ESSAYS ON THE ECONOMICS OF INFORMATION IN AUCTIONS

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Informational assumptions are an important aspect of the study of auctions in economic theory. However, there has been limited research into how the assumptions made by theorists impact their results. I explore two different aspects of the information available to bidders in auctions. The information that is important to the theoretical study of auctions can be divided into two types. First, there is information about the realized values of the bidders. I explore this through a model of an English auction with interdependent values where bidders are able to acquire the private information that is realized by other bidders. I find that the ability to costlessly acquire additional information about competitors does not impact the efficiency of the English auction. This is in line with other research into this type of information acquisition. I also briefly explore the revenue implications of bidders being able to acquire this type of information. The second type of information is about the overall structure from which the bidders values are drawn. In most theoretical treatments of auctions, it is assumed that bidders know this overall structure. I begin to relax this assumption by adding a small amount of uncertainty about the structure of one of the bidders valuation functions in auctions with interdependent values. Here I find that both the English auction and the second price auction are no longer efficient after the change in informational structure. I use data collected through economic experiments to test the theoretical predictions of this model and find that the English auction is more efficient both with the standard informational assumptions and with the change made in the informational

structure. Both of these results suggest that the open format of an English auction, where information is revealed over the course of the auction, may mean that theoretical results with stronger assumptions about informational structures at the beginning of the auction are somewhat robust to those assumptions.

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PREFACE

I would like to thank my committee members for their support and helpful comments through out the process of writing this dissertation. In particular, I would like to thank my primary advisor, Andreas Blume, for his guidance from the beginning of each of the projects. I am also thankful for various seminar participants at the University of Pittsburgh, St. Joseph's University and the Department of Justice who provided useful comments at various stages of the process. Finally, I would like to thank my family and friends who have given me a great amount of moral support.

1.0 INTRODUCTION

Auctions are an ancient form of markets. They date back to 500 B.C. in Greece where they were used to distribute wives to potential husbands with side payments included for less attractive women (Cassidy). They are still used around the world to distribute goods, from art to land to food to spectrum rights. The main characteristic that makes an auction is that there is a single seller and multiple potential buyers. There have been many changes in the way that auctions have been run over the years. They are run in large auction houses such as Sotheby's and are run over the internet on eBay. The most common formats studied are the first price auction, the second price auction, the English auction and the Dutch auction. The first and second price auctions are sealed bid auctions where the bidder submit their bid privately and the auctioneer determines the winner as the bidder who has submitted the highest bid. In the first price auction, the winner pays his bid. In the second price auction, the winner pays the second highest bid (the bid that is the highest bid other than his own). The English and Dutch auctions are open format auctions in which the price is announced in public and bidders decide whether or not to remain in the auction at a given price. In the English auction, the price is increased during the course of the auction and the auction continues until there is only one bidder remaining and that bidder is the winner. In the Dutch auction, the price starts at a very high level and is decreased over the course of the auction. Here the first bidder to leave the auction wins and pays the price at which she dropped out. It has been shown that the first price and Dutch auctions and the second price and English auctions are are outcome equivalent, respectively.

The study of auctions is one of the areas in which economic theory has been very successful. Theorists have been hired to help design auctions and to help bid in them. There has been research into optimal auctions where the auctioneer's goal is to maximize their profits. Vickery (1961) and later Myerson (1981) and Riley and Samuelson (1981) showed that under symmetric independent private values[1], the common auction formats mentioned above yield the same expected revenue for the auctioneer. This result is known as the Revenue Equivalence Theorem. There has also been research into achieving *efficient* auctions where the bidder who values the object being auctioned the most wins. In the following chapters, I will focus on this second question, exploring the efficiency of commonly used auction formats under various information structures. [2]

There are two types of information that may be important to the outcome of an auction. There is information regarding the realized value to the bidders. This includes both information affecting the bidder's value and that affecting the value of the other bidders. Then there is information about larger structure from which those values are derived. There are usually fairly strict assumptions made about this type of information in order to simplify the auction environment being studied.

There have been a number of papers focusing on bidders acquiring information about their rivals. It is primarily divided into two types: situations where the information is exogenously given to the bidders and situations where the bidders can decide whether to acquire information. The second chapter of the dissertation falls into the former category. The second chapter explores an English auction with interdependent values where there is a bidder who is given extra information about the values of other bidders. [3] It expands on previous literature, which had shown efficiency when bidders only know their own signals and also when some bidders know their valuations. The generalization comes in the form of the information structure. Here again there are two types of bidders: outsiders again

[1] Here bidders values are independently drawn from the same distribution.

[2] Although many auctioneers want to maximize their revenue, some auctioneers, specifically governments, may view efficiency as equally if not more important.

[3] Interdependent values model situations where bidders' values depend not only on their own tastes and information, but on those of the other bidders as well. This is in contrast to private values where bidders have independently determined values. Interdependent values are a specific type of *affiliated* values which were introduced in Milgrom and Weber (1982).

have no information, other than their own signals, and “partial-insiders” who know other bidders’ signals as well as their own, but do not necessarily have perfect information about their valuations. It is shown that efficiency continues to hold for this case. After showing that there is an efficient ex-post equilibrium, revenue comparisons are made between this case and the case when insider’s know their valuations. This varies from other research in this area which has concentrated on first and second price auctions, especially those with private values. There has also been some more generalized research done on the impact of information revelation in terms of the efficiency of auctions. Mikoucheva and Sonin (2004) find that it can not be generally shown that more information leads to more efficiency and in fact that depending on the format of the auction information revelation may lead to efficiency losses.

Another area that has not received a large amount of attention is the information that bidders have going into an auction. Most of the literature assumes that bidders have quite a bit of information both about their own value and about the information of the other bidders. In private value settings, where each bidder’s value for the object is determined independently, it is assumed that all of the bidders know the distribution function from which the values are drawn. In settings where values are not independent, some even stronger assumptions are sometimes made. The settings considered in this dissertation are ones with interdependent values. Here bidders’ values depend not only on the individuals’ private information, but on the information of the other bidders. In other research that investigates interdependent values, it is assumed that bidders not only know the distribution of bidders’ signals (the information that bidders get about their value before the auction) but also the way in which that information impacts all of the bidders’ values. This assumptions may be valid in cases where bidders frequently meet one another.[4] However, it is important to address situations where bidders do not have the opportunity to learn about their opponents. The third chapter focuses on these issues. It explores the English auction, this time along side the second price auction with interdependent values in order to determine which auction

[4]This would raise other questions about repeated auctions, which have been studied.

is efficient under two settings. First, as a baseline case, I explore the symmetric case, where all bidders have the same valuation function, comparing the auctions using data from experiments. Theoretically, the English auction and the second price auction are both efficient in the particular symmetric interdependent value case considered. I find that neither auction is fully efficient all of the time and that the English auction achieves efficiency with more frequency. Then I add the possibility that one of the bidders has two possible valuation functions. Only the bidder himself knows which valuation function he has in a particular auction. Once this uncertainty is introduced, theory predicts that the English auction will more frequently produce the efficient outcome. The experimental results are consistent with this prediction.

Generally I find that the English auction seems slightly less sensitive to informational assumptions than other auction forms.[5] This is most likely due to the fact that the English auction has an open format. This means that as the auction continues bidders acquire information. Thus the informational setting varies through out the auction. This is especially relevant to analyzing information about realized values since this information is usually revealed at some point during the auction. There are still a number of open questions to be resolved about the role of informational assumptions in auction theory. It is particularly important to address these questions as these theories are used to influence real policy decisions.

[5] Part of this is from comparing the results here to other papers which relax informational assumptions for the first and second price auctions.

2.0 AN ENGLISH AUCTION WITH PARTIAL-INSIDERS AND INTERDEPENDENT VALUES

2.1 INTRODUCTION

There are numerous situations in auctions where some bidders may have more or better information about the value of the object on which they are bidding. These include such instances as oil experts who are more familiar with a certain region (perhaps because they already have wells close by) or part of the management from a company that is now bidding in its sale. Intuition tells us that these differences in information with respect to other bidders should have some impact on the end results of the auction. One would also think that this impact would be in the favor of the bidders, since bidders frequently pay a premium to have an expert on their side. It is therefore important to develop an understanding of what kinds of influence "insiders" will have on the outcome of an auction, both to allow bidders to decide whether it is worth hiring an expert and also to allow auctioneers to decide how much information should be disseminated prior to an auction.

There are further nuances to the role of information in auctions. Even then most experienced of experts may not have perfect information about the actual value of what they are trying to buy. The oil expert may have better information because he has experience in the area, but this does not necessarily imply that he will know the exact value of the oil under particular piece of land. It merely means that his knowledge is in some way better. This also holds for insiders in a company, they quite obviously have better knowledge of how the company works and how it might improve, but they cannot be completely sure how their ideas will influence the value of the company. Therefore it makes sense to develop a model in which bidders can have more information than their competitors, but not necessarily perfect information. This is what I will focus on here.

When addressing this type of informational difference it is interesting to focus on situations where the information that bidders may have is valuable not only to themselves but also to their competitors. In the case of independent private values, the bidders are unlikely to care about the information of other bidders. However, in cases such as mineral rights auctions, bidders values are not independent or private. Rather they are interdependent, if the value of the object increases for one bidder it will also increase for the other bidders (although not necessarily by the same amount). For example, each of the bidders in an auction for a tract of land that may have oil would value the land based on the amount of oil under the ground. Therefore if they gained information that said that the amount of oil was higher than they previously believed, their value would go up. The amount that it would change would depend on their technological ability to get the oil out of the ground and their costs associated with doing so. Similarly, all of the bidders for a company care about the success of that company and information about the potential for increased revenues would increase the value to all bidders. Therefore the auction which is explored here is modeled as one with interdependent values.[1] Because of the interdependent nature of the valuations, the English auction will be the only one considered, since it has already been shown by Kim (2002) and Krishna (2001) that a second-price auction does not retain efficiency when the valuations of the bidders are interdependent.

2.2 LITERATURE REVIEW

As noted above, there has already been research done in this area. Krishna (2001) examines the equilibria of the English auction when bidders have interdependent values, are asym-

[1] I am focusing here on interdependent values rather than common values in order to allow the values of the bidders to be related to one another while not being exactly the same.

metric (i.e. their values are asymmetric) and have standard information. He extends the single-crossing condition of Maskin (1992) to the "average crossing" and "cyclical crossing" conditions, showing that if the valuation functions of the bidders follow either of these conditions then the auction will be efficient even when there are more than two bidders. Kim (2002) extends this finding to the case where some of the bidders, so called "insiders," have complete information about their valuation. This is an interesting problem because in actual auctions there are many times some of the bidders have more information. He shows that while the second and first price auctions may no longer be efficient the English auction retains this characteristic as long as the valuation functions meet "single crossing on indifference curves" and "average crossing on indifference curves".

This paper expands on Kim's work by examining the case where insiders may not know their valuations outright or have complete information about the other bidders' signals, but have some extra information about some of the other signals. These insiders will therefore be called "partial-insiders." It will be shown that the English auction is also efficient in this case.

Bergemann et al. (2008) analyze an interdependent value environment where the bidders can obtain information about their valuations only through a costly process. They find that bidders are likely to obtain more than the socially optimal amount of information and that this is increased when the bidders' values are more related. This is however a model where information acquisition is covert and thus differs from the model below where all of the bidders are aware of the insider status of the bidders. Compte and Jehiel (2008) also study information acquisition in auction settings and find that dynamic format auctions (such as the English auction) perform better than static auctions (such as the second price auction) in terms of revenue generation. They attribute this to the fact that information is acquired through out the auction in dynamic settings and therefore bidders have incentives to remain in the auction for a longer period of time in order to gain that information (thus driving up the price of the good being auctioned).

2.3 MODEL

The auctioneer sells one indivisible object to $N > 2$ potential buyers.[2] The auction used is an English auction, which as described above, is one in which the price increases as the auction continues. As the price, p, increases bidders decide to remain in the auction or to drop out of the auction.[3] As the auction progresses, it will be important to differentiate between the bidders who remain in the auction at a given price and those that do not. So, I will denote the set of active bidders by $A \in N$ and the inactive bidders as $\bar{A} \in N$. Once a bidder has dropped out they may not return. Once only one bidder remains, the auction ends and the remaining bidder wins. The winning bidder pays the price at which the last bidder to drop out left the auction.

The bidders will have interdependent values, so that their value will be a function of their own private information and the information of the other bidders. Each bidder's valuation function, denoted $v_i(\mathbf{s})$ with $i \in N$, will depend on an N-dimensional signal vector $\mathbf{s} \in \mathbb{R}^N$. So each bidder's value depends on the information of all of the bidders.[4] As a matter of notation, bidder i's signal is written as s_i and the signals of all bidders except bidder i are denoted as s_{-i}. The signals are independently drawn from the distribution $F\colon \mathbb{R}_+ \to [0, \omega]$ with density f. The distribution from which the signals are drawn are common knowledge to all of the bidders. The bidders' valuation functions are also assumed to be twice continuously differentiable with respect to $\mathbf{s}$. The bidders know the functional form of their valuation functions as well as those of their competitors. This is a key aspect of the equilibrium as the bidders will use this information to learn about the other bidders as they drop out of the auction. Some further restrictions on $v_i(\mathbf{s})$ will be made below in order to ensure the existence of an equilibrium and the efficiency of the outcome of the auction. An allocation

[2] I restrict my attention to $N > 2$ bidders, since the $N = 2$ bidders case is no different from that covered in Kim (2002).

[3] Bidders will want to remain in the auction as long as their expected payoff from winning the auction is positive.

[4] In order to find an equilibrium, it will be necessary that the each bidder's value is more influenced by their own signal than by the signals of other bidders.

is said to be *efficient* when the bidder with the highest realized valuation wins the object.

