OVERREACTION BEHAVIOR AND OPTIMIZATION TECHNIQUES IN MATHEMATICAL FINANCE

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Overreactions and other behavioral effects in stock prices can best be examined by adjusting for the changes in fundamentals. We perform this by subtracting the relative price changes in the net asset value (NAV) from that of market price (MP) daily for a large set of closed-end funds trading in US markets. We examine the days before and after a significant rise or fall in price deviation and MP return and find evidence of overreaction in the days after the change. Prior to a spike in deviation we find a gradual two or three day decline (and analogously in the other direction). Overall, there is a characteristic diamond pattern, revealing symmetry in deviations before and after the significant change. Much of the statistical significance and the patterns disappear when the subtraction of NAV return is eliminated, suggesting that the frequent changes in fundamentals mask behavioral effects. A second study subdivides the data depending on whether the NAV or market price is responsible for the spike in the relative difference. In a majority of spikes, it is the change in market price rather than NAV that is dominant. Among those spikes for which there is little or no change in NAV, the results are similar to the overall study. Furthermore, the upward spikes are preceded by one or two days of declining market price while NAV rises slightly or is relatively unchanged. This suggests that a cause of the spike may be due to over-positioning of traders in the opposite direction in anticipation.

We propose a mathematical model by combining an implementation of a state-of-the-art optimization algorithm, a dynamic initial parameter pool and a system of nonlinear differential equations to describe price dynamics. Given n-day period of MPs and NAVs from day i to day $i+n-1$, we get four optimal parameters in the Caginalp Differential Equations. Then, we solve the initial value problem to predict MP and return on day $i+n$ or later. The results of our statistical methods in real data confirm the model. We provide out-of-sample prediction that is more successful than random walk.

Keywords: numerical optimization, nonlinear optimization, overreaction, diamond pattern, over-positioning, price deviation, deviation model with partition, market price return prediction, computational finance, mathematical finance and economics, behavioral finance, differential equations, numerical solution of differential equations, data analysis, statistical methods in financial markets, market dynamics, bubble, algorithms, inverse problem of parameter estimation.

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PREFACE

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1.0 INTRODUCTION

There are different views, such as the efficient market hypothesis (EMH) and behavioral finance, about price dynamics and predictable patterns in stock prices. The former one has been a widespread theory for several decades. However, some recent studies have cast doubt on EMH for some time intervals or situations. The large price bubbles and market crashes, such as the internet/high-tech bubble, in recent years have been among the most dramatic examples [22]. Moreover, Bremer and Sweeney [10] find significant positive three day abnormal returns following the large price drop date, over the period between 1962 and 1986. They conclude that such a slow recovery is inconsistent with the notion that market prices fully and quickly reflect relevant information. Furthermore, Rosenberg et al. [45] and Zarowin [50] find evidence that stock prices overreact in the short run. They conclude that the stock market is inefficient since arbitrageurs, who detect the market's tendency to overreact, could earn huge returns by buying losers and selling winners. On the other hand, the set of assumptions and theory of behavioral finance have not been written down in the same precise manner as EMH. Therefore, more mathematical and statistical models and optimization techniques are needed.

We believe differential equations are powerful tools to understand price dynamics and the corresponding cognitive and emotional factors in financial markets. The marketplace is an outstanding laboratory to test it. We study overreaction behavior and computational optimization techniques for a large set of closed end funds such as Specialized Equity Funds (SEFs), General Equity Funds (GEFs) and World Equity Funds (WEFs), trading in US markets.

The remainder of the thesis is organized as follows. In Chapter 2, we present our deviation model (DM) and DM with partition. In Chapter 3, we obtain optimal parameters for the nonlinear differential equations by using two line search algorithms which are our customized designs and implementations for the problem. In Chapter 4, we perform out-of-sample prediction by employing past financial data and the optimal parameters. Chapter 5 concludes the thesis.

2.0 OVERREACTION BEHAVIOR

2.1 INTRODUCTION

During the past few decades, there has been an intense debate about the dynamics of stock prices. The prevalent theory has been the Efficient Market Hypothesis (EMH), which stipulates that stock prices move in accordance with the change in valuation. Since all participants quickly gain access to the same public information, there is a unique valuation about which the stock fluctuates randomly due to the presence of traders who are less informed. Thus, according to EMH, there is a unique price at each given moment that represents the value. Since a large number of traders are aware of this value, and eager to exploit any deviations from it, these deviations are not only temporary, but also random. If the deviations were biased in a particular way, the knowledgeable traders, argue the EMH theorists, would be aware of the bias and seek to exploit it, thereby eliminating it. The existence of systematic patterns in prices thus argues against the underlying assumptions of EMH.

In recent years, a new set of ideas, known as Behavioral Finance (BF), has gradually provided an alternative to EMH by stipulating that systematic biases exist in market dynamics. One aspect of this is that even experts are subject to the behavioral biases. Even if portfolio managers were not subject to these biases, they often do not have the latitude to reduce their exposure to stocks, or even a particular sector. For example, a manager may believe that almost all of the technology stocks are overvalued at a particular time. However, his fund prospectus may require that at least 95% of the assets be invested and that it be sector neutral (so that the percentage of technology stocks in his portfolio must match that of the S&P). The decision to buy the mutual fund itself is made by a less informed individual, but the manager can only mitigate that decision by an insignificant amount. To aggravate matters, any rise in the overvalued sector automatically increases their percentage ratio in the S&P, thereby forcing the manager to buy even more of the stocks that he believed to be overvalued.

Of course, EMH theorists would say that while a particular set of managers may be in this situation, there will be a large amount of capital, for example in hedge funds, that will take advantage of this by using short selling (see Appendix A for further discussion). Ultimately, these issues involve the quantities of assets and the behavior of investors controlling them. Hence the question of whether these assets are adequate to restore efficiency needs to be decided by an examination of the data. If the basic ideas of EMH are essentially correct, then the data would not exhibit any systematic biases, since the more informed traders would recognize and exploit them, thereby eliminating the effect.

A number of studies have shown systematic bias by examining either a long or short time horizon, as discussed below in the literature survey. A key idea in these studies involves comparing the return on a stock with the expected return based upon the overall market. In examining returns, there is an error or noise term specific to the stock or the sector, as discussed in classical finance (see Bodie et al. [7]). Essentially, this means that many factors can be expected to influence a particular stock. The randomness involved in these firm specific changes adds a significant amount of noise to any data analysis. For a given stock, if one has a reliable model for changes in valuation which could be subtracted from the trading price return, then this "noise" arising from the random events that alter valuation could be removed. This would leave behind either random fluctuations (as EMH would assert) or particular patterns reflecting systematic bias (as BF would assert). The difficulty here, for most stocks, is that there is no unique way to quantify changes in valuation. Data analysis utilizing a particular scheme for computing the valuation on a day-to-day basis would leave open the question of whether a different valuation procedure would lead to the same conclusions.

In order to circumvent these issues we consider a class of stocks, namely closed-end funds, for which the valuation is available based upon the underlying assets. Closed-end funds have been studied in numerous papers (see Anderson and Born [2] for survey), and are similar to other companies in that they are initiated by the pooling of a sum of money for the purpose of a particular type of investment. For example, suppose that \$300 million is raised for investment in Germany and the shares are priced (initially arbitrarily) at \$15, yielding 20 million shares. Once the fund is launched and the \$300 million is used to purchase German stocks, these investments will rise and fall along with these German stocks. The net asset value (NAV) is defined as the total value of the investments divided by the total number of shares and is computed daily. In our example, this would be \$15 initially, but would change with the German market subsequently. Meanwhile, once the initial public offering is concluded, the shares trade on the NYSE as any other stock. This means of course that there is no requirement that they trade at, or even near, the NAV. If they trade below the NAV, the stock is said to be trading at a discount, and analogously for a premium. Precisely, one defines the premium as

Premium = (Trading Price -NAV)/NAV.

The theoretical value of a closed-end fund is clearly related to its NAV. The NAV, plus or minus some percentage that varies very slowly in time, can be regarded as fundamental value. The major difference between the closed-end investment companies and most other companies is that the former is simpler, and its value is easier to establish. If the fund were liquidated at any point, the amount rendered for each share would be the NAV minus a small amount for the cost of the transactions. This is not only a theoretical possibility but also a reality for several funds that have been liquidated in this way. The fact that NAV is explicitly determined on a regular basis provides an opportunity to examine relative price changes and their relationship with valuation. Any inefficiency that is discovered in markets is usually labeled as an "anomaly", suggesting that it is an unusual aberration from the norm of efficient markets. Studies of closed-end funds that demonstrate inefficiency are often classified in this way, suggesting that similar phenomena do not occur with other stocks. An examination of some features of the closed-end fund data we have used suggests that the trading volume, ownership and exchange under which they are traded are similar to most other stocks. In particular, the daily trading volume in many closed-end funds is highly significant, usually in tens of thousands of shares, as with many mid-cap stocks. An examination of securities filings for closed-end funds shows ownership by a spectrum of institutions as well as individual investors. A large majority of these are traded on the

NYSE, so that the same rules apply. Given these similarities in trading volume, ownership and rules of trading (exchange mechanism), there is little to suggest that the short term price dynamics of closed-end funds would be significantly different from other stocks.

The vast majority of the studies of closed-end funds have focused on the long term issues.

Many of the closed-end funds have traded at discounts for prolonged times ([2], Chapter 6.). There have been various explanations advanced to account for this phenomenon, such as the structure of the fund, and the possibility that they will issue more shares, etc [1]. In some cases the discount may be compatible with EMH. For example, there may be a tax liability in the closed-end fund.

However, it is more difficult for EMH to justify systematic changes in the discount or premium that occur on a short term basis, which is our main interest in this chapter.

If the EMH were valid, the discount or premium would either be zero for all time, or slowly changing. Hence, the existence of a chronic discount or premium that may be due to tax related issues, for example, will not be relevant for our study. Even if there were some fundamental reason for an abrupt change in the discount or premium, it would not address the issue that we study in this chapter, namely, the precursors and aftershocks of this change.

In many cases, a premium or discount widens over a time period of weeks or months with relatively little change in the NAV. In the case of a large premium, the phenomenon appears to have the characteristics of a classical bubble. Sometimes, the origin of the bubble is due to a large interest in a particular country for which there are only a few ways to invest (Bosner-Neal et al. [8]). However, similar bubbles occur even when this is not the case. For example, the premium for the Spain Fund (SNF) grew to 50% in January of 2005, while the NAV was gradually declining, even though an exchange traded fund (EWP) could be purchased within 1% of its net asset value. Near the end of the Spain Fund bubble there were several days on which the trading price rose by several percent while the NAV was almost unchanged. The bubble burst as the trading price dropped by 19.32% on one day,

[1] Value based managers often say that some stocks (particularly those that are not in the limelight) are chronically undervalued. However, since there is no unique calculation to assess the value of a typical industrial corporation, the studies that can be done (e.g., using price-to-earnings ratios) are not as precise or convincing as the studies of closed-end funds.

again with little change in the NAV.

Utilizing 52 closed-end funds we begin by considering the set of days ("events") in which there is a significant deviation between the relative change in the market price and that of the NAV (see Section 2 for precise definition). This could occur in several ways; either there is a large change in the NAV and little corresponding change in the trading price, or there is a large change in the price without much change in the NAV. Alternatively, there could be a moderate change for both in opposite directions. For example, suppose there is a 1% increase in NAV on a given day (Day 0). If there is a 5% increase in the price, then we would have a 4% deviation. [Obviously, there is a strong relation between deviation and premium such that a positive deviation on Day 0 corresponds to a decrease in discount or an increase in premium. If the change in discount or premium is zero, then the deviation is zero as well.] We allow for the possibility that the excess change in price (on Day 0) could be due to some fundamental reason, such as a share buyback offer. The question is, what do we expect for the following day (Day 1)? If there were no systematic biases, then we would expect that the deviation of the following day would be zero. [Note that although there is a tiny drift term in both the NAV and the market price, the expected difference in drift will be zero. See main diagonal of Table 2.1.] If, on the other hand, we were to obtain a large sample of such events (Day 0), and find that, on average, there is a decrease in the difference between the relative change in the market price and that of NAV on Day 1, then this would be evidence of a systematic bias. Often the terminology "overreaction" is used when there is the change on a subsequent day in the opposite direction of the original day, and the term "underreaction" refers to subsequent change in the same direction. Using this procedure, we do not need to make a determination as to which market, say the closed-end fund in the NYSE, or the German market in the example above, is more efficient, and which market is overreacting. In many cases, we expect that it is the NAV representing the trading in a larger market that will be more efficient and less volatile. This is confirmed by a study of Pontiff [40] that showed a set of closed-end funds that were 64% more volatile than the underlying index. For example, the NAV of a fund investing in Japan is determined by a huge trading volume compared with the volume of the closed-end fund that invests in Japan. Consequently, one would expect, from the perspective of either EMH or BF, that the volatility would be greater in the smaller market, namely the closed-end fund. To examine this further, we have also performed a statistical testing using as "events" the days for which the NAV change exceeded particular threshold levels. Consistent with the study of Pontiff, our data suggests that a relatively small fraction of the events are characterized by large relative changes in NAV accompanied by small relative changes in the trading price. Most of the deviations occur with a relatively small change in the NAV that triggers a large change in trading price.

In both sets of statistical results we have found that there is evidence of an overreaction, i.e., on Day 1 there is a statistically significant change in the deviation that is in the opposite direction. Hence, a drop in the deviation on Day 0 is followed by a rise on Day 1, and analogously for a rise in the deviation. We have found overreaction for the market price returns as well. Unlike some of the studies on prices alone, these predictable changes on Day 1 are very substantial. Even more surprising, however, is the price movement in the opposite direction on the day prior to Day 0. In other words, a rise of the deviation on Day 0 is preceded by a dip. The key features of our results are displayed in Figure 2.1, in which the characteristic diamond pattern displays the gradual decline in deviations before the spike, and the decline after the spike. The opposite is true for a significant decline in deviations on Day 0. Figure 2.1 shows a symmetry between the upward and downward spikes, for low and medium threshold levels, on Day 0. But, more surprisingly, there is also an approximate symmetry between the days before and the days after the significant change (see Figure 2.1).

The presence of a decline before a sharp rise, from the perspective of EMH, is even more surprising than a subsequent decline. After all, one can attribute the decline after a sharp rise to an imperfect price adjustment process that has a time scale of a few days. However, the decline before a sharp rise indicates that there is a precursor of the deviation that is part of the cause. In the absence of an infinite amount of capital that is immediately available, one can explain this phenomenon as follows. On the day before the sharp rise there is an anticipation of negative news and, consequently, underinvestment on the part of the speculative traders. When the news is better than expected (e.g., a small rise in NAV instead of a sharp drop), there is an imbalance of cash/asset as the underinvested are rushing to buy. This initial and rapid price rise fuels further momentum buying that leads to a price at the end of Day 0 that is considerably higher than the previous day.

In other words, the overreaction happens because too many traders are caught short or underinvested, and there is a subsequent stampede to buy. The situation is analogous for downward spike on Day 0.

The perspective outlined above differs significantly from the EMH in that it invokes the concept of the finiteness of assets (see Caginalp and Balenovich [19]), rather than infinite arbitrage capital that is central to EMH. In order to examine the possible underlying causes we partition the data in Section 3 into four parts. We find that a majority of the spike events we consider are the result of market price returns rather than relative changes in NAV (see Figure 2.13, 2.14, 2.23 and 2.24). In a second study, we consider those spikes which occur while NAV is relatively unchanged. The data show that for upward spikes there is a gradual rise in the NAV accompanied by a gradual decline in market price (see Figure 2.15 and Figure 2.17). This is consistent with the concept (see Hypothesis 3) that traders with finite assets have been "caught short" or "underinvested" in anticipation of an event that turns out to be more positive than expected.

To the best of our knowledge, this is the first study to establish a precursor to significant short term changes. Another novel feature is the subtraction of the relative changes in fundamentals, thereby eliminating much of the noise that encumbers statistical testing.

2.1.1 Review of prior literature

The existence of an abnormal price reversal following a large price movement has been considered as evidence of the overreaction hypothesis. Several types of studies have discussed the existence and degree of overreaction or underreaction in the stock markets. While some of them consider overreaction or underreaction associated with momentum and reversal strategies over relatively long term, others examine it at the time of an extreme price change. The latter studies focus on daily market price adjustments to new information.

Positive (negative) cumulative abnormal returns following large positive (negative) price changes indicate underreaction, whereas reversals of returns suggest overreaction (Madura and Richie [34]).

Rosenberg et al. [45] and Zarovin [50] find evidence that stock prices overreact in the short run. They conclude that the stock market is inefficient since arbitrageurs who detect the market's tendency to overreact could earn huge returns by buying losers and selling winners.

Most of the latter studies define events as stock price changes in excess of $M\%$ (in either direction). A winner (loser) stock is a stock experiencing a one-day return at least $M\%$ ($-M\%$). Bremer and Sweeney [10] and Akhigbe et al. [1] used 10% trigger value to identify events.

Bremer and Sweeney examine the reversal of large price decreases for Fortune 500 firms. They find significant positive three day abnormal returns following the drop date, upon examining the period between 1962 and 1986. They conclude that such a slow recovery is inconsistent with the notion that market prices fully and quickly reflect relevant information. They suggest that this is incompatible with market efficiency. Moreover, they consider that one of the potential explanations for these remarkably large returns is market illiquidity.

Akhigbe et al. find a greater degree of overreaction within extreme positive price movements in technology stocks than within non-tech stocks, based on their subsequent stock price behavior, during the 1998-2000 period. Moreover, they detect a greater degree of underreaction within extreme negative changes in technology stocks than in non-tech stocks. They observe that the market is overoptimistic while evaluating technology stock prices in reaction to favorable and unfavorable information relative to a matched sample of non-technology firms.

Sturm [48] hypothesizes that post-event price behavior following large one-day price shocks is related to pre-event price and firm fundamental characteristics. He suggests that these characteristics proxy for investor confidence. He tests the relationship between pre-event long term returns and post-event short-term returns, for companies from the 2002 Fortune 500 index. He finds presence of a price shock effect whereby post-event reversals are smaller for larger price shocks.

More recently, Madura and Richie [34] find substantial overreaction of Exchange-Traded Funds (ETFs) during normal trading hours and after hours, giving opportunities for feedback traders. Their sample includes observation of daily opening and closing prices for AMEXtraded ETFs during the 1998-2002 period. The degree of overreaction is also more evident for international ETFs. They use three M values such as 5, 6 and 7, where $trigger > M\%$ for winners and $trigger < -M\%$ for losers.

Related to deviation of stock prices, Poterba and Summers [42] discuss the presence of transition periods when stock prices deviate from their fundamental values in illogical ways.

Financial markets are dynamic. Experimental economics has shown that even when there is no change or uncertainty in the expected payout of an asset, there is robust trading with dramatic changes (see Porter and Smith [41]), as there is always some uncertainty in the anticipation of the actions of other traders. For the closed-end funds we study, there is, of course, a stream of news that constantly readjusts the value of the fund. This is reflected in the NAV of the fund. However, the anticipation of strategies of other traders' actions and the inflow of information are also part of the market. As traders have access to faster and faster means of acquiring and processing information, it becomes possible to react on a more rapid time scale. While rapid dissemination of information could be a stabilizing force in the markets, the positive feedback strategies involved in trying to trade quickly on news or price movements could provide a destabilizing force that is often characterized by overreaction.

Moreover, studies involving long term behavior of prices (e.g., one or more years) tend to average over large disturbances, thereby hiding abnormal events. Hence, focusing on significantly large short term price changes can provide researchers with a tool to study these phenomena, and help decide the issues in an empirical manner. Of course, a large price change in itself does not necessarily indicate any abnormal investor reaction. A world event may drastically change the valuation of a closed end fund, for example. However, by subtracting out the NAV return of the fund, we can study changes that are predominantly exclusive of the changes in valuation. The closed-end funds comprise many stocks so that private information, etc., cannot provide an explanation for the rapid changes between the trading price return of the stock and the NAV return.

2.1.2 Possible theoretical reasons for overreaction or underreaction

1. Deviation of stock prices from their fundamental values (see [42]). For example, people tend to place too much emphasis on the strength of new information (see [31]). There may be overreaction to rumors or to facts ([34]).
2. Attribution theory. Weiner [49] gives a property of causal reasoning such that if an outcome is attributable to a non-stable cause, the future expectation will be either uncertain or different from the immediate past. Particularly, Sturm [48] suggests that if the price shock is attributed to a non-stable cause, the future outcome will either be uncertain or different from the price shock, leading to a reversal.
3. Stock price behavior is affected by feedback traders who trade based on recent price movements rather than fundamental factors (see [18] and [23]).
4. Affect and representativeness theories. If a particular market or sector is moving up rapidly, there is a positive image about it. Investors tend to flock to a particular investment, thereby increasing the price as they provide a posterior arguments to justify the ever higher price. For example, when the Spain Fund traded at a steep premium of about 100%, the justification for it was that it was difficult to buy Spanish stocks in the US in any other way. Yet if the potential for Spanish stocks is so great, why wouldn't the stocks already reflect that information?
5. Reference points in investments. Investors are often keenly aware of prices at which major turning points occurred. For example, if a closed-end fund touched \$20 and then retreated quickly, there is a general feeling of regret on the part of investors who wish they had sold at that point. The next time the stock reaches that point, it may be amply justified by the NAV; yet selling to avoid regret may be a cause of a larger deviation from NAV at that point. In other words, the selling near \$20 causes the price to lag behind the upward move in the NAV. This would be a negative deviation, as we define in the next section.

Moreover, Caginalp et al. [18] examine the relationship between momentum, fundamental value and overreaction based on a series of experiments to test the predictions of a momentum model using a dynamical systems approach.

The remainder of the chapter is organized as follows. In Section 2.2, we present our deviation model. In Section 2.3, the deviation model is handled with partition. In Section 2.4, we examine the Spain Fund Inc (SNF), as an illustrative example.

2.2 THE DEVIATION MODEL (DM)

In this section we examine the relative change in the market price to the relative change in the net asset value (NAV) price. Let P_t denote the market price at time t, and V_t denote the NAV price at time t. We define the deviation between the relative changes of these two quantities from day t to day $t+k$ (with k nonnegative) by

$$D_{t+k} = (P_{t+k} - P_t)/P_t - (V_{t+k} - V_t)/V_t. \tag{2.1}$$

2.2.1 Basic formalism

Table 2.1: **Basic formalism.** Interpretation of market price (MP) attitude using deviation operations. MP exhibits positive or negative reaction relative to the NAV.

		NAV			
	Deviation	*Large Decrease*	*Small Decrease*	*Small Increase*	*Large Increase*
MP	*Large Decrease*	*neutral*	*more negative*	*highly negative*	*highly negative*
	Small Decrease	*positive*	*neutral*	*highly negative*	*highly negative*
	Small Increase	*highly positive*	*highly positive*	*neutral*	*negative*
	Large Increase	*highly positive*	*highly positive*	*more positive*	*neutral*

In Table 2.1, we consider the D_{t+k} in terms of the relative changes to the NAV and the market price. For example, if there is a small decrease in NAV but a large decrease in market price, then D_{t+k} is negative, and we say that the market price exhibits negative sentiment relative to the NAV. That is, there is a relative pessimism of investor.

Before examining the statistics, we need to verify that the deviation formulation (2.1) introduced above is not biased. This is immediate from the definitions, and is summarized below in Proposition 1.

Proposition 1. *Let A be any array of market price returns and B be any array of NAV returns such that $A = B$. That is, $A(i)$ is an entry in the first column, $B(j)$ is an entry in the first row, and D_{t+k} is the corresponding deviation, in Table 2.1. Then, the double sum of all the possible deviation outcomes is zero, independent from the chosen threshold level.*

$$\sum_{i=1}^{n}\sum_{j=1}^{n} D_{t+k} = \sum_{i=1}^{n}\sum_{j=1}^{n}(A(i) - B(j)) = 0 \tag{2.2}$$

Also,

$$\sum_{i \neq j}^{n} D_{t+k} = \sum_{i \neq j}^{n}(A(i) - B(j)) = 0 \tag{2.3}$$

With a model that is not biased a priori, we can now determine if the deviations before and after days of significant change have zero mean, as would be predicted by the efficient market hypothesis, or whether there is a systematic tendency in the deviations.

2.2.2 Sample selection and descriptive statistics

To assess and analyze the overreaction or underreaction behavior of 52 closed-end funds (CEFs), we used not only Market Price (MP) but also Net Asset Value (NAV) data by using daily closing prices from CEFs such as 20 Specialized Equity Funds (SEFs), 15 General Equity Funds (GEFs) and 17 World Equity Funds (WEFs) during April 1, 1998-March 31, 2006.

Events are defined as abnormal deviations having threshold levels ($L < threshold \leqslant U$) for positive deviations where threshold is deviation in percent, $L > 0$ is the lower bound and $U > 0$ is the upper bound. Similarly, events for negative deviations are defined as abnormal deviations having threshold level ($-U \leqslant threshold < -L$). Given MP and NAV sequences for a fund and threshold level, we search successively for Day 0, a day experiencing an abnormal deviation. Then, we collect the deviations on 11-day window containing five pre-event days, Day 0 and five post-event days.

We group the threshold levels for large deviations into four groups for positive events

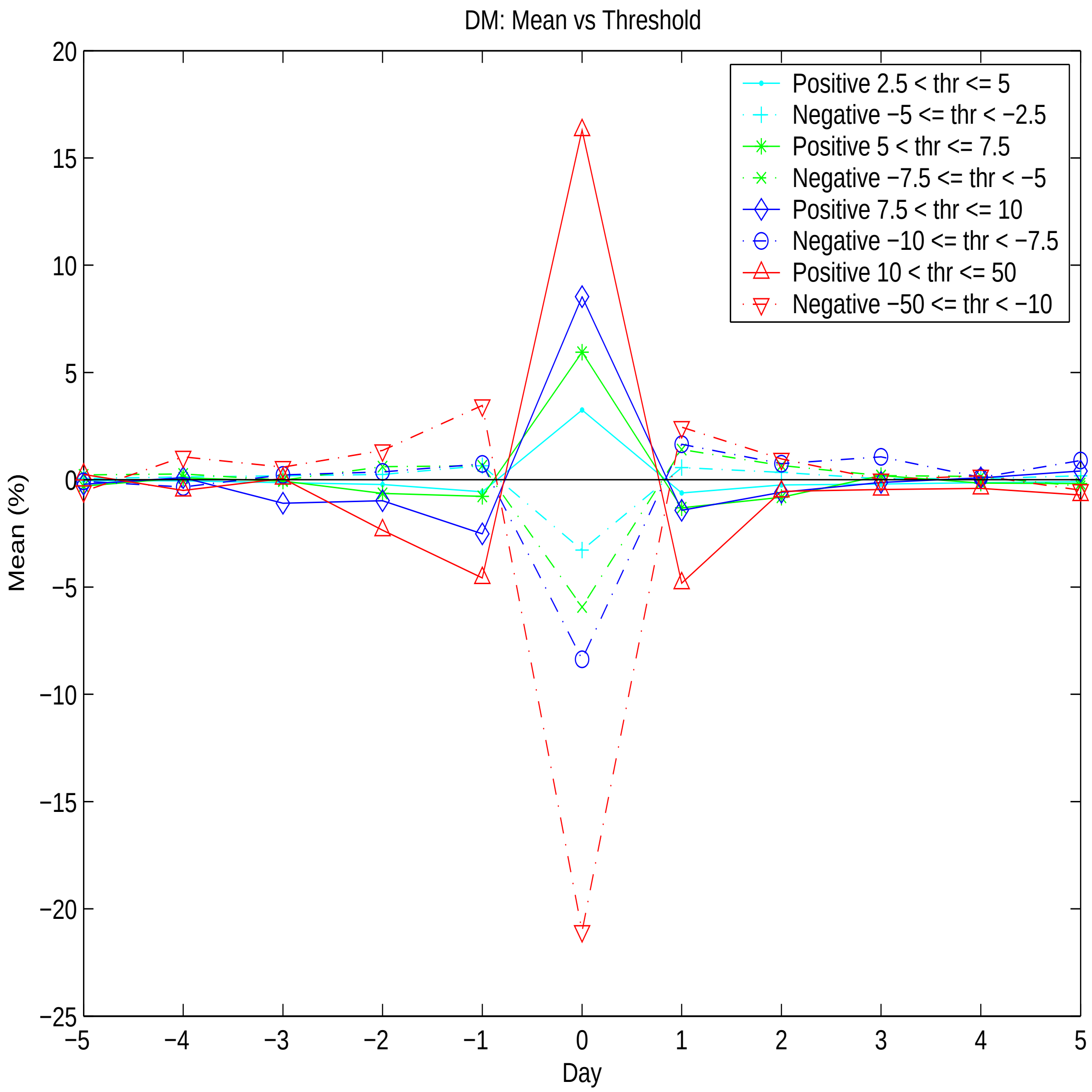

Figure 2.1: Mean deviation versus threshold ranges on 11-day window.

1. low ($2.5 < threshold \leqslant 5$),
2. medium ($5 < threshold \leqslant 7.5$),
3. high ($7.5 < threshold \leqslant 10$), and
4. very high ($10 < threshold \leqslant 50$) ,

and four groups for negative events

1. low ($-5 \leqslant threshold < -2.5$),
2. medium ($-7.5 \leqslant threshold < -5$),
3. high ($-10 \leqslant threshold < -7.5$), and
4. very high ($-50 \leqslant threshold < -10$).

Overreaction to minor changes (particularly recent ones) in valuation is emerging as a key concept in behavioral finance. In terms of our definitions, we examine the set of deviations between the market price returns and NAV returns (Day 0), and determine whether the following day (Day 1) is in the same or opposite direction.

Hypothesis 1. If there is a positive deviation on Day 0, there is a greater probability that there will be a negative deviation on Day 1. Similarly, a negative deviation on Day 0 is likely to be followed by a positive deviation on Day 1.

Hypothesis 2. If there is a positive deviation on Day 0, there is a greater probability that there will be a positive deviation on Day 1. Similarly, a negative deviation on Day 0 is likely to be followed by a negative deviation on Day 1.

In both cases **the null hypothesis (of the EMH)** is that the mean of relative changes on Day 1 is zero. Note that the drift term (average increase of a stock per day) is present in both of the quantities (market price and NAV) so that the subtraction eliminates this term.

2.2.3 Results for the deviation model

Figure 2.1 shows the mean deviation versus threshold ranges for positive and negative events on 11-day window. Prior to a spike in deviations we find a gradual two or three day decline (and analogously in the other direction). This suggests that a cause of the spike may be due to positioning of traders in the opposite direction. Overall, there is a characteristic diamond pattern, revealing a symmetry in the deviations before and after the significant

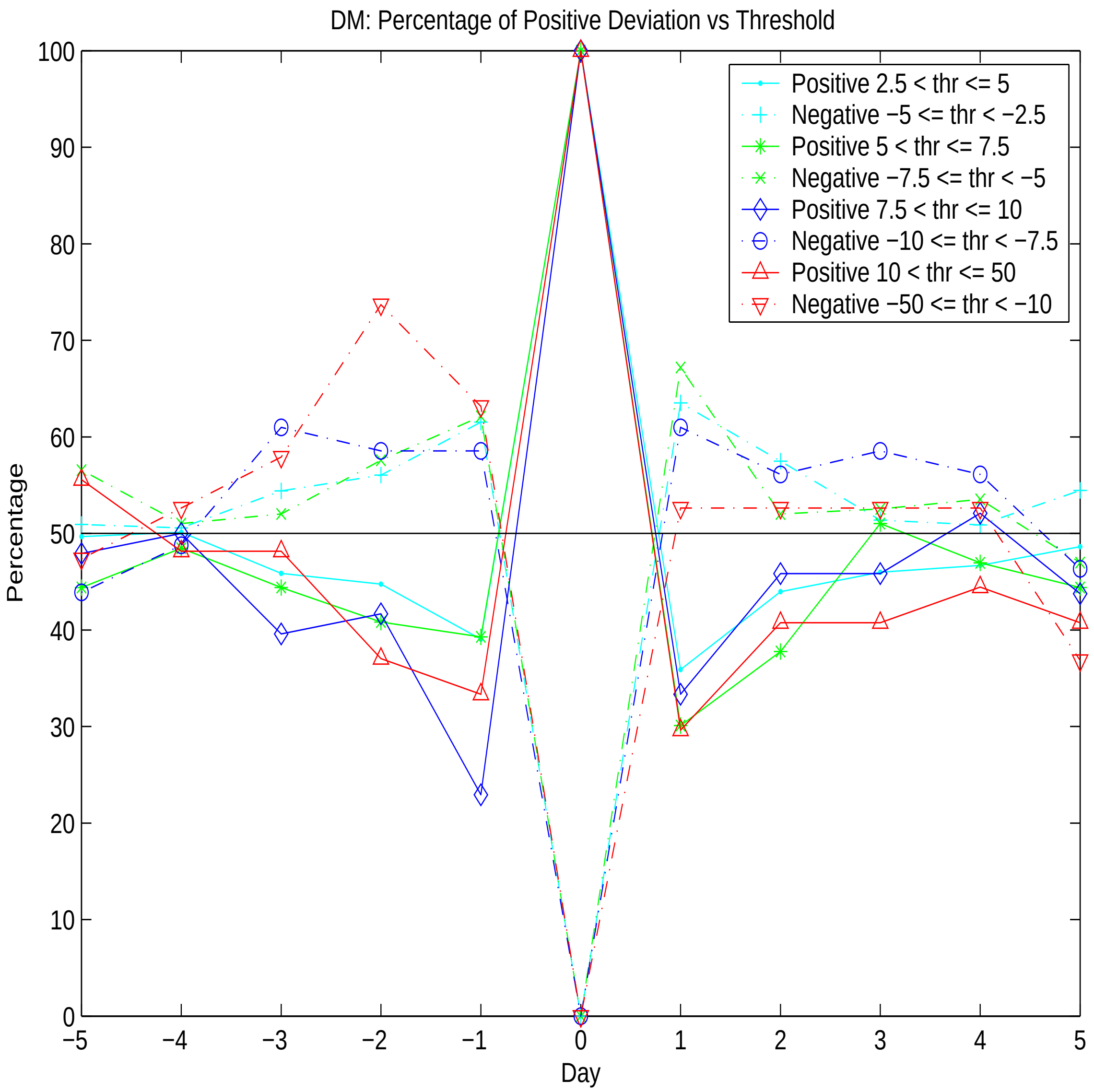

Figure 2.2: Percentages of positive deviations on 11-day window.

change. Figure 2.1 suggests overreaction for both directions because of the reversals during the post-event days. In addition, the magnitude of the reversal increases as the degree of shock increases. Moreover, the amounts on pre- and post-day are very close to each other for the low threshold levels, revealing another component of symmetry. Furthermore, the magnitude of the negative deviation is higher than that of positive deviation, only for the very high threshold level, on Day 0. This indicates that the effect of unfavorable information is higher than that of favorable information for this level, in the short term.

Figure 2.2 demonstrates the average percentage of positive deviations with respect to the large positive and negative deviations on Day 0. It supports the evidence of overreaction for both directions and all threshold levels. On Day 1, the percentages of positive deviations are less than 36%, indicating the reversal, for all positive threshold levels. In the negative direction the percentages of positive deviations are greater than 60%, indicating the reversal for the low, medium and high threshold levels on Day 1. During the two pre- and post-day, the percentages of positive deviations are less than 50%, for the large positive deviators. In the negative direction during the two pre- and post-day, the percentages of positive deviations are greater than 50% for the low and medium threshold levels.

Figure 2.3 and Figure 2.4 illustrate the volatility of the market on the 11-day window [29]. On Day 0, the variance approaches 0.40 for both directions for the low threshold level. They are another dimension of the symmetry. During pre- and post-event days, variance seems to be more stable for low threshold level, while the volatility becomes maximum around the shock day for very high threshold levels for both directions.

