted to exploit the expectational Phillips curve in an effort to achieve its goal of pushing unemployment below the natural rate. The private sector understands the incentives of the central bank and knows the central bank faces this temptation to inflate. The private sector, therefore, builds these inflationary expectations into its wage-setting decisions so that unemployment remains at its natural rate.

Alternatively, we can assume the central bank is able to precommit to a choice for μ_t before the private sector embeds its expectations into a particular choice of q_t. This policy can be viewed as a once-and-for-all choice of a policy rule. The central bank will then view the condition $F^e_{ps}(I_{t-1} \mid \mathcal{B}_{ps}) = \mu_t$ as a constraint that links its choice of μ_t to a subsequent choice for q_t.

Definition 3. *Assume that the central bank can precommit to a choice for μ before the private sector embeds its expectations into a particular choice of q. Its problem is then*

$$\max_{\mu_t} Z_{cb,t} \text{ subject to } q_t = \mu_t, \ (3.7)$$

and the solution thereof is called the Ramsey outcome. Following the literature we also call this the policy of a committed central bank.

The optimal monetary policy with commitment is

$$\mu^c_t = F^c_{cb}(I_{t-1} \mid \mathcal{B}_{cb}) = 0$$

When the central bank precommits to a choice for μ_t, it recognizes that it will lose ability it might otherwise have to surprise private-sector agents and thereby exploit the Phillips curve. Hence, under commitment, the central bank abandons any idea of pushing unemployment below the natural rate and, instead, focuses exclusively on achieving its goal of zero inflation.

A Ramsey outcome dominates a Nash outcome. Efforts to exploit the Phillips curve can lead only to a suboptimally high rate of inflation, $\mu^{nc}_t = b\alpha$, with no decrease in the unemployment rate.

3.3 LEARNING DYNAMICS

Now we assume that the central bank does not know (3.1) but believes that unemployment follows the process

$$U_t = \gamma_t z_t + \varepsilon_t$$

where γ is a vector of coefficients, z is a vector of regressors, and ε_t is a random variable orthogonal to z_t. The set of regressors will vary with the model that the central bank estimates. We assume two possible approximating models, a fully specified model;

$$U_t = \gamma_0 + \gamma_1 \pi_t + \gamma_2 w_t + \varepsilon_t \tag{3.9}$$

and a misspecified model;

$$U_t = \gamma_0 + \gamma_1 \pi_t + \varepsilon_t \tag{3.10}$$

Depending on their beliefs, the central bank and the private sector use either (3.9) or (3.10) to derive their policies. The second approximating model (3.10) is what Cho, Williams and Sargent (2002) used to explain the fluctuations in the US inflation rate. The omission of the private sector's expectation leads to a misperception of the shocks, which later leads to transitions between the Nash and Ramsey outcomes.

We suppose the central bank estimates γ by least squares regression of U on z in past data. Each period, the central bank updates its estimate of γ with the latest data and solves its optimization problem with the updated γ. In the standard least squares regression formula, the value of the coefficient vector γ is estimated by the formula

$$\gamma = \left(\sum_{1}^{T} zz'\right)^{-1} \left(\sum_{1}^{T} zU\right) \tag{3.11}$$

after T observations. This treats all data equally. More generally, γ can instead be computed using the formulas

$$\gamma_{t+1} = \gamma_t + a_t R_t^{-1} z_t \left(U_t - \gamma_t z_t\right) \tag{3.12}$$

$$R_{t+1} = R_t + a_t (z_t z_t' - R_t), \tag{3.13}$$

where a_t is a sequence of positive real numbers and R_t is an estimate of the moment matrix of z_t. Setting $a_t = 1/t$ gives back the standard least squares learning algorithm. Throughout this paper we will instead set $a_t = a$, employing what is known as a constant gain learning algorithm, which puts more weight on the recent observation and less weight on past observations. Constant gain learning is necassary to obtain the endogenous transitions between the Nash and Ramsey outcomes reported in Cho, Williams and Sargent (2002) and in section (3.4.1) of this paper. One justification for this constant gain algorithm is to formalize perpetual learning which is what we observe from policymakers.

With a constant gain algorithm the distribution of γ_t will not converge to a degenerate distribution since γ_t is nonnegligibly sensitive to random shocks even asymptotically. However, γ_t may converge to a limiting probability distribution. In the limit of small a, we can derive the limiting distribution.

3.4 LEARNING WITH THE SAME BELIEF SETS

First we assume that the central bank and the private sector have the same belief sets, $\mathcal{B}_{ps} \equiv \mathcal{B}_{cb}$. Later in the paper we assume the case where they have the same information set but different belief sets. The private sector wishes to forecast the decisions of the central bank. If they have the same information and belief sets they should find the same optimal behavior for the central bank. Depending on the shared belief set, there are three possible cases. In the first case, studied in section 3.4.1, the central bank misspecifies the economy, using the approximating model (3.10). In this approximating model the central bank ignores the expectations of the private sector. This will be very similar to what Cho, Williams and Sargent (2002) studied. Second, the central bank correctly incorporates the expectations of the private sector and uses (3.9) as its approximating model. In this case there are two possibilities. The central bank may move first and commit to a policy, section 3.4.2. Or the central bank may move after the private sector forms its expectations, section 3.4.3. This is the case where the central bank has no commitment technology and it is willing to exploit the expectations of the private sector.

The convergence analysis of least square learning depends on results from stochastic approximation theory. We will analyze the limiting behavior of the associated differential equations of the stochastic system. Similar work is done by Marcet and Sargent (1989) and

Woodford (1990). Further details of the convergence results of each of the following sections are given in Appendix 3.7.

3.4.1 Misspecified Central Bank Policy Rule

This section is a reproduction of Cho, Williams and Sargent (2002) with some minor differences. Their model has an unemployment target of 0 and it has equal weight on inflation and unemployment target in the objective function. But even with these minor differences the two models produce the same outcomes. Assume that the central bank does not know (3.1) but uses its own misspecified model

$$U_t = \gamma_0 + \gamma_1 \pi_t + \varepsilon_t \tag{3.14}$$

The commitment technology of the central bank is irrelevant since the central bank does not think the private sector matters. The central bank maximizes (3.4) with respect to (3.14) and (3.2). The resulting policy is

$$\mu_t = \frac{-b\gamma_1 \left(\gamma_0 - U^n + \alpha\right)}{1 + b\gamma_1^2} \tag{3.15}$$

With the misspecified model (3.14) the effects of expected inflation w_t are absorbed into the constant γ_0. Since μ_t and q_t are constant at the Nash equilibrium, the failure to include w_t as a regressor costs the central bank nothing in terms of statistical fit.

With a misspecified learning model the inflation rate makes recurrent cycles between the time-consistent Nash outcome and the time-inconsistent Ramsey outcome. Figure 2 shows a simulation of the system. In this model the central bank fails to include the private sector's expectation into its regression equation, the misspecification. Referring to Cho, Williams and Sargent (2002) we call the endogenous movement of the inflation rate to the Ramsey outcome an escape. Escapes occur when the algorithm is driven by an unusual sequence of random shocks. By these particular unusual sequence of random variables, γ_1 in (3.15) increases. This steepens the estimated Phillips curve which leads the central bank to lower the inflation rate. Discounting past observations helps this process along. But the system

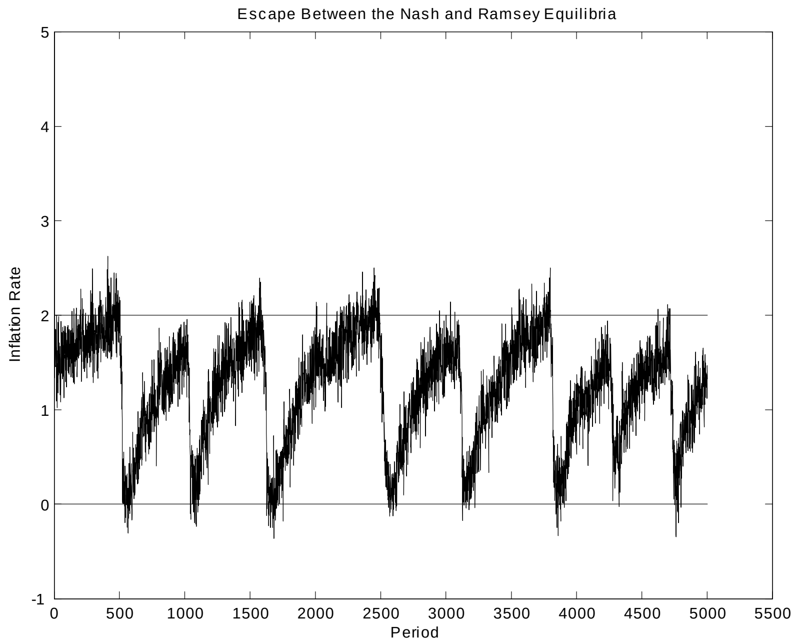

Figure 2: Nash Equilibrium is 2, Ramsey is 0.

cannot remain at the Ramsey outcome indefinitely since the Ramsey outcome is not a Nash equilibrium. Eventually the system will be drown back to the Nash equilibrium outcome.

3.4.2 A Committed Central Bank Learning the Economy with the Fully Specified Model

Learning with misspecified dynamics leads to escapes between the Nash outcome and the Ramsey outcome. It is of interest to see if the results change if the central bank considers the expectations of the private sector as a determinant of the unemployment rate. First let us suppose the central bank is committed. This adds one more condition to the maximization problem of the central bank: $q_t = \mu_t$. The central bank will maximize (3.4) with respect to (3.9), (3.2), (3.3) and $q_t = \mu_t$. The resulting policy rule is

$$\mu_t = \frac{-b\left(\gamma_1 + \gamma_2\right)\left(\gamma_0 - U^n + \alpha\right)}{1 + b\left(\gamma_1 + \gamma_2\right)^2} \tag{3.16}$$

Proposition 4. *When the central bank moves before the private sector and commits to a policy, the inflation rate, π_t, converges to a limiting probability distribution, a normal distribution with mean value equal to the Ramsey outcome.*

For the proof of this proposition refer to Appendix 3.7. Figure 3 is a simulation of this economy.

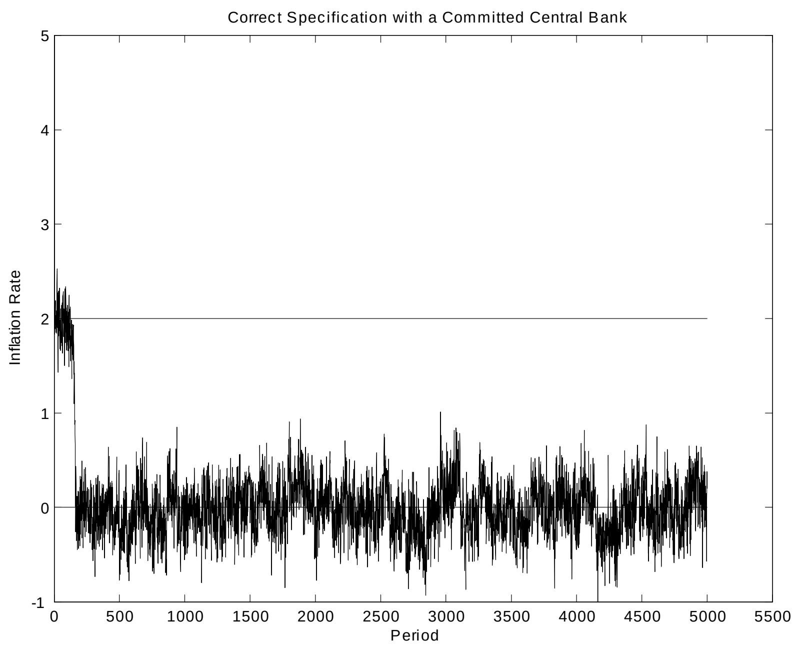

Figure 3: Nash equilibrium is 2, Ramsey is 0

This means that when the central bank plays the Ramsey plan every period, the inflation rate stays at the Ramsey equilibrium outcome even if this is not a Nash equilibrium outcome.

The associated differential equation, derived in Appendix 3.7, has a unique steady state with, $\gamma = \begin{pmatrix} U^n & -1 & 1 \end{pmatrix}$. When substituted into the central bank's policy function we get the Ramsey equilibrium outcome, $\mu_t = 0$.

3.4.3 A Non-Committed Central Bank Learning the Economy with the Fully Specified Model

In the previous section, the central bank moves first and commits to a policy. What if the central bank moves second? First the private sector forms its expectation, w_t, about the central bank's policy. Then the central bank chooses its policy, the targeted inflation rate μ_t. Assume the central bank does not know (3.1) but uses its own model

$$U_t = \gamma_0 + \gamma_1 \pi_t + \gamma_2 w_t + \varepsilon_t \tag{3.17}$$

We call this model the fully specified model since the expectation of the private sector is not omitted. The central bank will maximize (3.4) with respect to (3.17), (3.2) and (3.3). The resulting policy rule is

$$\mu_t = \frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_t}{1 + b\gamma_1^2} \tag{3.18}$$

Since the private sector can forecast this decision, it will set $q_t = \mu_t$. Then the policy rule of the central bank reduces to

$$\mu_t = \frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1 + b\gamma_1^2 + b\gamma_1\gamma_2} \tag{3.19}$$

Proposition 5. *When the central bank moves after observing the expectations of the private sector, the inflation rate, π_t, converges to a limiting probability distribution, a normal distribution with mean value equal to the Nash equilibrium inflation rate.*

Figure 4 is a simulation of the economy. The associated differential equation of this system has a unique steady state with $\gamma = \begin{pmatrix} U^n & -1 & 1 \end{pmatrix}$. When substituted into the central bank's policy function (3.19) we get the Nash equilibrium value, $\mu_t = \alpha b$. So at the equilibrium the central bank is playing the Nash equilibrium, the value that the inflation rate is converging. This shows us the Nash equilibrium is learnable if both agents believe the correct model specification.

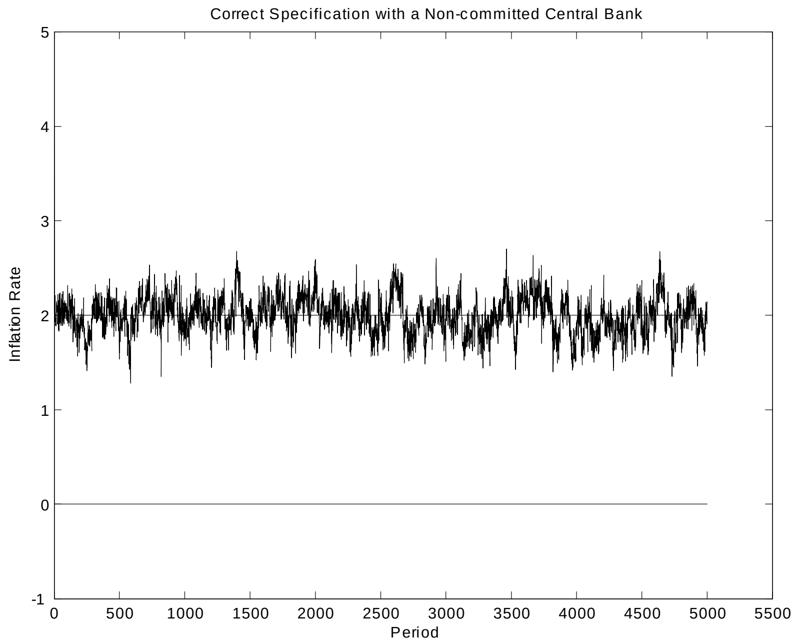

Figure 4: Nash Equilibrium is 2, Ramsey is 0.

3.5 TWO-SIDED LEARNING

In section 3.4 we assumed that the central bank and the private sector have the same belief sets, $\mathcal{B}_{ps} \equiv \mathcal{B}_{cb}$. Therefore the problem was reduced to a single agent problem. Having the same information and belief sets, the private sector is able to correctly predict, up to a noise, what inflation will be. But we know this is not always the case. In reality the central bank and the private sector may often have different views about how the economy works. In this section we assume the central bank and the private sector have different belief sets.

