Two Essays in Competitive Price Formation in Auctions

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ABSTRACT

In this work, I look at two competitive auction settings where a profit maximizing seller chooses auctions as a vehicle to sell to strategic bidders. In both essays, the auctioneer's problem is the selection of the optimal auction format. In the first essay, the auctioneer has a single item to sell while in the second essay, there are two items. In this work, I use game theoretic methods to derive the best course of action for the buyer and use this to arrive at the best course of action for the auctioneer.

In essay 1, I consider a hybrid (between English outcry and second price sealed bid) auction format where at any point in time, the identity of the highest bidder and the second highest price is known to all. I show that this format would generate higher revenues than the English outcry format if the bidders' valuations are interdependent. This is because of lesser risk of overpayment and winner's curse for the bidders in the hybrid auction and consequently, they are better off bidding their valuations earlier. Such behavior results in a quicker convergence of the outstanding price to the final price realized as the bidders can update their valuations with certainty. I test this claim by comparing objects auctioned in Yahoo! and eBay as eBay follows the hybrid action format while Yahoo! follows the English outcry format and do find that with interdependent object valuations revenue from the hybrid auction format is higher.

In the second essay, I consider an auctioneer who has two items to sell. These could be complements or substitutes or independent products. Given a pool of strategic bidders, I

investigate whether he is better off auctioning the items sequentially or as a bundle. To do so, I first solve the bidders’ optimization problem and use the solution to arrive at the implications for the seller. I find that with a moderate number of bidders (N>4), it is optimal to bundle strong complements only. On the other hand for substitutes, I find that bundling is optimal when the number of bidders is less than four.

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1. DISSERTATION OVERVIEW: COMPETITIVE PRICE FORMATION IN AUCTIONS

1.1. Introduction

Auctions as a mode of exchanging value have been in existence for a long time. For instance Shubik (1983) notes the use of auctions by the Roman army for distribution of war spoils. Cassady (1967) mentions the use of auctions in 193A.D. for selling the entire Roman Empire after the killing of Emperor Pertinax. Though auctions can be viewed as a special means of sales where the seller has a small number of objects, research within marketing concerning auctions is a relatively recent phenomenon. This is because the use of auctions prior to the success of online auctions sites was rather specialized. However, the above statement does not imply that research in auctions is recent. Rather, auction theory was used to tackle various problems by economics scholars in special settings. Some important applications include sale of treasury bills (Laffont 1997), sale of mineral rights (Hendricks and Porter 1988) and sale of broadband spectrum (Krishna and Rosenthal 1996). These specialized applications provided the background for the development of auction theory within a game theoretic framework.

With the popularity of the internet and online auction sites, auctions have attracted the attention of marketing scholars. This interest has resulted in work related to auctions in marketing where the approach to analysis is game theoretic (e.g., Rothkopf 1991, Rothkopf and Harstad 1994, Greenleaf, Rao and Sinha 1993) or involves psychology of consumer behavior (e.g., Greenleaf 2004, Kamins, Dreze and Folkes 2004, Dholakia and Simonson 2005). In the studies mentioned, we see that the interest in consumer behavior within auctions is a rather new development. This is primarily because online auctions predominantly involve settings where consumers (rather than firms) interact and trade between themselves.

In this work, I look at two settings that are applicable to online auctions. My focus in the two essays is the specific aspect of price formation and thus the revenue generated by different auction mechanisms. In the first essay, I contrast two online auction formats: English outcry and the hybrid auction formats. Existing auction theory (Milgrom and Weber 1982) shows that the revenue from the English outcry auction would be no less than the other classical auction formats. The English outcry auction involves bids being placed in increments till a point is reached where only one bidder remains who then pays the last (and the highest) bid placed. This is compared to the second price sealed bid and the first price sealed bid auction. In the second price sealed bid auction, each bidder places a single (sealed) bid and upon opening the bids, the auctioneer awards the object to the highest bidder at the second highest bid. In the first price sealed bid auction, the procedure is the same except the object goes to the highest bidder who pays the amount s/he bid.

The English outcry auction generates higher revenues than the sealed bid auctions in a setting where the bidders' object valuations are interdependent. This is because with an open ascending bidding sequence, the bidders can see the bid levels of the bidders and this allows the bidder to reassess her valuation of the object. Such a reassessment is not possible in any sealed bid auction where only one bid is placed.

The above reasoning however, does not take into account a hybrid auction format that combines the English outcry and the second price sealed bid auction and this is the focus of the first essay. Specifically, I consider a format where at any point in time, the highest bidder and the second highest bid are known. The winner of this new auction format is the highest bidder at the end of the auction duration where s/he pays the outstanding second highest amount. Since the auction involves multiple rounds of bidding, this setup combines features of English outcry and the second price sealed bid auction formats. In the first essay, I compare the revenue from this new hybrid format with that from the English outcry format. The hybrid format is used by the popular auction website eBay while Yahoo! employs the online English outcry format where the bidders place their bids in an increasing sequence. I first show that the hybrid format should generate higher revenues than the English outcry format and then test the claim empirically.

In the second essay, I look at the problem of an auctioneer who has two items to sell to a pool of bidders who are in the market for both objects. The objects themselves are complements (I later extend the model to look at substitutes) so that the willingness to pay for the bundle is greater than the sum of the willingness to pay for the individual objects. In a second price sealed bid auction framework, I look at the optimal strategy for bidders and arrive at consequent implications for the auctioneer. Specifically, I only consider two possibilities for the seller: sell the items as a bundle or sell the items sequentially. Contrary to expectations, I find that it is optimal for the seller to unbundle his offering even with moderate complements when there are a sufficient number of bidders. Mirroring the above, I also find that with sufficiently few bidders (specifically less than 4), it may be optimal to bundle substitutes. I also extend the model to look at the role of risk aversion and find that its relevance decreases as the number of bidders increases.

In both essays, I therefore look at how competitive forces shape the process of price formation and how this process is influenced by external factors. I choose an auction setting primarily due to the increased popularity of online auctions. Though I do not explicitly model the second essay along online auctions, the results map on to an online auction situation where the bidders place their bids in the last minute. As the title suggests, I look at the problem facing a strategic consumer. This is manifested in the framework and parameterization chosen for both essays that reflects the world of B2C and C2C marketplace. This is opposed to the bulk of the traditional auction literature that focuses on organizational buying situations (e.g., Krishna and Rosenthal 1996, McAfee and McMillan 1992, Hendricks and Porter 1988). However, the modeling approach adopted in both essays could be tailored to analyze B2B settings as well.

1.2. Overview

1.2.1. Research on Auctions in Marketing

Following Vickrey's (1961) seminal paper that developed the second price sealed bid auction, there has been a tremendous amount of work in economics regarding auctions. Till a few years back, most of the work in auctions was game theoretic in nature (e.g., Riley and Samuelson 1981, Maskin and Riley 1996, Myerson 1981) and considered the traditional auction format where the supplier and the bidders were required to be present physically at the same location for the auction to take place. Also, there had been no serious work in marketing literature studying auctions. A notable exception was Hoffman et.al. (1993) that looked at the use of auctions in a test market as a means to determine prices. By the beginning of the new century, online auctions had caught on in a big way. One implication of the popularity of the internet was a renewed interest in auctions not only among economists but also among marketing scholars. Thus, there have been studies looking at online auction behavior in economics (c.f. Lucking Reiley 2001, Roth and Ockenfels 2002) and also in marketing (e.g., Wilcox 2000, Haubl and Popkowski 2003, Ariely and Greenleaf 2000).

Apart from the popularity of online sites like eBay and Yahoo! there are two additional reasons for the surge in auction related work. First, there are subtle differences in the online auction rules when compared to the traditional auction setting. For instance, online auctions permit last minute bidding or sniping where it is optimal for the bidder to place his bid at the very last moment in the auction. This is because many online auctions have a “hard close” which allows the bidder to bid at the last moment. The reason behind this behavior is that by not participating in the auction till the very last moment, the bidder holds back information from other bidders which they may use to increase their bid for the object. A second reason for the increased interest in auctions is the availability of rich data about auctions of various kinds of objects which offers many opportunities to empirically test the various theories regarding bidder behavior.

The earlier discussion regarding research on auctions within economics and marketing centers around game theoretic models where the auctioneer and the bidders are surplus maximizing economic entities. There is limited work in auctions (e.g., Maskin and Riley 1984, Matthews 1987, Milgrom and Weber 1982) that considers risk aversion among the bidders and/or the auctioneer. A recent phenomenon is the incorporation of psychological aspects of bidder behavior. A few notable examples here are Wilcox (2000), Greenleaf (2004) and Haubl and Popkowski (2003). Greenleaf (2004) looks at the problem of setting an optimal reserve price and its behavioral implications. Haubl and Popokowski (2003) look at the psychological impact of a quick versus slow response to bidding in English outcry auctions on competing bidders. Specifically, they report that a quicker bid placed by the competition creates an impression of higher willingness to pay. Such an emphasis on the behavioral and psychological aspects of the auction setting is important in the current world as the online auction scenario is dominated by C2C activity and this is the backdrop for this work.

1.3. Scope and Summary of the First Essay

In the first essay, I compare two online auction mechanisms and their implications regarding the revenue generated. Specifically, I compare the online English outcry auction format and the hybrid auction format. In the online English outcry, the bidders place commonly observable bids in an increasing sequence for the object and the object goes to the highest bidder who pays the last amount s/he bid. The hybrid format can be visualized as a second price sealed bid auction with multiple rounds of bidding. Here, at any point in time, the bidders can see the highest bidder and the second highest bid placed. The object finally goes to the highest bidder at the second highest price. On one hand, since the bid observed is the second highest, this format has some similarity to the second price sealed bid auction. On the other hand, the bidding progresses in stages where the bids are placed in an increasing sequence. This gives the format some flavor of the English outcry auction.

For comparing revenue between the auction formats, I look at two leading online auctions of today-eBay and Yahoo! in the first essay. Here, I develop a model of bidder behavior to understand the conditions under which the eBay auction format would yield greater revenue. I develop the central claim theoretically showing the conditions[1] under which the revenue from the hybrid format would exceed the online English outcry format. This is an interesting result as the English outcry auction is known to generate higher revenues than the other auction formats (c.f. Milgrom and Weber 1982). However, the existing literature does not consider the hybrid auction format. The hybrid auction format however combines the desirable features of English outcry and the second price sealed bid auction. The propositions are tested empirically using data collected from the respective websites regarding the bid history for 200 matched pairs of objects sold at overlapping times[2]. I find that when the items sold are such that the consumer valuations are interdependent, eBay auction format is likely to yield higher revenue than the Yahoo! format.

[1] The conditions are absence of last minute bidding and proxy bidding. I discuss these in detail in chapter 2.
[2] Overlapping means that there was a period in time when both auctions were active. For example, if there was a Rolex presidential watch auctioned on May 15-22 2000 on eBay, there was another Rolex presidential watch whose auction in Yahoo was "alive" sometime between May 15-22 2000.

This effect is enhanced if there is lesser instance of last minute bidding. In both auctions, it is optimal to bid at the last moment. In the eBay format, if some bidders choose to bid before the last minute, the bid levels attracted are likely to be higher than in the Yahoo! format. This has two effects: First, the higher levels of earlier bids drives final prices upward and second, the eBay format also allows the bidders to update their valuations better resulting in quicker convergence of the bid levels to the final price.

1.4. Scope and Summary of the Second Essay

In the second essay, I look at the problem faced by an auctioneer who has two objects to sell and has to decide whether he should auction them together as a bundle or separately one after the other. While doing so, I adopt a second price sealed bid auction framework which is quite common in the multi object auction literature (see for instance Krishna and Rosenthal 1996; Rosenthal and Wang 1996; Chakraborty 2001). In addition to model tractability, the second price sealed bid auction resembles an online auction where the bids are placed at the very last minute. Upon identifying the optimal bidder behavior, I look at the implications of the bidding strategy for the seller. In the basic model, I look at complements that are of similar value *a priori*. I then ask the following questions. First, under what conditions is bundling the objects optimal for the auctioneer and when is it optimal to offer the objects sequentially? What is the role of risk aversion in sequential auction setting?[3] What is the effect of asymmetry in the nature of good sold? Does the reasoning in the case of complements extend to substitutes?

I find that contrary to expectation, the unbundled auction generates higher revenue with weak to moderate complements. I also show that as the extent of complementarity increases, the bundled auction does better. On the other hand, the number of bidders has the effect of making

[3] In the bundled auction, I show that the weakly dominant strategy of bidding one's valuation is invariant under the bidder's risk preferences.

the unbundled auction more attractive. This is primarily because bundling by its nature decreases the heterogeneity among the valuations of the bidders whereas there is no such effect in the unbundled auction. In the case of substitutes, I find that when the number of bidders is less than four, it may be optimal to bundle substitutes. In particular, when the number of bidders is two, it is always optimal to bundle and with three bidders, it is optimal to bundle provided the objects are not strong substitutes. The reason is that the optimal bidder strategy in the sequential auctions involves underbidding for the first object by all bidders and underbidding for the second object for the winner of the first auction. This has the effect of reducing the revenue for auctioneer and the effect is particularly severe with fewer bidders because the level of bids for the second object is greatly reduced. Another surprising result is that with increasing number of bidders, risk aversion increasingly becomes irrelevant. The reason is that with more bidders, there is greater risk to overbidding as the winner of the first object runs the risk of losing the second.

1.5. Essays 1 and 2: A Contrast

Although the essays examine the price formed due to competitive bidder behavior, they look at very different settings. In this section, I summarize the contrast between the two essays.

……………………………………..Insert Table 1 about here ……………………………………

As Table 1 shows, essay one is characterized by the auction of a single object while essay two looks at two items. In essay one, the primary result regarding greater revenue generated by the hybrid format is due to the item sold having interdependent valuation while essay two restricts its attention to the case of independent private valuation for the items. In essay 1, the number of bidders is exogenous and for the theoretical result, should be the same. In essay 2, the number of bidders is also exogenous but plays a crucial role in the auctioneer's bundling/unbundling decision. In essay2, I look at the impact of the number of bidders and as we shall see later, the number of bidders is a key variable in the analysis that not only influences the bid levels but also plays an important role in determining the optimal strategy for the auctioneer. Thus, essays 1 and 2 look at relevant problems for the seller in the online auction marketplace and arrive at optimal strategies given rational bidder behavior.

In addition to the above differences, essay 1 tests the key propositions empirically using data collected over the internet while essay 2 is entirely theoretical. Further, the consumer's problem in essay 1 is relatively straightforward while optimal bid choice for the consumer is relatively tricky in essay 2. This is because essay 2 considers the strategic bidder facing a sequential auction setting. In such a situation, the bidder has to decide on the bid for the first object while trading off the possibility of increased surplus if she wins both objects with the chances of losing the second object after aggressively bidding for the first. Thus, while both essays look at the seller's problem of choosing the optimal auction format given rational bidders, the consumer behavior varies greatly across both settings.

In the following sections, I first look at the first essay in chapter two followed by the second essay in chapter three. This is followed by chapter four that discusses the implications, key contributions and areas of future work.

2. ESSAY 1: REVENUE EQUIVALENCE IN ONLINE AUCTIONS: THE CASE OF YAHOO AND EBAY

2.1. Introduction

The past few years have seen Internet auctions emerge as an extremely popular way to purchase goods and services. For example, eBay.com had sold goods worth $8 Billion in the first quarter of 2004 (La Monica, *CNNmoney*, April 21, 2004), while on Oct. 25, 2003, Yahoo auctions had more than 17,000 antiques and art objects up for auction. The popularity of online auctions is largely due to the fact that in the physical world, both the bidders and the seller have to be present at the required time and place and the object has to be physically present. Online auctions eliminate such restrictions. Due to their popularity and the theoretical distinctions with respect to traditional auction theory, online auctions have begun to garner attention within marketing (e.g. Chakravarti et al. 2002; Lynch and Ariely 2000; Sinha and Greenleaf 2000; Wilcox 2000).

Online auctions have a somewhat different structure than the auction forms analyzed by traditional economic theory (Seidmann and Varkat 2000; Wilcox 2000). For instance, Yahoo.com allows bidders to bid on an object at any point in time in the specified auction duration. Thus, bidders are free to exit and re-enter, which is in contrast to the way traditional auctions are conceptualized in economic theory. EBay, on the other hand, is an auction that represents a hybrid of two types of auctions, the second-price sealed-bid and English outcry auctions (Shah, Joshi, and Wurman 2002; Wilcox 2000). In the second-price sealed-bid auction, each bidder presents a single sealed-bid for the object. The bids are opened simultaneously and the object goes to the highest bidder at the second highest bid. In the most common variant of the English outcry auction, the bidders bid for the object in an increasing sequence, that is – each bidder bids above the current highest price until a point is reached where no bidder is willing to bid above the current bid. The object is then awarded to the bidder with the highest bid at that price.

The auction format of eBay that I examine combines features of second price sealed bid auction and the English outcry auction. Here, the bidders submit bids for the object online. At any time, only the second highest bid is shown and the object goes to the highest bidder at the second highest bid at the end of the auction. However, like an English auction, the bidders can place more than one bid during the course of the auction. In this essay, I call the eBay format the *hybrid auction format*.

I use the properties of English outcry and second-price sealed-bid auctions to develop testable propositions. These are tested using data from 200 pairs of auctions of identical objects on Yahoo and eBay. I first show analytically that the revenue form the hybrid auction is no less than that of the English outcry auction with re-entry when the object being auctioned has some common value component. This means that for each bidder, the value of the object being auctioned depends on its value to other bidders. This is followed by an empirical study using a natural experiment where I compare the revenue generated by both auction formats.

I also test several ancillary propositions. First I propose that the revenue increases in both the above auction formats with the number of bidders. Second, I examine if convergence to the final price is likely to be faster for the hybrid auction format than that for the English outcry format. Third, I look at the impact of last minute bidding on each auction form. While last minute bidding is optimal for the bidders in both the auction formats, I argue that the resulting decrease in revenue (when compared to the case where there is no last minute bidding) is greater for the English outcry auction than that for the hybrid auction format. Finally I propose that on average, each bidder will bid fewer times in the hybrid auction format than in the English outcry format. The reason is that the bidders should bid higher in the hybrid auction than in the English outcry auction when they bid initially and thus the chances of revising their bid upwards should be lower.

2.2. Background and Research Issues

Auction theory comprises a large body of literature in economics beginning with Vickrey (1961), who outlined the second-price auction for the first time. He assumed that each bidder knows how much the object is worth to him/her (the reservation value) with certainty. However, this reservation value varies between bidders. This condition is known as the independent private value (IPV) condition. This situation is in contrast to auctions with a common value component where the valuation of each bidder may depend on the valuations of the other bidders. The concepts of independent private value (i.e., the case considered by Vickrey 1961) and common value were formalized within a game theoretic framework by Milgrom and Weber (1982), whose work has been the basis for subsequent auction theory research. Milgrom and Weber (1982) prove the revenue equivalence theorem where under the independent private value assumption; the revenue from the English outcry and the second-price sealed-bid auction formats is the same.

The basic results from the studies dealing with auctions are summarized by McAfee and McMillan (1987), Milgrom (1989) and more recently by Klemperer (1999). The key result is that for independent private value goods, there is revenue equivalence between the four auction forms (Dutch, first-price sealed-bid, second-price sealed-bid and English outcry). However, for goods with a common value component, the English outcry format generates the greatest revenue. I argue that in the online context, the hybrid auction format should generate even greater revenue than the English outcry format.

For this study, the main conclusion pertains to the situation where common value and buyer risk aversion hold. Both of these are relevant and highly realistic assumptions for online auctions. In the presence of risk aversion, bidders in the English outcry auction tend to bid closer to their reservation values (as they prefer smaller profits with greater certainty) and thus generate higher revenue for the seller. In contrast, for bidders in the second-price sealed-bid auction, it is optimal to bid the reservation price. However, there is no information gained regarding the valuations of the other bidders as each bidder bids only once. This is the central argument that leads to the result of Milgrom and Weber (1981, 1982) where they show that the English outcry auction format generates the greater expected revenue for the seller when compared to the second price sealed bid auction.

I examine the hybrid auction that can potentially generate higher revenues than the English outcry auction. I first theoretically establish the conditions under which the hybrid auction is likely to generate greater revenues than the English outcry auction in section 3.1. Having done this, I test this empirically by comparing the revenue generated by both formats for identical objects using a natural experiment.

2.2.1. Last Minute Bidding in Online Auctions

I look at last minute bidding in detail as this behavior is peculiar to online auctions and can play an important role in the final price realized by the auctioneer. Last minute bidding implies that bidders wait until the very end to place their bids. This is optimal for the bidder, as she does not reveal any information to the other bidders in the auction process. This means that other bidders do not get the opportunity to revise their valuations after inferring the bidder's valuation from his/her bid. Of course, this behavior would not impact revenue in auctions where bidders have independent private valuation, as the valuation of the bidders does not depend on the valuations of others. Last minute bidding behavior is peculiar to online auctions, because in a conventional English outcry auction, if there is a period of no bidding activity, the auction would end automatically.

The impact of last minute bidding in online auctions has been experimentally explored by Wilcox (2000) who finds that last minute bidding is optimal for the bidders in any ascending online auction, but reduces revenue for the seller. In an auction for a good having some common value component, it is optimal for the bidders to refrain from bidding and not reveal their valuation as this information could drive the competing bidders' valuation upwards. Wilcox (2000) found that bidders in eBay with greater experience tend to place their bids at the last moment. I build on Wilcox (2000) by comparing the role of last minute bidding for both the hybrid auction and the English outcry auction. One of the key aspects of this study is that I control for last minute bidding behavior and explain the reason for the different levels of influence exerted by this phenomenon on both auction formats.

Roth and Ockenfels (2000, 2002) develop an analytical model for explaining last minute bidding where they compare the bidding behavior in Amazon.com and eBay.com. They develop a game-theoretic framework where they consider two conflicting effects - the benefit and the risk of bidding at the last moment. The benefit is that the bidder does not reveal her private valuation to others. The cost is that the bidder risks her bid not being registered on time with the auction site. They find significant last minute bidding in both Amazon and eBay auction houses and report that last minute bidding is more pronounced for goods with a common value component than for independent private value goods. This result is intuitive, as the value of information revealed is greater in the case of goods with a common value component. I build on this by looking at the speed with which the outstanding price approaches the final price in both auction formats. This allows me to examine the impact of the information revealed by each bid in both the formats.

2.3. Propositions

2.3.1. Revenue Equivalence in Common Value Auctions

Analytical work in economics has shown that the expected revenue generated in the second-price sealed-bid and the English auction formats is the same in the case of independent private value objects (e.g. Milgrom and Weber 1982; Myerson 1981; Vickrey 1961). We can extend this result for the case of the hybrid auction and the English outcry formats. This is because in case of independent private value, bidders have an incentive to bid no more than their valuation. Specifically, in a hybrid auction, the bidders have no incentive to pay beyond their valuation and should thus never bid anything other than their valuations, and in an English auction bidders should bid up to their valuations. In both cases, the expected revenue realized would be the same as the valuation of the second highest bidder. The key here is that the bidding behavior of the other bidders does not affect the valuation of any bidder in any way.

However, in the case of goods with a common value component, the above equivalence may not hold. To see this, consider the auction of an object in a hybrid auction. The optimal bidding behavior in the hybrid auction (or any auction with free entry and exit) is to bid at the last moment. If a bidder chooses to bid anytime prior to the last moment, it is optimal to bid her valuation (or reservation price) at that point in time. This is because bidding lower only reduces the possibility of her bid being the high price and bidding higher is unnecessarily risky as she is faced with a possibility of paying an amount higher than her willingness to pay for the object.

It may appear that the bidders in the hybrid auction would bid low initially and then slowly increase their bid as in any English auction to either (a) distort the valuation (related to his/her own) of the other bidders or to (b) indulge in exploratory bidding. However, both are suboptimal. In the first case, the best way to distort is to not bid at all. Mathematically, if the valuation function of the bidders is of the form V (x_1,……..,x_n), n being the number of bidders, and x_n being the private signal of the n^{th} bidder, V (.) is non-decreasing in all of its arguments. Consequently, any bid would not decrease the valuation of any bidder. Also, any bid lower than the second highest bid in hybrid auction would not register at all. Thus, any such attempts at distortion are dominated by not bidding at all. In the case of exploratory bidding, due to the auction being second-price, the bidder gains little insight into the high valuation by bidding low values beforehand. Thus, the bidders wait till the last moment to place their bid. If they are unsure about their availability at the last moment, they would place their bid before the last moment during the duration of the auction.

If each bidder were better off bidding her valuation at that point in time, why would any bidder bid more than once in a hybrid auction? The answer has to be that the bidder has revised her valuation upwards upon observing the bidding process. Given this behavior, each bidder would not only bid more than once, but also bid his/her reservation price at that point in time. As such, from the earlier argument, it is easier for the bidders to assess the valuation of the other bidders because all bidders bid their reservation values for the object. The updating of the reservation value (due to the auction having a common value component) is also consequently more efficient when compared to the English outcry where it is assumed that the reservation values are inferred by looking at the bids. However, this updating is not possible in a second-price sealed-bid auction, as there is only one bid. Thus, the valuations of the other bidders are not revealed. Consequently, there is no updating as in the English outcry auction during the bidding process. Also, as there is only one bid (i.e., the reservation value of each bidder) in a second-price sealed-bid auction, there is no role for risk aversion in this auction type. The above discussion leads us to Proposition 1.1:[4]

> *Proposition 1.1*: If the object being sold has a common value component, the hybrid auction format generates higher revenue compared to the English outcry format with re-entry.