Figures 2.5 - 2.8 show that there is a decline before a sharp rise in MP return for all large positive deviators. Then there is reversal both in deviation and MP return. Figures 2.9 - 2.12 illustrate that there is at least one day rise before a sharp dip in MP return for the negative deviators. Then, there is reversal in MP return on Day 1 for the first three threshold levels. The reversal of a very large dip is slower because of the price effect.

2.2.3.1 Low thresholds In Table B1, the average deviation on Day 0 is 3.25% for the 1947 large positive events, after statistically significant three pre-day pessimism in the low threshold level. During the first five post-event days, there is reversal. In other words, MP

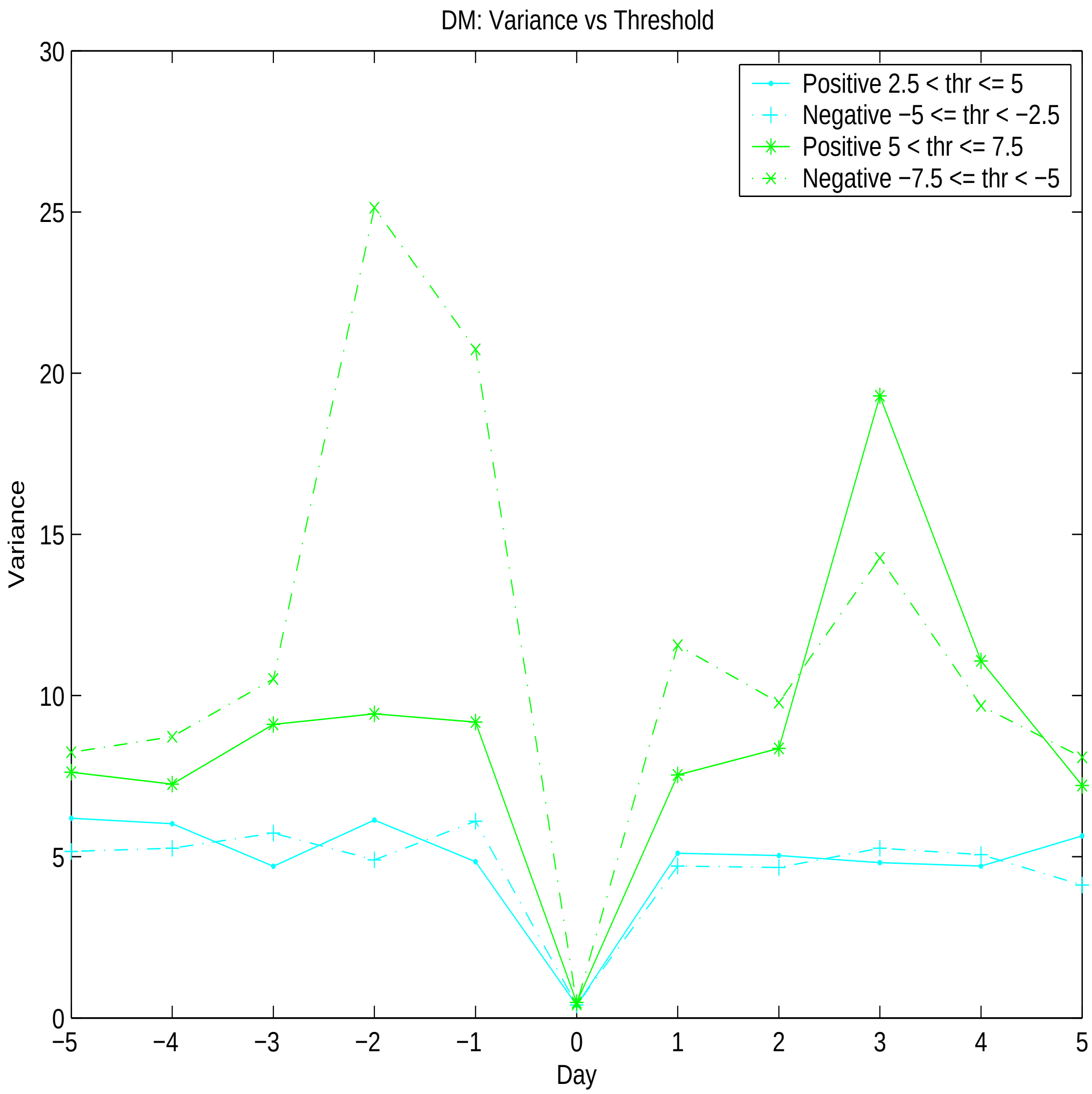

Figure 2.3: Variance versus low and medium threshold ranges.

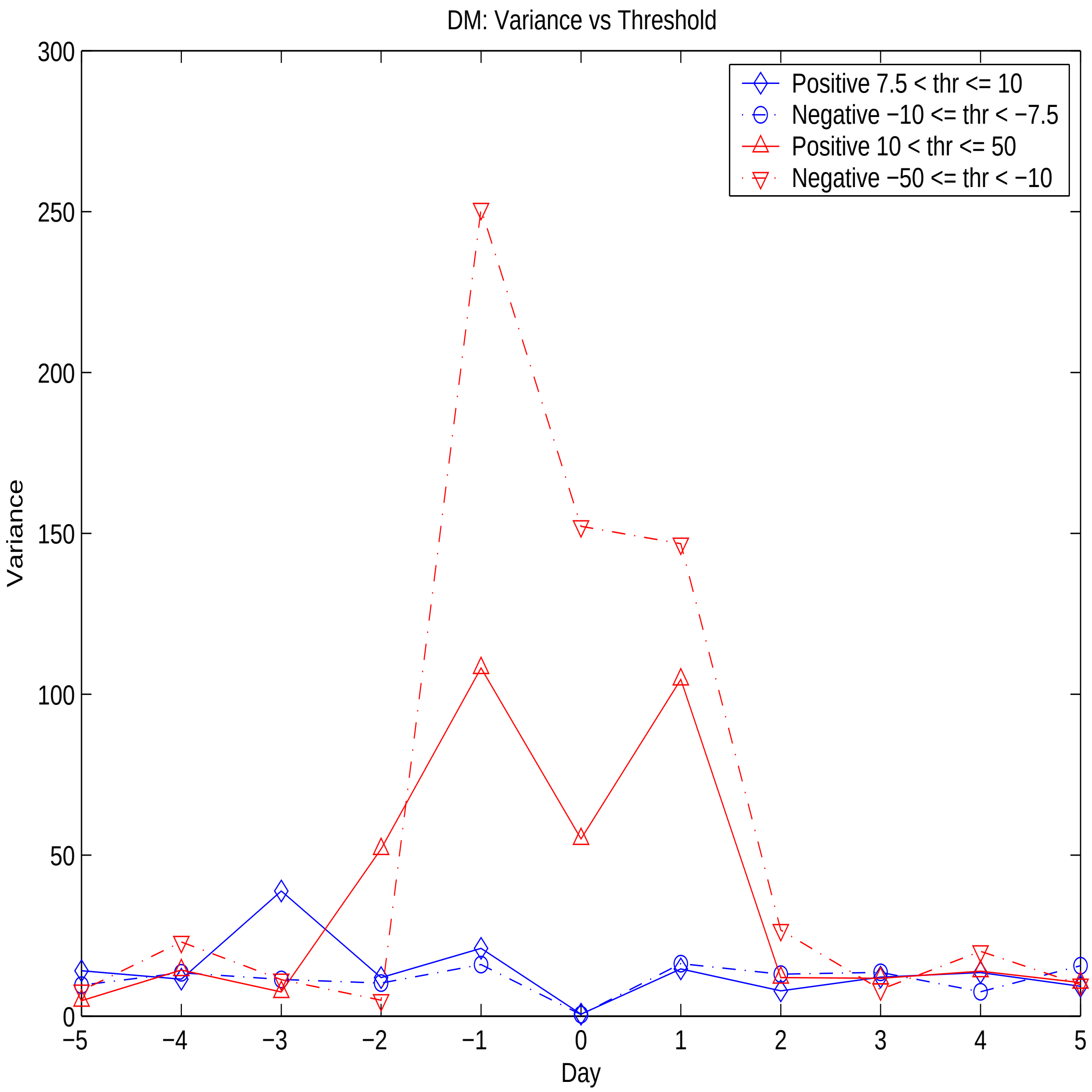

Figure 2.4: Variance versus high threshold ranges.

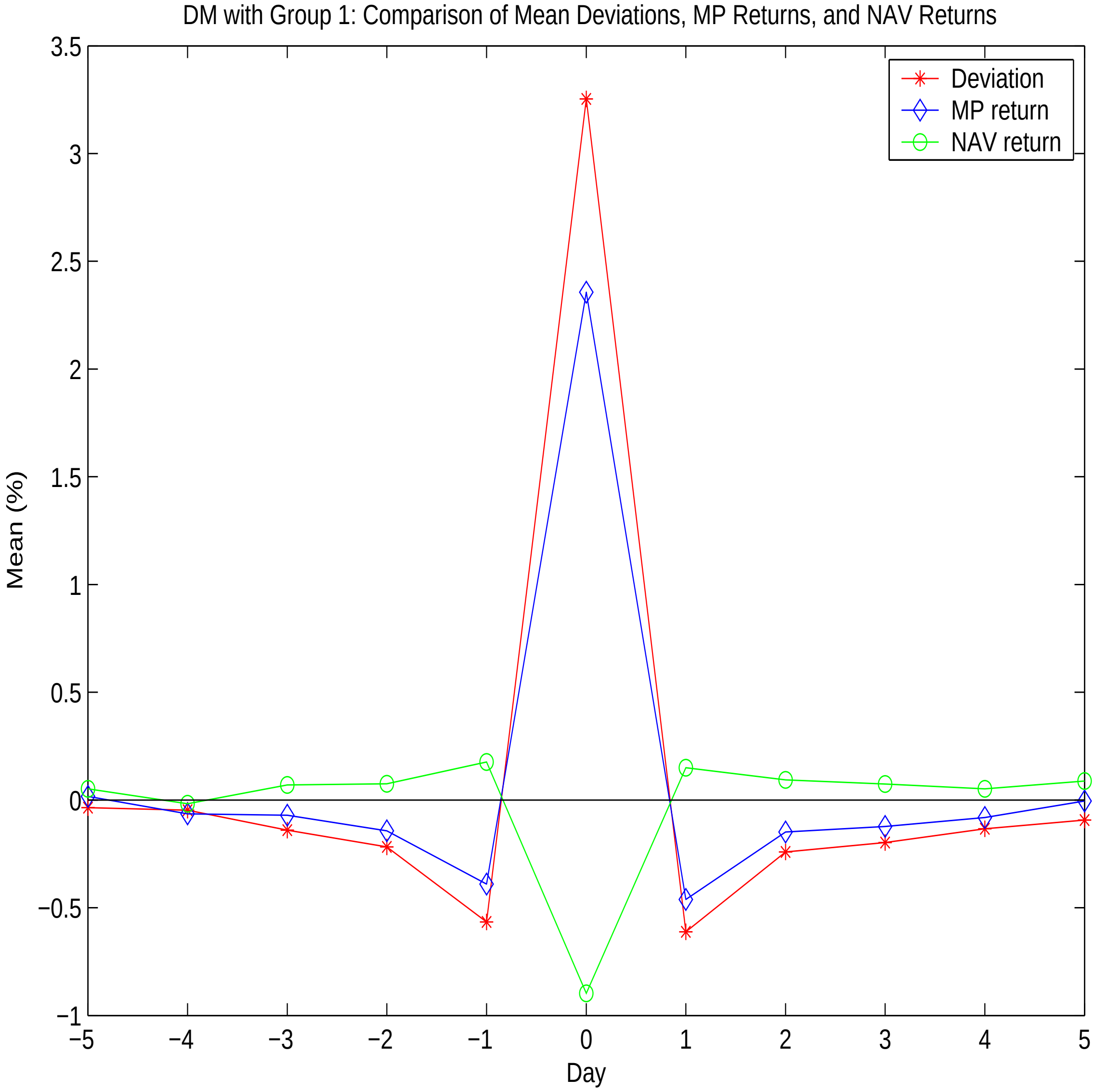

Figure 2.5: Relative optimism on Day 0 and the upper diamond pattern in the low threshold level.

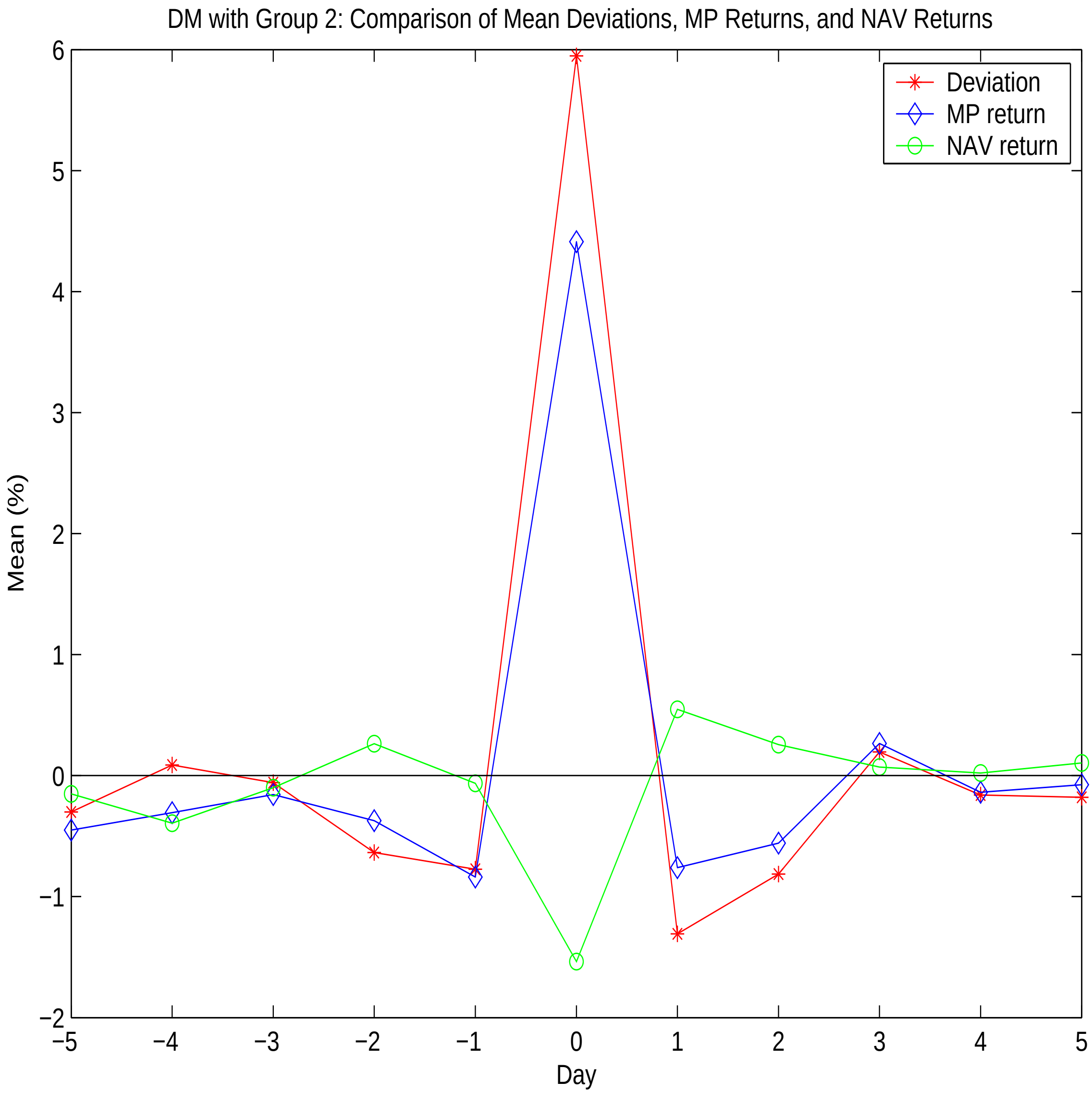

Figure 2.6: Precursor, relative optimism on Day 0, and aftershock in the medium threshold level.

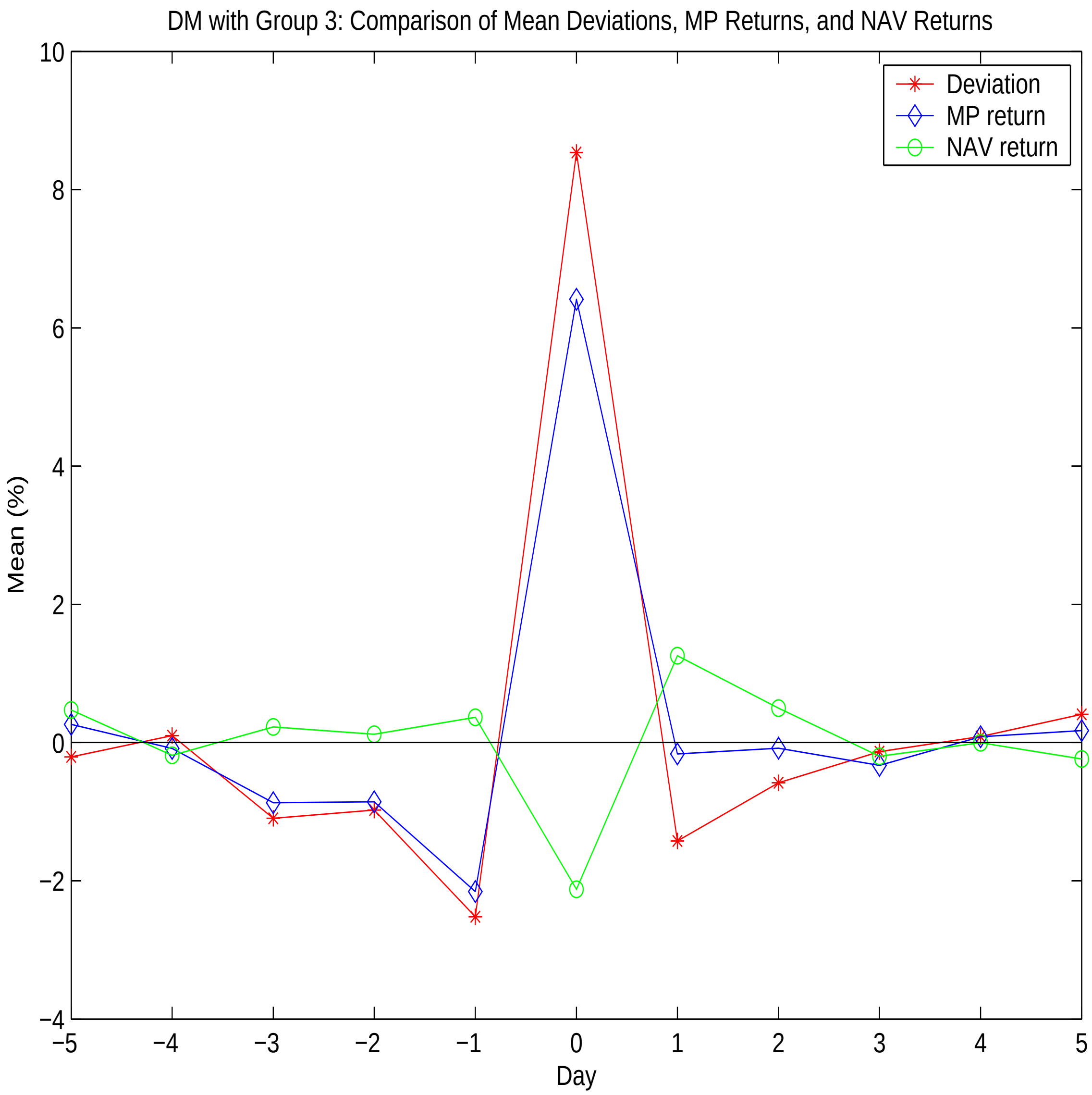

Figure 2.7: Precursor, relative optimism on Day 0, and the reversal during the post-event days in the high threshold level.

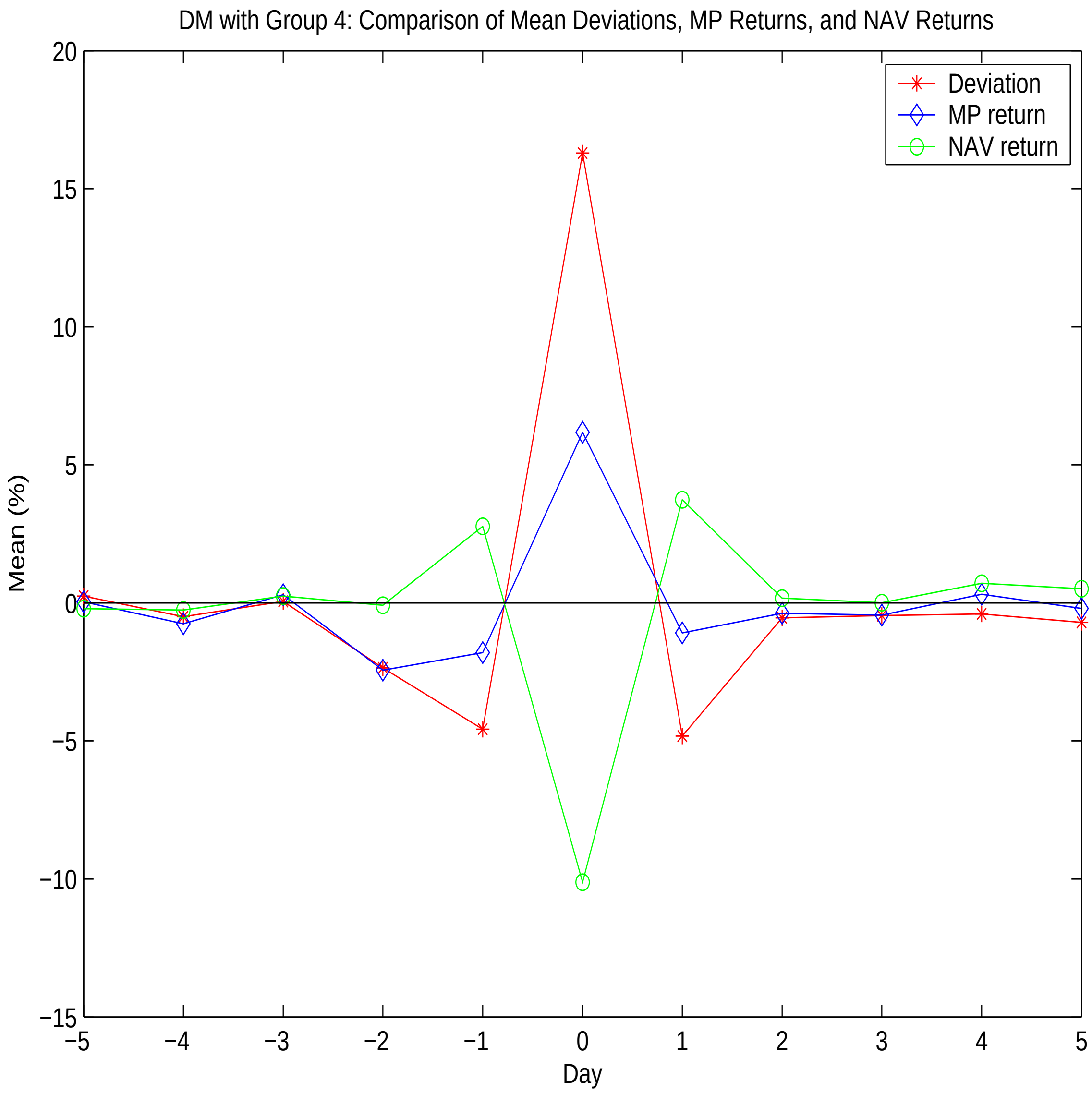

Figure 2.8: Relative optimism on Day 0 in the very high threshold level.

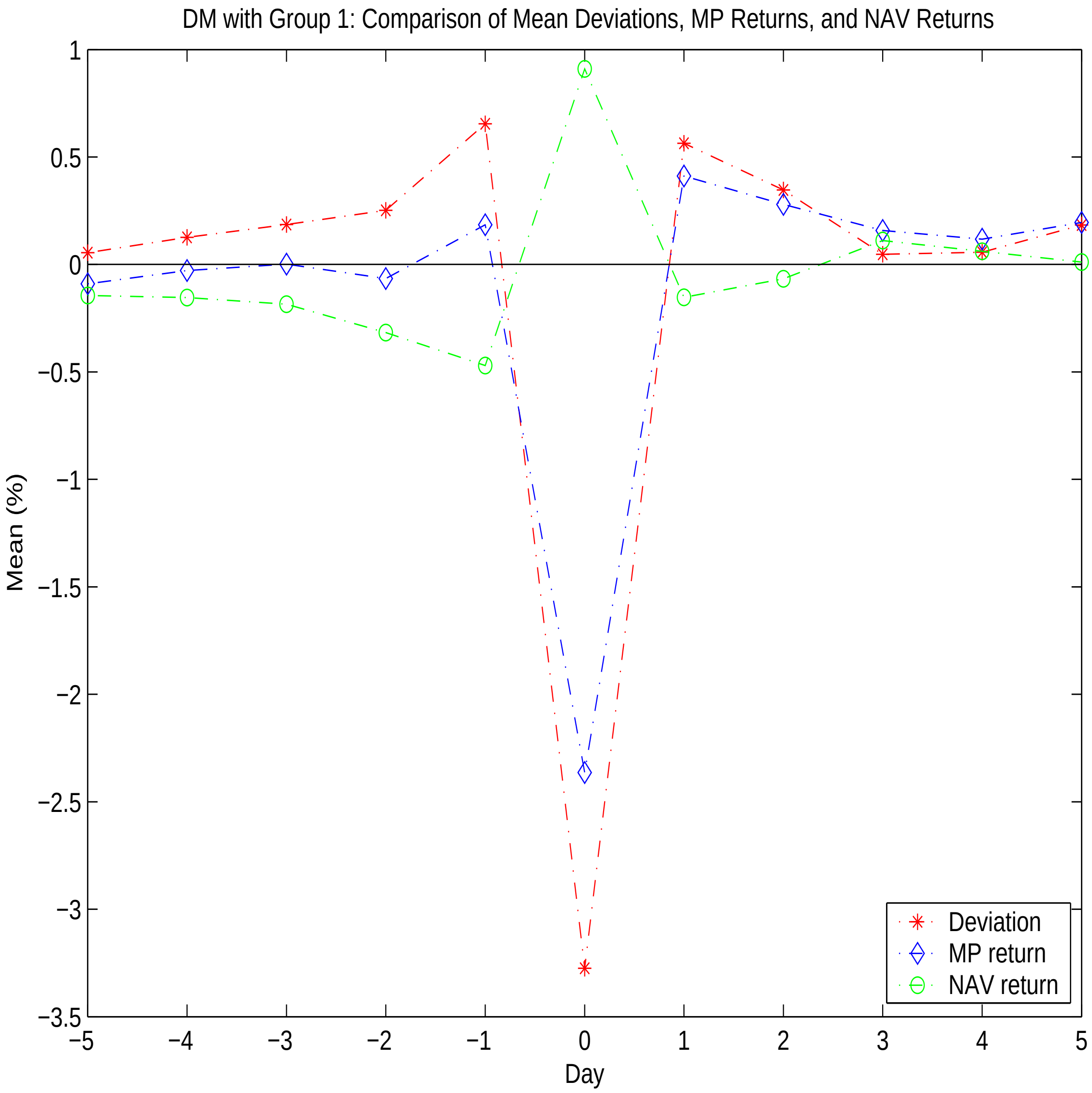

Figure 2.9: Precursor, relative pessimism on Day 0, and the post-event reversal in the low threshold level.

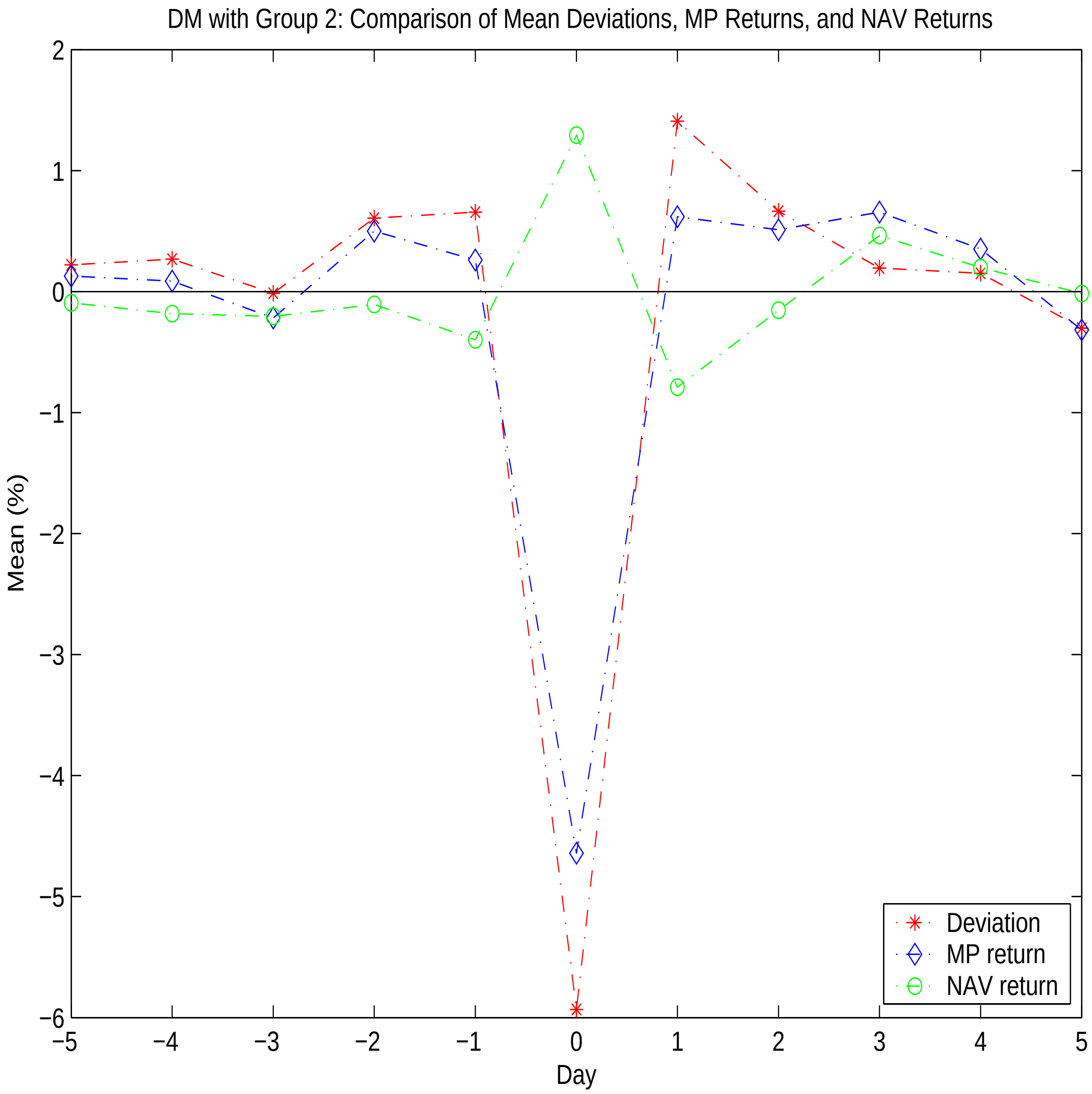

Figure 2.10: Precursor, relative pessimism on Day 0, and aftershock in the medium threshold level.

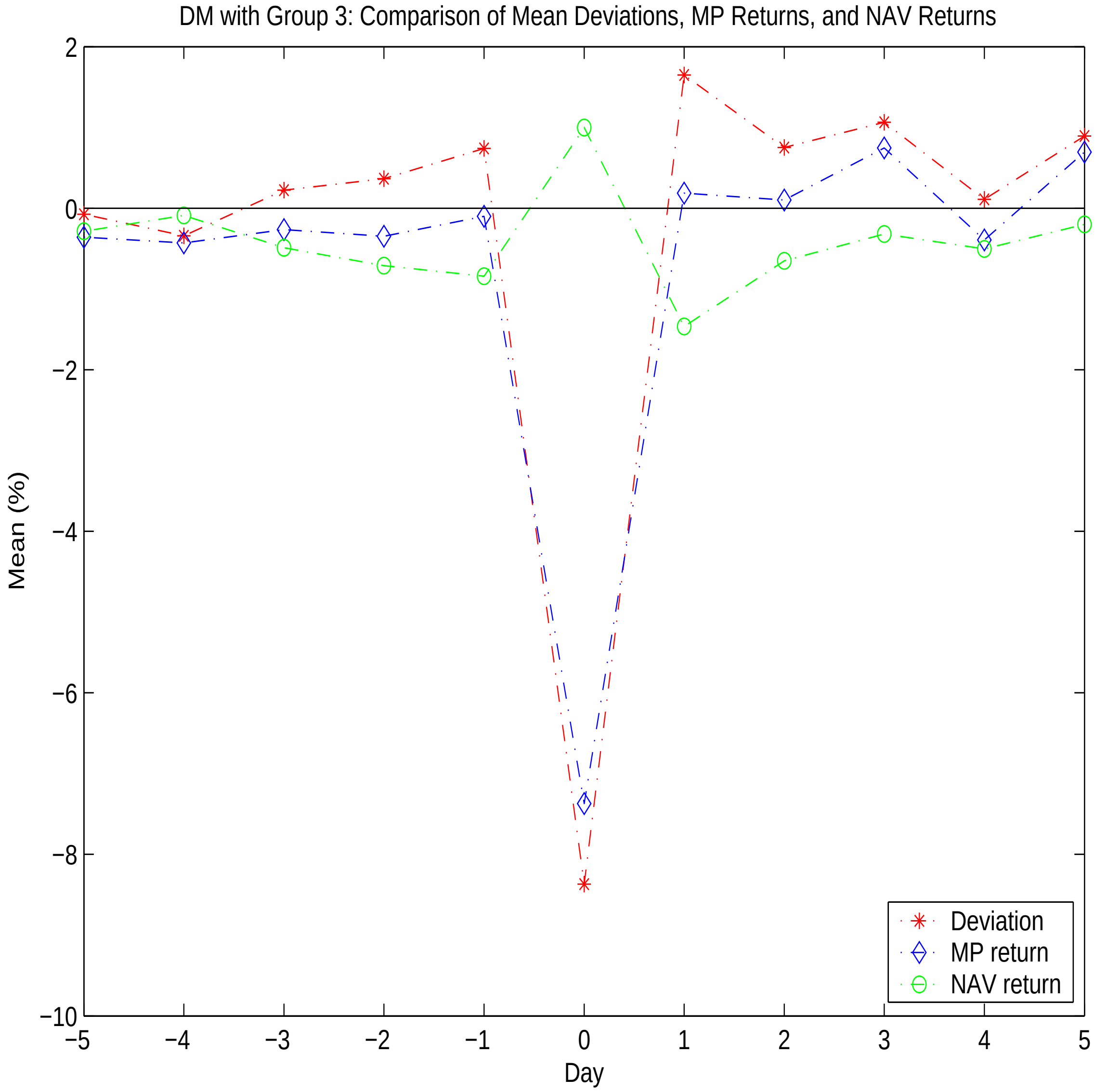

Figure 2.11: Pre-event relative optimism, relative pessimism on Day 0, and the post-event reversal in the high threshold level.

