AN EXPLORATION OF PRE-SERVICE ELEMENTARY TEACHERS' MATHEMATICAL BELIEFS

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The important role of beliefs in the learning and teaching of mathematics has been largely acknowledged in the literature. Pre-service teachers, in particular, have been shown to possess mathematical beliefs that are often traditional in nature (i.e. viewing teachers as the transmitters of knowledge and students as the passive recipients of that knowledge). These beliefs, which are formed long before the pre-service teachers enter their teacher education programs, often provide the foundation for their future teaching practices. An important role of teacher education programs, then, is to encourage the development (or modification) of beliefs that will support the kind of (reform) mathematics instruction promoted in these programs.

In this dissertation I explored the impact of different experiences within teacher education programs, particularly those related to mathematics courses, on the mathematical beliefs of pre-service elementary teachers. This exploration was structured around 3 interrelated strands of work.

The first strand drew from the existing literature to illuminate the concept of beliefs and identify ways in which teacher education programs may influence and promote change in the beliefs of pre-service teachers. This review also highlighted the need to further investigate the role and impact of mathematics courses for pre-service teachers.

The second strand introduced an analytic framework to examine the different views about mathematics promoted in textbooks used in mathematics courses. The findings demonstrated that the linguistic choices made by textbook authors may promote different views about mathematics and, as a result, create different learning opportunities for pre-service teachers. These findings may have several implications for textbook authors and those in teacher education programs who make decisions about textbook adoption.

Finally, the third strand investigated the impact of the curriculum materials and instruction in a research-based mathematics course on the beliefs of 25 pre-service elementary teachers. The findings showed that while beliefs are often highly resistant to change, it is possible to motivate change during a single mathematics course. Specifically, the nature of the curriculum materials and the role of the teacher educator in the course were found to have an important impact on the mathematical beliefs of the pre-service teachers.

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that are way too long in my absence. I'll miss our "mathie" lunches and the great conversations that filled them, but most of all I'll just miss YOU.

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1.0 CHAPTER 1: INTRODUCTION

1.1 OVERVIEW OF THE DISSERTATION

This dissertation investigates the relationship between the mathematical beliefs of pre-service elementary teachers (*PSETs*) and the experiences encountered within teacher education programs (*TEPs*) that may influence and promote change in those beliefs. Mathematics education research has suggested that a teacher's beliefs about mathematics may have a profound effect on the teaching practices adopted in the classroom (Cooney, 1985; Lester, Garofalo, & Kroll, 1989; Raymond, 1997; Raymond & Santos, 1995; Thompson, 1992), as well as on the beliefs of students in that classroom. With regards to the relationship between beliefs and the teaching of mathematics, Hersh (1979) noted:

> One's conception of what mathematics is affects one's conception of how it should be presented. One's manner of presenting it is an indication of what one *believes* to be most essential in it. … The issue, then, is not, What is the best way to teach? but, What is mathematics really about?…Controversies about … teaching cannot be resolved without confronting problems about the nature of mathematics (p.33).

In addition to investigating what pre-service teachers believe about the nature of mathematics, the learning of mathematics, and the teaching of mathematics, my work also aims to better understand the experiences in TEPs that may influence and promote change in those beliefs. Specifically, I focus much of my attention on mathematics content courses. Research on the beliefs held by future teachers is timely given educators' vision of a mathematics classroom, supported by the National Council for Teachers in Mathematics [NCTM], in which students achieve a full, flexible understanding and are “able to represent mathematics as a coherent and connected enterprise (NCTM 2000, p. 17).” In their ‘Principles and Standards for School Mathematics,’ the NCTM states that “effective teachers realize that the decisions they make shape students’ mathematical dispositions and can create a rich setting for learning” (NCTM, 2000, p. 18). These decisions are, either explicitly or implicitly, influenced by the beliefs held by the teacher. Therefore, achievement of the NCTM’s (1989, 1991, 2000) vision of the mathematics classroom necessitates serious consideration of the role that teacher beliefs play in the creation of such classrooms. This consideration also needs to extend to the role of TEPs in creating opportunities for future teachers to experience the kinds of mathematical activities they are expected to create for their own students.

The goal of this chapter is to acquaint the reader with this dissertation. I begin by providing an overview of the conceptual and theoretical considerations that guide the dissertation, including the relevant learning perspectives. Next, I present an outline of the main issues on which my research focuses and identify how I have worked to address these issues. After, I introduce and elaborate the research questions of the work. Finally, I discuss the structure of the dissertation.

1.2 CONCEPTUAL AND THEORETICAL CONSIDERATIONS

Although the terms ‘conceptual’ and ‘theoretical’ are often used interchangeably in reference to frameworks, I elaborate here on the distinctions I make between the two. I define a *conceptual framework* to be comprised of the concepts or variables that are operating within the explorations meant to inform a particular research question, whereas a *theoretical framework* refers to the specific learning theories in which the research is situated.

In this dissertation, the term *beliefs* is used to describe the central element of my conceptual framework that shapes the ways in which an individual interprets and responds to particular situations involving the learning and teaching of mathematics, and may guide behavior relating to those situations. In attempts to better understand beliefs and the specific mechanisms behind changes in them, I frame my discussions and analyses within different yet relevant theories. In much of the dissertation, I draw from the mutually supportive constructivist (e.g. Lerman, 1989; Savery & Duffy, 1996) and situated cognition (Brown, Collins, & Duguid, 1989; Lave & Wenger, 1991) theories. When applicable, I also draw from conceptual change theory, as well critical discourse analysis.

Supporters of constructivism value instructional strategies in which students are able to learn mathematics by actively constructing mathematical knowledge. A constructivist perspective also advocates instruction that emphasizes collaborative problem-solving, reflection, and exploration. Savery and Duffy (1996) characterize constructivism in terms of three propositions:

> (1) Understanding is in our interactions with the environment.
> (2) Cognitive conflict or puzzlement is the stimulus for learning and determines the organization and nature of what is learned.
> (3) Knowledge evolves through social negotiation and through the evaluation of the viability of individual understandings. (p. 136)

Learning within the complementary situated cognition perspective is seen "as a process of enculturation, or participation in socially organized practices, through which specialized skills are developed by learners as they engage in an 'apprenticeship in thinking' (Rogoff, 1990) or in 'legitimate peripheral participation' (Lave & Wenger, 1991)" (Scott, Asoko, & Leach, 2007, p.

45). As with constructivism, situated cognition presumes that knowing and learning are inseparable from actively doing (Brown, Collins, & Duguid, 1989) or participating (Lave & Wenger, 1991). By participating with legitimate (e.g. authentic) activities within a particular 'community of practice' (Wenger, 1998), novices become acquainted with the tools, language, and organizing principles of the community.

I am interested in using cognitive conflict as addressed in the second proposition offered by Savery and Duffy (1996) not as a stimulus for learning, but as a stimulus for addressing research questions about mathematical beliefs that will be outlined later in this chapter. The role of cognitive conflict here is similar to its use as described by Stylianides and Stylianides (2009a) in their work on proof, which is to help pre-service teachers reflect on their current mathematical beliefs, confront contradictions that arise in situations where some of these beliefs no longer hold, and recognize the importance of modifying these beliefs to resolve the contradictions.

In order to understand and explain mechanisms for belief change grounded in cognitive conflict, I also draw from conceptual change theory. With its origins tracing back to the prime of misconception research 30 years ago (Appleton, 2007) and associated with Piaget's (1985) ideas of cognitive conflict and disequilibrium, the theory of conceptual change describes the learning "required when the new information to be learned comes in conflict with the learner's prior knowledge usually acquired on the basis of everyday experiences" (Vosniadou & Lieven, 2004, p. 445). Conceptual change theory has primarily been applied to better understand the learning and teaching of mathematics and science (e.g. Appleton, 1997; Tirosh & Tsamir, 2006), yet it has been suggested that the theory may provide insights into changes in beliefs and the related difficulties (Pehkonen, 2006). In conjunction with the constructivist and situated cognition perspectives of learning, this theory allows for a deeper understanding of the process of belief change that this dissertation aims to achieve.

Finally, critical discourse analysis has been described as an approach to the study of discourse that is "interpretive and explanatory, [with] critical analysis implying a systematic methodology and a relationship between the text and its social conditions, ideologies, and power relations" (Wodak, 1996, p. 20). Critical discourse analysis draws from many different areas, including linguistic and social theory. In particular, I draw from the tools of systemic functional linguistics (Halliday, 1985), which views language as a social resource used by an individual to accomplish her purposes by expressing meanings in context.

1.3 ELABORATION ON THE ISSUES

Many efforts have been made to define the term and to make a distinction between related constructs such as 'attitudes,' 'conceptions,' and 'knowledge' (Rokeach, 1968), yet a lack of consensus remains (Furinghetti & Pehkonen, 2002). In the literature, one can find several overviews of beliefs research (e.g. Opt' Eynde et al. 2002; Pehkonen, 1994; Pehkonen and Törner, 1996; Schoenfeld, 1992; Thompson, 1992), with most concluding that interest in the area continues to grow, yet there are still many unanswered questions. Given the multiple characterizations of beliefs and the various contexts in which related research has been done, findings and results in this research domain have been wide-ranging. Grounded in an explicit characterization of beliefs (provided in the previous section) and the particular context of TEPs, this work aims to contribute to the findings in this research domain by addressing three specific issues which are outlined below.

The first issue deals with the origins of mathematical beliefs held by pre-service teachers. As a construct, a belief has been shown to be very personal and highly resistant to change, rooted deeply in the previous experiences of an individual (Kagan, 1992; Pajares, 1992). These beliefs have the potential to shape (explicitly or implicitly) interpretations of and decisions made relating to situations in which mathematical learning and teaching take place. Numerous studies into the mathematical beliefs of pre-service teachers reveal that their previous experiences have led to widespread alignment with more 'traditional' views of mathematics (Cooney, et al., 1998; Schuck, 1997). I define the *traditionalist view* as reflecting ideas that situate the role of the teachers as being transmitters of knowledge and of students as being passive recipients of that knowledge. The impact of these types of beliefs on future teachers' classroom practices, grounded in particular beliefs about what it means to do, learn, and teach mathematics, may be significant. Indeed, an alignment with a traditional transmission-model of teaching and learning can inhibit the development of a conception of mathematics as a humanistic, dynamic, problem-solving activity (Ernest, 1988) as supported by the NCTM's vision of reform.

This leads into the second issue addressed in this dissertation, namely the role of TEPs in the development and modification of beliefs held by PSETs. I define *TEPs* to include three components: (1) mathematics methods courses; (2) mathematics content courses; and (3) the student-teaching experience. Given the importance of beliefs, it seems essential that TEPs be made aware of their potential impact on and their means of developing mathematical beliefs that are consistent with the philosophy that underlies the constructivist perspective supporting the vision of reform (NCTM, 1989, 1991, 2000). This includes not only an awareness of interactions that take place between the pre-service teachers and teacher educators, but also of the curriculum materials that are used and provide the context for these interactions. From the elementary grades through the university, textbooks often constitute a significant part of what is involved in doing mathematics, providing frameworks for "what is taught, how it might be taught, and the sequence for how it could be taught" (Nicol & Crespo, 2006, p. 331). Given their importance and tenacity, it seems critical that mathematical beliefs be an area of concentration as early as possible in teacher education to encourage change away from the traditionalist perspective. One important question that currently remains unanswered is how textbooks used in TEPs (especially in mathematics content courses) may promote different views about the nature of mathematics.

This brings about the third and final issue addressed in this dissertation, which is the process of belief change. The process of belief change is a difficult one, yet it has been shown that it is not impossible (Pintrich, Marx, & Boyle, 1993; Schram, Wilcox, Lanier, & Lappan, 1988; Stuart & Thurlow, 2000). One obvious challenge for TEPs is the limited amount of time allotted to impact the beliefs of pre-service teachers (Hart, 2002). These programs have been shown to provide meaningful experiences related to both the learning and teaching of mathematics that can motivate change in pre-service teachers' mathematical beliefs towards a constructivist, reform perspective. It has been suggested, for instance, that change can come about from experiences that promote reflection (Kagan, 1992; Swars, Smith, Smith, & Hart, 2009) and that attempt to develop mathematical 'habits of mind,' a phrase used widely in the literature that describes the general process of thinking and problem-solving attributed to working mathematicians (e.g. Cuoco, 1998; Goldenberg, 1996). Change may also be motivated by learning opportunities that allow pre-service teachers to relate their mathematical experiences to the actual work of teaching through activities such as those utilizing case-based learning (e.g. Lampert & Ball, 1990; Markovits & Smith, 2008; Shulman, 1987) and viewing videos of actual elementary classrooms (Stylianides & Stylianides, 2010). Although this research shows that an understanding of mechanisms for belief change continues to improve, both the causes and the effects of that change remains an open question in educational research.

Now that I have outlined the three issues in the domain of beliefs that are relevant to this dissertation, I present the research questions grounded in these issues that guide the dissertation.

1.4 RESEARCH QUESTIONS

The overarching research question that guides the work is as follows:

> *What role do beliefs play in the learning and teaching of mathematics, and how might experiences within teacher education programs influence and promote change in the mathematical beliefs of pre-service elementary teachers?*

All experiences encountered in TEPs have the potential to impact pre-service teachers' beliefs about mathematics, but this dissertation concentrates primarily on experiences in the mathematics content course. The reasons for this focus are three-fold. First, the content course is traditionally taken before any methods course or the start of student teaching, and therefore provides the earliest opportunity to unveil and impact the mathematical beliefs held by PSETs. Second, the purpose of the mathematics content course is to attend predominantly to the 'mathematical knowledge needed for teaching' (e.g. Ball & Bass, 2000), and so places the PSETs firmly in the role of a mathematics student. It has been well-documented (Feiman-Nemser & Buchmann, 1986; Kagan, 1992; Lortie, 1975; Pajares, 1992) that the learning opportunities provided in their mathematics classroom, which includes the mathematical activities as well as student-student and student-teacher interactions, greatly impact the beliefs that pre-service teachers bring into their classrooms. Third, although there has been ample research on the impact of methods courses on the mathematical beliefs of pre-service teachers, considerably less has been done regarding the role of content courses. For these reasons, an analysis of the experiences of PSETs within the content course can provide broader insight into the development and potential modification of their beliefs. After all, the content course is likely the last opportunity for these future teachers to experience as students a primarily mathematical-learning atmosphere before entering the profession.

To investigate the overarching research question, I partition my work to address three primary research questions (RQs) that I outline below.

RQ1. *What are the experiences that impact pre-service teachers' beliefs?*

> 1.1. What beliefs relating to the nature of mathematics and the learning and teaching of mathematics have teachers been shown to possess?
>
> 1.2. How do beliefs influence teaching practices?
>
> 1.3. What kinds of experiences can influence mathematical beliefs, and what are some known mechanisms for belief change?
>
> 1.4. How can experiences within teacher education programs enact these mechanisms for belief change?

RQ2. *How may the linguistic choices made by authors of mathematics textbooks for pre-service elementary teachers promote different views about the nature of mathematics?*

> 2.1. What are components of a framework that could be developed and used to analyze the linguistic components found in textbooks to understand how the textbooks can promote different views about the nature of mathematics?
>
> 2.2. Using this framework, how do textbooks promote different views about the nature of mathematics in the particular areas of definitions and tasks?

RQ3. *Do pre-service elementary teachers' mathematical beliefs change as they progress through a research-based mathematics content course?*

> 3.1. What are the pre-service elementary teachers' initial mathematical beliefs?

3.2. What is the extent of belief change in the course and what are the specific mechanisms of this change?

Each of these questions attends to one or more of the three issues concerning the mathematical beliefs of PSETs profiled in Section 1.4: the origination of beliefs, the role and impact of TEPs, and the process of belief change. RQ1 aims to inform all three issues. RQ2 contributes to the first two issues. RQ3 serves primarily to investigate issues two and three. The way in which these research questions are addressed is outlined in the following section, where I describe the overall structure of the dissertation.

1.5 STRUCTURE OF THE DISSERTATION

Instead of addressing the questions described above through a single research study, I have adopted an alternative format that uses three distinct yet cohesive papers as the dissertation core. This structure allows for the use of a variety of methodologies as is necessitated by the diverse aspects of my overarching question.

Developed from the "bottom up," this structure was a natural outcome of several projects grounded in the common theme of mathematical beliefs. My early concentration on issues within this research domain led to preliminary investigations in various contexts, from textbooks to the classroom, and provided the ideal opportunity to adopt this multi-paper format. The dissertation consists of three separate but unified papers that are developed in Chapters 2, 3, and 4, with each addressing one of the three research questions outlined above.

The titles of these papers may help the reader get a sense of their focus. In Chapter 2 is Paper 1 which is titled, "The mathematical beliefs and experiences of pre-service teachers" and attends to RQ1. It serves to orient the reader to the diverse literature addressing the various conceptions of the term 'belief' and the complex relationship that exists between beliefs and the learning and teaching of mathematics. Also, this chapter discusses some possible mechanisms for belief change that may be enacted within TEPs. Besides attending to RQ1, Chapter 2 also provides the rationale for my decision to focus on the specific feature of the mathematics content course in TEPs to investigate mathematical beliefs of pre-service teachers (specifically, PSETs). The next two chapters do just that.

In Chapter 3 is Paper 2, which is titled, "Developing and implementing a critical discursive framework to analyze the views about mathematics being promoted by textbooks for pre-service elementary teachers," and attends to RQ2. Using systemic functional grammar (Halliday, 1985), I develop a framework to analyze particular linguistic components found in mathematics textbooks for PSETs (used in mathematics content courses) and relate them to three distinct views about the nature of mathematics: the Platonist, the instrumentalist, and the problem-solving views (Ernest, 1988). These views are introduced in Chapter 2 and serve as a common theme across all parts of the dissertation. The results of this textbook analysis can provide insights into the ways in which curriculum materials in mathematics content courses can support or inhibit particular views about the nature of mathematics. The analytic framework developed and employed in the chapter can be used to make suggestions about textbook characteristics that may better align with the reform vision of mathematics (NCTM, 1989, 1991, 2000) that reflects constructivist mathematical activity.

Finally, in Chapter 4 is Paper 3 which is titled "An exploration of the impact of instruction and activities in a research-based mathematics content course on the mathematical beliefs of pre-service elementary teachers," and attends to RQ3. In this chapter, I explore the mathematical beliefs held by PSETs and attempt to detail particular aspects of instruction and activities within their research-based mathematics content course at the University of Pittsburgh which appear to have promoted change in their beliefs. 'Research-based' is used to describe the course due to its development as a design experiment, which included "five research cycles of implementation, analysis, and refinement of a set of 'instructional sequences' (series of tasks and associated implementation strategies)" (Stylianides & Stylianides, 2009b, p. 241). Data collection included belief surveys, written responses to prompts, reflection journals, and individual interviews. As appropriate, specific mechanisms for change described in Chapter 2 are referenced in relation to the results of this chapter.

Chapter 5 is the concluding chapter and provides a summary of the major findings of Chapters 2, 3, and 4. This chapter also draws general conclusions from these interrelated studies that speak to the overarching research question, as well as address the implications of the three papers. I also discuss possible directions for future research.

Although either of the two empirical pieces described in Chapters 3 and 4 could have been extended to serve as the foundation for a more traditional, single-study dissertation, I felt that the multi-paper structure allowed me to provide a better understanding of the mathematical beliefs held by elementary pre-service teachers. The beliefs of these future teachers influence and are influenced by all experiences encountered as they make the progression from being students before entering their TEP, to being pre-service teachers during the program, and finally to being practicing teachers afterwards (see Figure 1.1 below). While I acknowledge that there are other factors (such as knowledge) that influence and are influenced by these experiences, I focus only on the factor of beliefs in this dissertation. In particular, the beliefs held as one proceeds through a TEP (the bolded center box) influence the acquisition of new experiences and knowledge that may either strengthen existing beliefs or encourage the adoption of new beliefs. In order to make TEPs more effective in addressing beliefs, it is crucial to understand as much about this progression as possible.

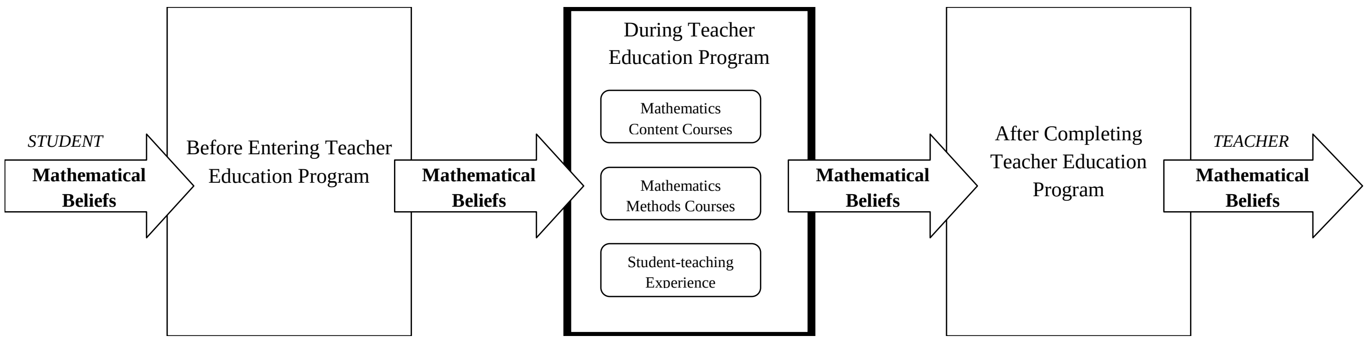

Figure 1.1: Progression from Student to Teacher

There are numerous features of TEPs to consider here, many of which are discussed in Chapter 2 and include those related to content courses, methods courses, and the student-teaching experience (see again the central bolded box in Figure 1.1). However, the emphasis of this dissertation is placed on the mathematics content course (see Figure 1.2 below). Chapter 3 focuses on better understanding the textbooks used in these courses and the different views

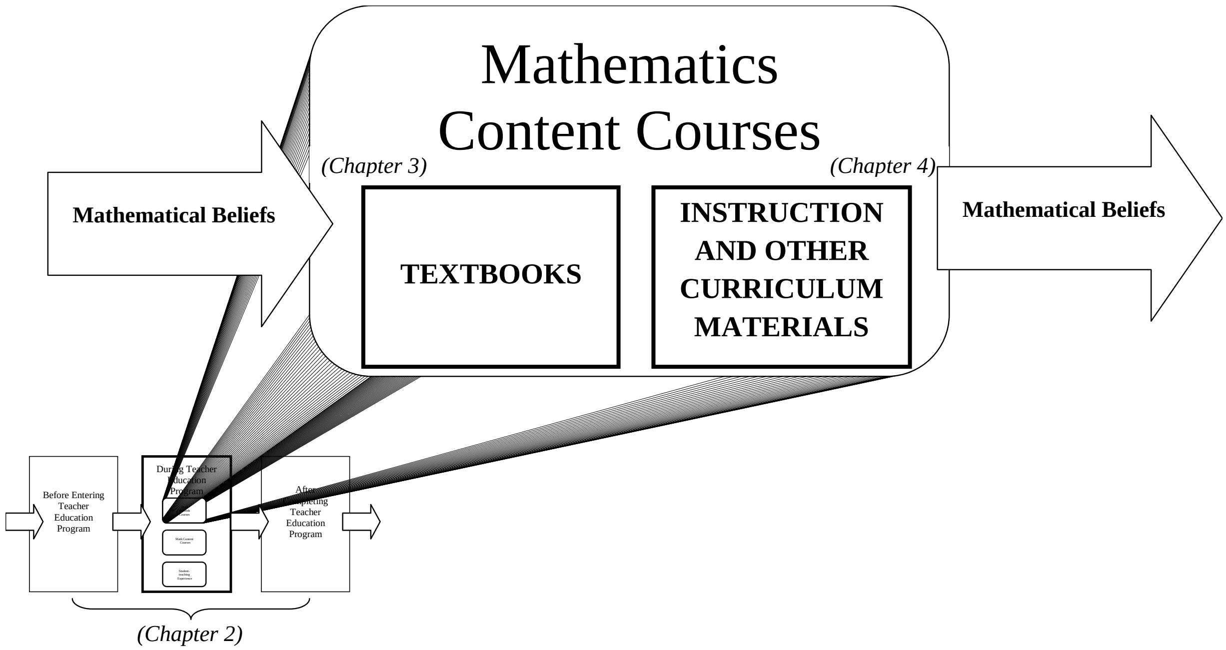

Figure 1.2: The Focus of Chapters 2 and 3

about the nature of mathematics being promoted there. Chapter 4 investigates the instruction and curriculum materials of a research-based content course that does not use a particular textbook. Here, I identify the specific activities and experiences prior to and within the course that were found to be influential to PSETs' mathematical beliefs. Indeed, an understanding of the learning opportunities provided within TEPs and their relationship with the mathematical beliefs of PSETs seems to be prerequisite to analyzing the experiences and instructional practices that come after them.

Each of the three papers is written so that it can stand alone and be read independently of the others. However, it is strongly recommended that Chapter 2 be read before Chapters 3 and 4. The primary purpose of Chapter 2 is to provide a broad and detailed synthesis of the literature relevant to the research questions, and therefore it provides the foundation for the chapters that follow. Also, both of the later chapters explicitly refer to Chapter 2 for a more elaborate discussion of the literature that provides background to the problems being investigated.

As is common with a dissertation employing this untraditional structure, a major challenge that I encountered was in trying to avoid repetition in the discussions of the relevant literature addressed in Chapters 3 and 4. I made the attempt to include only briefly that which was previously discussed in Chapter 2 relevant to the chapters, referring the reader to the synthesis for a more elaborated discussion when necessary. This was not an issue with respect to the methodologies of these later chapters, as I utilized different data sources and means of analysis. I have chosen to include a single reference list at the end of the dissertation as opposed to including three separate ones at the conclusions of Chapters 2, 3, and 4.

There are common themes that can be found across all chapters that further strengthen the connection between the chapters. As mentioned earlier, Ernest's (1988) three views about the nature of mathematics (Platonist, instrumentalist, and problem-solving) are prominently discussed throughout the dissertation in different contexts and are utilized in a variety of ways. The importance and use of the curriculum materials used in TEPs, specifically those in the mathematics content courses, are also consistently addressed.

2.0 CHAPTER 2: THE MATHEMATICAL BELIEFS AND EXPERIENCES OF PRE-SERVICE TEACHERS

2.1 INTRODUCTION

In 1989, the National Council of Teachers of Mathematics (NCTM) released its ‘Curriculum and Evaluation Standards for School Mathematics’ (referred to hereafter as *the Standards*) that initiated a major reform movement in school mathematics. The reform perspective outlined in this and other NCTM documents (1991, 2000) advocates an ideal image of the mathematics classroom that promotes a constructivist view of learning and teaching, with students actively engaged in problem-solving, reasoning, and communicating with both peers and the teacher. Fundamental change in many areas is necessary to realize such reform efforts, one of which is the development of mathematics teachers. In particular, the vision of the NCTM reform promoted by the Standards necessitates a change in the role of the mathematics teacher from that of a transmitter of knowledge to that of a facilitator of student-driven discussions (NCTM, 1991).

Teachers often act as filters through which curriculum materials are presented to students, and thus they play a central role in the implementation and success of reform efforts (Jegede, Taplin, & Chan, 2000) that encourage the creation of learning opportunities that promote a connected and flexible understanding of the discipline. Research has identified several factors that influence the development of teachers and their behaviors specifically within the domain of mathematics. One factor is teacher knowledge (e.g. Ball & Bass, 2000; Goulding, Rowland, & Barber, 2002; Koehler & Grouws, 1992; Ma, 1999; Mewborn, 2001), which includes knowledge of mathematical content, pedagogy, curriculum, and student learning. It has been suggested that teachers base many of their instructional decisions on this knowledge (Shulman, 1986), much of which is developed as a result of their training and experiences in teacher education programs (*TEPs*) (Foss & Kleinsasser, 1996). Although knowledge is certainly important, the factor at the heart of the issues discussed within this dissertation is one of the most researched factors influencing instructional practices of mathematics educators: beliefs. Specifically, this work investigates the beliefs of educators before they enter the classroom, as pre-service teachers progressing through TEPs.

It has been suggested that pedagogical decisions are deeply rooted in the beliefs held by the teacher (Ernest, 1989; Thompson, 1992; Schoenfeld, 2001). In the last 15 years alone, "there has been a considerable amount of research on teachers' beliefs based on the assumption that what teachers believe is a significant determiner of what gets taught, how it gets taught, and what gets learned in the classroom" (Wilson & Cooney, 2002, p. 128). Beliefs about students' abilities have been shown to greatly influence the instructional practices of mathematics teachers, affecting everything from classroom discourse to classroom assessments (Nathan & Koedinger, 2000). Specifically, teachers' beliefs about the nature of mathematics, learning mathematics, and teaching mathematics can greatly impact classroom performance (Borko, 1992), an idea supported by Hersh (1986) who made the claim that "a person's understanding of the nature of mathematics predicates that person's view of how teaching should take place in the classroom" (p.13). Many researchers that have come after him have attempted to establish and extend this idea.

The emergence of constructivism (outlined briefly in Chapter 1) as the foundation for mathematics education has shifted the notion of what it means to know and do mathematics, and as a consequence, what it means to learn and teach mathematics. Today, teachers are encouraged to embrace reform efforts by establishing classrooms focused on problem-solving and characterized by students' active engagement in meaningful mathematical activities involving investigation, inquiry, conjecturing, communication, and reasoning (NCTM, 1989, 1991, 2000). However, the instructional techniques advocated by mathematics reform are frequently in sharp contrast with the way that most teachers, both in-service and pre-service, learned mathematics themselves (Cohen & Ball, 1990). Often they have been exposed to the traditional style of mathematics teaching, in which students are treated not as constructors of knowledge, but as passive recipients of knowledge that are relegated to the memorization of facts, rules, and formulas and strings of routine problems. Several researchers (Frykholm, 1996; Raymond, 1997; Stipek, Givvin, Salmon, & MacGyvers, 2001; Thompson, 1992) have acknowledged the impact that these early mathematical experiences have on mathematical beliefs and classroom practices of teachers. Ma (1999) found that many practicing elementary teachers in the U.S., reflecting upon their own experiences as students, held the belief that mathematics was "an arbitrary collection of facts and rules in which doing mathematics means following set procedures step-by-step to arrive at answers" (p. 123). This traditional view of mathematics was found to be prevalent in both in-service and pre-service teachers (Benbow, 1993; Civil, 1993; Foss & Kleinsasser, 1996).

Under the premise that "teachers are key to students' opportunities to learn mathematics" (p. 1), the 15th Study of the International Commission on Mathematical Instruction [ICMI] (Ball & Even, 2008) aimed to develop a discourse in the research community about international programs and practices related to teacher education. The focus on teacher education is grounded in the belief that "no effort to improve students' opportunities to learn mathematics can succeed without parallel attention to their *teachers' opportunities for learning*" (p. 2), with one of two strands speaking directly to teacher preparation and their initial-teaching experiences.

Pre-service teachers enrolled in TEPs do not enter them as clean slates. They bring various beliefs about mathematics, what it means to do and teach mathematics, and visions of the mathematics classroom. Although not always explicit, the beliefs of pre-service teachers can be reflected in the content knowledge that they bring with them, their perceived pedagogical responsibilities in the classroom, and their embraced methods of learning. The beliefs held upon entering TEPs, shaped by past experiences with mathematics, have been shown to be tenacious (Stuart & Thurlow, 2000). These persistent (and often traditional) beliefs therefore have potential to influence the experiences within the program, as well as those that follow when an individual finally enters the profession. Due to their apparent importance, any attempt to improve teacher education necessitates an increased awareness of the previous experiences and resultant beliefs of pre-service teachers as they begin their coursework. Furthermore, teacher education should strive to better understand how they may provide future teachers with rich experiences that provide the opportunity to develop beliefs more consistent with the reform vision of mathematics.

As they progress through TEPs, pre-service teachers learn new concepts (possibly influenced by their held beliefs) associated with mathematics, the learning of mathematics, and the teaching of mathematics. The experiences gained within these programs are primarily meant to inform practices that promote the desired learning outcomes of their students, often guided by national and state standards, and the creation of a classroom culture that enables such learning to occur. However, research has shown that when pre-service teachers make the transition from the learning environment of their TEPs into the social and often political arena of teaching in schools, many often abandon the more progressive (e.g. reform) ideas and practices emphasized in their university program in favor of the more prevalent traditional values of either their mentor teacher or school district (Zeichner & Tabachnick, 1981). This has been referred to as the 'washing out' phenomenon. Darling-Hammond (2001) suggested that "teacher preparation (does) make a difference in both teachers' effectiveness and their likelihood of remaining in the profession" (p.1), yet little is known about the particular influences that these programs may have on the teachers' effectiveness in implementing reform-based mathematics. Before one can attempt to understand the factors that contribute to new teacher difficulties and the washing out phenomenon, one must first attempt to understand the beliefs that have been washed in that shape teachers' perspectives and practices. These beliefs are acquired both prior to and within the TEPs. It would also appear to be beneficial to determine and describe the kinds of experiences within the programs that have the potential to affect change in the beliefs held by pre-service teachers. The preparation of elementary school teachers in particular is arguably crucial, as the mathematics taught in elementary school forms the foundation on which students will build their future mathematical understandings. Whereas Chapters 3 and 4 will focus specifically on the beliefs of pre-service elementary teachers (*PSETs*), the discussions in this chapter refer to pre-service teachers more generally. Indeed, although there are many differences with regards to the preparation of elementary and secondary mathematics teachers, the beliefs held by both groups as they enter TEPs are grounded in similar experiences, and thus it may be suggested that their beliefs are similar. Moreover, an understanding of particular mechanisms of belief change that have been successfully enacted with both pre-service elementary and secondary teachers, as well as in-service teachers, is beneficial to this chapter because there is potential for all described mechanisms to be enacted specifically in TEPs for future elementary teachers. It is my intention to have this chapter serve as a foundation for the later chapters, where pre-service elementary teachers are the main focus.

This chapter addresses RQ1 and its related sub-questions:

RQ1. *What are the experiences that impact pre-service teachers' beliefs?*

1.1. What beliefs relating to the nature of mathematics and the learning and teaching of mathematics have teachers been shown to possess?

1.2. How do beliefs influence teaching practices?

1.3. What kinds of experiences may influence mathematical beliefs, and what are some known mechanisms for belief change?

1.4. How can different experiences within teacher education programs enact these mechanisms for belief change?

I organize Chapter 2 into two sections. The first section, Section 2.2, primarily attends to RQ 1.1, 1.2, and 1.3. In addition to a brief overview of the term 'beliefs' as it is used more broadly in educational research, this section also discusses particular studies that address the treatment of beliefs in the specific domain of mathematics education. This includes the various philosophies of mathematics described in the literature and their relationship to the beliefs found to be held by students, as well as both in-service and pre-service teachers of mathematics. Research addressing the relationship between mathematical beliefs and instructional practices follows, as well as a more complete overview of the washing out phenomenon mentioned earlier. Section 2.2 concludes with an overview of several mechanisms that have shown to be promising in the process of belief change and a summary of the section.

The second section of this chapter, Section 2.3, attends to RQ 1.4. It builds on the importance of pre-service teachers' mathematical beliefs and possible mechanisms for belief change as established in Section 2.2 and turns its focus to describing the experiences within TEPs that may both influence those beliefs and motivate change in them. I provide a framework that organizes this section depicting the relationships between mathematical beliefs and the three contexts of experience that will be addressed: (1) experiences prior to the TEP, as a student; (2) experiences within the mathematics methods course and student-teaching experience in the TEP, and; (3) experiences within the mathematics content course in the TEP.

By discussing the importance and influence of beliefs held by pre-service teachers and the ways in which those beliefs are in turn influenced by experiences both prior to and within TEPs, I make two claims. First, there needs to be an increased awareness of the beliefs held by pre-service teachers entering these programs. Second, in conjunction with the first claim, an understanding of how to both challenge and modify those beliefs in TEPs may positively influence the beginning teacher's ability to create and maintain a reform classroom environment that encourages the learning of "high-quality mathematics" (NCTM, 2000). Moreover, a better understanding of these factors may help to combat the washing out phenomenon that seems to affect the teaching practices of many new teachers. With regards to the widespread efforts to produce successful and effective teachers who embrace the vision of reform mathematics, "the most important obstacle is that teachers' beliefs and prior experiences of mathematics and mathematics teaching are not congruent with the assumptions of the Standards" (Ross, McDougall, and Hogaboam-Gray, 2002, p. 132). Therefore, that is the first obstacle that must be overcome before teachers even enter the classroom, in their TEPs. Though not an easy task to undertake, it is clear that these programs provide the only platform to provide "unique opportunities between the pre-service teacher's school experience and future teaching practice to pause and reconsider their affective dispositions towards mathematics teaching and learning" (Grootenboer, 2003, p. 42).

2.2 BELIEFS: WHAT ARE THEY AND WHAT IS THEIR IMPACT ON TEACHING PRACTICES?

Research on the beliefs of both in-service and pre-service teachers has developed into a significant field of study in mathematics education. This research has striven to understand such things as the general nature of beliefs, their development, and the ways in which beliefs influence teaching practices. There is much to be investigated in this arena, with many of the initial problems faced when studying beliefs rooted in the diverse and varied meanings of the term (Op't Eynde, de Corte, & Verschaffel, 2002; Pajares, 1992).

I begin this section by providing an overview of the different conceptualizations of the term 'belief' as offered by the relevant research and highlight some of the complexities of the belief construct. Next, I describe different philosophies about mathematics that emerged as a result of beliefs research. After, I summarize some main findings relating to young students' mathematical beliefs, as well as those found in relation to in-service and pre-service teachers, and make an effort to connect these findings to the different philosophies outlined previously. This is followed by a discussion of the research that has attempted to analyze the relationship between the mathematical beliefs held by teachers and their instructional practices. Focusing next on the instructional practices of beginning teachers, I provide an elaborated overview of the washing out phenomenon, followed by a section that describes some possible mechanisms that have been shown to elicit belief change in the research. I conclude Section 2.2 with a brief summary.

2.2.1 Characterizations of the term 'belief'

Most research would agree that all teachers hold certain beliefs about mathematics as a discipline, in addition to those related to what it means to learn and teach mathematics. Less agreement has been reached within the research with regards to clarifying a definition that efficiently captures what exactly 'beliefs' are (e.g. Leder, Törner, & Pehkonen, 2002; Op 't Eynde, de Corte & Verschaffel 2002; Pehkonen, 1994; Schoenfeld 1992; Thompson, 1992; Underhill, 1988). In response to a book written by Leder, Törner, and Pehkonen (2002) that focused on the impact of beliefs on both the teaching and learning of mathematics, Mason (2004) highlighted this difficulty in producing a universally accepted characterization in his presentation of an alphabet of terms caught up in the web of beliefs research:

> A is for attitudes, affect, aptitude, and aims; B is for beliefs; C is for constructs, conceptions, and concerns; D is for demeanor and dispositions; E is for emotions, empathies, and expectations; F is for feelings; G is for goals and gatherings; H is for habits and habitus; I is for intentions, interests, and intuitions; J is for justifications and judgments; K is for knowing; L is for leanings; M is for meaning-to; N is for norms; O is for orientations and objectives; P is for propensities, perspectives, and predispositions; Q is for quirks and quiddity; R is for recognitions and resonances; S is for sympathies and sensations; T is for tendencies and truths; U is for understandings and undertakings; V is for values and views; W is for wishes, warrants, words, and weltanschauung; X is for xenophilia (perhaps); Y is for yearnings and yens; and Z is for zeitgeist and zeal (p. 347).

According to Op 't Eynde, de Corte and Verschaffel (2002), "the diversity in the terms used to describe relevant beliefs, sometimes referring to the same, at other times to different beliefs, is symptomatic of the actual state of the research domain" (p. 15).

The relationship between beliefs and attitudes is a particularly challenging one. Although the two notions are often used interchangeably [as can be seen in the leading position of 'attitudes' in Mason's (2004) extensive list above], I offer some distinctions later that attempt to clarify the ways in which this dissertation defines and utilizes beliefs. McLeod (1992) has suggested that beliefs are fundamentally cognitive and personal in nature while their formation is subjective and emotional in nature. A belief is a cognitive construct "to which the holder attributes a high value, including associated warrants" (Goldin, 1999, p. 37) and does not require any formal justification.

Certain disagreements about the correct conceptualization of beliefs exist, yet there is far less divergence surrounding the idea that mathematical beliefs held by an individual are greatly influenced by her experiences with and observations of mathematics and the mathematics classroom. These experiences shape a composite of knowledge, conceptions, attitudes, and beliefs-- a "mathematical world view," as described by Schoenfeld (1985a). Given the interconnectedness of these components, any assessment of the mathematical knowledge held by a student or teacher (and actions relating to that knowledge) must be made with sensitivity to the impact of her held beliefs. For the purposes of this dissertation, I define *beliefs* as the implicitly or explicitly held subjective ideas about the nature of mathematics that influence the ways an individual conceptualizes, describes, and engages in the learning and teaching of mathematics.

Beliefs have long been aligned with the notion of subjectivity, while objective ideas are more closely associated with knowledge. Objective knowledge is defined as that which is considered formal and public, whereas subjective knowledge is more informal and often private, relating to what is called the affective domain (McLeod, 1992). In mathematics, objective knowledge can be seen as the universally accepted structure of the discipline, built on logically justified statements. Subjective knowledge, on the other hand, is unique to the individual and based on her own experiences. According to Nespor (1987), knowledge system information is semantically stored, whereas beliefs are drawn from experience or cultural sources of knowledge transmission. Therefore, beliefs have stronger affective and evaluative components than knowledge, and they typically operate independently of the cognition that is often associated with knowledge. Thompson (1992) also suggested that beliefs, unlike knowledge, can be held with varying degrees of conviction.

Attitudes, like beliefs, have many different characterizations in the literature. They are often thought of as affective responses that are influenced by beliefs and depict favorable or unfavorable feelings of moderate intensity and stability toward an object (Koballa, 1988; McLeod, 1992). Attitudes may be considered either as tendencies toward certain patterns of behavior, or propensities toward certain kinds of emotional feelings and reactions in specific domains--for instance in relation to mathematics. I provide an example here that aims to further illuminate the distinction between beliefs and attitudes. Suppose that a person holds the belief that the mathematical ability of an individual is fixed, and that this ability is fixed at a low level. This belief may encourage attitudes that reinforce avoidance behaviors toward mathematics and provide a reason for the student to expect failure through no fault of his own. Beliefs serve as the personal and subjective knowledge about an object or concept that influences behavior, while attitudes are reflected in the established ways of responding to particular situations and are based on held beliefs.

Since it is almost impossible to describe a single belief independent of others held by an individual, Shoenfeld (1985a) describes the widely-acknowledged notion of a ‘belief system,’ which includes beliefs about oneself (e.g., perceived mathematical ability), about the world (e.g., what goes up must come down because of gravity, not because of some magical force), and about the domain of mathematics (e.g., mathematics is all about calculations). Belief systems may act as a metaphor for examining and describing the make-up of the beliefs held by an individual (Green, 1971), much like a cognitive structure. As such, Thompson (1992) sees beliefs systems as "dynamic in nature, undergoing change and restructuring as individuals evaluate their beliefs against their experiences" (p. 130).

Green (1971) proposed three dimensions that organize an individual's belief system: (1) primary and derivative beliefs; (2) central and peripheral beliefs; and (3) belief clusters. The first dimension stresses the idea that no belief is independent of all others, and that direct experiences shape the more important and resistant primary beliefs that generate the less resistant derivative beliefs. For example, a teacher might hold the primary belief that students learn best by practicing learned procedures, with a related derivative belief that students do not benefit from working with others. Green's second dimension deals with the strength of the belief within the system, with central beliefs being very robust and resilient, while those on the periphery are more vulnerable to change. Lastly, given his claim that "beliefs are held in clusters, more or less in isolation from other clusters and protected from any relationship with other sets of beliefs" (p. 48), Green's third dimension provides some rationale for how it is possible for an individual to hold seemingly contradicting beliefs, as has been shown in several studies of beliefs professed to be held by teachers (e.g. Thompson, 1992). Working from this structure of beliefs systems, it is reasonable to suggest that all beliefs within an individual's system are initiated directly by the experiences in which they are engaged, or as a derivative of those primary beliefs. In relating this to Green's second dimension, the beliefs formed from first-hand experience would fall under the category of central beliefs, and are therefore very difficult to alter. In the case of teachers, it may be suggested that many of these central beliefs are formed not in the coursework or student-teaching experiences within the TEP; rather, it is likely that they are acquired much earlier in their experiences as students of mathematics. These central beliefs form the core of an individual's overarching philosophy about the nature of mathematics that may impact their learning and teaching of mathematics. In the next section, I describe different philosophies about the nature of mathematics that have emerged from the literature that are used in the sections that follow to discuss the beliefs of young students, as well as pre-service and in-service teachers.

2.2.2 Different philosophies about the nature of mathematics

The conceptions held by pre-service teachers about the nature of mathematics as a discipline, which are also referred to as 'mathematical epistemologies' (Schoenfeld, 1989), have a major influence on both their development as learners, and then later as teachers of mathematics (Ball, 1989). Ernest (1988) recognized three distinct philosophies of mathematics that emerged from empirical studies exploring the mathematical beliefs held by practicing teachers, namely the *Platonist* view, the *instrumentalist* view, and the *problem-solving* view. The first view, so named due to its roots in ideas of Plato, portrays mathematics as a static body of knowledge, "bound together by filaments of logic and meaning" (Ernest, 1988, p.10), waiting to be discovered as opposed to being created. Instrumentalists view mathematics as a collection of "unrelated but utilitarian" (Ernest, 1988, p.10) facts and procedures, primarily used by those trained with specific tools in order to accomplish a particular end. Finally, those embracing the problem-solving view feel that mathematics is dynamic, meaning that new mathematics is constantly being invented through a process of inquiry, and open to revision.

Similarly, Dionne's (1984) analysis of the perceptions of elementary teachers proposed that beliefs about mathematics are composed within either the 'traditional,' 'formalist,' or 'constructivist' perspective, respectively, while Törner and Grigutsch (1994) define their corresponding categories as representing the 'system,' 'toolbox,' or 'process' aspects. Even though the names are different, these notions have strong parallels to those set forth by Ernest (1988) as deduced by the ways in which they are described by the different authors.

Other researchers (e.g., Borasi, 1992; Lester, 1994) have related the notion of teachers' beliefs to their subjective knowledge about mathematics, describing their position of seeing mathematics (for example) as a finished product, or equating mathematical understanding with memorization and application of the correct formulas. Roulet (1998) offered a simpler dichotomy when describing one's view of mathematics, suggesting an alignment with either the 'absolutist' view or the 'constructivist' view of mathematics. Absolutists reflect the traditional view of mathematics and can be seen as a blend of Platonists and instrumentalists, professing that mathematics is a collection of fixed concepts and procedures, and a useful but unrelated collection of facts and rules (Ernest, 1989). The constructivist view, which lies at the heart of reform mathematics, emphasizes the practice of mathematics in the classroom that permits the student to construct, not just to absorb, mathematical knowledge. This view supports well Ernest's (1988) problem-solving view.

For the remainder of this chapter (and the dissertation at large), I adopt the three views about the nature of mathematics proposed by Ernest (1988) (the Platonist, instrumentalist, and problem-solving views) to help describe the literature on young students' and pre-service and in-service teachers' beliefs.

2.2.3 Young students' mathematical beliefs and philosophies

As discussed in Section 2.2.1, it is reasonable to suggest that all beliefs within an individual's belief system are initiated directly by the experiences in which they are engaged, or as a derivative of those primary beliefs (Green, 1971). With reference to the central beliefs associated with the learning and teaching of mathematics that most influence the practices used by teachers, Lortie (1975) suggested that they are formed during what he calls their 'apprenticeship of observation.' This phrase is used to describe the years of classroom experience undergone by pre-service teachers as students of mathematics. The beliefs that grow out of the apprenticeship of observation are mostly likely central beliefs, and therefore are at the heart of the beliefs found to be held when a student becomes a pre-service teacher in a TEP. Since these initial beliefs are essential to any discussion about the beliefs held by pre-service teachers and about the process of belief change within TEPs, I discuss them further below.

Ball (1991) provided a list of the mathematical beliefs often found to be held by elementary and middle school students that appear to primarily reflect the Platonist and instrumentalist views of mathematics. It is important to note that many of these beliefs were found to also be present in pre-service teachers, as is addressed in Section 2.2.4. The findings of Ball's study suggested that students indicated that:

(1) Mathematics is primarily a static collection of learned rules and facts.
(2) Knowing mathematics means knowing how to do it, i.e. correctly solving problems.
(3) Doing mathematics is the act of following learned, rote procedures in a step-by step fashion in order to arrive at a single, correct answer.
(4) Learning mathematics means memorization.
(5) Mathematical objects have little relevance or connection to real life objects and situations, so lend themselves only to symbolic representations.

These beliefs, which clearly conflict with the learning and teaching goals set forth by the current reform efforts (NCTM, 1989, 1991, 2000) and the problem-solving philosophy (Ernest, 1988), have been shown to be formed at an early age. Many researchers have investigated the mathematical abilities held by students in specific grades. Kloosterman and Coughan (1994) found that only 2 out 11 2^{nd} -graders interviewed believed that everyone has the ability to learn mathematics, and those same students felt that mathematical ability is a fixed one. Kloosterman, Raymond, and Emenaker (1996) tracked a group of first-graders over three years to investigate the development and stability of their mathematical beliefs, reporting almost universally a narrow view of the usefulness of mathematics. Moreover, the beliefs that these young students held about the role of group work and collaboration in the solving of mathematical tasks appeared to directly reflect their teachers' decision to incorporate group work into the class or not. Overall, these young students' beliefs appeared stable over the three year period of the study.

Similar beliefs to those described above appear to be maintained as students progress from the elementary school to middle and high school. Brown et al. (1988) described the results on the 1986 assessment of the National Assessment of Educational Progress (NAEP), which was administered to high-school students in the United States. This assessment explicitly addressed four specific categories of students' beliefs, including those relating to the nature of mathematics. It was discovered that students in the 7th and 11th grades held process or rule-oriented visions of mathematics. A large majority of these students reported that they believed they needed a known rule in order to solve mathematical problems, and that knowing how to use the rule requires practice in order to solve problems correctly. This instrumentalist interpretation of mathematics is not unique to the United States, as shown in Diaz-Obando et al.'s (2003) study of two students from Costa Rica and Spain, respectively. These students agreed that success in mathematics meant being able to use practiced, memorized procedures as illustrated by the teacher, and that mastery of these rote methods translated to mastery of the concepts. Another dichotomy expoused by Brown et al. showed either a static (i.e. unchanging) or dynamic (i.e. changing) view of mathematics, with approximately one-third of those assessed expressing the belief that new mathematical discoveries are rarely made. The results found by Garofalo (1989) on a similar population indicated that students typically described mathematics as the act of memorizing and practicing techniques which need to be taught by the teacher or illustrated in the textbook. Instead of seeing mathematics as a beautifully complex and interwoven conceptual web, the popular view seems to be that it is a subject made up of disjointed topics, completely separate and unrelated. The belief that only the most intelligent and creative people possess the ability to generate mathematics may encourage students to simply accept knowledge presented by those they view as an authority figure (e.g. the teacher, the textbook) blindly and without question (Schoenfeld, 1985b). By high school, students have usually developed their central beliefs about mathematics resulting from their personal experiences in mathematics classrooms (Fleener, 1996), and those views are predominantly traditional in nature and align closely with the Platonist and instrumentalist philosophies.