2.3.2. Number of Bidders

The number of bidders is likely to have a positive effect on the final revenue realized (Riley and Samuelson 1981; Vickrey 1961; Wilson 1977). Such an effect should occur due to two reasons. First, when there are more bidders, it is more likely that some bidders will have a greater valuation. Typically, the bidder valuations are modeled as drawn from a uniform distribution between the lowest and the highest possible value for the object (e.g., Milgrom and Weber 1982). Thus, probabilistically, as the number of bidders grows, so does the likelihood of some bidder's valuation being at the higher end of the distribution. Intuitively, the result should hold for any distribution as more bidders would increase the chances of a higher realization for the greatest valuation. Second, with risk aversion, the bidders recognize this fact and bid closer to their own valuation. Thus I propose:

> *Proposition 1.2:* All else being equal, the prices realized in the hybrid auction and English outcry auctions increase with the number of bidders.

2.3.3. Convergence to Final Price

From the discussion following proposition 1.1, we can see that it is easier for the bidders to update their valuation upwards in the hybrid auction format versus the English outcry format. Such bidding behavior allows rapid updating of the valuation by each bidder when compared to a situation where the bidder only observes incrementally increasing bids as in an English outcry

[4] I theoretically derive the conditions required for Proposition 1 in appendix A. In appendix A, I also derive conditions where the revenue from both formats should be the same. Specifically, I show that the expected revenue should be the same when there is no last minute bidding.

auction. Thus, it is likely that it would require fewer bids to push the outstanding price in the hybrid auction to the final price than in an English outcry auction. For example, if the second highest bidder's valuation were $3500, it may take many rounds of bidding for the outstanding price to reach $3500 in an English outcry auction. However, in the hybrid auction, this would be reached in the second round, as the bidders would bid their valuation at any point in time. Thus, we expect that the speed of convergence of the bid levels to the final price realized should be greater in the hybrid auction format followed by eBay. Formally:

> *Proposition 1.3:* The rate of convergence of the outstanding bid levels to the final price realized is greater for the hybrid auction format as compared to the English outcry format.

2.3.4. Impact of Last Minute Bidding

It is optimal for bidders in any ascending online auction to bid at the last minute. However, I argue that this effect is more pronounced in the case of English outcry than for the hybrid auction. Consider first the extreme case where all the bidders bid at the last minute (pure last minute bidding case). In the case of the English outcry auction, since the highest bidder pays the amount she bids and everybody can place only one bid, the situation is the same as in the first-price sealed-bid auction. On the other hand, in the hybrid auction if everyone bids only once at the last minute and the amount paid is the second highest bid, the situation is the same as in a second price sealed bid auction.

It has been shown analytically in Milgrom and Weber (1982) that in the presence of a common value component, the second price sealed bid auction generates higher expected revenue than the first price sealed bid auction. In our context, this means that the hybrid auction should generate greater revenues than the English outcry auction in the presence of common value and when every bidder bids at the last minute. When all bidders do not bid at the last minute, last minute bidding should decrease the revenue from English outcry. This is because, with fewer bids to condition upon, there is a smaller chance for any bidder to revise her valuation upwards. This effect is less intense for the hybrid auction, as even with few bids in the duration of the auction; the outstanding price is already quite high. In other words, in an online context last minute bidding should have a smaller influence on the hybrid auction than in English outcry auctions. This leads us to:

> *Proposition 1.4:* The presence of last minute bidding decreases the revenues from the English outcry and hybrid auction formats. However, the decrease is greater for the English outcry than for the hybrid auction.

2.3.5. Number of Bids per Bidder

As discussed earlier, the need for multiple bids by any bidder arises in the hybrid auction only if her reservation value has changed upwards. In contrast, this would not happen many times in the English outcry auction because there would be less opportunity for the bidders to revise their valuation. Thus the number of bids per bidder should be smaller in hybrid auction than in English outcry. To further clarify this argument, consider the car auction. Suppose Bidder A is uncertain only about the quality of the tires. If the tires were old, she would be willing to pay $4500. However, if the tires were new, she would pay $5000. Since Bidder A is risk averse, in a hybrid auction, s/he bids $4500. Now s/he spots another bidder (Bidder B), who knows the true quality of the tires. S/he knows that if B bids $4500, the tires would be of good quality. Upon seeing this bid of B (or discovering that her own bid is the second highest), Bidder A is then convinced that the true value of the car is $5000 and bids $5000. However, notice that such a process would take longer in English outcry because in English outcry, Bidder A would reach the price of $4500 only after many rounds of bidding, because Bidder A would not want to bid his/her reservation price to get the car. Thus, it would require a greater number of bids from A and B. Thus:

> *Proposition 1.5:* The number of bids made by any bidder will be smaller in the hybrid auction than in English outcry.

2.4. Research Setting

To test the propositions, I compare bidding on eBay.com and Yahoo.com. Both are very successful online-auction sites, though they use different formats. EBay has the same format as the hybrid auction and Yahoo.com has the same format as an English outcry auction with finite time duration. Before moving ahead, I look at some of the rules of the online counterparts of the hybrid auction and English outcry formats – Yahoo and eBay auctions.

In Yahoo, the bidders can place bids on the object for sale any number of times within the finite auction duration. The object then goes to the highest bidder at the end of the auction at the price bid by him. These characteristics make Yahoo an English auction with free entry. EBay is a format that has not been analyzed theoretically and due to arguments presented earlier should produce greater revenue than the English auction. In eBay, the bidders can place bids for the object any number of times during the course of the auction, but the bidders at any point in time only know the second highest bid. These are the characteristics of the hybrid auction as discussed earlier. Thus, eBay has the same format as a hybrid auction. Recall that the hybrid auction was defined earlier as an auction that combines the properties of the English outcry and the second-price sealed-bid auction formats. Here, the *second highest bid* and *the name of the highest bidder* are known at any point in time.

It should be noted that there are some special rules governing the two sites apart from the "regular" rules for English outcry and hybrid auctions. In both Yahoo and eBay auctions, the seller may specify a "buy price" and any bidder may claim the object for the buy price by bidding the buy price and the auction ends immediately with the bidder paying the buy price to the seller. Another special rule in Yahoo is the facility of proxy bidding for the players. Here, each player can specify the high bid till which she would be willing to compete given the current situation (in the bidding process) so that she need not place a fresh bid every time any other bidder tops her bid by the given bid increment. The auction site then automatically bids a higher price - outstanding price plus the bid increment by proxy. For both Yahoo and eBay, the auctions are of finite duration pre-specified by the seller. I include a dummy variable for the buy price to account for its effect. I code the auctions where the bidders exercise the option as a dummy variable similar to that in Yahoo.

2.5. Data and Analysis

2.5.1. Data Description

Auctions for 200 pairs of objects in Yahoo and eBay were compared such that the objects selected were identical in form and with respect to the promotional features. For example in eBay, if a photograph of the laptop accompanies the auction of a Dell™ laptop, care was taken that the auction for the same Dell™ laptop in Yahoo carried the same photograph and the same hardware and software configuration. I also ensured that the auctions took place at the same period. Specifically, the auctions included in the data were such that the duration of auction for the object in both sites overlapped. In the above example it means that if the auction for the laptop took place from August 23-August 30 in eBay, then the auction for the same laptop with the same features in Yahoo was 'alive' in at least one day of the above period (for example from August 17- 24).

Descriptive data from the auctions is shown in Table 2, in which three important comparisons are evident. First, there is no statistical difference between the auction duration in eBay and Yahoo. The average duration for eBay is 5.63 and for Yahoo it is 5.67. Second, the average number of bidders is slightly greater in eBay (7.00) than in Yahoo (5.26). I account for this in the analysis by including separate variables for these. Finally, last minute bidding is somewhat more prevalent in eBay (88 of the 200 auctions) than in Yahoo (62 of the 200 auctions).

…………………………………..Insert Table 2 about here ………………………………………

2.5.2. Variable Description

2.5.2.1. Presence or Absence of Common Value (COMM)

I infer the presence of common value for eBay auctions by examining the bid history for multiple bids made by two or more bidders in the course of the auction. If multiple bids are observed, then common value is inferred, as multiple bids in second-price auctions such as eBay indicate that the valuation of the object has changed for the particular individual. If common value was present, this dummy variable was coded as 1, otherwise 0.[5]

2.5.2.2. Presence of Jump Bidding (JUMP)

Jump bidding refers to the phenomenon where a bidder places a bid much higher than the existing outstanding bid when there is no clear compulsion to do so. For example, consider the auction of a car. In the English outcry format, if the current bid were $3000, with a bid increment (specified by the seller) of $200, a bid of $6000 (as opposed to say $3200 or $3300) would be a jump bid. Avery (1998) demonstrated analytically that preemptive jump bidding by one of the bidders is likely to reduce the final price realized in an English auction. Jump bidding signals to the other bidders that engaging in a bidding war would result in winner's curse for the other

[5] I also tried a coding scheme where extent of common value was coded as $\frac{\#\,Bidders}{\#\,Bids}$. The results were unchanged.

bidder. Note that this is not applicable in the case of the hybrid auction (or any second-price auction) where the jump bid is not observed. This takes away the advantage the jump bidder may be able to extract through such tactics. Thus, while testing the hypotheses, we need to control for the effect of jump bidding.

I infer jump bidding in Yahoo if any bid exceeded the previous high bid by 30% of the *final sale price*. This is a conservative coding scheme. For example, consider a Rolex Watch being sold in Yahoo with no reserve price. If the current high (outstanding) bid is $20 with a minimum bid increment of $5, a bid of $50 is a jump bid in a strict sense, but considering that the approximate value of the new watch would be much higher (say $1500) such a jump bid does not act as a preemptive jump bid in the spirit of Avery (1998). I also tried 20% and 40% cutoff levels and the results were unchanged. Because the highest bid is not observed in a hybrid auction (i.e., eBay), it is not possible for the bidder to signal aggression using pre-emptive jump bidding in the spirit of Avery (1998). Thus, I do not consider jump bidding for eBay.

2.5.2.3. Buy Price (BUYPRICE (Y) and BUYNOW (E))

This is a dummy variable that indicates whether the object was sold using the buy price option in Yahoo or eBay. For some auctions in Yahoo (eBay), the seller offers a "Buy-it-Now-Price" option so that any bidder bidding the buy price gets the object and immediately the auction ends. For example, if a laptop is being auctioned and the current price is $35 and the buy price is $500, any bidder can claim the object at $500 ending the auction immediately.[6]

2.5.2.4. First Bid/Reserve price (RESERVE (E) and RESERVE (Y))

The presence of a reserve price might affect the final selling price in the auction (McAfee and Vincent 1992; Milgrom and Weber 1982; Riley and Samuelson 1981). It is also intuitive to

[6] While coding jump bids in Yahoo, we excluded objects bought at the buy price. Instead of being a jump bid, this is deemed to be preemptive buying and thus is not coded as a jump bid.

see that a reserve price may automatically provide a higher start for the auction. Additionally, behavioral considerations may actually impact the choice of reserve price set by the auctioneer (Greenleaf 2004). I control for this variable by including the reserve price or the price at which the auction begins for Yahoo. In eBay, the reserve price is unknown to the bidders. However, the seller sets a minimum first bid which is known to the bidders and the role of this first bid is the same as that of the reserve price. I thus use the value of the stipulated first bid for eBay as the reserve.

2.5.2.5. Number of Bidders (BIDS (E) and BIDS (Y))

Proposition 1.2 predicts that the number of bidders will positively affect the final price realized in an auction. In the model, BIDS (E) and BIDS (Y) represent the number of bidders in eBay and Yahoo, respectively.

2.5.2.6. Auction Duration (DUR (E) and DUR (Y))

One implication of proposition 1.3 is that the duration of the auction will positively affect the revenue realized in the auction. This is because given any point in the auction duration, the price realized as a proportion of the final price would be greater for eBay than for Yahoo! Thus, any increase in the auction duration is likely to benefit more to the seller in Yahoo! than in eBay. Further, the auction duration is an important control variable for testing proposition 1.1. These are coded as DUR (E) and DUR (Y) and are the duration of the auction in days for eBay and Yahoo, respectively.

2.5.2.7 Last Minute Bidding (LAST (E) and LAST (Y))

Per Proposition 1.4, last minute bidding will adversely affect the revenue realized in both hybrid auction and English outcry auctions, more so for the English outcry format. Thus, I include dummy variables to represent last minute bidding. If there was a bid placed by any bidder in the very last minute of the auction, the dummy variable was coded as 1; otherwise it was coded as 0. LAST (E) and LAST (Y) represent these dummy variables for eBay and Yahoo, respectively.

2.6. Model Specification and Estimation

In this section I discuss the model specification and estimation. I first present the test used for the first (main) proposition in detail. The tests for the remaining propositions are presented later in this section. I test the first proposition using seemingly unrelated regression (Zellner 1962). I use seemingly unrelated regression (SUR) because we need to isolate the effect of the common value condition on the revenue realized. In order to do this, I account for the effect of the variables common to eBay and Yahoo by including them in the system of equations that I estimate. Further, while I infer the presence of common value in eBay, there is no such indicator for Yahoo. For testing, I model the prices realized as follows:

(1a) $EPrice=\alpha_1+b_1DUR\ (E)+b_2BIDS\ (E)+b_3LAST\ (E)+b_4RESERVE\ (E)+b_5\ BUYNOW(E)+b_6COMM+\varepsilon$

(1b) $YPrice=\alpha_2+\gamma_1DUR\ (Y)+\gamma_2BIDS\ (Y)+\gamma_3LAST\ (Y)+\gamma_4RESERVE\ (Y)+\gamma_5JUMP+\gamma_6BUYPRICE(Y)+\varepsilon$

where

- EPrice is the price realized for the object in eBay
- YPrice is the price realized in Yahoo
- DUR(E) and DUR(Y) represent the auction duration in eBay and Yahoo respectively in days
- BIDS(E) and BIDS(Y) are the number of bidders in eBay and Yahoo

- LAST(E) and LAST(Y) indicate the presence of last minute bidding in eBay and Yahoo respectively
- RESERVE(E) and RESERVE (Y) represent the stipulated first bid in eBay and the Reserve price set in Yahoo respectively
- BUYPRICE(Y) and BUYNOW(E) take the value 1 if the Buy price (Buy it Now) option in Yahoo (eBay) is exercised
- COMM represents the inferred presence of Common Valuation in eBay
- JUMP indicates the presence of Jump bidding in the Yahoo auction

I estimate two more models that relate the progression of price in the auction to the time elapsed in the auction. First, I formulate the outstanding price at any point in time as a quadratic function of the time elapsed. I call this Equation 2 and estimate the following regressions:

(2a) $EPrice=\gamma_1 t + \gamma_2 t^2 + \varepsilon$

(2b) $YPrice=\lambda_1 t + \lambda_2 t^2 + \varepsilon$

In the above, the coefficients γ_2 and λ_2 capture the relative speed with which the final price is reached and *t* represents the fraction of the *effective* time elapsed in the auction. This model is used to test Proposition 1.3, which predicts that increase in revenue due to increased auction duration is greater for the English outcry auction. In our context, I expect the prices in eBay price to converge more rapidly than in Yahoo. That is, we expect to see $\gamma_2 < \lambda_2$ and $\gamma_1 > \lambda_1$. The last model that I estimate is Model 3 that is similar to Model 2 but is used for testing Proposition 1.4. This is described below.

(3a) $EPrice' = \gamma'_1 t + \gamma'_2 t^2 + \varepsilon$

(3b) $YPrice' = \lambda'_1 t + \lambda'_2 t^2 + \varepsilon$

In model 3, EPrice' and YPrice' represent the prices in the *final stages* of the auction. To do this, I look at the bidding data after 80% of the auction duration is past. For example, if the auction duration is 5 days, the data considered for Model 3 is the last day only. Thus, I pay closer attention to the bidding behavior in the final stages of the auction. Consistent with Proposition 1.4 and the assumptions underlying Proposition 1.1, I expect both $\gamma'_1 > \lambda'_1$ and $\gamma'_2 < \lambda'_2$.[7]

I operationalize the variables in Models 2 and 3 as follows. In model 2, I normalize the values of price and time so that the data across all the auctions is comparable. I do this by specifying the price at any point in time as a fraction of the final price. Thus I also account for the role of reserve price, because it represents an artificially high starting point for the bids and therefore the ask price. To explain the adjustment, if the total duration of the auction is T and the first bid is placed at $t_0<T$, then, the effective auction duration=$T-t_0$. Now, if the final price realized is P and denoting the reserve price by R, the effective ask price is taken to be = $\beta_t = (b_t - R)/(P-R)$ where

- Bid placed at time $t \in [0, T]$: b_t
- Reserve Price: R
- First bid: b_0

Thus, I adjust the first bid so that it reflects the increase over the reserve price. Therefore, I now track the increase in the ask price net of the reserve. This ensures that all the auctions can be compared irrespective of the reserve price. Model 2 estimates how fast the outstanding ask price converges to the final selling price. I take the first bid as the starting point by considering the auction to have begun only after the first bid is placed. Thus, as shown above, the *effective*

[7] We conducted the above regressions with cutoffs where 70% and 90% of the auction duration was over and the results were identical.

auction duration is the time left in the auction after the first bid is placed. Thus, if the total number of bids is K, I can index the time of each bid b_t by the bid number $k \in \{1....K\}$ as

$b_t = \tau_t = (t(k+1)-t_0)/(T-t_0)$. When t=T, $\tau_T=1$, $b_T=P$ and $\beta_T=1$ and where, $b_t \in [0,1]$ and $\tau_t \in [0,1]$.

This normalization is done for each auction and the resulting τ_t and β_t form the dataset for the regression estimated in Model 2. A similar exercise achieves the required normalization for Model 3. The only difference is that in Model 3, the price at the time when four fifth (as opposed to initial time) of the auction is over is set to be zero and the final price is set to be 1. I then regress τ_t against β_t for all auctions. In Models 2 and 3, the intercept is set to zero as the first bid corresponds to the reserve price. In Model 2, $\gamma_2 < \lambda_2$ and $\gamma_1 > \lambda_1$ would imply that the convergence to the final price is more rapid. Model 2 is relevant to the propositions in two ways. First, I assume in Proposition 1.1 that each bidder bids his/her valuation in the hybrid auction and thus, multiple bidding by any bidder reveals the presence of common value. Greater concavity in Model 2a compared to 2b will support this thesis, as it implies that bids move towards the final value more rapidly in the hybrid auction than in the English outcry auction. Second, relevant to Proposition 1.2, $\gamma_2 < \lambda_2$ and $\gamma_1 > \lambda_1$ would show that the scope for increase in revenue with increased auction duration is smaller for the hybrid auction than for the English outcry auction. Finally, Model 2 with the parameters in the expected direction offers indirect evidence for Proposition 1.5 as greater concavity (or lesser convexity) in Model 2a compared to 2b will indicate that fewer bids are required to reach the final price in eBay than in Yahoo.

Model 3 tests proposition 1.4 in the following manner. In model 3, $\gamma'_1 > \lambda'_1$ and $\gamma'_2 < \lambda'_2$ implies that last minute bidding has a lower impact on eBay than for Yahoo. The inequalities

$\gamma'_1 > \lambda'_1$ and $\gamma'_2 < \lambda'_2$ would then imply that a greater proportion of the final price would be reached in eBay and in Yahoo. In other words, as we move towards the end of the auction, the difference between the final price realized and the outstanding bid price is lower in the case of eBay then for Yahoo. Now consider a bidder seeing the outstanding price and bidding in the last minute in Yahoo. The situation faced by the bidder is the same as in the first price sealed bid auction with a known reserve price and thus a low value of the reserve lowers the final bid when compared to a situation where the reserve is higher (Milgrom and Weber 1982). A greater number of bidders bidding at the last moment imply a lower outstanding price at the final minute. This then implies that the final price realized is thus lowered as more bidders bid at the last minute. Summarizing the discussion of the tests and the models developed, Table 3 shows the various restrictions and the propositions tested along with the relevant results observed.

…..............................Insert Table 3 about here…...................................

2.7. Results

I estimate a series of restricted models to test our propositions. These are detailed in Table 3. Proposition 1.1 predicts that revenue from the hybrid auction will be greater than that from the English outcry format. I test this proposition using Models 1a and 1b. First, I estimate a SUR between Models 1a and 1b while ignoring the variable COMM and test the restriction $\alpha_1 = \alpha_2$. I then re-estimate the SUR after including COMM and test the restriction $\alpha_1 + b_1 = \alpha_2'$. Here the variable α_2 refers to the intercept in Model 1b while estimating the SUR by *ignoring* the variable COMM in Model 1a, and α_2' refers to the intercept in Model 1b while estimating the SUR *after* including COMM in Model 1a.

I do the estimation described above using SUR where the coefficients for duration, number of bidders, buy price, reserve price and last minute bidding are restricted to be equal. In other words, the restriction, $\alpha_1 = \alpha_2$ is tested while restricting $b_2=\gamma_2$ (Number of Bidders), $b_3=\gamma_3$ (last minute bidding), $b_4=\gamma_4$ (reserve price), $b_5=\gamma_5$ (exercise of the buy price option) and, $b_6=\gamma_6$ (auction duration). These restrictions allow us to see the contribution of common value while keeping the other factors constant across both auction formats. Following this, I sequentially relax the above restrictions and repeat the estimation till I reach the case where no parameter is restricted to be equal across the system (the unrestricted case). Proposition 1 would be supported if the parameter corresponding to COMM(b_1) is positive and significant. I also estimate models 1a and 1b without including the variable COMM and contrast the results. This is done because there is no proxy for common value for yahoo auctions. Thus, the alternative specification (without including COMM) lets us see if there is any meaningful contribution by adding the common value term for eBay. For example, if we find that the revenue from eBay is increased if COMM is excluded, the proposition would not be supported (this would be indicated if the F-Value of rejection of $\alpha_1=\alpha_2$ is greater than the F-Value of the rejection of $\alpha_1+b_1 =\alpha_2'$).

..……………………………..Insert Table 4 about here…...………………………………

Table 4 reads as follows. The columns referred by Test 1-Test 6 indicate relaxation of the equality constraints sequentially where Test 1 refers to the situation where four parameters (duration, last minute bidding, reserve and the use of the buy price facility) are restricted and Test 6 is the unrestricted case where no parameters are restricted to be equal across the two models. The F-values and the corresponding significance of the difference are shown in the last two rows.

In Table 4, the estimate for common value is positive and significant for eBay. As we would expect, the coefficient for reserve (in eBay, this is the stipulated minimum first bid) is positive. However, it is not significant when the common value variable is included. One possible reason is that the influence of common value is so strong that it overshadows the impact of the reserve price. Another possible reason is that though reserve price insures the seller against very low prices, it does little in yielding a high final price. Thus, I conclude that using a reserve price is optimal for Yahoo but the gains are modest considering the final price realized. The estimates for duration, bids and last minute bidding have the expected signs for both auction forms except for the unrestricted case where the estimate for last minute bidding is positive in eBay.

The estimate for buy price for eBay is positive but again is not significant. In Yahoo!, the estimates are not significant but the sign varies with the nature of restrictions imposed. As such, the decision of setting the buy price is non-trivial and has benefits for buyers and the seller as this adds flexibility for both the auctioneer and the bidder by allowing direct purchase. Since I use buy price only as a control variable, I do not study this further. Finally, the estimate for jump bidding is positive but not significant. Given the role of jump bidding as studied by Avery (1998), we would expect that jump bidding would depress expected revenue (as pre-emptive jump bidding discourages other bidders) and thus expect the estimate to be negative. However, the estimate is non-significant. Likely a bigger sample would yield interpretable results.

As seen from table 4, Proposition 1.1 is supported. This is because the estimate for COMM is positive and significant (at $p<.05$ for all cases except when all the variables common to eBay and Yahoo! are restricted to be equal when $p<.1$). Also as required, the value of $\alpha_1 + b_1$ is always statistically greater than $\alpha_2{}'$. For example in Test 2 (Column B_2), $289.07^{***}+51.28^{**} > 249.68^{***}$ ($F = 26.27$). I also see that α_1 is always statistically greater than α_2. For example, in Column B_1 $323.81^{***}>250.05^{***}$ ($F= 22.64$). Thus, both restrictions are rejected with the F Value being greater for the former restriction than for the latter. This implies that the difference between (α_1 +b_1) and $\alpha_2^{'}$ is larger than the difference between α_1 and α_2 in terms of F-value (25.70>22.95). These results show that revenue in the hybrid auction format is greater when compared to the English outcry format and the difference is greater if common value is included. I also repeat the above process by relaxing the restrictions on the coefficients for Yahoo and eBay and find that Proposition 1.1 holds.

Proposition 1.2 predicts that the prices realized in the hybrid auction and English outcry auctions will be higher with more bidders present. This proposition would be supported by positive coefficients for BIDS (E) and BIDS(Y). The coefficient is non-significant for eBay, but significant for Yahoo is. Thus, Proposition 1.2 is only partially supported.

……………………………………..Insert Table 5 about here ……………………………………

Proposition 1.3 states that the rate of convergence of the outstanding bid to the final price realized would be greater for the hybrid format (eBay) than for English outcry (Yahoo!). This proposition is tested using Model 2 and the results are shown in Table 5. Proposition 1.3 would be supported by a greater convexity in the case of Yahoo in Model 2. Models 2a and 2b in Table 5 (see Figure 1 also) support Proposition 1.3. Specifically, γ_1> λ_1 (0.64>0.45) and γ_2 < λ_2 (0.36<0.55).

…….…………………………Insert Figure 1 about here …………………………………..