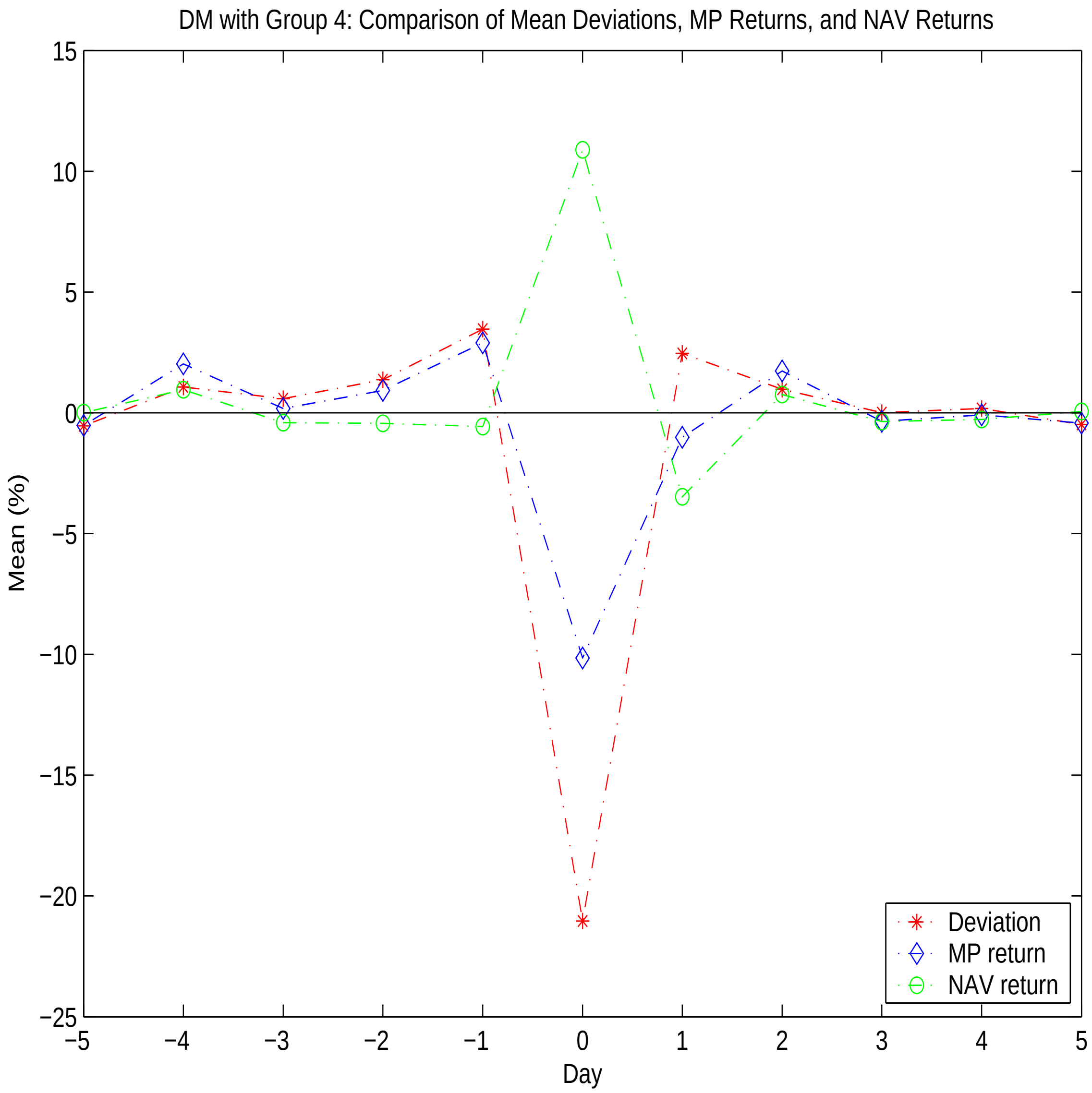

Figure 2.12: Relative pessimism on Day 0 in the very high threshold level with a small number of events ($n = 19$).

returns exhibit statistically significant pessimism relative to the percentage changes in NAV for this period.

In Table B2, after a four-day significant pre-day rise, the average deviation on Day 0 is -3.28%, close to that of positive events in magnitude, for the 1954 large negative events in the low threshold level. During the first two post-event days, there is statistically significant reversal. That is, MP returns show positive sentiment relative to the NAV returns for this period, while it is negative sentiment on Day 0.

2.2.3.2 Medium thresholds In Table B3, the average deviation on Day 0 is 5.95%, following two significant drops for the 196 large positive events in the medium threshold level. There is statistically significant two post-day reversal.

In Table B4, after two-day significant rise in relative optimism, the average deviation on Day 0 is -5.93%, close to that of positive events in magnitude for the 198 large negative events in the medium threshold level. During the first two post-event days, there is statistically significant reversal.

2.2.3.3 High thresholds In Table B5, the average deviation on Day 0 is 8.54% following two-day significant drop for the 48 large positive events in the high threshold level. Then, there is a statistically significant one day reversal. In other words, the relative positive sentiment on Day 0 is replaced by the negative sentiment subsequently.

In Table B6, the average deviation on Day 0 is -8.37% for the 41 large negative events in the high threshold level. On Day 1 and Day 3, statistically significant reversal takes place.

2.2.3.4 Very high thresholds In Table B7, the average deviation on Day 0 is 16.29% following two-day significant relative pessimism for the 27 large positive events in the very high threshold level. There is then a one day statistically significant reversal.

In Table B8, the average deviation on Day 0 is -21.04%, larger than that of positive events in magnitude, for the 19 large negative events in the very high threshold level. During the first four post-day, there is limited significant behavior due to the small sample size. Also, there may be price shock effects making the post-event reversals smaller in magnitude for the negative very high threshold levels. This suggests that the size of the threshold level on Day 0 affects the investor sentiment during the post-event days.

The statistically significant results thereby confirm Hypothesis 1, and reject both the null hypothesis and Hypothesis 2.

2.3 THE DEVIATION MODEL WITH PARTITION

In Section 2.2, we examined the spikes in the difference of daily MP returns and NAV returns. Now, we analyze the data by decomposing events into spikes in MP returns versus spikes in NAV returns. Partitioning in this way provides more detailed information.

The EMH involves another assumption, namely, that there is effectively an infinite amount of capital for arbitrage. An alternative set of ideas that explicitly utilizes the finiteness of assets of different groups has been the foundation of a mathematical approach to behavioral finance (see Caginalp and Balenovich [19] and references therein). This uses a price equation in which the transition between cash and asset can depend on other factors beyond valuation such as momentum trading (i.e., buying due to rising prices). Using models of this type, Caginalp et al. [18] were able to resolve some key issues in asset market experiments in which bubbles have been observed. One of the predictions of the differential equations has been that a larger bubble results if there is a larger total cash to asset ratio. Our current study allows us to test an important feature of this approach, namely the impact of finite assets, against the null hypothesis of EMH which stipulates infinite capital for arbitrage.

Hypothesis 3. Consider the subset of "events", (i.e., there is a significant deviation on Day 0) for which relatively little change occurs for NAV (as defined by BP1 in Section 3.1). Then on Day (-1) there is a deviation in the opposite direction.

In other words, suppose we consider the set of events in which there is little relative change in NAV on Day 0. If there is significant relative change in the market price on Day 0, what is the average change on Day (-1)? There is no reason for this to deviate from zero, according to **the default hypothesis of the EMH**. However, the asset flow approach in

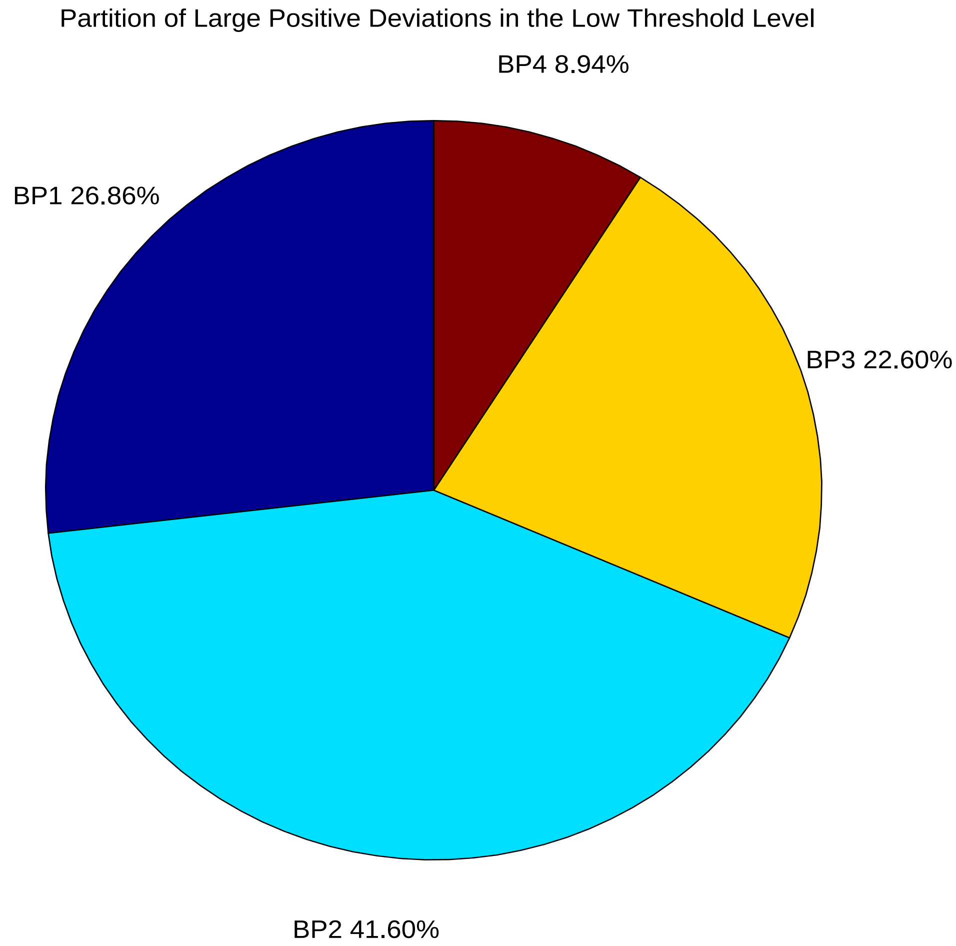

Figure 2.13: The percentage of large positive deviations influenced by large MP returns is 68.46% in the low threshold level.

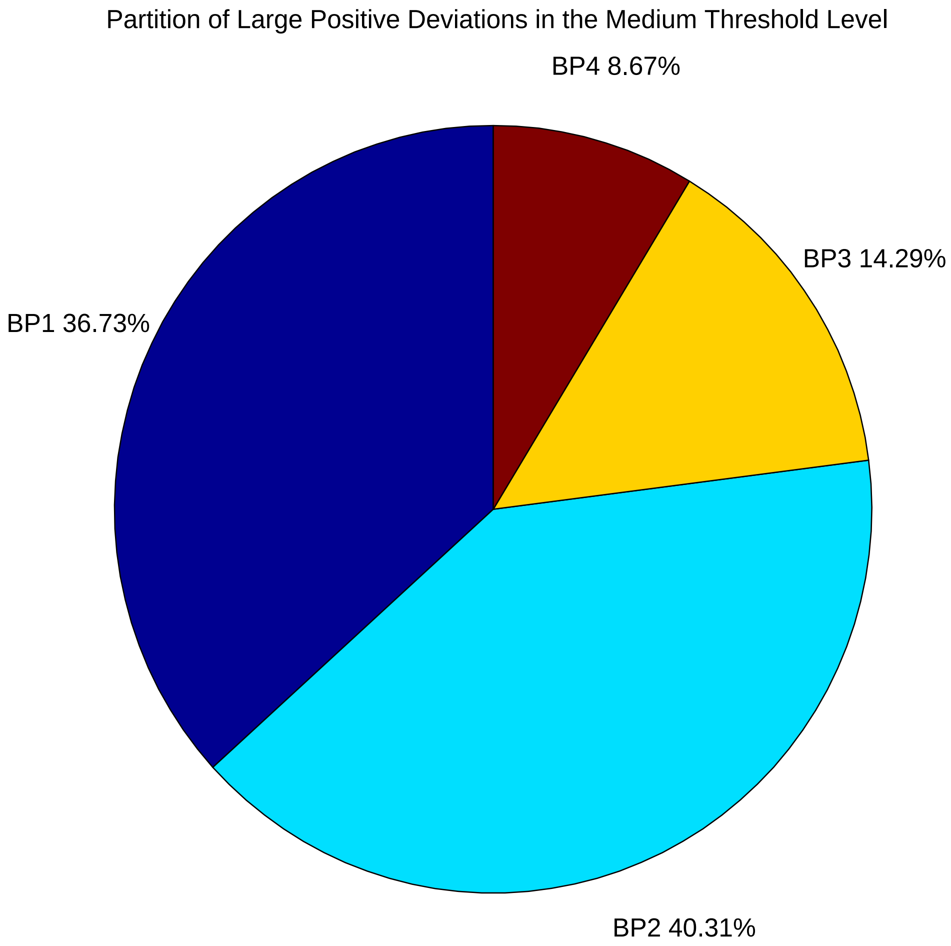

Figure 2.14: The percentage of large positive deviations influenced by large MP returns is 77.04% in the medium threshold level.

[19] stipulates that a cause of a significant change is the excess of cash that can be used to buy stock. If investors have an excess of cash due to net selling on Day (-1) there will be a significant rebound on Day 0.

2.3.1 Positive deviation with partition

Definition Let Ω_{RO} be the set of events for large positive deviations on Day 0. Then, a partition of Ω_{RO} is a collection $P_{RO} = \{BP_1, BP_2, BP_3, BP_4\}$ of nonempty subsets of Ω_{RO}, where BP_is are the blocks of the partition. They satisfy the following properties:

1. The blocks are pairwise disjoint
2. All of the Ω_{RO} is the union of the blocks.

In particular, we define "relatively unchanged" to mean that the change in one quantity is less than one-fifth of the other.

1. $BP_1 = \{$Large positive deviations | MP return spikes up while NAV is relatively unchanged on Day 0$\}$.
2. $BP_2 = \{$Large positive deviations | both MP return and NAV return are changed where the influence of MP return on Day 0 is greater$\}$.
3. $BP_3 = \{$Large positive deviations | both MP return and NAV return are changed where the influence of NAV return on Day 0 is greater$\}$.
4. $BP_4 = \{$Large positive deviations | NAV return spikes down while MP is relatively unchanged on Day 0$\}$.

Figure 2.13 and Figure 2.14 show that the vast majority of large positive deviations are influenced by large MP returns. Figure 2.15 - Figure 2.22 compare daily MP returns, NAV returns, and the deviations in the positive low and medium threshold levels for each block of partition on the 11-day window. Odd numbered tables from B9 to B39 represent the average deviations, MP returns, NAV returns, and reversals associated with large positive deviators of Day 0. The statistically significant results with the partitions BP1 and BP2 in the low and medium threshold levels and the partitions BP3 and BP4 in the low threshold level support Hypothesis 1, where the number of events is sufficiently large ($n \geq 30$). These subsets have

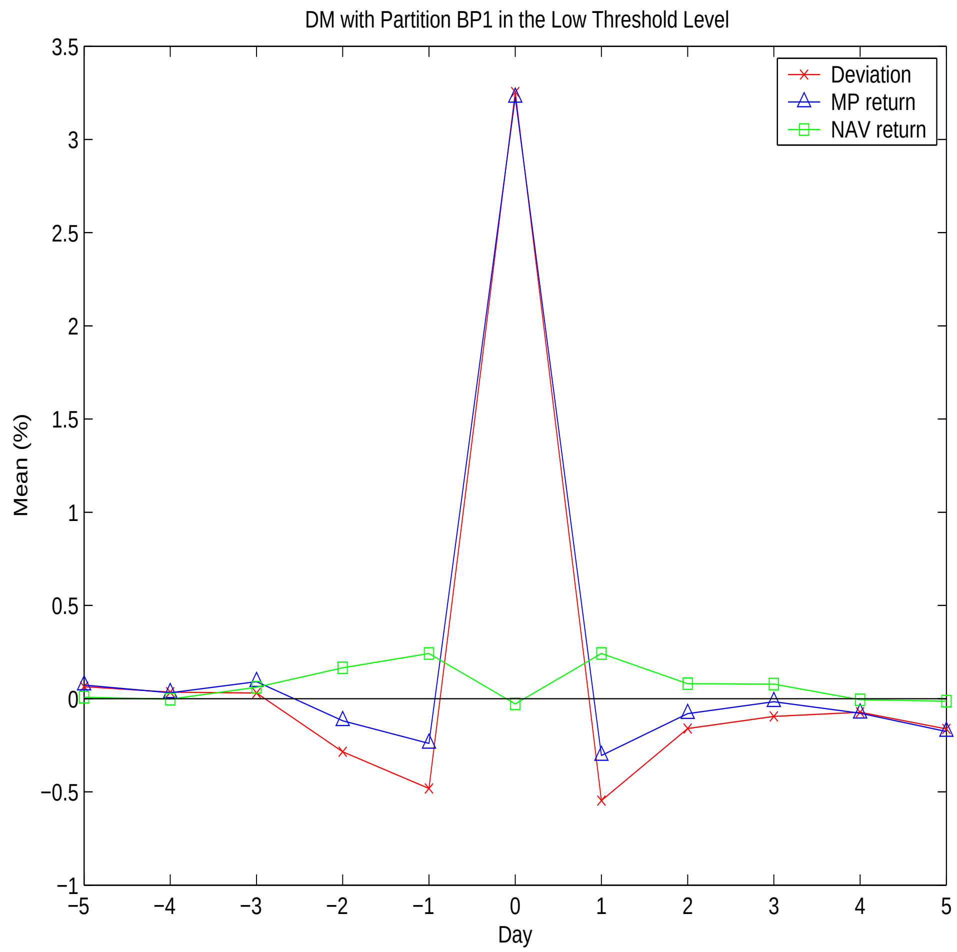

Figure 2.15: The comparison of daily MP returns, NAV returns, and the deviations shows overreaction upper diamond patterns for both deviation and MP return.

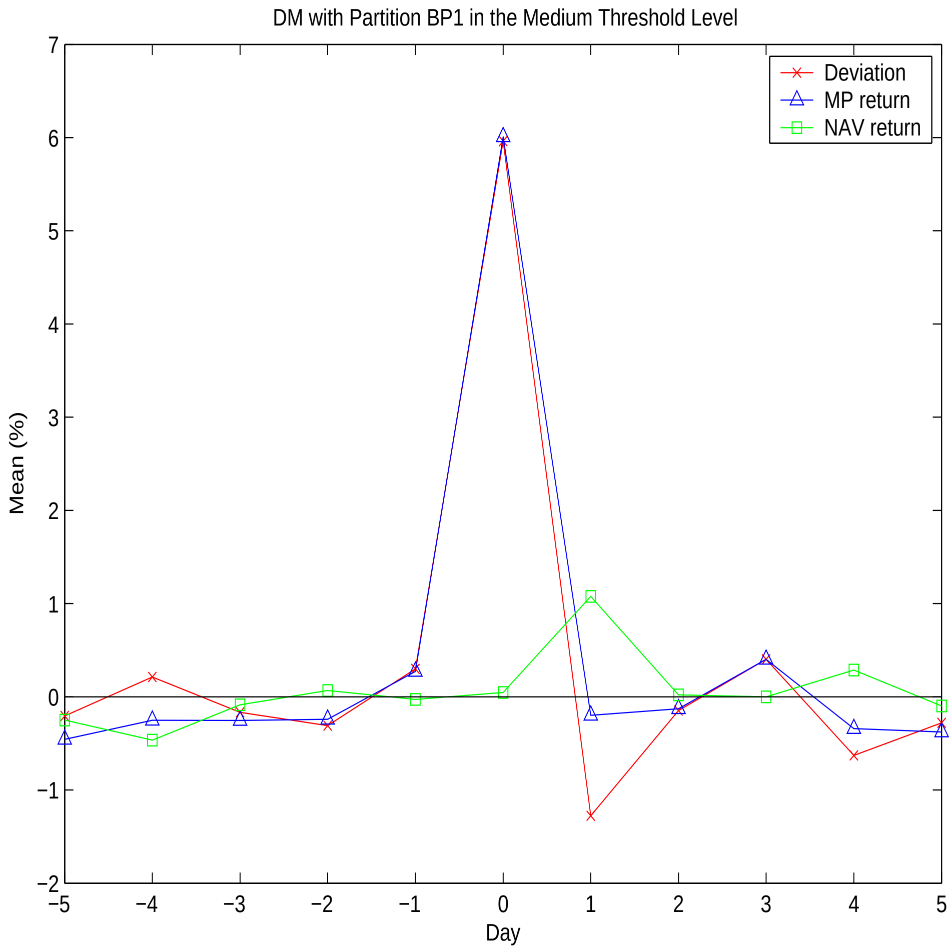

Figure 2.16: Comparison of daily MP returns, NAV returns, and the deviations in the positive medium threshold level.

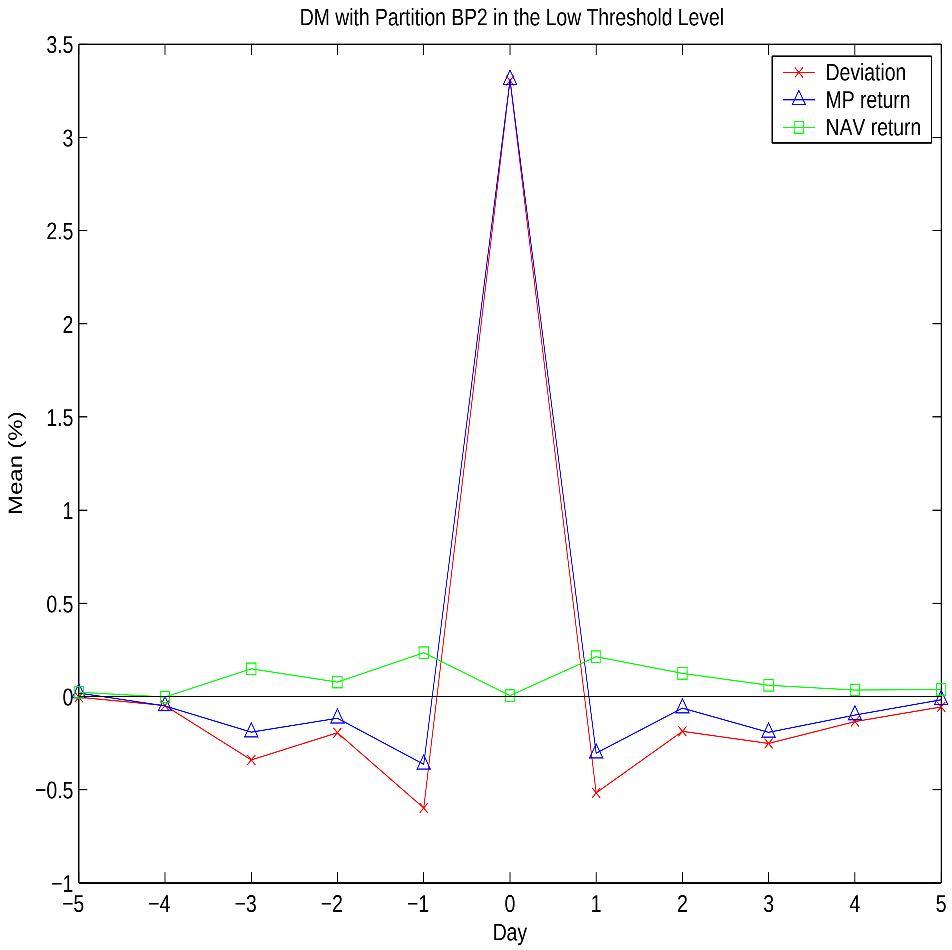

Figure 2.17: Comparison of daily MP returns, NAV returns, and the deviations in the positive low threshold level.

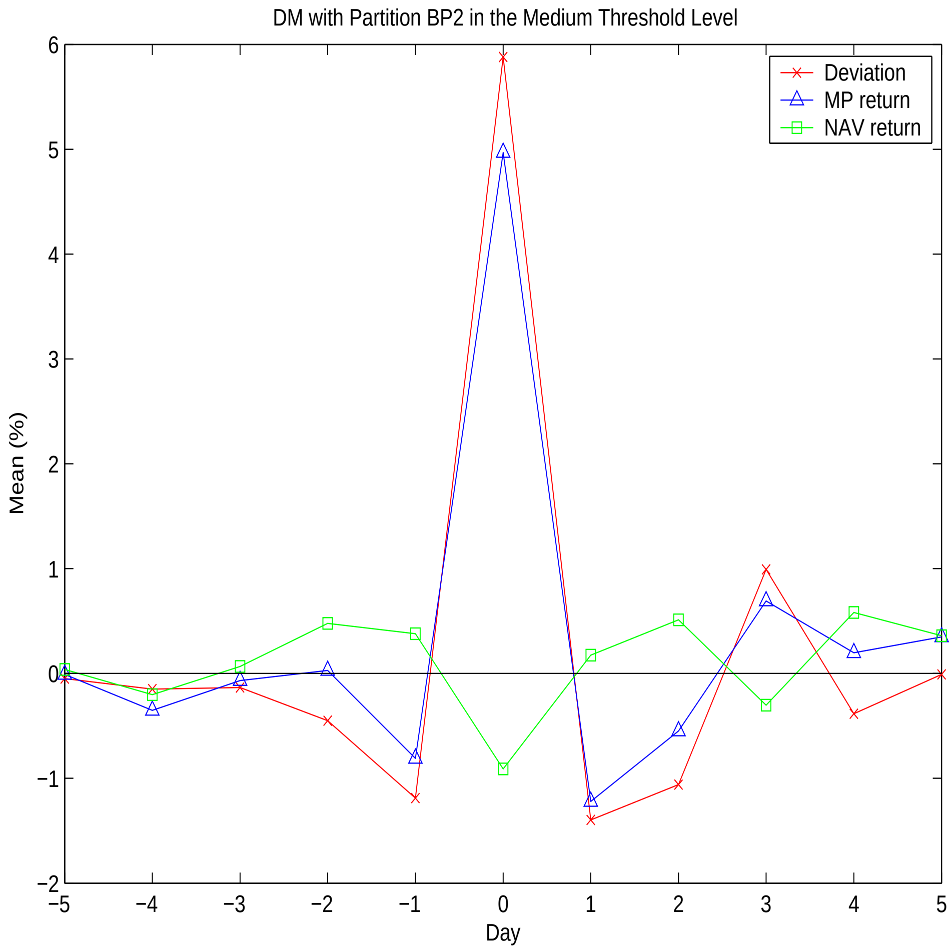

Figure 2.18: Comparison of daily MP returns, NAV returns, and the deviations in the positive medium threshold level.

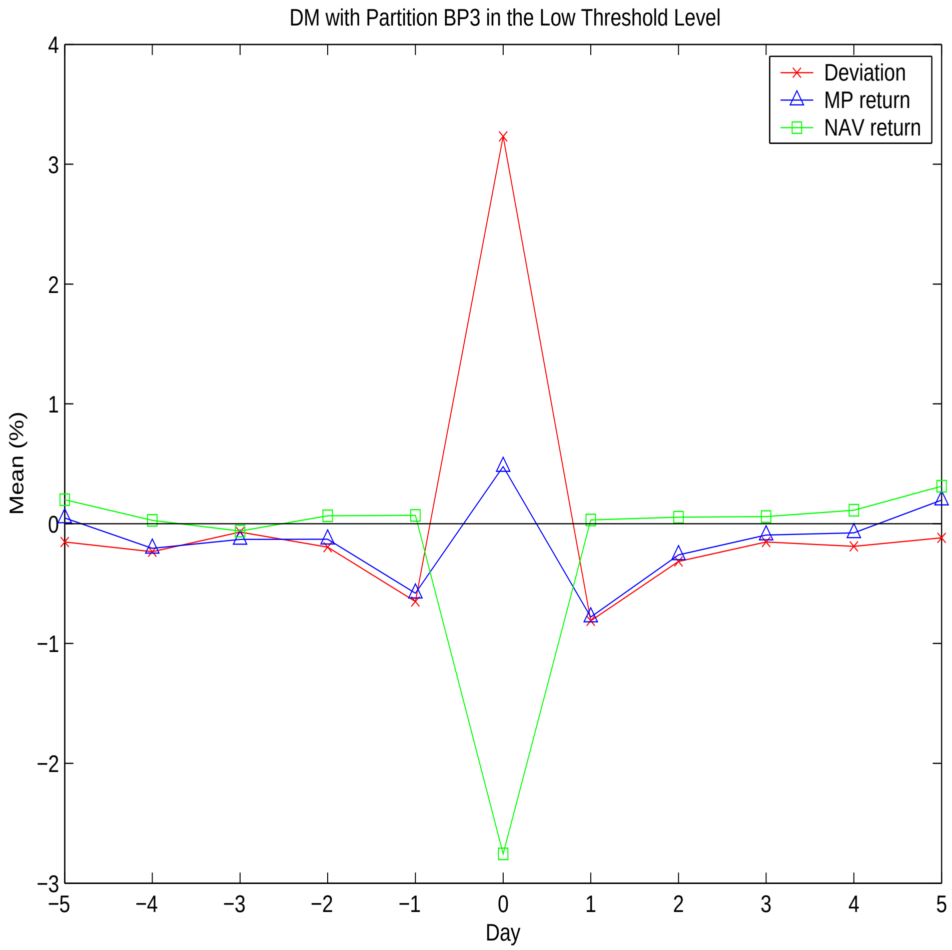

Figure 2.19: Comparison of daily MP returns, NAV returns, and the deviations in the positive low threshold level.

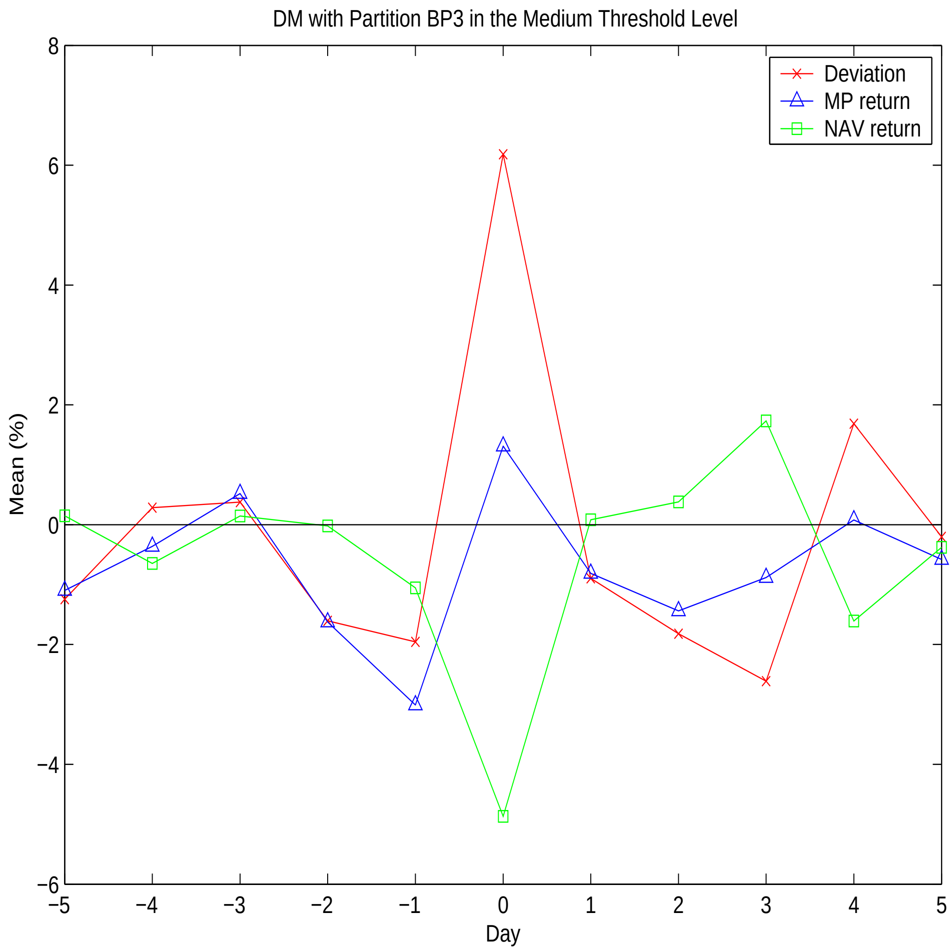

Figure 2.20: Comparison of daily MP returns, NAV returns, and the deviations in the positive medium threshold level with a small number of events ($n = 28$).

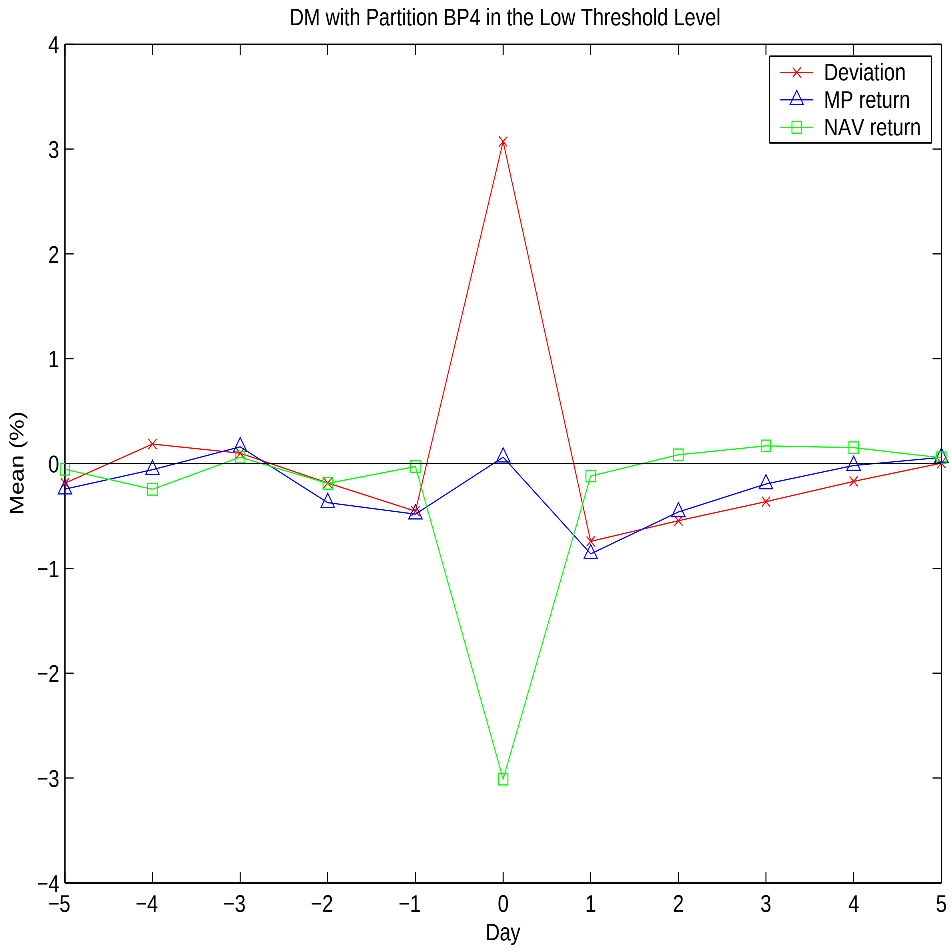

Figure 2.21: Comparison of daily MP returns, NAV returns, and the deviations in the positive low threshold level.

