

**APPROACHING THE PLANCK SCALE FROM A
GENERALLY RELATIVISTIC POINT OF VIEW: A
PHILOSOPHICAL APPRAISAL OF LOOP
QUANTUM GRAVITY**

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LOOP QUANTUM GRAVITY**

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University of Pittsburgh, 2006

My dissertation studies the foundations of loop quantum gravity (LQG), a candidate for a quantum theory of gravity based on classical general relativity. At the outset, I discuss two—and I claim separate—questions: first, do we need a quantum theory of gravity at all; and second, if we do, does it follow that gravity should or even must be quantized? My evaluation of different arguments either way suggests that while no argument can be considered conclusive, there are strong indications that gravity should be quantized.

LQG attempts a canonical quantization of general relativity and thereby provokes a foundational interest as it must take a stance on many technical issues tightly linked to the interpretation of general relativity. Most importantly, it codifies general relativity's main innovation, the so-called background independence, in a formalism suitable for quantization. This codification pulls asunder what has been joined together in general relativity: space and time. It is thus a central issue whether or not general relativity's four-dimensional structure can be retrieved in the alternative formalism and how it fares through the quantization process. I argue that the rightful four-dimensional spacetime structure can only be partially retrieved at the classical level. What happens at the quantum level is an entirely open issue.

Known examples of classically singular behaviour which gets regularized by quantization evoke an admittedly pious hope that the singularities which notoriously plague the classical theory may be washed away by quantization. This work scrutinizes pronouncements claiming that the initial singularity of classical cosmological models vanishes in quantum cosmology based on LQG and concludes that these claims must be severely qualified. In particular, I explicate why casting the quantum cosmological models in terms of a deterministic temporal evolution fails to capture the concepts at work adequately. Finally, a scheme is developed of how the re-emergence of the smooth spacetime from the underlying discrete quantum structure could be understood.

Keywords: Loop quantum gravity, Hamiltonian general relativity, canonical quantization, general covariance, diffeomorphism invariance, loop quantum cosmology, singularity, spacetime emergence.

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PREFACE

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1.0 INTRODUCTION

Theoretical physics at the outset of the 21st century is confronted with a quandary strangely reminiscent of the one Newton's predecessors faced in the 17th century: two incompatible theories quite successfully describe two different, but perhaps overlapping, domains of physical phenomena. There was the sub-lunar domain of the terrestrial physics of projectile motion to which Galileo so heavily contributed on the one hand, and the supra-lunar realm of a Copernican solar system with heavenly bodies moving in Keplerian orbits on the other. Galileo unsuccessfully attempted a (limited) unification of the two theoretical frameworks by means of circular inertia, but it was only Newton who formulated a mechanical theory capable of accounting for both celestial and terrestrial motion by using a consistent set of just a few simple axioms.

Today, as repeatedly emphasized by Carlo Rovelli,¹ physicists are faced with a similar challenge. The laws of various quantum theories (QTs) govern the small-scale phenomena of elementary particle physics, while the laws of general relativity (GTR) encode the large scale structure of the universe. The string theorist Brian Greene analogized the situation to a city with two different sets of traffic laws interfering with one another.² But since the fundamental forces responsible for the relevant effects studied by the two separate frameworks exhibit vastly different ranges and thus operate at very different scales, they seem to co-exist relatively peacefully. In Greene's loose talk, it is as if the two kinds of traffic to which the different sets of laws apply occur at very different levels such as to preclude (almost) every conflict between the laws belonging to different sets. Like in a cyclists' dream, it is as if there is a set of traffic laws applying to cars and one set applying to bicycles, with these two worlds rarely, if ever, clashing. But the impression of peace is deceptive. Rather than a prosperous marriage, the situation resembles an ill-conceived truce which could be broken anytime as new astrophysical data pours in. The 20th century revolution in theoretical physics, which has brought QT and GTR and thus smashed the conceptual coherence of classical physics, has so far failed to deliver a mathematically consistent and

¹Most recently in [Rovelli \(2006\)](#).

²In the PBS show "The Elegant Universe" (part I) aired on 28 October 2003.

conceptually unified successor framework. As Rovelli argues at the outset of his recently published treatise on quantum gravity (Rovelli 2004), each of the two separate frameworks assumes concepts contradicted by the other. In order to account for the evolution of quantum states as encoded in the Schrödinger equation, QT requires a time parameter external to the system at stake. In quantum field theory (QFT), the evolution of quantum fields similarly presupposes a fixed spacetime background independent of the fields. In a stark contrast, GTR dissolves these notions of external time and of a fixed background spacetime in favour of a dynamical spacetime which encapsulates the gravitational field. This dynamical structure co-evolves interdependently with the non-gravitational matter fields in accordance with the Einstein field equations coupled to the dynamical equations governing the matter fields. In return, however, in assuming a smooth, continuous metric field, GTR violates the dictum of QT that all dynamical fields must be quantized. While GTR has revolutionized the concepts of space and time, it has largely remained faithful to classical notions of matter and causality. Conversely, QT adheres to pre-(generally-)relativistic concepts of space and time, but completely reconceptualized matter, measurements, and causality.

The challenge of quantum gravity (QG) is the challenge of completing the 20th century revolution in physics by fusing the two incommensurable frameworks and of re-establishing a conceptual harmony at the fundamental level. Not only will a quantum theory of gravity (QTG) resolve the incompatibilities between QT and GTR, but it will also be an essential, and perhaps final, step to the theoretical unification of fundamental physics—or so it is hoped.³ As a result, QG is viewed by many as *the* challenge for theoretical physics in the 21st century. Today more physicists than ever are busy formulating such a new synthesis of quantum and relativistic physics. Their efforts have yielded a rich variety of approaches, as will be briefly outlined in Section 1.1. They most prominently include loop quantum gravity (LQG) and string theory (ST), along with, among others, non-commutative geometry, causal sets, Euclidean gravity, topological quantum field theory (TQFT), and more idiosyncratic approaches such as Regge calculus and Penrose’s twistor theory.

My thesis will concentrate on the conceptual foundations of canonical QG and of LQG. But why LQG? Apart from ST, LQG is the most mature and worked out approach to QG, having been initiated in 1988 by Jacobson, Smolin, and Rovelli based on Ashtekar’s connection formulation of GTR and has now grown into a respectable branch with an estimated 100 researchers worldwide. Contrary to ST, it starts out from the basic principles of GTR by formulating a non-perturbative theory presupposing background-independence. As will

³QT and GTR are *incompatible*, but they may not be *inconsistent*—or it may not be *decidable* whether they are inconsistent or not: it seems as if the mathematical frameworks of the two theories are too disparate to allow the derivation of a contradiction, which is what would be necessary to speak of *jointly inconsistent* theories in the proper sense of the term.

be explored in Chapter 5, it has celebrated its first successes—the calculation of the spectrum of the area and volume operators, as well as the derivation of the Bekenstein-Hawking black hole entropy—and is still actively developed. Its glowing lava, as it were, has not yet solidified into a mantric dogma and is still very much in flux. In particular, the problems of how the spin network states evolve and of the theory’s classical limit have not yet been resolved. [Rovelli \(2004\)](#) compares the task of solving the problem of QG with other seemingly insurmountable obstacles in the development of physical theories whose removal had always been guided by epistemological prejudices. He finds that for all cataclysmic resolutions of momentous conceptual challenges in the history of physics such philosophical guidance and nurture was essential. It is therefore an exciting prospect for me to probe the philosophical and conceptual commitments of LQG and to study how they are instrumental in resolving the tension between QT and GTR.

1.1 MAPPING QUANTUM GRAVITY

The quest for a quantum theory of gravity (QTG) starts at least as early as 1930⁴ with [Rosenfeld \(1930a,b\)](#) and has captured the imagination of physicists ever since. Rosenfeld’s original proposal envisioned a quantum field theory of quantum fluctuations of the metric on a Minkowski spacetime. Since then, many approaches have been developed. But I will make no attempt at recounting the early (or later) history of quantum gravity.⁵ Rather, this section is supposed to give some orientation in the field of QG.

Many different approaches to finding a quantum theory of gravity have been attempted. So many indeed that Fotini Markopoulou has repeatedly felt compelled to compare the situation in QG to the one in atomic theory a bit more than a century ago.⁶ Due to this

⁴The *need* for a QTG was, as far as I know, first expressed by Einstein in his 1916 remark that

“due to the inner-atomic movements of electrons, atoms would have to radiate not only electromagnetic, but also gravitational energy, if only in a tiny amount. As this is hardly true in nature, it seems as if quantum theory will have to modify not only Maxwellian electrodynamics, but also the new theory of gravitation.” ([Einstein 1916b](#), p. 696)

[“Gleichwohl müssten die Atome zufolge der inneratomischen Elektronenbewegung nicht nur elektromagnetische, sondern auch Gravitationsenergie ausstrahlen, wenn auch in winzigem Betrage. Da dies in Wahrheit in der Natur nicht zutreffen dürfte, so scheint es, dass die Quantentheorie nicht nur die Maxwellsche Elektrodynamik, sondern auch die neue Gravitationstheorie wird modifizieren müssen.”] He reiterated this stance two years later ([Einstein 1918](#), p. 164).

⁵For an account of the early history, see [Stachel \(1999\)](#). For a cursory overview of the (entire) history of quantum gravity, see [Rovelli \(2002a\)](#). For some illuminating remarks, cf. also [Ashtekar \(2005\)](#).

⁶She cites Boltzmann’s assessment: “Every Tom, Dick and Harry, felt himself called upon to devise his

excess of alternative approaches to QG, it will not be practicable to present all—or even most—of them on the present occasion. Apart from LQG, I will restrict myself to a few brief remarks on string theory in Section 1.2.⁷

This plethora of distinct approaches to QG reflects the state of a field in which there is no consensus as to what constitutes even the relevant departure points. Not only are we faced with the complete lack of a common axiomatic ground, but each camp views the others’ most important and most cherished vantage points as doubtful at best, and as irrelevant, incoherent, or almost trivially false in other cases. Alternative approaches to QG only seem to share the understanding that classical GTR and standard QFT should be reproduced from a QTG in the appropriate low-energy limits. Despite this disconcerting disarray, some camps live closer to one another than do others. Many authors of introductory texts to QG therefore attempt to group together different camps which depart from similar vantage points. A number of different classifications have been proposed in the literature, all from different points of view, stressing different aspects and all with different merits. Without thoroughly combing the entire range, let me briefly present two convincing proposals. For instance, Chris Isham (1994b, 1995) distinguishes four broad types of approaches: (i) quantize GTR, (ii) general-relativize QT, (iii) regard GTR as a low-energy limit of a QT constructed using conventional ideas, without a quantization of the gravitational field, and finally (iv) consider both GTR and QFT as low-energy limits of a radically new fundamental theory to be formed *ab initio*. Let me say a few words about each family in turn.

(i) Schemes of this family start out from classical GTR and attempt to obtain a quantum theory by applying so-called “quantization” techniques to the classical theory, i.e. they adopt some mathematical procedure which turns classical fields into quantum fields. I will follow Isham (1995, Sec. 2.2) in designating as *quantization* a diorthotic, i.e. corrective, scheme which starts out from a classical system and subjects it to a procedure of well-defined and justified steps in order to obtain a quantum system. For instance, the quantization of a classical field will yield a corresponding quantum field, described by a QT. Isham contrasts this procedure to formulating a QT without prior reference to a classical system, such as seeking a representation of an algebra on a Hilbert space. Quantization techniques involve a broad range of methods and have been successfully used to develop quantum electrodynamics (QED), quantum flavour dynamics (QFD) or Glashow-Salam-Weinberg electroweak theory, and quantum chromodynamics (QCD). Despite its success, it is believed by many to be reversing Nature’s order: because the world is quantum at heart, it is in principle wrong-

own special combination of atoms and vortices and fancied in having done so that he had pried out the ultimate secrets of the Creator.” Cf. e.g. Markopoulou (2003).

⁷For an extensive bibliographic review of approaches to understand the small scale structure of spacetime, cf. Gibb (1995).

headed to start from a theory of which we know it cannot be true in order to arrive at the fundamental theory. I will briefly return to this issue in Chapter 2.

Family (i) approaches come in two distinct genera: the canonical and the covariant. The latter tries to directly quantize the four-dimensional spacetime, but at the cost of splitting the gravitational field into a fixed background metric and “gravity proper” as a perturbation on this background. This genus has become nearly extinct since the realization that covariant quantizations of GTR are not perturbatively renormalizable in 1974.⁸ The canonical scheme casts GTR as a constrained Hamiltonian system in preparation to apply the so-called canonical quantization program and thus breaks GTR’s manifest general covariance by introducing a foliation of spacetime. However, this approach does not necessitate a splitting of the gravitational field, as we shall see in Chapters 3 and 4. My main focus will lie on this genus as it contains the species of quantum geometrodynamics and LQG.

(ii) Members of family (ii) try to use a quantum (field) theory as vantage point and then utilize some procedure performing a similar function as quantization did in family (i) approaches in order to make the quantum theory generally relativistic. Approaches of this family are sparse, Isham only mentions [Fredenhagen and Haag \(1987\)](#). More recent surveys of the most important effort in this camp can be found in [Brunetti, Fredenhagen, and Verch \(2003\)](#) and [Brunetti and Fredenhagen \(2006\)](#). The main idea of this main strand is to incorporate in a local sense general relativity’s principle of general covariance such as to obtain a locally generally covariant algebraic QFT. Interestingly, this seems to necessitate the simultaneous treatment of all admissible spacetimes as background on which the quantum fields live. Supposedly, gravitation is thought to enter the picture through the various background spacetimes which represent the gravitational field.

(iii) The third family, like family (ii), takes a quantum theory as its vantage point. But instead of “relativizing” the quantum theory, it attempts to extend the quantum theory using methods as conventional as possible with the goal that GTR will drop out of the extended theory as an appropriate low-energy limit. The most prominent species in this family is string theory (ST), which is sufficiently important in QG to warrant a section even in a dissertation on canonical approaches (see Section 1.2). Clearly, ST goes well beyond conventional QFT, both methodologically and in terms of its ambition. Isham nevertheless relates ST with family (iii) rather than (iv), presumably because it takes as its *vantage point*, both historically and systematically, conventional QFT and does not attempt to build a novel approach completely dissociated from “old” physics. Topological QFT ([Atiyah 1989](#); [Lawrence 1996](#)) is another genus of this family. In a topological QFT, the correlation functions between quantum observables are calculated from topological invariants, i.e. they do

⁸Cf. ’t Hooft and Veltman (1974) and Deser and Nieuwenhuizen (1974a,b).

not depend on the metric of the background spacetime. Topological QFTs known so far come in two subgenera, one containing the species of BF theory⁹ and Chern-Simons theory¹⁰ and the other consisting of Witten’s more subtle way of formulating topological invariants. Euclidean quantum gravity is another important genus in family (iii).¹¹

(iv) Finally, the last Ishamian family of approaches to QG is most aptly characterized by their iconoclastic attitude. No known physics serves as starting point for these approaches; rather, they consider radically novel perspectives and try to formulate a QTG *ab initio*, oftentimes axiomatically. All members of this family that I am aware of suggest only programmatic schemes and are rather remote from offering fully fledged theories of QG. The attraction of this family lies in the apparent incompatibility of the guiding principles behind GTR and any QT. The most prominent representatives of this family are the causal sets approach (Bombelli et al. 1987; Brightwell et al. 2003) and non-commutative geometry (Connes 2000; Schücker 2001, 2005). ST does not belong to this last category because it starts out from conventional QT and extends its methods and scope such as to include, in some sense, gravity. Despite its radically novel understanding of fundamental physics, it is not an attempt to formulate a QTG *ab initio*.

Naturally, one finds different—and inequivalent—classifications of the roads promising to lead to a QTG in the literature. A somewhat more standard classificatory scheme is found in Rovelli (2002a). He distinguishes three major families: (i) the sum-over-histories approaches, (ii) the covariant school, and (iii) the family of approaches based on the canonical quantization scheme. The third group is identical to Isham’s second genus of the first family. Rovelli’s second family, however, comprises species from Isham’s first genus of the first family, but also from his second and third families. The same holds for Rovelli’s first family, the sum-over-histories line, which takes Feynman’s path integrals as the methodical vantage point and quantizes GTR using his functional integrals. Both Hawking’s Euclidean quantum gravity programme as well as the spinfoam models, to be discussed below, have grown out of this idea.

Rovelli’s scheme does not really accommodate heterodox approaches such as causal sets and non-commutative geometry. But it provides a valuable classification for understanding the clash of the QG giants, ST and LQG. ST in all its versions has emerged from the covariant approach, while LQG is the most promising theory on the canonical side. The two sides reflect, to some extent, the divide between particle physicists trying to extend

⁹Interestingly, GTR can be formulated as a BF model with constraints. Thus, it might afford a close relation to LQG via a perturbative expansion of BF-type spinfoam models. For a self-contained introduction to spinfoams and BF theory and their relation, see Baez (1999).

¹⁰Originally published in Chern and Simons (1974).

¹¹The canonical reference for Euclidean QG is Gibbons and Hawking (1993).

the standard model to include gravity, and relativity theorists attempting to formulate a quantum theory of gravity based on GTR.

1.2 STRING THEORY

The debate between the covariant and the canonical camps is sometimes fought with humour, as Rovelli’s dialogue (Rovelli 2003a) between a string theorist and a student of LQG, emulating Galileo’s *Dialogue Concerning the Two Chief World Systems* of 1632, bears witness, and sometimes with bile, as some private reactions to Rovelli’s dialogue testify.¹² In any case, ST has managed to garner much more resources than LQG.¹³ In this section, the reader shall be briefed about the core ideas of ST. For an introduction to perturbative string theory, see Piran and Weinberg (1988), or the classical, but outdated, textbook by Green, Schwarz, and Witten (1987). A more recent textbook, which has quickly become the standard reference, is Polchinski (1998). For a popularization of ST including its non-perturbative aspects, see Greene (1999) and Susskind’s contribution (Susskind 2003) to the special issue of *Physics World* on QG. Zwiebach (2004) offers the most recent textbook on ST, which despite its technical parts seems quite accessible. The text is sufficiently recent to include a detailed chapter on how the black hole entropy is calculated in ST. See also the “official” string theory website: www.superstringtheory.com. For an introduction to duality theories, *M*-theory, *F*-theory, and other results in non-perturbative ST, see Sen (1999).

ST exists at two levels. At the perturbative level, on the one hand, ST consists of a set of well-developed mathematical techniques which define the string perturbation expansion over a given background metric. On the other hand, attempts at formulating the elusive non-perturbative theory, supposed to be capable of generating the perturbation expansion, have not succeeded so far. Such a theory, conventionally named *M*-theory, for “membrane,” “matrix,” or “mystical” theory, consists of but incipient formulations using non-perturbative compactifications of higher dimensional theories based on so-called duality symmetries, i.e. symmetries relating strong coupling limits in one string theory to a weak coupling limit in another (dual) string theory. Here, I will minimize my remarks on the technically contrived non-perturbative theory, and concentrate on giving a brief survey to the main ideas of perturbative ST.

¹²Carlo Rovelli, personal communication, June 2004.

¹³Lee Smolin estimates the number of researchers in ST to reach a total of 1,000, as compared to LQG which occupies roughly 100 researchers worldwide. The distribution of research funds, academic positions, and attention from the popular science press is equally disproportionate.

Perturbative ST (henceforth just ST) arose in the 1960s from attempts to describe the strong nuclear force. Physicists realized that its mathematical machinery to describe strong interactions dealt with extended objects called “strings,” rather than with the point-like particles of traditional approaches. Elementary particles, in this picture, were constituted by closed loops in different states of vibrations. But the initial enthusiasm died rather quickly when attempts to understand the theory’s requirement of a ten-dimensional spacetime and its stubborn prediction of a massless spin-2 particle failed. When it became clear that quantum chromodynamics offered a highly successful theory of the strong interaction, ST was about to sink into oblivion.

But the problems of ST can be turned into virtues, or at least to some degree, if it is no longer considered to be merely a theory of hadrons, the carriers of the strong interaction, but of all interactions. Specifically, the mysterious massless spin-2 particle is identified with the graviton, the interaction particle of gravitation. The extra-dimensions are interpreted as small, compact spatial dimensions resulting from the spacetime dynamics. No conflict with observations exists as long as these extra-dimensions are sufficiently small. In this picture, then, strings in different vibrational modes account for different families of particles, including quarks, leptons, Higgs, photons, gravitons, gluons, W - and Z -bosons. Interactions occur when strings break up or join together and thus, since all particles—including interaction particles—can be formed with strings in particular vibrational modes, ST accommodates all forces. The theory’s ambition of devising a grand unified theory is of course based on this claim. But in order to incorporate fermions, ST requires certain supersymmetries to obtain, now recognized as a generic feature of ST. Because at least some portion of supersymmetry needs to be realized if ST is to be mathematically consistent, supersymmetry (its existence, not the characteristic energy scale of its breaking) can be regarded as a prediction of ST quite regardless of which compactification is chosen. So far, however, supersymmetry has not been observed, despite numerous assurances from string theorists that it is “around the corner.” According to one educated guess ([Schwarz 2000](#)), the lightest supersymmetric particle (the “neutralino”), at a mass of at least 100GeV, might be detected by the Large Hadron Collider at CERN, which is planned to go into operation in 2007.

Perturbative ST is free of ultraviolet divergences. The “world lines” of the point particles as represented in Feynman diagrams of perturbative QFT become two-dimensional “world sheets” of an extended one-dimensional string who lives in a higher-dimensional spacetime. Analogously, the junctions of world lines in the Feynman diagrams used in the description of interactions turn into string world sheets of various topologies essentially imitating the Feynman sum-over-histories method. The n -th term of the ST perturbation expansion is associated with a Riemann surface of genus n . Since ST does not require for interactions to

be associated with world line junctions at given spacetime points, but only, it is claimed, with smooth changes of the topology of the two-dimensional world sheets of strings, the interaction amplitudes do not contain ultraviolet infinities. To repeat, strings are one-dimensional open or closed objects. But in principle, there can be any p -dimensional analogues of strings (called p -branes) sweeping out a $(p + 1)$ -dimensional “world volume.” However, for $p > 1$, non-renormalizable short-distance infinities arise.

Importantly, ST is not a background-independent theory. All strings, including those representing gravitons, live in a higher-dimensional background spacetime. In a purely bosonic ST, the dimensionality of the background spacetime is 26. If we add in fermions, and thus supersymmetry supplemented by constraints forming a so-called *super-Virasoro algebra*,¹⁴ only the specific choice of $d = 10$ cures otherwise prevalent mathematical anomalies. So in an apparent contradiction to any evidence, full supersymmetric ST requires a ten-dimensional background spacetime. As mentioned above, the contradiction can be resolved by curling up the six extra spatial dimensions to tiny scales inaccessible to experimentation. The geometrical structure of these extra dimensions behaves according to the dynamical equations of the theory and can, as a result of this, only assume the metric structure of so-called Calabi-Yau spaces.¹⁵ The process of rolling up the extra dimensions into a compact space too small to be observed is called *compactification*. Within the context of some STs, a Calabi-Yau compactification can then lead to an effective theory describing four-dimensional physics which resembles a supersymmetric extension of the standard model with gravity. Unfortunately, however, there are (at least) tens of thousands of Calabi-Yau spaces which meet the dynamical requirements and could therefore encode the metrical structure of the extra dimensions.¹⁶ Only if one picks a “right” specimen of Calabi-Yau space, the standard model is reproduced at low energies. Also, the topology of the Calabi-Yau space determines the number of lepton and quark families. But while suitable choices seem to reproduce a lot of the familiar four-dimensional physics, the number of these choices and the degree to which they determine the effective theory leads to an unacceptable level of immunity of the theory from empirical tests, enabling ST to absorb almost any experimental finding.

Only within the last ten years, during the so-called “second superstring revolution,”¹⁷ signs of a non-perturbative theory have emerged. So-called dualities relate the five perturbative superstring theories to one another, such that the five theories are but five dif-

¹⁴For a review of classical and quantum super-Virasoro algebras, cf. e.g. [Mansour \(2001\)](#).

¹⁵This was established, under some plausible conditions, in [Candelas et al. \(1985\)](#). The existence of these spaces was first proved by [Yau \(1977\)](#).

¹⁶Estimates for the number of stable compactifications range in the order of 10^{500} ([Carroll 2004](#)). We are not exaggerating, then, if we insist that the compactification of extra-dimensions is non-unique.

¹⁷The “first revolution” refers to the formulation of the five mathematically consistent, supersymmetric, perturbative string theories in ten dimensions during the mid-eighties.

ferent perturbative expansions about five different points, thus hinting at an underlying non-perturbative theory. Higher-dimensional p -branes are no longer plagued with infinities, as they were in the perturbative expansions. Three different kinds of dualities relate theories of strong coupling to those of weak coupling, theories compactified on a space of (relatively) large volume to those compactified on a space of small volume, and finally theories compactified on a space of large (small) volume to those of strong (weak) coupling.

In his contribution to PSA 19, Sean [Carroll \(2004\)](#) has offered an intuitive explication of how the gravitational force is supposed to be captured by ST. A transverse gravitational wave will exert a small tidal force on circularly arranged test masses. These then start to oscillate into ellipses along axes perpendicular to the direction of wave propagation. Because this pattern strongly resembles the propagation of an oscillating string, strings with the appropriate oscillation are believed to form gravitons, the interaction particles of gravity. Apart from the inclusion of the graviton, ST claims to solve the problem of QG for another reason. The world sheet theory must exhibit so-called Weyl invariance, a certain consistency condition on the background spacetime, in order for the perturbation expansion to be meaningful. This condition turns out to resemble the Einstein equations with source terms from an anti-symmetric tensor field and the dilaton ([Polchinski 1998](#), Vol. I, pp. 111f). It is an open question, however, whether the requirement of the Weyl invariance of the perturbation expansion will emerge from the dynamical equations of the non-perturbative theory. If it does not, it is unclear how the Einstein equations could emerge from ST in an appropriate low-energy limit.

Perhaps the most remarkable (theoretical) success of ST is its derivation of the Bekenstein-Hawking entropy of a black hole. Consider an amount of a hot gas that falls into a black hole. Respecting the Second Law of thermodynamics, of course, the combined system “black hole + gas” cannot decrease its total entropy in this process. Consequently, in absorbing the hot gas, the black hole must increase its entropy by at least the amount of entropy the gas had before its absorption. Along these lines, [Bekenstein \(1972, 1974\)](#) suggested that there is an entropy associated with black holes and that it should be proportional to the area of its horizon. Shortly thereafter, relying on calculations in QFT on curved spacetime backgrounds, [Hawking \(1975\)](#) and [Hartle and Hawking \(1976\)](#) showed that a black hole emits radiation, exactly at the temperature predicted by Bekenstein. Thus, a Schwarzschild black hole has the entropy

$$S = \frac{k}{4G}A, \tag{1.1}$$

in natural units (cf. Section [A.1](#)) where A is the surface area of the horizon of the black hole, with k the Boltzmann constant and G the Newton constant. Although neither the Bekenstein entropy nor the Hawking radiation have ever been observed, most physicists consider them

as part of established physics. This confidence is based on the many independent ways in which this result has since been derived from well-established theories.¹⁸

An entropy S is usually interpreted as the number of distinguishable states Ω , $S = k \ln \Omega$. Classically, the entropy of the black hole can be imagined as each element of the horizon of area ℓ_{Pl}^2 , where $\ell_{\text{Pl}} \approx 10^{-33}\text{cm}$ is the so-called *Planck length*, having one degree of freedom with a finite number of possible states. In ST, one essentially tries to come up with a function Ω based on some fundamental way of counting states corresponding to a black hole geometry. For instance, a stationary Schwarzschild black hole is related to a highly excited string without momentum. The entropy of the Schwarzschild black hole can then relatively easily “deduced” from the entropy of the string (Zwiebach 2004, Sec. 16.6). The “deduction,” however, for a case as general as the Schwarzschild solution remains rather loose. For the case of so-called extremal black holes, i.e. for black holes with $M^2 = a^2 + Q^2$ where M is the mass, a the angular momentum, and Q the charge of the black hole, string theorists (Maldacena 1996; Strominger and Vafa 1996) have been successful in deriving equation (1.1) more rigorously, including the correct proportionality factor. This result has been extended to some degree to higher-dimensional so-called near-extremal black holes (Horowitz and Strominger 1996). Although extremal black holes almost certainly do not exist (Wüthrich 1999, Secs. 2.1 and 3.5), the successful derivation of their entropy points to interesting connections between ST, QFT on curved spacetime and classical GTR.

Another recently claimed success of ST is that it apparently indicates that the de Sitter spacetime, a spacetime of an accelerating universe similar to our own, can be understood as a local minimum of vacuum energy Λ in the space of ST’s solutions (Kachru et al. 2003; Quevedo 2003). It is unclear, however, whether this result constitutes an independent success of the theory.

1.3 SYNOPSIS OF THE CHAPTERS

Chapter 2 starts out with discussing two questions which must be addressed prior to embarking upon an investigation into the foundations of quantum gravity: first, do we need a quantum theory of gravity at all, and second, if we do, does it follow that gravity should or

¹⁸For a critical renegade, however, see Helfer (2003). In this review, Helfer argues that no compelling theoretical case for (or against) Hawking radiation has been made so long as the two, in his view dubious, assumptions that first, old physics may be applied to vacuum fluctuations at arbitrarily large energy scales and that second, genuinely quantum-gravitational effects may be neglected, have not found a definite theoretical treatment. As an aside, the second issue, the so-called “trans-Planckian problem,” also appears in inflationary cosmology where length scales smaller than the Planck length might play a role.

even must be quantized? As I will argue in Chapter 2, these are two separate issues. The answer to the second question is less straightforward than is commonly assumed, since QG is a combination of two ancestor theories which both build on a continuous, four-dimensional, differentiable manifold with a Lorentzian metric. Yet a quantization of GTR is expected to dissolve the “classical” structure of spacetime. Despite this reservation, however, there are good reasons to indeed quantize gravity. Most of the material in this chapter has been published in [Wüthrich \(2005\)](#).

The following Chapters 3 and 4 prepare the stage for the quantization of GTR. This preparation comprises the discussion of the requirement of background independence as the towering feature of classical GTR, as well as the introduction of the postulate of general covariance and its relation to background independence. Chapter 3 discusses these foundational issues in GTR.

Once these essential physical ingredients have thus been cooked up, the mathematical formalism will have to be outlined. For this, I will remind the reader how to cast GTR in a Hamiltonian formalism and put together the recipe for the canonical quantization of a diffeomorphism-invariant system with constraints. This is mostly done in Chapter 4, also leading into Chapter 5. Chapter 4 will also address the motivation for and the legitimacy of casting GTR as a constrained Hamiltonian system. The justification of this step crucially depends on the interpretation of the invariance under active spacetime diffeomorphisms as a gauge invariance of GTR. It will be a central issue in Chapter 4 whether or not GTR’s invariance under active spacetime diffeomorphisms can be retrieved in the canonical formalism. Chapter 5 investigates, among others, how general covariance fares through the quantization process. I will argue that the full spacetime diffeomorphism invariance cannot be recovered in the canonical formulation of the theory, at least not as it stands. The chapter also includes a brief outline and discussion of the so-called problem of time and the related issue of finding observables, i.e. the physically fundamental magnitudes.

In Chapter 5, a brief outline of LQG will be presented. I will not attempt to be comprehensive in this survey, but rather to explicate its guiding ideas and concepts, particularly as they pertain to the issues at stake here. The presentation does not aspire to be mathematically rigorous. I shall sketch the main steps of choosing an algebra of canonical coordinates, solving the Gauss constraint to obtain the $SU(2)$ -invariant spin networks, solving the vector constraints to obtain a basis of spin networks invariant under spatial diffeomorphisms, i.e. the kinematical Hilbert space, and finally trying to solve the Hamiltonian constraint to arrive at the physical Hilbert space of the theory.

Apart from following the canonical recipe of Wheeler and DeWitt, the evolution of the basic spin network states can also be captured by formulating a covariant spinfoam model.

After a discussion of the results to date in the canonical camp, I will briefly outline the main ideas of the covariant approaches to find an appropriate dynamical regime. The covariant, Feynman-inspired sum-over-histories approach called spinfoams is motivated, to a large extent, by the sheer mathematical difficulties in solving the Hamiltonian constraint of the canonical theory. Unfortunately, its relation to the canonical quantization is unclear and it is therefore still an open question how, if at all, the covariant spinfoam model relates to canonical gravity. So far, the dynamical completion of the theory either by sticking to canonical methods or by extending the theory by covariant methods and results has not been achieved.

The last few years have also seen the arrival of what is usually referred to as “loop quantum cosmology” (LQC), the study of highly symmetric cosmological models in the context of LQG. One of the most intriguing features of these models is claimed to be the vanishing of the initial singularity in a Friedmann-type evolution. In order to appraise this claim, I build up my case over the course of three chapters. Chapter 6 describes the inevitability of the initial singularity in cosmological models based on classical GTR, which are briefly treated in Appendix C, and discusses cases where quantization has in fact washed away classical singularities. These are typically examples of singular behaviour in the dynamics of simple systems which are vanquished if these systems are properly quantized. These cases instill a hope that perhaps cosmological models based on a quantization of GTR might also overcome the big-bang singularity.

Chapter 7 then is dedicated to exposing LQC in some detail. LQC uses the same kinematical Hilbert space as does LQG, but freezes out all but one degree of freedom before solving the difficult Hamiltonian constraint equation. This symmetry reduction to kinematical states which correspond to isotropic and homogeneous spatial universes drastically simplifies the constraint equation and thus even allows to find solutions and to study the physical Hilbert space, assuming that the matter Hamiltonian is not too complicated, however. The next chapter, Chapter 8 offers an analysis of whether or not the claims that LQC vanquishes the initial singularity unavoidable in classical models withstand scrutiny. The singularity, as it will turn out, does vanish in the sense that the unphysical dynamical evolution so dear to loop quantum cosmologists effectively manages to penetrate back into a mirror world of “before the big bang.” At the same time, however, the singularity is not completely exterminated despite what is claimed. It survives in the sense that Laplacean determinism still fails at the big bang, although the failure is milder than in classical models. Chapter 8 also includes a discussion of some recent reactions to LQC.

One of the most important philosophical issues will be presented in Chapter 9. In particular, I shall treat the disappearance and the re-emergence of classical spacetime from QG.

Chapter 5 will illustrate how the continuous spacetime structure of classical GTR is lost in LQG—a circumstance which I will label as the “disappearance of spacetime.” A natural question to pose then pertains to how this continuous structure re-emerges in the classical limit. I will try to sketch what could be meant by the emergence of spacetime in the light of the deplorable absence of a physical resolution of the problem and will argue that the problem of understanding how classicality can emerge from the underlying quantum structure is very similar to the more familiar cases concerning ordinary QM. Essentially, selecting a rather special class of kinematical quantum states seems to permit to approximate the classical world. Some exact classical values of geometrical quantities can thus explicitly regained by a well-defined limiting procedure.

Finally, Chapter 10 presents some conclusions of my dissertation project.

2.0 WHY MUST GRAVITY BE QUANTIZED?

All major approaches to quantum gravity endorse, implicitly or explicitly, the view that gravity must be “quantized” in order to be amenable to a description at the fundamental level. To be sure, different approaches propose to execute this task in radically different manners. But they agree in that without a quantization of gravity no consistent quantum theory of gravity will be forthcoming. What such a quantization involves will vary with the approach taken. It may mean that the classical gravitational field is subjected to some definite quantization procedure which converts the classical field into a quantum field. Or it may involve extending the standard model of particle physics such as to include a spin-2 particle incorporating the gravitational interaction. I will also include among the “quantizers” those approaches which start out from a quantum structure of spacetime or of a quantum field of gravity *ab initio*. The reason for employing such a liberal notion of quantization is that for the purposes of the present chapter, differences in how a quantum spacetime is to be constructed are insignificant in determining whether the spacetime structure or the gravitational field exhibit, or should exhibit, a quantum nature *at all*. But rather than analyzing what is meant by “quantization” I shall focus on the intriguing foundational issues that arise in the context of motivating and justifying such a quantization of gravity. Before addressing the main question of the chapter, however, two brief remarks are in order.

First, one could oppose quantizing gravity, at least in a somewhat narrower sense of “quantizing” than just outlined, on the grounds that in fundamental physics starting out from a classical field in order to arrive at the quantum structure puts the cart before the horse. This mentality, whose slogan could be something like “quantum without quantization,” is expressed in the quote from Patton and Wheeler on the occasion of the 1975 Oxford symposium on quantum gravity: “However workable [the] procedure of ‘quantization’ is in practice [...], we know that in principle it is an inversion of reality. The world at bottom is a quantum world; and any system is ineradicably a quantum system. From that quantum system the so-called ‘classical system’ is only obtained in the limit of large quantum numbers.” (Patton and Wheeler 1975, p. 545) Some thus believe that proceeding by quantization is a principled mistake. Deutsch, an unconditional advocate of a quantum world through and

through, is equally adamant in his condemnation of quantization:

“in the [...] important matter of formalism we still know of no other way of constructing quantum theories than ‘quantization,’ a set of semi-explicit *ad hoc* rules for making a silk purse (a quantum theory) out of a sow’s ear (the associated classical theory) [...] I believe that quantization will have to go before further progress is made at the foundations of physics [...] To base the theory of quantum fields $\hat{\varphi}_i$ on that of classical fields φ_i is like basing chemistry on phlogiston or general relativity on Minkowski space-time: it can be done, up to a point, but it is a mistake; not only because the procedure is ill defined and the resulting theory of doubtful consistency, but because the world isn’t really like that. *No classical fields φ_i exist in nature.*’ (Deutsch 1984, p. 421f)

To be sure, most physicists agree with this assessment. In answering the question asked at the outset of this chapter, however, I shall not distinguish between the belief that gravity is quantum and the method of constructing a quantum theory of gravity by proceeding via a quantization of a classical field because I take the latter not to contradict the former. By lack of something better, quantization can thus be viewed as a means to get a glimpse at a complete and consistent quantum theory of gravity. For the purpose of this section, therefore, I am not concerned whether an ansatz approaches the problem “top-down” by quantizing a continuous gravitational field or “bottom-up” by postulating a quantum structure of gravity or of spacetime *ab initio*. With respect to the question addressed here, both will be regarded as giving the same answer, viz. that the gravitational field is, or even must be, quantum. What all views must concur in, of course, is that any quantum theory of gravity, regardless of how it might have been “discovered,” must produce the correct classical limits.

Second, one may be tempted to equate the quantization of spacetime with its discretization. This temptation is fuelled by the expectation that quantum spacetime will no longer exhibit the smoothness of the differentiable manifold of the classical level. It is important to recognize, however, that the quantization of spacetime is neither a necessary nor a sufficient condition for its discreteness. It is not a sufficient condition because quantization may lead to a theory of QG where the relevant observables have purely continuous spectra. Once the physical Hilbert space of the quantum theory is given, one can construct operators defined on this Hilbert space which encode the geometry of spacetime. Whether these operators turn out to have continuous or discrete spectra is an entirely contingent matter. Moreover, quantization is not a necessary condition for the discreteness of spacetime either: there exist approaches, most notably the causal sets approach, which postulate a discrete spacetime structure but fail to give a full quantum theory of spacetime.¹ So while in most approaches

¹The causal sets approach is still regarded as a classical theory so far, as it fails to provide a proper quantum dynamics. In a personal communication on 24 October 2003, causal sets theorist Fay Dowker has acknowledged the need to replace the classical probabilities involved in the dynamical evolution according to the causal sets theory with e.g. transition amplitudes in order to get a quantum theory. For more on causal

to QG, quantum spacetime might exhibit some form of discreteness, this discreteness neither implies nor is implied by the quantum nature of spacetime.

So why must gravity be quantized? Callender and Huggett (2001a,b) have sensibly suggested to distinguish between the two separate issues of motivating the quest for a quantum theory of gravity on the one hand and motivating the quantization of the gravitational field on the other. The first problem can be expressed by asking “why do we need a quantum theory of gravity at all?,” and the second, assuming that the first one was answered in the affirmative, by inquiring “why do we have to quantize gravity for the purpose of finding a quantum theory of gravity?.” It may seem that these two questions can hardly be kept separate since quantum field theory requires that all matter fields be quantized and general relativity teaches that those matter fields are the sources for the gravitational field. Whether or not the two questions collapse into one depends on whether it will turn out to be possible that quantum matter fields coexist with a classical, i.e. non-quantized, gravitational field. Since at least some approaches to gravity, the so-called *semi-classical* theories, insist that such coexistence is indeed possible, distinguishing the two questions will help to map the debate. Let me address both of these two separate questions in the course of the present chapter.

Notwithstanding some potential reservations regarding the empirical accessibility of the Planck scale, the need for a theory merging quantum theory and general relativity is universally acknowledged among physicists. Opinions part, however, when the floor is opened to discussions as to how the problem of constructing such a theory must be approached. This thesis contends that there are powerful and important arguments which suggest that gravity should indeed be quantized. Peres and Terno’s result (Peres and Terno 2001) in particular, to be discussed in Section 2.2, provides such an argument, which, however, turns out to be not entirely successful. The same section also deals with what is probably the most common litany defending the necessity of a discrete spacetime structure. I analyze one of its best expressions—the one due to Doplicher and collaborators—and conclude that it begs the questions. Common to both Peres and Terno’s and Doplicher’s arguments is their attempt of solely drawing on resources from physical theories and their mathematical apparatus rather than relying on metaphysical or aesthetic principles to be discussed in Section 2.1. These arguments favorable to quantization will be contrasted in Section 2.3 with two alternative approaches to quantum gravity which do not involve quantization and by their mere existence establish the contingency of quantization in attempts at formulating a quantum theory of gravity. Some conclusions for this chapter, which stands somewhat independently, follow in Section 2.4.

sets, see Section 5.3.2.

2.1 THE UNIFICATORY REFLEX IN FUNDAMENTAL PHYSICS AND ITS DISUNITIST CHALLENGERS

So why do we need a quantum theory of gravity? Although general relativity and quantum theory may be so disparate as to disallow the formal deduction of contradictions, they are generally taken to be incommensurable (families of) theories. A quantum theory of gravity is expected to remedy this theoretical schism and to bolster attempts at finding the Holy Grail of physics, a unified framework of all interactions. The argument from unification—unification for the sake of unification—does not, however, sway the sceptic. The “disunitist” would certainly be free to respond that at the very least, it may just as well be the case that the conceptual disunity of the two theories reflects a disunity in nature. In fact, she could claim, gravity’s stubborn refusal to be subsumed under the otherwise all-encompassing umbrella of the Standard Model may be interpreted as evidence for this disunity. Despite its rare explicit articulation and its questionable metaphysical strength, however, the unificatory impetus provides an extremely important motivation for attempts at quantizing gravity. In this section I shall also discuss the supposed implications of principles of unity for the second of Callender and Huggett’s question, i.e. whether the need of a QTG necessitates the quantization of gravity, because these principles claim to bear upon both issues. At the end of the section, I hope to have a clear conclusion what principles of unity can accomplish in both respects.

So a strong, but often nebulous, desire to present a unified theoretical framework at the level of fundamental physics populates the folklore of physicists and often fuels the search for a quantum theory of gravity. Arguments to this effect, if made explicit at all, typically elicit some principles of unity of nature, of theory, or of scientific method. An enquiry into these metaphysical motivations for pursuing a quantum theory of gravity must therefore offer at least some answers to the difficulty already encountered by [Oppenheim and Putnam \(1958\)](#): despite its omnipresence, the precise meaning of the unity or unities at stake is rarely, if ever, clear. The next few paragraphs are supposed to offer nothing more than a preliminary alleviation of this difficulty; for a much more thorough investigation of the different senses of unification operative in physics, see e.g. [Cat \(1998\)](#).

There can at least be three different kinds of unity, which all come in different shapes and sizes: a unity of nature, one of scientific method, and one of theory. The first kind of unity is surely expected to capture the idea that the “world” is a unified structure which does not disintegrate into causally disconnected substructures. This unified whole, furthermore, is expected to afford a systematic description at least of those aspects which are empirically accessible to us. It may mean that there ultimately exists only one kind of fundamental

entities, which stand in only one kind of relation to other fundamental entities, or that all fundamental entities live on an equal footing with one another, or something else entirely. But I will not pursue these ideas here, as I believe the other two notions of unity are more adequate to capture the unitist spirit at work in the context of interest.

Let me thus turn to the unity of method, i.e. the belief that there exists a unique, privileged scientific method capable of producing scientific knowledge. While the unity of theory as discussed below often motivated the search for a QTG in the first place, methodological unification has fuelled the idea that in such a QTG, gravity must be quantized. In the present context, methodological unification means something to the effect that all dynamical fields must be treated equally by being subjected to quantization. Methodological unification is then in turn justified by invoking Ockham's razor or some general idea of an "economy of thought." An example of such an attitude is expressed in [Duff \(1981, p. 81\)](#): "The idea of not quantizing gravity seems to be the very antithesis of the economy of thought which is surely the basis of theoretical physics." A methodological unification strategy based on an economy of thought, as [Mattingly \(2005\)](#) rightly points out, does not in itself imply the necessity to quantize the gravitational field, as Ockham's razor cannot decide between a full QTG and semi-classical QTG which does not contain extra entities absent in full quantum gravity. It is just that the gravitational field is not quantized. As [Mattingly \(2005, p. 336\)](#) concludes, "[a] unification strategy based on parsimony of ontology [as a methodological principle] thus affords no advantage to quantizing the Einstein tensor."

In the wake of the failure of attempted reductionist strategies, many philosophers of science have grown wary of such unconditional demands for methodological (and theoretical) unification. In fact, some philosophers have spearheaded a veritable witch hunt against what they regard as imperialist tendencies of physics (and economics) to claim metaphysical privilege among the sciences and to impose its method(s) upon other disciplines.² For instance, [Cartwright \(1999\)](#) argues that the nomological and conceptual diversity with which we describe the natural world should make us cautious in accepting the canon of received methodology according to which the laws of our fundamental physical theories command a universal application. While many of their claims and their arguments have not gone unchallenged,³ I take it that no one would seriously insist on a perfectly monolithic methodology

²See e.g. [Dupré \(1993\)](#); [Cartwright \(1995, 1999\)](#). Many of their objections to unification also apply to the unity of theory, to be discussed below.

³See e.g. [Hofer \(2003\)](#); [Sklar \(2003\)](#); and particularly [Teller \(2002\)](#) for a critical examination of [Cartwright \(1999\)](#). While Teller is sympathetic to the larger outlines of Cartwright's programme, he is highly dissatisfied with her arguments in support of this programme. [Sklar \(2003\)](#) and [Hofer \(2003\)](#) acknowledge the theoretical fragmentation at the level of fundamental physics, but insist that this does not imply that mathematical laws are not universal in their scope. [Teller \(2001, 2004\)](#), who agrees with Hofer and Sklar that foundational laws are universal in scope but also maintains a pluralist ontology, to be

of science. It thus seems as if invoking these very general philosophical principles does not work toward justifying the quantization of gravity, given that we need a QTG. Arguments relying on the resources of physical theories appear more promising. These arguments will be discussed in Section 2.2.

More pertinent to my enterprise is also the notion of unity of theory, which can be taken to express the idea that scientific theories must be terminologically, ontologically, or nomologically unified, either within or among special sciences. A *terminological unification* would limit the scientific vocabulary to expressions which are definable in terms of the vocabulary of the fundamental theory only. An *ontological unification* reduces the ontologies of all theories to the one monolithic ontology of what is considered to be the most fundamental theory. In one of its more radical incarnations, ontological unification requires that all basic entities are of one kind only. Even more radically, an ontological unitist could endorse a kind of Eleatic ontology consisting of but one individual. Such an extreme ontological unification, to be sure, would completely defy its own ambition if this one individual would be granted an arbitrarily complex structure. At the very least, however, it is clear that there must be *some* uniformity in the basic ontology, for otherwise, one might just regard the conjunction of the ontologies of all theories as the fundamental ontology—a construction surely in discord with unitist ambitions. Finally, a *nomological unification* is attained to the extent to which the laws of theories, whatever they are, are implied by those of the most fundamental theory. Again, a simple conjunction of propositions expressing the laws of the to-be-unified theories cannot be adequately considered as the fundamental law, on pain of violating the substance of unitist ideology.

Theories which are unified in some of these aspects need not be unified in others. For instance, as Morrison (1995) argues, while the electroweak theory has “unified,” i.e. has brought under the description of only one theoretical structure, both the electromagnetic and the weak nuclear forces, it has retained a disunity among the particles which carry the forces. Thus, it is well possible that a theory offers a nomological, but not an ontological unification. Conversely, Sklar (2003) rejects a pluralist ontology while acknowledging that even our fundamental theories are conceptually and explanatorily, and therefore arguably nomologically, fractured.

It is not entirely clear whether Morrison’s argument insisting on an ontological disunity in the case of electroweak theory is successful. She seems to base this claim on the distinction between the force carriers of the respective forces: electromagnetic interaction is mediated via photons, which are mass- and chargeless, while the weak nuclear force carriers, the W^\pm

discussed in the next paragraph, occupies a somewhat intermediate position in the debate. Hacking (1996) offers another pluralist view.

and Z bosons, are massive and, in case of the W^\pm , electrically charged. Mass and (electric) charge, if anything, must surely be counted as essential properties of elementary particles. Since the force carriers differ in their essential properties, she seems to argue, electroweak theory does not entertain a unified ontology. However, these force carriers, at least with these properties, only come into existence at energies below the unification energy. Above the unification energy of the order of 10 GeV, i.e. before the symmetry breaking which “generates” the masses of the W^\pm and Z bosons, the ontology of the theory arguably only consists of the gauge fields W_μ^a (where $a = 1, 2, 3$) and B_μ . These gauge fields, to be sure, also differ in what could be argued are their essential properties: the W_μ^a gauge field carries the weak isospin and the B_μ gauge field has weak hypercharge. I am no expert in field theoretic formulations of the standard model and will therefore refrain from further remarks, but it seems as if one lesson should be taken home: if electroweak theory can be charged with a failure to offer a unified ontology at the most fundamental level, it seems as if the charge should proceed along these lines and argue that and how the difference between carrying weak isospin or weak hypercharge marks an ontological fracture at the level of fundamental gauge fields, and not the difference between massive and massless “effective” constituents.

Let us assume that Morrison’s argument can be salvaged one way or another. An analogous stance, [Mattingly \(2005\)](#) proposes, could be taken as far as the second of Callender and Huggett’s questions, i.e. the quantization of the gravitational field, is concerned: the vision of a unified fundamental quantum theory prohibits the co-existence of classical and quantum structures in a fundamental theory of gravity. But since the question of whether a classical structure may be part of a QTG is precisely what is at issue, invoking a unificatory principle seems insufficient. For if it were not, one could similarly argue that for a unified ontology, any charged force carrier must be dismissed as disturbing ontological unity. Mattingly admits that it may be objected that quantization is not just like other properties such as charges and that this claim is correct. But, he insists, this claim “can hardly be taken as a principled objection to its own denial.” ([Mattingly 2005](#), p. 336) Crude principles of unity may be sufficient to motivate the search for a QTG, but they seem to fall short to imply the necessity of quantizing gravity in order to achieve this aim. From this and similar examples, Mattingly concludes that unificatory principles must be sharpened in order to deliver the work they are supposed to do here.

Without going further into the details of the debate over the unity or disunity of science, these brief remarks manifestly show that while there is a stable majority of philosophers critical of a prescriptive unitary methodological dogma, there is no consensus in the community as far as theoretical unification is concerned. A fortiori, philosophers cannot make unanimous recommendations for practicing physicists. What is evident, though, is that the supporters

of unification in QG fail to receive decisive philosophical support. Interestingly, however, this does not stop practitioners of QG, themselves for professional reasons unabashed unitists, to follow some of the allegedly implied prescriptions, albeit in a completely partisan manner!

Even though abstract principles of unity may, when scrutinized, offer little justification for the search for quantum gravity, they seem to be even less powerful as arguments establishing the necessity to quantize gravity. Perhaps the proponents of the search for a QTG and the quantizers of gravity must seek elsewhere. The next section, Section 2.2, will consider arguments in favour of quantization of gravity drawn from the resources of physics alone, rather than from metaphysical principles. These arguments, though still inconclusive, will offer a much more substantial support for quantizers.

Equally, it may suffice to mobilize the resources of physics itself to find perfectly valid objectives for pursuing quantum gravity. Extant theoretical physics strongly motivates the search for a quantum theory of gravity by itself and thus dispenses with the need to invoke metaphysical unitism. General relativity conceives of gravity as a dynamical field capturing the gravitational dance of *classical* matter, while quantum theory maintains that all dynamical fields must be *quantized*, i.e. that all matter is quantum. Although this tension may be mostly dormant at the energies that are currently experimentally accessible, there are situations where the interaction between the matter fields *qua* quantum fields and the gravitational field becomes pertinent. In particular, it is believed that exotic but crucial regimes such as the early universe or the (late) evolution of black holes require a QTG. In other words, there seems to be a class of phenomena whose explanation we have good reason to believe must include both generally relativistic effects as well as quantum effects. Against this, one might argue that to this day, no observations or experiments have been performed which directly probe energy levels at which generally relativistic and quantum effects become inextricably combined. First, despite the fact that the energies required to access the Planck scale directly continue to be out of reach, a growing number of research groups studies the phenomenology of quantum gravity and it is possible that impending empirical input might very soon substantiate the need for a quantum theory of gravity.

Second, there is a difference between observing phenomena which we believe must ultimately be explained by a QTG on the one hand, and directly identifying observational or experimental data as exhibiting a quantum-gravitational signature. For instance, it is one thing to have strong reasons for believing that in order to understand the formation and evaporation of black holes, one must have a grasp on their generally relativistic characteristics as well as on their quantum aspects. But it is quite another thing to be able to present an explicit catalogue of data for which we are certain can only account a QTG. Until we have completely formulated a QTG, of course, we may not even know what a quantum-

gravitational signature could look like. So it seems asking for way too much if we should be able to produce a catalogue of quantum-gravitational effects to be accounted for by a yet to be formulated QTG prior to even having started the first steps towards such a theory. But it surely cannot be that the full catalogue of empirical consequences of a theory must be offered as a justification to start its formulation. What *can* reasonably be asked for is a set of phenomena which existing theories have trouble accommodating and some justification why the resolution of these difficulties must be sought in a particular direction, and both of these demands seem to be satisfied in the case at hand. But to be sure, although we may well have strong reasons to believe that only a QTG will be able to offer a full understanding of the formation and evaporation of black holes even in the absence of data with unambiguous quantum-gravitational signature, it may turn out that we have been misled in this belief and that in fact there is a pair of theories, one a classical theory of gravity and the other a quantum theory not of gravity, but of something else, that can jointly account for everything we see. In other words, it might just be the case that there are two separate phenomenological domains which, despite deceiving appearances, have an empty intersection and that there are two theories which perfectly manage to account for their respective domains. In this case, the set of recalcitrant phenomena would neatly decompose into two (or more) domains reigned by different theories.

However, although the situation in which most physicists believe a QTG is required are phenomenologically exotic, and although many physicists and philosophers are highly critical of particular approaches to finding a QTG, no one seems to seriously deny the cogency and relevance of the enterprise of seeking a QTG. I will return to the unificatory rhetorics prevalent in the physics literature and the disunitist reflex recently found in philosophy pages, as well as to whether any morale can be drawn for this debate from quantum gravity in Section 2.4. Now, I turn to *physical* motivations for quantizing gravity.

2.2 WHY QUANTIZE GRAVITY?

Assuming then that we need a quantum theory of gravity, does gravity necessarily have to be quantized in such a theory? I concur with [Callender and Huggett \(2001a,b\)](#) and with [Mattingly \(2005\)](#) that this question must be answered in the negative. The mere existence of approaches to quantum gravity which do *not* involve the quantization of gravity implies that quantization is a contingent matter. Semi-classical theories of quantum gravity, which supply the wedge that Callender and Huggett drive between the two disparate questions of whether

we need a quantum theory of gravity and of whether such a need implies the quantization of gravity, constitute such approaches. Semi-classical quantum theories of gravity stipulate the coupling of the classical gravitational field, and therefore of the classical spacetime geometry, to the quantum fields of matter. The coupling of the gravitational field only occurs via the “classical” quantum magnitude of the expectation value of the stress-energy tensor of the quantum fields present in some quantum state of matter ψ . Thus, quantum matter can be coupled to the classical spacetime geometry via the “classical” quantity $\langle \widehat{T}_{\mu\nu} \rangle \doteq \langle \psi | \widehat{T}_{\mu\nu} | \psi \rangle$. The dynamics of the coupled system is governed by the coupled semi-classical Einstein and the quantum Schrödinger equations:

$$G_{\mu\nu}[g_{\mu\nu}] = 8\pi G \langle \widehat{T}_{\mu\nu} \rangle, \quad (2.1)$$

$$\widehat{H}[\psi, g_{\mu\nu}]|\psi\rangle = i\frac{\partial}{\partial t}|\psi\rangle, \quad (2.2)$$

where the $G_{\mu\nu}$, the so-called Einstein tensor, characterize the spacetime geometry and $\widehat{H}[\psi, g_{\mu\nu}]$ is the quantum Hamiltonian operator in which the gravitational field appears as external source. In (2.1), the classical Einstein equations are obviously modified by exchanging the classical energy-momentum density $T_{\mu\nu}$ with its expectation value $\langle \widehat{T}_{\mu\nu} \rangle$. Most importantly, equations (2.1) govern the so-called *quantum backreactions*, i.e. quantum fluctuations induced on the (classical) gravitational field by its coupling to the quantum fields of matter. It will be the subject of some debate in the part of Section 2.3 which discusses Peres and Terno’s no-go result whether this classical-quantum coupling, including the quantum backreactions, can be properly implemented by consistently combining the two dynamical sectors.

Typically, it is assumed that equations (2.1) offer a valid approximation to a full QTG in cases the backreactions are sufficiently small, at least locally. Unfortunately, as Wald (1994, p. 98) explains, the precise range of applicability of (2.1) is not known, no more than it is for the analogous semiclassical Maxwell equations $\nabla^\mu F_{\mu\nu} = -4\pi \langle j_\nu \rangle$ in QED, where $F_{\mu\nu} \doteq \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor, with A_μ the covariant vector potential of the electromagnetic field, and j_μ is the four-current. Furthermore, as he continues, anyone who wishes to calculate the quantum backreaction effects in this approach is confronted with at least three serious difficulties: at least one of the fundamental parameters required to define $\langle \widehat{T}_{\mu\nu} \rangle$ must put in by hand (or by experiment), the semi-classical Einstein equations (2.1) admit new solutions with “runaway” character, and the computation of $\langle \widehat{T}_{\mu\nu} \rangle$ itself is such a formidable task as to make it almost impossible.

Although these difficulties may very well turn out to be fatal for semi-classical theories with quantum backreactions, these theories offer, at least in principle, an escape from quantizing gravity. To be sure, only a few renegades actively pursue approaches in this camp, but

the recent results reviewed by [Brunetti and Fredenhagen \(2006\)](#) have reinvigorated semi-classical theories. But the justification for semi-classical theories is challenged by Peres and Terno’s no-go result, to be discussed in Section 2.2.3. Even if their argument is unsuccessful, however, semi-classical approaches have lost much of their appeal as a result of what seem to be almost insurmountable difficulties, as mentioned. Quite apart from the fact that quantization might offer an appealing escape from the dismal mood in the anti-quantizer camp, the question then becomes whether physics itself might dictate the quantization of gravity. After a brief glimpse at the prehistory of the current debate, the remainder of this Section will review what I take to be the two most promising explicit arguments in favour of quantization.

2.2.1 Early arguments pro quantization

So there is, overall, little doubt that in the final QTG gravity *will* be quantized, but the question will ultimately have to be settled on empirical grounds, as [Callender and Huggett \(2001a,b\)](#) remark. I concur with this assessment, although I want to resist giving the impression, at times created but ultimately dismissed by [Callender and Huggett \(2001a,b\)](#), that this conclusion is implied by the failure of those theoretical arguments discussed in this subsection.⁴ The argument to this conclusion rests on the premises that in an empirical science, such as physics, experiments and observations must decide between internally consistent, inequivalent competitor theories,⁵ and that QG is no exception in this respect. I see no reason to harbour principled doubts about both premises. The main conclusion of Callender and Huggett stands.

However, their investigation is too narrow in that it is confined to Eppley and Hannah’s classic, but somewhat obsolete argument. [Eppley and Hannah \(1977\)](#) have argued that gravity must be quantized on the basis that the interaction of a classical gravitational field with a quantum field leads to contradictions with trusted physical principles. The argument has the form of a reductio proof which opens a dilemma and purports to show that both horns of the dilemma lead to absurdity. The premise to be reduced to absurdity is, no surprise, the assumption that the gravitational field is a purely classical field. Another premise of their argument, *which is not meant to be reduced to absurdity, however*, is that quantum

⁴Note that I label Eppley and Hannah’s thought experiment as a *theoretical*, as opposed to experimental, pace [Mattingly \(2006\)](#). Thought experiments, to my mind, are nothing but theoretical arguments; appealing or persuasive at times, unconvincing at others, but never of the same *kind* of support as empirical confirmation. Consult [Norton \(2004\)](#) for a defence of this position. A proposed experiment with an anticipated outcome remains a thought experiment until it is actually performed.

⁵Given the enormous differences between the different approaches to QG, the simultaneous emergence of two empirically adequate and empirically equivalent theories seems vastly improbable.

mechanics is to be interpreted standardly: the wave function of a quantum state collapses immediately upon measurement into an eigenstate of the pertinent observable. For this substantial and far from uncontroversially true premise alone, which does important work in their argument, their thought experiment must be judged inconclusive. In a *reductio* proof, it is always the *conjunction of all premises* and not just an individual premise alone which is reduced to absurdity. Although thereby leaving the realm of logic, one may of course argue that a particular premise be abandoned. But such an argument can only succeed if all other candidate premises at stake can be shown to be uncontroversial.

Eppley and Hannah open the dilemma by presupposing what they take to be mutually exclusive and jointly exhaustive possibilities: either interaction with the gravitational field does or does not collapse the quantum state. If it does, Eppley and Hannah argue, then the conservation of energy-momentum must be violated, if the theory respects the uncertainty relation. On the other hand, if it does not, they allege to show, superluminal signalling becomes possible. Because either of these consequences is undesirable, they conclude, gravity cannot be classical.

Callender and Huggett (2001a,b) detect some significant loopholes in the argumentation in both horns of the dilemma and consequently discount Eppley and Hannah's argument as incomplete. Mattingly (2006) complains that the device proposed by Eppley and Hannah in order to measure the gravitation-quantum matter interaction would have to be so massive as to be confined to within its own Schwarzschild radius. I acknowledge all these counter-arguments. My main reservation against Eppley and Hannah, however, stems from their argument's dependence on the interpretation of quantum mechanics, as indicated above. The very structure of their thought experiment, I maintain, is fatally flawed in that they unjustly blame the absurdity on one particular among at least two disputed premises. Even if there were no loopholes at all, and even if their measuring device would be constructible, I urge, any argument based on one specific, highly controversial interpretation of quantum mechanics—a collapse interpretation in this case—suffers by extension and cannot yield a conclusive argument as to whether gravity must be quantized on physical grounds.

Before the Eppley and Hannah *gedanken* came to dominate the literature as the leading argument supposedly implying the quantization of gravity, an almost ancient argument by Bohr and Rosenfeld (1933) and refined by DeWitt (1962) populated the imagination of physicists.⁶ It questioned the adequateness of semi-classical theories mixing classical with quantum fields and allegedly showed how any classical field that is coupled to a quantum

⁶And before then, Einstein (1916b, 1918) and Klein (1927) defended the view that GTR will ultimately have to be modified in the sense that it must accommodate quantum aspects. For a discussion of the positions concerning the need for quantizing gravity or GTR held in the interwar period, consult Stachel (1999).

field must be quantized, as the uncertainty relations of the quantum field “infect” the classical field via “quantum disturbances.” [Brown and Redhead \(1981\)](#) have cast doubt on the argument by attacking the disturbance view of the indeterminacy. In the view of [Callender and Huggett \(2001a,b\)](#), this attack has been successful. However, in some sense at least, the attack launched by [Brown and Redhead \(1981\)](#) was against a strawman: [Rosenfeld \(1963\)](#) himself insisted in his reaction to DeWitt’s elaboration of what was generally taken to be his, Rosenfeld’s, argument implying the necessity to quantize gravity that the argument was never meant to be conclusive. In fact, Rosenfeld wrote, not only will empirical evidence eventually have to pronounce the verdict, but in the absence of such evidence, he stalled overly enthusiastic quantizers, “this temptation should be resisted.” ([Rosenfeld 1963](#), p. 354) Callender and Huggett concur with him, and I tend to agree, in that not even the most elegant and powerful formal apparatuses can yield conclusive recommendations concerning the quantization of gravity without the necessary empirical input. Or, as [Rosenfeld \(1963, p. 356\)](#) put it, “[e]ven the legendary Chicago machine cannot deliver the sausages if it is not supplied with hogs.”

2.2.2 Non-commutative Spacetime Operators

There exists an important tradition, going back to [Snyder \(1947a,b\)](#) and in fact even to von Neumann, which attempts the quantization of gravity by introducing non-commutative operators for spacetime coordinates. As in LQG, these operators have a discrete spectrum of eigenvalues and thus suggest a discrete interpretation of spacetime. In this case, but not in general as indicated above, this discreteness encapsulates the quantum nature of spacetime or, equivalently, of the gravitational field. Unlike in LQG, however, the discrete spectra are not results of the quantization procedure, but rather act as desiderata around which a quantum theory of spacetime must be constructed.

In a handwritten letter to Dirac dated 27 January 1934, von Neumann considers non-commuting “space coordinate operators” X , Y , and Z with a discrete spectrum as well as a time operator T with a continuous spectrum. Von Neumann starts out by stating what the desired behaviour of these operators should be: the spatial operators need to have discrete spectra; the temporal operator can have either discrete or continuous spectrum, but a continuous spectrum would be highly preferable;⁷ and last but not least, they must respect Lorentz symmetry. Note that the demand for Lorentz symmetry exonerates von Neumann from a potential accusation that demanding discrete spectra for spatial and a continuous spectrum for the temporal operator might introduce an inadmissible distinction

⁷“I would prefer a continuous spectrum for ‘time’.” (Cited from [Rédei \(2006, p. 100\)](#))

between space and time and a privileged observer. Motivated by the hope to exterminate singularities in electrodynamics which arise in approaching a point-like charge to within arbitrarily small distances, he was however soon discouraged by the lack of a proper physical justification for assuming the discreteness of physical space, as well as by Dirac's sceptical response of 28 February 1934. Four years later, on 17 March 1938, he wrote to a Hungarian physicist named Rudolf Ortvy that "I did not examine the model in deeper detail because I considered it very artificial and arbitrary—and so I still think today."⁸ A decade later, [Snyder \(1947a,b\)](#) proposed a similar scheme, from similar motivations, this time including the time operator among those with discrete spectrum.

The most popular argument for the non-commutativity of the spacetime coordinates and for the discreteness of spacetime has found a famous expression in [Doplicher, Fredenhagen, and Roberts \(1995\)](#). I think that it can be safely said that this article has now replaced [Eppley and Hannah \(1977\)](#) as the standard argument for quantization. Doplicher and collaborators have argued that the quantum uncertainty relations of the spacetime coordinates emerge from the combination of Heisenberg's principle with general relativity. The argument, as I interpret it, should contain two main parts: the first shows that continuous spacetime loses its operational meaning at small scales already at the semi-classical level; the second would extend the argument to full quantum gravity by defending that this operational limit is due to the truly (discrete) quantum nature of spacetime at Planck scale.

The first part of the argument encodes a rather common belief that the combination of the uncertainty relations and classical general relativity imposes restrictions on the operational meaning of classical spacetime. The reasoning runs along the following lines. If

- (1) the greater the accuracy (or, equivalently, the smaller the uncertainty) in the measurement of spatio-temporal coordinates, the stronger the gravitational field generated by the measurement, and
- (2) an increasing gravitational field eventually becomes strong enough as to collapse to a black hole, creating a closed trapped surface, and thus prevents any signal from leaving the region at stake, and
- (3) an operational meaning can only be attached to a spacetime localization in case signals can leave the region measured,

⁸Cited from [Rédei \(2006, p. 195\)](#). I am greatly indebted to Miklós Rédei for having pointed out this correspondence to me and having communicated the details of the letters, their references, and transcriptions (personal communications on 13 September 2005 and on 28 April 2006). The originals of the von Neumann-Dirac correspondence are to be found in the Library of Congress [no signatures] and the original letter to Ortvy is hosted by the Department of Manuscripts and Rare Books at the Library of the Hungarian Academy of Sciences [signature K 785/97]. Von Neumann's letters are all transcribed and, in the case of the letter to Ortvy, translated into English in [Rédei \(2006\)](#). Neither [Stachel \(1999\)](#) nor [Rovelli \(2004, Appendix B\)](#) seem to be aware of these pertinent von Neumann letters.

then

- (4) a localization measurement is operationally restricted to a certain maximum accuracy, or minimum uncertainty.

The third premise is merely a definition of operational meaning and does not seem to be either problematic or substantial. The first two premises, on the other hand, are those that procure the mileage. The second premise epitomizes how gravity determines the causal structure of spacetime. This is an immediate consequence of classical general relativity and deserves all the credit that general relativity does. The first premise is a rather direct consequence of combining Heisenberg’s uncertainty principle with the principles of general relativity, as [Doplicher, Fredenhagen, and Roberts \(1995\)](#) claim. According to the uncertainty principle, they explain, measuring a spacetime coordinate with accuracy $1/E$ generates an uncertainty E in the associated momentum. Thus, an uncontrollable energy of the order E is concentrated in the spacetime region to be measured. According to general relativity, the energy-momentum density $T_{\mu\nu}$ associated with this energy deforms the geometry of the spacetime.

But what does it mean to “measure a spacetime coordinate”? Unfortunately, I have not been able to find an explication of the measuring process anywhere in [Doplicher, Fredenhagen, and Roberts \(1995\)](#), nor in von Neumann’s correspondence, where the same language is used. It almost appears as if this tradition just posits the measurement of spacetime as a primitive concept of the theory. But this should not content us. What then would be an adequate explication of “measuring a spacetime coordinate” capturing its meaning? The terminology suggests that there exists a physical system (“spacetime”) which can be in different states (the eigenstates of the spatiotemporal operators). When we measure a particular spacetime coordinate, then, this measurement would amount to an application of the corresponding operator onto the state of the system and receive an eigenvalue in return. If the physical system is a chunk of spacetime, then these measurements would inform us what the spatiotemporal “size” of this chunk was. Clearly, such a metaphysically loaded concept of the measurement of spacetime coordinates must be unacceptable to an operationalist. For her, it should be preferable to conceive of spacetime measurements as measuring the spatiotemporal extension of a physical object.

Returning to the structure of the argument explicated above, note that the first two premises are implied by basic principles of well-trusted physical theories. While this certainly does not make them unassailable, our reasons for rejecting them would have to be substantial enough to challenge first principles of two of the most successful achievements of twentieth-century physics: quantum mechanics and general relativity.

One may argue, however, that these first principles will naturally be challenged at the

level of quantum gravity. The fact that attempts to combine quantum mechanics with general relativity unveil their conceptual incommensurability can be taken to *imply*, or at least strongly suggest, that some of their first principles must be violated in quantum gravity. In a sense, then, quantum mechanics and general relativity when combined already contain the seeds of their own destruction. Thus, one cannot expect that all principles that underpin the above premises will still be valid in full quantum gravity. But if not all premises hold, then the argument will of course collapse. Because the source energy associated with $T_{\mu\nu}$ is quantum and the argument as given is thus strictly semi-classical, the objection could conclude, it may very well turn out that the discreteness emerges only as an artefact of the manner in which quantum mechanics and general relativity were combined at the semi-classical level. Therefore, the argument as given so far must be complemented by a second part asserting that the operationally discrete spacetime at the semi-classical level results from an underlying discreteness at the fundamental Planck level.

Unfortunately, Doplicher and his collaborators do not provide such an addition. But neglecting to take this second leg seriously amounts to begging the question. It is undoubtedly true that if spacetime is discrete at the Planck level, it is reasonable to expect some signatures of this discreteness to surface at the semi-classical level. But the converse is not true, exactly because some or all of the premises made above may no longer obtain in full quantum gravity. The argument as it stands will hence not make any converts. Despite its appealing reliance on deeply entrenched physical principles, the argument thus falls short of proving that spacetime must be discrete (or, similarly in this case, that gravity must be quantized) from the resources of trusted physical theories alone.