The aspect of this model which differentiates this from previous papers is the presence of "partial-insiders." These bidders have more information than is typically assumed in models of auctions with interdependent values. Instead of simply knowing their own signal, they also know the signals of some subset of bidders other than themselves.[5] This information is gained at no extra cost to the bidders who are "partial-insiders" and bidders do not chose whether or not to gain this information, rather they are simply assumed to be more knowledgable than the other bidders.[6]Bidders who are not "partial-insiders" will be called outsiders and the set of outsiders will be denoted by $O \in N$ while $\bar{O} \in N$ will denote "partial-insiders." All of the bidders know which bidders are "partial-insiders" and know which information these bidders have. For "partial-insiders", $s_{K(i)}$ is the set of bidders whose signals bidder i knows before the bidding starts, other than her own. Therefore for all outsiders, $j \in O$, $K(j) = \emptyset$. To be clear about how the knowledge sets work; if bidder 2 knows bidder 1 and bidder 3's signals as well as his own, then $K(2) =\{1{,}3\}$ or $s_{K(2)} =\{s_1, s_3\}$. All of the other bidders know that bidder 2 knows bidder 1 and bidder 3's signals as well.

Finally, p_i^* denotes bidder i's drop-out price. This is the price at which bidder i leaves the auction and can no longer win. All bidders can see when a bidder drops out and know the price at which that bidder drops out. Therefore, bidders can glean information from the drop out prices of the other bidders.[7] In particular they are interested in determining the signals of the bidders that have dropped out before them, in order to update their information about their own valuation.

[5]This can include the set of all of the other bidders, in which case the insider is just like the insiders in Kim (2000).

[6]The costly acquisition of information is an interesting question, which has been addressed in a number of papers including Persico (2000).

[7]A key aspect of the bidders being able to gather information from other bidders' drop-out prices is that they know the functional form of these bidders valuation functions.

2.4 EQUILIBRIUM

The equilibrium is constructed in a very similar way to both Krishna and Kim. As the price rises, the bidders solve the following system of equations:

$v_j(\phi_A(p|A, p^*_{\bar{A}}), \phi_{\bar{A}\cap\bar{O}}(p|A, p^*_{\bar{A}}), s_{\bar{A}\cap O}){=}p$ for $j \in A \cap O$

$v_l(\phi_{A\setminus K(l)}(p|A, p^*_{\bar{A}}, s_{K(l)}), \phi_{\bar{A}\cap\bar{O}\setminus K(l)}(p|A, p^*_{\bar{A}}, s_{K(l)}), s_{\bar{A}\cap O\setminus K(l)}, s_{K(l)}){=}p$ for $l \in A \cap \bar{O}$

$v_t(\phi_{A\setminus K(t)}(p|A, p^*_{\bar{A}}, s_{K(t)}), \phi_{\bar{A}\cap\bar{O}\setminus K(t)}(p|A, p^*_{\bar{A}}, s_{K(t)}), s_{\bar{A}\cap O\setminus K(t)}, s_{K(t)}){=}p^*_t$ for $t \in \bar{A} \cap \bar{O}$

where the ϕ-functions are inverse bid functions.[8] The inverse bid functions are compared to the bidders actual signal. So that this system of equations shows that at each price the bidders calculate what their signal would have to be for them to just break-even if they were to win the auction at that price. This would mean that all of the other active bidders would have dropped out at that particular price, otherwise the bidder would not be winning at that price. So, the bidder, in calculating the signal at which they would break-even, assumes that the bidders who have not dropped out yet have a signal such that they would want to drop out at precisely that price. Therefore in each of the bidder's calculations, the inverse bid functions for the other active bidders are set to the signal such that they would break-even at the current price.[9] There are two types of inactive bidders, those that were insiders and those that were outsiders. Each of the active bidders can easily calculate the signals of the inactive outsiders based on when they dropped out of the auction. The calculation for the inactive insiders is more complicated since they may have dropped out of the auction prior to the bidders about whom they have information. It is not necessarily possible for the active bidders to be able to precisely determine the inactive insiders' signals, since the active bidders may not have the same information as the inactive insiders. So, the inactive insiders' signals are inferred as $\phi_{\bar{A}\cap\bar{O}}$-functions, which are increasing in p and are updated

[8]These are inverse bid functions because they are a function of the price that the bidder is bidding at that particular moment. An actual bid is a price. The inverse bid functions are related to the current price, the functional form of the bidder's valuation function and all of the information that he has at that particular price. This information includes which bidders have dropped out of the auction and the price at which they have dropped out.

[9]This calculation is feasible because the bidders know the forms of the valuation functions for all bidders.

until all of the signals that the particular insider knew at the beginning of the auction are revealed. [10] The active bidders then use the information gained from bidders dropping out to update their break-even signals, which for each $i \in A$ means $\phi_i(p|A, p^*_{\bar{A}}, s_{K(i)})$. The active bidders, $i \in A$, remain in the auction until the price where $\phi_i(p|A, p^*_{\bar{A}}, s_{K(i)})$=$s_i$.If after these calculations, the bidder's actual signal is higher than the one that would cause them to just break-even, then they will stay in the auction. If their signal equals or is less than the signal that they have calculated, then they will drop out of the auction.

The following proposition establishes the equilibrium bid functions:

Proposition 1 *For a set of bidders N with value functions such that $v'_{jj} > 0$ and $v'_{ij} \geq 0$, where $A \subseteq N$ is the set of active bidders, suppose that for all $i \in A$ and for all $s_{\bar{A}\cap 0}$ there exist a unique set of increasing and continuous functions $\phi_i: \mathbb{R}_+ \times p^*_{\bar{A}} \times s_{K(i)} \to [0, \omega_i]$ (for $j \in A \cap O$, $s_{K(j)} = \emptyset$ and therefore will be repressed below). Also suppose that there is a set of functions for $t \in \bar{A} \cap \bar{O}$, $\phi_t: \mathbb{R}_+ \times p^*_{\bar{A}} \times s_{K(i)} \to [0, \omega_i]$. These functions exist, such that for all $p \leq \min_{i\in A} \phi_i^{-1}(\omega_i \mid A, p^*_{\bar{A}})$, and for all $j \in A \cap O$, $l \in A \cap \bar{O}$, and $t \in \bar{A} \cap \bar{O}$:*

$v_j(\phi_A(p|A, p^*_{\bar{A}}), \phi_{\bar{A}\cap\bar{O}}(p|A, p^*_{\bar{A}}), s_{\bar{A}\cap O})$=$p$

$v_l(\phi_{A\setminus K(l)}(p|A, p^*_{\bar{A}}, s_{K(l)}), \phi_{\bar{A}\cap\bar{O}\setminus K(l)}(p|A, p^*_{\bar{A}}, s_{K(l)}), s_{\bar{A}\cap O\setminus K(l)}, s_{K(l)})$=$p$

$v_t(\phi_{A\setminus K(t)}(p|A, p^*_{\bar{A}}, s_{K(t)}), \phi_{\bar{A}\cap\bar{O}\setminus K(t)}(p|A, p^*_{\bar{A}}, s_{K(t)}), s_{\bar{A}\cap O\setminus K(t)}, s_{K(t)})$=$p^*_t$

Define $\beta^A_j: [0, \omega_j] \times p^*_{\bar{A}} \to \mathbb{R}_+$, for all $j \in A \cap O$, by $\beta^A_j(s_j, p^*_{\bar{A}})$=$\phi_j^{-1}(s_j \mid A, p^*_{\bar{A}})$

And define $\beta^A_l: [0, \omega_l] \times p^*_{\bar{A}} \times s_{K(l)} \to \mathbb{R}_+$, for all $l \in A \cap \bar{O}$

by $\beta^A_L(s_l, p^*_{\bar{A}}, s_{K(l)})$=$\phi_l^{-1}(s_l \mid A, p^*_{\bar{A}}, s_{K(l)})$

For any number N bidders these bid functions form an ex-post equilibrium.

Proof See Appendix A.

In order to show that the functions described above exist, some restrictions on the valuation function must be introduced. Both of the restrictions are typical in studies of auctions

[10] This means that not only do all the bidders know the shape of all the valuation functions, but they also know which signals are known by which insiders. This is only required for the general case. In the example below with three bidders, where only one has information about one other bidder, this does not necessarily need to be true.

with interdependent values and are from Maskin (1992) and Krishna (2002). The first is the *single crossing condition* which states that for all j and $j \neq i$

$$\frac{\partial v_j(s)}{\partial s_j} > \frac{\partial v_j(s)}{\partial s_i}$$

at every s such that $v_i(s)=v_j(s)=\max_{k\in N} v_k(s)$. The single crossing condition implies that a bidder's signal has a greater effect on his own valuation than on any other bidder.

For the second condition, the average of the values must be introduced. So define:

$\bar{v}(s)=\frac{1}{N}$

Then the *average crossing condition* states that for all j and $j \neq i$:

$$\frac{\partial \overline{v_j(s)}}{\partial s_j} > \frac{\partial v_j(s)}{\partial s_i}$$

In words the average crossing condition says that a bidder's signal effects the average valuation more than any single bidder's valuation. This of course means that the bidder's signal effects his own valuation more than it effects the average valuation. It is easy to show that the average crossing condition implies the single crossing condition when there are only two bidders left in the auction.

The following proposition shows that the ϕ_A and $\phi_{\bar{A}\cap\bar{O}}$-functions described in Proposition 1 exist and have the properties which are required in the equilibrium.

Proposition 2 *Suppose that the valuations satisfy the average crossing condition. Then for all $A \subseteq N$, for all s_N, there exists a unique set of differentiable and increasing functions, $\phi_i: \mathbb{R}_+ \times p^*_{\bar{A}} \times s_{K(i)} \to [0, \omega_i]$ for $i \in A$ and a unique set of differentiable functions, $\phi_t: \mathbb{R}_+ \times p^*_{\bar{A}} \times s_{K(i)} \to [0, \omega_i]$ for $t \in \bar{A} \cap \bar{O}$ such that for all $p \leq \min \phi^{-1}(\omega_i)$, and for all $j \in A \cap O$, $l \in A \cap \bar{O}$, and $t \in \bar{A} \cap \bar{O}$:*

$v_j(\phi_A(p|A, p^*_{\bar{A}}), \phi_{\bar{A}\cap\bar{O}}(p|A, p^*_{\bar{A}}), s_{\bar{A}\cap O})=p$

$v_l(\phi_{A\backslash K(l)}(p|A, p^*_{\bar{A}}, s_{K(l)}), \phi_{\bar{A}\cap\bar{O}\backslash K(l)}(p|A, p^*_{\bar{A}}, s_{K(l)}), s_{\bar{A}\cap O\backslash K(l)}, s_{K(l)})=p$

$v_t(\phi_{A\backslash K(t)}(p|A, p^*_{\bar{A}}, s_{K(t)}), \phi_{\bar{A}\cap\bar{O}\backslash K(t)}(p|A, p^*_{\bar{A}}, s_{K(t)}), s_{\bar{A}\cap O\backslash K(t)}, s_{K(t)})=p^*_t$

Proof See Appendix B.

The following example will help to clarify exactly how the bid functions work.

2.5 EXAMPLE

An example will make it more clear what the complexities of this situation are and how the mechanism of the English auction works in this case. For a simple valuation function, I will show that the English auction with 3-bidders, one of whom is an insider (he has information about one of the other bidder's signals), has an efficient ex-post equilibrium. The bidders' valuation functions will be $v_i = as_i + \sum_{j \neq i} s_j$ where $a > 1$. The average crossing condition is met for these valuation functions. All of the bidders know that these are the valuation functions for all of the bidders. Also the bidders' signals are drawn from the uniform distribution on [0,1].

Without loss of generality, let bidder 3 be the partial insider. Also, suppose that he knows bidder 2's signal, s_2. Each of the bidders know that bidder 3 knows bidder 2's signal. The other bidders only know their own signal and the distribution from which the other bidders' signals are drawn.

So, as the price rises the bidders solve the following system of equations to determine when they want to drop out of the auction:

$a\phi_1(p) + \phi_2(p) + \phi_3(p, s_2) = p$

$a\phi_2(p) + \phi_1(p) + \phi_3(p, s_2) = p$

$a\phi_3(p, s_2) + \phi_1(p) + \phi_2(p) = p$

The ϕ-functions, as mentioned above, are inverse bid functions. Before any bidder drops out, each of the bidders will want to leave the auction when their inverse bid function is equal to their own signal. Since they are determining whether they will break-even if they were to win, in their calculations the other bidders would also drop out at that price. This means that when the first bidder wants to drop out of the auction, they are assuming that the other bidders also have their signal. So, for a bidder, i, who is not an insider they would want to drop out when the price is equal to $(a+2)s_i$. The insider is slightly different in that he already knows one of the other bidder's signals so he would want to drop out when the price is equal to $(a+1)s_3 + s_2$.

Once one of the bidders drops out, the remaining bidders update their information about that bidder's signal. If the first bidder to drop out is not the insider, then the remaining bidders can perfectly update their information with that bidder's signal.[11] However, if the first bidder to drop out is the insider, it is not possible for both bidder 1 and bidder 2 to perfectly update their information about his signal. Bidder 2 will be able to perfectly infer bidder 3's signal, since it was her signal that he knew and she therefore has the same information that he did. Bidder 1 will not be able to do this, since he does not know bidder 2's signal. Instead, bidder 1 will have to continue to make the assumption that bidder 2 will have just dropped out in determining whether he wants to drop out at a given price. Therefore bidder 1 will be using this assumed signal for bidder 2 twice in his calculations, once as a stand in for bidder 2's inverse bid function and once in calculating bidder 3's signal.

Table 1 summarizes the various scenarios that may happen in the auction depending on the realization of the bidders signals. In each scenario, the bidders use the basic principle that they should drop out at the price where they would break-even if they were to win. This means that the first bidder to drop out assumes in their calculation that all of the bidders about whom they do not have information have the same signal as they do. The same goes for the second bidder to drop out. The table shows that the bidder with the highest valuation will always win the auction and pay a price lower than their valuation. Therefore the English auction is efficient in this example. Interestingly, the fact that bidder 3 knows bidder 2's signal instead of bidder 1's signal does not affect who wins the auction (although it can affect the price paid depending on which bidder has a higher signal).