Specifically, empirical research has largely focused on the beliefs of younger students as they relate to problem-solving (Garafolo, 1989; Greeno, 1991; Kloosterman, 1996; McLeod, 1992; Schoenfeld, 1992). This work spanned a variety of age groups, and exposed and described several emergent student beliefs about mathematics as a discipline and its relationship to mathematical problem-solving that are comparable to those found by Ball (1991) outlined above. Schoenfeld (1992) provided the following (non-exhaustive) register that highlights many of these convergent beliefs as detailed in this particular body of literature:

(1) Mathematics problems have one and only one right answer.
(2) There is only one correct way to solve any mathematics problem, usually the rule that the teacher has most recently demonstrated to the class.
(3) Ordinary students cannot expect to understand mathematics; they expect simply to memorize it and to apply what they have learned mechanically and without understanding it.
(4) Mathematics is a solitary activity, done by individuals in isolation.
(5) Students who have understood the mathematics they have studied will be able to solve any assigned problem in five minutes or less.
(6) The mathematics learned in school has little or nothing to do with the real world.

(7) Formal proof is irrelevant to processes of discovery or invention. (p. 359)

In addressing younger students' ideas about how mathematics should be taught in the classroom and the role of the teacher, the literature is less abundant. Frank (1988) found that middle-school students primarily believe that the role of the mathematics teacher is to simply convey the necessary mathematical knowledge to the students successfully, a view that was substantiated in an interview study by Kloosterman et al. (1996). This view is often carried with the student throughout her high-school career. In addition to those detailed above, this belief about the role of the mathematics teacher is likely to be carried even further as many of these students make the transition from a student of mathematics, to a pre-service teacher of mathematics and, eventually, to a teacher of mathematics.

2.2.4 Pre-service teachers' mathematical beliefs and philosophies

There is a growing body of research on pre-service teachers that suggests that as they enter their TEPs, they possess belief systems that also reflect a perspective of mathematics that is strongly Platonist or instrumentalist in nature. In a study of a group of PSETs, Benbow (1993) reported responses that clearly aligned with an instrumentalist view. The findings revealed that many of the future teachers claimed that there was always one (best) correct way to solve a mathematical problem, and that mathematics as a discipline is grounded in the memorization of correct procedures and facts with work dichotomized distinctly as right or wrong. This point of view was also found in the multi-case study of pre-service teachers by Civil (1990), with the idea that there is a single best way to solve a problem accompanied by an image of mathematics necessitating neat and quick responses. Foss and Kleinsasser (1996) surveyed and interviewed PSETs to unveil a dominant alignment between an instrumentalist view of mathematics that promotes memorization and practiced procedures and the often-cited belief that the mathematical ability of any individual is innate and static. The notion that mathematical knowledge is predetermined and fixed (which supports the Platonist view) was also uncovered by Frank (1990), where a large portion of pre-service teacher respondents agreed with the given statement, “some people have a mathematical mind, and some people don’t” (p. 11). Similar findings to those described above with regards to pre-service teachers have also been reported by Southwell and Khamis (1992), as well as Wood and Floden (1990).

Surveys administered by the Integrated Mathematics and Pedagogy (IMAP) Project focusing on pre-service teachers’ beliefs about mathematics found that many future teachers see mathematics as a web of unrelated procedures, and that these procedures can be successfully implemented without a deep understanding of the underlying concepts. Raymond and Santos (1995) agree that these strongly traditional beliefs (reflecting a Platonist and instrumentalist philosophy) are intrinsically constructed through the pre-service teachers’ own classroom experiences, which they describe as being comprised of the conventional lecture-practice-test pattern.

2.2.5 In-Service teachers’ mathematical beliefs and philosophies

According to Stipek, Givvin, Salmon, and MacGyvers (2001), “most American teachers have a conception of mathematics as a static body of knowledge, involving a set of rules and procedures that are applied to yield one right answer (p. 214).” Again, this conception parallels the instrumentalist philosophy found to be held by many pre-service teachers as well. The same teachers also report that teaching mathematics involves pre-existing information that is transmitted to (rather than created by) the student, supporting the Platonist philosophy. Aldridge and Bobis (2001) reported that the majority of the teachers they studied aligned with an instrumentalist view and professed mathematics to be characterized simply by correct or incorrect solutions found by rote learning procedures. This traditional view was also found by Nisbert and Warren (2000). After surveying and interviewing almost 400 practicing teachers in a variety of disciplines, Grossman and Stodolsky (1995) found that mathematics teachers, more than teachers in other subject areas, saw their discipline as highly sequential and static in nature. Reporting on the results of a long-term study on teachers' mathematical beliefs, Stipek, et al. found the most representative beliefs to be very similar to those found by Ball (1991) and Schoenfeld (1992) described in the previous two sections.

Beliefs found to be held by pre-service and in-service teachers are undeniably similar (Handal, Bobis, and Grimson, 2001; Middleton, 1992; Perry, Howard, & Tracey, 1999), yet it would be wrong to say that these common beliefs were the only ones reported to be held by in-service teachers. The variety in results relating to in-service teachers may be "partially the result of more flexible research designs allowing the collection of a broader set of responses" (Handal, 2003, p. 51), and partially a result of the broader scope of research questions investigated within this population. As an example of this variety in results, Howard, Perry, and Lindsay (1997) recognized not one, but two prevalent philosophies that the participant in their study of high school math teachers in Australia appeared to support. A larger group of teachers supported what the author called a 'transmission' view of mathematics, which valued both the Platonist and instrumentalist views that mathematical knowledge is transmitted and utilizes memorized procedures and facts. The smaller (yet significant) group supported a 'constructivist,' problem-solving view, and appeared to believe that mathematical knowledge was actively created by people and that knowledge was dynamic. Similar diversities in the beliefs of in-service teachers were reported by Middleton (1992) and Perry et al. (1999). It was rare to find empirical studies that reported a majority of teachers who saw mathematics as a dynamic, creative activity, open to revision and all learners (not just the highly intelligent). This is unfortunate given the fact that this is precisely the vision of mathematics that lies at the heart of the classroom envisioned by mathematical reform (NCTM, 1989, 1991, 2000).

2.2.6 Beliefs and their relation to teaching practices

In the previous subsections, I outlined different philosophies about the nature of mathematics and described how the beliefs of pre-service and in-service teachers align with those philosophies as reported in the literature. Specifically, I highlighted the three philosophies (or views) suggested by Ernest (1988): the Platonist, instrumentalist, and problem-solving views. These different views may have implications relating to both learning and teaching of mathematics and, as a result, to teaching practices. It is important to note that teachers in practice may combine elements of more than one of the three views. Several researchers (e.g. Ernest, 1988; Thompson, 1992) hypothesize that alignment with these different philosophies may have roots in the ways in which mathematics is conveyed to the future teachers as students in their mathematics classrooms. These experiences help to shape their own ideas about what it means to do mathematics (Brown, Cooney, & Jones, 1990) and the ways in which mathematics should be approached in the classroom (Cooney, 1985). I now attempt to make more explicit the potential relationships between these different mathematical views and corresponding ideas about learning and teaching mathematics.

A teacher working from the Platonist view will likely emphasize mathematical terminology and describe the connection between concepts that form existing explanations to the students as though the students were simply vessels of acquirement. A teacher holding the instrumentalist view of mathematics will likely teach mathematics as though it were a toolbox of rules and procedures, stressing the importance of systematic, rote exercises in order to promote precision and mastery of those tools, perhaps through the emphasis of applications. A teacher holding the instrumentalist view will not teach with an emphasis on meaning, as would a teacher holding the Platonist view. Finally, seeing mathematics from the problem-solving view may translate into engagement with students in the process of doing mathematics with a more constructivist philosophy. This type of instruction may encourage students to see mathematics as a creative and dynamic activity of which they are an integral participant. The teaching practices that may be advocated by teachers holding these three views are further elaborated below.

Kuhs and Ball (1986) investigated teachers' dominant conceptions of what they considered to be exemplary practices, and three of the conceptions that emerged from this research can readily be associated with Ernest's (1988) three views. They were: (1) the 'learner-focused' view; (2) the 'content-focused with an emphasis on conceptual understanding view; and (3) the 'content-focused with emphasis on procedures' view. Teaching within the learner-focused view would stress the students' individual construction of knowledge grounded in their own (active) experiences of doing mathematics. A teacher working from this view would also likely encourage the construction of that knowledge through social interactions and mathematical conversations. These practices would most likely be advocated by teachers having a problem-solving view of mathematics (van der Sandt, 2007), since they would see mathematics as dynamic and grounded in human-centered inquiry. Teaching within the content-focused view with an emphasis on conceptual understanding would support practices that are driven by the meaning of the mathematical content and would emphasizes conceptual understanding in the students (Thompson, 1992). Given that the content drives the instruction and classroom activities that stress "students' understandings of ideas and processes" (van der Sandt, 2007, p. 346), this view of teaching would likely be held by a teacher holding a Platonist view of mathematics. Finally, teaching within the content-focused view with an emphasis on performance highlights "student performance and mastery of mathematical rules and procedures, combined with stress on the use of exact, rigorous mathematical language" (van der Sandt, 2007, p. 346). A teacher working from this view would expect students to memorize and mimic procedures, and may not strive to understand any underlying reasons for student errors, "as futher instruction will result in appropriate learning" (Kuhs & Ball, 1986, as cited in van der Sandt, 2007, p. 346). Indeed, practice focused on fixed rules and procedures would likely occur when a teacher holds the instrumentalist view of mathematics. The fourth distinct view identified by Kuhs and Ball that did not readily align with Ernest's views adopted as the primary views for discussion in this dissertation was the 'content-focused' view. This view is based on the belief that effective teaching is grounded in one's knowledge of what comprises a successful classroom.

After analyzing the different conceptualizations of learning and teaching of mathematics held by in-service teachers, Renne (1992) offered a 'Purpose of Schooling/Knowledge Matrix' to display the four primary conceptualizations found. When looking at the purpose of schooling dimension of the matrix, two emergent groups were classified as either 'school-knowledge oriented' (SKO) or 'child-development oriented' (CDO). Teachers belonging to the former group believe that the act of teaching has the teacher distributing the important information to the students as dictated by the curriculum, and that learning encompasses the act of acquiring that information and imitating what was acquired (reflecting the Platonist view). As opposed to those who use the curriculum as their guide, those considered to be CDO allow the needs of the students to shape their instructional moves (reflecting the problem-solving view). The relation between a teacher's beliefs and her perceptions of knowledge is captured by the matrix's second dimension. SKO teachers utilize activities that focus on what is to be learned, and therefore promote memorization of rules and rote procedures which indicate that knowledge is fragmented. On the other hand, CDO teachers are concerned with making connections and promoting learning that encourages these connections.

In his empirical investigation into the beliefs of mathematics teachers, Ernest (1988) specified several significant features that influenced their teaching of mathematics, with the most prominent reported as:

> (1) The teacher's mental contents, or schema, particularly the system of beliefs concerning mathematics and its teaching and learning.
> (2) The social context of the teaching situation, particularly the constraints and opportunities it provides; and
> (3) The teacher's level of thought processes and reflection (p. 1).

Ernest asserted that a teacher's approach to teaching fundamentally rests on her belief system as noted in the first feature (where mental content includes mathematical knowledge), and in particular, ideas about the nature of mathematics and related ideas about learning and teaching mathematics. There is also documentation on the influence of belief systems on student performance. Work done by Schoenfeld (1985a) demonstrated that student-held beliefs had a strong influence on their mathematical problem-solving approaches.

When looking at the relationship between teaching practices and beliefs of the teacher, different studies have reported various degrees of consistency (Thompson, 1992). This should not be surprising due to the genuine complexity of the relationship, with numerous factors making it difficult to discern whether the beliefs influence the practice, or if it is the practice that influences the beliefs (McGalliard, 1983). It may be suggested that this influence is bidirectional, yet Raymond (1997) concluded that she believed teachers' beliefs influenced their practice more than their practice influenced their beliefs. This remains an open question in the research. The consistency of teachers' beliefs, however, has been largely discussed in the literature, and some of this research is further elaborated below.

On one hand, Thompson (1984) reported a relatively high degree of consistency between reported beliefs and teaching practices, a finding supported by other researchers in this area (e.g., Peterson, Fennema, Carpenter, & Leof, 1989). An elementary teacher in Thompson's study professed to hold and support the problem-solving view of mathematics, a claim that was clearly supported by the activities the teacher chose in the classroom to engage her students in actively doing mathematics. Moreover, this study found further evidence indicating the stability between teacher beliefs about the nature of mathematics and the practices in the classroom. Similar consistency was found by McGalliard (1983) in his small study of high school geometry teachers. He found, just as Foss and Kleinasser (1996) did with pre-service teachers, that the geometry teachers believed that "teaching is telling and learning is memorization" (p.19), and their authoritative approaches placed great emphasis on correct solutions and rarely offered or requested explanations for those solutions. By requiring little more from the students than active note-taking, these teachers appeared to be encouraging the students to construct the belief that the teacher is the authoritative creator and distributor of mathematical knowledge and that they themselves were mere receptors of that knowledge. Perceived limitations placed on the teachers by the education establishment (e.g. administration, assessments, and accountability) may create skepticism with regards to the strength of the connection between beliefs and practices. However, Kaplan (1991) suggested that greater consistency may be found in this research given the assumption that one makes a distinction between what he called ‘deep’ and ‘surface’ beliefs, in addition to differentiating between pervasive and superficial practices. Here, deep beliefs describe those beliefs that are fundamental and comprise the core of one’s belief system. Only experiences that penetrate to this core can change the view of mathematics in some essential way.

Contrary to these reported consistencies, Raymond (1997) found apparent inconsistencies between beginning teachers’ espoused beliefs and their actual classroom practices. One-third of the participants showed a strong connection between their professed beliefs and active practices, while another one-third showed a medium degree of correspondence. The remainder of the teachers displayed a low degree of correspondence. In terms of the directional relationship, it appeared that in all cases the beliefs of the participating teachers had a greater influence on their classroom practice than vice versa. Similar inconsistencies have been reported elsewhere (e.g. Brown, 1986; Shaw, 1990; Thompson, 1984). The teachers’ own beliefs, the abilities of their students, and the mathematical context were self-reported influences on teaching methods, yet Raymond concluded that there were many complex influences at play. In particular, she stated that mathematical beliefs played a central role. In addition to their previous and current classroom experiences and practices, Raymond suggested that one important influence on the teachers’ classroom practices was their teacher preparation courses (in addition to the social and cultural aspects of the classroom).

The variation in the reported consistency between professed beliefs and observed practices lies at the heart of much of the criticism aimed at beliefs research, particularly questioning the ways in which beliefs have traditionally been measured. As Thompson (1992) notes, "reliance on verbal responses to questions posed at an abstract level of thought as the only source of data is problematic [since] some of the beliefs professed by teachers are more a manifestation of verbal commitment to abstract ideas about teaching than of an operative theory of instruction" (p. 138). Therefore, it is methodologically inappropriate to take verbal dispositions as the sole evidence of a held belief. This claim is taken into account in Chapter 4, where the beliefs held by PSETs are explored and measured using a variety of data sources.

Given the breadth of the beliefs literature discussed in Section 2.2 thus far, one can see that beliefs have the potential to influence both the learning and teaching of mathematics. It has been shown that the beliefs held by many practicing teachers often result in practices that are not necessarily the most conducive to creating the constructivist mathematical experience supported by the reform vision of mathematics. Despite the fact that few teachers are shown to possess beliefs that closely align with the ideas of reform, it is suggested later in this chapter that all of the experiences provided within the TEP can provide opportunities to enact mechanisms of belief change that may promote a closer alignment with such beliefs. In the next section, I discuss an important piece of research that attempted to address this disconnect between learned theory and practice, or what Zeichner and Tabachnick (1981) called the 'washing out' phenomenon.

2.2.7 The 'washing out' phenomenon

Given the prevalence of (traditional) Platonist and instrumentalist beliefs about mathematics supported by teachers as discussed earlier, the assertion made by Zeichner and Tabachnick (1981) appears to remain a current concern in mathematics education and reform, namely that

> it now has become commonly accepted within the teacher education community that students become increasingly more progressive or liberal in their attitudes towards education during their stay at the university and then shift to opposing and more traditional views as they move into student teaching and in-service experience. (p. 7)

Here, I understand the term *liberal* to be similar in meaning to the ideas of reform-based mathematics (NCTM, 1989, 1991, 2000) that emerged several years after this assertion was made. At the time of Zeichner and Tabachnick's study, the 'washing out' phenomenon was suggested to be a result of the students being caught between the demands of their new school environment and of those promoted by their former TEPs, placing schools as the primary source of the 'socialization influence' and therefore absolving the TEPs of any responsibility.

However, Ziechner and Tabachnick (1981) situated themselves within a minority group that was not satisfied with this pardon of TEPs and sought to investigate the underrepresented importance of both the placement school and the university in this socialization process of new teachers. Ziechner and Tabachnick acknowledged the presence of these traditional views within teachers at the end of TEPs, yet they "account for their [the views] development in an entirely different way" (p. 8). One alternative view offered that is supported by much of the literature on the development of beliefs was illustrated nicely by Lortie (1975). He maintained that the classroom environment experienced as a student, prior to any active teaching experience, plays a vital role in a new teacher's adaptation initial-teaching experience. The hundreds of hours spent in their apprenticeship of observation internalized primary and central beliefs about mathematics teaching, stemming from direct observation of their own mathematics teachers, the majority of which probably promoted traditional views. Instead of seeing the university experiences as being washed out in the classroom, this line of literature fuels the argument that "the university is essentially impotent" (Zeichner & Tabachnick, 1981, p. 9) as far as impacting teaching practices, meaning that they had little impact in the first place.

Another, and arguably more provocative, alternative goes against the claim made by Lortie (1975) and his colleagues by assuming that the role of TEPs are far from impotent. However, researchers embracing this position do not share the most common assumption about the influence of these programs; indeed, they see them not as a "liberalizing influence, but one in quite the opposite direction" (Zeichner & Tabachnick, 1981, p. 9), in fact promoting the same traditional practices that are eschewed and criticized for triggering the washing out in the classroom in the first place. Bartholomew (1976) argued that while teacher educators encouraged the use of liberal terms and practices in both the elementary and high schools with their pre-service teachers, the social interactions practiced in their own classrooms with the future teachers reflected an emphasis on their demonstration of knowledge mastery, mostly through memorization of terms and procedures. Bartholomew's analysis highlights the disconnect between what universities say should be taught in the classroom and what is actually taught in the classrooms. It also provides a different lens through which to view the washing out of the university-advocated liberal views of education in the classroom in favor of more traditional practices. Namely, this widely accepted view of universities as being liberal establishments is maintained only by looking at what is embraced there in theory, not in practice, and "the change to conservative (traditional) attitudes merely expresses what was the position in practice all the time" (Bartholomew, 1976, p. 123). Teacher educators asking their students to know how to do something (i.e. demonstrating correct usage of a memorized procedure) without asking for explanations of why only serves to encourage future acceptance and adoption of the traditional values which are transplanted into classrooms in which they may teach. As summarized by Zeichner and Tabachnick (1981), this scenario tells us that "the effects of the university are not 'washed out' by school experience, but are in fact strengthened by school experience" (p. 9). The proposition that TEPs seem to strengthen the more traditional views of mathematics has been more recently established by other researchers (Brown & Rose, 1995; Day, 1996; Foss & Kleinsasser, 1996; Kagan, 1992).

These things considered, it seems unrealistic to view the role of TEPs on teaching practices as impotent. However, the review of the literature in this section has also underscored the fact that one cannot always assume that these programs are encouraging the learning and teaching of mathematics that supports the ideas set forth by the reform movement (NCTM, 1989, 1991, 1995, 2000). Instead of focusing on what is happening to new teachers as they first enter the classroom and placing all of the blame there, perhaps it would be more beneficial to focus on creating opportunities within TEPs that promote a culture of mathematics that the pre-service teachers can experience as students before attempting to create such a culture for their own students. By providing these future teachers with opportunities to develop the understandings and to experience the classroom norms that promote the high-quality mathematics emphasized by the NCTM, they will be better prepared to both manage and modify the existing norms of the classes in which they begin their teaching careers. Considering their influence on future teaching practices, mathematics educators must carefully consider the beliefs they are promoting in their classrooms, beliefs that are reinforced by their own classroom culture and practices.

The prevailing dominance of (traditional) Platonist and instrumentalist practices in the mathematics classroom suggests that a desire to produce teachers that promote the ideas of reform (NCTM, 1989, 1991, 2000) may be realized by better preparing pre-service teachers for the challenge of going against the grain in their potentially traditional placement schools. This also necessitates preparing pre-service teachers to combat the possibility of having reform-based ideas washed out by their initial-teaching experiences. If future teachers are expected to promote the mathematical practices and understandings supported by the reform vision with their students, it seems logical to expect that the teacher educators promote this vision of mathematics within courses for pre-service teachers in TEPs. In brief, beliefs that promote this vision of mathematics must be firmly washed in during the TEP in order to avoid being washed out as new teachers enter the classroom. Moreover, before one can attempt to study and understand the potential factors that contribute to new teacher difficulties and the washing out phenomena, it is imperative to first attempt to understand if the desired beliefs can be introduced to them in the first place.

In Section 2.2.8, I detail some of the research that has focused on the potential to change beliefs, as well as some mechanisms that have been found to be most successful in provoking that change. Using the described mechanisms for change as a foundation, I move on to the second major section of this chapter, where I attempt to address the different experiences within TEPs that can address and challenge the mathematical beliefs held by pre-service teachers, and how these programs can create opportunities that allow for change in those beliefs.

2.2.8 Changing beliefs: Some possible mechanisms

The mechanism for change in terms of beliefs seems to function as follows (Green, 1971; Stuart & Thurlow, 2000): A person compares her beliefs with new experiences and with the beliefs of others, and thus her beliefs are under continuous evaluation and change. When she adopts a new belief, it will be situated automatically within her belief system, since beliefs do not exist fully independently from one another. Otherwise, the original belief remains in the belief system, or may be reinforced. Figure 2.1 illustrates this process of belief change grounded in the ideas of Green (1971) and Stuart and Thurlow (2000), and includes the particular belief system related to the mathematical beliefs held by a pre-service teacher (which includes beliefs about the nature of mathematics as well as the learning and teaching of mathematics).

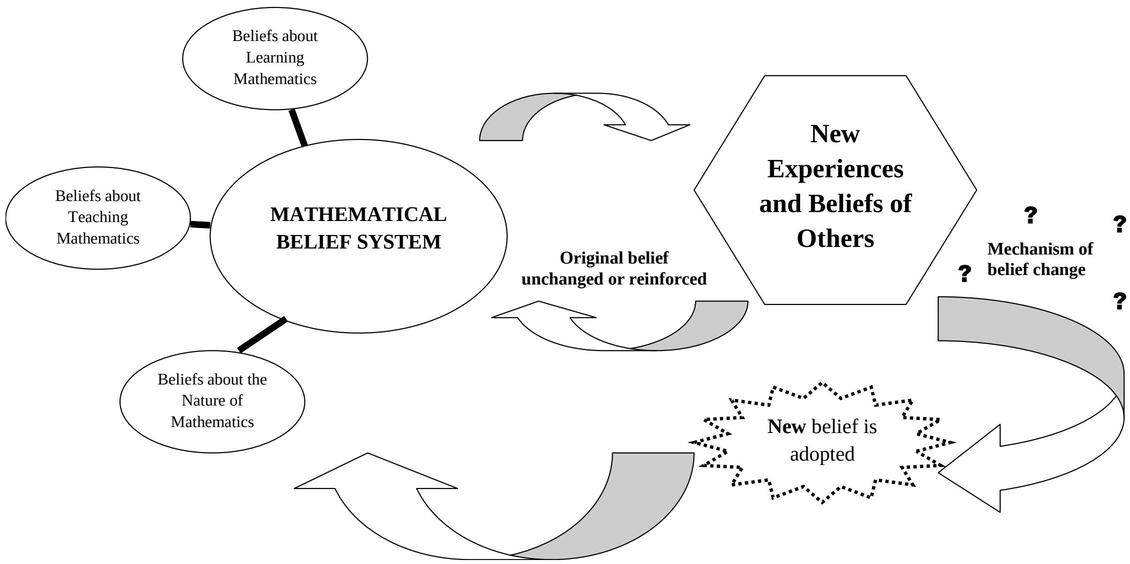

Figure 2.1: Process of Mathematical Belief Change

As substantiated by the literature reviewed in this chapter, it is important to address the beliefs held by pre-service teachers and how they are formed. Of equal importance is to address questions about the possibility of and mechanisms for belief change, specifically in the direction towards a reform-oriented, problem-solving view. One continuing issue in mathematics education research deals with the creation of TEPs that can influence this type of change and development. Section 2.2.8 provides evidence that change is possible, and also attends to the questionmarks in Figure 2.1 by outlining several successful mechanisms for change as described in the literature. These mechanisms are summarized at the end of the section.

To facilitate the process of belief change illustrated in Figure 2.1, one fundamental aspect of the methodology necessitates first that an individual be made aware of her current conceptions. This requires that one be provided opportunities that allow those beliefs to be challenged, perhaps by experiencing some kind of cognitive conflict, and potentially reevaluated. Additionally, Posner, Hewson, and Gertzog (1982) make the claim that in order for old beliefs to be changed or replaced, the new belief needs to be both plausible and intelligible. This idea was also identified by Pintrich, Marx, and Boyle (1993) to be a necessary condition in conceptual change. Other conditions described in their work can also be seen as necessary in order for not only conceptual changes, but also changes in beliefs. On top of having the new belief be plausible, Pintrich et al. also support the idea that the presence of an individual's dissatisfaction (or conflict) with her present belief is necessary, for she will feel no need to change her belief without reason. Using these conditions as a basis for investigation, I next describe studies that seem to have been able to both elicit and assess changes in the beliefs of pre-service teachers in particular, and highlight particular mechanisms present.

Collier (1972) used survey instruments to study the self-reported beliefs about mathematics of pre-service teachers as they progressed through their TEPs using a formal/informal spectrum of classification. He classified the formal end of the spectrum as describing those holding the traditional view of mathematics, with teachers' sole role being that of the lecturer and with emphasis placed on memorization of procedures. The informal part of the spectrum was characterized by an emphasis on experimentation and creativity, with teachers supporting a variety of student thinking. The study revealed that as the pre-service teachers moved through the programs, there was evidence that their beliefs experienced an evolution in the direction of the informal end of the spectrum, and therefore towards more reform-supported ideas about the teaching of mathematics. These early findings are encouraging in that they demonstrate that beliefs, which have already been shown to be tenacious when it comes to change, can be impacted even within a single preparation course. The research reporting evidence of belief change in pre-service teachers has primarily been situated within the United States, but such evidence has also been found at the international level. In Australia, Aldridge and Bobis (2001) cited a TEP as a source of change in mathematical beliefs of pre-service teachers towards a problem-solving perspective. Similar findings have been reported by Beswick and Dole (2001) and Grootenboer (2003).

Schram, Wilcox, Lanier, and Lappan (1988) provided evidence of change in pre-service teachers' beliefs about what it means to learn mathematics as they moved through a sequence of three mathematics courses. These courses placed heavy emphasis on development of conceptual understanding, collaboration and group work, and on activities classified as 'problem-solving' activities. These classroom characteristics were reported as being vital to influencing change in the mathematical beliefs of the pre-service teachers, as they represent those most common in the creation of opportunities for an individual to question and reevaluate current held beliefs. In a related study that focused on two pre-service teachers taking the first math course in the program sequence, Schram and Wilcox (1988) took a closer look at these students' views about what it means to know and learn mathematics. One student demonstrated a notable change in his beliefs, replacing his initial beliefs with new more desirable ones, but at the conclusion of the course the other student did not replace her original views as much as modify her belief structure in order to situate the beliefs brought forth from her new experiences within her existing conceptions about mathematics. Therefore, even when pre-service teachers participate in a course specifically designed to challenge held beliefs about mathematics, their resistant beliefs may have what Pajares (1992) called a 'filtering effect' on the new experiences in the classrooms. This can cause the student to 'filter' and misinterpret the overarching goals set forth by the teacher educator (Simon, et al., 2000), an effect that greatly influences the learning of the student, as well as the beliefs of the student.

Other researchers have embraced a social-constructivist view (e.g. Yackel & Cobb, 1996) in order to investigate changes in beliefs as situated within the socio-mathematical norms of the classroom. These norms may have some overlap with the classroom characteristics identified by Schram, et al. (1988) above. Several of the studies conducted within this framework (e.g. Erickson, 1993; Verschaffel et al., 2000) analyzed the effect of constructivist-influenced instruction on students' mathematical beliefs, situating the learning of mathematics within authentic and meaningful contexts. Those working from situated cognition theory may consider this kind of work grounded in authentic (teaching) contexts as providing opportunities for the pre-service teachers to engage in 'legitimate peripheral participation' (Lave & Wenger, 1991). The emergent argument from this collection of literature is that "if classroom practices are a major factor in the development of beliefs, it is plausible that significantly altering these environments can foster positive mathematics-related beliefs" (Muis, 2004, p. 355).

Feinman-Nemser and Featherstone (1992) attempted to motivate change in pre-service teacher beliefs by creating a constructivist environment in which the future teachers were involved as learners of both mathematics and related pedagogy. This proved to be successful for a couple of reasons. First, this environment allowed the pre-service teachers to experience the kind of constructivist learning that may have been absent during their early years as students; and second, it used "a teaching methodology that repeatedly has been proven effective in promoting construction of new knowledge, new ideas, and new beliefs" (Liljedahl, 2005, p. 2). Although this particular study analyzed the influence of a constructivist environment on the beliefs of pre-service teachers, a large majority of the corresponding literature has dealt with the beliefs of in-service teachers. Despite this difference in focus, the findings related to in-service teachers may be applied to the development of pre-service teachers' beliefs as well. Some of the research concentrating on in-service teachers is described below.

Erickson (1993) observed two teachers over the course of a year and a half long professional development program that aimed to help the teachers begin to implement the vision set forth by the NCTM (1989, 1991) advocating the constructivist view of learning and teaching in the classroom. The program strongly encouraged a shift in control from the teacher to the students by providing the students opportunities to collaborate with one another as the teacher promoted exploration and understanding over memorization and rote calculation. Although one teacher reported having a change in his beliefs about mathematics, evolving from a traditional view to one that was more consistent with the perspective of reform, his classroom practices did not reflect this change. As a result, his students' beliefs about mathematics were found to reflect the belief that the discipline is a set of memorized rules and procedures, and this belief showed no change over the course of the study. On the other hand, another teacher professed beliefs consistent with the reform-supported view of mathematics and consistently reflected these beliefs by creating a classroom practice that was rich with problem-solving activities. Her classroom also encouraged and supported small and large group discourse and exploration. Unlike the students that experienced the more traditional classroom, students in this second group displayed encouraging gains at the end of the study, professing that they saw mathematics as a sense-making activity. Furthermore, it was reported that they felt they did not need to be shown the "correct procedure" in order to successfully tackle a problem. Erickson's deeper analysis led him to conclude that the changes (or lack thereof) in students' beliefs about mathematics was directly influenced by the classroom culture and activities created by the teacher. He also concluded that the teacher likely had his or her own beliefs (if not necessarily his or her practices) influenced by participation in the professional development in the research study. These same mechanisms for belief change that have proved successful with in-service teachers may also apply to pre-service teachers in their TEPs.

Elaborating further on the activities created by the teacher mentioned above, Frykholm (2005) provided evidence from his research investigating a small group of pre-service teachers that indicated interaction with innovative curriculum material can have positive effect on pre-service teachers' beliefs about what constitutes good mathematics teaching and learning. Innovative curriculum materials here describe reform-oriented materials created in response to the Standards (NCTM, 1989, 1991, 2000). Indeed, a growing body of research has focused on the role of such materials in the process of teacher change (Lloyd, 2002; Lloyd & Wilson, 1998; Remillard, 2000) and has expanded to include their role in pre-service teacher change as well. The pre-service teachers in Frykholm's (2005) cohort reported that the innovative materials that they used challenged their ideas and beliefs about mathematics. This "catalyst for growth" (Frykholm, 2005, p. 31) triggered in them the realization that mathematics was more than just memorized procedures, and therefore teaching mathematics was more than just showing procedures to memorize. As a result of interacting with innovative curriculum materials, the pre-service teachers reportedly developed the desire to teach mathematics in a more connected and conceptual way.

Connecting to some of the literature already discussed in this section, Kagan (1992) identified three essential needs that appear to promote change in pre-service teachers' beliefs within the coursework of TEPs: (1) the pre-service teachers need to be given the opportunity to extensively work with and study students; (2) courses encountered in TEPs should reflect the activities and demands of the mathematics classrooms in which they will someday teach; and (3) the field experiences of the pre-service teachers should provide opportunities to work with in-service teachers that are willing to engage in self-reflection (in addition to promoting reflection in the pre-service teacher) and possible reconstruction of their own pedagogical beliefs. Similarly, Swars, Smith, Smith, and Hart (2009) suggested that in order to have "successful paradigmatic changes in (pre-service) teachers' beliefs and teaching practices" (p. 52), it is necessary for programs to create opportunities that: (1) generate interest in change; (2) problematize current practices and propose possible solutions; (3) allow experimentation with those possible solutions; and (4) reflect on the outcomes for students and teachers.

Related to both Kagan's (1992) second factor and several of those mentioned by Swars et al. (2009), Fenstermacher (1979) emphasized the responsibility of TEPs to provide pre-service teachers with a space to reflect on their initially held beliefs so that unconscious ideas may become more conscious, as well as helping to make connections between their beliefs. This prominent idea of challenging students' beliefs was central to the changing of those beliefs in the work of Feiman-Nemser, McDiarmid, Melnick, and Parker (1987), who claimed that such challenges are necessary for the student to make explicit the foundation of her beliefs in order to analyze and possibly modify those beliefs.

Liljedahl (2005) has shown that "pre-service teachers' experiences with mathematical discovery has a profound, and immediate, transformative effect on the beliefs regarding the nature of mathematics, as well as their beliefs regarding the teaching and learning of mathematics" (p. 2). The 'mathematical discovery' approach readily combines the two previously mentioned mechanisms by using small group work focused on problem-solving to challenge and make explicit the beliefs about learning and teaching held by the pre-service teachers.

The mechanisms described so far have been drawn from the literature that has explicitly addressed and analyzed beliefs in one way or another. Though these are clearly a valuable part of the present discussion, it is important to recognize that research conducted in different domains can be applied in order to better understand belief change. In their work using cognitive conflict as a mechanism for supporting change in PSET knowledge about explanation and proof, Stylianides and Stylianides (2009a) introduce the theoretical construct termed 'conceptual awareness pillars.' Broadly speaking, these pillars "describe instructional activities that aim to direct students' attention to their understandings or conceptions of a particular mathematical topic or idea" (p. 322) and can take place at both the individual and social level. These activities may include instructional activities such as tasks or reflections. Stylianides and Stylianides found that cognitive conflict experienced by PSETs was dependent on the extent to which instruction provided conceptual awareness pillars that permitted the PSETs to become more aware of their held conceptions. While this work provides support for the use of cognitive conflict as a mechanism for contributing to "developmental progressions in students' mathematical knowledge" (Stylianides & Stylianides, 2009a, p. 319), it is suggested here that this support can be extended to include progressions in (elementary) pre-service teachers' mathematical beliefs as well. An example of a conceptual awareness pillar that can take place at either level is to ask students to reflect on a prompt relating to a specific idea regarding mathematics, such as, "In order to solve a particular mathematics problem, I must first be taught the correct procedure." This prompt may be used to direct student attention to what they believe about what it means to do mathematics well, which in turn may reflect their held beliefs about what mathematics is as a discipline and what it means to teach mathematics. Chapter 4 will provide further discussion about conceptual awareness pillars and their impact on belief change within a research-based mathematics content course for PSETs.

Emerging from this review of the research are several ideas that form the foundation for changing mathematical beliefs of pre-service teachers, six of which are highlighted here. These mechanisms are summarized below in Table 2.1, along with the specific research that supports each.

Table 2.1: Summary of Mechanisms for Belief Change (MBC) from the Literature

MECHANISM FOR BELIEF CHANGE (MBC) FOR PRE-SERVICE TEACHERS	**RELATED RESEARCH**
(**MBC1**) Focus on problem-solving	Liljedahl, 2005; Schram, Wilcox, Lanier, & Lappan, 1988; Erickson, 1993; Verschaffel et al., 2000;
(**MBC2**) Opportunity for reflection	Fenstermacher, 1979; Kagan, 1992; Swars, Smith, Smith, & Hart, 2009
(**MBC3**) Collaboration/group work	Liljedahl, 2005; Schram, Wilcox, Lanier, & Lappan, 1988; Erickson, 1993; Verschaffel et al., 2000
(**MBC4**) Innovative curriculum materials/activities reflecting those supported by the Standards	Erickson, 1993; Frykholm, 2005; Lloyd, 1999; Lloyd & Wilson, 1998; Remillard, 2000
(**MBC5**) Coursework grounded in the work of teaching	Kagan, 1992; Swars, Smith, Smith, & Hart, 2009
(**MBC6**) Challenge beliefs (via cognitive conflict and conceptual awareness pillars)	Feiman-Nemser, McDiarmid, Melnick, & Parkerm 1987; Kagan, 1992; Pintrich, Marx, & Boyle, 1993.

These six mechanisms for belief change (referred to hereafter as *MBC*) are not the only ones that may promote belief change, but they do comprise a significant subset. In TEPs, it is important that pre-service teachers first and foremost be given the opportunity to be made aware of their belief. This opportunity is a precursor to each of these mechanisms. In Section 2.3, I focus on describing the different experiences of the pre-service teachers within the TEP and discuss how those experiences may support these particular mechanisms.

2.2.9 Summary

A teacher's beliefs about the nature of mathematics may impact the type of classroom activities in which students are engaged (Raymond, 1997), which in turn influence the students' experiences with mathematics. Positively changing pre-service teachers' mathematical beliefs is important and has shown to be achievable through several mechanisms outlined above. However, the more apparent issue is how to sustain those changes in order to avoid the washing out phenomenon upon entrance into a teacher's initial-teaching experiences.

Drawing from the six mechanisms for change outlined in Table 2.1 above as a foundation, in Section 2.3 I attempt to address the different experiences within TEPs that have influenced and can challenge the mathematical beliefs held by pre-service teachers. I also outline experiences within these programs that may provide opportunities to encourage change in those beliefs. In addition to briefly touching upon prior classroom experiences as students that shape the beliefs held by pre-service teachers as they enter their TEPs (reflected in the discussion of young students' beliefs in Section 2.2.3), these experiences are framed within three contexts common to the majority of teacher education: the mathematics methods course, the student-teaching experience, and the mathematics content course.

2.3 EXPERIENCES THAT IMPACT PRE-SERVICE TEACHERS' BELIEFS

It is important to acknowledge that not all TEPs in this country are the same. For example, PSETs at one mid-sized northeastern university are required to complete a five-year teacher certification program, wherein the first four years consist of coursework and the fifth year of field experience simultaneously with methods coursework. These students are required to take a single elementary mathematics content course, usually before their junior year, and this course serves as the prerequisite for their only elementary mathematics methods course taken during the fifth year. At another university in the southeast, PSETs can complete a four-year education degree that requires the completion of 12 semester hours of mathematics content courses, 9 of which are upper-division courses. These content courses and an elementary mathematics methods course are completed in the first two years of the program, after which the PSETs experience two years of integrated coursework (including additional methods courses and field "internships"). These two cases demonstrate the great variation among TEPs, but it is important to note that certain aspects such as content courses, methods courses, and the student-teaching experience are present almost in all TEPs in the United States.

Figure 2.2 below serves as a framework from which Section 2.3 is organized and depicts the possible relationships discussed here between PSETs' mathematical beliefs (numbered 1 in Figure 2.2) as elaborated in Section 2.2 and the different experiences of those pre-service teachers. These experiences include those held prior to the program (as a student of mathematics) (numbered 2 in Figure 2.2), and those within the different aspects of the TEPs [which includes the required mathematics course (numbered 4), and mathematics methods course with associated initial (student) teaching experiences (numbered 3)].

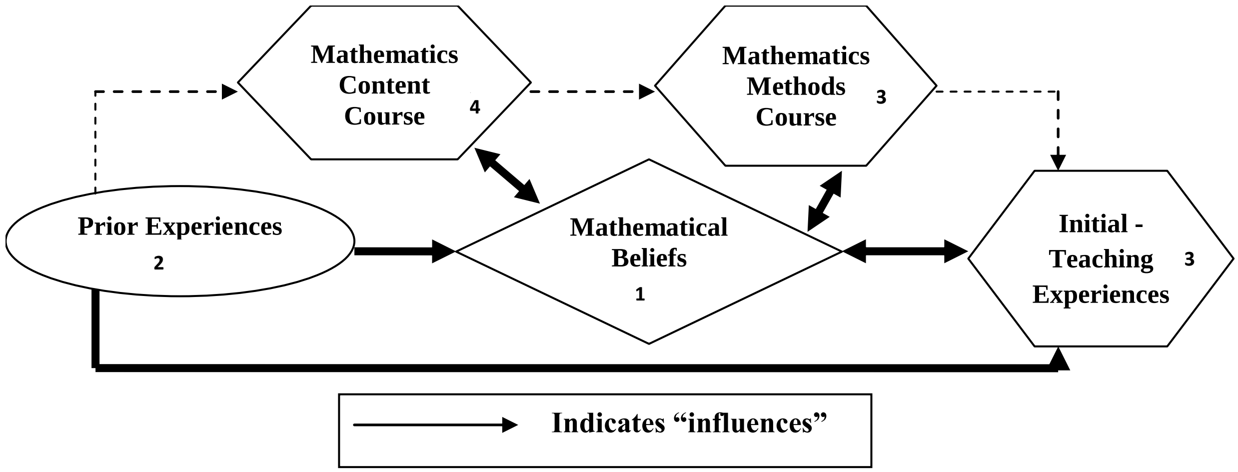

Figure 2.2: Framework that Organizes the Experiences in TEPs that Impact the Beliefs of Pre-service Elementary Teachers

A discussion of the mathematical beliefs found to be held by young students that are indicative of the prior experiences of pre-service teachers was given earlier in Section 2.2.3. These beliefs and corresponding experiences will not be revisited here, yet they are included in this framework because they represent the mathematical beliefs likely possessed by a pre-service teacher upon entering her TEP. I begin this section by describing and discussing the experiences related to the mathematics method course within the program and its corresponding student-teaching experience. Finally, the experience of the mathematics content course is elaborated.

As detailed later in this section, a large portion of the literature on the impact of TEPs on mathematical beliefs of pre-service teachers is concentrated on the role and impact of the methods course. There is also a clear emphasis on the importance of the student-teaching experience. Although there are significantly fewer studies addressing the role of the mathematics content courses, experiences associated with each of the three program components provide opportunities to address and provide mechanisms for change in pre-service teachers' beliefs. In the discussions that follow, particular mechanisms from Table 2.1 are referenced as *MBC*, followed by the corresponding number (e.g. MBC1, MBC2, etc.).

2.3.1 The mathematics methods course and student-teaching experience

Mathematics methods courses in TEPs are primarily designed to "develop pre-service teachers' understandings of mathematics, mathematics pedagogy, and children's mathematical development, and to cultivate a positive disposition toward teaching mathematics" (Wilkins & Brand, 2004, p. 226). Indeed, these courses are fundamental to any TEP. Whereas other education courses focus on theories of teaching and learning to support pedagogical knowledge, methods courses were purposely designed to teach effective pedagogical skills specifically for one content area (Ball, 1990). In addition to the coursework in the university setting, the mathematics methods course also intersect with another vital component of teacher development, namely the student-teaching experience. In this section, I address relevant research focusing on pre-service teacher beliefs as studied within these two contexts and connect it to particular mechanisms for beliefs change.

Wilkins and Brand (2004) utilized Hart's (2002) 'Mathematics Belief Instrument' with 89 pre-service teachers enrolled in a mathematics methods course centered around an investigative approach to thinking about and teaching mathematics, and followed the major principles of the Standards (NCTM, 1989, 2000). The instruction of the course was purposefully inquiry-based and utilized a textbook that could be classified as belonging to innovative curriculum materials. The enrolled pre-service teachers were "involved in reading, discussing, and writing about the philosophical underpinnings of different approaches to teaching with a focus on the role of the teacher and student" (p. 226). They also had the opportunity to collaborate with each other about their work and to create and implement lessons with actual elementary students. By creating this type of learning environment that evokes MBC1, MBC3, MBC4, and MBC5, the researchers found evidence suggesting that participation in the methods course influenced change in the participants' beliefs towards those more consistent with reform efforts and a problem-solving philosophy. A comparison of these results to those found by Hart, which investigated beliefs change over an entire TEP, show a comparable rate of change over a single course, and the authors hypothesize that the long-term impact of the entire program would probably be much greater.

Stuart and Thurlow (2000) focused their research efforts on another TEP with a primary goal of bringing about beliefs change in pre-service teachers within a different mathematics methods course. This research focused on the impact of the pre-service teachers' beliefs about the nature of mathematics, as well as on the impact that held beliefs about learning and teaching mathematics had on their teaching practices. Citing the opportunity to reflect as the most significant feature (MBC2), the authors provided support that belief change occurred over the duration of the course, with the reflection providing the pre-service teachers with the opportunity to be made more aware of their held beliefs as well as to evaluate them in light of their new classroom experiences. After tracking a pre-service teacher through both her mathematics methods course and student-teaching experience, Mewborn (2002) also attributed the positive change in the pre-service teacher's practices to her modified beliefs about the learning and teaching of mathematics brought on by the opportunity to participate in reflective thinking.

Positive change was also initiated in an in-service teachers' methods course based on 'Cognitively Guided Instruction' (CGI) (Carpenter et al., 1989), an integrated program of research that focused on, amongst other ideas related to student learning, in-service elementary teachers' knowledge and beliefs that influence their instructional practices. This type of instruction has many parallels to the constructivist and problem-solving classrooms described in Section 2.2.8, and therefore support the mechanisms for belief change found in such environments, namely MBC1 and MBC3. This CGI course also evoked MBC5 by encouraging pre-service teachers to make connections between the theories to which they were being exposed and actual classroom practices via examination of video clips showing teachers implementing CGI in classrooms with elementary students. Instruction of this sort is quite different from that experienced by most pre-service teachers when they were students from the elementary grades through university mathematics courses. On the contrary, these previous classes were likely guided by the textbooks and placed the teacher at the center of the learning environment, dictating the correct way to solve isolated problems and providing drill and practice problems (Nesbitt Vacc & Bright, 1999). A series of CGI related studies (e.g. Carpenter et al. 1989; Fennema et al. 1993; Fennema et al. 1996) found that learning to understand the development of children's mathematical thinking could result in fundamental changes in practicing teachers' beliefs and practices and that these changes were reflected within students' learning. For example, these teachers placed significantly greater emphasis on problem-solving in the classroom as opposed to rote computational skills, favored multiple solution strategies over single solution paths, listened more to their students' thinking and used it as the foundation of classroom discussions.

In a follow-up study conducted four years after the completion of a CGI course, it was reported that the majority of teachers not only sustained their beliefs and related practices, but learning had actually become 'generative.' The term 'generative' was used to describe their learning because these classrooms became a place where not only students were learning, but the teachers were also learning (Fennema et al., 1996). In particular, these teachers were found to: (1) view their students' mathematical thinking as central to their instruction; (2) have deeper understanding of their students' mathematical knowledge; (3) possess frameworks that allowed them to think about their students' thinking; (4) view themselves as the constructors and elaborators of their own knowledge about their students' thinking; and (5) make efforts to collaborate with fellow teachers about students' mathematical thinking and understanding. Although the increased appreciation for understanding children's thinking was indicated as a basis for belief change, that change only was only able to occur when teachers, both pre-service and in-service, were given the opportunity to apply their knowledge to real students and real classroom situations (Fennema et al., 1992; 1993), further supporting MBC5.

As a pre-service teacher takes on the dual role of a student teacher, she makes the transition from "school learner of mathematics to teacher of mathematics" (Brown et al., 1999, p. 301). Indeed, initial-teaching experiences have the potential to shape all teaching practices that follow. One may expect the transition from student to teacher to be an exciting culmination after years of training and preparation, yet many student teachers see this transition as a stressful and conflicting time (Veenman, 1984). Here, they are often faced with the unexpected realities of managing a classroom of boisterous children and an often unsupportive cohort of fellow teachers. Many of these difficulties relate to what Wubbels, Korthagen, and Brekelmans (1997) refer to as the 'gap' between theory and practice. This describes "the difficulty to use or apply theoretical notions in classroom practice" (p. 76), where the theories that drive many TEPs have little practical applications in real classrooms. This gap was also a major interest discussed in the work of Zeichner and Tabachnick's (1981) outlined in Section 2.2.7, and may be narrowed with closer attention paid to creating opportunities related to MBC5 in TEPs.

In his 2-year study of 44 pre-service teachers, Frykholm (1996) collected data from individual interviews, surveys, and lesson plans to find that they appeared to align themselves with the vision promoted by the NCTM recommendations (NCTM, 1989, 1991, 2000).

However, upon observing the implementation of the lessons in the classroom as student teachers, they "bore little or no resemblance to the values so highly espoused" (p. 665) initially by the participants, contributing to the body of research citing the frequent inconsistencies between professed beliefs and related classroom practices (Raymond, 1997). Frykholm reported that the pre-service teachers felt a marked tension between their TEPs and student-teaching placements with respect to implementing the kind of teaching supported by the Standards. In attempts to explain this disconnect, there has been a call for an increased effort on the part of TEPs to challenge the beliefs (MBC6) held by pre-service teachers in a classroom setting before it comes time for them to enter their own classrooms. Furthermore, these programs must pay particular attention to the role of the student-teaching experience that usually occurs within the methods course. As remarked by Samaras and Gismondi (1998), student teaching "has been viewed as an unmediated and unstructured apprenticeship which lacks relation to coursework and adequate supervision" (p. 716), therefore certainly lacking a structure that provides opportunities for student teachers to challenge their beliefs and reflect upon their experiences (MBC2).

Indeed, it is important to provide pre-service teachers with experiences and learning opportunities to evoke the different mechanisms for change outlined earlier in the chapter. These opportunities would ideally include those that challenge pre-service teachers' held beliefs, address the work of teaching, and allow the process of reflection in all aspects of their TEPs. Specifically, research has offered suggestions about the ways to address these goals within both the mathematics methods course and student-teaching experiences. Frequently within TEPs, many of the efforts to change pre-service teachers' beliefs are set in motion within methods courses, after the mathematics courses have been completed. These efforts may come too late to support pre-service teachers in developing the mathematical beliefs that will help them acquire a deep understanding of fundamental mathematics. In other words, the pre-service teachers are taught that in their classes devoted to pedagogy they should create a culture of learning, populated by a student community of shared inquiry and led by a teacher (themselves) that creates opportunities for these learners to make their own conjectures, be questioned by their peers and supported by strong mathematical arguments. This is a culture where both the students and the teacher are held accountable for the validity of the discoveries made. However, without the opportunity to engage in such a classroom, pre-service teachers may never have the chance to develop a belief that such a classroom can and will work, and hence may never try to mold their practices to create such an environment. Devoid of experiencing this kind of mathematics classroom as a student first, many future teachers may be overwhelmed when thrown into their first student-teaching classroom or first full-time classroom. These demanding new situations introduce a variety of factors that may cause the new teachers to revert back to the traditional techniques that go against everything for which the reform movement (and perhaps their TEP) stands. Seeing that "teaching itself is seen by beginning teachers as the simple and rather mechanical transfer of information" (Wideen et al., 1998, p. 143), without a mathematics-related course that challenges them and evokes the other mechanisms for belief change described in Section 2.2.8, pre-service teachers may not evaluate their mathematical beliefs about learning and teaching until after their initial-teaching experiences, after which it's almost too late.