But the question then becomes how *any* argument drawing solely on accepted physical theories can possibly establish that gravity must be quantized. If a quantum theory of gravity would be part of the established corpus of theories, the proof would be easy. But alas, it is not! The failure of current physics to offer a straightforward and unique path to a quantum theory of gravity strongly suggests that the formulation of such a theory will require new physics. In this case, however, one cannot accept an argument from the resources of *old* physics alone to the effect that gravity must be quantized.

2.2.3 Inconsistency of Quantum-Classical Dynamics

Despite this limitation of arguments relying on principles of accepted physics, some ways of constructing a quantum theory of gravity without quantization, it is claimed, can still be excluded. [Peres and Terno \(2001\)](#) and [Terno \(2006\)](#) have mounted an argument which seems to preclude the possibility of a consistent semi-classical quantum theory of gravity

which includes quantum backreactions. A somewhat peculiar idea underlies the enterprise: assuming that not all quantum dynamical variables are observable and some principled rules for distinguishing operators corresponding to observables from those associated with “unobservables,” a classical system is introduced as a quantum system for which all operators corresponding to observables commute. In general, there will be unobservables in the classical system which do not commute. It turns out that a formalism introduced in [Koopman \(1931\)](#) and refined in von [Neumann \(1932a,b\)](#) can suitably accommodate this idea. Consider, then, a system consisting of two sectors, one governed by quantum field theory (the quantum sector), and a classical sector described by so-called Koopmanian dynamics. Koopmanian dynamics offers a generalized formalism that allows to cast a classical system in the mathematical formalism of a quantum theory, i.e. as a Hamiltonian system living in a Hilbert space. The conjugate variables of the classical sector will be represented by commuting multiplication operators, as opposed to the non-commuting operators of the quantum sector. Furthermore, assume that the hybrid dynamics of the combined classical-quantum system is described by a unitary evolution on the joint Hilbert space $\mathcal{H} = \mathcal{H}_q \otimes \mathcal{H}_c$.⁹

Borrowing from the exposition of the Koopmanian formalism in [Mauro \(2002a,b\)](#),¹⁰ let me briefly outline the major ideas of Koopmanian mechanics. Koopmanian mechanics attempts to offer a mathematical framework in which to cast classical mechanics in order to make it more directly contrastable to quantum mechanics. The basic idea is to introduce a Hilbert space of complex and square-integrable functions $\psi(q, p)$ which can be thought of as “classical” wave functions. These wave functions ψ must be such that $\rho(q, p) \doteq |\psi(q, p)|^2$ can reasonably be interpreted as a measure for the probability of finding a particle at the point (q, p) in the phase space, which is parametrized by the independent and commuting variables q and p . In classical mechanics, the probability density $\rho(q, p)$ must evolve according to the Liouville equation

$$i \frac{\partial}{\partial t} \rho(q, p) = \widehat{L} \rho(q, p), \quad (2.3)$$

where \widehat{L} is the Liouville operator which assumes the role of classical analogue to the Hamiltonian operator present in the Schrödinger equation. The Liouville operator is a functional

⁹It might be objected here that the technical choice of using the tensor product of the two subspaces rather than the direct sum opens the argument to question. However, I think that this choice is justified given that it is customary in quantum mechanics to use the tensor product when combining two subsystems which individually continue to be in a state hosted by their sector, e.g. a two-particle system, and to opt for a direct sum of the smaller Hilbert spaces in cases where the joint system is either in a state living in one of the subspaces *or* in a state in the other subspace. Here, we are confronted by a physical system of interacting fields which maintain their classical or quantum character and not by a system which assumes *either* a classical *or* a quantum state altogether.

¹⁰But see also [Peres \(1993, pp. 317-319\)](#).

of the classical Hamiltonian function $H(q, p)$ defined on the phase space. Koopman and von Neumann found that if one postulates the same Liouville evolution for the classical wave function,

$$i\frac{\partial}{\partial t}\psi(q, p) = \widehat{L}\psi(q, p), \quad (2.4)$$

then equation (2.3) is automatically satisfied. Equation (2.4) can be considered the dynamical evolution equation for the classical Hilbert space, in close analogy to the standard Schrödinger equation of quantum mechanics. Koopman’s theorem (Koopman 1931, p. 316) shows that the dynamical evolution of $\psi(q, p)$ is unitary: given another Liouville wave function $\phi(q, p)$ which obeys (2.4), the scalar product $\langle\psi(q, p)|\phi(q, p)\rangle$ is invariant over time.

In quantum mechanics, however, the nexus between (2.4) and (2.3) no longer holds, as the quantum mechanical probability density spreads over time while it does not in classical mechanics. In Koopmanian mechanics, the wave functions do not spread over time, and the basic operators \widehat{q} and \widehat{p} , the configuration and conjugate momentum operators, commute with one another. Consequently, there is no uncertainty in the simultaneous measurement of q and p and no interference pattern in a “classical” double-slit experiment.

Let me return to the argument by Peres and Terno (2001) purporting to establish the inconsistency of quantum backreactions on the classical sector of the joint Hilbert space $\mathcal{H} = \mathcal{H}_q \otimes \mathcal{H}_c$. The following reconstruction follows Terno (2006) rather than Peres and Terno (2001) as the former offers a more detailed account than the latter. Since operators acting on different sectors of \mathcal{H} commute, classical and quantum operators will always commute, regardless of whether they correspond to observables or unobservables. If the interaction part of the unitary evolution operator is a function of observable and unobservable quantum operators on the one hand, but only of *observable* classical operators, then the interaction term will always commute with the classical observables, thus effectively decoupling the classical degrees of freedom from the quantum ones. In this case, therefore, quantum backreactions on the classical sector are precluded and consequently any interesting form of semi-classical theory of gravity is barred.

For this reason, the argument continues, the interaction term must also contain classical operators associated with the unobservable classical dynamical variables, as it is those onto which the quantum degrees of freedom latch. If one weaves such unobservable operators into the interaction part of the evolution operator of the joint system, however, then the equations of motion of the combined system will no longer formally be the same as for the purely classical equation (2.4) or the purely quantum equations of motion, i.e. the standard Schrödinger equation. In itself, such a structural dissimilarity would certainly not constitute grounds for rejecting the equations of motion of the combined system; after all, by introducing

an interaction between the classical and the quantum degrees of freedom, such dissimilarity is bound to occur. But the failure to exhibit such formal “isomorphism,” Peres and Terno argue, amounts to a violation of the correspondence principle, which is, of course, unacceptable. If I understand their argument correctly, which is not a trivially true antecedent condition, then the correspondence principle is also violated at the level of observable quantities only.

The failure of the correspondence principle is testified, as [Terno \(2006\)](#) elaborates, in attempts to obtain the correct classical limit that must hold for the combined system, i.e. in the limit where both sectors are classical. The reason for not obtaining the correct classical limit for hybrid systems with full interaction is that the evolution of the motion of observables differs if this formal correspondence does not apply. Since the presence of non-observable classical operators in the interaction term is responsible for the differences between the equations of motion, the argument continues, the interaction part can only contain observable classical operators on pain of violating the correspondence principle. But in this case, as explicated above, the quantum sector cannot influence the classical sector, thus precluding quantum backreactions! Even under these relatively weak assumptions, Peres and Terno conclude, it turns out to be impossible to define a mixed system with a hybrid dynamics that consistently includes the full interaction between the two sectors.

Although Peres and Terno’s result also inhabits the semi-classical realm, I maintain that this does not render it vulnerable to the above charges as it did the preceding argument. The reason is simple: it does not compound two theories on whose principles it relies, but only offers a no-go result using a general formalism and invoking the unitarity of the combined evolution as well as the requirement for a hybrid theory to possess the correct classical limit. These principles, it seems, constitute general desiderata for constructing theories rather than axioms of a specific theory that may become redundant as the theory is superseded. Although the validity of the principle of unitarity is controversial in QG, a full QTG is surely expected to comply at least with the correspondence principle. Insofar as Peres and Terno’s assumptions transcend particular theories, they are immune to the objections aired against [Doplicher, Fredenhagen, and Roberts \(1995\)](#).

Having said this, however, there are serious problems with their, Peres and Terno’s, argument. Quantum backreactions, in this scheme, can only be introduced if the interaction part of the joint evolution operator does not commute with the classical observables. Since any classical operators commute with the quantum ones, this coupling must be introduced via classical unobservables which must therefore be present in the interaction term. In order to be successful in this endeavour, the classical unobservables cannot commute with the classical observables. But what is the justification for this? Could it not equally well be argued that commutativity is the high watermark of classicality and that therefore *all* classical operators

must commute with one another? In the argumentative scheme proposed by Peres and Terno, this would imply that there is no backdoor for coupling the quantum degrees of freedom to the classical ones. But if that's the case, it becomes very questionable whether the formalism chosen is indeed capable of offering an adequate framework for considering a semi-classical QTG including quantum backreactions.

Another criticism, to be credited to James Mattingly,¹¹ arises from the consideration of semi-classical theories, which have actually been proposed, such as the QFT on curved space-times as discussed by Wald (1994). The fact that Wald's axioms, presented in Wald (1994, p. 89), are consistent and appear to satisfy the correspondence principle, or at least I see no reason why to think otherwise, suggests that either Peres and Terno's argument is not valid after all, or Koopmanian formalisms are not exhaustive of hybrid systems. Mattingly suspects that although Peres and Terno's argument might in fact undermine approaches which attempt to directly mix quantum and classical variables, it may not apply to approaches which operate by coupling the *expectation value* of quantum variables to the classical degrees of freedom. This, after all, is how semi-classical theories are typically characterized.

I thus conclude that while I accept Peres and Terno's line of thought as strong evidence that some ways of constructing semi-classical theories cannot be brought to fruition, it does not exhaust all, or even the major, possibilities to approaching the formulation of a semi-classical theory.

2.3 WHY *NOT* QUANTIZE GRAVITY?

Peres and Terno's result may preclude the possibility of formulating a consistent semi-classical quantum theory of gravity including full quantum backreactions by directly mixing classical and quantum variables. But there exist more promising alternative approaches that offer quantum theories of gravity which do not involve a quantization of the gravitational field and, I will argue, escape Peres and Terno. Typically, they understand gravity as an *induced* rather than a fundamental force. According to this view, gravity is not one of the four fundamental forces; instead, it emerges at a higher level as a result of the fundamental physics. In this section, I expose two proposals which, at least to some extent, do not regard gravity as a fundamental force. For them, since gravity is not fundamental, it does not have to be quantized. Thus, they provide quantum theories of gravity while denying the necessity of its quantization.

¹¹Mattingly expressed the following objection during our personal exchanges, most succinctly on 16 March 2006.

The reader may wonder why I wish to label these approaches as quantum theories of gravity. Obviously, they do not constitute theories of gravity qua quantum field. But even so, they offer a resolution of the conceptual tension between quantum theory and general relativity and promise to reconstitute an account of those phenomena which led us to quest for a quantum theory of gravity in the first place. Hence, they solve the problem that required a quantum theory of gravity, and shall thus be termed as such.

2.3.1 Sakharov’s Induced Gravity

Most prominently, perhaps, among these alternative approaches is Sakharov’s induced gravity theory. As it has recently been resurrected, it deserves particular attention.¹² It claims to implement the vision of Lorentz (1899-1900) contemplating the possibility of gravity as an effective force induced by residual electromagnetic forces. For Sakharov, gravity is thus not a fundamental physical field, but “induced,” i.e. emergent from quantum field theory like hydrodynamics emerges from molecular physics. Nota bene, since the interaction part of the action contains both classical and quantum terms, Sakharov’s account leads to a type of semi-classical quantum gravity.

The general framework for an induced gravity theory in Sakharov’s vein is set up by first assuming a Lorentzian manifold as a background on which to do quantum field theory. This background is a continuous, classical, *unquantized* spacetime. It is left free “to flap in the breeze,” i.e. no assumptions regarding its dynamical evolution are made. In particular, no Einstein equations—modified or not—enter the picture. When we do quantum field theory on this background spacetime, it turns out that the effective action at the one-loop level automatically contains terms proportional to the cosmological constant and to the Einstein-Hilbert action of general relativity, as well as higher order terms. Thus, it looks as if Einstein gravity is generated at the one-loop level from the interaction of quantum fields.

It may be instructive to note that gravity was not really created *ex nihilo*. At the bare minimum, a Lorentzian background manifold was assumed. Furthermore, the geometry of the background manifold acted as an external field for the quantum fields living on the background manifold. Terms encoding the geometry of the classical background thus cohabited in the Lagrangian of the effective action with the terms of the quantum fields. To the extent to which the background geometry was presupposed, arguably, the gravitational degree of freedom, but not its dynamics, was put into the theory from the start. Next, as the gravitational field is considered a degree of freedom, the variation of the effective action with

¹²The original paper is Sakharov (1967). There is a modern recast by Visser (2002). Sakharov’s original article had over 350 citations in high energy physics as of April 2006, of which 61 since January 2003 alone.

respect to its variables should automatically lead to the (semi-classical) Einstein equations.

While the fact that the Einstein-Hilbert action is mimicked at the one-loop level is very suggestive, it is unclear whether this framework is sufficient to fully recover gravity. But it is important to note that gravity, although arguably not fully “induced,” was not quantized, as the geometry was assumed to be classical. Furthermore, as other ansätze operating with the expectation value of $\widehat{T}_{\mu\nu}$, the approach avoids Peres and Terno’s prohibition of a semi-classical theory of gravity with quantum backreactions using direct mixing. Since no dynamical assumptions were made regarding the classical background (the classical sector), i.e. no classical dynamics imposed on the classical sector, the gravitational action is induced from the mutual interaction of the underlying quantum fields. The classical sector acts as a mere background and does not interact with the quantum sector. Therefore, no hybrid dynamics is necessitated in this approach.

Interestingly, there exist reverse efforts of constructing a fundamental quantum theory of gravity, subsumed under the heading “non-commutative geometry,” which assume only gravity to be a fundamental force and derive the standard model of electromagnetic, weak, and strong forces from gravity. However widespread and persuasive the belief in a fundamental theory containing gravity as a fundamental force may be, currently available observations and experiments at most license inferences regarding the same semi-classical realm that Sakharov’s approach describes. Not unlike the belief in the unity of nature, the belief in a fundamental theory including gravity is exposed by Sakharov as an additional commitment not warranted by the (currently available) resources of empirical physics alone.

2.3.2 Jacobson’s Gravitational Thermodynamics

More recently, [Jacobson \(1995\)](#) has offered a perspective that also cautioned against quantizing the Einstein equations. Rather than deriving the four laws of black hole thermodynamics from the classical Einstein equations, as did [Bardeen, Carter, and Hawking \(1973\)](#), Jacobson inverts the derivation by recovering the Einstein equations from the entropy’s proportionality to the horizon surface area of a black hole together with the fundamental thermodynamical relation $\delta Q = TdS$, connecting heat Q , temperature T , and entropy S . The heat is interpreted as the energy flux across a causal horizon and the temperature as the Unruh temperature relative to an accelerated observer just inside a local Rindler horizon. This heat manifests itself via the gravitational field it generates. As in conventional thermodynamics, where heat is interpreted as energy flux between unobservable degrees of freedom, the underlying mechanics of the energy flux is irrelevant. Assuming cosmic censorship, Jacobson formulates local gravitational thermodynamics for an observer by means of the boundary of

her past (her “causal horizon”), associating this boundary with entropy. The system that radiates heat is identified with the degrees of freedom behind the horizon, separated from the observer’s past by a causality barrier and is therefore unobservable.

As Jacobson shows, this interpretation imposes conditions on the curvature of spacetime such that the classical Einstein equations are implied. Therefore, he suggests that the Einstein equations can be more adequately analogized with the wave equation for sound in a medium, rather than interpreted as the dynamical equations for a fundamental field. These equations, he urges, as higher-level equations of state, should then not be quantized as if the gravitational field were fundamental, despite the fact that they may describe what is ultimately a quantum reality.

As in Sakharov’s approach, gravity does not represent a fundamental force. Rather, it emerges as a phenomenon supervenient on the energy flux from causally inaccessible degrees of freedom. Again, Peres and Terno’s result does not apply. However, Jacobson’s claim that gravity should not be quantized in this scheme *because* it represents a collective, higher-order degree of freedom is simply false. Physicists routinely quantize collective degrees of freedom such as sound (with “phonons” as quanta of sound). Whether or not a degree of freedom must be quantized or not does not depend on whether it is collective or individual, but on altogether different considerations. Hence, quantization cannot necessarily be escaped by Jacobsonian gravity; but it is not forced on it either.

Jacobson’s incipient program, unfortunately, has so far not been worked out in any detail. It remains to be seen, therefore, whether it will be able to offer a full picture of the workings of gravity. If it will be successful in this undertaking, then the quantizers of gravity may have met a challenge.

2.4 CONCLUSION OF THIS CHAPTER

This chapter has analyzed what I take to be the most influential arguments pertinent to the issue of whether gravity must be quantized in a final quantum theory of gravity, taking a closer look at two on each side of the debate. The argument exemplified by Doplicher and collaborators, prevalent in the physics community—as far as the community is concerned with the issue at all—, has been shown to be incomplete. It was charged with lacking a part establishing that the operationally discrete spacetime results from an underlying discreteness at the Planck scale. Next, I have discussed, but only partially accepted, Peres and Terno’s argument as a proof of the impossibility of constructing a consistent semi-classical theory

including truly hybrid dynamics. I insisted that it does not rule out accounts where either gravity is not a fundamental, but only an effective force, or the quantum degrees of freedom are coupled to the classical ones only via their expectation values, as is typical in semi-classical theories.

Sakharov's induced gravity program provides an effective, semi-classical approach to gravity that does not require a quantization of gravity. Similarly, Jacobson's gravitational thermodynamics conceives of gravity as emergent from the energy flux of unobservable degrees of freedom. Although regret must be expressed that these accounts are but nascent attempts to understand gravity and that they are therefore yet lacking in many respects, they establish that it is at least conceivable that the final theory of gravity may not involve quantization.

I have not addressed in this chapter the suggestion made by some that the fact that the classical theory of general relativity generically admits singularities in its solutions, in an attempt to exterminate these pathologies, necessitates the quantization of the gravitational field. This issue, in particular the fate of the "big bang" singularity in cosmological models based on LQG will be discussed at length in Chapters 6 through 8.

The question of whether the gravitational field must be quantized in a full quantum theory of gravity thus turns out to be more subtle than is commonly assumed. Once stripped from its endemic folklore, the field affords a wide variety of arguments drawing on the resources of physical theories. None of these arguments can, by itself, claim conclusiveness. Ultimately, I concur with [Mattingly \(2006\)](#) who takes the pragmatic view that different programs of constructing a QTG start out from different motivations and seek to solve different problems with different methods. It should not come as a surprise then that they sometimes also differ about whether gravity needs to be quantized, if so, how, and if not, why not. As far as LQG is concerned, it takes the view that quantizing is justified because quantizing gravity along the canonical program promises to resolve the nasty difficulties arising in perturbative approaches taken by semi-classical theories such as QFT on curved spacetime. So for proponents of LQG, the preference of a non-perturbative fundamental theory of gravity over a perturbative one offers plenty of reason to quantize.

Be this as it may, all these arguments in favour or against quantization address a multitude of important foundational issues encountered in the Herculean task of formulating a quantum theory of gravity. I have merely tried to scratch the surface of some of these issues in the hereby finished chapter.

3.0 PRINCIPLES OF GENERAL RELATIVITY

Although it has not been possible in Chapter 2 to establish the strict necessity of quantizing gravity, I shall venture forward with my main topic, LQG and its foundations and applications. As I have mentioned at the end of the previous chapter, LQG assumes the view that gravity must be quantized in order to avoid the difficulties that arise in perturbative approaches in semi-classical QG. These perturbative approaches operate with a fixed space-time background and will be seen in the course of this chapter as violating central tenets of GTR. For advocates of the loop approach, thus, LQG offers the dual attraction of following in a principled manner the insights of GTR and of avoiding disastrous consequences of perturbative approaches to understanding gravity in its quantum regime. This and the next chapter, however, will illustrate that the former claim must be qualified and that the latter is only purchased at the price of encountering several novel obstacles.

3.1 THE ORIGIN OF BACKGROUND INDEPENDENCE

The so-called *equivalence principle* constituted one of the two most important guiding principles for Einstein in his search for the general theory of relativity (GTR). To this date, it also stands tall in the “context of justification” of GTR. Due to its immense systematic relevance, it can justly be said to embody the heart and soul of gravitational theory, but by the same token, it is also responsible for most of the technical and conceptual difficulties encountered on the road to a QTG.

One can find numerous equivalent, “almost” equivalent, and inequivalent formulations of the equivalence principle in the literature. This variety can largely be explained by historical evolution: it should not come as a surprise that Galileo’s equivalence principle is inequivalent to Newton’s, as is Newton’s to Einstein’s. What is perhaps more surprising is that Einstein seems to have oscillated between different inequivalent expressions of his own version of the equivalence principle. Even worse, some of his alleged versions appear to degenerate into

trivial or even false statements, at least if one believes some rather influential interpreters of GTR.¹ The motivation for the present occasion is systematic rather than historical and I shall therefore not be concerned with finding a historically adequate reconstruction of Einstein’s—or anybody else’s—possibly evolving understanding of the equivalence at stake. The equivalence principle ordained below as Einstein’s should thus be considered as wearing a tag without a claimed precise historical ascription of intellectual property.

Newton claimed that a physical body’s property of “mass” is proportional to its “weight,” with the identical proportionality factor for all bodies.² The “mass” here refers to a body’s intrinsic resistance to any change of its kinematic state, i.e. to its inertial mass. “Weight,” on the other hand, designates a force acting on a body in a gravitational field. The body’s property to which the gravitational field couples, i.e. the source of its susceptibility to this force, is of course its gravitational mass. Newton’s claim thus amounts to the proportionality of the inertial and gravitational masses of all physical bodies, again with the proportionality factor being the same if the field strength of the gravitational field is held fixed. In fact, measurements indicate that the proportionality factor must be very close to one.³ This gives rise to the following equivalence principle:

Postulate 1 (Weak Equivalence Principle (WEP)). *The trajectory of a sufficiently small body falling freely in a gravitational field does not depend on the body’s internal structure or composition. The gravitational mass to which the gravitational field couples is equal to the inertial mass. This is true for all physical bodies.*

Postulate 1 (WEP) has a serious implication: it precludes the possibility of a physical procedure to directly determine inertial observers in the presence of a gravitational field. Forces which admit observers unperturbed by the corresponding force field⁴ lend themselves to a direct determination of their field: one just has to measure how the trajectories of a sufficient number of test objects subject to the force deviate from the trajectories of the unperturbed observers. But if—as Postulate 1 (WEP) claims—all observers are equally subject to the gravitational force as are the test objects, then they will move precisely in the same way and no deviation occurs. Without the introduction of ancillary constructions

¹The main culprit for the dissemination of this uncharitable reading of Einstein seems to be [Synge \(1960\)](#). For a recent account of the evolution of Einstein’s idea of an equivalence principle in a similar vein, see [Janssen \(2005\)](#); for a more charitable interpretation, see [Norton \(1985\)](#).

²Assuming, of course, a homogeneous gravitational field. Equivalently, it can be assumed that this proportionality factor is always measured “at the same spot” in the gravitational field or at least at spots of equal field strengths.

³See Section 2.1 of [Will \(2001\)](#) for the current status of the experimental confirmation of Postulate 1 (WEP).

⁴Electrically neutral observers without magnetic dipole moment may serve as an example, as their trajectories are not disturbed by the presence of an electromagnetic field.

to single out “inertial” observers, there exists no “background motion” to gauge the test objects’ motion. As a direct result of Postulate 1 (WEP), the gravitational field is thus not amenable to a straightforward physical determination.

In the absence of gravity and other forces, the special theory of relativity (STR) permits the construction of inertial observers via spatio-temporal measurements using rods and clocks. Minkowski spacetime, the manifold \mathbb{R}^4 equipped with the flat metric $\eta_{\mu\nu}$, serves as the inertial background in STR. But when gravity is switched on, inertial observers can no longer be constructed as the presence of a gravitational field influences the readings on clocks and rods. In this sense, the spacetime geometry cannot be considered flat anymore. Einstein identifies the trajectories of freely falling observers in a gravitational field with the geodesics of the now curved spacetime metric. Thus, the world lines of inertial observers coincide with the world lines of the test particles subjected to the gravitational force. The coincidence of inertial and gravitational trajectories suggests the identification of the fields responsible for inertial and gravitational phenomena. The following formulation of an equivalence principle captures this idea:

Postulate 2 (Einstein Equivalence Principle (EEP)). *Effects due to inertia and effects due to gravity are manifestations of the same structure. This structure is called the inertio-gravitational field. Depending on the kinematical state of an observer, the inertio-gravitational field may be split into inertial and gravitational components differently.*

Without promoting the extermination of one structure in favour of the other, as is often done in the literature, let us just say that according to GTR there exists one physical field which is responsible for both inertial and gravitational phenomena. Because this field is dynamical, the fixed inertial background of STR has been abolished.

Conventional QFT, however, requires a fixed inertial background structure. Typically, Minkowski spacetime offers such a background, but we may use any given fixed spacetime metric, including a curved one, on which to do QFT. The background metric enters in most equations of QFT; it underlies the formulation of the commutation relations, the operator product expansions, the propagators, etc. Regardless of which background metric is assumed, the important lesson is that *some* background spacetime must always be assumed in conventional QFT. Conventional QFT cannot, therefore, evade excising the inertial component from the inertio-gravitational field in order to get under way, and thereby assumes a split of the metric field,

$$g_{\mu\nu} = b_{\mu\nu} + h_{\mu\nu}, \quad (3.1)$$

where $b_{\mu\nu}$ is a non-dynamical background field and $h_{\mu\nu}$ are those perturbations on the inertial background field which we consider to be the gravitational field proper. The background $b_{\mu\nu}$

encodes spatio-temporal relations and thus the causal structure. On this smooth classical background $b_{\mu\nu}$ lives the quantum field $h_{\mu\nu}$, which must commute at spacelike separations as determined by the background geometry encoded in $b_{\mu\nu}$. A division of the gravitational field into a fixed background and a fluctuating quantum field stands in strident contradiction with Postulate 2 (EEP), according to which no fixed inertial background structure unaffected by local degrees of freedom exists. A theory following Postulate 2 (EEP) must be *background independent*, defined as follows:

Definition 1 (Background independence). *A theory is background independent iff it abides by Postulate 2 (EEP) in that it does not assume a split of the inertio-gravitational field into a fixed inertial and a gravitational component as proposed in equation (3.1).*

Background independence, so often invoked as a selling pitch by advocates of LQG, can thus be seen to essentially amount to a postulate demanding a generalized equivalence between inertial and gravitational masses.

3.2 GENERAL COVARIANCE

With the requirement for background independence in place, the challenge of its formal implementation arises. Many authors simply equate, without due explication, background independence with *general covariance*, which in turn is equated with invariance under active spacetime diffeomorphisms. The ambition of this section is to disentangle this cluster of related concepts by carefully investigating the relations they afford with one another. On its conception, Einstein (1916a, §3) introduces the notion of general covariance along the following lines:

Definition 2 (General Covariance). *A theory is generally covariant just in case its equations retain the same form in any coordinate system, i.e. they are covariant under arbitrary coordinate transformations.*

The *principle* of general covariance would then be the demand that a physical theory be generally covariant as defined in Definition 2. The precise formulation, which somewhat alters the demand for general covariance as understood by Definition 2, follows below in Postulates 3 and 4. Transformations as they are relevant to present purposes are described by *diffeomorphisms*, i.e. one-to-one and onto C^∞ -maps between differentiable manifolds which have a C^∞ -inverse, symbolically $\phi : \mathcal{M} \rightarrow \mathcal{N}$. In order to diffeomorphically transform tensor fields living in a manifold, such as $g_{\mu\nu}$ and $T_{\mu\nu}$, one introduces a mapping called the

“pullback” which is in a natural way associated with the diffeomorphism ϕ : ϕ “pulls back” a function $f : \mathcal{N} \rightarrow \mathbb{R}$ on \mathcal{N} to a function $f \circ \phi : \mathcal{M} \rightarrow \mathbb{R}$ on \mathcal{M} composed in a natural manner from ϕ and f (see also Figure 1). The function $\phi^* f$ on \mathcal{M} is then defined as the

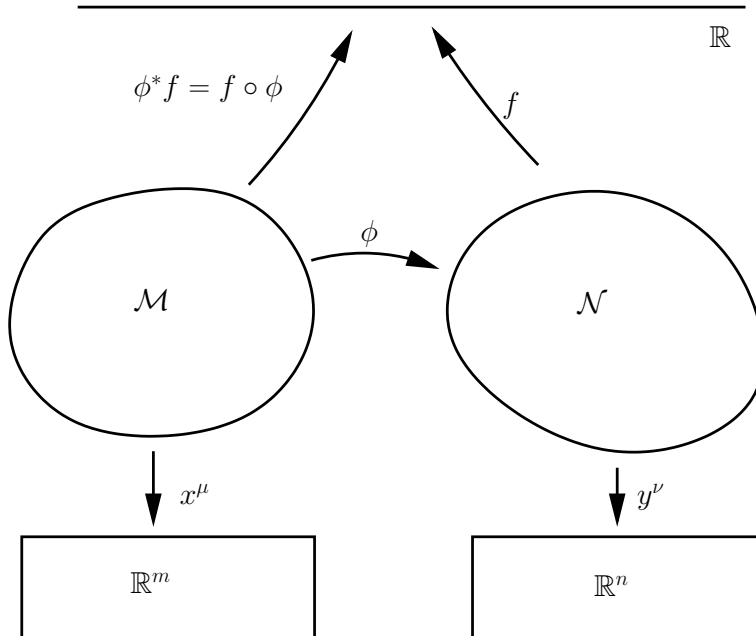


Figure 1: The *pullback* of f by ϕ , denoted ϕ^* , is defined as $\phi^* f \doteq (f \circ \phi)$.

function whose value at the point $p \in \mathcal{M}$ is the value of f at $\phi(p) \in \mathcal{N}$:

$$(\phi^* f)(p) \doteq (f \circ \phi)(p) = f(\phi(p)). \quad (3.2)$$

When ϕ maps points from \mathcal{M} to \mathcal{N} , then ϕ^* maps functions from \mathcal{N} to \mathcal{M} . Analogously, one can introduce a map ϕ_* called “pushforward” (or sometimes “carry along”) which maps functions from \mathcal{M} to \mathcal{N} .

To define the pullback more formally in the present context,⁵ a tensor T of type (r, s) at a point $p \in \mathcal{M}$ is formally defined as a multilinear function

$$T : \mathbf{T}_p^*(\mathcal{M}) \times \cdots \times \mathbf{T}_p^*(\mathcal{M}) \times \mathbf{T}_p(\mathcal{M}) \times \cdots \times \mathbf{T}_p(\mathcal{M}) \longrightarrow \mathbb{R} \quad (3.3)$$

⁵A more general definition of the pullback can be given by associating it with linear maps $\phi \in L(V, W)$ between vector spaces V and W . $L(V, W)$ is a more general group than $\text{Diff}(\mathcal{M})$. Most of the subsequent exposition is based on [Hawking and Ellis \(1973, Sec. 2.3\)](#), which offers a pleasingly clear and careful review of the subject. For notational consistency, I use a different notation and a slightly different terminology.

where there are r factors of the cotangent vector space of \mathcal{M} at p , $\mathbf{T}_p^*(\mathcal{M})$, and s factors of the tangent vector space of \mathcal{M} at p , $\mathbf{T}_p(\mathcal{M})$.⁶ The space of all such tensors is called $\mathbf{T}_s^r(p)$ and is given by the tensor product between r factors of $\mathbf{T}_p(\mathcal{M})$ and s factors of $\mathbf{T}_p^*(\mathcal{M})$. Particularly, $\mathbf{T}_0^1(p) = \mathbf{T}_p(\mathcal{M})$ and $\mathbf{T}_1^0(p) = \mathbf{T}_p^*(\mathcal{M})$. A *tensor field* of type (r, s) assigns each point $p \in \mathcal{M}$ a tensor (3.3).

From the introduction of the pullback of functions as defined in (3.2), one can define the (contravariant) vector mapping, or *pushforward for vector fields* v_i , $\phi_* v_i(f)$ evaluated at $\phi(p)$ as equal to $v_i(\phi^* f)$ evaluated at p . Similarly, one can introduce a vector mapping for covariant vectors w^i , the *pullback for vector fields*, by using the pushforward of functions.⁷ Upon completion of this step, we will have defined the maps

$$\begin{aligned}\phi_* & : \mathbf{T}_p(\mathcal{M}) \rightarrow \mathbf{T}_{\phi(p)}(\mathcal{N}), \\ \phi^* & : \mathbf{T}_{\phi(p)}^*(\mathcal{N}) \rightarrow \mathbf{T}_p^*(\mathcal{M}).\end{aligned}$$

From these constructions and from the definition of tensor fields in the previous paragraph, one can construct maps ϕ_* and ϕ^* associated with ϕ which map general contravariant tensors (i.e. tensors of type $(r, 0)$) from \mathcal{M} to \mathcal{N} and general covariant tensors (i.e. tensors of type $(0, s)$) from \mathcal{N} to \mathcal{M} respectively. So for a covariant tensor T of type $(0, s)$ defined on \mathcal{N} , the pullback ϕ^* of T by ϕ is given by a map $\phi^* : T \in \mathbf{T}_s^0(\phi(p)) \rightarrow \phi^* T \in \mathbf{T}_s^0(p)$ where for any contravariant vector $v_i \in \mathbf{T}_p(\mathcal{N})$

$$\phi^* T(v_1, \dots, v_s)|_p = T(\phi_* v_1, \dots, \phi_* v_s)|_{\phi(p)}. \quad (3.4)$$

Similarly, for a contravariant tensor S of type $(r, 0)$ defined on \mathcal{M} , the pushforward ϕ_* of S by ϕ is given by the map $\phi_* : S \in \mathbf{T}_0^r(p) \rightarrow \phi_* S \in \mathbf{T}_0^r(\phi(p))$ where for any covariant vector $w^i \in \mathbf{T}_{\phi(p)}^*(\mathcal{N})$

$$\phi_* S(w^1, \dots, w^r)|_{\phi(p)} = S(\phi^* w^1, \dots, \phi^* w^r)|_p. \quad (3.5)$$

Since it was stipulated at the outset that the map ϕ is always a diffeomorphism and therefore invertible, the associated mappings ϕ^* and ϕ_* also have an inverse, $(\phi^*)^{-1} = (\phi^{-1})^*$ and $(\phi_*)^{-1} = (\phi^{-1})_*$, respectively. This fact can be used to define a pullback (or a pushforward, as do [Hawking and Ellis \(1973\)](#)) which maps tensors of arbitrary rank from the tangent and cotangent spaces of \mathcal{N} to those of \mathcal{M} . This new, “extended” pullback is essentially composed

⁶The m -dimensional *tangent vector space* $\mathbf{T}_p(\mathcal{M})$ is defined as the space of all tangent vectors to \mathcal{M} at p . The m -dimensional *cotangent vector space* $\mathbf{T}_p^*(\mathcal{M})$ is defined as its dual space, containing all covariant vectors to \mathcal{M} at p corresponding to the contravariant tangent vectors \mathcal{M} at p . For more details, cf. [Hawking and Ellis \(1973, Sec. 2.2\)](#).

⁷The component indices are suppressed here; indices therefore indicate different vectors rather than different components of the same vector.

of the old pullback acting on the cotangent space \mathcal{N} and from the old inverted pushforward acting on the tangent space of \mathcal{N} . Using this composition and the reformulation of (3.4) as

$$T(v_1, \dots, v_s)|_{\phi(p)} = \phi^*T((\phi^{-1})_*v_1, \dots, (\phi^{-1})_*v_s)|_p,$$

we can define the generalized pullback as

$$T(w^1, \dots, w^r, v_1, \dots, v_s)|_{\phi(p)} = \phi^*T(\phi^*w^1, \dots, \phi^*w^r, (\phi^{-1})_*v_1, \dots, (\phi^{-1})_*v_r)|_p.$$

Since we will only be concerned with the case of diffeomorphisms as maps from one manifold into itself, i.e. $\mathcal{M} = \mathcal{N}$, given a single n -dimensional manifold \mathcal{M} , where n is typically four, we will regard as a diffeomorphism an invertible C^∞ -map from \mathcal{M} to \mathcal{M} . These diffeomorphisms constitute a group, $\text{Diff}(\mathcal{M})$, with the normal conjunction of mappings as group operation. The interpretation of diffeomorphisms as maps between manifolds which move around points of the manifold which does not make recourse to any coordinate system is called *active*. Diffeomorphisms may also be interpreted *passively*, as is done in Definition 2, i.e. in terms of transformations between coordinate systems. A coordinate system x^μ on \mathcal{M} is an invertible differentiable map from an open subset of \mathcal{M} to \mathbb{R}^n . For a tensor field T on \mathcal{M} , this coordinate map specifies the functions $t : \mathbb{R}^n \rightarrow \mathbb{R}$, i.e. the “field T in coordinates x .” A passive diffeomorphism is then an invertible differentiable map $\phi : \mathbb{R}^n \rightarrow \mathbb{R}^n$ which defines a new coordinate system y^μ on \mathcal{M} . The field value of T in the coordinates y^μ is then given by $t'(y^\mu) = t(\phi(y^\mu))$.

To appreciate the difference between active and passive diffeomorphisms, the reader is strongly encouraged to read Rovelli’s illustration of these distinct interpretations as given in [Rovelli \(2004, Sec. 2.2.4\)](#). In this wonderful illustration, Rovelli considers Earth’s surface and two points on it—the city of Paris and the village of Quintin in Brittany—as well as a temperature field defined everywhere on Earth’s surface. He associates a passive diffeomorphism with a re-expression of the temperature field at a fixed point as a consequence of a mere re-coordinatization of Earth’s surface. Covariance under passively interpreted transformations thus only means that nature does not care which coordinate system we impose upon her. Actively interpreted transformations, on the other hand, relate two a priori distinct fields with one another. For the sake of illustration, Rovelli considers a constant breeze which has uniformly transported yesterday’s temperature field to today’s. Yesterday’s temperature field and today’s temperature field are a priori distinct fields. Therefore, postulating covariance under actively interpreted transformations amount to more than just to claiming that nature does not mind about her coordinate dress. The two interpretations are often confused because for any two fields related by an active diffeomorphism, one can always find a passive transformation such that in the new coordinate system, one field is expressed by

the same functions as the other was in the old coordinate system. Imagine that the wind has moved the air from Quintin to Paris within 24 hours, and thus rotated the temperature field by a given angle α . If we think of this constant breeze which brings yesterday’s temperature field into today’s as an active diffeomorphism relating the two, then today’s temperature field at Paris can be given by the same expression as yesterday’s field at Quintin by rotating the coordinate system in which we write the temperature field by the same angle α .

This consideration can be generalized. For any two tensor fields defined on a manifold and related by an active diffeomorphism, we can find a corresponding passive diffeomorphism encoding a coordinate transformations such that the functions which represent one of the tensor fields in one coordinate system are identical with the ones that represent the other tensor field in the other coordinate system. If a theory is generally covariant according to Definition 2, then its equations of motion are the same in both coordinate systems. This implies, in turn, that for a dynamical tensor field which satisfies the equations of motion of a generally covariant theory, its relative by an active diffeomorphism must be a solution of the same equations too.⁸ Introducing the notion of “invariance under active diffeomorphisms”:

Definition 3 (Invariance under Active Spacetime Diffeomorphisms). *A theory is invariant under active spacetime diffeomorphisms just in case active spacetime diffeomorphisms map a solution of the dynamical equations to another solution of the same equations.*

In a theory whose dynamical equations can be derived from an action principle we call a *variational symmetry* those transformations which are members of a group \mathcal{G} of transformations which leave the theory’s action invariant. Every variational symmetry of the action is a dynamical symmetry of the equations of motion, but not vice versa as there can be scaling transformations which carry solutions of the dynamical equations into other solutions but are not variational symmetries of the action. Cf. Earman (2003, Sec. 2).

I am concerned with GTR and the Einstein fields equations as its dynamical equations. Given a manifold \mathcal{M} , pairs of tensor fields $(g_{\mu\nu}, T_{\mu\nu})$ solve these equations. Thus, the demand for general covariance as expressed in Definition 2 together with the uncontroversial codification of arbitrary coordinate transformations qua passive diffeomorphisms imply the

⁸It is unclear to what extent Einstein himself made a clean distinction between active and passive interpretations of transformations. As the present effort is not exegetical in nature, I shall not be concerned with this issue here. For detailed analyses, the reader should turn to Norton (1993) and Stachel (1980a). In a letter to the French mathematician Paul Painlevé in 1921, Einstein made a pertinent remark that “the coordinates themselves do not possess a physical significance, which means that they do not represent results of a measurement; only results obtained by the elimination of coordinates may pretend to objective significance.” (Cited after Biezunski (1989, p. 243), my translation) Earman (2006c, p. 448f) argues that this passage is sufficiently ambiguous to justify a reading according to which Einstein may have had in mind here the substantive version of general covariance, i.e. spacetime diffeomorphisms as gauge symmetries. Although the passage is not entirely free of ambiguities, I do not see much motivation for this reading.

invariance under active diffeomorphisms. Conversely, if a theory is invariant under active diffeomorphisms and arbitrary coordinate transformations are captured by passive diffeomorphisms, then the theory is generally covariant. General covariance and invariance under active diffeomorphisms will therefore, somewhat imprecisely, often be used interchangeably here—and elsewhere in the literature.

Let me introduce a more formal manner of speaking, inspired by Earman (1989). Consider a mathematical structure, a “theory,” and the associated set \mathfrak{M} of models. In GTR, the set \mathfrak{M} consists of triples $m = \langle \mathcal{M}, g_{\mu\nu}, T_{\mu\nu} \rangle$ as models of the theory. The “laws” L of the physical theory at stake, typically expressed as equations in the corresponding mathematical structure, pick out a subclass of models $\mathfrak{M}_L \doteq \text{mod}(L) \subset \mathfrak{M}$. For GTR, this subclass is constituted by those triples $\langle \mathcal{M}, g_{\mu\nu}, T_{\mu\nu} \rangle$ which satisfy Einstein’s field equations, $G_{\mu\nu}[g_{\mu\nu}] = 8\pi GT_{\mu\nu}$. In this idiom, a *symmetry operation* is usually defined as a map $\sigma : \mathfrak{M} \rightarrow \mathfrak{M}$ such that for any $m \in \mathfrak{M}_L$, we find $\sigma(m) \in \mathfrak{M}_L$. In other words, a symmetry of the laws L preserves \mathfrak{M}_L . The set \mathfrak{S}_T is the set of all those maps which satisfy this definition, or the set of all symmetries of theory T . Understood in this manner, a symmetry of GTR maps solutions of the field equations to solutions of the field equations. However, the present issue is more subtle: there are various potential notions of symmetry at stake here, the one just described as well as one that is based on invariance under spacetime diffeomorphisms. The first sense of symmetry, defined as a map $\sigma : \mathfrak{M} \rightarrow \mathfrak{M}$ such that \mathfrak{M}_L is preserved, implies that a solution of the field equations can be mapped to *any* other solutions of the field equations and the map will still qualify as a symmetry, as long as \mathfrak{M}_L is preserved. In other words, the map which brings, say, Kerr-Newman spacetime into Minkowski spacetime would be counted as a symmetry of the theory. Such a liberal conception of symmetry is undesirable since it will not offer any help in getting a fix on gauge in GTR. Instead, we will impose the further restriction that a map $\sigma : \mathfrak{M} \rightarrow \mathfrak{M}$ counts as a symmetry of the theory iff it arises from a corresponding spacetime diffeomorphism $\phi \in \text{Diff}(\mathcal{M})$. A map $\sigma : \mathfrak{M} \rightarrow \mathfrak{M}$ is considered to arise from a corresponding $\phi \in \text{Diff}(\mathcal{M})$ in case $\sigma : m = \langle \mathcal{M}, g_{\mu\nu}, T_{\mu\nu} \rangle \mapsto \sigma(m) = \langle \mathcal{M}, \phi^*g_{\mu\nu}, \phi^*T_{\mu\nu} \rangle$. These diffeomorphisms preserve the Einstein-Hilbert action of GTR and therefore carry a solution of the Einstein field equations into another solution of the same equations, which derive from the Einstein-Hilbert action by varying the metric field $g_{\mu\nu}$. If a symmetry transformation arising from $\phi \in \text{Diff}(\mathcal{M})$ acts on \mathfrak{M} in the described manner, then every element of $\text{Diff}(\mathcal{M})$ corresponds to a unique element in \mathfrak{S}_T . But while any map σ that is induced by such diffeomorphisms thus preserves \mathfrak{M}_L , the converse is not true. Thus, not every $\sigma \in \mathfrak{S}_T$ which preserves \mathfrak{M}_L should properly be considered as symmetry of the theory, but only the subset of \mathfrak{S}_T of those transformations which arise from a corresponding element in $\text{Diff}(\mathcal{M})$. I will denote this subset by $\mathfrak{S}\eta\mathfrak{m}_T$.

Formally, a theory T will be generally covariant just in case the set \mathfrak{Sym}_T consists of those mappings σ which correspond to active spacetime diffeomorphisms.

Definition 4 (Formal general covariance). *A theory T is formally generally covariant iff $\forall \phi \in \text{Diff}(\mathcal{M}) \exists! \sigma \in \mathfrak{Sym}_T [m = \langle \mathcal{M}, g_{\mu\nu}, T_{\mu\nu} \rangle \in \mathfrak{M}_L \rightarrow \sigma(m) = \langle \mathcal{M}, \phi^* g_{\mu\nu}, \phi^* T_{\mu\nu} \rangle \in \mathfrak{M}_L]$.*

From this definition, one can deduce in an obvious manner how the formal principle of general covariance, or, more accurately perhaps, the principle of invariance under active spacetime diffeomorphisms, formalizes as a postulate:

Postulate 3 (Formal principle of general covariance (FPGC)). $\forall \phi \in \text{Diff}(\mathcal{M}) \exists! \sigma \in \mathfrak{Sym}_T [m = \langle \mathcal{M}, g_{\mu\nu}, T_{\mu\nu} \rangle \in \mathfrak{M}_L \rightarrow \sigma(m) = \langle \mathcal{M}, \phi^* g_{\mu\nu}, \phi^* T_{\mu\nu} \rangle \in \mathfrak{M}_L]$.

Invariance under active spacetime diffeomorphisms is thus a symmetry of GTR. The laws of motion of GTR are of course Einstein’s field equations. I will therefore label the set of generally relativistic models as \mathfrak{M}_E . It turns out that in GTR, the Einstein-Hilbert action (4.1) is invariant under transformations $\phi \in \text{Diff}(\mathcal{M})$, and that therefore if $\langle \mathcal{M}, g_{\mu\nu}, T_{\mu\nu} \rangle$ solves the Einstein field equations, so does $\langle \mathcal{M}, \phi^* g_{\mu\nu}, \phi^* T_{\mu\nu} \rangle$. It is therefore possible to construct a transformation $\sigma \in \mathfrak{Sym}_T$ for each $\phi \in \text{Diff}(\mathcal{M})$ such that Postulate 3 (FPGC) is satisfied. GTR is formally generally covariant.

As a matter of fact, active spacetime diffeomorphism should be interpreted not only as a symmetry of GTR, but as a *gauge symmetry*. A gauge symmetry of a theory is a symmetry such that models of the theory related by the gauge transformation correspond to the same physical situation. This more substantive interpretation of general covariance is expressed in the following postulate:

Postulate 4 (Substantive principle of general covariance (SPGC)). *The group $\text{Diff}(\mathcal{M})$ of active spacetime diffeomorphisms is the gauge group of GTR.*

This postulate is by no means implied by Postulate 3 (FPGC), which is perfectly consistent with diffeomorphically related solutions representing physically different spacetimes. Although there seems to be a certain *résistance* in philosophy of physics against this gauge interpretation, most physicists happily embrace it.⁹ There are two main motivations in favour of Postulate 4 (SPGC). The more traditional, and, among philosophers, better known, motivation originates from the so-called “hole argument” and the related issue of determinism in GTR. The second reason for holding that spacetime diffeomorphisms should be taken

⁹There are also some philosophers, such as Earman (2006c), from whom the term “substantive general covariance” is borrowed. Weinstein (1999) has argued that GTR should not be considered a gauge theory because the group $\text{Diff}(\mathcal{M})$ is not a gauge group in the sense of particle physics, i.e. it is not the automorphism group of a principal fibre bundle. Although I agree that what is meant by the “physics” that remains invariant under gauge transformations is very different in GTR and in the standard model, this does not suffice to deny that it may be profitable to consider $\text{Diff}(\mathcal{M})$ as GTR’s gauge group, as I will argue below.

as gauge symmetry of GTR, which has more recently leaked into the philosophy of science literature, stems from the general analysis of symmetries in theories which obey an action principle. Let me briefly review both motivations.

The hole argument goes like this.¹⁰ Choose a $\phi \in \text{Diff}(\mathcal{M})$ such that it is the identity map everywhere except in a compact region $H \subset \mathcal{M}$, the “hole,” where ϕ is constructed such that it smoothly differs from the identity map. Given a model $\langle \mathcal{M}, g_{\mu\nu}, T_{\mu\nu} \rangle \in \mathfrak{M}_E$, then by Postulate 3 (PGC), $\langle \mathcal{M}, \phi^* g_{\mu\nu}, \phi^* T_{\mu\nu} \rangle$ will also be in \mathfrak{M}_E . But because of how ϕ was constructed, the two models coincide everywhere except in H . This implies that specifying the g - and the T -fields everywhere outside the hole does not suffice to uniquely determine the fields inside the hole H . In order to avoid a failure of determinism, the two mathematically distinct models $\langle \mathcal{M}, g_{\mu\nu}, T_{\mu\nu} \rangle$ and $\langle \mathcal{M}, \phi^* g_{\mu\nu}, \phi^* T_{\mu\nu} \rangle$ must be identified as representing the same physical situation, turning $\text{Diff}(\mathcal{M})$ into a gauge symmetry of GTR.

The second, and related, but more principled, motivation to see diffeomorphism invariance as a gauge symmetry stems from a general analysis of theories with so-called variational symmetries. Variational symmetries are just those transformations which leave the action of a theory invariant. These transformations form a group, usually denoted by \mathcal{G} . Theories whose dynamical equations can be derived from an action principle and are therefore cast as Euler-Lagrange equations fall under the reign of Noether’s theorems (Noether 1918). For these theories, gauge symmetries are just those transformations which absorb the underdetermination arising in the context of the second Noether theorem. This theorem states that the action of such a theory admits an infinite-dimensional Lie group $\mathcal{G}_{\infty s}$ of transformations which depend on s arbitrary functions of all independent variables and the first derivatives of these functions just in case there are s identities between the Lagrangean expressions, i.e. the left hand sides of the Euler-Lagrange equations, and their derivatives.¹¹ Note that the number s of these identities is equal to the number of arbitrary functions which define an element of the group $\mathcal{G}_{\infty s}$. The theorem implies that the Euler-Lagrange equations are not all

¹⁰The hole argument has originally been concocted by Einstein in late 1913 in order to absolve himself from producing a generally covariant set of field equations. As far as I am aware, it was first alluded to in correspondence to Ludwig Hopf (on 2 November 1913) and Paul Ehrenfest (on before 7 November 1913) and first appeared in print in Einstein (1914a, passim) and Einstein (1914b, p. 178). It has been resurrected by John Stachel (1980a), and brought into philosophical limelight by Earman and Norton (1987). I do not wish to cover the entire reaction of the philosophical community to Earman and Norton’s interpretation of the hole argument. Having said that, however, I would like to draw attention to Rickles (2005) who constructs a hole argument for LQG and to Iftime and Stachel (2006) who propose a generalization of the argument to any covariant theory.

¹¹A Lie group G is a differentiable manifold endowed with a group operation such that (1) $\cdot : G \times G \rightarrow G, (g_1, g_2) \mapsto g_1 \cdot g_2$, and (2) $^{-1} : G \rightarrow G, g \mapsto g^{-1}$. The dimension of the manifold is said to be the dimension of the Lie group. Sometimes, it is required in the literature that a Lie group be finite-dimensional. In this case, Noether’s second theorem would strictly speaking not be concerned with a Lie group, but instead with infinite-dimensional groups that share many properties of (finite-dimensional) Lie groups.

independent, but suffer from s “dependencies,”¹² and thus suffer from underdetermination in the sense that their solutions contain arbitrary functions of the independent variables. This, in turn, leads to an indeterministic dynamical evolution because these arbitrary functions that crop up in the solutions may be, for two solutions which only differ in the arbitrary functions they contain, identical up to a certain time and diverge thereafter. As long as any transformation between solutions of the Euler-Lagrange equations which differ only in these arbitrary functions is considered as a *gauge* transformation, however, the threat of indeterminism is immediately blocked as the solutions are no longer deemed physically inequivalent. Since the group $\text{Diff}(\mathcal{M})$ is a variational symmetry of the Einstein-Hilbert action (4.1), GTR falls under the spell of Noether’s second theorem. For GTR, the group $\mathcal{G}_{\infty s}$ is thus just $\text{Diff}(\mathcal{M})$ and $s = 4$, as four functions define a spacetime diffeomorphism. Therefore, we are faced with a case of underdetermination as solutions of the Euler-Lagrange equations of GTR—the Einstein equations—contain arbitrary functions. The underdetermination is taken care of if $\text{Diff}(\mathcal{M})$ is seen as the gauge group of GTR. Of the ten Einstein equations, six are thus seen to be independent.¹³

Both motivations behind Postulate 4 (SPGC), the hole argument and the more general considerations originating from Noether’s second theorem, thus offer reasons for endorsing the postulate based on a desire to avoid an indeterministic dynamical evolution. While this undoubtedly constitutes a sensible justification of Postulate 4 (SPGC), one should be aware of the fact that determinism does not universally rule in the context of generally relativistic spacetimes, not even modulo gauge. Suffice it to say on this occasion that deterministic evolution in GTR is limited to the extent to which a spacetime is globally hyperbolic.¹⁴

What is the relation between background independence as defined in the previous section and general covariance as parsed out above? Does a background independent theory automatically satisfy Postulate 4 (SPGC) and vice versa? That the two requirements are identical is certainly implied by the standard rhetoric found in canonical quantum gravity, where the two notions are often used interchangeably. However, as it will soon turn out, the exact identification is somewhat frivolous and, strictly speaking, false.

Let me first consider whether background independence is necessary for general covariance in the sense of Postulate 4 (SPGC). Assume a split of the metric field into a fixed inertial structure b and small perturbations h on this fixed background, “gravity proper.” Replace $g_{\mu\nu}$ in the Einstein-Hilbert action by $b_{\mu\nu} + h_{\mu\nu}$, i.e. perform the split as given in (3.1).

¹²“Abhängigkeiten” in Noether’s original wording.

¹³See also [Brading and Castellani \(2006, Sec. 6\)](#).

¹⁴This notion will be defined in Section 4.1. Cf. [Earman \(2004, Sec. 6\)](#) for a compact reference of the fate of determinism in GTR or [Earman \(2006a, Sec. 6\)](#) for an extended and magisterial discussion of the same material.

Instead of varying the action with respect to $g_{\mu\nu}$ in order to obtain a field theory of $g_{\mu\nu}$, only vary $h_{\mu\nu}$ and receive a field theory of $h_{\mu\nu}$ on the fixed inertial background spacetime $b_{\mu\nu}$. In the derivation of the dynamical equations of motion for a theory which follows an action principle, the resulting Euler-Lagrange equations will generally differ if the action is varied with respect to different fields. One can thus expect to see different dynamical equations arising from $\delta S[g_{\mu\nu}]/\delta g_{\mu\nu} = 0$ and from $\delta S[h_{\mu\nu}]/\delta h_{\mu\nu} = 0$. The exact difference, of course, depends on what is chosen as the fixed background in the second case. No doubt the split (3.1) can be made, and can be made in multiple ways. But for any particular choice of split, it will not be preserved under arbitrary spacetime diffeomorphisms, but only under those which preserve the particular splitting chosen. The mappings σ which should be considered symmetries of the theory are no longer exactly those induced by any spacetime diffeomorphisms, i.e. \mathfrak{Sym}_T , but only those corresponding to the subgroup of elements in $\text{Diff}(\mathcal{M})$ which preserve the splitting, denoted by \mathfrak{Sym}_T^b . Thus, the symmetry group of the theory will essentially be broken down to the symmetry group of the background spacetime, provided that the dynamical fields also obey the symmetry. For instance, if the background metric is $\eta_{\mu\nu}$, then the remaining symmetries of the theory will be those of Minkowski spacetime, viz. the Poincaré group. The larger group $\text{Diff}(\mathcal{M})$ will no longer be a symmetry of the theory.

The point becomes more obvious for a particular example. Bain (2004) discusses a version of Newton-Cartan gravity introduced by Christian (1997), which retains as fixed background a three-dimensional, spatially flat metric as well as a one-dimensional, temporal metric while treating the connection as a dynamical variable. The two metrics live orthogonally on one another and permit instantaneous causal signals. Bain finds that according to the identification suggested by Noether’s second theorem, the spacetime as well as the dynamical symmetries in the sense of the next paragraph are captured by the (extended) group $\text{Max}(\mathcal{M})$ of Maxwell transformations, which consists of transformations between rigid, non-rotating, linear accelerating, Euclidean reference frames.

There is a distinction used by Earman (1989, p. 45) and going back to Anderson (1967, Ch. 4) which nicely captures the issue at stake. If we introduce a distinction between fields B_i that represent the inertial background structure (the “spacetime”) on the one hand, and dynamical fields D_j (“gravity proper” and “matter”)—as we do when we insist on a splitting (3.1)—, then we should likewise introduce a distinction between spacetime and dynamical symmetries. A *spacetime symmetry*, then, is a mapping that leaves all the fields B_i invariant, i.e. a diffeomorphism $\phi : \mathcal{M} \rightarrow \mathcal{M}$ such that $\phi^* B_i = B_i$ for all i . A *dynamical symmetry* of a theory with laws L is a transformation of the dynamical fields D_j such that \mathfrak{M}_L is preserved and the transformations arise from corresponding diffeomorphisms. In other words, a map $\sigma \in \mathfrak{Sym}_T$ is a dynamical symmetry iff for any $m = \langle \mathcal{M}, B_1, \dots, B_m, D_1, \dots, D_n \rangle \in \mathfrak{M}_L$, it

is also the case that $\sigma(m) = \langle \mathcal{M}, B_1, \dots, B_m, \phi^* D_1, \dots, \phi^* D_n \rangle$ is an element of \mathfrak{M}_L . According to this terminology, then, standard GTR, which satisfies the requirement of background independence, has $\text{Diff}(\mathcal{M})$ as a dynamical symmetry and in some of its models, such as the Friedmann-Lemaître-Robertson-Walker spacetimes discussed on Appendix C, we additionally find spacetime symmetries. For a background-*dependent* theory, however, he will typically find Poincaré invariance as spacetime symmetry and whatever dynamical symmetries the QFT of the dynamical fields tell you. In this idiom, splitting the metric as in (3.1) breaks the diffeomorphism invariance down to Poincaré invariance (plus perhaps some dynamical symmetries), and changes the symmetry from a dynamical to a spacetime symmetry. Thus, a background-dependent theory violates Postulate 4 (SPGC) and general covariance can be seen as implying background independence.

The point can be put slightly differently yet again. The gauge group of GTR, $\text{Diff}(\mathcal{M})$ is an infinite-dimensional Lie group.¹⁵ If one introduces a fixed background spacetime on which the remaining fields propagate, then the group of variational symmetries of the action can at most be a finite-dimensional Lie group.¹⁶ Thus, we are no longer in the realm of Noether’s second theorem and have lost the important second justification of Postulate 4. This line of reasoning also suggests that it is not the distinction between spacetime and dynamical symmetries which does all the work for seeing $\text{Diff}(\mathcal{M})$ as a gauge symmetry. If this were the case, then the worry would surely arise that substantive general covariance has been introduced by hand rather than by principle. As can be gleaned from the fact that the variational symmetries form at most a finite-dimensional Lie group, the worry is allayed.

The claim that general covariance implies background independence can be challenged by invoking a toy theory described by Sorkin (2002) and discussed in Earman (2006c). Sorkin considers a classical scalar field Φ with mass m propagating on a Minkowski spacetime.¹⁷ The dynamics of this field is captured by the Klein-Gordon equation, which reads in a formally generally covariant variant

$$\eta^{\mu\nu} \nabla_\mu \nabla_\nu \Phi - m^2 \Phi = 0, \quad (3.6)$$

where ∇_μ is the covariant derivative operator determined by the Minkowski metric $\eta_{\mu\nu}$. The variation of the action

$$S[\Phi, \eta] = \frac{1}{2} \int d^4x \sqrt{-\eta} (\eta^{\mu\nu} \nabla_\mu \Phi \nabla_\nu \Phi + m^2 \Phi) \quad (3.7)$$

¹⁵But see footnote 11.

¹⁶Stachel (2006, Ch. 3).

¹⁷More precisely, he considers a *massless* scalar field. Earman (2006c) generalizes the toy theory such as to include scalar fields with non-vanishing mass m .

with respect to Φ while holding $\eta_{\mu\nu}$ fixed yields equation (3.6). The variational symmetry group of this action, and thus its gauge group, is the Poincaré group $P(1, 3)$. In order to render the toy theory of this action not only formally generally covariant, but also substantively so, we need to enlarge its gauge group such that it becomes $\text{Diff}(\mathcal{M})$. This enlargement will necessitate the following reformulation. First, we need to replace $\eta_{\mu\nu}$ by a general pseudo-Riemannian metric field $g_{\mu\nu}$. Second, the covariant derivative operator ∇_μ must now be determined by $g_{\mu\nu}$. Finally, equation (3.6) must be adjoined by a condition constraining $g_{\mu\nu}$ to be flat,

$$R_{\mu\nu\rho\sigma} = 0. \quad (3.8)$$

Any set of solutions of $g^{\mu\nu}\nabla_\mu\nabla_\nu\Phi - m^2\Phi = 0$ coupled with equations (3.8) is also a solution of (3.6), and vice versa. In order to remain faithful to the principled way of determining the gauge group of the new theory, the variational symmetries of its action must be studied. It turns out that if an auxiliary tensor field $\lambda^{\mu\nu\rho\sigma}$ with the same symmetries as $R_{\mu\nu\rho\sigma}$ is introduced and the action is rewritten such that the $\lambda^{\mu\nu\rho\sigma}$ play the role of a Lagrange multiplier,

$$S[\Phi, g_{\mu\nu}, \lambda^{\mu\nu\rho\sigma}] = \frac{1}{2} \int d^4x \sqrt{-g} (g^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi + m^2\Phi + \lambda^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}), \quad (3.9)$$

the gauge group of the toy theory becomes $\text{Diff}(\mathcal{M})$. Thus, the toy theory satisfies a substantive principle of general covariance. The variation of the auxiliary field $\lambda^{\mu\nu\rho\sigma}$ immediately gives the required flatness condition (3.8), while variation with respect to Φ gives the Klein-Gordon equation, as before. Furthermore, varying $g_{\mu\nu}$, which is now a dynamical field, leads to field equations expressing that the stress-energy tensor for Φ acts as a “source” for the auxiliary field.

I agree with Earman (2006c) that the two actions (3.7) and (3.9), which have different variational symmetries and contain different sets of fields, encode not two different formulations of the same theory, but two distinct theories. In this substantively generally covariant theory given by (3.9), every solution contains a metric field and a scalar field such that in each case, the metric field is the Minkowski metric $\eta_{\mu\nu}$ and the scalar field obeys the Klein-Gordon equation as determined by $\eta_{\mu\nu}$. Therefore, it could be argued, the theory (3.9) should count as background-*dependent*. If that were the case, the above claimed implication of background independence from (substantive) general covariance would no longer obtain.

I am not convinced that Sorkin’s toy theory (3.9) should be deemed as background-dependent, for the following reason. Background independence, as a reminder, required that a theory does not assume a split of the inertio-gravitational field into a fixed inertial field, the “background,” and a gravitational field propagating on this background. The demand

that the background remains “fixed” means that the theory’s action $S[\phi_1, \phi_2, \dots]$ is not varied with respect to the background field $b_{\mu\nu}$, which is not a dynamical variable of the theory, but only with respect to the gravitational field “proper” $h_{\mu\nu}$, and perhaps other dynamical fields. Translated to the case at hand, the Minkowski spacetime is presumably the fixed background field $b_{\mu\nu}$, while the scalar field Φ and the tensor field $\lambda^{\mu\nu\rho\sigma}$ are additional dynamical fields. Interpreted like this, there is no gravity proper in Sorkin’s toy example.¹⁸ Thus, we see that the dynamical equations of Sorkin’s toy theory are not gained from splitting the dynamical metric field $g_{\mu\nu}$ into a fixed component $b_{\mu\nu}$ and a dynamical component $h_{\mu\nu}$, re-expressing action (3.9), and varying it with respect to $h_{\mu\nu}$ and the other dynamical fields. It is just that the auxiliary field constrains the metric field to be the Minkowski metric. But the metric field $g_{\mu\nu}$ is perfectly dynamical and interacts with the other dynamical fields. So there is not much motivation for counting Sorkin’s example theory as background-dependent and I maintain that substantive general covariance entails background independence.

But is background independence also sufficient for substantive general covariance? No, as there could of course be background-independent theories of gravity with dynamical symmetries different from those found in GTR. Quite regardless of whether Sorkin’s toy theory is considered as background-dependent or not, therefore, background independence and general covariance are not equivalent notions and should thus not be used synonymously.

¹⁸This interpretation is incautious insofar as background independence demands that no split of the metric is made into background and gravity, which is what is implicitly done when one says that “there is no gravity in Sorkin’s toy example.”

4.0 HAMILTONIAN GENERAL RELATIVITY

4.1 GTR AS A HAMILTONIAN SYSTEM

Casting GTR as a Hamiltonian system with constraints has many advantages, as Earman (2003) affirmed: it gives the vague talk about “local” and “global” transformations a more tangible meaning, it explains how the fibre bundle formalism arises in cases when it does, it has a sufficiently broad scope to relate GTR to Yang-Mills gauge theories, it offers a formalization of the gauge concept, it connects to fundamental foundational issues such as the nature of observables and the status of determinism in GTR and in gauge theories in general. Moreover, the Hamiltonian formulation affords a natural affinity to the initial value problem in GTR.¹ The real gain of a Hamiltonian formulation, however, arises when one tries to quantize the classical theory. Typically, prescriptions to find a quantum theory from a classical theory require either a Lagrangian (e.g. for the path integral method) or a Hamiltonian (e.g. for canonical quantization) formulation of the theory. LQG relies on a canonical quantization procedure and thus uses a Hamiltonian formulation of GTR as a vantage point. This section offers an exhibition of the basic ideas involved in casting GTR as a Hamiltonian system, without claiming mathematical rigour.²

The Hilbert or Einstein-Hilbert action for GTR without matter is given by

$$S[g_{\mu\nu}] = \int_{\mathcal{M}} d^4x \mathcal{L} = \frac{1}{16\pi G} \int_{\mathcal{M}} d^4x \sqrt{-g} R, \quad (4.1)$$

where \mathcal{L} is the Lagrange density which induces the Lagrangian $L = \int_{\Sigma} d^3x \mathcal{L}$ and g the determinant of $g_{\mu\nu}$.³ This action leads to the (vacuum) field equations of GTR if one varies (4.1) with respect to the metric $g_{\mu\nu}$. Thus, Einstein’s field equations can be recognized as

¹Cf. Wald (1984, Appendix E.2). A *locus classicus* for the Cauchy problem in GTR is Choquet-Bruhat and York (1980), a more recent survey article is Friedrich and Rendall (2000).

²A useful introduction to the Lagrangian and the Hamiltonian formulation of GTR is given in Wald (1984, Appendix E). Wald’s textbook of 1984 only deals with the ADM version of Hamiltonian GTR and does naturally not treat Ashtekar’s version, pioneered in 1986.

³ $\Sigma \subset \mathcal{M}$ is a three-dimensional hypersurface of the four-dimensional manifold \mathcal{M} .

the equations of motion of the Lagrangian formulation of GTR, i.e. as the Euler-Lagrange equations, as indicated in Section 3.2. They are second-order differential equations. The solutions of the Euler-Lagrange equations will be uniquely determined by the independent variables and their derivatives just in case the Hessian of L , i.e. the matrix $\partial^2 L(q, \dot{q})/\partial \dot{q}^{n'} \partial \dot{q}^n$, is invertible. This is the case if and only if its determinant, sometimes also called “Hessian,” does not vanish. However, in case the determinant vanishes, and the second derivatives of the independent variables cannot be uniquely determined, the solutions of the Euler-Lagrange equations will contain arbitrary functions of time and will thus no longer be uniquely determined by the configuration variables and their first derivatives. The impossibility of inverting $\partial^2 L/\partial \dot{q}^{n'} \partial \dot{q}^n$ thus points to gauge freedom.

Finding a Hamiltonian formulation amounts to putting the Euler-Lagrange equations in the form of Hamiltonian equations of motion, $\dot{q} = \partial H/\partial p$ and $\dot{p} = \partial H/\partial q$, which are of first order. This can be achieved by the introduction of the so-called canonical momenta via

$$p_n = \frac{\partial L}{\partial \dot{q}^n}, \quad (4.2)$$

where $n = 1, \dots, N$, N being the number of degrees of freedom of the system at stake. These momenta are not all independent in cases we are faced with a system exhibiting gauge freedom—i.e. just in case the Hessian is singular, as follows. These dependencies are captured in the Hamiltonian formalism by relations between the configuration variables and the corresponding canonical momenta,

$$\phi_m(q, p) = 0, \quad m = 1, \dots, M, \quad (4.3)$$

where M is the number of dependencies. Thus, not all canonical variables are independent. The relations (4.3) between q and p are called *primary constraints* and define a submanifold smoothly embedded in phase space called the *primary constraint surface*. The phase space Γ is defined as the space of solutions of the equations of motion. Assuming that all equations (4.3) are linearly independent, which strictly speaking is not necessarily the case, this submanifold will be of dimension $2N - M$. Equations (4.3) imply that the transformation map between the Lagrangean phase space $\Gamma(q, \dot{q})$ and the Hamiltonian phase space $\Gamma(q, p)$ is onto but not one-to-one. Equations (4.2) define a mapping from an $2N$ -dimensional manifold of the q 's and \dot{q} 's to the $(2N - M)$ -dimensional manifold defined by (4.3). In order to render the transformation bijective and thus invertible, the introduction of extra parameters—“gauge fluff”—is required.⁴

⁴For more details on how the constraints arise in some Hamiltonian systems, see [Henneaux and Teitelboim \(1992, Ch. 1\)](#). My exposition largely follows this reference.