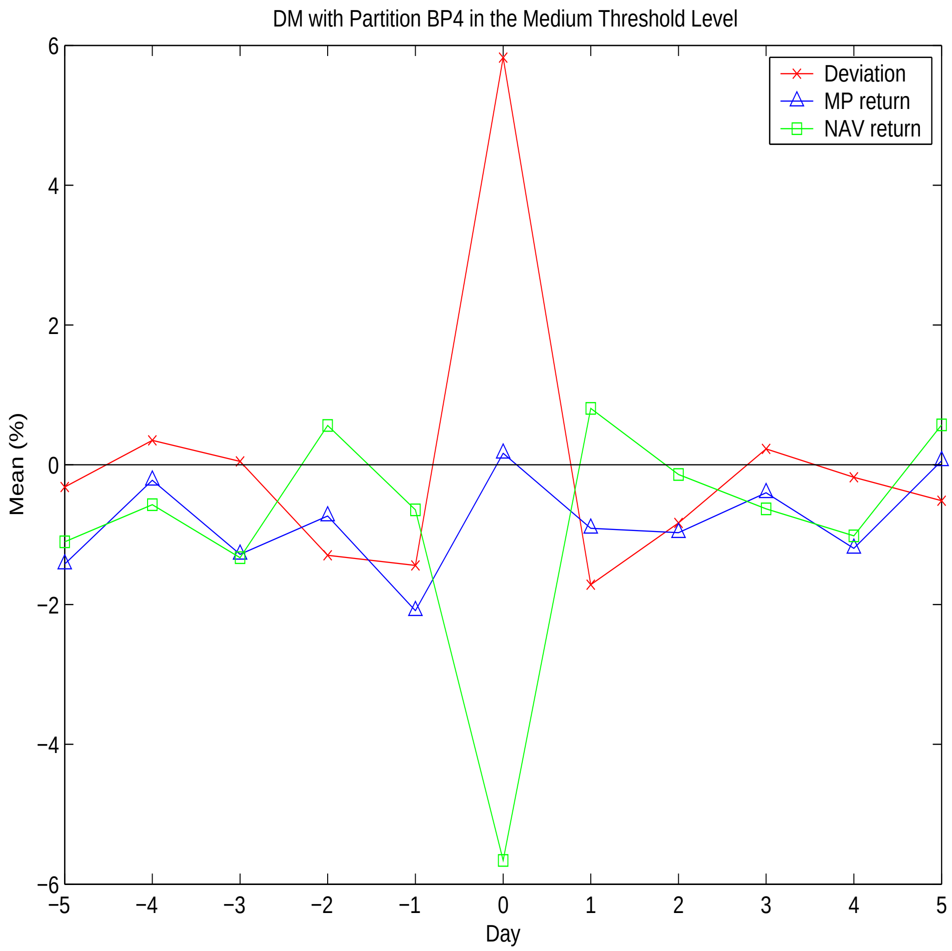

Figure 2.22: Comparison of daily MP returns, NAV returns, and the deviations in the positive medium threshold level with a small number of events ($n = 17$).

also statistically significant reversals in MP returns on Day 1. There are post-event reversals in the deviations for the other partitions also, but the number of events is small ($n < 30$) to make a conclusion.

Moreover, the subsets BP_1 in the low threshold level and BP_2 in the low and medium threshold levels confirm Hypothesis 3. All BP_is in the low threshold levels and BP_2 in the medium threshold level have statistically significant drop in MP return on Day (-1).

2.3.2 Negative deviation with partition

Definition Let Ω_{RP} be the set of events for large negative deviations on Day 0. Then, a partition of Ω_{RP} is a collection $P_{RP} = \{BN_1, BN_2, BN_3, BN_4\}$ of nonempty subsets of Ω_{RP}, where BN_is are the blocks of the partition. They satisfy the following properties:

1. The blocks are pairwise disjoint
2. All of the Ω_{RP} is the union of the blocks.

In particular,

1. BN_1 = {Large negative deviations | MP return spikes down while NAV is relatively unchanged on Day 0}.
2. BN_2 = {Large negative deviations | both MP return and NAV return are changed where the influence of MP return on Day 0 is greater}.
3. BN_3 = {Large negative deviations | both MP return and NAV return are changed where the influence of NAV return on Day 0 is greater}.
4. BN_4 = {Large negative deviations | NAV return spikes up while MP is relatively unchanged on Day 0}.

The results are similar to the positive deviations of the previous section and are displayed in Figures 2.23-2.32 and even numbered tables from B10 to B40. The statistically significant results with the partitions again confirm Hypothesis 1. Moreover, the subsets BN_1 in the low threshold level and BN_2 in the low and medium threshold levels confirm Hypothesis 3.

Furthermore, as the influence of NAV return on Day 0 increases (from BN_1 to BN_4), the magnitude of reversal in the MP return on Day 1 increases in the low threshold level.

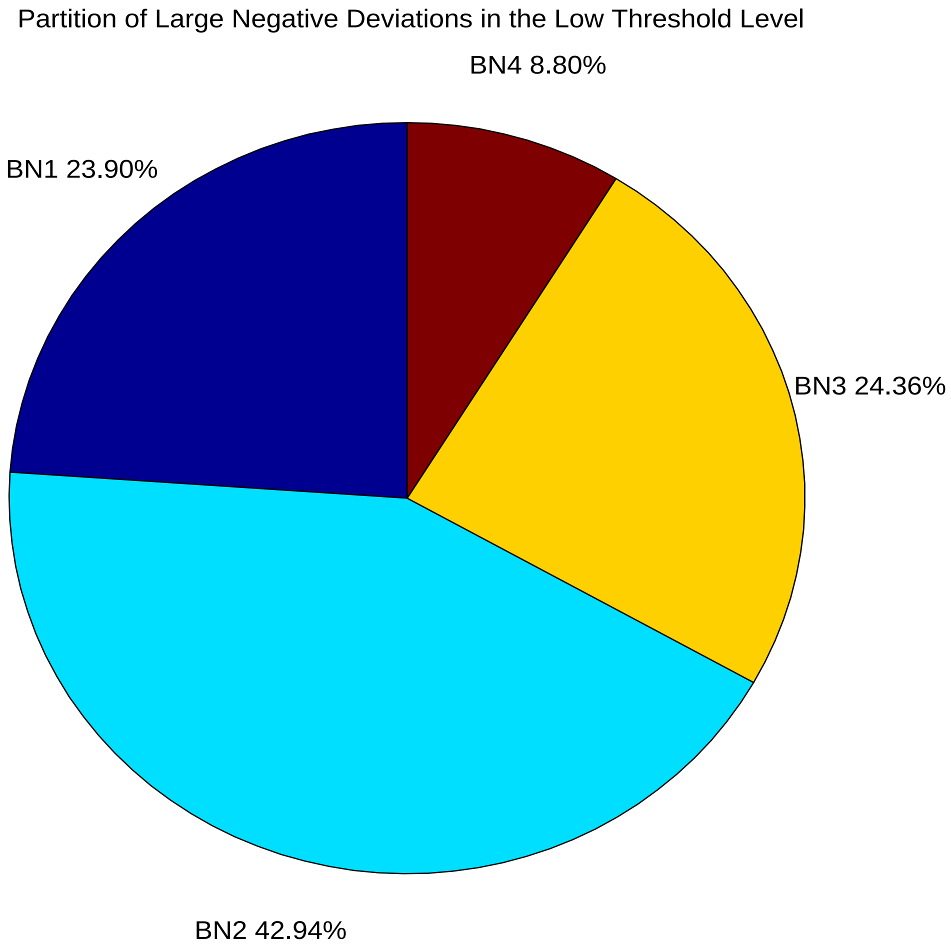

Figure 2.23: The percentage of large negative deviations influenced by large MP returns is 66.84% in the low threshold level.

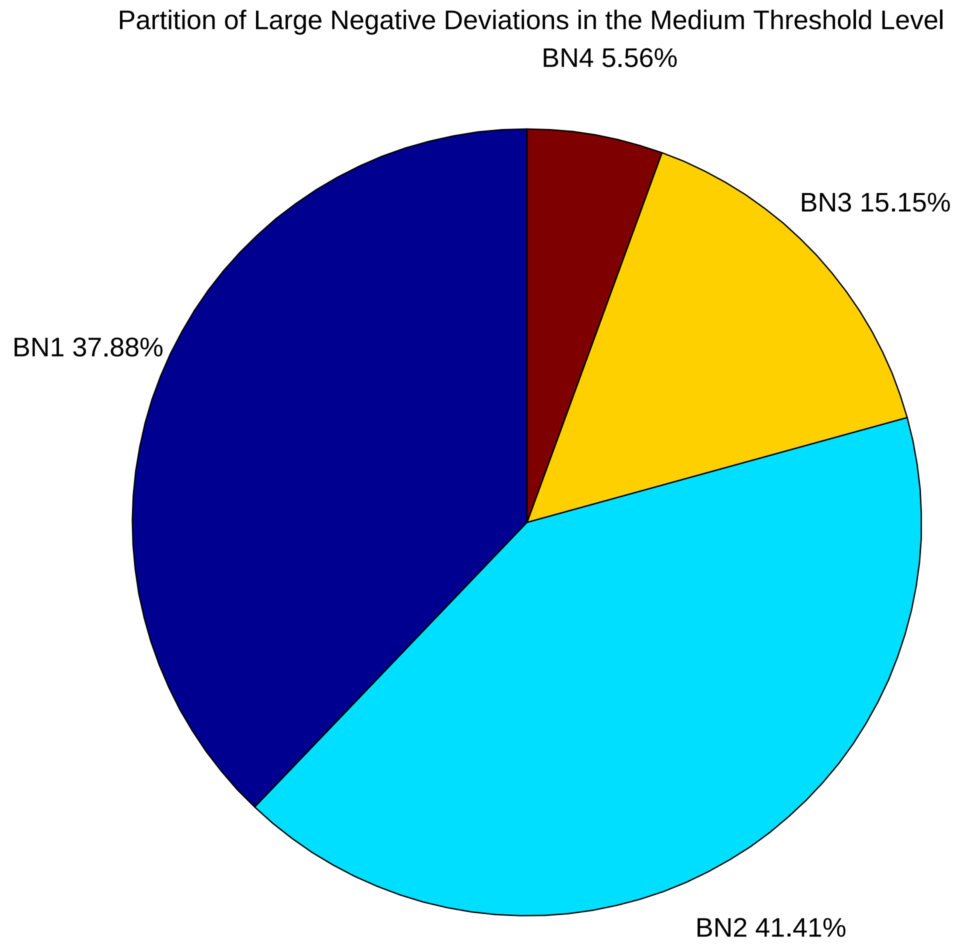

Figure 2.24: The percentage of large negative deviations influenced by large MP returns is 79.19% in the medium threshold level.

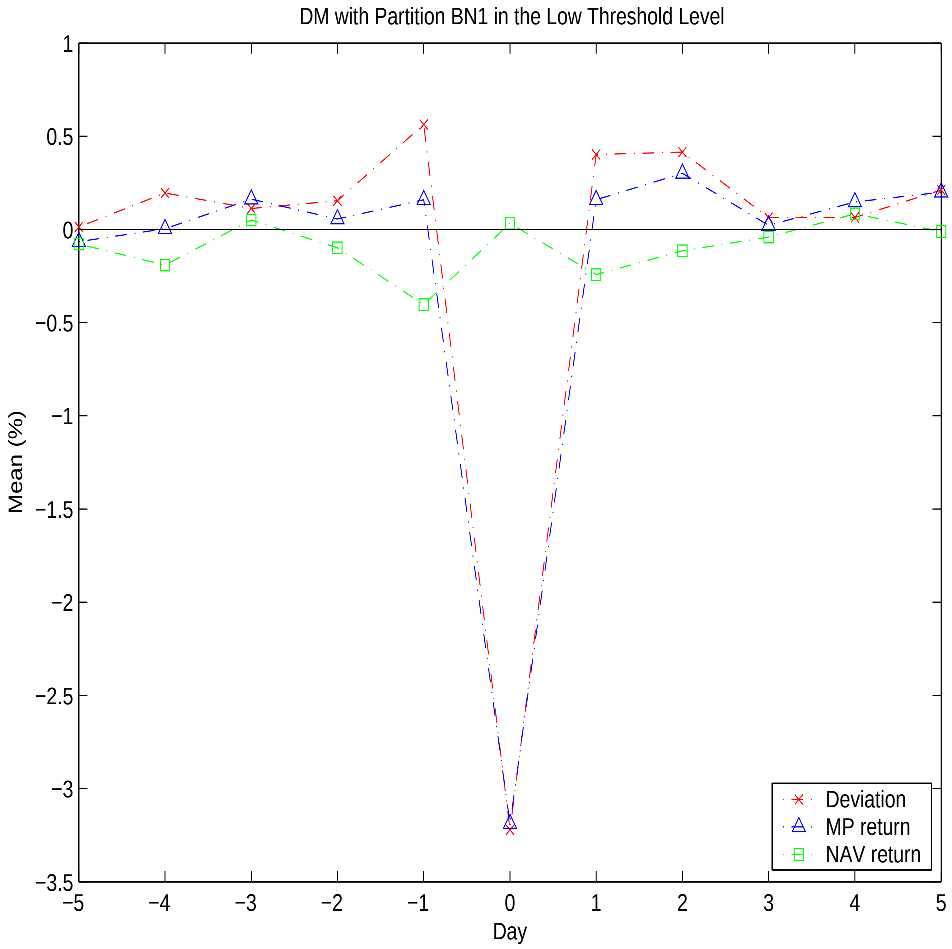

Figure 2.25: Comparison of daily MP returns, NAV returns, and the deviations in the negative low threshold level.

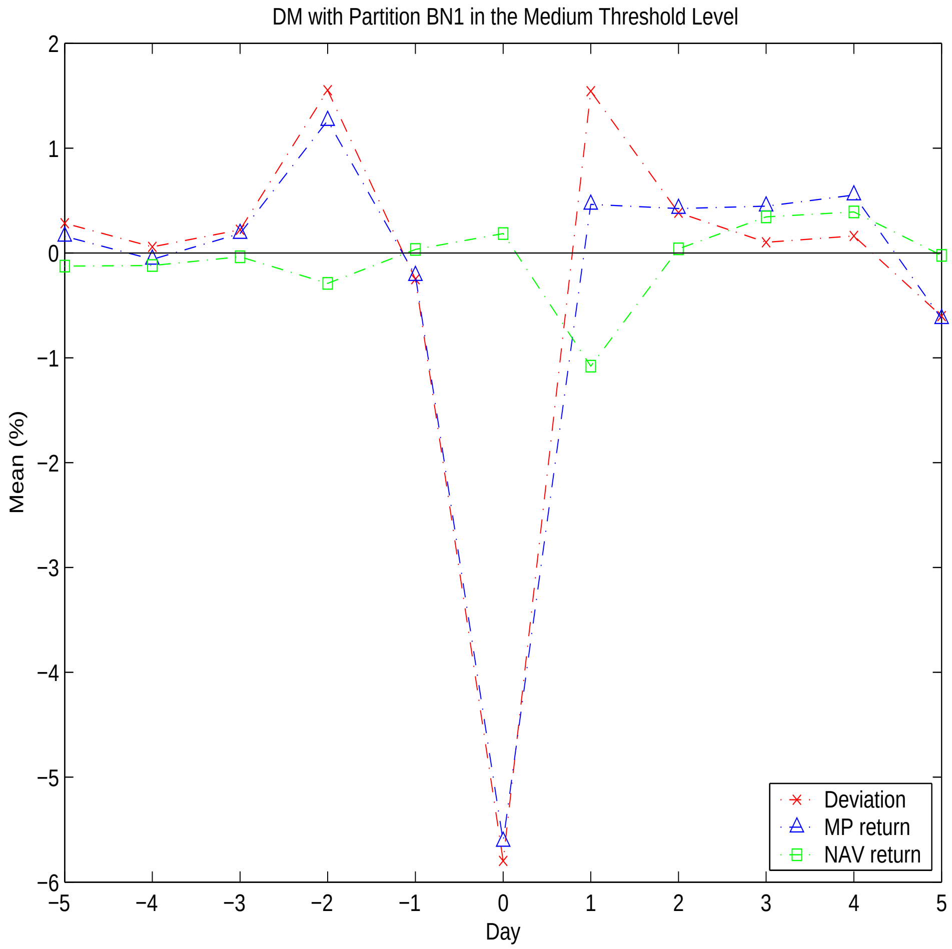

Figure 2.26: Comparison of daily MP returns, NAV returns, and the deviations in the negative medium threshold level.

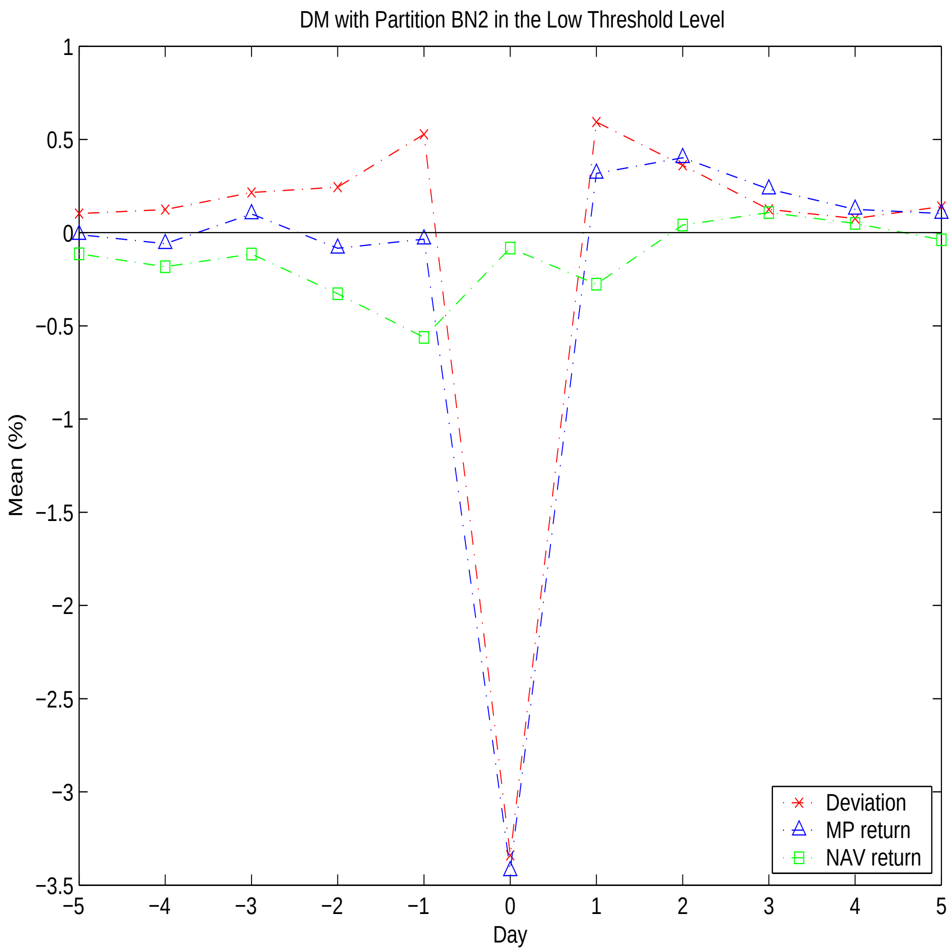

Figure 2.27: Comparison of daily MP returns, NAV returns, and the deviations in the negative low threshold level.

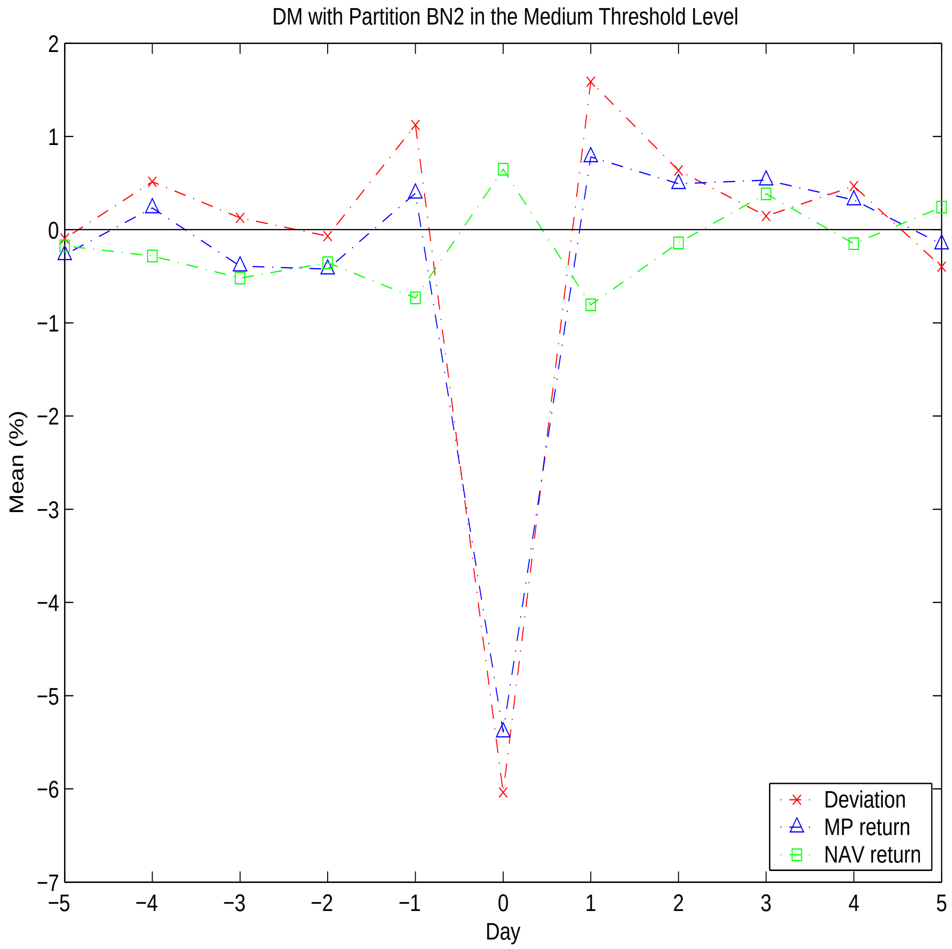

Figure 2.28: Comparison of daily MP returns, NAV returns, and the deviations in the negative medium threshold level.

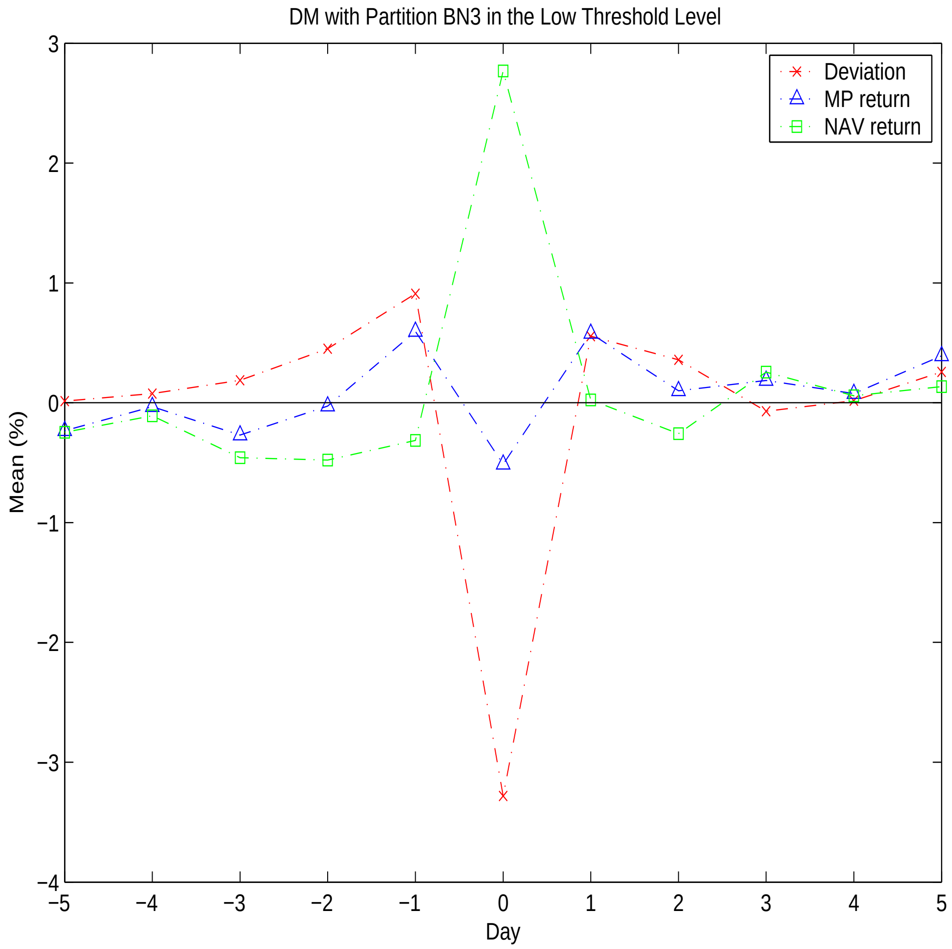

Figure 2.29: Comparison of daily MP returns, NAV returns, and the deviations in the negative low threshold level.

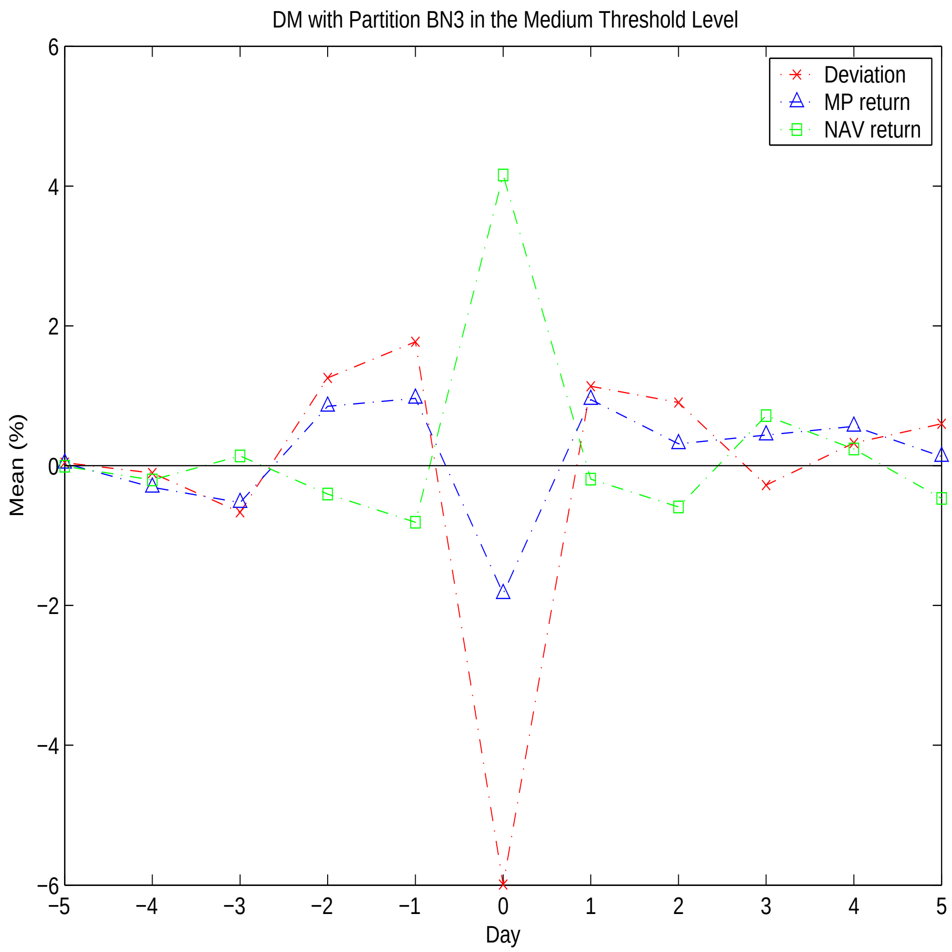

Figure 2.30: Comparison of daily MP returns, NAV returns, and the deviations in the negative medium threshold level.

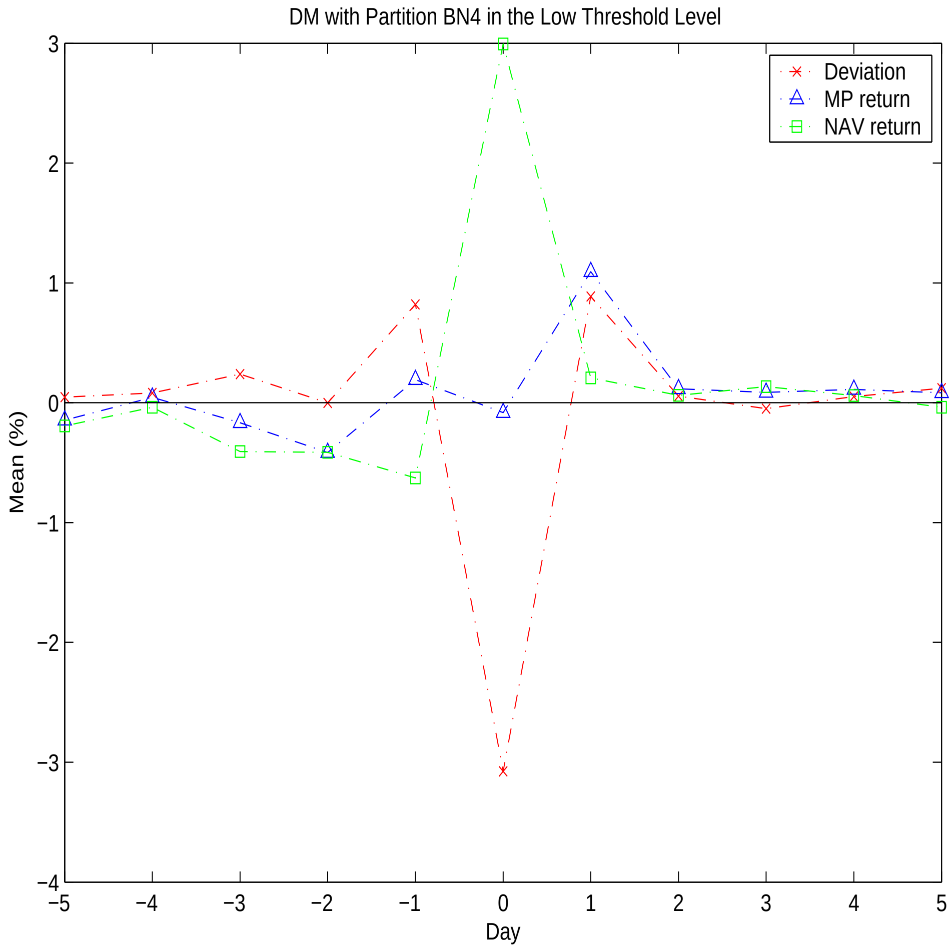

Figure 2.31: Comparison of daily MP returns, NAV returns, and the deviations in the negative low threshold level.

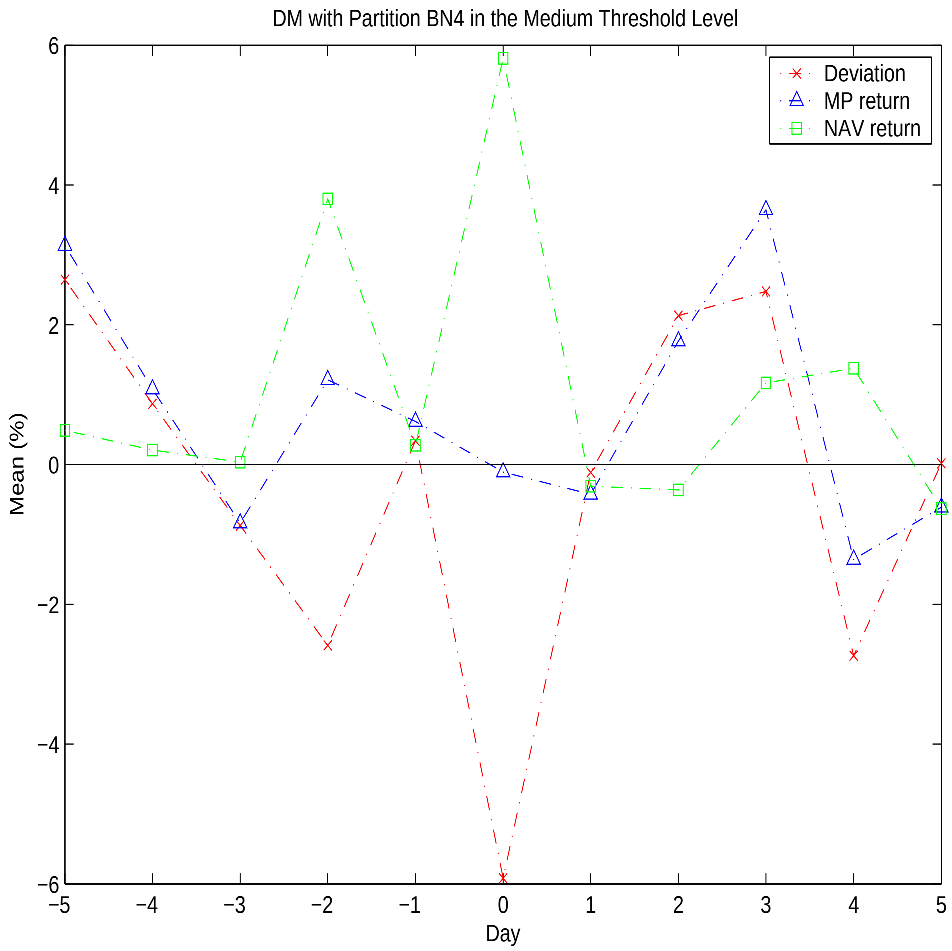

Figure 2.32: Comparison of daily MP returns, NAV returns, and the deviations in the negative medium threshold level with a small number of events ($n = 11$).

2.4 ILLUSTRATIVE EXAMPLES AND APPLICATION: BUBBLE

Overreaction played an important role in some recent financial market bubbles, and the methodology of our study examines this effect qualitatively and quantitatively.

The causes of the internet/high-tech bubble of the late 1990ś and the subsequent collapse are yet to be fully resolved.

One way to study bubbles is through experimental economics [41] where a bubble is defined as trading at prices above the fundamental value of an asset. Porter and Smith [41] summarize the results of laboratory asset market bubbles and discuss the effect of proposed changes in the asset market environment and institution to diminish bubbles. Caginalp, Porter and Smith [16, 17] report on a large number of laboratory market experiments indicating that a market bubble can be reduced under the following conditions: 1) a low initial liquidity level, 2) deferred dividends, and 3) a bid-ask book that is open to traders. Caginalp and Ermentrout [21] proposed a complete dynamical system for investor behavior resulting in a system of ordinary differential equations. The model assumes that investors have preferences based on a trend-based component or a fundamental value component. Too much emphasis on the price derivative (momentum) will generally result in large bubbles and subsequent crashes [19]. In the laboratory experiments of asset markets, one usually observes an initial period trading price, that is well below the realistic value, followed by rising prices that overshoot the fundamental value in the intermediate periods, resulting a characteristic "bubble" and a dramatic "crash" of prices near the end of the experiment.

Bubbles in the world's financial markets share many of the features of experimental asset bubbles. During the first half of the 1990's, some favorable developments, such as the end of the long Cold War, accelerated U.S. productivity growth, the inventions of the World Wide Web and the Internet browser, rapid commercialization of the Internet, and widespread use of computer networks and databases, stimulated interest in the stock market. The optimism of stock market investors encouraged entrepreneurs and firm managers to invest in capital assets. Consequently, overinvestment and malinvestment became common during the late 1990s [46]. The final movement in the stock market bubble appeared in telecommunications and information technology equipment manufacturing stocks, following the dot.coms peak in 1999. Twelve of the top 20 U.S. corporations by market capitalization were technology-related firms, and six of them had very high price earnings ratios in excess of 100. The Internet/high-tech bubble burst in the first quarter of 2000. The prices of information technology and telecommunications stocks experienced a steep drop during early 2000.

The concept of price changes based solely upon the classical self-optimization of agents is not enough to understand the recent internet/high-tech bubble. Schiller [47] examines the rise and fall of the internet/high-tech bubble and discusses the manner in which people had projected a relatively brief trend into the distant future.

The "overreaction diamond" pattern [27] has been shown to be statistically significant in our data set, and has demonstrated the systematic behavioral bias exhibited by the market price in relation to its fundamental value. This leads to the question of whether the methodology can be utilized as a tool in out-of-sample forecasts. The overreactions to positive developments in assessing the value of companies may be an important factor particularly in the emergence of the initial stage of a bubble that is subsequently aggravated by momentum trading (i.e., focus on market price increases). In many market situations the peak of a bubble is particularly frustrating to those who are attempting to time a market that is overvalued. As prices rise above fundamental value, traders who would like to exploit this overvaluation– as the efficient market hypothesis suggests they should – often find prices moving even further above the fundamental value. At some point the practical constraints (e.g., margin requirements) force them to buy the asset, which is now even more overvalued, at higher prices, thereby enhancing the bubble. Thus the issues of understanding the stages of the bubble are of enormous practical consequence, and overshadow such academic considerations as whether the trader motivations are rational or irrational – an issue that is almost philosophical by comparison.