2.3.2 The mathematics content course

In their review of TEPs around the world, Tatto, Lerman, and Novotna (2009) found that pre-service teachers' opportunities to engage with mathematical content were varied. Future teachers of elementary schools are often seen as generalists (having to teach all subjects), and therefore "the preparation they receive places low emphasis on mathematics content" (p. 19). Due to the growing body of research exposing the limitations of PSETs' content knowledge (Ball, 1990; Ma, 1999), many programs have attempted to design these content courses in order develop pre-service teachers' mathematical understandings. This is often done by addressing the mathematics the PSETs' encountered as students themselves in a new, conceptual way that better aligns with mathematical reform. Ideally, these courses would attempt to incorporate all six MBCs.

As Pajares (1992) said, "the earlier a belief is incorporated into a belief structure, the more difficult it is to alter, [whereas] acquired beliefs are more vulnerable to change" (p. 325). The traditional sequencing in TEPs of the primary mathematics-related experiences assumes that the mathematics content coursework comes first, as is followed by the mathematics methods course which often (but not always) parallels the student-teaching experience. It would seem to follow that efforts to incorporate the mathematical beliefs that would support practices that align with the ideas of reform should begin as soon as possible within the TEPs. Regarding the mathematical preparation of teachers, that would be within the mathematics content course.

An example of practices that support mechanisms for belief change that has been discussed earlier in this chapter relate to the CGI research (see Carpenter, et al., 1988, for further details) enacted in methods courses. Just as teachers in these classrooms experienced changes in their beliefs when given the opportunity to consider and challenge those beliefs (MBC6), pre-service teachers hypothetically may experience the same change when given the opportunity to participate in inquiry-based mathematics content courses. In addition to impacting beliefs, participating in inquiry-based classrooms may also contribute directly to the development in pre-service teachers 'habits of mind' (e.g. Cuoco, 1995; Goldenberg, 1996). As explained briefly in

Chapter 1, it is difficult to trace the origins of the phase habits of mind, as it generally describes the process of thinking and problem-solving attributed to working mathematicians. Relating to education, the Conference Board on Mathematical Sciences (CBMS, 2001) explicitly addresses its importance in their fourth recommendation for mathematics courses for future teachers, suggesting that these courses "develop the habits of mind of a mathematical thinker and demonstrate flexible, interactive styles of teaching" (p. 8). This requires that future teachers be provided with opportunities in their courses to ask thoughtful questions of themselves and others (MBC3), to explore interesting problems and different solution paths (MBC1), and to engage in meaningful mathematical communication.

In addition to permitting inquiry in the classroom, another feature of the mathematics course that has the potential to impact pre-service teachers' beliefs is the opportunity for reflection (MBC2). A large portion of the literature (Artzt, 1999; Artzt & Armour-Thomas, 1999; Nesbitt Vacc & Bright, 1999) has addressed the importance of allowing both students and teachers to be made aware of their own beliefs and behaviors in attempts to illuminate both connections and contradictions between the two. Artzt (1999) created a reflection model used by pre-service teachers both before and after implementation of a lesson during their student-teaching experience. This model required pre-service teachers to reflect on both their thinking regarding the lesson and their instructional practices during all phases of the lesson, from the planning phase to post-implementation. An increase in the awareness of beliefs and practices is needed in order to realize the potential for change to occur in those beliefs. Indeed, when studying teachers in particular, the ability to reflect has been directly connected to impacting change in teachers' practices and the various mechanisms of that change including reflection (MBC2). A successful teacher as envisioned by reform mathematics (NCTM, 1989, 1991, 2000) possesses the internal flexibility to manage classroom discussion, to question, and to elicit mathematical knowledge from students by probing for student explanations. The ability to reflect on these experiences and exchanges within the classroom and on the mathematical learning of the students is one avenue to advancing knowledge for both the teacher and the students.

There is comparatively less literature that focuses on the role of the mathematics content courses in TEPs in contrast to, for example, the role of methods courses and the student-teaching experience. Even though it is impossible to pinpoint the exact reason for this absence, one potential reason could be that in many programs, the required mathematics course(s) for pre-service teachers in North America and perhaps elsewhere are traditionally offered through the university's Mathematics Department (Davis & Simmt, 2006), not the Education Department.

In a presumably large number of universities, the mathematics preparation for pre-service teachers is centered upon the content found in mathematics textbooks that cover a massive amount of subject matter. These textbooks may not classify as innovative curriculum materials (MBC4). This traditional textbook design of teaching, often accompanied by direct lectures from the teacher, provides little opportunity for student interaction and engagement with the mathematics that is needed for teaching, and is likely void of any opportunity to reflect upon and possibly modify a student's associated beliefs. For example, several researchers (e.g. Lampert & Ball, 1990; Markovits & Smith, 2008; Shulman, 1987) have professed that pre-service teachers have much to gain from case-based learning, where they are given the opportunity to analyze and reflect upon real-life teacher case studies that exemplify theories, principals, and teacher decisions. Case-based learning is an example of activity that addresses what has been referred to as 'mathematical knowledge for teaching' (Ball & Bass, 2000), and focuses on the particular and specialized knowledge that one needs to possess in order to handle the mathematical issues that arise when teaching. Moreover, case-based learning directly relates to MBC5, as its use grounds pre-service teachers' work in the actual work of teaching. This specialized knowledge for teaching includes problems such as "offering mathematically accurate explanations that are understandable to students of different ages, evaluating the mathematical correctness of student methods, [and] identifying the mathematical correspondences between different student solutions for a problem" (Stylianides & Stylianides, 2010, p. 161). In attempts to address the learning opportunities provided within TEPs for the development of such knowledge, Stylianides and Stylianides (2010) introduced the notion of 'Pedagogy-Related mathematical tasks' (or 'P-R mathematical tasks') to describe tasks that support such development, and also can be identified as relating to MBC5. These kinds of tasks will be further discussed and utilized in Chapter 4. The kind of learning within the situated cognition perspective (Brown, Collins, & Duguid, 1989; Lave & Wenger, 1991), focusing on mathematical understanding grounded in the real work of teaching, has great potential to play an important role in the process of belief change in content courses. Such opportunities are rare in the more traditional mathematics classrooms, and yet these are precisely the opportunities needed to evoke belief change within TEPs.

In addition to grounding learning in the real work of teaching (MBC5) as discussed above, mathematics content courses also provide an ideal environment in which to implement conceptual awareness pillars (Stylianides & Stylianides, 2009a). These pillars can make students more aware of their held beliefs, and may therefore create cognitive conflict (MBC6) that challenges their beliefs. Recall that these pillars "describe instructional activities that aim to direct students' attention to their understandings or conceptions of a particular mathematical topic or idea" (p. 322), and may be utilized specifically to direct pre-service teachers' attention to their held beliefs about mathematics. Since the primary objective of these pillars is to focus on mathematical understandings, their implementation would likely be more successful in the content course than in the methods course, where greater attention is paid to pedagogical issues related to mathematics. Embedded within this particular mechanism are also components of others described earlier in this chapter, including reflection (MBC2). Stylianides and Stylianides (2009a) provided as example of a conceptual awareness pillar the opportunity for pre-service teachers to reflect on specific ideas or statements regarding mathematics that makes them more aware of their beliefs. Their reflections on the degree of agreement with statements such as "In order to solve a particular mathematics problem, I must first be taught the correct procedure," and "Mathematics exists independent of human activity" may help unveil their beliefs not only about mathematics as a discipline, but also related ideas about learning and teaching mathematics.

Recent research has described ways that teacher educators have begun to integrate innovative, Standards-based (NCTM, 1989, 1991, 2000) curriculum materials (MBC4) into coursework for pre-service teachers (Lloyd, 2002; Tarr & Papick, 2004) with positive effects. Many pre-service and student teachers enter classrooms to teach for the first time with Standards-based curriculum materials, and so early and maintained use of these kinds of materials is important. Considering the structure of most TEPs, this means that use would ideally begin in mathematics content courses and persist through all other aspects of the program since pre-service teachers traditionally enroll in content courses first. This is important given the considerable time needed for teachers to learn with these reform-based materials (Manouchehri & Goodman, 1998) and to develop new conceptual understandings of the mathematics presented in the materials (Ball & Feiman-Nemser, 1988). Moreover, interaction with these innovative curriculum materials in content course may provide future teachers with the chance to learn mathematics in a manner consistent with the reform effort that will likely guide their own teaching. Although the number of publications focused on in-service teachers' use of innovative curriculum materials continues to grow, many questions still remain about the interaction between curriculum materials in general and pre-service teachers. I suggest that one step towards better understanding this interaction may be to first attempt to understand the curriculum materials themselves. In particular, it may be beneficial to know more about the textbooks being used by pre-service teachers.

Textbooks are often the primary sources of mathematical activity in the mathematics classroom, from the elementary level to teacher education. It has been suggested that textbooks may play a substantial role in teachers' decisions about what is taught in K-12 classrooms (Stein, Remillard, & Smith, 2007), but considerably less is known about the ways in which textbooks used in TEPs may impact the mathematical beliefs and practices of the future teachers using them. Since these beliefs are central to investigating the ways that pre-service teachers approach and interact with curriculum materials (innovative or not), first as a student and then later as a teacher, it seems important to investigate the means by which those beliefs are developed. Just as textbooks influence what is taught and learned in elementary through high school mathematics classrooms, McCrory, Siedel, and Stylianides (2007) have claimed that they may "exert a major influence on the content and approach of courses for prospective elementary teachers (p. 5)" reaching over 80,000 students in TEPs each year. Indeed, it is possible that textbooks used in mathematics content courses, like mathematicians and mathematics educators, may promote underlying philosophies about the nature of mathematics as a discipline. These ideas "combine to create a distinct view of mathematics and mathematics learning that permeates each textbook"

(McCrory et al., 2007, p. 13), working from either one or a combination of several philosophies, that has the potential to influence the development of the beliefs of their readers. In Chapter 3, I adopt Ernest's (1988) three views (Platonist, instrumentalist, and problem-solving) to develop an analytic framework used to investigate the different views about the nature of mathematics promoted in textbooks used in content courses for PSETs.

Curriculum materials are undoubtedly vital to the experiences created in TEPs, as are the teacher educators that implement them. Although there has been an abundance of educational research related to teaching and learning in the elementary, middle, and high school grades, very little has been done at the university level. According to Bass (1998), university teaching is in critical need of systematic research. In particular, little is known about the knowledge that university mathematics faculty have related to mathematics pedagogy and its effect on their instruction (Selden & Selden, 2001). Even less is known about how that knowledge impacts the students in their classes, which often includes pre-service teachers. Mathematics content courses taught by mathematicians may not readily reflect ones that are problem-centered and driven by inquiry (MBC1), while encouraging small group work (MBC2) and reflection (MBC3) in order to develop deep and flexible mathematical understanding and beliefs that more closely align with the ideas promoted by reform.

One key factor that may help promote pre-service teacher beliefs that align with reform ideas about doing, learning, and teaching mathematics in pre-service teachers is the recognition of the important role played by teacher educators in the mathematics content course. Although it has been shown that there is much promise in influencing changes in pre-service teachers' ideas about what it means to do mathematics through different experiences, it cannot be ignored that it has also been found that changes in beliefs sometimes are only temporary. As described by

Wilson and Cooney (2002), the failure to maintain belief shifts is often related to beliefs about "what constitutes an appropriate role for the *teacher* of mathematics" (p. 142), not necessarily relating to the mathematics itself.

Stylianides and Stylianides (2010) make the claim that content courses require instructors that "need to have not only good knowledge of mathematics but also some pedagogical knowledge" (p. 170). What needs to be achieved is a balance between these two types of knowledge, regardless of which department (Mathematics or Education) offers the course. Indeed, a primary goal of the TEPs should be to "find ways to support the teaching practices of mathematicians and other instructors of mathematics content courses for prospective teachers" (Stylianides & Stylianides, 2010, p. 171). Courses created by educators with this balance between content and pedagogical knowledge would be more likely to promote mathematics as a creative human activity composed of a connected web of concepts. The pre-service teachers in this class would be more likely to participate then in problem-solving (MBC1), in P-R mathematical tasks (Stylianides & Stylianides, 2010) (MBC5), in sharing and discussing their different solutions with their classmates (MBC3), and in sharing mathematical authority with classmates and with the teacher. They would be provided with the opportunities to develop beliefs about the nature of mathematics, as well as the learning and teaching of mathematics, that are grounded in their own experiences and in the real work of teaching and less likely to be uprooted and washed away when they enter their own classrooms. They would hopefully create and maintain a classroom environment that encourages the learning of reform mathematics, which one would hope to be a major goal of all mathematics TEPs.

2.3.3 Summary

The literature reviewed the second section of this chapter leaves little doubt that different experiences, both prior to and within TEPs, have great influence on shaping pre-service teachers' beliefs about mathematics. Ross, McDougall, and Hogaboam-Gray (2002) reported, regarding reform, that "the most important obstacle is that teachers' beliefs and prior experiences of mathematics and mathematics teaching are not congruent with the assumptions of the Standards" (p. 132). Indeed, all components of the mathematics TEPs provide opportunities and experiences to change those beliefs. However, this change requires particular learning opportunities from the teacher educators shaping these experiences that elicit different mechanisms for belief change which include the six discussed here and summarized in Table 2.1.

In both the content and methods courses, the mathematics educator needs to simultaneously hold knowledge about mathematics, as well as knowledge about pedagogy. It may not be enough that only one of these be addressed, nor may it be enough that they are addressed independently. Teacher educators need to have the ability to demonstrate to pre-service teachers that mathematics is not about memorized procedures and facts, but is a creative human activity. Furthermore, these educators need to provide opportunities for the pre-service teachers that relate specifically to the pedagogical demands of the classroom as well as allow them to reflect upon their experience and related beliefs.

2.4 CONCLUSION

The relationship between mathematical beliefs, experiences, and teaching practices is a complex one, as each aspect involved is sure to influence all others in the relationship. For instance, the beliefs held by future teachers of mathematics can be shaped by their mathematical experiences, but their beliefs can also shape the ways in which they interpret and shape their different experiences as well as their practices. Even before taking their first course, students entering TEPs have "developed a web of interconnected ideas about mathematics, about teaching and learning mathematics, and about schools" (Ball, 1988, p. 22). These deeply-rooted beliefs stem from their own experiences as students of mathematics and have been shown to have the greatest impact on the practices these future teachers adopt in their own classrooms (Kagan, 1992; Stipek et al., 2001; Thompson, 1992). Stuart and Thurlow (2000) also reported that many future teachers, as well as many in-service teachers, see their role as merely a transmitter of knowledge in the form of learned processes and procedures, an "erroneous and simplistic belief about what it takes to be a successful teacher" (p. 114). Despite the fact that pre-service teachers often enter their TEPs with "misconceptions and negative attitudes towards mathematics, a subject they will soon be expected to teach" (Phillippou & Christou, 1998, p. 191), many courses in those programs fail to acknowledge the presence, role, and impact of those beliefs. Indeed, the beliefs brought by pre-service teachers influence not only what they learn, but also how they learn, and as such provide a target for potential change within the TEP. Here lies the greatest opportunity to enact mechanisms for change to reform mathematical beliefs, and as a result, the teaching practices of pre-service teachers.

In order to provide the learning opportunities that align with the reform image of mathematics for the students in their own classrooms, pre-service teachers need to be given the opportunity to check their beliefs against those that drive reform and have the opportunity to find those beliefs wanting. Though much of the research has focused on the change of pre-service teachers' beliefs within the mathematics methods course and related student-teaching experiences, it is important that an equal amount of attention be paid to the impact of the mathematics content course on teacher development. Chapters 3 and 4 will investigate this impact further, specifically with regards to PSETs.

Thompson (1992) stressed that understanding mathematical beliefs is essential to reform, emphasizing the impact of these beliefs on pre-service teachers' ability to expand upon, change, and develop teaching practices consistent with the reform movement. Indeed, more traditional (Platonist or instrumentalist) views of the nature of mathematics can work against the development of such a teacher. Future teachers holding different beliefs about the nature of mathematics will inevitably have very different visions about the best ways that mathematics should be taught and learned. The classroom experience created by a teacher believing mathematics to be a collection of irrefutable facts and rote procedures will look radically different in comparison to one created by a teacher who views mathematics as a creative and dynamic construct, driven by problem-solving and communication. Given the power of beliefs and their reported resistance to change, TEPs need to be actively aware of the beliefs held by their pre-service teachers. These programs should provide these future teachers with opportunities to reflect upon those beliefs and to engage in activities that have been shown to be mechanisms of change. This may result in the washing in of a reform-based view of mathematics that may potentially be more resistant to being washed out upon entering their initial-teaching experiences.

In addition to better understanding the learning opportunities and mechanisms for belief change present in the different components of TEPs, further research needs to be done with student teachers in their initial-teaching experiences. Such research would help to ascertain the greatest challenges that are faced in the classroom that contribute to the washing out phenomenon so that changes in programs can be made to address those particular challenges. This chapter has demonstrated that beliefs are certainly one of the most influential factors with regards to the development of reform-oriented teaching practices, shaped by one's experiences of mathematics, both the learning and teaching of it. At the heart of these matters is the idea that "one's conception of what mathematics is affects one's conception of how it should be presented. One's manner of presenting is an indication of what one believes is most essential in it . . . The issue, then, is not, What is the best way to teach? but, What is mathematics really all about? [Therefore], controversies about … teaching cannot be resolved without confronting problems about the nature of mathematics" (Hersh, 1986, p. 13).

3.0 CHAPTER 3: DEVELOPING AND IMPLEMENTING A CRITICAL DISCURSIVE FRAMEWORK TO ANALYZE THE VIEWS ABOUT MATHEMATICS BEING PROMOTED BY TEXTBOOKS FOR PRE-SERVICE ELEMENTARY TEACHERS

3.1 INTRODUCTION

Over the past 20 years, researchers in mathematics education have acknowledged the need to investigate the beliefs held by pre-service teachers along with their anticipated and eventually adopted teaching practices (Pajares, 1992; Richardson, 1996; Thompson, 1992; Wilson & Cooney, 2002). While these issues are independently significant, the study of pre-service teachers' beliefs and their influence on instructional practice has gained increased attention in the last decade. Recall from Chapter 2 that the term *belief* used in this dissertation means "the implicitly or explicitly held subjective ideas about the nature of mathematics that influence the ways an individual conceptualizes, describes, and engages in both the learning and teaching of mathematics." Emergent findings indicate that teachers' beliefs about mathematics are "a significant determiner of what gets taught, how it gets taught, and what gets learned in the classroom" (Wilson & Cooney, 2002, p. 128). In particular, Hersh (1986) claimed that "a person's understandings of the nature of mathematics predicates that person's view of how teaching should take place in the classroom" (p.13).

In the attempt to trace the origins of pre-service teachers' ideas about the nature of mathematics, this chapter explores the epistemological ideas promoted within the primary source of mathematical activity in the classroom: the textbook. Although it has been established that textbooks may play a substantial role in teachers' decisions about what is taught in K-12 classrooms (Stein, Remillard, & Smith, 2007), considerably less is known about the nature of mathematics being promoted in textbooks used in teacher education programs (*TEPs*). Just as textbooks influence what is taught and learned in elementary through high school mathematics classrooms, they "exert a major influence on the content and approach of courses for prospective elementary teachers" (McCrory, Siedel, & Stylianides, 2007, p. 5), reaching over 80,000 students in TEPs in the United States each year. I claim in this chapter that textbooks, like mathematicians and mathematics educators, may embody and promote (explicitly or implicitly) underlying philosophies about the nature of mathematics. Whether grounded in a single philosophy or a combination of several, textbooks have the potential to influence the development of the beliefs of their readers. This study does not provide a complete picture of that development, but rather provides a starting point by addressing pertinent questions about the kinds of mathematics being promoted within textbooks for pre-service elementary teachers (*PSETs*) in mathematics content courses, most particularly with regards to the treatment of definitions and tasks.

My decision to focus on the definitions and tasks in such textbooks was not an arbitrary one. As discussed in Chapter 2, all components of TEPs have the potential to impact the mathematical beliefs of pre-service teachers, yet there has been comparably less research focused on content courses. Since these content courses provide some of the last opportunities to experience as students the types of mathematical activities that will help them create classrooms envisioned by mathematical reform (NCTM, 1989, 1991, 2000), it is important that more research be done on this topic.

In addition to playing an important role in mathematical activity, the presentation of definitions in textbooks can reveal how textbooks promote different mathematical views. For example, one textbook may simply provide the reader with a single definition and present it as the only definition possible, while another may engage the reader in the creation of a definition that is open to modification. These two textbooks would be promoting vastly different views about the nature of mathematics, the first Platonist and the second problem-solving. These views (in addition to the instrumentalist view) were initially described in Chapter 2. Similar conclusions may be drawn when comparing the presentation of tasks.

This chapter addresses the following research question and related sub-questions:

RQ2. *How may the linguistic choices made by authors of mathematics textbooks for pre-service elementary teachers promote different views about the nature of mathematics?*

> 2.1. What are components of a framework that could be developed and used to analyze the linguistic components found in textbooks to understand how the textbooks can promote different views about the nature of mathematics?
>
> 2.2. Using this framework, how do textbooks promote different views about the nature of mathematics in the particular areas of definitions and tasks?

I first consider RQ 2.1. Drawing on the work of Morgan (1996, 1998, 2005, 2006) that was grounded in systemic functional linguistics (Halliday, 1973, 1985), I develop an analytic framework for describing the interaction[1] between the tools of functional linguistics and three different views about the nature of mathematics: the *Platonist view*, the *instrumentalist view*, and the *problem-solving view*. I then turn my attention to RQ 2.2 and use this framework to analyze

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[1] It is important to point out that *interaction* as it is used in this chapter should not be interpreted as indicating any kind of statistical relationship. The definition of interaction used here is to indicate "the act of some things interacting, or acting upon one another" (Wiktionary.com, 2010).

the views about the nature of mathematics being promoted in three textbooks for PSETs, focusing on the treatment of definitions and tasks within number theory chapters in the chosen texts.

3.2 RELATED LITERATURE AND BACKGROUND

In this section, I provide a brief review of the literature in several areas relevant to the goals of this chapter. Below, I briefly review some of the research discussed in Chapter 2 that is relevant to the work described in the current chapter. I review the different philosophies about mathematics proposed by various researchers and the ways in which these philosophies may influence teaching practices. Afterwards, I discuss the important role played by textbooks in mathematics classrooms and I outline some of the work done relating to textbooks in general. Together, this literature substantiates the need to investigate not only the beliefs held by PSETs, but also the particular factors that may have an impact on those beliefs.

3.2.1 Philosophies about mathematics and their relation to instructional practices

As discussed in Chapter 2, Ernest (1988) detailed three distinct philosophies of mathematics that emerged from his review of empirical studies about beliefs--the Platonist view, the instrumentalist view, and the problem-solving view. These views will be further elaborated upon in Section 3.3.1. Dionne (1984) proposed that views about mathematics are composed within one of the ‘traditionalist,’ ‘formalist,’ or ‘constructivist’ perspectives, while Törner and Grigutsch (1994) defined their categories as the ‘toolbox,’ ‘system,’ or ‘process’ aspect. These notions may have different names, but they do reflect in some ways those described by Ernest. Roulet (1998) had a more basic dichotomy in describing either the traditional, 'absolutist' view (which would include the Platonist and instrumentalist views) or the 'constructivist' view of mathematics (which would include the problem-solving view).

Recall that the literature suggested that a teacher holding the instrumentalist view of mathematics, which falls under the absolutist philosophy (Roulet, 1998), may teach mathematics as though it were a toolbox of rules and procedures. This teacher will likely stress the importance of systematic, rote exercises in order to promote precision and mastery of those tools. A teacher having the Platonist view may impress upon the students mathematical terminology and the connections between concepts that form explanations as though the students were simply vessels of acquirement. These two views about the nature of mathematics most naturally give rise to what Ernest (1999) calls a 'separated' view of school mathematics that will most likely lead to an authoritarian, traditional classroom. Conversely, a teacher having a problem-solving view of the nature of mathematics will likely translate into an engagement with students in the process of doing mathematics reflecting a more 'constructivist' philosophy.

In attempts to improve the quality of mathematics instruction, many teacher educators and mathematics education researchers have worked to understand the origins of the beliefs that pre-service teachers bring with them as they enter and progress through TEPs and the powerful effects of those beliefs on learning (Pajares, 1992) and eventual teaching practices. I have detailed much of that research in Chapter 2. Although there are numerous factors to consider in better understanding the origination and influence of pre-service teachers' beliefs, this chapter purposely restricts its focus to one particular aspect within the TEP: the textbook used in the mathematics content course. Next, I outline the importance and influence of textbooks to further provide rationale for my work.

3.2.2 The role and importance of mathematics textbooks

I define the term *textbook* to include the bound curriculum materials intended to be used by the PSETs in the mathematics content course. Research suggests that the majority of practicing teachers use mathematics textbooks as the foundation for their classroom activity, often reporting to have learned in this manner themselves as students (Brown, 1998; Romberg & Kaput, 1997). However, this is not universally true. Some teachers eschew the textbooks chosen by their districts in favor of creating their own instructional materials, primarily drawing on their own beliefs about what mathematics is and the best ways to teach and learn (Seeley, 2003). Regardless of whether or not these teachers choose to use their textbooks in class, this research suggests that this decision is grounded in the teachers' previous experiences with textbooks and their related beliefs about the role of textbooks in the classroom.

Given their pronounced role in both the teaching and learning of mathematics, textbooks have generated a vast amount of extensive research. Pepin and Haggarty (2001) suggested four categories in which most research on mathematics textbooks is located: (1) the mathematical intentions of the textbook (including the mathematical beliefs represented); (2) the pedagogical aspects of the textbook (ways in which the reader is supported); (3) the sociological contexts of the textbook; and (4) the cultural traditions represented in the textbooks. Each of these areas can be further broken into subcategories by either the student or teacher use, with the diverse methods of analysis corresponding to the even more diverse research questions posed.

According to Pepin and Haggarty, much of the empirical work done has focused on what is in the textbooks and how they are organized, referred to as the content and structure.

Many researchers report on the various content characteristics of a single textbook or curriculum, like 'Everyday Mathematics' (e.g. Fraivillig, Murphy, & Fuson, 1999), perhaps looking at the treatment of a particular topic, like division of fractions (e.g. Son, 2005) or proof (e.g. Stylianides, 2007, 2008, 2009). Others have attempted a more comprehensive overview. McCrory, Siedel, and Stylianides (2007) analyzed all textbooks written to be used by PSETs in mathematics content courses and compared the overall content of the textbooks, as well as the similarities and differences across the textbooks in three specific content areas. Although McCrory et al. reported a general consistency in the content of these textbooks as suggested by the chapter titles and the topics included under each, "the level of detail, depth and breadth of approaches, presentation of material, and functionality of the [text]books varies widely" (McCrory et al., 2007, p. 1). Many important lessons learned from their analysis were reported by the authors, one of which suggested that the differences found across the various textbooks in the presentation of mathematical concepts "are important and can be mathematically conflicting" (p. 29). Furthermore, it was suggested that an ongoing issue in need of attention regarding both teacher educators and textbook authors concerns the mathematical meaning being constructed by pre-service teachers as they engage with these textbooks when used in their content courses. This issue is of particular interest in the present chapter.

Regardless of whether one agrees that what is taught in the classroom comes predominantly from the textbook or that it depends solely on the teacher, both of these situations may have their foundations in the teachers' own experiences (Stipek et al., 2001). These reflect not only experiences as young children but also those as students progressing through their

TEPs. Specifically, the experiences associated with the mathematics textbooks used in mathematics content courses in these programs may influence pre-service teachers' own future textbook use and general instructional practices. In spite of the prominent role that textbooks have been shown to play in elementary and high schools (e.g. Apple, 1992; Schmidt et al., 1997), "little attention has been given to the role curriculum materials might play in teacher preparation and teacher development" (Nicol & Crespo, 2006, p. 331). For these reasons, this chapter explores the nature of mathematics promoted in textbooks used in mathematics content courses for PSETs.

3.3 THE ANALYTIC FRAMEWORK

The analytic framework described in this section has been developed to analyze the ways in which the linguistic choices made by textbook authors may promote different views about the nature of mathematics. To borrow from an article title of Sfard (2001), "there is more to discourse than meets the ears." A large portion of the existing research on the mathematical discourse has focused on the verbal exchanges in the social activity of classroom discussions and its impact on mathematical learning and teaching (Cobb, Yackel, & McClain, 2000; Lerman, 2001; Pimm, 1987), but that is not the focus in this particular chapter. I analyze the mathematical discourse that meets the *eyes* rather than the ears, in the discourse established by mathematics textbooks for PSETs. Discourse analysis is used in a variety of ways in the literature, depending on the underlying theoretical considerations of the researchers. These analyses are described as being "interpretive and explanatory, [with] critical analysis implying a systematic methodology and a relationship between the text and its social conditions, ideologies, and power relations" (Wodak, 1996, p. 20).

The critical discursive framework developed for the analysis in this chapter is shown in Figure 3.1. The framework's foundation is in the early work of Morgan (1996), whose ideas about critical discourse analysis closely reflect those used by linguist Norman Fairclough (1992). Morgan's work is grounded in Fairclough's assumption that every text somehow contributes to an individual's identity within her culture. Both Morgan and Fairclough root their ideas in the multifunctional linguistic theories of Halliday's (1985) systemic functional linguistics. The philosophy underlying systemic functional grammar is that language is a semiotic system that serves as a resource through which one creates meaning. Although symbols and illustrations undoubtedly affect the ways in which the reader responds to and interacts with the textbook, this analytic structure does not capture this interaction, and does not aim to do so. Nor do I wish to separate the mathematical content from the linguistic elements shaping the mathematical discourse. Rather, my goal is to investigate how the textbook author's language choices have the potential to promote three distinct views about the nature of mathematics.

The framework has two dimensions: the first represents different views about the nature of mathematics, and the second includes components of linguistic analysis. In the first dimension, I adopt the three distinct views Ernest (1988) detailed in his review of empirical studies focusing on teachers, namely the *Platonist view,* the *instrumentalist view*, and the *problem-solving view*. Other researchers have suggested different characterizations about the nature of mathematics, however my choice to use these three is supported by Thompson's (1984) findings that views similar to those suggested by Ernest parallel those most frequently observed in mathematics teaching. The second dimension consists of three linguistic components: (1)

actors; (2) *processes*; and (3) *modality*. The diamonds in Figure 3.1 represent the different interactions between actors and processes anticipated in textbooks promoting each of the three views, as well as the anticipated modality.

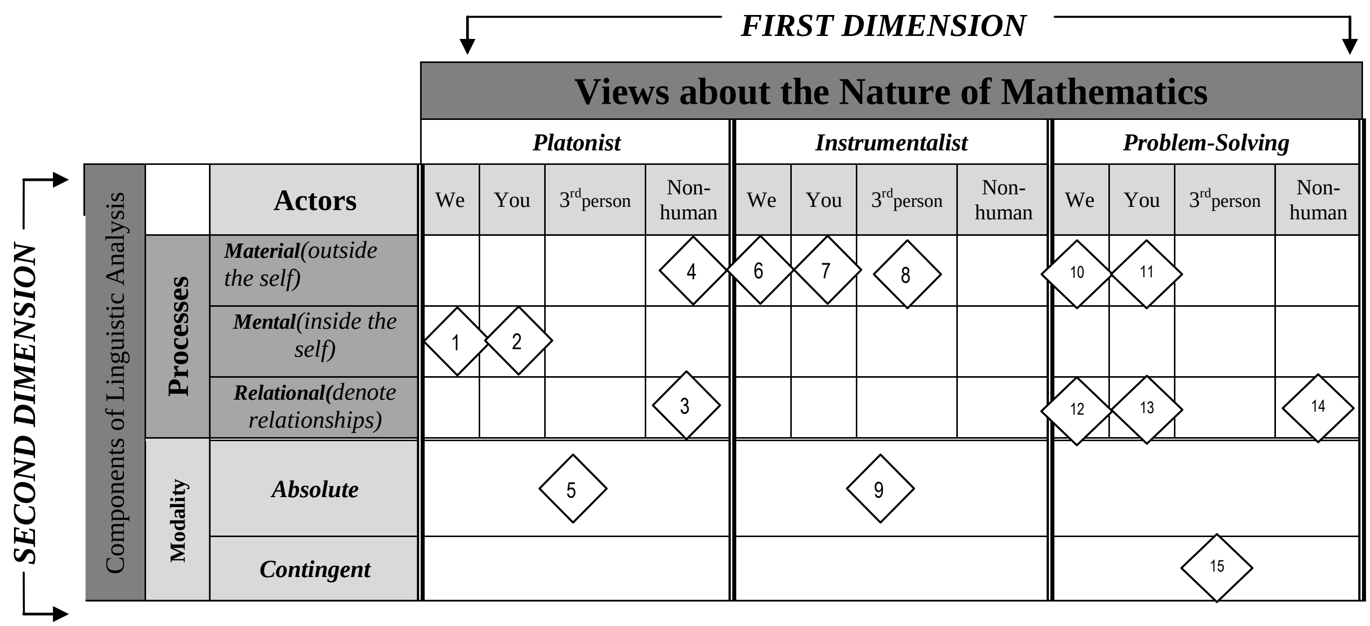

Figure 3.1: The Analytic Framework

These three views about the nature of mathematics, which were briefly discussed earlier, are elaborated upon in Section 3.3.1. The linguistic components are further detailed in Section 3.3.2. Finally, the ways in which these two dimensions interact are discussed in Section 3.3.3.

3.3.1 The first dimension: Three philosophies about the nature of mathematics

The *Platonist view* of mathematics, so named due to its roots in Platonic thought, portrays mathematics as a static body of knowledge "bound together by filaments of logic and meaning"

(Ernest, 1988, p.10). As exemplified by his famous metaphor of the cave (Shorey, 1930), Plato professed that the everyday world can only imperfectly approximate an unchanging, ultimate reality. Platonism supports the view that mathematics describes a non-sensual reality, existing independently of the human mind and is only perceived (probably incompletely) by the human mind. Mathematical objects are thought to be abstract and to "exist independently of mathematicians and their minds, languages, and so on" (Shapiro, 1997, p.37). To summarize these characteristics, Linnebo (2008) suggested three theses of the philosophy: (1) existence (there are mathematical objects); (2) abstractness (mathematical objects are abstract); and (3) independence (mathematical objects are independent of intelligent agents and their language, thought, and practices). In their analysis of students' ideas about mathematics, Halverscheid and Rolka (2006) offered the following as criteria typical of the Platonist view:

(1) A historical view of the nature of mathematics
(2) Mathematical objects are connected to one another
(3) A lack of mathematical activity is represented
(4) Topics are represented concretely

Within this philosophy, truth is seen as a definite outcome from the unified and connected structure of knowledge. Mathematics is not created but discovered by an individual in a predefined, set historical progression.

The *instrumentalist view* of mathematics supports the vision of the discipline as a collection of "unrelated but utilitarian" (Ernest, 1988, p.10) facts and procedures used by those trained with the tools in order to accomplish a particular end. Skills and procedures drive the act of doing mathematics, and these mechanisms exist independently of other mechanisms needed to perform other calculations with different objects. Given that mathematics is viewed as a tool to accomplish some particular end, an individual is not seen as a creator of those tools, but rather only a utilizer of those tools. Active participation is implicitly required, but the participation necessitates only following set directions as opposed to creating individual and personal pathways. Halverscheid and Rolka (2006) offer the following as criteria typical of the instrumentalist view:

(1) A disconnected vision of mathematical objects
(2) A static view of the nature of mathematics
(3) The utility of mathematics is the important thing

Mellin-Olsen (1981) points out that an instrumentalist view produces instrumental understanding of mathematics as opposed to relational understanding, "as the former usually is related to the practical use of the knowledge rather than to some deeper structure" (p. 351), in this case the structure of mathematics.

The *problem-solving view* supports the idea that new mathematics is constantly being invented through a dynamic process of inquiry, and that mathematical topics are connected and open to revision. This view heavily emphasizes the importance of doing mathematics as an activity, where problems can be approached and solved in a variety of acceptable ways, and where "patterns are generated and distilled into knowledge" (Koshy, Ernest, & Casey, 2000, p. 9). Supported by mathematical philosophers such as Lakatos (1976) and Polya (1957), this view maintains a mathematics that focuses on the investigation process, the connections among concepts, and the structure of mathematics as opposed to the correctness of answers. Halverscheid and Rolka (2006) offer the following as criteria typical of this view:

(1) Mathematics is a dynamic activity
(2) There is no single correct way to do mathematics
(3) Students participate actively in the creation of mathematics

3.3.2 The second dimension: Components of linguistics analysis

The second dimension of the framework is comprised of the linguistic components of *actors*, *processes*, and *modality*. The analytic framework suggests the ways in which interactions between these components can be related with the three views about the nature of mathematics discussed in the previous section. The proposed intersections are indicated by the 15 diamonds in Figure 3.1. The diamonds are numbered so that I may refer to specific parts of the framework in my discussions. For example, as indicated by diamonds 4 and 5, the Platonist view supports the interaction of *non-human* actors participating in material processes, as well as an absolute modality.

It is important at this point to note that the framework accommodates actor/process interactions that can be consistent with more than one view about mathematics. For instance, we see that diamond 6 tells us that *we* actor/material process interactions in a textbook may suggest an instrumentalist view. However, diamond 10 suggests that this same interaction can also be expected in a textbook promoting the problem-solving view. Although specific interactions can lead to two different interpretations about the particular view of mathematics, the framework proposes a collection of different interactions that relate to each view (i.e., each view is associated with several diamonds). Therefore, all interactions (not just those representing the single largest interaction) found in a particular textbook need to be considered. The analytic framework developed in this chapter can provide a rationale for determining the philosophy (or philosophies) underlying and promoted within a given textbook.

To further elaborate and clarify each component of the framework, I situate the discussions that follow within the context of Halliday's (1985) 'metafunctions' of language. Although Halliday suggests three such metafunctions, only two are utilized here: the ideational and interpersonal. These metafunctions serve as the foundation of the analytic tools used and are further explained below. In Section 3.3.2.1, I unpack Halliday's (1985) ideational function of language and describe the linguistic components of *actors* and *processes*. After, in Section 3.3.2.2 I explain Halliday's interpersonal function of language, which focuses on the interaction between the actors and processes. This section also introduces the third component of the linguistic dimension of the framework, *modality*, and describes its importance. In Section 3.3.3, I consider the ways in which elements of these metafunctions can promote different views about the nature of mathematics as either Platonist, instrumentalist, or problem-solving.

3.3.2.1 The ideational function: A focus on actors and processes

Although treated as separate entities in the analytic framework, it is difficult to effectively discuss *actors* and *processes* as separate components because both address the ideational function of language. The discussion that follows aims to clarify each of these notions as individual aspects of the analysis, and also the ways in which these notions work together.

At the heart of Halliday's (1985) assumptions about language is the conviction that language is functional and that it performs different metafunctions. In basic terms, the 'ideational function' refers to the ways in which language is used to construct human experiences. Halliday (1985) conjectures that the ways in which an individual describes her experiences of the world through the ideational function of language, in writing or otherwise, are shaped primarily by the acts of "doing, happening, feeling, and being" (p.101). He elaborates that "these 'goings-on' are sorted out in the semantic system of the language, and [are] expressed through the grammar of the clause" (Halliday, 1985, p.101). I define *clause* as a group of words that consists of a subject and a predicate.

The *processes* component of the framework describes "all phenomena…anything that can be expressed by a verb; even, whether physical or not, state, or relation" (Halliday, 1985, p. 159). Although Halliday defined six main types of processes, I describe here only the three utilized in my framework: material, mental, and relational processes.

Material processes are those that correspond to the participation of an actor in some kind of activity outside of her own mind in the physical world, and involve some other actor within the situation. Material processes relate to verbs such as "write," "draw," and "calculate," and they require the actor participating in these activities to interact with actors outside of herself. *Mental processes* relate to the inner experience of the participant and involve verbs that represent her perceptions, desires, and emotions such as "think," "observe," and "recall." Encoding the meanings of feeling or thinking, these processes differ from material processes which express concrete, physical processes of doing. Mental processes are internalized processes happening within the self, in contrast to the externalized processes of doing and speaking that happen outside of the self. Finally, *relational processes* are those that identify and associate one experience with other experiences, perhaps in a variety of different ways, using verbs such as "is," "connects," and "relates."

These processes are certainly important in determining the views about the nature of mathematics being promoted in texts. However, a discussion of the linguistic implications of their presence in mathematics textbooks is pointless without also considering both those *actors* involved with and affected by those processes. In textbooks, it is likely that both human and non-human actors are present. The presence of human actors in texts is indicated by the presence of the pronouns *we*, *you,* and *3rd person* participants (such as "one" or "the student"), while *non-human* actors take on the form of mathematical objects (like "the graph" or "the chart"). Analyzing personal pronouns helps to determine how textbook authors identify the actors present within the mathematical activity and how those actors are positioned with respect to that activity.

The relationships forged between the actors and the processes in which they are engaging in the text also point to where the agency lies within the mathematics. In this context, the agency would refer to the actors (human or not) responsible for establishing mathematical content and authority. The presence of pronouns such as *you* and *we* (as well as imperative verbs) indicates the presence of human actors and agency within the conversation created by the textbook, yet these are not the only possibilities. The emphasis often placed on the role of mathematical objects (like functions or graphs) within the discipline introduces the possibility of other (non-human) agency in the discourse. As explained in Morgan (1996), the linguistic term 'nominalization' is used to describe the transformation of processes and actions into objects. For instance, turning the action of multiplying into "multiplication," or changing permute into the object "permutation" demonstrates nominalization. Additionally, "the transformation of process into object removes the grammatical need to specify the actor in the process" (Morgan, 1996, p. 4). Depicting mathematical objects as performing actions thus also serves to obscure human agency.

To summarize this section, I posit that analysis of linguistic features such as the actors present within the written discourse provide a means to understand how mathematics textbooks promote ideas about the nature of mathematics. Also important are the types of processes (material, mental, relational) in which these actors are expected to engage. Indeed, the interactions between these features are just as revealing as the features are independently.

3.3.2.2 The interpersonal function: A focus on the interaction between actors and processes, and of the modality present

The interactions between actors and processes as described in the previous section and the modality of the text (all of which comprise the second dimension of the analytic framework) both speak to the 'interpersonal function' of language. The ideational function refers to the ways in which language is used to construe one's experience of the world (Halliday, 1985), while the interpersonal function is to construe the social relations being expressed between the author of the text and other participants.

Besides addressing the actors and processes independently, the analytic framework also indicates ways in which the two components interact. Analyses of these interactions prevalently rely on three linguistic forms: (a) personal pronouns; (b) imperatives; and (c) modality. Both personal pronouns and imperatives are captured in the two linguistic components of actors and processes, respectively, and so the analytic framework does not explicitly address the interpersonal function. It does, however, capture the interactions between the different actors and processes that correspond to this function and permits rich and revealing discussions in relation to the promoted mathematical views. I provide an illustration to help clarify this matter. For example, the first person pronoun *we* (an actor captured by the framework) indicates the author's "personal involvement with the activity portrayed in the text" (Morgan, 1996, p. 5). Although this provides important information about the textbook, more can be gleaned by analyzing the types of processes in which this actor is participating. From this, one can see that an analysis of the interactions between the actors and the processes (as captured by the diamonds in the framework in Figure 3.1) in which they engage is far richer than one that only looks at the components separately. These interactions speak directly to the interpersonal function. Having established this, I now say more about three linguistic forms of personal pronouns, imperatives, and modality.

In addition to serving as indicators of human actors in the text, personal pronouns have been used by several researchers as a means to explore the relationships associated with the interpersonal function (Fairclough, 1989; Pimm, 1987). As noted earlier, the presence of first person pronouns such as *we* indicate the presence of a human actor, namely the author who is personally involved in the discourse taking place within a text. On the surface, the pronoun *we* suggests that the author is interested in drawing the reader into the activities taking place as an active participant in a collective group. However, research (e.g. Pimm, 1987) has expressed concern over the potential vagueness with regards to whom exactly the *we* refers. Although it is possible that this pronoun is used so as to invite the reader to take an active role in both the development and understanding of knowledge, this is not always the case regarding the usage in mathematical texts. Morgan (1996) suggested that, when reading more academic mathematical writings, it is a common practice to write in the first personal plural form. It is customary to read statements such as "we will show this" and "we have seen that," despite the fact that oftentimes there is only a single author speaking to a passive reader. This use of *we* suggests that "the author is not speaking alone, but with the authority of a community of mathematicians" (Morgan, 1996, p. 4). The creation of the mathematics may not necessarily involve the readers at all, yet the author seems to expect them to take responsibility for understanding the mathematics. The presence of the personal pronoun *you* also reveals how the reader is connected to the mathematical activity taking place. In this situation, the author appears to be speaking directly to the reader, though it could be used in a more general sense, addressing no individual in particular (Rowland, 2000).

Herbel-Eisenmann and Wagner (2005) found two forms in which the pronoun *you* was used most prevalently in texts: (a) '*you* + (a verb)'; and (b) '(an inanimate object) + (a verb) + *you*.' The form '(an inanimate object) + (a verb) + *you*' permits nominalization as described earlier, where mathematical objects are created from processes and perform actions usually reserved for humans. The presence of nominalization not only obscures agency within the ideational function of language, but it also affects the relationships between the author, reader and mathematics. Of course, the inanimate object serving as the actor in Herbel-Eisenmann and Wagner's form (b) may be any *non-human* actors (not just nominalization), and may refer to what Morgan (1998) categorized as 'relational' or 'representational' objects. Such objects refer specifically to mathematical objects such as graphs, tables, functions and matrices that are non-human objects and may serve as actors within a text. Phrases using relational or representational objects, such as "the table shows you," place the authority in the hands of the inanimate table. In these situations, activity is being fueled by the mathematical objects themselves and is occurring without any participation from the reader. The form '*you* + (a verb)' was found to be most common by Herbel-Eisenmann and Wagner (2005), illustrated by examples such as "you find," "you recall," and "you calculate." Utterances like this suggest that the reader is being personally addressed by the authors, yet the authors maintain the authority and control over the ways in which they want the reader to interact with and construct the mathematics.

The student may also be addressed in this same capacity without the explicit presence of *you* by the presence of commanding imperative verbs. Although this could indicate (as when preceded by *you*) the author's "personal involvement with the activity portrayed in the text" (Morgan, 1996, p. 5), Rotman (1988) suggested that it is necessary to make the distinction between 'inclusive' and 'exclusive' verbs in order to describe the manner in which the author/reader relationship is shaped. *Inclusive* imperatives like “explain” and “describe” position the reader as a constructor of knowledge. On the other hand, *exclusive* imperatives such as “find” and “calculate” situate the reader as a follower of orders. As their names suggest, the inclusive imperatives include the readers within a larger mathematical community in which they are contributing members, while exclusive ones suggest that students simply perform the recommended actions independent of that community. I adopt the above descriptions of exclusive and inclusive for the analyses in this chapter.

Indeed, inclusive and exclusive imperatives create a major divergence in the ways in which the students see themselves in relation to the mathematical activity. The use of the actor *we* with an imperative process is often vague and “marks an author’s claim to be a member of the mathematical community which uses such specialist language and hence enables her to speak with an authoritative voice about mathematical subject matter” (Morgan, 1995, p.6). This also positions the reader as an active member of that community. Although this balance of experience is implied within the author-reader relationship of academic writing, this assumption is not true with regards to mathematical texts where students are still attempting to acclimate themselves to the basics of mathematical language and notation. The author’s intention may be to speak to the reader as an equal, but a reader unfamiliar with this convention may take away a very different message with regards to her role in mathematical activity. Instead of interpreting imperative commands like “find,” “list,” and “explain” as mathematical convention, the reader may interpret her role in relation to mathematics as only that of a follower of rules in a procedure-centered discipline.

The majority of the discussion in this section has addressed the linguistic forms of pronouns and imperatives mentioned at its opening and correspond to the linguistic components of actors and processes in the framework. The third linguistic form mentioned is also the final linguistic component of the framework. I define *modality* as that which describes the authority of the text, indicating the degree of likelihood of certain occurrences. Indicators of modality can be found in the "use of modal auxiliary verbs ('must,' 'will,' 'could,' etc.), adverbs ('certainly,' 'possibly'), or adjectives (e.g., 'I am sure that…')" (Morgan, 1996, p. 6). I dichotomize modality as either *contingent* or *absolute*, terms that have been used often in the literature but perhaps differently than the way they are used here. To clarify the present interpretations, *absolute modality* relates to expressions of a high likelihood or certainty, suggesting a picture of mathematics that is absolute and fixed, using words such as "must," "certainly," and "will." When the author uses phrases such as, "It is clear that the sum must be even," the reader is inclined to passively accept the knowledge unquestionably, and to believe that there is no possibility of alternatives. Conversely, I interpret *contingent modality* to be signified by terms such as "could," "may," and "possible," and it permits the possibility of alternatives. This understanding of contingent modality would point towards a view of mathematics that is dynamic and open to possible revision.

My goal for Section 3.3.2 was to show that the tools of systemic functional linguistics (Halliday, 1985) provide a sound basis with which to analyze the ideational and interpersonal functions of language found in mathematics textbooks. The analytic framework proposes the ways in which the linguistic components of actors, processes, and modality, as well as the interactions among them, correspond to different views about the nature of mathematics (i.e., the Platonist, instrumentalist, and problem-solving views). It takes the pieces of language developed by analyzing the ideational and interpersonal functions and puts them together to form a picture of the whole discourse, or in this case, a picture of the nature of mathematics being promoted by the textbook.

3.3.3 The interaction between the two dimensions of the analytic framework

Mathematics from the Platonist stance is introduced in a very matter-of-fact manner, with each new idea logically derived from the former in a linear fashion. This conception portrays mathematics as a static body of knowledge, "bound together by filaments of logic and meaning" (Ernest, 1988, p.10), waiting to be discovered as opposed to being created. A textbook working from the Platonist view would most likely indicate the presence of the reader as an actor by prevalently using the personal pronouns *you* and *we*. Phrases such as "you find" or "we see," would likely control the ways in which the mathematics is to be seen and understood (see diamonds 1 and 2 in the framework). Indeed, these actors would primarily be engaged in mental processes. For example, a Platonist text may use the phrase "you notice." This suggests that mathematics is not created by the reader; rather it is depicted as an entity that exists independent of the reader. A Platonist view would also be suggested by the frequent use of nominalization within the text, in which mathematical objects themselves are positioned as (non-human) actors in control of understanding and the human actors are positioned as passive recipients of the knowledge. The *non-human* actor may be connected with other objects in a rather abstract way (see diamond 4), independent of any human activity or influence. Morgan (1996) suggested that having a large portion of "mental processes (e.g. seeing, thinking) may suggest that mathematics is a pre-existing entity that is discovered" (p. 4), which is in alignment with the Platonist stance. Given the stance that mathematics is pre-existing and independent of the reader, this view also naturally promotes an increased use of exclusive imperatives within the text, as well as the use of absolute modality (see diamond 5).