Next, one introduces a Hamiltonian H as a function of position and momentum variables as

$$H(q, p) = \dot{q}^n p_n - L(q, \dot{q}). \quad (4.4)$$

This canonical Hamiltonian, however, is only uniquely defined on the primary constraint surface but can arbitrarily be extended to the rest of phase space. The Legendre transformations between the Lagrangean and the Hamiltonian phase spaces turn out to be invertible just in case $\det(\partial^2 L / \partial \dot{q}^{n'} \partial \dot{q}^n) \neq 0$. Should the Hessian vanish, as above, one can add extra variables u^m and thus render the Legendre transformations invertible. In this case, the Hamiltonian equations corresponding to the Euler-Lagrange equations become

$$\begin{aligned} \ddot{q}^n &= \frac{\partial H}{\partial p_n} + u^m \frac{\partial \phi_m}{\partial p_n}, \\ \dot{p}_n &= -\frac{\partial H}{\partial q^n} - u^m \frac{\partial \phi_m}{\partial q^n}, \\ \phi_m(q, p) &= 0. \end{aligned}$$

These Hamilton equations lead via arbitrary variations $\delta q^n, \delta p_n, \delta u^m$ (except for the boundary conditions $\delta q^n(t_1) = \delta q^n(t_2) = 0$) to the Hamiltonian equations of motion for arbitrary functions $F(q, p)$ of the canonical variables

$$\dot{F} = \{F, H\} + u^m \{F, \phi_m\}, \quad (4.5)$$

where $\{, \}$ is the usual Poisson bracket

$$\{F, G\} \doteq \frac{\partial F}{\partial q^i} \frac{\partial G}{\partial p_i} - \frac{\partial F}{\partial p_i} \frac{\partial G}{\partial q^i}.$$

It is clear that the notion of canonical coordinates and momenta should be appropriately generalized if one wishes to investigate a more general class of Hamiltonian generally relativistic systems. The notion of a symplectic structure on the phase space Γ suitably generalizes the structure captured by the canonical variables. The latter structure, encoded by the Poisson bracket as defined above, thus generalizes as follows. Let (Γ, ω) be a symplectic manifold, i.e. $\dim(\Gamma)$ is even and ω is a symplectic form.⁵ Given a Hamiltonian H as a smooth function on Γ , one can define a so-called *Hamiltonian vector field* X_H via the condition

$$\omega(X_H, \cdot) = dH,$$

⁵A *symplectic form* on the phase space Γ is a closed, skew-symmetric, non-degenerate two-form ω on Γ . A two-form on Γ is a bilinear function $T\Gamma \times T\Gamma \rightarrow \mathbb{R}$ which acts on pairs of vector fields on Γ . A two-form ω is *closed* just in case $d\omega = 0$, *skew-symmetric* just if $\omega(A, B) = -\omega(B, A)$, and *non-degenerate* just if $\forall B(\omega(A, B) = 0 \rightarrow A = 0)$.

where dH is the exterior derivative of H . The triple (Γ, ω, X_H) is then called a *Hamiltonian system*. The generalized Poisson bracket between smooth functions F and G is then given by

$$\{F, G\} \doteq \omega(X_G, X_F), \quad (4.6)$$

where X_F and X_G are vector fields defined in analogy to the Hamiltonian vector field X_H . Equivalently, one may define the generalized Poisson bracket as

$$X_{\{F,G\}} \doteq [X_F, X_G], \quad (4.7)$$

where $[.,.]$ is the *Lie bracket*, which is defined by $[X, Y]f = X[Y[f]] - Y[X[f]]$, where $X = X^\mu \partial / \partial x^\mu$ and $Y = Y^\mu \partial / \partial x^\mu$ are vector fields in \mathcal{M} and f is a function on \mathcal{M} . The Lie bracket satisfies bilinearity, skew-symmetry, and the Jacobi identity.

Consistency requires that the primary constraints ϕ_m be preserved over time, i.e. that $\dot{\phi}_m = 0$. As primary constraints are phase space functions, equation (4.5) then implies

$$\{\phi_m, H\} + u^{m'} \{\phi_m, \phi_{m'}\} = 0. \quad (4.8)$$

This equation has one of two possible forms: either it embodies a relation only between the q 's and p 's, without any u^m , or it results in a relation including the u^m . In the latter case, we just end up with a restriction on u^m . In the former case, however, (4.8) leads to additional constraints, called *secondary constraints*, on the canonical variables and thus on the physically relevant region of the phase space. These secondary constraints must also fulfill the consistency requirement of being preserved over time, which leads to new equations of the type of (4.8), which again are either restrictions on the u^m or constraints on the canonical variables, etc. Once the process is finished, and we have all secondary constraints⁶, denoted by $\phi_k = 0$ with $k = M + 1, \dots, M + K$, all constraints can be rewritten as $\phi_j = 0$ with $j = 1, \dots, J \doteq M + K$. The full set of constraints $\phi_j = 0$ defines a “subsubmanifold” in the phase space Γ , i.e. a submanifold of the primary constraint surface $\phi_m = 0$, called the *constraint surface* \mathcal{C} . The relevant difference between primary and secondary constraints is that primary constraints are direct consequences of equation (4.2), whereas the secondary constraints only arise once the equations of motion (4.5) are given.

Any two functions F and G in phase space that coincide on the constraint surface are said to be *weakly equal*, symbolically $F \approx G$. In case they agree throughout the entire phase space, their equality is considered *strong*, expressed with the usual characters as $F = G$. Above, I have introduced the qualification of constraints as primary. However, there is

⁶They are *not* referred to as tertiary, quaternary etc. constraints, but only collectively as “secondary” constraints.

a more important classification of constraints into first-class and second-class constraints, defined as follows:

Definition 5 (First-class constraints). *A function $F(q, p)$ is termed first class if and only if its Poisson bracket with every constraint vanishes weakly,*

$$\{F, \phi_j\} \approx 0, \quad j = 1, \dots, J. \quad (4.9)$$

A function in phase space is called second class just in case it is not first class.

The property of being first class is preserved under the Poisson bracket, i.e. the Poisson bracket of two first-class functions is first class again.

The fact that arbitrary functions u^m enter the Hamilton equations (or, equivalently, the Hamiltonian equations of motion) implies that a physical state is uniquely determined by a pair (q, p) , i.e. by a point in (Hamiltonian) phase space $\Gamma(q, p)$, but not vice versa. In other words, these arbitrary functions encode the gauge freedom which arises from systems with a singular Hessian. It can be shown that a dynamical variable F , i.e. a function on Γ , differs in value from time t_1 to time $t_2 = t_1 + \delta t$ by

$$\delta F = \delta v^a \{F, \phi_a\} \quad (4.10)$$

where the ϕ_a range over the complete set of first-class primary constraints and the v^a are the totally arbitrary part of the u^m , with $\delta v^a = (v^a - \tilde{v}^a)\delta t$ where v^a and \tilde{v}^a are two different choices of v^a at t_1 .⁷ In a deterministic theory, the transformation (4.10) does not modify the physical state and is thus considered a gauge transformation. In this sense, the first-class primary constraints generate gauge transformations. The famous ‘‘Dirac conjecture’’ attempts to extend this result to include all first-class constraints. In general, however, the conjecture is false as the existence of some admittedly contrived counterexamples implies.⁸ There is no harm for present purposes, however, if we assume that all first-class constraints generate gauge transformations. The restriction of a phase space function F to \mathcal{C} is gauge-invariant just in case $\{F, \phi_a\} \approx 0$, in which case (4.10) implies $\delta F \approx 0$. The first-class constraints are thus seen to generate motions within \mathcal{C} . In contrast, second-class constraints generate motions leading outside of \mathcal{C} .⁹ This distinction permits the explication of another important concept: the gauge orbit. A *gauge orbit* is a submanifold of \mathcal{C} which contains all those points in \mathcal{C} which form an equivalence class under a gauge transformation. The sets of these points are simply connected in \mathcal{C} since gauge transformations that connect these

⁷Cf. Henneaux and Teitelboim (1992, Sec. 1.2.1).

⁸Cf. Henneaux and Teitelboim (1992, Sec. 1.2.2).

⁹Cf. Belot and Earman (2001, Sec. 10.2.2).

points do not leave \mathcal{C} . They form a curve in \mathcal{C} . The gauge motion produced by the first-class constraints can thus be seen to be the tangents to these curves. The points of the gauge orbits in \mathcal{C} , equipped with a projection $\mathcal{C} \rightarrow \Gamma_{phys}$, constitute the so-called *reduced* or *physical phase space* Γ_{phys} . The physical phase space Γ_{phys} is defined as the set of points representing gauge equivalence classes of points in Γ . In other words, the physical phase space is obtained by identifying all points on the same gauge orbits. This means that the bundle of admissible dynamical trajectories passing through a particular point $x \in \mathcal{C}$ is mapped to the physical phase space such that the bundle is projected onto a single dynamical trajectory through the point in Γ_{phys} representing the gauge equivalence class in which x falls.

Assume a Hamiltonian system (Γ, ω, X_H) with constraints is given. Assume further that all constraints are first-class.¹⁰ Constraint equations are equations which the canonical variables must satisfy in addition to the dynamical equations of the system. If a set of variables were to pick one and only one physical state, then, given the existence and uniqueness of the solutions of the dynamical equations, one could plug the set of variables uniquely specifying the state into the dynamical equations and could thus obtain the full deterministic dynamical evolution of the physical degrees of freedom. If constraints are present, however, a set of variables does not uniquely describe a physical state. Solving the constraints thus means to use these additional equations to explicitly solve for a variable. This permits the elimination of this variable (and the now solved constraint equation). Solving the constraints of the constrained Hamiltonian system thus amounts to the reduction of the number of variables used to specify the physical state of the system. Once all constraint equations are solved and thus eliminated, the remaining canonical variables are ineliminable for the purpose of uniquely specifying a physical state. In this case, we are back to an unconstrained Hamiltonian system in the sense that its phase space is its *physical* phase space. In the absence of any second-class constraints, the total number of canonical variables ($= 2N$) minus twice the number of first-class constraints equals the number of *independent* canonical variables. Equally, the number of *physical* degrees of freedom is the same as half the number of independent canonical variables, or the same as half the number of canonical variables minus the number of first-class constraints.¹¹

¹⁰Second-class constraints can be regarded as resulting from fixing the gauge of a “larger” system with an additional gauge invariance. They can be replaced by a corresponding set of first-class constraints which capture the additional gauge invariance. Second-class constraints are thus eliminable. In fact, in some cases, it may prove advantageous to thus “enlarge” a system as this permits to circumvent some technical obstacles (Henneaux and Teitelboim 1992, Sec. 1.4.3), albeit at the price of introducing new “unphysical” degrees of freedom. Without loss of generality, we can thus consider a Hamiltonian system whose constraints are all first-class.

¹¹This manner of counting the physical degrees of freedom is well defined for any finite number of degrees of freedom, and perhaps for countably many too. For uncountably many degrees of freedom, new subtleties arise. Cf. Henneaux and Teitelboim (1992, Sec. 1.4.2).

Hamilton’s equations, at least in the narrower standard sense, explicitly solve for the time derivatives. This can only be achieved within GTR if its original 4-dimensional quantities are broken up into (3+1)-dimensional quantities, with time accruing in the one single dimension. A similar coercion must be exercised upon the four-dimensional structure of spacetime when we wish to consider an initial-value formulation of GTR. In order to find a Hamiltonian or an initial-value formulation, GTR must be regarded as describing the dynamical evolution *of something*. Breaking up spacetime into “space” that evolves in “time” in order to determine whether a well-posed initial-value formulation exists, i.e. whether the physical degrees of freedom enjoy a deterministic evolution, becomes manageable once we impose a gauge condition to weed out any unphysical degrees of freedom. As will become clear in Section 4.2.1, the traditional formulation of GTR as a constrained Hamiltonian system entertains twelve dynamical variables, the six independent components of the three-metric q_{ab} and the six independent components of the corresponding conjugate momentum π^{ab} . Half this number is six, and there are four first-class constraint equations (4.14) and (4.15), which leaves the gravitational field with two physical degrees of freedom per point in space. Fortunately, this is the same number of degrees of freedom as one gets for a linear spin-2 field propagating on a flat spacetime background, which can be considered as a weak-field limit of GTR.¹²

With a gauge condition enforced, Einstein’s field equations can be massaged into a form of hyperbolic second-order differential equations defined on manifolds which admit existence and uniqueness theorems. Even in an appropriate gauge fix, however, GTR allows for ways in which the field equations may fail to uniquely determine their solutions. Such failure in GTR is typically associated with the emersion of spacetime singularities or “holes” in the fabric of spacetime. In order to characterize the extent to which given initial data can be extended uniquely to a solution of the field equations, one imposes the condition of so-called *global hyperbolicity*, which requires the introduction of a few foregoing technical terms. The *causal future* of a set $\mathcal{S} \subset \mathcal{M}$, denoted by $J^+(\mathcal{S})$, consists exactly of those points in \mathcal{M} which can be reached from \mathcal{S} by a future-directed non-spacelike curve in \mathcal{M} . The *causal past* $J^-(\mathcal{S})$ is defined mutatis mutandis. The *strong causality condition* holds at a point $p \in \mathcal{M}$ just in case every neighbourhood of p contains a neighbourhood of p which no non-spacelike curve intersects more than once. In other words, a spacetime is strongly causal at a point p if p has arbitrarily small causally convex neighbourhoods. If strong causality holds in all points of a spacetime, this condition ascertains that the spacetime does not contain “almost” closed causal curves. With these definitions in place, a set $\mathcal{N} \subset \mathcal{M}$ is said to be *globally hyperbolic* just in case it satisfies two conditions: (i) for any two points $p, q \in \mathcal{N}$, $J^+(p) \cap J^-(q)$ is

¹²See Wald (1984, Sec. 4.4b); cf. also Wald (1984, p. 266) for a slightly different way of calculating the degrees of freedom of the gravitational field.

compact and fully contained in \mathcal{N} , and (ii) \mathcal{N} is strongly causal at all points $p \in \mathcal{N}$. A spacetime $\langle \mathcal{M}, g_{\mu\nu} \rangle$ is then defined to be *globally hyperbolic* just in case $\mathcal{N} = \mathcal{M}$, with \mathcal{N} globally hyperbolic. Roughly, this means that the spacetime does not suffer from “infinities,” “singularities,” or “holes.”

Another important notion is the *future domain of dependence* of a set $\mathcal{S} \subset \mathcal{M}$, denoted by $D^+(\mathcal{S})$, which is defined as the set of all points $p \in \mathcal{M}$ such that every past-inextendible causal curve through p intersects \mathcal{S} .¹³ Analogously, the *past domain of dependence* of a set $\mathcal{S} \subset \mathcal{M}$, denoted by $D^-(\mathcal{S})$ is the set of all points $p \in \mathcal{M}$ such that every future-inextendible causal curve through p intersects \mathcal{S} . The *domain of dependence* of \mathcal{S} is the union of the future and the past domains of dependence, symbolically $D(\mathcal{S}) = D^+(\mathcal{S}) \cup D^-(\mathcal{S})$.

Often, the subset $\mathcal{S} \subset \mathcal{M}$ is assumed to be an “achronal” set, in which cases I will denote it by Σ . A set $\Sigma \subset \mathcal{M}$ is *achronal* if there are no two points $p, q \in \Sigma$ such that $q \in I^+(p)$, where $I^+(p)$ designates the *chronological future* of p , i.e. the set of all points in \mathcal{M} which can be reached from p by a future-directed timelike curve. The *chronological past* $I^-(p)$ is defined analogously. Thus, a set Σ is achronal exactly if $I^+(\Sigma) \cap \Sigma = \emptyset$. An achronal set $\Sigma \subset \mathcal{M}$ is called a *Cauchy surface* in case $D(\Sigma) = \mathcal{M}$. As it can be shown (Geroch 1970), a necessary and sufficient condition that a spacetime has a Cauchy surface is that it be globally hyperbolic. According to a well-known theorem,¹⁴ a globally hyperbolic spacetime always admits a global time function t , i.e. a function such that each surface of constant t is a Cauchy surface of the spacetime. In this case, the spacetime can be foliated by Cauchy surfaces Σ_t and thus exhibits topology $\mathbb{R} \times \Sigma$, where the topology of the 3-spaces Σ_t is arbitrary, but must be the same for all Σ_t .¹⁵

Conversely, not every spacetime with topology $\mathbb{R} \times \Sigma$ affords a global time function or a Cauchy surface—at least not in the narrower sense as defined above. As a matter of fact, the spacetime may be foliated using a “flow of space” rather than a “flow in time,” cf. Figure 2. Both spacetimes in Figure 2 are topologically $\mathbb{R} \times \Sigma$, but only the one on the left with the foliation using a global time function has Cauchy surfaces and is globally hyperbolic as these terms have been defined in the context of the initial value problem in GTR. A suitably

¹³A curve $\gamma : I \rightarrow \mathcal{M}$, where I designates an interval of the reals, is called *inextendible* in case there exists no curve $\gamma' : I' \rightarrow \mathcal{M}$ such that $I \subset I'$ and $\gamma(r) = \gamma'(r)$ for all $r \in I$. A non-spacelike curve is termed *past-* (or *future-*)*inextendible* if it is inextendible in the past (or future) direction as defined by the time orientation.

¹⁴Theorem 8.3.14 in Wald (1984).

¹⁵At least for spatially compact Hausdorff spacetimes $\langle \mathcal{M}, g_{\mu\nu} \rangle$; this is essentially Geroch’s Theorem (Geroch 1967) which states that for a compact spacetime $\langle \mathcal{M}, g_{\mu\nu} \rangle$ whose boundary is the disjoint union of two closed spacelike three-manifolds, \mathcal{S} and \mathcal{S}' , \mathcal{S} and \mathcal{S}' are diffeomorphic if $\langle \mathcal{M}, g_{\mu\nu} \rangle$ admits a time orientation and does not contain closed timelike curves. Hence, in spatially open spacetimes, there may be topology change. For a penetrating discussion of topology change in general and of Geroch’s Theorem in particular, see Callender and Weingard (2000).

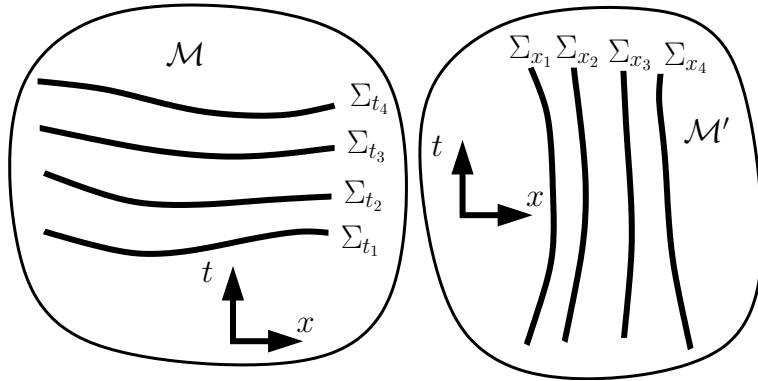


Figure 2: Foliation of spacetimes using a “flow in time” and a “flow in space.”

generalized Hamiltonian formalism might be able to accommodate spacetimes of topology $\mathbb{R} \times \Sigma$, regardless of whether the spacetime is foliated temporally or spatially. Carlo Rovelli’s favorite way of conceiving a suitably generalized Hamiltonian formalism focuses on the idea that data should be given on closed three-dimensional hypersurfaces that bound finite regions of four-dimensional spacetime, i.e. on bounds of four-dimensional “balls.”¹⁶ Equation (4.6) captures the structure of phase space without taking recourse to a particular coordinate system. In particular, it does not necessitate the coordinatization of Γ with the coordinates and canonical momenta of given spatial hypersurfaces. This liberation might help to clarify the important point that a point in the phase space of Hamiltonian general relativity is not a set of field variables and their time derivatives at some instant of time, but rather spacetimes with topology $\mathbb{R} \times \Sigma$ that solve the equations of motion. Thus, Hamiltonian formulations might apply to a larger class of models in GTR than does the initial value formulation, which only deals with globally hyperbolic spacetimes. There is a natural connection between these two formulations, but they need not coincide. What families of constraints would arise for posing data on timelike, rather than spacelike, hypersurfaces will need investigating.

The restriction to generally relativistic models with manifolds of topology $\mathbb{R} \times \Sigma$ is far from innocent, for at least two reasons. First, this restriction excludes models in \mathfrak{M}_E which may turn out to be of physical significance.¹⁷ Models with manifolds that permit a time-wise

¹⁶Personal communication, 3 July 2005. These balls, unlike those at the World Cup, need not be round.

¹⁷I disagree with Thiemann (2003, Sec. I.2) that the assumption of global hyperbolicity is forced on us, “at least classically.” At the very least, I would prefer to be only bound to the claim that we are in the business of studying globally hyperbolic domains of spacetimes which may not be maximal simpliciter and could possess non-globally hyperbolic extensions.

foliation into Cauchy surfaces of subsequent “moments in time” are globally hyperbolic and do therefore not admit closed timelike curves, thus effectively ruling out the possibility of time travel. Models with manifolds permitting a space-wise foliation into surfaces of adjacent “places in space,” as they can conceivably emerge in a more general Hamiltonian formulation, can contain closed timelike curves. In this sense, a Hamiltonian formulation of GTR does not a priori rule out the existence of closed timelike curves. It should be noted, however, in both known Hamiltonian formulations of GTR, spacetime is foliated using a global time function. This is hardly surprising since the canonical variables introduced afford a more natural interpretation for foliations reminiscent of the initial value problem, particularly in the so-called ADM formulation, to be discussed in the next section. Thus, although in principle all (vacuum) models in \mathfrak{M}_E with a topology $\mathbb{R} \times \Sigma$ can be cast in a Hamiltonian dress, we usually direct our attention to those models which disentangle “space” from “time” rather than those which separate one spatial dimension from the remaining three spatiotemporal dimensions or which distribute data on a three-dimensional hypersurface in an altogether different manner such as Rovelli’s three-dimensional bound of a four-dimensional ball. Models with a time of topology \mathbb{R} of course spoil the business of any time travel agency. The class of models in \mathfrak{M}_E with topology $\mathbb{R} \times \Sigma$ is certainly a physically important subclass, but there is no guarantee that it captures all physically relevant cases.

The second worry is much more threatening. For a spacetime theory manifestly obeying Postulate 4 in Chapter 3, i.e. it has *spacetime* diffeomorphism invariance as a gauge symmetry, cleaving the manifold into “space” and “time” breaks this manifest general covariance. The cleavage amounts to an introduction of a family of privileged observers, moving in the direction of the vector field normal to the folios of the foliation.¹⁸ But in GTR no observer enjoys a privileged status like this. It must thus be ascertained one way or another that we still, despite the splitting of the four-dimensional spacetime manifold in order to make the theory amenable to a Hamiltonian formulation, adhere to Postulate 4, faithful to the motto according to which no man shall pull asunder what Einstein has joined together. Unlike the restriction to globally hyperbolic spacetime, the restriction to spacetimes of topology $\mathbb{R} \times \Sigma$ *does* threaten Postulate 4. So I invite you to join me in tracing general covariance through the translation process of standard GTR with its manifest general covariance into Hamiltonian GTR where it is apparently lost. In the last section of the chapter, I study the fate of general covariance through the quantization procedure to which it is subjected in the path to LQG.

¹⁸This statement only applies to spacetimes which admit time-wise foliations; for those with space-wise foliations, no such physical observers can be introduced. Henceforth, I shall assume that the spacetimes under consideration allow the introduction of a global time and thus admit time-wise foliations.

Before I embark on this journey, another important limitation of Hamiltonian GTR must be explained, although this time the limitation seems purely practical and not for principled reasons. Relativists, of all people, sometimes use a peculiar distinction destined to confuse even some physicists, viz. the separation between gravity and matter, where a gravitational field can transpire even in the absence of matter. This is much less mysterious as it may at first sound. Relativists usually refer to the left hand side of Einstein’s field equations as “gravity” and to the right hand side as “matter.” In case $T_{\mu\nu} = 0$, the right hand side vanishes and “matter” is absent. The subclass of generally relativistic models $\langle \mathcal{M}, g_{\mu\nu}, T_{\mu\nu} = 0 \rangle \in \mathfrak{M}_E$ are the so-called *vacuum solutions* of Einstein’s field equations, sometimes also referred to as *Ricci-flat spacetimes*. These vacuum solutions build an important and comparatively well-understood subclass of models. Not much is known about the initial value problem for models containing matter, mostly because the application of pertinent theorems depends critically on the dynamical equations of the matter fields and on how they couple to Einstein’s equations. Most extant theorems guarantee the existence of a well-posed initial value formulation only in case the coupled equations constitute a hyperbolic system of equations satisfying certain additional conditions.¹⁹ Because matter is quantum, a full resolution can only be expected in a correspondingly full QTG.

These difficulties are avoided in most Hamiltonian formulations of GTR in the simplest conceivable way: matter is eliminated entirely and only vacuum solutions are studied. In the related Lagrangian formulation of GTR, the Lagrange density in (4.1) corresponds to the vacuum Einstein equations, i.e. to pure gravity and thus receives the honorary subscript “G.” If one desires to include matter, then the corresponding Lagrange density must be added to \mathcal{L}_G :

$$\mathcal{L}_{\text{total}} = \mathcal{L}_G + \mathcal{L}_{\text{matter}}. \quad (4.11)$$

It is clear at this point that studying pure gravity separately makes perfect sense and significantly simplifies the analysis. When the Legendre transformations are applied to switch to the Hamiltonian formulation, which we will need for LQG, only pure gravity is considered. Accordingly, both known Hamiltonian formulations of GTR, the ADM and the Ashtekar formalisms, only deal with Ricci-flat models of GTR. This is fine, as long as it is understood that matter will eventually have to be included in the full QTG. This may very likely turn out to be a rather formidable task; as of now, LQG only offers preliminary speculations at best as to how this could be achieved.

¹⁹If the matter content of the universe is exhausted by the fields ϕ_i satisfying themselves hyperbolic equations of motion and if $T_{\mu\nu}$ depends only of the fields ϕ_i , their first derivatives and the spacetime metric $g_{\mu\nu}$ and its first derivatives, then there exists a well-posed initial-value formulation. This is the case e.g. for the Einstein-Klein-Gordon, the Einstein-Maxwell (Wald 1984, p. 266), and the Einstein-Yang-Mills equations (Arns 1979).

4.2 OLD AND NEW VARIABLES

4.2.1 The ADM formalism

The first Hamiltonian formulation of GTR, relying on the action (4.1) and on a corresponding interpretation of the EFEs as statements about metrics, has been foreshadowed by Dirac (1958a,b) and was essentially completed by Arnowitt, Deser, and Misner in a series of articles which culminated in Arnowitt et al. (1962). Their Hamiltonian approach to GTR has come to be known as the “ADM formulation” of GTR. As explicated in the previous section, consideration is limited to spacetimes of topology $\mathbb{R} \times \Sigma$. Again, the initial-value problem is only relevant for this topology and since any globally hyperbolic spacetime necessarily has this topology, the limitation does not appear to be severe. However, as I have argued above, I consider the limitation at least noteworthy insofar as it excludes a significant class of generally relativistic models. For the rest of this thesis, it shall be assumed that spacetimes have topology $\mathbb{R} \times \Sigma$ unless stated otherwise.²⁰

The canonical ADM variables are the three-metric q_{ab} induced on the three-space Σ , the so-called lapse function N and shift vector field N^a and their conjugate momenta. The three-metric captures the geometry of the three-spaces Σ , and its momenta depend essentially on the extrinsic curvature and thus encode the information of how the three-space is embedded in four-dimensional spacetime. The lapse function and the shift vector, on the other hand, embody the gauge freedom, as they are arbitrary functions encoding time reparametrization invariance and spatial diffeomorphism invariance respectively. Their dynamical evolution is not determined by the theory.

The topology of the spacetimes considered implies that they admit a *foliation*, i.e. the three spaces Σ can be parametrized by a global “time” t . Thus, a foliated spacetime then entertains a one-parameter family of Cauchy surfaces Σ_t . So take one of the three-spaces and let it act as an initial hypersurface whose geometry evolves to later (or earlier) times via Einstein’s equations. This view of an evolving three-geometry evolving in time is often termed “*geometrodynamics*.” The action (4.1) is decomposed in (3+1)-terms and thus reexpressed as a functional of q_{ab}, N, N^a and their conjugate momenta. I will confine myself to merely stating the most important steps in constructing the ADM formulation of GTR. The reader should consult the literature in footnote 20, where all the results are derived.

The time function t chosen to parametrize the three-spaces is supplemented by a “time

²⁰ For a detailed introduction and discussion of the ADM formulation of GTR, see Arnowitt et al. (1962), Isham (1993, Sec. 3.3), Wald (1984, pp. 256ff, pp. 293ff, Appendix E.2), Isham and Kuchař (1985a, Sec. 2.3), Thiemann (2003, Sec. I.2.1), Thiemann (2001b, Sec. I.1.1), Gambini and Pullin (1996, Sec. 7.2), Baez and Muniain (1994, Ch. III.4). The subsequent brief outline is primarily based on Wald (1984).

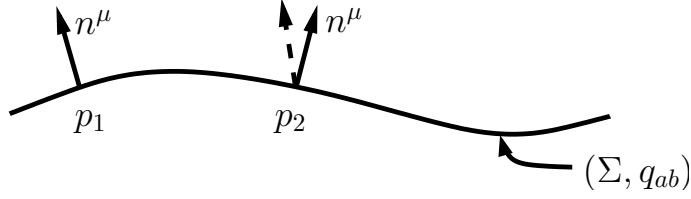


Figure 3: Geometric interpretation of K^{ab} .

flow” t^μ defined on all of \mathcal{M} . Neither time function nor time flow, however, have any physical meaning before the spacetime metric is known. But solving the Einstein equations for the unknown metric field is exactly the purpose of the entire spiel. The time flow can be decomposed into its normal and its tangential components, leading to the introduction of the *lapse function* N essentially representing the normal component and the *shift vector* N^a constituting the tangential component. Naturally, the configuration variable should be one that encodes the geometry of the three-spaces. The induced spatial metric q_{ab} satisfies this requirement. It is related to the four-metric of spacetime via $q_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$ where n_μ is the unit normal vector on Σ_t . One can show that the information contained in (q_{ab}, N, N^a) is equivalent to what is encoded in $g_{\mu\nu}$. The Lagrange density in (4.1) can then be re-expressed in terms of q_{ab}, N, N^a and their first derivatives:

$$\mathcal{L}_{ADM} = \frac{1}{16\pi G} \sqrt{q} N [{}^3R + K_{ab} K^{ab} - K^2], \quad (4.12)$$

where q is the determinant of q_{ab} , 3R the three-dimensional scalar curvature of the three-spaces, K_{ab} is the extrinsic curvature of Σ_t and $K = K^a_a$. The extrinsic curvature is related to the “time derivative” of q_{ab} , i.e. to the Lie derivative of q_{ab} in the direction of the time flow. It is defined by $K_{ab} = \frac{1}{2} \mathcal{L}_n q_{ab}$ where \mathcal{L}_n is the Lie derivative in the direction of n_μ . The Lie derivative of a vector field X in the direction of or “along” another vector field Y can be characterized via their Lie brackets as $\mathcal{L}_X Y = [X, Y]$.²¹ The extrinsic curvature encodes the information how Σ_t is embedded in the four-dimensional manifold and essentially constitutes the momentum canonically conjugate to q_{ab} . Figure 3 illustrates the geometric interpretation of the extrinsic curvature as a measure of difference between n^μ at p_2 and n^μ at p_1 parallel transported to p_2 along a the hypersurface Σ . More precisely, the canonically conjugate momentum density is defined as $\pi^{ab} = \partial \mathcal{L}_{ADM} / \partial \dot{q}_{ab} = \sqrt{q} (K^{ab} - K q^{ab})$. We can

²¹For a more thorough introduction and discussion of Lie derivatives, see Nakahara (2003, Sec. 5.3.2).

introduce a derivative operator associated with q_{ab} and use this to derive relations between the three-dimensional curvature tensor field ${}^3R^\mu{}_{\nu\rho\sigma}$ and the spacetime curvature $R^\mu{}_{\nu\rho\sigma}$. These relations are known as the *Gauss-Codacci relations*.

Since \mathcal{L}_{ADM} does not contain \dot{N} and \dot{N}^a , the canonical momenta P of N and P_a of N^a identically vanish. This means that we do not obtain an invertible relation between \dot{N} and \dot{N}^a on the one hand and their canonical momenta on the other. Constraints arise, and there is gauge freedom. The threatening indeterminism can in this case be overcome by not considering N and N^a and their conjugate momenta as dynamical variables and by redefining the configuration space as only containing the three-metrics q_{ab} on Σ_t and the momentum space accordingly. The dynamical variables of ADM are the six independent components of q_{ab} and the corresponding six independent conjugate momenta π^{ab} . Let me review how the constraints arise and what they imply.

A Hamiltonian density can then be introduced via

$$\mathcal{H}(q_{ab}, \pi^{ab}) = \dot{q}^{ab}\pi_{ab} - \mathcal{L}_{ADM}$$

which, in turn, defines the Hamiltonian H_{ADM} of the ADM formulation of GTR by $H_{ADM} = \int_{\Sigma} \mathcal{H} d^3x$.²² A straightforward calculation yields

$$\mathcal{H} = \sqrt{q}(NC + N^a V_a) \tag{4.13}$$

where

$$C = -{}^3R + q^{-1}\pi^{ab}\pi_{ab} - \frac{1}{2}q^{-1}\pi^2 \tag{4.14}$$

$$V_a = -2D^b(q^{-1/2}\pi_{ab}), \tag{4.15}$$

where $\pi = \pi^a_a$ and D^a is the unique, torsion-free covariant derivative operator on Σ_t associated with q_{ab} .²³ Unsurprisingly, the Hamiltonian density in (4.13) proportionally depends on terms involving the lapse function and the shift vector as its role is to generate the dynamical evolution of data on Σ and the lapse and shift push around the hypersurface Σ . But because the canonical momenta P and P_a vanish and consistency demands that this holds for all t , \dot{P} and \dot{P}_a vanish as well. From equations of motion (4.5), one obtains $\dot{P} = -C$ and $\dot{P}_a = -V_a$, which of course implies that $C = V_a = 0$. It turns out that C and V_a are primary constraints as they directly arise from the vanishing of the conjugate momenta of N and N^a , respectively. Equation (4.14) is called the *Hamiltonian* or *scalar* constraint function,

²²Here, I am glossing over some subtleties concerning the conditions under which total divergencies vanish.

²³Although C and V_a are *densities*, I desist from labelling them with calligraphic letters in order to follow more closely common notational practices.

while equations (4.15) are the *spatial diffeomorphism* constraint functions. The Hamiltonian constraint equation $C = 0$ relates the extrinsic curvature of a spacelike hypersurface to its scalar curvature and the diffeomorphism constraint equations $V_a = 0$ impose restrictions on the extrinsic curvature of the spacelike hypersurfaces. One can also find these constraints directly from the Gauss-Codacci relations, albeit in this case, they will not be functions of the canonical variables q_{ab} and π^{ab} , but rather of q_{ab} and K_{ab} .

Solving the constraints, as above, means to eliminate “unphysical” degrees of freedom, or the surplus description of physical states, such as to obtain a set of independent canonical variables, or, equivalently, to identify the physical degrees of freedom. Either the constraints are solved classically, and only the classical physical phase space with its symplectic structure is quantized, or the constrained system is first quantized according to Dirac’s procedure, to be described below, resulting in a (extended) Hilbert space of which the physical Hilbert space is a proper subset, as well as quantum constraint equations, i.e. quantum operators corresponding to the classical constraint functions acting on states in the extended Hilbert space. In principle, both options should lead to the same destination, but the technical difficulties along the way will generally be quite different. Typically, the second path is chosen, as will be explicated below.

The amazing consequence of the constraint equations together with (4.13) implies that the Hamiltonian is constrained to vanish in the ADM formulation of GTR. This is surprising, as one would expect a theory with zero Hamiltonian to exhibit a rather dull dynamics, yet the dynamics of GTR clearly seems more interesting. The constraint equations $C = V_a = 0$, it should be emphasized, correspond to the four Einstein equations which constrain the initial data on Σ_t . If one studies the dynamical evolution of the remaining degrees of freedom, however, it becomes clear that despite $H = 0$, something non-trivial can be said about the evolution of the data on Σ_t . If one computes the explicit expressions for $\dot{q}^{ab} = \{q^{ab}, H\}$ and $\dot{\pi}_{ab} = \{\pi_{ab}, H\}$, one recognizes that even on the physical phase space Γ_{phys} where $H = 0$ holds, Hamilton’s equations offer a non-trivial dynamical evolution.

As explicated above, the lapse and shift both contribute to the Hamiltonian and thus to time evolution by pushing the hypersurface Σ_t in its normal and tangent direction, respectively. If one sets the shift vector equal to zero, the Hamiltonian is only composed of its normal part given by

$$H_{\perp} \doteq C(N) = \int_{\Sigma} d^3x \sqrt{q} NC,$$

which can be seen as generating a motion of Σ in the direction of its normal n_{μ} . On the other hand, if one lets the lapse function vanish, one receives the tangent part of the Hamiltonian

as given by

$$H_{\parallel} \doteq V(\vec{N}) = \int_{\Sigma} d^3x \sqrt{q} N^a V_a,$$

which can be understood as pushing Σ in its tangent direction. Unfortunately, there exist some terminological imprecisions in the literature, as both H and H_{\perp} are sometimes called Hamiltonian and designated by the same letter. I will attempt to consistently use the word “Hamiltonian” and the letter “ H ” for the former and the term “Hamiltonian constraint” and the letter “ C ” for the latter.

From the expressions (4.14) and (4.15) it follows that the constraints form an algebra called *Dirac algebra*:

$$\begin{aligned} \{V(\vec{N}), V(\vec{N}')\} &= \kappa V(\mathcal{L}_{\vec{N}} \vec{N}'), \\ \{V(\vec{N}), C(N)\} &= \kappa C(\mathcal{L}_{\vec{N}} N), \\ \{C(N), C(N')\} &= \kappa V(q^{-1}(NdN' - N'dN)), \end{aligned} \tag{4.16}$$

where $\kappa = 1/16\pi G$ and $\mathcal{L}_{\vec{A}}B$ is the Lie derivative of B in direction of the vector field \vec{A} .²⁴ This algebra is closed. Thus, the commutator of the Hamiltonian H with any of the constraints can be written as a linear combination of constraints, which means that the condition $C = V_a = 0$ is preserved under dynamical evolution governed by the Hamiltonian. The evolution is thus consistent. The algebra is also sometimes called *constraint* or *hypersurface deformation algebra*. I will discuss the structure of algebra (4.16) in Section 4.4. This completes the discussion of the ADM formulation of GTR.

4.2.2 The Ashtekar-Barbero formalism

The formulation of LQG is based on Ashtekar’s “new variables” (Ashtekar 1986, 1987), a connection formulation of classical GTR.²⁵ Soon after the inception of these new variables, Jacobson and Smolin (1988) found an infinite number of loop-like, exact solutions of the Wheeler-DeWitt equation in Ashtekar’s formulation. These results provide the basis for the “loop representation of quantum general relativity” (Rovelli and Smolin 1988), and thus mark the birth of what matured into LQG.

The technical choice of what the basic variables should be consists of choosing an algebra of functions of field variables, which will later become quantum operators. Based on previous work by Sen (1982a,b), Ashtekar (1986, 1987) introduced what then became known as “new variables” of Hamiltonian GTR and today is called “Ashtekar” or “Ashtekar-Barbero”

²⁴Cf. Thiemann (2003, Eqs. I.2.1.12)

²⁵For a more recent textbook treating all the necessary concepts leading up to the new variables in an accessible manner, see Baez and Muniain (1994).

variables. Ashtekar’s reformulation was motivated by a desire to overcome what plagued the canonical programme for two decades, viz. the complicated dependence of the constraint equations on the ADM variables. These complications render the constraints insoluble. The Ashtekar-Barbero variables simplify the constraints considerably, although the direct geometrical meaning of the ADM variables is lost.

The first step to arrive at the Ashtekar-Barbero variables is to enlarge the phase space of GTR by switching from the ADM formulation to the *extended* ADM formulation of GTR. This step occurs by means of the introduction of a locally inertial frame in the form of a three-dimensional *triad field* $e_a^i(x)$ instead of the three-metric $q_{ab}(x)$ via $q_{ab}(x) = e_a^i(x)e_b^j(x)\delta_{ij}$. As a result of this, the variables of the extended phase space will carry tensorial as well as “internal” indices. These triads become the basic variables, with a conjugate momentum which essentially consists of the extrinsic curvature π_{ab} . The Einstein equations are then interpreted as statements about a connection rather than about a metric. The Ashtekar-Barbero variables are obtained from a transformation from the variables of the extended ADM formalism by

$$A_a^i = \Gamma_a^i + \beta\pi_{ab}e^{bi}, \quad (4.17)$$

where Γ_a^i is the so-called spin connection of the triad field e_a^i , defined by

$$\Gamma_a^i = \frac{1}{2}\epsilon^i{}_j{}^k e_k^b(\partial_a e_b^j - \partial_b e_a^j + e^{cj}e_{ai}\partial_b e_c^l), \quad (4.18)$$

and β the *Immirzi parameter*, the only free parameter of LQG²⁶ and by

$$E_i^a = \frac{\sqrt{\det q} e_i^a}{\beta}. \quad (4.19)$$

For a given three-space Σ , the configuration variable is thus the self-dual part A_a^i of the spin connection.²⁷ The conjugate momenta E_i^a are densitized triads, i.e. the triad e_i^a essentially multiplied by $\sqrt{\det q}$. These variables satisfy the “equal-time” Poisson bracket relations

$$\{A_a^i(x), E_j^b(y)\} \propto \delta_a^b \delta_j^i \delta^3(x, y) \quad (4.20)$$

²⁶The parameter $\beta \in \mathbb{C} \setminus \{0\}$ is often set either $\beta = i$, which makes the connection self-dual and thus more physically intuitive but at the same time inhibits quantization, as in large parts of [Rovelli \(2004\)](#), or as an element in \mathbb{R}^+ , which permits quantization but no longer bestows a direct spacetime interpretation on the connection, as in [Thiemann \(2003\)](#). The classical theory is not affected by the value of β , but the quantum theory is, where different values of β lead to different physical predictions, e.g. in the spectra of the area and volume operators and in the computation of the black hole entropy. The presence of this parameter in the quantum theory may well reflect a one-parameter quantization ambiguity, as is conjectured by [Rovelli \(2004, Sec. 4.2.3\)](#).

²⁷Self-dual means that the connection is invariant under the so-called Hodge star transformation. For more details, also on spin connections, see ([Rovelli 2004, Sec. 2.1.1](#)) and particularly ([Baez and Muniain 1994, pp. 439ff](#)).

on each spacelike hypersurface Σ , as well as $\{A_a^i(x), A_b^j(y)\} = \{E_i^a(x), E_j^b(y)\} = 0$.

The Ashtekar-Barbero variables, the connection variable A_a^i and its conjugate momentum, the triad variable E_i^a , offer the only successful path to a canonical formulation in terms of connections, all others produce second-class constraints (Gambini and Pullin 1996). Three different types of first-class constraints arise:

- The three *Gauss constraint equations* are

$$G_i \doteq D_a E_i^a = 0, \quad (4.21)$$

where D_a is the covariant derivative $D_a E_i^a \doteq \partial_a E_i^a + \epsilon_{ij}^k A_a^j E_k^a$. The Gauss constraints G_i are associated with the rotational gauge freedom of the triads, i.e. the Gauss constraints generate the infinitesimal $SU(2)$ transformation in the internal indices.

- The three (*spatial*) *diffeomorphism (or vector) constraint equations* are

$$V_a \doteq F_{ab}^i E_i^b = 0, \quad (4.22)$$

where $F_{ab}^i \doteq \partial_a A_b^i - \partial_b A_a^i + \epsilon^i_{jk} A_a^j A_b^k$ is the curvature of A_a^i . The vector constraints V_a generate the spatial diffeomorphisms in Σ .

- The *Hamiltonian (or scalar) constraint equation*, finally,

$$C \doteq F_{ab}^i E_j^a E_k^b \epsilon_i^{jk} = 0, \quad (4.23)$$

is related to the time reparametrization invariance.

Given the Poisson bracket relation (4.20) between the basic variables, one can determine the Poisson brackets among the constraints (4.21)-(4.23). Somewhat schematically, the result is the (extended) *Dirac or constraint algebra*:

$$\{V, V\} \propto V, \quad (4.24)$$

$$\{V, C\} \propto C, \quad (4.25)$$

$$\{C, C\} \propto V, \quad (4.26)$$

$$\{G, G\} \propto G, \quad (4.27)$$

$$\{V, G\} = \{C, G\} = 0. \quad (4.28)$$

From a purely classical point of view, the extension of Γ_{ADM} has complicated matters by replacing twelve variables q_{ab}, π^{ab} by eighteen variables A_a^i, E_i^a and by adding another family of constraints, the Gauss constraints. However, as will be further explicated in Section 4.4, the Gauss constraints remove the six “superfluous” degrees of freedom and thus establish that working in the extended phase space is really equivalent to working in Γ_{ADM} . By

introducing this additional gauge freedom, the theory becomes a gauge theory with a compact gauge group, viz. the group of triad rotations. This makes the system amenable to the powerful quantization techniques available for the canonical quantization of gauge theories. Now the system is ready to be ground through the Dirac mill of canonical quantization. Before we will do this, I discuss two immediate consequences of the Hamiltonian formulation of GTR: the problem of time and the freezing of the dynamics on the one hand, and the worry that foliating the four-dimensional spacetime breaks the manifest full four-dimensional diffeomorphism invariance and thus the gauge group of GTR.

4.3 THE END OF TIME?

The problem of time in canonical GTR arises relatively directly as a result of how it encodes its symmetries. The label of “problem of time” is often given to a number of related, but slightly different issues. Although it can—with invariable success—be used as a party gag to send philosophers of all but one persuasion into revolt or denial, one could argue that there is no problem of time at all. There simply *is* no time, and that is not a problem. But rather than merely shrugging it off like this, at least one would have to produce an account why we experience a flux of time so vividly that many philosophers insist on the objective reality of this flux. So while there is perhaps no problem of time, there seems to be a problem of no time. The problem of (no) time, then, consists of at least two aspects: the disappearance of time as a fundamental magnitude and the freezing of the dynamics. Let me address these issues in turn.

The first issue, then, pertains to the objective existence of time. In a so-called time-reparametrization-invariant theory, the action remains invariant under redefinitions of time $t' = f(t)$. In other words, time-reparametrization-invariant theories deem two descriptions of a physical system which only differ in their parametrizations of time as referring to the same physical situation. This gauge freedom, captured by the group $\text{Diff}(\mathbb{R})$, can be interpreted as the equivalence of different observers measuring coincidences of measurable (gauge-invariant) quantities with their clocks of different speeds. Consequently, time as an objectively measurable independent degree of freedom does not exist in theories of this type. However, values for events as measured by an observer can be uniquely related to the corresponding values of another observer. Thus, there exists an objective total ordering of events.²⁸ Although one

²⁸A bivalent relation R defines a *total order* on a set \mathcal{S} iff for all $a, b, c \in \mathcal{S}$ the following properties hold: (1) Raa (reflexivity), (2) $Rab \& Rba \rightarrow a = b$ (weak antisymmetry), (3) $Rab \& Rbc \rightarrow Rac$ (transitivity), and (4) $Rab \vee Rba$ (comparability).

might therefore wish to deny the objective existence of time already at this level, at least the possibility of a temporal flux or of objective becoming must still be acknowledged in the light of this total ordering of events. The relativity of simultaneity of STR, however, degrades this total ordering to a merely partial ordering.²⁹ This “partialness” of temporal orderings destroys all hope for absolute simultaneity and thus of objective becoming.³⁰ The monolithic time of absolutistic mechanics is replaced by the Lorentz times of inertial observers. All this is well known. But it gets worse once we move on to GTR.

Generically in GTR and unlike in STR, temporal ordering relations cannot be weakly antisymmetric—a necessary condition for a partially ordered set—due to the possibility of causal loops. In analogy to time-reparametrization-invariant theories, coordinate time is not a physical degree of freedom of GTR and evolution in it is thus not gauge-invariant. Any observable evolution must occur in a physical degree of freedom, a “clock” variable. Despite the various ways in which time surfaces in GTR, qua coordinate time, proper time, internal time, cosmological time, or ephemeris time, the theory can be formulated entirely without recourse to coordinate time.³¹ But a theory which admits a complete formulation without a time parameter thus allows the elimination of time as a fundamental physical magnitude. This disappearance of time is also testified by the fact that unlike the Schrödinger equation, the Wheeler-DeWitt equation (5.4) contains no time parameter and describes a dynamical evolution without recourse to time. *Sic transit gloria temporis*.

Importantly, the problem of time (and of change) arises for all canonical programmes of quantum gravity. In fact, both the issues of time and change appear in an unmitigated fashion already in classical GTR in its Hamiltonian formulation. It is sometimes said, however, that while the disappearance of time already occurs at the classical level, it must only truly be confronted at the quantum level.³² The reasoning behind this assertion relies on that although we cannot formulate the dynamics of GTR in terms of a physical time, we can introduce a handful of accessory times such as proper time, internal time, cosmological time etc. These cannot be used, however, to define physical time since they are either not globally definable, not observable, or defined only under specific conditions. The pseudo-Riemannian manifold of classical spacetimes makes these accessory notions possible. Since canonical QG no longer accommodates pseudo-Riemannian manifolds, the accessory times cannot be

²⁹If a bivalent ordering relation satisfies the first three axioms of the definition of a total order but not the fourth, it defines a *partial order*. Thus, comparability is lost in a merely partially ordered set.

³⁰At least for objective becoming as traditionally understood; there exists the possibility of defining a becoming relation in terms of the objectively given causal structure of Minkowski spacetime, cf. [Clifton and Hogarth \(1995\)](#).

³¹Cf. [Barbour \(1994a\)](#) or [Rovelli \(1991c, 2004\)](#), but see also Hájíček’s objection ([Hájíček 1991](#)) and Rovelli’s reply ([Rovelli 1991d](#)).

³²Cf. e.g. [Rovelli \(2004, Sec. 10.1.3\)](#).

transferred from the classical level into the QTG.³³ Thus no recourse can be made to any auxiliary notions of time and the dynamics must be defined in the absence of time. While I agree with the point that auxiliary notions of time can be introduced in classical GTR to allay the consequences of the disappearance of time, this will hardly console somebody interested in foundational issues. As far as the foundations are concerned, time disappears already at the classical level.

All this, however, is not quite as grave as it sounds, since physicists have learnt to formulate theories, including QTs, in the absence of time.³⁴ But the difficulties exacerbate when we address the aspect of the problem of time which should perhaps more adequately be called the *problem of change*. The issue was first recognized in print in Bergmann (1961),³⁵ but has been brought into prominence in the context of canonical quantizations of GTR mostly by Barbour (1994a,b, 1999).³⁶ Despite their difference, the issues of time and change are closely interrelated, so these articles typically treat both.

What is the *problem of change* and how does it arise? Most of the work to understand the issue has already been accomplished in the preceding sections of this chapter. In Definition 5, the distinction between first-class and second-class constraints has been introduced. The distinction becomes pertinent for the problem of change because it is used to define *Dirac observables*. Typically, these are defined to be functions on the constraint surface \mathcal{C} which are gauge invariant. More rigorously, an equivalence class of *Dirac observables*, sometimes considered as representing *one* Dirac observable, is defined as the set of those functions in phase space that have weakly vanishing Poisson brackets with all first-class constraints and coincide on the constraint surface. Alternatively, and equivalently, Dirac observables may be defined as functions in phase space which are constant along gauge orbits on the constraint surface. The equivalence obtains because it has been granted, for present purposes at least, that Dirac (1964) has established that the gauge transformations are generated by the first-class constraints of the system. It should be stressed that (Dirac) observables are whatever satisfies this technical definition, *not* the observable or measurable quantities of a theory. Observables exhaust the physical content of a theory. Their precise conceptualization, therefore, must indirectly offer an interpretation of the theory.

Since in GTR, according to Postulate 4, general covariance is interpreted as a gauge

³³Carlo Rovelli, personal communication, 21 July 2004.

³⁴Cf. Rovelli (1990) and Rovelli (2004, Secs. 3.2.4, 5.4.1 et passim).

³⁵And presumably the first time ever in Bergmann’s letter to Dirac on 9 October 1959, in which Bergmann clearly states that all field variables in canonical gravity suffer from “frozen dynamics.” Bergmann’s correspondence with Dirac is archived at Syracuse University, but unfortunately, I have not yet had the opportunity to peruse this archival resource.

³⁶See also Butterfield’s reaction (Butterfield 2002), Hájíček (1996, 1997), Isham (1993, 1994a), Kuchař (1991, 1992), and the papers mentioned earlier by Rovelli (1990, 1991a,c,d, 1995).

invariance under active spacetime diffeomorphisms, it seems as if Dirac observables should be invariant under spacetime diffeomorphisms. That this is far from obviously the case will be discussed in Section 4.4; the main reason for my reservations is that Dirac observables as defined above are magnitudes living in canonical formulations of gauge theories while general covariance is a concept from the standard formulation of GTR. In order to make this transition, then, we will need to understand how substantive general covariance is translated in the Hamiltonian formulation of GTR.

As is clear already prior to a more detailed study, invariance under spacetime diffeomorphisms leads in the constrained Hamiltonian formalism to spatial diffeomorphism and Hamiltonian constraint equations, such as $V_a = C = 0$ in the ADM formalism, where V_a and C are defined by (4.14) and (4.15). In other words, both the normal as well as tangential component of the Hamiltonian, which according to (4.5) generates the dynamical evolution of the three-spaces, are constrained to vanish. This implies that the Hamiltonian density (4.13) and therefore the Hamiltonian itself are constrained to vanish!³⁷ Because the Hamiltonian generates transformations with respect to “time,” this means that motion is pure gauge and the Dirac observables must all be constants of the motion. Equivalently, they must commute weakly with all first-class constraints, particularly including the Hamiltonian constraint. From equation (4.5), one can immediately recognize the equivalence of the two statements. Therefore, no Dirac observable of GTR suffers change. If the dynamics of spacetime is completely encoded in Dirac observables, then its dynamics is “frozen.” Not only do we live in a block universe, but it appears as if this block was a very dull one indeed.

The reaction in the quantum gravity community to the problem of change has been mixed. There are full endorsers such as Barbour (Barbour 1994a,b, 1999) who take it, combined with the aspect of the problem of time mentioned above, as evidence for the three-dimensionality of the world. Change, or the dynamics if you prefer, is fully encoded in the three-dimensional relational structure of spatial geometry. Rovelli (1991c,d, 2004), while supportive of the relational conclusion and himself a full endorser of the consequences of the “problems” of time and change, does not follow the rather idiosyncratic path of Barbour. Instead, he proposes the concept of *partial observables* (Rovelli 2002d), which designates physical magnitudes for which a measuring procedure can be defined leading to a measurement result in form of a number. According to Rovelli, although Dirac observables are constants of motion, correlations between partial observables change and can actually be measured.³⁸ Others again, like Kuchař (1991, 1992, 1993) have simply refused to accept

³⁷Generally covariant systems have weakly vanishing Hamiltonian constraints just in case the canonical variables transform as scalars under spacetime diffeomorphisms (Henneaux and Teitelboim 1992, Sec. 4.3.2).

³⁸For a very recent philosophical assessment of this response to the problem of change, see Rickles (2006). So recent in fact that I did not yet have the time to devote myself to Rickles’s article. He appears to attempt

this, as he calls it, Parmenidean conclusion. He associates observables with observability and thus considers the problem of change to be a *reductio* against the relevance of Dirac observables.³⁹ In their stead, he proposes to regard what I baptize *Kuchař observables*, i.e. dynamical variables which are invariant under the action of the $\text{Diff}(\Sigma)$ group but not of $\text{Diff}(\mathcal{M})$, as the physically most useful magnitudes to study. Kuchař observables commute with the spatial diffeomorphism constraint, but not with the Hamiltonian constraint and are therefore not constants of the motion. They capture changes in the spatial three-geometry. As Earman (2002a) has pointed out, the price to be paid for Kuchař’s manoeuvre is the loss of an elegant implementation of general covariance. Also, the daemon of indeterminism rears its ugly head. The gain of Kuchař’s proposal, however, is that it does not preclude the experience of change. Those who prefer to swallow the frozen pill, on the other hand, owe an explication of such experience and they will have to show how to do physics without time.

The causal spinfoam proposal of Markopoulou and Smolin,⁴⁰ to be presented in Section 5.3.2, can also be considered as a reaction to the problems of time and change.⁴¹ Their causal spinfoam models seems to allow a definition of observables which permits an un-freezing of the dynamics. These models encode the causal structure already at the Planck scale. They purport to avoid the problem by insisting that whatever is observable to any observer must reach them from within their causal past. Thus, the problem can allegedly be solved by re-establishing the broken link between observability and what the theory asserts regarding the physical reality, similar to Kuchař’s insistence on observability. In these causal spinfoam models, causal propagation is governed by a small set of rules regulating the temporal evolution of the quantized spatial three-geometry. Alternative approaches to formulate a quantum theory of gravity exist that take the causal structure to be paramount in overcoming or avoiding altogether the problem of time.⁴²

The philosophical reaction to the problem of change has been incomprehensibly meagre. One would expect that philosophers of physics would have jumped on the opportunity to

to give a structuralist basis to Rovelli’s idea of correlations between partial observables. I will return to the notion of partial observables in Section 8.1.

³⁹He calls Dirac observables “perennials”; see e.g. his (Kuchař 1993, Sec. 5). Usually, observables designate genuine physical magnitudes and do not imply observability. Another historical misnomer to cause abundant confusion, even among physicists! For a serviceable introduction to Kuchař’s programme, see Belot and Earman (2001, Sec. 10.6).

⁴⁰Markopoulou (2000a,b,c); Markopoulou and Smolin (1997, 1998).

⁴¹Kauffman and Smolin (1997) have proposed a more general class of quantum cosmological models with causality built in at the fundamental level. Smolin (2001a) has given a somewhat different reconstruction of how the problem of time arises and has suggested two principles which mount resistance to the consequences the problems of time and change are typically taken to entail. These principles consist of an insistence on connecting observables with observability and on the finite computability of the theoretical magnitudes.

⁴²This is the so-called *causal sets* approach by Sorkin and co-workers (Bombelli et al. 1987; Brightwell et al. 2003; Rideout and Sorkin 2000), also to be discussed in Section 5.3.2.

serve themselves not just to a free lunch of philosophical finger food, but to a royal banquet of exuberant foundational meals. Presumably, at least part of the answer to this puzzle lies in the indigestive mathematical dressing with which the meals come. Over the last few years, Earman (2002a,b, 2003) has spearheaded its introduction into philosophy of physics. Belot and Earman (2001) have studied the thorny conceptual issues in the wake of implementing general covariance as a gauge symmetry of GTR and thereby illuminated how these relate to technical and conceptual problems in the canonical programme of quantizing gravity. Earman (2002b) has explained how the problem of change in classical and quantum gravity is grist for McTaggart’s mill. If one insists that change in Dirac observables is a necessary condition for physical change and that physical change is a necessary condition for physical time, then one reaches the neo-McTaggartian conclusion that physical time cannot be real. Maudlin (2002) and Healey (2004) have strongly resisted this conclusion in print. Maudlin seems to be rather critical of the constrained Hamiltonian formalism and worries that the problem of change might be an artefact of this formulation of GTR. Quite apart from the motivations for applying the constrained Hamiltonian codification of the theory, Earman (2002b) cites the attempt by Ashtekar, Bombelli, and Reula (1991) to recast GTR in an alternative formulation—albeit of limited validity—in which the problem of change resurfaces. This, according to Earman, strengthens the claim that the problem of change is not a mere artefact of the formalism. Healey has opposed Earman’s conclusion by casting doubt on Earman’s premise that Dirac observables completely capture the physical content of a generally relativistic system. He suggests that change and its observability are restituted if one acknowledges that genuine physical magnitudes when observed must be frame-dependent quantities. By introducing frame-dependency, according to him, physical change supervenes on the deep structure of Dirac observables.

An awful lot thus depends on what one takes to be the genuine physical magnitudes in which the complete physical information is packed. In other words, the problem of change leads directly to an analysis of what we should take to be the observables of GTR. I have discussed or at least mentioned Dirac observables, partial observables, Kuchař observables, and observables as understood by Healey. Rovelli (1991b, 2002c) has shown how gauge-invariant observables can be introduced for GTR coupled to four particles. These observables are termed *GPS observables*, because they are defined by the physical reference frame actualized by the GPS technology. But the most important notion of observables that I have so far avoided are the so-called *Bergmann observables*. They were introduced by Bergmann (1961) and designate magnitudes that can be predicted uniquely from initial data. Observables, for Bergmann, encode the deterministic content of a theory. If observables must enjoy a deterministic evolution, as became clear in the discussion of the hole argument in Section

3.2, they must be confined to magnitudes which are invariant under active spacetime diffeomorphisms. Whether or not, and if so, to what extent, Bergmann observables coincide with Dirac observables will only be resolved once it becomes clear how the full spacetime diffeomorphism invariance is translated into the canonical approach. One would surely expect that the set of Dirac observables fully corresponds to the set of Bergmann observables. The next section will offer remarks towards this goal.⁴³

4.4 IMPLEMENTATION OF GENERAL COVARIANCE IN CANONICAL FORMULATIONS

The reader may have wondered above why I spend any spacetime presenting the ADM formalism when LQG uses the Ashtekar-Barbero connection formalism as its vantage point. The ADM formalism was discussed not only because of its intrinsic systematic and historical interest, but also because it will turn out to be of crucial importance to understand and assess the implementation of general covariance in the connection approach. The dilemma which prevents a direct evaluation of general covariance in the connection approach hinges on the value of the Immirzi parameter β : for $\beta \in \mathbb{R}$, which permits quantization, the connection acquires a spacetime meaning only after solving the Gauss constraints; for β purely imaginary, the covariant Lagrangian is the self-dual part of the Palatini action and the connection thus affords a spacetime interpretation—but the system is no longer amenable to quantization.⁴⁴ The analysis in this section will thus consist of two parts: first, it must be established that solving the Gauss constraints in the connection version of the generally relativistic Hamiltonian system with constraints leads back to the ADM phase space; second, it must be ensured that general covariance is taken into account in the ADM version of Hamiltonian GTR.

For the first part, referred to in the literature as the *symplectic reduction* of the constrained Hamiltonian system subject to the constraints (4.21)-(4.23) with respect to the Gaussian constraints, I defer to Thiemann (2001b, Sec. I.1.3) for the proof and contend myself with stating the pertinent theorem:

Theorem 1. *Consider the phase space Γ coordinatized by (A_a^i, E_i^a) with the Poisson brackets*

⁴³Another conceptualization of observables are the so-called *generalized coincidence observables*, which have first been constructed by Komar (1958) via the use of fields to coordinatize the spacetime manifold. Earman (2002b, p. 11) provides a very accessible discussion of Bergmann and generalized coincidence observables and their respective relation to Dirac observables.

⁴⁴I wish to thank Thomas Thiemann for pointing this out to me (on 3 August 2005). Cf. also footnote 26.

(4.20) and the constraints G_j, V_a, C of (4.21)-(4.23). Solving only the constraint equations $G_j = 0$ results precisely in the ADM phase space Γ_{ADM} with constraints C and V_a .

The symplectic reduction with respect to the Gauss constraints thus leads us back to Γ_{ADM} , alleviating any worries one might have had concerning the extension of the phase space by introducing the $SU(2)$ rotational degrees of freedom of the tetrads. The constraint equations (4.21) exhibit that these degrees of freedom are non-physical. This already terminates the first part of this section.

Let me turn to an examination of the fate of general covariance in Hamiltonian GTR à la ADM. The so-called “Dirac conjecture,” as mentioned in Section 4.1, maintains that all first-class constraints generate gauge transformations, i.e. the group of gauge transformations can be represented by the algebra of first-class constraints which generate the gauge transformations just as elements of the group $SO(3)$ can be represented by matrices which generate ordinary rotation in three-dimensional space. While Dirac’s conjecture is known to be false in the full generality just stated,⁴⁵ the counterexamples tend to be rather exotic and the adoption of the conjecture as a “working theorem” poses no threat for most practical purposes.

Let me briefly remind the reader about Lie groups and their associated Lie algebras, which are of eminent importance to the treatment of gauge theories as they provide the continuous groups on which all non-Abelian gauge theories are based.⁴⁶ A *Lie group* G is a differentiable manifold endowed with a group structure such that its group operations $G \times G \rightarrow G$ are differentiable. A group is constructed by its *generators*, i.e. a set of group elements such that possibly repeated application of these generators onto themselves and each other produces all the elements in the group. It is both usual and convenient to restrict attention to those groups which are generated by group elements arbitrarily close to the identity, the idea being that each group element can be reached by the iterated action of these infinitesimal generators. For our purposes, of course, where the differentiable manifold is \mathcal{M} endowed with the group structure of diffeomorphic transformations, these infinitesimal generators will be elements in $\text{Diff}(\mathcal{M})$ which are arbitrarily close to the identity map. The set of generators T^a of a Lie group must span the space of infinitesimal group operations. Therefore, the Poisson bracket of generators must be a linear combination of generators such that we have

$$\{T^a, T^b\} = i f_c^{ab} T^c, \quad (4.29)$$

where the numbers f_c^{ab} are called the *structure constants*. The set of vector fields \mathfrak{g} which

⁴⁵For a counterexample to the conjecture, consult [Henneaux and Teitelboim \(1992, Sec. 1.2.2\)](#).

⁴⁶For a more detailed discussion of Lie groups, their associated algebras, and their role in QFT, see [Peskin and Schroeder \(1995, Sec. 15.4\)](#). Cf. also [Nakahara \(2003, Sec. 5.6\)](#).

span the space of infinitesimal group transformations of a Lie group G with the Lie bracket $[\cdot, \cdot] : \mathfrak{g} \times \mathfrak{g} \rightarrow \mathfrak{g}$ as given by the commutation relations (4.29) constitute a *Lie algebra*, sometimes also denoted \mathfrak{g} . The commutation relations (4.29) completely determine the multiplication of group operations of the associated Lie group, but only sufficiently close to the identity. If one considers sufficiently large, but finite, transformations, additional global issues may arise: a well-trodden example concerns the two Lie groups $SU(2)$ and $O(3)$ which have identical Lie algebras $\mathfrak{su}(2) = \mathfrak{o}(3)$ but inequivalent global structure: the manifold of $O(3)$ is simply connected, whereas the one of $SU(2)$ is doubly connected.

Translated to the case at hand, Dirac's conjecture means that the constraints (4.21)-(4.23) are expected to generate the gauge transformations of the corresponding action. The Gauss constraints (4.21), which unproblematically generate the rotational gauge freedom of the tetrads, have already been eliminated by the symplectic reduction affirmed in Theorem 1. As can be seen from (4.28), the constraint algebra (4.24)-(4.28) is not simple as it has an invariant subgroup, constituted by the G_j 's. This is already an indication that the Gauss constraints can be divided out in a relatively straightforward manner, as it attested by Theorem 1. The remainder of this section is devoted to an analysis of the subalgebra (4.24)-(4.26) of the constraint algebra and of the transformations this subalgebra generates. According to Dirac's conjecture, which is not the source of difficulties here, it seems as if this subalgebra must generate the group of spacetime diffeomorphism $\text{Diff}(\mathcal{M})$. This, however, is not quite the case.

There is a clear sense in which the constraints V_a generate spatial diffeomorphisms. In order to see this, consider an arbitrary tensor field t_{ab} built from q_{ab}, π^{ab} defined on Σ . Such a tensor field then has the following Poisson bracket relations with the constraint functions:

$$\{V(\vec{N}), t_{ab}\} = \kappa \mathcal{L}_{\vec{N}} t_{ab} \quad (4.30)$$

$$\{C(N), t_{ab}\} = \kappa \mathcal{L}_N t_{ab} \quad (4.31)$$

where $\kappa = 1/16\pi G$ and $\mathcal{L}_{\vec{N}}$ and \mathcal{L}_N are the Lie derivatives in tangential and normal direction with respect to the hypersurface Σ , respectively. Thus, it can be seen from equation (4.30) that V can be interpreted as the generator of *spatial* diffeomorphisms, i.e. it generates the Lie algebra $\mathfrak{diff}(\Sigma)$ of the spatial diffeomorphism group $\text{Diff}(\Sigma)$, up to a caveat. The reservation to be raised at this point is that as mentioned above, a Lie algebra does not uniquely single out an associated Lie group in that its commutation relations only determine the multiplication of group operations of the associated Lie group in the neighbourhood of the identity operation of the group. This caveat becomes pertinent here for manifolds Σ whose

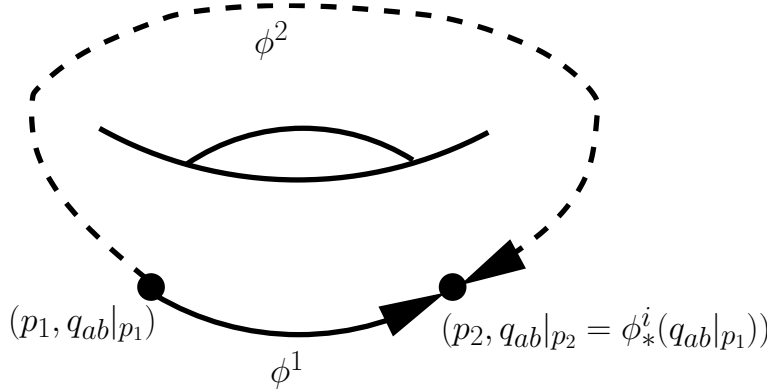


Figure 4: A manifold of genus one.

topology is of non-zero genus.⁴⁷ In these cases, the group $\text{Diff}(\Sigma)$ consists of more than one connected component, meaning that diffeomorphisms from different components are not homotopic⁴⁸ to one another. Figure 4 illustrates this with a manifold of genus one, where the two diffeomorphisms ϕ^1 and $\phi^2 \in \text{Diff}(\Sigma)$ cannot be smoothly deformed into one another and are therefore not homotopic. The group $\text{Diff}(\Sigma)$ where Σ is topologically of genus one, as in Figure 4, and the group $\text{Diff}(\Sigma)$ of a simply connected manifold Σ have the same associated Lie algebra $\mathfrak{diff}(\Sigma)$. Thus, the Lie algebra $\mathfrak{diff}(\Sigma)$ does not uniquely choose among inequivalent groups of diffeomorphisms acting on manifolds of different genera and does therefore not encode the entire information contained in the structure of the group. This is why it is sometimes said that Postulate 4 should be modified to the effect that only the connected part of $\text{Diff}(\mathcal{M})$ containing the identity should be considered as the gauge symmetry of GTR, denoted by $\text{Diff}_0(\mathcal{M})$.

Analogous to the case of V_a , C can be considered as the generator of time reparametrizations, or “temporal diffeomorphisms” $\text{Diff}(\mathbb{R})$. These transformations are often also called *deformations* of the hypersurface Σ *normal* to how it is embedded in \mathcal{M} .

Importantly, equations (4.30) and (4.31) only hold *on shell*, i.e. for generic functions in Γ_{ADM} when the vacuum Einstein equations hold. Off shell, the constraints generate different

⁴⁷The *genus* is a property of connected surfaces which can be used to characterize its topology. It is defined as the topologically invariant number $g \in \mathbb{N}_0$ of non-intersecting simple closed curves that can be drawn on the surface without separating the surface.

⁴⁸Two continuous maps $f, g : X \rightarrow Y$ are said to be *homotopic*, denoted by $f \sim g$, just in case there exists a continuous map $F : X \times I \rightarrow Y$ such that $F(x, 0) = f(x)$ and $F(x, 1) = g(x)$, where I represents a real interval. In this case, the map F is called a *homotopy* between f and g .

motions, implying that the set of gauge symmetries is *not* $\text{Diff}(\mathcal{M})$ everywhere in phase space. But this is not particularly worrisome in itself.⁴⁹

The really disturbing worry is different. The real problem is that the gauge group $\text{Diff}(\mathcal{M})$ disappears in the canonical formalism. Apparently, in the heyday of geometrodynamics, many people have been worried about this issue, as is testified by [Isham and Kuchař \(1985a,b\)](#). As a result of this pioneering work though, physicists in canonical GTR have ceased to be worried about the disappearance of $\text{Diff}(\mathcal{M})$ and either gloss over the issue entirely, or state that the constraints encode the full spacetime diffeomorphism invariance non-trivially, but *unproblematically*. It appears at least initially wrong to just gloss over the issue as the threatening loss of substantive general covariance clearly amounts to a serious challenge for the defender of the canonical approach. Deflating the worry by indicating that the full spacetime diffeomorphism invariance is still at work in the canonical formulation is of course acceptable, as long as one can offer valid reasons for doing so. These reasons, alas, are rarely offered in the literature. It almost seems as if [Isham and Kuchař \(1985a,b\)](#) have placated the community to the extent that its being sedated borders on local anesthesia. Let me attempt to initiate an awakening, or at least a sensitization.

A sufficient criterion for clearly seeing the full spacetime diffeomorphism invariance at work in the canonical approach would be if the Lie algebra $\mathfrak{diff}(\mathcal{M})$ of $\text{Diff}(\mathcal{M})$ were isomorphic to the Dirac algebra of constraints (4.16). If this condition were satisfied, this would offer convincing evidence that the symmetries of the theories were in both cases the same and that changing the action from the traditional (4.1) to (4.12) was a harmless transformation and not a change of theory. Changing the symmetries is typically taken to mean that one changes the theory, and not just its formulation. However, in the case of *gauge* symmetries, the issue is more subtle: what is relevant in this case, it seems, is whether the reduced phase space has been modified or not, modulo the well-understood extension of the ADM phase space in the Ashtekar-Barbero formulation which introduced the internal $SU(2)$ degrees of freedom. In order to find this out, one would have to solve the constraints, which is impossible in geometrodynamics because the Hamiltonian constraint has a highly non-trivial, because non-polynomial, form. Let me thus return to comparing the two algebras mentioned above: what is the relation between $\mathfrak{diff}(\mathcal{M})$ and the Dirac algebra (4.16)? Should it turn out that these algebras are not isomorphic, the canonical gravitist has the onus of producing a proof that the constraint algebra correctly codifies the gauge symmetry of GTR.

