# SOCIAL COMPARISONS AND REFERENCE POINTS IN GAME-THEORETIC MODELS

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## SOCIAL COMPARISONS AND REFERENCE POINTS IN GAME-THEORETIC MODELS

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This thesis analyzes how players' relative evaluation of the actions other agents choose affects individuals' strategic behavior, both in simultaneous and sequential-move games. First, in "The importance of foregone options: generalizing social comparisons in sequential-move games" (joint work with Ana Espinola-Arredondo), we examine a tractable theoretical model in which every individual compares other players' actions with respect to their foregone choices. We analyze the equilibrium prediction in complete information sequential-move games, and compare it with that of standard games where players are not concerned about unchosen alternatives. We show that, without relying on interpersonal payoff comparisons (i.e., assuming strictly individualistic preferences), our model predicts higher cooperation among the players than standard game-theoretic models. In addition, our framework embodies different behavioral models, such as those on social status acquisition as special cases. Finally, we confirm our results in different economic applications.

In "Social comparisons as a cooperating device in simultaneous-move games", I extend the above setting to simultaneous-move games. Specifically, I identify under what conditions introducing relative comparisons into players' preferences leads them to be more cooperative than in standard game-theoretic models. I show that this result holds under certain conditions on the reference point that players use in their relative comparisons (which determines when a particular action by other agent is considered kind or not) and on whether players' actions become more strategic complementary or substitutable. The model is then applied to different examples in public good games which confirm the intuition behind the results. Finally, in "Competition for status acquisition in public good games" I apply the above models of social comparisons to the context of status acquisition through contributions to public goods. I show that the simultaneous contribution order generates higher total contributions than the sequential mechanism only when donors are sufficiently homogeneous in the value they assign to status. Otherwise, the sequential mechanism generates the highest contributions.

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#### 1.0 INTRODUCTION

How do people evaluate each others' actions? How do individuals compare the actions of others with respect to certain reference points? And more importantly, how do these relative comparisons affect their strategic behavior?

My research in this Ph.D. Thesis examines how relative comparisons affect individuals' strategic behavior, both in simultaneous and sequential-move games. In particular, I identify under what conditions we can predict higher degrees of cooperation when individuals assign a value to relative comparisons than when they do not. Specifically, I show that this higher cooperation holds under certain conditions on the particular reference point that players use, and on whether players reciprocate or compensate each others' actions (i.e., whether players' actions become more strategic complementary or substitutable). I then apply this behavioral model to different economic contexts, ranging from bargaining games, to labor market (employer-employee relationship) and public good games; which enhance the intuition behind my results and suggest further economic applications. Finally, I demonstrate that this approach embeds several existing behavioral models as special cases: from social status acquisition to intentions-based reciprocity.

First, in "The importance of Foregone Options: generalizing social comparisons in sequential move games" (joint work with Ana Espinola-Arredondo), we rely on recent experimental evidence which supports the effect of a player's unchosen alternatives on other individual's actions. For example, Brandts and Solà (2001), Falk *et al.* (2003) and Charness and Rabin (2002) accumulate significant evidence supporting the importance of unchosen alternatives in the ultimatum bargaining game, while Andreoni, Brown and Vesterlund (2002) show the relevance of unchosen alternatives in public good games. Importantly, these experimental results cannot be rationalized using existing theories on inequity aversion or intentions-based reciprocity, as shown in the paper. Instead, any rationalization of this observed behavior must rely on a player's comparison between the actions that the proposer other individuals choose and those they do not (unchosen actions).

In this study we introduce a model that rationalizes these economic conducts in complete information sequential-move games within a general framework of economic behavior. Specifically, we assume that as in standard models, every player cares about her material payoff. Additionally, we consider that every individual compares other players' actually chosen actions with respect to a particular action that they could have selected (other players' foregone actions). Hence, this particular action is used by every individual as a reference point to measure the kindness she perceives from other players' choices. We then analyze the equilibrium prediction in complete information sequential-move games, and compare it with that of standard games where players are not concerned about unchosen alternatives. We show that, without relying on interpersonal payoff comparisons (i.e., assuming strictly individualistic preferences), our model predicts higher cooperation among the players than standard game-theoretic models. In addition, our framework embodies different behavioral models, such as those on intentions-based reciprocity and social status acquisition as special cases. Finally, we confirm our results in three economic applications: the ultimatum bargaining game, the labor market gift exchange game, and the sequential public good game.

In "Social comparisons as a cooperating device in simultaneous-move games", I extend the above setting to simultaneous-move games. Specifically, I identify under what conditions introducing relative comparisons into players' preferences leads them to be more cooperative than in standard game-theoretic models. I show that this result holds under certain conditions on the specific reference point that players use in their relative comparisons (which determines when a particular action by other agent is considered kind or not) and on whether players' consideration for relative comparisons leads them to regard each others' actions as more strategic complementary or substitutable.

Specifically, I show that when players consider other players' choices as relatively kind and players' actions become more strategically complementary, both players increase their equilibrium strategies beyond the equilibrium level in standard models. Similarly, this result is also applicable to the case in which players consider other agents' strategies as relatively unkind but actions become more strategically substitutable. Finally, I demonstrate that these results are not only valid for games where players' actions are regarded as strategic complements, but also for those in which these actions are strategic substitutes. Hence, this paper identifies under what conditions players' relative comparisons (evaluating other players' kindness) act as a device for cooperation that triggers higher strategy choices on both players.

Therefore, this paper's main contributions can be divided into two. First, from a general perspective, this paper shows that, under certain conditions, agents' consideration of relative comparisons may lead them to increase their actions with respect to those they choose in standard models (settings where players do make relative comparisons). Importantly, it examines this result applies even when players are not concerned about other players' material payoffs. Indeed, unlike models with inequity averse individuals where players do care about other individuals' payoffs (social preferences), this paper analyzes conditions under which agents choose higher (or lower) strategy levels than in standard models without the need to assume that they care about other people's payoffs, i.e., even when agents' preferences can be regarded as "strictly individualistic." Second, I show that the model this paper describes embeds as special cases existing behavioral models: from models on inequity aversion to those analyzing social status acquisition. Hence, this model offers a broader and more unifying explanation of agents' behavior than these models alone. The model is then applied to different examples in public good games which enhance the intuition behind the results. Furthermore, I show that many models from the behavioral game theory literature, such as inequity aversion and social status acquisition, can all be rationalized as special cases of this model.

In "Competition for status acquisition in public good games" I apply the above models of social comparisons to the context of public goods, and in particular, I assume that every donor evaluates other donors' contributions by comparing them with respect to her own. In this setting, donors hence acquire social status when their own contribution is higher than that of others. Intuitively, one may expect every donor's giving decision to be increasing in his value for social status, since this valuation might attenuate his incentives to free-ride on other donors' contributions. This intuitive prediction is indeed confirmed both in the simultaneous solicitation order (where both donors give simultaneously to the charity) and in its sequential version (in which one donor gives first and then the other gives second before the end of the game). Similarly, an individual's contribution should also be increasing in the value that other donors assign to status. Indeed, since an opponent with a higher value for status increases his contribution, individuals need to increase their donation to the charity in order to reduce as much as possible their loss of social status; this is confirmed in our model for both solicitation orders as well.

In addition, I analyze how total revenues raised by charities are affected by this competition among donors for higher social status, obtaining that total contributions are increasing in donors' concerns about status in both the simultaneous and sequential solicitation orders. A question of interest is which particular contribution order raises the highest total revenue to the charity. In particular, I provide a relatively simple answer to this questions which can be directly applied by practitioners. Specifically, populations of relatively homogeneous donors – in terms of the value they assign to status— induce a higher competition (and contributions) in the simultaneous public good game than in its sequential version. In contrast, groups of contributors with heterogeneous values to status submit higher total donations in the sequential contribution game than in its simultaneous counterpart. Hence, this paper contributes to the literature on public good games by analyzing which solicitation order raises the highest total revenue to the charity when players compete for social status.

Finally, I examine the possibility that donors' social status might be acquired from previous donations to the charity, or from any other sources. This is the case, for example, of famous philanthropists who start their competition for status with previously acquired levels of social status. In particular, I show that if this previous status enters additively into donors' status concerns, seniority may work as a strategic substitute for the status donors can acquire through current donations, reducing their contributions. In contrast, if currently acquired status emphasizes previously acquired rankings, then status acquired during different periods work as strategic complements, and current donations are increased.

## 2.0 THE IMPORTANCE OF FOREGONE OPTIONS: GENERALIZING SOCIAL COMPARISONS IN SEQUENTIAL-MOVE GAMES

Recent advances in behavioral economics allow for the possibility that individuals care about the payoffs of others. In particular, most of these advances suggest the existence of *social*, as opposed to *individual*, preferences reflecting individuals' predilection for fairness in the income distribution; see Fehr and Schmidt (1999) and Bolton and Ockenfels (2000). Despite the multiple situations that can be rationalized with these approaches, a recent literature suggests that individuals' behavior cannot be explained by theories on social preferences alone. Specifically, an agent's choices can only be supported by analyzing how she evaluates other players' chosen and *unchosen* actions. For example, Brandts and Solà (2001), Falk *et al.* (2003) and Charness and Rabin (2002) accumulate significant evidence supporting the importance of unchosen alternatives in the ultimatum bargaining game, while Andreoni, Brown and Vesterlund (2002) show the relevance of unchosen alternatives in public good games. In order to illustrate their results, let us briefly analyze Brandts and Solà's (2001) study. In particular, they examine an ultimatum bargaining game in which the proposer is called to choose among only two alternative divisions of the pie (which we normalize to a size of one) as the figures below illustrate.

Specifically, they consider two treatments. In the first one, represented in figure 1, the proposer chooses among two divisions of the pie, (0.2,0.8) and (0.125,0.875) —where the first and second component of every pair denote the receiver and proposer's payoff, respectively. In the second treatment, as figure 2 indicates, the first available division (0.2,0.8) is unchanged, while the second division becomes (0.875,0.125). Importantly, they show that, conditional on division 0.2 being offered to the responder (bold lines in the figures), the proportion of receiver's rejections is significantly higher when the unchosen division of the pie that



Figure 1: Responder accepts



Figure 2: Responder rejects

the proposer *did not* select was 0.875 (figure 2) than when it was 0.125 (figure 1). That is, for a given offer of 0.2 to the responder, the proportion of rejections increases in the share of the pie that the responder could have received. Intuitively, the receiver positively evaluates a given offer when the alternative division of the pie is below the actual offer that the proposer makes him (he infers "kindness"), and negatively otherwise (he interprets "unkindness"). Certainly, the receiver's pattern of rejections cannot be rationalized using inequity aversion. Indeed, once the offer (0.2,0.8) is made in both treatments, inequity in the payoff distribution is *constant* across treatments, and yet the receiver's behavior is *different* across treatments. The receiver's rejecting pattern cannot be explained using chosen actions either, since the proposer's chosen offer is constant across treatments but the receiver's behavior is not. Instead, any rationalization of the previous results must rely on the receiver's comparison between the actions that the proposer chooses and those he does not (*unchosen* actions).

References to unchosen actions are nevertheless not restricted to economic contexts alone. For instance, we frequently encounter references to unchosen alternatives in the way in which many national and international policies are announced to the media. Indeed, these public presentations are often accompanied with statements like "The government/organization/firm had to choose between policies A and B, and choosing A would have been so bad that we better selected B." These statements are certainly effective when they induce the listener to positively evaluate the chosen action B relative to the unchosen action A.

In this study we introduce a model that rationalizes this economic conduct in complete information sequential-move games within a general framework of economic behavior. Specifically, we assume that as in standard models, every player cares about her material payoff. Additionally, we consider that every individual compares other players' actually chosen actions with respect to a particular action that they could have selected (other players' foregone actions). Hence, this particular action is used by every individual as a *reference point* to measure the kindness she perceives from other players' choices.

This paper makes two main contributions. *First*, we identify conditions under which players' equilibrium actions are higher when individuals are concerned about these reference-

dependent comparisons than when they are not. In particular, this set of conditions allow for a direct prediction about whether players' cooperation rates when they are concerned about relative comparisons is either higher or lower than in standard game-theoretic models. Additionally, it examines players' cooperation rates even when they are not concerned about each other's material payoffs. Indeed, unlike models with inequity averse individuals where players do care about other individuals' payoffs (social preferences), this paper analyzes conditions under which agents choose higher strategy levels than in standard models without the need to assume that they care about other players' payoffs, i.e., agents' preferences can be regarded as "strictly individualistic." *Second*, we show that the model this paper describes embeds as special cases existing behavioral approaches: from models on inequity aversion to those analyzing social status acquisition. Finally, we apply our model to different economic applications where we enhance the intuition behind the results: the ultimatum bargaining game, the labor market gift exchange game, and the sequential public good game. Our equilibrium predictions are not only validated in these applications, but also confirmed by recent experimental data.

The structure of the paper is as follows. In the next section we discuss the literature on social preferences and intentions-based reciprocity, their relationship with our paper, and how it complements their approach. In section three, we describe the properties that players' utility function must satisfy in order to support our results in terms of higher degrees of cooperation. Furthermore, section four analyzes players' equilibrium strategy in these sequential-move games, and section five applies the model to three economic examples. Finally, the last section discusses some conclusions of the paper as well as its further extensions.

#### 2.1 RELATED LITERATURE

#### 2.1.1 Theoretical literature on social preferences

The literature on behavioral economics has extensively considered elements other than one's own payoff in individuals' utility function. This literature mainly deals with the so-called "other regarding preferences," since most of the papers in this area focus their attention on analyzing to what extent players care about the payoffs of his competitors, or about the distribution of payoffs in the entire population. In this respect, some papers on inequity aversion, such as Fehr and Schmidt (1999) and Bolton and Ockenfels (2000) play a prominent role. On one hand, Fehr and Schmidt (1999) consider in their two-player version the following utility function for player i

$$U_i(x_i, x_j) = x_i - \alpha_i \max\{x_j - x_i, 0\} - \beta_i \max\{x_i - x_j, 0\}$$

where  $x_i$  is player *i*'s payoff. Intuitively,  $\alpha_i$  represents the disutility from allocations that are disadvantageously unequal for player *i* (i.e., he may feel envy about player *j*'s payoffs), while  $\beta_i$  denotes the guilt feeling from being the agent with the highest payoff of the population.<sup>1</sup> Bolton and Ockenfels (2000) also develop a similar (yet more general) model of inequity aversion in which individuals' utility is assumed to be increasing and concave in their share of total income, i.e., people experience a positive but diminishing marginal utility from receiving a higher share of the total amount of social payoffs. These models of social preferences, however, cannot rationalize the puzzling experimental evidence presented in the introduction.<sup>2</sup> Indeed, any model which explains such results must necessarily complement the above specification by introducing the importance of unchosen alternatives into player *i*'s utility function, as this paper examines.<sup>3</sup>

<sup>&</sup>lt;sup>1</sup>Interestingly, Blanco *et al* (2007) present experimental evidence supporting inequity aversion at the *aggregate* level (across all participants of a particular game) but refuting it at the *individual* level (for a given participant across games). Their results can be confirmed by our model, whereby participants of a particular game exhibit concerns for unchosen alternatives, but they may use different foregone options across games as a reference point for comparison.

<sup>&</sup>lt;sup>2</sup>Another interesting experimental paper that also tests whether payoff distributions suffice to explain players' behavior in the ultimatum bargaining game is Bereby-Meyer and Niederle (2005). Specifically, they show that the responder is more likely to reject low offers when a rejection payoff is accrued to a third player —with no strategic role in the ultimatum bargaining game— than when such payoff is accrued to the proposer.

<sup>&</sup>lt;sup>3</sup>Some axiomatic approaches, such as Segal and Sobel (1999), examine what conditions on players' preferences must be satisfied in order to obtain utility functions which can be represented as a weighted average of a player's own material payoff as well as that of others. Despite their interest, our approach differs from theirs, since we not only include players' actually *chosen* actions in their utility function (as they do), but also players' *unchosen* actions.

#### 2.1.2 Models on intentions-based reciprocity

As suggested above, this paper is more in the line of Charness and Rabin (2002), whereby they analyze the intentions that players express with their actual choices along the game. In particular, they assume that agents evaluate multiple characteristics of the equilibrium allocation —including fairness and intentions— by establishing different comparisons between own and social payoffs (i.e., between  $x_i$  and  $x_j$ ). Specifically, when only intentions are considered, agent *i*'s utility function in Charness and Rabin's (2002) model reduces to

$$U_i(x_i, x_j) = \begin{cases} x_i + \theta(x_i - x_j) & \text{if player } j \text{ misbehaved} \\ x_i & \text{otherwise} \end{cases}$$

where player j's misbehavior can implicitly include player i's concern about player j's foregone options, and where  $\theta$  represents the importance of intentions-based reciprocity for player i. Note, however, that player i's disutility from player j's misbehavior is scaled up by the difference between player i and j's payoffs,  $x_i - x_j$ . Certainly, this confounds the elements triggering such perception of misbehavior (which implicitly includes unchosen alternatives), and how this misbehavior is then measured (by considering inequity aversion). Likewise, most of the experimental literature testing reciprocating behaviors triggered by kind intentions also considers that agent i measures player j's intentions by comparing  $x_i$ and  $x_j$ ; see Cox (2001, 2003).

Similarly, Falk and Fischbacher (2006) recently analyze how a given player i evaluates the kindness inferred from player j's actions by also comparing their payoffs. In particular, that study measures kindness by considering the product of two elements: the above interpersonal payoff comparison (what they refer as the "outcome term"), and a measure of other players' intentions which reflects the set of available choices for these players (the "intentions factor"). Hence, Falk and Fischbacher (2006) assume that the reference standard with which players compare their own payoff is that of other players, and then they scale up this payoff distribution according to the degree of freedom in the other players' available choices.

Finally, Cox, Friedman and Sadiraj (2007) construct a nonparametric model in which a player's preferences become more altruistic with respect to other players when she infers that these players have behaved generously with her. However, their notion of generosity is not equivalent to our definition of kindness, nor does their notion of altruism coincide with our definition of reciprocity, since they assume that players compare their payoffs with that of others in their group. Unlike these models, we do not introduce other people's payoffs into player i's evaluation of intentions or kindness. Instead, in our model player i measures the kindness in player j's actions by comparing player j's chosen and unchosen (foregone) actions. In the following section we describe how this comparison is made, and how it encompasses models on inequity aversion and intentions-based reciprocity as special cases.

#### 2.2 MODEL

Let us consider the following class of complete information sequential-move games with two players and two stages. Specifically, we focus on games in which: (1) players' actions work as strategic substitutes; and where (2) every player benefits from increases in other players' actions. In particular, let us consider games  $G = \langle S_i, S_j; u_i, u_j \rangle$ , in which a female leader (player j) selects an action  $s_j \in S_j \subset \mathbb{R}_+$ , and afterwards a male follower (player i) chooses an action  $s_i \in S_i \subset \mathbb{R}_+$ . The leader's action may represent, for instance, her wage offer to a worker, or her monetary contribution to a public good. Similarly, the follower's action may denote, respectively, his effort level in a labor market game, or his monetary donation to a charity in the sequential public good game. (Note that for simplicity we describe our model for continuous action spaces. Nonetheless, all our assumptions can be extended to discrete action spaces as well). Every action profile  $s = (s_i, s_j) \in S_i \times S_j$  is then mapped into the set of possible outcomes by function  $out : S_i \times S_j \to X$ . Note that an outcome, out(s), in the ultimatum bargaining game is a monetary amount, while in public good games is a pair composed of an amount of private goods and the total contributions to the public good. Finally, every player i assigns a utility value to every outcome through her utility function  $u_i: X \to \mathbb{R}.$ 

Note that the outcome function maps every action profile into a single outcome, i.e., there is a unique action profile leading to every terminal node of the game. Hence, for every outcome  $out(s) \in X$  we can identify the unique action profile  $s = (s_i, s_j)$  which induces that outcome. This allows utility function  $u_i : X \to \mathbb{R}$  to be represented over action profiles in the form  $U_i^{NC}: S_i \times S_j \to \mathbb{R}$ , i.e.,  $U_i^{NC}(s_i, s_j) \in \mathbb{R}$ . Specifically, superscript NC denotes that player *i* is "not concerned" about player *j*'s unchosen alternatives, as opposed to superscript C, which we use in the next section to refer to players who are "concerned" about each others' unchosen actions. Finally, let us henceforth denote by single (double) subscripts in the utility function its first (and second) order derivatives.

Assumption A1. Positive but decreasing marginal benefit from other players' actions,  $s_j$ . That is,  $U_{s_j}^{NC}(s_i, s_j) \ge 0 \ge U_{s_j s_j}^{NC}(s_i, s_j)$  for all  $s_i$  and  $s_j$ .

Thus, every player *i* benefits from increases in other players' actions, but at a decreasing rate. Note that we are deliberately vague about how  $U_i^{NC}(s_i, s_j)$  increases (or decreases) in her *own* action  $s_i$ . In this way, we can capture models where players' marginal utility from increasing her action is positive (e.g., contributions in public good games) as well as negative (e.g., effort in labor market games). Next, we assume that player *i*'s utility function is strictly concave in his own actions,  $s_i$ .

Assumption A2. Concavity.  $U_{s_i s_i}^{NC}(s_i, s_j) < 0$  for all  $s_i$  and  $s_j$ .

Note that concavity did not hold in the motivating example discussed in the introduction since players' action space was discrete and binary. Nonetheless, we introduce this assumption given that it guarantees the existence of a unique equilibrium when players' action space is continuous. In particular, uniqueness will facilitate the comparison of the equilibrium prediction in this case when players are not concerned about unchosen alternatives, and in the case when players are concerned.<sup>4</sup>

Assumption A3. Strategic Substitutability. Player j's (first mover) utility function satisfies  $U_{s_j s_i}^{NC}(s_i, s_j) < 0$  for all  $s_i$  and  $s_j$ .

<sup>&</sup>lt;sup>4</sup>In the case of discrete and binary action spaces, as those in the motivating example of the ultimatum bargaining game, concavity is not necessary. Instead, in order to facilitate the comparison of our results and those of standard models, we only need the subgame perfect equilibrium to be unique, both when players are concerned about foregone options and when they are not.

Thus, the first mover's marginal benefit from increasing her own action,  $s_j$ , decreases when the second mover raises her action,  $s_i$ . That is, the leader considers the follower's actions as strategic substitutes of her own. This assumption is sensible for a large class of games, where players try to free-ride each others' actions, e.g., the first mover's incentives to free-ride the second mover's donations to the public good or his effort decision. Therefore, A3 eliminates payoff structures such as those in the impunity game, whereby (in a variation of the ultimatum bargaining game) the first mover obtains exactly the same payoff regardless of the second mover's actions, i.e., unconditional on his acceptance or rejection of the first mover's offer. In contrast, A3 maintains the first mover's incentives to free-ride the second mover's action, since she considers players' actions as strategic substitutes.

#### 2.2.1 How kindness enters into players' preferences

As suggested in the motivating example from Brandts and Solà (2001), players' observed behavior is clearly inconsistent across the games in their example. The games they consider are nevertheless relatively similar, since only the set of available choices for the proposer is modified. In particular, we want to describe a single utility function which is general enough to be applicable to games maintaining "similar" properties, as the two treatments considered by Brandts and Solà (2001). Specifically, in this paper we regard games as being similar when the utility that player *i* obtains from every action profile *s* coincides across the games for which this action profile induces the same outcome out(s), and  $out(s) \in X$ . (In the previous example of the ultimatum bargaining game, if a given action profile induces the same outcome across different games then the utility that players obtain from this action profile coincide across these games.) In particular, let  $U_i^C(s_i, s_j)$  represent the utility function we apply to this class of games. Specifically,  $U_i^C(s_i, s_j)$  is player *i*'s utility function when he uses player *j*'s foregone options as a measure of the kindness behind her actions. Let us first describe how this kindness enters into player *i*'s utility function, and then analyze how players measure the kindness behind their opponent's actions. **Assumption A4.** Kindness. For any actions  $s_i \in S_i$  and  $s_j \in S_j$ , player *i*'s utility function satisfies

 $U_i^C(s_i, s_j) \geq U_i^{NC}(s_i, s_j) \text{ if kindness}$  $U_i^C(s_i, s_j) < U_i^{NC}(s_i, s_j) \text{ if unkindness}$ 

Therefore, this assumption determines when that player i is concerned about social comparisons and interprets kindness from player j's actions, his utility level is higher than when he is not concerned about these comparisons. Otherwise (when he infers unkindness), his utility level is lower. Let us next describe how this kindness affects player i's marginal utility.