Proposition 1.4 states that last minute bidding will have a negative impact on both eBay and Yahoo auctions but the impact will be smaller for eBay. The proposition would be supported by a greater degree of concavity (or lesser convexity) in Model 3a than in Model 3b. From Table 5, we see that this is indeed the case. Specifically, $\gamma^{'}_1$> $\lambda^{'}_1$ (0.35>0.02) and $\gamma^{'}_2$ < $\lambda^{'}_2$ (0.65<0.98). The intuition is made clear if we draw a line near the end of the auction duration in Figure 1. We see that in eBay, a greater outstanding ask price (as a percentage of the final price) is achieved in the final stages of the auction as compared to the Yahoo auction. This means that if the bidders do not bid at the last minute, the decrease in the final price would be smaller for eBay as compared to Yahoo.

Proposition 1.5 states that the number of bids made by any bidder is likely to be smaller in the hybrid auction than in English outcry. To test this proposition, I computed the ratio of the number of bids to the number of bidders for each auction in Yahoo and eBay separately. For our proposition to be supported, this ratio should be significantly greater for Yahoo than for eBay. I performed a one-tailed test of difference of means to examine the difference. The mean $\frac{Bids}{Bidders}$ proportion for eBay is 1.99 and it is 2.88 for Yahoo (Z=7.15, p<.01). Thus, I find that Proposition 1.5 is supported.[8]

2.8. Discussion

This study contributes to the existing literature in several ways. First, I analyzed an alternative auction format that is widely used and identify properties due to which it could yield higher revenues than the English outcry auction format. Second, I adopt a different approach from the predominant focus on independent private valued objects (Chakravarti et al. 2002; Laffont, Ossard, and Vuong 1995) in empirical work on auctions. One example of empirical research comparing English outcry and second-price sealed-bid auctions in a natural setting is the one by Lucking-Reiley (1999). However, this work confines itself to testing the revenue equivalence principle (for independent private value auctions) and is not concerned with online auctions. In contrast, I consider objects that have a common value component and control for a host of other characteristics peculiar to online auctions in a natural setting. Third, I propose that a

[8] For testing Proposition 5, we only had 162 observations. We had to discard some of the data due to missing bid information and the auctions where Buy price was exercised by the bidders, as this artificially reduces the number of bids.

second-price auction with a few modifications can dominate an English outcry auction. This is surprising, and implies that the eBay auction format merits further attention regarding its revenue properties.

I argue that eBay generates greater revenue than Yahoo or any other English auction under the conditions stated in Proposition 1.1. It would be interesting to see how eBay revenue compares with a slightly modified auction format like Amazon that takes away the advantage of last minute bidding. Roth and Ockenfels (2002) contrast the properties of eBay and Amazon auctions, but no study examines revenue equivalence in online auctions. However, it is difficult to imagine any auction setup that matches eBay in the sheer efficiency of information utilization by the bidders. Fourth, this is the first study that looks at revenue equivalence empirically (between the English outcry and the hybrid auction format) using the eBay auction format.

Fifth, I show how the hybrid auction works to mitigate the damaging effect (to the seller) of last minute bidding to the seller (Model 3). This is also seen in Figure 1: the outstanding price in the hybrid auction approaches the final price more rapidly than in the English outcry format. In an online auction, it is weakly dominant for any bidder to bid at the last moment because by doing so the bidder does not reveal his/her signal. If all bidders do the same, there is little opportunity for other bidders to update their valuation and this reduces the overall final price resulting to the seller. Roth and Ockenfels (2000, 2002) and Wilcox (2000) look at last minute bidding in auctions but do not relate it to the revenue realized. In contrast, I consider last minute bidding in a revenue equivalence context.

Finally, this study offers an interesting alternative (but not necessarily competing) explanation for the success of eBay. As we have seen, it is more likely that eBay fetches a better price than Yahoo, making it more attractive to potential sellers. This in turn drives greater demand because of increased choice for potential buyers in eBay (due to more sellers). Thus, I think that the greater supply in eBay generates its own higher demand, resulting in its popularity over Yahoo. Though other reasons such as first mover advantage (Cohen 2002) may also have benefited eBay, our results suggest that one reason for its continued popularity is fundamental to its auction mechanism. This is a key insight from our analysis.

Wolfstetter, Perry and Zamir (2000) look at another auction format with two rounds of second-price sealed-bid auctions where the second round has only the highest two bidders from the first round. With the constraint that they cannot bid lower than they bid earlier, they prove that this is equivalent to the English auction. The above result is interesting, as the format considered in Wolfstetter et al. (2000) is very similar to eBay. The only addition in eBay compared to the format they consider is that there are many such rounds of bidding with free entry and exit of all players at any time in the course of the auction. This gives eBay additional flexibility than the system proposed by Wolfstetter et al. (2000). It may be possible that it is this flexibility that results in the revenue realized from eBay higher than the format studied by Wolfstetter et al (2000).

It is also worthwhile dwelling on the results of Models 2 and 3. One may wonder why the price/time curves for Yahoo and eBay are so close. Looking at the bidding history of the various auctions, I feel that the following explanation is the most plausible. As already discussed, in any auction with free entry and exit, it is optimal for each bidder to bid at the last moment. If any bidder does choose to bid before that in the hybrid auction, s/he has nothing better to bid than his/her valuation at that point in time. However, this is not always the case and many bidders do resort to bidding incrementally above the ask price. Roth and Ockenfels (2002) refer to such behavior as “incremental bidding.” Thus, we can readily see that even if a few of the bidders engage in such behavior, the result is slower convergence of the bids to the final price. However, the presence of other bidders ensures that the final (higher) price is reached quickly (relative to an English auction where there is no incentive to bid one's own valuation). Also, in our study, I did not look at proxy bids that are a feature of the Yahoo auction. Proxy bidding in effect reduces the number of effective bids and thus increases the degree of concavity of price vs. time in an English outcry auction. Finally, the facility of proxy bidding in Yahoo auctions allows the bidders to bid higher than they would normally bid in an English outcry. In fact, if all bidders were to use proxy bidding in Yahoo, the yahoo auction format would be the same as the eBay auction format. Proxy bidding therefore increases the speed of convergence of bids to the final price in Yahoo making our test more conservative (as proxy bidding works in the opposite direction of our proposition).

2.9. Limitations and Future Research

While I took care to control for potential bias due to auction date, number of bidders, last minute bidding and specific auction rules, there are inevitably some limitations. First, I implicitly assume that the number of items available in a site for auction does not greatly impact the overall selling price achieved. This is because greater number of objects for sale implies greater number of buyers also. Thus, the increase in demand would be matched by increase in supply. However, I have taken care to see that there are as many bids as possible in each auction history analyzed to neutralize this effect.

Second, common value was coded as 0 or 1. This may be a bit drastic and a better measure may be one with a continuum of values between 0 and 1 to reflect extent of common value. However, it is unclear as to how this can be done objectively. As mentioned in footnote 2, I tried another measure by using the average number of bids placed by each bidder as a proxy for the extent of common value. Although our proposition holds in this case also, I feel that the theoretical justification of our assumptions for coding common value to be 0 or 1 is more robust given the broad nature of Proposition 1.1.

In conclusion, we see that in the presence of common values, the eBay auction tends to dominate the Yahoo auction. Such a study of bidding behavior can augment traditional test markets for arriving at the long-term price of the product for the marketer. This study could provide directions as to the possibility of increasing price over the long run with sustained usage. This is because the sustained usage would allow the users to reassess the value of the object. By identifying the categories/products (and the ranking of categories where the updating is more likely) where this is possible, the marketer can have useful inputs for long-term pricing. For example, if an object sells for a higher value in the eBay format than with Yahoo, then the marketer can infer the possibility of price increase after the introductory period. In an earlier study, Hoffman et al. (1993) look at auctions as a mechanism to replace test marketing in arriving at the price. This research offers another step in the same direction.

3. ESSAY 2: MULTI-OBJECT AUCTIONS OF COMPLEMENTS OR SUBSTITUTES: THE OPTIMALITY AND IMPLICATIONS OF BUNDLING VS. SEQUENCING

3.1. Introduction

Long regarded as the favored mode of exchange among connoisseurs and collectors of vintage wine and fine art, auctions have captured the imagination of the general public on the growing strength of online auction sites such as eBay and Yahoo. Revenues at eBay have jumped from \$86.1 Million in 1998[9] to \$1.53 Billion for the first two quarters of 2004 with a gross transaction value of \$16 Billion over the two quarters[10]. The volume of B2B auctions through exchanges such as Ariba and CommerceOne has been even more impressive; optimistic projections of the gross transaction value for 2004 in the US approach \$750 Billion (see Pekec and Rothkopf 2003; Sashi and O'Leary 2002).

These trends notwithstanding, auctions have received limited attention in the marketing literature. While the auction literature in economics is rich and diverse (see Klemperer 1999 for a review), even here the overwhelming focus has been on the auctions of *individual* objects. The few recent auction-related studies within marketing have delved into issues such as mechanism design and revenue implications in the context of such products as wine (Lynch and Ariely 2000), beef (Hoffman et al. 1993) and high value art (Greenleaf, Sinha, and Rao 1993). In contrast to these studies, we see many objects amenable to auctions as natural complements or substitutes of each other, and I find it entirely feasible to auction off a *bundle* of such objects. Examples would include telecast rights for the summer *and* winter Olympics, tickets for successive World Series baseball games, and a set of JFK's golf clubs. On the other hand, if a seller intends to auction such objects separately, it begs the questions whether a particular

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[9] eBay Profit and Loss Statement 1998
[10] eBay unaudited results http://investor.ebay.com/news/Q204/EBAY072104-139863.pdf and eBay.com.

auction sequence would be the best and what the revenue impact would be. Thus, the central issue of this study is whether, and under what conditions, should a revenue maximizing seller of a set of complements or substitutes auction them as a bundle or separately, in two sequential auctions.

To be sure, part of the motivation for this work comes from an emerging stream of research on B2B auctions involving multiple objects (e.g., Benoit and Krishna 2001; Chakraborty 2002; Levine 1997). While these articles do examine the relative attractiveness of sequential vs. simultaneous auctions for a seller of two or more products, this study builds on this stream by considering a broader gamut of product - and market-based variables: substitute, independent and complementary objects, asymmetry in valuations of the objects, and risk seeking propensity of the bidders. I see this orientation offering several fresh insights.

Conceptually, this study is motivated by the bundling literature that closely examines the interrelatedness (i.e., complementarity or substitutability) among product offerings (e.g., Guiltinan 1987). This research has defined two products as complements (or substitutes) when a consumer’s reservation price for the bundle of the two products is greater (or less) than the sum of the reservation prices for the two products taken individually (cf. Lewbel 1985; Venkatesh and Kamakura 2003). This literature suggests that the optimality of a particular bundling strategy depends on the number of offerings, the degree to which the products are substitutes or complements, and the level of marginal costs. (I will review the auction and bundling literatures in the next section and position the study and its contributions in relation to them.)

In this study I examine the multi-object auction problem in a second price, sealed bid format – from the standpoint of a revenue maximizing seller and address the following questions:

- Under what conditions (e.g., the degree of interrelatedness among products and the number of prospective bidders) should the portfolio of products be auctioned simultaneously - as a bundle, or sequentially - as individual objects?
- How should the seller modify the auctioning strategy if bidders are risk averse and not risk neutral?
- If the objects are valued differently by the market (e.g., a more valuable Super Bowl ticket plus a less valuable Bowl souvenir), should the higher valued object be auctioned first or second?

The bidders in my model are strategic. I address the following questions that relate to them:

- How aggressively (i.e., how much more than the reservation price for the object in question) should one bid? How would this strategy differ for complements and substitutes, and in the presence of value asymmetry and bidders' risk aversion?

My model yields closed form, pure strategy Nash equilibria of the bidders' strategies when the objects are offered separately or as a bundle. While these results suffice to answer the bidder-related questions, I rely on related simulation of a large number of auctions to infer the strategic guidelines for the seller. I find that the domain of optimality of auctioning a bundle of complements or substitutes is negatively related to the available pool of bidders. The optimality of bundling is not a given for all complements: even with a few bidders (at least four), sequential auctions are revenue maximizing for weak to moderate complements. In such cases, the seller's gain from the cascading effect of competition in sequential auctions can offset that from reduced buyer heterogeneity for the bundle. When the objects are asymmetric in their market value, it is revenue enhancing to auction the higher valued object first. The rationale is that the bidding is not only more aggressive for the first object but also positively related to magnitude of its perceived value.

It is optimal for all bidders to bid above their reservation prices in the first of sequential auctions of complements. Independent of the moderating factors I consider, the bid for the second object should be linked to the incremental value of this object and unrelated to the first bid. The bid for the bundle is straightforward and should equal the reservation price. For substitutes, on the other hand, it is best for the bidders to shade their bid downwards (bid less than their reservation price) for the first object. This is because the first round winner becomes less interested in the second object; the reduced competition increases the first round loser's odds of winning the second item.

The structure of the remainder of this paper is as follows. I position the study in section 2 in relation to representative articles on multi-object auctions and bundling. Next, I propose the model in section 3 and lay out five propositions in section 4. I then consider two key extensions of the main model in section 5 and conclude the paper with a discussion of the key findings, the study's limitations and future research directions.

3.2. Literature

As the motivation for our problem and modeling approach comes in part from the multi-object auctions literature and from the bundling literature on interrelated objects, I will draw attention to key previous studies and highlight the distinctive and complementary aspects of ours.

Studies on multi-object auctions have considered identical objects such as multiple bottles of vintage wine (e.g., Krishna and Rosenthal 1996; Lynch and Ariely 2000) as well as distinct objects (e.g., Avery and Hendershott 2000; Benoit and Krishna 2001). While the typical study has looked at independently valued objects (e.g., Chakraborty 2002; Palfrey 1983), recent exceptions have explored complements (cf. Benoit and Krishna 2001). The only substitutes considered are those that perfectly replace each other (e.g., Palfrey 1983). The Vickrey auction is the format of choice (e.g., Krishna and Rosenthal 1996; Rosenthal and Wang 1996). The seller's objective is typically to maximize revenues (e.g., Benoit and Krishna 2001; Chakraborty 2002) although auction efficiency has also been considered (e.g., Levin 1997;

Palfrey 1983). The articles are about evenly split between objects with independent private value (e.g., Krishna and Rosenthal 1996) and common value (e.g., Rosenthal and Wang 1996).

Along the above categorizing dimensions, I have chosen the following as some of our model's core characteristics:

- Multiple bidders (≥ 2)
- Two non-identical objects, given our focus on the bundling decision as opposed to the quantity decision
- Independent private value, as our primary thrust is on bidders who are users and not resellers of the objects.

I summarize the key findings of these studies and position this study in Table 6.

……………………………..Insert Table 6 about here…..……………………………

As discussed here, the distinguishing aspects of this study come from the consideration of the impact of the following additional characteristics:

- A *degree* of interrelatedness among the products that could vary from strong substitutability to strong complementarity
- The *interaction* between the above and the number of bidders
- Risk propensity of the bidders: risk neutral or risk averse; the latter might be more relevant in some B2C auction settings that attract infrequent bidders
- Both symmetric and asymmetric market valuations; the latter is intended to capture scenarios in which one of the objects is seen to be of "lesser" value (e.g., a DVD, as compared to a DVD player); this draws attention to the question whether a particular sequence matters to the seller.

I will highlight and elaborate on the strategic significance of the above in §3.4 and §3.5.

The literature on bundling contrasts the appeal of unbundling (i.e., pure components), pure bundling (akin to simultaneous sales) and mixed bundling strategies (i.e., sales of the objects individually and in combination). Bundling dominates unbundling for substitutes and complements when the marginal costs are negligible (cf. Bakos and Brynjolfsson 1999). The result is reversed when marginal costs are moderate relative to the market's willingness to pay, whereas – when the marginal costs are high – unbundling (or bundling) is more profitable for substitutes (or complements) (cf. Venkatesh and Kamakura 2003). Marginal costs are arguably absent in an auction setting as the seller already possesses the objects. So is bundling the revenue maximizing strategy in an auction setting? It is not always so, as we will see later.

Insights from extant bundling research do not readily extend to auctions for several reasons. First, traditional bundling is a form of second-degree price discrimination whereas auctions arguably map on to price discrimination of the more potent first-degree. Second, whereas the traditional bundling results apply typically for a setting devoid of supply constraints, limited supply is the hallmark of the auction problem. The restricted supply also means that the strategy of mixed bundling that balances penetration and profits might be irrelevant in an auctioning context. Third, under traditional bundling with a monopolist, the game structure is simpler because the seller only has to ensure that the bundle or the components meet the buyer's incentive compatibility and individual rationality constraints. Under auctions, buyers must act strategically to balance their individual valuations and the anticipated bidding behavior of competing bidders so as to maximize expected surplus. In sum, I see the problem of our study at the interface of bundling and standard auction literature, and contributing to both streams.

3.3. The Model

3.3.1. The Product Market

The seller in our model is also the auctioneer. S/he has one unit each of two objects '*A*' and '*B*' and seeks to maximize the expected revenue from selling them.[11] The objects are hard for interested consumers to find outside of the auction. The two-object formulation is consistent

[11] Maximizing expected revenue and expected profit are equivalent in a typical auction setting because seller is already in possession of the two objects and so the costs that s/he incurred in procuring them are sunk.

with the setting in several extant articles on auctions (e.g., Avery and Hendershott 2000; Hausch 1986) and bundling (e.g., Adams and Yellen 1976; Schmalensee 1984). Our seller has chosen the second price sealed-bid Vickrey auction format following earlier precedents (e.g., Krishna and Rosenthal 1996; Chakraborty 2002). Also the format closely resembles those on eBay and Yahoo.[12] The seller has to decide whether to offer the objects in a single bundled lot or separately in two sequential auctions. This decision of the seller will be common knowledge before the auction begins.

Totally, N+1 potential bidders are interested in purchasing the objects at incentive compatible prices. This number is known exogenously as in Krishna and Rosenthal (1996) and Chakraborty (2002). The minimum size of the bidder pool is 2 – the auction is meaningless otherwise. The bidders have private values for the objects being auctioned, and are risk neutral in that they expect their surplus from the auctions to be non-negative. The bidders see the two auctioned objects as complements of one another. (I consider substitutes in §5.) The degree of complementarity is parameterized as θ such that, if any bidder j's reservation prices for objects A and B taken separately are R_{jA} and R_{jB}, and that for the bundle is R_{jAB}, then:

$$\theta = \left(\frac{R_{jAB} - (R_{jA} + R_{jB})}{R_{jA} + R_{jB}} \right) \tag{1}$$

For a given object pair AB, the bidders in our model perceive the same degree of complementarity θ – a common assumption in the literature (e.g., Bakos and Brynjolfsson 1999; Moorthy 1988; Venkatesh and Kamakura 2003). For two independently valued objects, θ is zero. The greater the synergistic effect of complementarity, higher the θ value. Consistent with

[12] For example, both these real world auction sites involve proxy bidding in which it is it is optimal to bid at the last moment (cf. Roth and Ockenfels 2000, Wilcox 2000). Thus, if all bidders bid at the last moment, the auctions reduce to the second-price sealed bid format.

McAfee and Vincent (1997) and Riley and Samuelson (1981) in the auction literature and Carbajo et al. (1990) and Seidmann (1991) on bundling, I assume that the reservation prices of the bidders for each object taken alone are independent draws from a uniform distribution in [0, V]. This also means that the two products are symmetric in value at the market level. Further, as I consider two distinct objects, each bidder's reservation prices R_{jA} and R_{jB} for the objects taken alone are independent of each other (as in Bakos and Brynjolfsson 1999). As each bidder's valuation of the bundle is $(1+\theta)$ times the sum of the reservation prices of the two objects taken alone, it follows that the bidders' reservation prices for the bundle follow a triangular distribution, with support $[0, 2V(1+\theta)]$ and mean $V(1+\theta)$. All of the above information at the market level is common knowledge *except* the specific, individual-level realizations of reservation prices R_{jA} and R_{jB}, which are known only to each individual bidder j; i.e., R_{jA} and R_{jB} are private value as in Palfrey (1983).

3.3.2. The Bidding Process

The seller decides first whether to auction the objects separately or as a bundle. After this decision, the buyers decide and place their bids. I derive the equilibrium strategies for the bidders in the subgames where the bidders make their choices and, in turn, infer the seller's optimal strategy.

When the seller auctions the objects as a bundle, the bidding decision is straightforward as the offering is a single – albeit composite – object. In such a situation, each bidder's best bid, denoted b_{jAB}, can be no better than her reservation price for the bundle. In other words, bidding one's reservation price is a weakly dominant strategy. This is because reservation prices are independent (i.e., one bidder's reservation price does not affect another's) and private, and in a second price sealed bid auction it is then optimal to bid the valuation (Milgrom and Weber 1982;

Vickrey 1961). The revenue is thus the amount of the second highest bid which, as before, is the second highest bidder's reservation price.

If the seller offers the objects in two separate and sequential auctions, the bidding decisions turn out to be considerably trickier. As additional notation, I denote bidder *j*'s bid for object *A* as b_{jA}. If *j* is the winner of the first auction, then her bid for object *B* is b_{jB}; otherwise it is b'_{jB}. The bidders are strategic while bidding on the first object. Any given bidder considers the following factors:

- There are four possible outcomes: (i) the bidder could win the first object and go on to win the second object as well; (ii) the bidder could win the first object but end up losing the second; (iii) the bidder could lose the first object and yet win the second; (iv) the bidder could lose both objects. The bidder tries to maximize one's expected surplus given these possibilities
- The winner of the first object could gain an upper hand in the eventual bid for the second object. This is because none of the other bidders has any chance of realizing the benefit of complementarity (as they can only win the second object at best). In other words, there is some incentive in the first auction to bid aggressively – that is, bid more than one's reservation price for the first object
- The incentive to bid aggressively on the first object must be tempered by the realization that the winning bidder could lose the second object and end up with a negative surplus at the end of the day. This could happen if, for example, one of the first round losers has a high reservation price for the second taken alone and makes a strong bid.

With the first bid over and the results announced, some of the above uncertainties are resolved. Each bidder knows if she won or lost the object and the price paid by the winner. The winner is aware of her surplus from the first object. She would bid on the second object so as not to bring down – and possibly increase – her surplus from the first. All the first round losers bid their reservation prices for the second object. Doing so is a weakly dominant strategy for these bidders for reasons discussed under the earlier case of auctioning the bundle. The winner of the second auction and the price paid by her are subsequently revealed.

The complete game as it unfolds is shown in Figure 2.

...................................Insert Figure2 about here…................................

The seller's original decision of whether to auction the objects as a bundle or sequentially as two separate auctions must be based on how the game unfolds at the bidders' level. The equilibrium bids of the sellers are a critical driver of the seller's decision. I derive the Nash equlibria by backward induction.

3.3.3. The Equilibrium Bids

For reasons discussed earlier, the equilibrium bids are straightforward when the objects are auctioned as a bundle, and given by:

<u>Result 1</u>: *The pure strategy Nash equilibrium bid of bidder j for bundle AB is* $R_{jAB}\ \forall j$.

This is the weakly dominant strategy in a second price sealed bid auction with independent private valuations and follows from Vickrey (1961, p. 20) for a single object.

I will devote the rest of this section to the equilibria under sequential auctions. As the winning bidder pays the equivalent of the second highest bid, each bidder *j* maximizes the expected surplus from the auction. The bidder's expected surplus[13] is given by:

$$\text{Expected Surplus of } j = \text{I} + \text{II} + \text{III} \tag{2}$$

$$\text{where}\quad \text{I} = \begin{Bmatrix}\text{Probability of}\\ \text{Winning Both Objects}\end{Bmatrix} \times \{\text{Surplus from the Bundle}\}, \tag{3}$$

$$\text{II} = \begin{Bmatrix}\text{Probability of}\\ \text{Winning Object A}\\ \text{and Losing Object B}\end{Bmatrix} \times \{\text{Surplus from Object A}\}, \text{ and} \tag{4}$$

$$\text{III} = \begin{Bmatrix}\text{Probability of}\\ \text{Losing Object A}\\ \text{and Winning Object B}\end{Bmatrix} \times \{\text{Surplus from Object B}\}. \tag{5}$$

[13] The formulation in equation (2) is equivalent to the backward induction rule of a bidder maximizing P(Winning A)×(Surplus from A) + P(Winning B | Won A) × (Incremental surplus) + P(Winning B | Lost A) × (Surplus from B).

In the above, the probability of bidder *j* winning (or losing) an object refers to the probability that her bid is greater (or less) than the highest of the remaining bids, and the surplus from the object contingent on winning it represents the difference between the bidder's reservation price for the object less the second highest bid for that object. It is also evident that losing both objects leads to a surplus of zero. Based on the notation developed thus far, equation (2) can be represented mathematically as:

$$
Surplus=\left[\begin{array}{l}
\left\{\begin{array}{c}\mathrm{P}(\mathrm{Max}\{b_{iA}\}<b_{jA})\\ \mathrm{P}(\mathrm{Max}\{b'_{iB}\}_{i\neq j}<b_{jB})\end{array}\right\}\times\left\{R_{jAB}-E\left(Max\{b_{iA}\}+Max\{b'_{iB}\}\left|\begin{array}{l}\mathrm{Max}\{b_{iA}\}<b_{jA}\\ \mathrm{Max}\{b'_{iB}\}_{i\neq j}<b_{jB}\end{array}\right.\right)\right\}\\[2ex]
+\left\{\begin{array}{c}\mathrm{P}(\mathrm{Max}\{b_{iA}\}<b_{jA})\\ \mathrm{P}(\mathrm{Max}\{b'_{iB}\}_{i\neq j}>b_{jB})\end{array}\right\}\times\left\{R_{jA}-E\left(Max\{b_{iA}\}\,\middle|\,\mathrm{Max}\{b_{iA}\}<b_{jA}\right)\right\}\\[2ex]
+\left\{\begin{array}{c}\mathrm{P}(\mathrm{Max}\{b_{iA}\}>b_{jA})\\ \mathrm{P}(\mathrm{Max}\{\mathrm{Max}\{b'_{iB}\}_{i\neq j,k},b_{kB}\}<b_{jB})\end{array}\right\}\times\left\{\mathrm{R}_{\mathrm{jB}}-E\left(\begin{array}{l}(\mathrm{Max}\{\mathrm{Max}\{b'_{iB}\}_{i\neq j,k},b_{kB}\}<b_{jB})\\ \left|(\mathrm{Max}\{\mathrm{Max}\{b'_{iB}\}_{i\neq j,k},b_{kB}\}<b_{jB})\right.\end{array}\right)\right\}
\end{array}\right] \tag{6}
$$

The three multiplicative pairs on the right hand side of equation (6) correspond to those of equations (3), (4) and (5).