Unfortunately, however, not only do the commutation relations (4.16) not constitute a Lie algebra isomorphic to $\mathfrak{diff}(\mathcal{M})$, but it is not a representation of a Lie algebra at

⁴⁹If this issue is related to whether the constraint algebra closes only on or also off shell, then the worries of [Nicolai, Peeters, and Zamaklar \(2005\)](#), to be discussed in Section 7.2, apply.

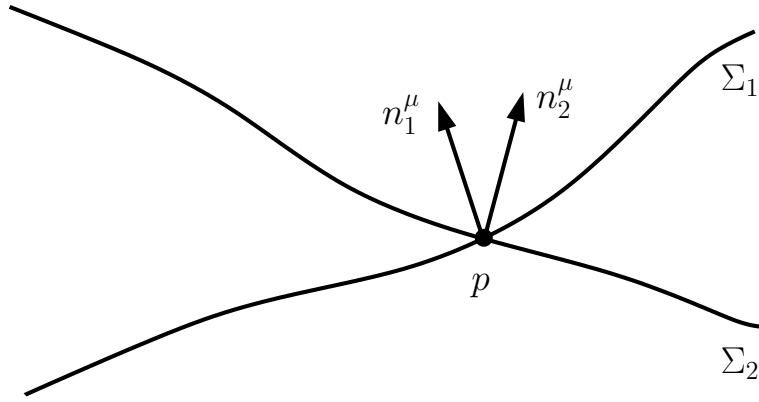


Figure 5: Dependence of pure deformations on the embedding.

all!⁵⁰ As can be gleaned from the last commutation relation in (4.16), the Poisson bracket of C with itself does not only explicitly depend on the pair of lapse functions, but also on the canonical variable q_{ab} . This means that the factor of the right-hand side of the relation is not just a structure constant, as it should be for a Lie algebra as explicated above, but involves an explicit dependence on the phase space. Essentially, as [Isham and Kuchař \(1985a, pp. 297f\)](#) explain, the decomposition of the generators of $\text{Diff}(\mathcal{M})$ into normal and tangential components introduces a reference to the metric $g_{\mu\nu}$ of \mathcal{M} with respect to which the embedding of Σ into \mathcal{M} is spacelike. Isham and Kuchař find it hardly surprising, then, that the induced three-metric q_{ab} appears in the last commutation relation in (4.16). The decomposition of diffeomorphisms into pure stretching in the direction of the shift vector N^a and pure deformations in the direction of the lapse function N thus does not constitute a Lie group because the pure deformations require for their definition that the equations of motion are solved beforehand. This implies that the reference to how the hypersurfaces on which the pure deformations act are embedded cannot be omitted. The dependence of the Lie derivative of an arbitrary tensor field t_{ab} living in Σ on the embedding of Σ is illustrated in Figure 5. On the other hand, the action of an element of $\text{Diff}(\mathcal{M})$ on $p \in \Sigma$ leads to a unique point in \mathcal{M} , unlike the generator of pure deformations which only delivers a unique action relative to a given embedding. In Figure 5, it is diagrammatically evident that the Lie derivatives of t_{ab} along two different normal vector fields arising from different embeddings

⁵⁰Cf. [Isham and Kuchař \(1985a, p. 297\)](#), [Isham \(1993, p. 191\)](#), [Thiemann \(2003, Sec. I.2.1\)](#). Thiemann states that the algebra defined by (4.16) is an “open” algebra in the BRST sense ([Henneaux and Teitelboim 1992, Sec. 3.2.5](#)) of whose representation only very little is known.

of Σ will, in general, differ: $\mathcal{L}_{n_1}t_{ab} \neq \mathcal{L}_{n_2}t_{ab}$.

Thus, the fact that $\mathfrak{diff}(\mathcal{M})$ and the constraint algebra defined by (4.16) are not isomorphic can be seen as arising from the canonical split of the four-dimensional spacetime structure into a three-dimensional “space” which is evolved in “time.” [Isham and Kuchař \(1985a, p. 297\)](#) warn, however, that this should not mislead one to argue superficially that the loss of full spacetime diffeomorphism invariance is inevitable in a canonical scheme. This argument, they explain, is flawed since there are counterexamples where a manifest four-dimensional invariance is broken by the imposition of a foliation of spacetime yet the full set of generators of the group encoding the full four-dimensional symmetry can be reproduced by dynamical variables on phase space. They cite Minkowski space field theory as a case in point, where the manifest Poincaré invariance is broken by the introduction of a specific foliation of Minkowski spacetime necessitated by the canonical formalism. The energy-momentum vector and the angular momentum tensor as dynamical variables on the phase space, however, represent the full set of generators of the Poincaré group $P(1, 3)$.

Not only is there the worry, now appeased, that forcing a foliation upon a four-dimensional spacetime irremediably breaks its four-dimensional symmetry group, but the fact that the Dirac algebra (4.16) is not Lie algebra also endangers the initial ambition of finding a quantization of GTR. According to [Isham and Kuchař \(1985a\)](#), no quantization techniques are known for constructing Hilbert space representations for non-Lie algebra commutation relations of the type (4.16). These difficulties can be overcome, apparently, in at least two different ways—or at least so it has been claimed. The older idea, original of [Bergmann and Komar \(1972\)](#), is to propose a much larger invariance group for geometrodynamics than $\text{Diff}(\mathcal{M})$: the group of mappings within the function space of the field variables $g_{\mu\nu}$, i.e. the group of functions from the set of all metrics $g_{\mu\nu}$ on \mathcal{M} into $\text{Diff}(\mathcal{M})$. This larger group involving a specific reference to the spacetime metric preserves the coordinate conditions which specify the imposed foliation. In this sense, it nicely captures the extension of the symmetry group from $\text{Diff}(\mathcal{M})$ to $\text{Diff}(\Sigma)$ plus pure deformations described above. Bergmann and Komar prove that $\text{Diff}(\mathcal{M})$ is a non-invariant subgroup of this newly proposed group.

The important result derived by [Bergmann and Komar \(1972\)](#) is that an invariant with respect to any of these three groups must be an invariant of all three. Therefore, they conclude, the invariants of GTR must be unaffected by a substitution of one of these groups by another one. This result bears consequences for the question of whether or not the set of Bergmann observables corresponds to the set of Dirac observables as introduced in the previous Section 4.3. Quantities which are invariant under transformations belonging to the third group constitute Dirac observables, while those invariant under transformations of the second group are Bergmann’s observables. Bergmann and Komar’s result, therefore, implies,

if true, if a quantity is a Dirac observable, it will also qualify under Bergmann’s criterion and vice versa. The set of observables, and therefore the physical content of the theory, remains the same, regardless of whether we consider the invariance group to be $\text{Diff}(\mathcal{M})$, the group associated with the Dirac algebra (4.16), or the extended group considered by Bergmann and Komar. The relevance of this result surely warrants more scrutiny and I hope to return to this issue on a future occasion. Without considering the details of their proposal, however, I am prepared to accept the conditional that if their claim is correct and the set of observables is indeed unaffected by the choice among the three groups, then the worry about the loss of full spacetime diffeomorphism invariance in canonical GTR is moot.

The more recent proposal, due to [Isham and Kuchař \(1985a,b\)](#), presents a scheme that can be considered “complementary” to the approach of Bergmann and Komar. Instead of enlarging the symmetry group, they seek to extend the phase space by adding embedding variables and by introducing corresponding constraints. This disallows the lapse and shift to be freely specifiable as they were in the approach described in Subsection 4.2.1. They are now definite functionals of the embedding variables, which permits Isham and Kuchař to claim that the Dirac algebra (4.16) now reproduces the Lie algebra $\mathfrak{diff}(\mathcal{M})$. This proposal, too, shall be the topic for another occasion.

One can seek to eliminate all the difficulties with representing the full four-dimensional diffeomorphism invariance that one encounters in the canonical formulation of GTR by choosing a radically different approach: avoiding a decomposition of spacetime into a “space” that evolves in “time” altogether. This can be achieved by renouncing the canonical phase space and by replacing it with a fully covariant one, i.e. one in which each point represents a spacetime solution of Einstein’s equation. The phase space, thus, consists of entire histories rather and does not involve references to particular moments in “time.” [Ashtekar, Bombelli, and Reula \(1991\)](#) have proposed such an approach which treats all spacetime diffeomorphisms on the same footing. Here, the four-dimensional symmetry remains manifest, unlike in the canonical approach. This covariant proposal, however, has not been worked out in detail. I suspect that the reason for this negligence must be found in the severe limitations of the covariant account proposed by [Ashtekar, Bombelli, and Reula \(1991\)](#): the dynamical behaviour of a quantum theory based on the covariant phase space of GTR would mimic the one of the classical theory. In particular, the account is expected to be incapable of avoiding the singularities of the classical evolution. Also, it appears as if such a quantum theory could not account for typical quantum effects such as tunneling. Furthermore, it does not admit strong gravitational fields and thus obliterates one of the primary motivations for seeking a QTG.⁵¹

⁵¹Cf. [Ashtekar et al. \(1991, Sec. 6\)](#).

5.0 A QUICK GUIDE TO LOOP QUANTUM GRAVITY

Importantly, LQG does not pretend to offer a unified field theory for all fundamental forces as ST does. Rather, it harbours the more modest goal of quantizing gravity while observing background independence. It should therefore be kept in mind that, unless a theory extends to include the other fundamental forces, it will not constitute a final, unified theory, even if it will correctly predict Planck-scale observations. What relation LQG might have to such a unified theory is unknown, although it has been suggested that it might be capable of offering a framework for such a theory.

The two major challenges for LQG remain to find a complete and consistent formulation of the *dynamics* of the theory and to derive its *(semi-)classical limit*. The former is attempted in both, the canonical (loop) formulation, as well as in its covariant (spinfoam) extension, to be addressed in Section 5.3. As I will argue, the rigorous establishment of the relation between the two formulations would greatly contribute to the understanding of the theory. I also will return to the problem of the semi-classical limit in chapter 9 when I discuss weave states and the emergence of spacetime.

This chapter gives a brief and non-technical account of the main ideas that define LQG. For a review of LQG, see [Rovelli and Gaul \(2000\)](#); [Rovelli \(1997, 1998a,b, 1999, 2004\)](#); [Rovelli and Upadhyaya \(1998\)](#); [Smolin \(1992, 2004b\)](#). For more technical reviews of LQG, see [Ashtekar and Lewandowski \(2004\)](#); [Gambini and Pullin \(1996\)](#); [Thiemann \(2001b, 2003\)](#). [Rovelli \(2003b\)](#) and [Smolin \(2001b, 2004a\)](#) give popular accounts.¹ The fact that leading proponents of LQG have recently been invited to describe LQG for the much wider audience of popular science journals may testify that the theory has reached a certain threshold of maturity and relevance also from the outsider's perspective. Exponents of LQG such as Markopoulou have almost gained celebrity status by having been portrayed by popular science journals ([Geftter 2002](#)), as well as in the general media ([Kindhauser 2003](#)). This makes it all the more urgent that philosophers of physics start to study LQG's conceptual

¹The bibliographical references in this chapter are far from complete, as I focused on outlining the theory rather than on attributing all the individual contributions. My exposition borrows from [Rovelli \(2004\)](#), which is also an excellent source for further references.

foundations, commitments, and implications.

The present chapter is organized as follows. Section 5.1 will outline the canonical quantization programme in general terms as well as applied to Hamiltonian GTR according to LQG. Section 5.2 sketches the kinematical level, i.e. the theory of “quantum space,” the so-called kinematical Hilbert space which results from solving the Gauss and spatial diffeomorphism constraints. Section 5.3, finally, briefly presents both the standard canonical dynamics of LQG, as well as the covariant extension of the theories, which has been proposed in order to resolve the problem of dynamics in the canonical approach. I close with a few remarks about the relation between the two. The reader should be warned that what follows will only serve badly as an independent introduction to LQG. It merely introduces the elements relevant for the further development of the argument.

On numerous occasions, LQG has been hailed as the full realization of relationism or of (physical) structuralism. It thus seems appropriate to remind enthusiastic structuralists of an argument, which has been developed in philosophy of mathematics and can be seen as a *reductio* of at least one common form of structuralism. Appendix B will offer this challenge to the spacetime structuralist. As it is somewhat peripheral to present concerns, it has been demoted to the appendix in order not to interrupt the continuity of the argument.

5.1 THE CANONICAL QUANTIZATION OF HAMILTONIAN GTR

Let me first address the question of how general covariance, and its codification in Hamiltonian GTR, fare through the quantization process. This is a crucial point: if LQG wants to claim that it is a quantization of GTR, then it better correctly quantizes the classical structure. The present section briefly reviews the main steps in the canonical quantization of Hamiltonian GTR according to LQG.

The so-called canonical quantization procedure for Hamiltonian systems with constraints was originally introduced by Dirac (1964), and subsequently explicated in virtually every review article on canonical QG.² The implementation of the procedure is highly non-trivial and has never, so far, been fully executed for a Hamiltonian formulation of GTR. Therefore, it offers a programmatic scheme rather than a ready-made recipe for solving the problem of quantizing GTR and it still awaits its fulfillment.

The first step was essentially completed in the last section, viz. the casting of the theory in Hamiltonian form and the identification of a pair of canonical conjugate variables in the

²Some recent examples include Gambini and Pullin (1996, Sec. 3.2.2), Henneaux and Teitelboim (1992, Sec. 13.3), Kuchař (1981, 1993), and references therein. My exposition mainly follows the first reference.

phase space and their Poisson bracket relation. The identification of the canonical variables and their Poisson relation amounts to the selection of an algebra of classical quantities. This is a rather technical choice which has a huge impact on the tractability of the mathematical challenges that arise in the course of steering through the procedure. This choice should in principle not incite philosophical worries, since it is merely a choice of a coordinate system in the phase space. The phase space, and therefore the theory, is the same in both cases. It must be noted, however, that a non-standard choice of classical observables will lead to non-standard quantization, and thus to inequivalent quantum theories.

Second, the algebra chosen at stage one is represented as a set of operators with the Poisson relations among the classical quantities promoted to commutation relations among these operators. The operators act on a functional space \mathfrak{H} of quantum states. Once one has chosen canonical variables q^n, p_n at the first stage, one can seek for a configuration representation of the functionals $\Psi[q]$ living in \mathfrak{H} with the fundamental operators defined by $\hat{q}\Psi[q] = q\Psi[q]$ and $\hat{p}\Psi[q] = -i(\delta\Psi[q]/\delta q)$ with the canonical commutation relation $[\hat{q}, \hat{p}] = i$.

Next, the constraint equations must be quantized to wave equations with the constraint operators acting on the space \mathfrak{H} . This must be performed such that the Poisson relations of the constraints are consistent with the commutation relations at the quantum level. This process depends on regularizations and factor orderings and does thus not yield a unique result, but gives rise to so-called quantization ambiguities.³ The space of solutions of the constraint equations is a subspace of \mathfrak{H} , let's call it $\tilde{\mathfrak{H}}$. The next step is to implement the evolution of the states via the Schrödinger equation or of the observables via the Heisenberg equation. For the case at hand, the Hamiltonian constraint equation will play the part of the Schrödinger equation. Finally, define an inner product on $\tilde{\mathfrak{H}}$. This step turns $\tilde{\mathfrak{H}}$ into a Hilbert space \mathcal{H} with normalized state vectors. With all this quantum structure in place, one can compute expectation values and make physical predictions. The choice of an inner product is not strictly speaking specified by the Dirac quantization scheme. Additional global symmetries such as Poincaré symmetry in QFTs on flat background spacetime uniquely determine the inner product. But in field theories of gravity, no such help is available. To find an inner product and thus a physical Hilbert space, as we will see, is perhaps the largest unresolved challenge for LQG.

As an example, let me briefly revisit the case of geometrodynamics. As a canonical algebra, one picks the 3-metric q_{ab} and its conjugate momentum π^{ab} and represents the 3-metric

³These ambiguities may incite philosophical worries as they imply the existence of inequivalent quantum theories with the same classical limit. One would thus hope for a physically well-motivated regularization and ordering procedure. According to Carlo Rovelli (personal communication, 30 June 2004), however, physicists should only start to worry about this once they have produced *one* consistent and complete QTG, which is, alas, not the case so far.

\widehat{q}_{ab} as a multiplicative operator and the momentum $\widehat{\pi}^{ab}$ as a derivative operator. The state vectors are required to be invariant under the symmetries of the theory, which are represented as constraints in the Hamiltonian approach. Hence, the state vectors must be annihilated by the constraint operators. The problem is to find the physical Hilbert space with an inner product. There are factor orderings such that one can solve the diffeomorphism constraint. The result will be that the state vectors must be functionals of the geometric properties of the 3-space which are invariant under spatial diffeomorphisms, rather than functionals just of q_{ab} . But the Hamiltonian constraint cannot be solved. This difficulty originates in the non-polynomial dependence of the constraint equations on the ADM variables. This means that the general procedure of promoting the constraint equation to a wave equation, apply some physically motivated factor ordering and regularization, and solve the resulting (Wheeler-DeWitt) equation cannot be accomplished and the program of quantum geometrodynamics grinds to a halt at this point. The problem of imposing a physically motivated inner product on the space of state vectors is compounded by the fact that no Dirac observable of the system is known. These obstacles lead to the search for new variables, which were indeed found by Sen and Ashtekar in the eighties. These new variables paved the way to LQG, where the situation is somewhat more hopeful.

The Ashtekar-Barbero variables, introduced in Section 4.2.2, are promoted to quantum operators acting on a functional space \mathfrak{H} of quantum states Ψ , abiding by the canonical commutation relation

$$[\widehat{A}_a^i(x), \widehat{E}_j^b(y)] = i\delta_a^b \delta_j^i \delta^3(x, y). \quad (5.1)$$

Choosing a configuration (i.e. connection) representation, the action of the operators can be represented by

$$\widehat{A}_a^i(x)\Psi[A] = A_a^i(x)\Psi[A] \quad (5.2)$$

and by

$$\widehat{E}_i^a(x)\Psi[A] = -i\frac{\partial}{\partial A_a^i(x)}\Psi[A], \quad (5.3)$$

where the $\Psi[A]$ are elements in \mathfrak{H} . The constraint operators necessary to turn the constraint equations into wave equations are based on these two operators \widehat{A} and \widehat{E} . It is in constructing the constraint operators that a choice of the orderings of basic operators creates quantization ambiguities. In actual practice, these ambiguities are resolved by making a “practical choice.”

Let us define the *physical Hilbert space* \mathcal{H} as the space of functions Ψ that solve all the constraints. Solving the Gauss law and the spatial diffeomorphism constraints is non-trivial, but can be done, and yields the *kinematical Hilbert space* \mathcal{K} of LQG, which corresponds to a quantization of three-space. The last step is to solve the Hamiltonian constraint, which is responsible for the evolution of the three-dimensional space and therefore for the dynamical

aspect of the system. Classically, the Hamiltonian constraint encodes the last step to space-time diffeomorphism invariance by reducing the phase space to the space of (equivalence classes of) solutions of the Einstein equations. In the quantum theory, the Hamiltonian constraint equation—the so-called *Wheeler-DeWitt equation*—projects the kinematical Hilbert space onto the physical one by excluding all quantum states which are not annihilated by the Hamiltonian constraint operator.

These are the rather abstract general steps toward a complete canonical QTG based on Ashtekar-Barbero variables. Now let’s look at some of the more concrete results at both levels, the kinematical as well as the dynamical one.

5.2 KINEMATICAL LOOP QUANTUM GRAVITY

Connections enable the parallel transport of objects such as tangent vectors along curves in the manifold. The result of parallel-transporting an object will generally depend on the path chosen, but preserves vector addition and scalar multiplication. Consider a smooth path γ from a point p to a point q in the manifold \mathcal{M} and a vector (or fibre) bundle structure E with a connection D defined on \mathcal{M} . For a given vector $u \in E_p$, one designates the result of parallel-transporting u from p to q along γ as $h[D, \gamma]u$. The linear map $h[D, \gamma] : E_p \rightarrow E_q$ is called a *holonomy* along the path γ . Only closed curves (i.e. loops) will be considered in what follows. In other words, we identify p and q .

At the quantum level, these holonomies, based on the spin connection, become the creation operator of the “loop states,” i.e. of states which represent an everywhere vanishing gravitational field except along the loop γ . That a loop representation, based on Ashtekar-Sen variables, can be used to find exact solutions of the quantum constraints was first discovered by [Jacobson and Smolin \(1988\)](#). This original approach, however, appeared to be incapable of solving all constraints simultaneously and was subsequently simplified by [Rovelli and Smolin \(1988, 1990\)](#), who laid the groundwork for the loop representation of quantum general relativity.

Many loops can be combined by knotting, linking, and kinking them into larger networks of loops. The study and classification of such knotted networks of loops is the subject of the branch of mathematics called *knot theory*.⁴ Linear combinations of these loop states span

⁴Apart from the early articles by Rovelli and Smolin ([Rovelli and Smolin 1988, 1990](#)), see also [Baez \(1996\)](#), who reaffirms with mathematical rigour how these knotted networks relate to knot theory. For a textbook on knot theory with a special emphasis on how it applies to physics, see [Kauffman \(2002\)](#).

a space. The so-called *spin network states* constitute an orthonormal basis of this space.⁵ The spin network states describe quantum space according to LQG, i.e. they result from a canonical quantization of the three-spaces Σ of the Ashtekar-Barbero version of GTR.

These spin networks are embedded in manifolds. They are represented by embedded graphs with nodes and edges linking the nodes. The spatial diffeomorphism invariance exacts the abstractness of the graphs. In other words, it does not matter how they are embedded into a background manifold, and deformations that do not change the number of nodes nor how they are linked together are considered physically meaningless. Thus, an abstract labelled graph represents an equivalence class of spin networks under the action of the spatial diffeomorphism group. This implies that the physical information contained in a spin network state does not depend on its embedding in a manifold.⁶

The (abstract) spin network states result after one has solved the Gauss and the spatial diffeomorphism constraints, but not the Hamiltonian constraint yet. These spin network states can be represented by abstract graphs with N nodes, where N is the number of “grains” of space to which they correspond. The links between the nodes represent a contiguity relation between the nodes so linked. Each link corresponds to a contribution towards the area of the surface that separates the adjacent quanta of space. The quantum state is characterized by both this graph as well as labels on the links and the nodes, where the label i_n on node n represent the quantum number of the volume and the label j_l on link l is the quantum number of the surface area between the two volumes represented by the nodes which the link connects. The resulting picture of space at the Planck level is thus a granular one, with indivisible Planck-scale chunks of space combining to form the three-dimensional macroscopic space. Spin networks do not represent quantum excitations *in* space. Rather, they constitute space and are quantum excitations *of* space. Figure 6 shows an example of a simple spin network. Figure 7 illustrates how spin network states are equivalence classes under spatial diffeomorphisms, i.e. they do not change under deformations of their embedded representations as shown in the figure.

The abstract spin network states form a complete orthonormal basis for the kinematical Hilbert space \mathcal{K} (Rovelli and Smolin 1995b). Rovelli (2004, Sec. 6.7), as many others in LQG, interprets the kinematical Hilbert space with its spin network basis as offering a picture of quantum space. According to this interpretation, the spatial geometry can be

⁵“Spin” because they carry spin $SU(2)$ representations on the nodes and links.

⁶And therefore not on how knots are embedded in manifolds either; this implies that whatever has physical significance must be invariant under what knot theorists call *Reidemeister moves*, which codify moves under which the interlacements are preserved. For an accessible approach to the theory of invariants in knot theory, see Kauffman (1988). Abstract spin network states are sometimes called “*s-knots*” in the literature. Whenever I use the term “spin network” or “spin network state,” I refer to the abstract ones, unless I explicitly state the contrary.

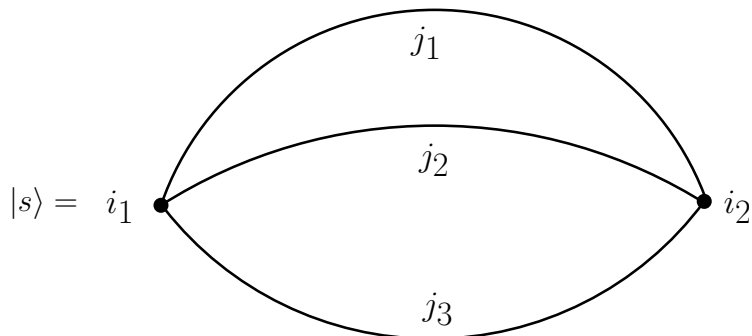


Figure 6: A simple spin network state $|s\rangle$ with two trivalent nodes.

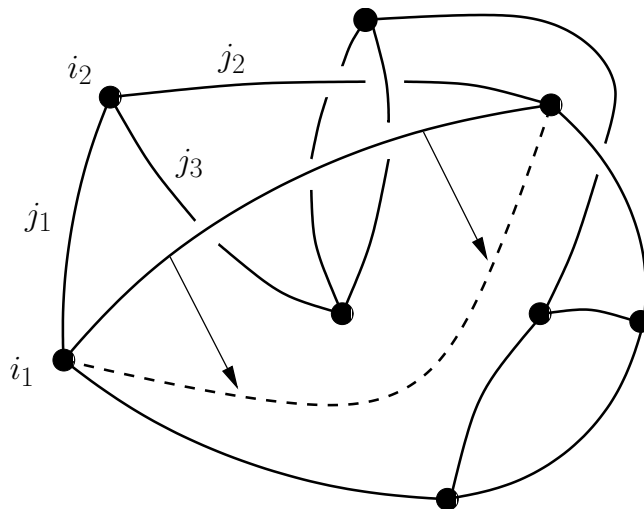


Figure 7: Deforming the (embedded) spin networks as shown does not change the state.

studied by analyzing the properties of the operators defined on \mathcal{K} corresponding to geometrical magnitudes, particularly the volume and area operators that can be constructed. These operators act on the spin network states. Their spectra yields important information concerning the geometrical interpretation of the spin network states. Since we study the properties of the gravitational field via the geometry of the physical space, the properties of (three-dimensional) gravitational fields are determined by the spectral properties of the volume and area operators. These operators, which will be discussed in more detail in Section 9.2.2, turn out to have discrete spectra (Ashtekar and Lewandowski 1997, 1998, 1999; Rovelli and Smolin 1995a,b). The granularity of the spatial geometry—the “polymer” geometry of space—follows from the discreteness of the spectra of the volume and the area operators. Essentially, each node (and only the nodes) in the network contributes a term to the sum of the volume of a region. On each node, there sits an “atom” of space with volume \mathbf{V}_n . These elementary grains of space are separated from each other by their surfaces of contiguity. Just as the volume operator receives contributions from the nodes of a region, the area operator acquires contributions from all the links that intersect the surface. For instance, the surface whose only intersecting link is a link with quantum number j_l has a surface area of $\mathbf{A}_l \propto \sqrt{j_l(j_l + 1)}$ (Rovelli 2004, Sec. 6.7). The problem, however, is that these operators do not correspond to Dirac observables and should therefore taken with a grain of salt. They are partial observables in the sense of Rovelli (2002d). I will return to this point in Sections 8.1, 8.2.4, and 9.2.2.

Physical three-space, in Rovelli’s interpretation, is a quantum superposition of spin network states, analogously to the physical electromagnetic field consisting of superposition of n -photon states. LQG *predicts* the existence of indivisible quanta of volume, area, and length, as well as their spectra (up to a constant). Importantly, this was a *result* of the loop quantization, rather than an *assumption*. According to LQG, measurements of the Planck geometry of space must therefore yield one of these values in the spectrum of the concerned operator.

5.3 DYNAMICAL LOOP QUANTUM GRAVITY

5.3.1 Canonical dynamics: Wheeler-DeWitt evolution

As was exhibited in chapter 4, the Hamiltonian of a generally covariant system vanishes weakly and thus generates a motion of pure gauge. The switch from the classical to the quantum theory involves the adaptation of the constraint equations as corresponding wave

equations with constraint operators acting on quantum states. The quantum Hamiltonian operator \widehat{H} is thus defined on the space of (abstract) spin network states \mathcal{K} because the other two families of constraints have already been solved at this stage. Replacing the Schrödinger equation as the quantum dynamical equation, the Hamiltonian constraint equation

$$\widehat{H}|s\rangle = 0, \tag{5.4}$$

the so-called *Wheeler-DeWitt equation*, captures the dynamics of canonical QG. The final goal of the procedure, establishing the physical Hilbert space \mathcal{H} is (at this stage) tantamount to solving this equation, as solving the Wheeler-DeWitt equation reduces \mathcal{K} to \mathcal{H} . Equivalently, one can try to find a projection operator $\widehat{P} : \mathcal{K} \rightarrow \mathcal{H}$ that projects spin network states onto the space of solutions of (5.4) (and the other constraint equations).

To construct a Hamiltonian \widehat{H} , or, equivalently, a projector \widehat{P} , poses a major challenge along the path to the complete formulation of LQG. There are currently several proposals on the market, but it is not yet clear which one of them, if any, captures the correct dynamical behaviour. [Thiemann \(1998a,b\)](#) has so far offered the most rigorous and complete construction of a Hamiltonian. As discussed in [Section 7.2](#), however, there are various difficulties with Thiemann’s construction. It is unclear, e.g., whether the resulting constraint algebra closes as it should ([Nicolai et al. 2005](#)). Quantization ambiguities similar to those encountered earlier seem to spoil the happy resolution. Generally, however, the Hamiltonian will only act on the nodes of the spin networks. For an outline of the concrete construction of the Hamiltonian operator, see [Rovelli \(2004, Sec. 7.1\)](#).

As in background-dependent QFT, the quantum dynamics of the theory is also fully coded by the transition amplitudes between spin network states. In fact, the matrix elements of the projector \widehat{P} are transition amplitudes between initial and final spin networks. Let us denote these transition amplitudes with $W(s_f, s_i)$. The matrix elements define an inner product of the physical Hilbert space:

$$W(s_f, s_i) \doteq \kappa \langle s_f | \widehat{P} | s_i \rangle_{\mathcal{K}} = \varkappa \langle s_f | s_i \rangle_{\mathcal{H}}. \tag{5.5}$$

The transition amplitudes between the spin network states is then just their physical inner product.

Since the gravitational field is not a denizen dwelling in a spacetime background, and in particular not evolving with respect to a (physical) fiducial time, unitary evolution is an ill-defined concept. Unitary evolution designates the conservation of a total probability amplitude *over time*. So if there is no time, unitary evolution becomes ill-defined. It is important to appreciate, however, that the transition amplitudes—sufficient to specify the dynamics—remain well-defined, despite the absence of time.

The construction of \widehat{H} or \widehat{P} completes the formal framework of the theory. The major difficulty is to actually calculate transition amplitudes (or, equivalently, construct a physical inner product). In order to circumvent these difficulties, physicists have started to search for alternative dynamical models and thus to study manifestly covariant Lagrangian models to obtain an understanding of the dynamics. Some of these models can be related to LQG and thus constitute an extension of the theory. Others arise from rather different contexts, but are still studied side-by-side with the LQG models due to their formal resemblance. Unfortunately, the relation between Hamiltonian LQG and these Lagrangian extensions is not yet well understood.

The codification of general covariance in LQG must be sought, as in classical Hamiltonian GTR, in the realization of the constraint algebra. LQG follows the substantive principle of general covariance as encoded in Postulate 4 in Chapter 3 insofar as it implements at the quantum level the classical constraint algebra (4.24)-(4.28). I have discussed the classical constraints and how they are supposed to capture general covariance in Section 4.4. I have briefly sketched the interpretation of *spatial* diffeomorphism invariance as a gauge symmetry in the quantum theory in Section 5.2, and in particular in Figure 7. How time reparametrization invariance will have to be understood in the quantum theory can only be adjudicated once the physical Hilbert space will be constructed.

5.3.2 Covariant dynamics: spinfoams

The Lagrangian spinfoam formalism, discussed in Rovelli (2004, Ch. 9), permits the explicit calculation of transition amplitudes $W(s_f, s_i)$ between initial spin networks $|s_i\rangle$ and final spin networks $|s_f\rangle$. The spinfoam idea follows Feynman in that it interprets the transition amplitudes as sums over paths or “world histories” of spin networks. The paths represent possible combinations of actions of \widehat{H} on the nodes of the spin network $|s_i\rangle$ such that it matches up with $|s_f\rangle$, and thus constitute possible world histories of spin networks. If the projection operator \widehat{P} is expanded perturbatively, the spinfoam approach suggests, in Feynman’s spirit, to interpret each term in the sum as a dynamically possible world history between two given spin networks.

The Hamiltonian can be concretely understood as acting on the nodes of a spin network state such that edges split. Its action is combinatorial in that it either multiplies an existing node into three or it collapses three nodes into one (see Figure 8; the numbers a, b depend on the labels of the spin network acted upon.). In this way, it affects the structure of the spin network and allows it to grow or to shrink.

A spinfoam σ , thus, can be understood as a labelled Feynman graph of spin networks

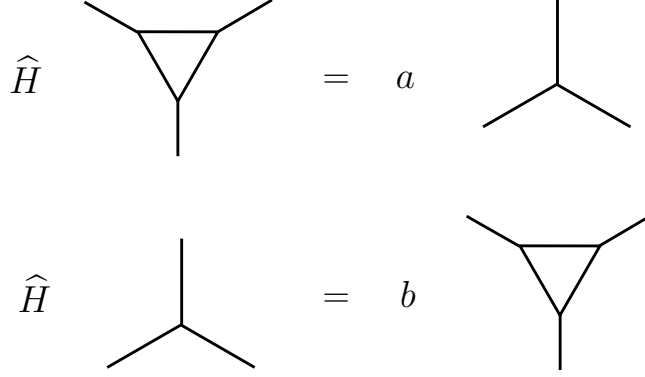


Figure 8: The basic action of the Hamiltonian on nodes of a spin network.

encoding the interactions occurring at the nodes, bounded by $|s_i\rangle$ and $|s_f\rangle$. In comparison with a Feynman graph which consists of vertices and of edges connecting the vertices, a spinfoam has an additional structure: it collects vertices, edges, and *faces*. Faces are the world histories of links in the spin networks, and they join at edges, the world histories of the nodes. Edges, in turn, meet at vertices, which represent the interactions among the nodes, i.e. the actions of the Hamiltonian as represented in Figure 8. The edges and the faces of a spinfoam are labelled with the corresponding quantum numbers for volumes and areas, respectively.

Expand the transition amplitude $W(s_f, s_i)$ perturbatively as a sum over all possible spinfoams bounded by the given spin networks $|s_i\rangle$ and $|s_f\rangle$. To be more precise, the expansion sums over the weighted amplitudes associated with each spinfoam. The idea is schematically illustrated in Figure 9. $\mathcal{A}(\sigma)$ represents the amplitude corresponding to a particular spinfoam σ . These spinfoams are endowed with quantum numbers on the vertices, edges, and faces. These numbers have been suppressed in the figure. Here, both $|s_i\rangle$ and $|s_f\rangle$ are the simple spin network of Figure 6. In order to calculate these spinfoam amplitudes $\mathcal{A}(\sigma)$, one decomposes the entire spinfoam into its vertices v . Each of these vertices owns a vertex amplitude $A_v(\sigma)$, which is determined by the matrix element of \hat{H} between the incoming and the outgoing spin networks. Similarly, one constructs the amplitudes of the faces $A_f(\sigma)$ and of the edges $A_e(\sigma)$. The amplitude $\mathcal{A}(\sigma)$ of a spinfoam σ is then obtained by the product of the amplitudes of all its individual vertices, edges, and faces:

$$\mathcal{A}(\sigma) = \mu(\sigma) \prod_f A_f(\sigma) \prod_e A_e(\sigma) \prod_v A_v(\sigma) \quad (5.6)$$

$$W(s_f, s_i) \sim \mathcal{A} \left(\text{Cylinder with 4 vertices and 4 edges} \right) + \mathcal{A} \left(\text{Cylinder with 6 vertices and 6 edges} \right) + \dots$$

Figure 9: Schematic example of how to calculate a transition amplitude $W(s_f, s_i)$.

where $\mu(\sigma)$ introduces the weight given to the amplitude of spinfoam σ . In fact, the amplitudes A_f, A_e, A_v do not depend on the entire spinfoam σ , but only on the quantum numbers of the adjacent faces and edges.⁷

The goal of a spinfoam model is to explicate equation (5.6) such that the transition amplitudes become calculable. Strictly speaking, some of these models do not systematically and did not historically arise from LQG, but from rather different contexts. However, due to their formal resemblance, they are studied side-by-side with models constructed in the context of Lagrangian LQG. The Ponzano-Regge model (Ponzano and Regge 1968), which dates back to well before the inception of LQG, relates to a discretization of three-dimensional GTR. These models significantly simplify the problem since GTR in three dimensions does not exhibit local degrees of freedom, i.e. the variables must satisfy a number of constraints which is equal to their local degrees of freedom. The so-called BF theory (Baez 1999) extends the formalism of the Ponzano-Regge models to four dimensions. The Barrett-Crane models (Barrett and Crane 1998; DePietri and Freidel 1999; Perez 2002) include local degrees of freedom, but do not exactly relate to GTR. There exist more models, but instead of babbling on, let me content myself with describing one in more detail, the Markopoulou-Smolin causal spinfoam model.

Penrose (1975) has argued, and has continued to argue ever since, that quantizing gravity bears the immense danger of quantum fluctuations of the spacetime imposing uncertainty on the causal structure of spacetime. This implied that there would be a non-vanishing

⁷For a more detailed discussion of spinfoam models and how they relate to LQG, see Rovelli (2004, Ch. 9). As far as I know, Reisenberger and Rovelli (1997) were the first ones to derive the formal framework that encapsulates the dynamics in terms of sums over histories from canonical LQG. They started out by expanding the exponential projection operator \hat{P} as a sum and gave each term of the sum a geometrical interpretation as a spinfoam. A rigorous introduction to spinfoams gives Baez (1998).

probability of causal interaction between originally spacelike-separated points. This in turn would threaten the foundation of QFT, since its commutation relations require an a priori causal structure such that spacelike-separated operators always commute. Penrose proposed to resolve the tension by assuming the causal structure to be fixed a priori and thus to remain “sharp.” This proposal is strangely reminiscent of past attempts in philosophy of science to formulate a causal theory of time in that it equally assumes causal relations to logically or metaphysically precede (spatio-)temporal relations.

Markopoulou and Smolin (1997, 1998) propounded a spinfoam model that realizes Penrose’s proposal. Their causal spinfoam model combines the kinematical states of LQG, the spin network states, with a discrete causal structure that captures the evolution of the kinematical states.⁸ This causal model is different from the original spinfoam proposals in that it does not try to find the explicit actions the Hamiltonian can perform on the nodes of the spin networks via a quantization procedure. Rather, it starts out from the kinematical states of LQG and combines these discrete “spatial slices” by “null” edges. These null edges offer a Planck-scale replica of the null geodesics we find in the continuous spacetimes of classical GTR. The dynamical rules of combining the spatial slices into a “spacetime network” are now such that information can only propagate in accordance with the discrete causal structure as encoded in the null edges. The resulting spacetime networks (or causal spinfoams) exhibit the structure of a causal set. *Causal sets* \mathcal{C} are endowed with a binary relation \prec such that for all $a, b, c \in \mathcal{C}$ (i) $a \prec b$ and $b \prec c$ imply $a \prec c$ (transitivity), (ii) $a \not\prec a$ (acyclicity),⁹ and (iii) all *past sets* $\mathcal{P}(a) \doteq \{b : b \preceq a\}$ are finite. Markopoulou and Smolin thus combine what are apparently the advantages of spinfoam models with the approach to QG based on causal sets (Bombelli et al. 1987, 1988; Brightwell et al. 2003; Moore 1988; Rideout and Sorkin 2000). Markopoulou (2000a,b,c) has generalized the approach to what she dubs *quantum causal histories*.¹⁰ Motivated by cosmology, these models are built to account for observers internal to the universe which they observe. It might be fruitful to exploit these models to formulate a theory of physical causation.

⁸Discrete models based on causal structures have also been proposed in the context of non-perturbative Lorentzian quantum gravity from path integral methods which are *not* related to LQG; cf. e.g. Ambjørn and Loll (1998). Livine and Oriti (2003, 2004) have developed a method for implementing causality in the framework of the Barrett-Crane model.

⁹This amounts to ruling out closed causal curves *a priori*. A theorem due to Malament (1977) establishes that if $\langle \mathcal{M}, g_{\mu\nu} \rangle$ and $\langle \mathcal{M}', g'_{\mu\nu} \rangle$ are both past and future distinguishing spacetimes, and if there exists a bijection f between \mathcal{M} and \mathcal{M}' such that both f and f^{-1} preserve the causal precedence relations, then f must be homeomorphism, i.e. a topological isomorphism between the manifolds. This implies that an approach encoding only the causal structure cannot allow closed causal curves. This means that such an approach does not command the resources to recover the metric structure of classical GTR in its full generality. The spacetimes that can be captured in the continuum limit by a causal-set approach thus represent a proper subset of those admitted by the Einstein equations.

¹⁰See also Hawkins et al. (2003).

5.3.3 Canonical vs. covariant formulations

How are we to interpret the spinfoams physically? The traditional interpretation of Feynman graphs conceives of the graphs as representing the dynamical possibilities of how a prepared initial state may interact such as to evolve into the subsequently measured final state. If we add up the weighted contributions of all graphs to the transition amplitude between the prepared and the measured state, we can predict the probability with which the final state will occur *given the prepared initial state*.

At first glance, imposing an analogous interpretation on spinfoams seems outright crazy because, after all, the spin network states that mark off the spinfoam are supposed to represent instantaneous three-spaces—and all of space. Preparing or measuring the entire quantum state of the universe is not only fiscally irresponsible, but impossible in principle. Disregarding the problem of coupling matter to spin networks, the resolution of which would certainly be necessary to understand the interaction between the material observer and the observed spatial geometry, it seems impossible to separate the system to be prepared and then measured from the observer, as the observer cannot be outside the spatial slice under scrutiny. Let us hence delimit the scope of the experiment to a “medium-sized” region, perhaps inside a detector or a similar measuring device, assuming the observer to suffer from no interactions with the region under the microscope. Of course, this assumption idealizes the true situation, where a chunk of “space” can hardly be prepared and measured and still remain completely isolated from the rest of the universe. Since spin networks are eigenstates of the area and volume operators, these delimiting states $|s_i\rangle$ and $|s_f\rangle$ can be interpreted as describing the results of geometrical measurements of these states in medium-sized regions. The area and volume operators do not commute with the Hamiltonian and are therefore not Dirac observables. They constitute, however, partial observables in Rovelli’s sense (Rovelli 2002d). The orthodox physical interpretation of spinfoams therefore maintains that the initial and final spin network states represent spatial regions and that the transition amplitudes predict the probability with which a prepared initial state evolves to the measured final one.

The Hamiltonian theory as exposed in Section 5.2 provides an interpretation of the initial and final states, i.e. of the spin networks. In order to obtain a complete physical interpretation, one would like to have a more solid understanding of the relationship between the Hamiltonian formulation of LQG and the covariant spinfoam models. Reisenberger and Rovelli (1997) have supplied the sketch of a formal derivation of a spinfoam model from Hamiltonian LQG. Filling in the details of such a derivation would amount to finding the equivalent of the Feynman-Kac formula in the context of a QFT with a spatio-temporal diffeomorphism invariance as a symmetry of the theory. The Feynman-Kac formula offers a

method of solving the wave equations of the canonical formalism by relating them to covariant path integrals. Establishing the link between LQG and spinfoams in a manner which is simultaneously explicit and rigorous, and which captures full-fledged four-dimensional gravity has as yet defied resolution. The most recent in a short series of partial successes has been achieved by [Noui and Perez \(2005\)](#). They explicitly construct the projector \widehat{P} from the kinematical Hilbert space \mathcal{K} of the canonical theory onto the physical Hilbert space \mathcal{H} , leading to a spinfoam model. Their construction of the physical inner product, however, only applies to the three-dimensional, Euclidean case. The extension of their approach to the Lorentzian case might be manageable.¹¹ However, different techniques would be needed in four dimensions, since the Hamiltonian and the spatial diffeomorphism constraints can no longer be subsumed as one single constraint, as they can in three dimensions. The general derivation of Lagrangian spinfoams from Hamiltonian LQG has so far eluded mathematically rigorous formulation.

In order to mathematically understand the relation between covariant and canonical formulations of a theory, it can be helpful to study the issue in the context of so-called *Gelfand-Naimark-Segal constructions*. A GNS construction establishes that any positive linear functional on a C^* -algebra \mathfrak{A} , called a *state*, determines a unique cyclic $*$ -representation of \mathfrak{A} , up to unitary equivalence. A C^* -algebra can be concretely enacted as a complex algebra \mathfrak{A} of continuous linear operators over a complex Hilbert space \mathfrak{H} amended by two additional properties: (i) \mathfrak{A} is topologically closed under the norm topology of operators, and (ii) \mathfrak{A} is closed under the $*$ -operation, which in this case is taking the adjoint of an operator. More generally, $*$ is a map from \mathfrak{A} to itself called involution. A *$*$ -representation* of a C^* -algebra \mathfrak{A} on a Hilbert space \mathcal{H} is a map $\pi : \mathfrak{A} \rightarrow \mathfrak{B}_{esa}(\mathcal{H})$, where $\mathfrak{B}_{esa}(\mathcal{H})$ is the algebra of bounded, essentially self-adjoint operators acting on \mathcal{H} , such that (i) π is a ring homomorphism which carries the involution on \mathfrak{A} into an involution on operators, and (ii) π is non-degenerate, i.e. if \mathfrak{A} has an identity, then π is unit-preserving.

The so-called (Streater-)Wightman ([Streater and Wightman 1964](#)), which is an application of this GNS construction, and Osterwalder-Schrader ([Osterwalder and Schrader 1973, 1975](#)) reconstruction theorems in background-dependent QFT address the challenge of recovering the canonical QFT from a covariant path integral formulation. [Streater and Wightman \(1964\)](#) and [Wightman \(1956\)](#) showed that the vacuum expectation value distribution together with the so-called *Wightman axioms* are sufficient to reclaim canonical QFT. These axioms, however, have only been shown to apply under simplifying assumptions such as lower dimensionality and no interaction. It remains open whether they can be satisfied in more generic cases. Wightman's reconstruction theorem applies to the Lorentzian case. Under

¹¹Alejandro Perez, personal communication, 7 May 2004.

some technical assumptions, the Osterwalder-Schrader reconstruction theorem asserts that a Euclidean QFT can be Wick-rotated into a Lorentzian QFT satisfying Wightman's axioms.

Since the inception of spinfoam models in LQG, results translating the reconstruction theorems into the context of diffeomorphism-invariant theories have been few and far between. [Perez and Rovelli \(2001\)](#) have constructed functions representing transition amplitudes between spin network states with a fixed number of “quanta of space” which can be calculated perturbatively, using the sums-over-four-geometries approach sketched above. The physical Hilbert space of the theory, they claim, can then be reconstructed from these functions, provided that some conditions parallel to Wightman's axioms obtain. In this sense the Perez-Rovelli construction is also an application of the GNS reconstruction theorem, one that generalizes Wightman's construction to background-independent QFTs. [Ashtekar, Marolf, Mourão, and Thiemann \(2000\)](#) have similarly extended Osterwalder and Schrader's general reconstruction scheme to include background-independent theories.

6.0 CLASSICAL SINGULARITIES AND THEIR FATE IN QUANTUM THEORY

6.1 SINGULARITIES IN THE CLASSICAL THEORY

The Hawking-Penrose singularity theorems in classical GTR have convinced many physicists that spacetime singularities, understood as geodesic incompleteness in the context of these theorems, generically occur for both gravitational collapse models as well as for a large class of cosmological models.¹ By *generically*, I mean that the singularities do not result from idealizations, including symmetry idealizations such as homogeneity and isotropy. This excludes the escape championed by Einstein who believed that the initial singularity in the standard cosmological models of GTR, the so-called Friedmann-Lemaître-Robertson-Walker (FLRW) spacetimes,² vanished in more realistic and therefore somewhat asymmetric spacetimes:

[...] one can seek to escape this difficulty [i.e. the occurrence of singularities] by pointing out that the inhomogeneity of the distribution of stellar material makes our approximate treatment illusory. (Einstein 1931, p. 237)³

A typical singularity theorem assumes that the spacetime satisfies everywhere an energy condition, a causality condition, and a boundary or initial condition and then proves that the spacetime is geodesically incomplete.⁴ The genericity of the geodesic incompleteness is

¹On singularities in GTR, see [Hawking and Ellis \(1973, Chs. 8-10\)](#) and [Wald \(1984, Ch. 9\)](#). For a philosophical discussion of singularities in GTR, see [Earman \(1995, Ch. 2\)](#).

²Appendix C offers a summary on the FLRW cosmological models of GTR.

³My translation. Original: “[Man kann] der Schwierigkeit durch den Hinweis darauf zu entgehen suchen, dass die Inhomogenität der Verteilung der Sternmaterie unsere approximative Behandlung illusorisch macht.”

⁴A curve $\gamma(t) : I \rightarrow \mathcal{M}$, where $t \in I \subseteq \mathbb{R}$, is called a *geodesic* with respect to a connection ∇ on \mathcal{M} just in case, for every point $p \in \gamma(I)$ in the image of I under γ , $\xi^b \nabla_b \xi^a = 0$ where ξ^a is the tangent field to $\gamma(t)$. Thus, a geodesic is a curve whose tangent vector is propagated along the curve itself. A parametrization of a curve $\gamma(t)$ yielding $\xi^b \nabla_b \xi^a = 0$ is termed an *affine parametrization*. The affine parameter of a geodesic curve is determined only up to a multiplicative and an additive constant expressing the freedom to renormalize the tangent vectors by a constant scale factor and to choose an initial point, respectively. An incomplete geodesic is then a geodesic which is inextendible in at least one direction despite its only having covered a finite range of the affine parameter. More formally, a curve $\gamma(t) : I \rightarrow \mathcal{M}$ is an *incomplete geodesic* iff $\gamma(t)$ is

inferred from the belief that the premises of the theorems must be satisfied in a wide class of physically realistic models of GTR.⁵ So GTR is plagued by the occurrence of singularities.

Einstein himself was deeply worried about the occurrence of singularities in solutions to his field equations. He insisted that the final unified field theory be free of singularities. His aversion to spacetime singularities was clearly expressed, albeit in a slightly different context, in a joint paper with Nathan Rosen:

A singularity brings so much arbitrariness into the theory [...] that it actually nullifies its laws. [...] Every field theory, in our opinion, must therefore adhere to the fundamental principle that singularities of the field are to be excluded. (Einstein and Rosen 1935, p. 73)

Similar reactions to singularities are very common, particularly among the older generation of relativists. The younger members of the community, to a degree at least, have grown up with singularities firmly in place and seem much less disturbed by their prevalence. Forty years after its inception, any scandal loses its shock value as people grow accustomed to its perpetual offense. However, this does not in itself make the infamy more acceptable. The infamy of singularities, Einstein and Rosen tell us, consists of the fact that they bring “arbitrariness into the theory.” What does this mean? At the 1979 centennial symposium held at Princeton in honour of Einstein, Peter Bergmann hinted at the answer when he aired what I take to be the community’s canonical reaction to the singularity theorems:

[Singularities] are intolerable from the point of view of classical field theory because a singular region represents a breakdown of the postulated laws of nature. I think one can turn this argument around and say that a theory that involves singularities and involves them unavoidably, moreover, carries within itself the seeds of its own destruction [...] (Bergmann 1980, p. 156)

Bergmann’s statement, however, equivocates on two common, but different, meanings of the term “singularity.” The first meaning is that of geodesic incompleteness. This is the sense in which the theory “involves [singularities] unavoidably,” since it is geodesic incompleteness which is established by the singularity theorems. But when Bergmann speaks of a singular region representing a “breakdown of the postulated laws of nature,” he must have in mind a different, albeit related, meaning. The laws of nature mentioned in the quotation, I take it, are the equations of motion which govern the physical system at stake. In this second sense

an inextendible geodesic and $I \neq \mathbb{R}$. Intuitively, in the case of a timelike geodesic, this means that a freely moving observer might ride on a world line which is exhausted after only a finite proper time. A spacetime $(\mathcal{M}, g_{\mu\nu})$ is *geodesically incomplete* iff it contains an incomplete geodesic. Finally, a spacetime is *geodesically complete* if it is not geodesically incomplete. At least timelike and null geodesic completeness are widely regarded as minimal conditions for a spacetime to be regular.

⁵However, the singularity theorems do not quite warrant such a strong conclusion as they suffer from a number of loopholes. For the identification and discussion of these loopholes, consult Senovilla (1997, particularly Sec. 6), which offers also the best account of the singularity theorems that I know.

then, a region of spacetime is termed *singular* in case the equations of motions do not permit to determine the dynamical evolution of the system in this region and thus fail to operate as valid dynamical laws. In relativistic cosmology, of course, the pertinent equations of motion are the Friedmann equations (C.5) and (C.6). These equations break down in singular regions because if components of the metric, the curvature, or the energy-mass density diverge, the equations become meaningless. Thus, singularities pose a threat to Laplacean determinism in that the evolution of the physical system, which is governed by the field equations, is no longer well-defined. It is not a priori clear that these two meanings that can be assigned to the term “singular” must coincide in all instances.

To what extent do singularities compromise determinism? Let us return to the terminology introduced in Chapter 4 for the purpose of answering this question. If the gravitational field is given on a spacelike hypersurface $\Sigma \subset \mathcal{M}$, then its value is determined (up to diffeomorphisms) within the whole domain of dependence of Σ . If there exists a spacelike hypersurface Σ such that the domain of dependence of Σ equals the entire spacetime, then data on Σ actually determines the physical state of the gravitational field through spacetime and the spacetime is globally hyperbolic. The dynamical evolution of FLRW models occurs regularly, i.e. for all specifications of k , Λ , and an “initial” value $a(t_0) = a_0$, there exists a unique solution of the Friedmann equation (C.5), at least for all $t > 0$. Thus, all FLRW models as they are defined by (C.1) in Appendix C, viz. for a manifold of topology 3+1 with time coordinate $t \in \mathbb{R}^+$, admit Cauchy surfaces. Therefore, the FLRW models are globally hyperbolic, with their surfaces of transitivity⁶ as spacelike hypersurfaces with the entire spacetime as their domain of dependence.

Thus, FLRW models guarantee Laplacean determinism, at least back to the big bang, where determinism fails. Equally, several physical magnitudes diverge as the big bang is approached going backward in time.⁷ What exactly happens at the “big bang”? For the FLRW models, the Einstein field equations imply an initial singularity, a point where the matter-energy density ρ blows up as the scale factor a approaches zero. But under reasonable assumptions,⁸ general relativity astonishingly predicts that for FLRW models, there was a time $t = 0$ in the finite past when $a(0) = 0$. Conservation of energy and momentum implicates via (C.4) that ρ grows as $a(t)$ decreases, which is what happens as we go back in time. Equation (C.2)

$${}^3R = \frac{k}{a_0^2 a^2(t)}$$

⁶See Appendix C.

⁷Of course, this “approaching” occurs in thought; i.e. it is not the case that physicists can make the time turn backwards.

⁸For details, see Appendix C.

shows that the same holds for the curvature scalar 3R of the three-spaces if $k \neq 0$. Since the leading term in the expression (C.7) for the four-dimensional curvature scalar 4R goes with $1/a$, 4R diverges for $a \rightarrow 0$ regardless of the values chosen for k . The Einstein field equations are not necessary to see that ρ , 3R , and 4R diverge, which is why I call this type of scalar singularity a *kinematic singularity*. If we conceive of the FLRW models as offering a picture of a universe evolving in time, which is not problematic as, to repeat, we have a preferred time coordinate defined by the isometries of the spacetime geometry, then we can call the instantaneous states which the universe can in principle assume *kinematic states*, analogous to the *façon de parler* in canonical general relativity.

In this picture, the trajectories of the galaxies will converge as they are tracked backward in cosmological time such as to finally intersect at the point $t = 0$. In contrast to an analogous scenario with a fixed Newtonian spacetime as background, where the trajectories of galaxies converge in one point as well, and where the density thus also diverges, *spacetime itself* becomes singular in the relativistic model as all distances between “points in space” tend to zero. This precludes that any physical law could be well-defined in this point of convergence. The singular point is therefore excluded from the spacetime. This means that the guarantee of Laplacean determinism cannot be extended through this initial singularity since the physical fields, including the gravitational field and thus spacetime itself, are no longer well-defined there. In this sense, the initial singularity is not part of spacetime and space and time themselves lose whatever meaning they had for regular spacetime points.⁹

At least for the cosmological models at stake, hence, the initial singularity cannot “nullify” determinism and the evolution of the gravitational field throughout all of (regular) spacetime, but it halts the (backward) dynamical evolution of the universe at $t = 0$. This can be seen from the Friedmann equation (C.5) which governs the dynamical evolution of the scale factor $a(t)$. For this equation, no regular solution $a = a(t)$ can pass through $a = 0$ and the evolution is ill-defined. As the Friedmann equation is nothing but one of the Einstein equations encoding the dynamics of the FLRW models, the singularity is *dynamical*. The impossibility to “dive through” the singular region is also captured by the geodesic incompleteness that this region exhibits. It is captured insofar as geodesics constitute permissible spatio-temporal paths of particles or radiation and their incompleteness means that for some finite value of the affine parameter, the curve is exhausted but has not met an endpoint.¹⁰

⁹One is tempted to say, with Wittgenstein, that “the limits of my language are the limits of my world.” (“Die Grenzen meiner Sprache sind die Grenzen meiner Welt.” *Tractatus* (5.6))

¹⁰A point $p \in \mathcal{M}$ is said to be a *future endpoint* of a future-directed non-spacelike curve $\gamma : I \rightarrow \mathcal{M}$ just in case for every neighbourhood $\mathcal{U} \subseteq \mathcal{M}$ of p there exists an $r \in I$ such that $\gamma(r') \in \mathcal{U}$ for every $r' \in I$ with $r' \geq r$. A *past endpoint* is defined analogously (where the “ \geq ” sign must be replaced by a “ \leq ” sign). An endpoint may or may not be on the curve, but it must be in \mathcal{M} . If a curve has an endpoint—future or past—, it is extendible.

The FLRW models are defined by a high degree of spacetime symmetries: they are all spatially isotropic and homogeneous. Since we have reason to believe that our universe approximately exhibits these symmetries at large scales, isotropic and homogeneous spacetimes must be taken physically seriously. Clearly, despite their empirical success as far as the large-scale structure is concerned, these spacetime symmetries constitute idealizations of the real cosmos. And it is these idealizations which may be taken to be responsible for the global hyperbolicity of the models. The real cosmos is likely to contain stars collapsing into black holes and similar events leading to singularities in the spacetime fabric, thus threatening to rescind global hyperbolicity. Global hyperbolicity, and thus Laplacean determinism, is reaffirmed, however, if the strong version of Roger Penrose’s cosmic censorship conjecture is true. The strong version of this conjecture claims that no singularity—except a possible initial singularity—can ever emanate causal signals to any observer living in a physically realistic spacetime. In other words, the strong cosmic censorship conjecture demands that all physically realistic spacetimes be globally hyperbolic. The weak companion hypothesis requires that singularities *resulting from gravitational collapse* are “hidden” behind their horizons such that they cannot send causal signals to any distant observer present in the asymptotically flat spacetime region. This means that no gravitational collapse of matter ever results in a “naked singularity,” but only in black holes.¹¹

Although no proof or disproof of the conjecture has so far been forthcoming in spite of vast efforts, there is evidence that it may be violated, even in its weak form, in non-generic situations of highly symmetric gravitational collapse.¹² Simultaneously, the corpus of evidence suggesting that the conjecture, at least in its weak form, may be true for generic physically realistic spacetimes has significantly grown over the last years.¹³ The violation of either or both of the cosmic censorship hypotheses has consequences for the validity of determinism. The existence and uniqueness theorems for the initial value problem for Einstein’s equations, such as Theorem 10.1.3 in Wald (1984, p. 251f), guarantee that there is a unique solution for which the initial value hypersurface Σ is a Cauchy surface. This solution, of course, is unique only up to diffeomorphisms. The question then arises whether this solution on the domain of dependence $D(\Sigma)$ of Σ is maximal or can be further extended beyond $D(\Sigma)$. If $D(\Sigma)$ is not maximal, then either a poor choice of initial value hypersurface was made, or else, intuitively, naked singularities have developed to prevent maximality

¹¹The strong version of the cosmic censorship conjecture is sometimes also said to demand that there be no “naked” singularities. In this context, “non-naked” of course means that the spacetime is globally hyperbolic. The weak conjecture does not imply the strong one, nor does the strong conjecture entail the weak one. For a more detailed account of both versions of the cosmic censorship hypothesis, see Wald (1984, Sec. 12.1).

¹²Cf. e.g. Shapiro and Teukolsky (1991).

¹³For a handy review, see Wald (1999).

of the deterministic evolution licensed by the existence and uniqueness theorems. Be this as it may, my main focus in this chapter is the big bang singularity and how it fares in quantum general relativity. I will therefore mostly ignore the ramifications brought about by gravitational collapse.

On a more general level, singularities may be considered as infecting the entire theory which breeds them. As exemplified in the Bergmann quote above, physicists often take the occurrence of singularities not so much as evidence against particular models of a theory, but as evidence against the theory itself. Later in his life, Einstein seems to have concurred with this attitude. In a departure from his earlier expectation that the singularity in FLRW models was an artefact of the unrealistically high symmetries of the models, Einstein seems to have shifted to placing his hopes in a unified field theory. He wrote in the appendix of *The Meaning of Relativity* added for the second edition in 1945:

[f]or large densities of field and of matter, the field equations and even the field variables which enter into them will have no real significance. One may not therefore assume the validity of the equations for very high density of field and matter, and one may not conclude that the “beginning of the expansion” must mean a singularity in the mathematical sense. (Einstein 1950, p. 129)

With Stachel (1980b), we can interpret this passage to mean that the very concept of $T_{\mu\nu}$ will break down near the singular region. Alternatively, we may take Einstein to make the weaker implication that the classical theory of general relativity will break down because *some* variables in the field equations become meaningless, i.e. leaving open the possibility that the breakdown originates in the gravity side of the field equations rather than in the matter side.

Two strategies can be pursued to rectify the problem: either GTR is modified such that some non-singular regime takes over in the vicinity of singularities while retaining the empirically confirmed low energy limit; or show that quantizing GTR washes out the singularities of the classical theory. As it has been argued in Chapter 2, there exist strong physical motivations to quantize GTR, which arise quite independently of the desire to control singularities. Furthermore, results from quantum theories have inspired an admittedly pious hope that a quantization of GTR may smooth out at least some of its singularities, including, perhaps, the initial singularity of FLRW models. This hope will be substantiated in the next section. Taken together, these two facts suggest that the second route should be pursued.

Note, however, that both the expectation that quantum effects will wash out the singularities as well as independent motivations for quantizing gravity are required to fully justify the second route. Many physicists, of which Kiefer (2005, p. 1f) is only a recent example,

believe that this justification can be cut short by just stating that the singularity theorems of classical GTR alone imply that a quantum theory of gravity is needed. But the dialectic of the situation is different: the singularity theorems only constitute a motivation for the *disjunction* of the two strategies mentioned above, not for one of them in particular. It would be a perfectly rational reaction to the singularity theorems to claim that GTR must only be modified or extended such that the singular regions also enjoy a regular regime, which of course does not imply quantization. That is why independent motivations for quantizing are needed to prefer the second strategy over the first.

6.2 DISSOLVING CLASSICAL SINGULARITIES

Before we embark upon an investigation on whether, and if so, to what extent, classical singularities may be “smoothed away” by quantum effects, a characterization of what should count as a *quantum singularity* is in order. Two distinct characterizations of quantum singularities are offered in the literature, somewhat parallel to curvature and dynamical singularities in the classical theory. There seems to be no consensus in the community as to how quantum singularities should be characterized, beyond the usual acceptance of the fact that there be a *disjunction* of two types. The first suggestion looks at the expectation values of “physical operators,” i.e. of operators which are believed to correspond to important physical observables, and checks whether they diverge as the spacetime region under examination is approached. If so, proponents of this suggestion say that the quantum system is singular. Whenever the distinction must be used in order to avoid confusion, I shall refer to this type of quantum singularity as *expectation-value singularity*. The second proposal, what I wish to call *dynamical (quantum) singularity*, speaks of a singular quantum system whenever the state of the system at one given instant does not uniquely determine the dynamical evolution of the system for all other times.¹⁴ This is the case if the quantum Hamiltonian operator \hat{H} , which governs the dynamical evolution of the quantum system, is *not* essentially self-adjoint on a C^∞ domain with compact support in L^2 , a Hilbert space of square integrable functions.¹⁵ Whenever the Hamiltonian operator or its closure is self-adjoint, then the evolution operator $\hat{U}(t) \doteq \exp(-i\hat{H}t)$ is unitary and well-defined for all t , avoiding a dynamical singularity.

The hope that the quantum may dissolve the singularities of classical GTR is nourished

¹⁴Horowitz and Marolf (1995, 5670) seem to have the same, or at least a sufficient similar, distinction in mind. Brunnemann and Thiemann (2006a) use a slightly different dichotomy between local and global singularities (see also Sec. 8.2.4), which goes back to Wald (1984, Ch. 9).

¹⁵A Hermitian operator \hat{A} is called *essentially self-adjoint* in case \hat{A} is closable and its closure is self-adjoint.

by three families of considerations. First, there are cases in ordinary quantum mechanics where quantum effects regularize the motion of point-like particles when it was singular at the classical level. Second, there exists ample evidence that quantum matter violates some of the energy conditions used to derive the singularity theorems. For instance, the strong energy condition may no longer be satisfied in the very early epochs of the universe. Third, in some curved spacetimes, the dynamics of quantum fields is well-defined and unique despite the appearance of singularities in the background spacetime. In these examples, the classical singularity does not bring about a dynamical quantum singularity in the relevant QFT in curved spacetime. Let me address the three families in turn.

For the first grounds for hope, consider a system of N point-like particles in three-dimensional Euclidean space \mathbb{R}^3 with interactions determined by a so-called *Kato potential*.¹⁶ The Kato potentials include attractive Coulomb potentials and potentials of the form r^{-m} for $|m| < 3/2$, i.e. they cover a rather general and important class of interactions. It has long been known that the N -body problem in Newtonian mechanics suffers from singularities. These singularities arise either from a collision of the point-like bodies or when bodies escape to infinity in finite time. A theorem by Hugo von Zeipel establishes that there exist no other possibilities.¹⁷ Ziaohong Xia (1992) proved the so-called *Painlevé conjecture* which surmises that there exist solutions of the Newtonian N -body problem with non-collision singularities for $N > 3$.

The reason why these singularities—both collision and non-collision—become pertinent for present purposes is that it turns out that the dynamics of N -body systems is less singular in ordinary quantum mechanics than in Newtonian mechanics. For interactions captured by Kato potentials, Hunziker (1966) has shown that every initial state with well-defined particle observables, such as position, momentum and energy, enjoys a regular global time evolution in these observables.¹⁸ Thus, the quantum evolution smooths the classical singularities. Whether this smoothing occurs generically for N -body quantum systems, including those with arbitrarily large N and with different interactions, is not known. Radin (1977) has proven that in case the number of particles involved is infinite and the interactions among them are sufficiently repulsive, then the quantum dynamics too is ill-defined. This result, to be sure, does not change the verdict that *in some cases* at least, classical singularities in the dynamical behavior of N -body systems are spirited away by the quantization of the system

¹⁶The original Kato condition is found in Kato (1951, p. 197). A different formulation gives Hunziker (1966, p. 300). I have no idea whether the two conditions are equivalent, as of course they should.

¹⁷Cf. Saari and Xia (1995).

¹⁸Kato (1951) previously established the significant assumption of Hunziker (1966) according to which the Hamiltonian which governs the evolution of the system of N bodies is *essentially self-adjoint*, i.e. that its closure is self-adjoint.

and its dynamics.

The second set of considerations concerns possible violations of one or several of the conditions invoked in the proofs of the singularity theorems. Could it be the case that in a semi-classical or full quantum theory of gravity, these conditions can no longer be expected to hold?¹⁹ There exists a multitude of local energy conditions often discussed in the context of classical GTR, only two of which shall be mentioned here.²⁰ The *weak energy condition* requires for any timelike vector field V^μ that the energy-momentum tensor satisfies $T_{\mu\nu}V^\mu V^\nu \geq 0$ everywhere in \mathcal{M} . Physically, the condition demands that the local energy density is positive as measured by any timelike observer. In a suitable orthonormal frame, the components of the energy-momentum tensor take the form of a perfect fluid defined by

$$T^{\hat{\mu}\hat{\nu}} = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & p_1 & 0 & 0 \\ 0 & 0 & p_2 & 0 \\ 0 & 0 & 0 & p_3 \end{bmatrix}, \quad (6.1)$$

where ρ is the rest energy density and the p_i are the three principal pressures. If the energy-momentum tensor can be written in the form (6.1), the weak condition requires that both $\rho \geq 0$ and $\forall i, \rho + p_i \geq 0$. A spacetime satisfies the *strong energy condition* iff, for any timelike vector field V^μ ,

$$\left(T_{\mu\nu} - \frac{T}{2} g_{\mu\nu} \right) V^\mu V^\nu \geq 0$$

holds anywhere in \mathcal{M} , where T is the trace of the energy-momentum tensor, $T = T_{\mu\nu}g^{\mu\nu}$.²¹ For a perfect fluid (6.1), this means that the strong condition is satisfied just in case $\forall i, \rho + p_i \geq 0$ and $\rho + \sum_i p_i \geq 0$.

The strong condition does not, in general, imply the weak condition—nor vice versa. For a perfect fluid, if $\sum_i p_i > 0$, then the weak condition will imply the strong one, whereas for $\sum_i p_i < 0$, the strong condition entails the weak one. In case $\sum_i p_i = 0$, the two conditions become identical. If the rest energy density and all principal pressures are positive, both conditions will automatically be satisfied. At the very least, the principal pressures cannot become too negative, i.e. there cannot be strong tensions equal in magnitude or larger than ρ , without violating the energy conditions.

¹⁹The initial singularity in FLRW models may be unacceptable because the models cannot be extrapolated back into the Planck regime of the early universe *for thermodynamic reasons*, as Bekenstein (1989) has shown for radiation-dominated FLRW universes. This result has been generalized by Schiffer (1991).

²⁰For a great review and further discussion of various energy conditions, see Visser (1995, Ch. 12).

²¹Some authors include in the definition of the strong condition that the inequality must also hold for null vector fields; see e.g. Senovilla (1997). I follow Visser (1995) who does not extend the condition to cover null vector fields as well. The strong energy condition was first introduced independently by Komar (1956) and by Raychaudhuri (1955, 1957).

It turns out that the more vulnerable candidate for violations appears to be the strong energy condition, which is assumed, at least in its averaged version, in most singularity theorems. Already at the classical level, the strong condition may be violated, e.g. for a classical massive scalar field.²² The classical scalar field satisfies all other energy conditions. Because an increasing number of singularity theorems require weaker energy conditions, it is thus not entirely clear whether singularities can be avoided altogether by having a massive scalar field violate the strong condition. The quantum cousin of the classical massive scalar field infringes the weak condition, as was first formally established for axiomatic QFT by [Epstein, Glaser, and Jaffe \(1965\)](#).²³ However, their proof has no stringent implications for the strong condition. If the spacetime contains a massive scalar field prepared in a coherent quantum state, it also violates the strong energy condition and the singularity can be avoided by a “big bounce.”²⁴ Because a classical scalar field also violates the strong energy condition, however, one is led to believe that “bouncing” may also be possible if the gravitational field is coupled to a purely classical scalar field.²⁵ As [Borde and Vilenkin \(1997\)](#) have argued, non-singular bouncing occurs generically in inflationary scenarios, which are likely to violate already the weak energy condition. It may be the case, they continue, that all averaged energy conditions are violated in these scenarios. This may well be interpreted as resulting from quantum effects, as inflation is expected to be powered by quantum mechanisms. [Vilenkin \(1983\)](#) has proposed an inflationary model without recurrence to bouncing: after its creation *ex nihilo*, it “quantum tunnels” into a de Sitter spacetime with finite curvature *ab initio*. The regularity seems to be achieved by virtue of the Higgs field contained in the model spacetime, and this field’s instanton effects. Further violations of energy conditions play on the Casimir effect, on Hawking radiation from evaporating black holes, on distortions of the quantum-electrodynamic vacuum called “squeezed vacuum,” on the Hartle-Hawking vacuum, and perhaps on other vacua ([Visser 1995](#), Sec. 12.3).