Textbooks working from an instrumentalist view of mathematics are likely see the discipline as a collection of "unrelated but utilitarian" (Ernest, 1988, p. 10) facts and procedures used by those trained with the tools in order to accomplish a particular purpose. The emphasis on procedures necessitates the presence of human actors (using pronouns *we* and *you*, as well as *third-person* actors) to carry out those procedures, yet the actors have a very restricted role in the creation of the knowledge being developed. The actors' actions would primarily be stressing the importance of systematic, rote exercises in order to promote precision and mastery of tools. Of the three views, the instrumentalist view is most likely to use the pronoun *you* with expressions of the form '*you* + (a verb)' [as identified by Herbel-Eisenmann and Wagner (2005)], such as in "you calculate" or "you find." Moreover, one would expect these expressions to be accompanied by temporal ("first," "next," "in the end") sequences that consequently restrict the reader's activity rather than suggesting contingency in modality or encouraging independent exploration. Such a focus on systematic procedures indicates the presence of a large proportion of material processes (see diamonds 6, 7, and 8) which "may be interpreted as suggesting a mathematics that is constructed by doing" (Morgan, 1996, p. 3), in which procedures take precedence over conjectures. The reader does have an active relationship to the mathematics; however, the processes in which the reader engages are purely material, constructing a mathematics which is about practical activity that is carried out in a procedural way. Since the text implies that the procedures are expected to be carried out in a particular way, it would promote absolute modality (see diamond 9).

Those embracing the problem-solving view are described as seeing mathematics as something that is constantly being invented through a process of inquiry and is open to revision. Given the emphasis placed on the reader's role as an active participant in the creation of knowledge, a textbook promoting this view would not only suggest the presence of human actors through the use of personal pronouns *we* and *you*, but also that the reader be actively included as a decision maker within those activities. As with the other two views, the processes in which the actors are involved are powerful indicators of the nature of mathematics being promoted. "A high proportion of material processes may be interpreted as suggesting a mathematics that is constructed by doing" (Morgan, 1996, p. 4), which is the view of mathematics promoted within the problem-solving view (see diamonds 10 and 11). Unlike the more absolutist conceptions, knowledge is not only created actively but also is open to possible revisions, with certain concepts having different yet mathematically equivalent descriptions and uses. If imperatives are used, it is likely that the commands are inclusive, welcoming the reader into the community as an active participant given the chance to construct her own knowledge instead of simply being presented with infallible and static knowledge. Additionally, engaging in a mathematics based on problem-solving would most likely allow the reader to participate in relational processes which make connections between concepts (see diamonds 12 and 13). This view also allows for the possibility of *non-human* actors, namely in the form of mathematical objects, to participate in relational processes that connect them to other objects (see diamond 14). This could occur with phrases such as "the table represents the ordered pairs in the graph." The presence of verbs, adverbs, and adjective phrases such as "could," "possibly," and "it is clear" point towards a contingent modality (see diamond 15), suggesting that mathematics is full of different possibilities and positions the reader to see a mathematics that is shaped by her own choices and actions, not just by the textbook author and the mathematics itself.

3.3.4 Limitations of the analytic framework

As with any analysis of textbooks in general, it is important to realize that textbooks form only one component of the curriculum, and it is impossible to make claims about how the material analyzed using this framework will be enacted in the mathematics content courses. That being said, I focus now on limitations of the framework itself.

First, although the two dimensions of the framework have clearly been described here and are grounded in the literature, the particular ways in which those dimensions interact (i.e., the ways in which particular interactions between the linguistic components correspond to the three views of mathematics) are a result of my own understandings and interpretations of that literature. I made decisions about where to place the diamonds within the framework as a result of comparing descriptions found in multiple sources relating to the different mathematical views and after having discussions with individuals more knowledgeable about systemic functional grammar. These are certainly not the only possible interpretations, and some may disagree with them. However, I have attempted to make transparent the reasons for these decisions, and it is crucial to recognize that the data and results in this chapter are valid only within my assumptions.

3.4 METHOD

The first step in my work has been the development of the analytic framework for describing the interaction between the tools of functional linguistics and different views about the nature of mathematics. This framework is an essential starting point for the analysis of the mathematical views being promoted by textbook authors. In this chapter, the framework is implemented through the analysis of three textbooks used in mathematics content courses for PSETs. Below I describe the rationale for the decision to analyze the language found in textbooks, the chosen textbooks, and the content focus of this analysis. Afterwards, I discuss the ways in which the textbooks were coded, as well as the process used to validate the coding.

3.4.1 Why is language important?

Given its highly organized symbolic structure, mathematics as a discipline is often described as a universal language, a means of communication impervious to cultural and physical divides. The significance of the relationship between language and mathematics was highlighted by the NCTM's (2000) 'Process Standard' on communication, which recommends that all students "use the language of mathematics to express mathematical ideas precisely" (p. 60).

Socio-cultural theory, with its contention that cognitive and learning processes are products of an individual's culture and the social interactions within it (Lerman, 2001), provides a meaningful framework for analyses of discourse as a mediating tool in mathematics education. Moreover, it aligns nicely with the constructivist commitment to "the idea that the development of understanding requires active engagement on the part of the learner" (Jenkins, 2000, p. 601). The language used in textbooks implicitly and explicitly promotes particular dispositions, understandings, values, and beliefs (Ochs, 1990). By analyzing the language choices authors make, one can make inferences about how the textbooks may promote different views about the nature of mathematics, which in turn have the potential to influence the reader's experience with that mathematics. In general, the language used in mathematics textbooks should attempt to "represent mathematics as an ongoing human activity" (NCTM, 1991, p. 25), engaging students in mathematical activity that reflects the vision of reform.

3.4.2 Textbook selection

Textbooks written specifically for PSETs are of primary interest in this chapter, given the focus on this particular population and component of TEPs within the dissertation. In addition to being written specifically for PETs, the textbooks chosen for analysis have been investigated and discussed in previous research and were found to possess particular qualities that make them desirable for the present work. Specifically, the textbooks included were chosen based on the work done by McCrory et al. (2007), which explored and compared the content of 14 textbooks used in mathematics content courses for PSETs (and was discussed earlier in Section 2.2).

After analyzing all the textbooks written for mathematics courses for PSETs, the authors concluded that the variation found across the different textbooks could result in rather different opportunities for PSETs to learn mathematics. It is important to acknowledge here that it is sometimes the case that instructors of these courses do not use a published textbook, but instead develop their own course materials or collect materials from different sources. An example of such a course provides the context for the work discussed in Chapter 4.

One of many dimensions discussed in McCrory et al.'s (2007) textbook study was the 'mathematical stance' of a textbook, which was described as "addressing the conception of mathematics that the book presents: What is important? What is the nature of mathematics? How does mathematics work as a discipline?" (p. 13). Revealingly, this dimension proved to be the most variable in their analyses. The authors classified mathematical stance, which accounted for things such as the role of definitions in mathematics, as 'explicit,' 'implicit,' or 'absent' from the texts. Using definitions as an example to illustrate these distinctions, McCrory et al. would categorize a book as having an explicit stance if definitions were used consistently throughout the book, and if the book discussed their use within the discipline of mathematics. If definitions were used consistently but their role in the discipline was unspoken, then the text would be considered to have an implicit mathematical stance. If definitions were not used or discussed consistently in either of these manners, the stance would be classified as absent. Given that one of my goals was to analyze the nature of mathematics being presented in definitions in textbooks, I concentrated my exploration on three of the four textbooks described as possessing explicit mathematical stance by McCrory et al. Indeed, this was a strategic choice since these explicit textbooks were already discovered to use definitions consistently throughout. Therefore, these textbooks provided the best opportunities to implement and test the applicability of the analytic framework I developed to investigate the various views about the nature of mathematics being promoted by the textbooks used by PSETs.

In addition to its relevance to the research questions in this chapter, the analysis of the chosen textbooks is also intended to contribute to the work done by McCrory et al. (2007). Although the three selected textbooks were found to be consistently explicit in describing the general function and importance of mathematical ideas, the analysis by McCrory and her colleagues was not able to describe precisely what the explicit textbooks authors' stance was regarding the general function and importance of mathematical ideas. The function and importance of ideas suggested by the authors serve as indicators of the view about the nature of mathematics being promoted, and it is my hope that the work of this chapter will contribute to a better understanding of these indicators by describing more precisely the mathematical views being promoted by the textbook authors. Written by Darken (2003), Parker and Baldridge (2004), and Wu (in preparation), the three textbooks will be referred to only by their authors' names from this point forward. I could not obtain a copy of the fourth book classified as explicit by McCrory et al. (2007) in time to include it in this analysis.

3.4.3 The focus on definitions and tasks in number theory

The choice to focus my analysis on a sample of definitions and tasks within the selected textbooks was not arbitrary. In addition to playing an important role in mathematical activity, the presentation of definitions in textbooks can capture how the textbook promotes different mathematical views. One textbook, for example, may simply provide the reader with a single definition and present it as the only definition possible, while another may engage the reader in the creation of a definition that is open to modification. These two textbooks would be promoting completely different views about the nature of mathematics, the first Platonist and the second problem-solving.

Researchers have given much attention to the role and structure of tasks in the mathematics curriculum over the past decade. Lesh and Kelly (1994) make the claim that tasks that focus on eliciting mathematical reasoning and promoting problem-solving are powerful tools of mathematics instruction, potentially improving students' understanding by enabling them to construct meaning and make connections between ideas. In regards to what makes a mathematical task worthwhile, Smith and Stein (1998) have heavily contributed to the literature by creating the 'Mathematical Tasks Framework,' which describes the cognitive demands of tasks and their relation to the potential learning opportunities of students. Smith and Stein's work, and all that has come after it, substantiates the need to better understand the nature of mathematical tasks with which students engage that may influence their experiences with mathematics. In particular, in this chapter I attempt to analyze tasks in order to determine the ways in which they position students to view the nature of mathematics. For instance, one text may consistently ask the reader to solve a task in a specific way, perhaps even referring to a step-by-step procedure used in a previous example, while another may pose a task that suggests no solution paths and may be solved in several different ways. These two textbooks would be promoting completely different views about the nature of mathematics, the first instrumentalist and the second problem-solving.

The specific content area under investigation is elementary number theory. Number theory is a customary topic covered in most mathematics courses for PSETs and one of the oldest and broadest branches of mathematics. Defined as the study of the natural numbers, number theory comprises much of the foundations of elementary school mathematics, "enriching one's understanding of multiplication and division" (Brown, 1999, p. 2) and aiding in the conceptual understanding of arithmetic in general. However, "many issues related to the structure of natural numbers and the relationships among numbers are not well grasped by learners" (Zazkis & Liljedahl, 2003, p. 3), which can potentially influence the teaching of these ideas when pre-service teachers enter their own classrooms. Often, the concepts of number theory have served as a means to investigate other topics relating to pre-service teachers. Martin and Harel (1989) used the concept of divisibility in their research on pre-service teachers' understanding of mathematical proof, while Lester and Mau's (1993) research employed prime factors to study their ideas about problem-solving. Although important and relevant to these and other topics, this study investigates textbooks' treatment of number theory as a topic that has great potential to promote a textbook's view about the nature of mathematics. Sections devoted to number theory are not only present in the selected textbooks, but are also very rich in definitions and tasks, enhancing the depth of my analysis.

One of the three textbooks analyzed (Parker and Baldridge) did not contain a chapter explicitly titled "Number Theory," and so the final decision about which chapter to investigate was based on the fact that it covered the following topics: factors, divisors, divisibility, even and odd numbers, prime numbers, least common multiple (LCM), and greatest common divisor or factor (GCD or GCF). At my request, Wu supplied an electronic copy of his number theory chapter for my analysis, as the complete textbook is still being revised for publication. Darken contained a chapter explicitly devoted to number theory. Every definition and task found in each of the chapters was coded for the analysis.

3.4.4 Coding

In this section, I discuss the coding and validation process used in the analysis. I first describe the ways in which both definitions and tasks were coded in my final analysis, providing examples as illustrations. After, I describe the processes followed before finalizing this coding method and the measures taken to ensure its validity.

3.4.4.1 Final coding

Each definition and task was coded for the three linguistic components that make up the second dimension of the analytic framework: (1) actors; (2) processes; and (3) modality. Although a primary goal of this analysis was to interpret the ways in which choices made relating to these linguistic components can promote the three views about the nature of mathematics that make up the first dimension of the framework, the definitions and tasks were not coded to directly capture the three views. Only after the actors, process, and modality were identified through the coding process did I attempt to make interpretations about how the code frequencies of the particular actor/process interactions and modality indicators may suggest these different views as indicated by the diamonds found in the framework in Figure 3.1. Recall that these diamonds depict the particular interactions between actors and processes and the indicated modality that I propose would be prominently found in each of the three views. These interactions and modality were directly captured by the coding and form the foundation of my analyses.

I begin by first defining the ways in which I use the terms *definition* and *task* in this chapter, and describe how I decided which portions of the text would be analyzed relating to each term. The codes employed in the final analysis are also outlined. After, I present two sample excerpts from the textbooks analyzed, with Figure 3.2 relating to a definition and Figure 3.3 to a task. I present these examples for a couple of reasons. First, they serve as illustrations for the ways in which I have defined the two terms, and second, they help demonstrate the ways in which all definitions and tasks analyzed were coded for the actors, processes, and modality.

For my final analysis of *definitions*, I made the decision to investigate the creation, statement, and related discussion of the definitions emphasized by the textbook authors. Oftentimes, these sections were comprised of several sentences. I determined these emphasized terms to be those obviously intended to be distinguished from others in the text, often by the consistent use of boxes labeled “Definition” or by bolding of the term within the text. For terms that were bolded, I further differentiated those terms to be included as a definition in my analysis to be those accompanied by a clear description of a newly-introduced term. I excluded bolded terms that were simply meant to remind the reader of a previously discussed term as well as those that were only mentioned in passing and not explicitly discussed further. Introductory text and exploration activities leading to the statement of the definition were also coded as part of the definition, as was any text immediately following the statement of the definition that related to the definition (e.g., alternative definitions, clarifications, etc.). I considered this material as important as the definition itself since it provided further insight into the ways in which the author engaged the reader with the mathematical ideas being introduced, and provided a greater wealth of actors and processes to analyze. Text that was not coded included any theorems, lemmas, or specific examples relating to the term being defined, which could have been embedded between the analyzed portions of the text. Figure 3.2 below illustrates what was considered to be a definition in the final analysis.

Definition of a Factor
Let A and B be whole numbers with $A \neq 0$. We say A is a factor of B written $A \mid B$, if and only if there is a whole number m such that $m \cdot A = B$.
The notation "A|B" can be read as "A divides B" or "A is a factor of B" or "A is a divisor of B." These synonyms result from the fact that division is the inverse operation of multiplication. Although the focus in this chapter is on multiplicative relationships, it is useful at times that you notice the connection between the factors of a number and the basic meaning of multiplication as repeated addition.

Figure 3.2: Sample Definition from Darken (2003, p. 477)

Figure 3.3 illustrates what was considered to be a *task* in the final analysis. Although some of the textbooks posed questions to be answered by the student within the discussion portions of the textbooks, not all did. For consistency, my analysis focuses only on the tasks posed at the end of each section within the selected chapters. Moreover, I counted as one task all parts that were associated with the initial marker indicating the task, which was primarily a number. For example, if a task was numbered as "1" and had two parts labeled (a) and (b), I would code all parts as relating to a single task, as would be the case in the example below. Again, there could be several sentences related to a single task. For each section, I coded only the tasks that related specifically to the concepts introduced and developed in that particular section, and did not include tasks that were distinguished as review of topics from previous sections or chapters.

Both definitions and tasks were coded in a similar manner. After identifying either a definition or a task, each sentence in the analyzed portions was coded to capture the actors present, the type of processes used, and explicit indicators of modality. It was common to find that there was more than one actor and one process present within a single sentence of the text, and each instance received its own actor and process code. This is illustrated further below.

1. Read 19-26 of Primary Math 4A. Notice how the ideas of factors and multiples are introduced, and how common multiples are defined on page 26. a) Use the method shown by the little girl in Problem 11 of page 26 and find a common multiple of 15 and 12. b) In Practice 1B of Primary Math 4A, do problems 1 and 4-7.

Figure 3.3: Sample Task from Parker and Baldridge (2004, p. 130)

As shown in Table 3.1, actors were coded according to personal pronouns used (*you*, *we*), and *third-person* participants (e.g., the student, someone) and *non-human* actors (e.g., mathematical objects) were coded accordingly. Each actor found in the text was engaged in some sort of process, with each of these processes falling into one of the categories described earlier in the chapter and outlined in Table 3.1. Therefore, each interaction was assigned two codes: (1) one code describing the actor (W, Y, T, or N, relating to *we*, *you*, *third-person*, and *non-human* actors, respectively), and (2) one code describing the process (M, N, or R, relating to a material, mental, or relational process, respectively). I hereafter refer to this two-code combination as a *double-code*.

Table 3.1: Final Codes Used in Analysis

LINGUISTIC COMPONENT	POSSIBILITIES	RELATED CODE/INDICATOR
(1) **Actors**	We	**W**
	You	**Y**
	Third-person	**T**
	Non-human	**N**
(2) **Process**	Material	**M**
	Mental	**N**
	Relational	**R**
(3)**Modality** (count indicators)	Absolute	always, must, will (when used in first and third person), has to (be), certain(ly), sure(ly)
	Contingent	may, can, might, sometimes, could, possible/possibly, maybe

Modality was coded in a different way than actors and processes. A count of absolute and contingent indicators found within the analyzed portions of the text was recorded and served as the foundation for the analysis related to modality. The exhaustive list of indicators I used is included in Table 3.1.

Definition of a Factor
Let A and B be whole numbers with $A \neq 0$**[CODED: N-R]**. We say **[CODED: W-M]** A is a factor of B **[CODED: N-R]**, written $A \mid B$, if and only if there is a whole number m such that $m \cdot A = B$.
The notation "A|B" can be read **[CODED: N-M]** as "A divides B" **[CODED: N-M]** or "A is a factor of B"**[CODED: N-M]**, or "A is a divisor of B"**[CODED: N-M].** These synonyms result from the fact **[CODED: N-R]** that division is the inverse operation of multiplication. **[CODED: N-R].** Although the focus in this chapter is on multiplicative relationships **[CODED: N-R]**, it is useful at times that you notice **[CODED: Y-N]** the connection between the factors of a number and the basic meaning of multiplication as repeated addition.

Figure 3.4: Sample Coding of a Definition from Darken (2003, p. 477)

Figure 3.4 provides an illustration of how I identified a definition in the text and the double-codes corresponding to the definition. In addition to the clearly marked box (or a bolded term, as was the case in some of the textbooks) alerting me to the fact that a definition was about to be addressed, my analyses included the portion of the text that immediately followed this definition, as it directly relates to ideas meant to further elaborate the defined concept. Examples embedded within the text were not coded as being part of a definition or related discussion. Figure 3.4 demonstrates that a single definition could be comprised of several sentences, each potentially containing several actor/process interactions. Therefore, it was common to have several double-codes associated with a single definition.

> 1. Read **[CODED: Y-M]** pages 19-26 of Primary Math 4A. Notice **[CODED: Y-N]** how the ideas of factors and multiples are introduced, and how common multiples are defined on page 26.
> a) Use **[CODED: Y-M]** the method shown by the little girl in Problem 11 of page 26 and find **[CODED: Y-M]** a common multiple of 15 and 12.
> b) In Practice 1B of Primary Math 4A, do **[CODED: Y-M]** problems 1 and 4-7.

Figure 3.5: Sample Coding of Task from Parker and Baldridge (2004, p. 130)

Figure 3.3 shown earlier illustrates what was counted as a single task in the analysis, which again could include several sentences that resulted in numerous double-codes. Figure 3.5 shows how a task was coded, utilizing the same manner as the coding of definitions.

Now that I presented the method of coding used in the final analyses, I describe in the three sections below the process that led to this method. This includes the three stages of the process, the changes made to the coding method that resulted from each stage, and the measures taken to ensure the validity of the coding. The following section describes the process chronologically.

3.4.4.2 Description of the three initial stages of coding

Stage 1

I initially decided that the coding of definitions and tasks in the textbooks should address and capture the three components of the linguistic dimension: (1) the actors present; (2) the processes in which they were engaged; and (3) the modality. For each of these components, I generated a code list as shown in Table 3.2 (with differences between it and the final code list shown in Table 3.1 highlighted).

Table 3.2: Code List Compiled in Stage 1

LINGUISTIC COMPONENT	POSSIBILITIES	RELATED CODE
(1) **Actors**	We	**W**
	You	**Y**
	Third-person	**T**
	Non-human	**N**
(2) **Process**	Material	**M**
	Mental	**N**
	Relational	**R**
(3)**Modality** (choose 1)	Absolute	**A**
	Contingent	**C**

Before the codes could be tested, I had to determine which portions of the text would be considered for analysis of the definitions and tasks. This was outlined in the previous section. Once these decisions were made, I selected a sample of definitions from the number theory chapter in each of the three textbooks, namely the definitions for factors, divisors, even and odd numbers, prime numbers, LCM, and GCD, and also a sample of tasks from the chapter. I based the selection of definitions on the fact that all of the analyzed texts defined these terms in their number theory chapters.

In this preliminary work, each sentence in the analyzed sections was coded according to the actors present, the type of process used, and the indicated modality. When there were more than one actor and one process interacting within a single sentence, each instance received its own actor and process code, as was described in my final coding section above. A major difference between Stage 1 and the final coding technique related to the coding of modality.

As indicated in Table 3.2, actors were coded according to the possible actors present (W, Y, T, N). The coding of actors was straightforward (as there was a clear presence of personal pronouns, *third-person* actors such as "one" or "the student," or *non-human* actors such as "the graph" or "the number"). Since there was greater variety in the processes found in the textbooks, I generated a list of all verbs used within the sections of interest during the Stage 1 analysis and classified the verbs as representing either a mental, material, or relational process and coded them correspondingly. At this time, I shared this process list and process descriptions with two other mathematics education doctoral students and asked them to independently classify and code only these processes. Afterwards, we discussed the processes and associated classifications until all three parties reached agreement, and these codes were used in this first stage of analysis.

Each actor/process interaction was also given a code relating to the modality of that particular interaction. Texts which indicated a high degree of certainty through words such as "must," "necessary," and "always" were coded as representing absolute modality, and those implying room for modification by using words like "may," "could," or "might" were coded as contingent. If these words were absent, I used my judgment to determine either an implied absolute or contingent modality, based on other linguistic elements present. Therefore, this first stage of coding produced not double-codes as described in the final analysis, but triplet-codes, with the first code relating to the actor present, the second to the process in which that actor was engaged, and the third to the indicated modality. Figure 3.6 below shows a comparison of Stage 1 coding (top part of the example) and final coding (bottom part of the example) of a portion of the definition of factor discussed earlier. Whereas this portion of the definition resulted in 6 double-codes in my final analysis, the Stage 1 work found 6 triplet-codes related to the definition. As this demonstrates, the number of codes remained the same in the move from triplet-codes to double-codes.

(STAGE 1 CODING) The notation "A|B" can be read **[CODED: N-M-C]** as "A divides B" **[CODED: N-M-A]**or "A is a factor of B"**[CODED: N-M-A]**, or "A is a divisor of B"**[CODED: N-M-A].** These synonyms result from the fact **[CODED: N-R-A]** that division is the inverse operation of multiplication. **[CODED: N-R-A].**
(FINAL CODING) The notation "A|B" can be read **[CODED: N-M]** as "A divides B" **[CODED: N-M]** or "A is a factor of B"**[CODED: N-M]**, or "A is a divisor of B"**[CODED: N-M].** These synonyms result from the fact **[CODED: N-R]** that division is the inverse operation of multiplication. **[CODED: N-R].**

Figure 3.6: Comparison of Stage 1 and Final Definition Coding from Darken (2003, p. 477)

The presence of explicit modality indicators such as "can" as in the first code in this sample ([CODED: N-M-C]) leads clearly to the third code of C, yet these indicators are not clearly established for every actor process interaction. Thus, I was forced to use my judgment and understandings grounded in the modality literature to assign modality to each of the other interactions. For the preliminary work in Stage 1, I analyzed the sample of definitions from the number theory chapter delineated earlier and a sample of tasks from that chapter, using triplet-codes. After the texts were coded, I constructed a table indicating the frequencies of each triplet-code so as to allow observations both within and across the textbooks based on the sample of definitions and tasks.

Stage 2

After the relatively informal analysis of a small sample of definitions and tasks in Stage 1 revealed some interesting findings, it was time to gain some insight into the usability of the coding schema and the validity of the coding process. I asked a mathematics education doctoral student unrelated to the study, yet casually familiar with it, to code a sample of the previously analyzed text (five definitions and five tasks from each textbook, 15 of each total). To introduce the second coder to the framework, I produced a detailed document which explained its two dimensions. Since the coding only dealt with the linguistics dimension of the framework and not the different views of mathematics, this document described briefly the three views utilized in the framework. The three linguistics components were more fully explained, and examples were offered to further clarify each. I also provided a blank excel document similar to the one I used to record the coding of my preliminary analysis from Stage 1. I did this to save time, but also to remind him (through the labeling set up in the spreadsheet) to associate with each analyzed portion of the text a triplet-code, i.e. an actor, a process, and the modality.

After the second coder had the opportunity to read through the prepared document and to code the 15 definitions and 15 tasks, he shared his spreadsheet with me. The goal was to gain inter-rater reliability by establishing high percentages of agreement in four areas: (1) the number of triplet-codes related to each definition; (2) the particular actors coded; (3) the particular processes coded; and (4) the modality coded. Before a complete comparison between actor, process, and modality codes took place, I initially noted that there was a distinct discrepancy with regards to the number of triplet-codes relating to each definition. My coding resulted in anywhere between 4 and 20 triplet-codes per definition, while the second coder returned no more than 2 for each. At this point, I engaged in a discussion with him to determine the reason for this difference, which turned out to be a disagreement about what was to be considered the introduction of the definition, as well as what should be included in the coding after the statement of the definition was given. In other words, more clarification was needed as to how to determine the portion of the text to be coded. In attempts to remedy this, the second coder and I sat down with a small sample of definitions, ones that were not included in this round of coding, in order to talk through what we believed constituted the selection that I had intended to include. Once we agreed on the parameters of making this choice (having pointed out the same portions of texts believed to comprise the introduction to, statement of, and discussion relating directly to the definition for a large portion of the sample), I gave the doctoral student one new definition and asked him to determine the text to be coded for my analysis. When his partitioning coincided with my own, we needed to discuss another issue that arose during the initial code comparison before I asked him to recode the original 15 definitions given to him.

The second coder reported few difficulties in locating the actors and processes interacting within the text, yet found it more difficult to establish the modality relating to each of these interactions. To initiate a more comprehensive discussion about this, we focused on a portion of the text related to one of the definitions coded. Although there was a definite presence of modality indicators such as “may,” “must,” and “could,” many of the actor/process interactions did not strongly suggest the modality in either the absolute or contingent direction. If one attempted to make an argument for an indicated absolute modality, the other could counter argue reasoning for judging it as contingent. At this time, I felt that requiring triplet-codes would greatly and negatively impact the validity of the coding given the possible variations in interpretations. Therefore, I enacted a major modification in the coding system. Instead of triplet-codes, the actor/process interactions would continue to be coded as originally outlined to elicit double-codes, meaning that each interaction would only be given two codes (the first for the actor, and the second for the process). As detailed in Section 3.4.4.1, these double-codes were adopted for the final analyses.

Since I believed that it was still important to somehow examine the modality promoted in the textbooks, I had to collect the related data in a way that was more objective. I decided to record the frequency of absolute and contingent indicators found within the analyzed portions of the textbooks. The two coders created a list of such indicators that would serve as the basis for this record, and independently analyzed a portion of text, recording the frequencies of these modality indicators and finding related percentages based on these frequencies. This more objective means of addressing the modality resulted in full agreement and was adopted for the remainder of my work. The exhaustive list of modality indicators created during Stage 2 can be found in Table 3.1.

Finally, although I had not taken any formal measures at this time in Stage 2 to check the agreement between our particular actor and process codes, I had noticed upon my initial examination that many of the processes I had coded as being material were given a relational code by the second coder. I wrote down three particular processes that fell into this category and asked him why he specifically considered each of these processes to be relational. I learned that he considered processes like “prove” and “find” to be relational because they often would produce a result relating to the information given. At this time, I tried to clarify that even though this was true, the actual processes themselves necessitated that the actor actually perform some kind of work outside of themselves to accomplish that result, and therefore should be coded as material. It is important to note that these discussions did not involve particular definitions that were being coded, rather just particular processes in question so as not to influence the reliability process. To further clarify, I revisited the coding document I prepared for him prior to his initial coding to review the examples provided and to include a few more specifically related to mathematics (as opposed to general linguistics examples that I had initially included that did not necessarily deal with mathematics). This helped tremendously and at the end of this conversation we were in complete agreement about these discrepancies. These clarifications are also reflected in the description of the framework presented in Section 3.3. I now felt confident in asking the second coder to code again the original 15 definitions and 15 tasks.

Stage 3

At this point, I was able to do a full comparison of the two sets of codes. Although this analysis revealed high agreement between my double-codes and those of the second coder, there were still some differences related to the number of double-codes associated with each definition. A new conversation clarified the reason for the discrepancy. To illustrate this issue, I offer a sample piece of text: "We ask that *you read the selection below, consider each pair of numbers, and find all the prime factors*." One can see that the italicized text relates to a single actor, *you*, and the processes in which that actor is engaged. For this, the second coder made the decision to code only the first actor/process interaction (you read), while my coding reflected three distinct actor/process interactions (you read, you consider, you find). To avoid future confusion, this situation was explicitly addressed within my coding document, with this example included to further clarify the ways in which the coder should code the different interactions. The final coding document can be found in Appendix A.

At this time, I performed an analysis of inter-rater reliability. Since I was interested in the level of agreement between the actors and processes we both coded, I considered only those instances in which we both provided a code. That is to say, for instances like those described in the previous paragraph, I would have three codes related to the second actor (you) in the sentence "We ask that you read [CODE: YM] the selection below, consider [CODE: YN] each pair of numbers, and find [CODE: YM] all the prime factors." In instances like this, the second coder had assigned a code to only the first interaction, "you read." Since I was made aware of this fact, I eliminated from my analysis of code agreements those codes where the actor was assumed to be carried to subsequent processes after the first. After this, the number of codes associated with the common definitions and tasks for the two coders were equal, and this data was used to calculate the inter-rater agreement relating to the actors and the processes coded.

This analysis revealed nearly unanimous agreement. To investigate the reliability in both the definitions and tasks, I divided the total number of agreements with the sum of the total agreements and disagreements. Of the 186 common double-codes relating to the 15 definitions (five from each textbook) analyzed, the inter-rater agreement relating to the actors present was 90%, while the inter-rater agreement relating to the processes coded was 93%. For the 15 tasks (five from each textbook) analyzed, there were far fewer codes in comparison to those relating to the definitions, and so I asked the second coder to code more tasks in order to establish reliability. In total, he coded 20% of all tasks analyzed, and this resulted in 43 common codes. The inter-rater agreement was complete relating to the actors present, and a mere 4 disagreements in processes resulting in 91% agreement. These numbers convinced me that the coding method was reliable, and I proceeded with the analysis of the whole sample.

3.5 RESULTS

In this section, I present the findings of the analysis using the analytic framework found in Figure 3.1. In the first part of this section, I present the results for the analysis of definitions and in the second the results for the analysis of tasks.

3.5.1 Definitions

Figure 3.7 summarizes the overall results of the analysis of definitions. The data corresponding to the interactions between actors and processes for each textbook can be found in the three-by-four numerical array below each textbook author. Each of the numbers in this array represents a percentage of the total number of codes. Immediately below this array are two rows that relate to the modality indicators for each textbook, where the top value represents the percentage of absolute modality indicators and the bottom the percentage of contingent modality indicators.

Each of the four rows in the three-by-four array under each textbook author corresponds to one process. These rows include four numbers (one for each possible actor), and by summing these four numbers you find the percentage of codes that show participation in each processes out of the total number of codes relating to all actors. For instance, under Parker and Baldridge in Figure 3.7, the sum of the entries in the first row (corresponding to material processes) mean that 62% (14 + 12 + 16 + 20) of all the codes showed all types of actors engaging in material processes. Each of the three rows of the array corresponds to a different process.

Each of the four columns in the three-by-four array under each textbook author corresponds to one actor. These columns include three numbers (one for each possible process), and by summing these three numbers you find the percentage of codes that show each actor engaging in all three types of processes. For instance, under Parker and Baldridge in Figure 3.7, the sum of the entries in the first column (corresponding to the *we* actor) means that 17% (14 + 3 + 0) of all codes showed the *we* actor engaging in all types of processes. Each of the four columns in the array corresponds to a different actor.

			Textbook											
			Darken				***Parker and Baldridge***				***Wu***			
Components of Linguistic Analysis		**Actors**	We	You	3^{rd} person	Non-human	We	You	3^{rd} person	Non-human	We	You	3^{rd} person	Non-human
	Processes	***Material***	13	21	8	11	14	12	16	20	33	2	4	9
		Mental	5	5	2	0	3	6	6	1	15	12	2	1
		Relational	0	5	0	30	0	0	2	20	1	1	0	20
	Modality	***Absolute***	20				21				50			
		Contingent	80				79				50			

Figure 3.7: Overall Results Relating to Definitions in the Three Textbooks

In this array you can also find the percentages of codes relating to the various interactions between the different actors and processes by finding where the process row and actor column intersect. For instance, in order to determine what percentage of codes related to *we* actors participating in mental processes, you find where the *we* actor column (the first column) intersects with the mental process row (the second row). Figure 3.7 shows that in Parker and Baldridge, 3% of the codes related to this particular type of interaction. The entries of the three-by-four array corresponding to each textbook sum to 100 to account for all codes in the analysis.

The results related to the modality can be found in the two rows immediately below the array. The sum of the two numbers in these rows also sum to 100 to account for all modality indicators.

Now that the reader knows how to read the results in the figures, I zoom in to discuss the results of each individual textbook, which can be found in Figures 3.8, 3.9, and 3.10 below. These figures show the results relating to the definitions in the number theory chapters of Parker and Baldridge, Wu, and Darken, respectively. The results related to the analyzed textbook can be found underneath the "Textbook" header in the figure, which also indicates the author. I made the decision to only consider the four highest percentages found in the actor/process interactions and the highest modality percentage (identified by the use of darker shading in the figures). These values form the foundation of my interpretations of the results and the related discussions. They enabled me to make connections between my findings and the different views about the nature of mathematics as described by the analytic framework in Figure 3.1.

The shading is used so that the reader may better see the ways in which the four highest percentages correspond to the appropriate diamonds of the related views from the framework which are presented alongside them. The two views identified as relating the most to the textbook are shown in their original place in the analytic framework as in Figure 3.1, with the textbook's data found in the place of the third (least-related) view. This was done to maintain consistency in the presentation of the framework and its relation to the results of this analysis. For example, the interactions with the highest percentages found in Parker and Baldridge as seen in Figure 3.8 led me to include the instrumentalist and problem-solving portions of the framework in relation to the results. These two views are presented in the portion of the analytic framework where the Platonist view (which is the least related to Parker and Baldridge) is described in Figure 3.1. This decision was made since the positions of the highest percentages of interactions related to diamonds 6, 7, and 8 under the instrumentalist view, and also diamonds 10, 11, and 14 under the problem-solving view. The high percentage of contingent modality indicators can be related to diamond 15 under the problem-solving view.

			Textbook				**Related views about the nature of mathematics**							
			Parker and Baldridge				***Instrumentalist***				***Problem-Solving***			
Components of Linguistic Analysis		**Actors**	We	You	3rd person	Non-human	We	You	3rd person	Non-human	We	You	3rd person	Non-human
	Processes	***Material****(outside the self)*	14	12	16	20	6	7	8		10	11		
		Mental*(inside the self)*	3	6	6	1								
		Relational*(denote relationships)*	0	0	2	20					12	13		14
	Modality	***Absolute***	21				9							
		Contingent	79								15			

Figure 3.8: Definitions in Parker and Baldridge and the Promoted Mathematical Views

As indicated by the darker shaded entries in Figure 3.8, the actor/process interactions with the highest percentages in Parker and Baldridge were: (1) *we*/material; (2) *you*/material; (3) *3rd person*/material; (4) *non-human*/relational; and (5) *non-human*/material. Five percentages were highlighted in Figure 3.8 since the fourth-highest percentage (20%) corresponded to two different interactions (4 and 5 in the previous sentence). This textbook had a large percentage of different actors engaged in material processes, which suggests that this text has elements from more than one view of mathematics. Indeed, none of the three textbooks in the sample was found to promote a single view about the nature of mathematics.

Based on the highest interaction percentages, the framework points to a possible alignment of Parker and Baldridge with both the instrumentalist and problem-solving views of mathematics. Interactions between *non-human* actors and material processes (the fifth interaction identified) suggest the Platonist view according to the analytic framework, but the other interactions present do not indicate a strong alignment with this view. The prevalence of the three actors participating in material process as shown in the first row corresponds with diamonds 6, 7, and 8 in the framework's description of the instrumentalist view. However, an interpretation of this text as solely promoting instrumentalist tendencies would not be a totally accurate one given that 20% of the total codes communicate a prominent presence of *non-human* actors employing relational processes. Indeed, such an interaction is absent from the framework's suggested description of an instrumentalist view. Although both the Platonist and problem-solving views would support such interactions, the fact that the Platonist view is completely devoid of human actors engaging with material processes (which are exactly the interactions Parker and Baldridge appear to promote) indicates that the problem-solving view is more appropriate. This is further supported by the high percentage of contingent modality indicators (79% of the total indicators found). All of the results presented in this section regarding the three textbooks are further elaborated upon in the Section 3.6 help to strengthen the interpretations of the textbooks.

The results in Figure 3.9 indicate far less variation regarding the interactions in Wu in comparison to Parker and Baldridge, with the four most prevalent interactions shown as: (1) *we*/material; (2) *we*/mental; (3) *you*/mental; and (4) *non-human*/relational. Based on these four percentages, the framework indicates a possible alignment with both the Platonist and instrumentalist views about the nature of mathematics. The prevalence of mental processes found as highlighted in the second row corresponds with diamonds 1 and 2 in the framework's description of the Platonist view. Indeed, of the three views described by the framework, the Platonist view is the only one in which I anticipated finding a strong presence of mental processes. However, an interpretation of this text as solely promoting a Platonist view would not be totally accurate given that 33% of the total codes suggest a prominent presence of the *we* actor engaging in material processes. As seen in Figure 3.1, this particular interaction is absent from my description of the Platonist view. Although both the instrumentalist and problem-solving

			Related Views about the Nature of Mathematics								**Textbook**			
			Platonist				***Instrumentalist***				***Wu***			
Components of Linguistic Analysis		**Actors**	We	You	3rd person	Non-human	We	You	3rd person	Non-human	We	You	3rd person	Non-human
	Processes	***Material****(outside the self)*				4	6	7	8		33	2	4	9
		Mental*(inside the self)*	1	2							15	12	2	1
		Relational*(denote relationships)*				3					1	1	0	20
	Modality	***Absolute***	5				9				50			
		Contingent									50			

Figure 3.9: Definitions in Wu and the Promoted Mathematical Views

views would support such interactions, the fact that Wu had 48% of the actors engaging in material processes (as compared to only 22% engaging in relational process) makes me to align the textbook more with the instrumentalist view. According to the framework, a large percentage of material processes more strongly supports this particular view. Moreover, more than any other author Wu seems to support an absolute modality (50% of total modality indicators), a finding that provides further support of an instrumentalist view.

As indicated by the darker shaded entries in Figure 3.10, the most prevalent interactions found in Darken were: (1) *we*/material; (2) *you*/material; (3) *non-human*/relational; and (4) *non-human*/material. As with Parker and Baldridge, the textbook had a large percentage of codes showing *non-human* actors engaged in material process. This particular interaction suggests the Platonist view, yet the high percentage of actors engaged in material processes discourages an alignment of this text with that view. Based on the three other percentages, the analytic framework indicates a possible alignment of Darken with both the instrumentalist and problem-

			Textbook				**Related Views about the Nature of Mathematics**							
			Darken				***Instrumentalist***				***Problem-Solving***			
Components of Linguistic Analysis		**Actors**	We	You	3rd person	Non-human	We	You	3rd person	Non-human	We	You	3rd person	Non-human
	Processes	***Material****(outside the self)*	13	21	8	11	6	7	8		10	11		
		Mental*(inside the self)*	5	5	2	0								
		Relational*(denote relationships)*	0	5	0	30					12	13		14
	Modality	***Absolute***	20				9							
		Contingent	80								15			

Figure 3.10: Definitions in Darken and the Promoted Mathematical Views

solving views of mathematics. The prevalence the *we* and *you* actors participating in material process as shown in the first row (totaling 13 + 21 = 34% of total codes) corresponds with diamonds 6 and 7 in the description of the instrumentalist view in the framework. These interactions also correspond to diamonds 10 and 11 relating to the problem-solving view. However, the fact that the interaction with the highest percentage of codes relates to *non-human* actors employing relational processes (30%) demonstrates a somewhat stronger alignment with the problem-solving view (represented by diamond 14). Indeed, this particular interaction is not expected in a textbook promoting an instrumentalist view. Although the Platonist view also supports the occurrence of *non-human* actors engaging in relational process (diamond 3), the relative infrequency of actors engaging in mental processes (comprising only 10% of the total) discourages a connection between Darken and the Platonist view. This connection is further discouraged when considering that 80% of the modality indicators found in the textbook suggest contingency. The analytic framework indicates that the Platonist view supports the presence of an absolute modality, and therefore the notable presence of contingent modality indicators suggests an alignment more closely overall with the problem-solving view.

3.5.2 Tasks

Figure 3.11 presents the results of the analysis on number theory tasks, with the numbers again related to the percentages of the total corresponding to each actor/process interaction in the three textbooks and to the modality indicators. Unlike the results described above regarding the definitions found in the textbooks, a single figure was used to simultaneously depict the findings related to all three textbooks. The reason for this is explained below.

The first thing that the reader may notice is the striking similarities between the most common interactions between the analyzed textbooks, once again considering only the four highest percentages. As described in the previous section, an analysis of definitions in the three textbooks led to distinct variations in the actors and processes present (and therefore distinct views that were being promoted). This was not the case with tasks. The majority of the actor/process interactions in all three textbooks fell into two particular categories (which can be identified by the use of the darkest shading in Figure 3.11): (1) *you*/material; and (2) *non-human*/relational. Although these results demonstrate that there are slight deviations related to the third and fourth highest percentages in the different texts, these two specific interactions mentioned above account for approximately three-quarters of all the codes from all three textbooks. Specifically, 82%, 72%, and 87% of all codes from Darken, Parker and Baldridge, and Wu corresponded to the two interactions, respectively.

			Textbook											
			Darken				***Parker and Baldridge***				***Wu***			
Components of Linguistic Analysis		**Actors**	We	You	3rd person	Non-human	We	You	3rd person	Non-human	We	You	3rd person	Non-human
	Processes	***Material***	2	58	4	2	1	48	9	9	1	51	0	10
		Mental	0	7	0	0	1	3	1	0	0	2	0	0
		Relational	0	2	1	24	0	1	3	24	0	0	0	36
	Modality	***Absolute***	43				33				50			
		Contingent	57				67				50			

Figure 3.11: Findings Related to Tasks in All Three Textbooks

Since the results related to tasks in these three textbooks were very similar, I examined the data as a whole (meaning all codes pertaining to tasks in the number theory chapters for all the textbooks were treated as a single data set) to create Figure 3.12. This figure presents the overall percentages of the different actor/process interactions found in the three textbooks.

Summing the values found in the three-by-four array under "Combined Percentages" will reveal 100% to show that all of the codes are accounted for in the figure. There was a distinguishable difference between the modality of these texts, however, and so the codes relating to the modality indicators were not considered in Figure 3.12 and are addressed separately later.

Components of Linguistic Analysis		Textbooks				Related views about the nature of mathematics							
		Combined Percentages				*Instrumentalist*				*Problem-Solving*			
	Actors	We	You	3rd person	Non-human	We	You	3rd person	Non-human	We	You	3rd person	Non-human
Processes	***Material***(outside the self)	1	53	4	7	6	7	8		10	11		
	Mental(inside the self)	0	4	0	0								
	Relational(denote relationships)	0	1	1	29					12	13		14

Figure 3.12: Combined Findings from Tasks in All Three Textbooks and the Promoted Mathematical Views

The four highest percentages darkened in Figure 3.12 suggest a possible alignment with both the instrumentalist and problem-solving views of mathematics. The prevalence of *you* actors participating in material process as shown in the first row corresponds with diamond 7 in the framework's description of the instrumentalist view, as well as diamond 11 relating to the problem-solving view. The large percentage of *non-human* actor/relational process interactions (29%) in conjunction with the high percentage of *you*/material process interactions (53%) may suggest alignment with only the problem-solving view. This conclusion is supported by the fact that the two highest percentages correspond to diamonds 11 and 14 under this view. Indeed, *non-human* actor/relational process interactions, which is the second highest percentage of interaction found, is not indicative of the instrumentalist view. However, the fact that 64% of the actors across all textbooks are engaged in material processes is indicative of the analytic framework's description of the instrumentalist view, whereas the majority of processes anticipated within a textbook strongly promoting the problem-solving view would be relational (as 31% of the codes are shown to do). Considering these relationships, the most appropriate interpretation of these results seem to necessitate an alignment with both the problem-solving and instrumentalist views.

As mentioned earlier, although there were strong similarities between the actor/process interactions found in all three textbooks in the task analysis, the same cannot be said about the modality. The modality findings that emerged through an analysis of tasks were similar to those found in the analysis of definitions, as can be seen by comparing the percentages of modality indicators displayed in Figure 3.12 with those shown in Figures 3.8, 3.9, and 3.10. Both Parker and Baldridge and Darken show a slight favor in the direction of contingent modality at 57% and 67% respectively, while Wu once again uses an equal amount of indicators suggesting absolute and contingent modality. Contingent modality further supports an alignment with the instrumentalist and problem-solving views of mathematics, as suggested earlier. Wu's use of absolute modality indicators, on the other hand, suggests the potential to promote a Platonist view as well.

3.6 DISCUSSION

I begin this section by discussing the general results of the definitions analysis, and follow with a discussion of the particular treatment of the definition of prime numbers in each of the three textbooks to further illustrate the general findings. I then discuss the general results related to the tasks, and follow with a discussion of three tasks (one task from each textbook) relating to the least common multiple (LCM) and greatest common divisor or factor (GCD or GCF) chosen purposefully to illustrate how different tasks may promote different views, as well as the related implications.

3.6.1 Discussion of results related to definitions

This section addresses each textbook separately and follows the same order as the presentation of results in Section 3.5. Thus, I begin by elaborating upon the results found in Parker and Baldridge, followed by Wu, and then Darken.

Considering the high proportion of both human and non-human actors participating in material processes (62% overall) as shown in Section 3.5.1, Parker and Baldridge appears to more strongly support an instrumentalist interpretation (see diamonds 6 and 7 in the framework). The very first sentence of the chapter supports this categorization and sets the tone for all of the activities that follow: "Mathematics is built on precise definitions and proceeds using clear reasoning" (p. 109). As with the case of prime numbers in this textbook, the majority of the discussions surrounding the various terms in the number theory chapter acknowledge the presence of human actors, usually the student. However, those actors may play a very restricted role in the creation of the knowledge being developed. The use of the *we* actor is a cause for ambiguity, as its presence may imply two different things. On one hand, the use of *we* may suggest that the author is establishing solidarity with the readers and placing them in equal positions of power within the text. On the other hand, the use of *we* may suggest that there is a separate community that holds the position of power within the text, and it is a community in which the reader is not included. These descriptions represent the exclusive and inclusive use of the *we* actor, respectively, and are useful in these discussions. The use of the two terms is similar to Rotman's (1988) use of the two terms regarding imperatives as described in Section 3.3.2.2.

For example, in the section dealing with divisibility tests, Parker and Baldridge precede the definition of 'divisible' with the following statement: "In the remainder of this chapter, *we* will often use letters A, B, …, k, l, … and a, b, … to represent whole numbers. At any time, *you* may assign them specific values (like A=20, k=5) to aid *your* understanding" (p. 113, italics added). The student is certainly present in this instance, yet the processes in which she is to engage are purely material. A prevalent use of material processes may construct a mathematics which is about practical activity that is primarily carried out in a procedural way. There was also a relatively high percentage (20%) of *non-human* actors participating in relational processes, which the analytic framework suggests would promote a problem-solving view of mathematics (see diamond 14). Indeed, this interaction stresses the connections and relationships between mathematical ideas. The problem-solving view is also supported by the high percentage (79%) of modality indicators suggesting contingency, or an increased likelihood of various possibilities.

As described in Section 3.5.1, the view about the nature of mathematics promoted by Wu suggests some similarities with the view promoted by Parker and Baldridge. Although *we* was the most prevalent actor in Wu's textbook (accounting for almost half of the total codes), there is strong evidence that the reader is not assumed to be actively participating in the knowledge being introduced. For example, when describing "*our* enduring interest in the primes" (p. 10), Wu places himself in the authoritative role. Seeing that prime numbers had only been defined in the previous sentence, the student certainly has not yet had a chance to fully understand what prime numbers are, let alone develop such a powerful interest in them. They are apparently excluded from the community comprising the collective *our* suggested by Wu. Further, Wu's mathematics is introduced in a very matter-of-fact manner, with each new idea logically derived from the former in a linear fashion. As in Parker and Baldridge, the presence of a *non-human* actor involved in material processes is evidence of what linguists call 'nominalization' and is a structure of language that serves to "obscure human agency" (Herbel-Eisenmann & Wagner, 2005, p.123). These *non-human* actors also relate to Morgan's (1998) notion of 'representational objects,' which describes non-human objects (in this case, mathematical objects) performing an activity usually related to humans.

Many definitions described in Wu's number theory chapter, and more generally the entire book, are followed by a collection of theorems using those definitions. It was common to find a phrase such as "the theorem tells us," or "the theorem says," which "depicts an absolutist image of mathematics, portraying mathematical activity as something that can occur on its own, without humans" (Herbel-Eisenmann & Wagner, 2005, p.123). Indeed, the student participates little in the creation of the definitions or the related mathematical knowledge. Primarily she is asked to act as a mere spectator of the activity, and to accept the knowledge which has been "clearly" outlined for her.