It is easily shown with a small amount of algebra that none of the bidders should wish to change their strategies. This is true because the bidder with the highest signal also has the highest value. No bidder can improve his payoff by deviating from the proposed strategy, since each bidder will stay in until their inverse bid function is equal to their signal. By staying in the auction longer, the other bidders will improperly think that the deviating bidder has a higher signal than they actually do. This will cause the price paid to increase

[11] This is because they know the shape of the bidder who has dropped out's valuation function and therefore know the system of equations that the bidder is solving in order to decide whether or not to drop out.

Table 1: Equilibrium Bidding Behavior in the Example

	Valuation Rank	**1st Drop Out Price**	**2nd Drop Out Price**
$s_3 > \max(s_1, s_2)$	$v_3 > \max(v_1, v_2)$	$(a+2)\min(s_1, s_2)$	$(a+1)\max(s_1, s_2)$ $+\min(s_1, s_2)$
$s_1 > s_2 > s_3$ 3 knows s_1 $(a+2)s_2 >$ $(a+1)s_3 + s_1$	$v_1 > v_2 > v_3$	$p_3 = (a+1)s_3 + s_1$	$p_2 = (a+1)s_2 + \frac{p_3 - s_2}{a+1}$
$s_1 > s_2 > s_3$ 3 knows s_1 $(a+2)s_2 <$ $(a+1)s_3 + s_1$	$v_1 > v_2 > v_3$	$p_2 = (a+2)s_2$	$p_3 = as_3 + s_1 + s_2$
$s_1 > s_2 > s_3$ 3 knows s_2	$v_1 > v_2 > v_3$	$p_3 = (a+1)s_3 + s_2$	$p_2 = (a+1)s_2 + \frac{p_3 - s_2}{a+1}$
$s_1 > s_3 > s_2$ 3 knows s_1	$v_1 > v_3 > v_2$	$p_2 = (a+2)s_2$	$p_3 = as_3 + s_1 + s_2$
$s_1 > s_3 > s_2$ 3 knows s_2	$v_1 > v_3 > v_2$	$p_2 = (a+2)s_2$	$p_3 = (a+1)s_3 + s_2$

and the price will increase above the deviating bidders' value. In other words, the bidders assume that the bidders who have not dropped out yet have their signal. So, a bidder with a lower signal will obviously end up with a negative payoff if they stay in longer than the equilibrium suggest that they should. Therefore this is one equilibrium of this auction.[12]

2.6 EFFICIENCY

Proposition 3 *Suppose that the valuations $v(s)$ satisfy the average crossing condition and β is an equilibrium of the English auction such that β_i^A are continuous, increasing functions whose inverses satisfy the break-even conditions. Then β is efficient.*

Proof For simplicity and without loss of generality we are going to assume that the bidders drop out in numerical order, starting with the highest number, so that $p_1^* \geq p_2^* \geq ... > p_N^*$. If the equilibrium is efficient, it must be that $v_1(s) \geq v_j(s)$ for all $j \neq 1$.

To perform a proof by contradiction, suppose that $v_i(s) > v_1(s)$ for some i. So that although bidder 1 wins the auction, bidder i actually has the higher valuation. Now assume that bidder 1 drops out at some p if bidder i remains in the auction beyond p_i^*. The break-even conditions for 1 and i at this p are:

$$v_1(s_1, \phi_i(p|1, i, p^*_{\bar{A}}, s_{K(1)}), \phi_{(\bar{A}\cap\bar{O})(1)}(p|1, i, p^*_{\bar{A}}, s_{K(1)}), s_{(\bar{A}\cap\bar{O})(1)}, s_{K(1)}) = p$$

$$v_i(s_i, \phi_i(p|1, i, p^*_{\bar{A}}, s_{K(i)}), \phi_{(\bar{A}\cap\bar{O})(i)}(p|1, i, p^*_{\bar{A}}, s_{K(i)}), s_{(\bar{A}\cap\bar{O})(1i}, s_{K(i)}) = p$$

(with $K(1), K(i) = \emptyset$ if bidder 1 or i are outsiders), because $\phi_1(p|1, i, p_{\bar{A}^*}) = s_1$. Also since bidder i has deviated from the equilibrium we know that $\phi_i(p|1, i, p_{\bar{A}^*}) > s_i$. Using the single crossing property we therefore have that $v_1(s) > v_i(s)$. Thus we contradict that $v_i(s) > v_1(s)$. □

[12]There could be a number of other equilibria of this auction.

2.7 PAYOFFS AND REVENUE

It is noteworthy that the above equilibrium implies that information does not improve the pay-offs of the information holder, because the auction continues to be efficient and in an English auction bidders can not influence the price that they pay. This seems to be somewhat counter-intuitive since in most cases firms hire experts in order to get more information, however these results imply that firms should not wish to pay to gain more information. One may consider that the hiring of experts may only improve the signal of the bidder. This type of situation is not covered in this paper however. Also in this setting bidders know which of the other bidders have additional information, this results may differ if the information acquisition can be done secretly.

It may also seem counterintuitive that the informed bidder seems to change his behavior and yet he is not positively impacted in anyway. This result comes from the fact that the information that the bidder receives would eventually have been revealed to him as a result of the open format of the auction. The information is also not costly to the bidder who receives it. They are simply given it. Given the result that the information does not actually improve the informed bidders outcome it is highly unlikely that a bidder would pay for this kind of information in this setting. However, this does not imply that a bidder would not want to acquire information in any setting as noted above.

Also, it can be shown that the revenue in the case with partial-insiders is less than that when the insiders are completely informed of their values. This coincides with Compte and Jehiel's findings that more information leads to higher revenues. More generally, comparisons can also be made between the different revenues earned under the Krishna, Kim and this particular information set up. Kim (2003) shows that more insiders provide more revenue. Therefore Kim's set up results in more revenue than that of Krishna. Furthermore Milgrom and Weber (1982) show that giving bidders more information about the winner increases the revenue of the auction. Thus, taking some of the information away from the insiders, as is done in this paper, will most likely have a negative effect on the revenue of the auction in comparison to the case when insiders have full information. This result implies that the seller should want to gather all the information possible for the bidders in order to improve his own revenue. In fact the seller should be the one who does all the information gathering the bidders. [13]

2.8 CONCLUSION

This paper has expanded the results of Krishna and Kim to any situation in which a bidder or bidders have information about other bidders. It is interesting to see that efficiency continues to hold. Also more interestingly that the requirements placed on the bid function are the weaker requirements of Krishna and not the stronger ones of Kim. This is most likely because of the way that the model was written. In Kim's case the insiders may know their valuation without knowing the specific signals of the other bidders. This paper requires that the signals of the other bidders are known, if the insider knows his valuation.

There are further things to examine within this more general problem of how information affects auctions. Another formulation of this problem might be that bidders with a common value do not know the valuation, but get some noisy signal of it. It would be interesting to know whether this formulation gives the same counter-intuitive result that having more information does not affect the payoff of the information holder and yet companies are willing to pay in order to get more information. Also it would be interesting to examine the case where some bidders have valuations that are no related to other bidders. This would reflect the case of art collectors who just want a piece of art for their homes. It would also be interesting to run an experiment on the above findings to discover whether bidders would actually follow the above suggested strategy. There are still interesting problems to be faced with respect to auctions.

[13] It should be noted that Board (2009) shows that this result may depend on the number of bidders in the auction. He finds that with two bidders in an English auction with independent private values that the revelation of information actually decreases revenue. Cremér et al (2009) also develop a mechanism by which a seller can fully extract surplus by determining how much information bidders acquire.

3.0 EXPERIMENTAL ANALYSIS OF AUCTIONS WITH UNCERTAIN VALUATION FUNCTIONS AND INTERDEPENDENT VALUES

3.1 INTRODUCTION

When a government is auctioning off cell-phone spectra or mineral rights it is concerned with revenue, but also it is concerned with efficiency.[1]In other types of auctions, the desire for efficiency is not so clear. However, as Cramton (1998) points out, "[r]evenue maximization and efficiency are closely aligned goals. Indeed, in ex ante symmetric settings, the seller's expected revenue is maximized by assigning the goods to those with the highest values." Therefore, even in non-governmental situations, auctioneers may still be concerned with the efficiency of the auction they use.

Many goods that are auctioned off both by the government and private auctioneers derive their value in two ways. First, each bidder has a private aspect to their value. This may be due to the technology of the particular firm or simply to tastes. Second, there is a common aspect. The common aspect may be the value of the minerals that can be extracted from a piece of land or the resale value of a work of art. Bidders for these goods have what are referred to as *interdependent values*[2]. Milgrom and Weber (1982) establish that, in a setting where values are symmetric and interdependent and where signals are affiliated, the English auction results in a higher price than the other commonly used auction formats. In these symmetric situations, Krishna (2003)shows all of the common auction formats are efficient

[1]By efficiency I mean that the auctioned object is sold to the individual who values it the most.

[2]Some authors refer to these types of values, ones with a private and a common component, as common values or affiliated values

under certain assumptions.[3]

The equilibria that are found in many papers about auctions with interdependent values rely on the assumption that the symmetry of the valuation functions is commonly known among the bidders. That is, knowing the functional form of one's own valuation function immediately allows the knowledge of the functional form of all other bidders valuation functions[4]. This implies that a firm's technologies (or an individual's personal tastes) are known to the entire bidding population. While this assumption may be somewhat realistic in situations such as the FCC spectrum auctions, where there were a few bidders who compete with one another regularly, there are a large group of auctions (e.g. art auctions) where it is less likely to hold. Thus, it seems as though the typical model of an auction with interdependent values is somewhat lacking. A better model would allow for some uncertainty about the precise valuation functions of the other bidders in the auction. One step towards allowing for such uncertainty would be to have bidders know a family of valuation functions from which the actual valuation function of each bidder is drawn.

However, allowing for bidders to be uncertain about the functional form of the other bidders' valuations may lead to a problem of multi-dimensional signals, since bidders would now receive private signals in the standard sense as well as signals about the functional form of their valuation. It has been well established in the theoretical auction literature, in particular by Jehiel and Moldovanu (2001), that auctions with interdependent values are not guaranteed to be efficent when the bidders' signals are multidimensional.[5]

Although the most commonly studied and implemented auctions are generally known to be inefficient in these situations, it is still worthwhile to ask which of these is most

[3]Krishna derives two criteria for efficiency. The first, *average crossing*, says that each bidder's signal influences his valuation more than the average of the bidders' valuations. The second, *cyclical crossing*, says that the influence of a bidder's signal on his own value is stronger than that on any other bidder's valuation and that these influences can be ranked.

[4]In later papers when symmetry does not hold, it is explicitly assumed that bidders know the form of the other bidders' valuation functions.

[5]Dasgupta and Maskin (2000) develop a generalized Vickery auction, which achieves a *constrained efficient* equilibrium when signals are multi-dimensional. (The notion of constrained efficiency is due to Holmstrom and Myerson (1983)). The generalized Vickery auction does this by having bidders submit bids that are contingent on the realization of the other bidders' valuations and then calculating a fixed point of these bid functions.

efficient. The English auction is commonly used in both mineral rights auctions and in art auctions (in both of which bidders seem to have interdependent values as described above). I will compare the English auction to the second price auction, since the second price auction has the similar characteristic of the price not being determined by the winner of the auction. In what follows, I will determine which auction, the English auction or the second price auction, will be efficient with the highest probability for a particular setting. I do this by finding a theoretical prediction for equilibrium bid functions and then testing these theoretical predictions in the laboratory.

3.2 LITERATURE REVIEW

The efficiency of the English auction has been well established in the theoretical auction literature. Maskin (1992) shows that when there are two bidders who have interdependent values the English auction (and therefore the second-price auction) is efficient when the values satisfy the *single-crossing condition* [6]. Krishna (2003) extends this result to three or more bidders,developing the average crossing condition and the cyclical crossing condition. The single crossing condition, the average crossing condition and the cyclical crossing condition are all flexible enough to allow for asymmetric valuation functions. In the example considered here, I use the simplest case of symmetric values.

The theoretical predictions of auction theory lend themselves nicely to experiments. Auctions have a concrete setting, which is easily replicated in the laboratory.

Most experiments involving auctions have concentrated on either the private value case or the pure common value case (although one of the more recent experiments involved asymmetric interdependent values, it is also reviewed below).

In the case of private values, Kagel et al. (1987) and others (Harstad 1990, Kagel and Levine 1993) find that the second price auction and English auction do not generate

[6] The single crossing condition requires that a bidders private information has more impact on his own value than it does on another bidder's value.

equivalent revenue as predicted by theory [7]. They find that the second price auction generally results in higher revenue that the English auction. This is because of rampant overbidding in the second price auction, while the English auction tends to converge toward dominant strategy[8] bidding.

In the case of common values, the winner's curse occurs in both open and closed bid auctions[9]. Kagel and Levine (1992) find that the winner's curse is more severe in the first price auction as compared to the English auction. They believe that the revelation of information in the English auction mitigates the winner's curse.

Andreoni et al. (2007) run an experiment comparing the first and second price auctions with private values when some of the bidders are informed about other bidder's valuations. They find that bidders behavior is close to theoretical predictions, although they do find some evidence for spiteful bidding by bidders in the second price auction when they know that they are not going to win.

Kirchkamp and Moldovanu (2004) is the only currently published experimental analysis of an English auction with interdependent values. They look at asymmetric interdependent values where bidder's values only depend on one other bidder's signal. They run auctions with 3 bidders and compare the English auction to the second-price auction. They find that the English auction is more efficient than the second-price auction. However, they find that subjects' behavior is not as efficient as the theoretical prediction. This may, in part, be due to the asymmetric valuations that are given to the bidders. They find that the major inefficiency comes from what they term "naive right bidder". That is bidders seem to have difficulty in calculating what to do when the bidder who dropped out does not

[7] The revenue equivalence theorem says that if values are independently and identically distributed and bidders are risk neutral, all the standard auction formats result in the same revenue for the auctioneer.