To determine the equilibrium bid for A (i.e., the first of the auctioned objects), each bidder *j* must assess how her reservation prices for the objects (taken separately) and for the bundle measure up against specific thresholds. Let us note parenthetically that this is an analytical requirement intended to assess the cumulative probabilities correctly. The bidders move simultaneously with their first bid, and we have:

<u>Result 2</u>: The pure strategy Nash equilibrium bids of j for objects A and B are:

When $(R_{jAB}-R_{jA})\leq V$:

$$b_{jA}^{*}=\begin{cases} R_{jA}+\dfrac{\left(R_{jAB}-R_{jA}\right)^{N+1}}{(N+1)V^{N}}-\dfrac{R_{jB}^{N+2}}{2(N+2)V^{N+1}\theta(1+\theta)} & \text{when } 0\le R_{jB}\le \theta V \\ R_{jA}+\dfrac{\left(R_{jAB}-R_{jA}\right)^{N+1}}{(N+1)V^{N}}-\dfrac{2N(N+2)R_{jB}^{N+1}-(N+1)(N+2)\theta VR_{jB}^{N}+2(\theta V)^{N+1}}{2N(N+1)(N+2)V^{N}(1+\theta)} & \text{when } \theta V< R_{jB}\le V \end{cases}$$

When (R_{jAB}-R_{jA}) > V:

$$b_{jA}^{*}=\begin{cases} R_{jAB}-\left(\dfrac{N}{N+1}\right)V-\dfrac{R_{jB}^{N+2}}{2(N+2)V^{N+1}\theta(1+\theta)} & \text{when } R_{jB}<\theta V \\ R_{jAB}-\left(\dfrac{N}{N+1}\right)V-\dfrac{2N(N+2)R_{jB}^{N+1}-(N+1)(N+2)\theta VR_{jB}^{N}+2(\theta V)^{N+1}}{2N(N+1)(N+2)V^{N}(1+\theta)} & \text{when } \theta V\le R_{jB}\le V \end{cases}$$

In all cases: $b_{jB}^{*}=R_{jAB}-R_{jA}$; $b_{jB}^{\prime *}=R_{jB}$.

The proof is in Appendix C. While I will draw on the equilibrium to state our propositions in section 3.4, I attention to the following points on sequential bids:

- The first bid of each bidder is greater than her reservation price for the first object in all cases, except when the bidder's reservation price for both objects is zero. The extent of such aggressive bidding on part of a bidder is related to her reservation price levels for the two objects. I will further examine this tendency to bid aggressively in section 3.4.
- The equilibrium first bid is more involved than the second. The solution representing the first bid has three components. Under Case I, for example, the first component is simply the reservation price of the first object. The second component is tied to the value boost from complementarity. The third term relates to the reservation for the second object. I will draw on related comparative statics in section 3.4.

For a given combination of the degree of complementarity (θ) and the number of bidders (N+1), I will draw on Results 1 and 2 to address the bidders' problem. To address the seller's problem, I must compare the revenues from these two results. As Result 2 is rather messy, it is difficult to address the seller's questions in closed form. However, given the closed form equilibrium bids, I can infer the seller's decision by simulating a large number of auctions for each combination of (θ, N+1). I perform this analysis and present normative guidelines in the form of proposition in the following section.

3.4. Propositions from the Model

With the model and equilibrium bids under the auction of the bundle and sequential auctions in place, I begin to address the study's questions. This section will rely on the closed form equilibrium bids (from section 3) to address the bidders' questions and related simulation (discussed below) to address the seller's questions.

In our framework, the number of bidders (N+1) and the degree of complementarity (θ) are common knowledge at/before the start of an auction. For *each* combination (N+1, θ), I simulated and analyzed 30,000 sets of auctions, where each set proceeds as follows. First, from a uniform distribution [0, 100] where 100 (an arbitrary upper limit) represents V, I randomly draw the reservation prices R_{jA} and R_{jB} for objects A and B of each of the N+1 bidders. The bundle reservation price R_{jAB} is then $(1+\theta)(R_{jA} + R_{jB})$ for each bidder j ($j \in \{1, \ldots, j, \ldots N+1\}$). The bids for objects A and B corresponds to the equilibrium bids in Results 1 and 2. For the auction of the bundle AB, I identify the highest bidder as the winner and the second highest bid as the seller's revenue. For the two sequential auctions, the winner of A (or B) is the highest bidder of that object. The revenue to the seller from the sequential auctions is the sum of the second highest bids from the two auctions. From across the 30,000 auctions for each (N+1, θ), I determine the seller's *expected* (i.e., mean) revenues from the auction of the bundle and from the sequential auctions. Comparing them tells us which of the two formats is optimal for the seller for that (N+1, θ) combination. I repeat this process for different (N+1, θ), varying N+1 in steps of 1 from 2 to 11 and θ from 0 to 0.8. This analysis has been synthesized in the form of a phase diagram in Figure 3A, with the domains of optimality of sequential and bundled auctions clearly demarcated. (I will refer to Figure 3B in a later subsection 3.4.2.)

...…………………………..Insert Figure3 about here…...……………………………

I present the normative guidelines below in the form of five propositions. The first three in section 3.4.1 below relate to the main model that assumes risk neutral bidders, and based on the phase diagram in Figure 3A and the earlier equilibria. The other two propositions in section 3.4.2 consider the impact of bidders' risk aversion and are based on additional analysis.

3.4.1. Guidelines on the Optimal Auction Format and Bidding Strategies

Our first proposition examines whether and when the sequential auctions are better or worse for the seller than the auction of the bundle.

Proposition 2.1: (a) Sequential auctions are optimal for the seller for weak to moderate complements if there are at least a few bidders (specifically, four or more). Auction of the bundle is optimal for strong complements only.

(b) The greater the number of bidders, the wider the domain of optimality of sequential auctions.

Comments: The phase diagram in Figure 3 contains the exact thresholds of θ and N+1. I treat complementarity as "weak" if $0 < \theta \leq 0.2$, "moderate" if $0.2 < \theta \leq 0.4$, and "strong" if $\theta > 0.4$. (By our definition, for moderate complements, each bidder is willing to pay as much as 40% more than the sum of her standalone reservation prices of the objects).

It sounds clichéd to say that a pair of complements offers the most value when a buyer can have them both. Thus, "conventional wisdom" would suggest that a bundled auction would be the best for the seller as s/he could extract this greater value from the bidders. Proposition 2.1(a) clarifies that this intuition applies for strong complements only. In an auction setting, a potentially counterbalancing advantage in favor of sequential auctions arises from the pool of bidders and is tied to its size. With weak to moderate complements, for which the value boost is limited, the seller benefits from playing the bidders against each other in two competitive events.

The seller gains from the first auction by tapping the bidders' desire to bid somewhat aggressively on object *A* in anticipation of also clinching object *B* later; the seller then gains from the second auction by playing off the winning bidder from the first auction against those who have a high value for object *B* alone.

A larger bidder pool has two influences. First, it makes the first round bidding more aggressive as the danger of losing the first object (and the resulting benefit of complementarity) is higher. Second, as more bidders are likely to value object *B* highly, the first round winner has to maintain an aggressive stance on the second round bid. This chain of competition boosts the attractiveness of sequential auctions (Proposition 2.1(b)).

Our preliminary review of eBay suggests that a large number of auctions have bidders numbering in the single digits (see Hossain and Morgan 2003). The sensitivity of Proposition 2.1 over this range of bidders is a sign that our guidelines may be of much practical relevance.

Our next two propositions consider the bidders' point of view in the context of sequential auctions. (Per Result 1, the bidding decision in the bundled auction is straightforward.)

Proposition 2.2: With sequential auctions, it is optimal to bid aggressively (i.e., more than the reservation price) on the first object. The degree of aggressive bidding is increasing in the reservation price for object A; i.e., $\frac{\partial}{\partial R_{jA}}\left(b - R_{jA}\right) > 0.$

Comments: The proof is in Appendix C. The first part of the proposition is easier to see – if winning the first object opens up the prospect of tapping the value boost of complementarity later, then the bidder should justify trying harder on the first object. The second part of the proposition means that the extent of aggression is climbing at an increasing rate in relation to the increase in the reservation price of the first object. A rationale is that the higher value of the first object is augmented by the effect of complementarity, encouraging a more aggressive first bid.

Proposition 2.3: With sequential auctions:

(a) *The first round winner's second round bid is (i) greater than her reservation price for the second object and (ii) unrelated to the magnitude of her first bid and to the number of competing bidders.*

(b) *First round losers' second round bids are equal to their respective reservation prices for the second object.*

Comments: The proof is in Appendix C. The significant point from part 3 (a(i)) is that the aggressive bidding on the first round (i.e., bidding more than one's reservation for the object in question) persists on the second round insofar as the first round winner is concerned. This is driven by the danger of losing the second object, which could then negate the advantage of winning the first object. Part (b) of the proposition highlights that first round losers are the ones who bid in the most neutral fashion for the second object. They have lost their chance of benefiting from complementarity and seek only to earn a non-negative surplus from the second auction. This is not to say that the bidding for the second object is tepid. The first round winner must contend with the prospect of a strong bid from those with a high reservation price for the second object *B*. Note that such a prospect is diminished when the bidders participate in a bundled auction in which bundling would work to decrease the heterogeneity among bidders, e.g., the effect of a high reservation price for object *B* could be neutralized by a low reservation price for object A.

Part a(ii) of the proposition formalizes an interesting result: the first round winner's second round bid is $b_{jB}^{*} = R_{jAB} - R_{jA}$ (i.e., independent of the first bid and the number of other bidders). As R_{jA} is the reservation price for A, the bid represents the gross increase in reservation price from owning the complement (i.e., the reservation price of B taken alone plus the value of complementarity). That is, although the winner's first round bid could potentially have left her with a negative surplus going into the second auction, she bids the most she could on the second object with the expectation that the second-price mechanism will win back the surplus.

It turns out in the equilibrium under sequential auctions that the *first round winner's combined total bid from the two rounds put together exceeds her reservation price for the bundle (i.e.,* $b^*_{jA} + b^*_{jB} > R_{jAB}$*).* Recall from Result 1 in section 3 that with the auction of the bundle the total bid was only R_{jAB}. This is partly behind Proposition 1 discussed earlier.

I now turn to relaxing an underlying assumption of the main model.

3.4.2. Impact of Bidders' Risk Aversion on Optimal Decisions

One could contend that our original assumption of risk neutrality on part of the bidders, which causes them to maximize expected surplus, is a driver of the somewhat aggressive bidding behavior in sequential auctions as outlined earlier. That is, bidders might be willing to risk a loss on the first round in anticipation of an offsetting gain on the second round.

While bidders' risk neutrality is the typical assumption in extant articles on multi-object auctions (e.g., Chakraborty 2002; Krishna and Rosenthal 1996; Levine 1997), I examine the impact of risk aversion here given its relevance in some choice settings. I first define a bidder to be *maximally risk averse* if she never wants to incur a negative surplus. Examining risk aversion is relevant only for sequential auctions as a bidder *must* win the first object to have any chance of tapping the benefit of complementarity. Under the bundled auction a bidder wins or loses both objects at her reservation price (i.e., with no risk of a negative surplus).

As before, a pure strategy Nash equilibrium exists in the bids for either object and is given in Appendix D. Related analysis of from additional simulation (carried out as before) yields the phase diagram in Figure 3B.

I now have the following proposition for the seller's optimal strategy when faced with risk averse bidders.

Proposition 2.4: In the presence of maximally risk averse bidders,

(a) The domain of optimality of sequential auctions shrinks relative to the risk neutral case. Sequential auctions are still optimal for weak to moderate complements if there at least a moderate number of bidders (i.e., at least eight).

(b) The domain of optimality of sequential auctions under bidder risk aversion approaches that under the risk neutrality when the number of bidders is large.

<u>Comments</u>: Refer to Figure 3(B) for proposition 2.4(a). Part (b) is proven analytically in Appendix D. While confirming the conjecture that risk aversion cuts down aggressive bidding, part (a) confirms that the optimality of bundled auction is not a given even in this case. What still lends supports to sequential auctions is the necessity on the bidders' part to win over two competitive rounds, instead of just one under the auction of the bundle. As before, cascading competition under sequential auctions adds to the driving influence of reduced buyer heterogeneity in the bundled auction. Part (b) makes the point that as the bidder pool is large, the increased effect of competition is sizable enough pushes the results closer to those under risk neutrality.

What then, is the optimal bidder strategy?

Proposition 2.5: Even with maximal risk aversion,

> *(a) It could be optimal for a bidder to bid more than one's reservation price for the first object on the first auction.*
>
> *(b) The bids for the second object of all bidders is identical to those under the risk neutral case.*

<u>Comments</u>: As shown in Appendix D, the equilibrium bids assuming maximal risk aversion are $b_{jA}^{*} = Max\{R_{jA}, R_{jAB} - V\}$ on the first round for all bidders *j*, meaning that the first bids could be greater than R_{jA}. As under the risk neutral case, $b_{jB}^{**} = R_{jAB} - R_{jA}$, the second round bid of the first round winner; and $b_{iB}^{'**} = R_{iB}$ for all first round losers. This proves the proposition. To explain part (a): As each bidder is aware that the bidders' reservation prices for object *B* taken alone fall below an upper limit *V*, keeping a reserve valuation of *V* (plus a trivial increment) from the first auction guarantees a win on the second auction. Thus, bidders may bid aggressively, up to ($R_{jAB} - V$), when the term is greater than R_{jA}, to increase their probability of winning the first object. To be sure, the extent of aggressiveness is less than that under the risk neutral case (i.e., $b_{jA}^{**} - R_{jA} < b_{jA}^{*} - R_{jA}$).

I find part (b) of proposition 2.5 interesting as it underscores the robustness of the result under risk neutrality. The rationale is that the second auction is indeed less risky from the bidders' point of view. After all, the first round losers do not gain from bidding more than their reservation prices. The first round winner is willing to bid the entire increase in reservation price from owning the complement B (i.e., $R_{jAB} - R_{jA}$), knowing fully well that she will not incur a loss.

In summary, we see that the decision to have a single (bundled) auction or sequential auctions for complements yields interesting and arguably counter intuitive insights.

3.5. Two Extensions: Impact of Value Asymmetry and Substitutability

Our main model and related analysis have focused on objects that are symmetric in their market level valuations (i.e., reservation prices for both are iid U[0, V]) and are either complements or independently valued objects. I relax these two assumptions in this section, first by considering asymmetry in valuation (3.5.1) and then by examining the case of substitutes (3.5.2). For the seller, asymmetry raises the important question of *sequencing*; i.e., which of the two objects should be auction first. The challenge with substitutes is that the winner of the first auction might lose interest in the second object and submit a weak bid for it, diluting the benefit of greater competition in sequential auctions.

3.5.1. Auctions of Asymmetrically Valued Objects

To recount an earlier example, a Super Bowl ticket that is valued highly by the target market and a related souvenir that is less valuable would make up an asymmetric pair. Bidders' reservation prices for the "more valuable" object *A* taken alone are distributed U [0, V] while those for the less valuable object *B* are U [0, 0.5V], the difference between V and 0.5V capturing the market level asymmetry[14]. The formulation allows stray bidders to have a higher reservation price for *B* than for *A*. Bidders are risk neutral. Other aspects of the main model remain unchanged. The seller's strategic alternatives are: (i) Auction the bundle; (ii) Auction object *A* first and then *B*; and (iii) Auction *B* first and then *A*. The equilibrium bids under alternative strategies are derived in Appendix E. I propose the following:

[14] The case of 0.5V chosen here is a representative scenario. The analysis and the resulting equilibrium are exactly the same for any kV where k in the interval (0, 1].

Proposition 2.6: Given significant asymmetry in market level valuations for A and B:

(a) *Sequential auctions are still optimal for the seller of weak to moderate complements if there are at least four bidders; auction of the bundle is optimal for strong complements only;*

(b) *When sequential auctions are optimal, it is best to auction the higher valued object first.*

Comment: The proof of the bidder's strategy is dealt with in Appendix E while the resulting simulation using the optimal strategy yields the above proposition. This is shown in Figure 4.

..……………………………..Insert Figure 4 about here…...………………………………

Part (a) of the proposition underscores the robustness of Proposition 2.1. Symmetry or lack thereof is not quite the reason why sequential auctions are appealing in the contexts they are. The multiple waves of competition and the resulting desire to bid aggressively, and the threat of losing out on the complementarity boost if the first object is not won are what favor the sequential auctions. Part (b) of the proposition makes the significant point that under the domain of optimality of sequential auctions, the specific sequence matters to the seller. Notice in the equilibrium that the first bid is positively related to the bidder's reservation price for the first object and the value boost of complementarity. As the first auction is decidedly the more competitive of the two auctions, putting up the higher valued object first helps in better extraction of the surplus.

3.5.2. Auctions of Substitutes

With substitutes, the reservation price for the bundle R_{jAB} of bidder j is less than the sum of her reservation prices for items A and B taken alone. Possible substitute pairs are single tickets for two successive baseball World Series games and two similar original drafts of Lincoln's Gettysburg address. The parameter θ is negative in these cases. I identify weak substitutes as those with θ in the region $0>\theta \geq -0.2$. For stronger substitutes, θ is less than -0.2, with the threshold θ for a pair of perfect substitutes being -0.5.[15]

I assume away the possibility of arbitrage but invoke the property of free disposal. For example, if a bidder has purchased the baseball tickets in the auction of the bundle, it is costless for her to discard one of them if she chooses to. She might choose to do so if she has a higher reservation price for either game but the overdose of watching two triggers disutility from the bundle. Correspondingly, I define a new valuation of the bundle $R^{\oplus}_{jAB} = \text{Max}\{R_{jA}, R_{jB}, R_{jAB}\}$. This incorporates the case when a single item may be preferable to a bundle.

I derive the pure strategy equilibria of the bids in Appendix F. Related simulations as before to answer the seller's questions yield the phase diagram in Figure 5.

..…..……………………………..Insert Figure 5 about here…...………………………………

Notice here that the plot for θ greater than zero is the same as in Figure 3A. The addition here is the demarcation for substitutes when $\theta < 0$. I propose the following:

Proposition2. 7: When the objects are substitutes:

(a) With just two bidders, auctioning the bundle is always optimal;

(b) With three bidders, auctioning the bundle is optimal for weak substitutes (i.e., with 0≥θ≥-0.2); sequential auctions are optimal for strong substitutes;

(c) With four or more bidders, sequential auctions are always optimal.

[15] This threshold of -0.5 is because R_{jA} and R_{jB} are equal for perfect substitutes, and the bundle reservation price R_{jAB} equals either R_{jA} or R_{jB}.

Comments: The specific thresholds are in Figure 5. The driver of part (a) of the proposition is that with two bidders, substitutability weakens the desire of the first round winner to pursue the second item. Even with weak substitutes, the competition for the second item is mild and the odds favor the first round loser. Interestingly, the anticipated mildness in the second round competition also brings down the first bid – each bidder sees an incentive in losing the first item to walk away with the second item. Of course, the bidders act strategically, and the combined revenue to the seller from sequential auctions is weakly lower than the auction of the bundle. On the other hand, in part (c), the bidder pool is large enough to trigger a fight on both rounds. A win is not a given. The benefit to the seller from these two rounds of competition is better for the seller than the auction of the bundle that is less appealing to the players. Part (b) represents a "compromise" between the two other scenarios; so the auction of the bundle is appealing for weak substitutes only.

Figure 4 is in a sense an "integrative summary" of our recommendations to the seller. It contains the domains of superiority of bundled and sequential auctions for both substitutes and complements. Notice in the figure that the domain of optimality of sequential auctions is monotonically increasing in the number of bidders (N+1) and decreasing in the degree of interrelatedness (θ). That is, the *combination* of these factors does matter to the seller.

With regard to the bidders' perspective, I turn to sequential auctions given their centrality for substitutes. I find that unlike with complements, *aggressive bidding on the first auction is not optimal* with two substitutes as there is *dis*-synergy from owning both. Indeed, the bidder tries to win the first object for less than her reservation price to hedge against the possible value drop from substitutability if she also wins on the second round. Indeed, the bidder might be better off raising the odds of losing both objects than winning them both. In view of these factors, bidding for substitutes under risk neutrality and maximal risk aversion are closer to each other than they are for complements. As before, the larger the bidder pool, the more alike are risk neutral and risk averse bids for the first of two substitutes.

For the second item, *the optimal bidding rule remains the same as for complements*; i.e., the first round winner j bids R'_{jAB}-R_{jA} and the first losers all bid their reservation prices for the second object.

3.6. Discussion

The boom in the auction market notwithstanding, the topic of multi-object auctions involving complements or substitutes has received surprisingly limited attention. The Marketing literature appears to have completely overlooked this important area. I have examined this problem from the view point of a revenue maximizing seller who has one unit each of two objects to offer. The objects are complements or substitutes of each other – such interrelatedness being a matter of degree and not a simple dichotomy. The seller is faced with a known pool of bidders (i.e., prospective buyers). Given this context, I have examined whether – and under what conditions – the seller gains by offering the objects as a bundle in a single auction or separately, in sequential auctions. I also offer guidelines to the bidders on optimal bidding strategies. While the conditions I establish relate primarily to the size of the bidder pool and the degree of interrelatedness between the two objects, I also examine the role of risk propensity of the bidders (risk neutral vs. risk averse) and the asymmetry in the market level valuations of the products.

Methodologically, I consider the second price sealed bid auction format and derive the pure strategy Nash equilibria of the bids in the different settings. These results suffice to address the bidders' problem. I conduct related simulations to address the questions facing the seller. I see our approach and findings to be of import to practitioners and academics.

3.6.1. Managerial Contributions

The first important implication of this study for sellers and auctioneers is that auctioning the objects as a bundle is not a given even for complements. Although bundling is well known to reduce heterogeneity in buyers' valuations (cf. Schmalensee 1984), sequential auctions have the advantage (from the seller's viewpoint) of creating a cascading (loosely, multiplicative) effect of competition. This effect is increasing in the size of the bidder pool. As a result, sequential auctions are the best for the seller even when the objects are moderate complements, so long as there are at least a few bidders (four or higher). Of course, the auction of the bundle prevails for strong complements.

Sellers are likely to be better off recognizing the importance of auction sequence when the products are asymmetric in their market level valuations (e.g., Super Bowl ticket plus souvenir). In general, the seller benefits from auctioning the more valuable object first. This is because the tendency to bid aggressively (i.e., more than the value of the object in question) is greater in the first of the two auctions.

The message to the bidders is that there is benefit from bidding aggressively on the first of two complements being auctioned as losing the first means forgoing the benefit of complementarity. The extent of such "aggressiveness" depends on the reservation price of the first object – those who value the first object more should bid more aggressively.

3.6.2. Theoretical Contributions

By considering a problem at the intersection of the topic areas of auctions and bundling, this study contributes to both areas. Adding to the earlier work on multi-object auctions, I have examined the impact of *both* complementarity and substitutability as well as the interactive role of the number of bidders. To the best of my knowledge, this study is also the first to examine the moderating role of the bidders' risk aversion and the role of auction sequence. Each of these dimensions has added new insights as noted under section 3.6.1.

The extant work on bundling draws strongly on second degree price discrimination. For this mechanism to be potent, the objects on sale must be in fairly abundant supply, giving the seller the flexibility to push the objects to or hold them back from specific segments. By considering two objects in limited supply I bring an added perspective. The strategic behavior of the prospective buyers (i.e., bidders) is also a distinctive feature of our study.

Nevertheless, our paper is restrictive in several ways. I discuss them below.

3.6.3. Research Limitations and Future Research Directions

I develop a stylized model that is based on some restrictive assumptions. I discuss the arising limitations of the study and propose directions for future research.

While I draw on precedents in the multi-object auction literature for choosing the second price, sealed bid format (see Chakraborty 2002 and Krishna and Rosenthal 1996), this format is only one of several available formats – first-price-sealed-bid and English outcry being two alternatives. Examining the impact of the auction formats on the optimality of bundled vs. sequential auctions is likely to generate interesting insights.

As I consider a seller who has decided to sell off the objects, I have not considered the role of reserve price. (I have in effect set it to zero.) I speculate that the reserve price might act as a *de facto* extra bid and add to the attractiveness of sequential auctions by bringing down the threshold number of bidders by one. This issue needs closer investigation.

Our assumption that the bidders' reservation prices are independent private value is arguably at odds with some industrial markets such as FCC auctions of wireless licenses and international bidding/leasing of oil fields. It may be more reasonable to assume that the objects have common value (as examined in Benoit and Krishna 2001). I urge future work relaxing our assumption of private value but retaining the other dimensions of our model such as the role of risk propensity and substitutability. I surmise that the motivation to bid aggressively might be blunted in the new setting as there is less surprise on what would constitute a winning bid.