At 50%, Wu's text has more than twice as many absolute modality indicators as Parker and Baldridge, using indicators such as "clearly," and suggesting a lack of alternative possibilities. Although material processes such as "check," "remove," and "define" were most commonly found throughout the chapter, Wu's text contains the largest percentage of mental processes among the three textbooks at 30%. Morgan (1996) claimed that a large number of mental processes "may suggest a mathematics that is a pre-existing entity that is discovered by mathematicians" (p. 2). Considering the prevalent modality and types of processes used, Wu's textbook suggests elements of the instrumentalist view (see diamonds 6, 7, and 8), but overall appears to more strongly promote a Platonist view. This characterization is grounded in the findings that show a restriction of student activity (the exclusive *we*) and that 12% of the total codes describe mental processes interacting with the student (the *you* actor). Mental processes represent the highest percentage of any of the processes used (see diamond 2). An alignment with the Platonist view is further supported by the absolute modality indicators found in Wu (see diamond 5) within phrases such as "it is immediately clear" and "it is easy to show."

On the other hand, Darken's *we* actor (which was found to be the least common human actor present at 18%) suggests the presence human agency in a way absent in Wu's use of the actor. Moreover, the ways in which it is used in Darken includes the student as a participant in mathematical activity. Most of the 21% of *you* actors participating in material activities in Darken were done using imperatives, and the commands were primarily inclusive. These types of imperatives welcome the student into a community as an active participant that is first given the chance to construct her own knowledge, with the formal definition coming afterwards. This is in sharp contrast to similar activities described in Parker and Baldridge, where definitions were often given with little student contribution and were followed by a procedure through which the student was led step-by-step. This contrast is made more apparent in Section 3.6.1.1, where the particular case of prime numbers is discussed. Whereas Parker and Baldridge employed a narrow-to-broad pathway (meaning they began with the definitions and then used a variety of problems to discuss the term), Darken's approach started broad, narrowed, and then broadened once again. This started the mathematical conversation with a student-driven activity that better prepared the reader for a more formal discussion of prime numbers. Although the use of imperatives implies a more absolute modality, the context of the imperatives in Darken is not as exclusive as in the other textbooks. Furthermore, Figure 3.10 indicates that the analysis of modality indicators produced an overwhelming amount supporting contingency (80%). Despite a clear emphasis on certain procedures, there is evidence to interpret Darken's text as promoting a problem-solving view of mathematics, with some instrumentalist tendencies. Figure 3.10 also shows that this textbook contained the largest percentage of relational processes (35%) found in any of the analyzed texts (see diamonds 12, 13, and 14). Also, most of the 52% material processes related to human actors (excluding the 11% with *non-human* actors) were classified as inclusive imperatives (see diamonds 10 and 11). More importantly, these processes are combined with a picture of the student who often appears to be invited to share in the mathematical activity of defining, as well as to be engaged in considering the evidence supporting the activity.

3.6.1.1 Discussion of the particular case of definitions of prime numbers

Although there would be merit in discussing each definition analyzed, the limited space does not permit such lengthy prose. Therefore, I have chosen to focus on just one particular definition (related to prime numbers) found to be common in each of the textbooks to provide an insightful illustration of a sample of the definitions analyzed. I also compare and contrast how the linguistic components there relate to different views about the nature of mathematics. Figures 3.13, 3.14, and 3.15 below represent the portions of the textbooks analyzed related to the definition of prime numbers in Darken, Parker and Baldridge, and Wu, respectively.

Although each textbook acknowledges that the student has a role within its discussion of prime numbers, the authors' choice of processes in which the student is involved seem to promote different ideas about what her role entails. In addition to using the *we* actor as a means to construct the student as an active member in the mathematics, Darken encourages the student to initially take responsibility for creating her knowledge about primes (instead of simply stating the definition for the student to accept without question). The section devoted to prime numbers opens with an activity meant to fuel the student's investigations into possible emergent patterns that move her towards the notion of prime numbers. The textbook gives instructions about how to use a chart introduced to fuel the activity, but the authority is quickly given to the student, where the author makes comments about "*your* work" and "*your* patterns" at the heart of the activity. Morgan suggested that by participating in the relational processes of investigating and observing patterns, the student may be presented with "a picture of mathematics as a system of relationships between objects or between objects and their properties" (Morgan, 1995, p. 3). In this particular case, the student is encouraged to see relationships between the concepts of natural numbers, their factors, and the property of being prime. This dynamic picture of mathematics resembles that promoted by the problem-solving view, which was one of the aligned views ascribed to Darken in Figure 3.10.

In contrast, Parker and Baldridge and Wu restrict the student's role as an active participant in the mathematics. While Darken asks the student to explore the mathematical terrain of prime numbers and to share in decision-making activity, the student using Parker and Baldridge is directed to carry out material tasks ("proceed," "circle," "cross out") related to a described procedure to follow, using imperative instructions. Darken suggests a view of mathematics that is dynamic and creative, whereas the focus on material processes in Parker and Baldridge "may be interpreted as suggesting a mathematics that is constructed by doing" (Morgan, 1995, p. 3). In the latter view of mathematics, procedures take precedence over explorations and conjectures.

Wu's student is involved to an even lesser degree. Besides including the student (through use of the actor *we*) in a mental processes within the phrase "we have already noted," Wu introduces the definition without any contribution on the part of the student. Although not immediately clear, the similar use of imperatives in Parker and Baldridge and Wu may be related to their more exclusive use of the *we* actor. Instead of interpreting imperative commands like "find," "list," and "explain" as mathematical convention, the student may interpret her (exclusive) role in relation to mathematics as only that of a follower of rules in a procedure-centered discipline. I now illustrate the difference between inclusive and exclusive imperatives as used in two of the textbooks (Darken on one hand, and Parker and Baldridge on the other), and discuss the corresponding implications.

Parker and Baldridge and Darken's main activities within the discussion of prime numbers appear at first glance to have some similarities based on the types of actors present and the kinds of processes in which they are engaged. Both textbooks seem to acknowledge the student as an active participant in demonstrating the relationship between the factors of a number and the attribute of being prime. However, there seem to be contrasting views about where the authority in the knowledge being constructed lies. Before any mention or definition of the term prime, Darken opens the section with a mostly blank chart (seen in Figure 3.13) that is meant to fuel the student's investigations into possible emergent patterns. Instructions are given about how to use the chart, but the authority is quickly given to the student, where the author makes comments about "*your* work" and "*your* patterns" at the heart of the activity. The commands are demonstrative of those considered inclusive, welcoming the student into the

Discover patterns among the factors of whole numbers in the following activity.

1. For each integer in the top row of the following chart, make each of it's factors. Use patterns formed by your X's to complete your work. [Hint: If there are any holes in your pattern, check to see if you have overlooked a factor.]

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	x	x	x	x																				
2		x		x																				
3			x																					
4				x																				
5																								
6																								
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2. Referring to your completed table, list the total number of factors for each of the following numbers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24

3. What is the only whole number with exactly one factor?
4. If N is a whole number greater than one, list two of its factors.
5. List the first five whole numbers that have exactly two factors.
6. Investigate whole numbers that have exactly three distinct factors.
   a. Collect some data.
   b. Observe a pattern and make a generalization.
7. a. Write 60 as a single produce with as many whole number factors as possible, not including one.
   b. Compare your answer with a classmate's answer. Are your answers different? Are they both right?

There are many interesting generalizations we can make when we look at whole numbers from the point of view of their factors. First of all, consider the number of factors a number may have. The number 1 is very special: it is the only number with exactly one factor (itself), and it is a factor of every other number. Every number N > 1 has at least two factors, itself and one. Numbers with exactly two factors are particularly interesting. In fact, such numbers are so interesting that they lead to a very interesting partitioning of the whole numbers.

Prime and Composite Numbers

A **prime number** is a whole number with exactly two whole number factors, itself and one.
Set of prime numbers = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, …}
(THE DEFINITION OF COMPOSITE NUMBERS NOT INCLUDED IN THIS DEFINITION)

Zero and one oare neither prime nor composite. As we have already observed, zero has an infinite number of factors, while 1 has only one factor.

Figure 3.13: Introductory Activity and Definition of Prime Numbers in Darken (2003, p. 487)

community as an active participant. Parker and Baldridge, on the other hand, acknowledge this relationship between factors and the condition of being prime (as seen in Figure 3.14), but do so in a vastly different way. Instead of promoting exploration on the part of the student, Parker and

Baldridge guide the student through a process driven by exclusive imperatives such as "proceed," "circle," "cross out," which leaves no other choice for the student to act otherwise or question the command. Even though both textbooks are grounded in the same mathematical ideas, the different processes engaged in by the student in discussing prime numbers have the potential to present two very different views about mathematical activity: the first situating them as creators of mathematics and the second as followers of procedures. In Wu, the student seems to be situated as the latter.

Most whole numbers can be expressed as products of smaller whole numbers. For example,

$$42 = 6 \times 7$$

The numbers 6 and 7 are called factors of 42, and 6 x 7 is a factorization of 42. (When discussing factorizations, order does not matter, so 6 x 7 and 7 x 6 are considered the same factorization.) The number 6 can be factored further, giving the factorization

$$42 = 2 \times 3 \times 7$$

The process stops there: none of the numbers 2, 3, 7 are products of smaller whole numbers—they are prime numbers.

DEFINITION 3.1. ***A prime number is a whole number $P > 1$ whose only factors are 1 and P. Whole numbers $N \geq 2$ which are not prime are called composite.***

The prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, and 19. The numbers 4, 6, 8, 9, etc. are composite because they can be written as products of smaller numbers. Note that, by definition, the numbers 0 and 1 are neither prime nor composite.

One can create a list of the prime numbers by writing down the whole numbers 2, 3, 4, 5, ...and successively checking each one. Of course, the even numbers larger than 2 are not prime, so there is no need to check them. Likewise the multiples of 3 after 3 needn't be checked. Thus we can proceed as follows. First, circle 2 and cross out ever multiple of 2 on the list. Next, circle 3 and cross out every third number (even if it has already been crossed out). Continue this procedure, at each step circling the first number that is not circled or crossed out, and then crossing out all of it's multiples.

In the end, the circled numbers are the list of primes. [Exclude historical discussion of Sieve of Erathosthenes] Notice that listing the numbers in rows of 12 makes it especially easy to cross out the multiples of 2 and 3.

Primes are important because they are "building blocks" from which all whole numbers are made. This "building" is done by multiplication. Thus discussions of primes involve the ideas of factors, multiples, products, and powers (all related to multiplication and division), but do not involve addition or subtraction.

Figure 3.14: Definition and Discussion of Prime Numbers in Parker and Baldridge (2004, p. 118)

We have already noted that some whole numbers have no proper divisors, e.g., 2, 3, 5, 7, 11, …A whole number which is greater than 1 and has no proper divisor is called a **prime**, or a **prime number**. Note that 2 is the only even prime. [Exclude definition of composite numbers.] By definition, 1 is neither a composite nor a prime.

There are many reasons for our enduring interest in primes—some of the simpler ones will surface presently—but a fundamental one is that they are the basic building blocks of the integers, in a sense that we shall make explicit in the next section.

Checking whether or not a whole number *n* is prime seems at first glance to be a formidable task: you have to check all the numbers from 2 to *n*-1 to see whether they divide *n*. [Exclude worked example]. We conclude, therefore, that if there are no divisors of a whole number n among numbers at least 2 but at most *n*/2, then there would be no divisors of *n* at all, and therefore such an *n* must be prime.

Figure 3.15: Definition and Discussion of Prime Numbers in Wu (in preparation, p. 10)

Pronouns and imperatives certainly contribute to the views about mathematics being promoted in the textbooks, yet they are certainly not the only influences. Modality also proves to be useful. The analyses discussed in Section 3.5.1 revealed few differences with regards to the modality. The modality in the textbooks was analyzed by recording the frequencies of indicators relating to either absolute or contingent modality. Indeed, although phrases such as "can be" or "may be" are classic indicators of contingent modality, they would not be classified as such when used in a sentence such as "any natural number *can be* written in this way." This use of "can be" speaks more to mathematical convention than to a degree of likelihood. Although not inundated with indicators of contingent modality such as "may," "possible," or "perhaps," Darken begins the section by saying that "there are many interesting *generalizations we can make* when we look at whole numbers from the *point of view* of their factors" (p. 488). These phrases open up the student to the possibility of alternative ways of looking at numbers from different perspectives, and suggests that there is a possibility that the student (in the 'inclusive' *we*) might be able to make choices. Wu and (to a lesser degree) Parker and

Baldridge, however, primarily present the definitions and related activities as fixed and unquestionable. They place the mathematical authority not in the hands of the student, but in the mathematics itself. By using phrases of absolute modality, a textbook may encourage the student not to understand the reasoning behind her actions, but rather to focus on following the unquestionable procedure correctly (even if it is not immediately clear and they do not want to admit their inability to see what should be obvious). This may suggest to the student that mathematics is all about absolute certainty and exists entirely outside of herself. In particular, Parker and Baldridge sequences the process of finding prime numbers ("first…, next…, in the end"), and consequently restrict the student's activity rather than suggesting contingency or encouraging exploration on her own. The same kind of restriction can be seen in Wu through the use of the phrase "you have to check."

3.6.2 Discussion of results related to tasks

An interesting observation with regards to the results related to tasks is the similarity between the highest-frequency actor/process interactions of the analyzed textbooks (you may refer back to Figure 3.11). As mentioned in the results section, a large percentage of all codes from the tasks analyzed from the three textbooks fell into two particular categories of interactions: (1) *you* /material; and (2) *non-human*/relational.

Considering that tasks are most likely to be written with the intention of engaging the student directly, it should not come as much of a surprise that the prevalent actor found here was *you*. The presence of this actor is vital in the creation of the types of tasks described by Lesh and Kelly (1994) that improve students' understanding by focusing on eliciting mathematical reasoning and promoting problem-solving. These tasks also position the student at the center of the task activity. As Lesh and Kelly suggested, these types of tasks are powerful tools in mathematics instruction because they enable students to construct their own meaning and to make connections between concepts. Therefore, to gain insight into whether or not tasks are creating these kinds of opportunities, it is necessary to not only have the student as the actor in the activities, but also to understand the kinds of processes in which she is engaged.

The *you* actor was clearly present in the textbooks, and a large majority of the codes related to this actor reflected an interaction with processes described through imperative verbs. As suggested by the results reported in Figures 3.11 and 3.12, these processes were usually material in nature (accounting for 53% of the total codes in the three textbooks combined), requiring the student to do things such as "find," "calculate," and "list." This heavy use of material processes may construct a view about mathematics which suggests that it is about practical activity carried out in a procedural way. This is especially true when the student is encouraged to review a process described in the section and to mimic that procedure for the assigned tasks. This does not appear to reflect the meaningful tasks described by Lesh and Kelly (1994) that promotes student-constructed understandings. For example, the first task posed in the last section of the number theory chapter in Parker and Baldridge asks the student to refer to a previous page, to "notice how the ideas of factors and multiples are introduced and….use the method…to find a common multiple" (p.130) for various pairs of numbers. Such tasks promote an instrumentalist view of mathematics, one where the student's primary role is to learn and imitate procedures described by the author. These exclusive imperatives do not encourage the student to feel like an active participant, a creator, in a community of mathematicians. Tasks that require students to merely follow a procedure are also unlikely to require a high level of cognitive demand (Smith & Stein, 1998), which automatically decreases the potential for high levels of student learning.

It is possible, however, that a *you* actor/material process interaction may be inclusive (Rotman, 1988). Unlike the situation described in the previous paragraph, this kind of interaction may allow the student engaging with the task to feel like a contributing member of a mathematical community, not merely following directions to a predetermined end. Instead of performing actions grounded in mathematical concepts and processes clearly defined within the textbook completely independently of the student, inclusive tasks encourage the student to focus on their own work and ideas in order to solve a problem. For example, on page 494, Darken asks the student to "make a conjecture about the form of the prime factorization of numbers with exactly seven factors and test your conjecture." In this instance, the imperative "make" implicitly addresses the *you* actor and has her engaging in the material process of "making" a conjecture. This may increase the cognitive demand required by the task (Smith & Stein, 1998) and could lead to an increase in student learning.

To illustrate this further, I draw compare the example from Darken given above and the example of an exclusive task from Parker and Baldridge shown in Figure 3.5. In Parker and Baldridge, the student is directly instructed to refer to a previously-demonstrated process, which suggests a more inclusive imperative. The student working with this task is engaging in material processes that are derivative of work already done previously by the textbook author. A student working with Darken is also engaged in material processes, but is presented with a greater opportunity to engage with a task that allows them to construct meaning and make connections between ideas, as recommended by Lesh and Kelly (1994). Not only does the task ask the student to make a conjecture without an implied path to take, it also instructs her to "test your conjecture," allowing her to position herself as an active participant, as a creator of mathematics who is playing an important role in the activity. Therefore, although it may be expected to find *you* actor/material process interactions via imperatives when investigating the linguistic elements of mathematical tasks (as I did within the three textbooks analyzed in the chapter), this discussion of the results suggests that the inclusive or exclusive nature of those processes may impact the ways that views about the nature of mathematics are promoted.

Interactions between *non-human* actors and relational processes also comprised a high percentage (29%) of all the coded data. This interaction, denoted by diamond 14 in the analytic framework in Figure 3.1, supports a problem-solving view of mathematics. Here, *non-human* actors (usually in the form of mathematical objects such as numbers, variables, or representations) participate in relational processes that connect them to other mathematical objects. In the sample of tasks analyzed, it was often the case that the most common *you* actor/material process interactions discussed earlier were immediately followed by these second types of actor/process interactions. This means that the student was often asked to do something (participate in some material process) that helped them demonstrate the relationship between mathematical objects (using a relational process). To help illustrate this, Wu asked the student to "prove that the product of two odd numbers is an odd number" (p. 9), which demonstrates this common combination of actor/process interactions. The imperative "prove" asks the student (actor *you*) to engage in the material process of proving something in order to demonstrate how another actor in the sentence (the non-human "product of two odd numbers") is related to a second mathematical object (an odd number). The implications of these interactions are further elaborated upon below.

Although a textbook having a large percentage of relational interactions between mathematical objects in tasks supports the vision of mathematics as being a web of interconnected ideas, it seems important to note the significance of the other actor/process interactions surrounding these relations. The example task from Wu in the previous paragraph requires that the student actively attempt to establish this relationship without strict guidance or suggestions, therefore holding her responsible for the construction of her own knowledge. This task appears to be cognitively demanding. Imagine now that a task focusing on the same content is stated as follows: "Refer to the worked example 2 on page 13 and use it to show that the product of two odd numbers is odd." The end result is still to illustrate the relationship between the product of two odd numbers, but the processes in which the task requires the student to engage deprives her from engaging in creative problem-solving. Instead, it places her in a role where she is to simply follow directions and to modify or reproduce work previously demonstrated. The low cognitive demand of the task automatically lowers the student learning that will result from the task (Smith & Stein, 1998). Therefore, this discussion suggests that it may be necessary not only to look at the individual interactions between actors and processes in tasks, but also to look at the various interactions present together.

As mentioned at the beginning of this section, the traditionally instructive statement of tasks may also account for the notable increase in the presence of absolute modality indicators in two of the three textbooks in comparison with those found in definitions. Parker and Baldridge showed an increase from 21% in definitions to 67% in tasks, and Darken went from 20% to 57%. The modality found in Wu remained consistent, having 50% of the modality indicators coded as absolute in both definitions and tasks.

The general findings related to the definitions discussed in Section 3.6.1 showed a noticeable variation in the actor/process interactions favored by the different authors. The related discussions illustrated the ways in which those interactions may impact the views about mathematics being promoted by the textbooks. This variation was clearly absent in the general findings relating to the tasks found in the very same chapters as those definitions. Although one may anticipate more consistency between the language choices made when creating definitions and the posed tasks relating to those definitions, perhaps the general nature of posing questions in mathematics does not allow for the diversity possible in the more discussion-oriented portions of the text. In Section 3.6.2.1, I use the analytic framework to describe and discuss the actor/process interactions within one particular task from each of the three textbooks' sections relating to the LCM and GCD.

3.6.2.1 Discussion of the particular case of tasks related to the LCM and GCD

The limited space does not permit lengthy prose on every task analyzed in this chapter. However, I believe there is value in using specific examples of tasks found in each of the textbooks to provide an insightful illustration of how tasks may promote different views about the nature of mathematics. Unlike the discussion in Section 3.6.1.1, the following discussion is not meant to illustrate the general findings related to the tasks analyzed in the three textbooks. Given the similar results found in the sample of tasks analyzed using the analytic framework, it was impossible to choose a "representative" task from each of the three textbooks to use as an illustration of the general findings. Rather, I have chosen one task from each textbook relating to the notions of LCM and GCD that provides the opportunity to demonstrate differences in the promoted views that may occur that are not fully captured by the framework. To maintain a level of consistency, I purposely selected tasks that were focused on the same mathematical content within the problem sets. I provide the three tasks that guide this discussion together in Table 3.3, as they appeared in the original textbooks.

As outlined in the discussion relating to the general findings, each textbook acknowledges that the student is the (*you*) actor with regards to these tasks. Tasks, by nature, are meant to engage the student in mathematical activity that moves her toward an objective or goal, and therefore necessitates the presence of this particular actor. However, in all three texts, this actor is not explicitly acknowledged; rather, the actor is implicitly found in processes being used by the author, which are all commanding imperatives.

Table 3.3: Sample Tasks and Coding Related to LCM and GCD in the Three Textbooks

TEXTBOOK (PAGE NUMBER)	**STATEMENT OF TASK [AND CODING]**
Darken (p. 505)	9. Examine **[Y-M]** the pattern to be found in numbers with three and five factors. a. Use **[Y-M]** this pattern to predict the smallest natural number with exactly seven factors. b. Test **[Y-M]** your prediction.
Parker and Baldridge (p. 130)	4. Use **[Y-M]** the method of Example 5.4 to find: a. GCF (28, 63) b. GCF (104, 132) c. GCF (24, 56, 180)
Wu (p. 34)	1. Find **[Y-M]** the gcd of each of the following pairs of numbers by listing all of the divisors of each number and comparing: 12 and 42, 34 and 85, 24 and 69, 18 and 117, 104 and 195.

The majority of the tasks found in the number theory chapters of the texts had the *you* actor participating in material processes (53% in Figure 3.12) as detailed in the results section, and this sample of three tasks is no exception. Indeed, this interaction the only one found in the three tasks (see Table 3.3). Wu asks the student to “find” the GCD of several pairs of natural numbers, while Darken and Parker and Baldridge require the student to "use" and "examine" in order to reach her objectives. Indeed, the analytic framework indicates that the *you* actor engaging in a material process relates to both an instrumentalist and problem-solving view of mathematics (as shown by diamonds 7 and 11, respectively). Although this is an appropriate analysis, this section aims to elaborate upon the ways in which the language choices made in these three tasks may differentiate them from one another in a way not fully captured by the framework.

Returning to Wu's task, the student is not only asked to "find" the GCD of several pairs of natural numbers, but the author also prescribes a method to do so which includes "listing" and "comparing." The student is not asked to do more than to follow the supplied solution path to the desired result. A similar occurrence can be found in Parker and Baldridge's tasks, in which the student is asked to "use *the method*" outlined for her in a previously worked example. In essence, the student could simply refer to this previous example, plug in the new numbers supplied in the task, and perform the procedure without necessarily understanding the rationale behind it. These tasks lie in contrast to the task found Darken, as demonstrated below.

Darken encourages the student to "examine the pattern" in order to reach the objective laid out for her, yet she is not given a specific path to follow or method with which to move towards that objective. The mathematical activity requires that the student take authority to investigate the numbers that have three and five factors in order to find emergent patterns that provide the foundation for the work that follows in the task. In the second part of the task, this pattern is used to "predict" what would happen in a situation involving numbers with seven factors. This process of predicting is relational in nature, as opposed to the strictly material processes found in the other two tasks. Just as the analysis of Darken's definition of prime numbers seemed to promote "a picture of mathematics as a system of relationships between objects or between objects and their properties" (Morgan, 1995, p. 3), this sample task also encourages the student to see those relationships. Unlike the sample tasks presented by Wu and Parker and Baldridge (which are discussed further below), Darken's task seems to be promoting the picture of mathematics as suggested by the problem-solving view as well as the instrumentalist view. The presence of this view is further substantiated by noting that part (b) of the task states that the student should "test *your* prediction." In this case, the student is directly acknowledged by the use of "your," and it positions her as an active participant in the task, a creator of mathematics who is playing an important role in the activity.

In contrast, Parker and Baldridge and Wu suggest more restriction with regards to the student's role in the tasks. Darken asks the student to explore and test her pattern, but a student engaged in the sample task from Parker and Baldridge is directed to carry a material process related to a described method outlined earlier in the chapter. Although both tasks taken from Darken and Parker and Baldridge employ the use of (material) imperatives, the way in which they are used and address the student are different. Therefore, the tasks may suggest different views about the nature of mathematics. Darken's emphasis on finding patterns and acknowledgement of the active student in the use of "your pattern" suggests a mathematics that is creative and human-driven. The material processes related to the "method of Example 5.4" in Parker and Baldridge, however, moves away from a creative view of mathematics towards one that is more dependent on prescribed procedures. Wu's task is similar to Parker and Baldridge in this respect, as the student is not only asked to "find" the GCD of given pairs of numbers, but is also instructed how to go about reaching this goal. As was often the case with the definitions discussed across all three textbooks, the utilization of imperative commands like "find" and "use" may not be interpreted as mathematical convention by the student. Rather, the student may interpret her role in relation to mathematics as only that of a follower of rules and methods as opposed to an active creator. Once again, the dichotomy of imperatives proposed by Rotman (1988) proves useful in making these important distinctions which lead to very different implications.

Using the analytic framework, these three sample tasks appear to be similar with respect to the actor/process interactions present (with all three found to have the *you* actor engaged in material processes). Indeed, this observation is to be expected given the general results displayed in Figures 3.11 and 3.12 in Section 3.5.2. All of the tasks acknowledge the student as an actor present in the textbook given the prevalent use of material processes to reach a particular goal related to factors, yet there appear to be contrasting views about where the authority lies in reaching that goal. Darken presents the task in a way meant to encourage the students' investigations into patterns without an imposed solution path. Instructions are given to "examine," but the authority is quickly given to the student, where Darken makes comments about making predictions and "*your* pattern" which guide the activity. The commands seem to be inclusive, as they treat the student engaged with the task as an active participant. Parker and Baldridge and Wu, on the other hand, present the task in such a way that the student is guided through exclusive material process with specific pathways given. In these two tasks, the student is not given the choice to act otherwise or question the command. All three tasks that are grounded in the same mathematical ideas, yet the processes present and placement of the authority in the tasks may promote very different views about the mathematical activity. Darken's task seems to situate the student more as a creator of knowledge and in more control of meeting the set objective, while Parker and Baldridge and Wu's tasks situate the student as a follower of set procedures.

In addition to the actors and processes present, the modality also plays an important role in understanding the mathematical views promoted by the tasks. As presented in the general results related to tasks in Section 3.5.2, this analysis revealed few differences between the three tasks with regards to the modality. The same can be said about these three specific tasks, where no indicators of either absolute or contingent modality are present. However, the tasks from Parker and Baldridge and Wu suggest a method or process by which the student achieves the task's objective, and as a consequence may restrict the student's activity rather than suggesting contingency or encouraging exploration on her own.

The three tasks discussed in this section, which are similar according to the analytic framework, have provided some important insight about the general applicability of the framework. Indeed, while the framework is able to capture important aspects of the tasks in the textbooks that can suggest the mathematical views being promoted, including interactions between actors and processes and modality indicators, it cannot capture everything. While a quantitative analysis of tasks is certainly a good starting point to begin understanding the promoted views, simultaneously conducting a more qualitative analysis may help to capture aspects of the textbook missed by a purely quantitative analysis using the framework.

3.7 CONCLUSIONS AND IMPLICATIONS

It has been suggested that textbooks play a substantial role in teachers' decisions about what is taught in K-12 classrooms (e.g. McCrory, et al., 2007). Considerably less is known about the ways in which mathematics may be promoted in the textbooks with which PSETs engage in their TEPs. The beliefs held by teachers impact all aspects of classroom activity and, as a result, student learning. Thus, it is important that researchers in mathematics education understand and aim to improve the means by which those beliefs are developed. The goal of this chapter has been to participate in both of these endeavors. First, by using the tools of systemic functional grammar (Halliday, 1985), I have developed an analytic framework that can be used to analyze the mathematical views being promoted by textbooks used in mathematics content courses for pre-service teachers that contributes to the understanding of how beliefs may be developed. Second, I implemented the analytic framework to analyze the different views about the nature of mathematics being promoted by three particular textbooks for PSETs. The findings of these analyses may be used to both understand and improve math textbooks regarding the mathematical views being promoted.

In addressing the mathematical views portrayed within textbooks, the NCTM (1991) suggests that they should "represent mathematics as an ongoing human activity" (p. 25). Moreover, their 'Communication Standard' (NCTM, 2000) clearly emphasizes the importance of using the language of mathematics to express mathematical ideas precisely, stressing that "students should understand the role of mathematical definitions and should use them in mathematical work" (p. 63). Given the importance of definitions and tasks in mathematical activity, I have attempted to address in this chapter critical questions about how their presentation in three textbooks used in mathematics content courses for PSETs may promote different views in relation to the nature of mathematics.

The three mathematics textbooks analyzed represent only a small sample of the textbooks available for PSETs, and the focus on just the number theory chapter narrows the scope even further. Therefore, it would be inappropriate to draw broad generalizations based on these findings either within individual textbooks or across the group of textbooks. These analyses do, however, make some observations about the ways in which definitions and tasks are presented within this particular sample and how that treatment can promote different views about the nature of mathematics and perhaps position the reader with respect to those views. Although these textbooks were chosen due to their similar stance as being explicit in their "meta-mathematical ideas" (McCrory et al., 2008), my investigation using the tools of systemic functional grammar (Halliday, 1985) has shown that their promoted views about the nature of mathematical activity are different. Each textbook was found to have elements of more than one view, and the discussion of the findings suggests that three different students using these three particular texts might have very different mathematical experiences with regard to the views suggested by the analytic framework.

Although the important role played by the teacher with regards to textbook use is certainly acknowledged, it was beyond the scope of the chapter to address this matter. The employed tools of functional linguistics allowed me to describe only the written features of the textbook, while helping me to understand and attempt to interpret the functions of those textbooks. I cannot speak to the ways in which the text could be modified once in the hands of the teacher, but this issue is certainly important and could provide the foundation for some future research. My analysis suggests that the ways in which definitions and tasks are presented in a mathematical textbook have the potential to impact the reader's experiences and broader vision about mathematics and mathematical activity. Textbooks, while not the only influential factor in the classroom, represent a significant factor that may influence what students have an opportunity to learn and the beliefs that result from that opportunity.

If the NCTM's (1989, 1991, 2000) vision of mathematics in the classroom is to become a reality, then students not only "need to have opportunities to learn to appreciate the roles definitions play in mathematical reasoning, but also to begin to see that doing mathematics involves more than following procedures or reproducing standard arguments" (Morgan, 2005, p. 9). In order to accomplish this, it is necessary that our future teachers experience the same type of opportunities in their own math courses during their TEPs. Moreover, the tasks used in these courses should focus on eliciting mathematical reasoning and promoting problem-solving that aim to improve pre-service teachers' understanding by enabling them to construct meaning and make connections between ideas (Lesh & Kelly, 1994).

The analysis and related discussions in this chapter have several implications. They can provide textbook authors, curriculum developers, and teacher educators with linguistic tools that can help them "anticipate the meanings, both substantive and positional" (Morgan, 2005, p. 9) relating to the views about mathematics being supported by textbooks for elementary school teachers. Additionally, the analytic framework developed can serve as a guide to create textbooks and other curriculum materials that construct more desirable relationships between the reader, author, and mathematics. These relationships may better support an overall vision about the nature of mathematics more purposefully in alignment with reform mathematics (NCTM, 1989, 1991, 2000).

In attempts to investigate consistencies in linguistic choices, an interesting direction for future research would be to examine other content areas in addition to number theory and compare the results with those found in this analysis. Moreover, as illustrated by the discussion of the three tasks in Section 3.6.2.1, the framework may not always capture some of the more nuanced distinctions between the textbooks by an analysis of actor/process interactions and modality indicators alone. Considering the usefulness of making distinctions between 'inclusive' and 'exclusive' (Rotman, 1988) imperatives in the discussion of the results in this chapter, it may be beneficial to incorporate them as elements of the framework. In particular, modifying the coding of material processes to indicate either inclusive material or exclusive material processes may help to better capture the differences in textbooks that result from their presence. Additionally, it may be beneficial to consider more explicitly elements such as authority and agency.

It may also be useful to investigate the ways in which pre-service teachers interact with the textbooks analyzed to better understand how they interpret the promoted view about the nature of mathematics and their roles in the mathematical activity given the linguistic choices made by the author. As suggested earlier, it would also be beneficial to attempt to understand how the different views being promoted by the textbooks might change as the material is implemented in the classroom.

Textbooks are not the only source that may influence PSETs' beliefs about the nature of mathematics, but they are certainly a prevalent feature in the majority of classrooms in this country, from the elementary to the university level. Therefore, whether intentional or not, the language choices made by textbook authors may promote particular views about the nature of mathematics that, in turn, may influence the reader's view of the nature of mathematics. Given their possible impact, those choices should be made both carefully and consciously.

4.0 CHAPTER 4: AN EXPLORATON OF THE IMPACT OF INSTRUCTION AND ACTIVITIES IN A RESEARCH-BASED MATHEMATICS CONTENT COURSE ON THE MATHEMATICAL BELIEFS OF PRE-SERVICE ELEMENTARY TEACHERS

4.1 INTRODUCTION

It has been established that teachers' beliefs about mathematics, which include those related to the nature of mathematics itself as well as the learning and teaching of mathematics, have significant influence on their instructional practices (Calderhead, 1996). As Hersh (1986) stated, "one's conception of what mathematics is affects one's conception of how it should be presented" (p.13). Therefore, a teacher's beliefs about mathematics may potentially have a strong impact on the beliefs of her students (Raymond, Santos, & Masingila, 1991; Thompson, 1984; Schoenfeld, 1991; Kloosterman, Raymond, & Emenaker, 1996). This research study contributes to a strand of recent research in teacher education that focuses on pre-service teachers, particularly those preparing to work at the elementary school level. Teacher education programs (*TEPs*) often concentrate on the knowledge acquired by pre-service elementary teachers (*PSETs*) that contributes to successful teaching, yet the need to both understand and to provide opportunities to modify the mathematical beliefs of those PSETs is of equal significance (Stuart & Thurlow, 2000).

Research has suggested that teacher beliefs about learning and teaching mathematics influence what happens in their classrooms. Much of this research was elaborated upon in Chapter 2. These beliefs usually develop long before engaging in any real teaching experience and oftentimes long before pre-service teachers enter a TEP. Indeed, "long before they enroll in their first education course or math methods course, [pre-service teachers] have developed a web of interconnected ideas about mathematics, about teaching and learning mathematics, and about schools" (Ball, 1988, p. 40) stemming from their past experiences as students. In her study of the beliefs of beginning elementary teachers, Raymond (1997) found that even more resistant to change than their general teaching beliefs were their beliefs about mathematics. This discovery motivates the need to address mathematical beliefs as soon as possible within TEPs in order to better understand those beliefs and to promote change towards the reform-oriented vision promoted by the NCTM (1989, 1991, 2000).

Mayers (1994) has suggested that is it of critical importance that TEPs both address and provoke change in pre-service teachers' beliefs about mathematics. Supporting this assertion after their own findings about the beliefs of pre-service teachers, Grouws and Shultz (1996) stated:

> Unless teacher educators realize that making an impact on prospective teachers [beliefs and attitudes] requires powerful interventions, it is unlikely that teacher educators will be able to alter the continuity of traditional mathematics teaching and learning. (p. 449)

Although not an easy task to undertake, it is clear that TEPs provide the only platform to provide "unique opportunities between the pre-service teacher's school experience and future teaching practice to pause and reconsider their affective dispositions towards mathematics teaching and learning" (Grootenboer, 2003, p. 42). Given that most methods courses focus on the pedagogical aspects of teaching and normally occur later within the program, it has been suggested that it is essential to address issues related to mathematical beliefs in the content course (Thompson, 1992).

This chapter addresses RQ3 and its related sub-questions:

RQ3. *Do pre-service elementary teachers' mathematical beliefs change as they progress through a research-based mathematics content course?*

3.1. What are the pre-service elementary teachers' initial mathematical beliefs?

3.2. What is the extent of belief change in the course and what are the specific mechanisms for this change?

I attempt to explore and document the mathematical beliefs held by PSETs in their research-based mathematics content course within a particular TEP. As indicted by RQ 3.1, one goal of this chapter is to gain a better understanding of the initial beliefs held by PSETs. The second goal of the chapter is to address RQ 3.2 and to capture any change in beliefs that occur within the course, as well as the specific aspects of instruction and classroom activities that may appear to promote this change. In attempts to meet this goal, I employ both quantitative and qualitative methods.

I acknowledge three important aspects of the literature on both in-service and pre-service teacher beliefs as I attempt to address my research questions. First, beliefs must be inferred from what the participating pre-service teachers say and what they do, and these beliefs must be evaluated through analysis that uses multiple data sources (Calderhead, 1996; Pajares, 1992; Schoenfeld, 2003). Second, beliefs are organized into complex systems, which can account for the existence and identification of what may appear to be conflicting or contradictory beliefs within an individual (Calderhead, 1996; Pajares, 1992; Thompson, 1992; Wilson & Cooney, 2002). Third, cognitive conflict provides a means by which pre-service teachers can reflect on their current mathematical beliefs, confront contradictions that arise in situations where some of these beliefs no longer hold, and recognize the importance of modifying these beliefs to resolve the contradictions (Calderhead, 1996; Pajares, 1992; Stylianides & Stylianides, 2009a). Cognitive conflict is particularly important as I attempt to explore RQ 3.2 regarding the PSETs' belief changes and the related mechanisms.

4.2 CONCEPTUAL AND THEORETICAL FRAMEWORK

In this section, I describe the conceptual and theoretical frameworks that guide the explorations in this chapter. As discussed in Chapter 1, although the terms 'conceptual' and 'theoretical' are often used interchangeably in reference to frameworks, I explain here the distinctions I make between the two. The conceptual framework is comprised of the concepts or variables that are operating within the explorations meant to inform the research questions outlined in the previous section. The theoretical framework, on the other hand, refers to the specific learning theories in which the research is situated. I summarize the elements of each of these frameworks in Figure 4.1 at the end of this section.

As in other parts of the dissertation, I use the term *beliefs* to refer to a concept that shapes the ways an individual conceptualizes, describes, and engages in certain situations. In particular, I have previously defined *beliefs* as "the implicitly or explicitly held subjective ideas about the nature of mathematics that influence the ways an individual conceptualizes, describes, and engages in both the learning and teaching of mathematics." Beliefs are often said to originate from previous (often traditional) learning experiences in the classroom, and are known to be highly resistant to change (Kagan, 1992; Nespor, 1987; Pajares, 1992). The beliefs held by a person provide a lens through which decisions are made and guide the behaviors and practices adopted in certain contexts, either consciously or unconsciously (Ernest, 1989; Nespor, 1987; Pajares, 1992). It is commonly said that beliefs rarely exist independently of others, and research has suggested that beliefs are structured and organized into some form of system (Green, 1971; Stuart & Thurlow, 2000).

Several educational researchers have attempted to describe the mathematical belief systems of teachers. Handel (2003) states that "most authors agree with a system mainly consisting of beliefs about: (a) what mathematics is; (b) how mathematics teaching and learning actually occurs; and (c) how mathematics teaching and learning should occur ideally" (p. 47) (Ernest, 1989; Thompson, 1991; as cited in Handel, 2003). According to Ernest (1991), the most important and influential aspect of a teacher's belief system relates to the first component about what mathematics is, and directly ties to the teacher's philosophy of mathematics. He argued that one's philosophy of mathematics lies at the heart of one's ideas about teaching and learning the subject and provide the foundation of classroom practices, contributing to the definition of the roles of both the learner (creator or receptor of knowledge) and the teacher (authoritarian or moderator of knowledge). As elaborated in previous chapters, Ernest (1988) proposed three main philosophical views about the nature of mathematics found prevalent among the teachers he studied. In the *Platonist view*, mathematics is "a static but unified body" (p. 250) of knowledge meant to be discovered, not created, by those engaging in the discipline. Those working from the *instrumentalist view* believe that mathematics is simply a collection of rules and procedures, with particular skills employed to reach a particular goal. Finally, the *problem-solving view* promotes a mathematics that is dynamic in nature, open to continuous revision at the hands of creative human beings. There is a growing body of research on pre-service teachers that suggests that students in TEPs have belief systems that reflect a perspective of mathematics that is strongly Platonist or instrumentalist in nature.

The studies on belief change have typically looked at changes that occurred within the mathematics methods course (Hart, 2002; Wilkins & Brand, 2004). Some earlier research reported little success in achieving this desired effect (e.g. Ball, 1989; Simon & Mazza, 1993), while more recent work has had more encouraging results (Lubinski & Otto, 2004; Spielman & Lloyd, 2004; Wilkins & Brand, 2004). Much of the literature about particular mechanisms that have been found to successfully promote belief change can be found in Chapter 2. In that chapter, I outlined six mechanisms for belief change (MBC) that may be found in the different components of TEPs, and are important elements of the conceptual framework. Table 4.1 summarizes those mechanisms and the literature that supports each of them. These mechanisms will be referenced during the discussions of the data in this chapter.

Table 4.1: Mechanisms for Belief Change Described in Chapter 2

MECHANISM FOR BELIEF CHANGE (MBC) FOR PRE-SERVICE TEACHERS	RELATED RESEARCH
(**MBC1**) Focus on problem-solving	Liljedahl, 2005; Schram, Wilcox, Lanier, & Lappan, 1988; Erickson, 1993; Verschaffel et al., 2000;
(**MBC2**) Opportunity for reflection	Fenstermacher, 1979; Kagan, 1992; Swars, Smith, Smith, & Hart, 2009
(**MBC3**) Collaboration/group work	Liljedahl, 2005; Schram, Wilcox, Lanier, & Lappan, 1988; Erickson, 1993; Verschaffel et al., 2000
(**MBC4**) Innovative curriculum materials/activities reflecting those supported by the Standards	Erickson, 1993; Frykholm, 2005; Lloyd, 1999; Lloyd & Wilson, 1998; Remillard, 2000
(**MBC5**) Coursework grounded in the work of teaching	Kagan, 1992; Swars, Smith, Smith, & Hart, 2009
(**MBC6**) Challenge beliefs (via cognitive conflict and conceptual awareness pillars)	Feiman-Nemser, McDiarmid, Melnick, & Parkerm 1987; Kagan, 1992; Pintrich, Marx, & Boyle, 1993.

An important assumption underlying this research is that contexts in which PSETs encounter instruction and activities [including 'conceptual awareness pillars' (Stylianides & Stylianides, 2009a)] that would purposefully create and support PSETs' experiencing cognitive conflict (MBC6 in Table 4.1) provide rich opportunities to explore and document the occurrence of and changes in their mathematical beliefs. Conceptual awareness pillars were introduced in Chapter 2, and are also an important element of the conceptual framework. Stylianides and Stylianides found that cognitive conflict experienced by students was dependent on the extent to which instruction provided conceptual awareness pillars that permitted them to become more aware of their held conceptions (in that case, conceptions about proof). This work provides support for the use of cognitive conflict as a mechanism for contributing to "developmental progressions in students' mathematical knowledge" (Stylianides & Stylianides, 2009a, p. 319), and I hypothesize that a similar use of cognitive conflict can provide a mechanism for progressions (or changes) in students' mathematical beliefs as well. I provide an illustration to strengthen this hypothesis. An example of a conceptual awareness pillar might ask students to reflect on a prompt relating to a specific idea regarding mathematics, such as, "In order to solve a particular mathematics problem, I must first be taught the correct procedure." This prompt may be used to direct student attention to what they believe about what it means to do mathematics, which in turn may reflect their held beliefs about what mathematics is as a discipline and what it means to teach mathematics. As in their discussions, I adopt Stylianides and Stylianides's (2009a) convention of referring to conceptual awareness pillars as *CAPs* hereafter.

The theoretical framework used in this chapter to inform the understanding and explanation of PSETs' belief changes and the mechanisms that promote them is grounded in the theories of conceptual change and situated cognition. Conceptual change theory supports the element of cognitive conflict in the conceptual framework and describes the process that occurs "when the new information to be learned comes in conflict with the learner's prior knowledge usually acquired on the basis of everyday experiences" (Vosnadiou & Lieven, 2004, p. 445). Swars, Smith, Smith, and Hart (2009) suggested that in order to have "successful paradigmatic changes in (pre-service) teachers' beliefs and teaching practices" (p. 52), it is necessary for TEPs to create opportunities that: (1) generate interest in change; (2) problematize current practices and propose possible solutions; (3) allow experimentation with those possible solutions; and (4) reflect on the outcomes for students and teachers.

The situated cognition theory (Lave & Wenger, 1991) of learning was briefly introduced in Chapter 1 and also supports an analysis utilizing the elements of the conceptual framework outlined above. From this perspective, learning is seen "as a process of enculturation, or participation in socially organized practices, through which specialized skills are developed by learners as they engage in an apprenticeship in thinking (Rogoff, 1990) or in legitimate peripheral participation (Lave & Wenger, 1991)" (Scott, Asoko, & Leach, 2007, p. 45). In this case, pre-service teachers experience enculturation into a community of mathematics educators through engagement with real aspects of the work of teaching through P-R mathematics tasks (as described in Chapter 2) that ask them to, for instance, understand and explain sample student work, judge the appropriateness of definitions in textbooks, and analyze common student errors.

ELEMENTS OF FRAMEWORKS

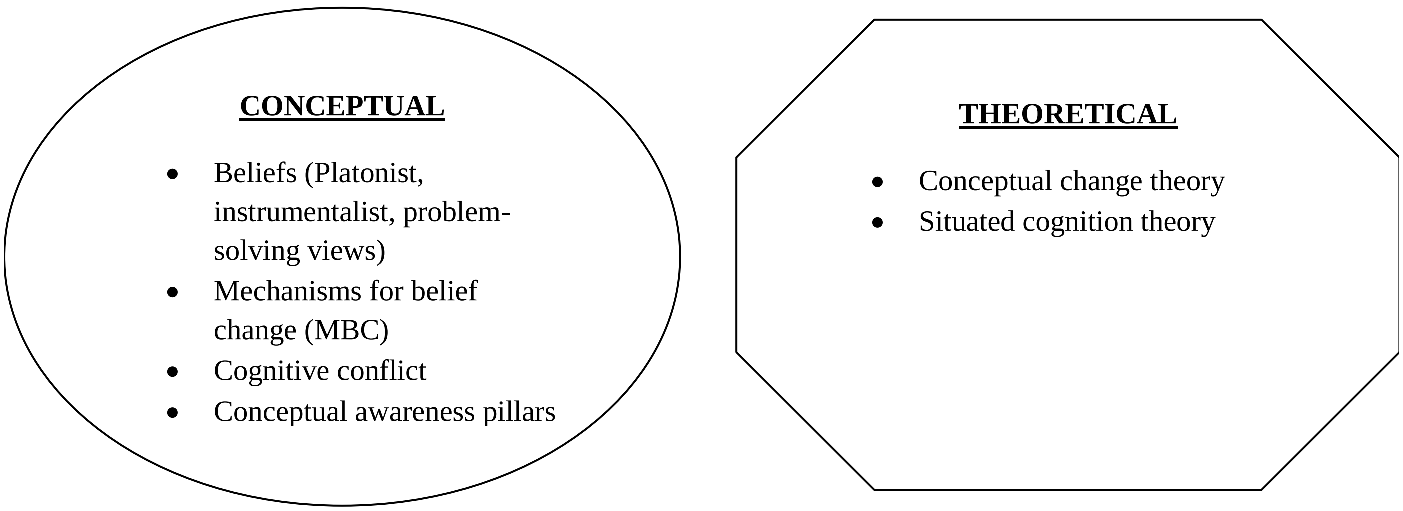

Figure 4.1: Elements of Conceptual and Theoretical Frameworks

4.3 METHODOLOGY

This chapter utilized both quantitative and qualitative data for its analyses. Broadly speaking, research using qualitative data "produces findings not arrived at by means of statistical procedures or other means of quantification" (Strauss & Corbin, 1990, p. 17), allowing for researchers in education to enhance the understanding of learning and teachers, whereas quantitative means may aim to correlate or predict. Typically more rich in detail, qualitative data is beneficial to research questions that aim to investigate descriptions of certain phenomena (addressing "what?") as well as those interested in the processes behind those phenomena (addressing “how?”) by using such data collection techniques as interviews, observations, and gathering of artifacts.

Considering the above description of qualitative research, it is appropriate for the present study to use both quantitative and qualitative data given the research questions outlined in Section 4.1. In order to examine complex entities such as beliefs and belief change, rich and detailed descriptions were needed that could not be captured fully with quantitative measures alone. Furthermore, several researchers in the area of beliefs (e.g. Richardson, Anders, Tidwell, & Lloyd, 1991) have openly criticized empirical studies that have relied primarily on quantitative measures, such as scored questionnaires or surveys, to identify and analyze the beliefs of their participants.

4.3.1 Research setting

The data described and analyzed in this chapter was collected during a one-semester research-based mathematical content course for PSETs taught by the author at a large university in the east. While the setting was certainly a convenient one, the rationale for its choice was far more deliberate and will be further elaborated in Sections 4.3.1.1 and 4.3.1.2. Although the university does not offer undergraduate degrees in education, the course serves as a prerequisite for undergraduates who plan on pursuing admission into an elementary teaching certification program, and is the only mathematics content course specified in the program requirements. This course offered the only opportunities for the participating PSETs to engage in the development of mathematical knowledge specific to the work of teaching, which contributed to it being an ideal context to investigate my research questions.

Developed by Stylianides and Stylianides (2009a, 2009b, 2010) to promote pre-service teachers' 'mathematical knowledge for teaching' (Ball & Bass, 2000) as discussed briefly in Chapter 2, the 3-credit course covered a variety of mathematical topics in the elementary mathematics curriculum, including algebra, geometry, and number theory. Furthermore, the course purposefully made connections between the mathematical knowledge being developed and the ways in which that knowledge can be applied to the work of teaching. The PSETs came from a variety of mathematical backgrounds (often weak) and majors, and had no previous mathematical pedagogy coursework. The classroom worked from a very specific instructional methodology (further described in Section 4.3.1.1), and was organized into small, flexible groups of 4-5 students and supported a problem-solving environment. Rich problems were often used in an effort to introduce the students to a variety of mathematical content relevant to the elementary grades. These problems were first attempted by individual PSETs, after which the PSETs shared and discussed their ideas within the small groups, and ultimately this would lead to a whole class discussion. There was no textbook used in the course. Rather, the curriculum materials used were specifically designed by Stylianides and Stylianides (2009a, 2009b, 2010) to address the mathematical knowledge needed for teaching and will be described in more detail in Section 4.3.1.1. Open communication and both student-instructor and student-student interaction was a common occurrence.