Let me turn to the third family of considerations fuelling the hope that a quantum theory may exhibit perfectly regular dynamics. [Horowitz and Marolf \(1995\)](#) have investigated the occurrence of dynamical quantum singularities in certain models of quantum field theory on curved classical spacetime with singularities. They have analyzed static spacetimes

²²Cf. [Hawking and Ellis \(1973](#), pp. 95-96) and [Visser \(1995](#), Sec. 12.3.1).

²³For a discussion of the validity of the so-called averaged energy conditions, which require the conditions to be valid only when averaged along sufficiently long null curves, see [Yurtsever \(1990\)](#), who conjectured that at least the averaged weak condition might hold for a large class of models of QFT on asymptotically flat spacetimes. [Klinkhammer \(1991\)](#) has shown that for spacetimes with non-trivial topologies, free massive scalar fields can violate even the averaged weak energy condition.

²⁴Cf. [Parker and Fulling \(1973\)](#). [Davies \(1977\)](#) has found that quantum-field theoretic effects avoid the initial singularity under a wide range of circumstances.

²⁵Cf. [Bekenstein \(1989](#), p. 971).

with timelike curvature singularities and have found that test particles exhibit non-singular dynamical behaviour in case they are treated quantum mechanically. According to their analysis, it turns out that the regularity of the evolution of the test wave packet results from an effective repulsive potential which shields the classical singularity. [Konkowski, Helliwell, and Wieland \(2003\)](#) have extended Horowitz and Marolf’s analysis by considering classically singular cylindrical and spherical spacetimes which are probed by quantum particles rather than by trajectories of classical observers. They reach a similar conclusion as Horowitz and Marolf, except that they find cases where the repulsion of the effective potential is not quite sufficiently large as to keep the probability of the quantum mechanical particle penetrating into the singular region small enough. Furthermore, it has been found that the Ellis-Schmidt type of the classical singularity does not matter and neither does—in most cases at least—the type of quantum probe, i.e. whether the quantum particle is a scalar field, a null vector field, or a spinor field.²⁶ However, quantum fields do not always evolve regularly on curved background spacetimes with classical singularities. Konkowski and her collaborators have studied the so-called “Levi-Civita spacetime,” some of which “contain” classical singularities and some of which do not ([Konkowski et al. 2004, 2005](#)). Those Levi-Civita spacetimes which are classically singular generically still suffer from a singularity when probed with a quantum field, while the classically regular Levi-Civita spacetimes remain regular with quantum fields evolving on them.

So do quantum effects generically wash out the initial singularity? There exists a plethora of further examples and counterexamples in the literature. This wealth of possibilities makes it impossible to draw any general conclusion. While some of the results discussed in this section can be taken as indication that quantum effects may generically smooth out classical singularities, at the bare minimum they induce the *hope* that a full QTG will be free of the singularities that so persistently plague the classical theory of general relativity. But this hope is a pious one, as the examples discussed here and elsewhere differ significantly from what can be expected to happen in a full QTG. First, whether or not classical cosmological singularities are smoothed out seems to depend on the quantization strategy. The typical strategy in canonical quantum cosmology is to freeze out degrees of freedom already at the classical level by restricting oneself to isotropic and homogeneous solutions of the Einstein equations. The highly symmetric system is then quantized à la Wheeler-DeWitt. It turns out in geometrodynamics that if one tries to construct a quantum operator corresponding to the scale factor starting from the basic ADM variables, the spectrum of this operator is not bounded above as it is in models based on LQG. The ADM quantization thus leads

²⁶[Ellis and Schmidt \(1977\)](#) have introduced a classification of classical singularities into quasi-regular, non-scalar curvature and scalar curvature.

to what I termed an expectation value singularity. [Bojowald and Morales-Técotl \(2004, Sec. 3\)](#) and [Ashtekar, Bojowald, and Lewandowski \(2003a, p. 246\)](#) claim that the indirect quantization procedure followed by LQC, as discussed in Section 7.2, is responsible for the fact that the eigenvalues of its scale factor operator will not go beyond a certain maximum. [Husain and Winkler \(2004\)](#) have indeed shown that if one opts for a quantization procedure akin to the one employed in LQC, then the cosmological singularity vanishes in quantum cosmology based on geometrodynamics as well. Hence, they conclude, it is not the nature of the classical variables which can be held responsible for the resolution of the singularity, i.e. the choice made between ADM and loop variables, but rather the quantization strategy pursued.

Even if the most promising quantization technique is chosen, however, success is not guaranteed. The second important point to be considered is that the extreme astrophysical situation encountered in the very early universe requires non-perturbative methods since we cannot assume the fields to be sufficiently small for a perturbative expansion to be well-behaved. Furthermore, in any theory in which the fields are embedded in the geometry, the fact that the classical geometry degenerates as the initial singularity is approached demands background independence. Canonical quantizations of GTR seem to be able to offer both. Let us turn to an analysis of cosmological models based on LQG then.

7.0 INTRODUCING LOOP QUANTUM COSMOLOGY

In cosmological models based on LQG, henceforth referred to as *loop quantum cosmology* (LQC), the novel quantization strategy indeed appears to lead to the dissolution of both the kinematic as well as the dynamical singularities of the classical theory.¹ The strategy pursued by Bojowald, who spearheaded many of the developments in LQC, and his fellow loop quantum cosmologists differs from the traditional Wheeler-DeWitt approach insofar as it retains the full theory in the early stages of the quantization procedure, and only then confines itself to quantum states in the kinematical Hilbert space which exhibit isotropy and homogeneity. More specifically, the strategy refrains from imposing any additional symmetry requirements on the classical models, then proceeds to solve the Gauss and vector constraint equations to obtain the kinematical Hilbert space \mathcal{K} of the full theory, just as in full LQG, and only then freeze out degrees of freedom by only considering highly symmetric kinematic states. Imposing these symmetries drastically simplifies the Hamiltonian constraint, which now becomes explicitly solvable and yields the physical Hilbert space of LQC. In this sense, LQC is not merely an exercise in cosmology, but can also be seen as an attempt to arrive at a toy model of the full theory, hopefully offering a few insights pertaining to the structure of the physical Hilbert space of the full theory. I believe that this latter goal cannot be attained due to the excessive symmetry demands which were imposed on the kinematical states. It will actually turn out, at least in one particular scenario discussed in Section 8.2.1, that the Hamiltonian constraint equation is only solved by a single state, which by itself constitutes the physical Hilbert space of LQC. This conclusion strongly suggests that too many degrees of freedom may have been frozen out.

In the classical theory, as is explicated in Appendix C, the density ρ , the three-dimensional spatial curvature 3R , and the four-dimensional spatio-temporal curvature 4R —all scalar quantities—tend to grow beyond all bounds as the scale factor a tends to zero. In LQC, however, it turns out that the inverse scale factor is bounded and that the universe thus has a

¹For an introduction to the application of LQG to cosmology, see [Bojowald and Morales-Técotl \(2004\)](#); but see also [Ashtekar \(2002\)](#) and [Rovelli \(2004, Sec. 8.1\)](#). For the mathematical foundations of LQC, see [Ashtekar et al. \(2003a\)](#). [Bojowald \(2004, 2005\)](#) are the most recent review articles.

maximal spatial (and spatio-temporal) curvature and a maximal density and thus avoids the kinematic singularity of the classical theory, although with an important qualification as will be discussed in Chapter 8. One can construct a self-adjoint curvature operator on the Hilbert space of spatially isotropic and homogeneous spin network states in the kinematic Hilbert space \mathcal{K} . This operator, it turns out, is vested with a spectrum bounded from above. This entails that even for a universe of vanishing size, the curvature does not blow up. In fact, like in the classical case, the curvature grows as the scale factor decreases. But unlike the classical universe, the quantum universe changes to a different regime after having reached a maximum curvature. This second regime actually tames the quantum curvature such as to drive it into zero as the scale factor vanishes. As this effect may testify, the mathematical properties of the loop quantization have indeed far-reaching physical consequences.

Furthermore, another, somewhat parallel, delight awaits us at the dynamical level: the dynamical singularity also vanishes in LQC, although again, this claim will be qualified in Chapter 8. The fact that the curvature did not diverge for arbitrarily small volumes of the universe may incline us to assume that the dynamical evolution of the LQC model remains non-singular—even at the big bang. This expectation is not entirely disappointed. As in LQG, the evolution is governed by the Hamiltonian constraint equation, which is a wave equation with the constraint operator acting on the isotropic, homogeneous states of kinematical LQC. These kinematical states depend on the scale factor a (and perhaps other physical fields). The scale factor a is used as a fiducial time with respect to which the isotropic, homogeneous spin network states evolve. But since a assumes discrete values, partial derivatives with respect to it must be replaced in LQC with finite difference operators. Thus, the Friedmann-Wheeler-DeWitt equation of evolution, aka the Hamiltonian constraint equation for the symmetry-reduced theory, becomes a difference equation, as opposed to a differential equation. Evolution occurs in discrete steps, although the notion of discreteness at work here will have to be clarified. Numerical studies of this evolution equation (Bojowald 2001a,b) have revealed that the dynamical behaviour of the LQC models on and around $n = 0$ is (almost) non-singular, where n symbolizes the iteration step of the discrete evolution, and that for sufficiently large n , the correct semi-classical limit follows in the form of the expected Friedmann evolution. The regularity of the evolution around the big bang permits a continuation of the model into a new realm of negative n . In other words, one can evolve right through what used to be a singularity into a mirror universe which did not exist classically as the classical evolution ground to a halt at the initial singularity. The n , which serve as a substitute for a time here, continue to decrease beyond the big bang as they adopt negative values. The dynamical singularity, despite claims to the contrary, does not vanish altogether as a singular residue survives the quantization.

Before I proceed to explicate these two aspects of how LQC washes out classical singularities, and in what sense and to what extent it does so, it should be noted that alternative approaches to QG may equally offer a resolution of singularities. While in classical GTR—as well as in cosmology in a Newtonian spacetime—the trajectories of any two galaxies must intersect in the finite past, including quantum effects according to the tastes of any QTG may prevent galaxies from coming arbitrarily close to one another and undergo a “big bounce” rather than a “big bang.” Thus, it seems as if QG quite generally might open up the possibility of pre-bang universes. The application of string theory, for example, to cosmology suggests that the big bang is either replaced by a transitional phase from an accelerated to a decelerated expansion of the universe or is the result of the collision of different “branes.” According to the first scenario, the so-called *pre-big bang scenario*, the universe has existed forever and was incredibly empty at first.² As the forces gradually gained in strength, matter began to clump and formed black holes in some regions and rebounded in big bangs, thus creating effectively distinct universes. The second scenario, the so-called *ekpyrotic scenario*, suggests that our universe occupies a multi-dimensional membrane or “brane” which lives in a higher-dimensional space. This brane might have collided with another brane, thereby releasing kinetic energy as matter and radiation and thus resulting in something with observational signatures very much like the big bang in standard cosmology.³ Veneziano (2004), who gives a popular introduction in cosmological models based on string theory, claims that these scenarios both afford observational signatures just sufficiently different from those of conventional inflationary models that they might be registered by the LIGO and VIRGO observatories. Furthermore, the frequency distributions predicted by the two scenarios also differ from one another sufficiently so that they should be distinguishable by the LIGO and VIRGO data.

According to Veneziano, both the pre-big bang and the ekpyrotic scenarios offer a perfectly regular dynamical evolution through what used to be the singular big bang. It is worth emphasizing, however, that at least the ekpyrotic universe still suffers from (a mild version of) the big bang singularity. Khoury (2004) concedes that a formal proof that the transition through the big bang can successfully be completed is still missing. The regularity of the dynamical evolution of the universe also at the big bang constitutes an *assumption* of the scenario, rather than a formally derived result. For LQC, the claim is stronger: given the theoretical framework of LQG, the regularity can be established under the usual assumptions of global symmetries that cosmological models must satisfy. As we will discover in paragraph

²Cf. Gasperini and Veneziano (2003) for a detailed review of the pre-big bang model.

³The original article on the ekpyrotic model is Khoury et al. (2001), a more recent review can be found in Khoury (2004).

7.3, however, this claim must also be enjoyed with an important qualification.

The remainder of this section is divided, like Gaul, into three parts: the first subsection gives a brief account of the foundations and the main technical choices made by LQC, followed by subsections on the fate of the kinematical and dynamical singularities respectively.

7.1 FOUNDATIONS OF LOOP QUANTUM COSMOLOGY

The main source for this subsection is [Ashtekar, Bojowald, and Lewandowski \(2003a\)](#). Its rather technical account of LQC shall be explained with less mathematical rigour, but with hopefully correspondingly wider appeal; it is abbreviated and adapted to my notation as introduced in Chapter 5.

Returning again to the terminology introduced in Section 3.2, where we introduced a distinction between spatio-temporal background fields B_i and dynamical fields D_j and an analogous distinction between spacetime and dynamical symmetries. We have seen in Chapters 4 and 5 how the dynamical symmetry of GTR, understood in the sense of Section 3.2, or something just like it, is encoded in both the classical and the quantum theories via constraint equations. Quantum cosmology, like generally relativistic cosmology, assumes certain *spacetime symmetries*, where the original definition of “spacetime symmetries” must be adjusted as follows. Originally, a spacetime symmetry was a mapping which leaves all the fields B_i invariant, i.e. a diffeomorphism $\phi : \mathcal{M} \rightarrow \mathcal{M}$ such that $\phi^* B_i = B_i$ for all i . Because GTR entertains no background fields B_i at all, however, it seems as if this definition has become vacuous. If we extend this definition to equally apply to the now dynamical spacetime structure, we can again contentfully speak of spacetime symmetries. Let us say, therefore, that a *spacetime symmetry* is a mapping which leaves dynamical field(s) D_j encoding the spacetime structure invariant, i.e. a diffeomorphism $\phi \in \text{Diff}(\mathcal{M})$ such that $\phi^* D_j = D_j$ for all j .

This re-definition works just fine for classical relativistic cosmology, but it will not be applicable *tel quel* to the quantum states. One might argue that such an application of the concept of spacetime symmetry to quantum states is unnecessary for the simple reason that one might as well pick the relevant symmetric spacetimes already at the classical level and then simply perform a quantization of this reduced theory. Such a resort, however, would suffer from two disadvantages: first, one would have to establish the equivalence of the state space obtained by a loop quantization of the classically reduced theory on the one hand and the subspace of symmetric states of full LQG on the other; second, even if that can be done—and in principle it should be possible—, the first road, unlike the second, would not

easily enable the interpretation of the quantum states of the reduced theory as symmetric states of the full quantum theory without recourse to the classical level. As a consequence, the reduction to symmetric states at the level of the full quantum theory appears much more attractive. However, this strategy presupposes a concept of *symmetry* at the level of the quantum theory.

To introduce such a concept, however, is not entirely trivial. Building on the symmetry concept as it has been developed in the context of the (classical) fibre bundle formalism for bundles with connections, [Bojowald and Kastrup \(2000\)](#) have proposed a definition of what should count as a symmetric state in the quantum theory.⁴ They repudiate as inappropriate for diffeomorphism-invariant theories the usual procedure for determining symmetric states via the invariance of the action of the symmetry group on the physical Hilbert space as this would lead to trivial results because the rotation group is a subgroup of the diffeomorphism group. Bojowald and Kastrup define *symmetric quantum states* invoking the theory of connections which are invariant under the action of symmetry groups on principal fibre bundles. With this definition at hand, they identify spaces of symmetric quantum states with certain spaces of spin-network states. Interestingly, it turns out that these spaces of symmetric quantum states can be equipped with a scalar product such that they form a Hilbert space again.

One problem arises in any attempt to implement this strategy: not all constraints have been solved in LQG, which means that effectively the physical Hilbert space containing those states amongst which one would like to select the states exhibiting the symmetry of interest is not yet available. The procedure must be carried out starting from the kinematic Hilbert space, then selecting the symmetric states, and then solve the reduced constraint equations on the spaces of symmetric states. This slight change of procedure bears advantages as well as disadvantages. First, given the collective failure of solving the Hamiltonian constraint and of thus finding the physical Hilbert space, one would hope that reducing the theory at the kinematic level might simplify the Hamiltonian constraint just enough to find the symmetry-reduced, but physical—as opposed to purely kinematic—Hilbert space. Furthermore, success with this modified strategy would instill the hope that one might receive pivotal insights concerning the quantization of the full theory from solving the reduced Hamiltonian constraint. On the other hand, however, one must be aware of the fact that there is no guarantee that these symmetric states in the kinematic Hilbert space which solve the reduced constraints coincide with the symmetric states in the physical Hilbert space of

⁴For the relevant definition of symmetric spaces in the context of the fibre bundle formalism as Bojowald and Kastrup apply it, see [Kobayashi and Nomizu \(1969\)](#), p. 225). The canonical introduction to fibre bundles is found in [Kobayashi and Nomizu \(1963\)](#).

the full theory. In particular, the reduction from a field theory to a mechanical system with one degree of freedom may be sufficiently radical to alter some of the characteristics of the full theory.

For open models, i.e. for classical cosmological models where the spatial curvature is negative or zero and Σ is diffeomorphic to \mathbb{R}^3 , the symmetry group, denoted by $\text{Sym}(\Sigma)$, is the Euclidean group.⁵ In this situation, as is usual in cosmology, one can find a fiducial metric ${}^0q_{ab}$. Given ${}^0q_{ab}$, one can fix an orthonormal triad ${}^0e_a^i$ and a co-triad ${}^0\omega_a^i$. In the full theory of LQG, the gravitational phase space is coordinatized by pairs (A_a^i, E_i^a) , representing fields on the three-dimensional manifolds Σ , viz. an $SU(2)$ -connection and a triplet of (densitized) vector fields, respectively. [Ashtekar, Bojowald, and Lewandowski \(2003a\)](#) declare a pair (A_a^i, E_i^a) of base fields on Σ to be *symmetric* just in case for every $\sigma \in \text{Sym}(\Sigma)$ there exists a local (Gauss) gauge transformation $g : \Sigma \rightarrow SU(2)$ such that

$$(\sigma^* A', \sigma^* E') = (g^{-1} A' g + g^{-1} dg, g^{-1} E' g). \quad (7.1)$$

[Ashtekar, Bojowald, and Lewandowski \(2003a\)](#) note that for every symmetric pair (A', E') which satisfies the Gauss and the spatial diffeomorphism constraints, there exists a unique pair (A, E) which is equivalent to (A', E') such that

$$A = \tilde{c} {}^0\omega^i \tau_i, \quad E = \tilde{p} \sqrt{{}^0q} {}^0e_i \tau^i, \quad (7.2)$$

where the τ s are essentially the Pauli matrices, 0q the determinant of the fiducial metric, and \tilde{c} and \tilde{p} are the constants which carry the non-trivial information contained in (A', E') . One can therefore change the base canonical variables from A and E to \tilde{c} and \tilde{p} . The fiducial metric ${}^0q_{ab}$ can be rescaled by a constant factor k^2 without consequence for the physical situation. Blowing up or shrinking the universe like this merely amounts to a rescaling of the only degree of freedom left by an arbitrary constant. Under such a rescaling the now canonical pair (\tilde{c}, \tilde{p}) becomes $(k^{-1}\tilde{c}, k^{-2}\tilde{p})$.⁶ As such rescaling cannot make a physical difference, \tilde{c} and \tilde{p} cannot have a direct physical interpretation. Physical meaning is only gained once new variables independent of the particular choice of the fiducial metric are introduced. A convenient choice with the required independence is

$$c = V_0^{\frac{1}{3}} \tilde{c} \quad \text{and} \quad p = V_0^{\frac{2}{3}} \tilde{p}, \quad (7.3)$$

⁵The Euclidean group $E(n)$ is the symmetry group of the n -dimensional Euclidean space. It is a subgroup of the affine group, and has as subgroups the group of translations T and the orthonormal group $O(n)$. Each element of $E(n)$ can be represented as a combination of translations and orthonormal transformations, i.e. of rotations and rotoinversions.

⁶Because of (7.2) and ${}^0q_{ab} = {}^0\omega_a^i {}^0\omega_b^j \delta_{ij}$ and ${}^0q^{ab} = {}^0e_i^a {}^0e_j^b \delta^{ij}$ respectively.

where V_0 is the volume of a “cell of space” in terms of the fiducial metric. For reasons of simplicity, this cell is typically assumed to be cubical with respect to ${}^0q_{ab}$. The symplectic structure of the gravitational symmetric phase space defined by (7.2) can then be expressed without recourse to the fiducial metric (or the unit cell volume):

$$\Omega_{\text{grav}}^s \propto dc \wedge dp. \quad (7.4)$$

Finally, as the Gauss and the spatial diffeomorphism constraint equations have already been solved, the only remaining constraint equation is the Hamiltonian constraint, which corresponds to a lapse constant across Σ , analogous to the N of the ADM formulation. Re-expressed in terms of (c, p) , it becomes

$$-6\beta^{-2} c^2 \operatorname{sgn} p \sqrt{|p|} \quad (+8\pi G C_{\text{matter}}) = 0, \quad (7.5)$$

where β designates the Immirzi parameter, sgn the signum function, and C_{matter} stands for the additional term in the constraint which would arise from a matter Hamiltonian. This completes the construction of the phase space for LQC.

7.2 THE DISAPPEARANCE OF THE CURVATURE SINGULARITY

What are the elementary functions of the basic canonical variables on the classical phase space which will translate unambiguously into quantum analogues? In full LQG, this role is played by functions of the $SU(2)$ -connections A —the holonomies—and the smeared triads. In the symmetry-reduced case of LQC, the almost periodic functions $g(c)$ of c make up the configuration variables,⁷ and the momentum variables are essentially given by p . The vector space of $g(c)$ corresponds to the space of cylindrical functions on the configuration space of the full theory and is called the *space of cylindrical functions of symmetric connections* in Ashtekar, Bojowald, and Lewandowski (2003a), denoted by Cyl_S . In terms of classical geometry, the momentum variable p determines the physical volume of an elementary cell via $V = |p|^{3/2}$. As LQC assumes homogeneity and isotropy, volume scales and length scales can be used interchangeably. Thus, $\sqrt{|p|}$ can naturally be interpreted as a length. Together, the configuration and momentum variables $g(c)$ and p build the holonomy-flux algebra such that the only non-vanishing Poisson bracket between these elementary functions is given by

$$\{g(c), p\} = \frac{8\pi\beta G}{6} \sum_j (i\mu_j \xi_j) e^{i\frac{\mu_j c}{2}}, \quad (7.6)$$

⁷And not the c 's themselves, as no operator directly corresponding to c can be defined on \mathcal{K}^S .

where $\mu_j \in \mathbb{R}, \xi_j \in \mathbb{C}$ and the j 's are labelling edges and run over a finite number of integers. As a matter of fact, the right-hand side of (7.6) is again a function in Cyl_S and the algebra of elementary variables is thus closed. As the canonical momenta are all proportional to p , they commute with one another. As a consequence, the momentum (or triad) representation conveniently exists.

As mentioned in the previous paragraph 7.1, the symmetric quantum states in the kinematic Hilbert space \mathcal{K} form a Hilbert space themselves, symbolized by \mathcal{K}^S . The Hilbert space \mathcal{K}^S is not separable, i.e. it does not have a countable basis. This misfortune complicates the concept of discreteness, as will become clear immediately.⁸ The canonical variables turned quantum operators work in analogy to the full theory: \hat{g} corresponds to the configuration operator and \hat{p} to the triad operator of the full theory. The almost periodic functions $\langle c|\mu\rangle = e^{i\mu c/2}$ constitute a complete orthonormal basis in \mathcal{K}^S .⁹ Both orthogonality and normalizability lead to important conceptual consequences, as to be discussed below. It turns out that these basis vectors are eigenstates of the momentum operator \hat{p} and that we have

$$\hat{p} |\mu\rangle = \frac{8\pi\beta\mu}{6} |\mu\rangle \doteq p_\mu |\mu\rangle. \quad (7.7)$$

This is the promised momentum representation.¹⁰ In order to attain a physical interpretation of μ , we can use the relation $V = |p|^{3/2}$ and rewrite (7.7) as

$$\hat{V} |\mu\rangle = \left(\frac{8\pi\beta|\mu|}{6} \right)^{3/2} |\mu\rangle \doteq V_\mu |\mu\rangle. \quad (7.8)$$

⁸However, there is also an important sense in which the non-separability of the Hilbert space *enables* discreteness. A Hilbert space representation which disallows operators that could be associated with a smooth spacetime geometry, such as a connection operator, only seems possible in a non-separable (kinematical) Hilbert space. I will return to this issue in Section 9.1 when I discuss the polymer representation proposed by [Ashtekar et al. \(2003b\)](#).

⁹The orthogonality follows from the construction of these functions as almost periodic functions on the Bohr-compactification on \mathbb{R} . While the details are somewhat too technical to review here, the integrator can be simplified by the following formula: $\int f(c)d\mu(c) = \lim_{T \rightarrow \infty} (2T)^{-1} \int_{-T}^T f(c)dc$ (courtesy of Martin Bojowald, 8 December 2005). Thus, $\langle \mu(c)|\mu'(c)\rangle = \lim_{T \rightarrow \infty} (2T)^{-1} \int_{-T}^T \exp(i(\mu' - \mu)c/2)dc = 2/(\mu' - \mu) \lim_{T \rightarrow \infty} T^{-1} \sin((\mu' - \mu)T/2) = 0$. The normalizability is a straightforward calculation. In total, we thus obtain $\langle \mu|\mu'\rangle = \delta_{\mu\mu'}$, where $\delta_{\mu\mu'}$ is the Kronecker delta rather than a distribution.

¹⁰In the reduced model of LQC, this representation can be shown to be unitarily equivalent to any one making reference to V_0 but it has the advantage that the symplectic structure to be quantized does not involve V_0 and that V_0 does thus not have to be fixed prior to quantization. As a result of this unitary equivalence, quantum physics is interpreted to be independent of the choice of V_0 . Hence, it makes sense to eliminate references to V_0 . In the full theory, the situation is more complicated, cf. [Ashtekar et al. \(2003a\)](#), p. 243).

Hence, for a universe in the quantum state $|\mu\rangle$, the physical volume of a cell of space in Planck units is given by $|\mu|^{3/2}$, up to a constant factor. For example, the physical volume of the cell in the quantum state $|\mu = 1\rangle$ is $(8\pi\beta/6)^{3/2}$ in terms of Planck units.¹¹

What is the spectrum of \widehat{p} ? Surprisingly, it is \mathbb{R} rather than a discrete subset of \mathbb{R} . In the full theory, the spectrum of the momentum operator and other geometric operators such as area and volume operators assume eigenvalues in a discrete subset of the real line. [Ashtekar, Bojowald, and Lewandowski \(2003a\)](#) explain that the high degree of symmetry imposed on the kinematic states leads to the collapse of the two quantum numbers of the full theory—the continuous label e denoting the edges and the discrete label j on the edges denoting their “spins”—into a single continuous label μ . But this does not imply that the spectrum of the momentum operator is continuous: since its eigenstates are normalizable—indeed the factor $(2T)^{-1}$ in footnote 9 was chosen such that $\langle\mu|\mu'\rangle = \delta_{\mu\mu'}$ —the spectrum of the operator is discrete by definition. Were the spectrum truly continuous, only distributionally “normalizable” eigenstates would exist. For *separable* Hilbert spaces, the general habit of calling a spectrum discrete in case the corresponding eigenstates are normalizable implies the usual understanding of a discrete spectrum of eigenvalues. However, for *non-separable* Hilbert spaces, i.e. for Hilbert spaces with non-denumerable bases, this habit can lead to discrete spectra which take all values in \mathbb{R} . Clearly, the present notion of discreteness offers a counterintuitive analysis in this case. For instance, the fact that the kinematic Hilbert space of LQC is non-separable together with the stated discreteness of the spectrum of \widehat{p} —where “discreteness” is understood according to the present notion—, this has the somewhat counterintuitive consequence that the identity operator on \mathcal{K}^S can be written as a “continuous sum” $\widehat{I} = \sum_{\mu} |\mu\rangle\langle\mu|$ rather than as an integral. However, this account of discreteness also has intuitive implications for the case at hand. Most importantly among these, the discreteness of the spectrum of \widehat{p} does not permit a straightforward introduction of an inverse operator \widehat{p}^{-1} necessary for investigating the behaviour of the curvature in the quantum regime, since this inverse is not densely defined around zero. This implication has conceptually important consequences. Let me explain.

In order to investigate whether the curvature, or whatever corresponds to the classical curvature in the quantum realm, suffers from a divergence at the big bang in the quantum theory as well, one must introduce a “curvature” operator defined on \mathcal{K}^S . Equation (C.2) illustrates that the curvature 3R of the three-spaces in FLRW models of classical cosmology is inversely proportional to $a^2(t)$, where $a(t)$ denotes the scale factor expressing the size of the universe. Since p is related to the physical volume of the three-spaces, an obvious choice

¹¹The Planck length is usually introduced as $\ell_{\text{Pl}} \doteq \sqrt{G\hbar c^{-3}}$. Sometimes, as in [Ashtekar et al. \(2003a\)](#), ℓ_{Pl} is set to equal $\sqrt{8\pi G\hbar c^{-3}}$. I will use the first convention, which means that $\ell_{\text{Pl}} = \sqrt{G}$ in natural units.

suggests $a = \sqrt{|p|}$. In order to determine the curvature, one thus needs to obtain an inverse operator of the scale factor operator. In general, given an eigenvalue problem $\widehat{X}|x\rangle = x|x\rangle$, it is straightforward to introduce powers of the operator \widehat{X} by repeated applications of \widehat{X} to $|x\rangle$ for positive powers and by (repeatedly) applying the inverse map \widehat{X}^{-1} to $|x\rangle$ for negative powers. The eigenvalues will then occur in the same power as the operator and we have $\widehat{X}^n|x\rangle = (x)^n|x\rangle$ where $n \in \mathbb{Q}$. If the spectrum of the eigenvalues x contains zero, this generalization must obviously proceed more cautiously for negative powers on pain of a divergent behaviour of the eigenvalues of the inverse operators. First, a necessary condition for \widehat{X} to be invertible, and thus for negative powers of \widehat{X} to have a well-defined operation, is that $\ker(\widehat{X}) \doteq \{|\phi\rangle \in \mathcal{H}; \widehat{X}|\phi\rangle = 0\} = \{|x = 0\rangle\}$, i.e. that the eigenvalue zero is non-degenerate, which is the case for \widehat{p} . Second, and more problematically, the catastrophe of divergence must be averted in some principled way. The most obvious strategy would be to assign by choice an action to \widehat{X} on $|x = 0\rangle$. Given, however, that the exact construction of this inverse operator and particularly its action on $|\mu = 0\rangle$ are of such crucial relevance to whether the inverse operator is bounded and the models free of kinematic singularities, this unsatisfactory ad hoc patchery would not be acceptable, since one would have little confidence that such ad hoc mending would produce the correct—as opposed to desired—behaviour for small volumes of the universe.

How, then, can the catastrophe for an inverse operator \widehat{X}^{-1} be avoided when zero is an eigenvalue of \widehat{X} ? We can apply \widehat{X}^{-1} on every state $|\Psi\rangle$ of the form $|\Psi\rangle = \sum_x k_x|x\rangle$ as long as $k_0 = 0$ and $\sum_x |k_x|^2$ exists. The operator \widehat{X}^{-1} would be densely defined if one could write $|x = 0\rangle$ as a suitable limit of such states $|\Psi\rangle$. If $|x = 0\rangle$ is an element of a Hilbert space basis, and thus a normalizable state, then it will be orthogonal on all other eigenstates of \widehat{X} . In this case, then, we would find for every state $|\Psi\rangle = \sum_x k_x|x\rangle$ that $\langle x = 0|\Psi\rangle = 0$. However, we should have $\langle x = 0|x = 0\rangle = 1$ in the limit. Since \widehat{X}^{-1} is not defined on $|x = 0\rangle$ and cannot be given by superpositions of other basis states, \widehat{X}^{-1} is not densely defined. This is exactly what happens if we are trying to introduce an operator \widehat{p}^{-1} in the present situation: since \widehat{p} admits a *normalizable* eigenvector $|\mu = 0\rangle$ with eigenvalue $\mu = 0$, the operator \widehat{p}^{-1} is not densely defined on \mathcal{K}^S . Thus, while the domain of \widehat{p} is dense in the Hilbert space with respect to the measure used, the domain of \widehat{p}^{-1} is not. Since the Hilbert space contains all the states of the quantum system considered, a description relying on operators which do not have a dense domain could not be used for all the states of the system and would thus be incomplete. Ideally, operators should be capable of acting upon *all* states. As experiments cannot be performed with arbitrary accuracy, dense domains are sufficient. If an operator were not densely defined, there would be situations in which the operator could not be applied, failing to predict a measurement outcome.

Since a dense definition of the action of the inverse operator on \mathcal{K}^S is thus required for quantization, one has to find a reformulation which avoids the gap, yet proceeds in a principled manner. Such a principled way, and the only *known* principled way, of constructing a densely defined operator can be found in Thiemann's construction of a Hamiltonian operator, which I will not review in detail here.¹² The procedure, which is sometimes called the *commutator-technique*, essentially consists of two steps: first, the problematic function of phase space variables is re-expressed as an unproblematic function of elementary variables as well as the volume function; second, the canonical quantization recipe is followed and the elementary variables and the volume function are replaced by their well-defined quantum counterparts. Without following the details of this construction, the procedure yields an inverse operator, which is densely defined, contains both configuration and momentum operators, and commutes with \widehat{p} and thus has simultaneous eigenstates with \widehat{p} , at least in the isotropic case. This inverse operator is called (*fundamental*) *triad operator* and can be written informally as \widehat{p}^{-1} , as opposed to \widehat{p}^{-1} . More precisely, the procedure yields the eigenvalue problem (compare with equation (26) in [Ashtekar et al. \(2003a\)](#)):

$$\left[\frac{\widehat{\text{sgn}(p)}}{\sqrt{|p|}} \right] |\mu\rangle = \sqrt{\frac{6}{8\pi\beta G}} \left(\sqrt{|\mu+1|} - \sqrt{|\mu-1|} \right) |\mu\rangle. \quad (7.9)$$

There are a few remarkable things about this spectrum. First, the operator admits an eigenstate $|\mu=0\rangle$ with zero eigenvalue. Coincidentally, the operator \widehat{p} admitted the same eigenstate also with an eigenvalue of zero. Classically, the spatial curvature is proportional to a^{-2} and thus to p^{-1} . In the quantum theory, therefore, the square of the eigenvalues as given by (7.9) corresponds to the curvature, up to proportionality factors as specified by (C.2). This means that a quantum universe of zero size has vanishing curvature.¹³ Second, the spectrum of (7.9) has a maximum value for $\mu=1$, which is $\sqrt{3/2\pi\beta}$ in units of one over Planck length or ℓ_{Pl}^{-1} . The kinematical curvature of the quantum universe, therefore, assumes a maximal value of $3/2\pi\beta$ in units of ℓ_{Pl}^{-2} . Incidentally, the kinematical curvature would diverge if either β or ℓ_{Pl} would go to zero, i.e. if either G or \hbar were zero, or c were infinite. Figure 10 shows a plot of the kinematic curvature in dependence of μ in units of ℓ_{Pl}^{-2} , where it is assumed that $\beta = 0.2375$ in rounded agreement with [Meissner \(2004\)](#), who determined β under the premise that the black hole entropy calculated from LQG yields the same result as the classic calculations by [Bekenstein \(1973, 1974\)](#) and [Hawking \(1975\)](#).

The entire literature on LQC, including [Ashtekar, Bojowald, and Lewandowski \(2003a\)](#), but with the notable exception of [Brunnemann and Thiemann \(2006a,b\)](#), interprets this

¹²Cf. [Thiemann \(1996, 1998a,b,c\)](#).

¹³This curvature will henceforth be termed *kinematical*, rather than spatial, curvature.

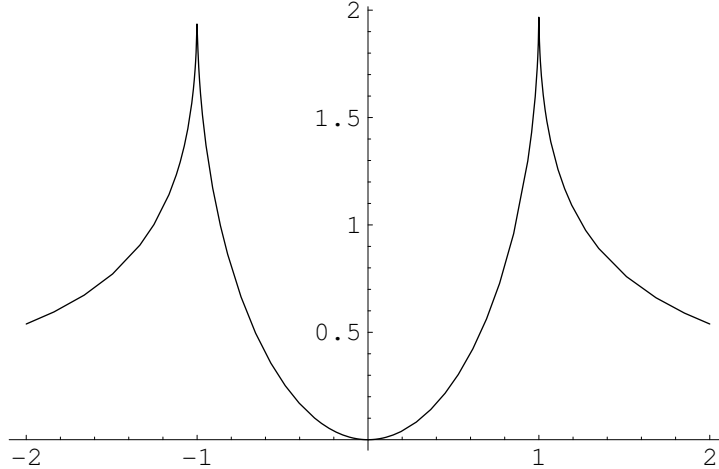


Figure 10: Kinematic curvature as function of μ (in units of ℓ_{Pl}^{-2}).

result such that curvature no longer grows beyond all bounds as the big bang is approached and that at least in this sense, the classical singularity is cured by quantum effects. Without introducing a “regulator” or some artificial cut-off, without massaging the classical expression into a suitable form or similar tricks, the advocates of LQC claim to have obtained the physically relevant result that in their approach, the kinematic curvature does not yield to singular behaviour in the Planck regime. To warrant this interpretation, we need to rewind once again and reconsider the most crucial steps in how this physical interpretation of equation (7.9) came to be.

The introduction of a quantum operator defined on \mathcal{K}^S which corresponds to the classical three-curvature depended critically on the fact that the eigenstates of \hat{p} were normalizable and that therefore the Hilbert space \mathcal{K}^S was spanned by a direct *sum* of the eigenstates of \hat{p} rather than by a direct integral. That the spectrum of \hat{p} was discrete in this sense implied that \hat{p}^{-1} was not densely defined on \mathcal{K}^S and thus prevented a direct construction of the triad, or inverse, operator. If this direct construction were permitted, the operator would not have been bounded from above. The inapplicability of the direct construction procedure meant that an alternative procedure had to be sought. At this point in the formulation of LQC, a procedure developed by Thiemann when he attempted to define an explicit Hamiltonian operator for the full theory of LQG comes into play. For this reason, the entire construction of LQG-based cosmological models depends on whether the ambition of Thiemann’s procedure to be a unique and privileged recipe for correctly constructing

operators which cannot be directly built because they are not everywhere densely defined in \mathcal{K} or \mathcal{K}^S is fulfilled. Without going into technical details of a method I have not closely studied, let me hasten to add a few remarks.

The first set of remarks lists the technical difficulties that Thiemann’s proposal faces. The first problem, identified by [Lewandowski and Marolf \(1998\)](#) and [Gambini, Lewandowski, Marolf, and Pullin \(1998\)](#), was that the Hamiltonian constraint operator as constructed by Thiemann was not capable of mimicking the classical Dirac constraint algebra (4.24)-(4.28) in a large class of models of the theory. [Nicolai, Peeters, and Zamaklar \(2005\)](#) discuss whether the quantum constraint algebra arising from Thiemann’s proposal actually closes, as it should, or not. The main difficulty here is that while it is clear what one means by a “closure” of an algebra for any classical constrained system, this notion is ambiguous for the quantized theory. The strongest version of closure, the so-called “off-shell” closure, which would obtain if the algebra closed on a comparatively large “habitat” of the Hamiltonian constraint operator which does not only contain diffeomorphically invariant states, has no prayer of being satisfied in the context of Thiemann’s proposal. The constraint algebra of LQG as it stands only closes for weaker notions of closure, such as “on-shell” closure which holds inside \mathcal{K} . Nicolai and collaborators argue that this spells bad news for Thiemann’s construction: if the constraint algebra only closes on-shell, i.e. after some of the constraints have already been “used,” but not off-shell, then one runs the risks of that the symmetries are anomalously implemented in the quantum constraints. The correct procedure, they insist, is to move ahead with solving the constraints only once the closure (and the anomaly-freeness) of the quantum constraint algebra is secured. They conclude by proposing off-shell closure of the algebra as a means to reducing the large number of ambiguities, and more importantly even, as the most promising way of imposing full spacetime covariance. It should be stressed, however, that these issues are technically subtle and no consensus has been reached so far. The jury, as it were, is still out on whether Thiemann’s construction is flawed or not.

Quite apart from these technically involved considerations, a necessary condition for the physical viability of the construction is its correct behaviour in the classical limit. Let us calculate the $|\mu| \gg 1$ limit then. The eigenvalue in (7.9) then undergoes the following modification:

$$\begin{aligned} \sqrt{\frac{6}{8\pi\beta G}} \left[\sqrt{|\mu+1|} - \sqrt{|\mu-1|} \right] &= \sqrt{\frac{6|\mu|}{8\pi\beta G}} \left[\sqrt{1+\frac{1}{\mu}} - \sqrt{1-\frac{1}{\mu}} \right] \\ &= \text{sgn}(\mu) \sqrt{\frac{6}{8\pi\beta G|\mu|}} + \mathcal{O}(\mu^{-3/2}). \end{aligned}$$

Thus, up to terms $\mathcal{O}(\mu^{-3/2})$, the eigenvalue for large $|\mu|$ is precisely equal to $\text{sgn}(p_\mu)/\sqrt{|p_\mu|}$ where p_μ is the eigenvalue of \hat{p} as given by (7.7). First, this justifies the somewhat excessive way of writing down the inverse operator in (7.9). Second, it agrees approximately with the classical relation between the momentum and the triad coefficient, given by $p \times (\text{sgn}(p)/\sqrt{|p|})^2 = 1$, which is another way of expressing the classical relation between the scale factor and the spatial curvature, essentially given by (C.2). The agreement is of course only valid in the limit for large universes, just where one expects the classical description to apply. Significant violations may only occur for sufficiently small $|\mu|$ and it turns out that they only transpire in the deep quantum regime of scales of less than a few Planck lengths.¹⁴

7.3 EVOLVING THROUGH THE BIG BANG

In order to complete the symmetry-reduced theory, the classical constraint (7.5) must be quantized and solved. This final step will or would yield the physical Hilbert space of LQC, denoted by \mathcal{H}^S . Without going into the details of the construction of the quantum Hamiltonian operator, which is intricate and surely not unique, a few brief remarks about the leading ideas behind the construction shall suffice.¹⁵ As was argued at the end of the foregoing section, the classical limit will provide an important constraint on how any such construction may proceed. The classical and quantum evolution are required to coincide for timelike infinity i^+ (and for i^-), but are expected to offer radically distinct pictures around the big bang. We will see in this section that LQC indeed satisfies—in fact: is constructed to satisfy—this requirement.

As in the full theory, the symmetry-reduced theory uses holonomies rather than the connection variables themselves as basic configuration variables. Because the classical constraint (7.5) is cast in terms of the connections c , it cannot directly be used to build the quantum Hamiltonian operator. One strategy to circumvent this obstacle would be to recast (7.5) in terms of holonomies in order to offer a vantage point for constructing the Hamiltonian oper-

¹⁴Bojowald (2001c) offers numerical calculations that estimate to which scale the classical geometry provides an adequate description. These numerical estimates in the symmetry-reduced models have to be taken with a grain of salt, numerically fully reliable results can, of course, only be expected from the full theory. These calculations, as all calculations in these simplified models, are nevertheless expected to offer insights into qualitative features and orders of magnitudes as they are built to concur with the full theory in these respects. Recently, physicists have started to more seriously study anisotropies and inhomogeneities in their cosmological models based on LQG. For the most up-to-date review of these efforts, see Bojowald (2005), who offers extensive discussions of how anisotropies and inhomogeneities are modifying the behaviour of the models.

¹⁵For the details, see Ashtekar et al. (2003a, Sec. 4).

ator of isotropic and homogeneous models in LQC. [Ashtekar, Bojowald, and Lewandowski \(2003a\)](#), however, choose a different approach in that they start out from the Hamiltonian of the full theory and simplify it by imposing isotropy and homogeneity. In principle, both procedures should yield the same result, but the second path has the advantage that it brings out the similarities with the full theory more closely. The pursued procedure parallels the one proposed by [Thiemann \(1996, 1998a,b,c,d,e,f\)](#) for the full theory. Thus, it inherits all the problems of Thiemann’s approach discussed above. On the up side, to repeat, this approach is the only known principled construction and is closely related to similar procedures followed in lattice gauge theories ([Ashtekar et al. 2003a](#)). The most important technical choices to be made concern quantization ambiguities related to the (continuous) length μ_0 of the sides of the triad in terms of the fiducial metric ${}^0q_{ab}$, which no longer drops out as it did in section 7.2. In addition to quantization ambiguities as they also arise in the full theory, LQC is confronted with the problem that the regulator can no longer be removed as a consequence. This difficulty is directly related to the fact that imposing homogeneity breaks the diffeomorphism invariance of the full theory.¹⁶ Within the context of the symmetry-reduced theory, the quantization ambiguity cannot be resolved in a natural way. The quantization ambiguity can be eliminated, however, by adopting considerations based on the full theory. The full theory, as discussed in chapter 5, predicts that space comes in small, indivisible chunks and that, therefore, it would be physically meaningless to consider scales μ_0 of arbitrary smallness. Assuming that μ_0 must be of the order of the minimum scale as predicted by the full theory, the quantization ambiguity can be removed. In consequence, while it would not be prohibited in the reduced theory to deal with arbitrarily small surface areas, a minimum area as suggested by the full theory is introduced in LQC.

With such a minimum scale at hand, the resulting “fundamental” vacuum Hamiltonian constraint operator for spatially flat¹⁷ models is given by

$$\widehat{C}_{\text{grav}} = \frac{96i}{\beta^3} \left(\frac{4}{\sqrt{3}} \right)^3 \sin^2 \frac{\sqrt{3}c}{8} \cos^2 \frac{\sqrt{3}c}{8} \left[\sin \frac{\sqrt{3}c}{8} \widehat{V} \cos \frac{\sqrt{3}c}{8} - \cos \frac{\sqrt{3}c}{8} \widehat{V} \sin \frac{\sqrt{3}c}{8} \right] \quad (7.10)$$

¹⁶The difficulty stems from the fact that triangulation of the manifold with tetrahedra of coordinate volume will no longer possess a well-behaved limit in homogeneous models, where an infinite number of tetrahedra equally contribute to the sum which replaced the integral of the (smeared) constraint function. Hence the sum diverges and the regulator cannot be eliminated. Cf. [Ashtekar et al. \(2003a\)](#).

¹⁷These are asymptotically open universes with spatial curvature $k = 0$. The formula becomes slightly, but insignificantly, more complicated for positive curvature, i.e. for closed universes. It does not apply for open universes with negative curvature, where a different method must be used ([Bojowald 2005](#), p. 43). For further discussion of closed models, see Section 8.2.

in units of ℓ_{Pl}^{-2} .¹⁸ Its action on the eigenstates of \hat{p} , which are also eigenstates of \hat{C}_{grav} , is as follows:

$$\hat{C}_{\text{grav}}|\mu\rangle = \frac{64}{\sqrt{3}\beta^3} \left(V_{\mu+\sqrt{3}/4} - V_{\mu-\sqrt{3}/4} \right) \left(|\mu + \sqrt{3}\rangle - 2|\mu\rangle + |\mu - \sqrt{3}\rangle \right), \quad (7.11)$$

again in units of ℓ_{Pl}^{-2} . LQC assumes the viewpoint that (7.11) correctly describes the gravitational part of the fundamental Hamiltonian constraint equation and that therefore its solutions will give us \mathcal{H}^S . This contribution together with the contribution from the matter Hamiltonian must add up to zero in order to satisfy the total Hamiltonian constraint¹⁹

$$\hat{C}|\Psi\rangle = (\hat{C}_{\text{grav}} + \hat{C}_{\text{matter}})|\Psi\rangle = 0. \quad (7.12)$$

Those states $|\Psi\rangle$ which solve this constraint equation are the physical states of the symmetry-reduced theory. As this reduced theory was intended to produce cosmological models based on LQG, each state $|\Psi\rangle$ which solves (7.12) can be considered as an admissible quantum cosmological model with the global symmetries “isotropy” and “homogeneity.” Obviously, these models will be constrained by the matter Hamiltonian \hat{C}_{matter} and potential coupling terms if matter is not minimally coupled to gravity. Typically in LQC, matter is assumed to have no curvature couplings.

As there exists a momentum representation, the physical states can be expanded in terms of this basis as

$$|\Psi\rangle = \sum_{\mu} \psi(\phi, \mu)|\mu\rangle, \quad (7.13)$$

where ϕ indicates the dependence of the coefficients ψ on the matter fields.²⁰ Since we know, courtesy of (7.11), how the gravitational constraint operator acts on eigenstates of

¹⁸According to Abhay Ashtekar (personal communication, 12 March 2005), this Hamiltonian is not self-adjoint, i.e. $\hat{C}_{\text{grav}} \neq (\hat{C}_{\text{grav}})^\dagger$. At the very least, because candidate physical states $|\Psi\rangle$ are really not necessarily normalizable states in the dual space of Cyl_S , this may have implications for picking the physical states and leads to complications in finding an inner product on the space of solutions of (7.14). See also footnote 20. On the occasion of the above mentioned conversation, Ashtekar promised that he and a collaborator were in the final steps of writing a paper giving a recipe for constructing a *self-adjoint* Hamiltonian operator for LQC. As far as I know, he has not yet archived or published such an article electronically or otherwise. There exist self-adjoint Hamiltonian operators for simplified models, cf. Section 8.2.

¹⁹Here, a constant factor has been absorbed into \hat{C}_{matter} , which is either $8\pi G$ in natural units or 8π in units of ℓ_{Pl}^{-2} , as in the following calculation.

²⁰In fact, the states $|\Psi\rangle$ live in Cyl_S^* , the space which is algebraically dual to the space Cyl_S of cylindrical functions of symmetric connections. Elements of this space need not be normalizable, a circumstance which can also be gleaned from the fact that the summation in (7.13) is over a *continuous* variable μ . Sometimes, as in Ashtekar et al. (2003a), states which may not be normalizable are written as $|\Psi\rangle$ rather than $|\Psi\rangle$. I will desist from introducing this notational subtlety.

the triad operator \widehat{p} , we can compute the total Hamiltonian constraint equation. This direct calculation will give

$$\sum_{\mu} \left[\psi(\phi, \mu) \left(V_{\mu+\sqrt{3}/4} - V_{\mu-\sqrt{3}/4} \right) \left(|\mu + \sqrt{3}\rangle - 2|\mu\rangle + |\mu - \sqrt{3}\rangle \right) + \frac{\sqrt{3}\beta^3}{64} \widehat{C}_{\text{matter}} \psi(\phi, \mu) |\mu\rangle \right] = 0.$$

There are obviously (sub-)terms within each term of this sum where the eigenstates have been shifted by $\sqrt{3}$, which results in this rather awkward way of writing down the sum. This can be rectified by regrouping the terms considering that the “contribution” of the $(\mu \pm \sqrt{3})$ -th term of the sum to the μ -th term is $\psi(\phi, \mu \pm \sqrt{3}) \left(V_{\mu \pm 5\sqrt{3}/4} - V_{\mu \pm 3\sqrt{3}/4} \right) |\mu\rangle$. Rearranging the sum with this mind yields a sum—the same sum of course—with each (sub-)term of the μ -th term ending in $|\mu\rangle$. A sufficient condition for this rewritten sum to vanish, and thus to accord with (7.12), is that each term vanishes by itself:

$$\begin{aligned} & \left(V_{\mu+5\sqrt{3}/4} - V_{\mu+3\sqrt{3}/4} \right) \psi(\phi, \mu + \sqrt{3}) - 2 \left(V_{\mu+\sqrt{3}/4} - V_{\mu-\sqrt{3}/4} \right) \psi(\phi, \mu) \\ & + \left(V_{\mu-3\sqrt{3}/4} - V_{\mu-5\sqrt{3}/4} \right) \psi(\phi, \mu - \sqrt{3}) = -\frac{\sqrt{3}\beta^3}{64} \widehat{C}_{\text{matter}}(\mu) \psi(\phi, \mu) \end{aligned} \quad (7.14)$$

In my opinion, this equation is not a *necessary* condition for the constraint (7.12) to be satisfied, just because it is not necessary for every term in a sum to individually vanish in order for the total sum to add up to zero. However, in [Ashtekar, Bojowald, and Lewandowski \(2003a\)](#), just as well as in the rest of the literature, the impression is given that (7.14) is both sufficient and necessary for (7.12) to obtain. Be this as it may, I should remind the reader that the V ’s in (7.14) are the eigenvalues of the volume operator and are just known real numbers. The matter Hamiltonian $\widehat{C}_{\text{matter}}$ only acts on the matter fields (encoded by ϕ) and is a function of μ insofar as metric components will generally arise in its expression. Importantly, equation (7.14) is not a differential equation, as would traditionally be expected, but a *difference* equation in the parameter μ , which is nevertheless continuous. According to [Ashtekar, Bojowald, and Lewandowski \(2003a\)](#), this is a direct consequence of the discreteness of the quantum geometry. As I have not followed the construction of the Hamiltonian constraint operator explicitly, I cannot authoritatively judge what the origin of this is, despite the plausibility of the explanation that it is intimately connected with the discreteness of the quantum geometry. The tandem fact that (7.14) is a difference equation together with the discreteness of μ will have a bearing on the interpretation of equation (7.14) *qua* codification of the dynamical evolution, to be discussed in section 8.1.

Let me restate the strict significance of (7.14): it is not primarily an evolution equation, but it constrains the coefficients $\psi(\phi, \mu)$ in the expansion of the triad basis $|\mu\rangle$ of any state $|\Psi\rangle$ which wishes to qualify as a physical state of the symmetry-reduced theory. States $|\Psi\rangle$ whose components in terms of the triad basis conform with (7.14) are the physical states sought and constitute the physical Hilbert space of the theory. This means that the situation should best be interpreted such that physical states do not undergo a dynamical evolution through the values of μ to be summed over, but rather as a *superposition* of kinematical states corresponding to spatial universes of different sizes, where the manner how these kinematic state superpose is constrained by (7.14). Agreed, (7.14) is as close as it gets to temporal evolution in the present setting in that a partial observable, encoded by the inverse scale factor operator, which is used as a cosmological clock and with respect to which an “evolution” can be determined. I urge, however, the need to be chary of this interpretation because the “evolution” governed by (7.14) fails to be deterministic. I will come back to this point in Section 8.1.

In order to finalize the construction of the physical Hilbert space \mathcal{H}^S , one would also have to introduce an inner product and thus select those states $|\Psi\rangle$ with finite norm. The physical Hilbert space, then, is spanned by those states of finite norm which satisfy the Hamiltonian constraint equation (7.14). The construction of the general physical Hilbert space of LQC *including the physical inner product* has not been achieved so far and remains an open issue. For some simplified models, the construction has been executed and led to the surprising result that the physical Hilbert space consists of only one single state, leaving us with just one cosmological model! I will return to this issue in section 8.2.1.

Another important feature of the constraint equation (7.14) is the fact that it will also impose conditions on coefficients $\psi(\phi, \mu)$ with negative μ . These coefficients accompany base states of negative μ in the construction of Ψ . Of course, they do not correspond to kinematic universes with negative volumes, but to universes of volume of the respective absolute values. As triad operator eigenstates and their corresponding coefficients of negative and positive μ are interlinked via (7.14), the states of negative μ are not only admissible kinematically, but will also participate in the construction of the physical states $|\Psi\rangle$. So in a sense, the radically distinct dynamical behaviour around the classical singularity suffices to open up the formerly closed and locked door to a mirror world, as can be seen in Figure 11. Thus, a second domain, call it *region II*, emerges as a result of evolving through the big bang and represents the mirror universe “of negative times.” In Figure 11, region I with $\mu > 0$ corresponds to the usual classical domain of a FLRW model. Unlike in the classical case, however, it also contains the big bang ($\mu = 0$) and the mirror region II where $\mu < 0$. Does the emergence of this mirror world imply that the denizens of region II experience their universe

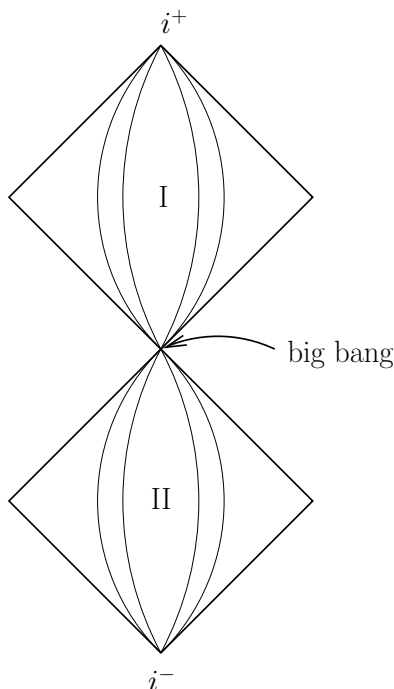


Figure 11: Conformal diagram of a FLRW model in LQC.

to collapse towards a non-singular big crunch? Not necessarily. It is perfectly possible that the direction of the physical time does not coincide with the direction of increasing values of μ , but opposes it. This would be the case, say, if the direction of physical time rests on a thermodynamical asymmetry which points in the direction of increasing absolute values of the scale factor. In this case, the citizens of II would also experience a universe evolving from a non-singular big bang. In this case, the scenario would resemble the birth of two twin universes rather than one single universe contracting and then expanding again. These remarks are highly speculative and should be taken with a grain of salt: their final validity must await the completion of the construction of the physical Hilbert space.

Two more remarks, both equally preliminary. First, while many members of the guild expected a quantization to wash away the singularities of the classical theory, the fact that LQC seems to predict an inflationary phase in the evolution of the early universe came as a surprise. The expansion follows a non-Friedmannian regime for small μ . Instead of a decelerating expansion from an initial singularity, this quantum regime imposes a rapidly accelerating expansion on the young universe.²¹ Unlike in inflationary scenarios, however,

²¹In fact, the expansion is powered by a^{12} instead of a^{-3} as in the Friedmann case! Cf. Rovelli (2004,

this inflation is not driven by an inflaton field or similar auxiliary constructions to account for the rapid expansion. In LQC, inflation seems to be generated merely by the quantum modifications of the dynamical equations of the gravitational field in the homogeneous and isotropic case (Bojowald 2002a; Bojowald et al. 2004a; Bojowald and Vandersloot 2003).

Second, recent results in Ashtekar and Bojowald (2006) show that the same technique applied to Schwarzschild spacetime leads to a similar resolution of the singularity there. Ashtekar and Bojowald (2005) have taken these results to indicate that perhaps all spacelike singularities of the classical theory might vanish in LQC. It may be preemptive to condense these findings to a claim that cosmology based on LQG, unlike the one based on classical GTR, is not tainted by the occurrence of singularities at all. But there have appeared by now clear indications that the quantum nature of LQC inoculates important cosmological models against incurring singularities.

In sum, then, in isotropic and homogeneous models of loop quantum cosmology the initial singularity of the classical model seems to disappear in two different senses: first, the curvature does not increase without bound for arbitrarily small scale factors; and second, there exists a principled—though perhaps not correct—way of extending the models through the initial singularity into a mirror world, thereby circumventing, to some extent at least, the classical singularity. Naïvely, interpreting the dynamical evolution of these models leads to a contracting and subsequently expanding universe. The classical and quantum evolution coincide for the timelike infinity i^+ (and for i^-), but offer radically distinct pictures around the big bang. Numerical analyses of very simple cases show that the classical and the quantum Friedmann models evolve indistinguishably, except in a very small region around the classical singularity where no corresponding semi-classical states seem to be available. The fact that the state vector becomes indeterminate for $\mu = 0$, as will be discussed in Section 8.1, suggests that the dynamical evolution around the classical big bang cannot be straightforwardly deterministic despite its regularity. This, and the general problem of entering a sector of the Hilbert space whose states do not afford corresponding semi-classical states and thus elude a classical interpretation, leads us to believe that a number of subtleties need to be clarified before we can meaningfully speak about what happened “before the big bang.” More than a mere subtlety is the fact that the operators \widehat{p}^{-1} and \widehat{p}^{-1} do not correspond to Dirac observables. This seriously undercuts their interpretation as physically relevant magnitudes, as will be pursued in Chapter 8.

pp. 216f).

8.0 EINSTEIN’S NEMESIS CONQUERED AT LAST?

8.1 PHYSICAL CURVATURE AND EPICYCLES OF DETERMINISM

Let us now take a more critical look at the issue of whether, and if so to what extent, the kinematic as well as the dynamical singularity indeed vanish in cosmological models based on LQG. As I have explicated in the previous section, the strategy pursued by LQC to solve the Hamiltonian constraint equation is to assume an arbitrary superposition of kinematical states (7.13), more specifically eigenstates of \hat{p} , and then obtain a difference equation of components $\psi(\phi, \mu)$ (7.14), which can be recursively solved for a given set of “initial conditions.” The standard interpretation of this difference equation, to repeat, is that of an evolution equation like the Schrödinger equation in ordinary quantum mechanics where the evolution is powered by the Hamiltonian constraint operator \hat{C} . To be sure, the physical states $|\Psi\rangle$ correspond to four-dimensional spacetimes. Trivially, therefore, these physical states do not undergo a dynamical evolution. The states whose evolution is governed by (7.14) are the kinematical states $|\mu\rangle$, via their components $\psi(\phi, \mu)$. The “four-dimensional” physical quantum states will then be constructed as a superposition of kinematical states, using the components constrained by (7.14). If this constraint equation is interpreted as an equation of dynamical evolution, as a “quantum Friedmann equation” as it were, then the scale parameter μ must be interpreted as a time variable with respect to which this evolution occurs. The scale operator, defined on the kinematical Hilbert space \mathcal{K}^S , does not correspond to a Dirac observable. Dirac observables must commute with all the first-class constraints generating gauge transformations, and hence with the Hamiltonian constraint as well. For a candidate operator \hat{O} , we must thus have

$$[\hat{O}, \hat{C}] |\Psi\rangle = 0 \tag{8.1}$$

for \hat{O} to qualify as a *physical operator*, i.e. an operator corresponding to a Dirac observable. As computed in appendix D, the commutator of the inverse scale factor operator and the Hamiltonian constraint operator does not vanish, except perhaps for very special choices of \hat{C}_{matter} . But this seems hardly acceptable: whether \hat{O} corresponds to a physical observable

depends on the matter content of the universe. Be this as it may, the inverse scale factor operator \widehat{p}^{-1} is not a physical operator and its spectrum can thus not be considered as an indication of possible measurements of the spatial curvature. Consequently, the fact that this spectrum is bounded from above does not issue a physically relevant warrant that the kinematical singularity is indeed avoided.¹ In order for this conclusion to be legitimately drawn, a physical operator corresponding to an observable encoding spatial curvature would be necessary.

The intuitive physical reason as to why both the inverse scale factor operator as well as the volume operator are not Dirac observables is that in order to obtain a volume of a three-dimensional universe from a four-dimensional cosmological model, one needs to introduce a particular gauge. The volume, and the curvature, of the three-dimensional slices will depend on the particular foliation introduced. There are good, but not compelling, reasons for defending a notion of privileged cosmological time at the level of the classical theory.² It is not so obvious, however, whether such a preferred spacetime foliation of cosmological models can be incorporated into models in quantum cosmology. Perhaps the so-called “group-averaging” technique can be used to introduce a coordinate time into the quantum theory, which, according to an analysis by [Bojowald, Singh, and Skirzewski \(2004b\)](#), is well-suited to emulate an evolution at least in the semi-classical regime. Around what corresponds to the initial singularity of classical models, the method does not yield a useful notion of time. Furthermore, it is unclear whether this coordinate time has any claim to offer a preferred notion of time. If it does not, then it seems as if there might be no justification for singling out the volume (and the inverse scale factor) operator as defined above as physically privileged.

Revisiting Rovelli’s idea of partial observables with a more mathematical bent, [Dittrich \(2004\)](#) has shown that for classical theories, if we have a phase space function which is invariant under a subalgebra $\mathfrak{C}_1 := \{C_{m+1}, \dots, C_n\}$ of $n - m$ constraints of the constraint algebra \mathfrak{C} containing a total of n constraints, then these functions form *partial observables* in the sense of [Rovelli \(2002d\)](#), i.e. they are physical quantities with an associated measuring operation resulting in a numerical outcome. Dittrich commences by assuming that any function in phase space represents such a physical quantity. Following Rovelli’s suggested procedure, she shows how these partial observables can be transformed into so-called *complete observables* with respect to the subalgebra $\mathfrak{C}_2 := \{C_1, \dots, C_m\}$ of the remaining m constraints. Complete observables in Rovelli’s sense are magnitudes which can be gained in some principled way from the (classical or quantum) theory at stake. Dittrich further proves two theorems (The-

¹The same point is made by [Brunnemann and Thiemann \(2006a\)](#), cf. Section 8.2.4 below.

²[Belot \(2005\)](#) has argued that two important arguments against such a preferred spacetime foliation of cosmological models in classical GTR fail to establish their purported conclusion.

orem 3.1 for a system with one constraint and Theorem 4.1 for a system with n constraints) which establish that, under some technical assumptions, complete observables as defined by Rovelli must be Dirac observables. The reader should be reminded at this point that the invariance group of these observables is not $\text{Diff}(\mathcal{M})$, but $\text{Diff}(\Sigma)$ combined with deformations in the direction normal to Σ (and rotations of the tetrads). In sum, Dittrich offers a detailed procedure how to construct Dirac observables from phase space functions which are invariant under a subalgebra \mathfrak{C}_1 of the constraint algebra.

Once one has the Poisson algebra of the Dirac observables, canonical quantization requires a representation of this Poisson algebra on a physical Hilbert space. This procedure turns the classical Dirac observables—if they have been identified—into operators of the quantum theory. As it is technically difficult to straightforwardly implement this strategy, LQG proceeds by first finding a representation of the algebra of partial observables on a kinematical Hilbert space, as explained in detail in Chapter 5 and as criticized by [Nicolai, Peeters, and Zamaklar \(2005\)](#). These partial observables are invariant under both rotations of the tetrads as well as spatial diffeomorphisms, as the Gauss and vector constraints have been solved in the construction of the kinematical theory. These partially invariant partial observables thus lead to the construction of non-physical operators defined on the kinematical Hilbert space \mathcal{K} . Examples of these non-physical operators are exactly the geometric operators such as the area and the volume operators in full LQG as well as the scale factor and the inverse scale factor operators in LQC.

Although the inverse scale factor operator does not commute with the Hamiltonian constraint and does therefore not correspond to a Dirac observable, its commutator with both the $SU(2)$ and the spatial diffeomorphism constraints vanishes. Thus, it constitutes an operator corresponding to a partially invariant partial observable as discussed by [Dittrich \(2004\)](#). As explained above, one can turn such a partial observable into a Dirac observable invariant under all constraints. The problem, however, [Brunnemann and Thiemann \(2006a\)](#) warn, is that the spectrum of the operator corresponding to the Dirac observable will in general differ from the kinematical spectrum, and sometimes drastically so. This means that the fact that the kinematical spectrum of the inverse scale factor operator is bounded from above cannot be used to produce reliable statements concerning the quantum fate of the classically divergent curvature. This would only be possible if the spectrum of the pertinent physical operator were known. For this, however, the physical Hilbert space would have to be known. The fact that the inverse scale factor is not a Dirac observable poses a real problem as it is unclear whether one can legitimately infer from the fact that the spectrum of the corresponding operator is bounded that the kinematical singularity is resolved. I will return to this point in Section 8.2.4 when I discuss Brunnemann and Thiemann’s argument.

What about the dynamical singularity? In order to evaluate the claim made in LQC that the dynamical singularity is completely eliminated, closer scrutiny must be applied to the quantum equations encoding the dynamical evolution, the “quantum Einstein equations” as it were. The role analogous to the Schrödinger equation in ordinary quantum mechanics, to repeat, is played by the Hamiltonian constraint equation. This equation is interpreted as a difference equation for the coefficients of the kinematical states in the superposition as which the physical state is assumed to be. This difference equation effectively offers a recursive relation for the coefficients. In order for such an equation to qualify as a dynamical equation, the label of these coefficients must play the role of a temporal parameter with respect to which the evolution of the system through the consecutive kinematical states occurs. Since the kinematical states used in LQC are the eigenstates $|\mu\rangle$ of the triad operator \hat{p} , which is simultaneously an eigenstate of the inverse scale factor operator, the dynamical label is μ . However, for the same reasons as above, this label is unphysical as the scale factor is not a Dirac observable. Hence, we have a dynamical evolution which covers the full range of the unphysical parameter μ .

In order to determine whether the singularity in the dynamical evolution with respect to μ evaporates in LQC, a brief reminder how the dynamical singularity came about in the classical theory is in order. In appendix C, I noted that Laplacean determinism breaks down at the big bang in the sense that the Friedmann equation (C.5) breaks down at $a = 0$. That’s why it turned out to be classically impossible to evolve the universe back to cosmological times before the big bang. Thus, determinism played a critical role in evaluating whether the dynamical behaviour of the universe was singular or not. The idea of testing for the remnants of a dynamical singularity in the context of the proposed models of LQC thus focuses on whether, given a post-big-bang “initial state” of the universe, the difference equation (7.14) permits the deterministic evolution past the kinematical state $|\mu = 0\rangle$ to earlier times.

Specifying an initial state to be fed into a differential equation only requires the specification of the physical system whose dynamical evolution is governed by the differential equation *at one particular instant in time*. This information suffices to determine the state of the system at all other times for which the system enjoys a deterministic evolution. For a difference equation, such as (7.14), obviously, specifying the state of the universe at only one “instant” in the unphysical time parameter does not suffice to even get started: at least two of the coefficients must be known in order to calculate the coefficients for the other μ ’s via (7.14). The eigenvalues V_μ of the volume operators are given real numbers, but the action of \hat{C}_{matter} on $|\mu\rangle$ will depend on what kind of matter is supplied. For present purposes, it must only be assumed that $|\mu\rangle$ are also eigenstates of \hat{C}_{matter} . If \hat{C}_{matter} is self-adjoint, then its eigenvalues will be real numbers as well.

Without loss of generality, let us position the two fixed coefficients ψ corresponding to the two “initial” “states” in the epoch a “long” “time” “before the big bang” and evolve the universe in the “forward” direction to “later” “times” closer and closer to the big bang. Let me briefly recall the reasons for the various quotation marks in the previous sentence: the “initial” “states” are of course by no means “initial,” there are infinitely many more both “before” and “after,” nor are they physical states, but only kinematical ones; furthermore, “long” is only long in terms of Planck units as the asymptotic region starts at $|\mu| \gg 10$ Planck times; also, the “time” parameter does not correspond to a physical notion of time, nor is there a physically justified direction of time in the world to the other side of the big bang. The last point was discussed above when I insisted that the model might just as well represent the birth of a twin universe rather than a contracting and then re-expanding universe. But I will drop the distracting quotation marks for the remainder of the section and only point out the time-reversal invariance of equation (7.14) before the argument proceeds.

Thus, we start evolving the universe at $\mu = -N\sqrt{3}$ for any large $N \in \mathbb{R}^+$. We need to fix the two coefficients $\psi(\phi, -N\sqrt{3})$ and $\psi(\phi, (-N+1)\sqrt{3})$ in order to recursively determine the coefficients $\psi(\phi, (-N+n)\sqrt{3})$ via (7.14) for all natural numbers $n > 1$ (and really $\forall n \in \mathbb{Z}$). In general, no difficulties will be encountered in this procedure because the discrete iterative steps one takes in evolving the universe in this manner generically step over the potential singularity at $\mu = 0$. Figure 12 schematically shows how this iterative determination of the coefficients $\psi(\phi, \mu)$ works, and how two legs to stand on (marked by a bar) are required to calculate the next step to be taken (marked by a cross). The gap between adjacent steps is $\sqrt{3}$ in fundamental units. It is clear, however, that the two coefficients which were given to us do not suffice to determine the full evolution across all values of $\mu \in \mathbb{R}$. Equivalently, the two coefficients are insufficient to fully specify the physical states $|\Psi\rangle \in \mathcal{H}^S$, they only determine a discrete subset of Lebesgue-measure zero. For the full specification, an entire half-open interval $\mu \in [-N\sqrt{3}, (-N+2)\sqrt{3}[$ of coefficients $\psi(\phi, \mu)$ must be fixed beforehand, i.e. we must know the state of the kinematical universe over the period of $2\sqrt{3}$ Planck times in order to be in the position to fully determine its evolution for all times.