3.5.1 A Robustness Check for Endogenous Fluctuations

In Cho, Williams and Sargent (2002), and in section 3.4.1 of this paper, the central bank learns the economy with a misspecified model, while the private sector has rational expectations. In this section we assume the private sector learns the economy with a fully specified model. It also correctly believes that the central bank is non-committed. The private sector believes the structure of the economy is described by

$$U_t = \eta_0 + \eta_1 \pi_t + \eta_2 w_t + \varepsilon_t \tag{3.20}$$

The vector of coefficients η will be used for the private sector while γ will continue to denote the vector of coefficients for the central bank. The problem the private sector thinks the central bank is solving is

$$\max_{\mu} Z_{cb,t} \text{ subject to (3.20), (3.2) and (3.3)}$$

The policy function the private sector forecasts is

$$F^e_{ps}(I_{t-1} \mid \mathcal{B}_{ps}) = \frac{-b\eta_1(\eta_0 - U^n + \alpha) - b\eta_1\eta_2 q_t}{1 + b\eta_1^2} \tag{3.21}$$

Given the information set and the belief set, $F^e_{ps}(I_{t-1} \mid \mathcal{B}_{ps})$ is what the private sector thinks the central bank's policy is. So the private sector will set wage inflation to

$$q_t = F^e_{ps}(I_{t-1} \mid \mathcal{B}_{ps}) = \frac{-b\eta_1(\eta_0 - U^n + \alpha)}{1 + b\eta_1^2 + b\eta_1\eta_2}$$

The central bank is the same central bank of section 3.4.1. The central bank uses the misspecified model, $U_t = \gamma_0 + \gamma_1 \pi_t + \varepsilon_t$. The policy function of the central bank is given in (3.15).

Proposition 6. *When the central bank misspecifies the economy where as the private sector learns with the fully specified model assuming a non-committed central bank, the inflation rate, π_t, converges to a limiting probability distribution which is normal with mean equal to the Nash equilibrium value.*

A simulation of this economy is given in figure 5, and proof of this proposition is in Appendix 3.7. When the private sector learns the economy with a correctly specified approximating model while the central bank has a misspecified model, we observe the convergence of the expectations of the private sector to the Nash equilibrium outcome and the inflation rate converges to the same mean also. This result is interesting in the sense that the escapes between the Nash and Ramsey outcomes of Cho, Williams and Sargent (2002) disappear. If, instead of assuming rational expectations for the private sector, we equip the private sector with a fully specified approximating model it can prevent the central bank from misinterpreting random shocks.

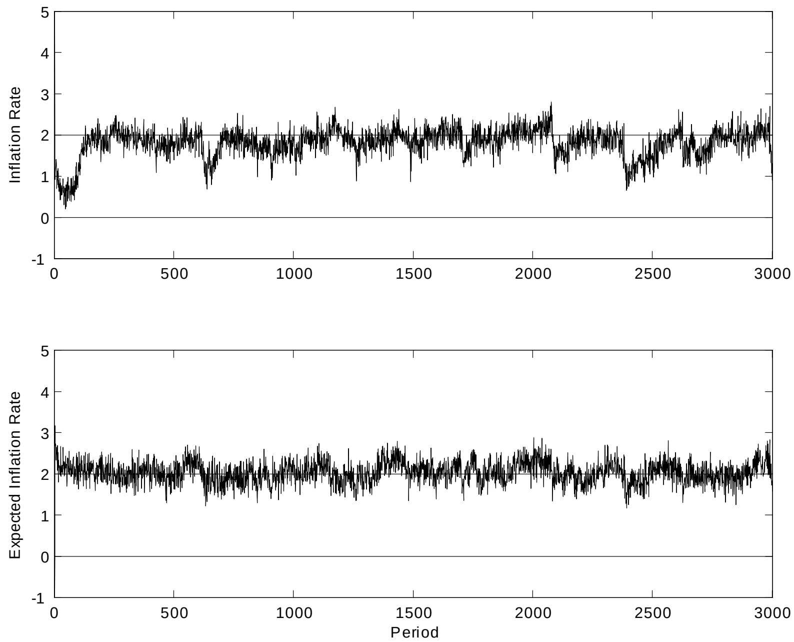

Figure 5: Nash equilibrium is 2, Ramsey is 0

3.5.2 Reverse Robustness Check

In the previous section we observed the disappearance of the endogenous fluctuations with a learning private sector even if the private sector learns the rational expectations policy. We would like to test the robustness of this result by considering the reverse case. Now a non-committed central bank learns the economy with a fully specified approximating model and the private sector learns the economy with a misspecified approximating model. This is a more plausible case since the central bank should be better informed than the private sector. The central bank's policy function is given in (3.18). Given the information set and the beliefs of the private sector, the policy function of the private sector will be similar to (3.15) but expressed in terms of the coefficient vector η:

$$q_t = F^e_{ps}(I_{t-1} \mid \mathcal{B}_{ps}) = \frac{-b\eta_1 (\eta_0 - U^n + \alpha)}{1 + b\eta_1^2} \tag{3.22}$$

Observing the wage inflation rate the central bank determines the inflation rate, up to a noise, using the policy function (3.18). The following proposition outlines the what happens.

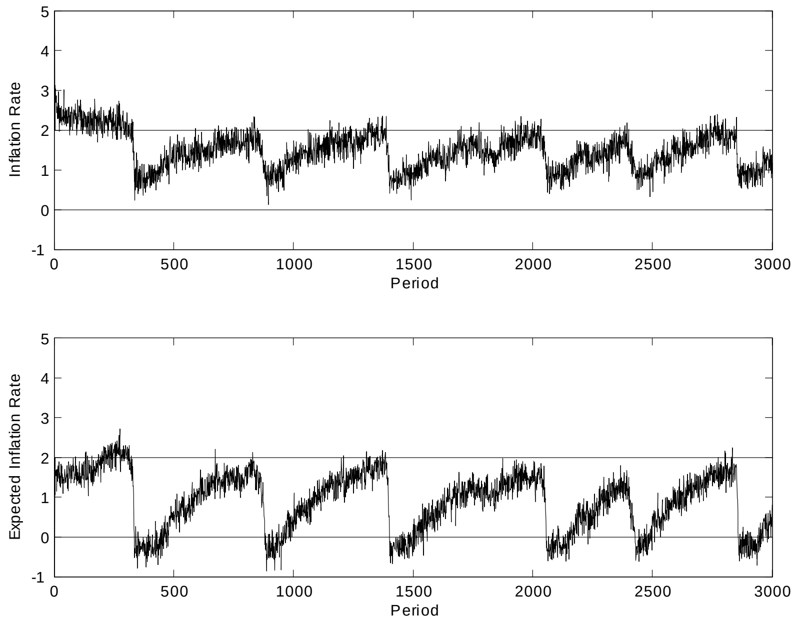

Figure 6: Nash equilibrium is 2, Ramsey is 0

When a non-committed central bank learns the fully specified model where as the private sector learns the misspecified model, the inflation rate, π_t, endogenously fluctuates with sudden escapes from the Nash equilibrium outcome. A simulation of this economy is given in figure 6. As can be seen in this figure, the inflation rate fluctuates together with wage inflation. This leads to endogenous fluctuations as seen in section 3.4.1. Since the central bank follows the private sector in its policy we observe similar fluctuations in the central bank determined inflation rate. This provides an alternative explanation for fluctuations in the inflation rate where this time the private sector is the cause of the fluctuations. But fluctuations in the inflation rate are not as wide spread as they are for wage inflation rate. Since the central bank has the ability to exploit the expectations of the private sector it can achieve better results.

Comparing with the previous case we observe the endogenous fluctuations in a different environment. This is a reproduction of the escapes in a setup where the private sector does not have rational expectations and the central bank is better informed than the private sector. In this scenario the cause of the escapes of Cho, Williams and Sargent (2002) is the private sector rather than the central bank. A central bank without a commitment to a particular policy choice determines an inflation rate that fluctuates with the expectations of the private sector.

3.5.3 Exploiting the Difference in Beliefs

In a two-sided learning environment we allow the central bank and the private sector to have different beliefs about the economy. Assuming a difference in beliefs opens up the possibility of exploiting these differences. In a natural rate model, if the central bank can keep the beliefs of the private sector lower than its actual policy, it may take advantage of this difference to achieve a lower than Nash equilibrium outcome level of inflation. Assume the private sector thinks the central bank is committed to a policy using a fully specified model. The private sector solves the maximization problem of the central bank

$$\max_{\mu_t} Z_{cb,t} \text{ subject to } q_t = \mu_t,\ U_t = \eta_0 + \eta_1 \pi_t + \eta_2 w_t + \varepsilon_t,\ (3.2) \text{ and } (3.3)$$

The policy function of the private sector is

$$q_t = F^e_{ps}(I_{t-1} \mid \mathcal{B}_{ps}) = \frac{-b\,(\eta_1 + \eta_2)\,(\eta_0 - U^n + \alpha)}{1 + b\,(\eta_1 + \eta_2)^2}, \tag{3.23}$$

The private sector forms its expectations before the central bank determines the inflation rate. Observing the expected wage inflation, the non-committed central bank determines the inflation rate using the policy function (3.18). The following proposition outlines the case.

Proposition 7. *When the central bank is not committed to a policy where the private sector learns the economy assuming a committed central bank the inflation rate, π_t, converges to a limiting probability distribution which is normal with mean equal to a restricted perceptions equilibrium.*

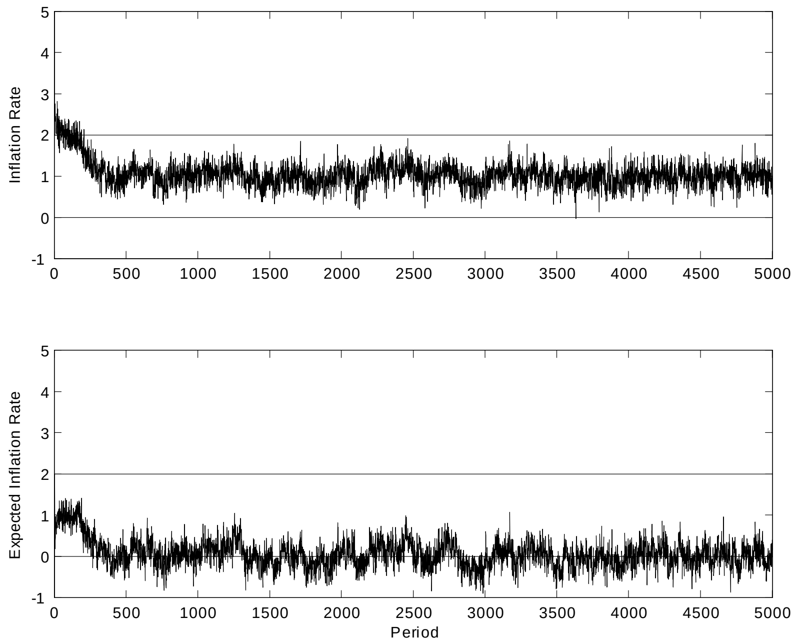

Figure 7: Nash equilibrium is 2, Ramsey is 0

A simulation of this economy is given in figure 7. The expectations of the private sector converge to the Ramsey equilibrium outcome where as the actual inflation rate converges to higher value. This is a restricted-perceptions equilibrium, an equilibrium that arises from the beliefs of agents rather than from the fundamentals of the model. This is an interesting result since if the difference in beliefs is maintained, the central bank attains a better result than the Nash equilibrium outcome, with an inflation rate between the Nash and the Ramsey outcomes.

It is well known that central banks make announcements to influence the private sector. The private sector pays attention to these announcements and they definitely have an important role in the formation of it expectations. According to rational expectations theory any kind of attempt to manipulate expectations should not work and the private sector should correctly predict the central bank determined inflation rate. But in reality the central bank does try to influence the beliefs of the private sector by making announcements, publishing more frequent policy forecasts and fuller statements explaining interest-rate policy are some examples to explain how the central bank is trying to influence the beliefs of the private sector to attain an advantage in determining the inflation rate. In this section we have seen these actions might work to the advantage of the central bank.

3.6 CONCLUSION

Expectations play an important role in the realization of the inflation rate, but different perceptions of the world lead to different expectations and policies. Whenever we consider models with multiple agents it is important to consider the implications of such differences. In this paper we test some previous results in the learning literature in a two-sided learning environment where two agents construct models and decision rules independently.

In his famous book "Conquest of American Inflation" Thomas Sargent analyzes the rise and fall of U.S. inflation after 1960. According to Sargent (1999) the role of expectations in economics was not well established before the 1970's. Policymakers of the time adopted methods derived from exploitation of the Phillips curve in the hope of lowering the inflation rate. As they learned from new data, they re-estimated their Phillips curve and adjusted their target inflation rate accordingly. But since they ignored the role of inflation expectations in the Phillips curve, fluctuations in the inflation rate resulted. First we show that with the inclusion of the expectations in a one-sided learning model the policymaker can achieve the results it targets.

In the second part of the paper we analyze the case where the central bank and the private sector have different views of the economy and they learn the economy with their own models. This allows us to test the robustness of the escapes of Cho, Williams and Sargent (2002) to two-sided learning. Our results show that the endogenous fluctuations of Cho, Williams and Sargent (2002) are not robust to a learning private sector. Even if the private sector learns the policy of the central bank we observe the disappearance of the fluctuations. But we show that it is possible to reproduce these endogenous fluctuations in a more plausible environment where the central bank uses a fully specified model and the private sector uses a misspecified model, so the central bank is better informed than the private sector. In this case the expectations of the private sector fluctuate, causing the inflation rate to fluctuate with it.

In another two-sided learning environment, the actual inflation rate and the expectations of the private sector converge to different values. Given its beliefs, the private sector is not capable of learning the policy of the central bank. The private sector updates its data set every period but this updating does not allow it to change its model specification. This is a weakness of the Evans-Honkapohja-Sargent learning mechanism. Since the unemployment rate decreases as much as the decrease in the expected rate of inflation, the regression coefficients do not respond to the divergence of the actual inflation rate from the expected inflation rate. The steady state that the inflation rate converges to is a restricted perceptions equilibrium, an equilibrium that arises from the beliefs of agents rather than from the fundamentals of the model. The existence of such a difference in beliefs lets the central bank achieve inflation lower than the Nash level.

We would like to see whether this result can be obtained in a less restrictive setting where the private sector learns via a mechanism (such as Bayesian learning) that does allow it to update its model specification. It is known that central banks make announcements or reveal information to affect the beliefs of the private sector. Is this because they actually can use their influence to manipulate the private sector's beliefs and achieve better than Nash outcome?

3.7 PROOFS

3.7.1 Proof of Proposition 4

We consider algorithms of the form

$$\theta_n = \theta_{n-1} + a\mathcal{H}\left(\theta_{n-1}, X_n\right) \tag{3.24}$$

$\theta_n \in \mathbb{R}^d, X_n \in \mathbb{R}^k$ with a starting point for θ_0. X_n is the vector of state variables. $\mathcal{H}(\cdot)$ is the functions describing the learning rule. Here n denotes discrete time so that we can use t below for continuous time.

We use Theorem 7.9 of Evans and Honkapohja (2001). Provided that the necessary assumptions of the theorem are satisfied the distribution of θ_n can be approximated, for small a and large n, by

$$\theta_t \backsim N(\theta^*, aC)$$

where θ^* is a globally asymptotically stable equilibrium point of the ODE $d\theta/dt = h(\theta)$, $h(\cdot)$ will be derived in a moment, and

$$C = \int_0^\infty e^{sB}\mathcal{R}\left(\theta^*\right)e^{sB'}ds$$

where $B = D_\theta h(\theta^*)$, $\mathcal{R}^{ij}(\theta) = \sum\limits_{k=-\infty}^{\infty} cov[\mathcal{H}^i(\theta, X_k^\theta), \mathcal{H}^j(\theta, X_0^\theta)]$.

We should derive the ordinary differential equation $d\theta/d\tau = h(\theta)$ first. The algorithm for updating γ_t is

$$\gamma_t = \gamma_{t-1} + aR_{t-1}^{-1}z_{t-1}(U_{t-1} - \gamma_{t-1}z_{t-1}) \tag{3.25}$$

$$R_t = R_{t-1} + a(z_{t-1}z_{t-1}' - R_{t-1}) \tag{3.26}$$

where $U_t = U^n - (\pi_t - w_t) + v_{1t}$, $z_t = \begin{pmatrix} 1 & \pi_t & w_t \end{pmatrix}'$ and $\gamma_t = \begin{pmatrix} \gamma_{0t} & \gamma_{1t} & \gamma_{2t} \end{pmatrix}'$, $\pi_t = \mu_t + v_{2t}$, $w_t = q_t + v_{3t}$, $\mu_t = \frac{-b(\gamma_1+\gamma_2)(\gamma_0-U^n+\alpha)}{1+b(\gamma_1+\gamma_2)^2}$, $q_t = \mu_t$.