**Assumption A5.** Reciprocity. For any actions  $s_i \in S_i$  and  $s_j \in S_j$ , player *i*'s utility function satisfies

 $U_{s_i}^C(s_i, s_j) \geq U_{s_i}^{NC}(s_i, s_j)$  if kindness  $U_{s_i}^C(s_i, s_j) < U_{s_i}^{NC}(s_i, s_j)$  if unkindness

Hence, A5 specifies that when player i interprets kindness from player j's actions, his marginal utility from increasing  $s_i$  when he is concerned about foregone options is weakly higher that when he is not. Otherwise, his marginal utility is lower. This property is illustrated in figure A1 (see appendix). In particular, this assumption leads player i to increase his action when he infers kindness (positive reciprocity), and to decrease it when he infers unkindness (negative reciprocity).

#### 2.2.2 How players measure kindness

Let us now describe how players evaluate the kindness behind other players' actions. In particular, we assume that player *i* measures kindness through the following distance function,  $D_i(s_i, s_j)$ , and that he infers kindness when the outcome of this distance function is positive, and unkindness otherwise.

$$D_i(s_i, s_j) = \alpha_i \left[ s_j - s_j^{R_i} \left( s_i, s_j \right) \right]$$

for any  $\alpha_i \in \mathbb{R}$ , where  $\alpha_i$  can be both positive or negative. Thus, player *i* evaluates player *j*'s kindness by comparing player *j*'s actually chosen action,  $s_j$ , and a particular reference action that player *i* uses for comparison,  $s_j^{R_i}(s_i, s_j) \in S_j$ , among player *j*'s available choices, as we define below.<sup>5</sup> For simplicity, this distance function was chosen to be linear. Nonetheless, from a more general perspective, player *i*'s distance function could be nonlinear, as long as it *increases* in player *j*'s actually chosen strategy,  $s_j$ , and *decreases* in the reference action that player *i* uses for comparison.

We consider that this reference-dependent measure is a natural way for player i to assess player j's actions, which is yet general enough to embed different behavioral models as special cases. In particular, this distance function is similar to that in the literature on referencedependent preferences, such as Köszegi and Rabin (2006). However, their model analyzes individual decision making, unlike this paper where we examine its strategic effects. On the other hand, our distance function differs from that in Rabin (1993) for simultaneous-move games and that in Dufwenberg and Kirchsteiger (2004) for sequential-move games. Indeed, these studies assume that player i compares his actual payoff with respect to the "equitable" payoff (his equitable share in the Pareto-efficient payoffs). In contrast, we allow player i to compare player j's actually chosen action with respect to any feasible action,  $s_j^{R_i}(s_i, s_j) \in S_j$ , leading to equitable or non-equitable payoffs. Let us next define the concept of reference

<sup>&</sup>lt;sup>5</sup>Note that, for simplicity, we assume that player i compares player j's actions, instead of the payoffs resulting from these action choices. Choosing the latter, however, would not modify our results, since player i's payoffs are increasing in player j's action choices (assumption A1). Hence, both a definition of kindness based on the payoffs that player i obtains from player j's choices and a definition directly based on these choices increase in player j's action choices.

action,  $s_j^{R_i}(s_i, s_j)$ , which player *i* uses as a reference point in order to evaluate the kindness that he perceives from player *j*'s actually chosen action,  $s_j$ .

**Definition 1.** Player *i*'s reference point function  $s_j^{R_i} : S_i \times S_j \to S_j$ , maps the pair  $(s_i, s_j)$  of both players' actually chosen actions, into a reference action  $s_j^{R_i} \in S_j$  from player *j*'s set of available choices. In addition,  $s_j^{R_i}(s_i, s_j)$  is weakly increasing in  $s_i$  and  $s_j$ , and twice continuously differentiable in  $s_i$  and  $s_j$ .

Hence, player i can use any of player j's available actions in  $S_j$  as a reference point.<sup>6</sup> That is,  $s_j^{R_i}(s_i, s_j)$  is allowed to be above/below/equal to player j's actually chosen action,  $s_j$ , which leads to negative/positive/null distances, respectively. Obviously, the particular sign of such distance affects player *i*'s utility function,  $U_i^C(s_i, s_j)$ , as described above. Additionally, note that when both players' strategy spaces are identical,  $S_i = S_j = S$ , player i's reference point function becomes  $s_j^{R_i}: S^2 \to S$ . In this context, the reference point function can be, for instance,  $s_j^{R_i}(s_i, s_j) = s_i$  for all  $s_j$ . In such case,  $D_i(s_i, s_j) = \alpha_i [s_j - s_i]$ , and player *i* compares player j's chosen action,  $s_j$ , with respect to her own,  $s_i$ . In particular, note two specific examples of this distance function. First, when  $\alpha_i > 0$ , it may represent the case that  $s_j > s_i$  is interpreted by player i as a signal of player j's kindness (e.g., her commitment to contribute high donations to the public good), whereas  $s_j < s_i$  is evaluated by player *i* as a sign of unkindness by her opponent (e.g., free-riding). The second example is related to players' concerns for status acquisition. Particularly, when  $\alpha_i < 0$ , player *i* makes the same comparison, but introduces the outcome of  $D_i(s_i, s_j)$  into her utility function negatively, i.e.  $D_i(s_i, s_j) = -\alpha_i [s_j - s_i] = \alpha_i [s_i - s_j]$  In these cases, player *i* may evaluate  $s_j > s_i$ negatively because the action space might represent the consumption of a given positional good that enhances social status.

Furthermore, we allow player *i* to modify the reference action he uses to compare player *j*'s actually chosen action, i.e.,  $s_j^{R_i}(s_i, s_j)$  is not restricted to be constant for all  $s_j$ . In

<sup>&</sup>lt;sup>6</sup>For simplicity, we restrict the range of reference points to player j's available choices,  $S_j$ . More generally,  $s_j^{R_i}(s_i, s_j)$  could take values outside  $S_j$ . We believe, however, that it is more natural to assume that player i compares player j's actions with respect to her foregone options than to actions which were not even available to her.

particular, we only assume that, for a given increase in player j's action,  $s_j$ , the reference point that player i uses,  $s_j^{R_i}(s_i, s_j)$ , does not increase as fast as player j's action, i.e.,  $1 \ge \partial s_j^{R_i}(s_i, s_j) / \partial s_j$ . Intuitively, this condition makes higher values of player j's action meaningful for player i, since they increase the outcome of his distance function, i.e.,  $\partial D_i(s_i, s_j) / \partial s_j = 1 - \partial s_j^{R_i}(s_i, s_j) / \partial s_j$ ; and as we described above, larger distances raise player i's utility level (kindness). As a remark, note that  $D_i(s_i, s_j)$  does not depend on any possible randomness over payoffs. Indeed, player i's utility level does not depend on the difference between payoffs he could have received from the outcomes of a certain lottery, but only on payoffs he could have obtained from alternative choices of the other players. This distinction differentiates our approach from regret theory, as in Loomes and Sugden (1982), since our model focuses on agent i's evaluation of other players' chosen and unchosen actions as a measure of their kindness. Finally, extending assumption A2 to the context of concerned players, we assume that  $U_i^C(s_i, s_j)$  is also strictly concave in all player i's actions,  $s_i$ .

#### 2.2.3 Best response function

Let  $s_i^C(s_j) \in \underset{s_i}{\operatorname{arg\,max}} U_i^C(s_i, s_j)$  denote player *i*'s best response function when he assigns a positive importance to player *j*'s foregone options, and  $s_i^{NC}(s_j) \in \underset{s_i}{\operatorname{arg\,max}} U_i^{NC}(s_i, s_j)$  his best response function when he does not. Let us next analyze the slope of player *i*'s best response function.

**Lemma 1.** The slope of player i's best response function when he is concerned about foregone options,  $s_i^C(s_j)$ , is higher than that when he is not,  $s_i^{NC}(s_j)$ . That is,

$$\frac{\partial s_i^C(s_j)}{\partial s_j} \ge \frac{\partial s_i^{NC}(s_j)}{\partial s_j} \text{ for any } s_j \in S_j$$

That is, when player i assigns a positive importance to foregone options he is more sensitive to increases in player j's actions than when he does not. In addition to being more sensitive, the next proposition shows that in fact he actually responds more (less) cooperatively when he perceives kindness (unkindness) from player j's actions compared to how he would react in the case of being unconcerned about player j's unchosen alternatives. **Proposition 1.** Player *i*'s best response function when he is concerned about foregone options is higher than that when he is not if player *i* infers kindness from player *j*'s actions; and lower if he infers unkindness. That is,

$$s_{i}^{C}(s_{j}) \geq s_{i}^{NC}(s_{j}) \text{ for all } s_{j} \text{ such that } D_{i}(s_{i},s_{j}) \geq 0$$
  
$$s_{i}^{C}(s_{j}) < s_{i}^{NC}(s_{j}) \text{ for all } s_{j} \text{ such that } D_{i}(s_{i},s_{j}) < 0$$

Intuitively, player *i* (when concerned about player *j*'s foregone options) responds more cooperatively to what he perceives as kind actions,  $D_i(s_i, s_j) \ge 0$ , than when he is unconcerned, i.e.,  $s_i^C(s_j) > s_i^{NC}(s_j)$ . The opposite happens when he interprets that player *j*'s actions are unkind, i.e.,  $s_i^C(s_j) < s_i^{NC}(s_j)$ . In other words, his interpretation of kind (or unkind) actions triggers a higher (lower) response when he is concerned about foregone options than when he is not. For example, the worker in the labor market gift exchange game, when perceiving kind actions from the firm manager, exerts a higher effort when he is concerned about the firm manager's unchosen alternatives (foregone wage offers) than when he is not, and a lower effort otherwise.

#### 2.3 EQUILIBRIUM ANALYSIS

Recall that player j represents the first mover in this complete information sequential-move game, and player i denotes the second mover. Note that player i's best response function,  $s_i^C(s_j)$ , in the subgame perfect equilibrium of this game was already described in the above lemma 1 and proposition 1. Let us now analyze player j's (first mover) equilibrium action in this sequential game.

**Lemma 2.** The leader's marginal utility from increasing her own action  $s_j$  is higher when the follower is concerned about her unchosen alternatives than when he is not. That is, for any action  $s_j \in S_j$  player j's (first mover) utility function satisfies,

$$\frac{\partial U_{j}^{NC}\left(s_{i}^{C}\left(s_{j}\right),s_{j}\right)}{\partial s_{j}} \geq \frac{\partial U_{j}^{NC}\left(s_{i}^{NC}\left(s_{j}\right),s_{j}\right)}{\partial s_{j}}$$

From this lemma, the following proposition is immediately derived.

**Proposition 2.** If assumptions A1-A5 are satisfied, then  $s_j^C \ge s_j^{NC}$ . That is, the leader's equilibrium strategy when dealing with a follower who is concerned about foregone options,  $s_j^C$ , is weakly higher than her equilibrium strategy when facing a follower not concerned about foregone options,  $s_j^{NC}$ .

Hence, in the subgame perfect Nash equilibrium strategy profile of the game with positive concerns for foregone options the leader chooses a higher equilibrium action than that in the game with no concerns for unchosen alternatives.<sup>7</sup> This result is especially relevant for certain games, such as the labor market gift exchange and the sequential public good game, where the introduction of concerns for foregone options leads to higher levels of cooperation among the players. In particular, as we show in section 5 for different economic applications, the fact that the follower is sensitive to the leader's unchosen alternatives attenuates the leader's incentives to shift most of the burden to the follower (reducing free-riding) which ultimately triggers higher actions from her than in standard game-theoretic models.<sup>8</sup> Furthermore, the profile of actions that players choose in equilibrium, as we also show in section 5, can better rationalize experimental results of players' observed behavior.

#### 2.3.1 Remarks on inequity aversion and reciprocity

In this paper we analyze how the consideration of foregone options affects players' equilibrium strategies. Nonetheless, in this subsection, we show that (under certain conditions) our model can also support the results of the literature on inequity aversion and intentionsbased reciprocity as special cases.

<sup>&</sup>lt;sup>7</sup>As a remark, note that the follower moves his action choice in the *opposite* direction than the first mover moves her when he regards actions as strategic substitutes (negatively sloped best response function); whereas he moves it in the *same* direction when actions are strategic complements (positively sloped best response function).

<sup>&</sup>lt;sup>8</sup>These results can be easily generalized to sequential-move games with N players. In such settings, every player measures the kindness he infers from the actually chosen strategies of every player who played before him. The outcome of each of these individual comparisons can then be added up (or scaled in a weighted average), in order to evaluate player *i*'s distance function. Despite the greater generality of such model, nonetheless, its results and intuition are already captured by the two-player setting we consider in this paper.

**Proposition 3.** Assume  $s_j^{R_i}(s_i, s_j) = s_i$  for all  $s_j$ . Then, player *i*'s preferences can be represented as a weighted average of her material payoffs and those of player *j*.

$$U_{i}^{C}\left(s_{i}, s_{j}\right) = \gamma_{i} U_{i}^{NC}\left(s_{i}, s_{j}\right) + \gamma_{j} U_{j}^{NC}\left(s_{j}, s_{i}\right) \quad where \ \gamma_{i}, \gamma_{j} \in \mathbb{R}$$

In particular, the above proposition uses Segal and Sobel's (1999) results to specify that, when player *i* compares player *j*'s actually chosen action,  $s_j$ , with that chosen by herself,  $s_i$ , her utility function  $U_i^C(s_i, s_j)$  can be represented as an (additively separable) weighted average of both players' material payoffs. Therefore, in such context our model captures players' concerns for inequity aversion (or altruism) as a special case. In addition, this model also captures the literature on intentions-based reciprocity as a special case. Indeed, the above utility representation embodies Charness and Rabin's (2002) model for the case that player *i* infers misbehavior from player *j*'s actions, and for  $\gamma_i = 1 - \theta$  and  $\gamma_j = -\theta$ . That is,

$$U_{i}^{C}(s_{i}, s_{j}) = (1 - \theta) U_{i}^{NC}(s_{i}, s_{j}) - \theta U_{j}^{NC}(s_{j}, s_{i})$$
  
=  $U_{i}^{NC}(s_{i}, s_{j}) + \theta \left[ U_{i}^{NC}(s_{i}, s_{j}) - U_{j}^{NC}(s_{j}, s_{i}) \right]$ 

Therefore, when players use their own action  $s_i$  as a reference point to compare other players' actually chosen action,  $s_j$ , our model embeds both inequity aversion and intentionsbased reciprocity as special cases.<sup>9</sup>

<sup>&</sup>lt;sup>9</sup>Clearly, this representation of player *i*'s utility function does not *completely* capture Charness and Rabin's (2002) model, since they analyze other facets of individuals' behavior, such as inequity aversion, in addition to reciprocity. However, when restricted to intensions-based reciprocity alone, and when player *i* infers misbehavior from player *j*'s actions, the above utility function coincides with that in Charness and Rabin (2002).

#### 2.4 APPLICATIONS

#### 2.4.1 Ultimatum bargaining game

Let us first apply our model to the ultimatum bargaining game where a (female) proposer j is called to choose how to divide a pie (of size normalized to one) between the (male) responder i and herself, and the responder either accepts or rejects the division suggested by the proposer,  $s_i \in \{A, R\}$ . In particular, let  $(s_j, 1 - s_j)$  represent the actual division offered by player j, where  $s_j$  denotes the share of the pie accruing to the responder (which coincides with his payoff,  $s_j = x_i$ ), and let  $1 - s_j$  be the remaining share of the pie that the proposer keeps for herself (which coincides with the proposer's payoff,  $1 - s_j = x_j$ ). Hence,  $x_i$  represents the offer that the proposer makes to the responder, and  $f_i$  denotes the foregone offer that the responder uses as a reference action,  $s_j^R$ . Specifically, the responder's utility function we use is given by the following expression<sup>10</sup>, for any  $x_i \in [0, 1]$ , and  $\alpha_i \geq 0$ ,

$$U_i^C(s_i, s_j) = s_j + \alpha_i \left( s_j - s_j^R \right) = x_i + \alpha_i \left( x_i - f_i \right)$$

Clearly, if  $x_i > f_i$ , the responder perceives kindness from the proposer, and gets his utility level increased in the second term. This additional utility is, furthermore, increasing in  $\alpha_i$ , the parameter reflecting the importance that the responder assigns to the distance  $x_i - f_i$ . Intuitively, perceiving kind actions has greater effects on a receiver who is highly concerned about foregone options than on a receiver with small concerns about them. In addition, when either  $\alpha_i = 0$  or  $x_i = f_i$ , the receiver's utility function just coincides with his utility when he is not concerned about the proposer's foregone options. In contrast, when  $x_i < f_i$  the second term becomes negative. Now, the responder gets his utility level decreased from the unkindness he perceives from the proposer's actual offer, since  $x_i < f_i$ . Next, we check that the responder's utility function satisfies all the assumptions we consider in the previous section.

<sup>&</sup>lt;sup>10</sup>Different functional forms for  $U_i^C(s_i, s_j)$  satisfy assumptions A1 through A5, leading to the results predicted in the previous section. Nonetheless, a simple expression is used here to emphasize intuition.

**Lemma 3**  $U_i^C(s_i, s_j)$  satisfies A1 through A5.

We now introduce an example, in order to illustrate the main intuition behind the above utility function. In particular, we focus on the comparison between those utility functions analyzed in the literature and that suggested above, by using the Brandts and Solà (2001) experimental results.

#### Example 1

Let us take an ultimatum bargaining game where the proposer chooses among two alternative divisions of the pie:  $(x_i, x_j) = (0.2, 0.8)$  versus  $(f_i, f_j) = (0.125, 0.875)$ , where the aforementioned experimental results observe an overall accepting behavior from the receiver, or  $(x_i, x_j) = (0.2, 0.8)$ versus  $(f_i, f_j) = (0.875, 0.125)$ , where the above experiments found several rejections. We first show that this pattern of rejections cannot be explained by Fehr and Schmidt's (1999) model on social preferences. When the receiver experiences inequity aversion, and the proposer offers  $(x_i, x_j) = (0.2, 0.8)$  instead of  $(f_i, f_j) = (0.125, 0.875)$ , the receiver accepts if

$$x_i - \alpha_i \max\{x_j - x_i, 0\} - \beta_i \max\{x_i - x_j, 0\} = 0.2 - \alpha_i \max\{0.8 - 0.2, 0\} = 0.2 - 0.6\alpha_i > 0, \text{ if and only if } \alpha_i < \frac{1}{3}$$

While, in the case of receiving an offer  $(x_i, x_j) = (0.2, 0.8)$  instead of  $(f_i, f_j) = (0.875, 0.125)$ the receiver rejects if

$$0.2 - \alpha_i \max \{0.8 - 0.2, 0\} = 0.2 - 0.6\alpha_i < 0$$
, if and only if  $\alpha_i > \frac{1}{3}$ 

which is not possible. Hence, this pattern of rejections cannot be explained by inequity aversion.

Let us now apply these payoffs to the utility function of the receiver with positive concerns about foregone options. In the case of receiving offer  $(x_i, x_j) = (0.2, 0.8)$  instead of  $(f_i, f_j) =$ (0.125, 0.875) the receiver accepts such offer if  $0.2 + \alpha_i(0.2 - 0.125) = 0.2 - 0.075\alpha_i > 0$ , i.e.,  $\alpha_i > -8/3$ , which is satisfied since  $\alpha_i \ge 0$ . Similarly, applying it to the case in which the proposer offers  $(x_i, x_j) = (0.2, 0.8)$  and foregoes  $(f_i, f_j) = (0.875, 0.125)$ , the receiver rejects it if  $0.2 + \alpha_i(0.2 - 0.875) = 0.2 - 0.675\alpha_i < 0$ , i.e.,  $\alpha_i > 0.29$ . Thus, this offer is rejected if and only if the receiver's concern about foregone options is sufficiently high,  $\alpha_i > 0.29$ .

Hence, the above utility function is then able to explain why a responder who has no concerns about social payoffs (i.e., an "individualistic" responder) accepts an offer when it is associated to kindness from the proposer,  $x_i > f_i$ , but can reject this *same* offer when he evaluates it as a signal of unkindness. From the above utility function, we obtain the following result, describing the responder acceptance rule in this example.

**Lemma 4.** In the ultimatum bargaining game with a responder who assigns a weight  $\alpha_i \geq 0$  to the proposer's foregone divisions of the pie,  $f_i$ , the responder accepts any offer  $x_i$  if and only if  $x_i \geq \bar{x}_i$ , where  $\bar{x}_i = \frac{\alpha_i}{1+\alpha_i}f_i$ .

Let us emphasize some interesting insights from the above lemma, illustrated in figure 3 below. Clearly, when  $\alpha_i = 0$  the responder's acceptance rule collapses to  $\bar{x}_i = 0$ . Indeed, when the responder does not assign any weight to the proposer's unchosen actions, then any positive division of the pie is accepted by the responder, as in standard ultimatum bargaining games. Furthermore, the responder's acceptance threshold  $\bar{x}_i$  is increasing in  $\alpha_i$ , the importance he associates to the proposer's unchosen alternatives, i.e., he becomes more demanding in  $\alpha_i$ . Finally,  $\bar{x}_i$  is increasing in  $f_i$ , the receiver's foregone option (represented by an upward shift in the figure). Thus, the more demanding the receiver becomes (higher  $f_i$ ) the more the proposer must offer him to induce his acceptance. Importantly, note that the minimum division that the receiver accepts,  $\bar{x}_i$ , is smaller than one (the total size of the pie) for any parameter values. Hence,  $\bar{x}_i$  leaves some strictly positive portion of the pie to the proposer even when the receiver is extremely demanding (high  $\alpha_i$  and  $f_i$ ).

Intuitively, the above acceptance rule of the responder shows that now the responder is not going to accept any positive offer, as the standard ultimatum bargaining game predicts when no concerns about the proposer's foregone options are considered. This fact clearly affects the proposer's optimal strategies. Certainly, if the proposer wants to obtain any



Figure 3: Cutoff  $\bar{x}_i$ , for  $f_i = 0.5$  and  $f_i = 0.8$ .

positive payoff from the game, she must make an offer which is accepted by the responder, as we show below.

**Proposition 4.** In the ultimatum bargaining game where the responder assigns an importance of  $\alpha_i \geq 0$  to the options that the proposer forwent, the following strategy profile describes the unique subgame perfect equilibrium of the sequential game.

Responder accepts any offer  $x_i$  such that  $x_i \ge \bar{x}_i$ , where  $\bar{x}_i = \frac{\alpha_i}{1+\alpha_i}f_i$ . Proposer offers  $x_i^* = \frac{\alpha_i}{1+\alpha_i}f_i$ , for any parameter values.

Unlike models where the receiver is not concerned about foregone options —where the proposer keeps the entire pie for himself— the distribution of equilibrium payoffs when the receiver assigns a positive importance to foregone options is less unequal, as the following corollary specifies.

**Corollary 1.** The distribution of equilibrium payoffs in the ultimatum bargaining game where the responder assigns importance  $\alpha_i$  to the proposer's foregone option,  $f_i$ , is

$$(x_i, x_j) = \left(\frac{\alpha_i}{1 + \alpha_i} f_i, 1 - \frac{\alpha_i}{1 + \alpha_i} f_i\right)$$

Indeed, note that this distribution of payoffs is more egalitarian than that of models where the receiver is not concerned about foregone options,  $(x_i, x_j) = (0, 1)$ , for any parameter
values. Hence, by considering the proposer's foregone options into the responder's utility function we obtain higher degrees of fairness in the equilibrium payoffs, as well as higher cooperation between the players.

Let us finally relate our theoretical results with those of the experimental literature. In particular, Falk *et al.* (2003) and Brandts and Solà (2001) show the existence of a relationship between the receiver's acceptance threshold and the particular foregone offer that the proposer did not make. Indeed, both of these studies show that, conditional on offer  $(x_i, x_j) = (0.2, 0.8)$  being made, the acceptance rate increases in the distance between the proposer's chosen and unchosen alternatives, as the following figures illustrate.