4. CONTRIBUTIONS, LIMITATIONS AND AREAS FOR FUTURE WORK

In this work, I look at competitive consumer price formation in two different auction settings. I look at how bidder strategies vary according to the specific auction mechanism adopted and the role of increased competition. Thus, we have the competition between consumers shaping the final price. In both essays, I look at the auctioneer who has to choose between different auction formats given rational consumer behavior. Thus in essay 1, the consumer's optimal bidding strategy across the eBay and Yahoo! auction formats drives the implications for the final price realized for the seller. This influences the seller's strategy regarding the particular auction format adopted. In essay 2, I derive the optimal bidding strategy for the consumer in two different settings – when the objects are auctioned as a bundle and when the objects are offered in sequence. I then derive the optimal bidding strategy and use this strategy to derive implications for the seller.

4.1. Key Contributions

In this section, I identify key theoretical contributions to the literature on auctions that follow from the two essays. First, this work adds to the theoretical work on last minute bidding in online auctions (Wilcox 2000, Roth and Ockenfels 2002) by examining the issue with respect to the specific format adopted. I find that the effect of last minute bidding is stronger (lower revenue) with the Yahoo! format than with eBay. This is because bid levels reach the final valuation faster with the eBay format than with the Yahoo! format. Second, we see the impact of proxy bidding in the English outcry auction format. As discussed earlier, the use of proxy bidding by all bidders would make the Yahoo! format identical to eBay. Thus, introducing the facility of proxy bidding makes the Yahoo! format more attractive for the seller. On the other hand for the bidders, this facility also brings convenience so that she saves the hassle of being present at the very last moment to place a bid.

The second essay extends the multi-object auction literature to dissimilar objects. The literature on multi-object auctions has till now restricted itself to either dissimilar but independent objects (e.g., Chakraborty 2001) or identical objects (e.g., Krishna and Rosenthal 1996). I model the relationship between the objects as a continuum ranging from strong complements to strong substitutes. I also allow the objects to be distinct and asymmetric. By considering substitutes and complements, we can see from proposition 7 in essay 2 that a unified picture regarding optimality of bundling/unbundling emerges from the analysis. In addition, I also look at the role of risk aversion in sequential auctions to get the rather surprising convergence of risk neutral and maximally risk averse bidding strategies in sequential auctions.

4.2. Methodology Adopted

In both essays, I use game theoretic methods to identify optimal actions for the players involved. This is relevant given the nature of competition and the opposing competitive forces involved. Indeed, use of game theory is fairly widespread in the analysis of auctions (e.g., Sinha and Greenleaf 2000, Greenleaf, Rao and Sinha 1993, Lynch and Ariely 2001).While the second essay is primarily theoretical, I test the propositions in the first essay using online data collected over the internet.

In essay 1, I estimate a system of linear models where the parameters are identified simultaneously. This is important as we have matched samples of objects that were sold at the same time over the internet using different formats. While the two formats considered (English outcry and hybrid) differ in terms of the bidding rules, I control for a number of factors that influence the final price realized in both. In spite of the difference in formats, we see that except two (jump bidding in Yahoo and common value dummy in eBay), the remaining factors are the same. This similarity in the nature of variables involved and the objects sold makes simultaneous estimation attractive. Another advantage of this methodology is that we can see the effects of controlling for various factors in isolation. This also allows us to look at the incremental impact of the presence of common value in the final price realized.

In Essay 2, I look at the choices of the seller and the consumer as outcomes of a sequential game. Here, the seller acts first followed by the consumer. Specifically, the bidders decide their strategy after the auctioneer decides on the auction method (bundled vs. sequential auction). As is customary in sequential games, I solve the game by backward induction so that the bidders' problem is solved first and this in turn is used to identify the optimal strategy for the seller. In both essays, the consumers are strategic and forward looking.

4.3. Implications for the Marketer

4.3.1. Implications of Essay1

In this section, I identify four specific implications for sellers using online auctions. First, the seller should appreciate the important difference between the traditional English outcry auction format and the hybrid format used by eBay. This is because, as we see from proposition 3; the hybrid auction format permits a speedier convergence to the final price. This in turn implies that fewer bids would be required to reach a given price level than with the English outcry format.

Second, the nature of the object sold is of crucial importance given the auction mechanism adopted. If the seller knows that the object sold is likely to have interdependent valuation, he can anticipate greater instances of last minute bidding in both auction formats. Also, the presence of interdependent valuations would imply greater possible revenue from the hybrid format when compared to the English outcry format. It should be noted that *a priori* inference of the presence and extent of interdependence in valuations across bidders for the product is not simple. Essay 1 offers preliminary guidelines for a seller to infer the presence of common valuation for the object (*ex post*) among the bidders by simply scanning the bid history for multiple bids in the hybrid auction format. The conclusion however is indicative of the presence of a common value component but not of the extent of interdependency in valuations.

Third, for experience goods, the eBay mechanism could reveal how the consumers assimilate information in the marketplace to develop their reservation prices for the product. Hoffman et. al (1993) look at price formation in an auction setting for packaged beef where they consider the fifth price auction (where the highest bidder gets than object at the fifth highest price), with a focus on the final valuation of the consumer. They further develop the sealed bid auction as a pretest tool to aid the marketer's decisions. My study on the other hand, could yield insights as to how the consumer arrives at her final reservation price and specifically, how valuable is the information. Another possibility is to modify the format as a multi-round mechanism where except the first round, the remaining rounds are probabilistic. This could yield insights as to the proportion and demographic characteristics of consumers who are unsure of the product (due to lack of experience with it) and use the bids of others as a proxy for experience to arrive at a valuation.

Finally, this study gives an additional (but not competing) explanation for the success of eBay. My results indicate that given the format, it is an attractive choice for the seller and thus, it is likely that sellers would be more attracted to eBay rather than Yahoo! This supply generates its own demand resulting in higher overall popularity. At the extreme, this might result in oversupply in eBay bringing down the number of bidders[16]. This also implies that although the eBay format itself is likely top generate higher revenue, buyers may not be as eager to purchase from eBay. Thus, the earlier conclusions regarding higher revenue from the eBay format has to be tempered by the possibility of lower demand.

4.3.2. Implications of Essay 2

In essay 2, the implications for the marketer/seller follow directly from the propositions developed in the model. First, with complements, we see that sequential auctions become more attractive with increased bidders. The reason is that due to the sequential nature of bids placed and objects sold, the heterogeneity of bids in sequential auctions (as discussed in Chapter 3) becomes strong enough to push revenue upwards. In the case of bundled auctions, heterogeneity actually reduces because of bundling. Thus, the auctioneer's decision to bundle or unbundle his offering depends both on the number of bidders and their risk preference. Thus, the auctioneer cannot infer greater revenue from bundling simply because of the complementary nature of the items sold. Specifically, bundling is always optimal for strong complements only.

In a similar vein, I show that unbundling substitutes is also not always optimal. Specifically, if the number of bidders is less than four, the seller may be better off bundling the offer. In our setting, this would imply that if the seller anticipates fewer bidders for both objects, he should seriously consider bundling the offer. On the other hand, if the number of bidders is greater than 4, unbundling is unambiguously better than bundling for substitutes. Finally, as per proposition 6 in essay 2, when the auctioneer has asymmetric objects to sell; he should sell the more valuable object first.

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[16] It is possible that this is already the case to some extent as on June 22nd 2005, Mangalindan notes in the *Wall Street Journal* that increasingly sellers are finding it difficult to get good buyer pools in eBay.

4.4. Implications for the Consumer

4.4.1. Implications of Essay 1

Looking at the results of essay 1, I can identify three implications for the internet savvy consumer of today. First, although Wilcox (2000) and Roth and Ockenfels (2002) discuss at length the motivation behind last minute bidding and its advantages, the consumer should know the nature of its impact on the final price realized and how it varies with auction forms. Essay 1 argues and reveals that last minute bidding is likely to have greater impact in the case of Yahoo! than with eBay. Although this supports the existing view regarding the advantages of last minute bidding, we can see that the potential benefit of bidding at the last minute is greater with Yahoo! than with eBay.

Next, it would be useful for consumers to understand the role of proxy bidding in Yahoo! This rule creates flexibility for consumers so that the auction website bids incrementally on their behalf. I argue that proxy bidding actually brings the Yahoo! format closer to eBay. In addition to flexibility, this rule reduces the hassle for the consumer (by avoiding late and multiple bids) but also implies a greater chance of paying a higher final price. It is for the consumer to tradeoff the benefits of proxy bidding with the benefits and risks[17] of last minute bidding.

Third, essay 1 argues that the presence or absence of a common value component in online auctions could be context driven rather than product driven. For instance, a computer engineer would make a more accurate assessment of the value of a computer by looking at its specifications than a consumer unfamiliar with computer terminology. This means that bids reveal different levels of information to different kinds of bidders. Therefore, it becomes more important for the uninformed bidders to scan the bid history before placing their bids at the last moment.

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[17] The main risk with last minute bidding is that the final bid may not be registered.

4.4.2. Implications of Essay 2

In essay 2, I identify three key implications that follow from the analysis. First, for sequentially sold complements, a risk neutral bidder is always better off by bidding higher than her willingness to pay for the first object. The risk of a possible loss is traded off by the possibility of synergy gains (if both objects are won) and the reduced intensity of competition for the second object after the first object is won. This results in the equilibrium bid for complements that drives the implications for the auctioneer.

Second, bidder strategy changes with increasing number of bidders. Specifically, with more bidders, there is a greater risk of losing the second object after the first object is won. With more bidders, the possibility of losing the second object after overbidding in the first round (resulting in a net loss) increases. At the limit, this risk of possible loss increases to such an extent, that risk neutral bidding converges to maximally risk averse bidding[18] as the number of bidders goes to infinity. Similarly in the case of substitutes, risk neutral bidding involves underbidding (bidding below the willingness to pay) for the first item. Again with very similar reasoning, increasing number of bidders also implies a convergence of maximally risk averse and risk neutral bidding. In the case of substitutes however, maximal risk aversion implies bidding the willingness to pay for the first item.

Paradoxically, my recommendation of aggressive bidding for the first object goes together with the usefulness of maximal risk aversion as a viable strategy. As I show in chapter three, maximally risk averse bidding also (weakly) permits bids above the willingness to pay for the objects. As seen earlier, the risk of making a loss is a restraining effect on the extent of overbidding by the risk neutral bidders. This risk increases with the number of bidders so that in the limit, risk neutral bid levels approach the bid levels with maximal risk aversion.

[18] As defined in essay 2, a maximally risk averse bidder is one who bids so that she never makes a loss.

Third, we see a phenomenon in an independent private value framework where bidders are confronted with the possibility of a loss with a second price sealed bid auction format. This is interesting as to the best of my knowledge, second price sealed bid auctions never expose the bidders to a risk of a loss in an independent private value framework. For instance, in a traditional single object auction, the bidder knows that she would pay less than her bid amount and thus can confidently bid her willingness to pay without risking a loss. In our case, the possibility of a loss arises as the bidder faces uncertainty about the realization of complementarity gains as it is possible that she loses the second auction after winning then first. This effect is different from the winner's curse (c.f., Kagel and Levin 1986 and Thaler 1988) where the possibility of loss arises due to the bidder's uncertainty regarding her valuation whereas in our setting, there is no such uncertainty.

4.5. Limitations and Areas for Future Work

In this section, I discuss the limitations of the research setting and the analysis of this work and thus simultaneously identify potential areas for future work. With respect to model assumptions, I use game theory in both essays to derive implications for both the marketer and the consumer thereby imposing perfect rationality and surplus maximizing behavior for all players involved. While I do consider risk aversion in essay 2 explicitly, real world consumers would not only be risk averse but also be heterogeneous with respect to their attitudes towards risk. Moreover, prospect theory (Kahneman and Tversky 1979) implies that consumers do not always maximize a risk averse utility function.

While incorporating prospect theory and/or risk aversion would be an improvement, the task is far from easy. For example, in a game theoretic setting, it is easy to model consumer surplus maximization as perfectly rational behavior and derive optimal strategies. Relaxing this allows infinite possibilities of behavior that is rational for the individual consumer but suboptimal from the researcher's point of view. One way to get around this could be to impose a distribution along the continuum of utility functions that attach varying weightage to the risk preferences or to the way consumers attach importance to the probabilities of success. I the context of the first essay, it may be possible to incorporate a concave utility function for the consumer and identify segments of consumers who exhibit different patterns of risk aversion and relate it to the extent of common value in the item sold. One way to do this could be a latent class analysis similar to Kamakura and Russell (1989). I think that these would be interesting areas for future work.

In both essays, the analysis assumes perfect information among bidders regarding certain object and auction characteristics. In essay 1, it is assumed that the number of bidders and the valuation functions are commonly known. Similarly in essay 2, the number of bidders, the distribution of valuations and the extent of complementarity are all common knowledge. While this assumption is common in the action literature (e.g. Milgrom and Weber 1982, Avery 1998, Levin and Smith 1996), relaxing it would certainly bring the analysis closer to the real world.

In an online marketplace where the seller can access the consumer without resorting to middlemen, I examine two specific contexts involving auctions. Indeed, auctions have grown in popularity in the recent years but this is largely attributable to the C2C market where consumers lacking the organization to sell their wares find auctions to be a great resource. While we do have implications for the marketer in terms of the kind of auction format to choose (in essay 1) and the optimal strategy regarding the bundling/unbundling problem (essay 2), I do not tackle the higher level question as to when should a marketer use auctions as a selling medium. Given a product, is it better to use direct sales to reach the consumer or is it better to use auctions to attract customers? There are merits for either approach. As noted in essay 2, auctions would allow strong price discrimination. However, if the auctioneer has many units to sell, theory has shown (e.g., Lynch and Ariely 2001) that the use of simultaneous auctions may result in a drop in price realized in later rounds before market clearance. This is referred to as the "free rider problem" where bidders strategically bid low in the early rounds as they anticipate lesser competition in the later rounds due to partial satiation of demand. Similarly, we have arguments going either way for and against direct sales. I think that this would be an important question to be tackled in later work.

4.6. Conclusions

In Summary, I feel that auctions offer great opportunities for research in marketing. Particularly, online auctions have great potential for research in the context where the seller and the consumer are human beings as opposed to businesses. In spite of a large body of existing work on auctions, I feel that research in auctions, particularly the C2C setting is relatively under researched. Given the emphasis on game theoretic modeling and consumer behavior, marketing scholars are in a great position to exploit the current popularity of online auctions to develop research streams where online consumers (as opposed to firms) are the decision makers. This work is a step in this direction and I expect further exciting research in this area.

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APPENDIX A

Proof of Proposition 1.1

Proposition 1.1: The expected revenue from the hybrid auction format is at least as great as that of the English outcry format with re-entry.

Proof:
Consider the situation where there are N bidders in both auctions. We first define the following notation.

Time duration for both auctions	:	T
Number of bidders	:	N
Signal received by bidder $i \in \{1...N\}$	:	x_i
Complete Vector of Signals	:	$X=\{x_i\}$
Expected Valuation of each bidder	:	$u_i=u(x_i,x_{-i})$

Additionally, $\frac{\partial u_i}{\partial x_i} > \frac{\partial u_i}{\partial x_j} \geq 0, i \neq j$

Thus, the object for auction is of common value. I also assume as in Milgrom and Weber (1982) that the signals are strictly affiliated.
Let $x_1>x_2>\ldots\ldots>x_N$ without loss of generality.
I assume that the signals are drawn from an everywhere continuous pdf f(.) defined over an interval with finite support and this is common knowledge as well.
I assume that the bidders are risk neutral.
First, I state three important results from Milgrom and Weber(1982) that would be used later.
Result A: The expected revenue from the second price sealed bid auction is at least as great as the revenue from the first price sealed bid auction when both auction formats have the same reserve price. In particular, the above relation holds when the reserve price is zero.
Result B: In the second price sealed bid auction with common values, the symmetric equilibrium bid strategy is as follows:

$$b_m^*=V(x_m, x_m) \qquad (A1)$$
where

V(x,y)=Expected Valuation of the bidder where x is the highest signal and y is the next highest signal. Thus, the bid function above implies that the dominant strategy for bidder m is to bid assuming that his/her signal is the highest and the next highest signal is tied to his/her signal. I use this result as the benchmark since the problem of bidding in the hybrid auction format is similar to that of bidding in a second price sealed bid auction with multiple rounds. If the remaining signals $X_2\ldots X_N$ are ordered as $Y_1>Y_2>..Y_{N-1}$, the above expression implies:

$$V(x_m, x_m)=E[u(X_1,Y_1,Y_2\ldots.Y_{N-1}) \mid X_1=x_m,Y_1=x_m] \qquad (A2)$$
In the above and in the following discussion, upper case denotes the random variable and the lower case denotes the actual realization of the random variable.

In the expression A2, y_i is the i^{th} order statistic of the remaining signals where the first order statistic among the N-1 signals is constrained to be equal to the signal of bidder m.

Result C: The symmetric equilibrium strategy in the English outcry auction is given by the threshold level

$$\beta_i = u\left(\underbrace{x_i,\ldots x_i}_{k-times}, x_{k+1},\ldots x_N\right) \qquad \text{(A3)}$$

up to which the bidder bids when N-k bidders have dropped out. Intuitively, this means that bidder i substitutes his/her own signal for all the bidders while constructing his/her expected valuation and once a bidder drops out, bidder i inverts the bid function to infer the signal of the bidder who dropped and updates the threshold to which the bidder will bid. Thus, the bidder i begins with $\beta_i = u\left(\underbrace{x_i,\ldots x_i}_{N-times}\right)$ and updates his/her threshold bid level as given by (A3) after N-k bidders have dropped out. Thus, if bidder 2 has the second highest signal, s/he would bid up to $\beta_2 = u(x_2, x_2, x_3, ..x_N)$ and drop out immediately after this as bidder 2 would infer the values of the signals $x_3 \ldots x_N$ when the other bidders drop out.

Case 1: Consider the situation where all the bidders bid at the last moment. This is the optimal strategy in common valued online auctions with a predetermined duration following Wilcox(2000). In this case, the hybrid auction and the English Outcry auction reduce to the second price sealed bid and the first price sealed bid auctions respectively. Using result A, the expected revenue from the second price sealed bid auction is at least as great as that of the first price sealed bid auction. Hence the result of the proposition follows.

Case 2: Consider the situation where 2 bidders deviate from the policy of last minute bidding and bid earlier. I assume that the bidders do not bid at the last minute only if they think that they will not be available at that time in the auction. Denoting these bidders by i and j, I assume without loss of generality that $x_i > x_j$. In the English outcry auction, these bidders compete as in a regular English outcry auction so that bidder j drops out when his/her expected valuation is reached. Bidder j's expected valuation here is simply $u(x_j,x_j)$ as given by equation (A3). On the other hand, bidder i only bids marginally above bidder j's valuation. Thus, for the remaining bidders, the signal x_j is known by inverting the valuation function for bidder j. The bidders also know that $x_i > x_j$ as i is the highest bidder.

Denote $x_i' = E[x_i | x_i > x_j]$ (A4)

In the hybrid auction, the bidders i and j bid as in the second price sealed bid auction. This is because the deviating bidders feel that they will not be available at the last moment and they know that the winning bidder pays the second highest bid. Thus from result B, bidder j bids $V(x_j,x_j) = E[u(x_1,y_1) \mid x_1 = x_j, y_1 = x_j] = u(x_j,x_j)$ and bidder i bids $V(x_i,x_i) = u(x_i,x_i)$ and since $x_i > x_j$; $V(x_j,x_j) < V(x_i,x_i)$. Since the second highest bid is posted, $V(x_j,x_j)$ is now known. This enables other bidders to invert the bid function and infer x_j. Also, the other bidders know that $x_i > x_j$ (as the highest bidder's identity is also posted). Thus, the other bidders can make an assessment of x_i as $E[x_i | x_i > x_j] = x_i'$ as in the English outcry auction.

Now consider the bidding at the last moment by all the bidders. In both auctions, a typical bidder k knows the following set of signals $\{x_k, x_j, x_i^{'}, E[x_n]$ where $n \neq j,i,k\}$. At the last minute therefore the bidding problem faced by the two sets of bidders is identical. However, in the English outcry auction with re-entry, the situation is that of a first price sealed bid auction and in the hybrid auction, the situation is the same as that of a second price sealed bid auction. From result A, we know that the revenue realized in this situation is greater in the second price sealed bid auction and hence the result follows. We also note that when the two bidders (j and i) bid earlier in the English outcry auction, bidder j (and consequently bidder i) would never bid above $u(x_j,x_j)=V(x_j,x_j)$ which is the bid placed by bidder j in the hybrid auction. Thus, the outstanding reserve price in the hybrid auction is the same as that in the English outcry auction and result A holds as the bidders in the hybrid auction bid as if they are in the second price sealed bid auction and the bidders in the English outcry auction bid as in the first price sealed bid auction.

The above reasoning is intact when the number of deviating bidders is more than 2. For example, consider 3 bidders i,j and k and $x_i>x_j>x_k$. When they bid before the last minute in the English outcry auction, bidder k drops out after his/her valuation is reached. Then bidders j and i continue bidding till j drops out after his/her valuation is reached. Thus the other bidders can infer x_k, x_j and $E[x_i|x_i>x_j]=x_i^{'}$. In the hybrid auction, bidders i,j and k 0bid $V(x_i,x_i)$, $V(x_j,x_j)$ and $V(x_k,x_k)$ respectively. Since $x_i>x_j>x_k$ by construction, $V(x_i,x_i)>V(x_j,x_j)>V(x_k,x_k)$. Since the highest bid is not revealed, the bidders infer x_k, x_j and $E[x_i|x_i>x_j]=x_i^{'}$. Thus at the final minute a typical bidder r has the following set of signals $\{x_k, x_j, x_r, x_i^{'}, E[x_n]$ where $n \neq j,i,k,r\}$. This is also exactly the same set of signals available to any bidder r in the English outcry auction. Now at the last minute, the bidders in the English outcry auction compete (as earlier) in a first price sealed bid auction and the bidders in the hybrid auction compete in a second price sealed bid auction. Since the bidders again have the same set of signals and the same outstanding price to beat, from result A we again see that the expected revenue from the hybrid auction is greater than that of the English outcry auction. I note that the outstanding price before then last minute is the same because of the following argument.
From result C and equation A3, we know that when bidder k drops out, bidder j bids up to $u(x_j,x_j,x_k)$ in the English outcry auction. In the hybrid auction, after inferring x_k, bidder j bids $V(x_j,x_j)=E(u(X_1,Y_1)\,|\,X_1=x_j,Y_1=x_j,X_k=x_k))= u(x_j,x_j,x_k)$

The above reasoning holds regardless of the *sequence* of bidding in the hybrid auction. I consider the possible cases below.

Case 2a: Bidder i bids first followed by bidder j and then bidder k in both auctions. Then s/he bids $V(x_i,x_i)$. Since the highest bid is not known, the next bidder (say bidder j) bids $V(x_j,x_j)$. Since $V(x_j,x_j)< V(x_i,x_i)$, $V(x_j,x_j)$ is observed and bidder k infers x_j. Now $V(x_k,x_k)= E(u(X_1,Y_1)\,|\,X_1=x_k,Y_1=x_k,X_j=x_k))<V(x_j,x_j)$ (in the preceding inequality, X_j is constrained to be x_k as X_j is always less than X_1) and thus s/he does not bid. In the English outcry auction also bidder j drops out after his/her valuation is reached. As $x_j>x_k$, valuation of bidder k is less than that of bidder j and s/he does not enter any bid and the situation is the same as the case where two bidders deviate and the proposition follows.

Case 2b: Bidder i bids first followed by bidder k and then bidder j in both auctions. Now, both signals x_k and x_j would be revealed as k would bid $V(x_k,x_k)<V(x_j,x_j)$ and consequently

bidder j also bids and since $V(x_j,x_j)< V(x_i,x_i)$=bid placed by i, both signals k and j are revealed leading us to case 2 discussed earlier.

Case 2c: Bidder j goes first. Now, if bidder i goes next in both auctions, k does not bid in both auctions as valuation of j is greater than the valuation of k. Thus, the only signal revealed in both auctions is x_j and the bidders assess $x_i^{'} = E[x_i|x_i>x_j]$ and the situation is the same for both acutions in the last minute as discussed in case 2. If bidder k goes first in both auctions, since $V(x_i,x_i)>V(x_j,x_j)>V(x_k,x_k)$, in the hybrid auction, k's bid is lower than j's bid and thus, his/her bid is known leading to bidders inferring x_k. Again since x_i is the highest signal among the three bidders, his/her bid is unseen and consequently $V(x_j,x_j)$ is revealed leading to bidders inferring x_j (and x_k due to earlier bids). Similarly the situation is the same in the English outcry auction as bidders j and k bid first leading to k dropping out after his/her valuation is reached. Then, after i enters, j drops out after his/her valuation is reached leading to signals x_j and x_k being revealed and the situation is the same as in case 2.

Case 2d: Bidder k goes first. Now if bidder i goes next in the hybrid auction, s/he bids $V(x_i,x_i)> V(x_k,x_k)$. Therefore the bid $V(x_k,x_k)$ is revealed since only the second highest bid is known. Now when bidder j bids $V(x_j,x_j)$, since $V(x_i,x_i)>V(x_j,x_j)>V(x_k,x_k)$, j's bid is the second highest and thus the bidders now infer x_k, x_j and $x_i^{'}$. Similarly in the English outcry auction, when bidders i and k bid, k drops out after his/her valuation is reached and bidder j enters and drops out after his/her valuation is reached. Thus the bidders infer x_k, x_j and $x_i^{'}$ and the situation is the same as in case 2. The situation is again the same if bidder j follows bidder k before bidder i. Only the order of revelation of signals changes in both auctions (x_k followed by x_j followed by $x_i^{'}$) in both the auctions and we are back to case 2.