The algorithm given in (3.25) and (3.26) is in the standard form of (3.24) when we define $\theta_t = \binom{\gamma_t}{vec(R_t)}$ and $X_t = \binom{z_{t-1}}{v_{1t}}$. The appropriate $\mathcal{H}$ function can be derived from (3.25) and (3.26). We can rewrite the algorithm in the following form

$$\gamma_t = \gamma_{t-1} + aR_{t-1}^{-1} z_{t-1}(U^n - (\pi_{t-1} - w_{t-1}) + v_{1t} - \gamma_0 - \gamma_1 \pi_{t-1} - \gamma_2 w_{t-1})$$

$$R_t = R_{t-1} + a(z_{t-1}z_{t-1}' - R_{t-1})$$

or

$$\gamma_t = \gamma_{t-1} + aR_{t-1}^{-1} z_{\gamma t-1} \left(\begin{array}{r} U^n - \gamma_0 - (\gamma_1+\gamma_2)\frac{-b(\gamma_1+\gamma_2)(\gamma_0 - U^n + \alpha)}{1+b(\gamma_1+\gamma_2)^2} + v_{1t-1}.. \\ .. - (1+\gamma_1)v_{2t-1} + (1-\gamma_2)v_{3t-1} \end{array} \right)$$

$$R_t = R_{t-1} + a(z_{t-1}z_{t-1}' - R_{t-1})$$

$$\gamma_t = \gamma_{t-1} + aR^{-1} \begin{bmatrix} 1 \\ \mu_{t-1} + v_{2t-1} \\ q_{t-1} + v_{3t-1} \end{bmatrix} \begin{bmatrix} U^n - \gamma_0 - (\gamma_1+\gamma_2)\frac{-b(\gamma_1+\gamma_2)(\gamma_0 - U^n + \alpha)}{1+b(\gamma_1+\gamma_2)^2} + v_{1t-1}.. \\ .. - (1+\gamma_1)v_{2t-1} + (1-\gamma_2)v_{3t-1} \end{bmatrix}$$

$$\dot{\gamma} = R^{-1} \begin{bmatrix} U^n - \gamma_0 - (\gamma_1+\gamma_2)\frac{-b(\gamma_1+\gamma_2)(\gamma_0 - U^n + \alpha)}{1+b(\gamma_1+\gamma_2)^2} \\ \mu_{t-1}\left(U^n - \gamma_0 - (\gamma_1+\gamma_2)\frac{-b(\gamma_1+\gamma_2)(\gamma_0 - U^n + \alpha)}{1+b(\gamma_1+\gamma_2)^2}\right) - (1+\gamma_1)\sigma_2^2 \\ q_{t-1}\left(U^n - \gamma_0 - (\gamma_1+\gamma_2)\frac{-b(\gamma_1+\gamma_2)(\gamma_0 - U^n + \alpha)}{1+b(\gamma_1+\gamma_2)^2}\right) + (1-\gamma_2)\sigma_3^2 \end{bmatrix} \tag{3.27}$$

There is a unique steady state of the differential equation (3.27) which is $\gamma^* = \begin{pmatrix} U^n & -1 & 1 \end{pmatrix}$. It is trivial to derive $h_R(\gamma, R)$ and its steady state. The steady state of $h(\theta)$ is a globally asymptotically stable equilibrium point since the eigenvalues of the 6×6 matrix $D_\theta h(\theta^*)$ have strictly negative real parts. Now we need to show that the assumptions of the theorem hold for our case.

Let $D = \{(\gamma, R) \mid \gamma \in \mathbb{R}^3, R \in (\zeta, \infty)^3\}$ for some fixed arbitrarily small $\zeta > 0$. Assume that z_t has support on some closed set and let $m^z = E(z_{t-1}z_{t-1}')$ be PSD. The polynomial bounds and Lipschitz conditions (A.2), (A.3) on $\mathcal{H}(\cdot)$ and $\partial\mathcal{H}/\partial X$ are met for compact sets

$Q \subset D$. Conditions (M.1)-(M.5) follow immediately from the assumptions that z_t and v_t are iid exogenous processes with bounded support. From the theorem of Coddington (1961, p. 248), it follows that $D_\theta h(\theta^*)$ is Lipschitz on D. The eigenvalues of $D_\theta h(\theta^*)$ are all negative which implies that θ^* is a globally asymptotically stable equilibrium point of the ODE. Hence assumptions (H.1)-(H.3) are met, the theorem applies to this case.

3.7.2 Proof of Proposition 5

We should derive the ordinary differential equation $d\theta/d\tau = h(\theta)$ and show that the unique steady state of this equation is globally asymptotically stable. The conditions of the theorem are similar to the first case.

For this case the adaptive system can be written in the form

$$\gamma_t = \gamma_{t-1} + aR_{t-1}^{-1} z_{t-1}(U_{t-1} - \gamma_{t-1} z_{t-1})$$

$$R_t = R_{t-1} + a(z_{t-1}z_{t-1}' - R_{t-1})$$

where $U_t = U^n - (\pi_t - w_t) + v_{1t}$, $z_t = \begin{pmatrix} 1 & \pi_t & w_t \end{pmatrix}'$ and $\gamma_t = \begin{pmatrix} \gamma_{0t} & \gamma_{1t} & \gamma_{2t} \end{pmatrix}'$, $\pi_t = \mu_t + v_{2t}$, $w_t = q_t + v_{3t}$, $\mu_t = \frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1 + b\gamma_1^2 + b\gamma_1\gamma_2}$, $q_t = \mu_t$.

We can write these equations as

$$\gamma_t = \gamma_{t-1} + aR_{t-1}^{-1} z_{t-1}(U^n - (\pi_{t-1} - w_{t-1}) + v_{1t} - \gamma_0 - \gamma_1\pi_{t-1} - \gamma_2 w_{t-1})$$

$$R_t = R_{t-1} + a(z_{t-1}z_{t-1}' - R_{t-1})$$

or

$$\gamma_t = \gamma_{t-1} + aR_{t-1}^{-1} z_{\gamma t-1} \begin{pmatrix} U^n - \gamma_0 - (\gamma_1 + \gamma_2)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1 + b\gamma_1^2 + b\gamma_1\gamma_2} + v_{1t-1} .. \\ .. - (1 + \gamma_1)v_{2t-1} + (1 - \gamma_2)v_{3t-1} \end{pmatrix}$$

$$R_t = R_{t-1} + a(z_{t-1}z_{t-1}' - R_{t-1})$$

$$\gamma_t = \gamma_{t-1} + aR^{-1} \begin{bmatrix} 1 \\ \mu_{t-1} + v_{2t-1} \\ q_{t-1} + v_{3t-1} \end{bmatrix} \begin{bmatrix} U^n - \gamma_0 - (\gamma_1 + \gamma_2) \frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1 + b\gamma_1^2 + b\gamma_1\gamma_2} + v_{1t-1} .. \\ .. - (1 + \gamma_1) v_{2t-1} + (1 - \gamma_2) v_{3t-1} \end{bmatrix}$$

$$\dot{\gamma} = R^{-1} \begin{bmatrix} U^n - \gamma_0 - (\gamma_1 + \gamma_2) \frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1 + b\gamma_1^2 + b\gamma_1\gamma_2} \\ \mu_{t-1} \left(U^n - \gamma_0 - (\gamma_1 + \gamma_2) \frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1 + b\gamma_1^2 + b\gamma_1\gamma_2} \right) - (1 + \gamma_1)\sigma_2^2 \\ q_{t-1} \left(U^n - \gamma_0 - (\gamma_1 + \gamma_2) \frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1 + b\gamma_1^2 + b\gamma_1\gamma_2} \right) + (1 - \gamma_2)\sigma_3^2 \end{bmatrix}$$

There is a unique steady state of this differential equation which is $\gamma = \begin{pmatrix} U^n & -1 & 1 \end{pmatrix}$. This steady state is a globally asymptotically stable equilibrium point of $h(\gamma, R)$ since the eigenvalues of $D_\theta h(\theta^*)$ have strictly negative real parts.

3.7.3 Proof of Proposition 6

We will show that the equilibrium point of the ordinary differential equation $d\theta/d\tau = h(\theta)$ is globally asymptotically stable. The other required conditions can be easily shown to be satisfied. The central bank is using 2 parameters, the private sector is using 3 parameters. Together with the adjustment matrices the system is represented by a 10×10 matrix. The eigenvalues of the matrix $B = D_\theta h(\theta^*)$ should have all negative real parts.

$$\gamma_t = \gamma_{t-1} + aR^{-1}_{\gamma t-1} z_{\gamma t-1}(U_{t-1} - \gamma_{t-1} z_{\gamma t-1})$$

$$R_{\gamma t} = R_{\gamma t-1} + a(z_{\gamma t-1} z'_{\gamma t-1} - R_{\gamma t-1})$$

$$\eta_t = \eta_{t-1} + aR^{-1}_{\eta t-1} z_{\eta t-1}(U_{t-1} - \eta_{t-1} z_{\eta t-1})$$

$$R_{\eta t} = R_{\eta t-1} + a(z_{\eta t-1} z'_{\eta t-1} - R_{\eta t-1})$$

where $U_t = U^n - (\pi_t - w_t) + v_{1t}$, $z_{\gamma t} = \begin{pmatrix} 1 & \pi_t \end{pmatrix}'$ and $\gamma_t = \begin{pmatrix} \gamma_{0t} & \gamma_{1t} \end{pmatrix}'$,$z_{\eta t} = \begin{pmatrix} 1 & \pi_t & w_t \end{pmatrix}'$ and $\eta_t = \begin{pmatrix} \eta_{0t} & \eta_{1t} & \eta_{2t} \end{pmatrix}'$, $\pi_t = \mu_t + v_{2t}$, $w_t = q_t + v_{3t}$, $\mu_t = \frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1 + b\gamma_1^2}$, $q_t = \frac{-b\eta_1(\eta_0 - U^n + \alpha)}{1 + b\eta_1^2 + b\eta_1\eta_2}$.

We can write these equations as

$$\gamma_t = \gamma_{t-1} + aR_{\gamma t-1}^{-1} z_{\gamma t-1}(U^n - (\pi_{t-1} - w_{t-1}) + v_{1t-1} - \gamma_0 - \gamma_1 \pi_{t-1})$$

$$R_{\gamma t} = R_{\gamma t-1} + a(z_{\gamma t-1} z'_{\gamma t-1} - R_{\gamma t-1})$$

$$\gamma_t = \gamma_{t-1} + aR_{\gamma t-1}^{-1} z_{\gamma t-1} \left(\begin{array}{r} U^n - \gamma_0 - (1+\gamma_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1+b\gamma_1^2} .. \\ .. + \frac{-b\eta_1(\eta_0 - U^n + \alpha)}{1+b\eta_1^2 + b\eta_1\eta_2} + v_{1t-1} - (1+\gamma_1) v_{2t-1} + v_{3t-1} \end{array} \right)$$

$$R_{\gamma t} = R_{\gamma t-1} + a(z_{\gamma t-1} z'_{\gamma t-1} - R_{\gamma t-1})$$

$$\gamma_t = \gamma_{t-1} + aR_{\gamma}^{-1} \left[\begin{array}{c} 1 \\ \mu_{t-1} + v_{2t-1} \end{array} \right] \left[\begin{array}{c} U^n - \gamma_0 - (1+\gamma_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1+b\gamma_1^2} .. \\ .. + \frac{-b\eta_1(\eta_0 - U^n + \alpha)}{1+b\eta_1^2 + b\eta_1\eta_2} + v_{1t-1} .. \\ .. - (1+\gamma_1) v_{2t-1} + v_{3t-1} \end{array} \right]$$

$$\dot{\gamma} = R_{\gamma}^{-1} \left[\begin{array}{c} U^n - \gamma_0 - (1+\gamma_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1+b\gamma_1^2} + \frac{-b\eta_1(\eta_0 - U^n + \alpha)}{1+b\eta_1^2 + b\eta_1\eta_2} \\ \mu_{t-1} \left(U^n - \gamma_0 - (1+\gamma_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1+b\gamma_1^2} + \frac{-b\eta_1(\eta_0 - U^n + \alpha)}{1+b\eta_1^2 + b\eta_1\eta_2} \right) - (1+\gamma_1)\sigma_2^2 \end{array} \right]$$

For the private sector:

$$\eta_t = \eta_{t-1} + aR_{\eta t-1}^{-1} z_{\eta t-1} \left(U^n - (\pi_{t-1} - w_{t-1}) + v_{1t-1} - \eta_0 - \eta_1 \pi_{t-1} - \eta_2 w_{t-1}\right)$$

$$R_{\eta t} = R_{\eta t-1} + a(z_{\eta t-1} z'_{\eta t-1} - R_{\eta t-1})$$

$$\eta_t = \eta_{t-1} + aR_{\eta t-1}^{-1} z_{\eta t-1} \left(\begin{array}{l} U^n - \eta_0 - (1+\eta_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1+b\gamma_1^2} .. \\ \quad .. + (1-\eta_2)\frac{-b\eta_1(\eta_0 - U^n + \alpha)}{1+b\eta_1^2 + b\eta_1\eta_2} + v_{1t-1} .. \\ \quad .. - (1+\eta_1) v_{2t-1} + (1-\eta_2) v_{3t-1} \end{array} \right)$$

$$\eta_t = \eta_{t-1} + aR_\eta^{-1} \begin{bmatrix} 1 \\ \mu_{t-1} + v_{2t-1} \\ q_{t-1} + v_{3t-1} \end{bmatrix} \begin{bmatrix} U^n - \eta_0 - (1+\eta_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha)}{1+b\gamma_1^2} .. \\ .. + (1-\eta_2)\frac{-b\eta_1(\eta_0 - U^n+\alpha)}{1+b\eta_1^2+b\eta_1\eta_2} .. \\ .. + v_{1t-1} - (1+\eta_1)v_{2t-1} + (1-\eta_2)v_{3t-1} \end{bmatrix}$$

$$\dot{\eta} = R_\eta^{-1} \begin{bmatrix} U^n - \eta_0 - (1+\eta_1)\frac{-b\gamma_1(\gamma_0-U^n+\alpha)}{1+b\gamma_1^2} + (1-\eta_2)\frac{-b\eta_1(\eta_0-U^n+\alpha)}{1+b\eta_1^2+b\eta_1\eta_2} \\ \mu_{t-1}\left(U^n - \eta_0 - (1+\eta_1)\frac{-b\gamma_1(\gamma_0-U^n+\alpha)}{1+b\gamma_1^2} + (1-\eta_2)\frac{-b\eta_1(\eta_0-U^n+\alpha)}{1+b\eta_1^2+b\eta_1\eta_2}\right) - (1+\eta_1)\sigma_2^2 \\ q_{t-1}\left(U^n - \eta_0 - (1+\eta_1)\frac{-b\gamma_1(\gamma_0-U^n+\alpha)}{1+b\gamma_1^2} + (1-\eta_2)\frac{-b\eta_1(\eta_0-U^n+\alpha)}{1+b\eta_1^2+b\eta_1\eta_2}\right) + (1-\eta_2)\sigma_3^2 \end{bmatrix}$$

There is a unique steady state of this differential equation, $\gamma = \begin{pmatrix} U^n + \alpha b & -1 \end{pmatrix}$, $\eta = \begin{pmatrix} U^n & -1 & 1 \end{pmatrix}$. The eigenvalues of $B = D_\theta h(\theta^*)$ for this steady state are all negative.