In particular, note that the first column of figure 4, where  $x_i - f_i = 0.2 - 0.5 = -0.3$ , represents a *negative* distance between the proposer's actual and foregone offer, from which the receiver infers "unkindness." On the other hand, column 3, where  $x_i - f_i = 0.2 - 0 = 0.2$ (and the distance is *positive*) denotes the case in which the receiver interprets "kindness" from the proposer's offer, since she could have offered him less than she actually did. Finally, column 2 illustrates the case in which the proposer has no degree of freedom in choosing her particular offer to the receiver. i.e., the proposer's offer is (0.2,0.8) and her alternative is also (0.2,0.8). In this case, the outcome of the distance function is zero, what leads the receiver to neither perceive "kindness" nor "unkindness" from the proposer's actions.<sup>11</sup>

Interestingly, the fact that the acceptance rate in the second column is exactly higher than when he perceives "unkindness" (column 1) but lower than when he infers "kindness" (column 3) supports our results.<sup>12</sup> A similar intuition is also applicable to Brandts and Solà's (2001) results as figure 5 suggests. Hence, both of these studies confirm our theoretical prediction about the proposer's offer. Indeed, proposers are observed to make low offers when kindness can be inferred from such offers (positive distances), and high offers when they are interpreted in terms of unkindness (negative distances).

<sup>&</sup>lt;sup>11</sup>According to Falk *et al.* (2003), the small (but positive) percentage of rejections in this case can be supported by players' inequity aversion, since they might dislike the unequal payoff distribution resulting from their acceptance of (0.2, 0.8). The fact that the responder does not attribute any responsibility to the proposer in settings where the latter does not have any choice to make has been extensively studied by psychologists with the use of attribution theory; see Ross and Fletcher (1985).

<sup>&</sup>lt;sup>12</sup>Despite the regularity of their results (acceptance rates which increase in the outcome of the distance function), both of these studies report relatively high acceptance rates when distances are highly negative. Nonetheless, such acceptance rates are still lower than in the case of positive distances.



Figure 4: Falk et al. (2003)

## 2.4.2 Labor market gift exchange game

We now apply the above model to a labor market gift exchange game, where the proposer is identified as a firm making a wage offer to a worker, who decides what level of effort to exert. In traditional models without considerations about unchosen options, since effort is costly and the worker is the last player to move, the worker's equilibrium strategy (in the subgame where the worker is called to move) is to exert zero effort regardless of the actual wage offer made by the firm. Operating by backwards induction, the subgame perfect equilibrium of this game predicts that the firm offers the lowest possible wage and that workers exert zero effort for any wage offered. These models have found however limited experimental evidence. Indeed, Fehr and Gachter (2000) summarize a series of experiments on labor markets where they confirm the existence of a positive correlation between the wage offered by the firm and the effort exerted by the worker.

We next suggest a utility function that satisfies the properties considered in section 3 and that can rationalize the above experimental results. As in previous sections, we assume that the firm chooses a wage offer  $x_i \in [0, 1]$  to the worker. Similarly, let  $f_i \in [0, 1]$  represent the foregone wage offer that the worker uses as a comparison against the actual wage offer  $x_i$ . In particular, let us consider the following utility function for the worker.

$$U_i^C(s_i, s_j) = x_i - e^2 + \alpha_i (x_i - f_i)e$$



Figure 5: Brandts and Sola (2001)

The above utility function coincides with the standard utility function of a worker who exerts costly effort when the parameter denoting the importance of foregone options,  $\alpha_i$ , approaches zero. The third term represents the relevance of the foregone options for the worker, i.e., the wage offers that the firm did not make when proposing the actual offer  $x_i$ . Note that when the foregone wage proposal is higher than the actual wage offered,  $x_i < f_i$ , then this third term becomes negative, and the worker experiences a disutility from each unit of additional effort exerted. Similarly, when  $x_i > f_i$ , this third term becomes positive, and the worker interprets that the intentions of the firm are cooperative. That is, the worker observes that the firm offered a wage level which is above its foregone option, which in turn increases the worker's utility since he feels treated generously. In particular, this utility function for the worker satisfies all the assumptions we considered in section 3, as the following lemma specifies.

## **Lemma 5.** $U_i^C(s_i, s_j)$ satisfies A1 through A5.

Intuitively, we should expect that, for proposals with a foregone option below the actual offer, the worker should feel pleased by the kindness of the firm, and responds by exerting a positive level of effort, in contrast to the standard game-theoretic model. These intuitions are confirmed in the following lemma.



Figure 6: An increase in  $a_i$ .

**Lemma 6.** In the gift exchange game where the worker assigns a value  $\alpha_i$  to the distance between the firm's actual wage offer and its forgone alternative, the worker's optimal effort level (in the subgame induced after the wage proposal) is given by

$$e(x_i) = \max\left\{\frac{1}{2}\alpha_i(x_i - f_i), 0\right\}$$

This optimal effort level is then positive if and only if the wage offer  $x_i$  is above the comparative foregone option,  $x_i > f_i$ , for any positive weight to foregone options,  $\alpha_i$ . In addition, an increase in the relative importance that the worker assigns to foregone options increases his optimal effort level, i.e.,  $e(x_i)$  weakly increases in  $\alpha_i$ . On the other hand, for a given weight on foregone options,  $\alpha_i$ , and for a given wage offer  $x_i$ , optimal effort  $e(x_i)$  increases as the comparative foregone option  $f_i$  decreases. Indeed, if the worker compares the actual wage he receives,  $x_i$ , with respect to the worst wage offer that the firm manager could ever pay him (e.g., the legal minimum wage), he is easily pleased by many positive wage offers. On the contrary, a worker who compares his relative position with respect to the best wage offer that the firm could afford to pay him certainly evaluates most of the wage offers he receives as a signal of unkindness from the firm manager.

This optimal effort level is illustrated in figures 6 and 7, which include in addition, the worker's effort level  $e^{NC}(x_i)$  in the case of assigning no importance to foregone options. Note that  $e^{NC}(x_i)$  is flat at zero for all  $x_i$ , since the worker exerts no effort for all wage offers. In both figures, the worker concerned about foregone options exerts positive efforts as long as



Figure 7: A decrease in  $f_i$ .

 $x_i > f_i$  for any positive weight on foregone options.<sup>13</sup> On the one hand, figure 6 indicates how the worker effort pivots upward —with center at  $x_i = f_i$ — when his concerns  $\alpha_i$  about the firms' unchosen alternatives increase. On the other hand, figure 7 represents how the worker effort shifts upwards when the firm's unchosen alternative decreases (leftward shift in the horizontal intercept).

Interestingly, these results are not only supported by the aforementioned experimental evidence, but also by recent empirical work. In particular, Mas (2006) shows that police arrest rates and average sentence length decline (and crime reports raise) when the wage increase that police unions obtain is lower than their wage demands, relative to when it is higher. Hence, police union wage demands would work as the reference point which they use in their negotiations for higher salaries with government officials.

Given the above optimal effort function, and operating by backwards induction, we can find the firm's optimal wage offer. Specifically, we assume the following (standard) utility function for the firm,  $V(s_j, s_i) = (v - x_i) e$ , where v represents the constant productivity of effort (e.g., how worker's effort is transformed into final output); and  $x_i$  denotes, as above, the actual wage offer made to the worker. Moreover, v > 1, since the productivity of effort is assumed to be higher than any of the wage offers,  $x_i \in [0, 1]$ . Inserting the worker's optimal effort function found above, and manipulating, we find the optimal offer made by the firm.

 $<sup>^{13}</sup>$ Note that our results in the labor market gift exchange game are similar to those in Akerlof (1982) since higher salaries induce higher effort levels. In particular, Akerlof's (1982) results are a special case of ours. when the foregone wage offer is exactly fixed at the "fair wage" level.

**Proposition 5.** In the gift exchange game where the worker assigns an importance of  $\alpha_i$  to the distance between the wage offer foregone by the firm and its actual offer, the subgame perfect equilibrium strategies are the following

Firm offers

$$x_i^* = \frac{v + f_i(x_i^*)}{2}$$

Worker accepts any offer  $x_i$  such that  $x_i > 0$ . In addition, the worker exerts an effort level of

$$e(x_i) = \max\left\{\frac{1}{2}\alpha_i\left(x_i - f_i(x_i)\right), 0\right\}$$

As the above proposition specifies, the firm's optimal offer  $x_i^*$  is higher than the worker's foregone option,  $f_i(x_i^*)$ , since v > 1. In addition,  $x_i^*$  is increasing in the foregone option,  $f_i(x_i^*)$ , that the receiver uses to make the comparison with respect<sup>14</sup> to  $x_i^*$ . In the standard models where concerns for foregone options are not considered, the subgame perfect equilibrium of the game predicts that the worker exerts no positive effort for any wage offer, and the firm, anticipating the worker's move, offers the lowest possible wage. In contrast, in the above environment including the importance of the foregone wage offers for the worker, we found that the firm makes a positive wage offer, since this offer can induce a higher level of exerted effort from the worker. That is, by showing kindness in high wage offers, the firm pleases the worker enough to induce him to exert higher efforts.

Clearly, the above equilibrium predictions are closer to the actual experimental results observed in the literature, Fehr and Gachter (2000), which find a positive correlation between the wage offered by the firm and the exerted effort levels from the worker. Many authors have rationalized the above findings by using the *efficiency wage theory* arguments. That is, if a worker is paid above the minimum wage, he has a greater opportunity cost of shirking, which induces him to work harder, and to exert effort levels that are increasing in his wage offer. This paper may thus complement this rationalization of the experimental results through efficiency wage theory. Nonetheless, the model we presented above can explain cooperative

<sup>&</sup>lt;sup>14</sup>Note that, for simplicity, we assume that the worker compares all wage offers with respect to the same foregone option, i.e.,  $f'(x_i^*) = 0$ . Similar results are nonetheless applicable for the more general case in which  $f'(x_i^*) \neq 0$ , and they are included in the proof of proposition 5 at the appendix.

behavior between employers and workers in the labor market without relying on the worker's opportunity cost of shirking, or his outside options if he is fired.

Finally, these results also provide an interesting explanation for the existence of *wage differentials* across industries. Indeed, as Krueger and Summers (1988) show, industry wage differentials are significant even after controlling for individual characteristics and firm quality; which suggests that these differentials are not just due to unobserved differences in labor quality. Our model then rationalizes this result by predicting that firms' equilibrium wage offer, after controlling for worker's productivity, may vary depending on the particular reference point that each worker uses for comparison.

### 2.4.3 Sequential public good game

The third game where we introduce the importance of the proposer's foregone options is the sequential public good game (PGG thereafter). Specifically, we consider a sequential solicitation game where a first mover is asked to submit a donation,  $s_j \in [0, 1]$ , for the provision of a public good, and observing her donation, a follower decides the contribution,  $s_i \in [0, 1]$ , he makes. In order to be consistent with the games defined above, the leader is assumed to not assign any weight to the follower's unchosen actions. In contrast, the follower assigns a relevance  $\alpha_i$  to a specific contribution that the leader did not select, and that the follower uses as a reference point for comparison (reference action,  $s_j^R$ ). In particular, leader and follower's utility functions are, respectively

$$U_{j}^{NC}(s_{j}, s_{i}) = z_{j} + [m(s_{i} + s_{j})]^{0.5}$$

$$U_{i}^{C}(s_{i}, s_{j}) = z_{i} + \left[m\left(s_{i} + s_{j}\right)\left[1 + \alpha_{i}\left(s_{j} - s_{j}^{R}\right)\right]\right]^{0.5}$$

Both of these functions are quasilinear in the private good, z, and their nonlinear part takes into account the utility derived from the total public good provision  $G = s_i + s_j$  (relevant for both players) and the distance  $\alpha_i (s_j - s_j^R)$ , which is only relevant for the follower. For simplicity, let us assume in this application that the follower uses the same reference action  $s_j^R$  for all action choices of the leader. Finally,  $m \ge 0$  denotes the return every player obtains from total contributions to the public good. Interestingly, note how foregone options are introduced into the follower's utility function. When the relevance he assigns to the leader's unchosen alternatives approaches zero,  $\alpha_i = 0$ , the follower only cares about the private and public good consumption. However, when he assigns a positive importance to foregone options, he experiences a higher utility from contributing to the public good when the leader's contribution is higher than the foregone option,  $s_j > s_j^R$ , or a lower utility otherwise,  $s_j < s_j^R$ . In addition, this utility function satisfies all the assumptions we consider in section 3, as the following lemma states.

## **Lemma 7.** $U_i^C(s_i, s_j)$ satisfies A1 through A5.

Since we are discussing a sequential game where the follower decides how much to give out of a continuous strategy choice, the second mover best response function is easily found by solving the follower's utility maximization problem. We summarize this result in the following lemma.

**Lemma 8.** In the sequential PGG, where the follower assigns weight  $\alpha_i$  to the distance between the leader's actual contribution,  $s_j$ , and the foregone contribution,  $s_j^R$ , the follower's best response function  $s_i^C(s_j)$  is given by

$$s_{i}^{C}(s_{j}) = \begin{cases} \frac{m(1-\alpha_{i}s_{j}^{R})}{4} - \left(1 + \frac{\alpha_{i}m}{4}\right)s_{j} & \text{if } s_{j} \in \left[0, \frac{m(1-\alpha_{i}s_{j}^{R})}{4-\alpha_{i}m}\right) \\ 0 & \text{if } s_{j} \ge \frac{m(1-\alpha_{i}s_{j}^{R})}{4-\alpha_{i}m} \end{cases}$$

Figure 8 compares the second mover's best response function when he is concerned about foregone options,  $s_i^C(s_j)$ , and when he is not,  $s_i^{NC}(s_j)$ .

Specifically, note that the introduction of the importance of foregone options into the second mover's utility function makes  $s_i^C(s_j)$  to pivot counterclockwise with respect to  $s_i^{NC}(s_j)$ , with center at  $s_j = s_j^R$ , making  $s_i^C(s_j)$  steeper than  $s_i^{NC}(s_j)$ . Hence, the second mover relatively "reciprocates" the first mover's contributions, since he reduces his donation when  $s_j < s_j^R$ , but increases it when  $s_j > s_j^R$ . After finding  $s_i^C(s_j)$ , and by sequential rationality, we can now find the first mover's equilibrium contribution in this game.



Figure 8: Comparing  $s_i^C(s_j)$  and  $s_i^{NC}(s_j)$ 

**Lemma 9.** In the sequential PGG, where the follower assigns a weight  $\alpha_i$  to the leader's foregone options, the leader's donation in the subgame perfect Nash equilibrium of the game is

$$s_j^* = \begin{cases} 0 & if \ \alpha_i < \bar{\alpha}_i \\ \frac{16(\alpha_i s_j^R - 1) + \alpha_i^2 m^2}{16\alpha_i} & otherwise \end{cases}$$

where  $\bar{\alpha}_i = \frac{16}{16s_j^R + m}$ 

Thus, the first donor submits a zero contribution when the second donor's concerns for foregone options are low enough,  $\alpha_i < \bar{\alpha}_i$ . Clearly, when  $\alpha_i = 0$  the first donor also submits a null donation, which coincides with the equilibrium prediction in standard PGGs. However, when the second donor's concerns for foregone options increase enough,  $\alpha_i > \bar{\alpha}_i$ , the first mover is induced to submit positive contributions that can trigger further donations from the second mover (given his reciprocating behavior described in the previous figure). Additionally, note that as expected, the leader's contribution is increasing in the follower's concerns for foregone options,  $\alpha_i$ , and in the foregone contribution that he uses as a reference point for comparison,  $s_j^R$ . **Proposition 6.** In the sequential PGG where the second mover assigns a weight  $\alpha_i$  to the first mover's unchosen alternatives, the following strategy profile describes the subgame perfect equilibrium of the game.

Proposer contributes

$$s_j^* = \begin{cases} 0 & if \ \alpha_i < \bar{\alpha}_i \\ \frac{16(\alpha_i s_j^R - 1) + \alpha_i^2 m^2}{16\alpha_i} & otherwise \end{cases}$$

And the second mover responds by contributing

$$s_i^C(s_j) = \begin{cases} \frac{m\left(1-\alpha_i s_j^R\right)}{4} - \left(1+\frac{\alpha_i m}{4}\right)s_j & \text{if } s_j \in \left[0, \frac{m\left(1-\alpha_i s_j^R\right)}{4-\alpha_i m}\right)\\ 0 & \text{if } s_j \ge \frac{m\left(1-\alpha_i s_j^R\right)}{4-\alpha_i m} \end{cases}$$

Particularly, the above results specify that by having a second mover concerned about the first mover's foregone options, the latter is induced to contribute (weakly) higher amounts than those she would donate in the case of facing a responder with no concerns about her unchosen alternatives. From a more general perspective, by introducing a follower concerned about the leader's foregone options, we are able to obtain (weakly) higher levels of cooperation in the public good provision.

## 2.5 CONCLUSIONS

Different experimental papers, such as Brandts and Solà (2001), Falk *et al.* (2003), and Andreoni *et al.* (2002), accumulate a significant evidence about the importance of a player's unchosen alternatives on other players' actions. Foregone options, in particular, may work as standards against which every individual evaluates the kindness of other players in the population. Importantly, these studies suggest that arguments on social preferences alone cannot explain their experimental results without complementing their approach by considering the importance of a players' unchosen alternatives inside his opponents' utility function.

This paper examines a tractable theoretical model that introduces these unchosen alternatives into individuals' preferences via a reference point. We first analyze the equilibrium prediction in complete information sequential-move games, and then compare it with that of standard games where players are not concerned about unchosen alternatives. We show that, without relying on interpersonal payoff comparisons (i.e., within "strictly individualistic" preferences), our model predicts higher levels of fairness in the resulting allocation, as well as higher cooperation among the players, than standard game-theoretic models. In addition, we demonstrate that this approach embeds as special cases many existing behavioral models: from inequity aversion to intentions-based reciprocity. Therefore, this model offers a broader and more unifying explanation of agents' conduct than these models alone. Furthermore, when applying our model to different sequential games, we obtain interesting results. First, the equilibrium allocation in the ultimatum bargaining game is fairer than that resulting from standard game-theoretic predictions. Second, worker's effort and firm's proposed wages are higher than in the usual labor market gift exchange model. Finally, equilibrium donations in the sequential public good game are higher than the predictions for standard models.

There are several natural extensions to the model introduced in this paper. First, it would be interesting to experimentally test under which payoff structures we can rationalize observed behavior using individuals' preferences over equitable payoffs, and in which environments human conduct is instead mainly explained by the players' "strictly individualistic preferences" suggested in this paper. One direct test of the dominance of these two behavioral motives is, for example, the following ultimatum bargaining game. The proposer is allowed to make only two divisions of the pie, of size normalized to one. In the first treatment she can offer (0.4, 0.6), giving 0.4 to the responder and keeping 0.6 for herself, or the equitable payoff (0.5, 0.5). In the second treatment, the first division of the pie is fixed in (0.4, 0.6), but the second division is now (0.6, 0.4) instead. Note that, conditional on the first offer, (0.4, 0.6), being made, the distance between the actual offer, 0.4, and the alternative offer is higher in the first treatment, 0.4 - 0.5 = -0.1, than in the second, 0.4 - 0.6 = -0.2. Hence, according to our equilibrium predictions, we should observe more rejections in the second treatment than in the first. However, if we observe higher percentage of rejections in the first than in the second treatment, it must be that responders in the first treatment evaluate the equitable payoffs that the proposer did not select as a more desirable goal than the higher individual payoff he could have received in the second treatment.

Second, in this paper the space of available alternatives was exogenously determined before the beginning of the game. However, it would be interesting to allow players to strategically select their available choices before the game starts, given that the kindness other players perceive from their chosen actions depends on which available strategies are *not* chosen. That is, by strategically selecting her set of available alternatives, a player may induce other players to infer a greater kindness from her actions. This strategic selection of available choices is observed in different contexts, where a player uses one of her unchosen alternatives as an excuse to support her actual choices, since the equilibrium payoff associated with that particular unchosen action would have been certainly worse than that from her chosen action. These extensions can certainly enhance our understanding of the role of players' foregone options on their opponents' incentives, and how such incentives can lead to higher degrees of cooperation from a strictly individualistic perspective.

# 3.0 SOCIAL COMPARISONS AS A DEVICE FOR COOPERATION IN SIMULTANEOUS-MOVE GAMES

## 3.1 INTRODUCTION

During the last decade several elements have been separately suggested to explain agents' behavior in experimental settings: from individuals' inequity aversion, as in Fehr and Schmidt (1999) and Bolton and Ockenfels (2000), to agents' preference for social status, as in Hopkins and Kornienko (2004) and Duffy and Kornienko (2005). Despite their ability to rationalize human conduct in specific economic environments, there is a substantial controversy about what particular facet most generally drives individuals' behavior in unrestricted environments. Or in other words, about the possibility to identify a *common element* connecting most of these experimental observations.

In this paper, I examine a model describing individual behavior that embeds many of these approaches as special cases of a broader explanation of human conduct in strategic settings. Specifically, this model is based on the common observation that people's choices are usually affected by the "kindness" they infer from the actions of the individuals they interact with, such as their neighbors, friends and relatives. Of course, the particular measure of "kindness" that each of us uses to evaluate other individuals' actions might be different. For instance, some people compare other agents' choices with respect to their own. Other individuals may instead evaluate other agents' actions with respect to some specific action they deem as "kind." Indeed many other examples abound; yet, they share a common pattern: in all of them individuals evaluate other agents' choices with respect to a particular reference action, which they use as a reference point for comparison.

Using this general definition of kindness, this paper examines the effects of social com-

parisons on strategic interaction. In particular, this study identifies under what conditions one can predict that individuals playing simultaneous-move games become more *cooperative* when they assign a positive importance to kindness, relative to when they do not. Particularly, this result holds under certain conditions on the reference point they use for comparison —which determines when a particular action by other agent is considered to be relatively kind or unkind— and on whether these considerations about kindness lead players to regard each others' actions as more strategically substitutable or complementary.

Specifically, I show that when players consider other players' choices as relatively kind and players' actions become more strategically complementary, both players increase their equilibrium strategies beyond the equilibrium level in standard models. Similarly, this result is also applicable to the case in which players consider other agents' strategies as relatively unkind but actions become more strategically substitutable. Finally, I demonstrate that these results are not only valid for games where players' actions are regarded as strategic complements, but also for those in which these actions are strategic substitutes. Hence, this paper identifies under what conditions players' relative comparisons (evaluating other players' kindness) act as a device for cooperation that triggers higher strategy choices by both players.

Therefore, this paper's main contributions can be divided into two. *First*, from a general perspective, this paper shows that, under certain conditions, agents' consideration of relative comparisons may lead them to become more cooperative than in standard models. Importantly, this result applies even when players are not concerned about other players' material payoffs. Indeed, unlike models with inequity averse individuals where players do care about other individuals' payoffs (social preferences), this paper analyzes conditions under which agents cooperate more than in standard models without the need to assume that they care about other players' payoffs, i.e., even when agents' preferences can be regarded as "strictly individualistic." *Second*, I show that the model this paper describes embeds as special cases existing behavioral models: from models on intentions-based reciprocity to those analyzing social status acquisition.

The paper is organized as follows. In the next section, I introduce the measure of kindness that players use and as how it enters into individuals' preferences. Sections three and four analyze players' equilibrium strategies when either both or only one of the parties assigns a positive weight to kindness in these simultaneous-move games. Then, in section five, I apply this model to different examples of public good games in which donors simultaneously contribute to a charity. Section six summarizes the main contributions of the paper.

#### 3.2 MODEL

Let us consider complete information simultaneous-move games in which every player i chooses an action from her strategy space  $S_i \in [\underline{s_i}, \overline{s_i}] \subset \mathbb{R}_+$ . This strategy may represent, for example, player i's voluntary contribution to a public good, or in the context of oligopoly games, its production decision in a Cournot model. In particular, let us use  $U_i^{NC} \equiv U_i^{NC}(s_i, s_j)$  to refer to player i's utility function when she is not concerned about relative comparisons. Since this utility function does coincide with those in the standard game-theoretic models, I alternatively refer to  $U_i^{NC}$  as player i's material payoff, where the superscript NC denotes the fact that player i is "not concerned" about relative comparisons. On the other hand, let  $U_i^C(s_i, s_j)$  be player i's utility function when she is "concerned" about relative comparisons. In the following subsection, I describe how players make their comparisons, and in subsection 3.2 how every player introduces the result of this comparison into her utility function.