When we run the recursive relation (7.14) for all values of μ in this half-open interval, we realize that for one choice, and for one choice only, we will encounter the presumed singularity at $\mu = 0$ “head-on.” If and only if N is a natural number, the evolution will step into the sink of the $\mu = 0$ singularity in the sense that there will be a $\psi(\phi, (-N+n)\sqrt{3})$ such that the combination of V_μ ’s vanishes. In general, this combination of volume operator eigenvalues is

$$V_{(-N+n+\frac{1}{4})\sqrt{3}} - V_{(-N+n-\frac{1}{4})\sqrt{3}}$$

for the coefficient $\psi(\phi, (-N+n)\sqrt{3})$ to be determined in the $(n-1)$ -th step of the iteration.

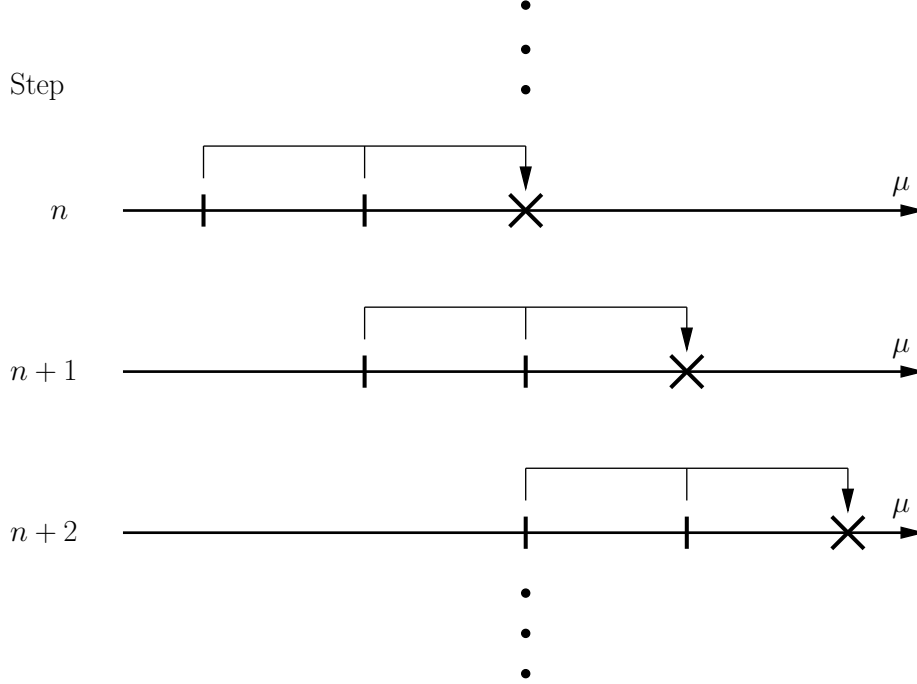


Figure 12: Recursive determination of the coefficients $\psi(\phi, \mu)$.

This combination is zero just in case $N = n$, i.e. iff $N \in \mathbb{N}$, since $V_\mu = V_{-\mu}$. When this happens, the corresponding coefficient just drops out of equation (7.14), which in this case just becomes an additional constraint on $\psi(\phi, -\sqrt{3})$ and $\psi(\phi, -2\sqrt{3})$:

$$\left[\frac{\sqrt{3}\beta^3}{64} \hat{C}_{\text{matter}}(-\sqrt{3}) - 2V_{-3\sqrt{3}/4} + 2V_{-5\sqrt{3}/4} \right] \psi(\phi, -\sqrt{3}) + \left(V_{-7\sqrt{3}/4} - V_{-9\sqrt{3}/4} \right) \psi(\phi, -2\sqrt{3}) = 0. \quad (8.2)$$

The attentive reader may argue that while the Hamiltonian constraint equation for $\mu = (-N + n - 1)\sqrt{3}$ (or, equivalently, the $(n - 1)$ -th step in the iteration) may fail to determine the coefficient $\psi(\phi, 0)$ in the same manner as it did for all other coefficients, the Hamiltonian constraint equation for $\mu = (-N + n)\sqrt{3} = 0$ (or, equivalently, the n -th step of the iteration) will do so. The difference is illustrated in Figure 13: while up to this point, I have only described how states have been determined, as in (a), by two earlier states, the constraint equation may also be used to specify the “sandwich state” at equal distances from two input states, an earlier and a later one, as shown in (b). Let me call the technique used in (a) *tripod technique* and the one in (b) the *sandwich technique*.

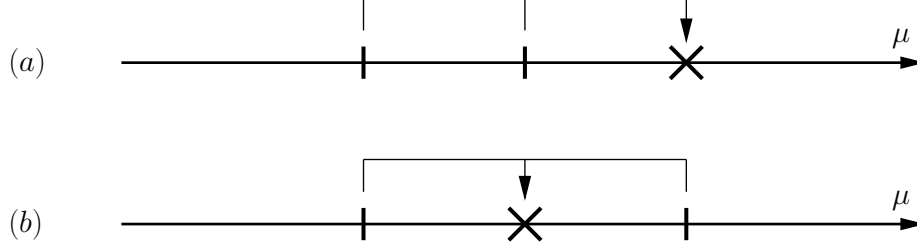


Figure 13: Two different ways of determining a third state from two given states.

The Hamiltonian constraint equation for $\mu = (-N + n)\sqrt{3} = 0$ reads

$$\begin{aligned} \left(V_{5\sqrt{3}/4} - V_{3\sqrt{3}/4} \right) \psi(\phi, \sqrt{3}) + \frac{\sqrt{3}\beta^3}{64} \widehat{C}_{\text{matter}}(0) \psi(\phi, 0) \\ + \left(V_{-3\sqrt{3}/4} - V_{-5\sqrt{3}/4} \right) \psi(\phi, -\sqrt{3}) = 0. \end{aligned} \quad (8.3)$$

Unless $\widehat{C}_{\text{matter}}(0)$ vanishes as well, it seems as if one might have an equation which determines $\psi(\phi, 0)$, using the sandwich technique if the tripod technique fails. Alas, this is not the case: an argument by [Bojowald \(2002b, Sec. 4.2\)](#) claims that generically for quantum matter, we have $\widehat{C}_{\text{matter}}(0) = 0$, implying, of course, the impossibility in LQC of determining the kinematical state of the universe at the big bang. This argument is either sound and we have $\widehat{C}_{\text{matter}}(0) = 0$ (call this situation I) or it is not and $\widehat{C}_{\text{matter}}(0) \neq 0$ (situation II),³ in both cases the state $\psi(\phi, 0)$ cannot be determined, neither by the tripod nor by the sandwich technique, for the following reason.

If Bojowald’s argument is sound, then (8.3) will determine the heretofore undetermined coefficient $\psi(\phi, \sqrt{3})$ by the tripod technique with no entry from the second “foot” and the deterministic evolution can continue through the big bang. In this case, all coefficients $\psi(\phi, \mu)$ will be determined by the half-open interval of initial conditions except the singularity $\psi(\phi, 0)$, which decouples. In this situation, we have an additional constraint (8.2), but also an additional freedom in choosing the decoupled coefficient $\psi(\phi, 0)$. The first restriction makes the solution space smaller than it would be otherwise, while the latter freedom enlarges it just as much. So in situation I, the full physical state $|\Psi\rangle$ is determined by the half-open interval of initial conditions, except for the kinematical state at the big bang, which can be chosen arbitrarily.

³I am neglecting here the logical possibility that the argument is not sound, but $\widehat{C}_{\text{matter}}(0)$ nevertheless vanishes. What is really of interest here is whether $\widehat{C}_{\text{matter}}(0)$ is zero or not.

If the argument is not sound, however, and it may be the case that $\widehat{C}_{\text{matter}}(0) \neq 0$, then (8.3) turns into an equation with two unknowns ($\psi(\phi, 0)$ and $\psi(\phi, \sqrt{3})$) and both the tripod as well as the sandwich technique strand. Thus, equation (8.3) fails to offer a deterministic evolution beyond the big bang. As $\psi(\phi, 0)$ and $\psi(\phi, \sqrt{3})$ are both not determined then, no $\psi(\phi, n\sqrt{3})$ for $n > N$ can be determined anymore, as one would have to use the tripod method, without any legs to stand on. To answer the attentive reader from a few paragraphs ago, the plan to apply the sandwich method for determining $\psi(\phi, 0)$ miscarries as there is no possibility at all to obtain $\psi(\phi, \sqrt{3})$ from the initial data. The evolution thus grinds to a halt at the singularity, at least for appropriately chosen initial conditions. What could save the situation II is an additional choice to fix one of the two unknowns $\psi(\phi, 0)$ or $\psi(\phi, \sqrt{3})$, thus enabling (8.3) to determine the other unknown by the tripod technique if $\psi(\phi, 0)$ is additionally fixed or by the sandwich method if $\psi(\phi, \sqrt{3})$ is given in addition. Thus, an additional choice of one coefficient allows the iteration to continue.⁴ Given such an additional choice, all the coefficients would be determined by the usual half-open interval of initial conditions, without exception, and the state $|\mu = 0\rangle$ would not decouple from the rest. Because we have such an additional freedom, the space of solutions would be larger than originally presumed.⁵ If such an enlargement is not tolerated, or, equivalently, no additional choice is made, the evolution runs aground at the big bang. In situation II, therefore, the half-open interval of initial conditions determines the physical state $|\Psi\rangle$ only up to a discrete subset of coefficients $\psi(\phi, n\sqrt{3})$ (where $\mathbb{N} \ni n \geq N$).

Situation II could be remedied, as mentioned, by adding another initially fixed coefficient. It would have to be one of the $\psi(\phi, n\sqrt{3})$ for $n \geq N$. But what a weird package of initial conditions: a set of kinematical states of the universe during a half-open interval of time long before the big bang plus a single kinematical state much later, either at the big bang or sometime thereafter. In this odd case, the total set of initial conditions required to determine the state of the physical system for all times would be topologically unconnected. The necessity to know the state of a system over some finite amount of time, as opposed to just one instant in time, may already appear unfamiliar from the usual deterministic laws of motion, which are typically differential equations. But at least we are still dealing with a connected set, which in principle corresponds to the situation in a physics laboratory, where today's state of a system, albeit over a finite period of time, is measured in order to make predictions of the system's state at one point in time tomorrow. Why, however, should it be necessary to know the system's state over a finite period of time today, plus its state at one

⁴In fact, it suffices to fix any one of the $\psi(\phi, n\sqrt{3})$ for $n \geq N$ to render the evolution fully deterministic.

⁵Of course, there still is the extra constraint (8.2), which diminishes the solution space accordingly. But this constraint exists quite independently of the other considerations.

moment in time next week in order to be in the position to predict its state at a particular time in two weeks' time? The question is rhetorical, as any attempt to insist on such a disconnected set of initial data appears grotesquely ad hoc.

Of course, the situation can be repaired by placing the half-open interval such that it contains the moment of the big bang. Here, the point $\mu = 0$ must be contained in the half-open interval of Lebesgue-measure $2\sqrt{3}$ in natural units. In this case, equation (8.2) no longer represents an additional constraint diminishing the space of physical states $|\Psi\rangle$, but a recursive relation helping in determining the coefficients backwards in time. If $\psi(\phi, 0)$ is given as initial data, then either $\psi(\phi, -\sqrt{3})$ is also given or it is not, depending on how exactly the half-open interval of initial data is chosen. If it isn't, then $\psi(\phi, \sqrt{3})$ must be given and $\psi(\phi, -\sqrt{3})$ can be obtained via (8.3), quite independently of whether $\widehat{C}_{\text{matter}}(0)$ vanishes or not. In any case, then, $\psi(\phi, -\sqrt{3})$ is given or can be obtained. With this information, (8.2) will be used to calculate $\psi(\phi, -2\sqrt{3})$, applying the tripod technique.

Regardless of whether $\widehat{C}_{\text{matter}}(0)$ is zero or not and of how the exact interval of initial data is chosen, all the coefficients $\psi(\phi, \mu)$ can be determined once the state of the universe at the big bang is among the initially known kinematical states. Under these circumstances, then, the dynamical singularity disappears. It does not, however, fully evaporate when $\psi(\phi, 0)$ is not initially fixed.

Despite the appealing consequence that the dynamical singularity is lost once one elects to include the kinematical state of the universe at the big bang, I maintain that this case should be excluded from serious considerations. We are essentially interested in finding out whether in the context of LQC, the dynamical singularity vanishes in the sense that one can deterministically evolve through the big bang. If it turns out that the only case in which determinism is fully valid for all values of μ occurs when the kinematical state of the universe at what corresponds to the classical singularity is fixed in advance, when, in other words, the delicate situation is taken care of by hand, then the case for the full dynamical regularity of the models of LQC is indeed very weak. For the remainder of the section, I shall assume that the set of initial conditions does not contain the state at $\mu = 0$.

Early in this chapter, I have proposed to use the criterion of whether the dynamical evolution occurs deterministically as the benchmark for whether or not it could be declared that there is an important sense in which the singularity disappears in the context of LQC. To be sure, the concept of deterministic evolution is already weakened in any quantum theory, but the Schrödinger evolution of an initial quantum state can nevertheless be considered fully deterministic, at least as long as the Hamiltonian is essentially self-adjoint, as defined in footnote 15.⁶

⁶For a more thorough discussion of the fate of determinism in quantum theories, cf. Earman (2004, Sec. 5)

Let me analyze to what extent this evolution can be considered regular, now that I have explicated in detail how models in LQC can be considered as dynamically evolving. To this end, let me cast the notion of Laplacean determinism in a more precise definition:

Definition 6 (Laplacean determinism). *The evolution of a physical system is (Laplace-) deterministic iff the state of the system at a time t together with the laws of nature that govern its dynamical evolution fix the state of the system at all times.*

Obviously, I assume the unproblematic existence of an external (physical) time in which the evolution of the physical system at stake occurs. To repeat, this is not the case for classical GTR, nor for LQG, nor for LQC. Definition 6, therefore, would really have to be appropriated for the generally covariant case in order to yield fully adequate categories for the situation that we are interested in. Let us, for the argument's sake, join most proponents of LQC in admitting the scale factor μ as a surrogate time as explicated above. In this case, Definition 6 is sufficient to frame the ensuing discussion.

Another deficiency of Definition 6, however, is immediately obvious. As explicated above at some length, the dynamical equation of interest is a difference rather than a differential equation. Offering the state of the system at only one point in time, therefore, will not suffice to get the evolution going. Not only must a half-open interval of initial data be specified, but the dynamical mechanism seems to be rather different: instead of seeking differentiable functions solving differential equations, one must drudge through uncountably many recursive relations for infinitely many iterations in order to evolve the system. It seems warranted, therefore, to denote the latter type of evolution by *evolution** in order to draw the distinction notationally. Let us customize the definition of determinism for present purposes as follows:

Definition 7 (Laplacean determinism for systems governed by difference equations). *The evolution* of a physical system is (Laplace-) deterministic iff the states of a system over a finite period of time together with the difference equation(s) that govern(s) its dynamical evolution* fix the state of the system at all times.*

Assuming, as I have, that initial conditions do not already specify the state of the universe at the big bang, determinism thus defined is violated in LQC for both situations (I) and (II). In situation (I), with $\widehat{C}_{\text{matter}}(0) = 0$, the coefficient $\psi(\phi, 0)$ decouples from the rest and cannot be determined by the Hamiltonian constraint equation (7.14). In situation (II), with $\widehat{C}_{\text{matter}}(0) \neq 0$ and not allowing the set of initial conditions to consist of several disjoint pieces, not only does (7.14) not determine the coefficient $\psi(\phi, 0)$, but leaves undetermined any $\psi(\phi, n\sqrt{3})$ for $n \geq N$ with $N \in \mathbb{N}$. Clearly, therefore, the unphysical evolution* typically and Earman (2006b).

present in LQC does not obey Laplacean determinism as introduced in Definition 7. It becomes also clear that what is responsible for the breakdown of determinism is still what corresponds to the classical singularity. In conclusion, the initial singularity of the classical theory has not completely been washed out by the quantum theory.

Having said that, however, the situation is not quite as bleak as this melancholy conclusion might suggest. Despite the failure of strict determinism, there is a weakened form of determinism which is realized in LQC. An appealing aspect of this weakening is that it is mathematically well-defined, and not just an ad hoc concept introduced to satisfy the philosophers' fancy.

Definition 8 (Quasi-determinism (for systems governed by difference equations)).

The evolution of a physical system is quasi-deterministic iff the states of a system over a finite interval of time together with the difference equation(s) that govern(s) its dynamical evolution* fix the state of the system at almost all times, i.e. they fix the state for all points in time except for a subset of points in time of Lebesgue-measure zero.*

Since the states undetermined by the initial data combined with the Hamiltonian constraint equation (7.14) are either just those of one moment in time ($\mu = 0$)—as in situation (I)—or of a discrete subset of the real number ($\mu = n\sqrt{3}, \forall n \geq N$)—as in situation (II)—, the undetermined states form a subset of all kinematical states through which the system evolves* with Lebesgue-measure zero. Thus, the dynamical evolution* of the universe as understood in LQC proceeds quasi-deterministically, even through the big bang.

Together with the above mentioned fact that the quantum states around the big bang have no semi-classical analogues and that it may thus be impossible to introduce a notion of physical time in the very early universe, the failure of strict determinism at the big bang may be taken as an indication that it was wrong-headed all along to insist on the pervasive rhetoric of evolution and determinism. In order to be able to meaningfully talk about how the universe looked like before the big bang, it seems, a globally applicable physical time would have to be introduced. I have observed with some satisfaction that proponents of LQC have considerably toned down their rhetoric of dynamical evolution over the last few months. At the Seventh International Conference on the History of General Relativity which took place on La Tenerife from 9-15 March 2005, Abhay Ashtekar has informed me that he and his collaborators have independently come to the conclusion that thinking about LQC in terms of dynamical systems evolving deterministically is problematic and that they are planning to drop, or at least significantly attenuate this language. In December 2005, Martin Bojowald has published a review on LQC including a brief section on determinism (Bojowald 2005, Sec. 7.3), which draws on exchanges we have had.

Of course, all these difficulties ultimately stem from the fact that we are dealing here with a generally covariant theory which does not admit an external time against which a dynamical evolution can be determined. Time, just like space, was part of the physical system which LQG originally set out to quantize!

8.2 THE INFIDELS: RECENT CRITIQUES OF LOOP QUANTUM COSMOLOGY

In this section, I investigate and discuss the most important objections which have been put forth against LQC. There are four, and they are ordered according to what I take to be their graveness, from mild to devastating. Some of them, discussed in Subsections 8.2.1-8.2.3, make a particular choice of matter and thus of \hat{C}_{matter} and then derive some problematic features. Others, such as Brunnemann and Thiemann, question the very ambition of LQC to represent the cosmological sector of LQG.

8.2.1 Noui, Perez, and Vandersloot: separable but small physical Hilbert space

Noui, Perez, and Vandersloot (2005) follow, like LQC, the recipe of imposing symmetry reduction in order to successfully construct the physical Hilbert space of the theory. As a classical vantage point, they use the self-dual Plebanski action rather than the standard Einstein-Hilbert action.⁷ The Plebanski action is a low-energy effective action for classical general relativity. The advantage of this approach is a simple Hamiltonian constraint function, whose quantum operator turns out to be self-adjoint—unlike the typical Hamiltonian constraint operators encountered in LQC. It is the self-adjointness of the Hamiltonian constraint operator which allows the application of a technique called *refined algebraic quantization* (Ashtekar et al. 1995) using “group averaging methods” in order to solve the quantum constraint equations and thus to find the physical Hilbert space. Noui, Perez, and Vandersloot show how this approach can be brought to bear fruit in the context of homogeneous and isotropic cosmological models based on canonical quantum gravity. Their approach, however, also comes at a price: it is only well-defined in the Riemannian sector of the theory, but not in the physically relevant Lorentzian sector.⁸

⁷For how the different actions for GTR in $(3 + 1)$ dimensions relate to one another and the constraints look like in each case, see Peldán (1994).

⁸By *Lorentzian quantum gravity*, one usually denotes a quantum theory of gravity whose partition function is given by an expression of the type $\int e^{iS[g_{\mu\nu}]}$, where $S[g_{\mu\nu}]$ is the Einstein-Hilbert (or the Plebanski) action for a Lorentzian metric $g_{\mu\nu}$. A theory with the same partition function is called *Riemannian quantum gravity*.

The model [Noui, Perez, and Vandersloot \(2005\)](#) present shares the kinematical Hilbert space with LQC as described above, but takes a different route in the construction of a Hamiltonian constraint operator. This operator, to repeat, is self-adjoint in their approach but not in standard LQC.⁹ This enables them to adopt the inner product of the kinematical Hilbert space and project it unto the physical Hilbert space. The states in physical Hilbert space are now those states which satisfy the quantum Hamiltonian constraint equation, i.e. those states which are annihilated by the Hamiltonian operator. Noui and collaborators find that in the case of a non-vanishing cosmological constant, the physical Hilbert space is reduced to one dimension. Physical states, as a result of the requirement that they must satisfy the Hamiltonian constraint equation, are exactly those states with a vanishing norm.

[Noui, Perez, and Vandersloot \(2005\)](#) claim that their main results transcribe into the Lorentzian sector of the theory and should therefore be taken seriously. Because of the symmetry relations of isotropy and homogeneity imposed, the relationship between the Riemannian and the Lorentzian sector of the theory become tractable. They then explicate how their approach and standard LQC share the same semi-classical limits. All this is quite unproblematic. But it does not suffice to make their point, as the regime at interest here is the irreducibly quantum regime around the big bang, where the differences between the self-adjoint Hamiltonian operator constructed from the self-dual Plebanski action and the non-self-adjoint Hamiltonian operator constructed from the Einstein-Hilbert action, both appropriately restricted to isotropic and homogeneous spacetimes, show up most prominently. This does not come as a surprise, given that the Plebanski action most closely mimics the Einstein-Hilbert action in the low energy limit of an effective field theory of gravity. In this sense, the Plebanski action does not seem adequate for investigations into the quantum regime governing the very early universe. Thus, I urge that the critique against LQC voiced by [Noui, Perez, and Vandersloot \(2005\)](#) must be received with this important limitation in mind.

in case $g_{\mu\nu}$ is a Riemannian metric with signature $(++++)$. Riemannian quantum gravity is easily confused with another term often found in the pertinent literature, *Euclidean quantum gravity*. The latter name refers to quantum theories of gravity which also use the Einstein-Hilbert (or Plebanski) action $S[g_{\mu\nu}]$ for a Riemannian metric, but have a different partition function, given by $\int e^{-S[g_{\mu\nu}]}$.

⁹However, self-adjoint Hamiltonian operators have been proposed in the context of standard LQC, e.g. by [Bojowald et al. \(2004b\)](#). While it is possible to define a group averaging procedure even for non-self-adjoint Hamiltonian operators, this may be unattractive from the numerical point of view as non-real eigenvalues may lead to numerical instabilities (ibid.).

8.2.2 Cartin and Khanna: Bianchi I model

A similar study by [Cartin and Khanna \(2005\)](#) investigates whether LQC produces reasonable results for “late times” in the Bianchi I model, a simple anisotropic cosmological model. They argue that at a safe distance from the classical singularity, i.e. for “late times” in the proposed sense of evolution, the models of LQC must satisfy certain restrictions in order to exhibit the expected classical behaviour in this regime and they show that this is not the case for the Bianchi I case. Cartin and Khanna conclude from this that either the method of quantization used in LQC does not yield useful results or else the conditions imposed in order to obtain the correct classical behaviour were overly restrictive. However, it could also be the case that the homogeneous and spatially flat Bianchi I model fails to represent the adequate model for early anisotropic perturbations of an increasingly isotropic universe. The second and third leg of this trilemma do not threaten the enterprise of LQC. It is an open question, therefore, whether Cartin and Khanna’s study poses a damaging challenge to LQC, and if so to what extent.

8.2.3 Green and Unruh: closed model

The presentation and discussion of LQC in Section 7.3 focused on quantum models of the flat FLRW-model with $k = 0$ and largely ignored the closed model with $k = 1$.¹⁰ [Green and Unruh \(2004\)](#) fill the gap by offering an analysis of the closed model, focusing on the case of a universe which is empty except for a massless scalar field. They find two difficulties with the description of the closed model as given by LQC.

First, for the closed model in particular, it turns out that the scale factor cannot serve as an adequate cosmological time parameter. Motivated by the classical analogy, advocates of LQC have taken the scale factor as a surrogate cosmological time, as there was no longer an explicit time dependence in the dynamical Hamiltonian constraint equation. Their idea was to then study the dynamical evolution of the remaining degrees of freedom with respect to this one. An immediate difficulty arises when one attempts to adopt this approach for recollapsing closed models: the scale factor will assume the same numerical value twice in the course of the history of the universe, whereas one would want to assign a different time to both instances of the same scale factor. Generally, the same volume of the universe would correspond to different times in its evolution. Thus, for recollapsing models, a bijective mapping between scales of the universe and cosmological times in its evolution is not possible.

¹⁰To repeat from footnote 17, the derivation as given in Section 7.3 does not cover the FLRW-models with negative curvature $k = -1$.

An advocate of LQC could, of course, rejoin to this objection that a bijective mapping will only be required for either the expansive or the collapsing epoch, but not for both. Whichever epoch will not be covered by the bijective mapping, can be described by a kind of “mirroring”: the covered epoch is just duplicated and both phases are then joined such as to complete the closed model. At the heart of the construction, of course, lies the assumption that both epochs are perfectly symmetric insofar as one is just the time-reversal of the other, including all physical processes within it. However, the fact that *all* physical processes will be time-reversed by the mirroring implies that physical time too will be time-reversed! This result stands independently of how a physical time might be defined, just as long as one accepts that the construction of a physical time can solely rely on some physical processes within the universe, adequately chosen for the purposes of introducing a physical time with a consistent direction. But it is exactly that a globally consistent direction can no longer be assigned for the entire closed model consisting of an expansive as well as a recollapsing epoch as the temporal direction will be opposite in both epochs to be conjoined. A way out of the quandary can only be found at the price of admitting that the construction of a physical time must proceed differently in both epochs in order to achieve a globally consistent direction of time. This move, I believe, belies the principles of a scientific endeavour which forgoes the services of ad-hoc reasoning.

However, the defender of LQC may retort by biting the bullet and admitting that if the “mirroring” technique is used to construct a closed quantum FLRW model, physical time and all physical processes will indeed be time-reversed in this approach. This may not be overly disquieting as LQC present a rather simple, reduced theory which cannot accommodate complex physical processes anyway, and which, in fact, was not *built* to accommodate them. So this may just be taken as an indication of the limits of LQC to offer a fundamental theory, rather than as a proof of its failure to capture important aspects of a quantum cosmological model based on GTR.

The second difficulty identified by [Green and Unruh \(2004\)](#) concerns the fact that a closed universe reaches a maximum size. Thus, any model capturing a closed universe, regardless of whether it is classical or quantum, should not permit sizes larger than this maximum. In the case of a quantum model, this means that whichever operators of the quantum theory contain the scale information must be bounded from above. For the models of LQC, more precisely, the scale or fundamental triad operator (7.9) must have in its spectrum an eigenvalue μ_{\max} such that all $\mu > \mu_{\max}$ vanish, or at least quickly converge to zero. If this is the case, then the expectation value of the scale operator will not exceed this maximum value. The second difficulty is just that: this convergence does not take place; quite the contrary, Green and Unruh have discovered some wildly diverging behaviour for large scales.

Essentially, [Green and Unruh \(2004\)](#) study the dependence of the state coefficients $\psi(\phi, \mu)$ of equation (7.13) on the scale μ . Instead of a rapidly decaying behaviour for $\mu > \mu_{\max}$, they find that the $\psi(\phi, \mu)$ grow exponentially, or show otherwise divergent behaviour for large μ for a wide variety of secondary parameters. They also explore different ansätze to quantize the closed model in the context of loop quantum gravity but conclude that at least the obvious ones fail. In sum, they argue, the closed model of classical cosmology cannot be accounted for in the framework of LQG.

Is this a problem? [Green and Unruh \(2004\)](#) insist that it is. They argue that although it is perfectly acceptable for cosmologists, including those working in LQC, to focus on the flat case as extant observations suggest that our Universe is very nearly flat, cosmology must be able to understand the flat model as a limiting case of the closed model. Such understanding, they continue, is necessary to adequately address the so-called “flatness problem,” according to which the fact that our Universe’s density is very close to the critical value, i.e. the value $\Omega = 1$ which amounts to an expansion rate that asymptotically tends to zero, must be explained by an acceptable cosmological theory.

The request for an explanation of the fact that our Universe is very nearly flat, however, does not deliver a fatal blow to any theory incapable of fulfilling this demand. The flatness problem is essentially a cosmological fine-tuning problem. Whether or not the flatness problem really requires, and affords, a scientific explanation, is an open question. If it does, there are two possibilities: either physicists manage to find a mechanism which forces the Universe to be nearly flat, or else one can attempt to acquit the explanatory debt by reference to a systematic observation selection bias, such as an anthropic principle. In the first instance, inflation can be constructed such as to be capable to sufficiently flatten any non-flat region present before the inflationary period ([Guth 1981](#)). However, since LQC also seems to be implying an inflationary phase, it can account for the observed flatness just as well as any other inflationary cosmology, and demanding that LQC must be capable of adequately describing the closed model in order to solve the flatness problem is unwarranted.

If these or similar explanations invoking some sort of physical mechanism eliminating non-flatness are seen as deficient, accepting the explanatory debt seems to force a retreat to invoking some kind of observation selection effects.¹¹ One such systematic bias, usually referred to in the literature as the *weak anthropic principle*, states that the observed values of the physical parameters of cosmological models cannot assume arbitrary values but are restricted by the requirements that there exist regions in the universe where carbon-based life can evolve and that the universe is sufficiently old for it to have already done so. In other

¹¹For a systematic account of observation selection effects, see [Bostrom \(2002\)](#); see [Bostrom \(2003\)](#) for a recent summary.

words, the weak anthropic principle demands that our observations are restricted by the necessary conditions for our existence.¹² Such a weak form of anthropic reasoning, however, while surely true, does not seem to do a lot of explanatory work.¹³ Requesting a scientific explanation of the observation that our Universe is almost flat seems to ask for more than being reminded of the fact that otherwise we would not be here to observe anything. The anthropic principle can of course be strengthened by insisting that our observations are not merely restricted by the necessary conditions for our existence, but that these conditions, furthermore, are necessary or at least highly probable themselves. While such a strong form of anthropic principle would certainly enjoy more explanatory potency, it seems scientifically untenable unless it is amended by a satisfactory explanation as to why these conditions should necessarily, or at least likely, obtain.

An explanation of this type would of course be susceptible to all criticisms against strong forms of anthropic reasoning, including the recent ones by [Smolin \(2006\)](#). In this article, Smolin offers a tentative explanation of why these life-enabling conditions are very likely to prevail. His account involves what he dubs “cosmological natural selection.” Cosmological natural selection is based on three premises, in close analogy to evolutionary biology, which, according to Smolin, is the only science which has successfully managed to *explain* the emergence and stability of complexity by offering a simple mechanism. These three assumptions are: (i) there exists a physical process which produces a multiverse with a long chain of descendants, (ii) for the space \mathcal{P} of N dimensionless parameters p_i , with $i = 1, \dots, N$, of the standard models of particle physics and cosmology, there exists a “fitness function” $F(p_i)$ on \mathcal{P} which encodes the average number of descendants of a universe with parameters p_i , and (iii) the parameters p_i for each universe differ from those of its immediate ancestor universe, on average by a random change sufficiently small as not to significantly change $F(p_i)$. If the multiverse, i.e. the population of universes, is sufficiently large, and is tested on sufficiently many random runs, then the population of universes will be peaked around local maxima of $F(p_i)$ after sufficiently many generations. These assumptions directly imply that if the parameters p_i are shifted around in \mathcal{P} , i.e. changed in their values, then $F(p_i)$ will be systematically decreased. Unlike other accounts of anthropic reasoning, cosmological natural selection offers predictions, which are in principle testable. Moreover, like inflation, it provides a mechanism which ascertains that our Universe exhibits the values of physical parameters that it does.

So either one can find such a mechanism, or else we must appease our explanatory desire

¹²I have borrowed this definition from [Smeenk \(2003, Sec. 5.5\)](#).

¹³Bostrom has argued, and I tend to agree, that not anthropic principles per se are problematic, but that whether or not they can play an explanatory role depends on the specific context to which they are applied. According to Bostrom, even the weak anthropic principle thus affords an explanatory value in *some* contexts.

by inviting anthropic reasoning. Depending on what one's take on anthropic reasoning is—and there seem to be significant differences, to say the least—, we have thereby gained a satisfactory explanation of the flatness of the Universe or we must content ourselves with acknowledging that we have reached the explanatory limits of scientific enquiry.

Let us return to Green and Unruh's claim that solving the flatness problem requires an understanding of how the flat model is a limiting case of the closed model, and that therefore the ability to adequately accommodate the closed model is for any cosmological theory a necessary precondition for solving the flatness problem. The last three paragraphs indicate, in my opinion, that first, it is not entirely clear to what extent the request for an explanation of flatness may be granted at all, and second, even in case it is, it seems as if the inflationary phase discovered by [Bojowald \(2002a\)](#) when the matter Hamiltonian \hat{C}_{matter} of LQC contains a massless scalar field can go at least some length toward producing the inflation required for accounting for the flatness ([Bojowald and Vandersloot 2003](#)). In sum, then, although I do not dispute Greens and Unruh's result that LQC cannot adequately accommodate closed FLRW models, I doubt that this failure presents an insurmountable and principled difficulty for it. It may well be the case that the flatness problem does not admit of a scientific explanation at all, or it may be that LQC yields an inflationary mechanism capable of accounting for the flatness without recourse to the closed model as a limiting case of the flat one.

Finally, there is perhaps a sense in which it should not come as a surprise that LQC is unable to properly handle closed FLRW models: after all, a closed model needs matter or energy to generate a sufficiently strong gravitational pull to recollapse. But LQC is based on LQG, which is a quantization of the vacuum sector of the phase space of Hamiltonian GTR and as such, does not admit matter in the universe. This suggests that introducing matter in LQC may be more subtle than just plugging a few matter fields into \hat{C}_{matter} . Obviously, this is a delicate point for which nobody has a good answer for the time being. But it is also a potentially disastrous point for LQC as one may react by exclaiming “so much the worse for LQC.” Matter, including all its garden varieties as well as all versions of dark and mysterious matter forms, is of paramount importance in cosmology in general and for closed FLRW models in particular. Without the confidence that the coupling of matter to gravity is correctly understood, it remains optimistic at best and naïve at worst to believe that the symmetry-reduced models of LQC adequately describe the cosmological sector of the physical Hilbert space of the correct quantum theory of gravity. What LQC can teach us about quantum cosmology, therefore, may be very limited indeed.

8.2.4 Brunnemann and Thiemann: cosmology or not?

The advocate of LQC may still recover from these objections, but the points raised in [Brunnemann and Thiemann \(2006a\)](#) pose a more serious threat. Also, the first three objections discussed cast doubt of the entire project of LQC, or of some particular aspects or models, but not specifically on its claim that the kinematical and the dynamical singularities disappear. Thus, they only indirectly cast doubt on the claimed resolution of the initial singularity. The article by Brunnemann and Thiemann, however, does both. Their paper builds on the important result obtained in [Brunnemann and Thiemann \(2006b\)](#) according to which the argument in LQC leading to the boundedness of the spectrum of the inverse scale factor operator does not translate into the full theory. More precisely, Brunnemann and Thiemann ask whether it is indeed the case in the full theory that at the classical singularity, i.e. for the kinematical quantum state corresponding to a universe of zero volume, the inverse of the triad operator has a finite eigenvalue which would correspond to finite curvature. It turns out that quite generically for a state of zero volume, the inverse scale factor is unbounded from above in the full theory. According to Brunnemann and Thiemann, inhomogeneous quantum excitations are responsible for the unboundedness of the inverse triad operator. These inhomogeneous excitations are of course admissible in the full theory, but they are precluded from making an appearance in the symmetry-reduced, strictly homogeneous LQC.

It is ironic, however, that it is the inhomogeneities which now endanger the extermination at least of the kinematic singularity of the classical models of cosmology when the presence of this singularity was first believed to be the artificial result of the only approximately valid symmetry of homogeneity, as was discussed at the outset of Chapter 6. As cited on page 103, Einstein initially believed that the occurrence of the big-bang singularity was an artefact of the unrealistic assumption of a perfectly homogeneous and isotropic universe and would vanish once small perturbations of these perfect symmetries were allowed to enter. Of course, the singularity theorems of Penrose, Hawking, and Geroch conclusively deposed this intuition. The calculation of [Brunnemann and Thiemann \(2006b\)](#) now suggests an intuition diametrically opposed to the one Einstein expressed: the inhomogeneities are not the remedy against, but the source of the singular behaviour, at least as far as the kinematical singularity is concerned.

As [Brunnemann and Thiemann \(2006a\)](#) continue, however, the unboundedness of the inverse triad operator should not be preemptively taken to imply that cosmological models based on the full theory of LQG suffer from a kinematical singularity. The main reason for this implication to fail is that although an operator may well be unbounded in general, it may become bounded when the Hilbert space is restricted to a particular subspace. This

means that the boundedness of the inverse triad operator is not necessary for the elimination of the kinematical singularity. Brunnemann and Thiemann argue that the restriction to the appropriately chosen cosmological sector of the full theory does indeed lead to the boundedness of the inverse triad operator at the big bang. They define the cosmological sector of the kinematical Hilbert space as the sector which describes a homogeneous and isotropic universe at large scales, thus admitting for small inhomogeneous and anisotropic perturbations. The idea then is to let the volume of the universe go to zero and check whether the expectation value of the inverse scale factor operator grows beyond all bounds. Brunnemann and Thiemann propose to regard the sector of the kinematical Hilbert space with those coherent states peaked at homogeneous and isotropic symmetries. If this proposal is accepted, then inhomogeneous and anisotropic excitations of the largely homogeneous and isotropic coherent states are still admissible, but confined to remain small.¹⁴ For such a coherent state of zero volume corresponding to the universe at the big bang, Brunnemann and Thiemann claim to have established that the expectation value of the inverse triad operator with respect to this state is bound from above. This calculation, unlike both the standard approach taken in LQC as well as the unboundedness of the inverse triad operator in the kinematical Hilbert space of the full theory, they claim, suggests that the kinematical singularity at the big bang is indeed avoided in cosmological models based on full LQG.

While this is promising, Brunnemann and Thiemann warn that it does not imply that the kinematical singularity vanishes. In this sense, the boundedness of the expectation value of the inverse triad operator with respect to a largely homogeneous and isotropic coherent state at the big bang is also not a sufficient condition for the elimination of the kinematical singularity. The reason for this is that the inverse scale factor—or the scale factor, for that matter—is not a gauge-invariant quantity and consequently not a Dirac observable. It is merely an operator defined on the kinematical Hilbert space which is used as a kind of auxiliary magnitude in LQC to construct a cosmological model. In order to make meaningful physical predictions, Brunnemann and Thiemann argue, one would need the full physical Hilbert space \mathcal{H}^S of LQC and operators densely defined on this Hilbert space.

Also as far as the dynamical singularity avoidance is concerned, [Brunnemann and Thiemann \(2006a\)](#) continue, the issue is far from resolved. They argue that although it may be the case either that kinematical states dynamically decouple and that additional constraints thus arise or that a violation of Laplacean determinism occurs or both, as outlined in [Section 8.1](#), the absence of both of these effects are neither sufficient nor necessary for the avoidance of dynamical singularities. It is not necessary, they claim, because even in the presence

¹⁴The particular set of coherent states at use here has been constructed by [Thiemann \(2001a\)](#), [Thiemann and Winkler \(2001a,b\)](#), and by [Sahlmann et al. \(2001\)](#).

of both effects, the physical Hilbert space may be sufficiently large as to accommodate all semi-classical cosmological models needed to emulate the classically relevant ones at large scales. It is also not sufficient because even in the absence of both effects, it might turn out that many of the dynamical states are not normalizable or have zero norm and thus not eligible as members of the physical Hilbert space. If many of the candidate states have to be excluded for these reasons, the physical Hilbert space may no longer be sufficiently large as to accommodate the important semi-classical sector describing cosmologically relevant models. Thus, Brunnemann and Thiemann conclude, for determining the avoidance of both the kinematical as well as the dynamical singularity, the issue must be cast in terms of physical states and physical observables.

While I accept the argument establishing the insufficiency, I wish to take issue with the one claiming non-necessity. Brunnemann and Thiemann argue that although there might be indeterministic behaviour of the quantum Friedmann equation as described in Section 8.1, the dynamical singularity might vanish *as long as the physical Hilbert space contains a semi-classical cosmological sector*. It appears as if they take the containment of such sector in \mathcal{H}^S as a sufficient condition for the resolution of the dynamical singularity, for otherwise the argument would not be valid. This interpretation is supported by a quote from much later into the article: “[...] the [dynamical] singularity is avoided if and only if there are sufficiently many semiclassical [physical states composed of spin network states that describe a sign flip with respect to the expectation value of the triad orientation sign operator].” (p. 1421) The particular states are required, according to Brunnemann and Thiemann, in order to correctly describe quantum geometries including a pre-big-bang regime. I am willing to grant that it may be a necessary condition, but because I insist that as long as only quasi-determinism, but not determinism, is satisfied at the “big bang,” there survives a residue of the dynamical singularity even if such a sector is present. The obedience to full determinism is also a necessary condition for the complete quantum evaporation of dynamical singularities. So we have (at least) two necessary conditions, none of which is individually sufficient.

According to [Brunnemann and Thiemann \(2006a, Sec. 5\)](#), the proper resolution of the issue would involve the construction of Dirac observables from partial observables which “evolve” with respect to another partial observable, a “clock” variable such as the scale factor, following the scheme proposed by [Dittrich \(2004\)](#). Once the physical states and some Dirac observables have been found, a semi-classical sector would have to be identified which correctly approximates the classically relevant cosmological models, and a physical Hamiltonian would have to be constructed in order to determine physical evolution, as opposed to the evolution with respect to a non-gauge invariant quantity. This will permit the resolution of the issue whether the dynamical singularity disappears: if the physical

Hamiltonian is essentially self-adjoint, it will deliver a regular physical evolution. Once all this is in place, one can take an operator defined on the physical Hilbert space and representing a physical observable which corresponds to a classically singular quantity such as curvature, density etc. Then, calculate the expectation value of this operator with respect to semi-classical states peaked around what corresponds to classically singular initial data and conclude that the kinematical singularity is avoided just in case this expectation value is finite. While Brunnemann and Thiemann admit that this programme is highly ambitious, they insist that it offers the only path to the full resolution of whether Einstein's nemesis is finally vanquished in the quantum theory.

This completes the part of my dissertation on the loop quantum avoidance of the initial singularity. So does the singularity vanish? The short answer is, particularly in the light of the discussion of [Brunnemann and Thiemann \(2006a\)](#), we do not know yet. There are many things that can be said about the evaporation of singularities, and I have tried to say a few in the preceding three chapters. But let me say one more thing: even if it will turn out that the singularities of the classical theory are not all avoided in the quantum theory, this by no means implies the failure of the quantum theory. In the classical theory, we have learnt that singularities only become really problematic when they havoc global hyperbolicity. Penrose's cosmic censorship hypothesis, if true, protects physically reasonable classical scenarios from breeding naked singularities, i.e. those singularities which wreck global hyperbolicity. It is still an open issue, however, whether the hypothesis is, in fact, true. Suppose that the hypothesis is not true. Does LQG command the resources to prevent the formation of naked singularities in the deep quantum regime? According to Bojowald,¹⁵ the only article which addresses this question is [Goswami, Joshi, and Singh \(2006\)](#). Goswami and collaborators investigate the quantum gravitational collapse of a scalar field which classically forms a naked singularity. They claim that a semi-classical analysis based on LQC shows that just as a naked singularity tends to form, a very strong outward energy flux will prevent the formation of such a singularity in LQG. One may wonder, of course, how it can be that LQG, which was expressly based on globally hyperbolic classical spacetime, may offer a promising investigation into how spacetimes hosting naked singularities might be modified by quantum effects. The basic idea of [Goswami, Joshi, and Singh \(2006\)](#) is the following: model the gravitational collapse by pasting together an interior and an exterior solution. The exterior region is assumed to be classical, but the interior of a homogeneous scalar field collapse, which

¹⁵Personal communication, 9 March 2006.

is classically described as an FLRW spacetime, can be treated by using techniques from LQC. In this case, of course, the cosmological time of the FLRW model is reversed from its usual direction such that we have classically a *final* singularity. In the classical model, this final singularity will be naked for particular choices of physical parameters. Similar considerations as those adduced above concerning the disappearance of singularities in LQC equally apply here. The situation is insofar different, though, as a particular matter field is assumed. With this in place, [Goswami, Joshi, and Singh \(2006\)](#) compute, using effective equations, that strong outward energy fluxes occur “due to supernegative pressures in the late regime” (p. 4) which prevent the collapse from completing and the singularity from forming. At least for the case at hand, Goswami and collaborators conclude, loop quantum effects thus uphold a quantum gravitational cosmic censorship. Some of the details of their argument have been questioned.¹⁶ Be this as it may, the study of the fate of naked singularities is but a nascent enterprise. It thus remains entirely open whether the formation of naked singularities is avoided altogether in the picture of quantum spacetime as drawn by LQG.

¹⁶Martin Bojowald, personal communication, 9 March 2006.

9.0 THE EMERGENCE OF SPACETIME

As we have seen in Chapter 5, the continuity of the classical spacetime is dissolved at the fundamental level in LQG and replaced by the combinatorial structure of spin networks and spinfoams. A relevant question then becomes how—and whether—the continuous spacetime structure can be recovered in the classical limit. Answering this question must, by necessity, remain speculative on the present occasion as the programme of constructing a complete and consistent canonical theory of quantum gravity has not been brought to its successful completion yet. But a few things, it seems, can still be said about the disappearance and the re-emergence of spacetime.

So spacetime disappears. How this disappearance of spacetime is to be understood in the present context will be clarified in Section 9.1. What is clearly *not* meant is the demise of space and time qua Newtonian containers in which physics unfolds, wrought by the advent of relativistic physics. This effect already appears at the classical level of relativistic physics and has been widely addressed by the philosophical and scientific communities. There exists a plethora of studies concerned with this consequence of GTR and the problem seems to survive the transition from the classical to the quantum theory unmodified. Except for a brief remark at the outset of Section 9.1, it shall not be covered here. More interestingly, Section 9.1 discusses a proposal due to Callender and Huggett (2001a). Callender and Huggett characterize the disappearance of spacetime via a presumed unitary inequivalence of the spin network basis with the basis of quantum three-metrics in quantum geometrodynamics. But the sense in which spacetime disappears I shall be most concerned with is the dissolution at the Planck scale of the smooth geometric structure of classical spacetime into the discrete structures which replace it in the quantum theory.

Section 9.2 will first discuss what the most promising approaches to recovering the classical structure from the discrete quantum world given the persistent failure to find such a semi-classical or classical limit so far are. Next, I shall issue a brief reminder of the most widely used notions of reductive relations between theories or between different explanatory levels in the contemporary literature on emergence and reduction in philosophy of science, most notably Butterfield and Isham (2001). These philosophical accounts shall then be ana-

lyzed with an eye toward which ones, if any, could be most fruitfully applied to the problem of finding a semi-classical limit for LQG. Conversely, QG might offer some relevant lessons for the philosophical literature. So the philosophical literature may serve as a toolbox for the specific problem of disappearance and re-emergence of spacetime in the context of canonical quantizations and may give guidance to physicists seeking to understand the classical limit of LQG. It would be exaggerated if one expected the philosophers to do the heavy-lifting, but it is encouraging to receive reassurances from physicists that philosophical guidance is welcome.¹ On the other hand, the present context may offer a case study which helps to refine the philosophers' toolbox.

Before I embark on the outlined programme, a couple of remarks. First, it may not be necessary that GTR can be fully recovered in the sense that all its models \mathfrak{M}_E have to re-appear in a well-justified classical limit. There might be valid quantum reasons why models with singular spacetimes do not re-appear when moving from the quantum to the classical theory. Given the self-imposed restriction to models with spacetime of topology $\mathbb{R} \times \Sigma$ necessitated by the canonical formalism, one would surely expect to see only models with spacetimes of this topology to re-emerge. This automatically precludes the possibility that models with acausal structure emerge from the quantum theory. Second, to the extent to which Hamiltonian GTR differs from GTR, one would expect to see an emergent theory different from GTR. In particular, if it is the case that the symmetries of have been changed in recasting GTR as a Hamiltonian system, the symmetries of the emergent classical theory must be modified as well. Generally, it would be immensely interesting to see whether the set of models of the emergent classical theory differs in any way from the set of models admitted by Hamiltonian GTR.

9.1 DISAPPEARANCE OF SPACETIME

In modern physics, space and time have suffered the loss of their privileged role as inert Newtonian background on which all physical phenomena must be understood.² General relativity has demanded that they give up their status as fixed stage for all physical processes and become part of the dynamical interaction among physical entities themselves. Thus, although there still exists a spacetime, it is no longer absolute, but it is demoted to being

¹As a prominent example, such reassurance has been repeatedly issued to me by Carlo Rovelli.

²Of course, Newtonian spacetime can only be considered as “inert” insofar as it is not “acted upon” by the matter content of the universe. The converse does not hold true as it acts upon the matter content by determining the geodesics along which matter cruises.

just another physical field. The fact that GTR has $\text{Diff}(\mathcal{M})$ as gauge symmetry means, due to the hole argument as discussed in Chapter 4, that the view which takes the spacetime manifold to be a substance has come under pressure. So there is a sense in which space and time have disappeared in the relativistic revolution, with its far-reaching philosophical consequences. Some of those who have resisted these consequences put their hopes into quantum physics. The idea behind this manoeuvre is to avoid bearing the consequences of relativity on the grounds that it does not constitute a fundamental theory. Therefore, the argument goes, the cataclysmic implications of GTR are merely tentative as all non-fundamental physics is subject to revision. Insofar as LQG claims to be a fundamental theory of gravity, its consequences can no longer be so lightly dismissed. LQG, just like any canonical quantization of GTR, starts out from GTR as a constrained Hamiltonian system, where the constraints encode the background independence and thus the generally relativistic innovation here at stake. As has been seen in Chapter 5, the constraints carry the background independence right through the quantization process over into the quantum theory. Modulo the reservations discussed in Chapter 4, canonical quantizations of GTR thus take the lack of an inert spacetime container very seriously. This means that LQG does not alter the verdict of GTR with respect to the Newtonian container *ex constructione*. So in this first sense of disappearance, spacetime no more and no less disappears in canonical quantum gravity than it does already at the classical level.

The second and third senses in which the spacetime disappears, however, constitute true innovations of quantum gravity. Let me start with the third reason for mourning the demise of spacetime, the conjectured unitary inequivalence between the bases of spin networks and of functionals of three-metrics in quantum geometrodynamics. Callender and Huggett (2001a, p. 21) seem to suggest to use this criterion of unitary equivalence to determine whether or not spacetime can still be regarded as fundamental in LQG. If the two bases were unitarily equivalent, in Callender and Huggett’s view, they would represent two different codifications of the same objects and it would not make sense to claim that one is more fundamental than the other. If, however, they were unitarily inequivalent, one would infer their physical inequivalence as unitary equivalence is considered a necessary condition for physical equivalence. From the perspective of LQG, Callender and Huggett conclude, this inequivalence implied that spin networks were the basic constituents and spacetime, encoded in the functionals of the three-metrics and their conjugate momenta, merely emergent or supervenient. More precisely, it is not *spacetime* which would be emergent in this case, but *space* tout court. If the three-metrics, and perhaps the spin networks, encode any aspect of classical spacetime, it will be its three-dimensional, “spatial” part and not the entire four-dimensional spacetime structure. Let me dismantle what I take to be Callender and Huggett’s idea.

The setting, of course, to introduce the notion of unitary equivalence vital to Callender and Huggett's argument is a Hilbert space theory.³ Typically, the concept is introduced in the context of operator representations of the canonical commutation relations. A *representation* of the canonical commutation relations is a pair $(\mathcal{H}, \{\widehat{O}_i\})$ consisting of a Hilbert space \mathcal{H} and a set $\{\widehat{O}_i\}$ of bounded, essentially self-adjoint operators $\widehat{O}_i \in \mathfrak{B}_{esa}(\mathcal{H})$ acting on \mathcal{H} and satisfying the canonical commutation relations. These commutation relations typically arise from the Poisson bracket structure of the corresponding classical theory. Two representations $(\mathcal{H}, \{\widehat{O}_i\})$ and $(\mathcal{H}', \{\widehat{O}'_i\})$ are said to be *unitarily equivalent* just in case there exists a bijective, linear, norm-preserving transformation, i.e. a *unitary map*, $U : \mathcal{H} \rightarrow \mathcal{H}'$ such that $U^{-1}\widehat{O}'_i U = \widehat{O}_i$ for all i . If one takes the operators to represent the observables of the quantum theory, then the physical content of this quantum theory is exhausted by the matrix elements of these operators. Thus, if $(\mathcal{H}, \{\widehat{O}_i\})$ and $(\mathcal{H}', \{\widehat{O}'_i\})$ are unitarily equivalent, then the two representations are physically equivalent in the sense that for any $|\Psi\rangle \in \mathcal{H}$, the state $U|\Psi\rangle \in \mathcal{H}'$ will have the same physical properties as does the state $|\Psi\rangle$.

This characterization of unitary equivalence involving representations of the canonical commutation relations, although predominant in the literature, is of no direct help for assessing Callender and Huggett's argument, which instead invokes the concept of unitary equivalence between different bases in different Hilbert spaces. But the concept can easily be translated to the case at hand: two bases $\{|a^{(k)}\rangle\}$ and $\{|b^{(l)}\rangle\}$ of two Hilbert spaces \mathcal{H} and \mathcal{H}' respectively are *unitarily equivalent* just in case there is a unitary map $U : \mathcal{H} \rightarrow \mathcal{H}'$ such that $U|a^{(k)}\rangle = |b^{(k)}\rangle$ for all k . It can be shown that one can easily construct such a unitary map using the orthonormality and the completeness of the bases: just use $U = \sum_k |b^{(k)}\rangle\langle a^{(k)}|$ and show by using orthonormality and completeness that U is indeed unitary.

Let me state this somewhat more rigorously. A complete inner product space is called a *Hilbert space*, denoted \mathcal{H} .⁴ Perhaps the most important example of a Hilbert space is the space l^2 of all sequences (x_1, x_2, x_3, \dots) of complex numbers such that $\sum_{k=1}^{\infty} |x_k|^2 < \infty$ and with an inner product of two sequences $x = (x_1, \dots)$ and $y = (y_1, \dots)$ in l^2 defined by $\langle x, y \rangle = \sum_{k=1}^{\infty} x_k \bar{y}_k$. A sequence of vectors in a Hilbert space \mathcal{H} which constitute an orthonormal system of vectors is termed an *orthonormal sequence*. A Hilbert space is said to be *separable* just in case it contains a complete orthonormal sequence, i.e. just if it admits a countable orthonormal basis. For example, l^2 is separable. Two Hilbert spaces \mathcal{H} and \mathcal{H}'

³For a presentation of the relevant concepts, see [Ruetsche \(2003, Sec. 2\)](#).

⁴A vector space E with a norm is called *complete* just in case every Cauchy sequence converges in E . A complex vector space E is called an *inner product space* iff it is equipped with a mapping $\langle \cdot, \cdot \rangle : E \times E \rightarrow \mathbb{C}$, a so-called *inner product* in E , which satisfies the following conditions for all $x, y, z \in E$ and $\alpha, \beta \in \mathbb{C}$: (i) $\langle x, y \rangle = \overline{\langle y, x \rangle}$ where the overbar denotes the complex conjugate, (ii) $\langle \alpha x + \beta y, z \rangle = \alpha \langle x, z \rangle + \beta \langle y, z \rangle$, and (iii) $\langle x, x \rangle \geq 0$, where $\langle x, x \rangle = 0$ implies $x = 0$. For a great textbook on Hilbert spaces, from which this paragraph has borrowed, see [Debnath and Mikusiński \(1999\)](#).

are *isomorphic* if and only if there exists a bijective, linear mapping $T : \mathcal{H} \rightarrow \mathcal{H}'$ such that $\langle T(x), T(y) \rangle = \langle x, y \rangle$ for every x and y in \mathcal{H} . A mapping which satisfies these conditions is called *unitary*, as introduced above. It can be shown (Debnath and Mikusiński 1999, Theorem 3.11.3(a)) that if \mathcal{H} is an infinite-dimensional separable Hilbert space, then it is isomorphic to l^2 . Isomorphisms among Hilbert spaces are an equivalence relation. Since it is therefore a transitive relation and since any infinite-dimensional separable Hilbert space is isomorphic to l^2 , any two infinite-dimensional separable Hilbert spaces must be isomorphic. As isomorphic Hilbert spaces by definition afford a unitary transformation among their elements, two bases in infinite-dimensional separable Hilbert spaces will always be unitarily equivalent.⁵ More generally even, it can be shown that two Hilbert spaces \mathcal{H} and \mathcal{H}' are isomorphic if and only if $\dim(\mathcal{H}) = \dim(\mathcal{H}')$, where $\dim(\mathcal{H})$, the *dimension* of \mathcal{H} , is defined as the cardinality of the bases of \mathcal{H} .⁶ It is thus a necessary and sufficient condition for \mathcal{H} to be separable that $\dim(\mathcal{H})$ is no greater than \aleph_0 , the cardinality of the natural numbers.

Let us return to the considerations offered by Callender and Huggett (2001a). To repeat, their idea was that if LQG turned out to be the correct theory of QG and if its kinematical base of spin networks is unitarily inequivalent to the geometrodynamical base of three-metrics, then spacetime cannot be considered as fundamental. Presumably, if the two bases were unitarily equivalent, the bases should be regarded as equivalent codifications of the same physical circumstances and spacetime would not lose its claim to fundamentality. This proposal is misguided in several respects. First, in order to establish unitary (in)equivalence between the spin network and three-metric bases, the physical Hilbert spaces of both quantum geometrodynamics and LQG would have to be given. On the one hand, thus, the construction of the Hilbert space of quantum geometrodynamics, i.e. the space of functionals of three-metrics which solve the diffeomorphism constraints, would be required. So far, no one has offered such a construction. The major roadblock for this approach seems to be the intractably difficult form of the constraints, which are non-polynomial. In all fairness, as was already lamented in Chapter 5, loop quantum gravitists have also failed so far to produce the physical Hilbert space of their theory. No Hilbert space, no basis. No basis, no checking for unitary equivalence.

But let us, for the argument's sake, assume that we have such physical Hilbert spaces of geometrodynamics and of LQG at our disposal. The construction of these Hilbert spaces

⁵But in the infinite-dimensional case, i.e. when an infinite number of degrees of freedom are considered, the Hilbert space may nevertheless carry unitarily inequivalent representations of the canonical commutation relations. The Stone-von Neumann theorem, which establishes the unitary equivalence of all (regular, irreducible) representations for finitely many degrees of freedom, does not apply to field theories with an infinite number of degrees of freedom.

⁶This is Theorem 3 in Halmos (1951, §16).

is only a necessary, but certainly not a sufficient condition to determine whether the bases are unitarily equivalent. So even if this construction would—unexpectedly—be achieved, it would be far from obvious that spacetime remained fundamental in case of unitary equivalence and emergent in case of unitary inequivalence. Let me elaborate.

The Callender and Huggett stance is faced with a quandary: either (i) the physical Hilbert spaces of quantum geometrodynamics and LQG are both separable, then their criterion is trivially satisfied; or (ii) one of the physical Hilbert spaces is separable, but the other is not, which implies that their criterion will not be satisfied; or (iii) the physical Hilbert spaces of both are non-separable, in which case the bases either (a) have the same cardinality or (b) they do not. In case (iii)(a), the Hilbert spaces are isomorphic and we are essentially back to situation (i). In case (iii)(b), the Hilbert spaces are not isomorphic and we are back to situation (ii). In reaction to (i), one could impose the additional requirement that the unitary transformation connecting the two bases must also preserve some set of algebraic relations characteristic of quantum geometrodynamics and also some set of algebraic relations characteristic of LQG. Characteristic algebraic relations would surely include, but need not be limited to, the canonical commutation relations. If the criterion would be thus amended, it would no longer be a foregone conclusion that there will exist such a unitary transformation.

Thus, it turns out that the question of whether bases of different Hilbert spaces are unitarily equivalent or not is not only not terribly exciting, but is also ill-suited to constitute a decisive criterion as to whether spacetime was fundamental or not. The imposition of the additional (or alternative) requirement demanding unitary equivalence of Hilbert space representations of the canonical commutation relations or some relevant algebras of observables at least renders the issue of whether spacetime is fundamental or not *contingent* and thus more interesting. Halvorson (2004), in an attempt to offer a rigorous formulation of Bohr’s notion of complementarity, studies representations of the canonical commutation relations⁷ for the one-dimensional particle. He constructs two representations, the “position” and the “momentum” representation. These representations, which are both set in a non-separable Hilbert space, are not unitarily equivalent to one another or to the regular Schrödinger representation. The position representation, for instance, contains exact position eigenstates, of which there are as many as there exist real numbers. Hence, the Hilbert space is not separable. This representation does not entertain a momentum operator, and in the analogous momentum representation, there exists no position operator.⁸ Although these two representations are empirically equivalent in that they command sufficient resources to fully

⁷More precisely: he investigates representations of the *Weyl algebra* arising from the canonical commutation relations.

⁸This is Theorem 1 in Halvorson (2004, p. 51).

capture any physical measurement procedure, Halvorson argues, the further claim that they must also be physically equivalent is not implied as the theoretical explanations they offer may differ significantly. Be this as it may, the issue of whether two different representations are unitarily equivalent is a contingent matter.

The matter would also be contingent, albeit for a very different reason, as long as the debate about separability in LQG lingers on and it thus remains an open question whether case (i) or (ii) applies. Nothing can be said about whether the Hilbert space of quantum geometrodynamics will be separable or not, and for this reason alone, the question must remain open. But let us look at least at the LQG side. In LQG, there exists an important controversy about the separability not of the yet to be found physical Hilbert space, but of the kinematical Hilbert space which admits a basis of spin network states, as explicated in Chapter 5. An influential discussion of the issue of separability has been offered in [Streater and Wightman \(1964, Sec. 2.6\)](#), who argue that the assumption that if one works in a field theory, one must only consider non-separable Hilbert spaces, is erroneous. In fact, they give strong motivations that most physically relevant circumstances will be described in separable Hilbert spaces, even in field theories. They mention two relevant situations when non-separability emerges. The first arises when one is faced with an infinite tensor product of Hilbert spaces, which is always non-separable. The second example, from statistical mechanics, concerns the expansion of a rigid box containing the physical system at stake such as to make the box arbitrarily large while maintaining the density. Both cases involve non-separable Hilbert spaces only because one considers actual infinities, which, according to Streater and Wightman, makes the systems difficult to deal with anyway. In these cases, one can only make sensible physical predictions if one restricts one's attention to separable subspaces. Therefore, they conclude, Hilbert spaces, as far as they are physically relevant, should really be separable.

The conviction that any physically relevant representation must be set in separable Hilbert spaces is not universally shared. In a close formal analogy to the position representation formulated in [Halvorson \(2004\)](#), [Ashtekar, Fairhurst, and Willis \(2003b\)](#) consider a so-called *polymer particle representation* of LQG, which also starts out from the usual Weyl algebra of exponentiated position and momentum operators. In this representation, states can be interpreted as polymer-like excitations of quantum geometry, hence the name. The polymer representation is set in a non-separable Hilbert space and is unitarily inequivalent to the regular Schrödinger representation. The basic variables of LQG are the holonomies of the gravitational connection A along one-dimensional curves and the fluxes of the conjugate momenta E through two-dimensional surfaces. It turns out that, similar to the example discussed by Halvorson, there exists a holonomy operator in the polymer representation,

but not a connection operator. Ashtekar and collaborators argue that this fact is welcome since it shows how well the mathematical structure of the representation captures essential features of quantum geometry, in particular its discreteness. The existence of a connection operator, which is constructed from derivatives and therefore seems to presuppose a smooth underlying manifold, would constitute an awkward result for a discrete quantum geometry. In this sense, the non-separability may be an advantage as it seems to be able to capture the discreteness of the quantum geometry.

Such a line of argument, however, appears to be circular: Ashtekar and collaborators make it sound as if when seeking a Hilbert space representation, one should hope to find a representation set in a non-separable Hilbert space which does not permit the construction of operators constitutive, or at least indicative, of a smooth spacetime geometry. But this already presupposes that the fundamental quantum geometry will turn out to be discrete, which, as argued in Chapter 5, was a *result* of the loop quantization process, not a preordained desideratum. In formulating the quantum theory of gravity, i.e. in constructing a physical Hilbert space which carries a representation of an algebra of observables, we have thus not yet arrived at the conclusion that quantum geometry is, in fact, discrete. This will only be determined once the formulation of the quantum theory is complete, and, hopefully, shown to be empirically adequate. To be sure, nothing in this paragraph goes against the possibility that the physical Hilbert space of LQG may turn out to be non-separable.

Regardless of whether one finds a separable or a non-separable Hilbert space more desirable, it is important to find out whether the Hilbert space of LQG in fact is or is not separable. Given the lack of a physical Hilbert space in LQG, one can investigate whether the kinematical Hilbert space is separable or not. After all, the kinematical Hilbert space contains those quantum states which are thought to “evolve” in “time” and thus has some parallels to the Hilbert space of a QFT. Alas, the kinematical Hilbert space \mathcal{K} of LQG is not separable. Without rehearsing the details for why this is so—they can be found e.g. in [Fairbairn and Rovelli \(2004\)](#)—let me briefly describe the intuitive reason. If the spin network states contain nodes with a sufficiently high valence, i.e. with a sufficiently high number of links joining at the node, the nodes are additionally labelled by certain continuous parameters, called *moduli* ([Rovelli and Smolin 1995b](#), p. 5749). These moduli do not appear to play any significant physical role in the theory such as weighing in relevantly in the spectra of operators representing partial observables. However, their emergence implies that the discreteness of kinematical quantum states is somehow spoilt. But more importantly, they render \mathcal{K} non-separable. [Fairbairn and Rovelli \(2004\)](#) explore the possibility of circumventing the emergence of these physically irrelevant but harmful moduli. This can be achieved by extending the spatial diffeomorphism group $\text{Diff}(\Sigma)$ such as to allow physical fields with

isolated points of non-differentiability. Fairbairn and Rovelli claim that such a modification in the gauge invariance group does not affect the physical predictions or consequences of the theory. What the modification of the gauge group changes, however, is the class of knot states formed by graphs invariant under the gauge group. Importantly, the equivalence classes of the spin network states are now countable and \mathcal{K} therefore separable.

The Fairbairn-Rovelli procedure to cure \mathcal{K} from a threatening non-separability has a precursor in the antidote given to nonseparable Hilbert spaces in QFT. The latter may for instance arise if one decomposes a free scalar field into an infinite number of harmonic oscillators, quantizes each degree of freedom individually to receive a separable Hilbert space, and then builds the (infinite) tensor product of the Hilbert spaces of the individual oscillators. The resulting Hilbert will be non-separable as its basis can be given by infinite sequences $|n_1, n_2, n_3, \dots\rangle$, where $n_i \in \mathbb{N}_0$, and these sequences are not countable. Vladimir Fock proposed to overcome the problem by selecting a subspace of the non-separable Hilbert space: rather than an infinite tensor product, one only considers sequences with an arbitrary, but finite number of non-vanishing n_i 's. These sequences can be used as a basis and they span a space, called *Fock space* and denoted \mathcal{F} , given by the (infinite) direct sum of (finite) tensor products of the Hilbert spaces codifying a single oscillator. Because the tensor products involved in the construction of this space are finite, \mathcal{F} is separable.

Even if it turns out that the Fairbairn-Rovelli manoeuvre is ultimately not legitimate, it would not be implied that the physical Hilbert space of LQG is non-separable. Since \mathcal{K} is only the kinematical Hilbert space, it must be projected onto the kernel of the Hamiltonian constraint operator in order to produce the physical Hilbert space and may become separable as a result of this projection. As long as the explicit construction of the physical Hilbert space has not been successfully achieved, however, this question must remain open. One may nevertheless speculate about what would happen if it turned out that both physical Hilbert spaces, the one of quantum geometrodynamics as well as the one of LQG, are non-separable. In this case, as stated above, whether or not Callender and Huggett's criterion is satisfied will depend on whether or not the bases of the two Hilbert spaces have the same cardinality. If they do, they will trivially be unitarily equivalent and if they do not, then they will trivially be unitarily inequivalent.

But be all of this as it may, the Callender-Huggett stance is still problematic for a more basic reason: it appears to presuppose that quantum geometrodynamics regards spacetime as fundamental while the approaches based on loop variables do not. It gives the metric codification of the geometry precedence over the connection codification in that it implicitly assumes only the first one to capture the geometrical essence of spacetime. At least at the classical level, both metric and connection description seem adequate and fully capture the

geometrical structure of spacetime. Regardless of whether one decides to use the metric or the connection codification of the classical geometry, the spacetime will dissolve into a quantum foam as the Planck regime is approached. Many (kinematical) quantum states, such as those near the initial singularity as discussed in Chapters 7 and 8, do not have a classical description which is approximately valid. At least as far as the kinematical states of LQG are concerned, the smooth manifold structure is replaced by a discrete geometry captured by abstract labelled graphs. As the kinematical Hilbert space of quantum geometrodynamics has not been found, one can only speculate how the quantum counterparts of the three-metrics look like. If the quantum three-metrics would turn out to be essentially unmodified by quantization, this would come as a huge surprise. But unless this unlikely event is certain to occur, we should assume that the smooth manifold character of the three-spaces also disappears in the quantization process. If that were not the case, of course, there would indeed be a relevant sense in which quantum geometrodynamics, unlike LQG, assume spacetime to be fundamental.

As argued above, Callender and Huggett’s criterion should thus be augmented by requiring that a unitary transformation connecting the bases of the two Hilbert spaces of quantum geometrodynamics and of LQG should also preserve some set of characteristic algebraic relations such as the canonical commutation relations or the Weyl algebra. The criterion could then be something like this: assuming that one has a Hilbert space representation of a set of algebraic relations characteristic of LQG, then if there exists a connection operator for this representation or other operators similarly constitutive of spacetime, then the smooth spacetime structure of classical GTR can still be considered as fundamental, whereas if such operators do not exist, the fundamental geometry is discrete.⁹ In the second case, but not in the first, the loop representation should come out as unitarily inequivalent to a geometrodynamical representation. Commitment to one construal over the other would correspondingly entail whether or not one must consider spacetime as fundamental. If that is the criterion, however, it seems to me as if the debate is not whether spacetime is fundamental or not, but whether it will turn out, at the fundamental level, to be smooth or discrete. As far as I am concerned, this demotes the issue to one which should not be decided by philosophical predilection, but by the complete and consistent quantum theory once we have it. The two accounts of quantum geometrodynamics and LQG are still far from this goal.

The more pressing issue on both accounts is whether, and how, the classical spacetime structure re-emerges from the quantum regime in some classical limit and what this would mean. Any QTG must be capable of reproducing classical GTR in the appropriate limit. At

⁹The polymer particle representation constructed by [Ashtekar, Fairhurst, and Willis \(2003b\)](#) clearly falls into the second category.

this point, one could also ask whether it would likewise be a condition of adequacy whether QFT emerges in some limit from QG. At least in a semi-classical limit, one would expect that the matter fields which feed into the right-hand-side of the semi-classical Einstein equations governed by the appropriate quantum-field-theoretic dynamics would re-emerge too. But since LQG is normally interpreted to exclusively deal with vacuum solutions, it is entirely unclear how, if at all, QFT could emerge in the semi-classical limit.