3.7.4 Proof of Proposition 7

We will show that the equilibrium point of the ordinary differential equation $d\theta/d\tau = h(\theta)$ is globally asymptotically stable. The other required conditions can be easily shown to be satisfied. The central bank and the private sector are using 3 parameters. Together with the adjustment matrices the system will be represented by a 12×12 matrix. The eigenvalues of the matrix $B = D_\theta h(\theta^*)$ should have all negative real parts.

$$\gamma_t = \gamma_{t-1} + aR_{\gamma t-1}^{-1} z_{\gamma t-1}(U_{t-1} - \gamma_{t-1} z_{\gamma t-1})$$

$$R_{\gamma t} = R_{\gamma t-1} + a(z_{\gamma t-1} z'_{\gamma t-1} - R_{\gamma t-1})$$

$$\eta_t = \eta_{t-1} + aR_{\eta t-1}^{-1} z_{\eta t-1}(U_{t-1} - \eta_{t-1} z_{\eta t-1})$$

$$R_{\eta t} = R_{\eta t-1} + a(z_{\eta t-1} z'_{\eta t-1} - R_{\eta t-1})$$

where $U_t = U^n - (\pi_t - w_t) + v_{1t}$, $z_{\gamma t} = \begin{pmatrix} 1 & \pi_t & w_t \end{pmatrix}'$ and $\gamma_t = \begin{pmatrix} \gamma_{0t} & \gamma_{1t} & \gamma_{2t} \end{pmatrix}'$, $z_{\eta t} = \begin{pmatrix} 1 & \pi_t & w_t \end{pmatrix}'$ and $\eta_t = \begin{pmatrix} \eta_{0t} & \eta_{1t} & \eta_{2t} \end{pmatrix}'$, $\pi_t = \mu_t + v_{2t}$, $w_t = q_t + v_{3t}$, $\mu_t = \frac{-b\gamma_1(\gamma_0 - U^n+\alpha) - b\gamma_1\gamma_2 q_t}{1+b\gamma_1^2}$ $q_t = \frac{-b(\eta_1+\eta_2)(\eta_0-U^n+\alpha)}{1+b(\eta_1+\eta_2)^2}$.

We can write these equations as

$$\gamma_t = \gamma_{t-1} + aR^{-1}_{\gamma t-1} z_{\gamma t-1}(U^n - (\pi_{t-1} - w_{t-1}) + v_{1t-1} - \gamma_0 - \gamma_1 \pi_{t-1} - \gamma_2 w_{t-1})$$

$$R_{\gamma t} = R_{\gamma t-1} + a(z_{\gamma t-1} z'_{\gamma t-1} - R_{\gamma t-1})$$

$$\gamma_t = \gamma_{t-1} + aR^{-1}_{\gamma t-1} z_{\gamma t-1} \begin{pmatrix} U^n - \gamma_0 - (1+\gamma_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_{t-1}}{1+b\gamma_1^2} .. \\ .. + (1-\gamma_2)\frac{-b(\eta_1+\eta_2)(\eta_0 - U^n + \alpha)}{1+b(\eta_1+\eta_2)^2} + v_{1t-1} .. \\ .. - (1+\gamma_1) v_{2t-1} + (1-\gamma_2) v_{3t-1} \end{pmatrix}$$

$$R_{\gamma t} = R_{\gamma t-1} + a(z_{\gamma t-1} z'_{\gamma t-1} - R_{\gamma t-1})$$

$$\gamma_t = \gamma_{t-1} + aR^{-1}_{\gamma} \begin{bmatrix} 1 \\ \mu_{t-1} + v_{2t-1} \\ q_{t-1} + v_{3t-1} \end{bmatrix} \begin{bmatrix} U^n - \gamma_0 - (1+\gamma_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_{t-1}}{1+b\gamma_1^2} .. \\ .. + (1-\gamma_2)\frac{-b(\eta_1+\eta_2)(\eta_0 - U^n + \alpha)}{1+b(\eta_1+\eta_2)^2} + v_{1t-1} .. \\ .. - (1+\gamma_1) v_{2t-1} + (1-\gamma_2) v_{3t-1} \end{bmatrix}$$

$$\dot{\gamma} = R^{-1}_{\gamma} \begin{bmatrix} U^n - \gamma_0 - (1+\gamma_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_{t-1}}{1+b\gamma_1^2} + (1-\gamma_2)\frac{-b(\eta_1+\eta_2)(\eta_0 - U^n + \alpha)}{1+b(\eta_1+\eta_2)^2} \\ \mu_{t-1}(U^n - \gamma_0 - (1+\gamma_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_{t-1}}{1+b\gamma_1^2} .. \\ .. + (1-\gamma_2)\frac{-b(\eta_1+\eta_2)(\eta_0 - U^n + \alpha)}{1+b(\eta_1+\eta_2)^2}) - (1+\gamma_1)\sigma_2^2 \\ q_{t-1}(U^n - \gamma_0 - (1+\gamma_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_{t-1}}{1+b\gamma_1^2} .. \\ .. + (1-\gamma_2)\frac{-b(\eta_1+\eta_2)(\eta_0 - U^n + \alpha)}{1+b(\eta_1+\eta_2)^2}) + (1-\gamma_2)\sigma_3^2 \end{bmatrix}$$

For the private sector:

$$\eta_t = \eta_{t-1} + aR^{-1}_{\eta t-1} z_{\eta t-1} (U^n - (\pi_{t-1} - w_{t-1}) + v_{1t-1} - \eta_0 - \eta_1 \pi_{t-1} - \eta_2 w_{t-1})$$

$$R_{\eta t} = R_{\eta t-1} + a(z_{\eta t-1} z'_{\eta t-1} - R_{\eta t-1})$$

$$\eta_t = \eta_{t-1} + aR^{-1}_{\eta t-1} z_{\eta t-1} \begin{pmatrix} U^n - \eta_0 - (1+\eta_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_{t-1}}{1+b\gamma_1^2} .. \\ .. + (1-\eta_2)\frac{-b(\eta_1+\eta_2)(\eta_0 - U^n + \alpha)}{1+b(\eta_1+\eta_2)^2} + v_{1t-1} .. \\ .. - (1+\eta_1)v_{2t-1} + (1-\eta_2)v_{3t-1} \end{pmatrix}$$

$$\eta_t = \eta_{t-1} + aR^{-1}_{\eta} \begin{bmatrix} 1 \\ \mu_{t-1} + v_{2t-1} \\ q_{t-1} + v_{3t-1} \end{bmatrix} \begin{bmatrix} U^n - \eta_0 - (1+\eta_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_{t-1}}{1+b\gamma_1^2} .. \\ .. + (1-\eta_2)\frac{-b(\eta_1+\eta_2)(\eta_0 - U^n + \alpha)}{1+b(\eta_1+\eta_2)^2} + v_{1t-1} .. \\ .. - (1+\eta_1)v_{2t-1} + (1-\eta_2)v_{3t-1} \end{bmatrix}$$

$$\dot{\eta} = R^{-1}_{\eta} \begin{bmatrix} U^n - \eta_0 - (1+\eta_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_{t-1}}{1+b\gamma_1^2} .. \\ .. + (1-\eta_2)\frac{-b(\eta_1+\eta_2)(\eta_0 - U^n + \alpha)}{1+b(\eta_1+\eta_2)^2} \\ \mu_{t-1}(U^n - \eta_0 - (1+\eta_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_{t-1}}{1+b\gamma_1^2} .. \\ .. + (1-\eta_2)\frac{-b(\eta_1+\eta_2)(\eta_0 - U^n + \alpha)}{1+b(\eta_1+\eta_2)^2}) - (1+\eta_1)\sigma_2^2 \\ q_{t-1}(U^n - \eta_0 - (1+\eta_1)\frac{-b\gamma_1(\gamma_0 - U^n + \alpha) - b\gamma_1\gamma_2 q_{t-1}}{1+b\gamma_1^2} .. \\ .. + (1-\eta_2)\frac{-b(\eta_1+\eta_2)(\eta_0 - U^n + \alpha)}{1+b(\eta_1+\eta_2)^2}) + (1-\eta_2)\sigma_3^2 \end{bmatrix}$$

There is a unique steady state of this differential equation, $\gamma = \begin{pmatrix} U^n & -1 & 1 \end{pmatrix}$, $\eta = \begin{pmatrix} U^n & -1 & 1 \end{pmatrix}$. The eigenvalues of $B = D_\theta h(\theta^*)$ for this steady state are all negative.

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4.0 THE CENTRAL BANKS' INFLUENCE ON PUBLIC EXPECTATION

4.1 INTRODUCTION

It is well known that the expectations of the public play an important role in the determination of the inflation rate. Central banks try to direct the expectations of the public by making announcements, releasing forecasts and explaining their policies. As an example to evidence of the Federal Reserve's attention to expectations, its chairman Bernanke recently said "Undoubtedly, the state of inflation expectations greatly influences actual inflation and thus the central bank's ability to achieve price stability"[1].

Understanding the relationship between policy actions and the formation of inflation expectations as well as determinants of the public's expectations of inflation is very important in monetary policy. This understanding may allow us to manipulate the public sector's inflation expectations to achieve better results which will be in the interest of any central bank. Besides open market operations, the central banks also engage in "open mouth operations". Public speeches, release of private information or reactions to unexpected market outcomes can be a considered as a way to influence inflation expectations with open mouth operations. So, do the expectations of the public follow the designation of the central bank? Probably "no". But then do these announcements have a point? Probably "yes".

In this paper we study the planned announcements of a central bank that tries to influence the public. This work is in parallel with the reform the FOMC is going through after the appointment of Mr. Bernanke which includes publishing more frequent policy forecasts and

[1] Inflation Expectations and Inflation Forecasting, at the Monetary Economics Workshop of the National Bureau of Economic Research Summer Institute, Cambridge, Massachusetts July 10, 2007

fuller statements explaining interest-rate policy.[2]

The paper is about central bank policies and announcements. It is known that the realization of the inflation rate depends on the expected value of inflation. By making regular announcements the central bank tries to influence the private sector to achieve better outcomes from its monetary policy.

New Keynesian models have been used extensively in the recent literature on monetary policy. These give rise to a Taylor-rule of optimal policy for the central bank under rational expectations[3]. Another positive side of this model is the central bank sets the nominal interest rate rather than the money supply which makes it more realistic than models with money supply setting. In this model, with rational expectations, announcements can have no effect on private sector expectations and add no information so they are effectively cheap talk. But assuming rational expectations for the private sector is a very strong assumption. Instead of assuming rational expectations for the private sector, alternatively, the private sector may learn the economy using a recursive least square type of learning (Berardi and Duffy (2007) and Evans and Honkapohja (2006)).

In this paper we include an announcement effect in the private sector's specification of the structural equation for the economy. Announcements do not matter, but the private sector does no know this, at least to begin with. The question is will they learn that announcements do not matter. If they do not learn the truth about announcements, this opens up the possibility of influencing private-sector expectations to achieve better results.

Previously Karaman (2007) or Berardi and Duffy (2007) considered models with the private sector learning a misspecified model that omits important factors. Here we consider what happens if the private sector includes an extanous factor in its specification. This specification of the economy is not a misspecification with an omission of some variables but a misspecification with an addition of an extranous variable.

In this paper we tweak the New Keynesian model to include another instrument, an announcement, to the central bank. With having this additional instrument the central bank has a role in open mouth operations besides its role in the open market operations.

[2] The Economist, March 23, 2006 "Bernanke ponders his course"

[3] See, e.g., Clarida, Gali and Gertler (1999) or Woodford (2003) for a complete exposition of this model and its micro-founded derivations.

With an additional instruments we expect the central bank to achieve better results. In this model the central bank first makes its announcement. Then the private sector observes these announcements and builds its expectations conditioning on the announcement. And finally, if the central bank has the discretion to do so, it revises its policy.

One important aspect of the model we used is the policy function of the central bank is a function of the private-sector expectations, unlike Walsh (1998). To analyze the effects of announcements Walsh (1998) linearizes the objective in the output gap to be able to derive a policy function free of private sector expectations. But in the model we use the policy of the central bank is still a function of the private sector expectations which is a more realistic assumption.

We would like to see how the central bank's announcements, which are essentially cheap talk, are taken by the private sector. Can the central bank manipulate the expectations of the private sector? Or equivalently, does the central bank have any credibility with the public? And will this change over time as the private sector learns about the economy?

4.2 MODEL

We first present the model and its solutions under two separate commitment technologies. Then we show the effects of the announcements on inflation expectations. The model is a New Keynesian model[4],

$$y_t = \widehat{E}y_{t+1} - \phi\left(r_t - \widehat{E}\pi_{t+1}\right) + g_t \tag{4.1}$$

$$\pi_t = \lambda y_t + \beta\widehat{E}\pi_{t+1} + u_t \tag{4.2}$$

$$v_t = (g_t, u_t)' = Fv_{t-1} + \epsilon_t$$

[4] We use a New Keynesian model where the realization of the state variables are dependant on inflation and output expectations. The model is developed by Clarida, Gali and Gertler (1999) as a science of monetary policy. But most of the papers that work on transparency and credibility avoid this paper and use variants of the model of Cukierman and Meltzer (1986). One reason why this model was avoided is the choice variable of the central bank (or the policy maker) is different than the variable which the expectations is taken. This brings some complications but with a new approach, we try to overcome this complication.

where $F = \begin{bmatrix} \mu & 0 \\ 0 & \rho \end{bmatrix}$, $\epsilon_t = (\epsilon_t^g, \epsilon_t^u)'$. π_t denotes the inflation rate, y_t the output level and $\widehat{E}$ is the private sector's expected inflation rate based on the previous period's information. $\widehat{E}$ does not necessarily represent rational expectations. y_t is the output gap. β is the discount factor. We assume that $\beta \in (0,1)$, $\lambda > 0$ and $\phi > 0$. The variables g_t and u_t represent demand and supply shocks respectively and it is assumed that $|\mu|, |\rho| \in [0,1)$, and $\epsilon_t^i \sim i.i.d.(0, \sigma_i^2)$, for $i = g, u$.

The objective of the central bank (CB) is to minimize its loss function

$$\min E_0 \sum_{t=0}^{\infty} \beta^t \mathcal{L}_t \tag{4.3}$$

where

$$\mathcal{L}_t = (\pi_t - \overline{\pi})^2 + \alpha (y_t - \overline{y})^2$$

4.2.1 Optimal Policy Under Discretion

We first consider the case where the CB cannot credibly manipulate beliefs in the absence of commitment. The CB takes private sector (PS) expectations as given in solving the optimization problem. Each period the CB chooses y and π to minimize

$$F_t = \min_{\{y_t, \pi_t\}} (\pi_t - \overline{\pi})^2 + \alpha (y_t - \overline{y})^2 + \widehat{E} F_{t+1}$$

subject to

$$\pi_t = \lambda y_t + \beta \widehat{E} \pi_{t+1} + u_t$$

taking as given $\widehat{E} F_{t+1}$, $\widehat{E}\pi_{t+1}$and u_t. Under discretion, future inflation and output are not affected by today's actions, and the CB cannot directly manipulate expectations. The first order condition from this minimization is

$$\lambda (\pi_t - \overline{\pi}) + \alpha (y_t - \overline{y}) = 0 \tag{4.4}$$

Using the optimality condition (4.4) in (4.2) we obtain a first order expectational difference equation for π_t

$$\pi_t = \frac{\lambda\left(\alpha\overline{y} + \lambda\overline{\pi}\right)}{\alpha + \lambda^2} + \frac{\alpha\beta}{\alpha + \lambda^2}\widehat{E}\pi_{t+1} + \frac{\alpha}{\alpha + \lambda^2}u_t \tag{4.5}$$

Using equations (4.4) and (4.5) we can obtain an expression for y_t:

$$y_t = \delta_1 - \delta_2\widehat{E}\pi_{t+1} - \delta_3 u_t \tag{4.6}$$

where $\delta_1 = \frac{\alpha\overline{y}+\lambda\overline{\pi}}{\alpha+\lambda^2}$, $\delta_2 = \frac{\lambda\beta}{\alpha+\lambda^2}$ and $\delta_3 = \frac{\lambda}{\alpha+\lambda^2}$

Finally, combining (4.6) and (4.1) we obtain the optimal interest rate target rule of the central bank:

$$r_t = -\frac{\alpha\overline{y} + \lambda\overline{\pi}}{\phi\left(\alpha + \lambda^2\right)} + \left(\frac{\lambda\beta}{\phi\left(\alpha + \lambda^2\right)} + 1\right)\widehat{E}\pi_{t+1} + \frac{1}{\phi}\widehat{E}y_{t+1} + \frac{1}{\phi}g_t + \frac{\lambda}{\phi\left(\alpha + \lambda^2\right)}u_t \tag{4.7}$$