[8] In both the English auction and second price auction with independent private values, it is a dominant strategy to bid your value.

[9] Open bid auctions are auctions where a price is either announced or bidders announce their bids publicly. The two most commonly known open auctions are the Dutch auction and the English auction. Closed bid auctions are auctions in which each bidder submits a sealed bid and the auctioneer determines the winner and the price which the winner will pay. The first price auction, where the winning bidder pays his bid, and the second price auction, where the winning bidder pays the second highest bid, are examples of closed bid auctions

directly effect their valuation. I use symmetric valuation functions in the first parts of my experiment to avoid these complicated calculations. This allows me to give both the English auction and second price auction a higher chance for success in achieving efficiency. However I find that even under these simpler conditions, there is remaining inefficiency. Kirchkamp and Moldovanu are also not focusing on differences in information structure as I am, so the results from the second part of my experiment are not comparable to their results.

3.3 EXPERIMENTAL DESIGN

In all of the auctions run during the experiment, there were three bidders, $i = 1, 2, 3,$, in each group. Each group participates in a separate auction in each period. Each bidder received a private signal (s_i), which was an integer drawn uniformly from the range [0,33].[10] Each signal was drawn independently.[11] In each session there were either nine or twelve subjects, so that there were 3 or 4 groups in a session. There were fifteen auctions run during a session. At the end of each auction (or period), the groups were randomly reassigned. At the end of each period, the subjects were told the signals of the other bidders from the period and the price at which the object was sold. [12] They were not told against which individuals in the room they were bidding.

I implemented both the English auction and the second price auction. During each session, only one type of auction was run. In the English auction, after the bidders were told their signals and their valuation functions, each bidder saw a price clock on their screen, which began at a price of zero and increased by five points every two seconds [13]. This continued for each bidder, until one of the bidders in the same group dropped out. At that

[10]The value of these signals were described in terms of points which were then converted into dollars at the end of the experiment at a rate of 1 point= 5 cents.

[11]For the purpose of being able to make comparisons across treatments, I randomly drew the signals for the sessions ahead of time. I did this four times and then used the same realizations across treatments.

[12]This allows subjects the greatest possibility of learning and is standard procedure in other auction experiments.

[13]I used this larger increase in points at the beginning of each period in the interest of time.

point, the screen briefly flashed and then showed the price at which that bidder dropped out. The price clock then slowed, increasing one point every two seconds. After the second bidder in a group dropped out, the auction ended for that group and the last remaining bidder won the object at a price equal to the second bidder's drop out price. In the second price auction, after the bidders were told their signals and their valuation functions, they were asked to submit an integer bid[14]. The computer then calculated the highest and second highest bid. The bidder who submitted the highest bid won the object and paid the second highest bid for it. If two bidders submitted the same bid, the computer flipped a fair coin to determine which bidder won the object.

There were two general settings which were run, one in which there was no uncertainty about the bidders' valuation functions and one in which there was uncertainty about one of the bidders' valuation functions.

The first setting is the standard setting of symmetric interdependent valuations[15]. In this setting, each bidder i was given the same valuation function $v_i(\mathbf{S}) = 3s_i + s_j + s_k$[16], where s_j and s_k are the other two bidders' signals. This setting was used as a baseline case. I chose this particular valuation function for ease of calculation and because, as shown below, the English auction and the second price auction are both efficient when bidders have this valuation function.

In the second setting, there is uncertainty about one of the bidders' valuation functions. The bidder about whom there is uncertainty will be known as the bidder 1. In each period, the individual who is bidder 1 changes as does the realization of their valuation function. Individuals who are bidder 1 are given two pieces of information at the beginning of the auction, they are told their signal and they are told which of the two possible valuation functions they have. The first possible valuation function is the same as the one above,

[14]This is different from Kirchkamp and Moldovanu (2004) who implement a second price auction using clocks. I use the more standard format to gather information about all of the bidders' bids. Also, in practice, the second price auction would not be implemented using clocks.

[15]This differs the setting of from Kirchkamp and Moldovanu (2004) which has asymmetric valuation functions $v_i(\mathbf{S}) = s_i + \alpha s_{i+1}$.

[16]This valuation function satisfies average crossing.

$v_1(\mathbf{S}) = 3s_1 + s_j + s_k$. The second is $v_1(\mathbf{S}) = s_1 + s_j + s_k$, where s_j and s_k are the same as above. Each of these valuation functions is realized with equal probability[17]. The other two bidders always have the valuation function $v_i(\mathbf{S}) = 3s_i + s_j + s_k$[18].

Table 2 summarizes the various experiments which were run. Appendix A gives an example of the instructions used in the English auction experiments, the other instructions were similar with changes due to the differences in the treatments.

3.4 THEORETICAL PREDICTIONS

In this section, I compute the theoretical predictions for equilibria in the English and second price auctions under both of the settings described above. For simplicity, I assume that the signals were uniformly distributed between 0 and 1, instead of 0 and 33 as they were in the experiment.

In all that follows I focus on symmetric equilibria. There are most certainly other types of equilibria for these auctions. However, most of the experimental literature focuses on finding and testing symmetric equilibria.

3.4.1 English Auction in Setting 1

Bids in the English auction can be thought of as drop out prices. Each bidder determines the highest price at which he is willing to be in the auction and drops out at that price.

There are also two stages to consider in the English auction. The first is when all three bidders are still active in the auction. The second is after a bidder has dropped out of the

[17]Given these particular valuation functions, there are two ways to think about the information which bidder 1 receives. The first is that bidder 1 receives a multi-dimensional signal which has both the valuation function and his signal. The second is that bidder 1 draws his signal from a different distribution where a heavier weight is placed on the first third of the signals. A future paper may explore a truly multidimensional case such as having a second valuation function be $v_1(\mathbf{S}) = 3s_1 + 2s_2 + s_3$

[18]The valuation functions for the second two bidders and the first valuation function for bidder 1 satisfy cyclical crossing. The second valuation function for bidder 1 only weakly satisfies cyclical crossing

Table 2: List of Experiments

Session No.	Setting	No. of Subjects	Auction Type
1	1	12	English
2	1	12	English
3	1	12	English
4	1	12	English
5	1	12	Second
6	1	12	Second
7	1	12	Second
8	1	12	Second
9	2	12	English
10	2	9	English
11	2	12	English
12	2	12	English
13	2	9	Second
14	2	12	Second
15	2	12	Second
16	2	12	Second

Setting=1 if no uncertainty about valuation functions.

Setting=2 if uncertainty about valuation functions

auction, so that there are only two bidders active in the auction. In the first stage, the bid function of each of the bidders depends only on their own signal. Whereas in the second stage, the bid function can depend on both the bidder's own signal and the price at which the bidder dropped out of the first stage, $\widehat{b}_0$. Each bidder will have a two part bid function consisting of a dropout price for the first stage, $b_0(s_i)$ and a dropout price for the second stage, $b_1(s_i, \widehat{b}_0)$.

Proposition 4 *Consider the bidding strategy defined by:*

$b_0(s_i) = 5s_i$

$b_1(s_i, \widehat{b}_0) = 4s_i + \frac{1}{5}\widehat{b}_0$

If all bidders follow this bidding strategy, it forms a Nash equilibrium.

Proof I will check to see if any of the bidders would want to deviate from their strategies. For concreteness I will assume that $s_1 > s_2 > s_3$. Since bidder 3 is the first bidder to drop out of the auction, the first drop out price is $\widehat{b}_0 = 5s_3$. So, the other two bidders know that $\frac{1}{5}\widehat{b}_0 = s_3$.

Bidder 2 would be the one who drops out of the auction and sets the price. Suppose that instead bidder 2 stays in the auction in order to win, which is the only reason that bidder 2 would stay in longer. Then bidder 1 will eventually drop out at a price of $p = 4s_1 + s_3$. Bidder 2 would then win and have a pay off of $\Pi_2 = 3s_2 + s_1 + s_3 - (4s_1 + s_3) = 3s_2 - 3s_1$. This is less than zero because $s_1 > s_2$. Therefore bidder 2 would not want to deviate, since the pay off from deviating is lower than the payoff from following the strategy.[19] By a similar argument, bidder 3 does not want to deviate from his strategy in order to win. Finally I must check that bidder 1 does not want to deviate from her strategy. If she does, the only way that would have an effect would be dropping out earlier than bidder 2. If she did this, she would have a payoff of zero. Bidder 1's payoff from following the above strategy is $\Pi_1 = 3s_1 + s_2 + s_3 - (4s_2 + s_3) = 3s_1 - 3s_2$. This is greater than zero, since $s_1 > s_2$. $\square$

3.4.2 Second Price Auction in Setting 1

For the second price auction, there is only one stage to consider. The bidders will bid in such a way as to ensure that they win only when their valuation is at least as large as the price which they will have to pay.

[19] In equilibrium, Bidders 2 and 3 get a payoff of zero since they do not win the object therefore do not pay anything.

Proposition 5 *There is a symmetric Nash equilibrium in the second price auction where each bidder bids* $b(s_i) = 4.5s_i$.

Proof Each bidder aims to maximize their expected utility. Expected utility is the difference between the bidder's value and the price that they would pay if they were to win the object. The expected utility for bidder i when bidding B while all the other bidders bid $b(\cdot)$, can be written as:

$$\int_0^{b^{-1}(B)} \left(\int_0^{s_j} (3s_i + s_j + s_k - b(s_j))ds_k + \int_{s_j}^{b^{-1}(B)} (3s_i + s_j + s_k - b(s_k))ds_k \right) ds_j$$

The bidder maximizes this by choosing B:

$$\frac{\partial U}{\partial B} = \frac{1}{2} b^{-1}(B)(3s_i + 6b^{-1}(B) - 2B) \frac{\partial b^{-1}(B)}{\partial B} = 0 \text{ gives has a solution } b(s_i) = 4.5s_i.$$

This forms a symmetric Nash equilibrium. □

This makes intuitive sense. Each bidder behaves as if the next highest bidder has the same signal as she does and the other bidder has a signal below that signal.

3.4.3 English Auction in Setting 2

With uncertainty about the valuation function of one of the bidders, there are still two stages to consider in the English auction. The first stage bids ($b_0^1(s_1)$ for bidder 1 and $b_0^2(s_i)$ for the other two bidders) will only depend on the signal of the individual bidder and the shape of their own valuation function, since the bidders have no other information. The second stage bids will depend on the signal of the bidder and the shape of that bidder's valuation function as well as the price at which the bidder dropped out of the first stage. It will also depend on whether the first bidder to drop out was bidder 1, so that the first drop out price would be ($\widehat{b_0^1}$), or one of the other bidders, so that the first drop out price would be ($\widehat{b_0^2}$).

Proposition 6 *The bidding strategies defined by:*

$b_0^1(s_1) = 3s_1$

$b_1^1(s_1, \widehat{b_0^2}) = 2s_1 + \frac{1}{5}\widehat{b_0^2}$

for bidder 1, when their valuation function is $v_1(\mathbf{S}) = s_1 + s_j + s_k$ *and*

$b_0^1(s_1) = 5s_1$

$b_1^1(s_1, \widehat{b}_0^2) = 4s_1 + \frac{1}{5}\widehat{b}_0^2$

for bidder 1 when their valuation function is $v_1(\mathbf{S}) = 3s_1 + s_j + s_k$ *and*

$b_0^2(s_i) = 5s_i$

$b_1^2(s_i, \widehat{b}_0^1) = 4s_i + \frac{4}{15}\widehat{b}_0^1$

if bidder 1 drops out of the auction first or

$b_1^2(s_i, \widehat{b}_0^2) = 4s_i + \frac{1}{5}\widehat{b}_0^2$

for bidders 2 and 3, forms an Nash equilibrium.

Proof The strategies put forth above follow the strategies from proposition 1. In the first stage, each of the bidders bids as if all the other bidders have the same signal as he does. They do this because it forms the highest price at which they would just break-even. Once one of the bidders drops out, the other two bidders must infer his signal. This is straightforward when the bidder who has dropped out is one of the bidders whose valuation function is known. However, when the bidder who drops out first is the bidder whose valuation function is not known, the other bidders are only able to form an expectation about her signal. They therefore must solve the equation:

$$E(s_1) = \frac{1}{2}(\frac{\widehat{b}_0^1}{3}) + \frac{1}{2}(\frac{\widehat{b}_0^1}{5})$$

which gives $E(s_1) = \frac{4}{15}\widehat{b}_0^1$. Given the other bidders' behaviors, before the valuation function of bidder 1 and all of the bidders' signals are reveled none of the bidders would want to stray from these strategies. □

3.4.4 Second Price Auction in Setting 2

With uncertainty about the valuation function of one of the bidders, a bid function in the second price auction will depend on the signal that the bidder receives and the shape of their valuation function.

Proposition 7 *The bidding strategies defined by:*

$b^1(s_1) = 2.5s_1$

for bidder 1 when their valuation function is $v_1(\mathbf{S}) = s_1 + s_j + s_k$ *and*

$b^1(s_1) = 4.5s_1$

when their valuation function is $v_1(\mathbf{S}) = 3s_1 + s_j + s_k$. *and*

$b^2(s_i) = 4.5s_i$

for bidder 2, forms an ex-ante Nash equilibrium.

Proof As in the 2nd price auction in setting 1, each bidder bids as though they are tied for having the highest signal with one other bidder and the other bidder has a signal below that. The bids in the second price auction do not depend on the valuation functions of the other bidders, since it is always optimal to bid your expected value. □

3.4.5 Efficiency

Efficiency is an outcome which is of particular interest to the auctioneer. This is true for governmental auctions since the government has an interest in ensuring that social welfare is maximized. It is also true in non-governmental auctions because of the relationship between efficiency and revenue.

Proposition 8 *In setting 1, both the English auction and the second price auction allocate the object efficiently.*

Proof Since the valuation functions of the bidders are all $3s_i + s_j + s_k$, the bidder with the highest signal also has the highest valuation.