Toward this end we consider a sample from our data set. We choose one with a particularly large bubble, namely the Spain Fund, to determine whether the daily deviations between the market price and net asset value have the potential to provide an indication of the peak of the bubble [28]. Figure 2.34 displays the cumulative market price changes and the cumulative NAV changes together with the difference, i.e., the cumulative daily deviations, which illustrates the emergence, expansion and bursting phases of the bubble. Of

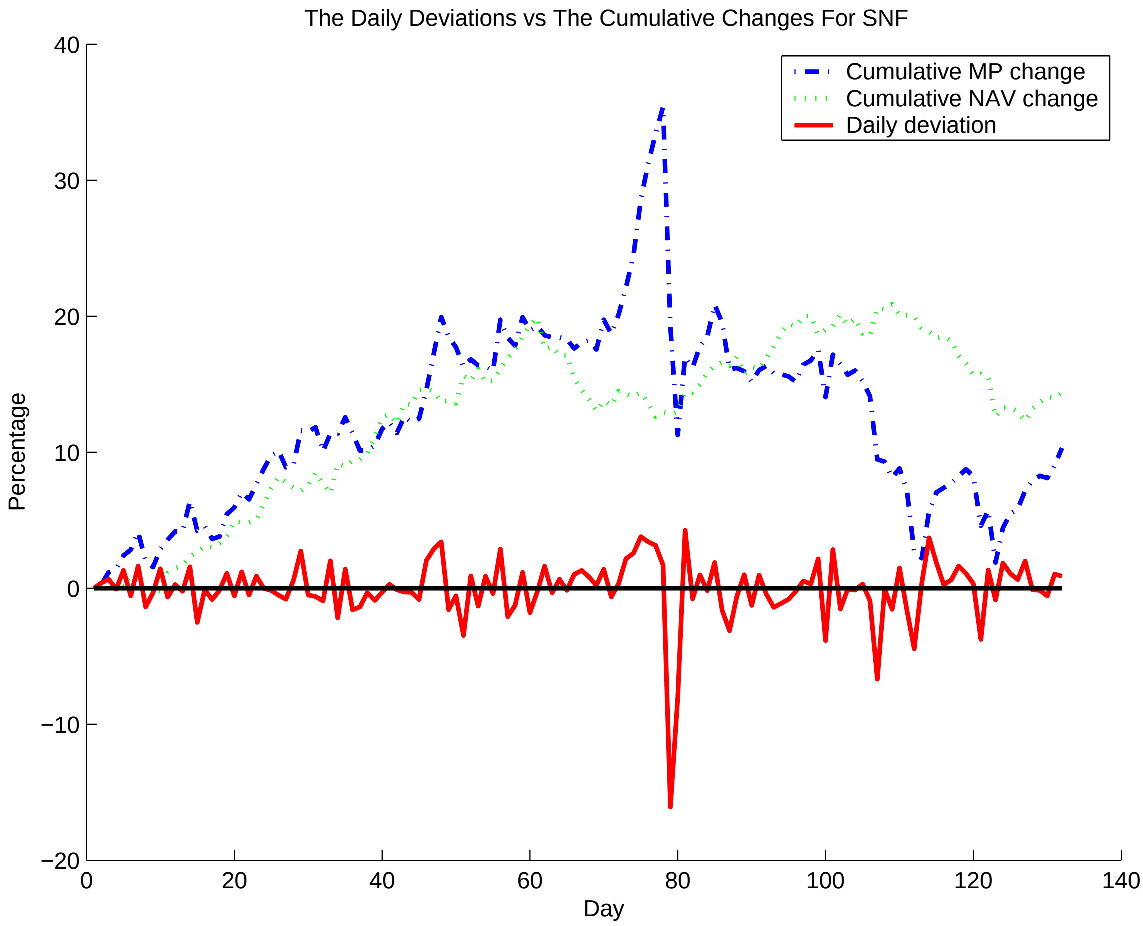

Figure 2.33: The daily deviations indicating many short term overreactions versus the cumulative MP and NAV changes in percent for SNF, between October 1, 2004 and April 13, 2005.

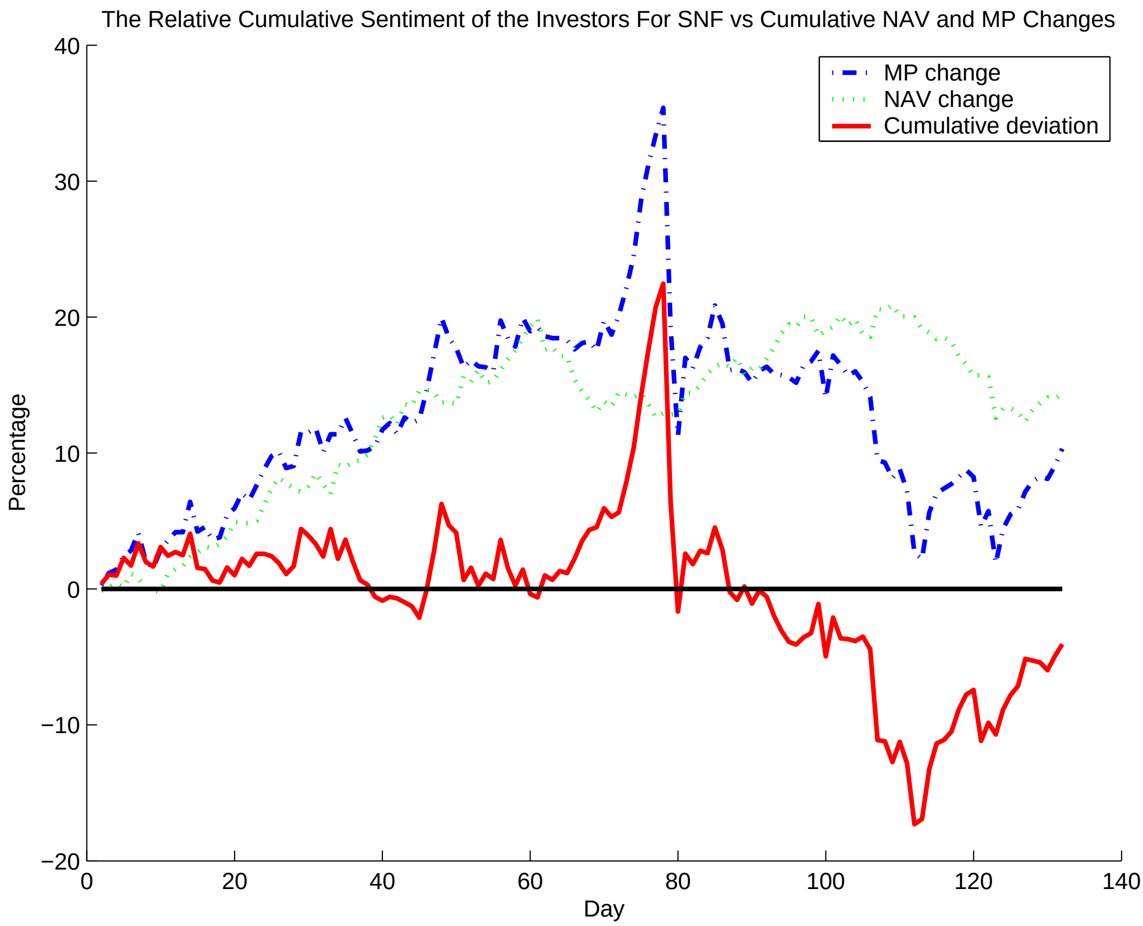

Figure 2.34: The cumulative daily deviations versus the cumulative MP and NAV changes in percent for SNF, between October 1, 2004 and April 13, 2005.

course, in retrospect it is clear when the bubble burst. However, examining the graph up to (but not beyond) the peak of the bubble, it is not as obvious that the market price would not move higher from this point.

Alternatively, as shown in Figure 2.33, we examine the cumulative market price change and the cumulative fundamental changes with simply the daily (not cumulative) deviations. The daily deviations remain within a fairly narrow band of a few percent throughout the 78 days of the sample, even though there is a substantial bubble (i.e., the cumulative difference becomes very large) between days 70 and 78, when it is overvalued by more than 20%. However, on Day 79 the daily deviation breaks clearly beyond this band and signals that the peak has been attained. Examining the period just before the peak we see that the integral of the daily deviations appears to be at the largest value, since the values are in the positive region for a number of consecutive days. In previous parts of the graph there is much more noise, in that positive values are followed by negative values.

In order to examine this possibility further, we return to the cumulative daily deviations which we denote by $f(t)$, displayed in Figure 2.34, and consider the integral $I(f)$, which is a measure of the magnitude of the bubble in that it sums the products of overvaluation and time. We examine this integral over time intervals with the lower limit t_0 defined by $f(t_0) = 0$ and upper limit t_1 defined by the next time at which $f(t_1) = 0$. A possible criterion for the peak of a bubble would be to establish a mean and standard deviation for these integrals, along with $f(t)$ itself and the daily deviations. A signal that the bubble is near its peak occurs when both $f(t)$ and $I(f)$ are outside of the 95% confidence interval of their respective means.

Out-of-sample tests on this idea can be performed on a large data set such as the one we have used. One can approximate $I(f)$ as $I_n(f)$ by using Newton-Cotes integration formulas [4] or some automatic numerical integration programs such as CADRE (Cautious Adaptive Romberg Extrapolation) and DQAGP from QUADPACK package.

3.0 DIFFERENTIAL EQUATIONS AND COMPUTATIONAL OPTIMIZATION WITH FINANCIAL APPLICATIONS

3.1 OPTIMIZATION PROBLEM

We use a nonlinear computational optimization technique successively to evaluate the vector K of four parameters ($c1,\ q1,\ c2,\ q2$) in the Caginalp Differential Equations (CDEs) (see [21] and [19]). That is, the inverse problem of parameter identification is converted into an optimization problem to minimize a function in four variables by using nonlinear least-square curve fitting via CDEs. We try to employ most of the data up to any given time in order to choose the parameters optimally. Then, we make a forecast for the next few days and compare the forecasts with the actual values.

In practice, optimization problems may have several local solutions. However, optimization methods which seek global minima can confuse whether a point K^* that has been found is a local minimum or a global minimum. There is no strategy that will guarantee the number of necessary iterations to discover the neighborhood of the global optimum (see [5], Chapter 23). Therefore, we use an initial parameter pool which has fixed initial vectors initially for each fund. The second part of the pool is updated via previously found optimal parameters and specific to the fund's price behavior. Then, we pick the minimum of the resulting relative minimum functional values and the corresponding optimal parameter to be used for the next day return prediction [24].

After presenting the proposed optimization algorithm in this chapter, we discuss the out-of-sample daily return prediction in Chapter 4.

3.1.1 The system of Caginalp differential equations (CDEs)

3.1.1.1 Notation

$P(t)$: The market price of the single asset.

$\frac{1}{P}\frac{dP}{dt}$: The relative price change.

$P_a(t)$: The fundamental value.

$V(t)$: The NAV price at time t.

k : The transition rate.

M : All the cash in the system.

N : The total number of shares.

$L := \frac{M}{N}$: The liquidity value. L is a fundamental scale for price.

B : The fraction of total funds in the asset.

$\zeta_1(t)$: The trend-based component of the investor preference.

$\zeta_2(t)$: The value-based component of the investor preference.

$\zeta(t)$: The investor sentiment function, which expresses the tendency to buy or sell. $\zeta := \zeta_1 + \zeta_2$.

3.1.1.2 Assumptions

(A) The demand D is the total cash supply times the transition rate k, or the probability that one unit of cash will be used to place an order. The supply S is $1-k$ times the fraction of total funds in the asset.

$$D = k(1-B), \quad S = (1-k)B, \quad \frac{D}{S} = \frac{k}{1-k}\frac{1-B}{B} \tag{3.1}$$

(B) The transition rate k is a weighted sum of the **current derivative** and **the valuation discount**,

$$k(t) = \frac{1}{2} + \frac{1}{2}\tanh(\zeta), \quad \zeta := \zeta_1 + \zeta_2, \tag{3.2}$$

where

$$\zeta_1 = \frac{q_1\tau_0}{P}\frac{dP}{dt}, \quad \zeta_2 = q_2\frac{P_a(t) - P(t)}{P_a(t)} \tag{3.3}$$

(C) The relative price changes linearly with excess demand

$$\frac{\tau_0}{P}\frac{dP}{dt} = \frac{D}{S} - 1 \tag{3.4}$$

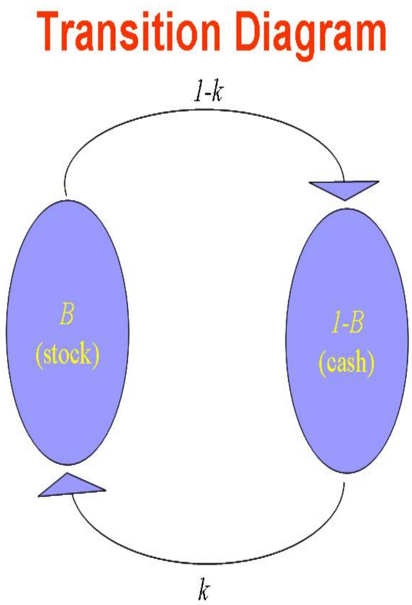

Figure 3.1: Transition.

3.1.1.3 CDEs The finiteness of assets, preference influenced by price momentum, and valuation (preference influenced by discount from fundamental value) are among several factors determining the price of an asset and its time evolution. "In the absence of a clear focus on fundamentals, the prices evolve into the liquidity value. Too much emphasis on the price derivative can generally result in larger bubbles and subsequent crashes. (see [19])"

The dependence of traders' preference on the price derivatives, deviation from fundamental value and the finiteness of traders' assets are involved in the following CDEs which are first published in [21] and improved in [19]. The subtraction of an exponential moving average value of a discount for the last equation is considered in [14].

1. The price equation:

$$\frac{dP}{dt} = Pr(\frac{k}{1-k}\frac{1-B}{B}) \tag{3.5}$$

where r is an increasing function satisfying that $r(1) = 0$ and taken as $r(x) = log(x)$.

2. "The finiteness of traders' asset" equation:
$$\frac{dB}{dt} = k(1-B) - (1-k)B + B(1-B)\frac{1}{P}\frac{dP}{dt} \tag{3.6}$$
B changes as the asset is bought and sold (the first two terms), and as the price changes (the last term). (See Figure 3.1)

3. Transition rate equation:
$$k(t) = \frac{1}{2} + \frac{1}{2}\tanh(\zeta_1 + \zeta_2) \tag{3.7}$$

4. "Change of trend-based component":
$$\frac{d\zeta_1}{dt} = c_1(\frac{q_1}{P}\frac{dP}{dt} - \zeta_1) \tag{3.8}$$

5. "Change of value-based component":
$$\frac{d\zeta_2}{dt} = c_2(q_2 A(t) - \zeta_2) \tag{3.9}$$

where $\sum_{k=1}^{10} e^{-0.25k} = 3.2318$ related to the normalization and the relative valuation change $A_t = \frac{V_t - P_t}{V_t} - \{\sum_{k=1}^{10}(3.2318)^{-1}\frac{V_{t-k}-P_{t-k}}{V_{t-k}}e^{-0.25k}\}$ for our discrete implementation. q_2 is multiplied by $A(t)$ which is the difference between the discount at t and the exponentially weighted average value of a discount that persists. For example, the discount for APB is often around 10%. The fact that the discount is 10% today does not mean people are eager to buy it due to undervaluation. However, if it goes to a 20% discount then some people look at that as bargain. Similarly, the Templeton Russia fund is usually at a 25% premium, so that a 10% premium looks like an undervaluation.

3.1.1.4 The functionality of the parameters The parameters $c1$, $q1$, $c2$, and $q2$ (see [19]) are the only parameters on the system of price evolution besides the scaling of time.

- c_1^{-1} is a measure of the "memory length". The numerical computations show that a very large value of c_1 may lead to unstable oscillation.
- Increasing q_1 tends to increase the importance of trend-based investing and amplitude of oscillations.
- A large value for c_2 means that investors take action very quickly when there is an over- or under-valuation.
- Increasing q_2 tends to drive prices closer to the fundamental value $P_a(t)$.

3.1.2 Non-linear least-square techniques with initial value problem (IVP) approach

Suppose we have a sequence of observed daily market prices $Y(K, t_i)$, $i = 1, .., m$ at times $t_1, .., t_m$. We solve the IVP (3.10) with CDEs (3.5-3.9) for U by using Runge-Kutta (RK4) method and an assumed value $\tilde{K}$ of the parameter K from the fund's initial parameter pool.

$$\frac{dU}{dt} = \begin{bmatrix} \frac{dP}{dt} \\ \frac{dB}{dt} \\ \frac{d\zeta_1}{dt} \\ \frac{d\zeta_2}{dt} \end{bmatrix} = f(U, K, t), \qquad U(t_1) = \begin{bmatrix} Y(K, t_1) \\ 0.5 \\ 0 \\ 0 \end{bmatrix} \tag{3.10}$$

where f is the right hand side of four differential equations in (3.5-3.9). We define $F[\tilde{K}]$ such that

$$F[\tilde{K}] := \sum_{i=1}^{n} W(i)\{P(\tilde{K}, t_i) - Y(K, t_i)\}^2 \tag{3.11}$$

where $F[\tilde{K}]$ represents the sum of exponentially weighted squared differences between the actual market price values $Y(K, t_i)$ and the computed market price values $P(\tilde{K}, t_i)$ obtained from the first row vector of the numerical solution U of IVP (3.10) by picking the values at time t_is. W is a positive weighting vector (for example, a vector of normalized exponentially increasing positive entries such as $W = (0.114050723751410, 0.14644402808844,$ $0.18803785418769, 0.24144538407642, 0.31002200989605)^T$ for $n = 5$) for the squared differences at time t_is. $\sum_{i=1}^{n} W(i) = 1$. We minimize $F[\tilde{K}]$ over $\Re^4$ by using line search algorithms. i.e., we seek a vector $\hat{K}$ such that $F[\hat{K}] \leqslant F[\bar{K}] \quad \forall \bar{K} \in \Re^4$.

The dynamical system of CDEs (3.5-3.9) has four first order ordinary differential equations and one algebraic equation. It is non-linear in terms of the dependent variables. Moreover, there are products of optimization parameters in the system (3.5-3.9) such that a product of c_1 and q_1 in the equation (3.8) and a product of c_2 and q_2 in the equation (3.9). The optimization problem is a non-linear least-squares problem since the subfunctions in the equation (3.11) are not linear in the components of K (i.e., c_1, q_1, c_2, and q_2). Furthermore, we assume $K > 0$ to be financially meaningful in the model (c_1 and c_2 are (positive) time scales. q_1 and q_2 are assumed to be positive in the original modeling).

3.2 ALGORITHMS

3.2.1 Main optimization algorithm to find optimal parameters via CDEs

Definition of constants, variables, and functions in the algorithm

$computedNLS$: The computed sum of squared terms in (3.11).

PER_1 : Period of event minus 1 over which optimal parameter vector is found.

PER_2 : Long period of most recent days before beginning of an event day. It is used to estimate the chronic discount which depends on a fund and time series of market price (MP) and net asset value (NAV).

$rkstsz$: RK4 step size.

$eventInd$: Day index from price list of a fund. It corresponds to the beginning of the current event period.

$firstEvent$: eventInd of the first event.

$parFixed$: The pool of initial parameters chosen via a set of grid points in a hyper-box defined by $L_i \leqslant K_i \leqslant U_i$.

y : A vector of candidate optimal parameters for the current event.

ϵ_1 : Threshold for the gradient, for example 10^{-6}.

ϵ_2 : Threshold for the computedNLS (3.11) according to the exponential weights. For example, $\epsilon_2 = 0.16$ means that the average error allowed per day for fitting (during optimization phase) is $sqrt(0.16) = 0.4$ corresponding to 40 cent.

QNewton: A function call to get candidate optimal parameter vectors by using quasi-Newton weak line search with BFGS formula and a dynamic initial parameter pool.

LSusingOneIC: A function call to obtain the computedNLS corresponding to a candidate optimal parameter vector

Inputs: fundNameMprice, fundNameNavPrice, fundNumber, and parFixed

Outputs: optimalParameters

1. Set PER_1, PER_2, $firstEvent$, $lastEvent = length(fundNameMprice) - PER_1 - 1$, $t1$, $t2$, $rkstsz$, ϵ_1 and ϵ_2

2. Set $param = parFixed$ and $optimalParameters = []$
3. for eventInd=firstEvent:lastEvent
   - $mpr = fundNameMprice(eventInd - PER_2 : eventInd + PER_1, 1)$
   - $npr = fundNameNavPrice(eventInd - PER_2 : eventInd + PER_1, 1)$
   - $newSum = zeros(t2, 1)$
   - (Below, $PER_1 + 1$ consecutive chronic discounts in equation (3.9) are computed by using the most recent PER_2 days for each one:)
   - for $t = t1 : t2$

     for $mem = 1 : PER_2$

       $prInd = t + PER_2 - mem$

       $newSum(t, 1) = newSum(t, 1) +$

       $(npr(prInd, 1) - mpr(prInd, 1))/(npr(prInd, 1) * exp(0.25 * mem))$

     $newSum(t, 1) = newSum(t, 1)/3.23180584357794$

     $discountNav(t, 1) = 1 - mpr(t + PER_2, 1)/npr(t + PER_2, 1)$

     $newDiscount(t, 1) = discountNav(t, 1) - newSum(t, 1)$
   - Reset local variables $locOptParam = []$, $locOptVal = []$, $locComputedNLSinit = []$, $locQNiter = []$
   - $lenparam = length(param)$
   - if $(lenparam >= MAXPOOLSZ)$

     $lenparam = MAXPOOLSZ$
   - $outerk = 0$
   - while $(outerk < lenparam)$

     $outerk = outerk + 1$

     $paramInit = param(outerk, :)'$

     $[y, checkQnewton, QNiter] = QNewton(paramInit, t1, t2, rkstsz,$

     $mpr(PER_2 + 1 : end, 1), npr(PER_2 + 1 : end, 1), newDiscount, \epsilon_1)$

     if $((length(y) \neq 0)\&(checkQnewton == 1))$

     – $computedNLS = LSusingOneIC(y, t1, t2, rkstsz, mpr(PER_2 + 1 : end, 1),$
       $npr(PER_2 + 1 : end, 1), newDiscount)$

     – if $((computedNLS < \epsilon_2)\&(y > 0))$

$locOptParam = [locOptParam; y']$

$locOptVal = [locOptVal; computedNLS]$

- $szLocOval = size(locOptVal)$
- $lenLocOval = szLocOval(1)$
- if $(lenLocOval > 0)$

$minval = min(locOptVal)$

$minValInd = find(locOptVal == minval)$

$globalOptParamCurrent = locOptParam(minValInd, :)$

if $(length(param) > MAXPOOLSZ)$

$param = [param(1 : INITPOOLSZ, :); globalOptParamCurrent;$

$param(INITPOOLSZ + 1 : MAXPOOLSZ, :)]$

else

$param = [param(1 : INITPOOLSZ, :); globalOptParamCurrent]$

$optimalParameters = [optimalParameters; globalOptParamCurrent]$

Given an n-day period of MPs and NAVs from day i to day $i+n-1$ where $n = PER_1+1$ and $i > PER_2$, we compute optimal parameter vector K_i for the period i. Then, we obtain $m - i + 1$ optimal parameters for the overlapping periods such as $[i, i + n - 1]$, $[i + 1, i + n]$, ..., $[m, m + n - 1]$ for the MP sequence of size $m + n - 1$.

We should choose n small enough so that the global error coming from the numerical solution of the ODEs becomes limited. Moreover, local price patterns which are related to 3 to 15 trading days on average can be exploited by small values of n during optimization and prediction processes. On the other hand, n should be large enough so that the parameter optimization process can capture the price trend reasonably. For example, we set $n = 5$.

We implement and compare two line search algorithms to get optimal parameters during optimization process [25]. The first algorithm uses a quasi-Newton method with weak line search for minimizing the sum of squares by using the CDEs, while the second one employs a refined random search technique ([11] and [37]) for this purpose. The former algorithm has a faster rate of convergence and it is more efficient.

Bremermann [11] proposed a useful optimization algorithm combining random directional line search (a coordinate descent method) and Lagrangian interpolation. Milstein [38] presents a method of biochemical kinetics parameter estimation for a system of nonlinear ODEs. Although the method [11] may converge rapidly at the beginning, it stagnates in a neighborhood of the relative minimum. Therefore, Milstein [37] adds cubic spline approximations and a Pseudo-Newton-Raphson step for the step length selection. However, it is still derivative-free algorithm for the search direction. Moreover, they are indeterministic methods because of the random directions. Furthermore, derivative-free algorithms which employ only functional values can be inefficient, since they should continue iterating until the search for minimizer is narrowed down to a small interval (see [39] for inefficiency of coordinate descent methods in practice). But, they have been supposed to be employed simply to optimize functions whose derivatives are unknown and cannot be approximated accurately. Finally, the step length selection via a fourth degree Lagrangian polynomial or a cubic spline requires five functional values at equidistant collinear five points to construct the Lagrangian polynomial or the cubic spline. After setting the first derivative of Lagrangian polynomial or the cubic spline zero, the minimum functional value is determined by evaluation of F at up to 3 or 8 points respectively depending on the number of roots at each parameter iteration where F is the square of the differences between the measured values and the computed values. These many function evaluations without guidance of derivative for search direction are more prone to fail during numerical solution of CDEs where there may be singularities at $k = 0$, $B = 0$, $k = 1$, or $B = 1$ for some initial parameters.

We mainly focus on the quasi-Newton method due to its advantages and our experience [26].

A line search method computes a search direction P_k and a step length s_k to move along that direction, at each iteration given by

$$K_{k+1} = K_k + s_k P_k. \tag{3.12}$$

Effective choices of P_k and s_k affect the success of the line search method. P_k needs to be a descent direction satisfying that $P_k^T \nabla F_k < 0$, so that it is guaranteed that F defined in equation (3.11) can be decreased along this direction. Also, P_k is of the form

$$P_k = -B_k^{-1} \nabla F_k, \tag{3.13}$$

where B_k is a symmetric and nonsingular matrix. In Newton's method B_k is the exact Hessian $\nabla^2 F(K_k)$.

Let

$$\phi(s_k) = F(K_k + s_k P_k). \tag{3.14}$$

Proposition 2. If s_k^* is the step that minimizes $\phi(s_k)$, then

$$P_k^T \nabla F(K_k + s_k^* P_k) = 0. \tag{3.15}$$

Proof After expanding $\phi(s_k)$ by using Taylor expansion, set $d\phi/ds_k = 0$ (see [5]).

Equation (3.15) implies that a perfect line search terminates at a point when the direction of search is perpendicular to the gradient vector.

3.2.1.1 Quasi-Newton method for minimizing the sum of squares

A quasi-Newton algorithm is used to minimize the sum of squares (3.11). The Newton method was not preferred because of the following drawbacks (see [5] and [39]).

1. It requires second derivative. Although it is possible to use finite difference expressions, the calculation of derivatives is one of the most time consuming parts. Moreover, the approximation can be inaccurate.
2. The search direction is obtained by solving an $n \times n$ linear system where n is the number of parameters and it is 4 in (3.11). Solving a linear system is costly and uses at least $O(\frac{1}{6}n^3)$ multiplications.
3. The Cholesky solution of $G_k P_k = -g_k$ may break down when Cholesky factorization is used and G_k is not positive definite where F is a function defined in equation (3.11) to be minimized, $g_k = \nabla F(K_k)$ and $G_k = \nabla^2 F(K_k)$.

In quasi-Newton method, the inverse of Hessian matrix $\nabla^2 F(K_k)^{-1}$ is approximated by using a positive definite matrix H_k, instead of computing exact second derivatives. The second derivative information is developed by updating the approximate matrix on each iteration. P_k is a descent direction, since H_k is positive definite and by using (3.13) we get $P_k^T \nabla F_k = -\nabla F_k^T H_k \nabla F_k < 0$.

Quasi-Newton method

1. Choose an initial parameter vector K_0 as an estimate of K that would minimize $F(K)$.
2. Choose initial symmetric positive definite matrix H_0 (Identity matrix I can be taken as H_0).
3. Set convergence tolerance $\epsilon_1 = 10^{-4}$ or set a maximum number of iterations.
4. While $\|\nabla F(K_{k+1})\| > \epsilon_1$
   - Set $g_k = \nabla F(K_k)$
   - Compute search direction $P_k = -H_k g_k$
   - Find candidate step length s_k by using backtracking line search algorithm where sufficient decrease condition is obtained for $F(K_k + s_k P_k)$.
   - Set $K_{k+1} = K_k + s_k P_k$, $\beta_k = g_{k+1} - g_k$, $\delta_k = K_{k+1} - K_k$
   - Get a new positive definite matrix H_{k+1}, by using the Broyden-Fletcher-Goldfarb-Shanno (BFGS) formula (3.19), such that

$$H_{k+1}\beta_k = \delta_k \tag{3.16}$$

5. End (while)

The gradient ($\nabla F(x)$) is approximated by using the central difference formula (see [39], Chapter 7)

$$\frac{\partial F}{\partial x_i}(x) \approx \frac{F(x + \epsilon_3 e_i) - F(x - \epsilon_3 e_i)}{2\epsilon_3} \tag{3.17}$$

for the partial derivatives, where

$$\frac{\partial F}{\partial x_i}(x) = \frac{F(x + \epsilon_3 e_i) - F(x - \epsilon_3 e_i)}{2\epsilon_3} + O(\epsilon_3^2), \tag{3.18}$$

$\epsilon_3 = u^{1/3}$, u is about 10^{-16} in double-precision arithmetic, and e_i is the ith unit vector.

Backtracking line search

The backtracking method provides either that the selected step length s is at least a fixed value ($\bar{s} = 1$), or that it is sufficiently short to satisfy the sufficient decrease condition but not too short ([39]).

- Set $\bar{s} = 1$ and choose σ, $\theta \in (0, 1)$
- Set $s = \bar{s}$

- Repeat until $F(K_k + sP_k) \leq F(K_k) + \theta s(\nabla F(K_k))^T P_k$
  - Set $s = \sigma s$
- End (repeat)
- Return with $s_k = s$.

The BFGS formula (see [12] and [13])

$$H_{k+1} = H_k - \frac{H_k \beta_k \delta_k^T + \delta_k \beta_k^T H_k}{\delta_k^T \beta_k} + (1 + \frac{\beta_k^T H_k \beta_k}{\delta_k^T \beta_k}) \frac{\delta_k \delta_k^T}{\delta_k^T \beta_k} \tag{3.19}$$

By using the formula (3.19), positive definite matrix H_{k+1} is obtained when the curvature condition $\delta_k^T \beta_k > 0$ is satisfied (see [5]). However, sometimes the curvature condition which rules out unacceptably short steps may not hold, even for the iterates close to the solution. In practice, to deal with the cases where $\delta_k^T \beta_k$ is negative or too close to zero, the BFGS update (3.19) is skipped by setting $H_{k+1} = H_k$. However, it should not be done often [39]. Experience suggests that we check the $\delta_k^T \beta_k$ and update H_{k+1} by identity matrix or set $H(i,i) = i/2$ rather than the skipping. We allow such cases limited times (at most five times) and try another initial parameter vector.

Each iteration of the quasi-Newton method can be done at a cost of $O(n^2)$ arithmetic operations in addition to the function and gradient evaluations (see [39]) where n is the number of parameters and it is 4 in (3.11). The algorithm has a super-linear rate of convergence [39]. Since there are no $O(n^3)$ operations which are seen in matrix-matrix operations or solving linear system, and calculation of second derivatives is not necessary, the quasi-Newton method is more advantageous than Newton's method. Although Newton's method converges quadratically, it is more costly per iteration because of the linear system. Moreover, rounding errors sometimes may prevent from observing such theoretical convergence rates in practice (see [5] and [32]). Although the errors in computed values of F, and the entries of ∇F and $\nabla^2 F$ in double precision arithmetic are usually negligibly small, they can be significant when ∇F is around zero (see [5]).

3.3 OPTIMIZATION RESULTS

Example 1. Given Actual MP $= (35.40, 35.62, 35.62, 35.68, 35.46)$ and NAV $= (43.95, 44.08, 44.75, 44.45, 43.93)$ for Alliance All-Market Advantage Fund (AMO) (a general equity fund (GEF)) over the five trading days vector Day $= (8.13.1999, 8.16.1999, 8.17.1999, 8.18.1999, 8.19.1999)$ beginning on Friday, and an initial pool having 56 parameter vectors, let us find the first optimal parameter vector with event index 11. After applying the main optimization algorithm in subsection 3.2.1, we get 56 candidate optimal parameter vectors via QNewton function calls. We allow only the positive candidate vectors satisfying the threshold condition with ϵ_2. Thus, we obtain a set of candidate vectors locOptParam and the corresponding set of minimized functional values locOptVal. Later, we find the minimum of locOptVal and the related optimal parameter vector globalOptParamCurrent. Figure 3.2 shows the curve fitting over the first 5-day period. The first optimal parameter vector will be used to predict next day (i.e. 8.20.1999) return. The optimal parameter vector is appended to the initial parameter pool so that the experience can be exploited for future optimizations.

Similarly, we get the second optimal parameter vector globalOptParamCurrent with event index 12 by using actual MP $= (35.62, 35.62, 35.68, 35.46, 35.79)$ and NAV $= (44.08, 44.75, 44.45, 43.93, 44.34)$ for AMO over Day $= (8.16.1999, 8.17.1999, 8.18.1999, 8.19.1999, 8.20.1999)$, and the initial pool having 57 parameter vectors. The second optimal parameter vector can be used to predict next trading day (8.23.1999) return. Figure 3.3 displays the curve fitting over the second 5-day period.

In this small example, eight consecutive optimization processes are shown by Figures 3.2-3.9. Table 3.1 summarizes the cost of the optimization process and the average maximum improvement factor (MIF) where

$$MIF = computedNLS_{min}/computedNLS_{init}. \tag{3.20}$$

While Table 3.2 illustrate the initial parameter vectors which could lead to optimal parameters for the events from 11 to 18, Table 3.3 shows the resulting optimal parameter vectors for these events.