4.3.1.1 Further background on the mathematics content course

Although every elementary TEP in this country contains at least one mathematics content course, the particular course that serves as the setting for this study is not necessarily typical. The result of Stylianides and Stylianides's (2009b) five-cycle 'design experiment' (see, e.g., Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Schoenfeld, 2006), this course was designed with the specific goal “to develop, implement, and analyze the effectiveness of instructional sequences to promote prospective teachers’ ‘mathematical knowledge for teaching’ (see, e.g. Ball & Bass, 2000; Hill et al., 2005),” (Stylianides & Stylianides, 2009b, p. 241), and is offered through the School of Education. In the United States, it is traditionally the case that such a course is offered through the Department of Mathematics (Davis & Simmt, 2006). The kind of specialized knowledge targeted in the course addresses content meant to better inform the future teachers’ practice by introducing them to mathematical issues that may take place in the classroom. Although textbooks in general are important sources of mathematical beliefs in content courses for PSETs, no textbook was used in this particular course. I provide below further details about the curriculum materials used in the course, including the notion of ‘Pedagogy-Related (P-R) mathematics tasks’ (Stylianides & Stylianides, 2010), which had a prominent place in the curriculum of the course.

Stylianides and Stylianides (2010) described three key features of P-R mathematics tasks: (1) a ‘primary mathematical object,’ such as the formation or validation of a conjecture or a discussion of correspondences between different solution methods; (2) a ‘focus on important elements of mathematical knowledge for teaching,’ such as proof and the language used with definitions, which have their grounding in the relevant literature about this specialized knowledge (Ball & Bass, 2000); and (3) a ‘secondary but substantial pedagogical object and a corresponding pedagogical space,’ which is the defining feature of P-R mathematics tasks. In these tasks, “the pedagogical object is substantial (i.e., it is an integral part of the task and important for its solution) and situates the mathematical object of the task in a particular pedagogical space, which relates to school mathematics and may derive from actual or fictional classroom records" (Stylianides & Stylianides, 2010, p. 164). This feature ensures that the task is grounded in the real work that the PSET will one day engage.

Given the various discussions about Ernest's (1988) three mathematical views (Platonist, instrumentalist, and problem-solving) used throughout this dissertation, the curriculum designed for this particular mathematics course appears to encourage instruction that aligns primarily with the problem-solving view. Indeed, the tasks comprising it intentionally engage the PSETs in the process of actively doing mathematics grounded within a constructivist philosophy.

Stylianides and Stylianides (2009b) describe several major aspects of the instructional approach that resulted from their repeated implementations of the course, with three of those aspects of particular interest for my work. First, they chose specific tasks and their related sequencing as a means to create cognitive conflict (Zazkis & Chernoff, 2008), offering the PSETs opportunities to engage with mathematical activities that "contradicted their current understandings and encouraged reflection on those contradictions" (Stylianides & Stylianides, 2009b, p. 242) that may promote change in those initial understandings. Recall that cognitive conflict was identified as one successful mechanism of belief change (MBC6) earlier in Table 4.1.

The second aspect of the approach used in this course deals with the classroom features and socio-mathematical norms that support the resolution of the cognitive conflicts. This support is offered by 2 different sources: (1) fellow PSETs within the class; and (2) the instructor, or teacher educator, of the course. PSETs are encouraged to actively engage in mathematical conversations with their peers, while the instructor serves as the "representative of the mathematical community in the classroom" (Stylianides & Stylianides, 2009b, p. 242). Enactment of this role is through the instructor's interactions with the PSETs during which the instructor may provide scaffolding for tasks when necessary and also support in developing necessary mathematical knowledge that may be impeded by the students' prior misconceptions and difficulties. This aspect permits the enactment of several mechanisms of belief change, such as the creation of a problem-solving environment grounded in inquiry (MBC1), the opportunity for reflection (MBC2), and group work (MBC3).

The third aspect of this course that is relevant to my work is that it was developed to "contextualize prospective teachers' mathematical work in realistic pedagogical situations" (Stylianides & Stylianides, 2009b, p. 242). Many of the activities in the curriculum of the course (e.g. P-R mathematics tasks) engaged the students at two levels. At the first level, the PSETs worked with the mathematical tasks themselves as learners of mathematics, while the second level shifted their perspective to that of the teacher and allowed them to connect their work on the task to the authentic work of teaching. This second level utilized tools such as videos of elementary students working similar mathematical content and sample student work, and directly reflects the activities grounded in the work of teaching (supporting the situated cognition theory of learning) shown to promote belief change (MBC5).

Although the research questions previously explored in the context of this course primarily investigated PSETs' experiences with reasoning-and-proving, conclusions drawn there by Stylianides and Stylianides (2009b) speak directly to the rationale for using the course as a context to study mathematical beliefs:

> The use of activities that ask students not only to express their mathematical ideas but also to evaluate these ideas *can have broader implications for the epistemic basis of belief in a classroom community*. In particular, such activities can shift the process of validating new knowledge in the community away from practices that rely on the authority of the instructor toward practices that promote students' active engagement in the validation process (p. 251).

The three instructional features of the content course outlined above (in addition to two others not mentioned here) have led to interesting and encouraging findings about the development and change in PSETs' understandings of proof. In Table 4.2 below, I summarize the three main features of the instructional approaches that are relevant to the explorations in this chapter and relate to them specific elements of the conceptual and theoretical frameworks described in Figure 4.1. These connections further support the hypothesis that the instructional approaches employed in this course make it an ideal environment in which to pursue investigations related to the research questions.

Table 4.2: Features of the Course Relevant to the Conceptual and Theoretical Frameworks

FEATURE OF THE COURSE	**RELEVANT ELEMENT(S) OF CONCEPTUAL FRAMEWORK**	**RELEVANT ELEMENT OF THEORETICAL FRAMEWORK**
Feature 1: Curriculum materials that create cognitive conflict	Cognitive conflict, CAPs, MBC1, MBC6	Conceptual change theory
Feature 2: The role of the teacher educator as a "representative of the mathematical community" and creator of a classroom environment to support the resolution of cognitive conflict (with help from peers)	MBC1, MBC3, MBC2	Situated cognition theory of learning
Feature 3: Curriculum materials (P-R mathematics tasks) grounded in the real work of teaching	MBC5	Situated cognition theory of learning

4.3.1.2 Personal history with the course

Since the instructional aspects of the course are fundamental to both the conclusions already made by Stylianides and Stylianides (2009a, 2009b) regarding PSETs' understanding of proof and to my investigations of belief change in the present study, it is important to provide the reader with critical background information relating to my history with the course, as well as provide evidence of my ability to provide instruction that closely aligns with the vision described by the creators of the course.

Although I played no role in the development and refinement of the instructional approach and the set of curriculum materials developed by Stylianides and Stylianides (2009a, 2010) for the course, that is not to say that my experience with it is limited or that my instruction of the course greatly deviates from the developers' description of the last research cycle of the design experiment. As a beginning doctoral student, I had the opportunity to observe three full enactments of the course with one of its developers as the instructor prior to teaching the course myself. During my observations, I kept detailed notes for my own records relating to the activities that shaped the classroom discussion, including the various solutions offered by the PSETs in the class and particular issues that were raised. I also paid special attention to the instructional moves and facilitation of the curriculum materials, as well as the various classroom interactions that included instructor-student and student-student exchanges (Features 1 and 2 in Table 4.2). In addition to my personal notes, I took field notes to capture the content of both small and large group discussions during each class at the request of the instructor as part of the data collection contributing to the research discussed in Stylianides and Stylianides (2009a, 2009b). These observations were accompanied by regular meetings with the instructor to debrief lessons, discuss any questions I had about the course, and to talk about the goals of his research.

Despite the fact that this work was not specifically geared to investigate the beliefs of the PSETs enrolled in the course, there were certain features of the data collection instruments that addressed those beliefs, specifically beliefs surveys administered and written responses to short-answer questions collected throughout the semester. The survey consisted of 25 Likert-items and several short-answer questions aimed at gaining a better understanding of the PSETs' thoughts about reasoning-and-proving. However, several items provided more general information about the PSETs' beliefs, and it was these items that drew my attention and fueled my research interests. I was given the opportunity to conduct end-of-the-semester interviews with the PSETs during which they discussed not only their ideas about reasoning-and-probing, but also the ways in which the course influenced their beliefs about what mathematics is and what it meant to learn and teach mathematics. As I moved from the role of the observer to that of the instructor of the course, my interest in the influence of the course on the beliefs held by the PSETs was further substantiated. PSETs in my classes consistently revealed to me (through their responses to belief surveys, written reflections about class activities, and personal conversations) evidence that belief change had occurred as a result of the course. The desire to further investigate the process and related mechanisms for this change forms the foundations of the research in this chapter.

Since first observing this course, I have had the opportunity to teach it a total of 17 times during the past four years. I have purposefully made every effort to implement the instructional method as described in Stylianides and Stylianides (2009b, 2010) and feel confident that my instruction of the course very closely aligns to that which I first observed. This is important considering that this instruction has shown to produce very promising results that have led me to believe this context ideal for my own research. During my own experiences as course instructor, I was able to pilot preliminary versions of some of the data collection instruments used in the present study, including variations in the beliefs survey modified to more clearly address issues related to mathematical beliefs. Considering my personal history with the course and the history of the course itself, I feel confident that it provided a unique and promising environment in which to study the mathematical beliefs of the PSETs enrolled in it and the impact that particular experiences and activities have on those beliefs.

It is important here to acknowledge the realization that it was impossible to implement instruction in a way that is completely devoid of my own personal beliefs about mathematics that may somehow impact the students in my class. Even though I consistently made efforts to avoid sharing my own thoughts and beliefs during class or individual discussions with the PSETs, I am aware that the decisions I made about the activities used and the ways in which I structured the PSETs' engagement with and discussions about those activities implicitly may have conveyed an alignment with particular mathematical beliefs.

4.3.2 Participants

The class consisted of 26 students, 5 males, and 21 females. There were 9 freshman, 5 sophomores, 5 juniors, 4 seniors, and 3 graduate students in the class. Out of the 26 PSETs, 7 had never taken a math class at the university before, 11 had taken one class, and the remaining 8 had taken two or more. The limited number of possible participants contributed to the decision to employ mostly qualitative methods since they allowed for a more detailed investigation of the beliefs and process of potential belief change in the participants. On the first day of class, I gave each PSET a handout that provided him or her with information about the research study which included a description of the purpose of the study, the methods to be used for the data collection, and the assurance of participant confidentiality (See Appendix B). Data collection measures included beliefs surveys, responses to prompts in homework assignments throughout the semester, and entries in reflection journals that were integrated into the PSETs' normal coursework. Since the data to be analyzed in this chapter was regarded as normal coursework, with regular feedback given to all PSETs as the materials were collected, I feel that I was able to maintain my integrity as both the researcher and the instructor of the course. The handout also indicated that I would be randomly requesting individual interviews with a sample of PSETs twice during the semester on a strictly voluntary basis, with a small stipend promised to those who completed both interviews. The PSETs were encouraged to let me know if they were not interested in participating in the individual interviews at this time, and their names were noted. Moreover, I invited everyone, not just the randomly selected PSETs, to speak to me about their experiences and beliefs at any time during the course if they so desired (without the promise of a stipend).

From the class of 26, I chose ten PSETs randomly for the initial individual interviews, and contacted these PSETs via email requesting participation. If one PSET chose not to participate, another was chosen randomly and a request for participation was made. This process continued until I secured ten willing participants. Out of these ten PSETs, six were able to meet with me for both individual interviews at the beginning and end of the semester, and only the interview data from these six PSETs were considered for use in my analysis. No other PSET took advantage of the opportunity to individually speak to me about his or her experiences and beliefs throughout the course.

Besides the interview data, written data from all PSETs was collected for analysis. Although the purpose of this chapter is to better understand the beliefs of the whole group of

PSETs enrolled in this particular mathematics content course and the influential activities impacting those beliefs, the individual interviews were helpful in my efforts to provide a more in-depth and detailed illustration of the trends found in the whole-class data.

4.3.3 Measures used in data collection

I employed a variety of data collection methods to allow the PSETs to express their views and describe their experiences in different ways. This was advantageous for a couple of reasons. First, it allowed me to engage with the PSETs in several, non-intrusive ways. Second, it provided a means of triangulation for the data. Adopting several of the measures used in previous studies on mathematical beliefs, the four primary measures employed were: (1) beliefs surveys; (2) responses to written prompts regarding beliefs and experiences with course activities; (3) reflection journals; and (4) individual interviews. In this section, I elaborate upon these data sources and describe the ways in which they inform RQ 3.1 and 3.2. Table 4.3 below displays the two research questions, the data sources that contribute to an understanding of each research question (which are identified as being either primary or illustrative data sources), and from what group of students the data was collected.

Table 4.3: Research Questions and the Relevant Data Sources Used to Inform Each

RESEARCH QUESTION	**DATA SOURCES (*PRIMARY OR ILLUSTRATIVE*)**	**COLLECTED FROM:**
RQ3.1: What are the participants' initial mathematical beliefs?	(1) Likert-items (*Primary*) (2) Responses to prompts (*Primary*) (3) Individual interviews (*Illustrative*)	(1) Whole class (2) Whole class (3) Six randomly selected PSETs (with data from three discussed in the chapter)
RQ3.2: What is the extent of belief change in the course and what are the specific mechanisms of this change?	1) Likert-items (*Primary*) (2) Responses to prompts (*Primary*) (3) Individual interviews (*Illustrative*) (4) Reflection journals (*Illustrative*)	(1) Whole class (2) Whole class (3) Six randomly selected PSETs (with data from three discussed in the chapter) (4) Whole class

4.3.3.1 Measures informing RQ 3.1

As indicated by Table 4.3, three of the measures were used to inform RQ 3.1: (1) Likert-items from the Beliefs Surveys and homework assignment 1; (2) written responses to prompts from the Beliefs Surveys and homework assignments; and (3) individual interviews. I provide more details about these three measures in turn below, and describe the ways in which they contributed to my investigation of RQ 3.1.

Surveys have been used extensively to gather information about large populations in all areas of research, including educational research in mathematics (e.g. Kuhs & Ball, 1986; Peterson, Fennema, Carpenter, & Leof, 1990; Raymond, 1997) and beliefs research in particular (e.g. Nesbitt Vacc, & Bright, 1999; Perry et al., 1999). Indeed, the population being studied presently is not large enough for meaningful analysis about the mathematical beliefs of the participating PSETs. However, the quantitative data collected via the Likert-items from the surveys and homework assignment 1 I describe in the following paragraphs provided helpful insights into the PSETs' initial mathematical beliefs (the focus of RQ 3.1).

Surveys are useful and appealing in their ease of distribution and analysis. There are several benefits to having research participants respond to the same exact items, independent of potentially influential factors such as personal dynamics between an interviewer and the participant and unintentional variations in the items that may result from these dynamics. Despite these benefits, it is also necessary to acknowledge the limitations that accompany the use of surveys. Although the written statements are identical for each participant taking the survey, it is impossible to know whether they will all interpret the statements in the same way.

The Initial and Final Beliefs Surveys (capitalized hereafter to denote my particular measure for data collection) were modified from a version utilized by Stylianides and Stylianides (2009a, 2009b), with several of their original Likert-items replaced with those previously established and validated in Kajander's (2007) 'Perceptions of Mathematics' (POM) survey. Likert-items are often used in educational research, as they are easy to both administer and complete. The Initial Beliefs Survey was piloted with the new items during my instruction of the course the semester before this data collection with promising results. For this study, the survey was administered twice, first during the initial class meeting (the Initial Beliefs Survey) and then on the second-to-last day of class (the Final Beliefs Survey). The Initial Beliefs Survey can be found in Appendix C.

The Initial Beliefs Survey consisted of 25 Likert-items, and the Final Beliefs Survey consists of the same 25 Likert-items, plus 12 more that were included in homework assignment 1. Therefore, there were a total of 37 Likert-items administered. Many of these items were meant to probe the PSETs' beliefs about what doing mathematics meant to them and what they believed was important in mathematics learning and teaching. Additionally, a selection of the items provided useful information about how the participants aligned with the three views about the nature of mathematics that was central to the analysis of this chapter. Each item was measured on a 4 point scale (1=very true; 2=sort of true; 3=not very true; 4=not at all true). Although an elaborated survey was used in the course, many of the Likert-items were not directly relevant to the research questions. I only present analysis and discussion of the 16 items that directly contributed to my investigations. These 16 items can be found in Appendix D. Items 1 through 4 provide information regarding the Platonist view, items 5 through 10 regarding the instrumentalist view, and items 11 through 16 to the problem-solving view. While other items included in the surveys provided information regarding the PSETs' more general beliefs and attitudes (e.g. "I like doing mathematics"), only those directly contributing to an understanding of their views with respect to the three views were analyzed in this chapter.

The second measure informing RQ 3.1 contributed data from written responses to three prompts in the Beliefs Surveys and homework assignments. Table 4.4 displays the prompts, which will be referred to by their numbers (e.g. Prompt 1, Prompt 2, Prompt 3) in later sections. I discuss only Prompts 1 and 3 in this section. I will discuss Prompt 2 in Section 4.3.3.2.

Table 4.4: Prompts Used in Data Collection and Discussions

PROMPT NUMBER	STATEMENT OF PROMPT	INCLUDED IN
Prompt 1	What is mathematics?	Initial Beliefs Survey, Final Beliefs Survey
Prompt 2	Identify three activities from the course that had the most significant contribution to your learning and impact on your beliefs about mathematics. Order the three activities beginning with the one that contributed the most. Write a paragraph (at least 100 words) for each activity, explaining why you chose these activities.	Homework Assignment 5
Prompt 3	If you become a teacher, you will want your students to have good experiences learning mathematics. What are three important features of the experiences you'd like to offer them? Elaborate on each feature and mention some examples of the experiences you consider important.	Initial Beliefs Survey, Homework Assignment 5

Present in the Initial and Final Beliefs Survey, Prompt 1 was used to gather information about the participants' views about the nature of mathematics in their own words. Participants' initial responses to the prompt in particular informed RQ 3.1, as they indicated their initial views about the nature of mathematics. This qualitative data complemented the quantitative data gathered about these views using the Likert-items. Prompt 3 was given in homework assignment 1 and homework assignment 5 (the final one in the course), and provided information related to RQ 3.1 from the Initial Beliefs Survey. Although there is no explicit mention of beliefs in this prompt, it did illuminate aspects of the mathematics classroom valued by the PSETs at the beginning of the course, which in turn was interpreted as reflecting their held beliefs about the learning and teaching of mathematics at the beginning of the course.

Finally, individual interviews represent the third measure used to inform RQ 3.1. Although measures such as surveys and written responses to prompts provided vital information about the research participants (as discussed earlier in Section 4.4.2.1), interviews proved to be a fruitful method of gathering qualitative data that could be analyzed in parallel to the quantitative data. Interviews permitted a small selection of the PSETs to speak about, elaborate on, and clarify their responses given on the Beliefs Surveys and to the various prompts in the Surveys and homework assignments. Speaking to the six PSETs not only allowed me to better understand the possible interpretations of the Likert-items from the Beliefs Surveys and homework assignment 1, but they also provided me with further details about their prior experiences and initial beliefs (which directly informed RQ 3.1). Since these details were not available for the entire class, these six participants served as possible illustrations and foundations for discussions regarding the whole-class trends found in the analysis. Specifically, interview data from three of these PSETs will be described in this chapter to serve as illustrations for three emergent trends regarding changes in mathematical views throughout the course.

I conducted interviews with the six PSETs at two different occasions: (1) immediately following the first week of class after the Initial Beliefs Survey and homework assignment 1 had been collected; and (2) during the last two weeks of the semester. No new questions were asked during this interview, and only the PSETs' previously submitted surveys and responses to prompts were discussed. Each interview was audio recorded with the participants' permission.

The first interview directly informed RQ 3.1 and lasted approximately 45 minutes. The major focus of this interview was on the responses from the Initial Beliefs Survey and homework assignment 1. This interview provided the opportunity for me to better understand the beliefs held by the PSETs at the beginning of the course and the previous mathematical experiences that may have contributed to the formation of these beliefs. It was imperative that these interviews be conducted as soon as possible after the first class, as the focus was to gather information about the beliefs and experiences of the students before having been exposed to the ideas and the culture of the course.

4.3.3.2 Measures informing RQ 3.2

As indicated by Table 4.3, all four measures were used to inform RQ 3.2: (1) Likert-items from the Beliefs Surveys and homework assignment 1; (2) written responses to prompts from the Beliefs Surveys and homework assignments; (3) individual interviews; and (4) reflection journals. I describe the ways in which all four measures contributed to my investigation of RQ 3.2, and provide further details about the reflection journals since this was the only measure not addressed in Section 4.3.3.1.

The quantitative data collected via the Likert-items from the Initial and Final Beliefs Surveys were used to monitor changes in those beliefs throughout the course. In particular, the means and differences of means of the whole class's responses to the 16 Likert-items found in Appendix D corresponding to the three different views about the nature of mathematics were calculated. These were then used to identify any changes that occurred as an outcome of the course.

The second measure informing RQ 3.2 contributed data from PSETs' written responses to Prompts 1, 2, and 3 in the Beliefs Surveys and homework assignments. Qualitative data was collected from these prompts given to the participants at different, specified points of time during the course with nearly identical phrasing used at each point. It was important that the prompts maintain their phrasing in order to monitor and establish any changes that occurred between their administrations, which corresponded to changes that were evidenced upon completion of the course. Prompt 1 was used to gather information about the participants' views about the nature of mathematics in their own words in the Initial and Final Beliefs Surveys, and therefore also helped me to identify changes in the beliefs of the PSETs. These responses also allowed me to identify specific trends in the changes of beliefs about the nature of mathematics found in the whole-class data.

Prompt 2 also informed RQ 3.2 and it was given at three occasions during the course: (1) on homework assignment 2; (2) on homework assignment 3; and (3) on (the final) homework assignment 5. Each occasion of this prompt asked the PSETs to discuss particular activities in the course up until the time the prompt was given that stood out to them and impacted their mathematical beliefs. The number of activities they were asked to choose varied with each administration. On the homework assignment 2, the PSETs were asked to choose two activities from a list provided to them detailing all course activities with which they engaged before completion of the assignment. For homework assignment 3, they were asked choose *at least* two from a new list that contained all the course activities up until that point in time (with repetitions from homework assignment 2 permitted). The final prompt given on homework assignment 5 is included in Table 4.4. I requested that the participants order these three activities in homework assignment 5 in order to better understand the value they placed on each activity. A complete list of the course activities can be found in Appendix E. Only the homework assignment 5 data was analyzed in this chapter.

Prompt 3 from the Initial Beliefs Survey and (final) homework assignment 5 also aided to my investigation of RQ 3.2. A comparison of the responses given at the beginning and at the end of the course helped to illuminate aspects of the mathematics classroom valued by the PSETs, which in turn can be interpreted as reflecting their changing beliefs about the learning and teaching of mathematics.

Individual interviews represent the third measure used to inform RQ 3.2. Since this research question related to understanding the process of belief change of the PSETs and the class activities that motivated that change, the information gathered in the first interview (described in the previous section) served as the foundation to making sense of that process for this sample of participants, with the influence of the course not yet a factor. A second and final interview with the six PSETs was conducted during a two-week period which included the last week of class and finals week before grades were submitted. Each interview lasted approximately 30 minutes. During the last week of class, the Final Beliefs Survey was administered to the whole class, and was an important focus of the second interview. Before these final interviews were conducted, the PSETs were informed that our conversation would be comparing differences in their responses to prompts from the beginning and end of the semester (see Appendix F for the document given to the participants prior to this interview), as well as discussion of differences that they noticed in their responses to the Likert-items. In preparation for the interview, the participants were given copies of all their relevant responses to review.

The fourth and final measure informing RQ 3.2 was reflection journals kept by the PSETs. As with surveys, journals have often been used as a data collection measure in studies similar to the one described in this chapter (e.g. Mewborn, 1999; Nesbitt Vacc, & Bright, 1999; Schuck, 1996). In addition to providing a means by which the PSETs could reflect on mathematical activities and their experiences with particular concepts, the reflection journals also provided me with insight into those experiences of primary interest to RQ 3.2. These journals, like the interviews, were not considered to be a major data source and were primarily used to illustrate findings from the whole-class data.

The PSETs made entries in the reflection journals regarding their experiences with individual activities in the course, as well as units that focused on a particular theme. While some activities were completed during a single class meeting, others took several classes to complete. In both cases, the journal entry associated with the activity or unit was completed at the end of the activity or unit. In each entry in the reflection journal, the PSETs were asked to share their general thoughts about the activity or unit by responding to two prompts in the journal: (1) *What particularly stood out to you in the activity/unit we've just completed?*; and (2) *Is there anything that is still unclear to you from our work with this activity/in this unit*? Oftentimes, the PSETs would identify a particular activity as they reflected on their experiences with an entire unit. Before each entry, the PSETs were reminded to attend to any changes in their mathematical beliefs that may have come about as a result of the activity or unit on which they were reflecting. In total, there were 5 entries in the journal. One entry in the reflection journal was different from the others. The entry corresponding to "The Blonde Hair Problem" (discussed in more detail in Section 4.5.3) consisted of two CAPs that elicited two reflections, one before and one after the completion of the problem. The first CAP used the following two prompts: (1) *Describe your initial reactions to "The Blonde Hair Problem"*; and (2) *Does this problem differ in any way from other problems you have encountered in the mathematics classes you have taken so far? If so, how?.* The second CAP used these two prompts: (1) *Describe your experiences working with "The Blonde Hair Problem"*; and (2) *Did this experience give you any new ideas that might be useful to you as a prospective teacher?*

The importance of reflection in TEPs has been discussed in the literature (Artzt, 1999; Nesbitt Vacc, & Bright, 1999), and has also been identified as a possible mechanism for belief change (MBC2 in Table 4.1). The reflection journals data often provided rich descriptions of the participants' thoughts and ideas in relation to specific activities they considered influential in the course, and impacted in some way their mathematical beliefs.

4.4 DATA ANALYSIS

Although emerging themes in the data primarily drove the coding for the qualitative data analysis, I also used pre-established codes rooted in the literature regarding different views about the nature of mathematics (Ernest, 1988) as well as those rooted in my own prior experiences in teaching the course. I organize this section by describing the data analysis related to each of the four measures for data collection described in Section 4.3.3.

4.4.1 Analysis of the Likert-items on the Beliefs Surveys and homework assignment 1

All participating PSETs' responses to the relevant Likert-items from the Initial and Final Beliefs Surveys and homework assignment 1 (found in Appendix D) were entered into a spreadsheet as the first step in the analysis of the quantitative data. Initially, I grouped the items according to the three views to which they related and simply explored the data for interesting patterns and findings related to individual items. The more formal analysis came after, where I calculated the statistical means and differences of the means of the whole-class responses to investigate changes between the responses given at the beginning of the course and those given at its conclusion corresponding to the three views. The findings related to these relevant items will be described in Section 4.5.1. The results of the analysis from the Initial Beliefs Survey informed RQ 3.1, while the analysis of both Surveys informed RQ 3.2.

4.4.2 Coding and analysis of responses to prompts in the Beliefs Surveys and homework assignments

There are two main types of coding that I performed, often sequentially, on the qualitative data related to responses to prompts (either in the Beliefs Surveys or from homework assignments): 'a priori' coding and 'inductive' coding. 'A priori' codes are described as those developed before the actual data was examined and grounded in theory and relevant literature. Revisions to a priori codes were made as necessary when explorations uncovered data that did not clearly fit into an existing category in the codes, and new codes were generated 'inductively' so that the categories could maximize mutual exclusivity and exhaustiveness (Weber, 1990). For the final analysis, both these a priori and inductive codes were used.

For example, the codes described in Table 4.5 below were developed a priori in order to analyze the PSETs' responses to Prompt 1 from the Initial and Final Beliefs Survey. I wanted to describe their self-reported views about the nature of mathematics as relating to either the Platonist, instrumentalist, or problem-solving view, and the codes were based on the descriptions of these views from the literature (elaborated in the previous 2 chapters and summarized in Section 4.2). Also included in Table 4.5 are sample responses to illustrate each of the three codes. It was common to find that a single response would have several statements that included ideas potentially related to different views, and so several codes were needed to capture them all, where each allusion to any of the views received a code. However, I discovered in my analysis of the data that in these situations, it appeared that one view was more strongly supported. In order to better capture this occurrence I introduced the notions of *primary* and *secondary* views. The primary view represents the view favored more strongly by the participant as indicated by the larger number of codes reflecting the particular view. The secondary view represents the view supported by the participant as indicated by the presence of the codes reflecting the view with the second-highest frequency. It was possible for a single response to contain codes from all three views, but I only considered these primary and secondary views.

Table 4.5: A Priori Codes Relating to PSETs Ideas about the Nature of Mathematics

CODES	LITERATURE DEFINITION	ILLUSTRATION
Platonist (PL)	Mathematics is characterized as a static but unified body of knowledge where interconnecting structures and truths play an important role. Mathematics is NOT created. Historical aspects may be highlighted.	"Math is just something that has always been done." "Math relates things in certain ways that need to be shown to you by somebody who already knows them."
Instrumentalist (IN)	Mathematics is seen as a useful but unrelated collection of facts, rules, formulae, skills and procedures. One important feature is the disconnectedness of mathematical objects.	"Math is made up of numbers and a bunch of formulas that plug those numbers into to figure something out about a particular problem." "You have to be taught the right mathematical procedures in order to solve the problems."
Problem-solving (PS)	Mathematics is considered as a dynamic and continually expanding field in which creative and constructive processes are of central relevance. The individual plays an important role in the creation of mathematical knowledge.	"Mathematics is a creative endeavor which helps you understand the world around you." "Math is about trying to solve a problem using a variety of different techniques and trying to make sense of it."

Figure 4.2 below provides a sample response to Prompt 1 from a PSET, an illustration of the coding, and the identified primary and secondary views.

> Mathematics is everywhere around us (**PL**). It is a part of our everyday lives, and it keeps things in a specified order (**PL**). It involves a collection of facts, formulas, and ideas, as well as procedures (**IN**).
>
> **PRIMARY VIEW**: Platonist (2 codes) **SECONDARY VIEW**: Instrumentalist (1 code)

Figure 4.2: Sample Response to Prompt 1 and Related Coding

Unlike responses to Prompt 1, the other prompts in the Beliefs Surveys and homework assignments permitted the occurrence of a greater diversity in responses. Although my previous experiences in teaching the class allowed me to anticipate several of the PSETs' responses to these prompts, it was more likely in these cases that explorations of the data uncovered responses not accounted for in the a priori codes, and so new categories were created and included in the final code list.

An example of this situation came during the coding related to Prompt 3. Before any analysis of the data collected, I created several a priori codes related to the various responses I anticipated hearing (e.g. "make math fun," "the teacher should be open to student questions," "students should be able to work together") given the responses I had received from PSETs in my past experiences teaching the course. As I reviewed the Initial Beliefs Survey data regarding this prompt, I discovered several features (e.g., "focus on the history of math," "the class should be grouped by ability") that I had not anticipated. Therefore, in addition to the a priori codes established before any analysis, I developed a code for each new feature I encountered in the data. The coding of Prompt 2 followed a similar process. The emerging findings from the qualitative data also compared with the quantitative data to search for common themes and for triangulation purposes. For example, the qualitative analysis of responses to Prompt 1 suggested the primary and secondary mathematical views held by the PSETs which could then be compared to the quantitative findings from the Likert-items corresponding to the different views to identify agreements or disagreements.

4.4.3 The use and analysis of individual interviews

Two often cited issues related to the process of transcribing interviews are the tremendous amount of time it takes and the vast amount of data produced at its conclusion. Although it may take up to 40 pages to capture everything said during an hour-long interview, it is often the case that only a very small portion of the transcription captures data that is both rich and relevant to the research questions under investigation. Additionally, the interview data were meant to provide illustrations of the general, whole-class results (in particular, for three emergent trends related to changes in the PSETs' primary and secondary mathematical views), and were not the primary source for analysis. For these reasons, my analysis of the interviews employed partial transcriptions and were not rigorously coded. Each interview was audio-recorded, and afterwards I listened to the interview in its entirety to review the exchange. Following this review, I played back the recording while keeping a record summarizing the particular matters being discussed every minute or so using the recorder counter. In addition to permitting me to become more familiar with the data, this technique also allowed me to highlight particular points in the interview that were directly relevant to that which I desired to capture for my analyses and discussions. I kept track of these instances that I thought may be important by using circled numbers within the summaries, where the numbers would correspond to the research question or questions which were touched upon in that segment. These segments would then later be transcribed in their entireties when necessary to be used as illustrations of trends found in the whole-class data. In addition to avoiding the time spent transcribing extraneous data, this method also made more transparent the experiences and the particular ideas and activities that the PSET valued. Interview data from three PSETs are described in the chapter as illustrations of three major whole-class trends.

4.4.4 The use and analysis of reflection journals

Like the interviews, the reflection journals were not considered a major data source and were used primarily for illustrative purposes. In particular, the journals were used to provide further insights into the PSETs' thoughts related to particular class activities cited in response to Prompt 2. Once the data from this prompt was analyzed, I identified the three activities that stood out the most to the PSETs and had the greatest impact on their beliefs. I then used entries in the reflection journals that corresponded to these three activities for elaborations and qualitative illustrations of the findings of Prompt 2. The journals were not rigorously coded. Specifically, I analyzed the reflection journals to find common themes in the reflections of the whole class related to the three activities about why the activities stood out to the PSETs. I did this by reading the entries of each PSET in turn, creating a list of each aspect of their experience identified by the PSETs as standing out to them and impacting their mathematical beliefs (adding new categories as they emerged), and keeping a count of frequencies for each aspect. Excerpts from PSETs' journals were used to provide illustrations of the themes found.

4.4.5 Establishing reliability and validity

Triangulation, which is described as the collection and evaluation of data collected from several different sources, was used in order to make the findings reported in this chapter more valid and reliable. By collecting multiple and varied data sources (both quantitative and qualitative), I was able to make comparisons to establish agreements or discrepancies in the data as I attempted to address the different research questions. The quantitative measures employed were relatively straightforward, as they only entailed a calculation of means and differences of means. These means and differences of means will be discussed in Section 4.6.1.

Analysis of responses to Prompt 1 utilized a priori codes grounded in the descriptions of the three views about the nature of mathematics (Platonist, instrumentalist, and problem-solving) as described in the literature. In order to establish reliability in the results of this coding (which required both an analysis and interpretation of the PSETs' responses), I enlisted the help of a second coder not related to the study. After an informal conversation about the views and what each entailed, the second coder was given a copy of Table 4.5 (found in Section 4.5.2) and was asked to code *all* of the participants' initial responses to the prompt from the Initial Beliefs Survey in order to identify the *primary* and *secondary* views present, as these classifications were crucial to my analysis and related discussions. It was made clear that each response provided the opportunity for several codes, and that each allusion to any of the views should receive a code. The view that had the largest number of codes for each response was to be identified by the second coder as the primary view, and the view with the second largest number of codes (if there were others) was to be identified as the secondary view. Each response was permitted a single primary view and a single secondary view. In order to establish inter-rater reliability, a comparison was made between the primary and secondary views I identified for each participant and those identified by the second coder. The reliability (in percent agreement) for the primary views in the initial responses was 92% and for the secondary views was 84% for the initial responses from all participants.

A second coder was not utilized for the coding of Prompts 2 and 3. For these prompts, there was not any interpretation or analysis to be done, as was the case for Prompt 1. This coding was very straightforward and based solely on the PSETs' responses, and entailed keeping track of characteristics mentioned by the PSETs in their own words. If a new characteristic was found in the responses, I simply created a new category for it and kept a count of frequencies.

As with Prompts 2 and 3, a second coder was not utilized for interviews and the reflection journals. Recall that these two data sources were illustrative rather than primary, meaning that they were only meant to provide illustrations of trends found in the whole-class data. The decision about which interview data to share was based on three emergent trends regarding changes in primary and secondary views found in the whole-class data, and how that data may provide insight into the reasons for those changes. I did not explicitly code the interview data, and therefore there was no need for a second coder. Similarly, the reflection journals were only used to illustrate findings in the whole-class data regarding important elements of the PSETs' experiences with specific activities in the course. Although I did code the reflection journals, this coding was very straightforward (similar to the coding of Prompts 2 and 3) and based solely on the PSETs' reflections. Again, I kept track of characteristics of the experiences mentioned by the PSETs in their own words. If a new characteristic was found in the journals, I simply created a new category for it and kept a count of frequencies. Therefore, no second coder was utilized for the reflection journals.

4.5 RESULTS

In Section 4.5.1, I provide the results of the analysis of the relevant Likert-items to inform: (1) the different views held by the PSETs about the nature of mathematics (Platonist, instrumentalist, and problem-solving); and (2) changes in the PSETs' views. These results inform both RQ 3.1 and RQ 3.2. In Section 4.5.2, I provide results from the analysis of the PSETs' initial and final responses to Prompt 1 from the Beliefs Surveys that also inform both RQ 3.1 and RQ 3.2. In addition to the analysis of Prompt 1, I revisit the relevant Likert-items from the Beliefs Surveys to identify and discuss individual items that demonstrated significant change (of interest to RQ 3.2).

Sections 4.5.3 and 4.5.4 focus on only RQ 3.2. In these sections, I present results relating to Prompts 2 and 3. These results speak to the specific activities and aspects of instruction cited by the participants as having the most impact on their mathematical beliefs.

4.5.1 Findings related to the Likert-items

The data for this section came from the PSETs' aggregated responses to the Likert-items on the Beliefs Surveys and homework assignment 1. One of the PSETs enrolled in the course did not complete the Initial Beliefs Survey, and so all of her data were excluded (therefore the final sample size for all results was n=25). Each Likert-item corresponded to one statement about mathematics (addressing either the nature of the discipline, the learning of mathematics, or the teaching of mathematics) and utilized a four-point scale (1=very true, 2=sort of true, 3=not very true, 4=not at all true). Lower scores indicate agreement (1 and 2) with the statement about mathematics, and the higher scores (3 and 4) indicate disagreement with the statement.

Below I present the findings related to the Likert-items corresponding to each of the three views about mathematics. The descriptive statistics, which are only indicative of central tendencies, are presented in Tables 4.6, 4.7, and 4.8 regarding the Platonist, instrumentalist, and problem-solving view, respectively. In Sections 4.5.1.1 and 4.5.1.2, I describe the ways in which these results inform RQ 3.1 and 3.2, respectively.

Table 4.6: Likert-items Related to the Platonist View (n=25)

SCALE: 1=very true, 2=sort of true, 3=not very true, 4=not at all true	**MEANS**		
SURVEY ITEM	PRE	POST	POST-PRE
1. Some people cannot be good at doing math no matter how hard they try.	2.68	2.76	0.08
2. Mathematical facts exist independent of human activity.	2.48	2.92	0.24
3. The mathematical body of mathematics is fixed, and always has been.	3.04	3.04	0
4. I think that all mathematical knowledge is interconnected.	1.72	1.68	-0.04

Note: Agreement with these statements indicate an alignment with the Platonist view.

Table 4.7: Likert-items Related to the Instrumentalist View (n=25)

SCALE: 1=very true, 2=sort of true, 3=not very true, 4=not at all true	**MEANS**		
SURVEY ITEM	PRE	POST	POST-PRE
5. Mathematics is a collection of facts, formulas, and procedures.	1.6	2.16	0.56
6. To do well in solving math problems, I have to memorize all the formulas that are relevant.	2.4	2.84	0.44
7. To do well in solving math problems, I have to be taught the right procedure.	1.92	2.64	0.72
8. It is the teacher's job to teach the steps in each new math method to the students before they have to use it.	1.52	2.16	0.64
9. Mathematics is a useful tool primarily used for particular calculations.	1.88	2.36	0.48
10. Doing mathematics means memorizing particular rules and processes.	1.96	2.56	0.6

Note: Agreement with these statements indicate an alignment with the instrumentalist view

Table 4.8: Likert-items Related to the Problem-Solving View (n=25)

SCALE: 1=very true, 2=sort of true, 3=not very true, 4=not at all true	**MEANS**		
SURVEY ITEM	PRE	POST	POST-PRE
11. Mathematics is a creative human activity. $^{\alpha}$	2.62	2.04	-0.58
12. In mathematics you can be creative and discover by yourself things you didn't already know .$^{\alpha}$	2.16	1.68	-0.48
13. Math problems can be done correctly in only one way. $^{\beta}$	3.32	3.76	0.44
14. There are many different equivalent ways to define correctly a mathematical concept. $^{\alpha}$	1.68	1.6	-0.08
15. There is usually one best way to write the steps in a solution to a math question.$^{\beta}$	2.16	2.88	0.72
16. I think that mathematics as a discipline can be revised. $^{\alpha}$	2.58	2.12	-0.36

Note: Agreement with items designated with "α" indicate an alignment with the problem-solving view. Disagreement with items designated with a "β" indicate this alignment.

4.5.1.1 Results relevant to RQ 3.1: The PSETs' initial beliefs

The findings regarding the PSETs' initial beliefs about mathematics can be found in the "PRE" columns of Tables 4.6, 4.7, and 4.8. The numbers here represent the whole-class means for the Likert-items on the Initial Beliefs Survey. The means found in Table 4.6 show both alignment and nonalignment with the Platonist view. The statements of these items were chosen because they correspond to the Platonist view, and therefore agreement with the items would suggest alignment with that view. Indeed, alignment can be seen in the responses to items 2 and 4 (with both having rounded means of "2=sort of true" on the response scale), while the general disagreement with items 1 and 3 suggests nonalignment with the view (with both having rounded means of "3=not very true" on the response scale).

The means found in Table 4.7 show a strong alignment with the instrumentalist view. Again, the statements of these items were chosen because they correspond to the instrumentalist view, and agreement with them would suggest alignment with the view. The class means show agreement with all 6 of the statements with the rounded means of each of these items found to be 2 (meaning that the PSETs believed the statements to be "sort of true").

Finally, the means in Table 4.8 show both an alignment and nonalignment with the problem-solving view. These statements were chosen because they correspond in some way to the problem-solving view. Agreement with items 11, 12, 14, 16 suggests an alignment problem-solving view, while disagreement with items 13 and 15 would suggest this same alignment. The means displayed in Table 4.8 show whole-class agreement with items 12 and 14, as well as disagreement with items 13, which shows some support for the problem-solving view. However, the means also provide evidence of nonalignment with the view. Indeed, the class as a whole reported that they felt the statement of item 15, "There is usually one best way to write up the steps in a solution to a math question," was sort of true (as indicated by the rounded mean 2 of the responses), which shows a nonalignment with the problem-solving view. Further evidence of this nonalignment was found in items 11 and 16, where the results show a general disagreement with the statements "Mathematics is a creative human activity" and "I think mathematics as a discipline can be revised."

4.5.1.2 Results relevant to RQ 3.2: Changes in the PSETs' beliefs

Although it is not valid to read too much into the information provided in these tables, they do show some interesting changes in the PSETs' responses. Before any formal analysis of the data, I defined a substantial shift in response to an item as either: (1) a difference of at least 2 points on the scale, or (2) a shift in the rounded means from 2 to 3 (or vice versa, from 3 to 2). These shifts are considered substantial because they demonstrate a major modification in the way a participant thinks about a particular item, going from agreement to disagreement (or disagreement to agreement, depending). Although this shift represents only 1 point on the scale, the shift from 2 to 3 represents a shift in the class from believing the statement is "sort of true" to believing that the statement is "not very true," which is an important shift.

Likert-items related to the Platonist view and the quantitative analysis of the items can be found in Table 4.6 above. The findings indicate a substantial shift in the whole-class responses to the statement of item 2, "Mathematical facts exist independent of human activity." In the Initial Beliefs Survey, the mean of all responses was calculated to be 2.48 (rounded down to 2), which reveals a level of agreement with this statement from the class (recall that "2= sort of true"). At the end of the class, there appeared to be a shift towards disagreement of the whole class with the statement as indicated by the mean score of 2.92 (rounded up to "3= not very true"). There was little change in the class means related to items 1, 3, and 4. The class maintained a disagreement with the statements in items 1 and 3 and an agreement with the statement in item 4.

Displayed in Table 4.7 are the results of the quantitative analysis of the Likert-items related to the instrumentalist view. Analysis revealed a substantial shift at the conclusion of the course in responses to items 6, 7, and 10. The pre- and post-means of responses to these items show a change in the direction of disagreement (a rounded mean of 2 changing to a rounded mean of 3) with the statements "To do well in solving math problems, I have to memorize all the formulas that are relevant," "To do well in solving math problems, I have to be taught the right procedure," and "Doing mathematics means memorizing particular rules and processes." The other four items shown in this table experienced less substantial changes. Items 5, 8, and 9 maintained a level of agreement with the corresponding statements, having rounded means of 2 (sort of true) for the PSETs.

Finally, displayed in Table 4.8 are the results on the quantitative analysis of the Likert-items relating to the problem-solving view. Items 11, 15, and 16 demonstrated a substantial shift in the mean responses from the Initial and Final Beliefs Surveys and homework assignment 1. The results show a shift in the direction towards a general agreement with the statements "Mathematics is a creative human activity" and "I think mathematics as a discipline can be revised," as well as a shift towards a general disagreement with the statement "There is usually one best way to write up the steps in a solution to a math question." Although not classified as substantial, there was noticeable change in the response means of item 13. In the Initial Survey, the calculated mean (rounded value of 3) suggests that the class somewhat disagreed with the statement "Math problems can be done correctly in only one way." In the Final Beliefs Survey, the mean indicates a movement of the class's responses towards a stronger disagreement with the statement (rounded mean of 4). Both 3 and 4 on the response scale represent a level of disagreement, yet movement towards 4 indicated that the disagreement with this item for the whole group was stronger at the end of the course than it was at the beginning. Items 12 and 14 experienced little change in that both maintained their level of agreement from the whole class.

The data presented in this section indicated several substantial shifts in the PSETs' responses to the Likert-items relating to the three views about the nature of mathematics over the period of the course. Specifically, substantial shifts were seen in items 2, 6, 7, 10, 11, 15, and 16. The shift seen in item 13, though not classified as substantial, was certainly notable and perhaps worthy of further discussion. Based on only this quantitative data, the findings suggest that several of the mathematical views held by the PSETs were impacted. The reasons for these changes (or lack thereof) will be explored and discussed in conjunction with the related findings of the qualitative data analysis in Section 4.6.

4.5.2 Findings related to Prompt 1

Using the descriptions of the three views about the nature of mathematics, each response to Prompt 1 in the Initial and Final Beliefs Survey was coded in order to capture all the different views that appeared to be held by each PSET. The results of this coding often suggested an alignment with more than one view for each participant, and therefore both a *primary* and *secondary* view was identified for each PSET (as described in more detail in Section 4.4.2). Table 4.9 reports the results related to responses to the two administrations of Prompt 1 to the whole group.

Table 4.9: Findings (in Percentages) Related to an Analysis of PSETs' Responses to Prompt 1 (n=25)

View about the Nature of Mathematics	**Percentage of Codes Relating to Each View (and the Percentage Corresponding to Primary and Secondary Views)**					
	Initial Response	*Primary view*	*Secondary view*	**Final Response**	*Primary view*	*Secondary view*
Platonist	26	69	31	31	68	32
Instrumentalist	60	90	10	43	73	27
Problem-Solving	14	0	100	26	25	75

The first column of the table identifies the three views about the nature of mathematics at the center of this study. The second and fifth columns with bolded headings report the overall percentages of codes found in the PSETs' responses to Prompt 1 corresponding to each of the views in the Initial Beliefs Survey and Final Beliefs Survey, respectively. For example, the "60" in the table indicates that 60% of all codes from the PSETs' initial responses to Prompt 1 were related to the instrumentalist view. Table 4.9 also relates the percentages of codes from each of the views corresponding to instances in which they were identified as either the primary or secondary view. These results can be found in columns with italicized headings, three and four (which sum to 100 to account for all codes related to each view) for the responses from the Initial Beliefs Survey, and columns six and seven (which also sum to 100) for those from the Final Beliefs Survey. For example, of the 60% of instrumentalist codes from the PSETs' initial responses to Prompt 1, 90% of those codes were associated with a primary instrumentalist view, while the remaining 10% were associated with a secondary instrumentalist view.

4.5.2.1 RQ 3.1: The PSETs' initial beliefs about the nature of mathematics

Overall, it appears that the instrumentalist view was most strongly supported by the whole class according to their initial responses to Prompt 1 with 60% of the statements coded as instrumentalist at the beginning of the course. This was more than twice those suggesting the Platonist view (26%) and more than four times the amount of problem-solving statements made (14%). Moreover, Table 4.9 indicates that the overwhelming majority (90%) of the instrumentalist statements corresponded to the participants' primary view about the nature of mathematics. A large portion of the Platonist codes in the initial responses to the prompt corresponded to the PSETs' primary views (69%). The statements coded as promoting a problem-solving view in the first response, on the contrary, never related to a primary view held by any of the PSETs. As the view least represented in the data set, problem-solving statements represented only 14% of the total codes.

4.5.2.2 RQ 3.2: Changes in the PSETs' beliefs about the nature of mathematics

The PSETs' responses were a bit more robust at the end of the course and resulted in slightly more codes overall. The numbers in the "Final Response" column of Table 4.9 show that the percentages of these codes were more evenly distributed among the three views about mathematics, although the strongest support was still situated with the instrumentalist view (43% of the total codes). The total number of instrumentalist codes decreased from 60% to 43% in the final response, with the percentage of those codes related to the PSETs' primary view of mathematics also decreasing (going from 90% in the Initial Beliefs Survey to 73% in the Final Beliefs Survey). There was a small increase in the percentage of the total codes related to the Platonist view (from 26% to 31%), and the percentages of Platonist codes depicting a primary or secondary view were almost identical between the two data sets. The problem-solving view, in particular, revealed many differences between the two collection periods. One of the greatest increases displayed in Table 4.9 can be seen in the difference in the percentage of overall codes suggesting a problem-solving view found in columns two and five, almost doubling from 14% to 26%. Moreover, even though no PSET initially responded to the prompt in a way that would suggest a primary problem-solving view, a quarter of the codes in the final response suggested this alignment. Although this view was the least likely of the three views to be held as the primary view in this group of PSETs, it was the strongest secondary view.

The same data described in Table 4.9 was also used to create Table 4.10 below. I found it useful to look at the changes of the primary and secondary views as suggested by the initial and final responses to Prompt 1 at the level of individual PSETs in addition to the whole-class level. The vertical dimension of Table 4.10 depicts all possible combinations of primary and secondary views (in the form "PRIMARY VIEW (SECONDARY VIEW") that could have been held by an individual PSET as indicated by the data from the Initial Beliefs Survey. In this table, "PL" represents the Platonist view, "IN" the instrumentalist, and "PS" the problem-solving view. For example, "PL(PS)" in Table 4.10 signifies a primary Platonist view, with a secondary problem-solving view. "PS(IN)" signifies a primary problem-solving view with a secondary instrumentalist view. If a PSET's response indicated an alignment with a single view, it was automatically classified as the primary view without another view in parentheses (e.g. "PL" denotes an alignment with only the Platonist view). These combinations of primary and secondary views also make up the horizontal dimension of the table and represent the view(s) found to be held in the Final Beliefs Survey. Each "x" in the table represents one participant and depicts each PSET's initial and final views about the nature of mathematics. The shaded diagonal in Table 4.10 corresponds to no change between the beginning and the end of the course.