In the English auction, the bidder with the lowest signal is the first to drop out, since each bidder would drop out when the price clock reaches $5s_i$. Then the bidder with the second lowest signal is the second to drop out, since each bidder would drop out when the price clock reaches $4s_i + \frac{1}{5}\widehat{b}_0$. Therefore the bidder with the highest signal wins the object in the English auction.

In the second price auction each bidder bids $4.5s_i$. Since the highest bidder wins the object, it is obvious that the bidder with the highest signal will win. □

Proposition 9 *In setting 2, there are signal realizations for which the English auction is efficient, but the second price auction is not.*

Proof In setting 2, both auctions remain efficient if $v_1(\mathbf{S}) = 3s_1 + s_j + s_k$. However, if $v_1(\mathbf{S}) = s_1+s_j+s_k$, then it is never efficient for Bidder 1 to win the object, since $s_1+s_j+s_k < 3s_j + s_1 + s_k$ always. However, there are certain signal realizations for which Bidder 1 will win. Bidder 1 will only win when $s_1 > \max s_2, s_3$. For concreteness, I will assume $s_2 > s_3$. In the English auction Bidder 1 will win when $2s_1 + s_3 > 4s_2 + s_3$, so Bidder 1 will win when $s_1 > 2s_2$. In the second price auction, Bidder 1 will win when $2.5s_1 > 4.5s_2$, so bidder 1 will win when $s_1 > 1.8s_2$. Obviously, Bidder 1 will win when they shouldn't with a higher probability in the second price auction. □

3.5 EXPERIMENTAL RESULTS

3.5.1 Raw Data

I will begin the analysis of the experimental data with a summary of the data. Table 3 summarizes the actual bids from the experiments($\widehat{b}$) and the theoretically predicted bids (b). Overbidding is evident in all of the auctions except the English auction under setting 1, which is explained below.

In all of the theoretical predictions, I look for a bid function that is monotonically increasing in the bidders' signals. Figure 1 shows relationship between the initial bid in the English auction (in the top graph) and bidder's signals and the bid in the second price auction and the bidder's signals (in the bottom graph), in the setting 1. The dots represent actual bids by subjects and the diagonal line represents the equilibrium prediction for the bids. If subjects were following equilibrium bid behavior, the actual bids would lie along the diagonal line. The actual bids, although not along the diagonal line, are obviously increasing in the bidder's signal. So, in the English auction a bidder should be able to infer that a higher first drop out price of a competitor implies a higher signal for that competitor. It is also reasonable to restrict equilibrium bids in the second price auction to ones that are monotonic in the bidders' signals.

Table 3: Summary Statistics

Variable	Mean	Std. Dev.	Min	Max
s_{it} *	16.044	8.80	1	33
English (Setting 1)				
$\widehat{b}_0$ †	57.524	28.07	0	150
b_0	80.22	43.98	5	165
$\widehat{b}_1$	72.651	31.76	0	160
b_1	66.477	35.45	4	138
2nd price (Setting 1)				
$\widehat{b}$	83.652	68.74	0	700**
b	70.737	39.32	4.5	148.5
English (Setting 2)				
$\widehat{b}_0$	50	30.637	0	175
b_0	64.610	30.65	6	135
$\widehat{b}_1$	67.452	26.14	10	175
b_1	62.608	28.26	10.67	133.33
2nd price (Setting 2)				
$\widehat{b}$	55.495	40.90	0	500**
b	38.715	22.98	4.5	117

*The signal realizations were the same for each treatment.

†Hats refer to actual bids whereas variables without hats refer to theoretical predictions.

**Bids above 700 were dropped as outliers, see note in discussion of 2nd price auction.

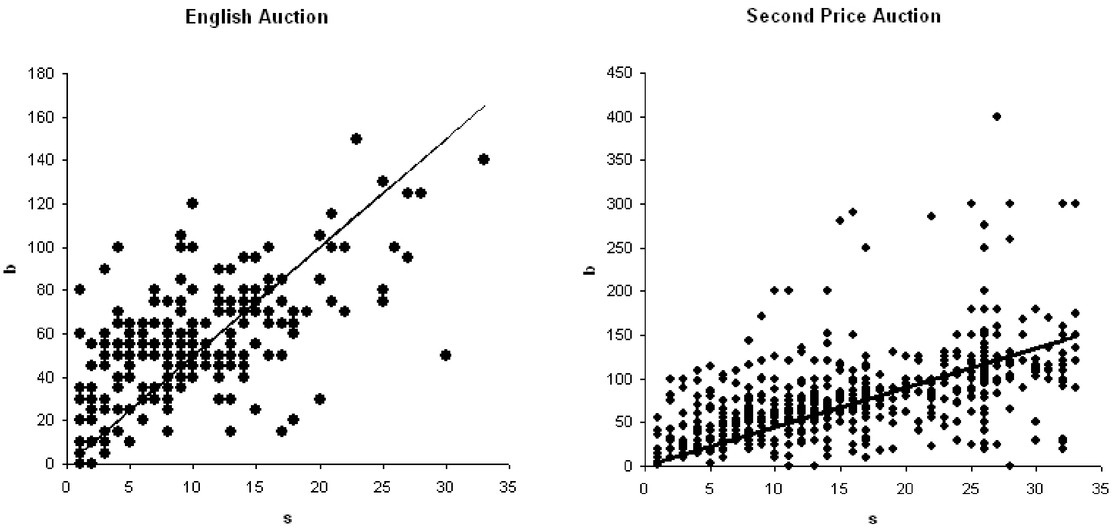

Figure 1: Monotonicity of first bid in the English Auction and bids in the 2nd price auction. Dots show actual bids and the line is the equilibrium bid.

3.5.2 Estimating bid functions

In all of the regressions below, there is the probability that any of the covariances in the experiment may be related (since there are multiple observations from each individual)[20]. So, the standard errors in the regressions must be replaced by the proper residuals (Rogers, 1993).

3.5.2.1 English Auction: Setting 1 In the English auction, since there are two separate parts of the equilibrium bidding function, there are two prices to compare to the theoretical predictions, which require two separate calculations. In the first stage of the English auction, bids depend on the bidder's signal alone. Therefore, I estimate $\widehat{b}_0 = \beta s_{it}$[21].

[20] It is reasonable to assume, however, that the covariances between each experiment is zero, since subjects were only allowed to participate in one session.

[21] In Appendix B, I report regressions similar to the ones performed in Kirchkamp and Moldovanu, so that a comparison may be made between the symmetric and asymmetric case.

Due to the nature of the English auction, only the lowest of the first drop out prices is actually observed in any given auction. All that I know about the first drop out prices of the other two bidders is that they are higher than the drop out price that is observed. Therefore when estimating the bid function for the first stage of the English auction, I must take into account that the 2 other bidders' first drop out prices are right censored. So, I use a censored normal regression (Tobin 1958; Amemiya,1973, 1984) to estimate the bid functions.

Table 4 reports the results from the estimation of the first bid in the English auction. There is significant overbidding, since the coefficient on s_{it} is significantly greater than the theoretical prediction of 5.

Table 4: 1st bid in the English Auction

Variable	**Coefficient**		**(Std. Err.)**
s_i	6.376^{**}		(0.177)
Significance levels :	† : 10%	$*$: 5%	$**$: 1%

Note: The test for significance was done for $\beta = 5$.

It is surprising that this coefficient is higher than the theoretical prediction, since in the summary statistics in Table 3 the first bids in the English auction under setting 1 are, on average, lower than the theoretical prediction for first bids. A possible reason for this is that the lower signaled bidders, for whom we have actual bids, are actually underbidding while the higher signaled bidders are overbidding. In order to test this possibility, I ran a censored normal regression of the difference between the actual and expected first bids ($y_{it} = \widehat{b}_0 - b_0$) on the bidder's signal (s_{it}). I find that the difference is increasing in the bidder's signal. This means that overbidding is worse for higher signaled bidders[22].

A possible alternative to the equilibrium bidding is that bidders bid as if the other two bidders simply have the expected signal, which in this case is $\frac{33}{2}$. If bidders were behaving this way, their first drop out price would be $\widehat{b}_0 = 3s_i + 33$ Table 5 reports the estimation of

[22] This is different from Kagel and Levine(1992) which found overbidding for low signaled bidders in English auctions with pure common values

Table 5: Overbidding in the First Stage of the English Auction

Variable	**Coefficient**	**(Std. Err.)**
s_{it}	4.134**	(0.292)
constant	-9.103*	(4.167)
Significance levels : $\dagger$: 10% $*$: 5% $**$: 1%		

Note: The test for significance was done for $\beta = 0$. Here I am measuring the amount of the overbidding.

$\widehat{b}_0 = \alpha + \beta s_{it}$. I find that this actually might be a plausible explanation, although there is still some overbidding.

Table 6: First bid in the English auction, allowing for an intercept

Variable	**Coefficient**	**(Std. Err.)**
s_{it}	3.461†	(0.226)
constant	38.197*	(2.943)
Significance levels : $\dagger$: 10% $*$: 5% $**$: 1%		

Note: The test for significance was done for $\beta = 3$ and $\alpha = 33$.

For the second stage of the English auction, I estimate $\widehat{b}_1 = \beta_1 s_{it} + \beta_2 \widehat{b}_0$. In the second stage of the English auction, I only observed the drop out price of one of the bidders since the final bidder does not actually drop out of the auction. So, I will have to use a censored normal regression again. Here the second drop out price of the bidder who dropped out first is left censored and the drop out price of the bidder who has not dropped out is right censored. I find that bidders put less weight on their own signal ($\beta_1 < 4$) and more weight on the first drop out price ($\beta_2 > .2$) than theory predicts.[23] The overall result is still overbidding

[23] It is possible that the bidders are best responding or nearly best responding to the behavior of the bidders in the first round of the auction. It is possible to test this theory using quantal response regressions, but this is beyond the scope of this project.

(see Appendix D).

Table 7: 2nd stage bid in the English Auction

Variable	Coefficient	(Std. Err.)
s_i	3.32**	(0.164)
bid in 1st stage	.399**	(0.042)
Significance levels : † : 10% ∗ : 5% ∗∗ : 1%		

Note: The tests for significance was done for $\beta_1 = 4$ and $\beta_2 = .2$.

3.5.2.2 Second Price Auction: Setting 1 Since, in the second price auction, all bidders submit a bid in all auctions, there is no need to use a censored normal regression when estimating the bids. Table 3 reports the summary statistics for the second price auction. [24]

For the second price auction, I estimate $\widehat{b} = \beta s_{it}$. I find that β is significantly different from the theoretical prediction of 4.5. This difference is less significant than the differences for the English auction. This makes sense given that there is a greater risk associated with overbidding in the 2nd price auction than in the 1st stage of the English auction. Overbidding in the 1st stage of the English auction is less risky because it is possible to correct for overbidding during the second stage.

3.5.2.3 English Auction: Setting 2 The same complications for estimating the bid functions with the English auction arise in the case of the second setting, where there is uncertainty about the valuation function of one of the bidders. Therefore censored normal regressions must be used for this case as well. The estimation of bid functions is complicated

[24]It should be noted that in one session for setting 1 and one session for setting 2, there was one subject who bid the maximum possible bid in multiple auctions. While this is part of an alternative Nash equilibrium (see Blume and Heidhues (2004), given the behavior of the other subjects it does not constitute equilibrium behavior. Given that these bids were driving the results of the regressions, they were dropped as outliers.

Table 8: Bids in the 2nd price auction

Variable	Coefficient	(Std. Err.)
s_{it}	6.359*	(1.15)
Significance levels : $\dagger$: 10%	$*$: 5%	$**$: 1%

Note: The test for significance was done for $\beta = 4.5$.

here by the fact that there are various theoretically predicted bid functions, depending on the bidder's type. Therefore, in order to avoid biasing the regressions by dropping certain observations, I simply explore the difference between the actual and predicted bids for all bidders. The results below show significant overbidding in both stages of the English auction.

Table 9: Overbidding in the first round of the English Auction

Variable	Coefficient	(Std. Err.)
s_{it}	-1.224**	(0.24)
intercept	36.223**	(3.32)
Significance levels : $\dagger$: 10%	$*$: 5%	$**$: 1%

Note: The test for significance was done for $\beta = 0$.

Table 9 reports the results for the 1st bid in the English auction. There is significant overbidding. The overbidding is also stronger for lower signaled bidders, which follows what Kagel and Levin (1992) found in English auctions with common values.

Table 10 reports the results for the 2nd bid in the English auction. There is significant overbidding that is less severe than the overbidding in the first stage. Again, this is probably due to the fact that overbidding bears a greater risk in the second stage than it does in the first stage.

Table 10: Overbidding in the second round of the English Auction

Variable	Coefficient	(Std. Err.)
s_{it}	-0.915**	(0.17)
intercept	16.586**	(2.71)
Significance levels : † : 10%	∗ : 5%	∗∗ : 1%

Note: The test for significance was done for $\beta = 0$.

3.5.2.4 Second Price Auction: Setting 2 As with the English Auction in setting 2, estimating specific bid functions for the second price auction in setting 2 is complicated by there being multiple theoretically predicted bid functions for the subjects. I therefore estimate overbidding by looking at the difference between the actual behavior of the subjects and the theoretically predicted bids. I also look for a relationship between the size of the bidder's signal and the size of overbidding. As in the English auction under this setting, I find that there is significant overbidding and that the overbidding is more severe for lower signaled bidders. It is also interesting to note that the magnitude of over bidding in the second price auction is between that of the first bid in the English auction and that of the second bid in the English auction.