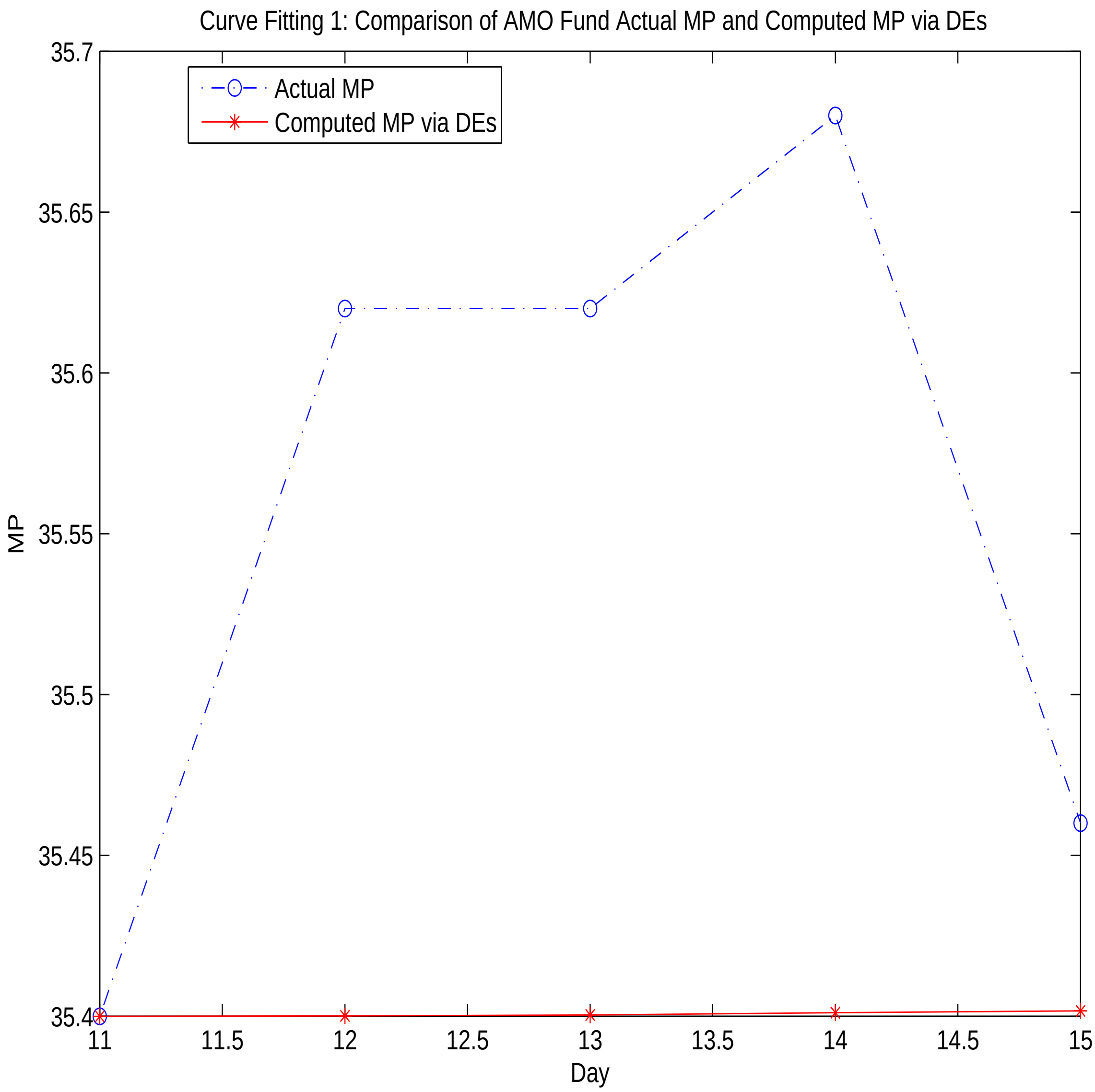

Figure 3.2: Curve fitting and getting optimal parameters for AMO MP's over the first 5-day period.

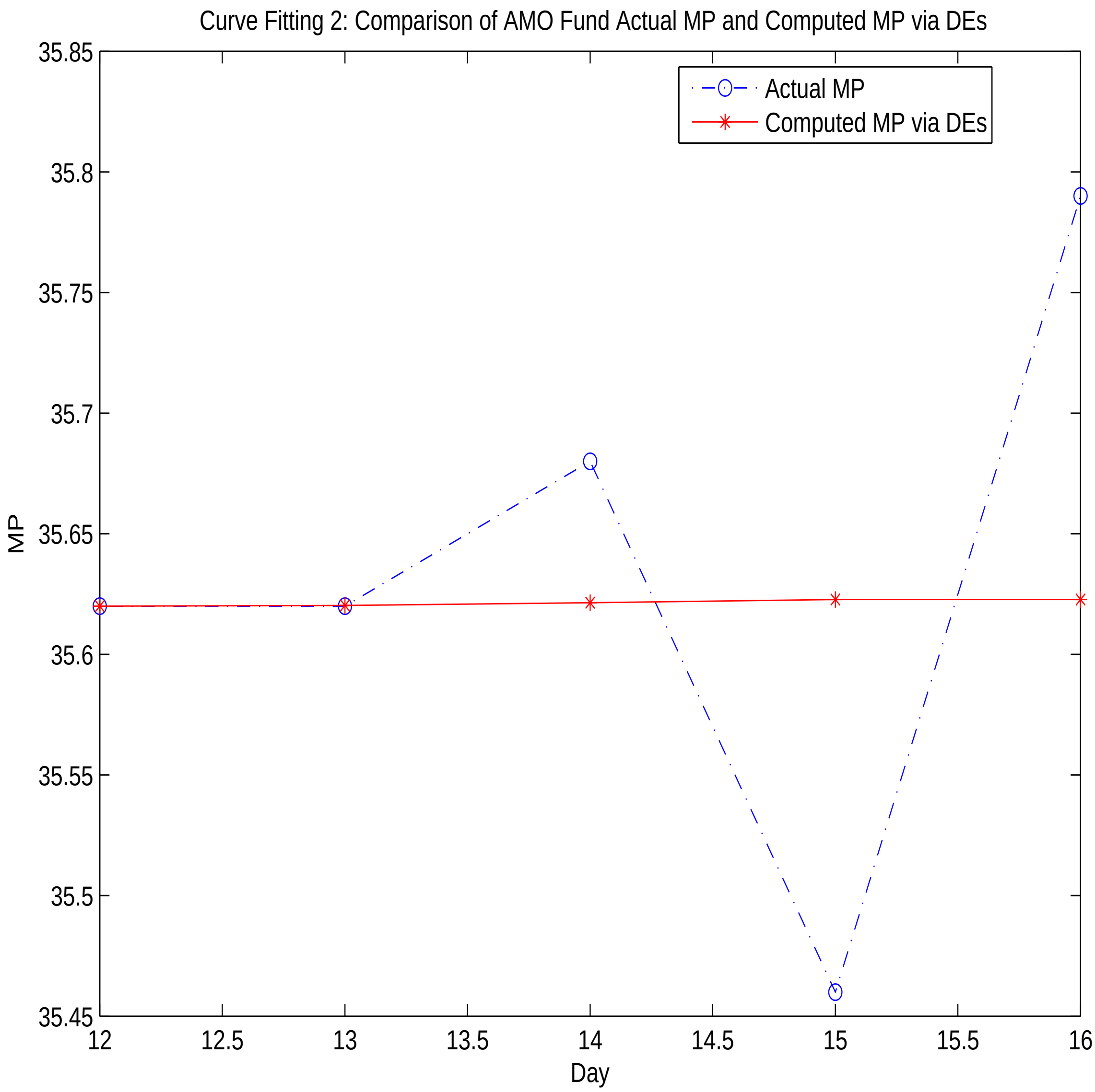

Figure 3.3: Curve fitting and getting optimal parameters for AMO MP's over the second 5-day period.

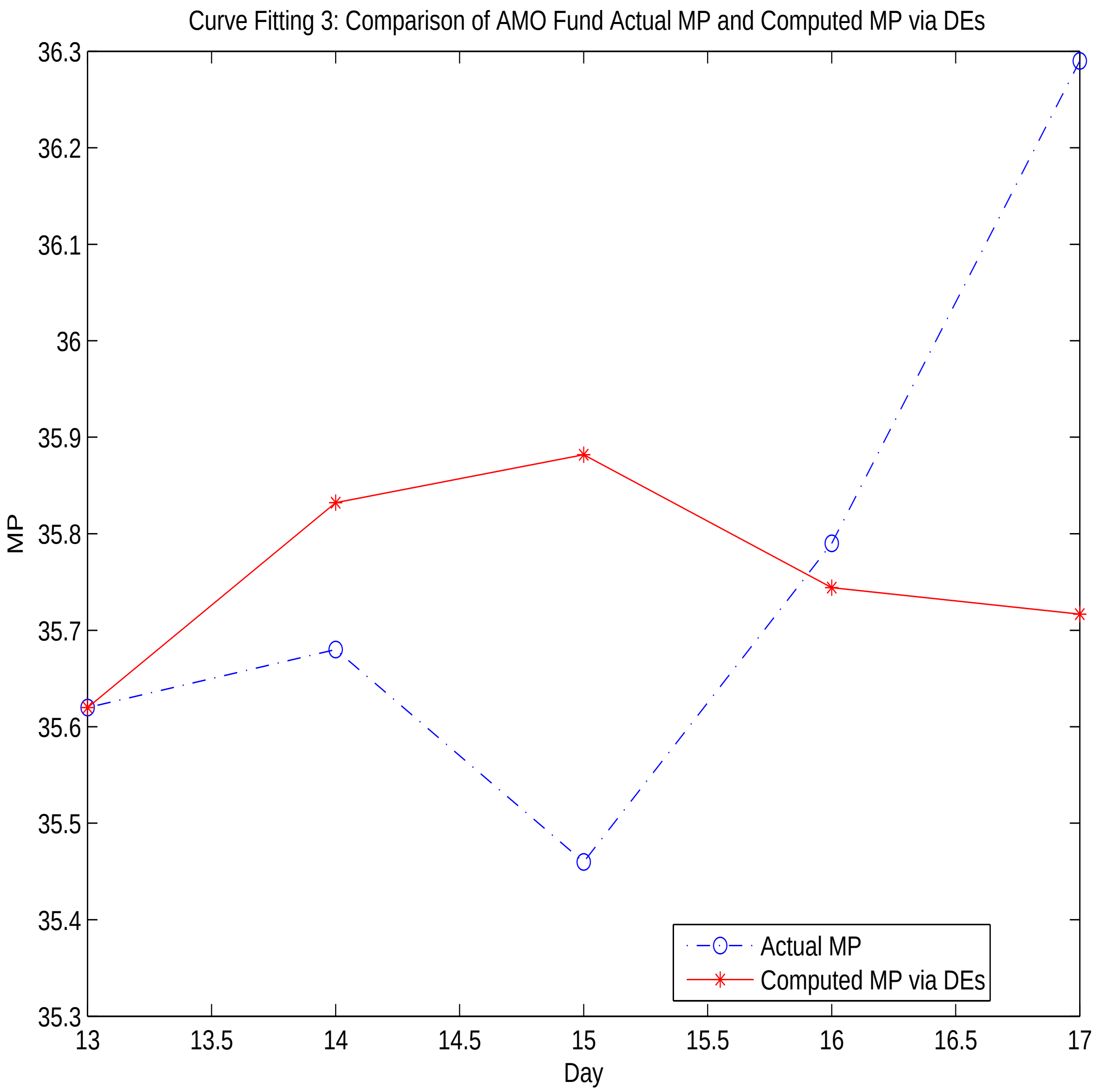

Figure 3.4: Curve fitting and getting optimal parameters for AMO MP's over the third 5-day period.

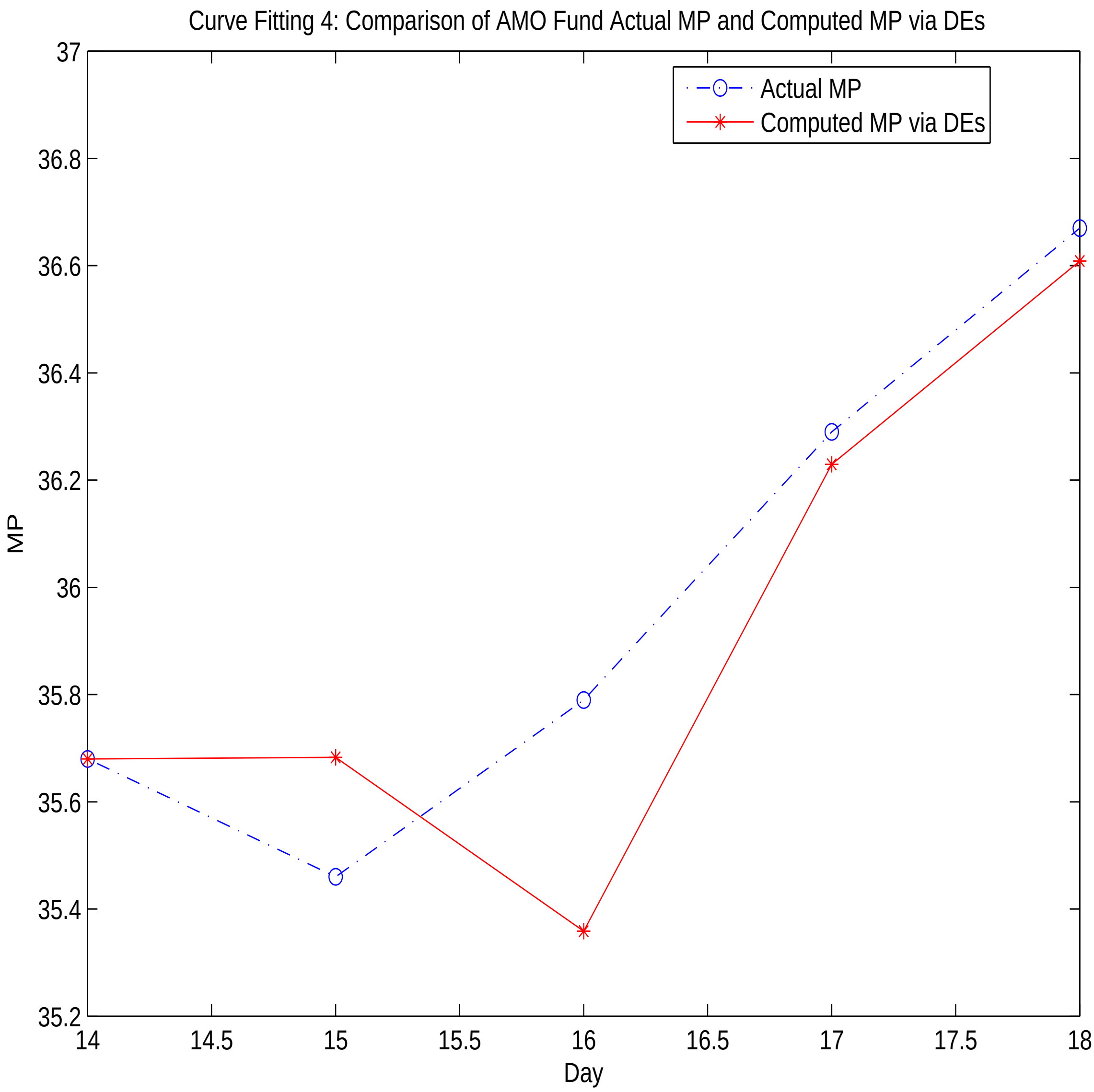

Figure 3.5: Curve fitting and getting optimal parameters for AMO MP's over the fourth 5-day period.

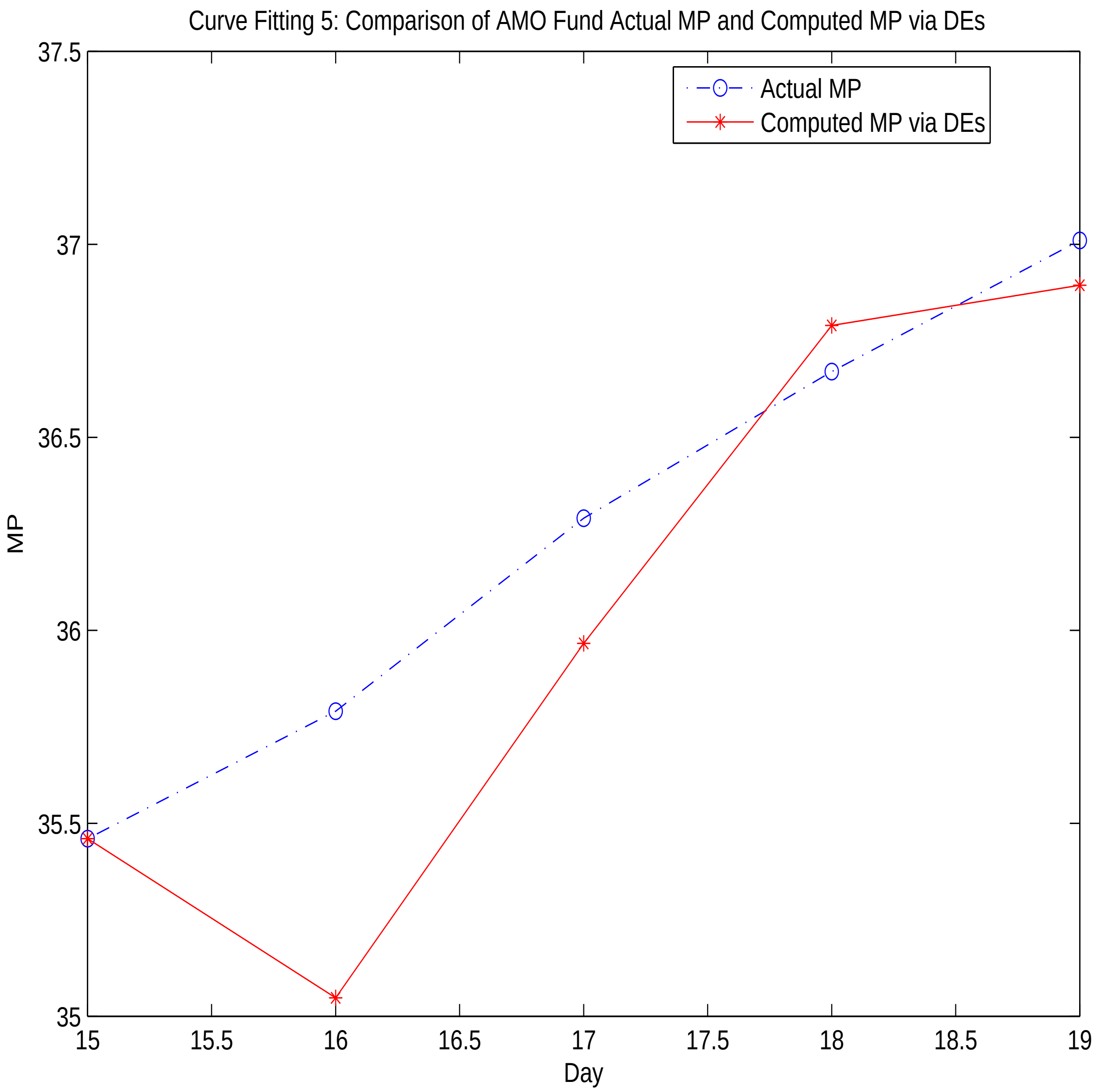

Figure 3.6: Curve fitting and getting optimal parameters for AMO MP's over the fifth 5-day period.

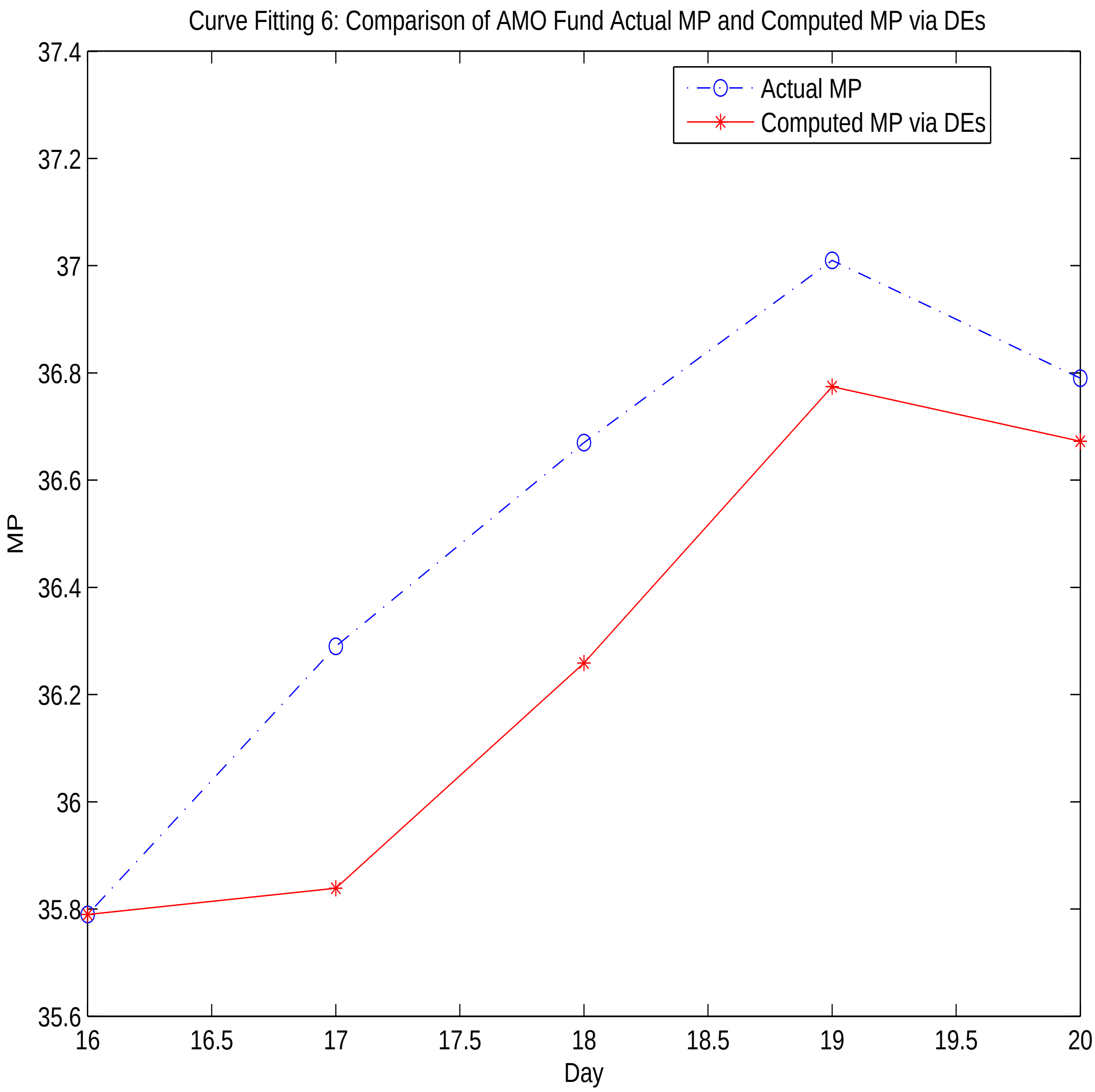

Figure 3.7: Curve fitting and getting optimal parameters for AMO MP's over the sixth 5-day period.

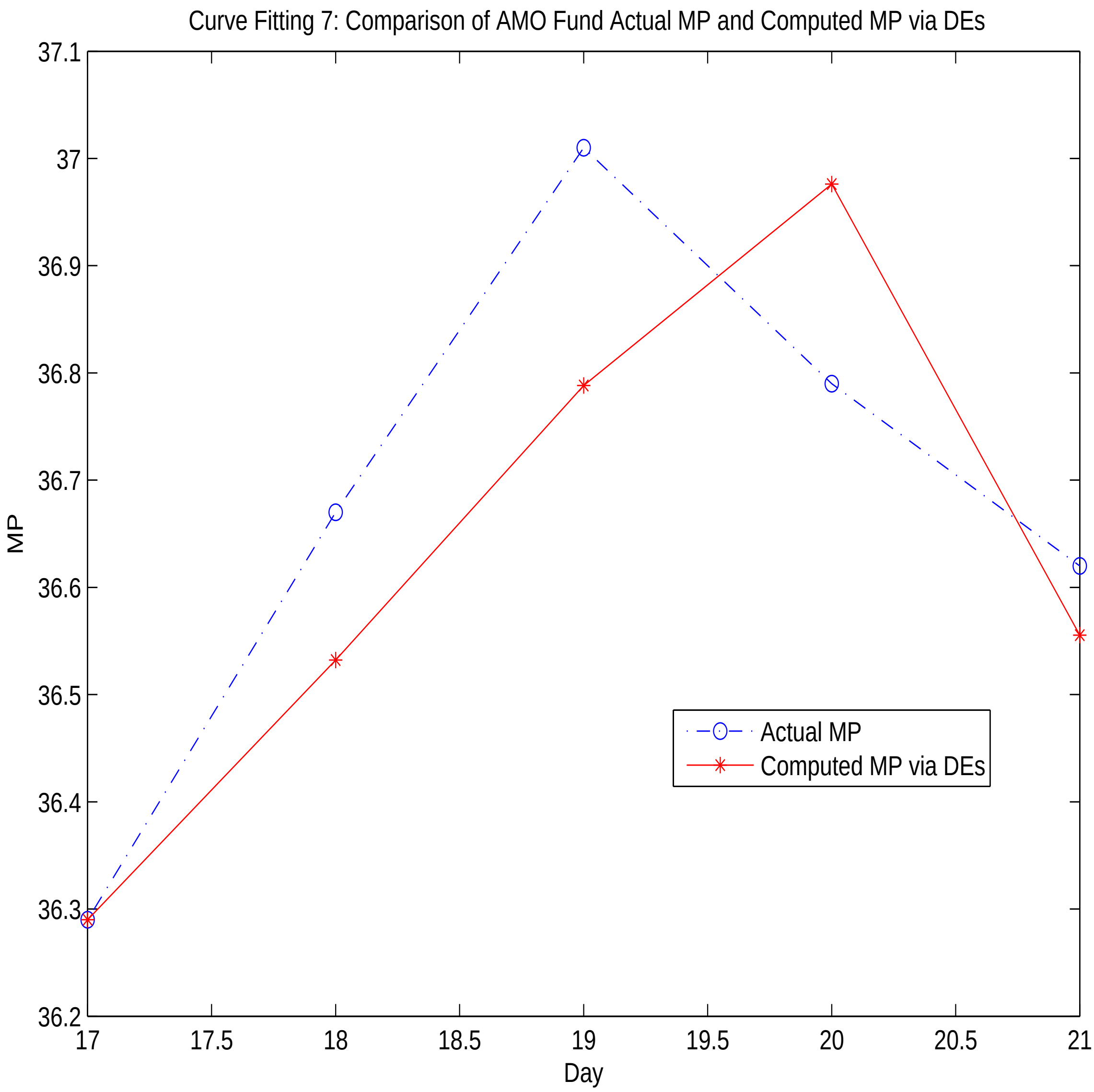

Figure 3.8: Curve fitting and getting optimal parameters for AMO MP's over the seventh 5-day period.

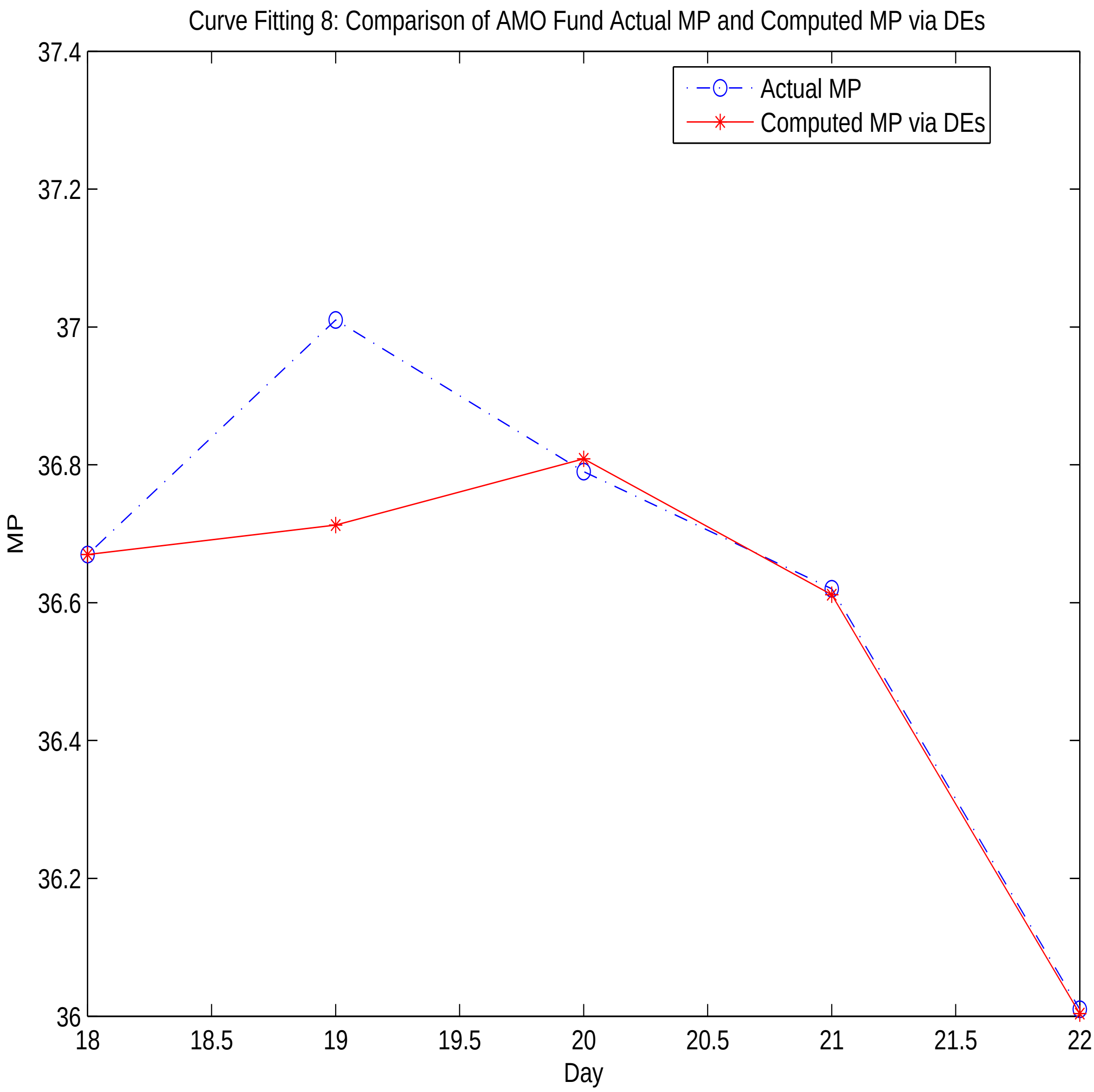

Figure 3.9: Curve fitting and getting optimal parameters for AMO MP's over the eighth 5-day period.

Table 3.1: **The computational optimization by finding parameters in the CDEs for a small example.** Quasi-Newton method with weak line search is applied for the AMO fund data during 8.13.1999-8.24.1999.

Number of Events	8
Event Period	5-day
Step Size for RK4	0.05
Number of Parameters in the Initial Pool	56
Maximum Number of Parameters in the Pool	80
Threshold for Gradient	10^{-6}
Prediction Attempt	100%
Average Number of QNw Iteration	132
Threshold for the Weighted Sum of Squares	0.16
Average Weighted Sum of Squares	0.0572
Average Maximum Improvement Factor	57.26 %

Example 2. We obtain the optimal parameters for a six sample CEFs with event period of 5-day related to the following Table 3.4. If we cannot get an optimal parameter satisfying the desired conditions, we skip the 5-day event and the next day prediction. So, the prediction attempt is 66.46%.

Example 3. We get optimal parameters for Asia Pacific Fund (APB) (a world equity fund (WEF)) with event period of 10-day by following the instructions in Table 3.5. If we cannot get an optimal parameter satisfying the desired conditions, we use the most recent computed optimal parameter so that we have 100 % prediction attempt.

Table 3.2: **Initial parameters.**

	Initial Parameters			
Event #	c_1	q_1	c_2	q_2
11	0.50100000000000	5.01000000000000	0.00500000000000	0.01000000000000
12	0.50151998114889	5.01012317937758	0.03341248949986	0.03893029114360
13	0.00100000000000	0.01000000000000	1.00500000000000	5.01000000000000
14	0.00100000000000	5.01000000000000	2.00000000000000	10.01000000000000
15	0.50100000000000	5.01000000000000	1.00500000000000	5.01000000000000
16	0.00100000000000	5.01000000000000	1.00500000000000	0.01000000000000
17	0.00100000000000	10.01000000000000	1.00500000000000	5.01000000000000
18	0.00100000000000	10.01000000000000	0.50500000000000	5.01000000000000

Table 3.3: **Optimal parameters.**

	Optimal Parameters			
Event #	c_1	q_1	c_2	q_2
11	0.50151998114889	5.01012317937758	0.03341248949986	0.03893029114360
12	0.50260637800436	5.01053711355921	0.04689133908317	0.05508322708754
13	0.00024815673835	0.00994432123190	0.56769323787362	4.93512909446886
14	0.001555126193	379.573676710672	52.749314817565	8.049115994692
15	0.002802342929	419.708990457345	52.957130549807	8.322994033815
16	0.70321773805002	5.11794551172671	1.11144330472582	1.45832582507956
17	0.003087412095	133.925346584385	0.005573025372	649.593725911804
18	0.002073137310	565.436200421300	0.004883175322	320.677184469970

Table 3.4: **The computational optimization by finding parameters in the CDEs for a large sample data set.** Quasi-Newton method with weak line search is applied for a six sample CEFs data during 1998-2006.

Number of Events	8411
Event Period	5-day
Step Size for RK4	0.05
Number of Parameters in the Initial Pool	56
Maximum Number of Parameters in the Pool	80
Threshold for Gradient	10^{-4}
Prediction Attempt	66.46 %
Average Number of QNw Iteration	80
Threshold for the Weighted Sum of Squares	0.16
Average Weighted Sum of Squares	0.0124
Average Maximum Improvement Factor	32.16 %

Table 3.5: **The computational optimization by finding parameters in the CDEs for 10-day event period.** Quasi-Newton method with weak line search is applied for the APB data during the trading days 1.17.2002-6.20.2003.

Number of Events	339
Event Period	10-day
Step Size for RK4	0.05
Number of Parameters in the Initial Pool	56
Maximum Number of Parameters in the Pool	80
Threshold for Gradient	10^{-5}
Prediction Attempt	100 %
Average Number of QNw Iteration	38
Threshold for the Weighted Sum of Squares	1.00
Average Weighted Sum of Squares	0.0491
Average Maximum Improvement Factor	66.64 %

4.0 MARKET PRICE RETURN PREDICTION

4.1 INTRODUCTION

During the past several decades, the dominant theory of finance has been the efficient market hypothesis (EMH). In its weak form the EMH asserts that any information relating to price cannot be used for excess profit since such information is readily available to anyone. In its stronger form EMH asserts similarly that all publicly available information cannot be used to increase profits beyond the risk premium inherent in that class of investments. Consequently, the best possible prediction that can be made for the price of a stock is given by

$$\frac{P_{t+1} - P_t}{P_t} = \beta r_M + \varepsilon_t. \tag{4.1}$$

In other words, the best predictor of tomorrow's price is today's price augmented by the tiny factor βr_M which represents the expected daily return for the overall market (i.e., a few percent divided by the 250 trading days per year) times the beta factor that scales the volatility of the stock relative to the overall market. The term ε_t is the excess return specific to the stock for day t. The mean of this term according to EMH must be zero for reasons stated above. Thus, we can state that neglecting a term of order $(10\%)(1/250) = 1/2500$, EMH asserts that

$$P_{t+1} = P_t + \varepsilon_t, \tag{4.2}$$

i.e., random walk (plus a tiny upward drift term), is the best forecast of tomorrow's price assuming knowledge of today's price.

Moreover, the EMH asserts that since all investors have information on the price history, such information cannot have any predictive value.

Caginalp and Laurent [20] performed the first scientific test to provide strong evidence in favor of any trading rule or pattern on a large scale. They applied a non-parametric statistical test for the predictive capabilities of candlestick patterns using daily data for each stock in the S&P 500 during the time period 1992-1996. The out-of-sample tests indicate statistically significant profit of almost 1% during a two-day holding period. Moreover, Caginalp and Balenovich [15] develop a foundation for the technical analysis of securities by using a dynamical microeconomic model. They deal with a broad spectrum of patterns that are generated by the presence of two or more trader groups with asymmetric information, besides the patterns generated by the activities of a single group.

Rapach et al. [43] employ in-sample and out-of-sample procedures related to data mining for international stock return predictability with macro variables.

Suppose we have a sequence of observed daily prices Y_i, $i = 1,..,m$ at times $t_1,..,t_m$. It is hard to model such erratic data Y_i (see Figure 4.1) by a smooth function such as a polynomial. In literature, generally the trend of the sequence is determined via the least squares line calculation. That is, K_1 and K_2 are computed to minimize

$$F(K) = \sum_{i=1}^{m} (Y_i - K_1 - K_2 t_i)^2 \tag{4.3}$$

It is a linear least-squares problem because the subfunctions in (4.3) are linear in K_1 and K_2.