Case 3: Only one bidder deviates from the strategy of last minute bidding

The above logic continues to hold as the deviating bidder (say bidder x_j) has an incentive to beat the reserve price only in the English outcry auction. In the hybrid auction, the highest bid is not known. Thus, the remaining bidders make identical inferences regarding signal x_j (the bidders infer $E[x_j|x_j>x_k$ where $V(x_k)$=R= Reserve price) in both auction forms.

Case 4: This is the case where the two highest signals happen to belong to the deviators who do not bid at the last moment. In this case, there is no bidding at the last minute as other bidders cannot beat the outstanding bid. As we have seen earlier, the outstanding price formed in this situation is identical for both the auction formats. Thus, in this particular case, the revenue realized from both auction formats is the same.

Thus, combining the above four cases, we see that the revenue realized from the Hybrid auction format is never less than that of the English outcry format with re-entry.

Remark: Case 4 shows that when there is no last minute bidding, the revenue from both formats is the same and otherwise the revenue is likely to be greater for the hybrid format. This result is in the same direction as Proposition 4.

APPENDIX B

Model 2 in Essay 1

First I introduce some notation

Total time duration for each auction :	T
Final price realized:	P
Bid placed at time $t \varepsilon [0,T]$:	b_t
Reserve Price:	R
First bid:	b_0
Time of first bid:	t_0

I now normalize as follows:

Effective auction duration:	$T-t_0$

For any bid b_t, the price is taken to be $(b_t-R)/(P-R)$
Corresponding time is taken to be $(t-t_0)/(T-t_0)$. Also, when t=T, $b_T=P$
Thus, we can see that
EPrice$\varepsilon[0,1]$
YPrice$\varepsilon[0,1]$
T$\varepsilon[0,1]$

For model 3a,

Total time duration for each auction :	T
Bid at time T=final bid	b_T
Final price realized:	P
Bid placed at time $t \varepsilon [0,T]$:	b_t
Price at that point	b_t/b_T
Thus, outstanding price when 80% of the auction is over	$b_{0.8}$
Bid placed at time t with t>0.8	b_t
Effective duration	T-0.8T
Normalized time (t')	(t-0.8T)/(T-0.8T)
Effective price increase	$b_t-b_{0.8}$
Effective increase as a fraction of total increase	$\frac{1}{b_T}\frac{(b_{t'}-b_{0.8})}{(1-b_{0.8}/b_T)}$

Thus,
EPrice$\varepsilon[0,1]$
YPrice$\varepsilon[0,1]$
T$\varepsilon[0,1]$

APPENDIX C

Main Model: Proofs of Equilibrium Bids for Complements and for Propositions 2.2 and 2.3

It follows directly from Milgrom and Weber (1982) and Vickrey (1961) that the optimal bid in the auction of a bundle of complements is the reservation price for the bundle. This appendix will focus on the equilibrium bids under the sequential auctions and related Propositions 2.2 and 2.3.

Without loss of generality, I derive the equilibrium bids from the standpoint of bidder *j*. For the first auction (of object *A*), each of the remaining *N* bidders is identified by the tag *i* such that $Max\{b_{iA}\}$ represents the highest of these *N* bids. If *j* is the first round winner, the tags *i* of the other bidders remain unchanged. However, if bidder *j* is a first round loser, then one of the other *N* bidders must have won object *A*. In that case, for the second auction (of object *B*), the first round winner alone is identified as bidder *k*, and each of the other *N*-1 bidders (excluding bidder *j*) will continue to be identified as bidders *i*. This clear identification of the first round winner is necessary as her second round bid is qualitatively different from those of the first round losers.

Now to restate equation (6), the bidder maximizes her expected surplus given by

$$Surplus=\left[\begin{array}{l} \left\{\begin{array}{c} \mathrm{P}(\mathrm{Max}\{b_{iA}\}_{i\neq j}<b_{jA}) \\ \mathrm{P}(\mathrm{Max}\{b'_{iB}\}_{i\neq j}<b_{jB}) \end{array}\right\}\times\left\{R_{jAB}-E\left(Max\{b_{iA}\}+Max\{b'_{iB}\}\left|\begin{array}{l}\mathrm{Max}\{b_{iA}\}<b_{jA} \\ \mathrm{Max}\{b'_{iB}\}_{i\neq j}<b_{jB}\end{array}\right.\right)\right\} \\ +\left\{\begin{array}{c} \mathrm{P}(\mathrm{Max}\{b_{iA}\}<b_{jA}) \\ \mathrm{P}(\mathrm{Max}\{b'_{iB}\}_{i\neq j}>b_{jB}) \end{array}\right\}\times\left\{R_{jA}-E\left(Max\{b_{iA}\}\middle|\mathrm{Max}\{b_{iA}\}<b_{jA}\right)\right\} \\ +\left\{\begin{array}{c} \mathrm{P}(\mathrm{Max}\{b_{kA}\}>b_{jA}) \\ \mathrm{P}(\mathrm{Max}\{\mathrm{Max}\{b'_{iB}\}_{i\neq j,k},b_{kB}\}<b_{jB}) \end{array}\right\}\times\left\{R_{jB}-E\left(\begin{array}{l}(\mathrm{Max}\{\mathrm{Max}\{b'_{iB}\}_{i\neq j,k},b_{kB}\}<b_{jB}) \\ \left|(\mathrm{Max}\{\mathrm{Max}\{b'_{iB}\}_{i\neq j,k},b_{kB}\}<b_{jB})\right.\end{array}\right)\right\} \end{array}\right] \quad \text{(C-1)}$$

In the above, the prime (') symbol as in b'_{iB} represents the bid of a first round loser. Also, in each term on the right hand side, P(.) represents the probability of the event of, say, bidder *j*'s first bid exceeding those of all other bidders.

I solve by backward induction. So I start with the equilibrium bids for the second object *B*.

<u>Equilibrium Bid for the Second Object *B*</u>:
Two cases matter: (a) When bidder *j* is the first round winner and (b) *j* is a first round loser.

Under case (a), to show that $b_{jB}=R_{jAB}-R_{jA}$ is the equilibrium bid for object B:

Suppose *j* has won the first object at a price *p* (i.e., the second highest bid for object *A* was *p*).

Bidder *j*'s surplus from winning the first object is $s=R_{jA}-p$. When bidding for the second object, *j* tries to improve upon this surplus. Accordingly, while bidding for *B*, she ensures that her total payout contingent on winning both items across the two auctions would not exceed R_{jAB}-s; i.e., not greater than $R_{jAB}-(R_{jA}-p) = R_{jAB}-R_{jA}+p$. Having already paid p on winning object A, j's maximum willingness to pay for object B is $R_{jAB}-R_{jA}$. Given that the auction of B is also of the second price sealed bid type, bidder j gains most by bidding the willingness to pay $R_{jAB}-R_{jA}$. *Q.E.D.*

Under case (b), when j is a first round loser, to show that $b'_{jB} = R_{jB}$ is the equilibrium bid for B.

As bidder j's reservation price for B is R_{jB} (i.e., no benefit of complementarity having lost the first auction), it is weakly dominant to bid one's valuation (cf. Milgrom & Weber 1982).

<u>Equilibrium Bid for the First Object A</u>:

The earlier equilibrium bids for object B are independent of the price at which the first object is won. This reduces the optimization problem in equation (C-1) to that of maximization over only one variable b_{jA} as there are weakly dominant bids for B that are independent of the price at which object A is won (or lost).

I will now begin to restate equation (C-1) by identifying the specific values of the various probabilities P(.) and the expected values E(.).

On $P\left(\text{Max}\{b_{iA}\}_{i \neq j} < b_{jA}\right)$: I assume that the maximum bid (i.e., the first order statistic of the distribution of first bids) for A follows the cdf G(.) that is everywhere differentiable. Its pdf is g(.). Therefore, $P\left(\text{Max}\{b_{iA}\}_{i \neq j} < b_{jA}\right)$ is G(b_{jA}) and $P\left(\text{Max}\{b_{iA}\}_{i \neq j} > b_{jA}\right)$ is (1- G(b_{jA})).

On $P\left(\text{Max}\{b'_{iB}\}_{i \neq j} < b_{jB}\right)$: As $b'_{iB} = R_{iB}$, the cdf of $\text{Max}\{R_{iB}\}_{i \neq j}$ is $F(x) = \left(\frac{x}{V}\right)^N$, the first order statistic of N realizations of R_{iB} that are uniformly distributed. Thus,

$$P(\text{Max}\{b'_{iB}\}_{i \neq j} < b_{jB}) = \begin{cases} \left(\dfrac{R_{jAB} - R_{jA}}{V}\right)^N & \text{when } 0 \le (R_{jAB} - R_{jA}) \le V \\ 1 \quad \text{when } (R_{jAB} - R_{jA}) > V \end{cases}$$

On $P\left(\text{Max}\{\text{Max}\{b'_{iB}\}_{i \neq j,k}, b_{kB}\} < b_{jB}\right)$: Denoting random variable $\text{Max}\{\text{Max}\{b'_{iB}\}_{i \neq j,k}, b_{kB}\}$ by z with associated cdf H(.) and pdf h(.), P(.) is H(R_{jB}) as R_{jB} is j's winning bid for B conditional on losing the first auction.

Recasting equation (A-1) for the surplus maximizing bidder, we have the objective function:

Maximize $\left\{ G\left(b_{jA}\right)\left(\frac{R_{jAB}-R_{jA}}{V}\right)^{N}\right\} \times \left\{ R_{jAB} - \left(\frac{1}{G\left(b_{jA}\right)} \int_0^{b_{jA}} tg(t)dt + \frac{1}{\left(\frac{R_{jAB}-R_{jA}}{V}\right)^{N}} \int_0^{R_{jAB}-R_{jA}} tN\left(\frac{t}{V}\right)^{N-1}\frac{1}{V}dt \right)\right\}$

$+\left\{ G\left(b_{jA}\right)\left(1-\left(\frac{R_{jAB}-R_{jA}}{V}\right)^{N}\right)\right\} \times \left\{ R_{jA} - \frac{1}{G\left(b_{jA}\right)} \int_0^{b_{jA}} tg(t)dt \right\}$

$+\left\{ \left(1-G\left(b_{jA}\right)\right).H(R_{jB})\right\} \times \left\{ R_{jB} - \frac{1}{H(R_{jB})} \int_0^{R_{jB}} zh(z)dz \right\}$

For simplicity, let this be: Maximize {I + II + III}

We must consider two cases. Case (i) is when (R_{jAB}-R_{jA})≤V: As R_{jAB}-R_{jA} is *j*'s bid for *B* conditional on *j* being the first round winner, (R_{jAB}-R_{jA})≤V means *j*'s probability of winning the *B* is below 100%. Case (ii) is when (R_{jAB}-R_{jA})>V: As V is the maximum possible bid of the first round losers, *j* is certain of winning object *B* with this bid.

I begin by considering **Case (i)**.

Notice that $\int_0^{b_{jA}} tg(t)dt = \int_0^{b_{jA}} tdG = tG(t)\Big|_0^{b_{jA}} - \int_0^{b_{jA}} G(t)dt = b_{jA}G\left(b_{jA}\right) - \int_0^{b_{jA}} G(t)dt$ and $G(0)=0$.

Using this relationship, simplifying term I through integration by parts and canceling like terms,

$$\text{I}=\left[G\left(b_{jA}\right)\left(\left(\frac{R_{jAB}}{N+1}\right)+\frac{N}{N+1}R_{jA}\right)-b_{jA}G\left(b_{jA}\right)+\int_0^{b_{jA}} G(t)dt\right]\left(\frac{R_{jAB}-R_{jA}}{V}\right)^{N}$$

Differentiating I with respect to b_{jA} and noting that $\frac{\partial}{\partial b_{jA}}\left(\int_0^{b_{jA}} G(t)dt\right) = G\left(b_{jA}\right)$,

$$\frac{\partial}{\partial b_{jA}}(I) = g\left(b_{jA}\right)\left[\left(\frac{R_{jAB}}{N+1}\right)+\frac{N}{N+1}R_{jA}-b_{jA}\right]\left(\frac{R_{jAB}-R_{jA}}{V}\right)^{N} \tag{C-2}$$

Similarly, with term II:

$$\frac{\partial}{\partial b_{jA}}(II) = g\left(b_{jA}\right)\left(R_{jA}-b_{jA}\right)\left(1-\left(\frac{R_{jAB}-R_{jA}}{V}\right)^{N}\right) \tag{C-3}$$

With term III: (Recall that $\text{Max}\{\text{Max}\{b'_{kB}\}_{k \neq j,i}, b_{iB}\} = z$ with associated cdf H(.) and pdf h(.).)

$$III = \left\{\left(1 - \mathrm{G}\left(b_{jA}\right)\right).H(R_{jB})\right\} \times \left\{\mathrm{R}_{\mathrm{jB}} - \frac{1}{H(R_{jB})}\int_0^{R_{jB}} th(t)dt\right\} = \left(1 - G\left(b_{jA}\right)\right)\left\{H\left(R_{jB}\right)R_{jB} - \int_0^{R_{jB}} tdH(t)\right\}$$

Noting that $\int_0^{R_{jB}} tdH(t) = tH(t)\Big|_0^{R_{jB}} - \int_0^{R_{jB}} H(t)dt$ and $H(0)=0$, and canceling like terms, we have

$$III = \left\{\left(1 - G\left(b_{jA}\right)\right)\int_0^{R_{jB}} H(t)dt\right\}, \text{ yielding}$$

$$\frac{\partial}{\partial b_{jA}}(III) = -g(b_{jA})\int_0^{R_{jB}} H(t)dt \qquad \text{(C-4)}$$

Now, to evaluate $\int_0^{R_{jB}} H(t)dt$, we must first state and prove the following lemma.

Lemma: z $\left(= \mathrm{Max}\{\mathrm{Max}\{b'_{iB}\}_{i \neq j,k}, b_{kB}\}\right)$ is distributed as follows in the interval [0, V]:

$$H(z) = \begin{cases} \dfrac{z^{N+1}}{2V^{N+1}\theta(1+\theta)} & \text{when } 0 \le z \le \theta V \\ \dfrac{2z^N - \theta V z^{N-1}}{2V^N(1+\theta)} & \text{when } \theta V < z \le V \end{cases}$$

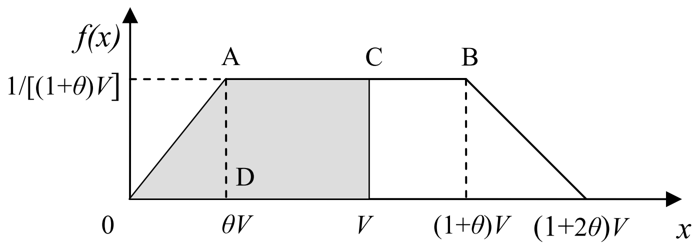
[Now although z can have a maximum value of $(1+2\theta)V$, I limit the lemma for the range where $z \le V$ as V is the upper bound of R_{jB}. This is the relevant range over which j has any chance of winning the second object having lost the first object.]

Proof of the Lemma:

From the equilibria of second round bids derived earlier, when bidder k is the first round winner, b'_{iB} (= R_{iB}) represents the second round bids of the N-1 first round losers (excluding j). The distribution of Max{R_{iB}} $\forall$ $k \neq i,j$ is the first order statistic w (i.e., the maximum of the N-1 bids) of R_{iB}. Then its cdf $T(w)$ is given by: $T(w) = \left(\dfrac{w}{V}\right)^{N-1}$;$0 \le w \le V$. Further, $b_{kB}=R_{kAB}-R_{kA}$.

To rewrite, $R_{kAB}-R_{kA}=(1+\theta)(R_{kA}+R_{kB})-R_{kA}=(1+\theta)R_{kB}+\theta R_{kA}$ (C-5)

As R_{kA}, R_{kB} are iid U [0, V], $(1+\theta)R_{kB}$ is distributed U [0, $(1+\theta)V$] and θR_{kA} is distributed U [0, θV]. Thus, the distribution of the random variable $X = (1+\theta)R_{kB}+\theta R_{kA}$ has pdf $f(x)$ as graphed below (with the shaded region denoting the relevant range R_{jB} which j has a possible chance of beating k):



$X=(1+\theta)R_{kB}+\theta R_{kA}$

In the graph, the pdf of x in [0, V] is:

$$f(x)=\begin{cases}\dfrac{x}{\theta(1+\theta)V^2} & \text{when } 0\le x\le \theta V\\ f(x)=\dfrac{1}{(1+\theta)V} & \text{when } \theta V< x\le V\end{cases}$$

Therefore, z $\left(=\text{Max}\{\text{Max}\{b'_{iB}\}_{i\neq j,k}, b_{kB}\}\right)$ is given by the cdf

$$H(Z\le z)=P(w\le z).P(x\le z)=T(z).F(z)=T(z).\int_0^z f(x)dx, \text{ yielding}$$

$$H(Z\le z)=\begin{cases}\left(\dfrac{z}{V}\right)^{N-1}\dfrac{z^2}{2\theta(1+\theta)V^2}=\dfrac{z^{N+1}}{2\theta(1+\theta)V^{N+1}} & \text{when } 0\le z\le \theta V\\ \left(\dfrac{z}{V}\right)^{N-1}\dfrac{2z-\theta V}{2(1+\theta)V}=\dfrac{2z^N-\theta V z^{N-1}}{2V^N(1+\theta)} & \text{when } \theta V< z\le V\end{cases} \tag{C-6}$$

This proves the lemma. We can now evaluate III and its partial derivative in equation (C-4).

$$\frac{\partial}{\partial b_{jA}}(III)=\begin{cases}-g(b_{jA})\displaystyle\int_0^{R_{jB}}\frac{z^{N+1}}{2V^{N+1}\theta(1+\theta)}dz=\frac{-g(b_{jA})R_{jB}^{N+2}}{2(N+2)V^{N+1}\theta(1+\theta)} & \text{when } 0\le R_{jB}\le \theta V\\ -g(b_{jA})\left[\dfrac{2N(N+2)R_{jB}^{N+1}-(N+1)(N+2)\theta V R_{jB}^{N}+2(\theta V)^{N+1}}{2N(N+1)(N+2)V^N(1+\theta)}\right] & \text{when } \theta V< R_{jB}\le V\end{cases} \tag{C-7}$$

F.O.C.: Setting $\dfrac{\partial}{\partial b_{jA}}(I+II+III)=0$ and solving for j's optimal first bid b^*_{jA} given (R_{jA},R_{jB},θ), we get that when $(R_{jAB}-R_{jA})\le V$:

$$b^*_{jA}=\begin{cases}R_{jA}+\dfrac{(R_{jAB}-R_{jA})^{N+1}}{(N+1)V^N}-\dfrac{R_{jB}^{N+2}}{2(N+2)V^{N+1}\theta(1+\theta)} & \text{when } 0\le R_{jB}\le \theta V\\ R_{jA}+\dfrac{(R_{jAB}-R_{jA})^{N+1}}{(N+1)V^N}-\dfrac{2N(N+2)R_{jB}^{N+1}-(N+1)(N+2)\theta V R_{jB}^{N}+2(\theta V)^{N+1}}{2N(N+1)(N+2)V^N(1+\theta)} & \text{when } \theta V< R_{jB}\le V\end{cases}$$

(C-8)

Notice that I reject $g(b_{jA})=0$ as a solution as this would imply an arbitrary first bid that is independent of j's reservation prices. The solution in equation (C-8) also satisfies the second order condition $\frac{\partial^2}{\partial b_{jA}^2}(I + II + III) < 0$ confirming that it is bidder *j*'s surplus maximizing first bid.

Now I turn to case (ii); i.e., when $(R_{jAB} - R_{jA}) > V$. If bidder *j*'s is the first round winner, her equilibrium bid of $b_{jB} = R_{jAB} - R_{jA}$ (>*V*) for object *B* will certainly win her *B* as competing bids are bounded in [0, V]. Therefore, in equation (A-1), $\mathrm{P}(\mathrm{Max}\{b'_{iB}\}_{i\neq j} < b_{jB}) = 1$. Using this and simplifying the maximization problem as with case (i) before, we get:

When $(R_{jAB}$-$R_{jA}) > V$:

$$b^*_{jA} = \begin{cases} R_{jAB} - \left(\frac{N}{N+1}\right)V - \frac{R_{jB}^{N+2}}{2(N+2)V^{N+1}\theta(1+\theta)} & \text{when } R_{jB} < \theta V \\ R_{jAB} - \left(\frac{N}{N+1}\right)V - \frac{2N(N+2)R_{jB}^{N+1} - (N+1)(N+2)\theta V R_{jB}^{N} + 2(\theta V)^{N+1}}{2N(N+1)(N+2)V^N(1+\theta)} & \text{when } \theta V \le R_{jB} \le V \end{cases}$$

(C-9)

Since I have already proved the optimality of bids b'_{jB} and b_{jB}, the equilibrium from the bidder's side in the sequential auctions is proved.

The equilibria are unique and symmetric. That is, they hold for each bidder and do not depend on the reservation prices of the other bidders. To appreciate the uniqueness, consider a case when the "other" bidders deviate in a systematic manner (tied to their valuations) so that the cdf of the first order statistic of the first bid is again differentiable everywhere. Then in the maximization problem, the term containing the pdf of the maximum first bid drops out, yielding the same equilibria confirming uniqueness.

Proof of Proposition 2.2:

To show that both $b^*_{jA} - R_{jA}$ positive (Proof of Proposition 2(a)):

In equations (C-7) and (C-8), note that $R_{jAB} = (1+\theta)(R_{jA} + R_{jB})$ and R_{jAB}-$R_{jA} = \theta R_{jA}+(1+\theta)R_{jB}$.

When $(R_{jAB}$-$R_{jA}) \le V$ and $R_{jB} \le \theta V$ (first part of equation (C-7)):

For this case: $b^*_{jA} - R_{jA} = \frac{(R_{jAB} - R_{jA})^{N+1}}{(N+1)V^N} - \frac{R_{jB}^{N+2}}{2(N+2)V^{N+1}\theta(1+\theta)}$

$$> \frac{1}{2V^N(N+1)(N+2)}\left[\frac{2(N+2)(1+\theta)\left[\left[(1+\theta)R_{jB} + \theta R_{jA}\right]^{N+1} - (N+1)R_{jB}^{N+1}\right]}{(1+\theta)}\right] \text{ as}$$

$R_{jB} \le \theta V$

>0 as $2(N+2)(1+\theta)>(N+1)$ and $(1+\theta)(R_{jA}+R_{jB})>R_{jB}$

Similarly, $b^*_{jA} - R_{jA} > 0$ can be established for the other cases in equations (C-8) and (C-9).

Now to show that $\frac{\partial}{\partial R_{jA}}\left(b^*_{jA} - R_{jA}\right)$ is positive (Proof of Proposition 2(b)):

When $(R_{jAB}-R_{jA}) \leq V$ (equation (C-7)):

$$\frac{\partial(b^*_{jA} - R_{jA})}{\partial R_{jA}} = \frac{\theta(N+1)\left[(1+\theta)R_{jB} + \theta R_{jA}\right]^N}{(N+1)V^N}$$. which is positive $\forall\ \theta>0$ (for the case $R_{jB} \leq \theta V$).

Likewise, $\frac{\partial(b^*_{jA} - R_{jA})}{\partial R_{jA}} = \frac{\theta(N+1)\left[(1+\theta)R_{jB} + \theta R_{jA}\right]^N}{(N+1)V^N} > 0\ \forall\ \theta>0$ (for $\theta V < R_{jB} \leq V$).

When $(R_{jAB}-R_{jA}) > V$ (equation A-8):

$$\frac{\partial}{\partial R_{jA}}\left(b^*_{jA} - R_{jA}\right) = \frac{\partial}{\partial R_{jA}}\left(\theta R_{jA}\right) = \theta$$ which is positive for all complements (for $R_{jB} \leq \theta V$).

Likewise $\frac{\partial}{\partial R_{jA}}\left(b^*_{jA} - R_{jA}\right) = \frac{\partial}{\partial R_{jA}}\left(\theta R_{jA}\right) = \theta > 0$ for all complements (for $\theta V < R_{jB} \leq V$).

Proof of Proposition 2.3:

The first round winner's second round bid is $b_{jB}=R_{jAB}-R_{jA}$ (from the equilibria already established).

Note that $b_{jB}-R_{jB} = R_{jAB}-R_{jA} - R_{jB} = \theta(R_{jA}+R_{jB})$ which is positive for all $\theta>0$. This proves Proposition 3 (a[i]). Part 3(a[ii]) follows immediately because $b_{jB}-R_{jB}$ depends only on j's valuations and θ, and not on b_{jA} or the bids of the other players.

As shown earlier, first round losers second round bids are $b'_{jB}=R_{jB}$. That is, the bids equal their individual reservation prices for B.

APPENDIX D

Bidding under Risk Aversion: Proofs of Equilibrium Bids and Propositions 2.4(b) & 2.5

In a market of maximally risk averse bidders, the equilibrium bids are (i) $b_{jA}^{**} = Max\{R_{jA}, R_{jAB} - V\}$ for all bidders for the first object *A*; (ii) $b_{jB}^{**} = R_{jAB} - R_{jA}$, the first round winner's bid for object B; and (iii) $b_{jB}^{'**} = R_{jB}$, the first round loser's second round bid.

As Proposition 5 pertains to the equilibrium bids, I will prove them jointly.

Maximal risk aversion is the case when the bidder never makes (and never intends to make) a loss.