#### **3.2.1** How players measure kindness

Let us now describe how players evaluate the kindness behind other players' actions. In particular, we assume that player *i* measures kindness through the following distance function,  $D_i(s_i, s_j)$ , and that he infers kindness when the outcome of this distance function is positive, and unkindness otherwise (see assumption 1 below).

$$D_i(s_i, s_j) = \alpha_i \left[ s_j - s_j^{R_i} \left( s_i, s_j \right) \right]$$

for any  $\alpha_i \in \mathbb{R}$ . Thus, player *i* evaluates player *j*'s kindness by comparing the difference between the action that player *j*'s chooses in equilibrium,  $s_j$ , and a particular reference action that player *i* uses for comparison,  $s_j^{R_i}(s_i, s_j) \in S_j$ , among player *j*'s available choices, as defined below.<sup>1</sup> I believe that this reference-dependent measure is a natural way for player *i* to assess player *j*'s actions, which is yet general enough to embed different behavioral models as special cases. In particular, this distance function is similar to that in the literature on reference-dependent preferences, such as Köszegi and Rabin (2006). However, their model analyzes individual decision making, unlike this paper where we examine strategic effects. On the other hand, the distance function suggested in this paper differs from that in Rabin (1993) for simultaneous-move games and that in Dufwenberg and Kirchsteiger (2004) for sequential-move games. Indeed, these studies assume that player *i* compares his actual payoff with respect to the "equitable" payoff (his equitable share in the Pareto-efficient payoffs). In contrast, I allow player *i* to compare the action that player *j*'s chooses in equilibrium with respect to *any* feasible action,  $s_j^{R_i}(s_i, s_j) \in S_j$ , leading to equitable or non-equitable payoffs. Let us next define the concept of reference action,  $s_j^{R_i}(s_i, s_j)$ , which player *i* uses as a reference point in order to evaluate the kindness that he perceives from player *j*'s chosen action,  $s_j$ .

**Definition 1.** Player *i*'s reference point function  $s_j^{R_i} : S_i \times S_j \to S_j$ , maps the pair  $(s_i, s_j)$  of both players' chosen actions, into a reference action  $s_j^{R_i} \in S_j$  from player *j*'s set of available choices. In addition,  $s_j^{R_i}(s_i, s_j)$  is weakly increasing in  $s_i$  and  $s_j$ , and twice continuously differentiable in  $s_i$  and  $s_j$ .

Hence, player *i* can use any of player *j*'s available actions in  $S_j$  as a reference point.<sup>2</sup> That is,  $s_j^{R_i}(s_i, s_j)$  is allowed to be above/below/equal to player *j*'s chosen action,  $s_j$ , which leads to negative/positive/null distances, respectively. Obviously, the particular sign of such

<sup>&</sup>lt;sup>1</sup>For simplicity, this distance function was chosen to be linear. Nonetheless, from a more general perspective, player *i*'s distance function could be nonlinear, as long as it *increases* in player *j*'s actually chosen strategy,  $s_j$ , and *decreases* in the reference action that player *i* uses for comparison. Note that in such setting, Bolton and Ockenfels' (2000) model (whereby agents' utility increases in their share of total income) could be embedded as a special case. For the sake of clarity, however, I henceforth use the above linear distance function.

<sup>&</sup>lt;sup>2</sup>For simplicity, I restrict the range of reference points to player j's available choices,  $S_j$ . More generally,  $s_j^{R_i}(s_i, s_j)$  could take values outside  $S_j$ . I believe, however, that it is more natural to assume that player i compares player j's actions with respect to her foregone options than to actions which were not even available to her.

distance affects player *i*'s utility function,  $U_i^C(s_i, s_j)$ , as we describe below. Additionally, note that when both players' action spaces are identical,  $S_i = S_j = S$ , player *i*'s reference point function becomes  $s_j^{R_i} : S^2 \to S$ . In this context, the reference point function can be, for instance,  $s_j^{R_i}(s_i, s_j) = s_i$  for all  $s_j$ . In such case, the distance function becomes  $D_i(s_i, s_j) = \alpha_i [s_j - s_i]$ , and player *i* compares the action that player *j* chooses in equilibrium,  $s_j$ , with respect to her own action,  $s_i$ .

In particular, note two specific examples of this distance function. First, when  $\alpha_i > 0$ , it may represent the case in which players' equilibrium actions satisfy  $s_j > s_i$ , and player *i* interprets kindness from player *j*'s choices (e.g., her commitment to contribute high donations to the public good), whereas  $s_j < s_i$  is evaluated by player *i* as a sign of unkindness by her opponent (e.g., free-riding). The second example is related to players' concerns for status acquisition. Particularly, when  $\alpha_i < 0$ , player *i* makes the same comparison, but introduces the outcome of  $D_i(s_i, s_j)$  into her utility function *negatively*, i.e.  $D_i(s_i, s_j) = -\alpha_i [s_j - s_i] =$  $\alpha_i [s_i - s_j]$  In these cases, player *i* may evaluate  $s_j > s_i$  negatively because the action space might represent the consumption of a given positional good that enhances social status.

Furthermore, we allow player *i* to modify the reference action he uses to compare player *j*'s chosen action, i.e.,  $s_j^{R_i}(s_i, s_j)$  is not restricted to be constant for all  $s_j$ . In particular, we assume that, for a given increase in player *j*'s action,  $s_j$ , the reference point that player *i* uses,  $s_j^{R_i}(s_i, s_j)$ , does not increase as fast as player *j*'s action, i.e.,  $1 \ge \partial s_j^{R_i}(s_i, s_j) / \partial s_j$ . Intuitively, this condition makes higher values of player *j*'s action meaningful for player *i*, since they increase the outcome of his distance function, i.e.,  $\partial D_i(s_i, s_j) / \partial s_j = 1 - \partial s_j^{R_i}(s_i, s_j) / \partial s_j$ . And as we describe below, positive distances ultimately raise player *i*'s utility level.

#### 3.2.2 How kindness enters into players' preferences

After examining how players evaluate other players' actions through the construction of a distance  $D_i$ , let us next analyze how this distance enters into players' utility function. First, I consider how a player prefers, for a given pair of chosen actions  $s_i$  and  $s_j$ , those pairs  $(s_i, s_j)$  associated to positive rather than negative distances.

Assumption 1. Kindness. For any actions  $s_i \in S_i$  and  $s_j \in S_j$ , player *i*'s utility function satisfies

$$U_i^C(s_i, s_j) \geq U_i^{NC}(s_i, s_j) \text{ for all } D_i(s_i, s_j) \geq 0$$
$$U_i^C(s_i, s_j) < U_i^{NC}(s_i, s_j) \text{ for all } D_i(s_i, s_j) < 0$$

Therefore, this assumption determines that player *i* interprets kindness from player *j*'s chosen actions when the outcome of her distance function is positive, and infers unkindness otherwise. That is, when player *i* is concerned about social comparisons and she interprets kindness from player *j*'s actions,  $D_i(s_i, s_j) \ge 0$ , her utility level is higher than when she is not concerned about these comparisons; and it is lower when she infers unkindness. Let us finally define when a player's relative comparisons are considered as relatively "demanding" with respect to other players' actions, and when they can be regarded as "not-demanding".

**Definition 2.** Player *i*'s relative comparisons are defined as "demanding" if and only if she infers unkindness (negative distance) from player *j*'s equilibrium action when players are not concerned about social comparisons,  $s_j^{NC}$ . That is,  $D_i^{NC} \equiv \alpha_i \left[ s_j^{NC} - s_j^R \right] < 0$ . Otherwise, player *i*'s relative comparisons are denoted as "not-demanding."

Intuitively, player *i* would be regarded as "demanding,"  $D_i^{NC} < 0$ , if the reference level she uses to compare player *j*'s actions is above  $s_j^{NC}$ , i.e., she sets a high standard to assess player *j*'s actions (demanding). On the contrary, player *i* would be regarded as "notdemanding,"  $D_i^{NC} > 0$ , if the reference level she uses to compare player *j*'s actions is below  $s_j^{NC}$ , setting a low standard to evaluate player *j*'s choices.

## 3.3 BEST RESPONSE FUNCTION

The previous section described the structure behind players' preferences, how they use the distance function to evaluate other players' actions, and how this distance enters into players'

utility function. In this section, I characterize players' best response function in this class of simultaneous-move games.

Let  $s_i^C(s_j) \in \arg\max_{s_i} U_i^C(s_i, s_j)$  denote player *i*'s best response function when she assigns a positive importance to relative comparisons, and let  $s_i^{NC}(s_j) \in \arg\max_{s_i} U_i^{NC}(s_i, s_j)$  represent her best response function when she does not assign any weight to such comparisons. For simplicity, both  $U_i^{NC}(s_i, s_j)$  and  $U_i^C(s_i, s_j)$  are assumed to be strictly concave in every player *i*'s own strategy,  $s_i$ , which guarantees that best response functions are uniquely defined. Additionally, in order to have a unique equilibrium in pure strategies, we consider the usual sufficient condition for best response functions to intersect only once.

Assumption 2. For any given strategy pair  $(s_i, s_j)$ , every player *i*'s best response function satisfies  $\left|\frac{\partial s_i^K(s_j)}{\partial s_i}\right| < 1$  where  $K = \{C, NC\}$ , i.e.,  $\left|\frac{\partial^2 U_i^K}{\partial s_i^2}\right| < \left|\frac{\partial^2 U_i^K}{\partial s_i \partial s_j}\right|$ , for all  $i \neq j$ .

That is, for players with positive concerns about relative comparisons,  $s_i^C(s_j)$  crosses  $s_j^C(s_i)$  from below, and similarly for players without concerns about comparisons. Let us henceforth denote by single (double) subscripts in the utility and distance functions their first (and second) order derivatives. Next, I start by specifying some properties about the *level* of the best response function, whereas lemma 2 determines properties about its *slope*. Thereafter, all proofs can be found in the appendix.

**Lemma 1.** Player i's best response function when she assigns a value to relative comparisons is above that when she does not,  $s_i^C(s_j) \ge s_i^{NC}(s_j)$ , for all  $s_j$ , if and only if the distance function that player i uses to evaluate kindness is increasing in her own strategy,  $s_i$ , for all  $s_i$  and  $s_j$ , i.e.,  $D_{s_i} \ge 0$  for all  $s_i$  and  $s_j$ .

Therefore, lemma 1 determines a necessary and sufficient condition  $(D_{s_i} \ge 0)$  which guarantees that player *i*'s best response function when she is concerned about relative comparisons is above that when she is not,  $s_i^C(s_j) \ge s_i^{NC}(s_j)$ , for any actions of player *j*. Graphically, lemma 1 can be interpreted as an upward shift in player *i*'s best response function, as figure 9 illustrates.



Figure 9: Best response function.

Intuitively, if an increase in player *i*'s strategy raises the outcome of her distance function (i.e., if  $D_{s_i} \ge 0$  for all  $s_j$ ) then player *i*'s best response function when she assigns a positive importance to relative comparisons is above that when she does not, i.e.,  $s_i^C(s_j) \ge s_i^{NC}(s_j)$ for all player *j*'s strategies. Interestingly, the case that lemma 1 describes is applicable, for instance, to games where players are concerned about status acquisition. Specifically, note that the distance function players use as a measure of the status they acquire,  $D_i(s_i, s_j) \equiv$  $-\alpha_i(s_j - s_i) = \alpha_i(s_i - s_j)$ , should clearly satisfy  $D_{s_i} > 0$ .

Finally, let  $U_{s_is_j}^{NC}$  represent the cross-derivative between player *i* and *j*'s strategies when players does not assign a value to social comparisons, and  $U_{s_is_j}^C$  be that when they do. Intuitively, an increase in this cross-derivative when players become concerned about social comparisons, from  $U_{s_is_j}^{NC}$  to  $U_{s_is_j}^C$ , implies that players' actions become more strategic substitutable. In contrast, a decrease in this cross-derivative means that players' actions become more complementary to each other.

**Lemma 2.** If  $\Delta_i = \frac{U_{s_i s_j}^C}{U_{s_i s_i}^C} - \frac{U_{s_i s_j}^{NC}}{U_{s_i s_i}^{NC}} \geq 0$ , then the slope of player *i*'s best response function increases when she assigns a value to social comparisons relative to when she does not; and decreases otherwise. That is,

If 
$$\Delta_i \ge (<) 0$$
 then  $\frac{\partial s_i^C(s_j)}{\partial s_j} \ge (<) \frac{\partial s_i^{NC}(s_j)}{\partial s_j}$  for all  $s_j$ 



Figure 10: Clockwise rotation, "Compensating" type of player,  $\Delta_i < 0$ .

Thus, lemma 2 specifies that, when player *i*'s utility function satisfies condition  $\Delta_i > 0$ , her best response function experiences a *anticlockwise* rotation from  $s_i^{NC}(s_j)$  to  $s_i^C(s_j)$ ; whereas this rotation is *clockwise* in the case that  $\Delta_i < 0$ , as the figures illustrate.

Graphically, when player *i*'s best response function is negatively sloped, these results imply that  $s_i^C(s_j)$  is steeper than  $s_i^{NC}(s_j)$  when  $\Delta_i < 0$ , as figure 10 illustrates; while it determines the opposite when  $\Delta_i > 0$  as figure 11 indicates. (In contrast, when player *i*'s best response function is positively sloped, lemma 2 specifies that  $s_i^C(s_j)$  is flatter than  $s_i^{NC}(s_j)$  when  $\Delta_i < 0$  is satisfied; and steeper otherwise.) In the figures, note that  $\bar{s}_j \in S_j$ represents the level of player *j*'s strategy for which  $s_i^C(s_j) = s_i^{NC}(s_j)$ .<sup>3</sup>

Intuitively, a clockwise rotation can be understood in terms of a greater necessity to *compensate* player j's actions as figure 10 illustrates: when  $s_j < \bar{s}_j$  player i chooses equilibrium levels of  $s_i$  above those in the game without concerns for relative comparisons, whereas when  $s_j > \bar{s}_j$  player i chooses lower levels of  $s_i$  in equilibrium. This is the case of the public good games presented in the example of section five, where player i considers her contributions to the charity more "necessary" when player j does not reach a minimum level,  $\bar{s}_j$ , but her contributions are less necessary when player j exceeds this level.

<sup>&</sup>lt;sup>3</sup>Note that in the case of a clockwise rotation, if  $\bar{s}_j$  takes a sufficiently high value, then  $s_i^C(s_j) \ge s_i^{NC}(s_j)$  for all  $s_j$ , leading to a similar result to that of lemma 1, illustrated in figure 2(a). Similarly, in the case of an anticlockwise rotation, if  $\bar{s}_j$  takes a sufficiently low value, then  $s_i^C(s_j) \ge s_i^{NC}(s_j)$  for all  $s_j$ .



Figure 11: Anticlockwise rotation, "Reciprocating" type of player,  $\Delta_i > 0$ .

An opposite argument is applicable to anticlockwise rotations of player *i*'s best response functions (i.e., when  $\Delta_i < 0$ ) where player *i* can be interpreted to *reciprocate* player *j*'s actions. Indeed, player *i* reduces her strategy choice below that in standard models when player *j* does not reach threshold  $\bar{s}_j$ . In contrast, when  $s_j > \bar{s}_j$  player *i* "rewards" player *j* for exceeding such level. Because of this underlying intuitive reasoning, I define the reciprocating and compensating types of players as follows.<sup>4</sup>

**Definition 3.** Player *i*'s behavior is defined as "compensating" if and only if her best response function rotates clockwise (i.e.,  $\Delta_i < 0$  holds). Otherwise, her behavior is "reciprocating".

### 3.4 EQUILIBRIUM ANALYSIS

From our previous analysis, one can anticipate that player *i*'s equilibrium strategies in this model,  $s_i^C$ , are higher than in models without concerns about distances,  $s_i^{NC}$ , when  $s_i^C(s_j) > s_i^{NC}(s_j)$  for all  $s_j$ . That is, when  $D_{s_i} \ge 0$  is satisfied, as specified in lemma 1. Indeed, in such

<sup>&</sup>lt;sup>4</sup>These intuitions also hold when players' best response functions are positively sloped. Indeed, when  $\Delta_i < 0$  one can interpret  $s_i^C(s_j)$  being flatter than  $s_i^{NC}(s_j)$  as that player *i* "compensates" player *j*'s actions. On the contrary, when condition  $\Delta_i > 0$  holds, and  $s_i^C(s_j)$  becomes steeper than  $s_i^{NC}(s_j)$ , one can infer that player *i* "reciprocates" player *j*'s strategy.

cases the consideration of distances shifts upwards player i's best response function along *all* player j's strategies, what leads player i to choose higher equilibrium strategy levels. The following proposition confirms this result.

**Proposition 1.** If condition  $D_{s_i} \ge 0$  holds for all  $s_i \in S_i$  and  $s_j \in S_j$ , then  $s_i^C \ge s_i^{NC}$ , for any reciprocating or compensating behavior of players *i* and *j*.

Hence, proposition 1 determines that player *i*'s equilibrium strategy when she is concerned about relative comparisons is weakly higher that when she is not, if  $D_{s_i} \ge 0$  holds. In that case, player *i*'s Nash equilibrium strategy increases for any type of player (compensating or reciprocating), and for any distance function players might use (demanding or not-demanding). This is indeed a useful result, since it allows for a prediction about the ranking between equilibrium strategies  $s_i^C$  and  $s_i^{NC}$  just by checking whether condition  $D_{s_i} \ge 0$  holds. As commented above, condition  $D_{s_i} > 0$  is specially relevant in the case of those players who are concerned about status acquisition. Indeed, as the example of section five illustrates,  $s_i^C \ge s_i^{NC}$  is satisfied for any parameter values when players assign a positive importance to status, confirming the above result of proposition 1.

One may ask, however, if the above result still holds when condition  $D_{s_i} \geq 0$  is not satisfied for all  $s_j$ , i.e., when the best response function  $s_i^C(s_j)$  is above  $s_i^{NC}(s_j)$  for some values of  $s_j$  but below for others. Indeed,  $D_{s_i} \geq 0$  is a relatively strong condition, which we henceforth relax. (In particular, we assume that  $D_{s_i} \geq 0$  holds only for some values of  $s_i$ , whereas  $D_{s_i} < 0$  is satisfied for others, which leads to best response function  $s_i^C(s_j)$  to be above  $s_i^{NC}(s_j)$  for some values of  $s_j$  but below for others). For expositional clarity, let us first analyze the case in which both players are concerned about relative comparisons. Then, section 4.2 examines the case where player i is the only individual who assigns a value to these comparisons.

#### 3.4.1 Both players are concerned about comparisons

In this section I examine how the above ranking of equilibrium strategy choices varies when both players assign a positive importance to the outcome of their distance function. For simplicity, let us assume that both players' relative comparisons are symmetric:  $\Delta_i \times \Delta_j > 0$ , i.e., both players are relative reciprocators or compensators, although the "intensity" of these effects does not need to coincide  $\Delta_i \neq \Delta_j$ .

**Proposition 2.** Every player i's equilibrium strategy satisfies  $s_i^C \ge s_i^{NC}$  if player i is either:

- 1. a compensator using a demanding distance function; or
- 2. a reciprocator using a not-demanding distance function.

In addition, this result holds both for strategic substitutes and strategic complements.

The figures illustrate the results behind proposition 2 analyzing the ranking of players' equilibrium strategies. In particular, the type of player is represented in rows and the kind of distance function she uses is in columns. Specifically, figure 12 describes the results for negatively sloped best response functions (strategic substitutes), while 13 summarizes proposition 2 for the case that players' best response functions have a positive slope (strategic complements).

	Not demanding D <sup>NC</sup> >0	Demanding D <sup>NC</sup> <0
Compensators $\Delta < 0$	$s_i^C < s_i^{NC}$	$s_i^C > s_i^{NC}$
	Sj <sup>C</sup> <sj<sup>NC</sj<sup>	$S_j^C > S_j^{NC}$
Reciprocators $\Delta > 0$	$s_i^C > s_i^{NC}$	$s_i^C < s_i^{NC}$
	$s_j^C > s_j^{NC}$	$S_j^C < S_j^{NC}$

Figure 12: Strategic substitutes.

Interestingly, for the case of strategic complements but also for strategic substitutes,  $s_i^C > s_i^{NC}$  and  $s_j^C > s_j^{NC}$  are satisfied either when: (1) players are compensators with relatively demanding distance functions; or (2) when players are reciprocators with notdemanding distance functions. Intuitively, in the first case player *i* evaluates player *j*'s

	Not demanding D <sup>NC</sup> >0	Demanding D <sup>NC</sup> <0
Compensators $\Delta < 0$	$s_i^C < s_i^{NC}$	$s_i^C > s_i^{NC}$
	$S_j^C < S_j^{NC}$	$S_j^C > S_j^{NC}$
Reciprocators $\Delta > 0$	$s_i^C > s_i^{NC}$	$s_i^C < s_i^{NC}$
	<b>s</b> j <sup>C</sup> > <b>s</b> j <sup>NC</sup>	$S_j^C < S_j^{NC}$

Figure 13: Strategic complements.

actions as relatively low given that she uses a demanding distance function. Additionally, since she is a compensating type of player, she increases her equilibrium strategy. In contrast, in the second case, player i evaluates player j's actions as relatively high, given that she uses a not-demanding distance function. Since, in addition, she is a reciprocating type of player, she raises her strategy in equilibrium.

Note an interesting implication of these results. In particular, if players compare each others' actions with respect to the *highest* choice available to each other (i.e., both players are extremely "demanding"), then further cooperation among the players can only be predicted when individuals are regarded as compensators, e.g., they compensate each others' lack of contributions to the public good. In contrast, if players compare each others' actions with respect to the *lowest* available choice of the other player (and players can then be regarded as "not-demanding"), stronger cooperation occurs only when players are reciprocators.

#### **3.4.2** Only player *i* is concerned about comparisons

Let us now analyze the case in which player i is the only individual concerned about the outcome of her distance function, i.e.  $\Delta_i \neq 0$  and  $\Delta_j = 0$ .

**Proposition 3.** Consider that  $\Delta_i \neq 0$  and  $\Delta_j = 0$  for all  $j \neq i$ , then

- 1. Player i's equilibrium strategy satisfies  $s_i^C \ge s_i^{NC}$  if and only if he is either: (1) a compensator using a demanding distance function; or (2) a reciprocator using a not-demanding distance function. This result holds both for strategic substitutes and complements.
- 2. Player j's equilibrium strategy satisfies  $s_j^C \ge s_j^{NC}$  if and only if  $s_i^C < s_i^{NC}$  in the case of strategic substitutes, and if  $s_i^C > s_i^{NC}$  in the case of strategic complements.

$\Delta_j = 0$	Not demanding D <sup>NC</sup> <sub>i</sub> >0	Demanding D <sup>NC</sup> i<0
Compensator $\Delta_i < 0$	$s_i^C < s_i^{NC}$	$s_i^C > s_i^{NC}$
	$S_j^C > S_j^{NC}$	$S_j^C < S_j^{NC}$
Reciprocator $\Delta_i > 0$	$s_i^C > s_i^{NC}$	$s_i^C < s_i^{NC}$
	Sj <sup>C</sup> <sj<sup>NC</sj<sup>	$S_j^C > S_j^{NC}$

Figure 14: Strategic substitutes.

$\Delta_j = 0$	Not demanding $D^{NC}_{i} > 0$	Demanding D <sup>NC</sup> i<0
Compensator $\Delta_i < 0$	$s_i^C < s_i^{NC}$	$s_i^C > s_i^{NC}$
	$S_j^C < S_j^{NC}$	$S_j^C > S_j^{NC}$
Reciprocator Δ <sub>i</sub> >0	$s_i^C > s_i^{NC}$	$s_i^C < s_i^{NC}$
	$S_j^C > S_j^{NC}$	$S_j^C < S_j^{NC}$

Figure 15: Strategic complements.