Let K_1^* and K_2^* be minimizers of (4.3). Then, de-trended data ($V_i = Y_i - K_1^* - K_2^* t_i$) which cannot be modeled by the trend-line is handled with the following autoregressive model (see [9])

$$V_i = \eta_1 V_{i-1} + \eta_2 V_{i-2} + \eta_3 \tag{4.4}$$

and the coefficients $\eta_i s$ are determined.

Instead of two-stage approach (4.4), one can use a single-stage model (see [5])

$$Y_i = K_1 + K_2 t_i + K_3 (Y_{i-1} - K_1^* - K_2^* t_{i-1}) + K_4 (Y_{i-2} - K_1^* - K_2^* t_{i-2}) \tag{4.5}$$

and minimize

$$F(K) = \sum_{i=1}^{m} (Y_i - K_1 - K_2 t_i - K_3 (Y_{i-1} - K_1^* - K_2^* t_{i-1}) - K_4 (Y_{i-2} - K_1^* - K_2^* t_{i-2}))^2. \tag{4.6}$$

The single stage approach to fit coefficients provides a much closer estimate to the original data than the two-stage process (see [5]).

Example 4. Given daily closing prices of TRF

$$MP = (10.12, 9.81, 9.87, 9.63, 10.06, 10.97, 11.09, 11.03, 11.34, 12.01,$$

$$12.07, 12.19, 12.07, 11.21, 10.48, 11.34, 11.15, 11.46, 11.21, 11.09),$$

and initial parameter vector $K = (1, 1, 1, 1)$, the single-stage non-linear least-squares model (4.5) is employed and the comparison between actual data and the least square model is displayed in Figure 4.1. $F(K)$ in (4.6) is minimized and the optimal parameters $K_1 = 10.9975$, $K_2 = 0.0191$, $K_3 = 0.9143$, and $K_4 = -0.2158$ are obtained. The sum of squared errors from t_3 to t_{20} is 2.9740. So, the corresponding mean square error is 0.1652. Since the method combines only trend and de-trend phases without microeconomic model, the actual MPs are repeated by the computed MPs with one day delay.

In this chapter, we study price forecast with the system of ODEs (3.5-3.9) for an arbitrary day independent from a pattern. However, there are several factors affecting the success of the prediction.

- Forecasting is often difficult in many disciplines. For example, weather forecasting [35] has been studied extensively for many decades with some success, and yet there are still many surprises. In the case of markets, forecasting is especially difficult since one is trying essentially to make a forecast that is better than the aggregate forecast of the market participants. As noted earlier, the efficient market hypothesis asserts that this is not possible.
- There is a wide range of variability in getting optimal parameters. That is, the residual values may change between 10^{-1} and 10^{-14}.
- There are also difficulties arising from the numerical methods to solve the nonlinear ODEs having singularities for some initial parameters. For example, we have methods which are efficient for certain stiff and non-stiff applications (see [3]). For an arbitrary day price prediction via CDEs, we meet with both stiff problems (having widely varying time scales where the standard numerical methods may require extremely small step size

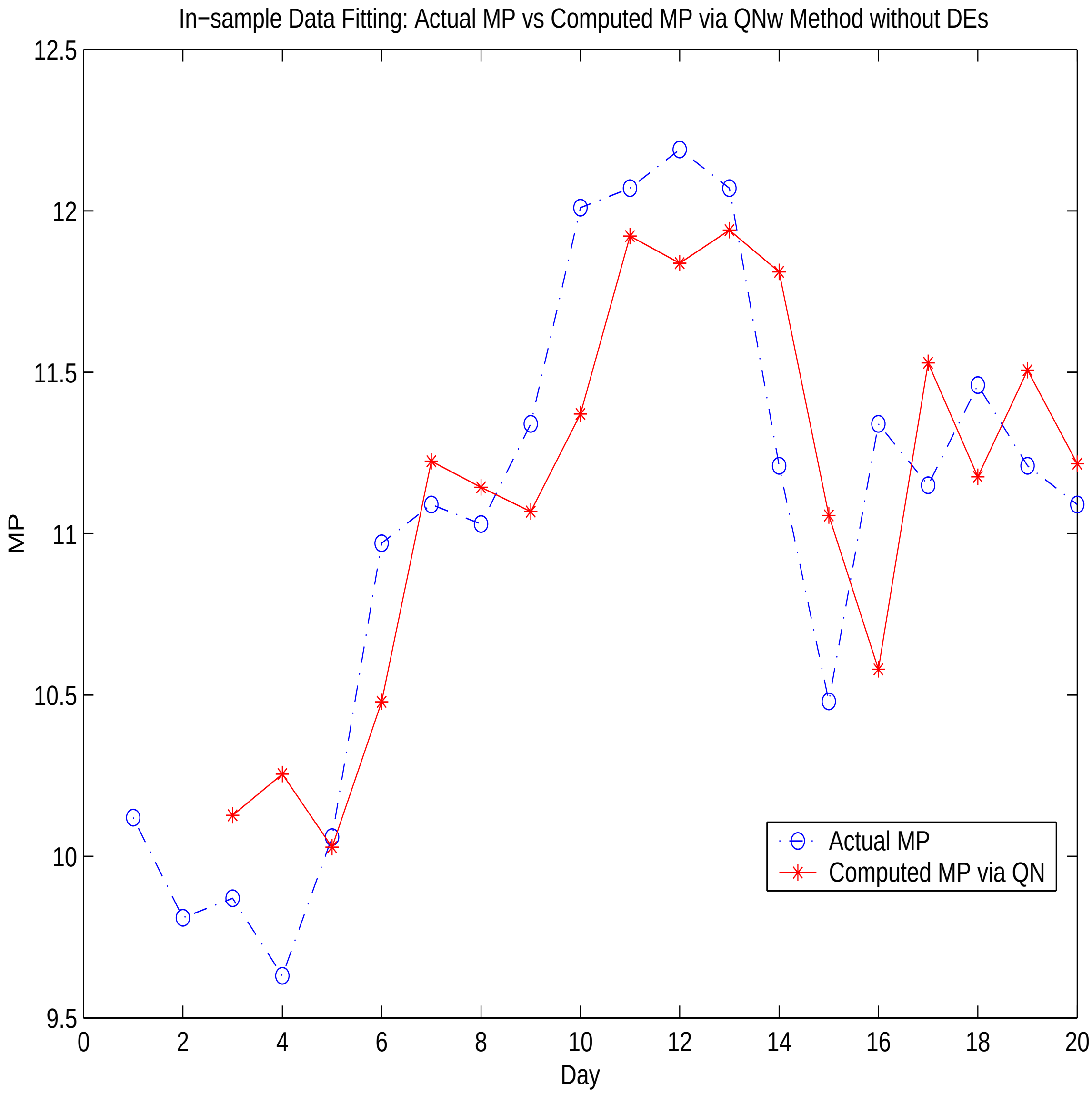

Figure 4.1: A single-stage non-linear least-squares model (4.6) is applied to the Templeton Russia Fund (TRF) data from March 4th, 1999 to March 31st, 1999.

h) and non-stiff problems. However, we experienced failures during Newton iterations for BDF2 method which is suitable for stiff problems. Therefore, we need a general-purpose code combining advantages of several algorithms.

We employ the dynamical microeconomic model (3.5-3.9) which provides valuable constraints as if conservation laws in physics, rather than the classical time series analysis with the single stage approach explained above. Despite the difficulties, we provide out-of-sample predictions which are more successful than EMH.

4.2 METHOD DESCRIPTION

The proposed out of sample prediction is performed in the following way. Given MPs and NAVs for an n-day period from day i to day $i+n-1$ and the corresponding optimal parameter vector K_i for the i'th period computed via an optimization method in Chapter 3, we solve the initial value problem (IVP) with CDEs (3.5 - 3.9) to predict MP value and return on day $i+n$.

4.3 SUCCESS TESTS

4.3.1 Absolute Difference of Predicted Return and Actual Return

We compare two columns of paired sequences. The first column $|return_{PredDe} - return_{Actual}|$ consists of the absolute values of differences between the actual daily returns and the predicted daily returns via the differential equations. The second column $|return_{PredRw} - return_{Actual}|$ has the absolute values of differences between the actual daily returns and the predicted returns via random walk. Then, we apply the Mann-Whitney U test [36] and the Wilcoxon rank sum test [36] to column 1 and column 2. They are non-parametric tests of the hypothesis that two independent samples come from distributions with equal medians. We use non-parametric tests because we make no assumptions about the distribution of the data. The Mann-Whitney U test is equivalent to the Wilcoxon rank sum test for equal medians (see [30] and [33]).

Null Hypothesis, H_0 : The median absolute value of difference $|return_{PredDe} - return_{Actual}|$ and the median absolute value of difference $|return_{PredRw} - return_{Actual}|$ are equal.

Alternative Hypothesis, H_1 : The median absolute value of difference $|return_{PredDe} - return_{Actual}|$ is less than the median absolute value of difference $|return_{PredRw} - return_{Actual}|$.

Depending on the p-value of a non-parametric test, we may get a conclusion.

4.3.2 Prediction of Relative Price Change Direction

Now, we get relative price changes for the actual MP and the predicted prices via the proposed method. Then, we count the number of the right matches corresponding to the daily relative price increase or decrease. We get a new sequence such that the sequence element is 1 if there is a right match. Otherwise, the sequence element is -1. We apply z-test to the sequence. According to EMH, the mean value of the sequence would be 0 as null hypothesis. The alternative hypothesis states that the mean value of the sequence is different from zero.

4.4 PREDICTION RESULTS

Example 5. By using the 8 optimal parameters obtained in Example 1, we solve the initial value problem (IVP) with CDEs (3.5 - 3.9) to predict MP value and return for the next days from day 16 to day 23. In Figure 4.2, actual MP $= (35.79, 36.29, 36.67, 37.01, 36.79, 36.62, 36.01, 35.84)$ and predicted MP via CDEs $= (35.46, 35.79, 36.49, 37.08, 36.65, 36.50, 35.96, 35.70)$ are compared for the trading days Day $= (8.20.1999, 8.23.1999, 8.24.1999, 8.25.1999, 8.26.1999, 8.27.1999, 8.30.1999, 8.31.1999)$.

In Figure 4.3, for the same days as in Figure 4.2, actual return $= (0.0093063, 0.0139704, 0.0104712, 0.0092719, -0.0059443, -0.0046208, -0.0166576, -0.0047209)$ and predicted return via CDEs $= (0.0000001, 0.0000069, 0.0055841, 0.0112044, -0.0096853, -0.0079270, -0.0180671, -0.0086843)$ are shown.

In Figure 4.4, the absolute errors for the predicted returns via CDEs are $|return_{DE} - return_{actual}| = (0.0093062, 0.0139635, 0.0048871, 0.0019325, 0.0037410, 0.0033062, 0.0014095, 0.0039633)$ and the absolute errors for the predicted returns via RW are

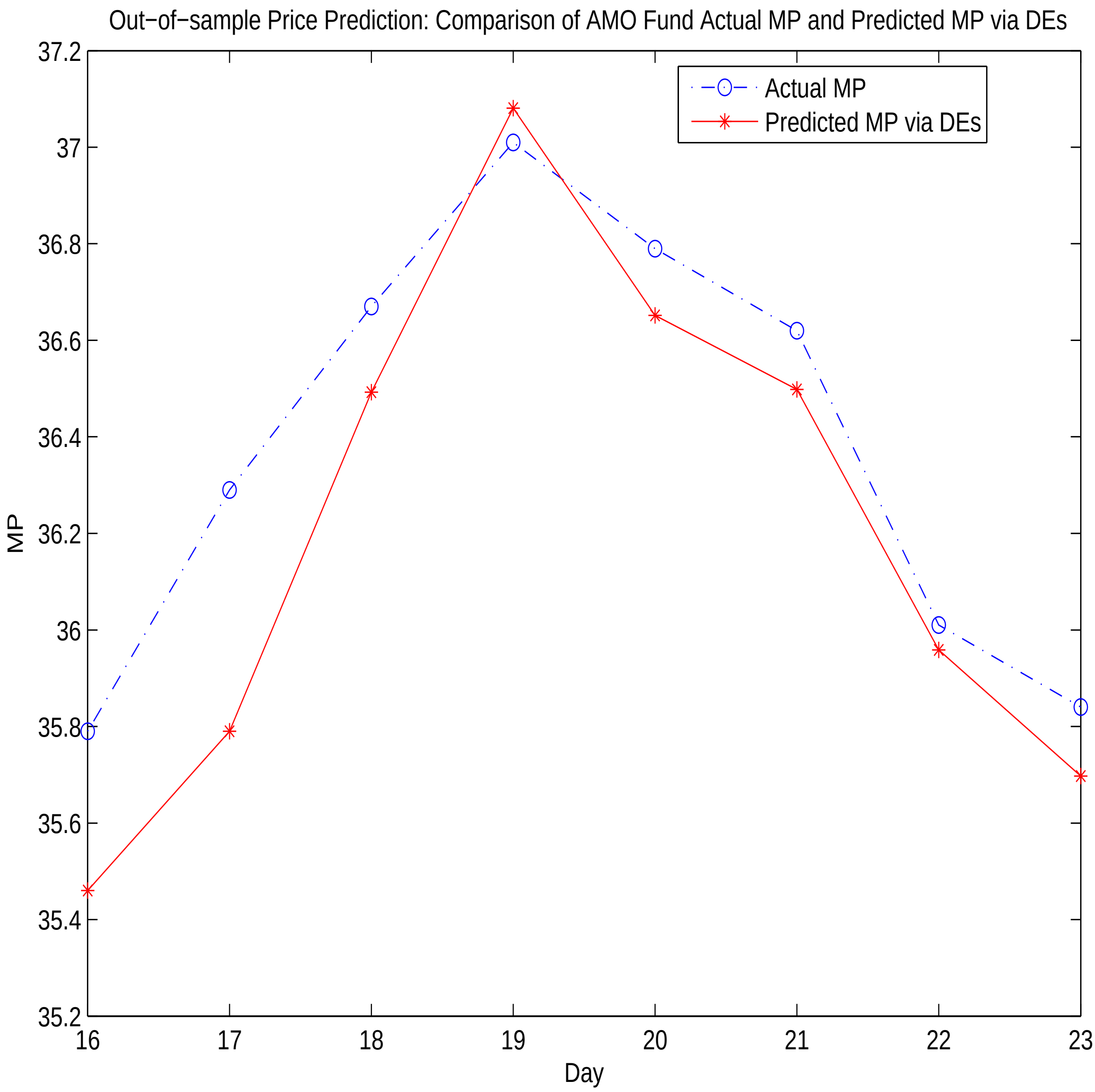

Figure 4.2: Prediction of AMO MPs over 8-day

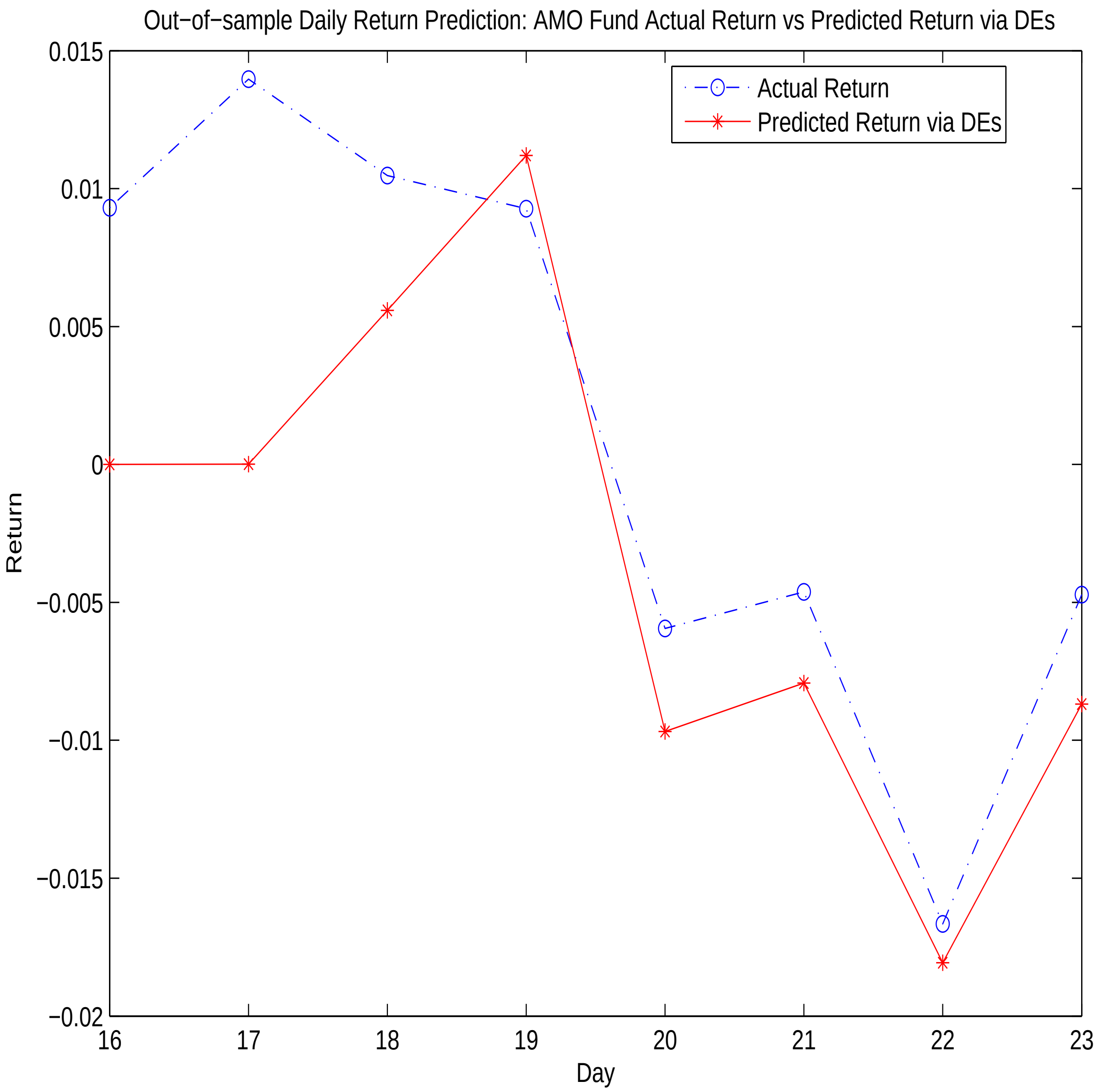

Figure 4.3: Prediction of AMO fund daily returns over 8-day

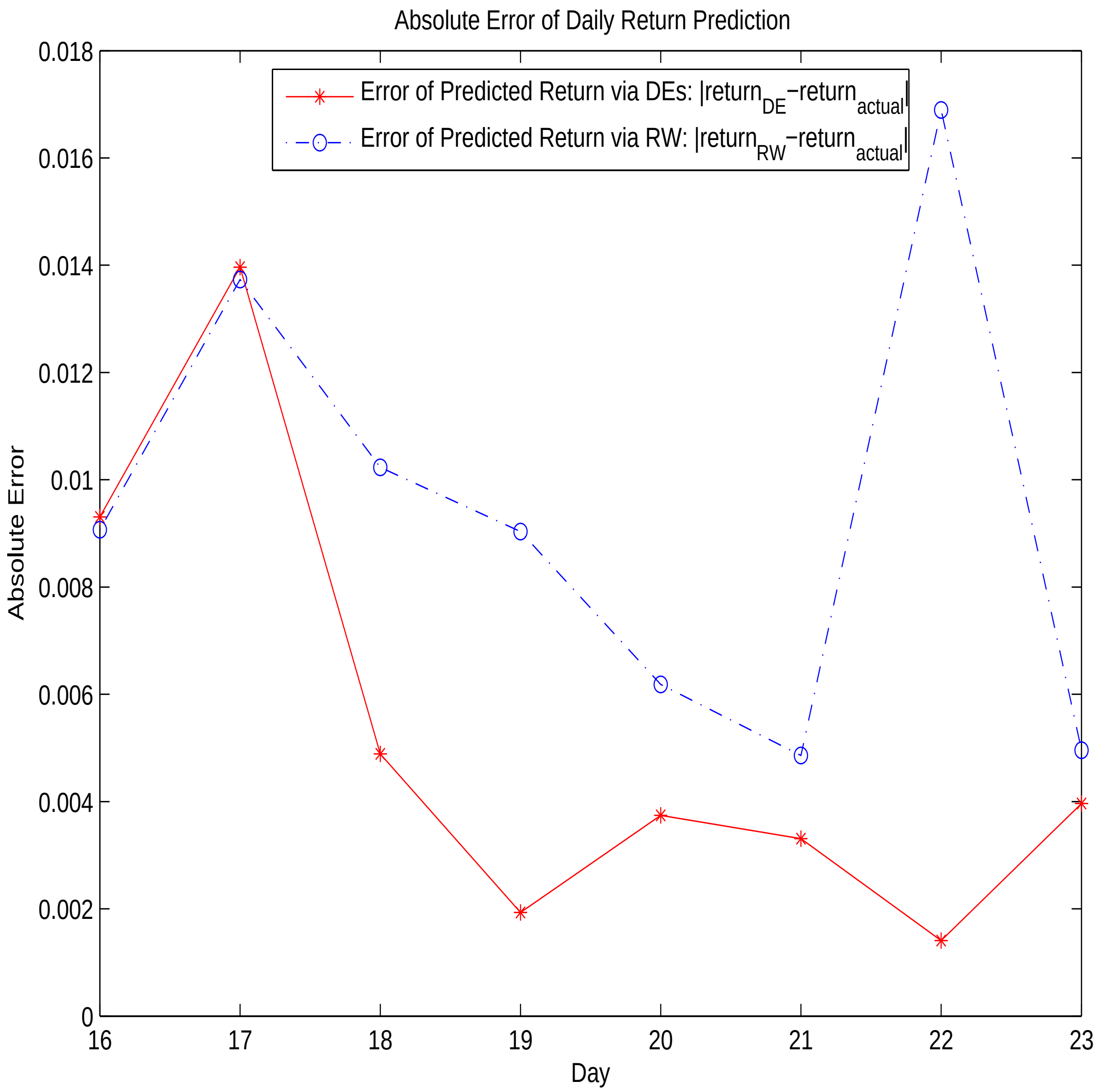

Figure 4.4: Absolute error of AMO fund predicted returns over 8-day.

$|return_{RW} - return_{actual}| = (0.0090682, 0.0137323, 0.0102331, 0.0090338, 0.0061824, 0.0048589, 0.0168957, 0.0049590)$. After day 17, the absolute errors for the predicted returns via CDEs are less than that of RW.

For example, the MP and return on day 20 is predicted (see Figure 4.2 and Figure 4.3 respectively) by using initial conditions on day 19 and the computed optimal parameter vector $(0.002802342929, 419.708990457345, 52.957130549807, 8.322994033815)$ for 5-day period from day 15 to day 19 as in Figure 3.6 and Table 3.3. It is remarkable to predict such a reversal in MP and sign of return on day 20 after a 3-day rise trend in MP. This successful prediction cannot be expected from a prediction via pure curve fitting.

By using Mann-Whitney U test for 8 events, we get
$median(|returnPredDe - returnActual|) = 0.00385$ and
$median(|returnPredRw - returnActual|) = 0.00905$. That is, the error via CDEs is less than half of the error via RW. Point estimate for $ETA_1 - ETA_2$ is -0.00422. But, computed $ETA_1 - ETA_2$ is -0.0052. 95.9 % CI for $ETA_1 - ETA_2$ is $(-0.00883, 0.00003)$. The rank sum $W = 49.0$. Test of $ETA_1 = ETA_2$ vs $ETA_1 < ETA_2$ is significant at 0.0260. Since $0.0260 < 0.05$, we can reject the null hypothesis at the 0.05 level for this small example. Moreover, the prediction success of MP return direction by CDEs is 100 %.

Example 6. We predict the next day MP return by using the optimal parameters obtained in Example 2. We apply Mann-Whitney U test and have
$median(|returnPredDe - returnActual|) = 0.00554$ and
$median(|returnPredRw - returnActual|) = 0.00577$ for the 5590 prediction attempts. Point estimate for $ETA_1 - ETA_2$ is -0.00018. 95.0% CI for $ETA_1 - ETA_2$ is $(-0.00036, -0.00001)$. The rank sum $W = 30898021.0$. Test of $ETA_1 = ETA_2$ vs $ETA_1 < ETA_2$ is significant at 0.0193. The test is significant at 0.0193 also when adjusted for ties. Therefore, we can reject the null hypothesis at the 0.05 level for this sample portfolio.

When we apply Wilcoxon rank sum test, p-value is 0.0386, z-val is -2.0681, and rank sum is 30898023.0. Thus, we can reject the null hypothesis at the 0.05 level by using Wilcoxon rank sum test, as well.

The prediction success of relative price change direction by CDEs is 63.33% (with 3540 right direction matches out of 5590 prediction attempts) which is greater than 50%. When we apply z-test to the direction match sequence of -1 and 1, the mean value is 0.2666, p-value is 0, 95.0% CI is $(0.2403, 0.2928)$, and z-val is 19.9288. Therefore, we can reject the null hypothesis. Moreover, the success of prediction that the price will be non-increasing or non-decreasing is 69.84% with 3904 right matches out of 5590 prediction attempts. According to z-test, the mean value is 0.3968, p-value is 0, 95.0% CI is $(0.3706, 0.4230)$, and z-val is 29.6658. Again, we can reject the null hypothesis.

While the success of this method is encouraging, more large scale studies are needed before concluding that this procedure in itself can be used profitably.

Example 7. We get MP and return prediction of APB for 10-day event period by using optimal parameters obtained in Example 3 and Table 3.5. We compare the predicted returns with the actual returns during 2.19.2002-6.23.2003. According to Mann-Whitney U test for 339 events, $median(|returnPredDe - returnActual|) = 0.00846$ while $median(|returnPredRw - returnActual|) = 0.00871$. Point estimate for $ETA_1 - ETA_2$ is 0.00020. 95.0 % CI for $ETA_1 - ETA_2$ is $(-0.00089, 0.00129)$. The rank sum $W = 116154.0$. For the test of $ETA_1 = ETA_2$ vs $ETA_1 < ETA_2$, we cannot reject the null hypothesis for this example by using the method of full prediction attempt via 10-day event period since W is > 115090.5, although

$$median(|returnPredDe - returnActual|) < median(|returnPredRw - returnActual|).$$

The prediction success of relative price change direction by CDEs is 54% which is smaller than that of Example 6 because of the rate of prediction attempt, larger event period and larger ϵ_2. But, it is still greater than 50%.

5.0 CONCLUSIONS

The issues of overreaction and underreaction are central to the debate on behavioral finance, but are often difficult to establish statistically through data analysis. We have performed a study in which the relative change in the fundamental value is subtracted from that of the trading price, so that the difference provides a clearer picture of the underlying dynamics of trading price. In particular, we found that for a set of closed end funds (CEFs) over a long period, any significant deviation between the market price return and the fundamental value return on a particular day is likely to be followed by a reversal the next day. More surprisingly, however, was the discovery that prior to such "event" days, there is a tendency to move gradually in the opposite direction during the previous two or three days. This precursor for the significant changes is also very different from the results one would expect from the efficient market hypothesis. There is no reason for the spike from a traditional finance perspective. However, with different groups interacting and maneuvering to find an edge, it seems that if one group is positioned, for example, as a short in anticipation of negative news, a small amount of good news is reason to buy aggressively to cover the short. The aggressive buying then pushes the price far above the levels justified by the change in fundamentals.

Within the framework of EMH, a market price is a highly stable equilibrium value that is established by traders having common information. However, another viewpoint incorporated into the asset flow theory in [19] is that there are two or more large groups that have widely differing assessments of value. At a particular time, the market receives either a small amount of new information, or a small amount of additional traders. The traders are aware of other viewpoints as the information or resources arrive. However, there is uncertainty created by the strategies (and resources) of others. Consequently, there is a price movement that can be far in excess of any new information. As discussed in the asset flow references, overreaction (Hypothesis 1) is a natural consequence of this approach within a particular time scale that must be established by the data. While overreaction can have several other explanations, it is difficult to justify within the context of EMH.

The statistics have confirmed our viewpoint that the random changes in fundamentals obscure most of the behavioral effects in price movements. When the same tests are done without subtracting the net asset value, much of the statistical significance disappears. This is at the heart of the debate between behavioral finance and the efficient market advocates. The latter argue that overreactions and underreactions should not be systematically distinguishable. Augmenting earlier studies ([1], [34] and [48]) we find that our "event" criteria, described as a deviation between market price return and net asset value return, stipulate sufficient conditions for overreaction. The magnitude of the overreaction we find is quite significant even for the lower threshold levels (i.e., when the deviation is only a few percent). The presence of a precursor to such events is even more difficult to explain from an efficient market perspective. There is also remarkable symmetry between the pre-event and post-event days, as well as for the positive and negative deviations.

Closed end funds provide a good avenue to test ideas of market dynamics. In some ways the situation is similar to options trading. The value of an option is related to the trading price of the underlying stock, and one can examine the efficiency of the option price relative to the stock price, without making an a priori assumption on the efficiency of the stock price. In a similar way, one can examine the efficiency of the closed end fund relative to the net asset value. A previous study [40] had shown that the volatility of the closed end fund is much greater than the volatility of the underlying index. Our study confirms this from a different perspective, and it is consistent with the concept of finite assets (rather than infinite capital for arbitrage) that underlies Hypothesis 3. In other words, if one compares a large, widely followed market such as Japan with a relatively small closed end fund investing in Japan, then the assumption of infinite arbitrage capital is much less likely to be valid for the closed end fund. The reason for this is not so much due to a closed end fund's structure, but rather to its size, visibility and trading volume. After all, if there is a trading volume of tens of thousands in a particular closed end fund, the potential profit on deviations of a few percent is too small for all but the tiniest hedge funds. Thus one would expect the closed end fund to be more volatile than the underlying assets, even from the EMH perspective. However, one would expect the level of deviations to be much smaller and less systematic than we have found.

A large part of the patterns we have found disappear when the relative change in NAV is not subtracted from the relative change in market price. This may explain why many data studies of markets show fairly small deviations from efficiency. As noted earlier, the valuation is influenced by many factors that can be regarded, from the perspective of traders, as stochastic. Hence any effort to show systematic behavioral bias that does not account for these changes in valuation encounters a great deal of "noise" so that obtaining statistical significance is difficult. It has been noted by Black [6], an EMH advocate, that "noise makes it very difficult to test either practical or academic theories about the way economic or financial markets work." He adds that a reasonable definition of efficiency is that the market price is "more than half of value and less than twice value." The methodology we have used helps overcome this obstacle of "noise" in understanding market dynamics.

One aspect of our study focuses on those events in which there is relatively little change in NAV during the occurrence of a significant relative change (e.g., increase) in market price. A new phenomenon discovered in our analysis is that there is a dip during the two or three days prior to the upward spike. It would be difficult to concoct any explanation of this based upon the EMH, or any of the prevalent ideas in finance. However, this phenomenon is perfectly consistent with the asset flow approach in which the classical price theory is augmented with the concepts of finiteness of assets and trading decisions based upon momentum as well as valuation.

A key challenge to behavioral finance has been the development of a paradigm– such as the risk/reward criterion of classical finance– on which a quantitative theory can be developed. This is more difficult than the paradigm for classical finance since the latter is essentially a default theory based on an idealization. A necessary first step then is the establishment of key phenomena that can be used to develop a theory. One of the main arguments of efficient market theorists has been the absence of obvious systematic biases in market prices. Early statistical studies indicated that prices were close to a random walk.

While subsequent studies have shown some short term biases, they have often been dismissed as too small to be profitable. The omnipresence of random events that influence valuation as well as the wealth of traders tends to introduce a sufficient amount of noise into the system that makes it difficult to uncover deterministic influences in price dynamics.

Both parts of our study in Chapter 2 eliminate the randomness inherent in valuation. In particular, one of the data sets comprises significant relative changes in market price that occur in the absence of much change in valuation. This has allowed us to examine the remaining influences on price dynamics, and identify patterns in prices that can be used to test the validity of new theories and methodologies in behavioral finance.

We propose a nonlinear computational optimization algorithm combining a quasi-Newton weak line search with BFGS formula and a dynamic initial parameter pool to obtain the vector of four optimal parameters in the Caginalp Differential Equations (CDEs). Given an n-day period of MPs and NAVs from day i to day $i+n-1$, we compute optimal parameter vector K_i for the period i. Then, we solve the initial value problem (IVP) with CDEs (3.5 - 3.9) to predict MP and return on Day $i+n$. That is, we use the optimal parameters for the next day out-of-sample return prediction. Thus, we obtain $m-i+1$ optimal parameters for the overlapping periods such as $[i, i+n-1]$, $[i+1, i+n]$, ..., $[m, m+n-1]$ for the MP sequence of size $m+n-1$. And also, we can predict MP returns on Days $(i+n, i+n+1, ..., m+n)$.

It is known in literature that the improvement of the quasi-Newton methods over steepest descent and derivative-free algorithms are remarkable [39]. According to our experience, the quasi-Newton method is more efficient than Newton method for CDEs because second derivatives are not required.

The threshold for the gradient should be sufficiently small. But, decreasing the threshold from 10^{-4} to 10^{-6} just increases the average number of quasi-Newton iterations from 89 to 156 without significant improvement in minimization for a large sequence of data. So, we believe that the threshold values between 10^{-4} and 10^{-5} are reasonable for gradient without perfect line search, in practice.

Another novel and important component of the proposed algorithm is the dynamic initial parameter pool. The fixed part of the pool consists of the expected initial vectors. The dynamic part of the pool is updated via previously found optimal parameters and it is specific to the fund's price behavior. The overall pool provides a stable number of quasi-Newton iterations because the experience is employed and the impact of most recent events are dominated.

By reactive evaluation of the financially meaningful optimal parameters employing most of the data up to any given time, we get a stable 32% average maximum improvement factor and a reasonable average daily deviation in market price return during the curve fitting for a sample large data set.

We need a reasonable minimization during the preceding period for a successful next day price return prediction. Sometimes it is possible to get a better curve fitting locally if one were to ignore the intrinsic constraints. However, it does not imply there would be a better prediction always. For example, some vectors with negative parameters may provide smaller sum of squares. But, the negative parameters are not meaningful financially in the model. Moreover, while minimizing the sum of squares, we place exponential weights on the most recent price changes which is financially important. Furthermore, there is a trade off between trend curve fitting and de-trended curve fitting. As shown in Chapter 2 and [15], there are various price return patterns which are relevant for 3 to 10 trading days. They can be caught by de-trended curve fitting. On the other hand, trend curve fitting should not be neglected because the percentage of momentum traders are significant. There are other constraints such as finiteness of traders' asset [19] as well. Also, the time scalings to reflect the current reaction speed of momentum traders and value based traders should be handled automatically. Therefore, the dynamical microeconomic model (3.5-3.9) which combines several factors is more successful than pure curve fitting.

The absolute error of predicted return via CDEs decreases or becomes at least stable for a sample portfolio of CEFs over a long period. We can reject the null hypothesis at the 0.05 level by applying non-parametric tests such as the Mann-Whitney U test and the Wilcoxon rank sum test for the column of the absolute values of differences between the actual daily returns and the predicted daily returns via CDEs and the column of the absolute values of differences between the actual daily returns and the predicted returns via random walk. Moreover, we find that the prediction success of relative price change direction for a fund is stable along six years. According to z-test for the sequence containing 1 for right direction match and -1 for mismatch, we can reject the null hypothesis at the 0.05 level.

To the best of our knowledge, this is the first study to find the next day price return direction for an arbitrary day with a significant right match probability greater than 50% by using the power of differential equations.