Using equations (4.1), (4.2) and (4.7) we can write this system in a matrix form as:

$$x_t = A + B\widehat{E}x_{t+1} + Du_t \tag{4.8}$$

where $x_t = (\pi_t, y_t)'$, $A = \begin{pmatrix} \lambda\delta_1 \\ \delta_1 \end{pmatrix}$, $B = \begin{pmatrix} \beta - \lambda\delta_2 & 0 \\ -\delta_2 & 0 \end{pmatrix}$, $D = \begin{pmatrix} \alpha\lambda^{-1}\delta_3 \\ -\delta_3 \end{pmatrix}$

The steady state of the model under discretion is

$\pi_{ss} = \frac{\lambda\delta_1}{1-\beta+\lambda\delta_2}$ $\quad y_{ss} = \delta_1 - \frac{\lambda\delta_1\delta_2}{1-\beta+\lambda\delta_2}$

4.2.2 Optimal Policy Under Commitment

We now consider the case where the central bank can credibly commit to future policies. Thus, private sector expectations are not taken as given but are instead considered as variables that can be influenced to achieve policy objectives. Optimal monetary policy in this commitment case amounts to minimization of (4.3) subject to (4.2) holding in every period. The first order conditions from this minimization problem can be rearranged to yield

$$\lambda\left(\pi_t-\overline{\pi}\right)+\alpha\left(y_t-y_{t-1}\right)=0 \tag{4.9}$$

From (4.2) and (4.9) we get

$$y_t=\frac{\lambda}{\alpha+\lambda^2}\left(\overline{\pi}+\frac{\alpha}{\lambda}y_{t-1}-\beta\widehat{E}\pi_{t+1}-u_t\right)$$

which combined with (4.1) gives the policy rule

$$r_t=-\frac{\lambda\overline{\pi}}{\phi\left(\alpha+\lambda^2\right)}-\frac{\alpha}{\phi\left(\alpha+\lambda^2\right)}y_{t-1}+\left(\frac{\lambda\beta}{\phi\left(\alpha+\lambda^2\right)}+1\right)\widehat{E}\pi_{t+1}+\frac{1}{\phi}\widehat{E}y_{t+1}+\frac{1}{\phi}g_t+\frac{\lambda}{\phi\left(\alpha+\lambda^2\right)}u_t \tag{4.10}$$

Equations (4.1), (4.2) and (4.10) represent the economic system under commitment, given private sector expectations. We can rewrite this system in matrix form

$$x_t=A+B\widehat{E}x_{t+1}+Cx_{t-1}+Du_t \tag{4.11}$$

where $x_t=(\pi_t,y_t)'$ and $A=\begin{pmatrix}\frac{\lambda^2\overline{\pi}}{\alpha+\lambda^2}\\ \frac{\lambda\overline{\pi}}{\alpha+\lambda^2}\end{pmatrix}$, $B=\begin{pmatrix}\frac{\alpha\beta}{\alpha+\lambda^2} & 0\\ \frac{-\lambda\beta}{\alpha+\lambda^2} & 0\end{pmatrix}$, $C=\begin{pmatrix}0 & \frac{\alpha\lambda}{\alpha+\lambda^2}\\ 0 & \frac{\alpha}{\alpha+\lambda^2}\end{pmatrix}$, $D=\begin{pmatrix}\frac{\alpha}{\alpha+\lambda^2}\\ \frac{-\lambda}{\alpha+\lambda^2}\end{pmatrix}$

The steady state values for inflation and output are $\pi_{ss}=\overline{\pi}$, $y_{ss}=\frac{1-\beta}{\lambda}\overline{\pi}$ respectively.

4.2.3 Stages of the Game

In this model, in addition to its policy, r_t, the CB has another instrument, an announcement, i_{t+1}, which can be used to influence the private sector's formation of $\widehat{E}\pi_{t+1}$. These announcements can be in the form of a speech, release of forecasts or any communication and need not correspond to any actual economic variable.

First the CB makes its announcement, i_{t+1}. The CB makes this announcement every period. As derived later in the paper the announcement will be a function of some constants, the supply shock and part of PS expectations. The PS in this model does not have rational expectations but uses a simple learning model which structurally includes the announcments. It is known that the PS pays attention to the announcements. Effective or not, we assume that the PS considers the announcements by including them in its learning model. The CB is aware of the fact that the PS conditions on its announcement and therefore optimizes for the best announcement it can make[5]. Second, observing i_{t+1}, the PS builds their inflation expectations, $\widehat{E}\pi_{t+1}$, conditional on the announcement of the CB. And third, observing the expected rate of inflation, the CB makes a policy. The policy of the CB will depend on the commitment technology it has. Depending on this, it will either make its policy with (4.7) or with (4.10).

4.3 DYNAMICS

4.3.1 Expectational Stability

In this model the realization of the state variables depend on the expected future value of those variables. This means that we need to define how the expected value of these variables are determined. The strongest possible assumption, rational expectations, may not be reasonable for a dynamic world. We frequently observe shocks or sometimes structural changes which prevent agents from predicting the true inflation rate. A weaker assumption is to use learning models to see if agents can learn the rational expectations equilibrium over time. In the context of Evans and Honkapohja (2003) agents do not know the true parameters

[5] We also consider a case where the CB is using an ad-hoc rule for the announcement.

of the structural model but try to deduce them via regressions of past data. The current iteration of the regression is used to make decisions in each period. Next period when new information (data) is available to agents, they take another regression. If the model converges to the rational expectations equilibrium, the equilibrium is said to be e-stable (Evans and Honkapohja (2001)).

4.3.1.1 Stability Under Discretion It is known that (Evans and Honkapohja (2003) and Berardi and Duffy (2006)) the rational expectations equilibria of this model is determinate and expectationally stable if both eigenvalues of B lie inside the unit circle. The perceived law of motion used in their model is

$$\begin{aligned} \widehat{E}\pi_{t+1} &= \gamma_0^\pi + \gamma_1^\pi u_t \\ \widehat{E}y_{t+1} &= \gamma_0^y + \gamma_1^y u_t \end{aligned}$$

This is the minimal state variable (MSV) solution, following McCallum, the solution with the minimum number of variables.

Now consider the case where the PS learns the economy and it is potentially influenced by the announcements of the CB. The innovation of this paper is to add a extraneous instrument to the model and see if the CB can use this to influence the PS to achieve better results. Every period the CB makes an announcement, i_{t+1}, that influences the PS's expectation of the next period's inflation rate. The PS thinks that the announcement is only effecting the inflation rate (The formation of the expected value of output does not depend on the announcement). Assume that the agents do not know (4.8) but believe

$$\widehat{E}x_{t+1} = \Gamma_0 + \Gamma_1 u_t + \Gamma_2 i_{t+1} \tag{4.12}$$

where $\widehat{E}x_{t+1} = (\widehat{E}\pi_{t+1}, \widehat{E}y_{t+1})'$, $\Gamma_0 = \binom{\gamma_0^\pi}{\gamma_0^y}$, $\Gamma_1 = \binom{\gamma_1^\pi}{\gamma_1^y}$ and $\Gamma_2 = \binom{\gamma_2^\pi}{0}$ are the coefficient matrices to estimate. The announcement i_{t+1} can be a constant or a function of the supply shock u_t. Assume that i_{t+1} is in the form

$$i_{t+1} = \omega_0 + \omega_1 u_t + v_t \tag{4.13}$$

where $v_{t+1} = v_t + n_t$, $n_t \sim N(0, \sigma_n^2)$. The variable v_t in the announcement is a noise in the reception of the announcement by the PS. It is always possible to have misunderstandings or misinterpretations in the talk of the central banks. So the CB is choosing ω_0 and ω_1 but the realization of the effective announcement is dependent on the announcement noise, v_t. We will consider different ways of choosing ω_0 and ω_1 but we will keep the functional form of the announcement i_{t+1} the same. There are a couple of reasons for this. First of all when the CB optimizes for the best i_{t+1}, this optimization will be explained later, it derives the announcement in this functional form. Besides this, with this functional form the perceived law of motion reduces to the MSV solution which is desirable in the learning literature. Substituted into (4.12) and (4.8) we get

$$x_t = A + B\Gamma_0 + B\Gamma_2\omega_0 + (B\Gamma_1 + B\Gamma_2\omega_1 + D)\, u_t + B\Gamma_2 v_t$$

We iterate this one period forward and then apply the expectation operator to get the actual law of motion

$$\widehat{E}x_{t+1} = A + B\Gamma_0 + B\Gamma_2\omega_0 + (B\Gamma_1 + B\Gamma_2\omega_1 + D)\, \rho u_t + B\Gamma_2 v_t \tag{4.14}$$

The T-map is from (4.12) to (4.14). The announcement i_{t+1} in (4.12) can be a constant or a function of u_t. In the first case the e-stability condition of the system doesn't change where in the later case it may change. The steady state of this system is derived in appendix 4.6.1.

4.3.1.2 Stability Under Commitment The e-stability of the policy under commitment is shown by Evans and Honkapohja (2006). The private sector uses laws of motion for inflation and output that are specified as

$$\begin{aligned} \pi_t &= \gamma_0^\pi + \gamma_1^\pi u_t + \gamma_2^\pi y_{t-1} \\ y_t &= \gamma_0^y + \gamma_1^y u_t + \gamma_2^y y_{t-1} \end{aligned}$$

or in a compact form

$$x_t = \Gamma_0 + \Gamma_1 u_t + \Gamma_2 y_{t-1}$$

We assume that the learning equation of the PS is

$$\widehat{E}x_{t+1} = \Gamma_0 + \Gamma_1 u_t + \Gamma_2 x_{t-1} + \Gamma_3 i_{t+1}{}^{6} \tag{4.15}$$

where i_{t+1} is the announcement of the CB and $\Gamma_0, \Gamma_1 : 2\times1$, $\Gamma_2 = \begin{pmatrix} 0 & \gamma_2^{\pi} \\ 0 & \gamma_2^{y} \end{pmatrix}$, $\Gamma_3 = \begin{pmatrix} \gamma_3^{\pi} \\ 0 \end{pmatrix}$. This form is in the MSV form if the announcement is in the following form

$$i_{t+1} = \omega_0 + \omega_1 u_t + \omega_2 y_{t-1} + v_t \tag{4.16}$$

We included the lagged value of the output for the announcement under commitment. Again this functional form of the announcement under commitment is derived in the optimization of the CB. Please see appendix 4.6.3 for this derivation.

Substitute (4.16) into (4.15) to get the PLM

$$\widehat{E}x_{t+1} = \Gamma_0 + \Gamma_3\omega_0 + (\Gamma_1 + \Gamma_3\omega_1) u_t + Lx_{t-1} + \Gamma_3 v_t \tag{4.17}$$

where $Lx_{t-1} = \Gamma_3\omega_2 y_{t-1} + \Gamma_2 x_{t-1}$. Therefore L is a two by two matrix. Substitute (4.17) into (4.11) to get

$$x_t = A + B\left[\Gamma_0 + \Gamma_3\omega_0 + (\Gamma_1 + \Gamma_3\omega_1) u_t + Lx_{t-1} + \Gamma_3 v_t\right] + Cx_{t-1} + Du_t$$

$$x_t = A + B\Gamma_0 + B\Gamma_3\omega_0 + (B\Gamma_1 + B\Gamma_3\omega_1 + D) u_t + (BL + C) x_{t-1} + B\Gamma_3 v_t$$

Iterate one period forward and take the expectation to get

$$\widehat{E}x_{t+1} = A + B\Gamma_0 + B\Gamma_3\omega_0 + (B\Gamma_1 + B\Gamma_3\omega_1 + D) \rho u_t + (BL + C) x_t + B\Gamma_3 v_t$$

Substitute (4.11) and (4.17) into the previous equation to get

$$\begin{aligned} \widehat{E}x_{t+1} &= A + B\Gamma_0 + B\Gamma_3\omega_0 + (B\Gamma_1 + B\Gamma_3\omega_1 + D) \rho u_t \\ &\quad + (BL + C)\left(A + B\widehat{E}x_{t+1} + Cx_{t-1} + Du_t\right) + B\Gamma_3 v_t \end{aligned}$$

[6] There is a difference in which variable is learned, x_{t+1} or $\widehat{E}x_{t+1}$. But our results do not change with whichever is used.

$$
\begin{aligned}
\widehat{E}x_{t+1} &= A + B\Gamma_0 + B\Gamma_3\omega_0 + (BL+C)\,A \\
&\quad + \left[(B\Gamma_1 + B\Gamma_3\omega_1 + D)\,\rho + (BL+C)\,D\right] u_t \\
&\quad + (BL+C)\,Cx_{t-1} + (BL+C)\,B\widehat{E}x_{t+1} + B\Gamma_3 v_t
\end{aligned}
$$

$$
\begin{aligned}
\widehat{E}x_{t+1} &= A + B\Gamma_0 + B\Gamma_3\omega_0 + (BL+C)\,A \\
&\quad + \left[(B\Gamma_1 + B\Gamma_3\omega_1 + D)\,\rho + (BL+C)\,D\right] u_t \\
&\quad + (BL+C)\,Cx_{t-1} \\
&\quad + (BL+C)\,B\left[\Gamma_0 + \Gamma_3\omega_0 + (\Gamma_1 + \Gamma_3\omega_1)\,u_t + Lx_{t-1} + \Gamma_3 v_t\right] + B\Gamma_3 v_t
\end{aligned}
$$

$$
\begin{aligned}
\widehat{E}x_{t+1} &= A + B\Gamma_0 + B\Gamma_3\omega_0 + (BL+C)\,A + (BL+C)\,B\Gamma_0 + (BL+C)\,B\Gamma_3\omega_0 \\
&\quad + \left[(B\Gamma_1 + B\Gamma_3\omega_1 + D)\,\rho + (BL+C)\,D + (BL+C)\,B\,(\Gamma_1 + \Gamma_3\omega_1)\right] u_t \\
&\quad + \left[(BL+C)\,C + (BL+C)\,BL\right] x_{t-1} \\
&\quad + \left[(BL+C)\,B\Gamma_3 + B\Gamma_3\right] v_t
\end{aligned}
$$

The last equation is the ALM. The steady states of this system is derived in appendix 4.6.2.

4.4 DETERMINATION OF THE ANNOUNCEMENT, I_{T+1}

At the beginning of the period the CB makes its announcement. We only consider announcements in the form of (4.13). With this functional form the CB is choosing a pair of values for ω_0 and ω_1. After this choice v_t is realized and the PS observes i_{t+1}. We present a few different ways of making the announcement. First we consider the case where the CB is using an ad-hoc rule to choose the announcement. Even though the rule is fixed, it is a function of the supply shock. Then we consider the cases where the CB optimizes for the best possible announcement. In the first optimized case the CB has full knowledge of how the PS forms its expectations. The PS is learning the expected rate of inflation using (4.12). So full information corresponds to knowing all Γ values of this learning equation. In the following case the CB does not know how much its announcements will be weighted by the PS, or does not know the coefficient γ_2^π. And in the last case the CB is using its own model to estimate the PS's expected inflation rate. In this case we use two-sided learning introduced by Karaman (2007).

4.4.1 Ad-hoc Announcement Rule

Suppose the CB chooses an arbitrary announcement rule where ω_0 and ω_1 are just constants, for example

$$i_{t+1} = a + bu_t$$

With this rule the CB aims to offset the effects of the supply shock.

For the simulations of the paper we would like to use the calibration of McCallum and Nelson (1999).