The above two figures describe the results of proposition 3, emphasizing the ranking of player i and j's equilibrium strategies when only player i is concerned about relative comparisons. In particular, note that the ranking of equilibrium strategy choices for the concerned individual (player *i*) coincides with that when both players assign a positive value to relative comparisons. That is,  $s_i^C \ge s_i^{NC}$  holds in the same contexts regarding player *i* for figures 12 and 14 in the case of strategic substitutes, and for figures 13 and 15 in the case of strategic complements.

On the other hand, player j's equilibrium strategy moves in the *opposite* direction of player i's when actions are strategic substitutes, whereas it moves in the *same* direction when they are strategic complements. Intuitively, when players' actions are strategic substitutes, player j decreases her equilibrium strategy when she knows that player i increases hers, as figure 14 indicates. In contrast, when players' actions work as strategic complements (as in figure 15), player j raises her strategy choice when she predicts that player i increases hers in equilibrium.<sup>5</sup>

We can extract two main conclusions from the above results. First, a single individual with positive concerns about social comparisons suffices for higher strategy choices in equilibrium  $s_i^C \ge s_i^{NC}$  (at least for that player) under certain contexts; and it is valid for both players if their actions are strategic complements. Second, when both individuals assign a positive importance to social comparisons, players' equilibrium strategies move in the same direction, i.e., they experience a "coordinating effect." Importantly, this result is not only valid when players' actions are strategic complements, but also when they are strategic substitutes.

#### 3.4.3 Connection with the literature

In this section, I analyze how the model presented in this paper encompasses certain models on social preferences and intentions-based reciprocity as special cases, as the following proposition shows.

<sup>&</sup>lt;sup>5</sup>Finally, note that these results can be easily generalized to simultaneous-move games with N players. In such settings, however, every player measures the kindness he infers from the actually chosen strategies of each of the other N - 1 players. The outcome of each of these individual comparisons can then be added up (or even scaled in a weighted average), in order to evaluate player *i*'s distance function. Despite the greater generality of such model, nonetheless, its results and intuition are already captured by the two-player setting I consider in this paper.

**Proposition 4.** Assume  $s_j^{R_i}(s_i, s_j) = s_i$  for all  $s_j$ . Then, the player *i*'s preferences over player *j*'s actions can be represented by

$$U_{i}^{C}\left(s_{i}, s_{j}\right) = \gamma_{i} U_{i}^{NC}\left(s_{i}, s_{j}\right) + \gamma_{j} U_{j}^{NC}\left(s_{i}, s_{j}\right) \quad where \ \gamma_{i}, \gamma_{j} \in \mathbb{R}$$

In particular, the above proposition specifies that when player *i* compares player *j*'s chosen action,  $s_j$ , with that chosen by her,  $s_i$ , her utility function  $U_i^C(s_i, s_j)$  can be represented as a weighted average of her material payoffs and those of player *j*. Therefore, in such context our model captures players' concerns for inequity aversion (or altruism) as a special case, such as in Fehr and Schmidt (1999) and in Bolton and Ockenfels (2000). In addition, this model also captures certain concerns about intentions-based reciprocity as a special case. For example, the above utility representation embodies Charness and Rabin's (2002) model<sup>6</sup> for the case that player *i* infers misbehavior from player *j*'s actions, and for  $\gamma_i = 1 - \theta$  and  $\gamma_i = -\theta$ . That is,

$$U_{i}^{C}(s_{i}, s_{j}) = (1 - \theta) U_{i}^{NC}(s_{i}, s_{j}) - \theta U_{j}^{NC}(s_{j}, s_{i})$$
  
=  $U_{i}^{NC}(s_{i}, s_{j}) + \theta \left[ U_{i}^{NC}(s_{i}, s_{j}) - U_{j}^{NC}(s_{j}, s_{i}) \right]$ 

Finally, note that the model presented in this paper also encompasses contexts in which players care about social status. Indeed, as commented in section 3, this occurs when players compare others' actions with respect to her own and they introduce the outcome of this comparison negatively into her utility function. In particular, the distance function becomes  $D_i(s_i, s_j) \equiv -\alpha_i(s_j - s_i) = \alpha_i(s_i - s_j)$ , where player *i*'s utility increases when  $s_i > s_j$  and decreases otherwise.

<sup>&</sup>lt;sup>6</sup>Clearly, this representation of player *i*'s utility function does not capture Charness and Rabin's (2002) complete model, since they analyze other facets of individuals' behavior, such as inequity aversion, in addition to reciprocity. However, when restricted to intensions-based reciprocity alone, and when player *i* infers misbehavior from player *j*'s actions, the above utility function coincides with that in Charness and Rabin (2002).

#### 3.5 APPLICATION TO PUBLIC GOOD GAMES

In this section, I construct a simple example in which the above general model is applied to a public good game (PGG). Specifically, let us first assume that player i's utility function coincides with those in standard public good games,

$$U_i^{NC}(s_i, s_j) = [w - s_i]^{0.5} + [m(s_i + s_j)]^{0.5}$$

where w represents the amount of money available for contributions to the public good,  $s_i \in \mathbb{R}_+$ . Hence,  $w-s_i$  denotes the remaining units of money which have not been contributed and that can be used for consumption of private goods. Finally, let  $m \in \mathbb{R}_+$  be the (constant) return from the total contributions to the public good,  $s_i + s_j$ . Let us now introduce players' concerns about relative comparisons. In order to be consistent with the above model, let us first construct an example of a distance function that increases in player *i*'s strategy, i.e.,  $D_{s_i} > 0$  for all  $s_j$ , as in the case in which players care about status acquisition. Second, I analyze an example of a distance function that is not increasing for all player *i*'s strategy, i.e.,  $D_{s_i} > 0$  does not hold for all  $s_j$ .

#### 3.5.1 An example about status acquisition

Let us first consider that players increase their perception of social status when their contribution to the public good is above that of the other donor, i.e., when  $s_i > s_j$ . For simplicity, let us construct a linear distance function  $D_i \equiv -\alpha_i(s_j - s_i) = \alpha_i(s_i - s_j)$ , where player *i* compares her equilibrium contribution,  $s_i$ , with that of player *j*'s,  $s_j$ . Therefore, player *i*'s utility function becomes

$$U_i^C(s_i, s_j) = [w - s_i]^{0.5} + [m(s_i + s_j) + \alpha(s_i - s_j)]^{0.5}$$

where  $\alpha_i = \alpha_j = \alpha$  for simplicity. The next proposition describes player *i*'s equilibrium contribution in this context, and below I compare it with respect to hers in the standard PGG.

**Proposition 5.** In the simultaneous PGG game where players assign a value to status, every player  $i = \{1, 2\}$  submits a Nash equilibrium contribution of  $s_i^C = \frac{(\alpha+m)^2 w}{2m+(\alpha+m)^2}$ .

Specifically, the following corollary shows that, indeed, player *i*'s equilibrium contribution in this model is strictly higher than when she is not concerned about status acquisition (and generally about distances such that  $D_{s_i} > 0$  for all  $s_j$ ).

**Corollary 1.** Every player i's equilibrium contribution in the simultaneous PGG game,  $s_i^C$ , when all players assign value to status,  $\alpha > 0$ , is (strictly) higher than her contribution when they do not,  $\alpha = 0$ .

Interestingly, this result could be anticipated by directly using proposition 1. Indeed, since player *i* can increase the outcome of the distance function by increasing her own strategy (i.e.,  $D_{s_i} > 0$  for all  $s_j$  as in this case) then the ranking result  $s_i^C > s_i^{NC}$  could be predicted without the need to find reduced form solutions for the players' equilibrium contributions.

### **3.5.2** An example where comparisons are defined over $s_i$

Let us now construct a similar example in order to gain a clearer intuition about proposition 2's results. Particularly, let us assume that player *i* makes relative comparisons with a distance function that is *not* increasing in player *i*'s own strategy choice, i.e.,  $D_{s_i} > 0$  does not hold for all  $s_j$ . For example, if player *i* wants to evaluate player *j*'s commitment with the provision of the public good, she might use distance function  $D_i \equiv \alpha_i(s_j - s_j^{ref})$ , where  $s_j$  represents player *j*'s equilibrium contribution, and  $s_j^{ref} \in (0, 1)$  denotes a particular contribution to the public good that players may have agreed upon before the beginning of the game, and that player *i* uses as a reference point to compare  $s_j$ . Thus, player *i*'s utility function in this model becomes,

$$U_i^C(s_i, s_j) = [w - s_i]^{0.5} + [m(s_i + s_j) + \alpha(s_j - s_j^{ref})]^{0.5}$$

Specifically, note that player *i*'s utility level increases when player *j* contributes to the public good above her reference level  $s_j > s_j^{ref}$  (for example, more than what she committed

to), since player i might infer that player j's chosen strategy is a signal of a strong commitment with the provision of the public good. Let us next analyze player i's best response function.

**Proposition 6.** In the simultaneous PGG game, where every player  $i = \{1, 2\}$  assigns a value to the distance  $s_j - s_j^{ref}$ , player i's best response function,  $s_i^C(s_j)$ , is given by

$$s_{i}^{C}(s_{j}) = \begin{cases} \frac{\alpha s_{j}^{ref} + m^{2}w}{m(1+m)} - \frac{\alpha + m}{m(1+m)}s_{j} \text{ if } s_{j} \in \left[0, \frac{\alpha s_{j}^{ref} + m^{2}w}{\alpha + m(2+m)}\right] \\ 0 \text{ if } s_{j} > \frac{\alpha s_{j}^{ref} + m^{2}w}{\alpha + m(2+m)} \end{cases}$$

Comparing it with player i's best response function when she assigns no importance to distances,

$$s_i^{NC}(s_j) = \begin{cases} \frac{mw}{1+m} - \frac{1}{1+m}s_j \text{ if } s_j \in \left[0, \frac{mw}{2+m}\right] \\ 0 \text{ if } s_j > \frac{mw}{2+m} \end{cases}$$

one can clearly observe two main differences between these best response functions, from which we can conclude that player *i* is a "compensator". First, the vertical intercept of  $s_i^C(s_j)$  is higher than that of  $s_i^{NC}(s_j)$  for any  $\alpha > 0$  and  $s_j^{ref} > 0$ , i.e.,  $\frac{\alpha s_j^{ref} + m^2 w}{m(1+m)} > \frac{mw}{1+m}$ . And second,  $s_i^C(s_j)$  is steeper than  $s_i^{NC}(s_j)$ , i.e.,  $\frac{\alpha+m}{m(1+m)} > \frac{1}{1+m}$ . Therefore, player *i*'s best response function experiences a clockwise rotation from  $s_i^{NC}(s_j)$  to  $s_i^C(s_j)$  similar to that figure 2(b) illustrates. In contrast, when  $\alpha < 0$  player *i* becomes a "reciprocator." Indeed, the vertical intercept of  $s_i^C(s_j)$  is now lower than that of  $s_i^{NC}(s_j)$  for any  $\alpha < 0$ ; in addition,  $s_i^C(s_j)$  is now flatter than  $s_i^{NC}(s_j)$  since  $\frac{m-\alpha}{m(1+m)} < \frac{1}{1+m}$ . Hence, when  $\alpha < 0$  player *i*'s best response function experiences an anticlockwise rotation from  $s_i^{NC}(s_j)$  to  $s_i^C(s_j)$  similar to that illustrated in figure 2(c). Given the above results about player *i*'s best response function, let us now determine player *i*'s equilibrium contribution to the public good for any value of  $\alpha$ .

**Proposition 7.** In the simultaneous PGG game, every player *i*'s contribution when both players assign a value to the distance  $s_j - s_j^{ref}$  is given by  $s_i^C = \frac{\alpha s_j^{ref} + m^2 w}{\alpha + m(2+m)}$ .

Let us finally compare, alike in the previous example, every player i's donation in this model with respect to hers in the (standard) case when she assigns no value to distances.

**Corollary 2.** In the simultaneous PGG game, every player i's Nash equilibrium contribution when she assigns a value to the distance  $s_j - s_j^{ref}$ ,  $s_i^C$ , is strictly higher than hers when she assigns no weight to such distance,  $s_i^{NC}$ , if

- 1. players are "compensators" using a demanding distance function, i.e., conditions  $\alpha > 0$ and  $s_j^{NC} < s_j^{ref}$  hold; or
- 2. players are "reciprocators" using a not-demanding distance function, i.e., conditions  $\alpha < 0$  and  $s_j^{NC} > s_j^{ref}$  hold.

This result confirms proposition 2 in the general description of the model. Indeed, it specifies an alternative procedure to check whether  $s_i^C > s_i^{NC}$  without the need to find reduced form solutions for player *i*'s equilibrium contribution level. In particular, one just needs to check the conditions it describes: when players can be regarded as "compensators,"  $s_i^C > s_i^{NC}$  holds if these players use demanding distance functions,  $s_j^{NC} < s_j^{ref}$ . Otherwise, when players are regarded as "reciprocators,"  $s_i^C > s_i^{NC}$  is satisfied only if players use notdemanding distance functions,  $s_j^{NC} > s_j^{ref}$ ; as proposition 2 showed.

#### 3.6 CONCLUSIONS

This paper analyzes the effect of players' relative comparisons on their equilibrium strategies in simultaneous-move games. In particular, I show that when players relative comparisons lead them to regard each others' actions as more strategically complementary (players are regarded as "reciprocators"), and when they are not-demanding on the actions that they expect from each other, predicted levels of cooperation among the players are higher when they care about these comparisons than when they do not. Similarly, when players' considerations for relative comparisons lead their actions to become more strategically substitutable (players are regarded as "compensators"), and they demand high actions from each other, players' cooperation is stronger than when they do not. Interestingly, these results are not only valid for games where players' actions are regarded as strategic complements, but also for those in which they are strategic substitutes. Therefore, this paper shows the role of social comparisons as devices of cooperation in a relatively general class of simultaneous-move games. Specifically, these results explain why individuals choose to cooperate even when they do not assign any value to each others' payoffs; a common assumption in the literature predicting cooperation, which this paper does not consider.

Furthermore, I demonstrate that the results of this paper embed some existing behavioral models: from intentions-based reciprocity and status acquisition. Hence, this paper furthers our understanding of the facets explaining players' observed cooperation in multiple experiments. Let us finally remark some of the several extensions to the model introduced in this paper. Particularly, note that the action space was exogenously determined before the beginning of the game. However, it would be interesting to allow players to strategically select their available choices (their action space) before the game starts, given that the kindness other players perceive from their own choices depends on which actions are *not* chosen. This strategic selection of available choices is observed in different contexts, where a player uses one of her unchosen alternatives as an excuse to support her actual behavior. Further research in the effect of relative comparisons in individuals' strategic interaction will indeed improve our understanding of economic behavior in a greater variety of settings.

## 4.0 COMPETITION FOR STATUS ACQUISITION IN PUBLIC GOOD GAMES

## 4.1 INTRODUCTION

The effect of status on individuals' consumption of private goods has been extensively analyzed from a theoretical perspective, and confirmed by multiple studies. Indeed, many authors, starting from Smith (1759) and Veblen (1899), have examined agents' incentives to consume certain positional goods (such as luxury cars) for the only purpose of acquiring social status among their neighbors, co-workers or friends; see Ball *et al.* (2001), Frank (1985) and Hopkins and Kornienko (2004).

Despite the extensive analysis of status in private good settings, there is yet a limited literature on how social status acquisition may influence individuals in public good contexts, and specifically in their private contributions to charitable organizations. Nonetheless, the importance of status as a motive for individual donations cannot be overemphasized. For example, both *BusinessWeek* and *Slate* magazines recently created rankings of the most generous U.S. philanthropists. More generally, publicizing the list of donors, as well as the size of their contributions to the charity, constitutes a common practice of many charitable organizations, what suggests that many donors are indeed concerned about how their contribution is ranked relative to others'. In the same spirit, recent experimental literature, such as Kumru and Vesterlund (2005) and Duffy and Kornienko (2005), have also confirmed the role of status as an individual incentive affecting donors' giving behavior in different experimental settings.

This paper contributes to this literature by constructing a theoretical model that analyzes how individual (and total) contributions to a charity are affected by players' competition for social status. Intuitively, one may expect every donor's giving decision to be increasing in his value for social status, since this valuation might attenuate his incentives to free-ride on other donors' contributions. This intuitive prediction is indeed confirmed both in the simultaneous solicitation order (where both donors give simultaneously to the charity) and in its sequential version (in which one donor gives first and then the other gives second before the end of the game). Similarly, an individual's contribution should also be increasing in the value that other donors assign to status. Indeed, since an opponent with a higher value for status increases his contribution, individuals need to increase their donation to the charity in order to reduce as much as possible their loss of social status; this is confirmed in our model for both solicitation orders as well.

A question of interest is which particular contribution order raises the highest total revenue to the charity. In particular, I provide an answer to this questions which can be directly applied by practitioners. Specifically, populations of relatively *homogeneous* donors —in terms of the value they assign to status— induce a higher competition (and contributions) in the simultaneous public good game than in its sequential version. In contrast, groups of contributors with *heterogeneous* values to status submit higher total donations in the sequential contribution game than in its simultaneous counterpart. Hence, this paper contributes to the literature on public good games by analyzing which particular solicitation order raises the highest total revenue to the charity when players compete for social status.

Finally, I examine the possibility that donors' social status might be acquired from previous donations to the charity, or from any other sources. This is the case, for example, of famous philanthropists who start their competition for status with previously acquired levels of seniority. In particular, I show that if this previous status enters additively into donors' status concerns, seniority may work as an strategic substitute for the status donors can acquire through current donations, reducing their contributions. In contrast, if currently acquired status emphasizes previously acquired rankings, then status acquired during different periods work as strategic complements, and current donations are increased.

The structure of the paper is as follows. In the next section, I discuss the literature dealing with status, both in a private or public good setting. In section three the model is presented, and sections four and five describe the results in terms of the players' equilibrium contributions in the simultaneous and sequential games, respectively. In section six, given the previous results, I find the contribution mechanism that maximizes the charity's total revenue. Section seven presents an extension of the previous results, in which I consider the effect of seniority on current donations. Finally, section eight summarizes the main results of the paper and comments about its further extensions.

## 4.2 RELATED LITERATURE

## 4.2.1 Relative status acquisition

Let us address two main points regarding relative status acquisition as a motive for voluntary giving to public goods. First, different papers in the literature on status seeking (or status acquisition) have dealt with individual's behavior when consuming *private* goods which may enhance their relative status over other individuals of their group; see Congleton (1989), Frank (1985), Ball and Eckel (1998), Ball *et al.* (2001), and Hopkins and Kornienko (2004). Most of this literature, however, considers that an individual consuming a private good can only acquire status if he is the subject consuming the *highest* amount of that good among all individuals of his group. Importantly, the status an agent acquires does not depend on the distance between his consumption and the other individual's consumption. In this paper I use a less extreme assumption about how status enters into the players' utility function. Specifically, status is increasing in the difference between an individual's contribution and the donation of the other subject submitting donations to the charity. That is, this paper introduces the traditional status concerns in private good consumption into a public good setting, but also modifies the usual assumption about how relative status can be acquired. Hence, every player i is not only concerned about ranking (because the difference between his contribution to the charity and that of player i is positive), but also about how "intense" is this difference.

The second point regarding the consideration of relative status as a motive of voluntary giving to public goods deals with recent experimental studies on this topic. For instance, Kumru and Vesterlund (2005) and Duffy and Kornienko (2005), introduce individual con-
cerns about status in the contributor's utility function. In the first of these papers, relative status is considered to be *exogenous*, i.e., a player experiences a higher utility derived from status if he contributes to the same charity as the player with highest status in the group. In contrast, status acquisition in this paper is *endogenous*, as in Duffy and Kornienko (2005), since every player seeks to acquire a greater relative status in his group by contributing more than other donors. Their experimental evidence strongly supports our theoretical results.

### 4.2.2 Strategic role of charities

Until recently, most of the studies in public good games usually consider contributors as the sole active players of the game, and limit charities to the only role of administering the funds raised from contributions and the final production of the public good. This simplifying assumption was probably necessary, in order to clearly understand the motives for voluntary giving in simplified models. Some recent papers, nevertheless, have begun to notice the prominent and strategic role that charities can play in voluntary contributions games, for example when deciding whether contributions should be received simultaneously or sequentially. In fact some papers even allow the charity to decide between an exogenously determined contribution order and an endogenous one, where contributors are asked to unanimously vote about the time structure of the game they prefer; see Potters, Sefton and Vesterlund (2005). This paper goes more in the line of this recent literature since it assigns charities an strategic role by allowing them to decide which is the optimal solicitation order, depending on the donors' preferences for relative status acquisition.

#### 4.3 MODEL

Let us consider a public good game (PGG) where N = 2 agents privately contribute to the provision of a public good. Let  $g_i$  denote subject *i*'s voluntary contributions to the public good, and let  $x_i \ge 0$  represent his consumption of private goods. Additionally, I assume that the marginal utility individual *i* derives from his consumption of the private good is one. Specifically, I use the following quasilinear utility function,<sup>1</sup> where private goods enter linearly, while both total contributions, G, and relative status,  $status_i$ , are included in the nonlinear function  $v(\cdot)$ .

$$U_i(x_i, G, status_i) = x_i + v(G, status_i)$$

As noted above, the term  $status_i$  represents the utility that individual i gets from relative status. In particular, I assume that the status subject i acquires by contributing  $g_i$  is given by the difference between his contribution and that of the other player. That is,

$$status_i = \alpha_i (g_i - g_j)$$
 for any  $i, j \in \{1, 2\}$  and  $i \neq j$ 

First, note that subject *i* enhances his relative status if his contribution is greater than individual *j*'s; otherwise, if subject *j* contributes more than he does, then subject *i* perceives himself as an individual with lower status than subject j.<sup>2</sup> In addition, this difference is scaled by  $\alpha_i$ , indicating the importance of relative status for subject *i*, where  $\alpha_i \in [0, +\infty)$ . As commented in the previous section, this is a game of complete information. Hence, in the equilibrium of the PGG, player *i* correctly conjectures donor *j*'s contribution,  $g_j$  for all  $j \neq i$ , and as a consequence he knows whether he acquires status through his contribution,  $g_i > g_j$ , or if he does not,  $g_i < g_j$ . Furthermore, all the elements of the game, including the particular values of  $\alpha_i$ , are assumed to be common knowledge among the players.

For simplicity, I assume the nonlinear function  $v(G, status_i) = \ln [mG + status_i]$ , where  $m \in [0, +\infty)$  denotes the return player *i* obtains from total contributions to the public good. Finally, let *w* represent every player's endowment of monetary units that can be distributed

<sup>&</sup>lt;sup>1</sup>This quasilinear specification eliminates wealth effects, which may nonetheless exist in some real cases. Such quasilinear utility function was chosen, however, because: (1) it isolates the effect of status on charitable contributions (without confounding it with wealth effects); and (2) the use of alternative utility functions provides similar results to those in this paper without adding significant intuitions.

<sup>&</sup>lt;sup>2</sup>Note that this public good game can be easily generalized to N players. In such setting, every donor measures the status he acquires by comparing his contribution and that of the other N-1 players. The outcome of each of these comparisons can then be added up (or scaled in a weighted average) in order to evaluate player *i*'s acquired status. Despite the greater generality of such model, nonetheless, its results and intuitions are already captured by the two-player setting I consider in this paper.

between private and public goods consumption. Therefore, the representative contributor's maximization problem is given by

$$\max_{x_i,G} U_i(x_i, G, status_i) = x_i + \ln\left[mG + \alpha_i \left(g_i - g_j\right)\right]$$

subject to 
$$x_i + g_i = w$$
  
 $g_i + g_j = G$   
 $g_i, g_j \ge 0$ 

Using  $x_i = w - g_i \ge 0$ , we can simplify the above program to

$$\max_{g_i \ge 0} w - g_i + \ln \left[ m(g_i + g_j) + \alpha_i (g_i - g_j) \right]$$

In particular, the first term,  $w - g_i$ , represents the utility derived from the consumption of the remaining units of money that have not been contributed to the public good<sup>3</sup>. The second term denotes, on the one hand, the utility that individual *i* gets from the consumption of the total contributions to the public good  $g_i + g_j$ , and on the other hand, the utility derived from relative status acquisition.