We are left with the second of the three issues listed at the beginning of this section, i.e. the dissolution of the classical continuous spacetimes into combinatorial structures of labelled graphs. In order to appraise the disappearance—and the re-emergence—of spacetime, quantum states of spin networks must be associated with classical spacetimes. These two levels are linked by two procedures: *quantization* starts out from the classical level and results in a corresponding quantum theory, and *taking the classical limit* leading from a quantum theory to its classical approximation. In some sense, quantization provides the “context of discovery” of QG in that it leads the physicist’s hands in guiding her from known low-energy physics to unknown Planck-scale physics. It offers the obvious direction for searching, and hopefully arriving at, a QTG. As discussed in Chapter 2, however, it is systematically speaking the false direction. The quantum theory of higher energies captures the fundamental reality, whereas the classical theory serves merely as a low-energy effective theory, as an “approximation to the truth.” Therefore, if quantization helped in the context of discovery, its inverse direction proffers the relevant “context of justification.” It ascends (or descends if you prefer) from the fundamental quantum reality to theories closer to direct experience. The inverse operation, taking the classical limit, should have as its effect the re-emergence of the continuous spacetime with its pseudo-Riemannian manifold.

9.2 RE-EMERGENCE OF SPACETIME

9.2.1 General considerations

So for LQG, pace [Dreyer \(2006\)](#), spacetime is not a fundamental concept, but one which is expected to arise in an appropriate classical limit.¹⁰ An analysis of the relation between the fundamental objects (the spin networks) and their classical counterparts thus becomes pertinent. It seems as if a necessary condition for maintaining that spacetime is fundamental were to establish a bijective mapping between the set of classically admitted spacetimes

¹⁰And, again pace [Dreyer \(2006\)](#), spacetime is not fundamental in string theory neither, at least not in its non-perturbative expression ([Horowitz 2005](#)).

and the quantum states in the physical Hilbert space of the QTG. In this case, one could argue that although spacetime may not be wearing its usual classical dress, it remains the same entity even in its quantum gown, and this entity exhibits both quantum and classical features depending on the energy scale one is studying. Due to the lack of a complete theory of QG, and the resultant absence of a physical Hilbert space, the criterion must for present purposes be revamped as demanding that there be a one-to-one onto correspondence between the three-spaces of the classical phase space and the spin network states. The criterion thus recast, however, cannot claim to be necessary with the same authority as did the original version, for a critical mind could always reply that even if the three-dimensional or kinematical states do not entertain a bijective mapping between them, this would by no means imply that the same must hold for the full physical states, depending on how the dynamics are encoded in the theory. Of course, if the QTG had a physical Hilbert space whose states could be brought into a bijection with the states of the state space of the corresponding classical theory, we would be seriously surprised. Consider the n -body problem: while the phase space of states of an n -particle system in a physical space of m dimensions is topologically \mathbb{R}^{2mn} and therefore finite-dimensional in classical mechanics, the corresponding quantum space of states is the infinite-dimensional Hilbert space $L^2(\mathbb{R}^{mn})$, the space of square-integrable functions on \mathbb{R}^{mn} .

Finding the semi-classical and the classical limits of LQG has so far resisted substantive understanding. Relating the quantum geometry of spin network states to smooth classical geometries requires a few steps, as is sketched by [Rovelli \(2004, Sec. 6.7.1\)](#). Discrete weave states $|S\rangle$ that approximate a given metric, as seen in equations (9.4) and (9.5), play an important role in this connection. But the quantum-classical correspondence is very much the subject of ongoing research. Suffice it to say at this point that the correspondence will likely relate many spin network states to a single classical metric, not unlike in thermodynamics where macrostates can in general be realized by different microstates, as would be expected. Furthermore, many quantum states will not correspond to classical states at all, and only rather special quantum states can be related to classical states. LQG thus seems to entail that space(time) is not fundamental, but emerges somehow from the discrete Planck-scale structure. In order to render the relationship between quantum states of LQG and classical states of the gravitational field intelligible, the meanings of “somehow” and “emergence” in the preceding sentence need to be clarified. The remainder of this chapter attempts to sketch a possible answer to the following questions: How does (classical) spacetime emerge from the structures at the fundamental level? And is there a systematic way to reduce the notorious vagueness involved in “emergence,” at least in this context? Will any of the prevalent notions of emergence or supervenience in the literature help in this endeavour?

Emergence is of course a prominent subject in the philosophy of science literature, particularly in the context of considering inter-theoretic relationships. Under no circumstances shall I try to add to this literature on the level of general discussions of reduction and emergence. Silberstein (2002), who offers a useful review of emergence in philosophy of science, displays how emergence is typically understood as the denial of reductionist claims. According to him, an emergentist is someone who defends the view that there is no robust sense in which our scientific theories about the macroscopic world can be reduced to or identified with more fundamental theories of the world.¹¹ Emergentism is thus understood as the denial of inter-theoretic reduction.¹² In contrast to this characterization, I wish to remain non-committal in this debate, at least to the extent to which the subsequent remarks still permit the suspension of judgment. Thus, our use of terminology will be different, as will be explicated below. If the classical theory, or an aspect thereof, emerges from the more fundamental quantum level, this does not imply that the classical theory cannot in an important sense be reduced to the quantum theory. Having said this, however, it will turn out that some features of the classical theory are idiosyncratic to this level and cannot in a narrow sense be reduced to the quantum theory. Whether or not these non-fully-reducible aspects of the classical theory have the same claim to reality as do the features of the quantum theory, must be the topic for another occasion.

As I have a rather specific context in mind, I will orient my terminology predominantly toward the extant literature in this narrower context. The only philosophical literature pertinent to emergence in the context of QG consists, as far as I know, of Butterfield and Isham's contributions (Butterfield and Isham 1999, 2001). They propose to regard quantization and emergence as two distinct, somewhat inverse, and *independent* strategies for solving the problem of QG. Of course, there can exist pairs of theories such that the classical spacetime structure of one emerges from the other without the second being a quantized version of the first. Moreover, the quantization of a classical theory might not guarantee the re-emergence of the classical structure from the resulting quantum theory, due to interpretational issues (Butterfield and Isham 2001, p. 80). Apart from the lack of a well-understood classical limit of LQG, I shall neglect these wider meanings of emergence and use it to designate the result of the inverse process of quantization. I find this limitation justified since I am only dealing with the relation between LQG and classical GTR.

The main labour is to try to determine, in the absence of a classical limit, the sense in which classical spacetime could or could not emerge from the quantum structure. One

¹¹I do not distinguish, as Silberstein does, between the ontological and the epistemological brand of emergentism. The version used here is essentially his epistemological variety.

¹²Similarly, but inequivalently, emergentism can be seen as the denial of *supervenience*, a possibility explicitly allowed by Howard (2007).

sense in which emergence cannot be applied in this context bears a temporal connotation, i.e. in the sense of something becoming manifest over a finite period of time or coming into existence through some sort of evolution. This temporal connotation of emergence is present in both, everyday language as well as the philosophical literature.¹³ Quite regardless of the absence of time, which was discussed in Section 4.3, emergence here signifies a systematic relation between theories or between entities postulated by them rather than a process in some time.

Given the richness and diversity of the literature on reductive relations between theories, Butterfield and Isham (1999) conclude that this should be taken to sustain the conclusion that there may not be a single concept of reduction to fit all instances considered, not even if the analysis is confined to physics.¹⁴ Since the primary goal of the present chapter is to illuminate the relation between GTR and LQG, and not to contribute to this vast literature in general philosophy of science, this richness will be mined for concepts of reduction that most appropriately describe this relation. My analysis should not be restricted to existing categories in the philosophical discourse, but nevertheless tie in with extant concepts of reduction which have been considered in physics.

As explicated above, due to fact that the full physical Hilbert space is not known yet, I will limit the subsequent remarks to the kinematical Hilbert space and how classical three-spaces could be considered as emerging from this quantum structure. This strategy has the huge advantage that, unlike Butterfield and Isham, I can avoid the problem of time altogether, which makes the task considerably simpler. On the other hand, however, a confinement to considerations at the kinematical level also bears the considerable risk that not all conclusions reached at that level translate into the final form of the theory. Concretely, this means that the path sketched below suggesting how to return from quantum states living in the kinematical Hilbert space to the classical three-states may be an entirely different one when one attempts to understand how the full four-dimensional classical spacetime re-emerges from the physical quantum states.

Butterfield and Isham (1999) distinguish three ways in which theories (or their concepts, entities, laws, or models) can stand in a reductive relation to one another: definitional extension, supervenience, and emergence. The first typically assumes a syntactic understanding of theories, i.e. it understands a theory as a deductively closed set of propositions. Applying Butterfield and Isham's definition of it to the case at hand, one could say that GTR is a *definitional extension* of LQG iff it is possible to add to LQG definitions of all non-logical

¹³E.g. in Pepper (1926).

¹⁴No attempt shall be made to substantially consider the wider literature on the topic. Cf. Spector (1978) for an analysis of various proposals for reduction as an inter-theoretic relation, with a particular eye on the physical sciences.

symbols of GTR such that every theorem of GTR can be proven in LQG thus augmented. The concept of definitional extension is attractive because it gives us a clear understanding of how two theories, one of which is a definitional extension of the other, relate to another. Thus, definitional extension goes a long way to explain why the predecessor theory was as successful as it was and why it breaks down where in fact it does. However, we do not expect the relation between GTR and LQG to be as clear-cut as it is between Newtonian mechanics and special relativity, where the concept of definitional extension admits a rather straightforward application. In order to determine whether or not GTR is a definitional extension of LQG, one needed to know how to recover the classical limit. Unless there is at least some progress in the recovery of the classical limit of LQG, the concept of definitional extension cannot be usefully applied to the case at stake. One would expect, to be sure, that relating LQG to GTR will involve approximations such that propositions of GTR only hold approximately in LQG, and only under certain conditions. More specifically, one first extends the definitions of LQG such as to make it conceptually sufficiently potent to be able to prove all theorems of an intermediate theory, from which GTR can, in a well-understood way, be recovered as an approximation. This process of approximation could for instance consist of taking specific limits of certain parameters of the intermediate theory, and perhaps in a particular order, or it could involve the negligence of some quantities or states in order to recover the theory to be approximated from a subset of the phase space of the approximating theory. But all of this goes beyond the concept of definitional extension and shall be discussed below when I will discuss approximation as a form of emergence.

The second relation considered by Butterfield and Isham is supervenience. *Per definitionem*, GTR supervenes on LQG iff all its predicates *supervene* on the predicates of LQG, with respect to a fixed set \mathfrak{A} of objects on which both predicates of GTR and of LQG are defined. The set of predicates of GTR is said to *supervene* on the set of predicates in LQG, given a set of objects \mathfrak{A} , iff any two objects in \mathfrak{A} that differ in what is predicated of them in GTR must also differ in what is predicated of them in LQG. The fact that supervenience requires a stable set \mathfrak{A} of objects underlying both theories, i.e. an identical ontology on which the ideologies of both theories are defined, renders it rather useless in the present case. In a very rough way, the ontology of both theories of course contains the gravitational field. But the finer structure of the ontologies of both theories do not resemble each other: in LQG, one might perhaps find loops, or spin networks, or more generally the inhabitants of the physical Hilbert space in its ontology, while in GTR, no such objects can be found. Hence, supervenience, at least as defined above, does not offer any help in understanding the relation between GTR and LQG. Of course, the requirement that the set \mathfrak{A} must underlie both theories can be relaxed: one could instead demand that the set \mathfrak{A} of objects on which

the sets of properties \mathfrak{P}_1 and \mathfrak{P}_2 of the two theories are defined must be closed under compositional operations such as mereological sums or the formation of sets. The sets \mathfrak{P}_1 and \mathfrak{P}_2 would then be defined with respect to some base individuals, forming subsets \mathfrak{A}_1 and \mathfrak{A}_2 of \mathfrak{A} . Typically, these predications would induce some properties on the non-basic composite objects. Conceivably, this relaxation might be sufficient to overcome the disjointness of the sets \mathfrak{A}_1 and \mathfrak{A}_2 .

So GTR can neither be understood as a definitional extension of LQG, nor should it be seen as supervenient on LQG. However, it turns out to emerge from LQG if one admits a sufficiently liberal notion of emergence. And this is exactly what Butterfield and Isham do: they introduce emergence in somewhat broader—and vaguer—terms than the previous two notions. For them, a theory T_1 is often said to *emerge* from another theory T_2 iff there exists either a limiting or an approximating procedure to relate the two theories. A *limiting procedure* is taking the mathematical limit of some physically relevant parameters, in general in a particular order, of the underlying theory in order to arrive at the emergent theory. I do not see an interesting manner in which GTR could be regained from LQG by means of subduing the latter to something as simple as a limiting process, at least not without approximation. The reason for this will be made explicit in the next paragraph. An *approximating procedure*, as Butterfield and Isham imply, designates the process of either neglecting some physical magnitudes, and justifying such neglect, or selecting a proper subset of states in the state space of the approximating theory, and justifying such selection, or both, in order to arrive at a theory whose values of physical quantities remain sufficiently close to those of the theory to be approximated. Landsman (2006) argues that the classical world only emerge from the quantum theory if some quantum states and some observables of the quantum theory are neglected, *and* some limiting procedure is executed. According to his view, to be discussed below, relating the classical with the quantum world thus takes both, the limiting as well as the approximating procedure.

Rovelli (2004, Sec. 6.7.1) delivers an account of how limiting procedures alone are incapable of establishing the missing link. He relates how loop quantum gravitists have not even suspected that quantum space might turn out to have a discrete structure during the period from the discovery of the loop representation of GTR around 1988 to the derivation of the spectra of the area and volume operators in 1995. He reminisces how during this period researchers believed that the classical, macroscopic geometry could be gained by taking the limit of a vanishing lattice constant of the lattice of loops. This limiting procedure was taken to run analogously to letting the lattice constant of a lattice field theory go to zero and thus define a conventional QFT. With this model in mind, something remarkable happened when people tried to construct so-called weave states which are characterized as

approximating a classical metric: when the quantum states were defined as the limit one gains when the spatial loop density grows to infinity, i.e. when the loop size is assumed to go to zero, it turned out that the approximation did not become increasingly accurate as the limit was approached. This can be taken as a clear indication that taking this limit was physically inappropriate. What was observed instead was that eigenvalues of the area and volume operators increased. This, of course, meant that the areas and volumes of the spatial regions under consideration also increased. In other words, the physical density of the loops did not increase when the “lattice constant” was decreased. The physical density of loops, it turned out, remains unaffected by how large the lattice constant is chosen, it is simply given by a dimensional constant of the theory itself. This dimensional constant is precisely Planck’s constant. This result, as discussed in Chapter 5, is interpreted to mean that there is a minimal physical scale. Or, in Rovelli’s words, “more loops give more size, not a better approximation to a given [classical] geometry.” (ibid.) The loops, it turns out, have an intrinsic physical size.

According to the traditional understanding of emergence, as described in e.g. [Silberstein \(2002\)](#), the classical metrics cannot simply be reduced to the underlying quantum structure, for the latter carries properties which the former do not. And the case here seems more serious as when attempted to regain Newtonian mechanics from special relativity by taking the limit $c \rightarrow \infty$. Although it can be argued, and I believe justly so, that although taking this limit is mathematically well-controlled, one transgresses, strictly speaking, the confines of special relativity, which explicitly assumes that the speed of light was finite. While this was delicate enough, in the present case, one can even take the limit of letting the lattice constant go to zero, but the physical loop density is not affected by taking this limit at all! Taking this limit, then, does not change the structure from discrete quantum states to smooth manifolds. It just does not change anything in the physics. As some of the features of the classical geometry such as smoothness cannot be reduced to or identified with properties of the quantum states of the more fundamental theory, GTR in toto does not reduce to LQG. According to the traditional understanding, this would imply that the classical spacetime “emerges” from the quantum theory. But the way I have fixed the terminology, following [Butterfield and Isham \(1999\)](#), we will have to be able to establish either a limiting or an approximating procedure, or a combination of the two, in order to relate the two theories before we can say that one emerges from the other. These remarks go to show that a limiting procedure, at least if used in isolation, will just not do the trick.

Considering approximation then, a series of theories the last of which will mimic classical spacetimes via approximations needs to be constructed. First of all, let us be clear what the “approximandum,” the classical theory to be approximated, should be. In LQG as discussed

so far, a funny distinction has been made, viz. the one between gravity and matter. For a general relativist, these are two distinct concepts with very different roles; essentially, gravity is the left-hand side of the Einstein equations, while matter constitutes the right-hand side. In the quantization that led to LQG, no matter was assumed to be present: LQG results from a vacuum quantization of GTR. So states in the physical Hilbert space of LQG as sketched here should lead to semi-classical states which yield emergent classical spacetimes that are vacuum solutions of the Einstein equations. This leaves us with the problem of how to include matter in the picture—a controversial issue which I have tried to avoid. Some claim that as long as the construction of a Fock-space in LQG has not been completed, we do not know how to weave matter into the spin network states, while others simply add additional representations to the nodes of the spin networks in order to represent matter, and yet others protest that all of this is entirely unnecessary as matter is implicitly included *ab initio*.¹⁵ Be this as it may, for LQG-cum-matter, the classical limit to be achieved by approximation would be GTR coupled to all known matter. Indeed, such a theory, if we had it, might qualify as a theory offering a full unification. But the current version of LQG does not include matter and should therefore lead to vacuum spacetimes in the classical limit.

The approximandum suffers from another limitation, also mentioned several times in the course of this thesis: rather than full GTR, we should expect to obtain Hamiltonian GTR, and therefore only spacetime models of topology $\Sigma \times \mathbb{R}$ with constraints forming a Dirac algebra (4.24)-(4.28). It may be argued that these departures from GTR are not essential in the sense that the core of what is believed to be physically relevant models has been maintained in switching from traditional GTR to Hamiltonian GTR as outlined in Chapter 4. But more importantly, how is this approximation to be concretely implemented? The answer to this question is wide open. Some remarks toward possible answers will be given in Subsection 9.2.2. Before then, however, let me briefly consider an important pertinent article by Landsman (2006).

Landsman (2006) discusses three major manners in which classical physics is typically held to relate to quantum physics: (i) by a limiting procedure involving the limit $\hbar \rightarrow 0$ for a finite system, (ii) by a limiting procedure involving the limit $N \rightarrow \infty$ of a large system of N degrees of freedom while \hbar is held constant, and (iii) either by decoherence or by a consistent histories approach. Landsman defends the point of view that while none of these manners is individually sufficient to understand how classicality emerges from the quantum world, they jointly suggest that it results from ignoring certain states and certain observables from the quantum theory. Ultimately, he contends, the classical world does not exist.

The origin of the idea that taking the mathematical limit $\hbar \rightarrow 0$ will return the classical

¹⁵Winston Fairbairn, personal communication, 29 June 2004.

world from the quantum realm, Landsman convincingly suspects, can be found in [Einstein \(1905\)](#), who discusses Planck’s radiation law capturing the spectral energy density function \mathbf{u} of a radiating black body at temperature ϑ , which was given in [Planck \(1901, eq. 12\)](#) as

$$\mathbf{u} = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/k\vartheta} - 1} \quad (9.1)$$

in SMI-units (cf. [Appendix A.1](#)), where h is what later became known as “Planck’s constant,” ν is the frequency, c the velocity of light, and k another new constant introduced by Planck and now known as “Boltzmann’s constant.” As [Einstein \(1905, §2\)](#) has found, taking the limit $h\nu/k\vartheta \rightarrow 0$ returns the classical equipartition law. To be precise, Einstein takes the limit $\vartheta/\nu \rightarrow \infty$ of large wave lengths, which is mathematically equivalent if h and k are held constant. Physically, this makes more sense as h and k are of course assumed to be *constants*. There is an important conceptual difference: by taking the limit $\vartheta/\nu \rightarrow \infty$ one does not leave the realm of quantum physics but only studies what quantum mechanics will predict for large wave lengths; by taking the limit $\hbar \rightarrow 0$ (or $h \rightarrow 0$), however, one changes the theory itself and studies how the theory would be changed in the modified regime. In the former case, one seeks to find out how quasi-classical states behave in quantum theory, while in the latter case, one tries to relate quantum mechanics to a corresponding classical theory.

Landsman argues, and I have no ambition to challenge his argument, that the limit $\hbar \rightarrow 0$ by itself does not suffice to fully account for the classical world with its classical denizens behaving classically if a quantum theory, any quantum theory, is used as a systematically correct vantage point. Landsman also admits, however, that the many mathematically suggestive results when taking this limit should not be ignored and certainly go some way towards explicating how the classical world emerges. According to Landsman, a similar argument can be made for a limiting procedure involving the limit $N \rightarrow \infty$ of arbitrarily large systems. Considering this limit, one realizes that strictly classical behaviour will only emerge in the case of an infinitely large system, which of course is an idealization. Although the size of systems for which classical behaviour can be observed is “much smaller” than in this idealization, the fact that there exist large systems with well-behaved classical behaviour suggests that the limit offers a valid way of getting a grasp on the relation classical-quantum world. Mathematically, it turns out ([Landsman 2006, Sec. 6](#)) that taking the limit $N \rightarrow \infty$ is a special case of taking the limit $\hbar \rightarrow 0$. This can be interpreted to mean that the idealization of assuming a system to be infinite is a case of a diminishing quantum of action in a finite system. So the first two ways in which the classical relates to the quantum involved the limits $\hbar \rightarrow 0$ and $N \rightarrow \infty$.

Taking limits like this, however, is confined to studying arising classical behaviour in particular states with respect to a given observable. If the observables show an incorrect limiting behaviour, or if one starts out from superposition states, then no classical physics can be gained from the underlying quantum physics. This incites a grave worry: why should it be that the macroscopic world never happens to be in superposition states? Or, in other words, what principled reasons do we have for excluding from consideration a vast class of quantum states and of observables when we wish to understand how classicality emerges? If the fact that it is exactly those states and observables which are capable of reproducing classical physics in the appropriate limits that we have used in our analysis is taken as a justification of our having done so, the smell of a post factum reasoning becomes forbidding. It is exactly the ambition of the third group of techniques to regain classicality to offer such principled justifications of the necessary omissions while avoiding any post factum reasoning. As discussed in [Landsman \(2006\)](#), this third group most prominently includes, but may not be limited to, decoherence and consistent histories.

The main idea of the program of decoherence—I shamelessly ignore the consistent histories approach—is that the generically assumed presence of interference in quantum states is suppressed by the system’s interaction with the “environment,” such as occurs in a measurement process.¹⁶ In other words, while a quantum state is generally a superposition state, the interaction with the environment, which is assumed to be macroscopic, destroys this superposition very rapidly. The interaction with a macroscopic environment thus disentangles entanglement. In order to make these statements more principled and rigorous, a *theory* of decoherence is required. This theory will have to specify what is meant by “macroscopic environment,” under what circumstances the interaction between environment and system leads to a suppression of interference, and how this suppression is supposed to work in detail. Without going into the details here, I state that *if* a theory of decoherence makes good on these requirements, then we are equipped with a principled way to justify the elimination of certain quantum states and certain observables in the quest to recover classicality from a quantum theory. I will return to the question of whether decoherence might help with the issue of emergence of classical spacetime from LQG towards the end of Section 9.2.2.

The main contention of [Landsman \(2006\)](#) claims that understanding the emergence of the classical world, which strictly speaking does not exist, from the quantum realm is an intricate business and will require a combination of the techniques described: the neglect of some observables, and justifying such neglect, and selecting some subset of states in the quantum state space, and justifying such selection, as well as limiting processes which relate the values of some parameters of the quantum theory to those of a corresponding classical

¹⁶For introductory reviews into decoherence, see [Bacciagaluppi \(2005\)](#) and [Zeh \(1996\)](#).

theory. Relating the quantum to the classical world will typically require two steps. The first step can be thought of as preparing the quantum system for the transition, as it occurs still within the boundaries of the quantum world, while the second step effectively performs the transition from a particular vantage point in the quantum realm to the classical world. Thus, the first step will essentially consist of neglecting some observables and of selecting some states, i.e. of an approximating procedure as described by [Butterfield and Isham \(1999\)](#). The second step is then the limiting procedure which brings the values of some quantum parameters into those of the classical ones. In the literature, the first step is often said to result in *semi-classical* states via a *semi-classical* limit, while the *classical* limit constitutes terminologically the second step.

9.2.2 Remarks towards a concrete implementation

Turning now to see to what extent, if at all, such a scheme has been or could be implemented in the case at hand, let me first remind the reader that all approaches to find the semi-classical and classical limits of LQG are confined to use the kinematical Hilbert space \mathcal{K} as its vantage point. The classical states corresponding to the kinematical spin network states of the quantum theory will thus be three-metrics on three-manifolds. Obviously, this cannot be the end of the story as ultimately, we want to understand how fully four-dimensional classical spacetimes emerge from the physical states of the quantum theory. But this does by no means imply that the endeavour of studying how classicality emerges from the kinematical Hilbert space is fruitless; quite the contrary: in the light of the difficulties in constructing a Hamiltonian constraint operator, an important test for any candidate is that it must reproduce, in a principled way, the classical Hamiltonian constraint function in some appropriate limit. Unless this test is fulfilled, it seems impossible for the physical states, which are defined to be those kinematical states which live in the kernel of the Hamiltonian constraint operator, to correctly relate to the corresponding classical states. Getting a grasp on what it means to draw the classical limit of the background-independent QFT as it stands now is therefore of decisive importance for at least two reasons. First, it is an eminent, and perhaps indispensable, help in the construction of the physical Hilbert space itself. The Hamiltonian constraint operator is not only not polynomial in the basic variables A and E , but it is not even an analytic function of them. This is never the case in familiar QFTs, and new techniques will be needed. Second, only once a consistent quantum theory is completed and it has been established that it possesses the correct classical limit, can the canonical quantization programme be considered fulfilled. To be sure, possessing the correct classical limit is not a sufficient condition for qualifying as *the* QFT, but is merely necessary as one

would expect there to be many consistent quantum theories with one particular classical limit. Which quantum theory will eventually turn out to be the best can only be determined empirically. At this point, however, it would be considered a huge success if *one* consistent and complete quantum theory with the correct limiting behaviour could be formulated.

The rough idea of constructing semi-classical states from the kinematical Hilbert space \mathcal{K} is to find those kinematical states which correspond to almost flat three-metrics, i.e. to three-geometries where the quantum fluctuations are believed to be negligibly small. Two major approaches to construct semi-classical theories dominate the extant literature, the so-called weave state approach and the ansatz using coherent states. One major approach to semi-classical QG from LQG leads via *coherent states* and has been pioneered by Thiemann and Winkler (Sahlmann et al. 2001; Thiemann 2001a; Thiemann and Winkler 2001a,b,c). The main difficulty for this ansatz is that the powerful coherent state machinery developed for the usual perturbative Fock spaces of QFT on a given background spacetime is no longer available for the non-perturbative kinematical Hilbert space of LQG.¹⁷ Other proposals include Varadarajan’s “photon Fock states” and generalizations thereof (Ashtekar and Lewandowski 2001; Varadarajan 2000), and the Ashtekar group’s shadow states (Ashtekar et al. 2003b). Most of the remainder of this chapter shall be dedicated, however, to the most prominent approach of constructing semi-classical states, the so-called “weave states.”

The idea of a *weave state* originally introduced in Ashtekar, Rovelli, and Smolin (1992),¹⁸ revolves around selecting spin network states that are eigenstates of the geometrical operator for the volume of a region \mathcal{R} with eigenvalues which approximate the corresponding classical values for the volume of \mathcal{R} as determined by the classical gravitational field. Simultaneously, these selected spin network states are eigenstates of the geometrical area operator for a surface \mathcal{S} . More technically, consider a macroscopic three-dimensional region \mathcal{R} of spacetime with the two-dimensional surface \mathcal{S} and the three-dimensional gravitational field $e_a^i(\vec{x})$ defined for all $\vec{x} \in \mathcal{R}$. This gravitational field defines a metric field $q_{ab}(\vec{x}) = e_a^i(\vec{x})e_b^j(\vec{x})\eta_{ij}(\vec{x})$ for which it is possible to construct a spin network state $|S\rangle$ such that $|S\rangle$ approximates the metric q_{ab} for sufficiently large scales $\Delta \gg \ell_{\text{Pl}}$ in a yet to be rigorously specified sense.¹⁹ Clas-

¹⁷For details, see Thiemann (2001b, Sec. II.3). The coherent states approach is discussed in Section II.3.2, the weave states in II.3.1 and the photon Fock states in II.3.3.

¹⁸For an intuitive introduction, see Rovelli (2004, Sec. 6.7.1). The picture is that of the gravitational field like a (quantum cloud of) fabric(s) of weaves which appears to be smooth if seen from far but displays a discrete structure if examined more closely. Hence *weave states*.

¹⁹The “upper case” spin network states $|S\rangle$ live in \mathcal{K}^* , the *pre-kinematical Hilbert space*, i.e. the Hilbert space containing all spin network states which solve the Gauss constraints, but not necessarily the spatial diffeomorphism constraints. Thus, the spin network states in \mathcal{K}^* are not represented by abstract graphs, as are those in the full kinematical Hilbert space \mathcal{K} , but as embedded graphs on a background manifold. This choice is just conveniently following the established standard in the literature on weave states; we will see below that this poses no problem as everything can be directly carried over to the spatially diffeomorphically

sically, the area of a two-dimensional surface $\mathcal{S} \subset \mathcal{M}$ and the volume of a three-dimensional region $\mathcal{R} \subset \mathcal{M}$ with respect to a fiducial gravitational field ${}^0e_a^i$ are given by (Rovelli 2004, Sec. 2.1.4)

$$\mathbf{A}[^0e, \mathcal{S}] = \int |d^2\mathcal{S}|, \quad (9.2)$$

$$\mathbf{V}[^0e, \mathcal{R}] = \int |d^3\mathcal{R}|, \quad (9.3)$$

where the relevant measures for the integrals are determined by ${}^0e_a^i$. This fiducial metric is typically, but not necessarily, chosen to be flat. The requirement that the spin network state $|S\rangle$ must approximate the classical geometry for sufficiently large scales is made precise by demanding that $|S\rangle$ be a simultaneous eigenstate of the area operator $\widehat{\mathbf{A}}$ and the volume operator $\widehat{\mathbf{V}}$ as introduced in Section 5.2 with eigenvalues equal to the classical values as given by (9.2) and (9.3), respectively, up to small corrections of the order of ℓ_{Pl}/Δ :

$$\widehat{\mathbf{A}}(\mathcal{S})|S\rangle = (\mathbf{A}[^0e, \mathcal{S}] + \mathcal{O}(\ell_{\text{Pl}}^2/\Delta^2)) |S\rangle, \quad (9.4)$$

$$\widehat{\mathbf{V}}(\mathcal{R})|S\rangle = (\mathbf{V}[^0e, \mathcal{R}] + \mathcal{O}(\ell_{\text{Pl}}^3/\Delta^3)) |S\rangle. \quad (9.5)$$

If a spin network state $|S\rangle$ satisfies these requirements, then it is called a *weave state*. In fact, the length scale Δ , which is large compared to the Planck length ℓ_{Pl} , characterizes the weave states, which are for this reason sometimes denoted $|\Delta\rangle$ in the literature. At scales much smaller than Δ , the quantum features of spacetime would become relevant, while at scales of order Δ or larger, the weave states exhibit a close approximation to the corresponding classical geometry in the sense that it determines the same areas and volumes as the classical metric q_{ab} . In this sense, the weave states are semi-classical approximations.

Please permit two remarks. First, it should be noted that the correspondence between weave states and classical spacetimes is many-to-one. In other words, equations (9.4) and (9.5) do not determine the state $|S\rangle$ uniquely from a given three-metric q_{ab} . The reason for this is that these equations only put constraints on values averaged over all of \mathcal{S} and \mathcal{R} , respectively, and we have assumed *ex constructione* that these regions are large compared to the Planck scale. Of course, there are many spin network states with these averaged properties, but only one classical metric which exactly corresponds to these averages values. The situation can be thought of as somewhat analogous to thermodynamics, where a physical system with many microscopic degrees of freedom has many different microscopic states with the same averaged, macroscopic properties such as temperature.

invariant level.

Second, the weave states as introduced above have merely been defined at the pre-kinematic level, i.e. they are not formulated in terms invariant under spatial diffeomorphisms.²⁰ The reason for this choice lies mostly in that this is the canonical choice in the literature, but also because in this way, the weave states can be directly related to three-metrics, rather than equivalence classes of three-metrics. This, however, does not constitute a problem whatsoever, as the characterization of weave states can be carried over into the context of diffeomorphically invariant s -knot states $|s\rangle$, i.e. spin network states in \mathcal{K} , as follows. If we introduce a map $P_{\text{diff}} : \mathcal{K}^* \rightarrow \mathcal{K}$ which projects states in \mathcal{K}^* related by a spatial diffeomorphism unto the same element of \mathcal{K} , then the state $\mathcal{K} \ni |S\rangle = P_{\text{diff}}|S\rangle$ is a *weave state* of the classical three-geometry $[q_{ab}]$, i.e. the equivalence class of three-metrics q_{ab} under spatial diffeomorphisms, just in case $|S\rangle$ is a weave state of the classical three-metric q_{ab} as defined above.

There arises a serious difficulty in constructing the semi-classical regime. As Minkowski spacetime is a tremendously accurate approximation for most of our actual spacetime, one expects that the quantum theory entertains a state, call it $|0_M\rangle$, capable of mimicking Minkowski spacetime in large scales. At a fixed time given a fiducial time parameter, “Minkowski space” is captured by a flat three-metric $\eta_{ab} = \delta_{ab}$. It is not the case, however, that $|0_M\rangle$ is a weave state of η_{ab} , for the following reason. In classical Hamiltonian GTR, the three-metric q_{ab} is build entirely from momentum variables, i.e. from densitized triads. Because weave states are typically rather concentrated around eigenvalues close to the fiducial metric, it may be the case that operators built from configuration variables will show highly delocalized behaviour with respect to these weave states. Thus, $|0_M\rangle$ cannot be an eigenstate of the gravitational field, which is constructed from the triad variables, for such an eigenstate would have maximal spread in operators built from connection variables. $|0_M\rangle$ must be “peaked” around weave states corresponding to the fiducial metric, with a mean value exactly corresponding to the fiducial metric, and with minimal spreads with respect to both basic operators $\widehat{A}_a^i(\vec{x})$ and $\widehat{E}_i^a(\vec{x})$. It can be, and has been (Corichi and Reyes 2001), shown that it is possible to construct weave states which are peaked in both the connection and the spin network basis.

It thus seems that the notion of approximation as outlined by Butterfield and Isham might bear fruits in schematizing the semi-classical approaches involving weave states. At least when these are taken to be simultaneous eigenstates of the area and volume operators, as they are in (9.4) and (9.5), some physical quantities must be neglected, viz. all those operators constructed from connection operators, as the “geometrical” eigenstates are maximally spread in these operators, and the kinematical states must be carefully selected

²⁰Cf. also footnote 19.

to only include those which are peaked around the geometrical values determined by the fiducial metric. It is at least questionable, however, whether the neglect of connection-based operators can be justified. If it cannot, then only semi-classical states which are peaked in both the connection and the triad basis, and are peaked in such a manner as to approximate classical states, should be considered. In this case, we would still only have a selection of states, but perhaps no operators which must be ignored. Once we have completed this stage, and we have found semi-classical states which approximate classical states, then a limiting procedure can be executed to see classicality fully emerge. Such a limiting procedure will involve taking the limit $\ell_{\text{Pl}}/\Delta \rightarrow 0$, which will make the small corrections in (9.4) and (9.5) disappear. This limit can be performed by either have Δ go to infinity, or ℓ_{Pl} go to zero (or both). The first choice corresponds to letting the size of the spatial region \mathcal{R} grow beyond all limits, and thus resemble the limit $N \rightarrow \infty$ as discussed in Section 9.2.1, which considered how classicality can generally emerge from ordinary quantum mechanics. The second choice, letting the Planck size go to zero, corresponds then to the case $\hbar \rightarrow 0$ from Section 9.2.1. With the second choice, but arguably not the first, we leave the realm of the quantum theory and arrive at a strictly classical description of the spatial geometry.

Generically, of course, kinematical states do not satisfy (9.4) and (9.5). How can the selection of kinematical states with such fine-tuned properties then be justified? Unless the scheme relating (kinematical) LQG to classical (Hamiltonian) GTR offers such a justification, it is hard to see how such a scheme could answer one of the key questions: why does classical GTR work as well as it does and why does it break down where in fact it does? In attempts to relate ordinary QM to its classical counterpart, as discussed above, more often than not reference is made to decoherence through the quantum systems's interaction with its environment. Would perhaps decoherence offer a justification for the selection of very special kinematical states? Could it be the case that the kinematical quantum state is somehow, in a principled way, forced into a weave state when it interacts with its environment? It seems dubious, at the very least, that in the present context decoherence can play the role it does for the traditional problem of emergence of classicality because here the spin network states are supposed to be the quantum account of space—and all of it. Introducing an environment with which these spin network states can interact seems to imply that there must be something outside of quantum space.

Does it really? Not if we conceive of areas and volumes as local properties of the quantum gravitational field, just as these geometrical properties were local in GTR. As was explicated in Chapter 5, given a region \mathcal{R} of quantum space, e.g. a chunk of space in our laboratory, each node of the spin network state represents a grain of such a space as it contributes to the eigenvalue of the volume operator. Similarly, each link from a node within \mathcal{R} to a

node outside of \mathcal{R} , i.e. each link which intersects the boundary \mathcal{S} of \mathcal{R} , contributes to the eigenvalue of the area operator. If we had measurement devices at our disposal with Planck-scale accuracy, we could measure the volume and the surface area of a region of space. Such a measurement would essentially amount to counting the nodes within a region as well as counting the links which leave the region. The area and volume operators correspond to partial observables in the sense of Rovelli (2002d).²¹ If, however, the region \mathcal{R} considered does not encompass all of space, but only a delimited piece of it, then finding an environment for such a region is straightforward.²² This re-opens the door for invoking decoherence as a potential justification of the selection of kinematical states related to classical geometry. Thus, it could be the case that if we perform an area or volume measurement on surface \mathcal{S} or region \mathcal{R} , respectively, then the measurement interaction forces the quantum state of the corresponding piece of space into a weave state. The price of admitting decoherence back in, however, is exorbitant: the infamous measurement problem rears its ugly head again.

Recently, the notion of decoherence has been generalized such as to also cover cases in which the system that decoheres does not depend for its decoherence on an interaction with its environment.²³ In fact, a sort of “self-induced” decoherence seems to allow closed systems to entirely decohere on their own. Without going into the details of the proposal, it is clear that if it works, then it will be attractive for quantum cosmological settings, and spin network states seem to be perfect candidates to decohere in a self-induced manner without having to rely on an environment. With a notion of self-induced decoherence, it might even be possible to avoid the measurement problem, at least for this particular step.

Many details still have to be filled in, but the emerging picture is sufficiently clear. Perhaps disappointingly, perhaps reassuringly, the way in which classical GTR emerges from LQG according to this most promising semi-classical approach does not fundamentally differ from how classical mechanics is most promisingly thought to emerge from ordinary QM. At least not if the move to use decoherence by arguing that although we are dealing here with a quantum theory of space, it is possible to have an external observer. In other words, unlike in traditional QM, a *case* must be made that the application of the traditional measurement concept is appropriate at all, at least for scenarios not depending on self-induced decoherence. This will only be the case if we can successfully argue that a medium-sized region of spacetime can be measured “from the outside.” But the above considerations show, the situation as to how classicality emerges will be essentially the same as it is for traditional QM.

²¹See Rovelli (2004, Sec. 6.7) for a detailed discussion of the physical interpretation of the spin network states.

²²This is again the “medium-sized” region of spacetime which was already submitted to measurement in Section 5.3.3.

²³Cf. Castagnino and Lombardi (2005), Castagnino et al. (2006)

10.0 ALL'S WELL THAT ENDS WELL OR LOVE'S LABOUR'S LOST?

While theoretical and mathematical physicists working in fundamental physics usually have a clear preference for one of the disjuncts in the title over the other, I hope to have shown that loop quantum gravity and the entire programme of canonical quantization of classical general relativity enjoy, and suffer from, both. On the one hand, as I have most pronouncedly argued in the context of the codification of the full spacetime diffeomorphism invariance in the Hamiltonian formalism and the problem of constructing the physical Hilbert space of the quantum theory, the loop approach has still foundational chores to do. In all likelihood, these two issues will turn out to be deeply interwoven. Physicists still face the task of finding the precise expression of the Hamiltonian operator and of completing the theory by concretely constructing a physical Hilbert space with an inner product. Likewise, they face the challenge of establishing that the theory has the correct semi-classical and classical limits. Furthermore, the relation between the covariant extension of the theory and its covariant core is not yet well understood. Philosophers, on the other hand, should be encouraged to investigate the more interpretative questions at both the classical as well as the quantum level and address issues such as the nature of the observables of the classical and the quantum theories, whether, and if so, to what extent, relationism is supported by loop quantum gravity, can the concept of determinism be suitably generalized to quantum spacetime, and can decoherence play an important role in understanding the emergence of classical spacetime. There are still many issues wide open and the success of the entire approach will stand or fall with the successful resolution of these difficulties.

On the other hand, loop quantum gravity and classical Hamiltonian general relativity have greatly contributed to the clarification and advancement of our understanding of the fundamental nature of spacetime, and in particular, to the interpretation of classical general relativity. They have forced us to clarify the issue of symmetries, and in particular of spacetime diffeomorphism invariance, and of their role as gauge symmetries of the theory. Also, they have given us at least an abstract scheme for determining observables and for extracting the physical content of a theory. Many techniques developed to deal with loop quantum gravity have also enriched quantum field theory, some of whose concepts have been

generalized to apply to background independent theories as well. And finally, on a somewhat different note, the relation between the underlying, still incomplete, quantum theory and the classical theory emerging from it have let me onto paths less trodden and to positions more speculative than those encountered in the quantization process. Without failure, all of these studies have encountered a wealth of important foundational issues and thus instill the desire to continue them.

There is no doubt that the conclusive philosophical appraisal of loop quantum gravity must await the completion of the theory. Although this may be disappointing news to anyone who demands final and immovable answers to foundational questions, and although an investigation into the foundations of loop quantum gravity is met by a pervasive open-endedness, which may be frustrating at times, the canonical approach is sufficiently rich and sufficiently solidified to offer a rewarding harvest for any foundational enquiry, as I hope to have illustrated. This promise of abundance stands in a stark contrast to the surprising paucity of discussion of canonical quantum gravity in the philosophical literature. With a few notable exceptions in the technical philosophy of physics literature, quantum gravity has largely been met by the philosophical community with neglect or disinterest. But this is an opportune moment for philosophers to develop an interest in quantum gravity: with loop quantum gravity a serious contender among theories of quantum spacetime has arisen, with its practitioners willing collaborators of philosophers, and the theory has sufficiently matured while it is still under construction and thus hospitable to more speculative endeavours.

APPENDIX A

NOTATION AND CONVENTIONS

A.1 GENERAL REMARKS

I try to follow, not necessarily with complete success, the notational and terminological conventions of [Rovelli \(2004, pp. xviii-xxiii\)](#). With a few exceptions as indicated, natural units ($\hbar = c = 1$) are used and the Einstein summation convention is assumed.

A.2 INDEX NOTATION

The now standard abstract index notation is used, as e.g. in [Wald \(1984\)](#), without following Wald in his particular choices. The index notation here follows the most common conventions in the field, as does [Rovelli \(2004\)](#). These conventions are:

- Greek letters from the middle of the alphabet, μ, ν, \dots , represent four-dimensional spacetime tangent indices. They are part of the notation for the tensorial object itself and do *not indicate its components*. If an equation only holds with respect to a particular basis and must thus be understood as an equation holding only between or among tensor components, an indication will be made in the text.
- Upper-case Latin letters from the middle of the alphabet, I, J, \dots , designate four-dimensional internal or Lorentz tangent indices. In the context of STR, no distinction between Lorentz or spacetime tangent indices is made.
- Lower-case Latin indices represent three-dimensional indices; the beginning of the alphabet, a, b, \dots , is used for tensorial indices, the middle of the alphabet, i, j, \dots , for internal ones.

A.3 FURTHER NOTATIONS AND CONVENTIONS

- The Minkowski metric will be written as η_{IJ} or $\eta_{\mu\nu}$, with signature $[-, +, +, +]$. This metric is used to raise and lower four-dimensional Lorentz indices I, J, \dots . The metric of \mathbb{R}^3 is δ_{ij} , the Kronecker delta. This metric is used to raise and lower three-dimensional internal indices i, j, \dots .
- ϵ_{IJKL} , or $\epsilon_{\mu\nu\rho\sigma}$, and ϵ_{ijk} , or ϵ_{abc} , are the totally antisymmetric objects with $\epsilon_{0123} = 1$ and $\epsilon_{123} = 1$ in four or three dimensions, respectively.

A.4 ACRONYMS

GTR	General Theory of Relativity
LQC	Loop Quantum Cosmology
LQG	Loop Quantum Gravity
QED	Quantum Electrodynamics
QFT	Quantum Field Theory
QG	Quantum Gravity
QTG	Quantum Theory of Gravity (any theory combining QT and GR)
QM	Quantum Mechanics (“classical” theory around 1930)
QT	Quantum Theory (any quantum theory, or all)
STR	Special Theory of Relativity
ST	(Super-)String Theory (any string theory, or all)

APPENDIX B

CHALLENGING THE SPACETIME STRUCTURALIST

“Structuralism” or “structural realism,” which I shall use synonymously, can be, and has been, interpreted in a number of inequivalent ways. Its core thesis, however, is some variation or other of the idea that whatever scientific theories reveal of the external world is *structural* knowledge of this world. Structural realism has been developed as a response to the pessimistic meta-induction according to which it is incautious, at least, to commit oneself to any particular scientific theory, given the fact that so far, in any given domain of enquiry, all but one theories have been superseded by their successor theories. Structural realism reduces the realist commitment in that it takes on a realist attitude toward those aspects of a theory which are preserved through the course of a scientific revolution. It defends a non-vacuous position in that it positively maintains that whatever is preserved through the course of revolutions is exactly the “structure” embedded in a theory. It is therefore toward this structure that we should make our realist commitment.

This characterization, of course, remains empty unless some precision of the term “structure” is offered. Paraphrasing [Esfeld \(2004\)](#), the following definition can be assumed:

Definition 9 (Structure). *A structure \mathfrak{S} is a pair $\langle \mathfrak{D}, \mathfrak{R} \rangle$ which consists of a non-empty set of relations \mathfrak{R} (“ideology”) as well as a non-empty set of relata \mathfrak{D} (“ontology”), the domain of \mathfrak{S} .*

In this definition, I have already made the choice of not considering some varieties of structuralisms according to which there are only relations, without underlying or accompanying relata. At least the rhetoric of some British structuralists suggests that they defend this extreme, and in my opinion untenable, position. Given the unintelligibility of this position, I suspect that such an attribution is plainly false and the position really defended by these British structuralists differs from the one painted by their rhetoric. But let me press on.

The structuralism discussed in this section, as indicated by Definition 9, will assume that both the relations and the relata are ex aequo necessary constituents of the structural knowledge the structuralist hopes to attain.¹ The structuralist position defended by Esfeld (2004) amounts to an endorsement of the following definition:

Definition 10 (Structural Realism). *The “structural” aspects of a scientific theory, toward which we should entertain a realist attitude, relevantly capture the structure of the external world. The fundamental scientific theories thus reveal the structure of the external world, where structure is used in the sense of Definition 9. The objects $x \in \mathfrak{D}$, i.e. the things which exemplify the relations $R \in \mathfrak{R}$, do not have any intrinsic properties, but only relational ones.*

So what is really there according to the structural realist is a network of relations among objects which do not possess any intrinsic properties but are purely defined by their “place” in \mathfrak{S} . How these structural aspects are identified in a given theory is, of course, a highly non-trivial matter and will largely depend on one’s interpretation of the theory at stake. For present purposes, I shall assume that these structural aspects of the theory have been successfully identified.

Let me attempt to capture the idea of structuralism more formally. What the structural realist characterized by Definition 10 demands is that the world is fundamentally described by a structure \mathfrak{S} with objects which exemplify the intra-structural relational properties, but no other properties. Any other properties would introduce a unwanted reference to something beyond the purely structural. These intra-structural relational properties are exactly those which are invariant under automorphisms of \mathfrak{S} .² Thus, the set \mathfrak{R} , which figures in the definition of a structure, and consequently in the definition of structural realism, contains only automorphically invariant relational properties.

It can be shown that structural realism characterized as in Definition 10 and applied to the present context suffers from serious difficulties in accommodating highly symmetric spacetimes. For these particular spacetime solutions with a high degree of symmetry, a devastating argument in full analogy to the one run by Keränen (2001) against structuralism in the philosophy of mathematics can be given. To this end, consider the highly symmetric cosmological standard model in classical GTR, to Friedmann-Lemaître-Robertson-Walker

¹This also implies that the subsequent discussion will not follow Stachel (2006) in assuming that either the objects or the relations should be primary, where the disjunction is exclusive. For present purposes, it is presumed that an attractive version of structuralism can be presented without insisting either on privileging relations over relata or on eliminating objects altogether. I am not alone: Esfeld (2004) shares this presumption.

²An *automorphism* f is a map from a domain A onto itself which preserves the structure of A (i.e. is an isomorphism of a set onto itself). A property is *invariant under an automorphism* when any element $a \in A$ has the property iff its image $f(a)$ has it.

(FLRW) spacetimes, discussed in Appendix C. These cosmological models realize the cosmological principle, according to which no position in space is privileged in any way, if this principle is interpreted as requiring spatial homogeneity, as it usually is. Spatial homogeneity demands that the universe be the “same in every location.” This idea can be formalized by

$$\forall p, q \in \Sigma \subset \mathcal{M}, \forall F \in \mathfrak{P} (Fp \leftrightarrow Fq), \quad (\text{B.1})$$

where \mathfrak{P} is the set of admissible physical properties with respect to which points in Σ must be the “same” and Σ is a folio of manifold \mathcal{M} when foliated by the cosmological time as foliation parameter. Proposition (B.1) is valid for any spacelike folio of such a foliation of a FLRW-spacetime. The spacetime structuralist of the persuasion as captured by Definition 10 reformulates the spatial homogeneity of cosmological models as

$$\forall x, y, z \in \Sigma = \mathfrak{D}, \forall R \in \mathfrak{R} (Rxx \leftrightarrow Ryz). \quad (\text{B.2})$$

The individuality of the objects in \mathfrak{D} must be ascertained by an identity criterion with whose help objects can be distinguished. If we had such a criterion, we could count how many objects are contained in the structure which best describes the world. The most prominent such criterion is Leibniz’s Principle of the Identity of Indiscernibles, or a modernized version thereof. The core idea of this family of principles is to utilize distinction between objects in terms of the properties they exemplify as a criterion to individuate them. In more formal words,

$$\forall F \in \mathfrak{P} (Fx \leftrightarrow Fy) \rightarrow x = y. \quad (\text{B.3})$$

Varieties of this Principle of the Identity of Indiscernibles (PII) typically differ in what is taken to be in \mathfrak{P} (French 2006): (i) $\forall F$ ranges over all possible properties, (ii) $\forall F$ ranges over all possible properties except spatio-temporal ones, and (iii) $\forall F$ ranges only over intrinsic properties. The properties here at stake, even in the weakest version (i), are all so-called qualitative properties. A property is called *qualitative* just in case its exemplification does not depend upon the existence of any particular individual. Haecceistic properties, i.e. properties based on a particular object’s being that particular individual, or its “thisness,” are thus excluded. Furthermore, disjunctive properties such as “being featherless or being biped” are to be excluded, as well as perhaps properties with otherwise “pathological” behaviour. *Intrinsic* are all and only those qualitative properties whose exemplification is independent of the existence of other contingent objects. Intrinsic properties can thus be attributed independently of accompaniment or loneliness. All non-intrinsic properties are either *extrinsic* if they are monadic, or else relational.

The strongest version of PII, viz. PII(iii), claims that no two individuals can possess the same intrinsic properties. This principle is clearly violated in classical physics, where distinct particles may be regarded as indistinguishable as far as intrinsic properties are concerned. Black (1952) has proposed a counterexample against such a strong version of PII by placing two indistinguishable spheres in an otherwise empty universe. Assuming that we placed *two* individual spheres into the vacuum universe, Black's example not only violates PII(iii), but also PII(ii). Does it also violate PII(i)? Not as long as the universe into which the spheres are put is interpreted as a *fixed* background. In this case, the set of properties attributed to the two spheres are not entirely identical: at least properties based on their spatio-temporal location will not coincide.³ So at least the weakest form of PII is usually taken to be valid in classical physics, where spatio-temporal trajectories of rigid bodies do not overlap.

The structural realist must re-interpret PII to adapt it to her purpose. The only acceptable version of PII for her, clearly, holds that $\forall F$ ranges over \mathfrak{R} . So for the structural realist, (B.3) thus becomes

$$\forall z \in \mathfrak{D}, \forall R \in \mathfrak{R} (Rzx \leftrightarrow Ryz) \rightarrow x = y, \quad (\text{B.4})$$

where the outermost universal quantifiers of x and y have been omitted for clarity's sake, just as in (B.3). Now the analogue of Keränen's argument (Keränen 2001) can be derived easily: for spatially homogeneous spacetimes, we get from (B.1) and (B.4) by modus ponens that $x = y$. But since x and y have been arbitrary elements in $\Sigma = \mathfrak{D}$, all points of Σ coincide and there is only one point in Σ . In other words, the universe consists of one point only! Since the group of automorphisms of Σ are the isometries of Σ , and the group of isometries of Σ is transitive, i.e. any point in Σ can be moved into any other point in Σ , all points in Σ must share the same properties.⁴ But if all points in Σ share the same automorphically invariant properties, they can only constitute one individual.

The structural realist cannot distinguish between the elements of \mathfrak{D} . But if the objects in \mathfrak{D} cannot be distinguished, they must be identified according to (B.4). However, when every other object that there might have been must be identified with one particular object, then we say that there *is* only one object. In other words,

$$\exists!x [x \in \mathfrak{D}] \iff_{\text{def}} \exists x [x \in \mathfrak{D} \wedge \forall y (y \in \mathfrak{D} \rightarrow x = y)].$$

Can the spacetime structural realist hope to have a comeback after this reductio of her position? Yes, and there are at least five defensible strategies to meet this challenge:

1. dismiss Definition 10 and redefine structural realism;

³This point assumes that the topology of the space into which the spheres are placed is \mathbb{R}^3 . One could re-interpret the situation as one sphere which is put into a non-Euclidean space.

⁴Cf. footnote 1 in Appendix C.

2. deny the relevance of homogeneous cosmological models (and similarly symmetric spacetimes);
3. reject PII(i) *even for classical physics* and urge its replacement with another criterion of individuation;
4. claim that PII(i) is inapplicable to the case at hand because no criterion of individuation is needed at all; and
5. claim that PII(i) is inapplicable to the case at hand because bare points in Σ as such need not be individuated.

The first strategy can easily be chosen, but for each novel characterization, one will have to ascertain that it does not run afoul of the Keränen move. It transcends the limits of the present investigation to offer an analysis which is sufficiently comprehensive to permit generic conclusions about whether structural realism *per se* must face this challenge. I suspect, however, that most versions of structural realism which deserve this name will be subject to a similar counter-argument. Let me briefly consider the other four exit-strategies.

The second path, which denies the relevance of these highly symmetric spacetimes on which the challenge relies, will most promisingly do so by invoking a measure-theoretic consideration. The symmetric spacetimes of the type FLRW, this defence will run, are arguably of measure zero in the space of solutions of the classical field equations. Since the Einstein equations have not been solved in their full generality and since this space is therefore not explicitly known, this line of argument incurs a promissory note recording the debt of producing this space including a natural measure defined on it. Even if we accept this promissory note for the time being, the contention that the disturbing conclusion will only arise in almost no possible worlds is not implied. The structural realist will still have to establish that the displeasingly symmetric spacetimes are in fact of measure zero. But let us grant this, and let us even grant that the defence translates into the quantum case, i.e. that highly symmetric solutions are also of measure zero in the physical Hilbert space of full LQG. Even then would this defence be rather inelegant, as it is simply not the case that e.g. the FLRW models are irrelevant. This train of thought, however, opens up the possibility of running it in the “return” direction. But the inverted argument belongs to the third class of defence strategies.

The third strategy rejects PII(i) as a valid criterion of individuation and seeks to supplant it. The first answer in this vein builds on an inversion of the objection aired in the previous paragraph and denies that PII(i) is necessarily true. It insists that its truth is contingent and if its application in the context of fundamental physics leads to absurd results, it should be given up as a criterion of individuation and be replaced by a more sensible alternative. Thus, the above argument shows that PII(i) implies a disturbing and *obviously false* conclusion for

some highly symmetric spacetimes. This fact can be taken as *evidence* that the principle must be rejected as a criterion of individuation in those possible worlds in which it thus fails. This is exactly what it means for the principle to be only contingent: while it is true in almost all possible worlds, it is false in some possible worlds, which happen to be of measure zero. To find out whether the principle is true in the actual world is thus a matter of empiricity. As we arguably live in a spatially inhomogeneous universe, it will likely turn out as true. This escape, however, must explain why a metaphysical principle of individuation should not be universally valid no matter what. Why should it be acceptable to entertain different such criteria even within a single physical theory?

Saunders (2003) offers another variant of the third strategy to counter the challenger. He proposes to find, or construct, a symmetric, but irreflexive relation $S \in \mathfrak{R}$ such as to render the objects “weakly discernible” and hence save them from being identified.⁵ The attraction of this approach is that this irreflexivity can ground the object’s individuality without recourse to some sort of primitive thisness. However, it faces at least two other difficulties. First, it may not be so trivial to find such a relation defined on a homogeneous spacelike hypersurface Σ without disturbing its homogeneity, if homogeneity is understood as defined in (B.1). Second, it appears as if in order to appeal to such relations, an individuation of objects must already be presupposed: how can I know that there are at least two objects such that an irreflexive relation can be exemplified on the elements of Σ ? I do not see how this suspicion of circularity can be dispelled.

Of course there remains at least a third possibility for enacting an exit-strategy of type three. Esfeld (2004, p. 603) explicitly leaves open the possibility that the objects have non-qualitative properties such as primitive thisness. These haecceities seem to deliver a last resort if everything else has failed. If the individuation of objects is based on them, they become ineliminable members of the set of properties that the structural realist must admit. But haecceities are non-relational properties, and certainly not automorphically invariant. Therefore, haecceities cannot be an attractive option for the structural realist. Most philosophers, with the notable exception of Oliver Pooley, would consider a haecceistic account of individuation as having a strong family resemblance with typical forms of substantivalism, and therefore as opposed to the spirit of structural realism understood here as a kind of relationism. More importantly, perhaps, unobservable haecceities re-open the “gap between metaphysics and epistemology” that many structural realists such as Esfeld are so anxious to close.

Esfeld (2004) himself endorses an escape along the fourth line of defence, i.e. denying

⁵A relation R defined on a set \mathfrak{D} is *irreflexive* just in case no element of \mathfrak{D} is thus related to itself: $\forall x \in \mathfrak{D}(\neg Rxx)$.

that *any* criterion of individuation is needed at all, when he states that “[t]here is no need for the [objects] of which [relational] properties can be predicated to be distinct individuals.” (p. 611) He reaffirms his this attitude two pages later when he writes that “[t]he argument of this paper accepts that relations require things that stand in the relations (although these things do not have to be individuals, and they need not have intrinsic properties).” (p. 613) Insisting that objects which stand in relations need not be individuals seems highly counterintuitive: should there not be a fact of the matter whether two “places” x and y in a structure \mathfrak{S} are identical or not? In other words, should the statement $x = y$ not have a definite truth value? Although intuitions may deceive, and they often do at the level of fundamental physics, it nevertheless seems as if the advocate of the no-criterion-of-identity-is-needed position owes an account of why such a criterion is not needed and why our intuitions must be overturned in this case.

In a brief reaction to my challenge, [Esfeld and Lam \(2006, Sec. 1\)](#) criticized what they take to be my metaphysical position as motivating my questionable demand that objects must carry intrinsic properties as only such properties can ground the individuality of the relata. This metaphysical position, which they seem to attribute to me, assumes that the objects enjoy a metaphysically prior position over the relations. I respond with an emphatic “no.” I insist that even for structural realists à la Definition 10, i.e. for someone who puts relata and relations on an equal footing, identity criteria will be required to individuate both objects and relations. In general, no intrinsic properties are required at all in order to individuate objects. Thus, I also maintain that individuating objects and relations is in general possible within the structural realist programme, but runs into trouble in highly symmetric cases.⁶ The highly symmetric cases may only go to show that we have not correctly identified the structure \mathfrak{S} , as I try to sketch in the description of the sixth strategy.

The final reply to the challenge seems to most promising route to me: claim that PII(i) does not apply to the case at hand since the points of the bare manifold Σ as such need not be individuated. This is a much milder form of response four because it does not deny that a criterion of individuation is required for the places of a fundamental structure but still claims that the points in Σ need not be individuated. This strategy is also minimally radical: structural realism as typically understood, and as characterized by Definition 10, can be maintained, while the individuation need not be governed by haecceities. As it maintains

⁶I have not discussed the individuation of properties. It seems obvious to me, however, that this will be equally necessary for a structure \mathfrak{S} to be intelligible. But it might seem less problematic if relata are only related by *one* relation. For a structural realist, however, this would resurrect a similar worry as the one exposed by the challenge: will it still be possible in general to individuate objects in the ontology with the help of only one relation? The answer is yes, although in order to be able to individuate objects, symmetric structures must be excluded.

that the objects in \mathfrak{D} of a structure \mathfrak{S} must be individuated, and since the manifold points in Σ cannot be individuals as shown by the challenge, it must consequently deny that the manifold points in Σ can play the role of the relata in \mathfrak{S} . Therefore, manifold points should not be interpreted as places in the fundamental structure describing reality according to GTR. In other words, $\Sigma \neq \mathfrak{D}$. Arguably, this resolution of the challenge posed to structural realism in this section receives independent support from the hole argument discussed in Section 3.2. Of course, this sketch of a proposed resolution is entirely programmatic. In particular, it will have to be fleshed out how one can identify the individuals which take the place of the relata in the structure at hand. But this is the topic for another occasion.

APPENDIX C

CLASSICAL COSMOLOGY: FRIEDMANN-LEMAÎTRE-ROBERTSON-WALKER SPACETIMES

This appendix gives a quick summary of the most important points on the cosmological standard model in GTR, the so-called *Friedmann-Lemaître-Robertson-Walker (FLRW) spacetimes*. In the main body of the text, it is assumed that the reader is familiar with this material, for which I claim no originality whatsoever. Those unfamiliar with FLRW spacetimes, or those in need of a reminder should consult this appendix before moving on to Chapter 6. Reader interested in studying the FLRW models in depth, should consult the pertinent literature, most notably [Hawking and Ellis \(1973, Sec. 5.3\)](#), [Weinberg \(1972, Sec. 14.2 and Ch. 15\)](#), and [Wald \(1984, Ch. 5\)](#) and references therein or [Cheng \(2005, Chs. 7, 8\)](#) for a more recent account and up-to-date references.

Modern cosmology uses the so-called *Copernican* or *cosmological principle*, according to which no position in space—including ours—is privileged in any way. It is generally interpreted to mean that the universe must exhibit spatial homogeneity, at least approximately so. Spatial homogeneity is mathematically encoded in the action of a group of isometries on \mathcal{M} with spacelike hypersurfaces as the group's surfaces of transitivity.¹ Any point on any spacelike hypersurface is thus equivalent to any other point on the same hypersurface. Appendix B exploits this homogeneity to challenge the spacetime structural realist.

Spatial homogeneity is only expected to obtain at sufficiently large scales. Unfortunately, testing spatial homogeneity seems impossible, for we have access to only a tiny portion of the entire universe. What we can observe, however, are isotropies and anisotropies. Observations have so far corroborated that the universe is approximately spherically symmetric about our location. Anisotropies in the cosmic background radiation, for example, have been

¹The surface of transitivity of a group G acting on the manifold \mathcal{M} is the set $\Omega \subset \mathcal{M}$ of all points such that the group action $G \times \Omega \rightarrow \Omega$ possesses only a single group orbit, i.e. for every pair of elements $x, y \in \Omega$, there exists a group element $g \in G$ such that $gx = y$.

observationally confined to be small. Hence, exact spherically symmetric models of GTR are good approximations to the large-scale structure of spacetime—at least in the region that we can observe. Because of the spherical symmetry about our location, the Copernican principle would have it that the universe is spherically symmetric about every point in spacetime. The close relationship between isotropy and homogeneity of space has been made explicit by a theorem due to [Walker \(1944\)](#):

Theorem 2 (Walker). *If a spacetime $(\mathcal{M}, g_{\mu\nu})$ exhibits exact spherical symmetry about each of the points $p \in \mathcal{M}$, then it is spatially homogeneous and admits a six-parameter group of isometries whose surfaces of transitivity are spacelike hypersurfaces Σ of constant curvature.*

Spacetimes $(\mathcal{M}, g_{\mu\nu})$ with exact spherical symmetry about every point are exactly the FLRW spacetimes mentioned above. [Theorem 2](#) establishes that there exists a preferred foliation of a FLRW spacetime. This foliation into spacelike hypersurfaces of constant three-dimensional curvature is determined by the isometry group and can be labelled by a time coordinate t . A time t thus privileged is called a *cosmological time*. In conveniently chosen coordinates t, r, θ, ϕ , where $t \in \mathbb{R}^+, r \in \mathbb{R}^+, \theta \in [0, \pi], \phi \in [0, 2\pi]$, FLRW spacetimes are given by the metric

$$ds^2 = -dt^2 + a_0^2 a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right], \quad (\text{C.1})$$

where $a(t)$ is a yet to be determined positive, dimensionless function of the cosmological time, a_0 is the “present size of the universe,” and k is a constant which can assume the values $-1, 0$ and $+1$ in appropriately chosen units for r . This constant captures the sign of the spatial curvature. The spatial geometry of the spacelike hypersurfaces Σ privileged by the isometry group is given by the part of [\(C.1\)](#) in square brackets, which is independent of t . When $k = 0$ or -1 , the hypersurfaces Σ are diffeomorphic to \mathbb{R}^3 (“open”) and r runs from 0 to ∞ . For $k = 1$, the Σ are diffeomorphic to a three-sphere S^3 , in which case Σ is compact or “closed” and r runs from 0 to 2π . Cosmological time is diffeomorphic to \mathbb{R} and assumes values in the open interval $]a, b[\subseteq \mathbb{R}$.

The three-dimensional curvature scalar of spacetimes with metric [\(C.1\)](#) is given by

$${}^3R = \frac{k}{a_0^2 a^2(t)}, \quad (\text{C.2})$$

i.e. the curvature of the three-spaces is inversely proportional to $a^2(t)$. For $k = -1$ (the hyperbolic case) and for $k = 0$ (the Euclidean case), the Σ s are infinite, while for $k = 1$ (the spherical case), they are unbounded, but finite, with circumference ${}^3U = 2\pi a_0 a(t)$ and proper volume ${}^3V = 2\pi^2 a_0^3 a^3(t)$. In the latter case, the Σ s can be regarded as spheres of

radii $a_0 a(t)$ in \mathbb{E}^4 , in which case $a(t)$ is the size of the universe in terms of the present size of the universe, i.e. in terms of the radius of the spherical three-space at some fiducial time. For the hyperbolic and Euclidean geometries, this direct interpretation of $a(t)$ fails. As it nevertheless scales the three-spaces, $a(t)$ is called the (dimensionless) *scale factor* in all three cases.

As a result of the symmetry of (C.1), the energy-momentum tensor takes the form (Hawking and Ellis 1973, p. 70)

$$T_{\mu\nu} = (\rho + p)V_\mu V_\nu + p g_{\mu\nu}, \quad (\text{C.3})$$

which is the form of a perfect fluid with density $\rho(t)$ and pressure $p(t)$ which only depend on time, but must be the same anywhere in a given Σ because of the Copernican principle. The flow lines of the fluid assume constant values of r, θ and ϕ . The unit vectors V_μ are tangents to these flow lines. This means that the coordinate system which yielded the simple form in which (C.1) is cast is a co-moving frame. The factor $a_0 a(t)$ thus scales the separation of flow lines, i.e. the separation between neighbouring galaxies. From the conservation of energy and momentum $\nabla^\mu T_{\mu\nu} = 0$ (where “ ∇ ” designates the covariant derivative) and (C.3), one obtains

$$\dot{\rho} = -3(\rho + p)\frac{\dot{a}}{a}. \quad (\text{C.4})$$

Together with an equation of state $p = p(\rho)$, (C.4) can be used to determine the density ρ as a function of the scale factor a . For instance, for a radiation-dominated universe, we have $p = \rho/3$ (“radiation”) which, when inserted into (C.4), leads to $\rho \propto a^{-4}$. For a universe dominated by non-relativistic matter, the pressure is negligible ($p \ll \rho$, “dust”) and (C.4) is solved by $\rho \propto a^{-3}$. Thus, all physical parameters have been reduced to a single one, the scale factor, which captures the remaining dynamical degree of freedom.

In order to find the dynamical evolution of the remaining degree of freedom, the Einstein field equations must be used. The cosmological principle, via the FLRW metric (C.1), fixes the left hand side of the equations. But it also constrains the right hand side: the simplest choice of $T_{\mu\nu}$ satisfying the requirement of isotropy and homogeneity is the perfect fluid (C.3). The Einstein equations thus relate the geometric parameters of the scale factor $a(t)$ and of the curvature signature k to those describing the cosmic fluid, i.e. to the density $\rho(t)$ and the pressure $p(t)$. As mentioned above, the state equation $p = p(\rho)$ and (C.4) reduce the three unknown functions to a single one— $a(t)$ —which is determined by the Einstein equations. The symmetry requirements of the cosmological principle (and of diffeomorphism invariance) reduce the independent components of the Einstein equations from ten to two

(Wald 1984, Sec. 5.2). They are given by

$$\frac{\dot{a}^2}{a^2} = \frac{8G\pi}{3}\rho - \frac{k}{a_0^2 a^2} + \frac{\Lambda}{3}, \quad (\text{C.5})$$

$$\frac{\ddot{a}}{a} = -\frac{4G\pi}{3}(\rho + 3p) + \frac{\Lambda}{3}, \quad (\text{C.6})$$

and are called the first and second *Friedmann equation*, respectively. The three equations (C.4), (C.5), and (C.6) are not functionally independent: the two Friedmann equations (C.5) and (C.6) are related to (C.4) via the Bianchi identities $\nabla^\mu(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}) = 0$. Because (C.4) affords such a straightforward interpretation as statement of energy conservation and is mathematically simple, it is often used in conjunction with (C.5).² These equations in two variables a and ρ are of first order, and thus govern one single degree of freedom.

Even without specifying an equation of state, some statements about the FLRW models can be made. First, provided that $\Lambda = 0$ and that $(\rho + 3p)$ is positive, which is not entirely unreasonable to assume, by (C.6), the scale factor $a(t)$ decelerates. The universe, according to these models, can only be static in case $\Lambda = 4\pi G(\rho + 3p)$. This may or may not be the case. Because it is reasonable to assume that $\rho > 0$ and $p \geq 0$, the rate of expansion can only be increasing if $\Lambda > 0$.³ Barring the possibility of quintessence or other strange forms of matter, and insisting—perhaps erroneously—that $\Lambda < 4\pi G(\rho + 3p)$, the rate of expansion of the universe is slowing down. Furthermore, since a is per definitionem presently greater than zero and because the presently observed red shift of distant galaxies suggests $\dot{a}/a > 0$, it must be the case that there was a time t in the finite past such that $a(t) = 0$. Let us calibrate the origin of the t -coordinate such that $a(0) = 0$.

Thus, assuming $\Lambda < 4\pi G(\rho + 3p) > 0$, interpreting the observed red shift as evidence for $\dot{a}/a > 0$, demanding exact isotropy about every point of the cosmological model and excluding strange forms of matter, general relativity makes the astounding prediction that there was a time in the finite past when the distance between all points of space was zero and thus all flow lines coincided in this point. Since the flow lines are interpreted as idealized world lines of galaxies, this means that going back in time toward this point of coincidence, the galaxies must have been closer and closer. Therefore, it does not come as a surprise that (C.4) implies that the density ρ must increase as the scale factor decreases (and conversely), and in fact must increase without bound as $a \rightarrow 0$. Equation (C.2) shows that the same holds for the curvature scalar 3R of the three-spaces if $k \neq 0$. In fact, the Einstein equations

²E.g. Cheng (2005, p. 137f) uses (C.5) and (C.6) to derive (C.4), while Hawking and Ellis (1973, p. 137) explain that whenever $\dot{a} \neq 0$, (C.5) can be derived as a first integral of (C.4) and (C.6).