Calibration	β	ϕ	λ	α
McCallum and Nelson (1999)	0.99	0.164	0.3	0.5

We also assume that $\overline{\pi} = 2$, $\overline{y} = 2$, $\rho = \mu = 0.35$. With these parameter values the steady state values of inflation rate and output gap under discretion and under commitment are

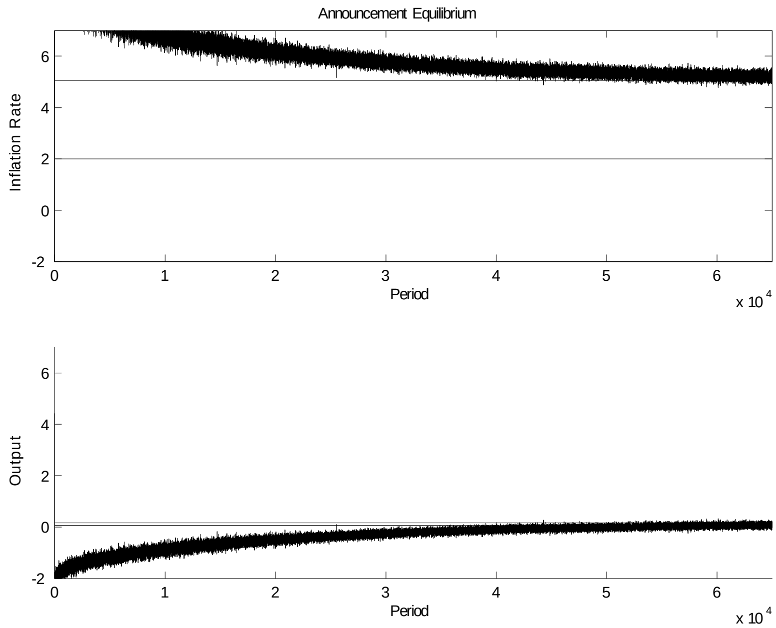

Figure 8: Inflation rate and output gap with an ad-hoc announcement rule, $i_{t+1} = 2 + 0.9u_t$

	Inflation	Output
Under Discretion	5.053	0.168
Under Commitment	2	0.067

The horizontal lines in the graphs represent the inflation and output levels under discretion and under commitment. In the output section of the graphs, the two horizontal lines almost coincide since the output values under discretion and under commitment are very close to each other.

As it can be seen in figure (8), the inflation rate and output gave converge to their steady state values. The coefficient on the announcement, γ_3^π, converges to 0. The agent learns to ignore the announcements but this takes some time.

4.4.2 Optimized Announcement Rules

For the following announcement rules (next three subsections) the CB determines the announcement with an optimization. In these optimizations (in the first stage) the CB does not ignore the fact that it can influence the PS expectations. In other words, it does not take the PS expectations as given. But in the third stage after the PS expectations are formed, the CB takes PS expectations as given.

4.4.2.1 Full Information Case Rather than using an ad hoc rule, the CB may optimize to get the best announcement level. This announcement is a second instrument to the CB to achieve its inflation and output goals. First with making the announcement the CB tries to influence the expectations of the PS. After the PS builds its expectations the CB makes its policy. To determine the optimal announcement level at first stage we need to work backward from the final stage of the game. Given the announcement level (4.13) and the PS expectations (4.14), the CB makes its policy according to (4.7). Then the CB minimizes the loss function (4.3) with respect to the announcement level, i_{t+1}. We assume there is no announcement for the output gap. The minimization problem is

$$\min_{\{i_{t+1}\}} \mathcal{L}_t = E\left\{(\pi_t - \overline{\pi})^2 + \alpha\,(y_t - \overline{y})^2\right\}$$

subject to

$$\begin{aligned}
\pi_t - \lambda y_t - \beta \widehat{E}\pi_{t+1} - u_t &= 0 \\
y_t - \widehat{E}y_{t+1} + \phi\left(r_t - \widehat{E}\pi_{t+1}\right) - g_t &= 0 \\
\widehat{E}x_{t+1} - \Gamma_0 - \Gamma_1 u_t - \Gamma_2 i_{t+1} &= 0 \\
r_t - \zeta_0 - \zeta_1 \widehat{E}\pi_{t+1} - \zeta_2 \widehat{E}y_{t+1} - \zeta_3 g_t - \zeta_4 u_t &= 0
\end{aligned}$$

The first order condition simplifies to

$$\overline{\pi} - \lambda \overline{y} - \beta \widehat{E}\pi_{t+1} - u_t = 0 \tag{4.18}$$

Therefore we get

$$i_{t+1} = \frac{1}{\gamma_2^{\pi}}\left(\frac{\overline{\pi} - \lambda\overline{y} - u_t}{\beta} - \gamma_0^{\pi} - \gamma_1^{\pi} u_t\right) \tag{4.19}$$

which implies $\omega_0 = \frac{1}{\gamma_2^{\pi}}\left(\frac{\overline{\pi}-\lambda\overline{y}}{\beta} - \gamma_0^{\pi}\right)$ and $\omega_1 = \frac{-1}{\gamma_2^{\pi}}\left(\frac{1}{\beta} + \gamma_1^{\pi}\right)$. The first order condition given in (4.18) implies that the CB will set an announcement to achieve its inflation and output targets, $\overline{\pi}$ and $\overline{y}$. Since the supply shock and the γ are available to the CB when determining the announcement level, it is able to set $\widehat{E}\pi_{t+1}$. This might look like a very strong assumption. So we also tried some other ways of determining the announcement.

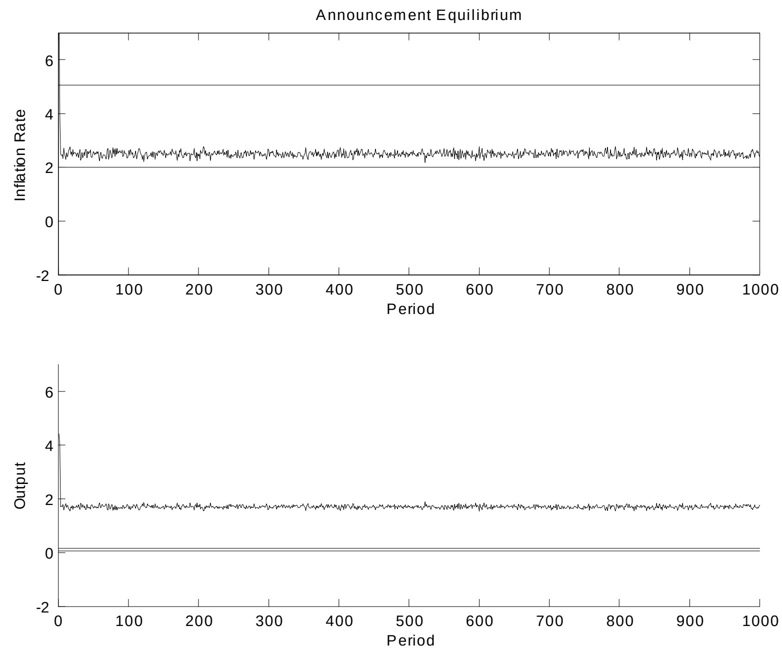

Figure 9: Inflation rate and output level with optimized announcement

Figure (9) is a simulation of the dynamics. With optimized announcements the CB is making the necessary announcements to keep the inflation rate stable around 2.5 which is below the Nash equilibirum value but above the equilibrium value under commitment. In this case the expected value of output converges to 2, which is the target value.

Discussion

Adding a second instrument, with the full knowledge of the formation of expected inflation rate, allows the CB to set the expected inflation rate. This looks like a very strong assumption. Therefore we would like to see what happens when the CB does not have full knowledge of the PS expectation formation.

4.4.2.2 Announcement with Incomplete Information Assume that the CB is aware of the fact that announcements matter but doesn't know to what extent they matter. In the previous section we assumed that the CB observes all the γ values which is a strong assumption. This time we assume that the CB believes in the following equation

$$\widehat{E}\pi_{t+1} = \gamma_0^\pi + \gamma_1^\pi u_t + ci_{t+1} \tag{4.20}$$

where c is constant. This means that the CB believes that its announcement i_{t+1} will be weighted by the PS but does not know the true value of this weight. The CB makes policy as if the PS is using (4.20) when setting its expectations. But the PS is using the model given in (4.12). This means that even if the CB announces and behaves as if its announcements will be completely considered (the coefficient of i_{t+1} will be taken to be unity), the PS evaluates the effectiveness of these announcements by using equation (4.12). Pay attention to the fact that the CB is using the same constant and supply shock coefficients here, γ_0^π and γ_1^π respectively. In the next section we will consider the case where the CB is using its own estimates for these coefficients: η_0^π and η_1^π. Assume that the CB uses the same minimization given above and derives (4.18). Therefore the announcement level is given by

$$i_{t+1} = \frac{1}{c}\left(\frac{\overline{\pi} - \lambda\overline{y}}{\beta} - \gamma_0^\pi\right) - \frac{1}{c}\left(\frac{1}{\beta} + \gamma_1^\pi\right) u_t$$

Figure (10) is a simulation of the system.

The inflation rate and the output gap are converging to their Nash equilibrium values. This means that the CB is not able influence the PS permanently: PS learns to discount the announcements.

4.4.2.3 Announcement with Incomplete Information, Two-Sided Learning Case

Assume that the CB uses its own model to estimate (4.20). The structure of the CB's learning function as follows

$$\widehat{E}\pi_{t+1} = \eta_0 + \eta_1 u_t + ci_{t+1}$$

Instead of using the estimates of the PS, the CB is using its own learning model. We use the letter η for the CB's estimate. Together with (4.18) the CB derives the following announcement

$$i_{t+1} = \frac{1}{c}\left(\frac{\overline{\pi} - \lambda\overline{y}}{\beta} - \eta_0\right) - \frac{1}{c}\left(\frac{1}{\beta} + \eta_1\right) u_t$$

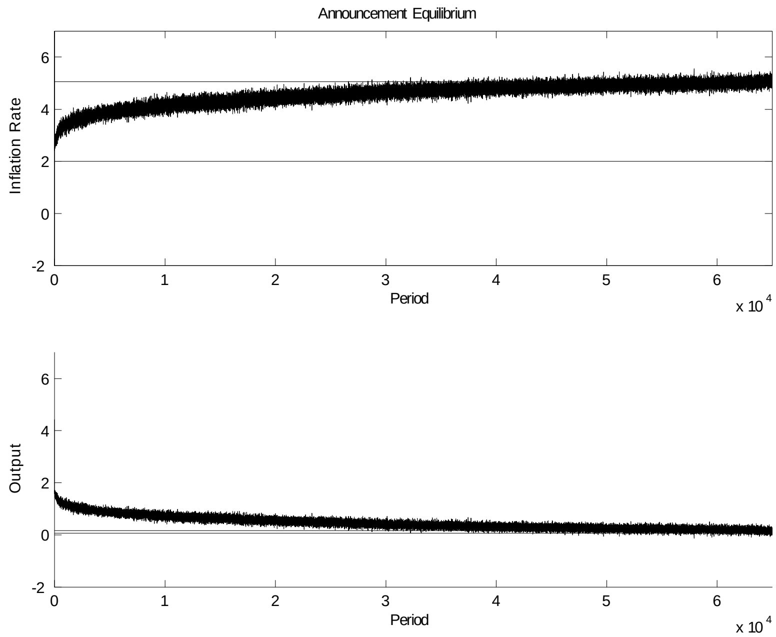

Figure 10: The inflation rate and output gap when the CB has credibility concerns

We observe convergence to the discretion equilibrium. Again the CB is not able to influence the PS permanently. The next proposition is the summary of the first section.

Proposition 8. *The central bank cannot influence the private sector inflation expectations to achieve lower than Nash equilibrium level of inflation rate and output gap unless it has the full knowledge of formation of the private sector inflation expectations. Using an extraneous instrument in addition to its policy instrument does not help the central bank to achieve better results.*

4.5 CONCLUSION

Expectations of future variables are very important when making monetary policy. This is the most important reason why Economics is not simply a field of engineering. You cannot engineer the economy as you engineer a building or an electrical circuit, for they are governed by known deterministic laws. Economics deals with human behavior, which

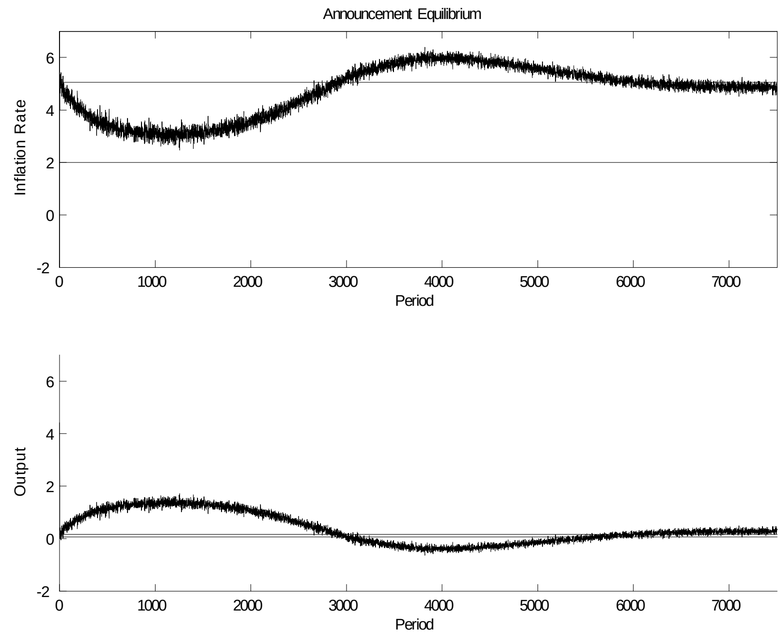

Figure 11: The inflation rate and the output gap when there is 2-sided learning

is often compared with chaotic behavior. That is why considering inflation expectations is very important. In this paper we explore different ways of influencing people.

In this paper we show that it is not possible to influence the people unless you have perfect knowledge of their behavior. But even if you do not have that perfect knowledge, which method you use matters in terms of speed of convergence to the Nash equilibrium.

We would like to extend this work by using a Kalman filter to determine the expected inflation. There has been much work on central bank policies and for sure there will be much more coming. The way monetary policy is made is much different than how it was made 30 years ago and it will be different 30 years from now too.

4.6 STEADY STATES

4.6.1 Steady States of the Discretion Case

The T-map is from (4.12) to (4.14). Assume that the announcement is made in the form (4.13). With this announcement the perceived law of motion becomes to

$$\widehat{E}x_{t+1} = \Gamma_0 + \Gamma_2\omega_0 + (\Gamma_1 + \Gamma_2\omega_1)\, u_t + \Gamma_2 v_t \tag{4.21}$$

From the T-map we get the following equalities.

$$\begin{aligned}
\gamma_0^\pi + \gamma_2^\pi\omega_0 &= \lambda\delta_1 + (\beta - \lambda\delta_2)\,(\gamma_0^\pi + \gamma_2^\pi\omega_0) \\
\gamma_0^y &= \delta_1 - \delta_2\,(\gamma_0^\pi + \gamma_2^\pi\omega_0) \\
\gamma_1^\pi + \gamma_2^\pi\omega_1 &= \big((\beta - \lambda\delta_2)\,(\gamma_1^\pi + \gamma_2^\pi\omega_1) + \alpha\lambda^{-1}\delta_3\big)\,\rho \\
\gamma_1^y &= (-\delta_2\,(\gamma_1^\pi + \gamma_2^\pi\omega_1) - \delta_3)\,\rho \\
\gamma_2^\pi &= (\beta - \lambda\delta_2)\,\gamma_2^\pi
\end{aligned}$$

From the fifth equation we get $\gamma_2^\pi = 0$ which implies

$$\begin{aligned}
\gamma_0^\pi &= \frac{\lambda\delta_1}{1 - \beta + \lambda\delta_2} \\
\gamma_1^\pi &= \frac{\rho\alpha\lambda^{-1}\delta_3}{1 - \rho\,(\beta - \lambda\delta_2)}
\end{aligned}$$

These steady state values are no different than case with no announcement.

4.6.2 The Steady State Under Commitment

$$x_t = A + B\widehat{E}x_{t+1} + Cx_{t-1} + Du_t \tag{4.22}$$

where $x_t = (\pi_t, y_t)'$ and $A = \begin{pmatrix} \frac{\lambda^2\overline{\pi}}{\alpha+\lambda^2} \\ \frac{\lambda\overline{\pi}}{\alpha+\lambda^2} \end{pmatrix}$, $B = \begin{pmatrix} \frac{\alpha\beta}{\alpha+\lambda^2} & 0 \\ \frac{-\lambda\beta}{\alpha+\lambda^2} & 0 \end{pmatrix}$, $C = \begin{pmatrix} 0 & \frac{\alpha\lambda}{\alpha+\lambda^2} \\ 0 & \frac{\alpha}{\alpha+\lambda^2} \end{pmatrix}$, $D = \begin{pmatrix} \frac{\alpha}{\alpha+\lambda^2} \\ \frac{-\lambda}{\alpha+\lambda^2} \end{pmatrix}$.