Table 4.10: Individual Changes Found in Responses to the Prompt 1 (n=25)

		Primary View (Secondary View) in Final Response								
Primary View (Secondary View) in Initial Response		PL	PL(IN)	PL(PS)	IN(PL)	IN	IN(PS)	PS(PL)	PS(IN)	PS
	PL	x		xx						
	PL(IN)		x	x*x*[1]	*x*	*x*				
	PL(PS)									
	IN(PL)	x	x		x				x	
	IN		x		xx	xx	x			x*x*
	IN(PS)		x				xx*xx*			
	PS(PL)									
	PS(IN)									
	PS									

[1]The bolded and italicized ***x*** represents a participant randomly chosen for individual interviews

Investigating this data at the level of the individual alongside the whole-class data displayed in Table 4.9 led to three interesting trends. First, a noticeable subset of the PSETs demonstrated a change towards the problem-solving view (especially away from the instrumentalist view) at the end of the course, either as a primary or secondary view. Of the 16 PSETs that displayed some kind of change (represented by the “x”s not found in the shaded diagonal), 8 PSETs either demonstrated an emergence of the problem-solving view, or a change from a secondary to a primary problem-solving view. A second trend relates to those who demonstrated no change in their primary Platonist view from the beginning of the course to the end, although the emergence or change in the secondary view may have occurred (6 of the 25 participants). The third and final trend (which can be most clearly seen in Table 4.10) describes those who appear to have demonstrated no change at all in their belief about the nature of mathematics during the course (9 of the 25 participants), represented by the “x”s in the shaded diagonal.

4.5.3 Responses to Prompt 2

The results displayed in Table 4.11 show how many PSETs cited each activity in their final homework response, as well as the number of times each activity was discussed first by the PSETs (and therefore relates to the activity they reported as being the most influential). Below, I illustrate these results with excerpts from homework assignment 5, as well as illustrations of emergent themes from the whole-class entries in the reflection journals (see Section 4.3.3.2 for elaboration on the prompts used for the reflection journal). I also highlight how these three tasks address features of my conceptual framework (e.g. cognitive conflict, CAPs, MBC).

Table 4.11: Influential Activities Cited with the Highest Frequencies and Relevant Features of Conceptual Framework (n=25)

ACTIVITY	BRIEF DESCRIPTION OF ACTIVITY	TIMES MENTIONED IN H.A. 5*	TIMES MENTIONED 1ST IN H.A. 5	RELEVANT FEATURES OF CONCEPTUAL FRAMEWORK
“The Blonde Hair Problem”	PSETs are asked to solve a word problem that many initially deem impossible by breaking it down and working step-by-step.	17	1	CAPs, cognitive conflict
“The Squares Problem”	PSETs explore and attempt to explain a pattern, understanding that it is not enough just to check a few cases to be sure the pattern will always work.	14	8	CAPs, cognitive conflict
“Work with the Two Considerations”	Given the importance of definitions, the PSETs use the Two Considerations in attempts to achieve balance between mathematics as a discipline and students as learners when creating and using definitions in the classroom.	11	7	MBC5 (through the use of a P-R mathematics task)

***Note**: “H.A. 5” stands for “homework assignment 5,” the final homework assignment

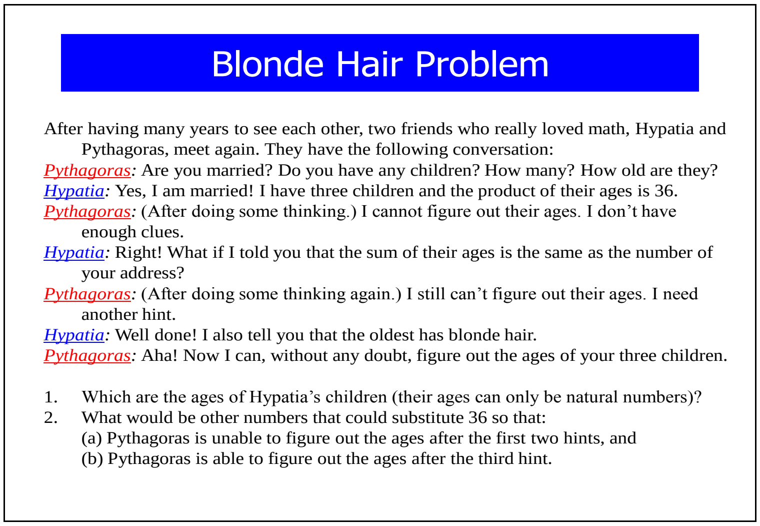

Figure 4.3: "The Blonde Hair Problem"

"The Blonde Hair Problem" was mentioned with the most frequency in homework assignment 5, although only one participant listed it first in response to the prompt. As indicated in Table 4.11, "The Blonde Hair Problem" was a task that utilized two CAPs (Stylianides & Stylianides, 2009a) as entries in the reflection journal to direct the PSETs' attention to their current beliefs, thereby facilitating students to experience cognitive conflict (MBC 6 in Table 4.1). As described in Section 4.3.3.2, one CAP for this activity was implemented immediately after the PSETs were first shown the problem (found in Figure 4.3), and asked the PSETs to respond to two prompts: (1) *Describe your initial reactions to "The Blonde Hair Problem"*; and (2) *Does this problem differ in any way from other problems you have encountered in the mathematics classes you have taken so far? If so, how?* The second CAP was implemented after the completion of the activity and also asked the PSETs to respond to two prompts: (1) *Describe your experiences working with "The Blonde Hair Problem"*; and (2) *Did this experience give you any new ideas that might be useful to you as a prospective teacher?* The second CAP specifically aimed to have the PSETs reflect on their reactions to and beliefs about the problem (presumably that the problem was "impossible" due to a lack of information). These CAPs were intended to help students become more aware of their beliefs and conceptions about problem-solving, and made it more likely that they would experience a cognitive conflict when their beliefs were challenged as they discovered that they could successfully complete the problem after all. After this conflict was experienced and the activity completed, the PSETs were asked to reflect a second time, this time describing their experiences with the problem and their thoughts about using such a problem in their own future classrooms. Asking the PSETs to complete these two reflections may have directly contributed to the high number of citations that the activity received regarding the course activities that "stood out" to them.

The PSETs reflected upon their experience working on "The Blonde Hair Problem" in their responses to the CAPS in the reflection journals, as well as in their homework assignment 5. There were several features that emerged from an analysis of the whole-class data from these sources that the PSETs cited as being important in their engagement, the majority of which focused on different aspects of problem-solving. The four themes related to their experiences that were most prevalently discussed were: (1) the importance of breaking down a problem step-by-step; (2) the importance of not giving up, even when a problem initially seems impossible; (3) math problems can be both fun and challenging; and (4) the value of group work.

Approximately one-third of the PSETs reported in their reflection journals that their engagement with "The Blonde Hair Problem" encouraged them to emphasize the importance of approaching a big problem by breaking down the different parts systematically and approaching one piece at a time. Many of these same students also described their initial reaction to the problem as being impossible as they reflected upon their experiences after having solved it. PSETs that focused on their original assumptions that the problem could not be solved often identified a feeling of success and accomplishment after having successfully solved it. For example, one student wrote:

> Fascinating! This problem really captured my interest. Before we started it, I thought there was no way I could solve this, but it showed me that you can solve things even when you really feel it's not possible. It felt great!

Similar comments could be found in homework assignment 5. One PSET stated:

> "The Blonde Hair Problem" made an impact on me in that it forced me to break a seemingly overwhelming problem down into its smallest components—which ended up only requiring very basic math knowledge and skills. I never would have probably even attempted it before. It helped me to look at the way I solve problems in a new way, and I think that it could be a great way to teach students to of mathematics to think about solving problems in this way.

Another PSET reflected on her own ideas related to the second theme regarding impossible or difficult problems and included her desire to share similar experiences with her own students:

> This just reinforces the idea that sometimes on the surface, a problem may seem too difficult to do, with parts that you think don't make any sense. But it's best not to panic and just think about it, step by step. This will be very useful to share with my students.

In homework assignment 5, one PSET cited "The Blonde Hair Problem" as the activity that stood out the most to her in the course. She discussed the reasoning behind her choice:

> Approaching the problem all together made it seem impossible to solve. Most of us thought that we weren't given enough information to get a solution, but in fact we just needed to give ourselves the chance to really look at the problem in a different light to get it. I hated the problem at first, but after making myself work through it and understanding the purpose of it, it's now the problem that I appreciate the most in the course, and the one that made me think different not just about doing math, but about teaching it. You need to try.

The third theme of the PSETs' discussions about "The Blonde Hair Problem" bridged their own experiences with the experiences they would like to create for their own students in the future (as many of the PSETs described above did). An excerpt addressing the realization that math problems can be both fun and challenging from one PSET's reflection journal is shared below:

> I thought this was a very interesting and fun problem! I see now that you don't necessarily need numbers to get answers, and logic is just as big of a part of math. When I finally solved the problem and it really made sense to me, it opened my eyes to the way that math problems can be challenging and fun. I can show students that it is important to focus on what we know from a problem and not just what we don't know. And not to give up. Fun problems are problems that make you think.

Another PSET focused her reflection on this feature of the problem and its influence on her future role as a teacher:

> Working with this problem allows me to think I'll be able to encourage my students not to give up, and to encourage them to have fun and be open with a problem that seems too difficult. It also teaches me to be patient and that some problems really aren't that hard as you might first think, if you just look at it harder and try. This is important, to teach the students patience. And to be patient as a teacher by allowing them to work though more difficult problems.

Many responses on homework assignment 5 echoed the PSET above. One participant discussed her reasons for choosing "The Blonde Hair Problem" as the second most influential activity in the course. She wrote:

> I initially didn't like this problem, and the easiest solution seemed to be to give up on it. I think this is a really common feeling among students in math classes, that's it's just easier to give up. However, what I really got out of the problem is that something challenging doesn't necessarily mean impossible, and there's a good chance that it can be broken down to become much less challenging—even fun! I think that's an important concept to teach to young math students, so that it makes math less scary. That's what I hope to do, to give them problems like this and show them it's nothing outside of their realm of knowledge.

The final theme discussed prominently with regards to experiences with "The Blonde Hair Problem" is a feature identified in reference to other problems described later in this section and deals with the importance of group work. One PSET reflected on her experiences working in her small group in her reflection journal, and its influence on her beliefs about the nature of mathematics away from the instrumentalist perspective:

Working through the problem with my group members made solving the problem seem a bit easier and more feasible than if I had to work alone. The Blonde Hair Problem taught me that working through a problem oneself instead of just learning the answer or being told how to find the answer really can help develop critical thinking skills. It'd be great to use something like this with my students.

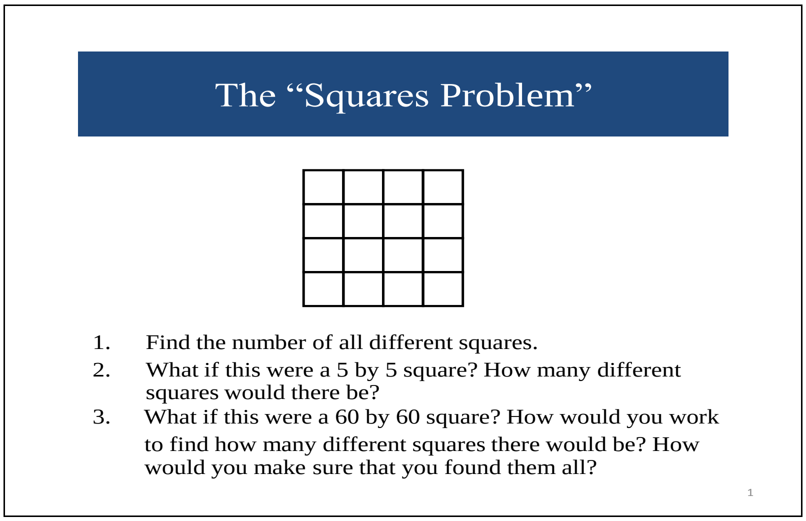

Figure 4.4: "The Squares Problem"

"The Squares Problem" was the activity mentioned with the second-highest frequency in response to prompt, yet it was the activity that was mentioned first the most often. It was the first activity in which the PSETs engaged in the course. As with "The Blonde Hair Problem," "The Squares Problem" (an activity that is elaborated in more detail in Stylianides and Stylianides, 2009a) was a task that used the important features of CAPs and cognitive conflict. As described by Stylianides and Stylianides, the first CAP was implemented after the PSETs' initial work with the problem (as shown in Figure 4.4) and asked them to discuss (in writing) if they could be sure that the pattern they identified about the number of different squares in a 60 x 60 square based on an examination of two smaller cases was true. I collected these individual responses and used them to set up a second CAP, where I shared with the whole class the common responses provided regarding the validity of the pattern. These two CAPs "were intended to help students become more aware of their conceptions about their validation methods, thereby making it more likely that they would experience a cognitive conflict when these methods would prove problematic later on" (Stylianides & Stylianides, 2009a, p. 329). In particular, the PSETs later experienced cognitive conflict when they encountered an activity (not discussed in this chapter but discussed in Stylianides and Stylianides, 2009a) that challenged their reliance on empirical arguments as a validation method. This activity engaged the PSETs with a numerical pattern that was shown to fail, with a counterexample discovered at the n=6 case. CAPs 1 and 2 in "The Squares Problem" that preceded this activity "were expected to increase the likelihood for the counterexample to become pivotal for the students" (Stylianides & Stylianides, 2009a, p. 330), and the PSETs could then resolve the cognitive conflict by acknowledging that it was not enough to simply check a finite number of cases in order to be sure that a pattern would always work.

In addition to the discussions in homework assignment 5, the PSETs were asked to reflect upon their experiences of engaging with the first instructional sequence of the course in their reflection journals, which included "The Squares Problem" as its opening and closing activity (this sequence is described in detail in Stylianides and Stylianides, 2009a). An analysis of the whole class data from these sources revealed a number of different aspects of the problem that stood out to the students, many of which related to their previous experiences in doing mathematics, as well as their mathematical beliefs about teaching and learning. The most frequently mentioned feature of the experience related to an elevated awareness in the importance of explanations in mathematics, also referred to as "understanding the why" in problems, especially when dealing with patterns. One PSET wrote in her journal:

> What I took away from this problem is that explaining WHY something works is essential. It takes more time and effort to actually understand the reasoning behind a problem, but in teaching it seems essential to have that knowledge. Formulas and patterns can be helpful, but first you must understand the basis of them. Understanding the WHY gives you confidence that you know what you're talking about.

Having listed "The Squares Problem" first (and therefore identifying it as the most influential activity on their beliefs), another PSET explained:

> "The Squares Problem" was the first problem we worked on in this class, and I think it contributed the most to my learning and had the most impact on my beliefs. By learning to explain the problem so thoroughly, it set a precedent for the way to explain any of the other activities that we did, and the way we should have our own students explain. I had never really had to explain anything thinking about how it would sound to a child, or whether the child could understand it or not. I now see that as a teacher, you have to think about these things.

Another PSET also described the importance of explanation in reference to her ideas about teaching in her journal:

> You need to be able to explain what you're doing in the basic form. When working with children who are just learning the basics, you as a teacher need to provide answers and explanations of why you are doing certain procedures. It's not good enough anymore to just get an answer, but to be able to explain it in terminology elementary students can understand.

Other PSETs focused on the importance of explanation and described how difficult it was for them to do considering that they were never expected to provide such detailed explanations in their other math classes (and certainly not at the elementary level). The majority of PSETs described their work with "The Squares Problem" as challenging, but appeared to enjoy the challenge. One PSET recalled her experiences and described her frustrations about a lack of experience producing explanations herself. Moreover, she discussed this in reference to her previous teachers:

> What stood out to me with this problem was the great details of why things worked out the way they do. Over the years I have had many math teachers assume we know information, and that's when I get confused, frustrated, and angry. I never want to do that. With this, I have understood every step of the way. I can say I know and understand "The Squares Problem."

Although not as prevalent as responses discussing the impact of the problem on their beliefs about the role of explanation in the learning and teaching of mathematics, a small number of PSETs (approximately one-fifth) described in their reflection journals how small group work stood out to them in their experience with "The Squares Problem," and how valuable it can be. At the end of her journal entry regarding the problem, one PSET wrote:

> Working in groups showed how different everyone thinks and explains things. I also realized how important it is to understand why you need to know why you know something.

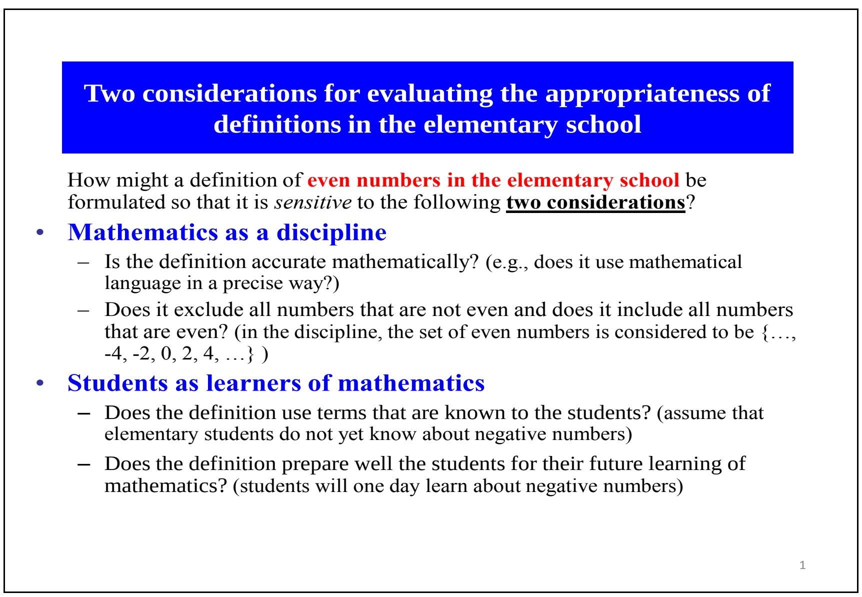

Figure 4.5: "The Two Considerations"

The third and final activity that was reported as impacting the most PSETs' mathematical beliefs was "Work Related to the Two Considerations for Definitions" (referred to hereafter as

"The Two Considerations"). Figure 4.5 describes "The Two Considerations" as they were introduced to the course, and were used by Stylianides and Stylianides (2010) to help illustrate their notion of P-R mathematics tasks outlined in Section 4.3.1.1. Included in "a series of tasks that highlighted the importance of definitions in both school mathematics and the discipline of mathematics" (Stylianides & Stylianides, 2010, p. 167), "The Two Considerations" were used in a variety of (P-R mathematics) tasks related to the definitions of even and odd numbers in the elementary grades, in particular with regards to how the PSETs may judge the appropriateness of definitions of even numbers found in textbooks. My instruction followed that outlined by Stylianides and Stylianides (2010) and consisted of a 'typical mathematics task' that was followed by a P-R mathematics task. The typical mathematics task (which was purely mathematical in nature and devoid of pedagogical issues) asked PSETs to classify a list of numbers as being even or not, and to come up with their own definition of even numbers. I asked the PSETs to share their classifications of the numbers and definitions in small groups of 2 or 3. This was meant to draw their attention to the diversity in the definitions created and to further rouse their interest about appropriate definitions. As I prepared to introduce the P-R mathematics task, I discussed the challenge of creating a mathematically appropriate definition of even numbers (as supported by the discipline) in the elementary grades that simultaneously attends to the knowledge of young students (who, the class was to assume, did not know about negative numbers). I then presented to the PSETs "The Two Considerations," which served as the introduction to the P-R mathematics task that followed. This task asked them to evaluate the appropriateness of seven definitions for even number for elementary school students (these definitions are provided in Stylianides and Stylianides, 2010, p. 167). These activities were explicitly grounded in the real work of teaching (MBC5).

I use both the responses from homework assignment 5 and excerpts from the reflection journals to illustrate the findings. The reflection journals contained an entry from each PSET describing his or her experiences with the entire unit on definitions which discussed the importance of definitions in both the discipline and in the work of teaching. From those PSETs that described their experiences with "The Two Considerations" in particular, the data suggested a number of important aspects of the activity that appeared to make an impact. The first feature (discussed by approximately half of the PSETs that discussed "The Two Considerations" in their reflection journals) demonstrated an increased awareness that multiple correct definitions could exist. As one PSET put it:

> Whenever I previously thought about definitions, I always thought about them as one, concrete statement that applies to everything. I know realize that mathematical definitions are similar to English definitions: there are different definitions based on use and context. It's important to know there is more than one definition.

Another PSET chose "The Two Considerations" as the third most influential activity in the course, and mentioned her revelation that there could be multiple definitions:

> I know that I usually looked at definitions and thought that it was the only definition, so I'd copy it down and be done with it. I never realized that teachers could alter definitions for the level that they were teaching, I just figured that they used the ones from the (text)book. This work with definitions and "The Two Considerations" will allow me to take any definition now and alter it for the level of class and students that I'm teaching.

Related to this feature was the occasional mention of past experiences in which teachers demanded that the participant use a specific definition in his or her work, even if the participant tried to create and utilize an analogous definition that made more sense to him or her. Below is an excerpt from one PSET's reflection journal:

> The first thing I thought about with regards to this work was my experience with mathematics when I was growing up. If there was even something I did not understand I'm sure that it could have been taught to me through different terminology or definition.

> When I think of my past teachers I can remember them almost scolding us to use THE definition.

The final feature that was prevalently discussed in the data was associated with the role of definitions in classroom discussions, and in particular the role of the teacher in understanding how to deal with different definitions. One PSET stated in his homework assignment 5:

> It stood out to me just how much teachers have to make sense of and evaluate definitions used by their students. I knew kids would think of their own form of the definitions, but the teacher has to think of a way to disprove the students relatively quickly. The thing with this is that the teacher never really knows what a student might come up with so there's no way to prepare!

Oftentimes, entries like the one above also indicated that the PSETs were surprised about how difficult it appeared to be to find an appropriate definition to use in their classrooms, although many suggested that "The Two Considerations" made them more confident in their ability to do so.

4.5.4 Responses to Prompt 3

After coding the data related to the PSETs' responses to Prompt 3, three themes emerged regarding the features they associated with good experiences of mathematics: (1) socio-mathematical norms; (2) the instructor of the class; and (3) features of the curriculum. These emergent themes reflect closely Features 2 and 3 of the course highlighted earlier in Table 4.2. The findings from the coding of both the initial and final responses are presented in Table 4.12. This table includes only the features that were mentioned by at least three PSETs. Many of the aspects in the table also reflect certain features discussed by the PSETs in relation to the three activities that "stood out" to them and impacted their beliefs.

The table shows that the features identified by the PSETs at the end of course relating to socio-mathematical norms corresponded to a classroom that: (1) supports a learning environment that is fun and enjoyable for the students; (2) supports an environment that encourages creativity in the students; and (3) encourages open discussion. Enjoyable classrooms appeared to be important to many (9 out of 25) PSETs at the beginning of the course, yet there was an obvious decrease (from 9 to 4) in the perceived importance of this feature at the course's conclusion. On the other hand, the number of PSETs who mentioned the creation of a learning environment that encourages creativity in students tripled by the end of the course (from 2 to 6). The most evident change that occurred within the socio-mathematical norm theme as shown in Table 4.12 is the increase in the desire to create a classroom that is open to discussion, with citations from 6 PSETs at the end of the course in comparison to only a single PSET at the beginning of the course. Indeed, these findings are consistent with the results related to Prompt 2 regarding features of the three identified activities.

Table 4.12 shows that the features related to the instructor of the class most valued by the PSETs were: (1) the encouragement of group work and collaboration; (2) teacher approachability; and (3) knowledge of students' future learning. Cited with one of the highest frequencies, the importance of encouraging group work and collaboration was mentioned by more than five times as many PSETs in their concluding response to the prompt as in the initial response. This finding reinforces the importance that small group work had on these future teachers as identified in their reflections on both "The Blonde Hair Problem" and "The Squares Problem." Another prominent feature identified was the importance of teacher approachability

Table 4.12: Emergent Themes and Corresponding Features from Responses to Prompt 3 (n=25)

THEME	FEATURE	INITIAL COUNT	CONCLUSION COUNT
Socio-mathematical norms	Learning environment is fun and enjoyable for the students	9	4
	Environment encourages creativity in the students	2	6
	Environment should be open to discussion	1	6
	Create a culture that abolishes fear of math.	1	3
	Create a culture of explanations (not just calculations)	0	3
The instructor of the class	Encourage group work/collaboration	2	11
	Teacher should be available/students should be comfortable approaching teacher	6	7
	Clearly present concepts and methods to solve problems, promote memorization, repetition	8	2
	Know what students need to know for future learning	1	5
	Teacher should be enthusiastic about math	1	3
	Tell the students that everyone can succeed in math	3	0
	Teacher should be patient	2	1
Features of the curriculum	Different solution paths possible for different kinds of learners	6	7
	Use problems that show application to real-life	9	2
	Use challenging problems	2	6
	Use problems that encourage exploration/"hands-on"	3	5
	Activities should be interactive/use games	3	2

and the ability to create a comfortable learning environment (a feature that did not change much over the duration of the course). This characteristic was not explicitly identified in the data corresponding to the three influential activities, yet other changes revealed in the table do align with the results described in the previous section relating to those activities. For example, there was a clear increase in the number of PSETs that believed a teacher should have an understanding of not only the current knowledge of her students, but also an understanding of their future learning. This idea was found in the reflection journals of several PSETs as they described the impact of their work with “The Two Considerations” on their beliefs about teaching mathematics. These results also suggest a decrease in the number of PSETs that value an instrumentalist view of teaching (from 8 to 2 with regards to the feature “Clearly present concepts and methods to solve problems, promote memorization, repetition”) in their notions of good experiences in doing mathematics. This movement away from an instrumentalist view of mathematics was substantiated by the results relating to changes found in Likert-items, responses to Prompt 1, as well as those described earlier in this section regarding the significant course activities from Prompt 2.

Finally, Table 4.12 displays the features identified by the PSETs relating to the third emergent theme, features of the curriculum. The PSETs described features of the curriculum that: (1) provided problems with different solution paths for different kinds of learners; (2) used challenging problems; and (3) used problems that showed real-life applications. The desire to use different solution paths to provide learning opportunities for different kinds of learners remained consistent, but there was an increase in the number of PSETs that felt the use of challenging problems would aid in the creation of good mathematical experiences. A small increase was also shown regarding the use of exploration activities. The feature of a good experience in doing mathematics identified by the PSETs that had the greatest decrease in citations (from 9 to 2) related to the importance placed on using problems grounded in real-life situations.

4.6 DISCUSSION

In this section, I discuss the general results described in Section 4.5 and accompany this discussion with interview data from three PSETs that help to illustrate three general trends found in the whole class data. I begin by providing an overview of the impact that the course in this study had on the mathematical beliefs of the PSETs, including three trends that emerged regarding their views about the nature of mathematics. Next, I choose three of the six PSETs interviewed to illustrate these trends and provide further insights into the reasons for the changes that occurred in these three participants. Finally, I describe features of the course that appeared to promote the changes found.

4.6.1 Overview of the impact of the course on mathematical beliefs

The results of this study showed that it is possible for a mathematics content course to impact the mathematical beliefs of PSETs, with approximately 1/3 of the participants moving towards a more problem-solving view. When the PSETs entered the focal course, they likely had already formed a range of mathematical beliefs grounded in their prior experiences (Lortie, 1975). At the beginning of the course, the PSETs shared their beliefs about mathematics as a discipline and the learning and teaching of mathematics through responses to Likert-items and other prompts. Consistent with much of the literature reviewed at both the beginning of this chapter and in Chapter 2, those beliefs largely reflected an instrumentalist and Platonist view of the discipline. These views are not aligned with the vision of reform (NCTM, 1989, 1991, 2000), and therefore unfavorable for teachers of mathematics. Over the duration of the course, the participants' beliefs were challenged in different ways and they were constantly encouraged to critically reflect on those beliefs.

The quantitative and qualitative data revealed a substantial change in many PSETs' views about the nature of mathematics as they progressed through the course. Although support of the instrumentalist view was still strong, there was evidence that supported the emergent trend that many PSETs moved towards a more problem-solving view of mathematics, either as a primary or secondary view (referred to as *Trend 1* hereafter). Of Ernest's (1988) three views, this problem-solving view most closely reflects that promoted by the NCTM (1989, 1991, 2000) and mathematical reform, and therefore is a desirable view to be held by future teachers of mathematics. The shift towards this view found in this study is consistent with past studies interested in analyzing the impact of TEPs on beliefs, although those studies have primarily focused on the impact of mathematics methods courses (e.g. Hart, 2002; Wilkins & Brand, 2004). The changes found in the beliefs of a substantial subset of the PSETs participating in this study were indeed positive, yet it is clear that there is room for improvement.

The findings indicate that many of the central beliefs about the nature of mathematics are resilient to change (Green, 1971). The other (related) trends that emerged from the whole-class, in addition to the one discussed in the previous paragraph, suggest that: (1) it was particularly difficult to encourage change in a primary Platonist view, although the emergence or change in a secondary view may have occurred (referred to hereafter as *Trend 2*); and (2) it was difficult to encourage change in beliefs about the nature of mathematics in general, as many PSETs appeared to have demonstrated no change at all in their (primary or secondary) view (referred to hereafter as *Trend 3*). Although responses to Prompt 1 revealed little change in the Platonist stance of many of the participants (relating to Trend 2), further distinctions may need to be made regarding Platonist tendencies with regards to mathematics as a discipline and those with regards to the learning and teaching of mathematics. This is further clarified in the next section where interview data from three PSETs is described, with Max's data in particular providing support for the need for such distinctions. It is clearly a difficult and continuing task to motivate belief change towards a problem-solving view in pre-service teachers, yet the changes in this direction evidenced in the beliefs of some of the PSETs were encouraging.

4.6.2 Interview data from three PSETs to help illustrate the three trends

In this section, I will draw heavily from the data provided by three of the PSETs that completed individual interviews in attempts to provide illustrations and additional insight into the general results of the whole class. As outlined in the previous section, analysis of whole-class responses to Likert-items and Prompt 1 given at the beginning and conclusion of the course revealed three major trends. To gain further insight into these trends and the reasons for the belief change (or lack thereof), I have chosen to elaborate upon the data from three participants from the six interviewed. Gillian, Max, and Maria (the names are pseudonyms) were chosen because they each represent one of the three trends identified (Trend 1, Trend 2, and Trend 3, respectively), and their data provided me with the best opportunity to better understand these trends. Of the other three students interviewed (whose data is not discussed in this chapter), one fell under Trend 3, while the other two represented changes not representative of the whole class. Specifically, the student that fell under Trend 3 experienced no change in their primary instrumentalist and secondary problem-solving views. One of the students not representative of the whole class showed a change from a primary Platonist and secondary instrumentalist view to a primary instrumentalist and secondary Platonist view, while the other showed a change from a primary Platonist and secondary instrumentalist view to a primary instrumentalist view (with no secondary view present).

4.6.2.1 Gillian: A change towards the problem-solving view (Trend 1)

The content course providing the context for this study coincided with the end of Gillian's freshman year. She had taken many math classes in her educational career, including a calculus class during her first semester at the university. Both her father and brother possess degrees in mathematics, and therefore she reported engaging in mathematical discussions regularly outside of the school setting, and expressed a general comfort with the discipline. In her Initial Beliefs Survey, she indicated a strong desire to teach elementary students, with her feelings about teaching mathematics to elementary students slightly less positive.

Table 4.13: Gillian's Initial and Final Response to Prompt 1

PSET	INITIAL RESPONSE	END-OF-SEMESTER RESPONSE
Gillian	Mathematics is formulas, equations, and numbers. It is a way to calculate information in a way people can measure. Math is used to help predict information based on past trends.	Mathematics is a collection of formulas, procedures, and facts all related, but it's more than that too. You need to be able to build upon previous knowledge in order to continue on. If you don't know set rules, it doesn't really matter. In most cases you can be doing the work based off previous knowledge or create something new and actually be proving the rule you didn't know.

The data from Gillian's responses to the Likert-items from the two Beliefs Surveys and homework assignment 1, as well as her initial and final responses to Prompt 1 (shown in Table 4.13 above) indicated a shift in her beliefs from an instrumentalist view to a problem-solving one. Gillian described her most memorable past experiences in mathematics classrooms as reflecting "a game." During our initial interview, she elaborated upon these memories, and in them grounded her initial instrumentalist belief about the nature of mathematics:

> It was always like a game. I remember the teacher breaking us up into 2 groups and putting problems on the board and whoever would solve them the fastest would win a small prize. But it was a lot of repetition, which when you get older you think it sucks, but when you're younger, it sets it in. You need to have that repetition.

As Gillian described her more recent mathematical experiences, she admitted that there were always elements of these early classroom experiences present, with the teachers clearly outlining procedures and formulas to use, making her feel "like a parrot" as she reproduced their work. As she discussed her initial response to Prompt 1 she solidified her primary classification as an instrumentalist:

> In mathematics, you always have your end product. And you know that this is what you have to do to get to it. I definitely see it as a tool, something to get you to the end result.

In addition to describing mathematics from the instrumentalist perspective in her own words, Gillian agreed with statements such as, "To do well in solving math problems, I have to memorize all the formulas that are relevant," "Learning to follow 'the steps' to generate correct answers is important," and "Mathematics is a useful tool primary used for calculations," all of which suggest alignment with the instrumentalist view.

At the conclusion of the course, the data provided evidence that Gillian's view regarding the nature of mathematics had shifted. During the final interview, I asked her to reflect upon the differences in her responses to Prompt 1:

> Having taken the course and dealt with all of the different aspects of math, I think my description [of what math is] became a little more detailed. I really think that's because of the different activities picked, they really showed us the different aspects of math. It's really not as straightforward as I thought, there are different parts. I think that formulas are important, but they're not so important that you have to memorize them anymore, what I wrote here is that even if you don't know the set rules, and most of the time you don't, but in the work we were doing we were proving the rules, we were saying this is why this works.

At this part of the conversation, she recalled a comment she had made during our initial interview regarding mathematics as a creative endeavor:

> When I see the word creative, I think more of arts, so I think of people drawing and painting and stuff like that. And I don't see somebody who is creative in that sense as being mathematically inclined.

After being reminded of the wording of the statement, she was quick to point out that she no longer felt the same way. This was supported by the change found in her response to the statement "Mathematics is a creative human activity," which shifted from a 3 (not very true) to a 2 (sort of true), a substantial shift found in the whole class data as well. Gillian reported that almost every problem in which she engaged in the course provided her with the opportunity to "think outside of the box" and be creative. She elaborated further:

> I think that I said it's more creative due to the explanation aspect of things, the challenge of explaining your discoveries. In class, you needed to know the meanings of words, but you never told us "this is exactly how you need to do this," and when we all had to think about it, we actually could do it. And if you pulled anyone off the street and made them sit down with it, they could probably do it. So I think that was a discovery of myself, you know, I really CAN, if I take the time, explain what I'm doing.

The importance of explanations over procedures in the classroom was a major focus of our final interview, and Gillian's shift towards the problem-solving perspective was becoming more apparent. The shift away from the instrumentalist view was enhanced as we discussed differences seen in her responses to the Likert-items between the Initial and Final Beliefs Survey. Whereas Gillian initially agreed with the item in the Initial Beliefs Survey, "To do well in solving problems, I have to memorize all the formulas that are relevant," her response at the conclusion of the course showed a strong disagreement with the statement. As she reflected on this change, she also suggested a change in her beliefs about teaching procedures to her own students:

> This all relates to explanations—let them [the students] go out and figure it out themselves. We obviously have to give them some decent tools to start off with, but for the most part you learn by messing around, seeing what works and what doesn't work, give them a few minutes then bring the class together to see what everyone has come up with. Then, building off of what they've come up with, then you can give them the formulas that relate to their own work

Reflecting a substantial shift away from the instrumentalist perspective found in the whole-class data, Gillian touched upon the changes in her responses to the item "Doing mathematics means memorizing particular rules and procedures." Her reasoning regarding this statement reflected her comment above about the elevated importance of explanations in her mind. She also made a point to comment on the lack of rules and procedures used to solve the problems posed in the course, particularly "The Squares Problem" and "The Blonde Hair Problem." As the final interview drew to a close, Gillian further clarified how important the work done with definitions and the characteristics of a good explanation were to her, and the impact on her beliefs about learning and teaching:

> The definitions, like "The Two Considerations," and explanations aspect, how important they are because of how damaging they could be to children's learning. That is the biggest change that stands out to me from having this course. When I think about explaining something now, I realize how I'm breaking things down, making sure every little thing is understood, whereas before I would be more…general. Just focusing on the answer, really. And that's not the most important thing.

4.6.2.2 Max: No shift from a primary Platonist view (Trend 2)

Max entered the class as a sophomore, having a strong background in mathematics that included university-level calculus and statistics. On the Initial Beliefs Survey, he indicated that he thought both of the statements "I would like to be an elementary teacher" and "I would like to teach mathematics to elementary school students" were "sort of true." These responses were clarified during our first interview where he told me that the focal course was his first education class, and that he was still in the process of making the decision to become an elementary school teacher or not. He cited his other option as becoming a high school history teacher.

Max described his past experiences in mathematics classrooms as being fairly negative, with those relating to his high-school years appearing to be the strongest:

> I didn't have the best experience in high school, the math department was really understaffed or something…I had a teacher and he didn't speak English very well …well, I get that a lot here (at the university) too, but he would just put a PowerPoint up, not even a PowerPoint, just transparencies and he would just read them to us. And then he would give us worksheets. And that was how we learned.

His description greatly reflected the traditional classroom, with the teacher acting as the transmitter of knowledge and the students as passive recipients. When asked to talk more about his earlier experiences in the elementary school, he grew more positive, if only for a moment:

> In the elementary school, it was different with the groups and the stuff we would talk about, I liked it better. I actually loved math up until about 8th grade, I was really good at it and in the gifted and talented program. And then I hit geometry and it was just…anything in high school it was terrible. I just had a lot of bad teachers in high school. It made me a little bit bitter.

Table 4.14: Max's Initial and Final Response to Prompt 1

PSET	INITIAL RESPONSE	END-OF-SEMESTER RESPONSE
Max	Mathematics is the study of using numbers and it explain happenings in our daily life, it's all around us. Math is used by everyone everyday and is an integral part of our lives. Math can range from simple addition to calculus and physics, but the same concepts apply to all forms of math, they're all related.	Mathematics is the study of numbers and how they can affect our lives. Math is everywhere around us and can give us explanations for things and can open doors to new things. Math isn't just something that occurs in the classroom or the lab, it is used by everyone every day in the world.

These earlier experiences gave rise to Max's primarily Platonist view about the nature of mathematics, with strong secondary instrumentalist tendencies. In his initial response to Prompt 1 (as found in Table 4.14 above), he described mathematics as representing the happenings around us, seemingly independent of humans, reflecting a Platonist view. This was further evidenced by Max's agreement with the statement "Everything important about mathematics is already known by mathematicians" and an even stronger agreement with "The mathematical body of knowledge is fixed and always has been." Describing mathematics in his own words, he also mentioned the utility of mathematics as supported by the instrumentalist view. Max's secondary view was further established by very strong agreement (1 on the scale) to Likert-items from the Initial Beliefs Survey and first homework assignment "Mathematics is a collection of facts, formulas, and procedures," "To do well in solving math problems, I have to be taught the right procedures," and "It is the teacher's job to teach the steps in each new math method to the students before they have to use it." As he talked during our first interview about the features of good experiences in doing mathematics that he would like to offer his students, he mentioned that he simply thought of his own experiences in math classrooms and simply described the opposite of those experiences.

The relevant data collected at the beginning and conclusion of the course suggested that Max experienced no change in his primary Platonist view as a result of the course. Yet, there was evidence that he did experience a shift in his secondary view from the instrumentalist to the problem-solving view. His final response to Prompt 1 was very similar to his initial response, and was still focused on a Platonist description of mathematics. He described mathematics as an entity that exists all around us in a way that suggests independence from human beings. However, during our final interview Max made a few comments that suggested that saying that there was absolutely no change in this primary view would not be entirely accurate (despite the fact that he even mentioned Plato by name). Although Max initially agreed with the Likert-item stating that "Mathematical knowledge is fixed and always has been," his response in the Final

Belief Survey showed a complete change, going from a 1 (“very true”) to a 4 (“not at all true”). He elaborated on his reasons behind this change:

> People are always finding new things…like, Stephen Hawking, how many new things has he found in what, 50 years since he’s been doing work? There’s always going to be people who look at the world and think of math and see something new, so it’s not fixed by any means. I mean, how many jumps have we had in the last couple hundred years, let alone thousands with Plato and Aristotle, so I don’t think it’s fixed at all. At the same time, I guess maybe the procedures, or the basics of mathematics, I guess I don’t really think they’ll change that much. But I think the mathematical body will always be changing.

Max acknowledged that his overall notion regarding mathematics as a discipline remained fairly constant, yet he reported that his ideas about teaching and learning mathematics had been modified. In comparing his two responses, he noted:

> I don’t think that my idea of mathematics has changed, you know, mathematics as a discipline, but I would definitely say that my ideas about mathematics and teaching are drastically different. I always thought of mathematics as just this thing that I would have to teach instead of thinking about it as something I’d want to teach, but this class kind of opened up the door a bit…you know, it’s not that scary, math’s not that bad! I actually enjoyed the problems…some of the problems were frustrating, to say the least, but it changed my opinion about teaching mathematics. It can be engaging and fun and it can explain.

These comments help to illustrate the secondary problem-solving view that seemed to emerge from an analysis of Max’s data at the conclusion of the course, replacing the secondary instrumentalist view he held initially. In reference to the change in the Likert-item “To do well in solving math problems, I have to be taught the right procedure,” a substantial change also found in the whole-class data, Max described the process in which he had engaged with respect to the scale used for all Likert-items:

> I went from a 1 [very true] to a 3 [not very true] during our first interview and to a 4 [not at all true] here in the final survey, so that was quite the process I went through! At the beginning, I was just thinking about the way I’ve been learning, with calculus and everything before it, they’d give us a procedure, tell us about it, give us a worksheet to hand in, and we’d never talk about it again. But being taught the right procedure is not the most important part of learning…I think working out the problem and figuring things

out for yourself, and then learning the procedure. Because if you figure it out on your own, you'll know it that much better from figuring it out your own way. And then when you learn the procedure, you can connect the 2…it's just more of a process than just giving them the formula.

The change in the secondary view towards the problem-solving view was also demonstrated in Max's responses to particular Likert-items. Also paralleling a substantial change in the whole-class data, he noted that his initial disagreement with the statement "There is usually one best way to write the steps in a solution to a math question" had switched to reflect agreement, as most of the problems featured in the course could be approached in a multitude of "equally good" ways.

One aspect of the instruction in the course that appeared to particularly impact Max's notion of a good experience in doing math was the use of group work. In his initial interview, Max had mentioned that he usually liked working by himself because he liked to follow his own "train of thought," something that he could not do when working with others. He indicated that it was a lot of work, trying to "keep up with other peoples' thoughts" and attempting to understand their thinking. However, when talking about a change he noticed in his response to a particular item ("I like solving math problems only when I can work them out easily"), his belief about the value of group work (both as a teacher and student) appeared to transform:

> Group work was really important in this class, a huge part of teaching, sometimes you just need somebody else's opinion—this idea really changed at the beginning of the class. They might think a different way that you do, and that's a good thing. You can do this in teaching too, you're surrounded by other teachers doing the same exact thing you're doing, why not get their opinion or ideas?

Overall, the class affected Max in many positive ways. Towards the end of the final interview, he confided in me that he made the decision to go into elementary education after all. He mentioned that he was glad to take a class that really opened the door to an understanding of what teaching mathematics could be like, and through engagement with problems like "The Blonde Hair Problem," it could be fun. Max also reported that he came to the realization that "knowing a subject is not the same as knowing how to teach it." When asked if he had any final comments or questions as our interview drew to a close, he smiled and stated:

> Teaching math isn't going to be the worst thing ever. That was one of the things holding me back, and it's not anymore.

4.6.2.3 Maria: No shift in either of her primary or secondary (instrumentalist and problem-solving) view about the nature of mathematics (Trend 3)

Like Max, Maria was a sophomore when she entered the math class, and also came from a strong mathematical background. Her mother is currently a high school math teacher, although she started her career in the elementary school and has always encouraged Maria to pursue a career in early education. Maria's most recent experiences in a math classroom were all prior to enrolling in the university, although she received university credit for her high school calculus course. Also similar to Max (and despite her mother's enthusiasm), Maria indicated that she "sort of" wanted to be an elementary teacher at the beginning of the course, a feeling that extended to her desire to teach mathematics to elementary students in particular.

Maria described her past experiences in mathematics classrooms as fairly positive, and had the very unique experience of having the same math teacher from grade 4 to grade 8. She described some of the benefits of this arrangement, as well as the routine of the classroom during our first interview:

> We were able to move a bit faster, and so we covered more material than a regular class. It was almost like…it was so comfortable because we were together for so long, knew each others' strengths and weaknesses. At the class, the way it was run, it was very structured. She'd [the teacher] write notes, we'd copy them, she'd give us examples you'd have the chance to explain it and ask you questions, so it was very interactive, very memorable.

Maria also had good memories of her experiences in high school. She spoke of her calculus teacher in particular who strongly emphasized the history of mathematics with regards to the material being taught, something Maria said she had never gotten before and found very interesting. Discussions in this class permitted her to see “where things in math came from,” and encouraged her to include historical discussions as one of the features of good experiences in doing math she would like to create for her future students, as identified on the Initial Beliefs Survey.

Table 4.15: Maria’s Initial and Final Response to Prompt 1

PSET	INITIAL RESPONSE	END-OF-SEMESTER RESPONSE
Maria	Mathematics is a way of solving number problems which can be applied to everyday life using a variety of formulas and ways of solving.	Mathematics is a cumulative skill in which individual, related skills are learned to build a foundation for working with numbers, variables, shapes, and everyday life.

These and other experiences gave rise to a primary instrumentalist and secondary problem-solving view, as suggested by the data collected at the beginning of the course (her initial and final responses to Prompt 1 is found in Table 4.15 above). In her own words, she described mathematics as a way of solving problems “using a variety of formulas,” evidencing the instrumentalist view. This primary view was further supported by her strong agreement with the statements “Mathematics is a collection of facts, formulas, and procedures,” “To do well in solving math problems, I have to be taught the right procedures,” and “Mathematics is a useful tool primarily used for particular calculations.” A secondary problem-solving view also emerged in her written response to Prompt 1 as she suggested the possibility of solving problems in many different ways, pointing to a mathematics that permits creativity. Indeed, the Initial Beliefs Survey showed an agreement with statements such as “Mathematics is a creative human activity” and “I think mathematics as a discipline can be revised.” Maria further revealed an alignment with the two views during our first interview, as she started by talking about the importance of procedures, but hinted at some problem-solving undertones:

> I think to be able to do math, you have to know the formula, but some things in math you can just KNOW how to do, there’s always another way to do it. I feel it’s knowing the alternate ways to do something is what makes somebody good at math. Being able to see everything and figure it out on your own rather than learning a formula and applying it. It goes a longer way if you can understand it fully…There’s always different ways to approach it, a problem.

Maria illustrated the third trend found in the whole-class data, demonstrating no change in either her primary or secondary mathematical view as a result of the course. In her case, analysis of the initial and final responses to Prompt 1 and responses to particular Likert-items both identified an instrumentalist primary view of mathematics, with a secondary problem-solving view. The discussions during our final interview, however, did suggest that she had experienced some change with regards to her beliefs about good experiences in teaching and learning mathematics as a result of experiences in the content course. Maria emphasized the role of the teacher as she described the features of good experiences she wanted to offer her students, citing teacher enthusiasm as a central component. She connected another feature (incorporating fun problems) to the teacher, suggesting that only a teacher with enthusiasm for mathematics could successfully implement fun problems that allow students to both learn and feel successful in doing mathematics.

Although there appeared to be no change in Maria’s primary and secondary views about mathematics overall, she did demonstrate some changes in her beliefs as reflected by responses to specific Likert-items. The item “To do well in solving math problems, I have to memorize all the formulas that are relevant” experienced a substantial shift, paralleling the substantial shift found in the whole-class data, and showed a slight shift away from her primary instrumentalist view. As she discussed this change, she evidenced agreement with the problem-solving view while maintaining her primary instrumentalist view:

> I think it's still an important part, random formulas that you build on year after year are valuable. I think they're good things to have in your toolbox, but I think it's important when you give a formula, like we've done in this class, to prove the formula to see why the formula exists. I remember in elementary school, some formulas just had no relevance to anything we were learning, you just memorized it, and now I see that it's important to realize where the formulas come from because figuring that out leads to understanding.

Additionally, although Maria still showed an agreement with statements such as "Mathematics is a collection of facts, formulas, and procedures" and "Mathematics is a useful tool primarily used for particular calculations," the agreement was not as strong at the end of the course as it was at the beginning, with her responses moving from 1 ("very true") to 2 ("sort of true") in both instances.

In describing the activities and experiences of the course that had the most impact on her mathematical beliefs, watching videos from a real 3rd grade class came to the forefront:

> I think it was really valuable, even at this point, to get in-class experience, and you can even get it from watching a video, so that really stuck out to me. I'm trying to get as much experience as possible…I think it…just observing any classrooms, I get ideas from teachers, how they treat their students and their curriculum, how they deal with their classroom. What I remember from the video was that the teacher allowed the students to be really open and she valued their explanations and their ideas and their conjectures. I think that's important, because what I'm learning about in a lot of my education classes is valuing the students and dialogue in class, valuing what they say and learning yourself from that. It was important to see the video, because you can see that the teacher can learn from the students. I hope I can create a classroom like that someday.

She also placed heavy emphasis on the importance of group work in her experiences in solving problems and the related explanations:

> I think the group work was very beneficial, I think it was beneficial to develop ideas in your head first too, and then share it in groups, so your ideas aren't muffled by other people's ideas. It was also important because I learned so much by having other group members explain things to me. Some of the things…my brain just didn't work that way, and I just didn't understand at first what they were talking about. We really worked

> together well on it. It was especially useful trying to explain something to someone. Explaining myself, I definitely got better at it.

The conversation about the importance of group work and explanations led us to the final point that Maria wanted to make with regards to the impact of her experiences in the course on her beliefs, particularly with regards to her beliefs about the role of the math teacher and the learning of her students:

> Thinking about explanations instead of just procedures and how I'd want to teach…I think this was really relevant in this course because we did figure out a lot of the problems ourselves. I mean, sometimes you [the teacher] helped us a bit if we were having problems, but we could always get it. Even when we were working in groups, you could figure it out yourself. I think it's really interesting that you don't always have to be taught something. The teacher doesn't always have to teach it. There are things that you know that you don't know that you know!

4.6.2.4 Summary of illustrations of the three trends

There are undoubtedly many stories that could be told from the data collected during this study. The three PSETs selected for elaborated discussion represented prominent aspects of the study's general findings related to Trends 1, 2, and 3, and their individual stories provided rich insights into the changes that occurred in their own beliefs (and perhaps those of other participants) as a result of their experiences in the course. The qualitative data analyzed in the previous three subsections provides facets of the process of belief change not captured though analysis and discussion of only the quantitative data. When discussed together, the use of a variety of data proved to be more valuable than when discussed in isolation.

4.6.3 Factors influencing the changes in mathematical beliefs

The various data collected throughout the study revealed that the participating PSETs did experience change in their beliefs about learning and teaching mathematics, in addition to their beliefs about the nature of mathematics, during the course. There were several factors found that appear to have contributed to this change with regards to specific features of the course: (1) the activities (or curriculum materials) used in the course; and (2) the teacher educator.