³Or if there exists a strange form of matter, the so-called “quintessence” (Earman 2001). Cf. Section 12 of this reference for a review of the evidence for $\Lambda \neq 0$ up to 2001.

show that because $\rho \rightarrow \infty$ for $a \rightarrow 0$, there must be *some* scalar which diverges for $a \rightarrow 0$. The four-dimensional scalar curvature, which is given by

$${}^4R = -\frac{6}{a^2} \left(a\ddot{a} + \dot{a}^2 + \frac{k}{a_0^2} \right), \quad (\text{C.7})$$

grows beyond any bound as $a = 0$ is approached from above. This singular behaviour of 3R and 4R is what I refer to in Chapter 6 as a *kinematic singularity*. In contrast to the analogous situation in Newtonian spacetime, where galaxies also converge in one point and where thus all particles intersect in this point and the density equally diverges, spacetime itself becomes singular in the relativistic model as all distances between “points in space” vanish. No physical laws could be well-defined there. The point of convergence is therefore excluded from the spacetime and the cosmological time t assumes only strictly positive values.

One physical law which breaks down at $a = 0$ is the Friedmann equation (C.5), as no regular solution $a = a(t)$ can pass through $a = 0$. This means that the (backward) evolution of the universe is ground to a halt at the “big bang.” In this sense, the FLRW models with the specified parameters are also *dynamically singular*. The dynamically singular character of these models can also be expressed by pointing out that they are geodesically incomplete at the “initial” singularity, because geodesics can be interpreted as representing admissible trajectories of matter or radiation.

Initially, many physicists believed that the initial singularity could be avoided when the strictly isotropic FLRW models would be replaced with more realistic models including anisotropies in the distribution of matter and energy. This hope was based on the idea that these local irregularities in the density distribution would grow when traced back in time and could thus prevent the singularity by causing the universe to “bounce,” i.e. to grow larger again after (or really: before) a minimal size was attained. Those who championed this idea must have felt a heartbreaking disappointment when Penrose and Hawking proved their singularity theorems, establishing that the initial singularity occurs generically in a wide class of physically realistic models.

APPENDIX D

COMMUTATOR OF THE INVERSE SCALE FACTOR AND HAMILTONIAN CONSTRAINTS OPERATORS

In this appendix, I shall compute the commutation relation of the inverse scale factor operator and the Hamiltonian constraint operator applied to an eigenstate $|\mu\rangle$ of the volume operator, as used in Section 8.1. The calculation of

$$\left[\widehat{\left(\frac{\text{sgn}(p)}{\sqrt{|p|}} \right)}, \widehat{C}_{\text{grav}} \right] |\mu\rangle$$

is straightforward, given equations (7.9) and (7.11). We get (in Planck units)

$$\begin{aligned} \widehat{\left(\frac{\text{sgn}(p)}{\sqrt{|p|}} \right)} \widehat{C}_{\text{grav}} |\mu\rangle &= \frac{32}{\sqrt{\pi\beta^5}} \left(V_{\mu+\sqrt{3}/4} + V_{\mu-\sqrt{3}/4} \right) \cdot \\ &\quad \left[\left(\sqrt{|\mu + \sqrt{3} + 1|} - \sqrt{|\mu + \sqrt{3} - 1|} \right) |\mu + \sqrt{3}\rangle \right. \\ &\quad - 2 \left(\sqrt{|\mu + 1|} - \sqrt{|\mu - 1|} \right) |\mu\rangle \\ &\quad \left. + \left(\sqrt{|\mu - \sqrt{3} + 1|} - \sqrt{|\mu - \sqrt{3} - 1|} \right) |\mu - \sqrt{3}\rangle \right], \end{aligned}$$

and

$$\begin{aligned} \widehat{C}_{\text{grav}} \widehat{\left(\frac{\text{sgn}(p)}{\sqrt{|p|}} \right)} |\mu\rangle &= \frac{32}{\sqrt{\pi\beta^5}} \left(V_{\mu+\sqrt{3}/4} + V_{\mu-\sqrt{3}/4} \right) \cdot \\ &\quad \left(\sqrt{|\mu + 1|} - \sqrt{|\mu - 1|} \right) \left(|\mu + \sqrt{3}\rangle - 2|\mu\rangle + |\mu - \sqrt{3}\rangle \right). \end{aligned}$$

Obviously, the coefficients for the $|\mu + \sqrt{3}\rangle$ - and the $|\mu - \sqrt{3}\rangle$ -terms differ in the two equations. Combining them yields

$$\left[\left(\frac{\widehat{\text{sgn}(p)}}{\sqrt{|p|}} \right), \widehat{C}_{\text{grav}} \right] |\mu\rangle = \frac{32}{\sqrt{\pi\beta^5}} \left(V_{\mu+\sqrt{3}/4} + V_{\mu-\sqrt{3}/4} \right). \quad (\text{D.1})$$

$$\left[\left(\sqrt{|\mu + \sqrt{3} + 1|} - \sqrt{|\mu + \sqrt{3} - 1|} - \sqrt{|\mu + 1|} + \sqrt{|\mu - 1|} \right) |\mu + \sqrt{3}\rangle \right. \\ \left. + \left(\sqrt{|\mu - \sqrt{3} + 1|} - \sqrt{|\mu - \sqrt{3} - 1|} - \sqrt{|\mu + 1|} + \sqrt{|\mu - 1|} \right) |\mu - \sqrt{3}\rangle \right],$$

which does not, in general, vanish. The inverse scale factor operator does therefore not represent a Dirac observable.

BIBLIOGRAPHY

- Jan Ambjørn and Renate Loll. Non-perturbative Lorentzian quantum gravity, causality, and topology change. *Nuclear Physics B*, **536**:407–434, 1998. URL <http://arxiv.org/hep-th/9805108>.
- James L Anderson. *Principles of Relativity Physics*. Academic Press, New York, 1967.
- Judith M Arms. Linearization stability of gravitational and gauge fields. *Journal of Mathematical Physics*, **20**:443–453, 1979.
- Richard Arnowitt, Stanley Deser, and Charles W Misner. The dynamics of general relativity. In Louis Witten, editor, *Gravitation: An Introduction to Current Research*, pages 227–264. Wiley, New York, 1962. URL <http://arxiv.org/gr-qc/0405109>.
- Abhay Ashtekar. New variables for classical and quantum gravity. *Physical Review Letters*, **57**:2244–2247, 1986.
- Abhay Ashtekar. New Hamiltonian formulation of general relativity. *Physical Review D*, **36**:1587–1602, 1987.
- Abhay Ashtekar. Quantum geometry and gravity: recent advances. In Nigel T Bishop and Sunil D Maharaj, editors, *General Relativity and Gravitation (2001)*, Singapore, 2002. World Scientific. URL <http://arxiv.org/gr-qc/0112038>.
- Abhay Ashtekar. Gravity and the quantum. *New Journal of Physics*, **7**:198, 2005. URL <http://arxiv.org/gr-qc/0410054>.
- Abhay Ashtekar and Martin Bojowald. Black hole evaporation: a paradigm. *Classical and Quantum Gravity*, **22**:3349–3362, 2005. URL <http://arxiv.org/gr-qc/0504029>.
- Abhay Ashtekar and Martin Bojowald. Quantum geometry and the Schwarzschild singularity. *Classical and Quantum Gravity*, **23**:391–411, 2006. URL <http://arxiv.org/gr-qc/0509075>.
- Abhay Ashtekar, Martin Bojowald, and Jerzy Lewandowski. Mathematical structure of loop quantum cosmology. *Advances in Theoretical and Mathematical Physics*, **7**:233–268, 2003a. URL <http://arxiv.org/gr-qc/0304074>.

- Abhay Ashtekar, Luca Bombelli, and Oscar Reula. The covariant phase space of asymptotically flat gravitational fields. In Mauro Francaviglia, editor, *Mechanics, Analysis and Geometry: 200 Years After Lagrange*, pages 417–450. Elsevier, Amsterdam, 1991.
- Abhay Ashtekar, Stephen Fairhurst, and Joshua L Willis. Quantum gravity, shadow states, and quantum mechanics. *Classical and Quantum Gravity*, **20**:1031–1062, 2003b. URL <http://arxiv.org/gr-qc/0207106>.
- Abhay Ashtekar and Jerzy Lewandowski. Quantum theory of geometry I: Area operators. *Classical and Quantum Gravity*, **14**:A55–A81, 1997. URL <http://arxiv.org/gr-qc/9602046>. Online version entitled “Quantum theory of gravity I: area operators”.
- Abhay Ashtekar and Jerzy Lewandowski. Quantum theory of geometry II: Volume operators. *Advances in Theoretical and Mathematical Physics*, **1**:388–429, 1998. URL <http://arxiv.org/gr-qc/9711031>.
- Abhay Ashtekar and Jerzy Lewandowski. Quantum field theory of geometry. In Tian Yu Cao, editor, *Conceptual Foundations of Quantum Field Theory*, pages 187–206, Cambridge, 1999. Cambridge University Press. URL <http://arxiv.org/hep-th/9603083>.
- Abhay Ashtekar and Jerzy Lewandowski. Relation between polymer and Fock excitations. *Classical and Quantum Gravity*, **18**:L117–L128, 2001. URL <http://arxiv.org/gr-qc/0107043>.
- Abhay Ashtekar and Jerzy Lewandowski. Background independent quantum gravity: A status report. *Classical and Quantum Gravity*, **21**:R53–R152, 2004. URL <http://arxiv.org/gr-qc/0404018>.
- Abhay Ashtekar, Jerzy Lewandowski, Donald Marolf, José Mourão, and Thomas Thiemann. Quantization of diffeomorphism invariant theories of connections with local degrees of freedom. *Journal of Mathematical Physics*, **36**:6456–6493, 1995. URL <http://arxiv.org/gr-qc/9504018>.
- Abhay Ashtekar, Donald Marolf, José Mourão, and Thomas Thiemann. Constructing Hamiltonian quantum theories from path integrals in a diffeomorphism invariant context. *Classical and Quantum Gravity*, **17**:4919–4940, 2000. URL <http://arxiv.org/quant-ph/9904094>.
- Abhay Ashtekar, Carlo Rovelli, and Lee Smolin. Weaving a classical metric with quantum threads. *Physical Review Letters*, **69**:237–240, 1992.
- Abhay Ashtekar and John Stachel, editors. *Conceptual Problems of Quantum Gravity*. Birkhäuser, Boston, 1991.
- Michael Atiyah. *The Geometry and Physics of Knots*. Cambridge University Press, Cambridge, 1989.

- Guido Bacciagaluppi. The role of decoherence in quantum theory. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. 2005. URL <http://plato.stanford.edu/archives/sum2005/entries/qm-decoherence/>.
- John C Baez. Spin networks in nonperturbative quantum gravity. *Proceedings of Symposia in Applied Mathematics*, **51**:167–203, 1996.
- John C Baez. Spin foam models. *Classical and Quantum Gravity*, **15**:1827–1858, 1998.
- John C Baez. An introduction to spin foam models of quantum gravity and BF theory. *Lecture Notes in Physics*, **543**:25–94, 1999. URL <http://arxiv.org/gr-qc/9905087>.
- John C Baez and Javier P Muniain. *Gauge Fields, Knots and Gravity*. World Scientific, Singapore, 1994.
- Jonathan Bain. Theories of Newtonian gravity and empirical indistinguishability. *Studies in History and Philosophy of Modern Physics*, **35**:345–376, 2004.
- Julian Barbour. The timelessness of quantum gravity: I. The evidence from the classical theory. *Classical and Quantum Gravity*, **11**:2853–2873, 1994a.
- Julian Barbour. The timelessness of quantum gravity: II. The appearance of dynamics in static configurations. *Classical and Quantum Gravity*, **11**:2875–2897, 1994b.
- Julian Barbour. *The End of Time: The Next Revolution in Physics*. Oxford University Press, Oxford, 1999.
- James M Bardeen, Brandon Carter, and Stephen W Hawking. The four laws of black hole mechanics. *Communications in Mathematical Physics*, **31**:161–170, 1973.
- John Barrett and Louis Crane. Relativistic spin networks and quantum gravity. *Journal of Mathematical Physics*, **39**:3296–3302, 1998.
- Jacob D Bekenstein. Black holes and the second law. *Lettere al Nuovo Cimento*, **4**:737–740, 1972.
- Jacob D Bekenstein. Black holes and entropy. *Physical Review D*, **7**:2333–2346, 1973.
- Jacob D Bekenstein. Generalized second law of thermodynamics in black-hole physics. *Physical Review D*, **9**:3292–3300, 1974.
- Jacob D Bekenstein. Is the cosmological singularity thermodynamically possible? *International Journal of Theoretical Physics*, **28**:967–981, 1989.
- Gordon Belot. Dust, time and symmetry. *British Journal for the Philosophy of Science*, **56**:255–291, 2005.
- Gordon Belot and John Earman. Pre-Socratic quantum gravity. In [Callender and Huggett \(2001a\)](#), pages 213–255.

- Peter G Bergmann. Observables in general relativity. *Reviews of Modern Physics*, **33**:510–514, 1961.
- Peter G Bergmann. Remarks made included in “Open discussion, following papers by S.W. Hawking and W.G. Unruh”. In Harry Wolf, editor, *Some Strangeness in the Proportion: A Centennial Symposium to Celebrate the Achievements of Albert Einstein*, page 156. Addison-Wesley, Reading, MA, 1980.
- Peter G Bergmann and Arthur Komar. The coordinate group symmetries of general relativity. *International Journal of Theoretical Physics*, **5**:15–28, 1972.
- Michel Biezunski, editor. *Correspondances Françaises*, volume 4 of *Albert Einstein: Œuvres Choisis*. Éditions du Seuil, Éditions du CNRS, Paris, 1989.
- Max Black. The identity of indiscernibles. *Mind*, **61**:153–164, 1952.
- Niels Bohr and Leon Rosenfeld. Zur Frage der Messbarkeit der elektromagnetischen Feldgrößen. *Mat.-fys. Medd. Dan. Vid. Selsk.*, **12**(8), 1933. English translation by Aage Petersen (“On the question of the measurability of electromagnetic field quantities”) in John A Wheeler and Wojciech H Zurek, editors, *Quantum Theory and Measurement*, pages 479-522. Princeton: Princeton University Press, 1983.
- Martin Bojowald. Absence of a singularity in loop quantum cosmology. *Physical Review Letters*, **86**:5227–5230, 2001a. URL <http://arxiv.org/gr-qc/0102069>.
- Martin Bojowald. Dynamical initial conditions in quantum cosmology. *Physical Review Letters*, **87**:121301, 2001b. URL <http://arxiv.org/gr-qc/0104072>.
- Martin Bojowald. Inverse scale factor in isotropic quantum geometry. *Physical Review D*, **64**:084018, 2001c. URL <http://arxiv.org/gr-qc/0105067>.
- Martin Bojowald. Inflation from quantum geometry. *Physical Review Letters*, **89**:261301, 2002a. URL <http://arxiv.org/gr-qc/0206054>.
- Martin Bojowald. Isotropic loop quantum cosmology. *Classical and Quantum Gravity*, **19**:2717–2741, 2002b. URL <http://arxiv.org/gr-qc/0202077>.
- Martin Bojowald. Loop quantum cosmology: recent progress. *Pramana*, **63**:765–776, 2004. URL <http://arxiv.org/gr-qc/0402053>.
- Martin Bojowald. Loop quantum cosmology. *Living Reviews in Relativity*, **8**(11), 2005. URL www.livingreviews.org/lrr-2005-11. Cited 9 December 2005.
- Martin Bojowald and Hans A Kastrup. Symmetry reduction for quantized diffeomorphism-invariant theories of connections. *Classical and Quantum Gravity*, **17**:3009–3043, 2000. URL <http://arxiv.org/hep-th/9907042>.

- Martin Bojowald, James E Lidsey, David J Mulryne, Parampreet Singh, and Reza Tavakol. Inflationary cosmology and quantization ambiguities in semi-classical loop quantum gravity. *Physical Review D*, **70**:043530, 2004a. URL <http://arxiv.org/gr-qc/0403106>.
- Martin Bojowald and Hugo A Morales-Técotl. Cosmological applications of loop quantum gravity. *Lecture Notes in Physics*, **646**:421–462, 2004. URL <http://arxiv.org/gr-qc/0306008>.
- Martin Bojowald, Parampreet Singh, and Aureliano Skirzewski. Coordinate time dependence in quantum gravity. *Physical Review D*, **70**:124022, 2004b. URL <http://arxiv.org/gr-qc/0408094>.
- Martin Bojowald and Kevin Vandersloot. Loop quantum cosmology, boundary proposals, and inflation. *Physical Review D*, **67**:124023, 2003. URL <http://arxiv.org/gr-qc/0303072>.
- Luca Bombelli, Joochan Lee, David Meyer, and Rafael D Sorkin. Space-time and a causal set. *Physical Review Letters*, **59**:521–524, 1987.
- Luca Bombelli, Joochan Lee, David Meyer, and Rafael D Sorkin. Reply to ‘Comment on “Space-Time as a causal set”’. *Physical Review Letters*, **60**:656, 1988.
- Arvind Borde and Alexander Vilenkin. Violation of the weak energy condition in inflating spacetimes. *Physical Review D*, **56**:717–723, 1997.
- Nick Bostrom. *Anthropic Bias: Observation Selection Effects in Science and Philosophy*. Routledge, New York, 2002.
- Nick Bostrom. The mysteries of self-locating belief and anthropic reasoning. *Harvard Review of Philosophy*, **11**:59–74, 2003.
- Katherine Brading and Elena Castellani, editors. *Symmetries in Physics: Philosophical reflections*. Cambridge University Press, Cambridge, 2003.
- Katherine Brading and Elena Castellani. Symmetries and invariances in classical physics. In [Butterfield and Earman \(2006\)](#). Forthcoming.
- Graham Brightwell, H Fay Dowker, Raquel S García, Joe Henson, and Rafael D Sorkin. “Observables” in causal set cosmology. *Physical Review D*, **67**:08403, 2003. URL <http://arxiv.org/gr-qc/0210061>.
- Harvey R Brown and Michael L G Redhead. A critique of the disturbance theory of indeterminacy in quantum mechanics. *Foundations of Physics*, **11**:1–20, 1981.
- Romeo Brunetti and Klaus Fredenhagen. Towards a background independent formulation of perturbative quantum gravity. 2006. URL <http://arxiv.org/gr-qc/0603079>.

- Romeo Brunetti, Klaus Fredenhagen, and Rainer Verch. The generally covariant locality principle—a new paradigm for local quantum field theory. *Communications in Mathematical Physics*, **237**:31–68, 2003.
- Johannes Brunnemann and Thomas Thiemann. On (cosmological) singularity avoidance in loop quantum gravity. *Classical and Quantum Gravity*, **23**:1395–1428, 2006a. URL <http://arxiv.org/gr-qc/0505032>.
- Johannes Brunnemann and Thomas Thiemann. Unboundedness of triad-like operators in loop quantum gravity. *Classical and Quantum Gravity*, **23**:1429–1484, 2006b. URL <http://arxiv.org/gr-qc/0505033>.
- Jeremy Butterfield, editor. *The Arguments of Time*. Oxford University Press, Oxford, 1999.
- Jeremy Butterfield. Critical notice for Julian Barbour, *The End of Time: The Next Revolution in Physics*. *British Journal for the Philosophy of Science*, **53**:289–330, 2002.
- Jeremy Butterfield and John Earman, editors. *Handbook of the Philosophy of Science. Vol. 2: Philosophy of Physics*. Elsevier B.V., Amsterdam, 2006. Forthcoming.
- Jeremy Butterfield and Chris Isham. On the emergence of time in quantum gravity. In [Butterfield \(1999\)](#), pages 111–168. URL <http://arxiv.org/gr-qc/9901024>.
- Jeremy Butterfield and Christopher Isham. Spacetime and the philosophical challenge of quantum gravity. In [Callender and Huggett \(2001a\)](#), pages 33–89.
- Craig Callender and Nick Huggett, editors. *Physics Meets Philosophy at the Planck Scale*. Cambridge University Press, Cambridge, 2001a.
- Craig Callender and Nick Huggett. Why quantize gravity (or any field for that matter)? *Philosophy of Science*, **68**:S382–S394, 2001b. Supplement.
- Craig Callender and Robert Weingard. Topology change and the unity of space. *Studies in History and Philosophy of Modern Physics*, **31**:227–246, 2000.
- Philip Candelas, Gary T Horowitz, Andrew Strominger, and Edward Witten. Vacuum configurations for superstrings. *Nuclear Physics B*, **258**:46–74, 1985.
- Sean Carroll. Why three dimensions of space just aren't enough. URL <http://pancake.uchicago.edu/~carroll/dimensions04.pdf>. Talk given at the 19th Biennial Meeting of the Philosophy of Science Association, Austin, TX, 2004.
- Daniel Cartin and Gaurav Khanna. Absence of preclassical solutions in Bianchi I loop quantum cosmology. *Physical Review Letters*, **94**:111302, 2005. URL <http://arxiv.org/gr-qc/0501016>.
- Nancy Cartwright. The metaphysics of the disunified world. In David Hull, Micky Forbes, and Richard M Burian, editors, *PSA 1994: Proceedings of the 1994 Biennial Meeting of*

- the Philosophy of Science Association*, volume 2, pages 357–364. Philosophy of Science Association, East Lansing, MI, 1995.
- Nancy Cartwright. *The Dappled World: A Study of the Boundaries of Science*. Cambridge University Press, Cambridge, 1999.
- Mario Castagnino, Roberto Laura, and Olimpia Lombardi. Decoherence in open and closed systems. 2006. URL <http://arxiv.org/quant-ph/0601132>.
- Mario Castagnino and Olimpia Lombardi. Self-induced decoherence and the classical limit of quantum mechanics. *Philosophy of Science*, **72**, 2005. URL <http://philsci-archive.pitt.edu/archive/00001883/>. Forthcoming.
- Jordi Cat. The physicists' debates on unification in physics at the end of the 20th century. *Historical Studies in the Physical and Biological Sciences*, **28**:253–299, 1998.
- Ta-Pei Cheng. *Relativity, Gravitation and Cosmology: A Basic Introduction*. Oxford Master Series in Particle Physics, Astrophysics, and Cosmology. Oxford University Press, Oxford, 2005.
- Shiing-Shen Chern and James Simons. Characteristic forms and geometric invariants. *The Annals of Mathematics*, **99**:48–69, 1974.
- Yvonne Choquet-Bruhat and James W York, Jr. The Cauchy problem. In [Held \(1980\)](#), pages 99–172.
- Joy Christian. Exactly soluble sector of quantum gravity. *Physical Review D*, **56**:4844–4877, 1997.
- Rob Clifton and Mark Hogarth. The definability of objective becoming in Minkowski space-time. *Synthese*, **103**:355–387, 1995.
- Alain Connes. A short survey of noncommutative geometry. *Journal of Mathematical Physics*, **41**:3832–3866, 2000. URL <http://arxiv.org/hep-th/0003006>.
- Alejandro Corichi and J Manuel Reyes. A Gaussian weave for kinematical loop quantum gravity. *International Journal of Modern Physics D*, **10**:325–338, 2001. URL <http://arxiv.org/gr-qc/0006067>.
- Paul C W Davies. Singularity avoidance and quantum conformal anomalies. *Physics Letters B*, **68**:402–404, 1977.
- Lokenath Debnath and Piotr Mikusiński. *Introduction to Hilbert Spaces with Applications*. Academic Press, San Diego, second edition, 1999.
- Roberto DePietri and Laurent Freidel. $SO(4)$ Plebanski action and relativistic state sum models. *Classical and Quantum Gravity*, **16**:2187–2196, 1999. URL <http://arxiv.org/gr-qc/9804071>.

- Stanley Deser and Peter van Nieuwenhuizen. One-loop divergences of quantized Einstein-Maxwell fields. *Physical Review D*, **10**:401–410, 1974a.
- Stanley Deser and Peter van Nieuwenhuizen. Nonrenormalizability of the quantized Dirac-Einstein system. *Physical Review D*, **10**:411–420, 1974b.
- David Deutsch. Towards a quantum theory without “quantization”. In Steven M Christensen, editor, *Quantum Theory of Gravity: Essays in Honor of the 60th Birthday of Bryce S DeWitt*, pages 421–430. Hilger, Bristol, 1984.
- Bryce S DeWitt. Definition of commutators via the uncertainty principle. *Journal of Mathematical Physics*, **3**:619–624, 1962.
- Paul A M Dirac. Generalized Hamiltonian mechanics. *Proceedings of the Royal Society London*, **A246**:328–332, 1958a.
- Paul A M Dirac. The theory of gravitation in Hamiltonian form. *Proceedings of the Royal Society London*, **A246**:333–343, 1958b.
- Paul A M Dirac. *Lectures on Quantum Mechanics*. Belfer Graduate School of Science Monograph Series, New York, 1964. Reprinted by Dover Publications, Mineola, NY (2001).
- Bianca Dittrich. Partial and complete observables for Hamiltonian constrained systems. 2004. URL <http://arxiv.org/gr-qc/0411013>.
- Sergio Doplicher, Klaus Fredenhagen, and John E Roberts. The quantum structure of space-time at the Planck scale and quantum fields. *Communications in Mathematical Physics*, **172**:187–220, 1995. URL <http://arxiv.org/hep-th/0303037>.
- Olaf Dreyer. Emergent general relativity. In Oriti (2006). URL <http://arxiv.org/gr-qc/0604075>. Forthcoming.
- Michael J Duff. Inconsistencies of quantum field theory in curved space-time. In Chris J Isham, Roger Penrose, and Denis W Sciama, editors, *Quantum Gravity 2: A Second Oxford Symposium*, pages 81–105. Clarendon Press, Oxford, 1981.
- John Dupré. *The Disorder of Things: Metaphysical Foundations of the Disunity of Science*. Harvard University Press, Cambridge MA, 1993.
- John Earman. *World Enough and Space-Time: Absolute versus Relational Theories of Space and Time*. MIT Press, Cambridge, MA, 1989.
- John Earman. *Bangs, Crunches, Whimpers, and Shrieks*. Oxford University Press, New York, 1995.
- John Earman. Lambda: the constant that refuses to die. *Archive for History of Exact Sciences*, **55**:189–220, 2001.

- John Earman. Gauge matters. *Philosophy of Science*, **69**:S209–S220, 2002a. Supplement.
- John Earman. Thoroughly modern McTaggart or what McTaggart would have said if he had read the general theory of relativity. *Philosophers' Imprint*, **2**(3), 2002b. URL <http://www.philosophersimprint.org/002003/>.
- John Earman. Getting a fix on gauge: An ode to the constrained Hamiltonian formalism. In [Brading and Castellani \(2003\)](#), pages 140–162.
- John Earman. Determinism: what we have learned and what we still don't know. In Joseph Keim Campbell, Michael O'Rourke, and David Shier, editors, *Freedom and Determinism*, pages 21–46. MIT Press, 2004. URL <http://www.ucl.ac.uk/~uctytho/Determinism.pdf>.
- John Earman. Aspects of determinism in modern physics. In [Butterfield and Earman \(2006\)](#). Forthcoming.
- John Earman. Quantum determinism, quantum indeterminism, and quantum befuddlement. 2006b. Manuscript.
- John Earman. Two challenges to the requirement of substantive general covariance. *Synthese*, **148**:443–468, 2006c.
- John Earman and John D Norton. What price spacetime substantivalism? The hole story. *British Journal for the Philosophy of Science*, **38**:515–525, 1987.
- Albert Einstein. Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt. *Annalen der Physik*, **17**:132–148, 1905. URL <http://arxiv.org/quant-ph/190503001>.
- Albert Einstein. Bemerkungen. *Zeitschrift für Mathematik und Physik*, **62**:260–261, 1914a. These comments have been appended to a reprint of “Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation” in the same journal.
- Albert Einstein. Prinzipielles zur verallgemeinerten Relativitätstheorie und Gravitationstheorie. *Physikalische Zeitschrift*, **15**:176–180, 1914b.
- Albert Einstein. Die Grundlage der allgemeinen Relativitätstheorie. *Annalen der Physik*, **49**:769–822, 1916a.
- Albert Einstein. Näherungsweise Integration der Feldgleichungen der Gravitation. *Sitzungsberichte der preussischen Akademie der Wissenschaften, physikalisch-mathematische Klasse*, pages 688–696, 1916b.
- Albert Einstein. Über Gravitationswellen. *Sitzungsberichte der preussischen Akademie der Wissenschaften, physikalisch-mathematische Klasse*, pages 154–167, 1918.

- Albert Einstein. Zum kosmologischen Problem der allgemeinen Relativitätstheorie. *Sitzungsberichte der preussischen Akademie der Wissenschaften, physikalisch-mathematische Klasse*, pages 235–237, 1931.
- Albert Einstein. *The Meaning of Relativity*. Princeton University Press, Princeton, NJ, third edition, 1950.
- Albert Einstein and Nathan Rosen. The particle problem in the general theory of relativity. *Physical Review*, **48**:73–77, 1935.
- Jean Eisenstaedt and Anne J Kox, editors. *The Universe of General Relativity*, volume 11 of *Einstein Studies*. Birkhäuser, Boston, 2005.
- George F R Ellis and Bernd G Schmidt. Singular space-times. *General Relativity and Gravitation*, **8**:915–953, 1977.
- Kenneth Eppley and Eric Hannah. The necessity of quantizing the gravitational field. *Foundations of Physics*, **7**:51–68, 1977.
- Henri Epstein, Vladimir J Glaser, and Arthur Jaffe. Nonpositivity of the energy density in quantized field theories. *Il Nuovo Cimento*, **36**:1016–1022, 1965.
- Michael Esfeld. Quantum entanglement and a metaphysics of relations. *Studies in History and Philosophy of Modern Physics*, **35**:601–617, 2004.
- Michael Esfeld and Vincent Lam. Moderate structural realism about space-time. *Synthese*, 2006. URL <http://philsci-archive.pitt.edu/archive/00002778/>. Forthcoming.
- Winston Fairbairn and Carlo Rovelli. Separable Hilbert space in loop quantum gravity. *Journal of Mathematical Physics*, **45**:2802–2814, 2004. URL <http://arxiv.org/gr-qc/0403047>.
- Klaus Fredenhagen and Rudolf Haag. Generally covariant quantum field theory and scaling limits. *Communications in Mathematical Physics*, **108**:91–115, 1987.
- Steven French. Identity and individuality in quantum theory. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. 2006. URL <http://plato.stanford.edu/archives/spr2006/entries/qt-idind/>.
- Steven French, Dean Rickles, and Juha Saatsi, editors. *The Structural Foundations of Quantum Gravity*. Clarendon Press, Oxford, 2006. Forthcoming.
- Helmut Friedrich and Alan Rendall. The Cauchy problem for the Einstein equations. *Lecture Notes in Physics*, **540**:127–224, 2000.
- Rodolfo Gambini, Jerzy Lewandowski, Donald Marolf, and Jorge Pullin. On the consistency of the constraint algebra in spin network quantum gravity. *International Journal of Modern Physics D*, **7**:97–109, 1998. URL <http://arxiv.org/gr-qc/9710018>.

- Rodolfo Gambini and Jorge Pullin. *Loops, Knots, Gauge Theories and Quantum Gravity*. Cambridge University Press, Cambridge, 1996.
- Maurizio Gasperini and Gabriele Veneziano. The pre-big bang scenario in string cosmology. *Physics Reports*, **373**:1–212, 2003. URL <http://arxiv.org/hep-th/0207130>.
- Amanda Geffer. Throwing Einstein for a loop. *Scientific American*, **287**(6):40–41, 2002.
- Robert Geroch. Topology in general relativity. *Journal of Mathematical Physics*, **8**:782–786, 1967.
- Robert Geroch. Domain of dependence. *Journal of Mathematical Physics*, **11**:437–449, 1970.
- Phil E Gibb. The small scale structure of space-time: a bibliographical review. 1995. URL <http://arxiv.org/hep-th/9506171>.
- Gary W Gibbons and Stephen W Hawking, editors. *Euclidean Quantum Gravity*. World Scientific, Singapore, 1993.
- Rituparno Goswami, Pankaj S Joshi, and Parampreet Singh. Quantum evaporation of a naked singularity. *Physical Review Letters*, **96**:031302, 2006. URL <http://arxiv.org/gr-qc/0506129>.
- Daniel Green and William G Unruh. Difficulties with recollapsing models in closed isotropic loop quantum cosmology. *Physical Review D*, **70**:103502, 2004. URL <http://arxiv.org/gr-qc/0408074>.
- Michael B Green, John H Schwarz, and Edward Witten. *Superstring Theory*. Cambridge University Press, Cambridge, 1987. Two volumes.
- Brian Greene. *The Elegant Universe: Superstrings, Hidden Dimensions, and the Quest for the Ultimate Theory*. W.W. Norton, New York, 1999.
- Alan H Guth. Inflationary universe: A possible solution to the horizon and flatness problems. *Physical Review D*, **23**:347–356, 1981.
- Ian Hacking. The disunities of the sciences. In Peter Galison and David J Stump, editors, *The Disunity of Science: Boundaries, Contexts, and Power*, pages 37–74. Stanford University Press, Stanford, 1996.
- Petr Hájíček. Comment on “Time in quantum gravity: An hypothesis”. *Physical Review D*, **44**:1337–1338, 1991.
- Petr Hájíček. Time evolution and observables in constrained systems. *Classical and Quantum Gravity*, **13**:1353–1375, 1996.

- Petr Hájíček. Time evolution of observable properties of reparametrization-invariant systems. *Nuclear Physics B (Proc. Suppl.)*, **57**:115–124, 1997. URL <http://arxiv.org/gr-qc/9612051>.
- Paul R Halmos. *Introduction to Hilbert Space and the Theory of Spectral Multiplicity*. Chelsea Publishing Company, New York, 1951.
- Hans Halvorson. Complementarity of representations in quantum mechanics. *Studies in History and Philosophy of Modern Physics*, **35**:45–56, 2004.
- James B Hartle and Stephen W Hawking. Path integral derivation of black hole radiance. *Physical Review D*, **13**:2188–2203, 1976.
- Stephen W Hawking. Particle creation by black holes. *Communications in Mathematical Physics*, **43**:199–220, 1975.
- Stephen W Hawking and George F R Ellis. *The large scale structure of space-time*. Cambridge University Press, Cambridge, 1973.
- Eli Hawkins, Fotini Markopoulou, and Hanno Sahlmann. Evolution in quantum causal histories. *Classical and Quantum Gravity*, **20**:3839–3854, 2003. URL <http://arxiv.org/hep-th/0302111>.
- Richard Healey. Change without change, and how to observe it in general relativity. *Synthese*, **141**:1–35, 2004. URL <http://philsci-archive.pitt.edu/archive/00001162/>.
- Alan Held, editor. *General Relativity and Gravitation: One Hundred Years After the Birth of Albert Einstein*. Plenum Press, New York, 1980.
- Adam D Helfer. Do black holes radiate? *Reports on Progress in Physics*, **66**:943–1008, 2003. URL <http://arxiv.org/gr-qc/0304042>.
- Marc Henneaux and Claudio Teitelboim. *Quantization of Gauge Systems*. Princeton University Press, Princeton, 1992.
- Carl Hoefer. For fundamentalism. *Philosophy of Science*, **70**:1401–1412, 2003.
- Gerard 't Hooft and Martinus Veltman. One-loop divergencies in the theory of gravitation. *Annales de l'Institut Henri Poincaré A*, **20**:69–94, 1974.
- Gary T Horowitz. Spacetime in string theory. *New Journal of Physics*, **7**:201, 2005. URL <http://arxiv.org/gr-qc/0410049>.
- Gary T Horowitz and Don Marolf. Quantum probes of spacetime singularities. *Physical Review D*, **52**:5670–5675, 1995.
- Gary T Horowitz and Andrew Strominger. Counting states of near-extremal black holes. *Physical Review Letters*, **77**:2368–2371, 1996.

- Don Howard. Emergence in the physical sciences: lessons from the particle physics and condensed matter debate. In Nancey Murphy and William R Stoeger, editors, *Evolution and Emergence: Systems, Organisms, Persons*. Oxford University Press, Oxford, 2007. Forthcoming.
- Don Howard and John Stachel, editors. *Einstein and the History of General Relativity*, volume 1 of *Einstein Studies*. Birkhäuser, Boston, 1989.
- Walter Hunziker. On the space-time behavior of Schrödinger wavefunctions. *Journal of Mathematical Physics*, **7**:300–304, 1966.
- Viqar Husain and Oliver Winkler. On singularity resolution in quantum gravity. *Physical Review D*, **69**:084016, 2004. URL <http://arxiv.org/gr-qc/0312094>.
- Mihaela Iftime and John Stachel. The hole argument for covariant theories. *General Relativity and Gravitation*, 2006. URL <http://arxiv.org/gr-qc/0512021>. Forthcoming.
- Chris J Isham. Canonical quantum gravity and the problem of time. In Luis Alberto Ibort and Miguel Ángel Rodríguez, editors, *Integrable Systems, Quantum Groups, and Quantum Field Theories*, pages 157–288. Kluwer, Dordrecht, 1993. URL <http://arxiv.org/gr-qc/9210011>.
- Chris J Isham. Canonical quantum gravity and the question of time. In Jürgen Ehlers and Helmut Friedrich, editors, *Canonical Gravity: From Classical to Quantum. Proceedings of the 117th WE Heraeus Seminar Held at Bad Honnef, Germany, 13-17 September 1993*, pages 150–169. Springer Verlag, Berlin, 1994a.
- Chris J Isham. Prima facie questions in quantum gravity. In Jürgen Ehlers and Helmut Friedrich, editors, *Canonical Gravity: From Classical to Quantum. Proceedings of the 117th WE Heraeus Seminar Held at Bad Honnef, Germany, 13-17 September 1993*, pages 1–21. Springer Verlag, Berlin, 1994b. URL <http://arxiv.org/gr-qc/9310031>.
- Chris J Isham. Structural issues in quantum gravity. 1995. URL <http://arxiv.org/gr-qc/9510063>.
- Chris J Isham and Karel V Kuchař. Representations of spacetime diffeomorphisms. I. Canonical parametrized field theories. *Annals of Physics*, **164**:288–315, 1985a.
- Chris J Isham and Karel V Kuchař. Representations of spacetime diffeomorphisms. II. Canonical geometrodynamics. *Annals of Physics*, **164**:316–333, 1985b.
- Ted Jacobson. Thermodynamics of spacetime: the Einstein equation of state. *Physical Review Letters*, **75**:1260–1263, 1995. URL <http://arxiv.org/gr-qc/9504004>.
- Ted Jacobson and Lee Smolin. Non-perturbative quantum geometries. *Nuclear Physics B*, **299**:295–345, 1988.

- Michel Janssen. Of pots and holes: Einstein's bumpy road to general relativity. *Annalen der Physik (Leipzig)*, **14**:58–85, 2005. Supplement.
- Shamit Kachru, Renata Kallosh, Andrei Linde, and Sandip P Trivedi. De Sitter vacua in string theory. *Physical Review D*, **68**:046005, 2003. URL <http://arxiv.org/hep-th/0301240>.
- Tosio Kato. Fundamental properties of Hamiltonian operators of Schrödinger type. *Transactions of the American Mathematical Society*, **70**:195–211, 1951.
- Louis H Kauffman. New invariants in the theory of knots. *American Mathematical Monthly*, **95**:195–242, 1988.
- Louis H Kauffman. *Knots and Physics*. World Scientific, Singapore, 2002. Third edition.
- Stuart Kauffman and Lee Smolin. A possible solution for the problem of time in quantum cosmology. Published online at www.edge.org/3rd_culture/smolin/smolin_p1.htm, 1997. URL <http://arxiv.org/gr-qc/9703026>.
- Jukka Keränen. The identity problem for realist structuralism. *Philosophia Mathematica*, **9**:308–330, 2001.
- Justin Khoury. A briefing on the ekpyrotic/cyclic universe. In Katsuhiko Sato and Shigehiro Nagataki, editors, *Frontier in Astroparticle Physics and Cosmology*, RESCEU International Symposium Series No. 6 (RISS-6), Tokyo, 2004. Universal Academy Press. URL <http://arxiv.org/astro-ph/0401579>.
- Justin Khoury, Burt A Ovrut, Paul J Steinhardt, and Neil Turok. Ekpyrotic universe: Colliding branes and the origin of the hot big bang. *Physical Review D*, **64**:123522, 2001. URL <http://arxiv.org/hep-th/0103239>.
- Claus Kiefer. Quantum gravity: General introduction and recent developments. *Annalen der Physik*, **15**:129–148, 2005. URL <http://arxiv.org/gr-qc/0508120>.
- Ernst Kindhauser. Fotini Markopoulou Kalamara. *Weltwoche*, **71**(17):30, 2003.
- Oskar Klein. Zur fünfdimensionalen Darstellung der Relativitätstheorie. *Zeitschrift für Physik*, **46**:188–208, 1927.
- Gunnar Klinkhammer. Averaged energy conditions for free scalar fields in flat spacetime. *Physical Review D*, **43**:2542–2548, 1991.
- Shoshichi Kobayashi and Katsumi Nomizu. *Foundations of Differential Geometry*, volume 1. Interscience Publishers, New York, 1963.
- Shoshichi Kobayashi and Katsumi Nomizu. *Foundations of Differential Geometry*, volume 2. Interscience Publishers, New York, 1969.

- Arthur Komar. Necessity of singularities in the solution of the field equations of general relativity. *Physical Review*, **104**:544–546, 1956.
- Arthur Komar. Construction of a complete set of independent observables in the general theory of relativity. *Physical Review*, **111**:1181–1187, 1958.
- Deborah A Konkowski, Thomas M Helliwell, and Christopher Wieland. Quantum singularities. In José Alberto Lobo Gutierrez, editor, *Gravitation and cosmology: Proceedings of the Spanish Relativity Meeting 2002*, page 193. Publicacions de la Universitat de Barcelona, Barcelona, 2003. URL <http://arxiv.org/gr-qc/0408036>.
- Deborah A Konkowski, Thomas M Helliwell, and Christopher Wieland. Quantum singularities of Levi-Civita spacetimes. *Classical and Quantum Gravity*, **21**:265–272, 2004. URL <http://arxiv.org/gr-qc/0401009>.
- Deborah A Konkowski, Cassidi Reese, Thomas M Helliwell, and Christopher Wieland. Classical and quantum singularities of Levi-Civita spacetimes with and without a positive cosmological constant. In Lorenzo Fatibene, Mauro Francaviglia, Roberto Giambó, and Giulio Magli, editors, *Dynamics and Thermodynamics of Black Holes and Naked Singularities: An International Workshop (Politecnico di Milano, 13-15 May 2004)*, pages 247–259. 2005. URL <http://arxiv.org/gr-qc/0410114>.
- Bernard O Koopman. Hamiltonian systems and transformations in Hilbert space. *Proceedings of the National Academy of Sciences of the United States of America*, **17**:315–318, 1931.
- Karel Kuchař. Canonical methods of quantization. In Chris J Isham, Roger Penrose, and Denis W Sciama, editors, *Quantum Gravity 2: A Second Oxford Symposium*, pages 329–376. Clarendon Press, Oxford, 1981.
- Karel Kuchař. The problem of time in canonical quantization of relativistic systems. In [Ashtekar and Stachel \(1991\)](#), pages 141–68.
- Karel Kuchař. Time and interpretations of quantum gravity. In Gabor Kunstatter, Dwight E Vincent, and Jeffrey G Williams, editors, *Proceedings of the 4th Canadian Conference on General Relativity and Relativistic Astrophysics*, pages 211–314. World Scientific, Singapore, 1992.
- Karel Kuchař. Canonical quantum gravity. In Reinaldo J Gleiser, Carlos N Kozameh, and Osvaldo M Moreschi, editors, *General Relativity and Gravitation (1992)*, pages 119–150. IOP Publishing, Philadelphia, 1993. URL <http://arxiv.org/gr-qc/9304012>.
- Nicolaas P Landsman. Between classical and quantum. In [Butterfield and Earman \(2006\)](#). Forthcoming.
- Ruth J Lawrence. An introduction to topological field theory. *Proceedings of Symposia in Applied Mathematics*, **51**:89–128, 1996.

- Jerzy Lewandowski and Donald Marolf. Loop constraints: A habitat and their algebra. *International Journal of Modern Physics D*, **7**:299–330, 1998. URL <http://arxiv.org/gr-qc/9710016>.
- Etera R Livine and Daniele Oriti. Implementing causality in the spin foam quantum geometry. *Nuclear Physics B*, **663**:231–279, 2003. URL <http://arxiv.org/gr-qc/0210064>.
- Etera R Livine and Daniele Oriti. Causality in spin foam models for quantum gravity. In *Proceedings of the 15th SIGRAV Conference on General Relativity and Gravitational Physics*. IOP Conference Series, 2004. URL <http://arxiv.org/gr-qc/0302018>. Forthcoming.
- Hendrik A Lorentz. Considerations on gravitation. *Koninklijke Akademie van Wetenschappen te Amsterdam. Proceedings*, **2**:559–574, 1899-1900. Reprint. Originally published as “Beschouwingen over de zwaartekracht”, *Koninklijke Akademie van Wetenschappen te Amsterdam. Verslagen van de Gewone Vergaderingen* **8**: 603-620.
- David B Malament. The class of continuous timelike curves determines the topology of spacetime. *Journal of Mathematical Physics*, **18**:1399–1404, 1977.
- Juan M Maldacena. *Black Holes in String Theory*. PhD thesis, Princeton University, 1996. URL <http://arxiv.org/hep-th/9607235>.
- Mostafa Mansour. On the quantum super Virasoro algebra. *Czechoslovak Journal of Physics*, **51**:883–888, 2001.
- Fotini Markopoulou. An insider’s guide to quantum causal histories. *Nuclear Physics B* (Proc. Suppl.), **88**:308–313, 2000a. URL <http://arxiv.org/hep-th/9912137>.
- Fotini Markopoulou. The internal description of a causal set: what the universe looks like from the inside. *Communications in Mathematical Physics*, **211**:559–583, 2000b. URL <http://arxiv.org/gr-qc/9811053>.
- Fotini Markopoulou. Quantum causal histories. *Classical and Quantum Gravity*, **17**:2059–2072, 2000c. URL <http://arxiv.org/hep-th/9904009>.
- Fotini Markopoulou. Planck-scale models of the universe. In John D Barrow, Paul C W Davies, and Charles L Harper, editors, *Science and Ultimate Reality: Quantum Theory, Cosmology and Complexity (Explorations Celebrating the Vision of John Archibald Wheeler)*, pages 550–563. Cambridge University Press, Cambridge, 2003. URL <http://arxiv.org/gr-qc/0210086>.
- Fotini Markopoulou and Lee Smolin. Causal evolution of spin networks. *Nuclear Physics B*, **508**:409–430, 1997. URL <http://arxiv.org/gr-qc/9702025>.
- Fotini Markopoulou and Lee Smolin. Quantum geometry with intrinsic local causality. *Physical Review D*, **58**:084032, 1998. URL <http://arxiv.org/gr-qc/9712067>.

- James Mattingly. Is quantum gravity necessary? In Eisenstaedt and Kox (2005), pages 325–337.
- James Mattingly. Why Eppley and Hannah’s experiment isn’t. *Physical Review D*, **73**:064025, 2006. URL <http://arxiv.org/gr-qc/0601127>.
- Tim Maudlin. Thoroughly muddled McTaggart or how to abuse gauge freedom to create metaphysical monstrosities. *Philosophers’ Imprint*, **2**(4), 2002. URL <http://www.philosophersimprint.org/002004/>. With a Response by John Earman.
- Daniilo Mauro. On Koopman-von Neumann waves. *International Journal of Modern Physics A*, **17**:1301–1325, 2002a. URL <http://arxiv.org/quant-ph/0105112>.
- Daniilo Mauro. *Topics in Koopman-von Neumann theory*. PhD thesis, Università degli Studi di Trieste, 2002b. URL <http://arxiv.org/quant-ph/0301172>.
- Krzysztof A Meissner. Black hole entropy in loop quantum gravity. *Classical and Quantum Gravity*, **21**:5245–5252, 2004. URL <http://arxiv.org/gr-qc/0407052>.
- Cristopher Moore. Comment on “space-time as a causal set”. *Physical Review Letters*, **60**:655, 1988.
- Margaret Morrison. Unified theories and disparate things. In David Hull, Micky Forbes, and Richard M Burian, editors, *PSA 1994: Proceedings of the 1994 Biennial Meeting of the Philosophy of Science Association*, volume 2, pages 365–373. Philosophy of Science Association, East Lansing, MI, 1995.
- Mikio Nakahara. *Geometry, Topology and Physics*. Graduate Student Series in Physics. Institute of Physics Publishing, Boston and Philadelphia, 2003.
- John von Neumann. Zur Operatorenmethode in der klassischen Mechanik. *Annals of Mathematics*, **33**:587–642, 1932a.
- John von Neumann. Zusätze zur Arbeit “Zur Operatorenmethode...”. *Annals of Mathematics*, **33**:789–791, 1932b.
- Hermann Nicolai, Kasper Peeters, and Marija Zamaklar. Loop quantum gravity: an outside view. *Classical and Quantum Gravity*, **22**:R193–R248, 2005. URL <http://arxiv.org/hep-th/0501114>.
- Emmy Noether. Invariante Variationsprobleme. *Nachrichten von der Königlichen Gesellschaft der Wissenschaften zu Göttingen. Mathematisch-physikalische Klasse*, pages 235–257, 1918. Reprinted in Nathan Jacobson, editor, *Emmy Noether: Gesammelte Abhandlungen—Collected Papers*. Springer-Verlag, Berlin, 1983.
- John D Norton. What was Einstein’s principle of equivalence? *Studies in History and Philosophy of Science*, **16**:203–246, 1985. Reprinted in Howard and Stachel (1989), 5–47.

- John D Norton. General covariance and the foundations of general relativity: Eight decades of dispute. *Reports on Progress in Physics*, **56**:791–858, 1993.
- John D Norton. On thought experiments: Is there more to the argument? *Philosophy of Science*, **71**:1139–1151, 2004.
- Karim Noui and Alejandro Perez. Three dimensional loop quantum gravity: physical scalar product and spin foam models. *Classical and Quantum Gravity*, **22**:1739–1762, 2005. URL <http://arxiv.org/gr-qc/0402110>.
- Karim Noui, Alejandro Perez, and Kevin Vandersloot. On the physical Hilbert space of loop quantum cosmology. *Physical Review D*, **71**:044025, 2005. URL <http://arxiv.org/gr-qc/0411039>.
- Paul Oppenheim and Hilary Putnam. Unity of science as a working hypothesis. In Herbert Feigl, Michael Scriven, and Grover Maxwell, editors, *Minnesota Studies in the Philosophy of Science II: Concepts, Theories, and the Mind-Body Problem*, pages 3–36. University of Minnesota Press, Minneapolis, 1958.
- Daniele Oriti, editor. *Towards Quantum Gravity—Approaches to a New Understanding of Space and Time*. Cambridge University Press, Cambridge, 2006. Forthcoming.
- Konrad Osterwalder and Robert Schrader. Axioms for Euclidean Green’s functions. *Communications in Mathematical Physics*, **31**:83–112, 1973.
- Konrad Osterwalder and Robert Schrader. Axioms for Euclidean Green’s functions, II. *Communications in Mathematical Physics*, **42**:281–305, 1975.
- Leonard Parker and Stephen A Fulling. Quantized matter fields and the avoidance of singularities in general relativity. *Physical Review D*, **7**:2357–2374, 1973.
- Charles M Patton and John A Wheeler. Is physics legislated by cosmogony? In Chris J Isham, Roger Penrose, and Denis W Sciama, editors, *Quantum Gravity: An Oxford Symposium*, pages 538–605. Clarendon Press, Oxford, 1975.
- Peter Peldán. Actions for gravity, with generalizations: a review. *Classical and Quantum Gravity*, **11**:1087–1132, 1994.
- Roger Penrose. Twistor theory, its aims and achievements. In Chris J Isham, Roger Penrose, and Denis W Sciama, editors, *Quantum Gravity: An Oxford Symposium*, pages 268–407. Clarendon Press, Oxford, 1975.
- Stephen C Pepper. Emergence. *Journal of Philosophy*, **23**:241–245, 1926.
- Asher Peres. *Quantum Theory: Concepts and Methods*. Kluwer Academic Publishers, Dordrecht, 1993.

- Asher Peres and Daniel R Terno. Hybrid classical-quantum dynamics. *Physical Review A*, **63**:022101, 2001. URL <http://arxiv.org/quant-ph/0008068>.
- Alejandro Perez. Spin foam quantization of Plebanski's action. *Advances in Theoretical and Mathematical Physics*, **5**:947–968, 2002. URL <http://arxiv.org/gr-qc/0203058>.
- Alejandro Perez and Carlo Rovelli. Observables in quantum gravity. 2001. URL <http://arxiv.org/gr-qc/0104034>.
- Michael E Peskin and Daniel V Schroeder. *An Introduction to Quantum Field Theory*. Advanced Book Program. Addison-Wesley Publishing Company, Reading, MA, 1995.
- Tsvi Piran and Steven Weinberg, editors. *Strings and Superstrings*. World Scientific, Singapore, 1988.
- Max Planck. Über das Gesetz der Energieverteilung im Normalspectrum. *Annalen der Physik*, **4**:553–563, 1901.
- Joseph Polchinski. *String Theory*. Cambridge University Press, Cambridge, 1998. Two volumes.
- Giorgio Ponzano and Tullio Regge. Semiclassical limit of Racah coefficients. In Felix Bloch, editor, *Spectroscopy and Group Theoretical Methods in Physics*. North-Holland, Amsterdam, 1968.
- Fernando Quevedo. The string-theory landscape. *Physics World*, **16**(11):21–22, 2003.
- Charles Radin. The dynamical instability of nonrelativistic many-body systems. *Communications in Mathematical Physics*, **54**:69–79, 1977.
- Amalkumar Raychaudhuri. Relativistic cosmology I. *Physical Review*, **98**:1123–1126, 1955.
- Amalkumar Raychaudhuri. Singular state in relativistic cosmology. *Physical Review*, **106**:172–173, 1957.
- Miklós Rédei, editor. *John von Neumann: Selected Letters*. American Mathematical Society, Providence, RI, 2006.
- Michael P Reisenberger and Carlo Rovelli. “Sum over surfaces” form of loop quantum gravity. *Physical Review D*, **56**:3490–3508, 1997. URL <http://arxiv.org/gr-qc/9612035>.
- Dean Rickles. A new spin on the hole argument. *Studies in History and Philosophy of Modern Physics*, **36**:415–434, 2005.
- Dean Rickles. Time and structure in canonical gravity. In French et al. (2006). URL <http://philsci-archive.pitt.edu/archive/00001845/>. Forthcoming.
- David P Rideout and Rafael D Sorkin. A classical sequential growth dynamics for causal sets. *Physical Review D*, **61**:024002, 2000. URL <http://arxiv.org/gr-qc/9904062>.

- Leon Rosenfeld. Über die Gravitationswirkungen des Lichtes. *Zeitschrift für Physik*, **65**: 589–599, 1930a.
- Leon Rosenfeld. Zur Quantelung der Wellenfelder. *Annalen der Physik*, **5**:113–152, 1930b.
- Leon Rosenfeld. On the quantization of fields. *Nuclear Physics*, **40**:353–356, 1963.
- Carlo Rovelli. Quantum mechanics without time: a model. *Physical Review D*, **42**:2638–2646, 1990.
- Carlo Rovelli. Is there incompatibility between the ways time is treated in general relativity and in standard quantum mechanics? In [Ashtekar and Stachel \(1991\)](#), pages 126–140.
- Carlo Rovelli. What is observable in classical and quantum gravity? *Classical and Quantum Gravity*, **8**:297–316, 1991b.
- Carlo Rovelli. Time in quantum gravity: An hypothesis. *Physical Review D*, **43**:442–456, 1991c.
- Carlo Rovelli. Quantum evolving constants. Reply to “Comment on ‘Time in quantum gravity: An hypothesis’ ”. *Physical Review D*, **44**:1339–1341, 1991d.
- Carlo Rovelli. Analysis of the different meaning of the concept of time in different physical theories. *Il Nuovo Cimento B*, **110**:81, 1995.
- Carlo Rovelli. Half way through the woods. In John D Norton and John Earman, editors, *The Cosmos of Science: Essays of Exploration*, pages 180–223. Pittsburgh University Press, Pittsburgh, 1997.
- Carlo Rovelli. Loop quantum gravity. *Living Rev. Rel.*, **1**(1), 1998a. URL <http://www.livingreviews.org/Articles/Volume1/1998-1rovelli/>. Cited on 25 May 2005.
- Carlo Rovelli. Strings, loops and others: a critical survey of the present approaches to quantum gravity. In Naresh Dadhich and Jayant Narlikar, editors, *Gravitation and Relativity: At the Turn of the Millennium. Proceedings of the GR-15 Conference, held at IUCAA, Pune, India, during December 16-21, 1997*, pages 281–331. Inter-University Centre for Astronomy and Astrophysics, Pune, 1998b. URL <http://arxiv.org/gr-qc/9803024>.
- Carlo Rovelli. Quantum spacetime: what do we know? In [Callender and Huggett \(2001a\)](#), pages 101–122. URL <http://arxiv.org/gr-qc/9903045>.
- Carlo Rovelli. Notes for a brief history of quantum gravity. In Vahe G Gurzadyan, Robert T Jantzen, and Remo Ruffini, editors, *Proceedings of the 9th Marcel Grossmann Meeting*, pages 742–768. World Scientific, Singapore, 2002a. URL <http://arxiv.org/gr-qc/0006061>.
- Carlo Rovelli. GPS observables in general relativity. *Physical Review D*, **65**:044017, 2002c. URL <http://arxiv.org/gr-qc/0110003>.

- Carlo Rovelli. Partial observables. *Physical Review D*, **65**:124013, 2002d. URL <http://arxiv.org/gr-qc/0110035>.
- Carlo Rovelli. A dialog on quantum gravity. *International Journal of Modern Physics D*, **12**:1509–1528, 2003a. URL <http://arxiv.org/hep-th/0310077>.
- Carlo Rovelli. Loop quantum gravity. *Physics World*, **16**(11):37–41, 2003b.
- Carlo Rovelli. *Quantum Gravity*. Cambridge University Press, Cambridge, 2004.
- Carlo Rovelli. Unfinished revolution. In [Oriti \(2006\)](#). Forthcoming.
- Carlo Rovelli and Marcus Gaul. Loop quantum gravity and the meaning of diffeomorphism invariance. *Lecture Notes in Physics*, **541**:277–324, 2000. URL <http://arxiv.org/gr-qc/9910079>.
- Carlo Rovelli and Lee Smolin. Knot theory and quantum gravity. *Physical Review Letters*, **61**:1155–1158, 1988.
- Carlo Rovelli and Lee Smolin. Loop representation of quantum general relativity. *Nuclear Physics B*, **331**:80–152, 1990.
- Carlo Rovelli and Lee Smolin. Discreteness of area and volume in quantum gravity. *Nuclear Physics B*, **442**:593–622, 1995a. URL <http://arxiv.org/gr-qc/9411005>. Erratum: *Nuclear Physics B*, **456**:734.
- Carlo Rovelli and Lee Smolin. Spin networks and quantum gravity. *Physical Review D*, **52**:5743–5759, 1995b. URL <http://arxiv.org/gr-qc/9505006>.
- Carlo Rovelli and Peush Upadhyaya. Loop quantum gravity and quanta of space: a primer. 1998. URL <http://arxiv.org/gr-qc/9806079>.
- Laura Ruetsche. A matter of degree: putting unitary inequivalence to work. *Philosophy of Science*, **70**:1329–1342, 2003.
- Donald G Saari and Zhihong Xia. Off to infinity in finite time. *Notices of the American Mathematical Society*, **42**:538–546, 1995.
- Hanno Sahlmann, Thomas Thiemann, and Oliver Winkler. Coherent states for canonical quantum general relativity and the infinite tensor product extension. *Nuclear Physics B*, **606**:401–440, 2001. URL <http://arxiv.org/gr-qc/0102038>.
- Andrei D Sakharov. Vacuum quantum fluctuations in curved space and the theory of gravitation. *Doklady Akademii Nauk SSSR*, **177**:70–71, 1967. *Soviet Physics Doklady* **12** (1968): 1040–1041. Reprinted in *General Relativity and Gravitation*, **32** (2000): 365–367.
- Simon Saunders. Physics and Leibniz’s principles. In [Brading and Castellani \(2003\)](#), pages 289–307.

- Marcelo Schiffer. Does thermodynamics rule out the existence of cosmological singularities? *International Journal of Theoretical Physics*, **30**:419–436, 1991.
- Thomas Schücker. Forces from noncommutative geometry. 2001. URL <http://arxiv.org/hep-th/0110068>.
- Thomas Schücker. Forces from Connes' geometry. *Lecture Notes in Physics*, **659**:285–350, 2005. URL <http://arxiv.org/hep-th/0111236>.
- John H Schwarz. Introduction to superstring theory. 2000. URL <http://arxiv.org/hep-ex/0008017>.
- Amitabha Sen. Gravity as a spin system. *Physics Letters B*, **119**:89–91, 1982a.
- Amitabha Sen. Quantum theory of a spin 3/2 system in Einstein spaces. *International Journal of Theoretical Physics*, **21**:1–35, 1982b.
- Ashoke Sen. An introduction to non-perturbative string theory. In David Olive and Peter C West, editors, *Duality and Supersymmetric Theories*, pages 297–413, Cambridge, 1999. Cambridge University Press. URL <http://arxiv.org/hep-th/9802051>.
- José M M Senovilla. Singularity theorems and their consequences. *General Relativity and Gravitation*, **29**:701–848, 1997.
- Stuart L Shapiro and Saul A Teukolsky. Formation of naked singularities: the violation of cosmic censorship. *Physical Review Letters*, **66**:994–997, 1991.
- Michael Silberstein. Reduction, emergence and explanation. In Peter Machamer and Michael Silberstein, editors, *The Blackwell Guide to the Philosophy of Science*, pages 80–107. Blackwell Publishers, Malden (MA) and Oxford, 2002.
- Lawrence Sklar. Dappled theories in a uniform world. *Philosophy of Science*, **70**:424–441, 2003.
- Christopher J Smeenk. *Approaching the Absolute Zero of Time: Theory Development in Early Universe Cosmology*. PhD thesis, University of Pittsburgh, 2003.
- Lee Smolin. Recent developments in nonperturbative quantum gravity. In Juan Pérez-Mercader et al., editors, *Quantum Gravity and Cosmology*, pages 3–84. World Scientific, Singapore, 1992. URL <http://arxiv.org/hep-th/9202022>.
- Lee Smolin. The present moment in quantum cosmology: Challenges to the arguments for the elimination of time. In Robin Durie, editor, *Time and the Instant*, pages 112–143. Clinamen Press, Manchester, 2001a. URL <http://arxiv.org/gr-qc/0104097>.
- Lee Smolin. *Three Roads to Quantum Gravity*. Basic Books, New York, 2001b.
- Lee Smolin. Atoms of space and time. *Scientific American*, **290**(1):56–61, 2004a.

- Lee Smolin. An invitation to loop quantum gravity. In Philip C Argyres, Timothy J Hodges, Freydoon Mansouri, Joseph J Scanio, Peter Suranyi, and L C Rohana Wijewardhana, editors, *Quantum theory and symmetries: Proceedings of the 3rd International Symposium (Cincinnati, 10-14 September 2003)*, pages 655–682. World Scientific, Singapore, 2004b. URL <http://arxiv.org/hep-th/0408048>.
- Lee Smolin. Scientific alternatives to the anthropic principle. In Bernard Carr, editor, *Universe or Multiverse*. Cambridge University Press, Cambridge, 2006. URL <http://arxiv.org/hep-th/0407213>. Forthcoming.
- Hartland S Snyder. Quantized space-time. *Physical Review*, **71**:38–41, 1947a.
- Hartland S Snyder. The electromagnetic field in quantized space-time. *Physical Review*, **72**:68–71, 1947b.
- Rafael D Sorkin. An example relevant to the Kretschmann-Einstein debate. *Modern Physics Letters A*, **17**:695–700, 2002.
- Marshall Spector. *Concepts of Reduction in Physical Science*. Temple University Press, Philadelphia, 1978.
- John Stachel. Einstein’s search for general covariance. Paper read at the Ninth International Conference on General Reality and Gravitation, Jena, 1980a.
- John Stachel. Remarks made included in “open discussion, following papers by S.W. Hawking and W.G. Unruh”. In Harry Wolf, editor, *Some Strangeness in the Proportion: A Centennial Symposium to Celebrate the Achievements of Albert Einstein*, page 157. Addison-Wesley Publishing Company, Reading, MA, 1980b.
- John Stachel. Early history of quantum gravity (1916-1940). In Bala R Iyer and Biplab Bhawal, editors, *Black Holes, Gravitational Radiation and the Universe; Essays in Honor of C.V. Vishveshwara*. Kluwer, Dordrecht, 1999.
- John Stachel. Structure, individuality and quantum gravity. In [French et al. \(2006\)](#). Forthcoming.
- Raymond F Streater and Arthur S Wightman. *PCT, Spin and Statistics and All That*. Mathematical Physics Monograph Series, Benjamin Cummings, New York, 1964. Reprinted by Addison-Wesley, Advanced Book Classics, Redwood City, 1989 and by Princeton University Press, Landmarks in Mathematics and Physics, Princeton, 2000.
- Andrew Strominger and Cumrun Vafa. Microscopic origin of the Bekenstein-Hawking entropy. *Physics Letters B*, **379**:99–104, 1996. URL <http://arxiv.org/hep-th/9601029>.
- Leonard Susskind. Superstrings. *Physics World*, **16**(11):29–35, 2003.
- John L Synge. *Relativity: The General Theory*. North-Holland, Amsterdam, 1960.

- Paul Teller. Twilight of the perfect model model. *Erkenntnis*, **55**:393–415, 2001.
- Paul Teller. Essay review of *The Dappled World*, by Nancy Cartwright. *Noûs*, **36**:699–725, 2002.
- Paul Teller. How we dapple the world. *Philosophy of Science*, **71**:425–447, 2004.
- Daniel R Terno. Inconsistency of quantum-classical dynamics, and and what it implies. *Foundations of Physics*, **36**:102–111, 2006. URL <http://arxiv.org/quant-ph/0402092>.
- Thomas Thiemann. Anomaly-free formulation of non-perturbative, four-dimensional Lorentzian quantum gravity. *Physics Letters B*, **380**:257–264, 1996. URL <http://arxiv.org/gr-qc/9606088>.
- Thomas Thiemann. Quantum spin dynamics QSD. *Classical and Quantum Gravity*, **15**:839–873, 1998a. URL <http://arxiv.org/gr-qc/9606089>.
- Thomas Thiemann. Quantum spin dynamics QSD: II. the kernel of the Wheeler-DeWitt constraint operator. *Classical and Quantum Gravity*, **15**:875–905, 1998b. URL <http://arxiv.org/gr-qc/9606090>.
- Thomas Thiemann. Quantum spin dynamics QSD: III. quantum constraint algebra and physical scalar product in quantum general relativity. *Classical and Quantum Gravity*, **15**:1207–1247, 1998c. URL <http://arxiv.org/gr-qc/9705017>.
- Thomas Thiemann. Quantum spin dynamics QSD: IV. 2 + 1 Euclidean quantum gravity as a model to test 3 + 1 Lorentzian quantum gravity. *Classical and Quantum Gravity*, **15**:1249–1280, 1998d. URL <http://arxiv.org/gr-qc/9705018>.
- Thomas Thiemann. Quantum spin dynamics QSD: V. quantum gravity as the natural regulator of the Hamiltonian constraint of matter quantum field theories. *Classical and Quantum Gravity*, **15**:1281–1314, 1998e. URL <http://arxiv.org/gr-qc/9705019>.
- Thomas Thiemann. Quantum spin dynamics QSD: VI. quantum Poincaré algebra and a quantum positivity of energy theorem for canonical quantum gravity. *Classical and Quantum Gravity*, **15**:1463–1485, 1998f. URL <http://arxiv.org/gr-qc/9705020>.
- Thomas Thiemann. Gauge field theory coherent states (CGS): I. General properties. *Classical and Quantum Gravity*, **18**:2025–2064, 2001a. URL <http://arxiv.org/hep-th/0005233>.
- Thomas Thiemann. Introduction to modern canonical quantum general relativity. 2001b. URL <http://arxiv.org/gr-qc/0110034>. Submitted to Living Reviews in Relativity.
- Thomas Thiemann. Lectures on loop quantum gravity. *Lecture Notes in Physics*, **631**:41–135, 2003. URL <http://arxiv.org/gr-qc/0210094>.

- Thomas Thiemann and Oliver Winkler. Gauge field theory coherent states (CGS): II. Peakedness properties. *Classical and Quantum Gravity*, **18**:2561–2636, 2001a. URL <http://arxiv.org/hep-th/0005237>.
- Thomas Thiemann and Oliver Winkler. Gauge field theory coherent states (CGS): III. Ehrenfest theorems. *Classical and Quantum Gravity*, **18**:4629–4681, 2001b. URL <http://arxiv.org/hep-th/0005234>.
- Thomas Thiemann and Oliver Winkler. Gauge field theory coherent states (CGS): IV. Infinite tensor product and thermodynamical limit. *Classical and Quantum Gravity*, **18**:4997–5054, 2001c. URL <http://arxiv.org/hep-th/0005235>.
- Madvahan Varadarajan. Fock representations from $U(1)$ holonomy algebras. *Physical Review D*, **61**:104001, 2000. URL <http://arxiv.org/gr-qc/0001050>.
- Gabriele Veneziano. The myth of the beginning of time. *Scientific American*, **290**(5):30–39, May 2004.
- Alexander Vilenkin. Birth of inflationary universes. *Physical Review D*, **27**:2848–2855, 1983.
- Matt Visser. *Lorentzian Wormholes: From Einstein to Hawking*. AIP Press, New York, 1995.
- Matt Visser. Sakharov’s induced gravity: a modern perspective. *Modern Physics Letters A*, **17**:977–992, 2002. URL <http://arxiv.org/gr-qc/0204062>.
- Robert M Wald. *General Relativity*. The University of Chicago Press, Chicago, 1984.
- Robert M Wald. *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics*. The University of Chicago Press, Chicago, 1994.
- Robert M Wald. Gravitational collapse and cosmic censorship. In Bala R Iyer and Biplob Bhawal, editors, *Black Holes, Gravitational Radiation and the Universe; Essays in Honor of C.V. Vishveshwara*. Kluwer, Dordrecht, 1999. URL <http://arxiv.org/gr-qc/9710068>.
- A G Walker. Completely symmetric spaces. *Journal of the London Mathematical Society*, **19**:219–226, 1944.
- Steven Weinberg. *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*. John Wiley & Sons, New York, 1972.
- Steven Weinstein. Gravity and gauge theory. *Philosophy of Science*, **66**:S146–S155, 1999. URL <http://philsci-archive.pitt.edu/archive/00000834/>.
- Arthur S Wightman. Quantum field theory in terms of vacuum expectation values. *Physical Review*, **101**:860–866, 1956.

- Clifford M Will. The confrontation between general relativity and experiment. *Living Rev. Rel.*, 4(4), 2001. URL <http://www.livingreviews.org/Articles/Volume4/2001-4will/>. Cited on 20 May 2005.
- Christian Wüthrich. On time machines in Kerr-Newman spacetime. Master's thesis, University of Bern, 1999. URL http://www.itp.unibe.ch/diploma_thesis/wuthrich/wuthrichLiz.pdf.
- Christian Wüthrich. To quantize or not to quantize: fact and fiction in quantum gravity. *Philosophy of Science*, 72, 2005. URL <http://philsci-archive.pitt.edu/archive/00001972/>. Forthcoming.
- Zhihong Xia. The existence of noncollision singularities in newtonian systems. *Annals of Mathematics*, 135:411–468, 1992.
- Shing-Tung Yau. Calabi's conjecture and some new results in algebraic geometry. *Proceedings of the National Academy of Sciences of the United States of America*, 74:1798–1799, 1977.
- Ulvi Yurtsever. Does quantum field theory enforce the averaged weak energy condition? *Classical and Quantum Gravity*, 7:L251–L258, 1990.
- H Dieter Zeh. The program of decoherence: ideas and concepts. In Domenico Giulini, Erich Joos, Claus Kiefer, Joachim Kupsch, Ion-Olimpiu Stamatescu, and H Dieter Zeh, editors, *Decoherence and the Appearance of a Classical World in Quantum Theory*, pages 5–34. Springer, Berlin, 1996.
- Barton Zwiebach. *A first course in string theory*. Cambridge University Press, Cambridge, 2004.