We assume that the learning equation of the PS is

$$\widehat{E}x_{t+1} = \Gamma_0 + \Gamma_1 u_t + \Gamma_2 x_{t-1} + \Gamma_3 i_{t+1} \tag{4.23}$$

where i_{t+1} is the announcement of the CB and $\Gamma_0, \Gamma_1 : 2\times 1$, $\Gamma_2 = \begin{pmatrix} 0 & \gamma_2^\pi \\ 0 & \gamma_2^y \end{pmatrix}$, $\Gamma_3 = \begin{pmatrix} \gamma_3^\pi \\ 0 \end{pmatrix}$. This form is in the MSV form if the announcement is in the following form

$$i_{t+1} = \omega_0 + \omega_1 u_t + \omega_2 y_{t-1} + v_t \tag{4.24}$$

We included the lagged value of the output for the announcement under commitment.

Substitute (4.24) into (4.23) to get the PLM

$$\widehat{E}x_{t+1} = \Gamma_0 + \Gamma_3\omega_0 + (\Gamma_1 + \Gamma_3\omega_1) u_t + Lx_{t-1} + \Gamma_3 v_t \tag{4.25}$$

where $Lx_{t-1} = \Gamma_3\omega_2 y_{t-1} + \Gamma_2 x_{t-1}$. Therefore L is a two by two matrix. Substitute (4.25) into (4.22) to get

$$x_t = A + B\left[\Gamma_0 + \Gamma_3\omega_0 + (\Gamma_1 + \Gamma_3\omega_1) u_t + Lx_{t-1} + \Gamma_3 v_t\right] + Cx_{t-1} + Du_t$$

$$x_t = A + B\Gamma_0 + B\Gamma_3\omega_0 + (B\Gamma_1 + B\Gamma_3\omega_1 + D) u_t + (BL + C) x_{t-1} + B\Gamma_3 v_t$$

Iterate one period forward and take the expectation to get

$$\widehat{E}x_{t+1} = A + B\Gamma_0 + B\Gamma_3\omega_0 + (B\Gamma_1 + B\Gamma_3\omega_1 + D) \rho u_t + (BL + C) x_t + B\Gamma_3 v_t$$

Substitute (4.22) and (4.25) into the previous equation to get

$$\begin{aligned}\widehat{E}x_{t+1} &= A + B\Gamma_0 + B\Gamma_3\omega_0 + (B\Gamma_1 + B\Gamma_3\omega_1 + D)\,\rho u_t \\ &\quad + (BL + C)\left(A + B\widehat{E}x_{t+1} + Cx_{t-1} + Du_t\right) + B\Gamma_3 v_t\end{aligned}$$

$$\begin{aligned}\widehat{E}x_{t+1} &= A + B\Gamma_0 + B\Gamma_3\omega_0 + (BL + C)\,A \\ &\quad + \left[(B\Gamma_1 + B\Gamma_3\omega_1 + D)\,\rho + (BL + C)\,D\right]u_t \\ &\quad + (BL + C)\,Cx_{t-1} + (BL + C)\,B\widehat{E}x_{t+1} + B\Gamma_3 v_t\end{aligned}$$

$$\begin{aligned}\widehat{E}x_{t+1} &= A + B\Gamma_0 + B\Gamma_3\omega_0 + (BL + C)\,A \\ &\quad + \left[(B\Gamma_1 + B\Gamma_3\omega_1 + D)\,\rho + (BL + C)\,D\right]u_t \\ &\quad + (BL + C)\,Cx_{t-1} \\ &\quad + (BL + C)\,B\left[\Gamma_0 + \Gamma_3\omega_0 + (\Gamma_1 + \Gamma_3\omega_1)\,u_t + Lx_{t-1} + \Gamma_3 v_t\right] + B\Gamma_3 v_t\end{aligned}$$

$$\begin{aligned}\widehat{E}x_{t+1} &= A + B\Gamma_0 + B\Gamma_3\omega_0 + (BL + C)\,A + (BL + C)\,B\Gamma_0 + (BL + C)\,B\Gamma_3\omega_0 \\ &\quad + \left[(B\Gamma_1 + B\Gamma_3\omega_1 + D)\,\rho + (BL + C)\,D + (BL + C)\,B\,(\Gamma_1 + \Gamma_3\omega_1)\right]u_t \\ &\quad + \left[(BL + C)\,C + (BL + C)\,BL\right]x_{t-1} \\ &\quad + \left[(BL + C)\,B\Gamma_3 + B\Gamma_3\right]v_t\end{aligned}$$

The last equation is the ALM. For a better reading we write it again in the following form

$$\begin{aligned}\widehat{E}x_{t+1} &= A + (BL + C)\,A + B\,(\Gamma_0 + \Gamma_3\omega_0) + (BL + C)\,B\,(\Gamma_0 + \Gamma_3\omega_0) \\ &\quad + \left[(B\,(\Gamma_1 + \Gamma_3\omega_1) + D)\,\rho + (BL + C)\,D + (BL + C)\,B\,(\Gamma_1 + \Gamma_3\omega_1)\right]u_t \\ &\quad + (BL + C)\,(BL + C)\,x_{t-1} \\ &\quad + \left[(BL + C)\,B\Gamma_3 + B\Gamma_3\right]v_t\end{aligned}$$

The steady states can found from the following equalities

$\Gamma_0 + \Gamma_3\omega_0 = A + (BL + C)\, A + B\, (\Gamma_0 + \Gamma_3\omega_0) + (BL + C)\, B\, (\Gamma_0 + \Gamma_3\omega_0)$

$\Gamma_1 + \Gamma_3\omega_1 = (B\, (\Gamma_1 + \Gamma_3\omega_1) + D)\, \rho + (BL + C)\, D + (BL + C)\, B\, (\Gamma_1 + \Gamma_3\omega_1)$

$L = (BL + C)\, (BL + C)$

$\Gamma_3 = (BL + C)\, B\Gamma_3 + B\Gamma_3$

The last equation implies $\gamma_3^\pi = 0$. Then we get the value of γ_2^π and γ_2^y from the third equation, γ_1^π and γ_1^y from the second equation, γ_0^π and γ_0^y from the first equation.

4.6.3 Determination of the Announcement Under Commitment

$$\min_{\{i_{t+1}\}} \mathcal{L}_t = E\left\{(\pi_t - \overline{\pi})^2 + \alpha\,(y_t - \overline{y})^2 + \beta\,(\pi_{t+1} - \overline{\pi})^2 + \alpha\beta\,(y_{t+1} - \overline{y})^2\right\}$$

subject to

$$\begin{aligned}
\pi_t &= \lambda y_t + \beta \widehat{E}\pi_{t+1} + u_t \\
y_t &= \widehat{E}y_{t+1} - \phi r_t + \phi \widehat{E}\pi_{t+1} + g_t \\
\pi_{t+1} &= \lambda y_{t+1} + \beta \widehat{E}\pi_{t+2} + u_{t+1} \\
y_{t+1} &= \widehat{E}y_{t+2} - \phi r_{t+1} + \phi \widehat{E}\pi_{t+2} + g_{t+1} \\
r_t &= \zeta_0 + \zeta_1 y_{t-1} + \zeta_2 \widehat{E}\pi_{t+1} + \zeta_3 \widehat{E}y_{t+1} + \zeta_4 g_t + \zeta_5 u_t \\
r_{t+1} &= \zeta_0 + \zeta_1 y_t + \zeta_2 \widehat{E}\pi_{t+2} + \zeta_3 \widehat{E}y_{t+2} + \zeta_4 g_{t+1} + \zeta_5 u_{t+1} \\
\widehat{E}x_{t+1} &= \Gamma_0 + \Gamma_1 u_t + \Gamma_2 x_{t-1} + \Gamma_3 i_{t+1}
\end{aligned}$$

where $\zeta_0 = \frac{-\lambda\overline{\pi}}{\phi(\alpha+\lambda^2)}$, $\zeta_1 = \frac{-\alpha}{\phi(\alpha+\lambda^2)}$, $\zeta_2 = \frac{\lambda\beta}{\phi(\alpha+\lambda^2)} + 1$, $\zeta_3 = \frac{1}{\phi}$, $\zeta_4 = \frac{1}{\phi}$, $\zeta_5 = \frac{\lambda}{\phi(\alpha+\lambda^2)}$, Γ_0, $\Gamma_1 : 2 \times 1$, $\Gamma_2 = \begin{pmatrix} 0 & \gamma_2^\pi \\ 0 & \gamma_2^y \end{pmatrix}$, $\Gamma_3 = \begin{pmatrix} \gamma_3^\pi \\ 0 \end{pmatrix}$

The first order condition of this minimization is

$$(\pi_t - \overline{\pi})\frac{\partial \pi_t}{\partial i_{t+1}} + (y_t - \overline{y})\,\alpha\frac{\partial y_t}{\partial i_{t+1}} + (\pi_{t+1} - \overline{\pi})\,\beta\frac{\partial \pi_{t+1}}{\partial i_{t+1}} + (y_{t+1} - \overline{y})\,\alpha\beta\frac{\partial y_{t+1}}{\partial i_{t+1}} = 0$$

$\frac{\partial y_t}{\partial i_{t+1}} = -\phi\frac{\partial r_t}{\partial i_{t+1}} + \phi\gamma_3^\pi = -\phi\varsigma_2\gamma_3^\pi + \phi\gamma_3^\pi = \phi\gamma_3^\pi\,(1 - \varsigma_2)$

$\frac{\partial \pi_t}{\partial i_{t+1}} = \lambda\frac{\partial y_t}{\partial i_{t+1}} + \beta\gamma_3^\pi = \lambda\phi\gamma_3^\pi\,(1 - \varsigma_2) + \beta\gamma_3^\pi$

$\frac{\partial y_{t+1}}{\partial i_{t+1}} = -\phi \frac{\partial r_{t+1}}{\partial i_{t+1}} = -\phi \varsigma_1 \frac{\partial y_t}{\partial i_{t+1}} = -\phi^2 \varsigma_1 \gamma_3^{\pi} (1 - \varsigma_2)$
$\frac{\partial \pi_{t+1}}{\partial i_{t+1}} = \lambda \frac{\partial y_{t+1}}{\partial i_{t+1}} = -\lambda \phi^2 \varsigma_1 \gamma_3^{\pi} (1 - \varsigma_2)$
$1 - \varsigma_2 = \frac{-\lambda\beta}{\phi\left(\alpha+\lambda^2\right)}$, $\phi (1 - \varsigma_2) = \frac{-\lambda\beta}{\alpha+\lambda^2}$
$\lambda\phi (1 - \varsigma_2) + \beta = \frac{-\lambda^2\beta}{\alpha+\lambda^2} + \beta = \frac{\alpha\beta}{\alpha+\lambda^2}$
$-\phi^2 \varsigma_1 (1 - \varsigma_2) = -\phi^2 \frac{-\alpha}{\phi\left(\alpha+\lambda^2\right)} \frac{-\lambda\beta}{\phi\left(\alpha+\lambda^2\right)} = \frac{-\alpha\beta\lambda}{\left(\alpha+\lambda^2\right)^2}$
$-\lambda\phi^2 \varsigma_1 (1 - \varsigma_2) = -\lambda\phi^2 \frac{-\alpha}{\phi\left(\alpha+\lambda^2\right)} \frac{-\lambda\beta}{\phi\left(\alpha+\lambda^2\right)} = \frac{-\alpha\beta\lambda^2}{\left(\alpha+\lambda^2\right)^2}$

The FOC can be rewritten in the following form

$$(\pi_t - \overline{\pi}) \frac{\alpha\beta}{\alpha + \lambda^2} + (y_t - \overline{y}) \frac{-\alpha\beta\lambda}{\alpha + \lambda^2} + (\pi_{t+1} - \overline{\pi}) \frac{-\alpha\beta^2\lambda^2}{\left(\alpha + \lambda^2\right)^2} + (y_{t+1} - \overline{y}) \frac{-\alpha^2\beta^2\lambda}{\left(\alpha + \lambda^2\right)^2} = 0$$

$$\pi_t - \overline{\pi} + (y_t - \overline{y})(-\lambda) + (\pi_{t+1} - \overline{\pi}) \frac{-\beta\lambda^2}{\alpha + \lambda^2} + (y_{t+1} - \overline{y}) \frac{-\alpha\beta\lambda}{\alpha + \lambda^2} = 0$$

$\pi_{t+1} = \lambda y_{t+1} + \beta \widehat{E} \pi_{t+2}$

$$\pi_t - \overline{\pi} + (y_t - \overline{y})(-\lambda) + \left(\lambda y_{t+1} + \beta \widehat{E} \pi_{t+2} - \overline{\pi}\right) \frac{-\beta\lambda^2}{\alpha + \lambda^2} + (y_{t+1} - \overline{y}) \frac{-\alpha\beta\lambda}{\alpha + \lambda^2} = 0$$

$$\pi_t - \overline{\pi} - \lambda y_t + \lambda \overline{y} + \frac{-\beta\lambda^3}{\alpha + \lambda^2} y_{t+1} + \frac{-\beta^2\lambda^2}{\alpha + \lambda^2} \widehat{E} \pi_{t+2} + \frac{\beta\lambda^2}{\alpha + \lambda^2} \overline{\pi} + \frac{-\alpha\beta\lambda}{\alpha + \lambda^2} y_{t+1} + \frac{\alpha\beta\lambda}{\alpha + \lambda^2} \overline{y} = 0$$

$$\pi_t + \left(\frac{\beta\lambda^2}{\alpha + \lambda^2} - 1\right) \overline{\pi} - \lambda y_t + \left(\frac{\alpha\beta}{\alpha + \lambda^2} + 1\right) \lambda \overline{y} + \left(\alpha + \lambda^2\right) \frac{-\beta\lambda}{\alpha + \lambda^2} y_{t+1} + \frac{-\beta^2\lambda^2}{\alpha + \lambda^2} \widehat{E} \pi_{t+2} = 0$$

$$\pi_t + \left(\frac{\beta\lambda^2}{\alpha + \lambda^2} - 1\right) \overline{\pi} + \left(\frac{\alpha\beta}{\alpha + \lambda^2} + 1\right) \lambda \overline{y} - \lambda y_t - \beta\lambda y_{t+1} + \frac{-\beta^2\lambda^2}{\alpha + \lambda^2} \widehat{E} \pi_{t+2} = 0$$

$y_{t+1} = \widehat{E} y_{t+2} - \phi r_{t+1} + \phi \widehat{E} \pi_{t+2}$

$$\pi_t + \left(\frac{\beta\lambda^2}{\alpha + \lambda^2} - 1\right) \overline{\pi} + \left(\frac{\alpha\beta}{\alpha + \lambda^2} + 1\right) \lambda \overline{y} - \lambda y_t - \beta\lambda \left(\widehat{E} y_{t+2} - \phi r_{t+1} + \phi \widehat{E} \pi_{t+2}\right) ..$$

$$.. + \frac{-\beta^2\lambda^2}{\alpha + \lambda^2} \widehat{E} \pi_{t+2} = 0$$

$$\pi_t + \left(\frac{\beta\lambda^2}{\alpha+\lambda^2} - 1\right)\overline{\pi} + \left(\frac{\alpha\beta}{\alpha+\lambda^2} + 1\right)\lambda\overline{y} - \lambda y_t - \beta\lambda\widehat{E}y_{t+2} + \beta\lambda\phi r_{t+1}..$$

$$.. - \left(\beta\lambda\phi + \frac{\beta^2\lambda^2}{\alpha+\lambda^2}\right)\widehat{E}\pi_{t+2} = 0$$