The second round bids $b_{jB}^{**} = R_{jAB} - R_{jA}; b_{jB}^{'**} = R_{jB}$ from Appendix A are weakly dominant and independent of the bidders' risk preferences; it was shown that the bidder can *not* bid any better for the second object. Thus the results hold with maximal risk aversion, proving Proposition 5(b).

On the first round bids: Suppose bidder *j* bids higher than $Max\{R_{jA}, R_{jAB} - V\}$. For instance consider a bid $b^{\oplus} = b_{jA}^{**} + \delta = Max\{R_{jA}, R_{jAB} - V\} + \delta$.

If bidder *j* wins the first object at a price $p^{\oplus} \in \lfloor b_{jA}^{**}, b^{\oplus} \rfloor$ and loses the second object (or wins the object at the amount bid for the second object), her net surplus is negative. Therefore, as a maximally risk averse bidder, any bid greater than b_{jA}^{**} is suboptimal. Now how about a lower bid?

Consider a bid $b^{\otimes}$ less than b_{jA}^{**}. I show that the bidder can never be better off but could be worse off. If a highest bid $p^{\otimes}$ from among the other *N* bidders is between $b^{\otimes}$ and b_{jA}^{**}, then *j* lost the opportunity to win *A* and earn a positive surplus. If $p^{\otimes}$ is below both $b^{\otimes}$ and b_{jA}^{**}, then *j* got no additional benefit by bidding $b^{\otimes}$ and not b_{jA}^{**}. Thus, b_{jA}^{**} is weakly dominant.

When $Max\{R_{jA}, R_{jAB} - V\}$ equals $R_{jAB} - V$, it means that *j*'s first bid is greater than her reservation price R_{jA}, proving proposition 5(a).

Further, conforming to maximal risk aversion, bidder *j* will not make a loss with the above bid combination $\{b_{jA}^{**}, b_{jB}^{**}, b_{jB}^{'**}\}$. When $Max\{R_{jA}, R_{jAB} - V\}$ is R_{jA}, *j* makes a non-negative surplus at the end of the first round irrespective of winning or losing *A*. If *j* wins A with this bid, then a second round win at R_{jAB}-R_{jA} means a total payout no greater than a total payout no greater than R_{jAB}. When $Max\{R_{jA}, R_{jAB} - V\}$ is $R_{jAB} - V$, it means *j* has a reserve amount *V* for the second auction. If *j* wins the first auction, a win on the second is assured – again for a total payout no greater than R_{jAB}. When *j* loses the first round, then her second round bid is her valuation R_{jA}; i.e., no chance of ending up with a negative surplus.

Proof of Proposition 2.4(b):
Here I show that the equilibrium bidding strategy when the bidders are maximally risk averse converges to that with risk neutral bidders when the number of bidders is large. Since the optimal bids for the second object are the same under both cases, our proof is limited to the first bid. Mathematically, I need to show that $\lim_{N+1\to\infty} b^{*}_{jA} = b^{**}_{jA}$ or $\lim_{N+1\to\infty}(b^{*}_{jA} - b^{**}_{jA}) = 0$ where the superscript * and ** represent the risk neutral and maximally risk averse situations respectively.
I consider compare each of the four cases across equations (C-7) and (C-8) with the corresponding maximally risk averse bids.

When (R_{jAB}-R_{jA})<V and $\theta V>R_{jB}$:

$$R_{jAB} - R_{jA} - V < 0 \Rightarrow R_{jAB} - V < R_{jA} \Rightarrow b^{**}_{jA} = Max\{R_{jA}, R_{jAB} - V\} = R_{jA}$$

$$b^{*}_{jA} = R_{jA} + \frac{(R_{jAB} - R_{jA})^{N+1}}{(N+1)V^{N}} - \frac{R_{jB}^{N+2}}{2(N+2)V^{N+1}\theta(1+\theta)} \text{(when } \theta V>R_{jB}\text{)}$$

$$\text{When } (N+1) \to \infty: (b^{*}_{jA} - b^{**}_{jA}) = \frac{(R_{jAB} - R_{jA})^{N+1}}{(N+1)V^{N}} - \frac{R_{jB}^{N+2}}{2(N+2)V^{N+1}\theta(1+\theta)}$$

$$= \frac{(R_{jAB} - R_{jA})}{N+1}\left(\frac{R_{jAB} - R_{jA}}{V}\right)^{N} - \left(\frac{R_{jB}}{\theta V}\right)\left(\frac{R_{jB}}{V}\right)^{N+1}\left(\frac{1}{2(1+\theta)}\right) \to 0$$

(as each term in the last expression tends to zero individually as $\theta V>R_{jB}$ and (R_{jAB}-R_{jA})<V.)

When (R_{jAB}-R_{jA})<V and $\theta V>R_{jB}$: (Again, $b^{**}_{jA} = R_{jA}$ as above; b^{*}_{jA} is as in the 2nd part of eqn. (C-8))
Thus as N goes to infinity,

$$\text{When } (N+1) \to \infty: (b^{*}_{jA} - b^{**}_{jA}) = -\frac{R_{jB}}{(N+1)}\left(\frac{R_{jB}}{V}\right)^{N}\left(\frac{1}{1+\theta}\right) + \frac{\theta V}{2N}\left(\frac{R_{jB}}{V}\right)^{N}\left(\frac{1}{1+\theta}\right)$$

$$- \frac{V\theta^{N}}{N(N+1)(N+2)(1+\theta)} + \frac{(R_{jAB} - R_{jA})}{N+1}\left(\frac{R_{jAB} - R_{jA}}{V}\right)^{N} \to 0$$

as each individual term goes to zero because $\theta V \leq R_{jB} \leq V$ and (R_{jAB}-R_{jA})<V.

When (R_{jAB}-R_{jA})>V and $\theta V<R_{jB}$: (Now $b^{**}_{jA} = R_{jAB} - V$; b^{*}_{jA} is as in the 1st part of eqn. (C-9))

$$\text{When } (N+1) \to \infty: (b^{*}_{jA} - b^{**}_{jA}) = \left(\frac{V}{N+1}\right) - \left(\frac{R_{jB}}{\theta V}\right)\left(\frac{R_{jB}}{V}\right)^{N+1}\left(\frac{1}{2(1+\theta)}\right) \to 0$$

When $(R_{jAB}$-$R_{jA})$>V and $\theta V<R_{jB}$: $(b_{jA}^{**}=R_{jAB}-V$ as above; b_{jA}^{*} is as in the 2nd part of eqn. (C-9))

$$\text{When}\,(N+1)\rightarrow\infty:(b_{jA}^{*}-b_{jA}^{**})=\left(\frac{V}{N+1}\right)-\frac{R_{jB}}{(N+1)}\left(\frac{R_{jB}}{V}\right)^{N}\left(\frac{1}{1+\theta}\right)+\frac{\theta V}{2N}\left(\frac{R_{jB}}{V}\right)^{N}\left(\frac{1}{1+\theta}\right)$$

$$-\frac{V\theta^{N}}{N(N+1)(N+2)(1+\theta)}\rightarrow 0$$

APPENDIX E

Asymmetry in Market Level Valuations: Proofs of Equilibrium Bids and Proposition 2.6(b)

The reservation prices for the higher valued object, *A* (say), are distributed U [0, *V*] and those for the lower valued object *B* are distributed U [0, *v*] with $v<V$. In 2.5.1, *v* is set to 0.5*V*. The model is otherwise similar to the main model. The equilibrium bids when the objects are auctioned as a bundle equal the reservation prices for the bundle, as before. I focus here on sequential auctions.

The proof is analogous to that in Appendix C. So while omitting a formal derivation here, I draw attention to the following points:

- The equilibrium second bids correspond to those under symmetry as the decision problem facing the bidders at the start of the second auction is identical in both cases. The strategic considerations of the bidders are similar as the winner of the first bid is the only one with the possible opportunity to tap the value of complementarity
- The presence of two "parameters" *v* and *V* increases the number of different thresholds we must consider and makes the analytical derivation more involved.

The equilibria when object A is auctioned first are as follows:

For object *B*: $b_{jB}=R_{jAB}-R_{jA}$ when *j* is the first round winner; $b'_{jB}=R_{jB}$ when *j* is the first round loser.

For object *A*:

When $(R_{jAB}-R_{jA})<v$ and $\theta V<(1+\theta)v$:

$$b^*_{jA}=\begin{cases} R_{jA}+\dfrac{\left(R_{jAB}-R_{jA}\right)^{N+1}}{(N+1)v^N}-\dfrac{R_{jB}^{N+2}}{2(N+2)Vv^N\theta(1+\theta)} & \text{when } R_{jB}<\theta V \\ R_{jA}+\dfrac{\left(R_{jAB}-R_{jA}\right)^{N+1}}{(N+1)v^N}-\dfrac{2N(N+2)R_{jB}^{N+1}-(N+1)(N+2)\theta VR_{jB}^{N}+2(\theta V)^{N+1}}{2N(N+1)(N+2)v^N(1+\theta)} & \text{when } \theta V\le R_{jB}<V \end{cases}$$

When $(R_{jAB}-R_{jA})<v$ and $\theta V>(1+\theta)v$:

$$b^*_{jA}=R_{jA}+\frac{\left(R_{jAB}-R_{jA}\right)^{N+1}}{(N+1)v^N}-\frac{R_{jB}^{N+2}}{2(N+2)Vv^N\theta(1+\theta)}$$

When $(R_{jAB}-R_{jA})\ge v$ and $\theta V<(1+\theta)v$:

$$b_{jA}^{*}=\begin{cases} R_{jA}-\left(\dfrac{N}{N+1}\right)v-\dfrac{R_{jB}^{N+2}}{2(N+2)Vv^{N}\theta(1+\theta)} & \text{when } R_{jB}<\theta V \\ R_{jA}-\left(\dfrac{N}{N+1}\right)v-\dfrac{2N(N+2)R_{jB}^{N+1}-(N+1)(N+2)\theta VR_{jB}^{N}+2(\theta V)^{N+1}}{2N(N+1)(N+2)v^{N}(1+\theta)} & \text{when } \theta V\le R_{jB}<v \end{cases}$$

When $(R_{jAB}$-$R_{jA})\geq v$ and $\theta V>(1+\theta)v$:

$$b_{jA}^{*}=R_{jA}-\left(\frac{N}{N+1}\right)v-\frac{R_{jB}^{N+2}}{2(N+2)Vv^{N}\theta(1+\theta)}.$$

Proof of Proposition 2.6(b):
To show that with sequential auctions involving asymmetrically valued objects, it is revenue maximizing for the seller to auction the higher valued object first.

I prove the proposition graphically. I consider θ values of 0.1 and 0.2 over which sequential auctions are typically better for the seller than the auction of the bundle. For each combination (N+1, θ) in the range shown below, I simulate 30,000 auctions each for two scenarios: when the higher valued object is auction first or second. I compare the expected revenues from the two sequences. The results are shown in figure 5.

..……………………………..Insert Figure 5 about here…...………………………………

The % revenue gain along the vertical axis below is the percentage increase in revenue to the seller by auctioning the higher valued object first and the lower valued object second, and not vice versa.

The graph is positive everywhere confirming the proposition. Some higher order effect can also be seen. Notably, the significance of sequencing is higher when the degree of complementarity is greater. The importance of sequencing is diminishing in the size of the bidder pool.

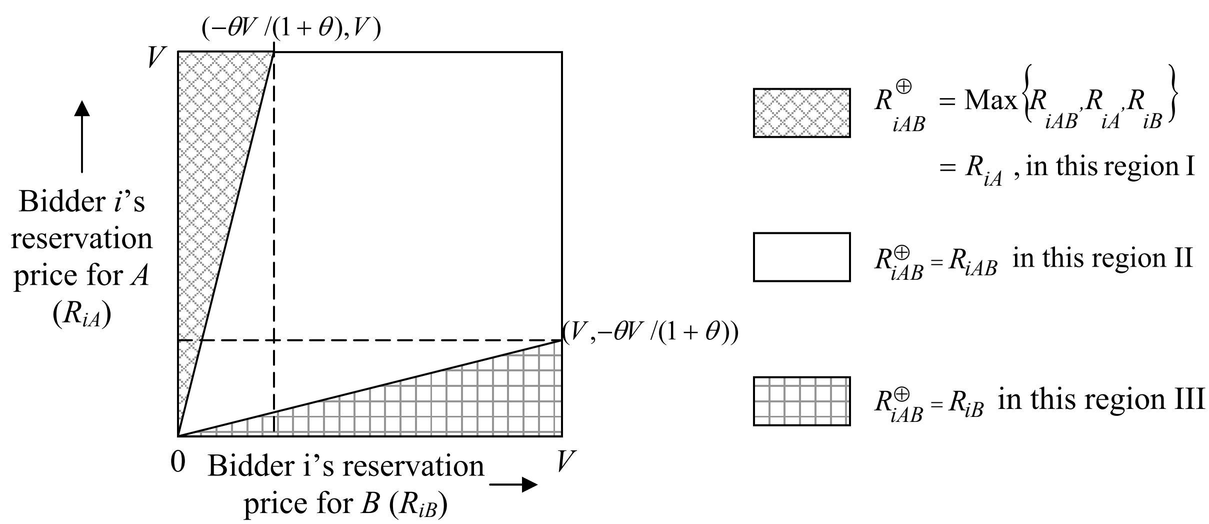
APPENDIX F

Bidding for Substitutes: Proofs of Equilibrium Bids

The proofs closely parallel those in Appendix C. The key difference is that the sub-additive valuation of the bundle (reflected by the negative θ) may make one item unattractive to the holder of the other item. Thus, in modifying Appendix A for substitutes, I replace R_{jAB} in equation (C-1) to $R^{\oplus}_{jAB} = \text{Max}\{R_{jAB}, R_{jA}, R_{jB}\}$. This reflects the free disposal property discussed in section 3.5.2. When the objects are auctioned as a bundle (i.e., a single composite unit), bidding $R^{\oplus}_{jAB}$ is a weakly dominant strategy. The following proof relates to sequential auctions.

Akin to that in Appendix A for complements (for similar reasons), the bidder j's optimal second bid when she is the first round winner is $b^*_{jB} = R^{\oplus}_{jAB} - R_{jA}$; if the first auction is lost, she bids $b'^*_{jB} = R_{jB}$.

In arriving at the equilibrium first bid, three possibilities must be identified (see Figure below):



Now consider the case when $R^{\oplus}_{jAB} = R_{jA}$; that is, A taken alone is the most preferred. Therefore, j gains by bidding this amount for it. If j wins the first round, then she should bid an amount of zero for B (as $b^*_{jB} = (R^{\oplus}_{jAB} - R_{jA}) = 0$). If j loses on the first round, then $b'^*_{jB} = R_{jB}$.

When $R^{\oplus}_{jAB} \neq R_{jA}$, I assume as before that the highest bid for the first object has an everywhere differentiable cdf. The optimal bid function is given by:

$$b_{jA}^{*} = R_{jA} + \frac{\left(R_{jAB} - R_{jA}\right)^{N+1}}{(N+1)V^{N}} - \int_{0}^{R_{jB}} H(t)dt \qquad \text{(F-1)}$$

We need to evaluate the term H(z) where $z = Max\{Max\{b'_{iB}\}_{i \neq j,k}, b_{kB}\}$, to identify the equilibrium.

The bidder's problem in the case of substitutes is thus largely the same as with complements. However, there are some differences. First, with substitutes, R_{jAB}-$R_{jA} < V$. Second, there is the issue of free disposal. The latter plays a role in the evaluation of H(z) when bidder j decides the bid for the second object after losing the first. To do so, we need the distribution of the random variable $b_{kB} = R^{\oplus}_{kAB} - R_{kA} = x$ where k is the winner of A. I obtain this from the above diagram in which $R^{\oplus}_{kAB} - R_{kA} = 0$ in region I, $R^{\oplus}_{kAB} - R_{kA} = R_{kAB} - R_{kA}$ in II and $R^{\oplus}_{kAB} - R_{kA} = R_{kB} - R_{kA}$ in III.

This figure allows us to evaluate the following probabilities:

$P[R_{kAB} - R_{kA} < x \mid R^{\oplus}_{kAB} = R_{kAB}].P[R^{\oplus}_{kAB} = R_{kAB}]$

$P[R_{kB} - R_{kA} < x \mid R^{\oplus}_{kAB} = R_{kB}].P[R^{\oplus}_{kAB} = R_{kB}]$ and

$P[R_{kA} - R_{kA} < x \mid R^{\oplus}_{kAB} = R_{kA}].P[R^{\oplus}_{kAB} = R_{iA}] = P[R^{\oplus}_{kAB} = R_{kA}]$.

This allows us to evaluate $Max(Max\{b'_{iA, i \neq i,k}\}, b_{kB})$ and solve for b^{*}_{jA} in equation (F-1). This yields us the following equilibrium for the first round bid:

1. When $R^{\oplus}_{jAB} = R_{jA}$: $\quad b_{jA}^{*} = R_{jA},$

2. When $R^{\oplus}_{jAB} = R_{jB}$:

Case 2a: $0 \leq R_{jB} \leq (1+2\theta)V$

$$b_{jA}^{*} = R_{jA} + \frac{\left(R_{jB} - R_{jA}\right)^{N+1}}{(N+1)V^{N}} - \frac{1}{(1+\theta)}\left[\frac{\theta R^{N}_{jB}}{2NV^{N-1}} + \frac{(1+2\theta)R^{N+1}_{jB}}{(N+1)V^{N}} - \frac{3\theta^{2} R^{N+2}_{jB}}{2(N+2)V^{N+1}(1+2\theta)}\right]$$

Case 2b: $(1+2\theta)V \leq (R_{jB}-R_{jA}) \leq (1+2\theta)V/(1+\theta)$

$$b_{jA}^{*} = R_{jA} + \frac{\left(R_{jB} - R_{jA}\right)^{N+1}}{(N+1)V^{N}} - \frac{(2-3\theta)}{2(1+\theta)}\frac{R^{N}_{jB}}{NV^{N-1}} - \frac{\theta}{2(1+2\theta)}\left(\frac{R^{N+2}_{jB}}{(N+2)V^{N+1}}\right)$$

Case 2c[19]: $V(1+2\theta)/(1+\theta) < R_{jB} - R_{jA} < V$

$$b_{jA}^{*} = R_{jA} + \frac{\left(R_{jB} - R_{jA}\right)^{N+1}}{(N+1)V^{N}} - \frac{R^{N}_{jB}}{NV^{N-1}} + \frac{R^{N+2}_{jB}}{2(N+2)V^{N+1}} - \frac{R^{N+1}_{jB}}{(N+1)V^{N}}$$

[19] In this expression, the value of $P[(1+2\theta)V \leq R_{jAB} - R_{jA} \leq (1+2\theta)V/(1+\theta)] = [(1+2\theta)/(1+\theta)](\theta/2)$ is very small and we have set it to zero.

3. When $R^{\oplus}_{jAB} = R_{jAB}$

Case 3a: $0 \le R_{jB} \le (1+2\theta)V$

$$b_{jA}{}^{*} = R_{jA} + \frac{\left(R_{jAB} - R_{jA}\right)^{N+1}}{(N+1)V^{N}} - \frac{1}{(1+\theta)}\left[\frac{\theta R_{jB}^{N}}{2NV^{N-1}} + \frac{(1+2\theta)R_{jB}^{N+1}}{(N+1)V^{N}} - \frac{3\theta^{2} R_{jB}^{N+2}}{2(N+2)V^{N+1}(1+2\theta)}\right]$$

Case 3b: $(1+2\theta)\, V \le R_{jB}\text{-}R_{jA} \le (1+2\theta)V/(1+\theta)$

$$b_{jA}{}^{*} = R_{jA} + \frac{\left(R_{jAB} - R_{jA}\right)^{N+1}}{(N+1)V^{N}} - \frac{(2-3\theta)}{2(1+\theta)}\frac{R_{jB}^{N}}{NV^{N-1}} - \frac{\theta}{2(1+2\theta)}\left(\frac{R_{jB}^{N+2}}{(N+2)V^{N+1}}\right)$$

Table 1

Essay 1 and Essay 2: A Contrast

	Essay 1	***Essay 2***
Object Nature	Independent and interdependent values	Independent private valuation only
Auction Format	Comparison of English outcry and hybrid auction formats	Second price sealed bid auction
Number of bidders	Exogenous	Exogenous but considered as a variable
Number of objects	Single item	Two items
Seller Problem	Choice between auction formats	Choice between bundling/unbundling

Table 2

Essay 1: Summary of Data Collected for eBay and Yahoo

(n=200 matched auctions)

Data	Yahoo	eBay	Difference
Number of Bidders (Average)	5.26	7.00	Significant**
Presence of Common value	NA	138	
Auction Duration (Average)	5.55 (Mode = 7 Days)	5.63 (Mode =7 Days)	Not Significant
Presence of Last Minute Bidding	62 out of 200	88 out of 200	Significant*

* indicates significance at p<. 05
** indicates significance at p<. 01

Table 3

Essay 1: Summary of Restrictions Tested for Propositions 1,2,3 and 4

Short Summary of Propositions	Constraints Tested	Result if proposition is supported	Observed Result
Proposition 1.1: With Common Values, Revenue Greater for eBay than for Yahoo.	$\alpha_1=\alpha_2$ (1a) $\alpha_1+b_1=\alpha_2'$ (1b) Model 1a and 1b	Reject (expect $\alpha_1>\alpha_2$) Reject (expect $\alpha_1 + b_1> \alpha_2'$). Also expect first inequality ($\alpha_1>\alpha_2$) to be stronger than the second ($\alpha_1 + b_1> \alpha_2'$).	Both restrictions rejected and second inequality stronger as seen by F statistic. Parameter for Common value in eBay positive and significant.
Proposition 1.2: Revenue Increases with number of bidders.	$b_2=0$ $\gamma_2=0$ Model 1a and 1b	Reject (expect b_2, $\gamma_2 >0$)	Proposition partially supported for Yahoo! only as coefficient (b2) for eBay not significant. (Details in Table 3)
Proposition 1.3: Rate of increase of revenue with duration greater for Yahoo than for eBay.	$\gamma_1 = \lambda_1$, $\gamma_2 = \lambda_2$ Models 2a and 2b	Reject (expect $\gamma_1 > \lambda_1$ and $\gamma_2 > \lambda_2$) in models 2a and 2b	We see $\gamma_1 > \lambda_1$ and $\gamma_2 > \lambda_2$ Both results in the expected direction and proposition supported
Proposition 1.4: Impact of Last Minute Bidding greater in eBay than in Yahoo.	$\gamma'_1 = \lambda'_1$, $\gamma'_2 = \lambda'_2$ Models 3a and 3b	Reject (expect $\gamma'_1 > \lambda'_1$ and $\gamma'_2 < \lambda'_2$) in models 3a and 3b	We see $\gamma'_1 > \lambda'_1$ and $\gamma'_2 < \lambda'_2$ Both results in expected direction and proposition supported

Table 4

Essay 1: Results of sequentially relaxing constraints on coefficients of Model 1a and 1b for testing proposition 1.1

	Test 1		Test 2		Test 3		Test 4		Test 5		Test 6	
Restrictions (refer Models 1a and 1b)	$\gamma_2=b_2$, $\gamma_3=b_3$, $\gamma_4=b_4$, $\gamma_5=b_5$, $\gamma_6=b_6$		$\gamma_3=b_3$, $\gamma_4=b_4$, $\gamma_5=b_5$, $\gamma_6=b_6$		$\gamma_3=b_3$, $\gamma_5=b_5$, $\gamma_6=b_6$		$\gamma_3=b_3$, $\gamma_6=b_6$		$\gamma_3=b_3$		Unrestricted	
Restriction Tested	$\alpha_1=\alpha_2$	$\alpha_1+b_1=\alpha_2$	$\alpha_1=\alpha_2$	$\alpha_1+b_1=\alpha_2$	$\alpha_1=\alpha_2$	$\alpha_1+b_1=\alpha_2$	$\alpha_1=\alpha_2$	$\alpha_1+b_1=\alpha_2$	$\alpha_1=\alpha_2$	$\alpha_1+b_1=\alpha_2$	$\alpha_1=\alpha_2$	$\alpha_1+b_1=\alpha_2$
	Without CV	With CV	Without CV	With CV	Without CV	With CV	Without CV	With CV	Without CV	With CV	Without CV	With CV
Ebay	A_1	A_2	B_1	B_2	C_1	C_2	D_1	D_2	E_1	E_2	F_1	F_2
Intercept (α_1)	323.93***	295.53***	323.81***	289.07***	323.75***	288.41***	320.63***	285.53***	320.67***	285.60***	309.84***	274.97***
COMM(b_1)		42.07*		51.28**		52.16**		51.87**		51.80**		51.65**
BIDS (b_2)	4.53	3.35	3.30	0.62	2.93	0.12	3.31	0.52	3.20	0.44	2.76	.02
LAST(b_3)	-7.90	-7.24	-7.53	-6.35	-7.59	-6.42	-6.02	-4.88	-5.87	-4.78	23.21	2.46
RESERVE (b_4)	0.22**	0.22**	0.23***	0.23***	0.16**	0.15*	0.15*	0.13	0.15*	0.13	0.15*	0.13
BUYNOW (b_5)	7.37	9.78	6.62	8.92	8.99	11.81	80.62	82.61	77.77	80.61	56.38	59.42
DUR (b_6)	2.98	2.74	3.33	3.41	3.69	3.83	3.62	3.77	2.55	3.03	1.99	2.46
Yahoo												
Intercept (α_2)	250.53***	295.53***	250.05***	249.68***	249.53***	249.05***	251.02***	250.51***	250.91***	250.44***	255.72***	255.17**
JUMP(γ_1)	5.87	2.25	8.55	6.85	11.10	9.98	8.16	7.10	8.82	7.56	14.43	13.10
BIDS (γ_2)	4.53	3.35	5.75*	5.56*	6.06**	5.94**	6.41**	6.28**	6.58**	6.40**	7.42**	7.23**
LAST(γ_3)	-7.90	-7.24	-7.53	-6.35	-7.59	-6.43	-6.02	-4.88	-5.87	-4.78	-24.08	-22.71
RESERVE (γ_4)	0.22**	0.22**	0.23***	0.23***	0.24***	0.24***	0.25***	0.25***	0.25***	0.25***	0.25***	0.25***
BUY PRICE (γ_5)	7.37	9.78	6.62	8.92	8.99	11.81	-7.02	-3.95	-7.14	-4.03	-9.37	-6.27
DUR (γ_6)	2.98	2.74	3.33	3.41	3.69	3.84	3.62	3.77	3.89	3.96	3.84	3.91
F Value for Restriction	22.95	25.70	22.64	26.27	21.55	25.11	17.81	21.52	17.83	21.51	7.58	11.01
P-value	.005	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0062	.0010

Table 5

Results for Regression 2

Model 2a $EPrice=\gamma_1 t + \gamma_2 t^2 + \varepsilon$

Model 2b $YPrice=\lambda_1 t + \lambda_2 t^2 + \varepsilon$

t➔time elapsed from first bid as a fraction of total duration

Model	Parameter	Estimate	Adjusted R^2
2a	γ_1	0.64***	0.78
	γ_2	0.36***	
2b	λ_1	0.45***	0.71
	λ_2	0.55***	

* indicates significance at P<. 10
** indicates significance at p<.05
***indicates significance at P<. 01

Results for Regression 3

Model 3a $EPrice'=\gamma'_1 t + \gamma'_2 t^2 + \varepsilon$

Model 3b $YPrice'=\lambda'_1 t + \lambda'_2 t^2 + \varepsilon$

t➔time elapsed after 80% of the auction duration is over.
EPrice, YPrice: Price above the price when 80% of the auction was over in eBay and Yahoo respectively..