Table 11: Bids in the Second Price Auction

Variable	Coefficient	(Std. Err.)
s_{it}	-1.323**	(0.28)
Intercept	26.772**	(4.25)
Significance levels : † : 10%	∗ : 5%	∗∗ : 1%

3.5.3 Learning

In previous experimental analyses of auctions (Kagel et al. 1987, Kagel and Levine 1993, Harstad 1990), including English auctions and second price auctions, it has been found that experience (either during the experiment or across sessions) causes behavior to converge towards the equilibrium predictions. I therefore ran regressions to test whether overbidding relative to the theoretical predictions decreased over time. For the English auction without uncertainty about valuation functions, there is no significant evidence of learning over the auction periods. In fact overbidding does not seem to follow any pattern.

Table 12: Learning in the English Auction: Setting 1

Variable	**Coefficient**	**(Std. Err.)**
auction2	4.276	(14.472)
auction3	5.428	(14.559)
auction4	2.217	(14.577)
auction5	5.171	(14.522)
auction6	6.630	(14.521)
auction7	6.732	(14.464)
auction8	4.165	(14.455)
auction9	2.993	(15.721)
auction10	6.803	(15.543)
auction11	9.629	(15.623)
auction12	13.045	(15.526)
auction13	10.387	(15.809)
auction14	11.726	(15.517)
auction15	8.704	(15.582)
constant	52.295	(10.488)

For the second price auction in setting 1, there are some significant period effects. However, there is no particular pattern to the coefficients. If there was learning, the coefficients would decrease as the subjects move through the session. However, the coefficients rise and fall through out the sessions. The same holds true in the English auction and the second price auction in setting 2.

Table 13: Learning in the 2nd Price Auction: Setting 1

Variable	Coefficient	(Std. Err.)
auction2	24.163**	(8.698)
auction3	45.538*	(17.687)
auction4	30.063**	(8.830)
auction5	41.15**	(9.507)
auction6	44.163**	(10.625)
auction7	298.01	(247.32)
auction8	75.613**	(28.119)
auction9	65.488**	(22.468)
auction10	57.788**	(18.454)
auction11	60.4**	(22.602)
auction12	60.775**	(23.355)
auction13	36.75**	(8.543)
auction14	36.925**	(8.815)
auction15	41.038**	(9.968)
constant	-26.175**	(6.417)

Significance levels : † : 10% $*$: 5% $**$: 1%

3.5.4 Efficiency

Theoretically, given the experimental set up, the English auction and the second price auction should both always achieve efficiency in setting 1. However, in the experiment, neither the English auction nor the second price auction achieve efficiency 100% of the time. In fact, the English auction achieves efficiency more often than the second price auction. In the second setting, the English auction should be more efficient than the second price auction. This efficiency ranking is found to hold true in the experimental data. [25]

Table 14: Efficiency of the English and 2nd price auction

	English Auction	**2nd Auction**	z	**Prob** $> \lvert z \rvert$
Setting 1	.7734	.667	2.515	0.01
Setting 2	.734	.618	2.607	0.009

To a certain extent, inefficiency can be considered "worse" when there is a large difference in the value of the winning bidder and the bidder with the highest value. In the first setting, this difference in values can be measured as a difference in signals. Table 15 shows that, in both the English auction and the second price auction, there is a higher probability for efficiency the larger the difference between the bidders signals.[26] This effect is stronger for the English auction than for the second price auction.

Another way to measure the inefficiency caused by the highest valued bidder not winning the auction is to look at the differences between the highest value and the value of the winning bidder. In looking at setting 1, the highest value a a bidder could have would be 165. [27] I find that in the circumstances where the English auction is inefficient the average difference between the value of the highest valued bidder and that of the winning bidder is 14.86. In the

[25] Efficiency is a dummy variable equalling one when the winner has the highest realized value and zero otherwise.

[26] Table 15 shows results from a probit regression of efficiency on the signal difference between the winner's signal and the signal of the bidder to drop out last in the English auction. It also shows the results from a probit regression on efficiency and the next highest bidder's signal in the second price auction.

[27] This would be the rare circumstance where all bidders draw the same signal of 33.

Table 15: Efficiency and Signal Differences

	Variable	Coefficient	(Std. Err.)
English Auction	*signaldifference*	0.229^{**}	(0.019)
	Intercept	-0.431^{**}	(0.125)
2nd Price Auction	*signaldifference*	0.106^{**}	(0.016)
	Intercept	-1.919^{**}	(0.357)

Significance levels : † : 10% $*$: 5% $**$: 1%

second price auction, the average difference between the value of the highest valued bidder and that of the winning bidder is 17.93. In both cases this represents an approximately 18% difference between the value of the highest bidder and the value of the winning bidder.

3.6 CONCLUSION

First I have found, as Kirchkamp and Moldovanu (2004) found, that in both the English auction and the second price auction, bidders tend to overbid relative to the theoretical predictions and this remains even when bidders are presented with symmetric interdependent values. This does not appear to be alleviated with experience in the short run. Perhaps, if subjects were able to return after having some experience, some of the overbidding would disappear. This is an area for continued research.

The English auction is frequently used to auction off items which have interdependent values. Given what I have found with respect to efficiency, it makes sense that this happens. The English auction under the standard theoretical assumptions, where bidders have symmetric valuation functions, yields higher efficiency both theoretically and in the laboratory. Efficiency is more crucial when there is a large difference between the bidder's valuations and the English auction is more likely be efficient in this scenario. I have also found that even when bidders do not know the valuation function of one of the bidders, which is a more complicated situation, the English auction more strongly yields higher efficiency. This seems to imply that the English auction does better than the English auction as the bidding environment becomes more complex.

There are a few things which may explain the higher efficiency of the English auction. The first is the fact that some information is revealed during the English auction. As the auction continues, each bidder gets a clearer picture of their valuation. Even if bidders are not behaving as equilibrium would predict, they are able to get some sense of the other bidder's signals as the auction progresses. Also, the decision to drop out at a certain price (which is the decision facing a bidder in the English auction) is an easier decision than choosing a price to bid. At each price in the English auction, the bidder simply has to decide whether or not to stay in the auction. The English auction also makes the danger of paying a price higher than the object's value more obvious. The bidder can see the price they will have to pay if the auction ends at that moment.

There are a number of avenues for further research into auctions with interdependent values. First, it would be interesting to see how bidders behave in the first price auction, relative to the English and second price auctions, especially since the first price auction is a commonly used auction format. Also, it would be interesting to see how behavior changes when the bidder who has two parts to their signal has a multidimensional signal that cannot be collapsed into a single signal.

APPENDIX A

PROOF OF PROPOSITION 1

The following two conditions are jointly sufficient for this to be an equilibrium. Since the winner can not influence the price that she pays, one condition is that she will not suffer a loss by winning. The second is that a bidder who is supposed to lose will not make a profit by deviating in order to win. To show that these two conditions hold, we will only need to consider the sub-auction where A=1,2. For simplicity of notation, in each of the cases below bidder 1 is the winning bidder and bidder 2 is the only other remaining bidder. WLOG we assume that bidders drop out in numerical order, so that bidder 3 is the one to drop out immediately before bidder 2 is supposed to drop out. Also for simplicity we assume that no two bidders drop out at the same time.

There are a number of situations to consider.

First consider the case where both bidders 1 and 2 are insiders. Within this case there are four sub-cases.

The first is the case where, as part of their inside information, each of the insiders already know the other's signal. In this case, they already know their valuations because they have been able to induct all the signals of the bidders who have dropped out before them from the prices at which the bidder's have dropped out and from their own inside information. They are able to do this because the above system of equations has been reduced to:

$v_1(\phi_1(p_3^*|A, p_{\bar{A}}^*, s_{K(1)}), \phi_2(p_3^*|A, p_{\bar{A}}^*, s_{K(2)}), \phi_{(\bar{A}\cap\bar{O})\setminus K(1)}(p_3^*|A, p_{\bar{A}}^*, s_{K(\bar{A}\cap\bar{O})}),$

$s_{(\bar{A}\cap O)\setminus K(1)}, s_{K(1)}) = p_3^*$

$v_2(\phi_1(p_3^*|A, p_{\bar{A}}^*, s_{K(1)}), \phi_2(p_3^*|A, p_{\bar{A}}^*, s_{K(2)}), \phi_{(\bar{A}\cap\bar{O})\setminus K(2)}(p_3^*|A, p_{\bar{A}}^*, s_{K(\bar{A}\cap\bar{O})}),$
$s_{(\bar{A}\cap O)\setminus K(2)}, s_{K(2)}) = p_3^*$

Since $\phi_1(p_3^*|A, p_{\bar{A}}^*, s_{K(1)})$ is related related to 1's signal, which he already knows, he is only concerned with $\phi_{\bar{A}\cap\bar{O}(1)}(p_3^*|Ap_{\bar{A}}^*, s_{K(1)})$. These signals can also now be solved for because the bidder knows when the other bidders dropped out and the signal of the last remaining bidder other than himself; therefore he is solving a system of $((\bar{A}\cap\bar{O})(1))$ equations with at most $((\bar{A}\cap\bar{O})(1))$ unknowns. Once he has solved these equations he knows all the signals of the other bidders and thus knows his own valuation. The same argument can be made for bidder 2. Therefore this case is the same as a Standard English auction where the bidders know their valuations and it has already been shown that it is an equilibrium for them to drop out at their valuation, which is what the above bidding function tells the bidders to do.

The second is where neither bidder knows the others signal. In this case the break-even conditions after bidder 3 has dropped out at p_3^* are:

$v_1(\phi_1(p_3^*|A, p_{\bar{A}}^*, s_{K(1)}), \phi_2(p_3^*|A, p_{\bar{A}}^*, s_{K(2)}), \phi_{(\bar{A}\cap\bar{O})\setminus K(1)}(p_3^*|A, p_{\bar{A}}^*, s_{K(\bar{A}\cap\bar{O})}),$
$s_{(\bar{A}\cap O)\setminus K(1)}, s_{K(1)}) = p_3^*$

$v_2(\phi_1(p_3^*|A, p_{\bar{A}}^*, s_{K(1)}), \phi_2(p_3^*|A, p_{\bar{A}}^*, s_{K(2)}), \phi_{(\bar{A}\cap\bar{O})\setminus K(2)}(p_3^*|A, p_{\bar{A}}^*, s_{K(\bar{A}\cap\bar{O})}),$
$s_{(\bar{A}\cap O)\setminus K(2)}, s_{K(2)}) = p_3^*$

At this point bidder 2 is supposed to drop out at $p_2^* = \phi_2^{-1}(p|1, 2, p_{\bar{A}}^*, s_{k_2})$. Since $\phi_1(p|A, p_{\bar{A}}^*, s_{k_1})$ is increasing in p, $s_1 = \phi_1(p_1^*|1, 2, p_{\bar{A}}^*, s_{k_1}) > \phi_1(p_2^*|1, 2, p_{\bar{A}}^*, s_{k_1})$. Therefore since $v_{11}' > 0$, this implies that:

$$v_1(s) \geq v_1(\phi_1(p_2^*|1, 2, p_{\bar{A}}^*, s_{k_1})) = p_2^*$$

Thus bidder 1 does not suffer a loss by remaining in the auction.

The third sub-case is when bidder 2 knows bidder 1's signal but bidder 1 does not know bidder 2's. Here bidder 2 will drop out at his valuation. Since bidder 1 is supposed to win the auction $v_1 > v_2$ and therefore bidder 1 does not suffer a loss by remaining in the auction.

The fourth is the case where bidder 1 knows bidder 2's signal but bidder 2 does not know bidder 1's. This is essentially the same case as above.

The next case also has four sub-cases, it is when one bidder is an insider and one is an outsider.

Let's first consider the sub-case where bidder 1 is the insider and she knows bidder 2 (an outsider's signal). If this is the case, she knows her valuation and therefore will drop out if the bidding goes above it. So she does not suffer a loss in this case.

The next sub-case is where bidder 1 is again an insider but this time does not know outsider bidder 2's signal. In this case the break-even conditions, after bidder 3 drops out at p_3^*, are:

$v_1(\phi_1(p_3^*|1,2,p_{\bar{A}}^*,s_{K(1)}),\phi_2(p_3^*|1,2,p_{\bar{A}}^*,s_{K(1)}),\phi_{\bar{A}\cap\bar{O}}(p_3^*|1,2,p_{\bar{A}}^*,$
$s_{K(1)}),s_{\bar{A}\cap O},s_{K(1)}) = p_3^*$

$v_2(\phi_1(p_3^*|1,2,p_{\bar{A}}^*),\phi_2(p_3^*|1,2,p_{\bar{A}}^*),\phi_{\bar{A}\cap\bar{O}}(p_3^*|1,2,p_{\bar{A}}^*),$
$s_{\bar{A}\cap\bar{O}}) = p_3^*$

At this point bidder 2 is supposed to drop out when $p_2^* = \phi_2^{-1}(p|1,2,p_3^*)$ (or when $s_2 = \phi_2(p|1,2,p_{\bar{A}}^*)$). Since $\phi_1(p|A,p_{\bar{A}}^*,s_{K(1)})$ is increasing in p,
$s_1 = \phi_1(p_1^*|1,2,p_{\bar{A}^*},s_{K(1)}) > \phi_1(p_2^*|1,2,p_{\bar{A}^*},s_{K(1)})$. Therefore since $v_{11}^{'} > 0$, this implies that:

$$v_1(s) \geq v_1(\phi_1(p_2^*|1,2,p_{\bar{A}}^*,s_{k_1}),s_{\bar{A}\cap\bar{O}\setminus K(1)},s_{\bar{A}\cap O\setminus K(1)},s_{K((1)}) = p_2^*$$

Thus bidder 1 does not suffer a loss by remaining in the auction.