$r_{t+1} = \zeta_0 + \zeta_1 y_t + \zeta_2 \widehat{E}\pi_{t+2} + \zeta_3 \widehat{E}y_{t+2}$

$$\pi_t + \left(\frac{\beta\lambda^2}{\alpha+\lambda^2} - 1\right)\overline{\pi} + \left(\frac{\alpha\beta}{\alpha+\lambda^2} + 1\right)\lambda\overline{y} - \lambda y_t - \beta\lambda\widehat{E}y_{t+2} \quad ...$$

$$+\beta\lambda\phi\left(\zeta_0 + \zeta_1 y_t + \zeta_2\widehat{E}\pi_{t+2} + \zeta_3\widehat{E}y_{t+2}\right) - \left(\beta\lambda\phi + \frac{\beta^2\lambda^2}{\alpha+\lambda^2}\right)\widehat{E}\pi_{t+2} = 0$$

$$\pi_t + \left(\frac{\beta\lambda^2}{\alpha+\lambda^2} - 1\right)\overline{\pi} + \left(\frac{\alpha\beta}{\alpha+\lambda^2} + 1\right)\lambda\overline{y} + \beta\lambda\phi\zeta_0 + \left(\beta\phi\zeta_1 - 1\right)\lambda y_t \quad ...$$

$$+\left(\phi\zeta_3 - 1\right)\beta\lambda\widehat{E}y_{t+2} + \left(\beta\lambda\phi\zeta_2 - \beta\lambda\phi - \frac{\beta^2\lambda^2}{\alpha+\lambda^2}\right)\widehat{E}\pi_{t+2} = 0$$

$\zeta_0 = \frac{-\lambda\overline{\pi}}{\phi\left(\alpha+\lambda^2\right)}$, $\zeta_1 = \frac{-\alpha}{\phi\left(\alpha+\lambda^2\right)}$, $\zeta_2 = \frac{\lambda\beta}{\phi\left(\alpha+\lambda^2\right)} + 1$, $\zeta_3 = \frac{1}{\phi}$, $\zeta_4 = \frac{1}{\phi}$, $\zeta_5 = \frac{\lambda}{\phi\left(\alpha+\lambda^2\right)}$

$$\pi_t + \left(\frac{\beta\lambda^2}{\alpha+\lambda^2} - 1\right)\overline{\pi} + \left(\frac{\alpha\beta}{\alpha+\lambda^2} + 1\right)\lambda\overline{y} + \frac{-\beta\lambda^2\overline{\pi}}{\alpha+\lambda^2} + \left(\frac{-\alpha\beta}{\alpha+\lambda^2} - 1\right)\lambda y_t \quad ...$$

$$+\left(\phi\frac{1}{\phi} - 1\right)\beta\lambda\widehat{E}y_{t+2} + \left(\phi\left(\zeta_2 - 1\right) - \frac{\beta\lambda}{\alpha+\lambda^2}\right)\beta\lambda\widehat{E}\pi_{t+2} = 0$$

$$\pi_t - \overline{\pi} + \left(\frac{\alpha\beta}{\alpha+\lambda^2} + 1\right)\lambda\overline{y} + \left(\frac{-\alpha\beta}{\alpha+\lambda^2} - 1\right)\lambda y_t = 0$$

$\pi_t = \lambda y_t + \beta\widehat{E}\pi_{t+1} + u_t$

$$\lambda y_t + \beta\widehat{E}\pi_{t+1} + u_t - \overline{\pi} + \left(\frac{\alpha\beta}{\alpha+\lambda^2} + 1\right)\lambda\overline{y} + \left(\frac{-\alpha\beta}{\alpha+\lambda^2} - 1\right)\lambda y_t = 0$$

$$\left(\frac{\alpha\beta}{\alpha+\lambda^2} + 1\right)\lambda\overline{y} - \overline{\pi} + \frac{-\alpha\beta\lambda}{\alpha+\lambda^2}y_t + \beta\widehat{E}\pi_{t+1} + u_t = 0$$

$y_t = \widehat{E}y_{t+1} - \phi r_t + \phi\widehat{E}\pi_{t+1} + g_t$

$$\left(\frac{\alpha\beta}{\alpha+\lambda^2}+1\right)\lambda\overline{y}-\overline{\pi}+\frac{-\alpha\beta\lambda}{\alpha+\lambda^2}\left(\widehat{E}y_{t+1}-\phi r_t+\phi\widehat{E}\pi_{t+1}+g_t\right)+\beta\widehat{E}\pi_{t+1}+u_t=0$$

$$\left(\frac{\alpha\beta}{\alpha+\lambda^2}+1\right)\lambda\overline{y}-\overline{\pi}+\frac{-\alpha\beta\lambda}{\alpha+\lambda^2}\widehat{E}y_{t+1}+\frac{\alpha\beta\lambda\phi}{\alpha+\lambda^2}r_t+\frac{-\alpha\beta\lambda\phi}{\alpha+\lambda^2}\widehat{E}\pi_{t+1}+\frac{-\alpha\beta\lambda}{\alpha+\lambda^2}g_t+\beta\widehat{E}\pi_{t+1}+u_t=0$$

$$\left(\frac{\alpha\beta}{\alpha+\lambda^2}+1\right)\lambda\overline{y}-\overline{\pi}+\frac{\alpha\beta\lambda\phi}{\alpha+\lambda^2}r_t+\left(\beta-\frac{\alpha\beta\lambda\phi}{\alpha+\lambda^2}\right)\widehat{E}\pi_{t+1}+\frac{-\alpha\beta\lambda}{\alpha+\lambda^2}\widehat{E}y_{t+1}+\frac{-\alpha\beta\lambda}{\alpha+\lambda^2}g_t+u_t=0$$

$r_t=\zeta_0+\zeta_1 y_{t-1}+\zeta_2\widehat{E}\pi_{t+1}+\zeta_3\widehat{E}y_{t+1}+\zeta_4 g_t+\zeta_5 u_t$

$$\left(\frac{\alpha\beta}{\alpha+\lambda^2}+1\right)\lambda\overline{y}-\overline{\pi}+\frac{\alpha\beta\lambda\phi}{\alpha+\lambda^2}\left(\zeta_0+\zeta_1 y_{t-1}+\zeta_2\widehat{E}\pi_{t+1}+\zeta_3\widehat{E}y_{t+1}+\zeta_4 g_t+\zeta_5 u_t\right)\quad ...$$
$$+\left(\beta-\frac{\alpha\beta\lambda\phi}{\alpha+\lambda^2}\right)\widehat{E}\pi_{t+1}+\frac{-\alpha\beta\lambda}{\alpha+\lambda^2}\widehat{E}y_{t+1}+\frac{-\alpha\beta\lambda}{\alpha+\lambda^2}g_t+u_t=0$$

$$\left(\frac{\alpha\beta}{\alpha+\lambda^2}+1\right)\lambda\overline{y}-\overline{\pi}+\frac{\alpha\beta\lambda\phi}{\alpha+\lambda^2}\zeta_0+\frac{\alpha\beta\lambda\phi}{\alpha+\lambda^2}\zeta_1 y_{t-1}+\frac{\alpha\beta\lambda\phi}{\alpha+\lambda^2}\zeta_2\widehat{E}\pi_{t+1}+\frac{\alpha\beta\lambda\phi}{\alpha+\lambda^2}\zeta_3\widehat{E}y_{t+1}\quad ...$$
$$+\frac{\alpha\beta\lambda\phi}{\alpha+\lambda^2}\zeta_4 g_t+\frac{\alpha\beta\lambda\phi}{\alpha+\lambda^2}\zeta_5 u_t+\left(\beta-\frac{\alpha\beta\lambda\phi}{\alpha+\lambda^2}\right)\widehat{E}\pi_{t+1}+\frac{-\alpha\beta\lambda}{\alpha+\lambda^2}\widehat{E}y_{t+1}+\frac{-\alpha\beta\lambda}{\alpha+\lambda^2}g_t+u_t=0$$

$\zeta_0=\frac{-\lambda\overline{\pi}}{\phi\left(\alpha+\lambda^2\right)}$, $\zeta_1=\frac{-\alpha}{\phi\left(\alpha+\lambda^2\right)}$, $\zeta_2=\frac{\lambda\beta}{\phi\left(\alpha+\lambda^2\right)}+1$, $\zeta_3=\frac{1}{\phi}$, $\zeta_4=\frac{1}{\phi}$, $\zeta_5=\frac{\lambda}{\phi\left(\alpha+\lambda^2\right)}$

$$\left(\frac{\alpha\beta}{\alpha+\lambda^2}+1\right)\lambda\overline{y}+\left(\frac{\alpha\beta\lambda\phi}{\alpha+\lambda^2}\frac{-\lambda}{\phi\left(\alpha+\lambda^2\right)}-1\right)\overline{\pi}+\frac{\alpha\beta\lambda\phi}{\alpha+\lambda^2}\frac{-\alpha}{\phi\left(\alpha+\lambda^2\right)}y_{t-1}\quad ...$$
$$+\left(1+\frac{\alpha\beta\lambda\phi}{\alpha+\lambda^2}\frac{\lambda}{\phi\left(\alpha+\lambda^2\right)}\right)u_t+\left(\beta+\frac{\alpha\beta\lambda\phi}{\alpha+\lambda^2}\left(\zeta_2-1\right)\right)\widehat{E}\pi_{t+1}=0$$

$\frac{\alpha\beta\lambda\phi}{\alpha+\lambda^2}\left(\zeta_2-1\right)=\frac{\alpha\beta\lambda\phi}{\alpha+\lambda^2}\frac{\lambda\beta}{\phi\left(\alpha+\lambda^2\right)}=\frac{\alpha\beta^2\lambda^2}{\left(\alpha+\lambda^2\right)^2}$

$$\left(\frac{\alpha\beta}{\alpha+\lambda^2}+1\right)\lambda\overline{y}+\left(\frac{-\alpha\beta\lambda^2}{\left(\alpha+\lambda^2\right)^2}-1\right)\overline{\pi}+\frac{-\alpha^2\beta\lambda}{\left(\alpha+\lambda^2\right)^2}y_{t-1} \qquad \cdots$$

$$+\left(1+\frac{\alpha\beta\lambda^2}{\left(\alpha+\lambda^2\right)^2}\right)u_t+\left(\beta+\frac{\alpha\beta^2\lambda^2}{\left(\alpha+\lambda^2\right)^2}\right)\widehat{E}\pi_{t+1}=0$$

$$\left(\frac{\alpha\beta+\alpha+\lambda^2}{\alpha+\lambda^2}\right)\lambda\overline{y}+\left(\frac{-\alpha\beta\lambda^2-\left(\alpha+\lambda^2\right)^2}{\left(\alpha+\lambda^2\right)^2}\right)\overline{\pi}+\frac{-\alpha^2\beta\lambda}{\left(\alpha+\lambda^2\right)^2}y_{t-1} \qquad \cdots$$

$$+\left(\frac{\left(\alpha+\lambda^2\right)^2+\alpha\beta\lambda^2}{\left(\alpha+\lambda^2\right)^2}\right)u_t+\left(\frac{\left(\alpha+\lambda^2\right)^2+\alpha\beta\lambda^2}{\left(\alpha+\lambda^2\right)^2}\right)\beta\widehat{E}\pi_{t+1}=0$$

$$\left(\alpha\beta+\alpha+\lambda^2\right)\lambda\overline{y}+\left(\frac{-\alpha\beta\lambda^2-\left(\alpha+\lambda^2\right)^2}{\alpha+\lambda^2}\right)\overline{\pi}+\frac{-\alpha^2\beta\lambda}{\alpha+\lambda^2}y_{t-1} \qquad \cdots$$

$$+\left(\frac{\left(\alpha+\lambda^2\right)^2+\alpha\beta\lambda^2}{\alpha+\lambda^2}\right)u_t+\left(\frac{\left(\alpha+\lambda^2\right)^2+\alpha\beta\lambda^2}{\alpha+\lambda^2}\right)\beta\widehat{E}\pi_{t+1}=0$$

Let's rewrite the previous equation like

$$\tau_0\overline{y}+\tau_1\overline{\pi}+\tau_2 y_{t-1}+\tau_3 u_t+\tau_4\widehat{E}\pi_{t+1}=0$$

where $\tau_0=\left(\alpha\beta+\alpha+\lambda^2\right)\lambda$, $\tau_1=\left(\frac{-\alpha\beta\lambda^2-\left(\alpha+\lambda^2\right)^2}{\alpha+\lambda^2}\right)$, $\tau_2=\frac{-\alpha^2\beta\lambda}{\alpha+\lambda^2}$, $\tau_3=\left(\frac{\left(\alpha+\lambda^2\right)^2+\alpha\beta\lambda^2}{\alpha+\lambda^2}\right)$, $\tau_4=\left(\frac{\left(\alpha+\lambda^2\right)^2+\alpha\beta\lambda^2}{\alpha+\lambda^2}\right)\beta$

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5.0 CONCLUSION

Diamond's (1965)overlapping-generations model with productive capital and money is used by many researchers. The question of whether the equilibria of this model are learnable by adaptive agents who do not initially possess rational expectations has not been previously explored. In particular, one might hope to use learning to reduce the set of rational expectations equilibria and in particular, to rule out the possibility of dynamically inefficient equilibria. Our results suggest that stability analysis under adaptive learning does not provide a means for selecting from among the multiple equilibria in this model. In particular, we find that dynamically inefficient equilibria *are* learnable. While the finding that learning does not work as a selection device in this model might be viewed as a negative result, the finding that dynamically inefficient equilibria are learnable might be viewed (positively or negatively!) as rationalizing some kind of government intervention, e.g. fiat money or social security transfer schemes that restore the economy to one of dynamic efficiency.

Expectations play an important role in the realization of the inflation rate, but different perceptions of the world lead to different expectations and policies. Whenever we consider models with multiple agents it is important to consider the implications of such differences. In this paper we test some previous results in the learning literature in a two-sided learning environment where two agents construct models and decision rules independently.

In his famous book "Conquest of American Inflation" Thomas Sargent analyzes the rise and fall of U.S. inflation after 1960. According to Sargent (1999) the role of expectations in economics was not well established before the 1970's. Policymakers of the time adopted methods derived from exploitation of the Phillips curve in the hope of lowering the inflation rate. As they learned from new data, they re-estimated their Phillips curve and adjusted their target inflation rate accordingly. But since they ignored the role of inflation expectations in the Phillips curve, fluctuations in the inflation rate resulted. First we show that with the inclusion of the expectations in a one-sided learning model the policymaker can achieve the results it targets.

In the second part of the paper we analyze the case where the central bank and the private sector have different views of the economy and they learn the economy with their own models. This allows us to test the robustness of the escapes of Cho, Williams and Sargent (2002) to two-sided learning. Our results show that the endogenous fluctuations of Cho, Williams and Sargent (2002) are not robust to a learning private sector. Even if the private sector learns the policy of the central bank we observe the disappearance of the fluctuations. But we show that it is possible to reproduce these endogenous fluctuations in a more plausible environment where the central bank uses a fully specified model and the private sector uses a misspecified model, so the central bank is better informed than the private sector. In this case the expectations of the private sector fluctuate, causing the inflation rate to fluctuate with it.

In another two-sided learning environment, the actual inflation rate and the expectations of the private sector converge to different values. Given its beliefs, the private sector is not capable of learning the policy of the central bank. The private sector updates its data set every period but this updating does not allow it to change its model specification. This is a weakness of the Evans-Honkapohja-Sargent learning mechanism. Since the unemployment rate decreases as much as the decrease in the expected rate of inflation, the regression coefficients do not respond to the divergence of the actual inflation rate from the expected inflation rate. The steady state that the inflation rate converges to is a restricted perceptions equilibrium, an equilibrium that arises from the beliefs of agents rather than from the fundamentals of the model. The existence of such a difference in beliefs lets the central bank achieve inflation lower than the Nash level.

We would like to see whether this result can be obtained in a less restrictive setting where the private sector learns via a mechanism (such as Bayesian learning) that does allow it to update its model specification. It is known that central banks make announcements or reveal information to affect the beliefs of the private sector. Is this because they actually can use their influence to manipulate the private sector's beliefs and achieve better than Nash outcome?

Expectations of future variables are very important when making monetary policy. This is the most important reason why Economics is not simply a field of engineering. You cannot engineer the economy as you engineer a building or an electrical circuit, for they are governed by known deterministic laws. Economics deals with human behavior, which is often compared with chaotic behavior. That is why considering inflation expectations is very important. In this paper we explore different ways of influencing people.

In this paper we show that it is not possible to influence the people unless you have perfect knowledge of their behavior. But even if you do not have that perfect knowledge, which method you use matters in terms of speed of convergence to the Nash equilibrium.

We would like to extend this work by using a Kalman filter to determine the expected inflation. There has been much work on central bank policies and for sure there will be much more coming. The way monetary policy is made is much different than how it was made 30 years ago and it will be different 30 years from now too.