Intuitively, note that in our model an increase in player j's contribution,  $g_j$ , imposes both a positive and a negative externality on player i's utility level. The *positive* externality from  $g_j$  on player i's utility is just the usual one arising from the public good nature of player j's contributions. Player j's donations, however, impose also a *negative* externality on player i since this donation reduces the status perception of player i, i.e., higher  $g_j$ decreases  $\alpha_i (g_i - g_j)$ , for any given  $g_i$ . Finally, note that we do not make any additional assumption on the quasilinear part of player i's utility function in order to guarantee that it is positive for any parameter values. Indeed, as we show in the next sections, this term is never negative in equilibrium, since low contributions by player i correspond to those cases for which  $\alpha_i$  is close to zero.

<sup>&</sup>lt;sup>3</sup>Note that allowing for asymmetric monetary endowments,  $w_i \neq w_j$ , would not change our results, since players' utility function is quasilinear in w.

#### 4.3.1 Best response function

In order to gain a clearer intuition of the results, let us analyze player i's best response function. Henceforth, all proofs are relegated to the appendix.

**Lemma 1.** In the simultaneous PGG with player who assign a value to status acquisition, player i's best response contribution level,  $g_i(g_j)$ , is

$$g_i(g_j) = \begin{cases} 1 + \frac{\alpha_i - m}{\alpha_i + m} g_j & \text{if } g_j \in \left[0, \frac{m + \alpha_i}{m - \alpha_i}\right], \text{ and} \\ 0 & \text{if } g_j > \frac{m + \alpha_i}{m - \alpha_i} \end{cases}$$

if  $\alpha_i < m$ . And in the case that  $\alpha_i > m$ ,  $g_i(g_j) = 1 + \frac{\alpha_i - m}{\alpha_i + m} g_j$  for all  $g_j$ .

Clearly, when  $\alpha_i < m$ , player *i*'s best response function is decreasing in  $g_j$ , while  $\alpha_i > m$  implies a positively sloped best response function, as figures 16 and 17 indicate.



Figure 16: Best response  $g_i(g_j)$  when  $a_i < m$ 

In particular, when  $\alpha_i < m$  the positive externality that player j's donations impose on player i's utility dominates the negative one, and player i considers player j's contributions as strategic *substitutes* of his own, as in the usual PGG models without status. On the other hand, when  $\alpha_i > m$  the negative externality resulting from player j's contributions is higher than the positive externality originated from the public good nature of his contributions. In



Figure 17: Best response  $g_i(g_j)$  when  $a_i > m$ 

this case, player *i* considers player *j*'s donations as strategic *complements* to his own, which leads to the positively sloped best response function depicted in figure 17. In addition, from the above lemma and discussion, it is easy to infer that the slope of player *i*'s best response function increases in his value to status,  $\alpha_i$ . The following lemma states this result, which it is applicable both in the simultaneous and sequential PGG.

**Lemma 2.** Player *i*'s best response function,  $g_i(g_j)$ , is (weakly) increasing in his value to status acquisition,  $\alpha_i$ , and (weakly) decreasing in *m*, for any parameter values.

This result is clear from the above figures. Indeed,  $g_i(g_j)$  pivots upward, with center at  $g_i = 1$ , as  $\alpha_i$  increases: from a negative slope when  $\alpha_i < m$  to a positive slope when  $\alpha_i > m$ .

#### 4.4 SIMULTANEOUS CONTRIBUTIONS

After analyzing player *i*'s best response function and its interpretation, we can now examine player *i*'s optimal contribution in this simultaneous-move game.

**Proposition 1.** In the simultaneous PGG with players who value status acquisition, player  $i = \{1, 2\}$  submits the following Nash equilibrium contribution level

$$g_i^{Sm} = \begin{cases} 1 & \text{if } \alpha_i > 0 \text{ and } \alpha_j = 0\\ \frac{\alpha_i(\alpha_j + m)}{(\alpha_i + \alpha_j)m} & \text{if } \alpha_i > 0 \text{ and } \alpha_j > 0\\ 0 & \text{if } \alpha_i = 0 \text{ and } \alpha_j > 0 \end{cases}$$

and  $g_i^{Sm} + g_j^{Sm} = 1$  if  $\alpha_i = \alpha_j = 0$ 

Figure 18 illustrates the set of parameter values that support the above different contribution levels. In particular,  $g_i^{Sm} = 1$  on the vertical axis of the figure where  $\alpha_j = 0$ ;  $g_i^{Sm} = 0$  on the horizontal axis, where  $\alpha_i = 0$ ; and  $g_i^{Sm} = \frac{\alpha_i(\alpha_j + m)}{(\alpha_i + \alpha_j)m}$  when  $\alpha_i, \alpha_j > 0$ . Intuitively, player i submits  $g_i^{Sm} = 1$  when he assigns a value to status and player j does not; submits a zero contribution when he does not assign any value to status,  $\alpha_i = 0$ , and player j does,  $\alpha_j > 0^4$ ; and finally submits  $g_i^{Sm} = \frac{\alpha_i(\alpha_j + m)}{(\alpha_i + \alpha_j)m}$  when both players assign a value to status.



Figure 18: Equilibrium contributions in Sm

In addition, figure 18 includes the  $45^0$ - line, where  $\alpha_i = \alpha_j$ , what divides equilibrium contribution levels into two parts: an upper division where  $\alpha_i > \alpha_j$  and as a consequence  $g_i^{Sm} > g_j^{Sm}$ , and a lower division where  $\alpha_i < \alpha_j$  and  $g_i^{Sm} < g_j^{Sm}$ . This result is very intuitive given that both players' equilibrium strategies are symmetric up to their individual value to

 $<sup>^{4}</sup>$ Note that zero donations can be alternatively interpreted as players who decide not to participate in the contribution mechanism.

status. Hence, in this simultaneous game, the player who assigns the highest value to status submits the highest donation. Next, the following lemma presents the comparative statics of player i's equilibrium donation.

**Lemma 3.** In the simultaneous PGG, player i's equilibrium contribution,  $g_i^{Sm}$ , is weakly increasing in his value to status acquisition,  $\alpha_i$ , and in player j's value,  $\alpha_j$ , for any parameter values. Furthermore,  $g_i^{Sm}$  is weakly decreasing in the return, m, that every donor obtains from total contributions.

That is, a player who values status competes more ferociously when he becomes more concerned about the status he can acquire through his contributions, but also when his opponent becomes more concerned about status. Indeed, since his opponent increases his donation, player i must increase his own as well if he pretends to maintain his level of social status unchanged. Finally, note that individual donations are decreasing in the return that every donor obtains from total contributions to the public good. That is, for a given value of status among donors, individual contributions decrease as his benefits from total contributions to the public good (free-riding effects) dominate his benefits from an increase in his individual contribution (status effects).

These results might be specifically vivid in the case of donors helping charities with low returns from total contributions, such as those operating in distant countries. Indeed, according to our previous results, a donor would donate *more* to charities with goals he does not directly benefit from (low returns) than from those he does (high returns), for a given value of the status he acquires from his donations to either charity. As a consequence of the above individual giving decision from players i and j, total contributions are the following.

**Lemma 4.** In the simultaneous PGG total contributions induced from Nash equilibrium play,  $G^{Sm}$ , are

$$G^{Sm} = \begin{cases} 1 & \text{if } \alpha_j = 0 \text{ and } \alpha_i > 0\\ 1 + \frac{2\alpha_i \alpha_j}{(\alpha_i + \alpha_j)m} & \text{if } \alpha_i > 0 \text{ and } \alpha_j > 0\\ 1 & \text{if } \alpha_i = 0 \text{ and } \alpha_j \ge 0 \end{cases}$$

Figure 19 represents total contributions in this simultaneous PGG for any  $\alpha_i$  and  $\alpha_j$ ; and figure 20 illustrates the three areas in which total contributions can be divided. In particular, making use of figure 20, it is immediate to conclude that: (1) when player *i* assigns no importance to status but player *j* does, on the horizontal axis of figure 17, player *j* submits  $g_j^{Sm} = 1$ ; (2) when the opposite happens,  $\alpha_j = 0$  and  $\alpha_i > 0$  on the vertical axis, it is player *i* who submits  $g_i^{Sm} = 1$ ; and finally (3) when both players are positively concerned about status,  $\alpha_i, \alpha_j > 0$  in the interior points of the figure, both players give positive amounts and their total contributions are  $1 + \frac{2\alpha_i \alpha_j}{(\alpha_i + \alpha_j)m}$ .



Figure 19: Total contributions  $G^{Sm}$ 

Finally, note that players' total contributions when *either* of them does not value status coincides with total contributions when *none* of them does,  $G^{Sm} = 1$ . Together with its increasing pattern in  $\alpha_i$  and  $\alpha_j$ , we can conclude that  $G^{Sm}$  is higher when players' value of status acquisition are relatively homogeneous (45<sup>0</sup>-line) than when they are heterogeneous.

# 4.5 SEQUENTIAL CONTRIBUTIONS

Let us next examine donors' contributions in the sequential PGG, where player i is the first donor solicited to contribute (and he can only give once)<sup>5</sup>. Observing his contribution,

<sup>&</sup>lt;sup>5</sup>The assumption that charities only allow donors to give once could be criticized because of being unrealistic. Nevertheless, note that this assumption is equivalent to considering that charities allow players to donate more than once, but they do not reveal donations until the end of the game. Indeed, any of these interpretations generates the same individual and total contributions.



Figure 20: Total contributions

player j (the follower) determines his donation using his best response function from lemma 1,  $g_j(g_i)$ . By sequential rationality, player i can insert  $g_j(g_i)$  into his utility function to decide which is the optimal contribution that maximizes his utility.

**Proposition 2.** In the sequential PGG the following contribution level describes the subgame perfect equilibrium strategy for player i (first mover)

$$g_i^{Seq} = \begin{cases} 0 & \text{if } \alpha_i \in [0, \bar{\alpha}_i] \text{, and} \\ \frac{\alpha_i \alpha_j + 3\alpha_i m + \alpha_j m - m^2}{2m(\alpha_i + \alpha_j)} & \text{if } \alpha_i \in (\bar{\alpha}_i, +\infty) \end{cases}$$

where  $\bar{\alpha}_i = \frac{m(m-\alpha_j)}{3m+\alpha_j}$ . Similarly, for player *j* (second mover)

$$g_{j}^{Seq} = \begin{cases} 1 \quad if \quad \alpha_{i} \in [0, \bar{\alpha}_{i}), \\ \frac{1}{2} \left( \frac{\alpha_{i} \alpha_{j}}{(\alpha_{i} + \alpha_{j})m} + \frac{m}{\alpha_{i} + \alpha_{j}} + \frac{4\alpha_{j}}{\alpha_{j} + m} - 1 \right) \quad if \quad \alpha_{j} < m \text{ and } \alpha_{i} \in [\bar{\alpha}_{i}, \hat{\alpha}_{i}), \\ or \quad if \quad \alpha_{j} > m \text{ and } \alpha_{i} \in [\bar{\alpha}_{i}, +\infty), \text{ and} \\ 0 \quad if \quad \alpha_{j} < m \text{ and } \alpha_{i} \in [\hat{\alpha}_{i}, +\infty) \end{cases}$$

where  $\hat{\alpha}_i = \frac{m(3\alpha_j^2 + m^2)}{-\alpha_j^2 - 4\alpha_j m + m^2}$ .

Let us first analyze player i's decision about contributing positive amounts. From the above proposition, we know that player i submits a strictly positive contribution if and only



Figure 21: Player *i*'s equilibrium contribution  $g_i^{Seq}$ 

if  $\alpha_i > \bar{\alpha}_i$ . Figure 21 represents player *i*'s equilibrium contribution for different values of  $\alpha_i$  and  $\alpha_j$ , and figure 22 illustrates cutoff level  $\bar{\alpha}_i$  for different values of *m*.

**Corollary 1.** In the sequential PGG, the first mover's equilibrium contribution,  $g_i^{Seq}$ , satisfies:

1.  $g_i^{Seq} > 0$  when  $\alpha_i = 0$ , if and only if  $\alpha_j > m$ . 2.  $g_i^{Seq} > 0$  when  $\alpha_i > m$ , for any  $\alpha_j$ .

That is, when the first mover does not assign any value to status,  $\alpha_i = 0$ , he submits a positive contribution when the second mover's best response function is positively sloped, i.e.,  $\alpha_j > m$ ; otherwise, when  $\alpha_j < m$ , he submits a zero contribution. Intuitively, a first mover with no value for status (as in the first point of the above corollary) free-rides the responder's contribution when  $\alpha_j < m$  as usual in sequential PGGs without any considerations about status. The above intuition is illustrated in figure 21, and in particular at the  $\alpha_j$ -axis, where  $\alpha_i = 0$ . Note that for any value of  $\alpha_j$  such that  $\alpha_j < m$ , player *i*'s optimal contribution is zero, while for any  $\alpha_j > m$ , player *i* submits positive donations.

On the other hand, the second result of corollary 1 specifies a condition on player *i*'s value of status,  $\alpha_i > m$ , that leads him to submit positive contributions regardless of the value that the second mover may assign to status acquisition,  $\alpha_i$ . Graphically, this result is



Figure 22: Cutoff level alpha *i* for m = 0.5 and m = 0.8.

obvious from figure 21. In particular, any  $(\alpha_i, \alpha_j)$ -pair satisfying  $\alpha_i > m$  is above the cutoff level  $\bar{\alpha}_i$  for any parameter values, that leads to strictly positive contributions from the first mover. Let us next examine some comparative statics about  $g_i^{Seq}$  in this sequential game.

**Lemma 5.** In the sequential PGG with players who assign a value to status,  $g_i^{Seq}$  is weakly increasing both in his own value for status acquisition,  $\alpha_i$ , and in player j's value,  $\alpha_j$ , for any parameter values.

The intuition behind these results coincides with that arising from the comparative statics of  $g_i^{Sm}$  in the simultaneous game, and I refer to section four for a discussion of its interpretation. Let us now analyze under which parameter values player *i* decides to contribute a donation that cannot be exceeded by player *j*, and guarantees himself, as a consequence, a greater relative status.

**Lemma 6.** In the sequential PGG with players who assign a value to status, the first mover contributes a strictly higher donation than the second mover,  $g_i^{Seq} > g_j^{Seq}$ , if and only if his value for status,  $\alpha_i$ , satisfies  $\alpha_i > \alpha_i^*$ , where  $\alpha_i^* = \frac{\alpha_j^2 + m^2}{2m}$ .

Specifically note that, as the figure 23 illustrates, pairs of parameter values above the isolevel curve  $\alpha_i^*$  support  $g_i^{Seq} > g_j^{Seq}$ , while those below support  $g_i^{Seq} < g_j^{Seq}$ , keeping the return from total contributions, m, constant.



Figure 23: Isolevel curve  $a_i^*$ .

Intuitively, when the first mover assigns high values to status and the second does not  $(\alpha_i > \alpha_i^* \text{ and } \alpha_j < \alpha_i^*)$  the first donor induces the second mover to "give up" from the competition by submitting a sufficiently high contribution that the second donor cannot profitably exceed. On the contrary, when the second mover is the only player with a relatively high value for status ( $\alpha_i < \alpha_i^*$  and  $\alpha_j > \alpha_i^*$ ) the first donor submits a relatively low contribution, what "tempts" the second mover with the possibility of winning the competition for status by contributing a high donation to the charity. Let us next analyze the charity's total revenues in this sequential solicitation mechanism.

**Lemma 7.** In the sequential PGG total contributions induced from the subgame perfect Nash equilibrium of the game,  $G^{Seq}$ , are

$$G^{Seq} = \begin{cases} 1 & \text{if} \quad \alpha_i \in [0, \bar{\alpha}_i) \\ \frac{2\alpha_j}{\alpha_j + m} + \frac{\alpha_i(\alpha_j + m)}{(\alpha_i + \alpha_j)m} & \text{if} \quad \alpha_j < m \text{ and } \alpha_i \in [\bar{\alpha}_i, \hat{\alpha}_i), \text{ or if} \quad \alpha_j > m \text{ and } \alpha_i \in [\bar{\alpha}_i, +\infty) \\ \frac{\alpha_i \alpha_j + 3\alpha_i m + \alpha_j m - m^2}{2m(\alpha_i + \alpha_j)} & \text{if} \quad \alpha_j < m \text{ and } \alpha_i \in [\hat{\alpha}_i, +\infty) \end{cases}$$

Interestingly, when player *i* assigns a sufficiently low value to status acquisition,  $\alpha_i < \bar{\alpha}_i$ , he does not contribute and player *j* responds by contributing one. In this case,  $G^{Seq} = 1$ , and the results resemble those in sequential PGG models without status considerations,  $\alpha_i = \alpha_j = 0$ . In contrast, when player *i* assigns a sufficiently high value to status,  $\alpha_i \in [\bar{\alpha}_i, \hat{\alpha}_i)$ , and  $\alpha_j > m$ , he contributes positive amounts which are then reciprocated by the positive contributions of player j, leading to the total contributions specified above. Finally, if  $\alpha_i > \hat{\alpha}_i$ and player j's best response function is negatively sloped,  $\alpha_j < m$ , player i contribution crowds-out all profitable donations by player j, and he is the only donor contributing to the charity.

**Corollary 2.** Total contributions in the PGG where players assign a value to status acquisition, are weakly higher than when players do not, both in the simultaneous and sequential mechanism, and for any parameter values.

Intuitively, the private benefit from status arising from a player's individual contribution introduces further incentives to give to the charity, in addition to the usual incentives to the public good provision. Interestingly, this result is related with that of Morgan's (2000), in which he shows that when charities use lotteries in their fundraising campaigns, total revenues are higher under both solicitation mechanisms.

#### 4.6 COMPARING CONTRIBUTION MECHANISMS

Different questions naturally arise from the above results. For example, given a particular pair of players' values for status,  $(\alpha_i, \alpha_j)$ , under what contribution order does player *i* (or player *j*) contribute more? Or, what contribution order maximizes total donations received by the charity? Let us first compare individual contributions, and then extend our results to the total revenues received by the charity.

**Lemma 8.** Player *i*'s equilibrium contributions in the simultaneous and sequential PGG satisfy  $g_i^{Sm} > g_i^{Seq}$  if and only if: (1)  $\alpha_i > m$  and  $\alpha_j > m$ , or (2)  $\alpha_i < m$  and  $\alpha_j < m$ , for all  $i = \{1, 2\}$  and  $j \neq i$ . Similarly, for player j,  $g_j^{Sm} > g_j^{Seq}$ , if and only if  $\alpha_i > m$ .

That is, when players' value of status is relatively homogenous, i.e.,  $\alpha_i > m$  and  $\alpha_j > m$ (or if  $\alpha_i < m$  and  $\alpha_j < m$ ), the first mover contributes more in the simultaneous PGG than in its sequential version. This result is indicated in figure 24, where the first and third quadrant illustrate parameter combinations for which player *i* submits higher contributions in the simultaneous than in the sequential game, i.e.,  $g_i^{Sm} > g_i^{Seq}$ . If, on the contrary, players' value of status is relatively *heterogeneous*, i.e., if  $\alpha_i > m$  and  $\alpha_j < m$  for all  $j \neq i$ , then the above inequality is reversed, i.e.,  $g_i^{Sm} < g_i^{Seq}$ , represented in the second and fourth quadrants.



Figure 24: Comparison of individual contributions



Figure 25: Revenue comparisons.

In the case of player j, note that he submits  $g_j^{Sm} > g_j^{Seq}$  if player *i*'s best response function is positively sloped,  $\alpha_i > m$ . Intuitively, when  $\alpha_i > m$  player *i* (the first mover in the sequential game) induces player *j* to "give-up" from the competition for social status by submitting a sufficiently high donation. In contrast, when  $\alpha_i < m$  player *i* "tempts" player *j* to win the competition for social status by submitting a sufficiently low contribution which can be easily exceeded. After describing the ranking of individual contributions in the simultaneous and sequential mechanisms, let us now analyze how it translates into total contributions.

**Proposition 3.** Total contributions under the simultaneous PGG are higher than under the sequential PGG,  $G^{Sm} > G^{Seq}$ , if and only if

> $\alpha_i > m \text{ and } \alpha_j > m, \text{ or}$  $\alpha_i < m \text{ and } \alpha_j < m$

The results from this proposition are graphically illustrated in figure 25. Shaded areas indicate sets of parameters values for which the simultaneous contribution mechanism provides higher revenues to the charity than the sequential game,  $G^{Sm} > G^{Seq}$ , whereas unshaded areas support the contrary, i.e.,  $G^{Sm} < G^{Seq}$ .

Let us first elaborate on those parameter values supporting  $G^{Sm} > G^{Seq}$ , where  $\alpha_i > m$ and  $\alpha_j > m$  (or where  $\alpha_i < m$  and  $\alpha_j < m$ ), i.e., players' assign a relatively homogenous value to status. In the first case, when both donors assign a high value to status,  $\alpha_i > m$ and  $\alpha_j > m$ , competition for social status between players is so intense in the simultaneous version of the game that  $G^{Sm} > G^{Seq}$ . On the other hand, when both players assign a low value to status, i.e.,  $\alpha_i < m$  and  $\alpha_j < m$ , we find equilibrium predictions resembling those in PGGs where players do not care about status. In particular, now both players consider each others' contributions as strategic substitutes, since the benefits they experience from the public good dominate those of acquiring social status. Therefore, the first mover reduces his contribution anticipating that the second donor will increase his, what he then free-rides. Since, in addition, the second mover does not increase his donation enough to compensate for such a decrease, we observe  $G^{Sm} > G^{Seq}$ .

Let us now analyze those parameter values for which  $G^{Sm} < G^{Seq}$ , what occurs when donors are relatively *heterogeneous* in the value they assign to status acquisition, i.e.,  $\alpha_i > m$  and  $\alpha_j < m$  for all  $i = \{1, 2\}$  and  $j \neq i$ . As described in the previous section, when  $\alpha_i > m$ and  $\alpha_j < m$  the first donor (who assigns a relatively high value to status) induces the second mover (who does not) to "give up" from the competition by submitting a sufficiently high contribution, which cannot be exceeded by the second donor.

On the other hand, when the second mover is the only player who assigns a relatively high value to status,  $\alpha_i < m$  and  $\alpha_j > m$ , the first donor submits a low contribution, that "tempts" the second mover with the chance of winning the competition by contributing a higher donation to the charity. Ultimately, the first mover's incentives to induce the second donor to "give up" the competition for status (or to be "tempted" to win) leads to  $G^{Sm} < G^{Seq}$ . Finally, note that when both players assign the same value to status acquisition,  $\alpha_i = \alpha_j$ , as illustrated in the 45<sup>0</sup>-line of figure 25, total contributions satisfy  $G^{Sm} > G^{Seq}$ , for any parameter values.<sup>6</sup> This particular revenue ranking result uses Romano and Yildirim's (2001) model, who provide a general framework with which to compare total revenues in simultaneous and sequential public good games. Unlike Romano and Yildirim (2001), this paper provides a ranking for *individual* contributions between the simultaneous and sequential solicitation mechanisms.

#### 4.7 EXTENSION: SENIORITY IN STATUS

Previous sections considered that individuals can only acquire status through their donations while playing the PGG. Donors, however, were not allowed to start the voluntary contribution game with some previous status arising, for example, from their prior donations to the charity during past solicitation mechanisms, or from any other source (i.e., previous seniority in status). In this section, I analyze how our results would change when allowing for such seniority in status.<sup>7</sup> In particular, assuming that players i and j start the PGG with previous

<sup>&</sup>lt;sup>6</sup>This result constitutes a generalization of that in Varian (1994) who determines that  $G^{Sm} > G^{Seq}$  when  $\alpha_i = \alpha_i = 0$  in the standard public good game where players do not assign any value to status acquisition.

<sup>&</sup>lt;sup>7</sup>Note that I consider previously acquired status (seniority) in this extension of the paper in order to analyze how such seniority would modify our results regarding players' competition for status. However, it would be interesting to examine donors' contribution decisions during different periods in an intertemporal model of charitable contributions.

seniority levels of  $S_i$  and  $S_j$  respectively, their utility function becomes

$$U_{i} = w - g_{i} + \ln \left[ m \left( g_{i} + g_{j} \right) + \alpha_{i} \left( S_{i} + g_{i} - g_{j} \right) \right]$$

Let us first examine players' individual contributions in both the simultaneous and the sequential-move game.