As elaborated upon in Section 4.3.1, the focal course was chosen for this analysis for several reasons. Feature 3 described in Table 4.2 identified as an important feature of the course the P-R mathematical tasks that were developed as part of the curriculum materials. Engagement with this type of instruction and these activities allowed the PSETs to expose their held beliefs, reflect on those beliefs in light of their experiences with the activities in the course, and perhaps engage in the process of belief change. Additionally, they encouraged the development of new understandings of mathematical content specifically aimed at the work of teaching.

Interview data from the three PSETs illustrated that their previous experiences as students of mathematics played a significant role in the development of their beliefs in mathematics. Discussions about these past experiences often exposed negative aspects of their mathematics classrooms which included both the activities in which they engaged and the teachers of the classes. Feature 2 in Table 4.2 identified as another important feature of the course the role of the teacher educator as a "representative of the mathematical community" and creator of a supportive and discussion-based classroom environment. Indeed, this vision of a mathematics teacher is often in sharp contrast with teachers found in more traditional classrooms. In the next sections, I elaborate upon each of the two factors that appeared to influence changes in the PSETs' beliefs in turn.

4.6.3.1 The curriculum materials used in the course

Recall that the participating PSETs were asked to identify the specific activities in the course that stood out to them the most and had the most impact on what they believed about mathematics, teaching mathematics, and learning mathematics. Many of the activities used in the curriculum of the course included implementations of CAPs and were purposefully designed to create cognitive conflict (MBC6) in the participants as they engaged with them (see Features 1 and 3 in Table 4.2). As first described in Chapter 2 and revisited here in Section 4.2, CAPs are described as "instructional activities that aim to direct students' attention to their understandings or conceptions of a particular mathematical topic or idea" (Stylianides & Stylianides, 2009a, p. 322). In this case, I was interested in the results of implementing CAPs in relation to P-R mathematics tasks with the intention of directing the PSETs' attention to their mathematical beliefs. An exploration of the PSETs' descriptions of and reflections about "The Blonde Hair Problem," "The Squares Problem," and "The Two Considerations" activities showed an enjoyable engagement with these activities, and this enjoyment was often cited as the reason why the activity stood out and made an impact on their beliefs. The findings provide evidence that these activities, often grounded in the real work of teaching (MBC5), altered PSETs' beliefs about mathematics, especially in regards to the ways in which they believed it should be learned and taught. These activities, which challenged the students and purposefully created cognitive conflict (Stylianides & Stylianides, 2009a) frequently had several different solution paths, and focused on explanations rather than just correct solutions, were often in sharp contrast to the experiences in their prior math courses. Indeed, the curriculum materials used in the focal course have shown to enact several of the mechanisms for belief change introduced in Chapter 2 with encouraging results.

4.6.3.2 The role of the teacher educator

The PSETs used as illustrations of the three trends ascribed to their past math teachers a significant role in the development of their (often Platonist and instrumentalist) mathematical beliefs. I, as the teacher educator in the course and the classroom experience I created, may have played a central role in their process of belief change. Many of the features that emerged under the theme related to the instructor of the class in the PSETs' responses to Prompt 3 about good experiences in doing mathematics reflected elements of my instruction. Indeed, in their discussion of the theoretical framework underpinning the development of the course's curriculum, Stylianides and Stylianides (2009a) describe major features as the "means for supporting the resolution of cognitive conflict and the role of the instructor" (p. 317). The data described in this chapter appears to provide evidence that the latter had an impact on the PSETs' beliefs about the learning and teaching of mathematics. In addition to visualizing the teacher educator in their course as "the representative of the mathematical community in the classroom" (p. 317), Stylianides and Stylianides described the educator's significant role in implementing CAPs and scaffolding when necessary in order to help students resolve their cognitive conflict (see Feature 2 in Table 4.2). Without a teacher educator embracing these multiple roles, a successful enactment of the curriculum materials as discussed in the previous section would be highly improbable.

In the data describing the features of what the PSETs associated with a good experience in doing mathematics, recall that three themes emerged from their responses, one of which was the instructor of the class. Although it cannot be said for certain, the changes that occurred with regards to these features may be associated with the PSETs' experiences regarding me as the teacher educator in the focal course. For instance, the feature in this theme that experienced the largest increase in citations by the PSETs regarding a good experience in doing mathematics was the encouragement of group work and collaboration. While only two PSETs mentioned the importance of group work on the first day of the course, 11 PSETs identified the feature at the end of the course (more than five times as many than at the beginning of the course). Indeed, an emphasis on group work was an important feature of my instruction, and may have had an impact on the PSETs' own ideas about elements of instruction that relate to good mathematical experiences.

Although the other two themes (the socio-mathematical norms and features of the curriculum) were considered separately in the presentation of the results related to Prompt 3, they can be seen as extensions to the role of the instructor of the class. Indeed, the support for such norms and the implementation of curriculum materials relies heavily on the educator. For instance, as identified in the previous paragraph, the PSETs reported an increased appreciation for the value of collaboration and small group work (MBC3), a feature that was prevalent during the individual interviews with both Max and Maria. The whole-class data further suggested an appreciation for a fun classroom environment that encourages creativity, discussion, and explanation. Such classroom features are not possible unless they are valued and by the teacher, who then creates learning opportunities that support such features in the classroom. Good experiences, according to the data, are also supported by problems that challenge students and encourage hands-on explorations, activities that must be chosen and presented by the teacher. As found in the cases discussed, experiences in the focal course reflected many of these features and challenged some of the students' beliefs regarding both the learning and teaching of mathematics, and therefore may have helped to change their beliefs about what constitutes a good experience in doing mathematics. In particular, engagement with activities in the course (e.g. "The Squares Problem," "The Blonde Hair Problem") resulted in the belief that mathematics can be simultaneously challenging and fun.

The interview data suggested that some PSETs may have viewed the experience of the mathematics content course as a positive experience of mathematics that contrasted with negative previous experience they had in elementary, middle, and high school. This did not come as a surprise, as the finding that pre-service teachers often report negative past experiences with regards to mathematics has been widely reported in the literature (e.g. McLeod, 1992). This perception was reinforced in the comparisons often drawn by the three PSETs between their previous teachers and the teacher educator instructing the mathematics course. Some features that were cited with increased frequency regarding good experiences in doing mathematics by the three PSETs at the end of the course identified were teacher approachability, teacher enthusiasm, and the importance of being able to prepare students for future learning. These qualities necessitate that the educator be knowledgeable about both the content and the students, and that they possess excitement about mathematics, as well the ability to creating a problem-solving environment (MBC1) that can promote excitement in the students.

4.6.3.3 Summary

In summary, both the curriculum materials of the course and the teacher educator appear to have played important roles in the process of belief change in the participating PSETs in the focal course. Although these two features are independently important, their impacts and resultant successes are inextricably intertwined. Stylianides and Stylianides (2009a) identified the critical role of teacher educator as an important feature in supporting the PSETs' cognitive conflict arising as a result of engagement with CAPs. Moreover, their development of the curriculum materials envisioned a teacher educator whose role "involved providing scaffolding strategies such as asking probing questions to… [engage] students in activities intended to help them become (more) aware of their conceptions of particular mathematical topics" (p. 324). The results described in this chapter have shown that the teacher educator provided opportunities not only to encourage an awareness of these conceptions, but also to help the participants become more aware of their held beliefs about mathematics. Specific activities were identified by the PSETs as impacting their previous beliefs regarding the learning and teaching of mathematics, and these activities may be seen as those that created the highest levels of cognitive conflict through the use of CAPs and P-R mathematical tasks. The descriptions provided by the PSETs about their renewed visions of good experiences in doing mathematics may also reflect their own experiences in the course. Indeed, it is possible that the role and actions of the instructor implementing the activities that the PSETs observed may have had an impact on their visions of good experiences, in addition to the impact of the activities themselves. Indeed, the PSETs may have viewed the teacher educator as a "representative of the mathematical community" (p. 324), specifically the community of teachers. The participating PSETs were given the opportunity to observe the kind of instruction that led to an environment that values collaboration, discussion, and explanation. These features of instruction appeared to be associated with the PSETs' beliefs about what would constitute good experiences doing mathematics in their own classrooms someday as found in the data.

4.7 CONCLUSIONS

Many interesting findings have emerged from the analysis undertaken in this chapter. First, there was evidence that the research-based mathematics content course described in this study provided opportunities that promoted belief change in some of the participating PSETs towards a problem-solving view. This view is desirable for future teachers of mathematics as, out of the three views about the nature of mathematics described by Ernest (1988), the problem-solving view most closely aligns with the vision promoted by mathematical reform (NCTM, 1989, 1991, 2000).

As suggested by Lortie (1975), the PSETs' initial beliefs about mathematics had their roots in their early experiences as students. This resulted in a strong initial alignment with the Platonist and instrumentalist views of mathematics, which are both associated with the traditionalist perspective that does not align well with reform mathematics. These views were shown to be deeply-rooted and resilient to change over the duration of the mathematics course. Considering the tenacity of these beliefs, it seems crucial that educational researchers and teacher educators better understand and attempt to address them as early as possible in and within all components of TEPs. The Platonist view was shown to be particularly resilient to change. However, interview data from one PSET maintaining the Platonist view suggests that a Platonist view about the nature of mathematics may not necessarily result in Platonist views about learning and teaching mathematics.

Overall, the data suggested that there were several features of the course that appeared to promote belief change. The course provided the PSETs with opportunities that: (1) made them aware of their held beliefs through the use of CAPs, cognitive conflict, and reflection; and (2) allowed them to experience curriculum materials and instruction that supported a constructivist, problem-solving perspective. The important features of the course that appeared to impact the PSETs fell into two categories that were separate yet connected. The first category corresponded to the instruction and activities guiding the course, and the second to the teacher educator. The features of a classroom associated with good experiences in doing mathematics were described as those which promote opportunities for reflection, collaboration, and discussion in a fun yet challenging problem-solving environment, all created by the instructing educator. The PSETs appeared to move towards the belief that mathematics was about more than just memorized facts and formulas, and that discussions of explanations within a problem-solving environment promoted mathematical understanding. Moreover, they appeared to develop more reform-oriented beliefs regarding the role of the mathematics teacher as a facilitator of student-constructed knowledge as opposed to a mere transmitter of knowledge. Indeed, the course exposed the PSETs to instruction grounded in a reform perspective, and they were given opportunities to apply what they learned to the real work of teaching. The activities related to "The Two Considerations" discussed in this chapter provide an example of a P-R mathematics task (Stylianides & Stylianides, 2010) that was grounded in the real work of teaching and was shown to impact the beliefs of the participating PSETs. These tasks, which attend to both mathematics and pedagogy, create for PSETs "opportunities to not only develop their knowledge about definitions, mathematical language, and even numbers, but also to see and appreciate how this knowledge may be applicable in relevant teaching contexts" (Stylianides & Stylianides, 2010, p. 168). In other words, these tasks promote learning opportunities to develop the PSETs' mathematical knowledge for teaching (Ball & Bass, 2000) and allow for legitimate peripheral participation (Lave & Wenger, 1991). Indeed, the features of the course that appeared to have successfully promoted belief change overlap greatly with the mechanisms for belief change outlined in Chapter 2.

There are several implications of the results and related discussion of this chapter. Even though the results are encouraging given the evidence that many PSETs experienced change in their beliefs about mathematics during a single course, it cannot be ignored that many of the Platonist and instrumentalist views held by the participants were resilient to change. However, the findings do suggest that if TEPs acknowledge the presence of these resistant beliefs and their potential impact on the creation of reform-oriented classrooms and make an active effort to address them, these programs can make a difference. Yet in order to make a difference, all components of TEPs need to explicitly address the importance of beliefs and provide learning opportunities that allow pre-service teachers to become aware of their held beliefs, to promote positive change in those beliefs, and to sustain those changes. This would ideally include a purposeful enactment of the mechanisms for belief change identified in Chapter 2 (and revisited in this chapter) in content courses, methods courses, and during the student-teaching experience. These mechanisms recommend the creation of problem-solving environments utilizing innovative curriculum materials and conceptual awareness pillars grounded in the introduction of cognitive conflict. This environment would also value reflection, perhaps through the use of reflection journals as employed for data collection in this study, and collaboration.

There are also implications with regards to the analytic framework described in Chapter 3. An analysis of, for instance, "The Squares Problem" described in Section 4.5.3 using the framework identifies as the most common actor/process interaction the *you* actor engaging in material processes (e.g. you find, you work). According to the framework, this interaction suggests the promotion of either the instrumentalist or problem-solving view of mathematics (relating to diamonds 7 and 11 in Figure 3.1). "The Squares Problem" was cited the second most influential activities by the PSETs regarding their mathematical beliefs, with the most frequently mentioned feature of their experience with the problem relating to an elevated awareness in the importance of explanations in mathematics. This emphasis on understanding "the why" and not just “the how” seems to suggest a closer alignment with the problem-solving view than the instrumentalist view, yet this is not captured by an analysis of the statement of the activity using the framework alone. In order to better understand the ways in which particular tasks or activities promote different views about mathematics (as well as changes in those views), it seems important to also analyze the ways in which the activities are implemented in the classroom. As will be elaborated in Chapter 5, the framework may also be used in this type of analysis to investigate, perhaps, transcripts of classroom discourse in addition to the written statements of the activities.

I acknowledge that there are a number of limitations in the study discussed in Chapter 4 due to the context of the study, the relationship between the participants and myself, and my own personal beliefs. First, the findings of the study relate to a relatively small number of PSETs (25) in a very specific mathematics course, that is not representative of content courses in general. However, that is not to say that the features found to impact belief change in this particular group of PSETs cannot be implemented in other content courses across the country. Second, I was also the instructor in the content course that served as the research context, and I naturally forged a relationship with the PSETs which was mutually respectful and amiable. The PSETs were informed and well-aware that their responses to surveys and prompts, as well as their entries in a reflection journal, were not only a part of their normal coursework but also data for my research. It is possible that they desired to be of assistance, so to speak, by responding in a manner that they thought I would prefer regarding their beliefs about mathematics and the activities within the course I was instructing. In the same vein, I realize that while I made efforts to refrain from explicitly informing the PSETs about my own held beliefs about mathematics, the choices I made with regards to the activities in the course and the point of view most clearly communicated may implicitly have suggested those beliefs. Also, the relationship between me as the researcher and my students as the research participants may have influenced to some extent my perceptions of them and their data. This is unavoidable, however I was conscious of these issues and made explicit efforts and took precautions to minimize them. Third, as highlighted by Max's interview data regarding his Platonist view, having a particular view of mathematics as a discipline may not necessarily imply that an individual has the same view of teaching and learning of mathematics. By focusing the analyses in this chapter primarily on the PSETs' views of mathematics, it is possible that some of the changes in the participants' beliefs might have been missed. Perhaps the PSETs lacked the language necessary for describing how they had come to view mathematics that was more readily available to them as they described their ideas about teaching and learning. Finally, although the findings suggest that many of the PSETs experienced changes in their beliefs as a result of the course, there is no evidence that these changes will be sustained in the long run. It would have been beneficial to have continued the data collection for an extended period through the remainder of the TEP and perhaps even into their initial-teaching experiences. This longitudinal collection of data experiences would certainly have added to an understanding of the stability of the PSETs' beliefs and strengthened the related findings. Furthermore, while the various measures of data collection employed in this chapter richly captured the PSETs' experiences and beliefs, all data was self-reported. Additionally, the quantitative data were analyzed only in terms of differences and comparison of means. The inclusion of some data collection that involved observations possibly relating to enactment of beliefs, as well as stronger tests of statistical significance, may have added to the rigor of the data and increased reliability in the findings.

5.0 CHAPTER 5: CONCLUSION

5.1 INTRODUCTION

The overarching research question in this dissertation has been the following:

What role do beliefs play in the learning and teaching of mathematics, and how might experiences within teacher education programs influence and promote change in the mathematical beliefs of pre-service elementary teachers?

To address this question, this work has been structured around three separate yet interconnected papers which have addressed the question through three distinct but interwoven lines of work. Figure 5.1 provides a map of how the overarching research question was investigated through three specific research questions in Papers 1, 2, and 3. The dashed arrows in the figure represent how different aspects within each individual paper informed other aspects of the paper, while the solid arrows represent the ways in which the three papers informed each other. The solid arrows are numbered for ease in discussion of the interconnectedness of the papers, which will be elaborated upon in Section 5.2.

Paper 1 provided a broad synthesis of the beliefs-related literature, which included different characterizations of the term and established their importance with regards to the learning and teaching of mathematics, specifically in the context of teacher education programs

(*TEPs*). It also outlined some successful mechanisms for belief change, as well as described the ways in which such change can be supported in different components of TEPs (methods courses, content courses, and student-teaching experiences). Paper 2 elaborated upon the role of textbooks and their possible impact on the beliefs of pre-service elementary teachers (*PSETs*) in mathematics content courses. It went on to investigate the different views about the nature of mathematics being promoted in textbooks used in those courses by implementing an analytic framework developed for the analysis. Paper 3 was informed by and expanded upon the ideas established in the first two papers. It investigated the features of instruction and the curriculum materials developed within a research-based mathematics content course and their impact on the mathematical beliefs of the enrolled PSETs.

The method employed to investigate the overarching research question by means of these three papers was grounded in the key idea that mathematical beliefs play a vital role in both the learning and teaching of mathematics. Therefore, this dissertation proposes that questions and concern about learning and teaching cannot be fully addressed without first understanding the origins and impact of beliefs. In particular, the beliefs of PSETs are often based on their own experiences as students, and provide the foundation for their own future instructional practices. Given that these past experiences commonly reflect the Platonist and instrumentalist views of traditional classrooms, it is critical that TEPs acknowledge the existence of such beliefs and make efforts to provide opportunities that promote change in beliefs towards a problem-solving view of mathematics. This problem-solving view more closely reflects that supported by reform mathematics.

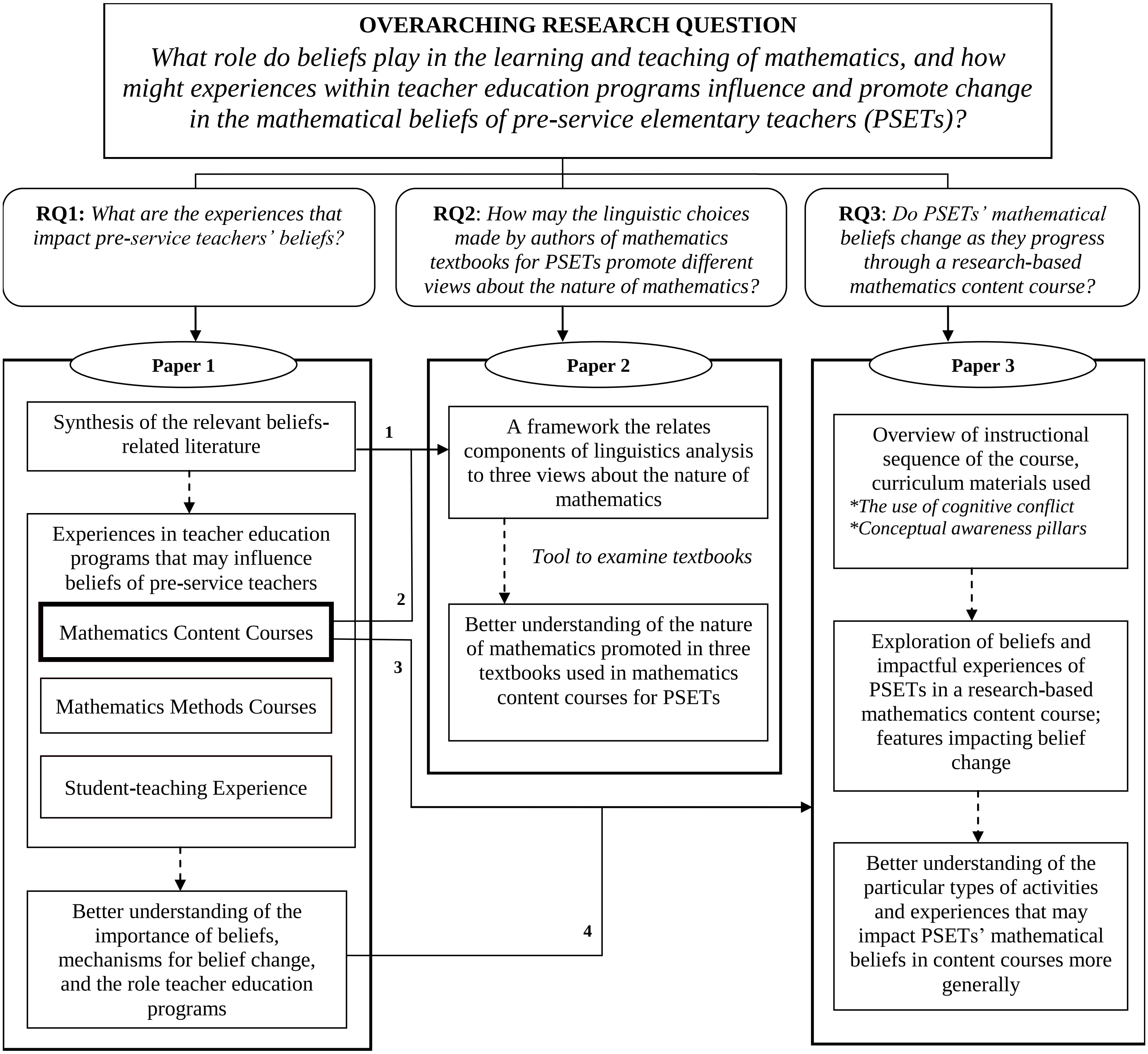

Figure 5.1: Map of How the Three Papers Contributed to the Overarching Research Question and How They are Interrelated

Next, I summarize the three papers as discussed in Chapters 2, 3, and 4. After, I discuss implications of the research and possible directions for future research. Finally, I make some concluding remarks.

5.2 SUMMARIES OF THE THREE PAPERS

5.2.1 Paper 1: The mathematical beliefs and experiences of pre-service teachers

Paper 1 in Chapter 2 provided insights into the various conceptualizations of the term 'belief' as used in the mathematics education research, and how those conceptualizations relate to the beliefs found to be held by young students, as well as pre-service and in-service teachers. The literature established the fact that beliefs of pre-service and in-service teachers (although shown to be highly-resistant to change) can be changed, and in the first section of the paper I described six successful mechanisms for belief change identified in that literature: (1) a focus on problem-solving and exploration; (2) the opportunity for reflection; (3) collaboration and small group work; (4) the use of innovative curriculum materials supporting the Standards; (5) having coursework grounded in the real work of teaching; and (6) the need for beliefs to be challenged (e.g., through the use of cognitive conflict). These mechanisms provided the foundation for the second section of the paper, where I described the different experiences within TEPs that may promote belief change in pre-service teachers.

Despite the fact that pre-service teachers often enter TEPs with beliefs that reflect traditional views of mathematics, many programs may fail to acknowledge the presence, role, and impact of those beliefs. In spite of this, all components of the programs (which typically include content courses, methods courses, and a student-teaching experience) have the potential to provide rich opportunities to enact the various mechanisms for change as outlined in Paper 1 to impact mathematical beliefs. Yet, much of the research exploring the impact of learning opportunities in TEPs on the beliefs of future teachers has concentrated on the impact of mathematics methods courses. Paper 1 suggests that further research regarding the impact of mathematics content courses need to be made to the existing body of beliefs research. This suggestion is further supported by the fact that mathematics content courses, which focus on developing content knowledge rather than pedagogical knowledge, are traditionally taken before any methods course or the start of student teaching, and therefore provide the earliest opportunity in TEPs to unveil and impact the mathematical beliefs held by pre-service teachers.

5.2.2 Paper 2: Developing and implementing a critical discursive framework to analyze the views about mathematics being promoted by textbooks for pre-service elementary teachers

Paper 2 in Chapter 3 discussed research that has shown that the content of textbooks is crucial in determining what is taught in classrooms from the elementary to high-school level. The amount of available research describing the content and impact of textbooks used in TEPs, however, is substantially less by comparison. Paper 2 contributes to this research and also builds on the work in Paper 1 (see arrows 1 and 2 in Figure 5.1) by considering one way in which the mathematics content course may promote different mathematical views in the textbooks utilized, which may in turn influence the beliefs of PSETs. In order to investigate the promoted views, I developed an analytic framework that drew from scholarly work on different philosophies of mathematics and critical discourse analysis. According to this framework, different interactions between the linguistic components of *actors* and *processes* and the presence of different *modality* indicators suggest alignment with the *Platonist*, *instrumentalist*, and *problem-solving* views about the nature of mathematics. In particular, the analysis of tasks and definitions in the number theory chapters of three textbooks used in content courses for PSETs illustrated how the framework can serve as a tool to examine the different views about mathematics promoted in each textbook, and described how these different views may provide different learning opportunities for PSETs.

5.2.3 Paper 3: An exploration of the impact of instruction and activities in a research-based mathematics content course on the mathematical beliefs of pre-service elementary teachers

Using different measures such as surveys, written prompts, reflection journals, and individual interviews, Paper 3 in Chapter 4 also extended on the work of Paper 1 (see arrows 3 and 4 in Figure 5.1) attempting to better understand: (1) the initial beliefs held by PSETs; and (2) how the instruction and activities experienced in their (research-based) content course may have promoted change in those beliefs. Although the findings suggested that the majority of PSETs initially held Platonist and instrumentalist views about mathematics (both associated with the 'traditional' view described in Chapter 1), many of them experienced change towards the problem-solving view through engagement with the course. Even though numerous PSETs appeared to have rather resilient Platonist or instrumentalist *primary* views, the data suggested that some of these PSETs did experience a shift in their *secondary* mathematical view. Moreover, this shift was often in the direction of the problem-solving view. This movement is promising considering that the problem-solving view most closely aligns with the vision of reform mathematics advocated by the NCTM.

The results in Paper 3 suggested that the changes in the mathematical beliefs of the PSETs may have been impacted by the activities included in the curriculum materials used in the course, as well as the instruction of the teacher educator. Overall, the findings suggest that it is certainly possible to successfully enact several mechanisms for belief change within a mathematics course, and not just methods courses (where the majority of the prior research has focused). Yet, more work needs to be done in order to investigate the sustainability of the belief change over a long period of time. This includes investigating beliefs over the duration of an entire program as well as the transition from the TEP to the actual classroom.

5.3 IMPLICATIONS

Based on the results and discussions related to the three papers, I propose some implications that fall into two areas. The first area is that of teacher education, whereas the second area corresponds to beliefs research more generally.

5.3.1 Implications for teacher education

One basic yet fundamental recommendation is that TEPs actively acknowledge the important role played by beliefs in the development of mathematics teachers. A growing amount of evidence has shown that a large portion of teachers, both in-service and pre-service, possess beliefs that align with (traditional) Platonist and instrumentalist perspectives, beliefs that may prevent the kinds of classrooms desired by the (constructivist) problem-solving vision of reform. It seems important to intervene in the development of such beliefs before teachers enter the profession, and TEPs appear to be an ideal setting for such an intervention to occur. This necessitates that all aspects of these programs, from the student-teaching experience to the content and methods courses, provide consistent opportunities that not only allow pre-service teachers to become aware of their held beliefs, but also to challenge and change those beliefs as necessary.

I propose that this can be accomplished by enacting specific mechanisms for belief change that have been described in Chapter 2. This enactment would involve the creation of problem-solving environments based in inquiry that challenge future teachers and allow them to construct their own mathematical understandings. This environment would also ideally allow for reflection and collaboration around innovative activities that are grounded in the real work of teaching.

In addition to the general classroom environment, particular attention also needs to be paid to the curriculum materials used in TEPs. As shown by the analysis in Chapter 3, the textbooks used in content courses may promote different views about the nature of mathematics. The textbooks chosen for these courses should be selected carefully and purposefully. Also, given that pre-service teachers are likely to rely on their textbooks once they enter their own classrooms, TEPs may consider providing future teachers with opportunities and tools to explore and reflect on a variety of both traditional and innovative curriculum materials that reflect the different materials that they may someday use. This type of interaction may provide the future teachers with the opportunity to analyze and discuss the benefits and shortcomings of the different curricula (including those relating to the different views about mathematics promoted in the curricula), which may in turn impact their own future textbook use.

Although it is vital that initial changes in beliefs occur, perhaps even more critical is that changes experienced by PSETs be maintained as they progress through their TEPs and eventually enter their own classrooms. It has been suggested that the beliefs and experiences aligning with the vision of reform held by new teachers can often be washed out as they enter the complex arena of their first classroom, especially when fellow teachers or the school district at large do not share the vision espoused by the new teachers and their TEPs. An acknowledgement of these future obstacles in TEPs may better prepare pre-service teachers for their often-difficult first years in the classroom. This may also provide the conviction needed to avoid having any positive beliefs washed away. TEPs should also consider carefully those selected as mentor teachers during the student-teaching experience, as mentor teachers have been shown to serve as influential factors on pre-service teachers when they make the transition into the classroom.

5.3.2 Implications for beliefs research

As with the research described in Chapter 4, much of the previous work done in the area of beliefs has reported the results of a single course or a sequence of courses utilizing data that was self-reported by the participants. It was advantageous to the work in Chapter 4 to use both quantitative (e.g. surveys) and qualitative (e.g. reflection journals, interviews) data to identify trends that emerged during a relatively short period of time and to clarify the beliefs of three particular PSETs whose interview data was discussed. Indeed, using both qualitative and quantitative data could prove to be fruitful in other studies as well. This approach provides depth that is likely lacking when using each type of data separately. Moreover, it allows the research questions about the beliefs of pre-service (as well as in-service) teachers to be approached in a variety of ways and helps to establish reliability of explanations through triangulation. Qualitative data such as interviews and observations may clarify and supplement quantitative data, as they provide the opportunity to address the "why?" and "how?" that is not frequently captured via purely quantitative means.

5.4 DIRECTIONS FOR FUTURE RESEARCH

There are several directions for future research that can be considered based on the findings from the three papers. Some of these directions were briefly mentioned at the ends of Chapters 2 and 3, as they directly related to the limitations of each paper outlined there.

The design of the studies undertaken in Chapters 3 and 4, while leading to fruitful outcomes, focused on relatively small samples (three textbooks in Chapter 3 and 25 students in Chapter 4), and future research concentrating on larger samples may capture more richness in the data. Along these lines, an extension of the work described in Chapter 4 may entail data collected over a longer period of time in an effort to monitor changes or stability in beliefs beyond a single course. With respect to the textbook analysis in Chapter 3, the framework may also be used to analyze the views about mathematics being promoted in textbooks not only used in TEPs, but those used across different grade levels, from the elementary to the high school. Future research may consider analyzing the mathematical beliefs of a specific cohort of PSETs as they move from their content course to their methods course, as well as into their student-teaching and initial-teaching experiences. As discussed in Section 5.3.2, it seems important that such long-term research using both quantitative and qualitative means be enlisted in an effort to monitor and document the activities and experiences that most influence and shape the beliefs of pre-service teachers so as to better inform teacher (and perhaps even professional) development. Moreover, this research needs to move beyond investigations of reported beliefs to further investigate the relationship between reported beliefs and their influence on actual classroom practices.

Making a bridge between the empirical studies in Chapters 3 and 4, many interesting directions for future research may involve exploring the ways in which the implementation of textbooks found to promote particular mathematical views impact the mathematical beliefs of both PSETs and teacher educators in content courses. This includes explorations of the ways in which PSETs interpret their roles in mathematical activity outlined by the written curriculum given the different views about the nature of mathematics being promoted by textbook authors. It would also be interesting to investigate the ways in which the views being promoted may change through implementation of teachers with varying beliefs. In other words, the framework has the potential to investigate not only the written curriculum, but aspects of the implemented and attained curriculum as well. Transcriptions of classroom discourse may be analyzed (in a very similar manner as were the textbooks) using the analytic framework to understand how linguistic choices made by teachers may promote different views about the nature of mathematics. This type of analysis may also inform the ways in which the views supported by particular textbooks are maintained (or not) through implementation. Finally, the framework may be used to analyze elements of the attained curriculum. To help determine what PSETs have learned and how that learning may have impacted their mathematical views, the framework could be used to analyze reflections and responses to prompts centered on mathematical beliefs (such as the prompt "What is mathematics?" discussed in Chapter 4) with respect to the various actors, processes, and modality indicators that they chose to use.

5.5 CONCLUDING REMARKS

The findings and discussions within this dissertation suggest that although the beliefs held by PSETs are often less than desirable and resistant to change, there is certainly reason for optimism. Providing PSETs with experiences that involve various mechanisms (e.g. problem-solving environments, opportunities for reflection, innovative curriculum materials grounded in the work of teaching) creates a promising doorway for belief change. It may be suggested that continued engagement with such experiences throughout the TEP may allow the process of belief change to continue and for beliefs to become refined and more central in an individual's belief system. Exposure to these types of experiences in a single course is certainly beneficial to the process of belief change, but would appear to be inadequate to inspire and maintain the level of change desired by the reform movement. Various experiences are needed to help PSETs develop beliefs about mathematics that support the reform movement, experiences that should be provided in all components of TEPs from the content and methods courses to the student- and initial-teaching experiences. This requires an acknowledgment of the importance of beliefs in every aspect of these experiences, from the textbooks and other curriculum materials used to the role of the teacher educators and their methods of instruction. Although research continues to contribute to our understanding about the importance, influences, and impact of mathematical beliefs on both the learning and teaching of mathematics, many questions remain, and these questions may be viewed as central to the improvement of mathematics education as a whole.

APPENDIX A

CODING DOCUMENT USED IN CHAPTER 3

How to Identify *Definitions* and *Tasks* for the Analysis

Here I describe the ways in which I identify *definitions* and *tasks* to be analyzed with regards to the linguistic components of the framework (which are described below) for you to follow.

(i.) For to *definitions*, I made the decision to investigate the creation, statement, and related discussion of the definitions emphasized by the textbook authors. Oftentimes, these sections are comprised of several sentences. Definitions are the emphasized terms obviously intended to be distinguished from others in the text, often by the consistent use of boxes labeled "Definition," or by an intentional bolding of the term within the text. For terms that were bolded, I further differentiated those terms to be included as a *definition* in my analysis to be those accompanied by a clear description of a newly-introduced term. I **excluded** bolded terms that were simply meant to remind the reader of a previously discussed concept as well as those that were only mentioned in passing and not explicitly discussed further. Introductory text and exploration activities leading to the statement of the definition were also coded as part of the definition, as was any text immediately following the statement of the definition that related to the definition (e.g., alternative definitions, clarifications, etc.). I did **not** code any theorems, lemmas, or specific examples relating to the term being defined, which could have been embedded between the analyzed portions of the text.

(ii.) I counted as one *task* all parts that were associated with the initial marker indicating the task, which was primarily a number. For example, if a task was numbered as "1" and had two parts labeled (a) and (b), I code all parts as relating to a single task. Again, there could be several sentences related to a single task. For each section of the textbook, I coded only the tasks that related specifically to the concepts introduced and developed in that particular section, and did **not** code tasks that were distinguished as being related to review of previous topics.

The Second Dimension: Descriptions of the Linguistic Components of the Framework

Here I describe in detail each of the three linguistic components utilized in my framework (actors, processes, and modality) that are used to code definitions and tasks, as well as provide illustrative examples of each.

(i.) *Actors*: The role of the actor is that of the doer, the one that makes the action happen. Human actors are coded according to personal pronouns used (*you*, *we*). If no explicitly stated actor precedes a commanding verb (which we call the process here), it is assumed that the actor is *you* and is coded as such. *Third-person participants* (e.g. the student, someone, one) and *non-human actors* (e.g. mathematical objects) are coded as belonging to one of the two groups.

CHOOSE ONE ACTOR PER INTERACTION (See Table 1 below)

(ii.) *Processes*: Processes, as explained by Halliday (1985), are described to "cover all phenomena…anything that can be expressed by a verb; even, whether physical or not, state, or relation" (p. 159). In order to help you better understand the different processes utilized here, I draw from "An Introduction to Systemic Functional Linguistics" by Suzanne Eggins (2004).

a. *Material*: These are processes of doing, of concrete and tangible actions. One identification criterion for material processes is that they can be probed by asking: *What did x do?* (ex. What has Diana done? Diana *donated* blood. Diana *went* to Georgia. Diana *carried* the bag. [material processes italicized]). Here I have provided a list of verbs which provide examples of material processes that may be found in the mathematics textbooks: *find*, *list*, *calculate*, *compare*, *write*, *multiply*.

b. *Mental*: These processes encode meanings of thinking or feeling. One identification criterion for material processes is that they can be probed by asking: *What do/does [actor] think/feel/know about x?* (ex. What do you think about cats? I *hate* them. What did she think about the excuse? She *believed* it. [mental processes italicized]). Mental processes can be ones reflection cognition (verbs of thinking, knowing, understanding, e.g. *know*), affection (verbs of liking, fearing, e.g. *hate*), or perception (verbs of seeing, hearing, e.g. *heard*). These are all coded the same. Here I have provided a list of verbs which provide examples of material processes that may be found in mathematics textbooks: *think*, *understand*, *recall*, *note*, *consider*.

c. *Relational*: These processes cover the many different ways in which *being* can be expressed. There are further subcategories of relational processes which are not addressed in this work. Here I have provided an extensive list of verbs which provide examples of relational processes that may be found in mathematics textbooks: *be, is, equals, adds up to, make, signify, mean, define, spells, indicate, express, suggest, act as, symbolize, represent, stands for, refers to, exemplify*.

CHOOSE ONE PROCESS PER INTERACTION (See Table 1 below) (iii.) *Modality*: Modality is the degree of certainty or obligation expressed in the clause/sentence. Halliday (1985) describes the modality as being where the speaker expresses judgments as to the likelihood or probability of something happening or being. A count of the frequency of modality indicators (described below) is a part of the coding process.

a. *Contingent modality*: Suggests a possibility for alternatives. Indicators include words such as: *perhaps, may, could*.

b. *Absolute modality*: Suggests certainty, no room for alternatives. Indicators include words such as: *always, never, must, absolutely, certainly*.

KEEP A COUNT OF ALL MODALITY INDICATORS YOU COME ACROSS IN THE TEXT

Table 1: Final Codes to be Used in Textbook Analysis

LINGUISTIC COMPONENT	**POSSIBILITIES**	**RELATED CODE/INDICATOR**
(1) Actors	We	**W**
	You	**Y**
	Third-person	**T**
	Non-human	**N**
(2) Process	Material	**M**
	Mental	**N**
	Relational	**R**
(3)Modality (count indicators)	Absolute	always, must, will (when used in first and third person), has to (be), certain(ly), sure(ly)
	Contingent	may, can, might, sometimes, could, possible/possibly, maybe

Important things to remember when coding:

(1) Every definition and task can be comprised of several sentences, and each sentence may contain several actor/process interactions. You should code EVERY interaction for (a) the actor present and (b) the process in which they are engaged. For example, you may find a statement like this: "We ask that *you read the selection below, consider each pair of numbers, and find all the prime factors.*" The portion of the text in italics explicitly depicts the *you* actor in the first interaction (with the process *read*), but this actor is also implicitly interaction with two processes later in the statement (*consider* and *find*). Therefore, this italicized portion of the text would actually have THREE interactions to code: *you*/*read*; *you*/*consider*; *you*/*find*. Please consider this example as illustrative of how such statement should be coded.

APPENDIX B

RESEARCH STUDY OVERVIEW GIVEN TO PSETS

IL 1473
OVERVIEW OF RESEARCH STUDY

The purpose of this research study is to investigate the mathematical beliefs that you have brought to this course, Mathematics for Elementary School Teachers, as well as the past experiences with mathematics that have influenced these beliefs. Moreover, the study also aims to understand the ways in which the activities and experiences within this course may impact those beliefs. For these reasons, I am interested in further analyzing the surveys, reflection journals, and short-answer responses that are already integrated into the required coursework for this class.

From those willing to participate, I will ask for volunteers to meet with me twice during the semester (during the second week of class and before grades are due) for individual interviews. These interviews will last approximately 30 minutes and will focus on simply clarifying and elaborating upon responses you've already given previously about your experiences and mathematical beliefs in your normal coursework. A willingness to allow your regular coursework to be analyzed does not necessitate your participation in the interviews. There are no foreseeable risks associated with this project, nor will your participation influence your grade for this course. Each participant that completes all parts of the study, including both individual interviews, will receive a $25 stipend as a token of my appreciation. All responses used in this study are confidential, and results will be kept under lock and key. Any results reported relating to the data collected will use pseudonyms to protect the identity of the participants. Your participation is voluntary, and you may withdraw from this project at any time. Also, if you are not randomly selected for the individual interviews, you are still invited to speak to me at any time during the semester regarding your beliefs and experiences (without the promised stipend). If you are under the age of 18, you are not eligible to participate in this research study. This study is being conducted by Leah Shilling, who can be reached at 814.937.7398, or at les31@pitt.edu, if you have any questions. Thank you for your consideration.

APPENDIX C

INITIAL BELIEFS SURVEY

Please answer the questions that follow about your ideas and experiences related to mathematics. The information you provide here will help me adjust my teaching to better meet your individual needs and experiences. I want to emphasize that *there are no right or wrong answers* to the questions below. So, please tell me what you *really* think!

Part A.

Instructions: For each statement, circle the number under the answer that best describes what you think or feel.

	very true	sort of true	not very true	not at all true
1. I would like to become an elementary school teacher.	1	2	3	4
2. I would like to teach *mathematics* to elementary school students.	1	2	3	4
3. Everything important about mathematics is already known by mathematicians.	1	2	3	4
4. Mathematics is a collection of facts, formulas, and procedures.	1	2	3	4
5. I am afraid to make an attempt to solve a math problem that seems difficult even though it may actually be accessible to me.	1	2	3	4
6. I like solving math problems *only* when I can work them out easily.	1	2	3	4
7. Mathematics is a creative human activity.	1	2	3	4
8. Doing math can be fun.	1	2	3	4

9. To do well in solving math problems, I have to memorize all the formulas that are relevant.	1	2	3	4
10. I find it useful working with others to solve math problems.	1	2	3	4
11. I don't want to consider multiple solutions for a math problem, because I worry that I will get confused.	1	2	3	4
12. If I cannot solve a math problem in 5-10 minutes, then I know I cannot solve it.	1	2	3	4
13. In mathematics you can be creative and discover by yourself things you didn't already know.	1	2	3	4
14. Math problems can be done correctly in only one way.	1	2	3	4
15. To do well in solving math problems, I have to be taught the right procedure.	1	2	3	4
16. It is really important to me to really understand how and why math procedures work.	1	2	3	4
17. There are many different equivalent ways to define correctly a mathematical concept.	1	2	3	4
18. Learning to follow "the steps" to generate correct answers is very important.	1	2	3	4
19. It is the teacher's job to teach the steps in each new math method to the students before they have to use it.	1	2	3	4
20. I like doing math.	1	2	3	4
21. A person who has taken advanced mathematics courses in college has the *mathematical* knowledge required to teach elementary school mathematics.	1	2	3	4
22. Some people cannot be good at doing math no matter how hard they try.	1	2	3	4
23. I feel confident about my math skills.	1	2	3	4
24. There is usually one best way to write the steps in a solution to a math question.	1	2	3	4
25. I am afraid of doing math.	1	2	3	4

Part B.

Instructions: Answer each of the following prompts/questions in the space provided.

1. I am a:

(a) freshman (b) sophomore (c) junior (d) senior (e) graduate student

2. What mathematics classes did you take in high school? What mathematics courses did you take in college?

3. If you become a teacher, you will want your students to have <u>good</u> experiences learning mathematics. What are <u>three</u> important features of the experiences you'd like to offer them? Elaborate on each feature and mention some examples of the experiences you consider important.

4. Write a paragraph in response to the following question: *What is mathematics?*

5. What are your expectations for this course? What do you hope to learn?

6. Do you have any concerns about this course? Is there anything else you would like to share with me at this point?

APPENDIX D

THE 16 LIKERT-ITEMS RELEVANT TO CHAPTER 4

		MEANS		
	SURVEY ITEM	PRE	POST	POST-PRE
Platonist	1. Some people cannot be good at doing math no matter how hard they try.	2.68	2.76	0.08
	2. Mathematical facts exist independent of human activity.	2.48	2.92	0.24
	3. The mathematical body of mathematics is fixed, and always has been.	3.04	3.04	0
	4. I think that all mathematical knowledge is interconnected.	1.72	1.68	-0.04
Instrumentalist	5. Mathematics is a collection of facts, formulas, and procedures.	1.6	2.16	0.56
	6. To do well in solving math problems, I have to memorize all the formulas that are relevant.	2.4	2.84	0.44
	7. To do well in solving math problems, I have to be taught the right procedure.	1.92	2.64	0.72
	8. It is the teacher's job to teach the steps in each new math method to the students before they have to use it.	1.52	2.16	0.64
	9. Mathematics is a useful tool primarily used for particular calculations.	1.88	2.36	0.48
	10. Doing mathematics means memorizing particular rules and processes.	1.96	2.56	0.6
Problem-Solving	11. Mathematics is a creative human activity.	2.62	2.04	-0.58
	12. In mathematics you can be creative and discover by yourself things you didn't already know.	2.16	1.68	-0.48
	13. Math problems can be done correctly in only one way.	3.32	3.76	0.44
	14. There are many different equivalent ways to define correctly a mathematical concept.	1.68	1.6	-0.08
	15. There is usually one best way to write the steps in a solution to a math question.	2.16	2.88	0.72
	16. I think that mathematics as a discipline can be revised.	2.58	2.12	-0.36

NOTE: Items 1-4 correspond to the Platonist view, items 5-10 to the instrumentalist view, and items 11-16 to the problem-solving view.

APPENDIX E

LIST OF ALL COURSE ACTIVITIES IN CHRONOLOGICAL ORDER

1) The “Squares Problem”
2) The “Circle and Spots” Problem
3) The expression $1+1141n^2$
4) Discussion of student solutions in the Zack’s paper
5) Portfolio entry on Zack’s paper
6) The “Paper/Cuts Problem”
7) Number domains and their relations (natural numbers, whole numbers, integers, rational and irrational numbers, real numbers)
8) Decimals: terminating and non-terminating decimals
9) Representing relationships between the different number domains using Venn diagrams
10) Portfolio entry on what counts as a “good explanation”
11) Discussion on what counts as a “good explanation”
12) Begin work on definitions, with particular reference to even and odd numbers
13) Introduction of the two considerations when working with definitions
14) Begin analysis of textbook definitions of even and odd numbers
15) More work on definitions and analysis of textbook definitions on even and odd numbers
16) Looking at equivalent definitions
17) View video clip of 3rd graders dealing with definitions, “Shea numbers”
18) Some true statements about even and odd numbers
19) Video clip of 3rd graders discussion “provability” of “odd+odd=even”
20) Proving “odd+odd=even”
21) Discussion of a 3rd grader’s proof of the statement “odd+odd=even”
22) Algebra and its connections to geometry
23) Begin discussion of properties of arithmetic

24) Handout on the distributive property and FOIL -- working with areas of rectangles
25) Deriving the formula $1 + 2 + 3 + \ldots + (N - 1) + N = \frac{1}{2} \cdot N \cdot (N+1)$ using geometry
26) Gauss method
27) Discussion of different types of sequences: arithmetic and geometric sequences
28) Deriving the formula $1 + 3 + 5 + \ldots + (2N-1) = N^2$ using algebraic and geometric ways
29) The "Blonde Hair Problem"
30) Definitions of factors and multiples and the relationship between the two
31) The "Locker Problem"
32) Defining prime and composite numbers
33) The Fundamental Theorem of Arithmetic and different methods for prime factorization (*trail division method* and *factor tree method*)
34) The notions of Greatest Common Divisor (GCD) and Least Common Multiple (LCM)
35) Divisibility tests
36) Further work in geometry
37) "Area and Perimeter Problem" and discussion
38) Definitions of convex and nonconvex (or concave) polygons
39) Definitions of interior and exterior angles of convex polygons
40) Finding the formula for the sum of the interior angles and exterior angles of any convex polygon with *n* sides
41) Discussion of student explanations for the sum of the exterior angles
42) The "Floors Problem"
43) The "Sushi Problem"

APPENDIX F

FINAL INTERVIEW DOCUMENT

IL 1473 End-of-Class Interview

I would like to thank you once again for your participation in my research project. As the semester comes to an end, I would like to meet with you individually one more time to discuss your experiences with the work we've done together in the course and to give you an opportunity to share any final thoughts or questions you may have regarding both the course and the research project of which you've been such a vital part. Please read the instructions below in preparation of this final meeting.

Instructions

1. Time:

   The exit interview should take approximately 30 minutes. Please check your calendars to determine at least three half-hour blocks during which you are available to meet with me between (date) and (date). Please email me these times by (date), and I will let you know the specific date and time of our interview.
2. Location:

   This final interview will take place in my office, which is located in 5519 WWPH. Please be sure to arrive on time.
3. Preparation:

   As with all interviews up until this point, this final interview in no way will affect your final grade for the course. It is intended to contribute to the data I am collecting for my research on which you have previously been debriefed about. In order to make this interview a smooth and fruitful one, I ask you to please find attached a description of the three topics the interview will be addressing and things you can do in advance to prepare so that your interview will be completed in a timely manner.

Preparing for your Exit Interview

Below are the three topics final interview will cover and things you can do in advance to prepare so that your session will be completed in a timely manner. To refresh your memory of the experiences you had in the course, please find attached a <u>summary table of the major activities we did in each class meeting</u>. Please bring this summary table with you to the interview.

1. *Comparing your responses to the following prompt at the beginning and end of the semester:*

> *"If you become a teacher, you will want your students to have <u>good</u> experiences learning mathematics. What are <u>three</u> important features of the experiences you'd like to offer them? Elaborate on each feature and mention some examples of the experiences you consider important."*

You responded to this prompt twice: the first time was at the beginning of the semester (Initial Survey, Part B, question #3) and the second time was at the end of the semester (HW #5, task #4). Copies of your responses to this question have been returned to you.

<u>In preparation for your debriefing session:</u>

a. Compare your responses to this prompt at the beginning and end of the semester.

b. Identify changes in the kinds of experiences you said you would like to offer to your students if you become a teacher, and try to think what might have caused these changes. Can you explain any of these changes in terms of specific experiences you had in the course?

2. *Comparing your responses to the following prompt at the beginning and end of the semester:*

> *"Write a paragraph in response to the following question: <u>What is mathematics</u>?"*

You responded to this prompt twice: the first time was at the beginning of the semester (Initial Survey, Part B, question #4) and the second time was at the end of the semester (End-of-semester Survey, Part B, question #2). Copies of your responses to the two surveys have been returned to you.

<u>In preparation for your debriefing session:</u>

a. Compare your responses to this prompt at the beginning and end of the semester.

b. Identify changes in your conception of what is mathematics and try to think what might have caused these changes. Can you explain any of these changes in terms of specific experiences you had in the course?

3. *Comparing your responses to the multiple choice items of the Initial Survey and End-of-semester Survey.*

Part A of the Initial Survey you completed at the beginning of the semester had 25 multiple choice items. Part A of the End-of-semester Survey you completed at the end of the semester had the exact same items. Copies of your responses to the two surveys have been returned to you.

<u>In preparation for your debriefing session:</u>

a. Compare your responses to the multiple choice items at the beginning and end of the semester.

b. Identify 5-6 items where your responses changed the most and try to think what might have caused these changes. Can you explain any of these changes in terms of specific experiences you had in the course?

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