Model	Parameter	Estimate	Adjusted R^2
2a	γ'_1	0.35***	0.78
	γ'_2	0.65***	
2b	λ'_1	0.03	0.71
	λ'_2	0.97***	

* indicates significance at P<. 10
** indicates significance at p<.05
***indicates significance at P<. 01

Table 6

Essay 2: Positioning Essay 2 vis-à-vis Extant Articles on Multi-Object Auctions

Study	Scope/Key Findings	Distinctive Aspects of Our Study
Benoit & Krishna (2001)	• In an English auction setting, when bidders are budget constrained and have common value of two complements or substitutes, B&K demonstrate anecdotally that a bundled (sequential) auction is revenue maximizing for strong complements (otherwise)	• Our bidders have independent private values and are not budget constrained, but follow incentive compatibility & individual rationality • Unlike B&K, I derive closed form Nash equilibrium bidding strategies; results underscore a wide domain of optimality for sequential auctions even w|o budget constraints
Bernhardt & Scoones (1994)	• Given a two-object case with at least three bidders in a Vickrey sealed-bid auction setting, the seller's expected revenues are greater when the object for which buyers' valuations are more dispersed is auctioned first	• Interrelatedness and bidders' risk propensity that I model are not within the scope of the B&S study • I assume that the dispersions in valuations for the two objects are similar
Chakrabo-rty (1999, 2002)	• With additive valuations for two non-identical objects, in a Vickrey auction when bidder have independent private value, unbundling (bundling) is more profitable when the number of bidders is below (exceeds) a critical number • When bidders have common values, bundling additionally reduces the problem of winner's curse	• Our model considers complements and substitutes with independent objects as a special case. • Though the objects are IPV, there is an analogous situation as that of the winners curse as it is optimal for the bidders to bid more than their valuations so that there is a possibility for the bidder to incur losses.
Feng & Chatterjee (2002)	• When each of several risk neutral bidder seeks only one good at the most in a Vickrey auction, the price decline anomaly is possible • Sequential (or simultaneous) auctions are optimal when the bidder to good ratio is small (or large)	• F&C do not examine interrelatedness and bidders' risk propensity that I model • Issues such as bidders' uncertainty over the number of auctions and product quantity, and signaling are not within our purview
Hausch (1986)	• The optimality of sequential vs. simultaneous auctions is examined for two identical objects and with two bidders, within on a signaling framework • Sequential (or simultaneous) auctions increase seller's expected revenue when the first auction is (not) expected to be "non-revealing"	• The impact of interrelatedness among objects (complements vs. substitutes), the number of bidders and their risk propensity is not within Hausch's purview • Bidders in our setting have unambiguous and independent private value. The scope and nature of our results are different
Levin (1997)	• In a mechanism design/optimal auctions context, given a pair of complements, a seller is always better off selling them as a bundle. In this study, the valuations of the objects are	• Our model is flexible in that the valuations are independent realizations so that there is reduction of heterogeneity in the valuation of bundles yielding different

	driven by a common iid signal.	results.
Palfrey (1983)	• For a monopolist with one unit of each of several independent goods, a bundled auction is preferable in a Vickrey auction setting when there only a "small" number of buyers.	• Unlike Palfrey, I examine the products' interrelatedness and bidders' risk propensity (although I do not examine welfare issues) • I demonstrate the unbundling has greater appeal for the seller than suggested in Palfrey
This Study	• Sequential, unbundled auctions maximize seller's revenue even for moderate complements when there are at least a few bidders (four or more) • The domain of sequential auctions is larger when the bidders are risk neutral (and not risk averse) • When object valuations are asymmetric, the more valuable good should be auctioned first	Not Applicable

Figure 1

Essay 1: Progression of Price With Respect to Time in eBay and Yahoo! for Model 2 and Model 3[20]

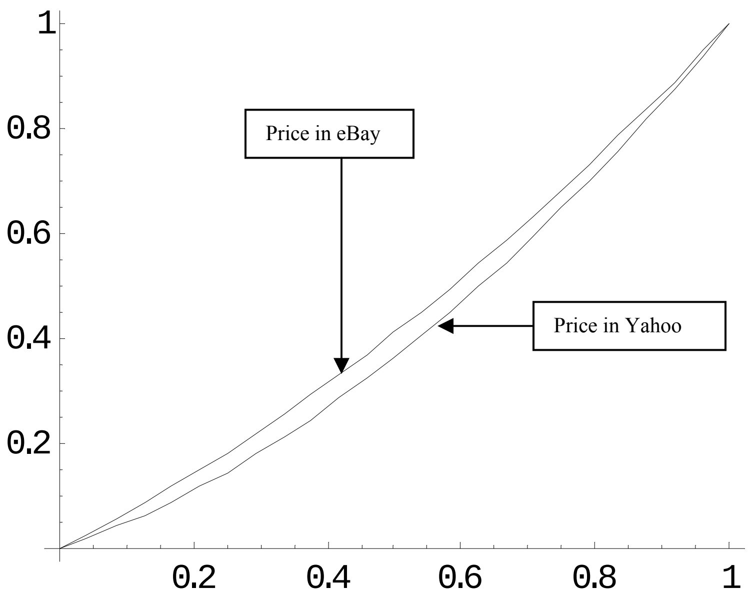


Figure 1a

X axis: Time elapsed as a fraction of total time
Y axis: Price reached at corresponding time as a fraction of final price

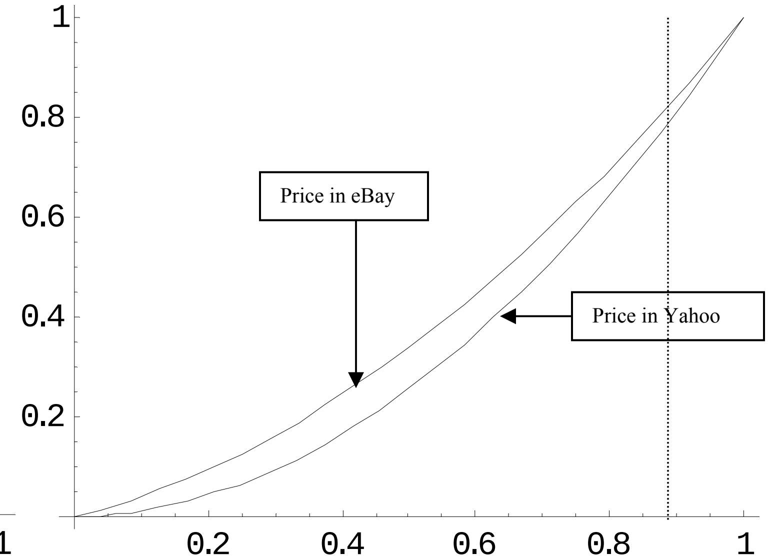


Figure 1b

X axis: Time elapsed after 80% of the auction is over as a fraction of the total time (last 20%)
Y axis: Price beyond the price at the 80% stage of the auction duration as a fraction of final price

[20] The X-axis is time as a fraction of total time and the Y-axis is the outstanding price as a fraction of , the point (0.50,0.75) would indicate that at the halfway mark in the auction, the outstanding price attained is ¾ of the final price realized. Again, in 1b, (0.9,0.5) would indicate that after 98% of the time is over (as (0.98-0.8)/0.20=0.9), the outstanding price is halfway between the final price and the price when 80% of the auction was over. The first graph (a) is for the entire auction duration. The second graph (b) is for the final fifth of the duration (i.e. price progression after 4/5 of the auction is over). These plots show that (i) In general, the prices in eBay move towards the final price faster than that in Yahoo and (ii) The difference in the speed of convergence towards the final price in eBay is more
pronounced during the final stages of the auction.

Figure 2

Essay 2: The Sequential Bidding Game in Essay 2

Note: 1. Degree of interrelatedness θ and number of bidders N+1 are common knowledge.
2. Individual level reservation prices R_{jA}, R_{jB} for objects A and B are private value.
3. Seller moves first and decides whether to bundle or not.

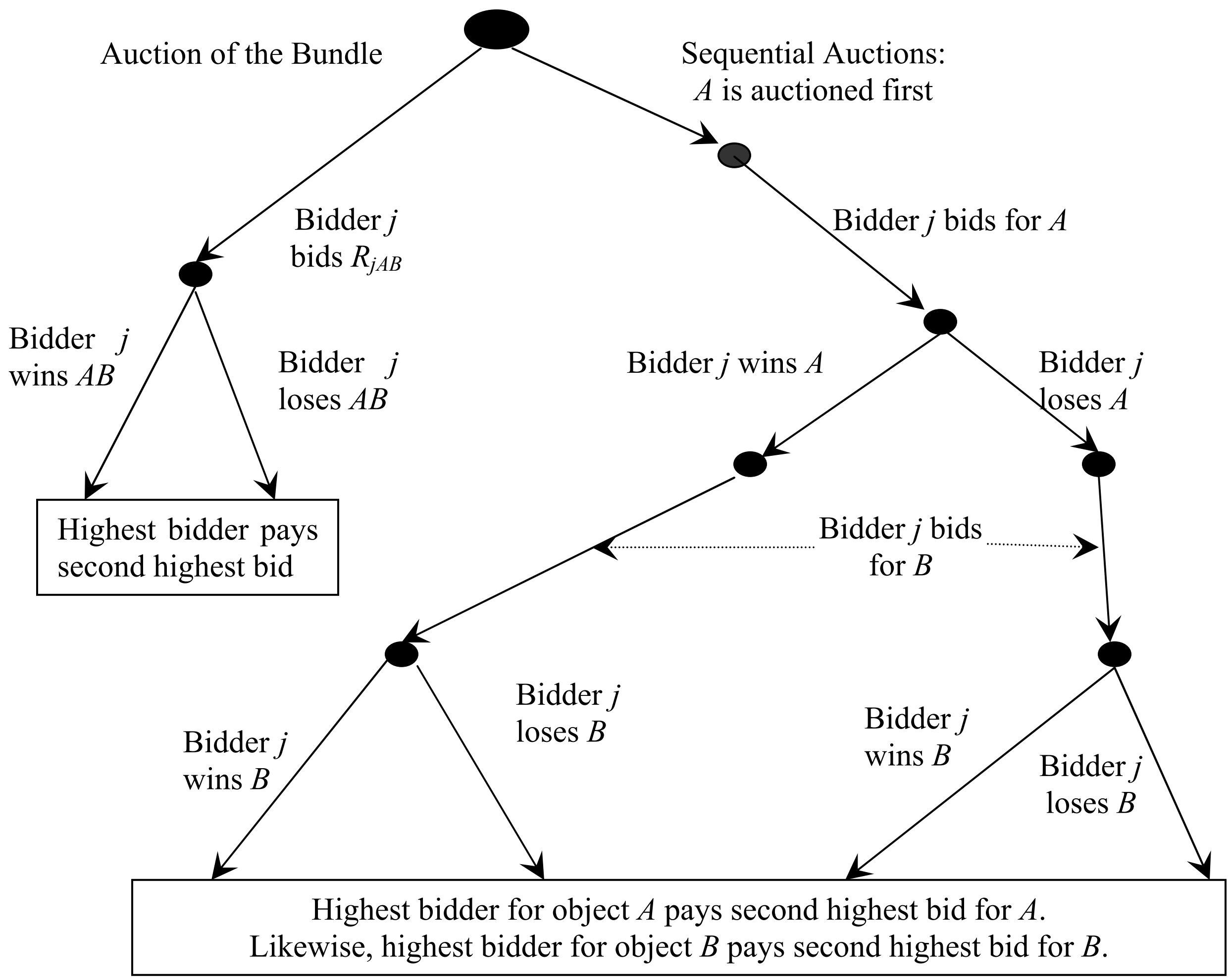
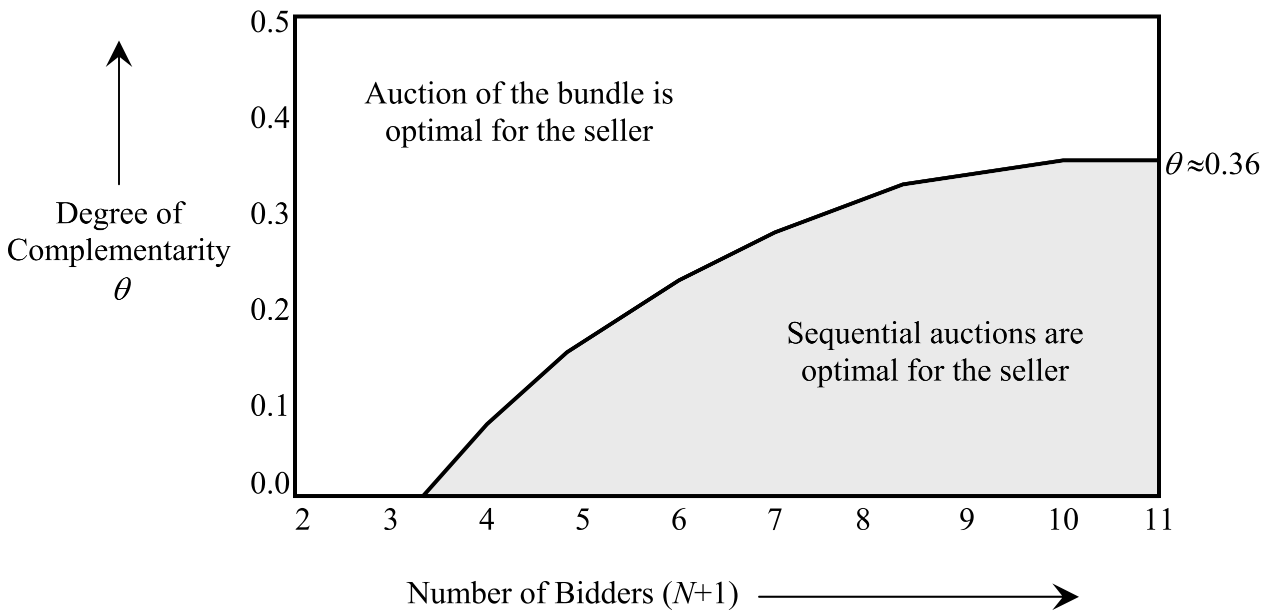


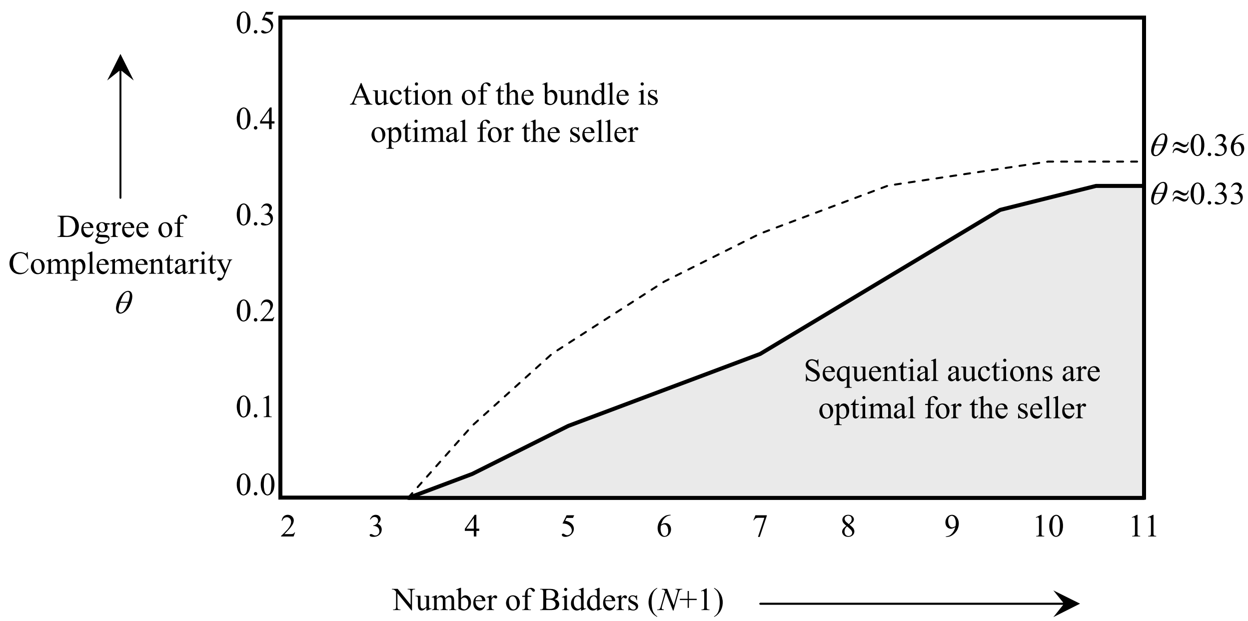
Figure 3

Essay 2: Optimality of Sequential and Bundled Auctions for Complements

<u>Figure 3A</u>: Main Model with Risk Neutral Bidders



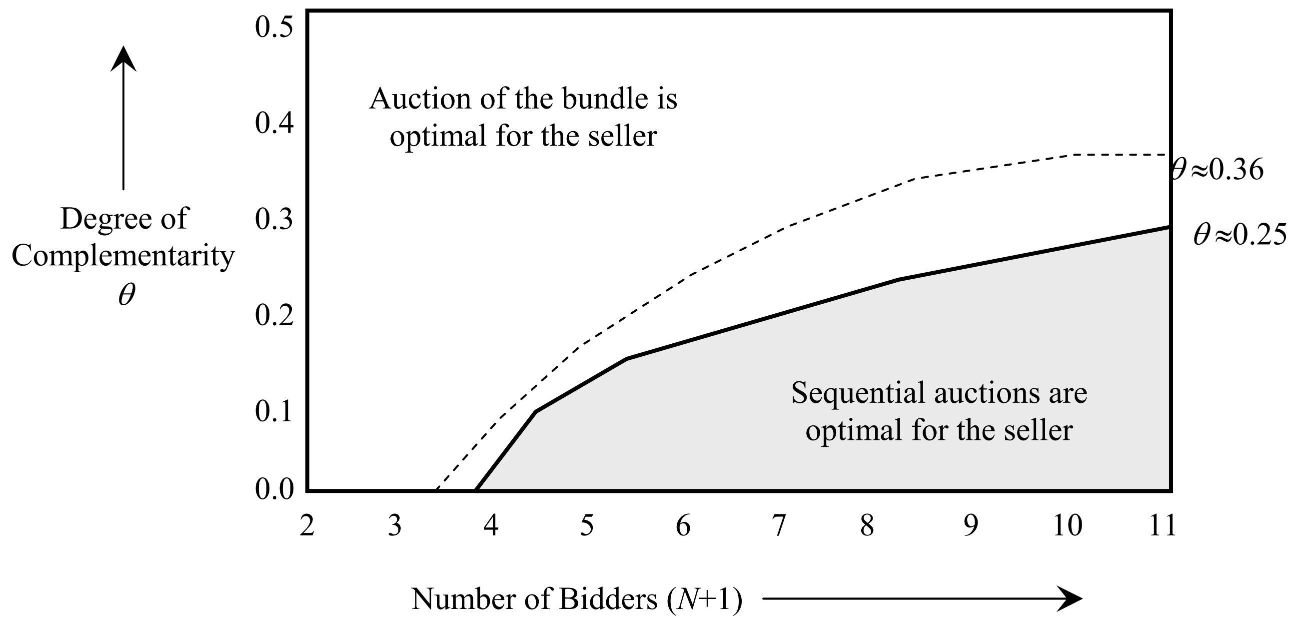
<u>Figure 3B</u>: Extension with Maximally Risk Averse Bidders



(<u>Note</u>: The dotted line in Figure 3B represents the indifference frontier for the risk neutral case from Figure 3A.)

Figure 4

Essay 2: Asymmetric Complements: Optimality of Sequential and Bundled Auctions (Bidders are Risk Neutral)



(<u>Note</u>:
1. The dotted line in the figure represents the indifference frontier for the symmetric case from Figure 4A.
2. Reservation prices for the more valuable object are distributed U [0, V] while those for the other object are U [0, 0.5V]. The higher valued object is auctioned first.)

Figure 5

Essay 2: Asymmetric Objects: Optimal vs Suboptimal Sequencing

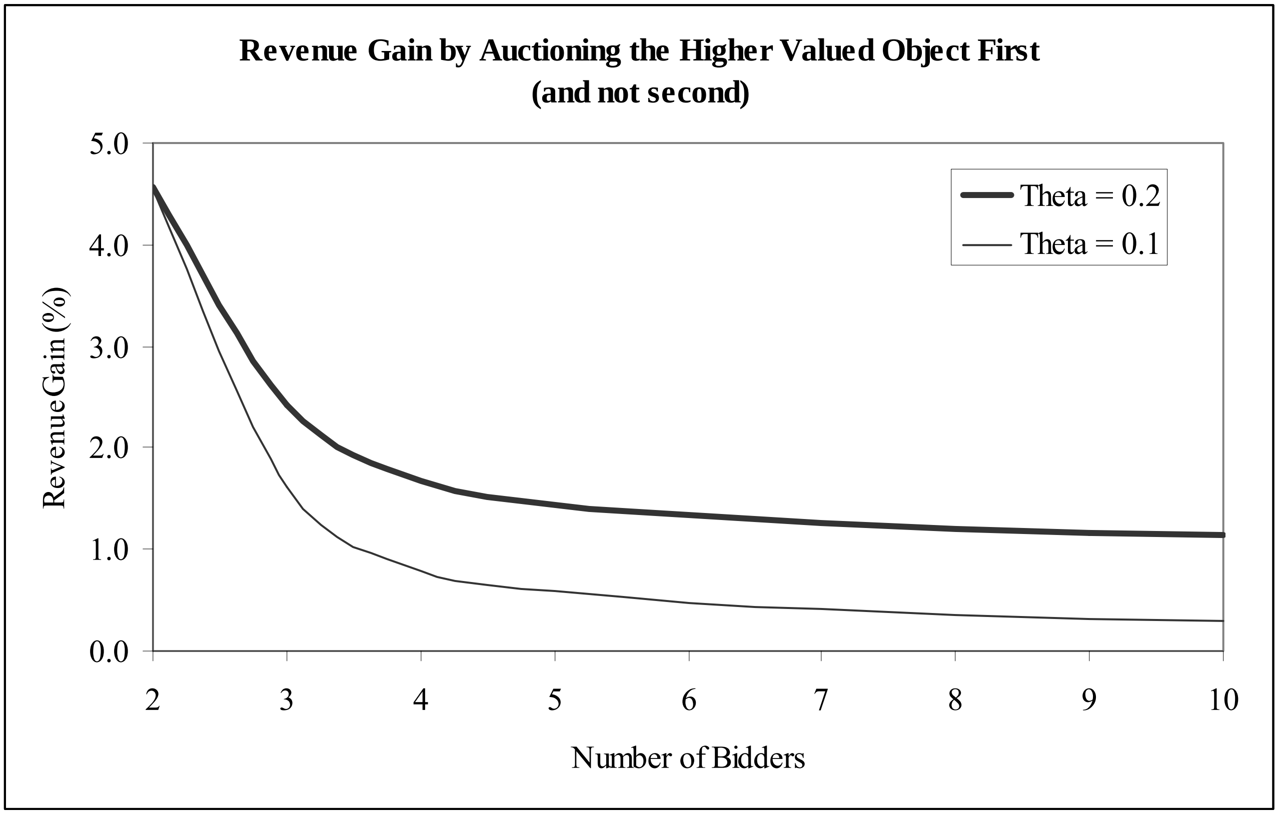


Figure 5: The graph is positive everywhere confirming the proposition. Some higher order effect can also be seen. Notably, the significance of sequencing is higher when the degree of complementarity is greater. The importance of sequencing is diminishing in the size of the bidder pool.

Figure 6

Essay 2: Substitutes vs Complements: Optimality of Sequential and Bundled Auctions (Risk Neutral Bidders)