**Proposition 4.** In the simultaneous and sequential PGG, with players who assign a value to status acquisition,  $\alpha_i, \alpha_j > 0$ , and seniority in status is given by  $S_i$  and  $S_j$ , player i's equilibrium contribution is weakly decreasing in his own seniority in status,  $S_i$ , for any parameter values. In addition, player i's equilibrium contribution is also weakly increasing in the other player's seniority in status,  $S_j$ , if and only if  $\alpha_i < m$ .

That is, the seniority player *i* acquires in previous rounds of the game works as a substitute of the status that he can acquire today by raising his contribution to the charity<sup>8</sup>. Nonetheless, a greater seniority of player *j*,  $S_j$ , leads player *i* to increase his contribution only if his own concern for status satisfies  $\alpha_i < m$  and thus his best response function is downward sloping, i.e., he decreases his donation as a result of player *j*'s contributions. Since, in addition, an increase in player *j*'s seniority,  $S_j$ , reduces her contributions today, then we obtain that higher seniority levels for player *j* finally increase player *i*'s donation. Let us now finally determine which solicitation order generates the highest revenue for the charity. Since our results from section six are not modified, we refer to that section and to figure 24 for a discussion of their intuition.

**Proposition 5.** Total contributions under the simultaneous PGG are higher than under the sequential PGG,  $G^{Sm} > G^{Seq}$ , when seniority is considered,  $S_i, S_j > 0$ , if and only if  $\alpha_i > m$  and  $\alpha_j > m$ , or  $\alpha_i < m$  and  $\alpha_j < m$ .

<sup>&</sup>lt;sup>8</sup>This result is a consequence of how seniority in status enters into players' utility function. If seniority entered scaling up the difference between individual contributions, i.e.,  $\alpha_i (g_i - g_j) S_i$ , an increase in  $S_i$  would have the same effect in player *i*'s equilibrium donations as a raise in  $\alpha_i$ . That is, status acquired during previous and current periods would work as strategic complements, and an increase in  $S_i$  would lead player *i* to *raise* her contribution  $g_i$ . More empirical research is needed, however, to exactly determine how seniority in status enters into donors' preferences.

#### 4.8 CONCLUSIONS

Recent experimental evidence (as well as casual observation) support status acquisition as an individual incentive for charitable giving. Nonetheless, and despite its interest, relatively few studies have analyzed this topic from a theoretical approach. This paper analyzes relative status acquisition as an additional incentive to contribute to PGGs, and unlike recent papers on this literature, I define players' status as the difference between their own donation and that of others. As expected, contributors' giving decisions are increasing both in their own concerns for status,  $\alpha_i$ , and that of the other donor,  $\alpha_j$ . This pattern clearly reflects donors' competition in their contributions with the objective of acquiring higher social status, which is confirmed both in the simultaneous and sequential solicitation mechanisms. In addition, I identify what parameter values induce the charity to choose a simultaneous over a sequential contribution order. In particular, I show that the charity prefers simultaneous PGGs when players are sufficiently homogeneous in the relative value they assign to status acquisition<sup>9</sup>. Otherwise, the charity prefers the sequential mechanism.

In an extension, I analyze how the above results would be modified if we allow donors to start their competition for status acquisition with previously acquired "stocks" of status, i.e., seniority in status. In particular, the results in terms of what contribution mechanism is more profitable for the charity are not changed. However, several insights about the role of seniority in status are obtained. Specifically, I demonstrate that when previous status enters additively into donors' concerns, seniority may work as an strategic substitute for the status that donors can acquire through current donations, reducing their contributions. In contrast, if currently acquired status emphasizes previously acquired rankings, then status acquired during different periods works as strategic complements, and current donations are increased.

Different extensions of this paper would enhance our understanding of the role of status acquisition in PGG. First, it would be interesting to analyze how a charity can influence donors' concerns about status, by inducing on them higher or lower preferences for status acquisition. Similarly, another extension of this paper could go in the direction of considering

<sup>&</sup>lt;sup>9</sup>This result is also to that of Dixit (1987) for contests where players expend effort to win a certain prize.

status gathering in PGGs with incomplete information. In such settings contributors do not know each other's preferences for status, which is closer to many real-life situations, where donors may have a common understanding of the return from the public good, but may not know each others' preferences for status acquisition. Further research in this area can certainly provide additional insights about donors' incentives to contribute to charities, how status acquisition affects their giving decisions and, finally, how does it lead them to compete in their contributions.

# 5.0 APPENDIX

# 5.1 APPENDIX TO CHAPTER 1

# 5.1.1 Figure 26



Figure 26: Assumption 5(a)

#### 5.1.2 Figure 27



Figure 27: Assumption 5(b)

# 5.1.3 Proof of Lemma 1

Since  $s_i^{NC}(s_j)$  and  $s_i^C(s_j)$ , then

$$\frac{\partial s_{i}^{C}\left(s_{j}\right)}{\partial s_{j}} = \frac{\partial s_{i}^{NC}\left(s_{j}\right)}{\partial s_{j}} + \left[1 - \frac{\partial s_{j}^{R_{i}}\left(s_{i}, s_{j}\right)}{\partial s_{j}}\right]$$

where  $\frac{\partial D_i(s_i,s_j)}{\partial s_j} = 1 - \frac{\partial s_j^{R_i}(s_i,s_j)}{\partial s_j}$  given that  $D_i(s_i,s_j) \equiv s_j - s_j^{R_i}(s_i,s_j)$ . Hence,  $\frac{\partial s_i^C(s_j)}{\partial s_j} \ge \frac{\partial s_i^{NC}(s_j)}{\partial s_j}$  since  $1 \ge \frac{\partial s_j^{R_i}(s_i,s_j)}{\partial s_j}$  by definition.

# 5.1.4 Proof of Proposition 1

We first show that player *i*'s best response functions when she is concerned about player *j*'s foregone options and when she is not, respectively,  $s_i^C(s_j) \in \underset{s_i \in S_i}{\operatorname{arg\,max}} U_i^C(s_i, s_j)$ , and  $s_i^{NC}(s_j) \in \underset{s_i \in S_i}{\operatorname{arg\,max}} U_i^{NC}(s_i, s_j)$ , contain a single point. Then, we show the result stated in proposition 1.

Note that player *i*'s utility function when she is concerned about player *j*'s unchosen alternatives,  $U_i^C(s_i, s_j)$ , is strictly concave in  $s_i$  and it is defined over a strictly convex domain. This guarantees that player *i*'s best response function  $s_i^C(s_j) \in \underset{s_i \in S_i}{\operatorname{arg\,max}} U_i^C(s_i, s_j)$ contains a single point. A similar argument is also applicable for player *i*'s utility function when she does not assign any relevance to player *j*'s foregone options,  $U_i^{NC}(s_i, s_j)$ , since it is also strictly concave in  $s_i$  and it is defined over a strictly convex domain. Hence,  $s_i^{NC}(s_j) \in \underset{s_i \in S_i}{\operatorname{arg\,max}} U^{NC}(s_i, s_j)$  also contains a single point.

Once we know that player *i*'s best response function is unique, we just have to compare them in the intervals where  $D_i(s_i, s_j) \ge 0$  and  $D_i(s_i, s_j) < 0$  in order to check if proposition 1 is satisfied. Let us show it by contradiction. Hence, let us assume that  $s_i^C(s_j) < s_i^{NC}(s_j)$ when  $s_j \ge s_j^{R_i}(s_i, s_j)$  (i.e.  $D_i(s_i, s_j) \ge 0$ ). Then, for a function  $\tilde{s}_i(s_j) \in S_i$  sufficiently close to  $s_i^{NC}(s_j)$ ,

$$U_{i}^{C}\left(s_{i}^{NC}\left(s_{j}\right), s_{j}\right) - U_{i}^{C}\left(\tilde{s}_{i}\left(s_{j}\right), s_{j}\right) \leq U_{i}^{NC}\left(s_{i}^{NC}\left(s_{j}\right), s_{j}\right) - U_{i}^{NC}\left(\tilde{s}_{i}\left(s_{j}\right), s_{j}\right) = 0$$

That is, player *i*'s marginal utility of raising her strategy when evaluated at the maximizer when she is unconcerned about foregone options,  $s_i^{NC}(s_j)$ , is below the marginal utility she could achieve by using this same strategy  $s_i^{NC}(s_j)$  when she is not concerned about player *j*'s unchosen alternatives, which is by definition zero. But this would violate assumption A5 (reciprocity), which states that, when  $D_i(s_i, s_j) \ge 0$ ,

$$U_{s_i}^C(s_i, s_j) \ge U_{s_i}^{NC}(s_i, s_j)$$

must hold for any action  $s'_i$  sufficiently close to  $s_i$ , including  $s^{NC}_i(s_j)$ . Hence,  $s^{C}_i(s_j) < s^{NC}_i(s_j)$  when  $D_i(s_i, s_j) \ge 0$  cannot be true. Similarly for  $s^{C}_i(s_j) > s^{NC}_i(s_j)$  when we have

that  $D_i(s_i, s_j) < 0$ . Hence, it can only be true that

$$s_{i}^{C}(s_{j}) \geq s_{i}^{NC}(s_{j}) \text{ for all } D_{i}(s_{i}, s_{j}) > 0$$
  
$$s_{i}^{C}(s_{j}) < s_{i}^{NC}(s_{j}) \text{ for all } D_{i}(s_{i}, s_{j}) \leq 0$$

#### 5.1.5 Proof of Lemma 2

From proposition 1 we know that the difference between player *i*'s best response function when she is concerned and unconcerned about foregone options,  $s_i^C(s_j) - s_i^{NC}(s_j)$ , is weakly increasing in the distance  $D_i(s_i, s_j)$ . In addition, by assumption A1 we have that player *j*'s utility function  $U_j^{NC}(s_j, s_i)$  is strictly increasing in  $s_i$ . Hence,

$$U_{j}^{NC}\left(s_{j}, s_{i}^{C}\left(s_{j}\right)\right) - U_{j}^{NC}\left(s_{j}, s_{i}^{NC}\left(s_{j}\right)\right) \text{ is weakly increasing in } D_{i}\left(s_{i}, s_{j}\right)$$

Therefore, for two actions  $s_j, s'_j \in S_j$  such that  $s'_j > s_j$ , we have that  $D_i(s_i, s'_j) > D_i(s_i, s_j)$ , what implies that

$$U_{j}^{NC}\left(s_{j}', s_{i}^{C}\left(s_{j}'\right)\right) - U_{j}^{NC}\left(s_{j}', s_{i}^{NC}\left(s_{j}'\right)\right) \ge U_{j}^{NC}\left(s_{j}, s_{i}^{C}\left(s_{j}\right)\right) - U_{j}^{NC}\left(s_{j}, s_{i}^{NC}\left(s_{j}\right)\right)$$

and rearranging,

$$U_{j}^{NC}\left(s_{j}', s_{i}^{C}\left(s_{j}\right)\right) - U_{j}^{NC}\left(s_{j}, s_{i}^{C}\left(s_{j}\right)\right) \ge U_{j}^{NC}\left(s_{j}', s_{i}^{NC}\left(s_{j}'\right)\right) - U_{j}^{NC}\left(s_{j}, s_{i}^{NC}\left(s_{j}\right)\right)$$

#### 5.1.6 Proof of Proposition 2

Let us  $s_j^C$  and  $s_j^{NC}$  denote the leader's equilibrium strategies when dealing with a concerned and not concerned follower, respectively. Let us prove  $s_j^C > s_j^{NC}$  by contradiction. Hence, assume that  $s_j^C < s_j^{NC}$ . If this is the case, then the leader's marginal utility from raising her action must be higher when the follower is unconcerned about foregone options than when he assigns a positive importance to them. But this contradicts lemma 2. In particular, recall that lemma 2 states that the marginal utility of raising the proposer's action is higher for the first mover when the second mover is concerned about unchosen alternatives than when he is not. Hence  $s_j^C < s_j^{NC}$  must be false, and proposition 2 is satisfied.

#### 5.1.7 Proof of Proposition 3

Using Segal and Sobel (1999), we know that player i's preferences over player j's actions can be represented by

$$U_{i}^{C}\left(s_{i},s_{j}\right)=\gamma_{i}U_{i}^{NC}\left(s_{i},s_{j}\right)+\gamma_{j}U_{j}^{NC}\left(s_{j},s_{i}\right) \text{ where } \gamma_{i},\gamma_{j}\in\mathbb{R}$$

if preferences satisfy continuity and independence, as well as Segal and Sobel's (1999) condition  $(\bigstar)$  which states that

if 
$$U_i^{NC}(s_i', s_j) = U_i^{NC}(s_i, s_j)$$
, then  $s_i' \sim_i s_i$   $(\bigstar)$ 

which are all satisfied in our model.

### 5.1.8 Proof of Lemma 3

Let us consider the receiver's utility function when he assigns a positive importance to the proposer's foregone options and when he does not, respectively,  $U_i^C(s_i, s_j, S_j) = x_i + \alpha_i(x_i - f_i)$ , and  $U_i^{NC}(s_i, s_j) = x_i$ . First, assumption A1 is satisfied since  $U_i^{NC}(A, s'_j) \ge U_i^{NC}(A, s'_j)$  and  $U_i^{NC}(R, s'_j) \ge U_i^{NC}(R, s_j)$  for all  $s_j$  since  $x'_i > x_i$  if and only if  $s'_j > s_j$ . Additionally, A2 (concavity) is satisfied since

$$\frac{\partial^{2}U_{i}^{C}\left(U_{i}^{NC},D_{i}\right)}{\partial s_{i}^{2}} = \frac{\partial^{2}U_{i}^{NC}\left(s_{i},s_{j}\right)}{\partial s_{i}^{2}} = 0$$

A3 is trivially satisfied by player *i*. Regarding assumption A4 (kindness) is satisfied since

$$\begin{array}{lll} U_{i}^{C}\left(s_{i},s_{j}\right) &> & U_{i}^{NC}\left(s_{i},s_{j}\right) \mbox{ since } x_{i} + \alpha_{i}(x_{i} - f_{i}) > x_{i} \mbox{ if } x_{i} > f_{i} \\ U_{i}^{C}\left(s_{i},s_{j}\right) &= & U_{i}^{NC}\left(s_{i},s_{j}\right) \mbox{ since } x_{i} + \alpha_{i}(x_{i} - f_{i}) = x_{i} \mbox{ if } x_{i} = f_{i} \\ U_{i}^{C}\left(s_{i},s_{j}\right) &< & U_{i}^{NC}\left(s_{i},s_{j}\right) \mbox{ since } x_{i} + \alpha_{i}(x_{i} - f_{i}) < x_{i} \mbox{ if } x_{i} < f_{i} \end{array}$$

Finally, A5 (reciprocity) is also satisfied, since in their model,  $s'_i > s_i$ , if and only if  $s'_i = A$  and  $s_i = R$ , what implies that for all  $D_i(s_i, s_j) \ge 0$  (i.e.,  $x_i \ge f_i$ )

$$U_i^C(s'_i, s_j) - U_i^C(s_i, s_j) \ge U_i^{NC}(s'_i, s_j) - U_i^{NC}(s_i, s_j)$$

$$\iff x_i + \alpha_i (x_i - f_i) - 0 \ge x_i - 0 \text{ for any } x_i < f_i$$

and when  $D_i(s_i, s_j) < 0$  (i.e.,  $x_i < f_i$ ), since

$$[x_i + \alpha_i (x_i - f_i)] - 0 < x_i - 0$$
 for any  $x_i < f_i$ 

# 5.1.9 Proof of Lemma 4

Let  $(x_j, x_i)$  denote the proposed allocation that the proposer offers to the responder. We know that the responder will accept any offer  $x_i$  if and only if the utility he gets by accepting is weakly above than the (zero) utility he gets by rejecting. That is,  $x_i + \alpha_i(x_i - f_i) = 0$  $\iff x_i = \frac{\alpha_i}{1 + \alpha_i} f_i.$ 

Let us now check for sufficiency. Note that the responder does not to accept any offer  $x_i < \bar{x}_i$ . Instead, accepting any offer  $x_i < \bar{x}_i$  would imply negative utility levels, and the responder would be better off by rejecting such an offer, obtaining zero utility. Thus,  $x_i < \bar{x}_i$  cannot be an equilibrium strategy.

Finally we need to check that the responder does not reject any offer above  $\bar{x}_i$ . Let us assume that the responder sets an acceptable threshold  $\hat{x}_i > \bar{x}_i$ . Then, any offer  $x_i$  such that  $\bar{x}_i < x_i < \hat{x}_i$  would be rejected, and the responder would find that accepting it constitutes a profitable deviation. Therefore, the acceptance threshold cannot be strictly above  $\bar{x}_i$ . Hence, the responder does not accept any offer  $x_i \in [0, \bar{x}_i)$ , but accepts any offer weakly above this threshold level  $\bar{x}_i$ .

#### 5.1.10 Proof of Proposition 4

From lemma 8 we know the responder's acceptance threshold. Since the proposer wants to maximize the remaining portion of the pie which is not offered to the receiver –and guarantees that the receiver accepts such division– he offers  $\frac{\alpha_i}{1+\alpha_i}f_i$ . This is preferred by the proposer rather than not participating when his remaining share of the pie  $1 - \frac{\alpha_i}{1+\alpha_i}f_i > 0$ . That is, the proposer makes the minimal offer  $\frac{\alpha_i}{1+\alpha_i}f_i$  if and only if  $f_i < \frac{1+\alpha_i}{\alpha_i}$ . Since  $f_i \in [0, 1]$  and  $1 < \frac{1+\alpha_i}{\alpha_i}$  for any  $\alpha_i \ge 0$ , then the previous condition  $f_i < \frac{1+\alpha_i}{\alpha_i}f_i$  is satisfied for any  $\alpha_i \ge 0$ . Therefore, the proposer makes the minimal offer  $\frac{\alpha_i}{1+\alpha_i}f_i$  for any parameter values.

#### 5.1.11 Proof of Lemma 5

Let us consider the worker's utility function when he assigns a positive importance to the proposer's foregone options and when he does not. Respectively,  $U_i^C(s_i, s_j) = x_i - e^2 + \alpha_i(x_i - f_i)e$  and  $U_i^{NC}(s_i, s_j) = x_i - e^2$ . Therefore, assumption A1 is satisfied since  $U_i^{NC}(s_i, s'_j) \geq U_i^{NC}(s_i, s_j)$  for any  $s_i$  and any  $s'_j > s_j$  since  $\frac{\partial U_i^{NC}(s_i, s_j)}{\partial s_j} = 1 \geq 0$ . Additionally, A2 (concavity) is also satisfied since

$$\frac{\partial^2 U_i^C\left(s_i, s_j\right)}{\partial e^2} = \frac{\partial^2 U_i^C\left(s_i, s_j\right)}{\partial e^2} = -2 < 0$$

A3 is trivially satisfied by player i. Additionally, A4 (kindness) holds since

$$U_{i}^{C}(s_{i}, s_{j}) > U_{i}^{NC}(s_{i}, s_{j}) \text{ since } x_{i} - e^{2} + \alpha_{i}(x_{i} - f_{i})e > x_{i} - e^{2} \text{ for any } x_{i} > f_{i}$$
$$U_{i}^{C}(s_{i}, s_{j}) = U_{i}^{NC}(s_{i}, s_{j}) \text{ since } x_{i} - e^{2} + \alpha_{i}(x_{i} - f_{i})e = x_{i} - e^{2} \text{ for any } x_{i} = f_{i}$$
$$U_{i}^{C}(s_{i}, s_{j}) < U_{i}^{NC}(s_{i}, s_{j}) \text{ since } x_{i} - e^{2} + \alpha_{i}(x_{i} - f_{i})e < x_{i} - e^{2} \text{ for any } x_{i} < f_{i}$$

On the other hand, A5 (reciprocity) as well since  $s_i = e$  and  $\frac{\partial U_i^C(s_i,s_j)}{\partial e} = -2e + \alpha (x_i - f_i)$ and  $\frac{\partial U_i^{NC}(s_i,s_j)}{\partial e} = -2e$ , then

$$\frac{\partial U_{i}^{C}\left(s_{i},s_{j}\right)}{\partial e} \geq (<)\frac{\partial U_{i}^{NC}\left(s_{i},s_{j}\right)}{\partial e} \text{ if } D_{i}\left(s_{i},s_{j}\right) \geq (<) 0$$

# 5.1.12 Proof of Lemma 6

The worker's optimal amount of effort to exert as a function of the wage proposal offered by the firm,  $e(x_i)$ , can be obtained from solving the following utility maximization problem

$$\max_{e \in \mathbb{R}_+} U_i^C(s_i, s_j) = x_i - e^2 + \alpha_i (x_i - f_i(x_i)) e^{-\alpha_i (x_i - f_i(x_i))} e^$$

Differentiating with respect to e, and manipulating, we have

$$e(x_i) = \begin{cases} \frac{1}{2}\alpha_i \left(x_i - f_i(x_i)\right) & \text{if } x_i > f_i(x_i) \\ 0 & \text{otherwise} \end{cases}$$

For sufficiency, just note that the worker will never respond to an offer  $x_i$  by exerting a higher effort level than the one specified in  $e(x_i)$ . Indeed, on the one hand, if he exerts higher

effort levels, he will have more disutility from such effort than the utility he derives from the third term of the above utility function for  $x_i > f_i(x_i)$ . On the other hand, if he exerts less effort, then the marginal utility from exerting additional effort when  $x_i > f_i$  (third term of the utility function) would be greater than the marginal disutility from exerting effort (second term). Hence, the worker would be better off by exerting more effort. Hence, the above effort level  $e(x_i)$  is optimal for the worker when the wage offered is  $x_i$ .

#### 5.1.13 Proof of Proposition 5

As shown in the above lemma 2, the worker's optimal effort level is given by

$$e(x_i) = \max\left\{\frac{1}{2}\alpha_i\left(x_i - f_i(x_i)\right), 0\right\}$$

Regarding the employer offer, we know that the employer inserts the above best response function into his utility function, in order to find the optimal wage offer.  $\max_{x_i \in [0,1]} (v - x_i)e(x_i)$ . Hence,

$$x_i^* = \frac{v\left(1 - f_i'(x_i^*)\right) + f_i(x_i)}{2 - f_i'(x_i^*)} \in \arg\max\left(v - x_i\right)e(x_i)$$

Note that the employer prefers to offer  $x_i^* = \frac{v(1-f'_i(x_i^*))+f_i(x_i)}{2-f'_i(x_i^*)}$ , where  $x_i^* > f_i(x_i^*)$  since v > 1 and  $f'_i(x_i) < 1$ , and induce a positive effort level from the worker, rather than offering any wage level  $\hat{x}_i < f_i(\hat{x}_i)$  which induces no effort; see  $e(x_i)$ . Indeed, the employer's equilibrium utility level from offering  $x_i^*$  is  $V = (v - x_i^*) \frac{1}{2} \alpha_i (x_i^* - f_i(x_i^*))$ , which is positive for any parameter values. Instead, if the employer makes any offer  $\hat{x}_i < f_i(\hat{x}_i)$ , the worker exerts no effort, and  $U_F = 0$ . Hence,  $x_i^*$  is indeed the equilibrium wage offer.

Finally, in order to check for the worker's voluntary participation, we need to find what is the minimum offer to be accepted by the worker. That is, we must find a wage offer  $s_j = x_i$ such that  $U^C(s_i, s_j, S_j) = 0$ .

$$x_i - e(x_i)^2 + \alpha_i(x_i - f_i(x_i))e(x_i) = 0$$

$$\iff x_i - \left(\max\left\{\frac{1}{2}\alpha_i(x_i - f_i(x_i)), 0\right\}\right)^2 + \alpha_i(x_i - f_i)\max\left\{\frac{1}{2}\alpha_i(x_i - f_i(x_i)), 0\right\} = 0$$

In the case in which the foregone option  $f_i(x_i) > x_i$ , then the above expression is reduced to  $x_i = 0$ . That is, any wage offer is accepted. On the other hand, in the case in which  $f_i(x_i) < x_i$ , then, we can reduce the above expression to  $x_i = \frac{-2+\alpha_i^2 f_i(x_i)+2\sqrt{1-\alpha_i^2 f_i(x_i)}}{\alpha_i^2}$ , which is always negative, for any values of  $\alpha_i$  and  $f_i(x_i)$ . Therefore, the minimum offer to be accepted by the worker in both cases  $(x_i > f_i(x_i)$  and  $x_i < f_i(x_i))$  will be  $\bar{x}_i = 0$ , since we are assuming that the firm cannot make any negative offers. Note that in the case that  $f'_i(x_i) = 0$  then  $x_i^*$  becomes  $x_i^* = \frac{v+f_i(x_i)}{2}$ .