The next sub-case is where bidder 1 is an outsider and bidder 2 is an insider with knowledge of bidder 1's signal. Here the break-even conditions after bidder 3 drops out at p_3^* are:

$v_1(\phi_1(p_3^*|1,2,p_{\bar{A}}^*),\phi_2(p_3^*|1,2,p_{\bar{A}}^*),\phi_{\bar{A}\cap\bar{O}}(p_3^*|1,2,p_{\bar{A}}^*),s_{\bar{A}\cap O}) = p_3^*$

$v_2(\phi_2(p_3^*|1,2,p_{\bar{A}}^*,s_{K(2)}),\phi_{\bar{A}\cap\bar{O}\setminus K(2)}(p_3^*|1,2,p_{\bar{A}}^*,s_{K(2)}),s_{\bar{A}\cap O\setminus K(2)},s_{K(2)}) = p_3^*$

Bidder 2 drops out when $p_2^* = \phi_2^{-1}(p|1,2,p_{\bar{A}}^*,s_{K(2)})$. Since $\phi_1(p|A,p_{\bar{A}^*})$ is increasing in p, $s_1 = \phi_1(p_1^*|A,p_{\bar{A}^*}) > \phi_1(p_2^*|A,p_{\bar{A}^*})$. Therefore since $v_{11}^{'} > 0$, this implies that:

$$v_1(s) \geq v_1(\phi_1(p_2^*|1,2,p_{\bar{A}}^*),s_{\bar{A}\cap\bar{O}},s_{\bar{A}\cap O}) = p_2^*$$

Thus bidder 1 does not suffer a loss by remaining in the auction.

The final case to consider is when both bidder 1 and bidder 2 are both outsiders. In this case the break-even conditions, after bidder 3 drops out at p_3^* , are:

$v_1(\phi_1(p_3^*|1,2,p_{\bar{A}}^*), \phi_2(p_3^*|1,2,p_{\bar{A}}^*), \phi_{\bar{A}\cap\bar{O}}(p_3^*|1,2,p_{\bar{A}}^*),$
$s_{\bar{A}\cap O}) = p_3^*$

$v_2(\phi_1(p_3^*|1,2,p_{\bar{A}}^*), \phi_2(p_3^*|1,2,p_{\bar{A}}^*), \phi_{\bar{A}\cap\bar{O}}(p_3^*|1,2,p_{\bar{A}}^*),$
$s_{\bar{A}\cap O}) = p_3^*$

At this point bidder 2 is supposed to drop out when $p_2^* = \phi_2^{-1}(p|1,2,p_{\bar{A}}^*)$, which implies that $s_2 = \phi_2(p_2^*|1,2,p_{\bar{A}}^*)$. Since $\phi_1(p|A,p_{\bar{A}^*})$ is increasing in p, $s_1 = \phi_1(p_1^*|A,p_{\bar{A}^*}) >$ $\phi_1(p_2^*|A,p_{\bar{A}^*})$. Therefore since $v_{11}^{'} > 0$, this implies that:

$$v_1(s) \geq v_1(\phi_1(p_2^*|1,2,p_{\bar{A}}^*), s_{\bar{A}\cap\bar{O}}, s_{\bar{A}\cap O}) = p_2^*$$

Thus bidder 1 does not suffer a loss by remaining in the auction.

Next we will check that bidder 2 doesn't want deviate in order to win when he is supposed to lose. Again we will go through similar cases and sub-cases to the ones above. First consider the case where both bidders 1 and 2 are insiders. Within this case there are four sub-cases.

The first is the case where, as part of their inside information, each of the insiders already know the other's signal. In this case, they already know their valuations because they know all the signals of the other bidders in the auction, either by induction from the price at which they dropped out or from their own information. Therefore this case is the same as a standard English auction where the bidders know their valuations and it has already been shown that it is an equilibrium to drop out at your valuation.

The second is where neither bidder knows the others signal. In this case the break-even conditions after bidder 3 has dropped out at p_3^* are:

$v_1(\phi_1(p_3^*|1,2,p_{\bar{A}}^*, s_{K(1)}), \phi_2(p_3^*|1,2,p_{\bar{A}}^*, s_{K(1)}), \phi_{\bar{A}\cap\bar{O}\setminus K(1)}(p_3^*|1,2,p_{\bar{A}}^*, s_{K(1)}),$
$s_{\bar{A}\cap O\setminus K(1)}, s_{K(1)}) = p_3^*$

$v_2(\phi_1(p_3^*|1,2,p_{\bar{A}}^*, s_{K(2)}), \phi_2(p_3^*|1,2,p_{\bar{A}}^*, s_{K(2)}), \phi_{\bar{A}\cap\bar{O}\setminus K(2)}(p_3^*|1,2,p_{\bar{A}}^*, s_{K(2)}),$
$s_{\bar{A}\cap O\setminus K(2)}, s_{K(2)}) = p_3^*$

Since bidder 1 is supposed to win, $p_1^* > p_2^*$, which implies that $\phi_2(p_1^*|1,2,p_{\bar{A}}^*, s_{K(2)}) >$ $\phi_2(p_2^*|1,2,p_{\bar{A}}^*, s_{K(2)})$ since $\phi_2(p|1,2,p_{\bar{A}}^*, s_{K(2)})$ is increasing in p. Since bidder 2 is supposed to drop out at p_2^*, $s_2 = \phi_2(p_2^*|1,2,p_{\bar{A}}^*, s_{K(2)})$. This combined with the above statement implies

$\phi_2(p_2^*|1,2,p_{\bar{A}}^*,s_{K(2)}) > s_2$. Also since bidder 1 is supposed to drop out at p_1^*, we know that $s_1 = \phi_1(p_1^*|1,2,p_{\bar{A}}^*,s_{K(1)})$. Bidder 2 is able to perfectly infer this signal if he remains in beyond the point where $\phi_2(p_2^*|1,2,p_{\bar{A}}^*,s_{K(2)}) = s_2$, violating the bidding rule. Since v_2 is increasing in signals, we know that:

$$v_2(s_1,\phi_2(p_3^*|1,2,p_{\bar{A}}^*,s_{K(2)}),s_{\bar{A}\cap\bar{O}\setminus K(2)},s_{\bar{A}\cap O\setminus K(2)},s_{K(2)}) > v_2(s_1,s_2,s_{\bar{A}\cap\bar{O}\setminus K(2)},s_{\bar{A}\cap O\setminus K(2)},s_{K(2)})$$

Therefore bidder 2 would lose by remaining in the auction beyond the point where $s_2 = \phi_2(p_2^*|1,2,p_{\bar{A}}^*,s_{K(2)})$.

The third is the case where bidder 1 knows bidder 2's signal but bidder 2 does not know bidder 1's. It is easy to see that this is pretty much the same case as the one above, and therefore bidder 2 would lose by staying in beyond the point where $s_2 = \phi_2(p_2^*|1,2,p_{\bar{A}}^*,s_{K(2)})$.

The fourth sub-case is when bidder 2 knows bidder 1's signal but bidder 1 does not know bidder 2's. Here the bidding strategy requires bidder 2 to drop out at his valuation. Thus it is obvious that he will suffer a loss by remaining in and winning the auction.

The next case also has four sub-cases, it is when one bidder is an insider and one is an outsider.

Let's first consider the sub-case where bidder 1 is the insider and she knows bidder 2 (an outsider's signal). The break-even conditions after bidder 3 drops out at p_3^* are:

$v_1(\phi_1(p_3^*|1,2,p_{\bar{A}}^*,s_{K(1)}),\phi_2(p_3^*|1,2,p_{\bar{A}}^*,s_{K(1)}),\phi_{\bar{A}\cap\bar{O}\setminus K(1)}(p_3^*|1,2,p_{\bar{A}}^*,s_{K(1)}),$
$s_{\bar{A}\cap O\setminus K(1)},s_{K(1)}) = p_3^*$

$v_2(\phi_1(p_3^*|1,2,p_{\bar{A}}^*),\phi_2(p_3^*|1,2,p_{\bar{A}}^*),\phi_{\bar{A}\cap\bar{O}}(p_3^*|1,2,p_{\bar{A}}^*),$
$s_{\bar{A}\cap O}) = p_3^*$

Since bidder 1 is supposed to win, $p_1^* > p_2^*$, which implies that $\phi_2(p_1^*|1,2,p_{\bar{A}}^*) > \phi_2(p_2^*|1,2,p_{\bar{A}}^*)$ since $\phi_2(p|A,p_{\bar{A}}^*)$ is increasing in p. Since bidder 2 is supposed to drop out at p_2^*, $s_2 = \phi_2(p_2^*|1,2,p_{\bar{A}}^*)$. This combined with the above statement implies $\phi_2(p_1^*|1,2,p_{\bar{A}}^*) > s_2$. Also since bidder 2 remains in to win the auction, bidder 1 drops out at $p = \phi_1^{-1}(p|1,2,p_{\bar{A}}^*,s_{K(1)})$. Since v_2 is increasing in signals, we know that:

$$v_2(s_1,\phi_2(p_1^*|1,2,p_{\bar{A}}^*),s_{\bar{A}\cap\bar{O}},s_{\bar{A}\cap O}) > v_2(s_1,s_2,s_{\bar{A}\cap\bar{O}},s_{\bar{A}\cap O})$$

Therefore bidder 2 would suffer a loss by remaining in the auction in order to win.

The next sub-case is where bidder 1 is again an insider but this time does not know outsider bidder 2's signal. In this case the break-even conditions, after bidder 3 drops out at p_3^*, are:

$v_1(\phi_1(p_3^*|1,2,p_{\bar{A}}^*, s_{K(1)}), \phi_2(p_3^*|1,2,p_{\bar{A}}^*, s_{K(1)}), \phi_{\bar{A}\cap\bar{O}\setminus K(1)}(p_3^*|1,2,p_{\bar{A}}^*, s_{K(1)}),$
$s_{\bar{A}\cap O\setminus K(1)}, s_{K(1)}) = p_3^*$

$v_2(\phi_1(p_3^*|1,2,p_{\bar{A}}^*), \phi_2(p_3^*|1,2,p_{\bar{A}}^*), s_{\bar{A}\cap O}) = p_3^*$

At this point bidder 2 is supposed to drop out when $p = \phi_2^{-1}(p|1,2,p_{\bar{A}}^*)$. Since bidder 1 is supposed to win, $p_1^* > p_2^*$, which implies $\phi_2(p_1^*|1,2,p_{\bar{A}}^*) > \phi_2(p_2^*|1,2,p_{\bar{A}}^*)$. This means that $\phi_2(p_1^*|1,2,p_{\bar{A}}^*) > s_2$. Bidder 1 drops out at $p = \phi_1^{-1}(p|1,2,p_{\bar{A}}^*, s_{K(1)})$. Since v_2 is increasing in signals, we know that:

$$v_2(s_1, \phi_2(p_1^*|1,2,p_{\bar{A}}^*), s_{\bar{A}\cap\bar{O}}, s_{\bar{A}\cap O}) > v_2(s_1, s_2, s_{\bar{A}\cap\bar{O}}, s_{\bar{A}\cap O}) = v_2(s)$$

Therefore bidder 2 would suffer a loss by remaining in the auction in order to win.

The next sub-case is where bidder 1 is an outsider and bidder 2 is an insider with knowledge of bidder 1's signal. Here bidder 2 knows his valuation and the bidding strategy requires that he drops out at his valuation. Obviously remaining in the auction beyond his valuation will cause bidder 2 to suffer a loss.

The final sub-case is where bidder 1 is an outsider and bidder 2 is an insider without knowledge of bidder 1's signal. Here the break-even conditions after bidder 3 drops out at p_3^* are:

$v_1(\phi_1(p_3^*|1,2,p_{\bar{A}}^*), \phi_2(p_3^*|1,2,p_{\bar{A}}^*), s_{\bar{A}\cap O}) = p_3^*$

$v_2(\phi_1(p_3^*|1,2,p_{\bar{A}}^*, s_{K(2)}), \phi_2(p_3^*|1,2,p_{\bar{A}}^*, s_{K(2)}), \phi_{\bar{A}\cap\bar{O}\setminus K(2)}(p_3^*|1,2,p_{\bar{A}}^*, s_{K(2)}),$
$s_{\bar{A}\cap O\setminus K(2)}, s_{K(2)}) = p_3^*$

At this point bidder 2 is supposed to drop out when $p = \phi_2^{-1}(p|1,2,p_{\bar{A}}^*, s_{K(2)})$. Since bidder 1 is supposed to win, $p_1^* > p_2^*$, which implies $\phi_2(p_1^*|1,2,p_{\bar{A}}^*, s_{K(2)}) > \phi_2(p_2^*|1,2,p_{\bar{A}}^*, s_{K(2)})$. This means that $\phi_2(p_1^*|1,2,p_{\bar{A}}^*, s_{K(2)}) > s_2$. Bidder 1 drops out at $p = \phi_1^{-1}(p|1,2,p_{\bar{A}}^*)$. Since v_2 is increasing in signals, we know that:

$$v_2(s_1, \phi_2(p_1^*|1, 2, p_{\bar{A}}^*, s_{K(2)}), s_{\bar{A}\cap\bar{O}}, s_{\bar{A}\cap O}, s_{K(2)}) > v_2(s_1, s_2, s_{\bar{A}\cap\bar{O}}, s_{\bar{A}\cap O}, s_{K(2)}) = v_2(s)$$

Therefore bidder 2 would suffer a loss by remaining in the auction in order to win.

The final case to consider is when both bidder 1 and bidder 2 are both outsiders. In this case the break-even conditions, after bidder 3 drops out at p_3^*, are:

$v_1(\phi_1(p_3^*|1, 2, p_{\bar{A}}^*), \phi_2(p_3^*|1, 2, p_{\bar{A}}^*), s_{\bar{A}\cap O}) = p_3^*$

$v_2(\phi_1(p_3^*|1, 2, p_{\bar{A}}^*), \phi_2(p_3^*|1, 2, p_{\bar{A}}^*), s_{\bar{A}\cap O}) = p_3^*$

At this point bidder 2 is supposed to drop out when $p = \phi_2^{-1}(p|1, 2, p_{\bar{A}}^*)$. Since bidder 1 is supposed to win, $p_1^* > p_2^*$, which implies $\phi_2(p_1^*|1, 2, p_{\bar{A}}^*) > \phi_2(p_2^*|1, 2, p_{\bar{A}}^*)$. This means that $\phi_2(p_1^*|1, 2, p_{\bar{A}}^*) > s_2$. Bidder 1 drops out at $p = \phi_1^{-1}(p|1, 2, p_{\bar{A}}^*)$. Since v_2 is increasing in signals, we know that:

$$v_2(s_1, \phi_2(p_1^*|1, 2, p_{\bar{A}}^*), s_{\bar{A}\cap\bar{O}}, s_{\bar{A}\cap O}) > v_2(s_1, s_2, s_{\bar{A}\cap\bar{O}}, s_{\bar{A}\cap O}) = v_2(s)$$

Therefore bidder 2 would suffer a loss by remaining in the auction in order to win.