APPENDIX A

SHORT SELLING AND MARKET EFFICIENCY

Much of the classical finance arguments are a consequence of the theoretical possibility of selling short, whereby an investor sells shares of an overvalued asset that he does not own, and uses the cash from the proceeds to invest in a stock that is undervalued. Since this would garner a profit without the requirement of any capital, the investment can be increased without bound, thereby eliminating any deviation from the true valuation. There are several serious practical problems with this argument. One is that short sales are often not possible, particularly in large quantities, since they must be borrowed. Another is that there are strict limits on the net amount of short sales. For example, a typical investor with \$100,000 can buy about \$250,000 worth of stock using margin borrowing. However, he can only short about \$40,000. Furthermore, for most investors, a short position implies that he must **pay** – not receive –interest, contrary to the theoretical hypotheses. Thus, shorting a stock that is overvalued by 20%, for example, does not imply a profit unless the short seller can be assured that the overvaluation will disappear before his interest expenses exceed 20% of the stock's value. If the cost of carrying short position is 7%, which is typical currently, then one will not have a profit on a short sale of a 20% overvaluation if the premium does not disappear entirely within three years. The differences between the theoretical finance and reality is further complicated by the fact that the huge capital in many types of accounts including retirement accounts cannot be used to short sell. Thus, it is not at all clear that the capital of hedge funds and expert individuals is adequate to offset the buying of a huge number of individuals through mutual funds and other accounts.

Applying these concepts to typical closed-end funds, we see that a premium of, say 20%, over NAV may not lead to short selling under conditions where an investor must pay a 7% interest charge. Since many of these premiums and discounts have persisted over years, there is no assurance for the short seller that a sharp increase in the premium will be eliminated due to a quick return to fundamental value. Furthermore, borrowing shares for short selling is not always possible.

In EMH, the assertion is not that all participants are rational and unbiased, but that there is a sufficiently large pool of funds controlled by rational and value oriented investors, so that the dynamics of the market is essentially the same as if all investors were free of bias. In the absence of significant short selling, however, there is no mechanism whereby the actions of a biased group of traders to be neutralized quickly by more value oriented investors.

APPENDIX B

TABLES FOR THE DEVIATION MODEL (DM) AND THE DM WITH PARTITION

Eight tables for the DM in Section 2.2 and thirty two tables for the DM with partition in Section 2.3 are included. We do not have the assumption of normality (see [44], p. 240-244). Let $\overline{x}$ be sample mean and s be the sample's standard deviation. We use t-statistic, $t = \frac{\overline{x}-\mu}{s/\sqrt{n}}$ with $(n-1)$ degrees of freedom, when the number of observations (n) is less than 30 where the expected mean μ is zero as stated earlier in the null hypothesis.

When the sample size is sufficiently large (for example $n \geq 30$), $\overline{x}$ and $z \approx \frac{\overline{x}-\mu}{s/\sqrt{n}}$ have approximately normal distributions (see [36], p. 246-248 and 363-367).

The values in the tables are represented in the form of two decimal digits after rounding. Statistical significance is denoted by stars at the 0.01 (***), 0.05 (**), and 0.1 (*) levels using a 1-tailed test for significance in all tables.

TABLES FOR THE DM

Table B1: **Positive low threshold level for the DM.** Average deviations, in percent, associated with 1947 large positive deviators of Day 0 for $2.5 < \mathit{threshold} \leqslant 5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	-0.05	-0.14	-0.22	-0.57	3.25	-0.61	-0.24	-0.20	-0.13
Z$-$Statistic	-0.84	-2.86	-3.88	-11.34	228.99	-11.94	-4.74	-3.97	-2.71
Significance		***	***	***	***	***	***	***	***
Percentage > 0	50.13	45.87	44.74	39.03	100.00	35.90	43.97	45.97	46.69
Variance	6.03	4.70	6.14	4.84	0.39	5.11	5.04	4.82	4.71

Table B2: **Negative low threshold level for the DM.** Average deviations, in percent, associated with 1954 large negative deviators of Day 0 for $-5 \leqslant \mathit{threshold} < -2.5$ during 1998-2006

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.13	0.19	0.25	0.66	-3.28	0.56	0.35	0.05	0.06
Z$-$Statistic	2.40	3.44	5.02	11.72	-229.40	11.50	7.09	0.92	1.11
Significance	***	***	***	***	***	***	***		
Percentage > 0	50.56	54.40	56.04	61.51	0.00	63.51	57.47	51.38	50.87
Variance	5.27	5.73	4.91	6.10	0.40	4.71	4.67	5.26	5.06

Table B3: **Positive medium threshold level for the DM.** Average deviations, in percent, associated with 196 large positive deviators of Day 0 for $5 < threshold \leqslant 7.5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.09	-0.06	-0.64	-0.77	5.95	-1.31	-0.81	0.19	-0.16
Z−Statistic	0.46	-0.27	-2.90	-3.57	120.30	-6.67	-3.94	0.62	-0.68
Significance			***	***	***	***	***		
Percentage > 0	48.47	44.39	40.82	39.29	100.00	30.10	37.76	51.02	46.94
Variance	7.26	9.11	9.44	9.18	0.48	7.54	8.36	19.29	11.08

Table B4: **Negative medium threshold level for the DM.** Average deviations associated with 198 large negative deviators of Day 0 for $-7.5 \leqslant threshold < -5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.27	-0.01	0.61	0.66	-5.93	1.41	0.66	0.19	0.15
Z−Statistic	1.30	-0.06	1.70	2.03	-128.16	5.83	2.99	0.73	0.69
Significance	*		**	**	***	***	***		
Percentage > 0	51.01	52.02	57.58	62.12	0.00	67.17	52.02	52.53	53.54
Variance	8.72	10.51	25.13	20.73	0.43	11.56	9.78	14.27	9.68

Table B5: **Positive high threshold level for the DM.** Average deviations associated with 48 large positive deviators of Day 0 for $7.5 < threshold \leqslant 10$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.10	-1.10	-0.98	-2.52	8.54	-1.42	-0.58	-0.13	0.09
Z$-$Statistic	0.21	-1.22	-1.95	-3.81	78.19	-2.58	-1.43	-0.26	0.17
Significance			**	***	***	***	*		
Percentage > 0	50.00	39.58	41.67	22.92	100.00	33.33	45.83	45.83	52.08
Variance	11.48	38.88	11.99	21.03	0.57	14.63	7.87	12.13	13.56

Table B6: **Negative high threshold level for the DM.** Average deviations associated with 41 large negative deviators of Day 0 for $-10 \leqslant threshold < -7.5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	-0.34	0.22	0.37	0.74	-8.37	1.65	0.75	1.10	0.11
Z$-$Statistic	-0.60	0.43	0.73	1.19	-81.14	2.62	1.33	1.85	0.26
Significance					***	***	*	**	
Percentage > 0	48.78	60.98	58.54	58.54	0.00	60.98	56.10	58.54	56.10
Variance	13.47	11.38	10.27	15.98	0.44	16.29	13.03	13.59	7.58

Table B7: **Positive very high threshold level for the DM.** Average deviations associated with 27 large positive deviators of Day 0 for $10 < threshold \leqslant 50$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	-0.50	0.05	-2.36	-4.57	16.29	-4.82	-0.54	-0.45	-0.40
T$-$Statistic	-0.68	0.10	-1.70	-2.28	11.41	-2.45	-0.82	-0.69	-0.55
Significance			*	**	***	**			
Percentage > 0	48.15	48.15	37.04	33.33	100.00	29.63	40.74	40.74	44.44
Variance	14.30	7.49	51.98	108.16	55.07	104.65	11.95	11.78	13.92

Table B8: **Negative very high threshold level for the DM.** Average deviations associated with 19 large negative deviators of Day 0 for $-50 \leqslant threshold < -10$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	1.10	0.59	1.37	3.46	-21.04	2.46	0.97	0.01	0.18
T$-$Statistic	0.97	0.76	2.69	0.95	-7.43	0.88	0.82	0.01	0.18
Significance			***		***				
Percentage > 0	52.63	57.89	73.68	63.16	0	52.63	52.63	52.63	52.63
Variance	23.04	11.31	4.92	250.86	152.22	146.79	26.76	8.29	20.12

TABLES FOR THE DM WITH PARTITION

Table B9: **The DM with partition BP1 in the low threshold level.** Average deviations, MP returns, and NAV returns, in percent, associated with 523 large positive deviators of Day 0 for $2.5 < threshold \leqslant 5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.04	0.03	-0.29	-0.48	3.25	-0.55	-0.16	-0.10	-0.07
Significance			***	***	***	***	**		
Mean MP Return	0.03	0.09	-0.12	-0.24	3.23	-0.31	-0.08	-0.02	-0.09
Significance				***	***	***			
Mean NAV Return	-0.01	0.06	0.17	0.24	-0.03	0.24	0.08	0.08	-0.01
Significance			***	***	***	***	*	*	

Table B10: **The DM with partition BN1 in the low threshold level.** Average deviations, in percent, associated with 467 large negative deviators of Day 0 for $-5 \leqslant threshold < -2.5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.20	0.11	0.15	0.56	−3.22	0.40	0.42	0.06	0.06
Significance	*		*	***	***	***	***		
Mean MP Return	0.00	0.16	0.06	0.16	−3.19	0.16	0.30	0.02	0.15
Significance		*		**	***	*	***		*
Mean NAV Return	−0.19	0.05	-0.10	−0.40	0.03	−0.24	−0.11	-0.04	0.08
Significance	***		*	***	***	***	**		

Table B11: **The DM with partition BP1 in the medium threshold level.** Average deviations, in percent, associated with 72 large positive deviators of Day 0 for $5 < threshold \leqslant 7.5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.21	-0.17	-0.31	0.30	5.96	−1.28	-0.15	0.40	-0.63
Significance					***	***		*	***
Mean MP Return	-0.25	-0.25	-0.24	0.27	6.01	-0.20	-0.13	0.40	-0.34
Significance					***			*	
Mean NAV Return	−0.46	-0.09	0.07	-0.03	0.05	1.08	0.02	-0.00	0.29
Significance	**					***			*

Table B12: **The DM with partition BN1 in the medium threshold level.** Average deviations, in percent, associated with 75 large negative deviators of Day 0 for $-7.5 \leqslant threshold < -5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.06	0.22	1.55	-0.25	-5.80	1.54	0.38	0.10	0.16
Significance			***		***	***	*		
Mean MP Return	-0.06	0.18	1.26	-0.22	-5.61	0.46	0.42	0.45	0.55
Significance			**		***	**	*	*	**
Mean NAV Return	-0.12	-0.04	-0.29	0.04	0.19	-1.08	0.04	0.34	0.39
Significance			*		***	***		**	**

Table B13: **The DM with partition BP1 in the high threshold level.** Average deviations, in percent, associated with 21 large positive deviators of Day 0 for $7.5 < threshold \leqslant 10$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.46	-2.63	-1.01	-1.26	8.59	-1.51	-0.25	-0.72	-0.05
Significance		*			***	*			
Mean MP Return	0.16	-2.87	-0.73	-1.28	8.28	0.15	0.64	-0.90	-0.21
Significance		*			***			*	
Mean NAV Return	-0.30	-0.25	0.28	-0.03	-0.31	1.66	0.89	-0.18	-0.16
Significance					**	***	**		

Table B14: **The DM with partition BN1 in the high threshold level.** Average deviations, in percent, associated with 24 large negative deviators of Day 0 for $-10 \leqslant threshold < -7.5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.18	0.42	0.26	-0.04	-8.43	2.09	0.81	1.60	-0.48
Significance					***	**		***	
Mean MP Return	-0.51	-0.11	-0.08	-0.42	-8.48	0.26	0.11	1.20	-0.94
Significance					***			**	**
Mean NAV Return	-0.69	-0.54	-0.33	-0.38	-0.04	-1.83	-0.70	-0.46	-0.46
Significance	***	**				***	**	*	*

Table B15: **The DM with partition BP1 in the very high threshold level.** Average deviations, in percent, associated with 4 large positive deviators of Day 0 for $10 < threshold \leqslant 50$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	1.30	1.50	-10.5	0.24	12.69	-1.05	-4.60	0.64	-0.42
Significance					***		***		
Mean MP Return	0.38	2.31	-10.20	0.56	12.60	1.92	-2.30	-0.14	-0.39
Significance					***				
Mean NAV Return	-0.87	0.81	0.31	0.32	-0.10	2.97	2.30	-0.78	0.02
Significance						*			

Table B16: **The DM with partition BN1 in the very high threshold level.** Average deviations, in percent, associated with 10 large negative deviators of Day 0 for $-50 \leqslant threshold < -10$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.52	1.21	1.32	-1.64	-16.14	1.85	2.10	1.30	-0.46
Significance			**		***			*	
Mean MP Return	1.80	0.63	0.40	0.82	-16.27	1.29	2.60	0.45	-0.80
Significance	**				***		*		
Mean NAV Return	1.20	-0.58	-0.92	2.46	-0.13	-0.55	0.53	-0.82	-0.34
Significance			*						

Table B17: **The DM with partition BP2 in the low threshold level.** Average deviations, in percent, associated with 810 large positive deviators of Day 0 for $2.5 < threshold \leqslant 5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	-0.05	-0.34	-0.19	-0.60	3.30	-0.52	-0.19	-0.25	-0.13
Significance		***	**	***	***	***	**	***	**
Mean MP Return	-0.05	-0.19	-0.12	-0.36	3.31	-0.30	-0.06	-0.19	-0.10
Significance		***	*	***	***	***		**	*
Mean NAV Return	-0.00	0.15	0.08	0.24	0.01	0.21	0.12	0.06	0.04
Significance		***		***		***	***		

Table B18: **The DM with partition BN2 in the low threshold level.** Average deviations, in percent, associated with 839 large negative deviators of Day 0 for $-5 \leqslant threshold < -2.5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.12	0.22	0.24	0.53	−3.34	0.59	0.36	0.13	0.08
Significance	*	***	***	***	***	***	***	**	
Mean MP Return	-0.06	0.10	-0.08	-0.04	−3.42	0.32	0.40	0.23	0.12
Significance		*			***	***	***	***	*
Mean NAV Return	−0.18	−0.12	−0.33	−0.56	-0.08	−0.28	0.04	0.11	0.05
Significance	***	**	***	***	*	***		**	

Table B19: **The DM with partition BP2 in the medium threshold level.** Average deviations, in percent, associated with 79 large positive deviators of Day 0 for $5 < threshold \leqslant 7.5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	-0.15	-0.13	-0.45	−1.19	5.88	−1.40	−1.10	0.99	-0.38
Significance				***	***	***	***	***	
Mean MP Return	-0.35	-0.07	0.03	−0.81	4.97	−1.22	-0.55	0.69	0.20
Significance				***	***	***	*	**	
Mean NAV Return	-0.20	0.07	0.48	0.38	−0.91	0.18	0.51	-0.30	0.58
Significance			**	**	***		**	*	***

Table B20: **The DM with partition BN2 in the medium threshold level.** Average deviations, in percent, associated with 82 large negative deviators of Day 0 for $-7.5 \leqslant threshold < -5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.51	0.13	-0.07	1.12	-6.04	1.59	0.64	0.15	0.47
Significance	**			***	***	***	**		*
Mean MP Return	0.23	-0.39	-0.42	0.39	-5.39	0.78	0.49	0.53	0.32
Significance			*		***	**	*	*	
Mean NAV Return	-0.28	-0.52	-0.36	-0.73	0.65	-0.81	-0.14	0.38	-0.15
Significance	*	**	**	***	***	***		**	

Table B21: **The DM with partition BP2 in the high threshold level.** Average deviations, in percent, associated with 17 large positive deviators of Day 0 for $7.5 < threshold \leqslant 10$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	-0.65	-0.04	-1.26	-3.31	8.63	-2.08	-1.00	1.10	-0.21
Significance			*	***	***	***	*		
Mean MP Return	-0.46	0.55	-1.49	-2.70	6.76	-0.88	-1.10	0.36	0.20
Significance			*	**	***	*	*		
Mean NAV Return	0.19	0.59	-0.23	0.61	-1.87	1.20	-0.10	-0.69	0.41
Significance		*			***	**		*	

Table B22: **The DM with partition BN2 in the high threshold level.** Average deviations, in percent, associated with 11 large negative deviators of Day 0 for $-10 \leqslant threshold < -7.5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	-0.84	0.00	1.34	2.15	-8.35	1.25	1.00	1.10	1.30
Significance				**	***				*
Mean MP Return	-0.67	-0.49	-0.48	-0.28	-7.96	-0.12	-0.29	0.28	0.83
Significance					***				
Mean NAV Return	0.17	-0.49	-1.82	-2.43	0.39	-1.37	-1.30	-0.80	-0.49
Significance			**	***		*	*		

Table B23: **The DM with partition BP2 in the very high threshold level.** Average deviations, in percent, associated with 7 large positive deviators of Day 0 for $10 < threshold \leqslant 50$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	-0.36	-0.99	-1.75	-2.77	16.57	-5.63	0.39	-0.72	-2.60
Significance				*	***	*			
Mean MP Return	-0.88	-0.13	-3.05	-2.95	13.71	-1.85	0.03	-0.49	-1.60
Significance			*	*	***	*			
Mean NAV Return	-0.52	0.86	-1.30	-0.18	-2.86	3.78	-0.36	0.23	1.10
Significance			**		**	*			

Table B24: **The DM with partition BN2 in the very high threshold level.** Average deviations, in percent, associated with 2 large negative deviators of Day 0 for $-50 \leqslant threshold < -10$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	2.60	0.74	2.28	2.50	-13.81	8.27	1.20	-2.60	-2.80
Significance				*	*	***			
Mean MP Return	2.70	1.48	3.39	3.53	-9.75	4.40	2.10	-0.44	-0.70
Significance				*	*	*			
Mean NAV Return	0.10	0.73	1.10	1.03	4.06	-3.87	0.86	2.10	2.10
Significance					*	*			

Table B25: **The DM with partition BP3 in the low threshold level.** Average deviations, in percent, associated with 440 large positive deviators of Day 0 for $2.5 < threshold \leqslant 5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	-0.23	-0.07	-0.20	-0.65	3.23	-0.81	-0.32	-0.15	-0.19
Significance	*		**	***	***	***	***	*	*
Mean MP Return	-0.21	-0.13	-0.13	-0.58	0.48	-0.78	-0.26	-0.09	-0.08
Significance	*	*		***	***	***	**		
Mean NAV Return	0.03	-0.06	0.07	0.07	-2.76	0.03	0.05	0.06	0.11
Significance					***				

Table B26: **DM with partition BN3 in the low threshold level.** Average deviations, in percent, associated with 476 large negative deviators of Day 0 for $-5 \leqslant threshold < -2.5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.08	0.19	0.45	0.91	-3.28	0.56	0.36	-0.07	0.02
Significance		**	***	***	***	***	***		
Mean MP Return	-0.03	-0.27	-0.03	0.60	-0.51	0.58	0.10	0.19	0.08
Significance		***		***	***	***		**	
Mean NAV Return	-0.11	-0.46	-0.48	-0.32	2.77	0.02	-0.26	0.26	0.06
Significance		***	***	***	***		***	**	

Table B27: **DM with partition BP3 in the medium threshold level.** Average deviations, in percent, associated with 28 large positive deviators of Day 0 for $5 < threshold \leqslant 7.5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.28	0.38	-1.60	-1.96	6.18	-0.90	-1.80	-2.60	1.70
Significance			***	***	***	**	***	*	*
Mean MP Return	-0.36	0.52	-1.62	-3.01	1.31	-0.81	-1.40	-0.88	0.08
Significance			***	***	***	**	***		
Mean NAV Return	-0.65	0.15	-0.02	-1.05	-4.87	0.08	0.38	1.70	-1.60
Significance	**			**	***				*

Table B28: **DM with partition BN3 in the medium threshold level.** Average deviations, in percent, associated with 30 large negative deviators of Day 0 for $-7.5 \leqslant threshold < -5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	-0.10	-0.67	1.26	1.77	-5.99	1.14	0.90	-0.28	0.32
Significance			***	***	***	**			
Mean MP Return	-0.31	-0.53	0.85	0.96	-1.83	0.95	0.31	0.44	0.56
Significance			*	*	***	**			*
Mean NAV Return	-0.20	0.14	-0.41	-0.81	4.16	-0.19	-0.59	0.72	0.24
Significance				**	***			*	

Table B29: **DM with partition BP3 in the high threshold level.** Average deviations, in percent, associated with 6 large positive deviators of Day 0 for $7.5 < threshold \leqslant 10$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.57	-0.11	-0.57	-4.57	8.19	-0.73	-0.01	-1.30	2.90
Significance				*	***				*
Mean MP Return	-0.16	0.96	-0.76	-3.56	3.18	0.25	0.66	-0.41	2.30
Significance				*	***				
Mean NAV Return	-0.72	1.07	-0.19	1.01	-5.01	0.98	0.67	0.93	-0.61
Significance		*			***	*			

Table B30: **DM with partition BN3 in the high threshold level.** Average deviations, in percent, associated with 4 large negative deviators of Day 0 for $-10 \leqslant threshold < -7.5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	-3.80	0.15	-1.04	-0.25	-8.31	-0.11	-0.34	-1.20	0.74
Significance					***				
Mean MP Return	-0.94	-1.34	-2.07	-0.49	-2.29	-0.18	0.97	0.36	0.41
Significance			*		***		*		
Mean NAV Return	2.90	-1.49	-1.04	-0.24	6.02	-0.07	1.30	1.60	-0.32
Significance		*			***		**		

Table B31: **DM with partition BP3 in the very high threshold level.** Average deviations, in percent, associated with 9 large positive deviators of Day 0 for $10 < threshold \leqslant 50$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	-1.90	-0.35	-0.98	-6.16	19.00	-7.46	0.03	-0.83	0.21
Significance			**		***	*			
Mean MP Return	-1.60	-0.18	-0.46	-1.04	1.96	-0.40	0.34	-0.32	1.50
Significance	**				**				**
Mean NAV Return	0.32	0.17	0.52	5.11	-17.04	7.06	0.32	0.52	1.30
Significance					***	*			**

Table B32: **DM with partition BN3 in the very high threshold level.** Average deviations, in percent, associated with 2 large negative deviators of Day 0 for $-50 \leqslant threshold < -10$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	5.60	-2.64	4.44	15.50	-17.66	-3.51	-0.92	-3.70	9.00
Significance					*				
Mean MP Return	10.00	-2.47	2.60	18.80	-5.67	-3.51	-0.80	-4.40	4.70
Significance					***				
Mean NAV Return	4.50	0.16	-1.83	3.37	11.99	0.00	0.12	-0.71	-4.40
Significance	*		**	*					

Table B33: **The DM with partition BP4 in the low threshold level.** Average deviations, in percent, associated with 174 large deviators of Day 0 for $2.5 < threshold \leqslant 5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.19	0.10	-0.19	-0.46	3.07	-0.74	-0.55	-0.36	-0.17
Significance			*	***	***	***	***	**	
Mean MP Return	-0.06	0.16	-0.37	-0.48	0.06	-0.86	-0.46	-0.19	-0.02
Significance			***	***	***	***	***		
Mean NAV Return	-0.25	0.06	-0.19	-0.03	-3.01	-0.12	0.08	0.17	0.15
Significance	**		*		***				

Table B34: **The DM with partition BN4 in the low threshold level.** Average deviations, in percent, associated with 172 large deviators of Day 0 for $-5 \leqslant threshold < -2.5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.08	0.24	-0.00	0.82	-3.08	0.89	0.05	-0.05	0.05
Significance		*		***	***	***			
Mean MP Return	0.05	-0.17	-0.42	0.19	-0.08	1.09	0.12	0.09	0.11
Significance			***		***	***			
Mean NAV Return	-0.04	-0.41	-0.41	-0.63	2.99	0.21	0.06	0.13	0.06
Significance		***	***	***	***	*			

Table B35: **The DM with partition BP4 in the medium threshold level.** Average deviations, in percent, associated with 17 large deviators of Day 0 for $5 < threshold \leqslant 7.5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.35	0.05	-1.30	-1.44	5.83	-1.72	-0.83	0.23	-0.18
Significance			*	**	***	***			
Mean MP Return	-0.22	-1.28	-0.73	-2.09	0.17	-0.91	-0.97	-0.40	-1.20
Significance		*		***	*				**
Mean NAV Return	-0.57	-1.33	0.56	-0.65	-5.66	0.81	-0.14	-0.63	-1.00
Significance					***				*

Table B36: **The DM with partition BN4 in the medium threshold level.** Average deviations, in percent, associated with 11 large deviators of Day 0 for $-7.5 \leqslant threshold < -5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.87	-0.87	-2.59	0.34	-5.92	-0.12	2.10	2.50	-2.70
Significance					***		***		
Mean MP Return	1.10	-0.84	1.21	0.62	-0.11	-0.43	1.80	3.60	-1.40
Significance							**		*
Mean NAV Return	0.21	0.03	3.80	0.28	5.81	-0.31	-0.36	1.20	1.40
Significance					***				

Table B37: **The DM with partition BP4 in the high threshold level.** Average deviations, in percent, associated with 4 large deviators of Day 0 for $7.5 < threshold \leqslant 10$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.69	0.96	-0.24	-2.74	8.37	0.81	-1.20	-0.22	-2.20
Significance				*	***	**			**
Mean MP Return	0.32	0.85	1.02	-2.34	0.00	0.62	-0.50	-0.14	-2.20
Significance		*							**
Mean NAV Return	-0.38	-0.11	1.26	0.40	-8.37	-0.19	0.72	0.08	-0.01
Significance			*		***		*		

Table B38: **The DM with partition BN4 in the high threshold level.** Average deviations, in percent, associated with 2 large deviators of Day 0 for $-10 \leqslant threshold < -7.5$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	3.10	-0.77	-0.88	4.39	-7.84	2.11	0.82	-1.20	-0.71
Significance					***				
Mean MP Return	2.90	1.33	0.61	5.52	-1.03	1.71	0.43	-0.97	-2.10
Significance	**	**							
Mean NAV Return	-0.22	2.10	1.48	1.13	6.81	-0.39	-0.39	0.23	-1.40
Significance					**				**

Table B39: **The DM with partition BP4 in the very high threshold level.** Average deviations, in percent, associated with 7 large deviators of Day 0 for $10 < threshold \leqslant 50$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	0.15	0.79	-0.09	-7.09	14.60	-2.78	0.12	-0.33	1.10
Significance		*		*	***				
Mean MP Return	-0.24	0.18	0.05	-2.97	0.40	-2.93	-0.61	-0.74	1.00
Significance				*					
Mean NAV Return	-0.39	-0.61	0.14	4.11	-14.20	-0.14	-0.73	-0.41	-0.05
Significance					***				

Table B40: **The DM with partition BN4 in the very high threshold level.** Average deviations, in percent, associated with 5 large deviators of Day 0 for $-50 \leqslant threshold < -10$ during 1998-2006.

Day	-4	-3	-2	-1	0	1	2	3	4
Mean Deviation	-0.26	0.56	-0.11	9.26	-35.08	3.74	-0.56	-0.00	-0.88
Significance					***				
Mean MP Return	-0.94	-0.17	0.34	0.43	0.13	-6.81	0.86	-0.30	-0.32
Significance							*		
Mean NAV Return	-0.68	-0.73	0.45	-8.83	35.21	-10.60	1.40	-0.29	0.56
Significance					***	*		**	

APPENDIX C

ABBREVIATIONS

AMO	Alliance All-Market Advantage Fund
APB	Asia Pacific Fund
BF	Behavioral finance
BN1	Block of large negative deviations such that MP return spikes down while NAV is relatively unchanged on Day 0
BN2	Block of large negative deviations such that both MP return and NAV return are changed where the influence of MP return on Day 0 is greater
BN3	Block of large negative deviations such that both MP return and NAV return are changed where the influence of NAV return on Day 0 is greater
BN4	Block of large negative deviations such that NAV return spikes up while MP is relatively unchanged on Day 0
BP1	Block of large positive deviations such that MP return spikes up while NAV is relatively unchanged on Day 0
BP2	Block of large positive deviations such that both MP return and NAV return are changed where the influence of MP return on Day 0 is greater
BP3	Block of large positive deviations such that both MP return and NAV return are changed where the influence of NAV return on Day 0 is greater
BP4	Block of large positive deviations such that NAV return spikes down while MP is relatively unchanged on Day 0

CDEs	Caginalp differential equations
CEF	Closed-end fund
DM	Deviation model
EMH	Efficient market hypothesis
ETF	Exchange-Traded Fund
EWP	iShares MSCI Spain Index Fund
GEF	General Equity Fund
MIF	Maximum improvement factor
MP	Market price
NAV	Net asset value
NYSE	New York Stock Exchange
P_t	Market price at time t
SEF	Specialized Equity Fund
SNF	Spain Fund Inc
S&P 500	Standard & Poor's Composite 500
TRF	Templeton Russia and East European Fund
V_t	NAV price at time t
WEF	World Equity Fund

BIBLIOGRAPHY

[1] A. Akhigbe, S.J. Larson, and J. Madura, Market underreaction and overreaction of technology stocks, *J. of Psychology & Financial Markets* 3 (2002), 141-151.

[2] S.C. Anderson and J.A. Born. *Closed-End Fund Pricing: Theories and Evidence*, Boston: Kluwer Academic Publishers, 2002.

[3] U.M. Ascher and L.R. Petzold, *Computer Methods for Ordinary Differential Equations and Differential-Algebraic Equations*, Philadelphia, PA: SIAM, 1998.

[4] K.E. Atkinson, *An Introduction to Numerical Analysis*, 2nd ed., New York: John Wiley & Sons, Inc., 1988.

[5] M. Bartholomew-Biggs, *Nonlinear Optimization with Financial Applications*, Boston: Kluwer Academic Publishers, 2005.

[6] F. Black, Noise *J. of Finance* 41 (1986), 529-543.

[7] Z. Bodie, A. Kane and A.J. Marcus, *Investments*, 6th ed., Boston, MA: Irwin McGraw-Hill, 2005.

[8] C. Bosner-Neal, G. Brauer, R. Neal and S. Wheatley, International restrictions and closed-end country fund prices, *J. of Finance* 45 (June 1990), 523-547.

[9] G.E. Box, and G.M. Jenkins, *Time-series Analysis for Forecasting and Control*, Holden Day, 1970.

[10] M. Bremer, and R.J. Sweeney, The reversal of large stock price decreases, *J. of Finance* 46 (1991), 747-754.

[11] H. Bremermann, A method of unconstrained global optimization, *Mathematical Biosciences* 9 (1970), 1-15.

[12] C.G. Broyden, The convergence of a class of double rank minimization algorithms, part 1, *J. Inst. Maths. Applications* 6 (1970), 76-90.

[13] C.G. Broyden, The convergence of a class of double rank minimization algorithms, part 2, *J. Inst. Maths. Applications* 6 (1970), 222-231.

[14] G. Caginalp and V. Ilieva, Hybrid methods for testing support and resistance, (2006).

[15] G. Caginalp and D. Balenovich, A theoretical foundation for technical analysis, *J. Technical Analysis* Winter/Spring 59 (2003), 5-22.

[16] G. Caginalp, D. Porter, and V. Smith, Financial bubbles: Excess cash, momentum, and incomplete information, *J. of Psychology Financial Markets* 2(2) (2001), 80-99.

[17] G. Caginalp, D. Porter, and V. Smith, Overreactions, momentum, liquidity, and price bubbles in laboratory and field asset markets, *J. of Psychology & Financial Markets* 1 (2000), 24-48.

[18] G. Caginalp, D. Porter, and V. Smith, Momentum and overreaction in experimental asset markets, *Int. J. of Industrial Organization* 18 (2000), 187-204.

[19] G. Caginalp and D. Balenovich, Asset flow and momentum: Deterministic and stochastic equations, *Phil. Trans. R. Soc.* 357(1758) (1999), 2119-2133.

[20] G. Caginalp and H. Laurent, The predictive power of price patterns, *Applied Mathematical Finance* 5 (1998), 181-206.

[21] G. Caginalp and G.B. Ermentrout, A kinetic thermodynamic approach to the psychology of fluctuations in financial markets, *Appl. Math. Lett.* 3(4) (1990), 17-19.

[22] www.pitt.edu/~caginalp

[23] D.M. Cutler, J.M. Poterba, and L.H. Summers, Speculative dynamics and the role of feedback traders, *American Economic Review* 80 (1990), 63-68.

[24] A. Duran and G. Caginalp, Overreaction and optimization in stock markets, minisymposium presentation at *SIAM Conf. on Financial Mathematics and Engineering*, Boston, MA, July 9-12 (2006).

[25] A. Duran and G. Caginalp, A Comparison of numerical optimization techniques for financial markets, presentation at *SIAM Annual Meeting*, Boston, MA, July 10-14 (2006).

[26] A. Duran and G. Caginalp, Differential equations and computational optimization for closed end funds, presentation at *AMS Joint Mathematics Meetings*, San Antonio, TX, Jan. 12-15 (2006), Abstracts of papers presented to the *AMS* 27(1) (2006) 190.

[27] A. Duran and G. Caginalp, Overreaction diamonds: Precursors and aftershocks for significant price changes, (2005), 30 pages.

[28] A. Duran and G. Caginalp, Data mining for overreaction in financial markets, *Proceedings of the IASTED International Conf. on Software Engineering and Applications (SEA)*, Phoenix, AZ, Nov. 14-16 (2005), 28-35.

[29] A. Duran and G. Caginalp, Overreaction and risk for closed end funds, presentation at *SIAM Conf. on Mathematics for Industry: Challenges and Frontiers*, Detroit, MI, (2005).

[30] J.D. Gibbons, *Nonparametric Statistical Inference*, 2nd ed., M. Dekker, 1985.

[31] D. Griffin, and A. Tversky, The weighing of evidence and determinants of confidence, *Cognitive Psychology* 24 (1992), 411-435.

[32] N.J. Higham, *Accuracy and Stability of Numerical Algorithms*, SIAM, Philadelphia, 1996.

[33] M. Hollander and D.A. Wolfe, *Nonparametric Statistical Methods*, Wiley, 1973.

[34] J. Madura, and N. Richie, Overreaction of exchange-traded funds during the buble of 1998-2002, *J. of Behavioral Finance* 5 (2004), 91-104.

[35] D.K. Mak, *The Science of Financial Market Trading*, New Jersey: World Scientific, 2003.

[36] W. Mendenhall, R.J. Beaver and B.M. Beaver *Introduction to Probability and Statistics*, 11th ed., Pacific Grove: Brooks/Cole, 2003.

[37] J. Milstein, Spline and weighted random directions method for nonlinear optimization, *Mathematical Biosciences* 74 (1985), 247-256.

[38] J. Milstein, The inverse problem: Estimation of kinetic parameters, *Modelling of Chemical Reactions Systems: Proceedings of an Int. Workshop* Heidelberg Series: Springer Series in Chemical Physics 18 (1981), 92-101.

[39] J. Nocedal and S.J. Wright, *Numerical Optimization*, Springer Series in Operations Research, New York: Springer-Verlag, 1999.

[40] J. Pontiff, Excess volatility of closed-end funds *American Economic Review* 87 (March 1997), 155-167.

[41] D. Porter, V.L. Smith, Stock market bubbles in the laboratory, *Applied Mathematical Finance* 1 (1994), 111-128.

[42] J.M. Poterba, L.H. Summers, Mean reversion in stock prices: Evidence and implications, *J. of Financial Economics* 22 (1988), 27-59.

[43] D.E. Rapach, M.E. Wohar, and J. Rangvid, Macro variables and international stock return predictability, *Int. J. of Forecasting* 21(1) (2005), 137-166.

[44] S. Roman, *Int. to the Mathematics of Finance*, New York: Springer, 2004.

[45] B. Rosenberg, K. Reid, and R. Lanstein, Persuasive evidence of market inefficiency, *J. of Portfolio Management* 11 (1985), 9-16.

[46] J. Saxton, *Economic repercussions of the stock market bubble*, A joint economic committee study, United States Congress, July 2003.

[47] R.J. Shiller, *Irrational Exuberance*, Princeton University Press, 2000.

[48] R.R. Sturm, Investor confidence and returns following large one-day price changes, *J. of Behavioral Finance* 4 (2003), 201-216.

[49] B. Weiner, Attributional thoughts about consumer behavior, *J. of Consumer Research* 27 (2000), 382-387.

[50] P. Zarovin, Short-run market overreaction: Size and seasonality effects, *J. of Portfolio Management* 15 (1989), 26-29.