Since the bidder who is supposed to drop out second does not want to deviate, no other bidder further down in the drop out order would want to deviate in order to win also.

Thus we have proved that the bidding strategy given above is an equilibrium. □

APPENDIX B

PROOF OF PROPOSITION 2

A solution to the system of break even conditions exists by the Cauchy-Peano theorem. Now we will show that $\phi_A(p)$ is the unique solution and that it is increasing. Begin by rearranging the last $|\bar{A} \cap \bar{O}|$ lines of the system of equations and getting:

$$(D_{\phi_A}, v_{\bar{A}\cap\bar{O}}(\phi'_A) = -(D_{\phi_{\bar{A}\cap\bar{O}}}, v_{\bar{A}\cap\bar{O}}(\phi'_{\bar{A}\cap\bar{O}})$$

Rearranging we get:

$$\phi'_{\bar{A}\cap\bar{O}} = -(D_{\phi_{\bar{A}\cap\bar{O}}}, v_{\bar{A}\cap\bar{O}})^{-1}(D_{\phi_A}, v_{\bar{A}\cap\bar{O}})(\phi'_A)$$

Then we substitute this into the first $|A|$ lines of the system of equations and rearranging again we get:

$$(D_{\phi_A} v_A - (D_{\phi_{\bar{A}\cap\bar{O}}} v_A)(D_{\phi_{\bar{A}\cap\bar{O}}})^{-1}(D_{\phi_A} v_{\bar{A}\cap\bar{O}}))(\phi'_A) = e$$

It is easy to see that the average crossing condition implies that: $(D_{\phi_A} v_A - (D_{\phi_{\bar{A}\cap\bar{O}}} v_A)$ $(D_{\phi_{\bar{A}\cap\bar{O}}})^{-1}(D_{\phi_A} v_{\bar{A}\cap\bar{O}}))$

satisfies the dominant average condition and therefore Lemma 5 applies again. □

Definition 10 *An $m \times m$ matrix satisfies the dominant average condition iff:*

$\frac{1}{m}\sum_{k=1}^{m} a_{kj} > a_{ij}$ *for all* $i \neq j$

and

$\sum_{k=1}^{m} a_{kj} > 0$ *for all* j

Lemma 11 *(borrowed from Krishna (2002)): Suppose A is an matrix that satisfies the dominant average condition. Then A is invertible. Also there exists a unique $x > 0$ such that: $Ax = e$*

For a proof of this lemma see Krishna (2002), page 274.

APPENDIX C

INSTRUCTIONS: ENGLISH AUCTION

This is an experiment in decision making. The Department of Economics has provided funds for this research. During the course of the experiment, you will make a series of decisions. If you follow the instructions carefully and make good decisions, you can earn a considerable amount of money, which will be paid to you in cash at the end of the experiment. Please do not talk for the duration of the experiment.

Groups

In this experiment we will create a market in which you will act as the buyer of a fictitious object in a sequence of trading periods. In each trading period you will be randomly grouped with two other bidders. One object will be auctioned off to each group, each period. Your grouping will change over a series of periods and will remain anonymous. Since there are 3 people in a group, each person has two partners. One person in your group will be called “Partner 1” and the other will be called “Partner 2”. As you will see, they will affect you in the same way. These names are purely used for clarity.

Values

In each period your value for the object will change. At the beginning of each period you will be told your private signal for that period (only you will be told your signal) and the equation that determines your value of the object. You can think of your value as a redemption value for the object, if you were to win it. That is, if you win the object you will be able to redeem it for the dollar amount equal to your value.

Your value of the object depends on your private signal and the private signals of the other members of your group. Your value equation for the entire experiment is: $3 * a + b + c$, where a is your private signal, b is the private signal of “Partner 1” and c is the private signal of “Partner 2”. This is the same for every member of your group. That is, from your perspective, “Partner 1”’s value equation is $3 * b + a + c$ and “Partner 2”’s value equation is $3 * c + a + b$. You should think about this as follows:

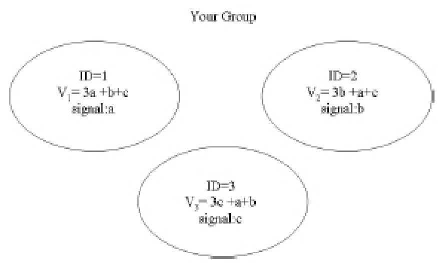

Figure 2: Bidding Group

The signals in the experiment are drawn from the numbers between 1 and 33. They are in terms of points. At the end of the experiment your points will be converted into money at the rate of 1 point = 5 cents. Any amount between 1 point and 33 points is equally likely to be chosen for each person in the experiment. The signals are also drawn independently. For example, if your signal draw is 15, it is no less and no more likely that any signal is drawn for another person.

The table below shows some potential realizations of your value, if your signal is 25 and therefore your value function is $75 + b + c$.

Of course there are many other possible realizations of your value, each depending on the signals drawn for your partners.

Table 16: Value Examples

Your signal	3*a	b	c	Your value
25	75	20	15	110
25	75	20	33	128
25	75	5	15	95
25	75	25	25	125
25	75	10	15	100

How the Auction Works

The first screen that you see (Figure 3) tells you your signal and your value equation. For example in Figure 3, the private signal is 11, therefore the value equation is 33 + b + c. While you are on this screen, you also have the opportunity to use a calculator, if you would like. You can do this by clicking on the calculator button in the bottom right hand corner.Once you are ready to move on to the auction you will press the continue button.

After everyone has pressed the continue button the auction will begin. You will see a screen like figure 4. At the top of the screen is the current price, by current price we mean the price that you would "pay" for the object if you were to win. This number will increase by five points every 2 seconds. Below this you are reminded of your signal and your value equation. Below this is the red Drop Out button. When you press this button you will no longer be participating in the auction for that period.

If one of your partners drops out of the auction, your screen will flash briefly and then the price at which they have dropped out will show up below the drop out button (see Figure 5). At this point the price clock will slow down. It will increase by 1 point every 2 seconds.

Once two members of a group have dropped out the auction will end and the member of the group who has not dropped out is the winner of the auction. **If you drop out of the auction, your payoff for the period is zero.** If you are the last remaining member

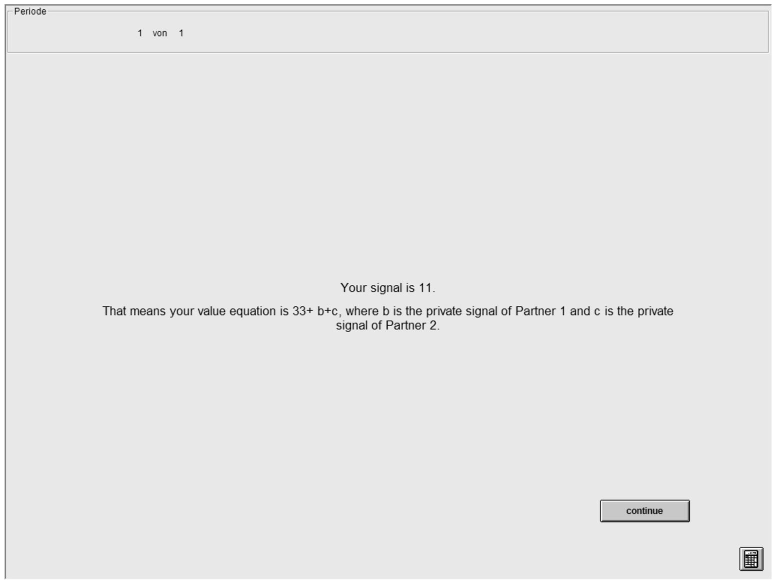

Figure 3: First screen

Figure 4: Second screen

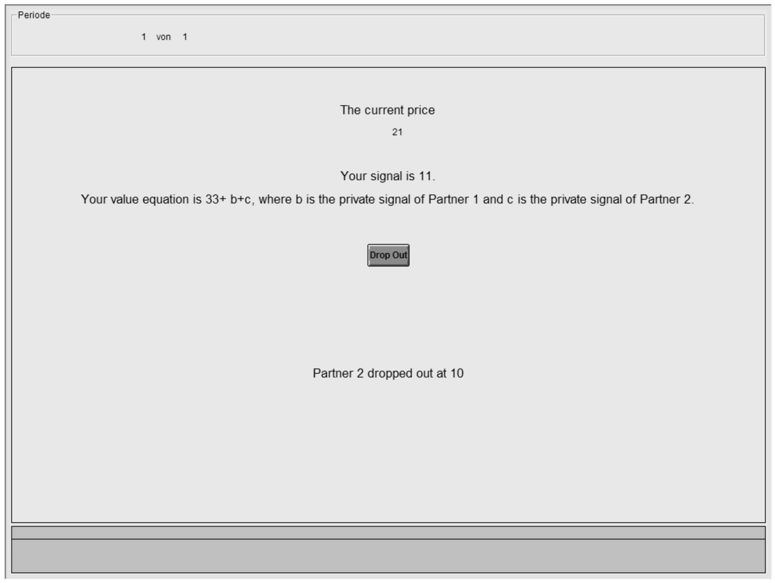

Figure 5: Third screen

of the group, then you win the auction. If you win, your payoff is your **value minus the price at which the last member of your group dropped out of the auction**. **This amount can be negative**, if you drop out after the price reaches your value.

Finally, at the bottom of the screen, after the first period, there is a history of the auctions in which you have participated. This will tell you your signal and the signals of the other members of your group from last period. (Remember that your group and your signal, as well as everyone else's group and signal, change every round.) You are also told the price at which the fictional object was sold and your profit.

Bankruptcy

As stated above, there is a chance that you will earn a negative payoff in a period. The first time you earn a negative payoff, the computer will ask you if you want to invest your show up fee (unless you have already won more than the negative amount). If you decide that you want to continue, your negative earnings will be removed from your show up fee of $5.

An Example

Let's consider an example. You are told that your signal is 30, therefore your value equation is $90 + b + c$. Furthermore, let's suppose that $b = 25$ and $c = 15$. Suppose your "Partner 1" drops out when the price clock reaches 9, then your screen will show that he did this. "Partner 1" is no longer in the auction. Suppose then that "Partner 2" drops out when the price clock reaches 15. This means that you have won the fictional object at a price of 15. The payoff for Partners 1 and 2 is zero and your payoff is $3*30+25+15-15 = 130-15 = 115$ points (which converts into $5.75), in this period.

This will be run 15 times. Again your partners will be reassigned at the end of each period. Once all 15 periods are finished you will see a screen that tells you your payoffs for each of the periods and your total payoffs for the experiment. Again, it is possible that in a given round you earn a negative payoff, this will be subtracted from your $5 show up fee.

We will go through a few more examples at the end of the instructions and then we will begin the auction.

Summary

- there are four groups of three people participating in the experiment.
- the groups are randomly reassigned each period.
- there is a fictional object auctioned off to each group, in each period.
- your value of the object is determined by your private signal and the private signals of each of the other two members of your group.
- the equation that determines your value is : $3 * a + b + c$, where a is your signal, b is Partner 1's signal and c is Partner 2's signal.

- this equation is the same for every member of your group for every period.
- signals are drawn from the interval from 1 point to 33 points.
- at the end of the experiment points are converted into money at a rate of 1 point = 5 cents.
- the auction run is a clock auction, where the price rises every 2 seconds.
- until the first member of your group drops out, the price will increase by 5 points every 2 seconds.
- after the first member of your group drops out, the price will increase by 1 point every 2 seconds.
- once you drop out of the auction you can not rejoin the auction and your payoff for the round is zero.
- once one of the other group members drops out you will see the price at which they dropped out.
- if you are the last remaining member of your group in the auction, you win the auction and pay the price at which the final member of your group dropped out.
- your pay off for the period if you win the auction is your value minus the price. ** Please note this amount can be negative. If it is the negative amount will be subtracted from your $5 show up fee.

Do you have any questions?

Understanding Quiz

1. Suppose that your value equation is $3 * 20 + b + c$. Also, $b = 5$ and $c = 25$. Partner 1 drops out at 20 and Partner 2 drops out at 60. You have not dropped out of the auction What is your payoff?

2. Suppose that your value equation is $3 * 20 + b + c$. Also, $b = 5$ and $c = 25$. Partner 1 drops out at 20 and Partner 2 drops out at 400. You have not dropped out of the auction. What is your payoff?

3. Suppose that your value equation is $3 * 20 + b + c$. Also, $b = 5$ and $c = 25$. You drop out at 20 and Partner 2 drops out at 225. You have dropped out of the auction. What is your payoff?

APPENDIX D

ALTERNATE FORMULATION OF BID REGRESSIONS

In their paper, Kirkchamp and Moldovanu use a different regression to compare actual bids to the bids predicted by theory. They regress the actual bid on the expected bid, forcing no constant. They use the censored normal regression for both the English auction and the second price auction, because they used a clock in the second price auction as well as in the English auction. In this regression, if the coefficient is different from one, there is over bidding. The tables below show that I arrive at the same result found above, using this formulation.

Table 17: Alternate Formulation of Bid Functions

Auction	**Coefficient**	**(Std. Err.)**
English: First Bid	1.540^{**}	(0.077)
English: Second Bid	1.144^{**}	(0.023)
Second Price	1.166^{*}	(.0623581)
Significance levels : $\dagger$: 10% $\quad *$: 5% $\quad **$: 1%		

Note: The significance tests were done for $\beta = 1$

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