# 5.1.14 Proof of Lemma 7

Let us consider the responder's utility function when he assigns a positive importance to the proposer's foregone options and when he does not, respectively,

$$U_{i}^{C}(s_{i},s_{j}) = z_{i} + \left[m\left(s_{i}+s_{j}\right)\left[1+\alpha_{i}\left(s_{j}-s_{j}^{R}\right)\right]\right]^{0.5} \text{ and } U_{i}^{NC}(s_{i},s_{j}) = z_{i} + \left[m(s_{i}+s_{j})\right]^{0.5}$$

Therefore, assumption A1 is satisfied since  $U_i^{NC}(s_i, s'_j) \ge U_i^{NC}(s_i, s_j)$  for any  $s_i$  and  $s_j > s_j$  given that  $\frac{\partial U_i^{NC}(s_i, s_j)}{\partial s_j} = \frac{m}{2[m(s_i+s_j)]^{0.5}} > 0$  for any parameter values.

Similarly, A2 (concavity) is also satisfied since

$$\frac{\partial^2 U_i^C\left(s_i, s_j\right)}{\partial s_i^2} = -\frac{m^2}{4\left[m\left(s_i + s_j\right)\right]^{3/2}} \le 0$$

$$\frac{\partial^2 U_i^C(s_i, s_j)}{\partial s_i^2} = -\frac{m^2 (1 + \alpha_i (s_j - s_j^R))^2}{4 \left[ m(s_i + s_j) \left[ 1 + \alpha_i (x_i - s_j^R) \right] \right]^{3/2}} \le 0$$

A3 is trivially satisfied by player i. In addition, A4 (kindness) is satisfied given that

$$U_i^C(s_i, s_j) > U_i^{NC}(s_i, s_j) \text{ for any } s_j > s_j^R$$
$$U_i^C(s_i, s_j) = U_i^{NC}(s_i, s_j) \text{ for any } s_j = s_j^R$$
$$U_i^C(s_i, s_j) < U_i^{NC}(s_i, s_j) \text{ for any } s_j < s_j^R$$

On the other hand, A5 (reciprocity) holds as well since

$$\frac{\partial U_i^C\left(s_i, s_j\right)}{\partial s_i} = -1 + \frac{m(1 + \alpha_i(s_j - s_j^R))}{2\left[m(s_i + s_j)\left[1 + \alpha_i(s_j - s_j^R)\right]\right]^{0.5}} \text{ and}$$

$$\frac{\partial U_i^{NC}\left(s_i, s_j\right)}{\partial s_i} = -1 + \frac{m}{2\left[m(s_i + s_j)\right]^{0.5}}$$

then it is easy to show that

$$\frac{\partial U_i^C\left(s_i, s_j\right)}{\partial s_i} \ge (<) \frac{\partial U_i^{NC}\left(s_i, s_j\right)}{\partial s_i} \text{ if } s_j \ge (<) s_j^R$$

# 5.1.15 Proof of Lemma 8

The responder's utility maximization problem is just given by

$$\max_{z_{i},G} U_{i}^{C}(s_{i},s_{j}) = \max_{z_{i},G} z_{i} + \left[ mG \left[ 1 + \alpha_{i} \left( s_{j} - s_{j}^{R} \right) \right] \right]^{0.5}$$

subject to 
$$z_i + s_i = w_i$$
  
 $s_i + s_j = G$   
 $s_i, z_i \ge 0$ 

Using  $z_i = w_i - s_i$  and  $s_i + s_j = G$  in  $U_i^C(s_i, s_j)$ , we can reduce the above program to

$$\max_{s_i} U_i^C(s_i, s_j) = \max_{s_i} w_i - s_i + \left[ m\left(s_i + s_j\right) \left[ 1 + \alpha_i \left(s_j - s_j^R\right) \right] \right]^{0.5}$$

Differentiating with respect to  $s_i$ , and manipulating, we find the best response function for the second mover concerned about the first mover's foregone options.

$$s_i^C(s_j) = \begin{cases} \frac{m\left(1-\alpha_i s_j^R\right)}{4} - \left(1 + \frac{\alpha_i m}{4}\right) s_j \text{ if } s_j \in \left[0, \frac{m\left(1-\alpha_i s_j^R\right)}{4-\alpha_i m}\right) \\ 0 \text{ if } s_j \ge \frac{m\left(1-\alpha_i s_j^R\right)}{4-\alpha_i m} \end{cases}$$

# 5.1.16 Proof of Lemma 9

Regarding the first mover (player *i*), we know that he inserts the above best response function of the follower into his utility function,  $U_j^{NC}(s_j, s_i) = w - s_j + [m(s_i + s_j)]^{0.5}$  which is maximized at  $s_j^* = \frac{16(\alpha_i s_j^R - 1) + \alpha_i^2 m^2}{16\alpha_i}$ . However, this expression is positive only for sufficiently high values of  $\alpha_i$ . In particular,  $\frac{16(\alpha_i s_j^R - 1) + \alpha_i^2 m^2}{16\alpha_i} > 0$  if and only if  $\alpha_i > \frac{16}{16s_j^R + m} = \bar{\alpha}_i$ . Therefore,

$$s_j^* = \begin{cases} 0 & \text{if } \alpha_i < \bar{\alpha}_i \\ \frac{16(\alpha_i s_j^R - 1) + \alpha_i^2 m^2}{16\alpha_i} & \text{otherwise} \end{cases}$$

#### 5.2 APPENDIX TO CHAPTER 2

#### 5.2.1 Proof of Lemma 1

I first show that player *i*'s best response functions when she is concerned about distance  $D_i(\cdot)$ and when she is not, respectively,  $s_i^C(s_j) \in \underset{s_i}{\operatorname{arg max}} U_i^C(s_i, s_j)$ , and  $s_i^{NC}(s_j) \in \underset{s_i}{\operatorname{arg max}} U_i^{NC}(s_i, s_j)$ , contain a single point. Then, I show the result stated in lemma 1.

Note that player *i*'s utility function when she is concerned about  $D_i(\cdot)$ ,  $U_i^C(s_i, s_j)$ , is strictly concave in  $s_i$  and it is defined over a strictly convex domain  $S_i \times S_j$ . This guarantees that player *i*'s best response function

$$s_i^C(s_j) \in \underset{s_i}{\operatorname{arg\,max}} U_i^C(s_i, s_j)$$

contains a single point. A similar argument is also applicable for player *i*'s utility function when she does not assign any relevance to distance  $D_i(\cdot)$ ,  $U_i^{NC}(s_i, s_j)$ , since it is also strictly concave in  $s_i$  and it is defined over a strictly convex domain  $S_i \times S_j$ . Hence,

$$s_i^{NC}(s_j) \in \underset{s_i}{\operatorname{arg\,max}} U_i^{NC}(s_i, s_j)$$

also contains a single point.

Next, I want to show that  $U_{s_i}^C(s_i, s_j) \ge U_{s_i}^{NC}(s_i, s_j)$  holds if and only if  $s_i^C(s_j) \ge s_i^{NC}(s_j)$ for all  $s_j$ . First, suppose by contradiction, that  $U_{s_i}^C(s_i, s_j) \ge U_{s_i}^{NC}(s_i, s_j)$  but  $s_i^C(s_j) < s_i^{NC}(s_j)$  for all  $s_i$  and  $s_j$ . Let us then take a linear combination  $\hat{s}_i(s_j)$  of these two best response functions,  $s_i^C(s_j)$  and  $s_i^{NC}(s_j)$ , such that

$$\hat{s}_i(s_j) = \theta s_i^C(s_j) + (1 - \theta) s_i^{NC}(s_j)$$
 for all  $s_j$ , where  $\theta \in (0, 1)$ 

When  $U_{s_i}^{NC}(s_i, s_j)$  is evaluated at  $\hat{s}_i(s_j)$ , we must have  $U_{s_i}^{NC}(\hat{s}_i(s_j), s_j) > 0$ . However, if  $s_i^C(s_j) < s_i^{NC}(s_j)$ , then  $U_{s_i}^C(\hat{s}_i(s_j), s_j) < 0$ . Therefore,

 $U_{s_i}^C(\hat{s}_i(s_j), s_j) < U_{s_i}^{NC}(\hat{s}_i, s_j)$ , which is a contradiction.

Hence, if  $U_{s_i}^C(s_i, s_j) \ge U_{s_i}^{NC}(s_i, s_j)$  for all  $s_i$  and  $s_j$  then  $s_i^C(s_j) \ge s_i^{NC}(s_j)$  for all  $s_i$  and  $s_j$ . Let us next show that if  $s_i^C(s_j) \ge s_i^{NC}(s_j)$  for all  $s_i$  and  $s_j$ , then  $U_{s_i}^C(s_i, s_j) \ge U_{s_i}^{NC}(s_i, s_j)$  for all  $s_i$  and  $s_j$ . Suppose by contradiction that  $s_i^C(s_j) \ge s_i^{NC}(s_j)$  for all  $s_j$ , but  $U_{s_i}^C(s_i, s_j) < U_{s_i}^{NC}(s_i, s_j)$  for some  $s_i$  and  $s_j$ . Then,  $s_i^C(s_j) < s_i^{NC}(s_j)$  would hold for some  $s_i$  and  $s_j$ , which is a contradiction. Thus,  $U_{s_i}^C(s_i, s_j) \ge U_{s_i}^{NC}(s_i, s_j)$  holds if and only if  $s_i^C(s_j) \ge s_i^{NC}(s_j)$  for all  $s_j$ .

Applying this condition to player *i*'s utility function, we have  $U_{s_i}^{NC}(s_i, s_j) = U_{s_i}$  and  $U_{s_i}^C(s_i, s_j) = U_{s_i}^C(U_i^{NC}, D_i)$ . Hence,  $U_{s_i}^C(s_i, s_j) = U_{s_i} + U_{D_i}D_{s_i}$ . Thus,  $U_{s_i}^C(s_i, s_j) \ge U_{s_i}^{NC}(s_i, s_j)$  in this context means  $U_{s_i} + U_{D_i}D_{s_i} \ge U_{s_i}$ , which reduces to  $U_{D_i}D_{s_i} \ge 0$ . Finally, since  $U_{D_i} \ge 0$  given that positive distances increase players' utility level (kindness assumption), condition  $U_{D_i}D_{s_i} \ge 0$  can be reduced to  $D_{s_i} \ge 0$ . Hence,  $s_i^C(s_j) \ge s_i^{NC}(s_j)$  is satisfied for all  $s_j$  if and only if condition  $D_{s_i} \ge 0$  holds for all  $s_i$  and  $s_j$ .

#### 5.2.2 Proof of Lemma 2

Let us first find the slope of player *i*'s best response function in the standard game without concerns about distances. Applying the implicit function theorem, we have  $\frac{\partial s_i^{NC}(s_j)}{\partial s_j} = -\frac{U_{s_is_j}^{NC}}{U_{s_is_i}^{NC}}$ . Let us now compare it with the slope of player *i*'s best response function when player *i* is concerned about distances. Applying the implicit function theorem again,  $\frac{\partial s_i^C(s_j)}{\partial s_j} = -\frac{U_{s_is_j}^{SC}}{U_{s_is_i}^C}$ .

Comparing the absolute value of both slopes,  $\frac{\partial s_i^C(s_j)}{\partial s_j} > \frac{\partial s_i^{NC}(s_j)}{\partial s_j}$  holds if and only if  $\Delta_i = \frac{U_{s_is_j}^C}{U_{s_is_i}^C} - \frac{U_{s_is_j}^{NC}}{U_{s_is_i}^{NC}} > 0.$ 

### 5.2.3 Proof of Proposition 1

From Lemma 1 we know that if  $D_{s_i} \ge 0$  holds for all  $s_j$ , then  $s_i^C(s_j) > s_i^{NC}(s_j)$  for all  $s_j$ .

Now we want to show that if  $s_i^C(s_j) \ge s_i^{NC}(s_j)$  for all  $s_j$ , then  $s_i^C \ge s_i^{NC}$ . Suppose by contradiction that  $s_i^C(s_j) \ge s_i^{NC}(s_j)$  for all  $s_j$ , but  $s_i^C < s_i^{NC}$ . Since this counterpositive statement must be true for any slopes of player *i* and *j*'s best response functions, it must also be true when  $s_i^C(s_j)$  and  $s_i^{NC}(s_j)$  are both negatively sloped, and when player *j* is not concerned about distances, i.e.,  $s_j^C(s_j) = s_j^{NC}(s_j)$ . In this case, if  $s_i^C < s_i^{NC}$  then, either

1. 
$$s_i^C(s_j) < s_i^{NC}(s_j)$$
 for all  $s_j$ , and  $\left|\frac{\partial s_i^K(s_j)}{\partial s_j}\right| < 1$  for all  $i \neq j$  and  $K = \{C, NC\}$ , or  
2.  $s_i^C(s_j) \ge s_i^{NC}(s_j)$  for all  $s_j$ , and  $\left|\frac{\partial s_i^K(s_j)}{\partial s_j}\right| > 1$  for all  $i \neq j$  and  $K = \{C, NC\}$ ,

which are both a contradiction. Thus, if  $s_i^C(s_j) \ge s_i^{NC}(s_j)$  for all  $s_j$ , then  $s_i^C \ge s_i^{NC}$ .

#### 5.2.4 Proof of Proposition 2

Let us first find an useful result about player *i*'s best response functions when evaluated at  $s_j = s_j^{NC}$ .

**Lemma A.** If assumptions 1 and 2 are satisfied, then for every player  $i = \{1, 2\}$ ,

$$\left[\Delta_i \times D_i^{NC}\right] \times \left[s_i^C(s_j^{NC}) - s_i^{NC}(s_j^{NC})\right] > 0$$

Proof of Lemma A:

We want to show that if  $\Delta_i \times D_i^{NC} > 0$  then  $s_i^C(s_j^{NC}) > s_i^{NC}(s_j^{NC})$ . Notice that:

- 1. If  $\Delta_i < 0$  and  $D_i^{NC} < 0$ , then  $s_i^C(s_j)$  rotates clockwise and  $s_j^{NC} < \bar{s}_j$ . Then,  $s_i^C(s_j) > s_i^{NC}(s_j)$  for all  $s_j < \bar{s}_j$ , including  $s_j^{NC}$  (see figure 1a).
- 2. If  $\Delta_i > 0$  and  $D_i^{NC} > 0$ , then  $s_i^C(s_j)$  rotates anticlockwise and  $s_j^{NC} > \bar{s}_j$ , as in figure 1b Then,  $s_i^C(s_j) > s_i^{NC}(s_j)$  for all  $s_j > \bar{s}_j$ , including  $s_j^{NC}$ .

Therefore,  $\left[\Delta_i \times D_i^{NC}\right] \times \left[s_i^C(s_j^{NC}) - s_i^{NC}(s_j^{NC})\right] > 0.$ 

Thus, lemma A specifies a ranking for player *i*'s best response functions when evaluated at  $s_j = s_j^{NC}$ . In particular, it determines that  $s_i^C(s_j^{NC}) > s_i^{NC}(s_j^{NC})$  if either: (1) player *i* is a compensator using a relatively demanding distance function,  $\Delta_i < 0$  and  $D_i^{NC} < 0$ ; or if (2) player *i* is a reciprocator using a not-demanding distance function,  $\Delta_i > 0$  and  $D_i^{NC} > 0$ . With this interesting result, we can now prove Proposition 2.

#### First result

From the above Lemma A we know that

$$\Delta_i \times D_i^{NC} > 0 \Longrightarrow s_i^C(s_j^{NC}) > s_i^{NC}(s_j^{NC}) = s_i^{NC}$$

Let us now show that, for a given player j's best response function,  $s_j^C(s_i) = s_j^{NC}(s_i)$ ,

$$s_i^C(s_j^{NC}) > s_i^{NC} \Longrightarrow s_i^C > s_i^{NC}$$

In order to show the above result, assume by contradiction that for a given  $s_j^{NC}(s_i)$ ,  $s_i^C(s_j^{NC}) > s_i^{NC}$  implies  $s_i^C < s_i^{NC}$ . First, take two negatively sloped best response functions, and assume  $s_i^C < s_i^{NC}$ . Then, when evaluated at  $s_j = s_j^{NC}$ , player *i*'s best response function must satisfy  $s_i^C(s_j^{NC}) < s_i^{NC}(s_j^{NC}) = s_i^{NC}$ , which is a contradiction. Thus, if  $s_i^C(s_j^{NC}) > s_i^{NC}$ then  $s_i^C > s_i^{NC}$ . Therefore, for a given  $s_j^{NC}(s_i)$ , and using Lemma A we have that

$$\Delta_i \times D_i^{NC} > 0 \Longrightarrow s_i^C(s_j^{NC}) > s_i^{NC} \Longrightarrow s_i^C > s_i^{NC}$$

#### Second result

From Lemma A we know that

$$\Delta_j \times D_j^{NC} < 0 \Longrightarrow s_j^C(s_i^{NC}) < s_j^{NC}(s_i^{NC})$$

Then, we now want to show that if  $s_j^C(s_i^{NC}) < s_j^{NC}(s_i^{NC})$  is satisfied, then

$$s_i^C > s_i^{NC}$$
 when  $s_i^{NC}(s_j)$  is negatively sloped

and  $s_i^C < s_i^{NC}$  otherwise. Then, assume that  $s_j^C(s_i^{NC}) < s_j^{NC}(s_i^{NC})$  and that  $s_j^C(s_i)$  is negatively sloped. Therefore,  $s_j^C(s_i) < s_j^{NC}(s_i)$  for all  $s_i > \bar{s}_i$ , including  $s_i^{NC}$ . Since, in addition,  $s_j^C(s_i)$  must cross  $s_i^{NC}(s_j)$  from below by assumption 3, then  $s_i^C > s_i^{NC}$ . Similarly, when  $\Delta_j \times D_j^{NC} > 0$  holds and players' best response functions have positive slope,  $\frac{U_{s_is_j}^C}{U_{s_is_i}^C} < 0$ , then we have an analog reasoning,

$$\Delta_j \times D_j^{NC} > 0 \Longrightarrow s_j^C(s_i^{NC}) > s_j^{NC}(s_i^{NC})$$
 from Lemma A

Therefore,  $s_j^C(s_i^{NC}) > s_j^{NC}(s_i^{NC})$  for all  $s_i < \bar{s}_i$ , including  $s_i^{NC}$ . Finally, assumption 2 for the context of positively sloped best response functions implies that  $s_j^C(s_i)$  must cross  $s_i^{NC}(s_j)$  from above. Hence,  $s_i^C > s_i^{NC}$ .

#### 5.2.5 Proof of Proposition 3

From Lemma A above we know that

$$\Delta \times D^{NC} > 0 \Longrightarrow s_i^C(s_j^{NC}) > s_i^{NC}(s_j^{NC}) \text{ for all } i \neq j$$

Now we want to show that,

$$s_i^C(s_j^{NC}) > s_i^{NC}$$
 for all  $i \neq j \Longrightarrow s_i^C > s_i^{NC}$  for all  $i = \{1, 2\}$ 

In order to show the above claim, assume by contradiction that  $s_i^C(s_j^{NC}) > s_i^{NC}(s_j^{NC})$ holds for all  $i \neq j$  but  $s_i^C < s_i^{NC}$  for at least some *i*. For simplicity, let us take two best response functions with negative slopes, and consider that  $s_i^C < s_i^{NC}$ . Then, when evaluated at  $s_j = s_j^{NC}$ ,  $s_i^C(s_j^{NC})$  must be below  $s_i^{NC}(s_j^{NC})$ . Applying the same reasoning to player *j*, we conclude that

$$s_i^C(s_j^{NC}) < s_i^{NC}(s_j^{NC}) \text{ and } s_j^C(s_i^{NC}) < s_j^{NC}(s_i^{NC})$$

which is a contradiction. Therefore, if  $s_i^C(s_j^{NC}) > s_i^{NC}(s_j^{NC})$  for all  $i \neq j$ , then  $s_i^C > s_i^{NC}$  for all  $i = \{1, 2\}$ . Thus, using Lemma A we have that

$$\Delta \times D^{NC} > 0 \Longrightarrow s^C_i(s^{NC}_j) > s^{NC}_i \Longrightarrow s^C_i > s^{NC}_i$$

# 5.2.6 Proof of Proposition 4

Using Segal and Sobel (1999), we know that the second mover's preferences over the first mover's actions can be represented by

$$U_{s_{i}}^{C}\left(s_{i},s_{j}\right)=\gamma_{i}U_{i}^{NC}\left(s_{i},s_{j}\right)+\gamma_{j}U_{j}^{NC}\left(s_{i},s_{j}\right) \text{ where } \gamma_{i},\gamma_{j}\in\mathbb{R}$$

if preferences satisfy continuity and independence, as well as Segal and Sobel's (1999) condition  $(\bigstar)$  which states that

if 
$$U_i^{NC}(s_i', s_j) = U_i^{NC}(s_i, s_j)$$
, then  $s_i' \sim_i s_i$   $(\bigstar)$ 

which are all satisfied in our model.

#### 5.2.7 Proof of Proposition 5

Both players are asked to simultaneously submit their voluntary contributions to the public good. Fixing subject j's contribution,  $s_j$ , we have that

$$s_i(s_j) = \begin{cases} \frac{\alpha + m}{1 + \alpha + m} w & \text{if } s_j = 0\\ \frac{\alpha + m}{1 + \alpha + m} w + \frac{\alpha - m}{(\alpha + m)(1 + \alpha + m)} s_j & \text{if } s_j \in \left(0, \frac{(\alpha + m)^2}{m - \alpha}\right]\\ 0 & \text{if } s_j \in \left(\frac{(\alpha + m)^2}{m - \alpha}, +\infty\right) \end{cases}$$

if  $\alpha < m$ . In contrast, when  $\alpha > m s_i(s_j)$  does not become zero or negative for any value of  $s_j$ . The corresponding best response function for player *i* in this case is

$$s_i(s_j) = \begin{cases} \frac{\alpha + m}{1 + \alpha + m} w & \text{if } s_j = 0\\ \frac{\alpha + m}{1 + \alpha + m} w + \frac{\alpha - m}{(\alpha + m)(1 + \alpha + m)} s_j & \text{if } s_j > 0 \end{cases}$$

Regarding the equilibrium contributions, note that symmetry eliminates corner solutions in this case. Hence, the only equilibrium contribution is that resulting from the crossing point of player *i*'s and *j*'s best response functions (interior solution). Solving for  $s_i$  and  $s_j$ in a system of two equations, we obtain  $s_i^C = \frac{(\alpha+m)^2 w}{2m+(\alpha+m)^2}$ , as the interior Nash equilibrium contribution level.

Finally, if both players are equally *not* concerned about status,  $\alpha_i = \alpha_j = 0$ , we obtain the interior solution in standard public good games, where every player i's Nash equilibrium contribution level is given by  $s_i^{NC} = \frac{mw}{2+m}$ .

### 5.2.8 Proof of Corollary 1

Recall that player *i*'s equilibrium contribution in the model without status acquisition is  $s_i^{NC} = \frac{mw}{2+m}$ . Comparing it with the equilibrium contribution level in the model with status considerations,  $s_i^C = \frac{(\alpha+m)^2w}{2m+(\alpha+m)^2}$ ,

$$s_{i}^{C} - s_{i}^{NC} = \frac{(\alpha + m)^{2} w}{2m + (\alpha + m)^{2}} - \frac{mw}{2 + m} = \frac{2\alpha(\alpha + 2m)w}{(2 + m)\left[2m + (\alpha + m)^{2}\right]}$$

which is positive for any  $\alpha > 0$ , reflecting that  $s_i^C > s_i^{NC}$ .
#### 5.2.9 Proof of Proposition 6

In this public good game, both players are asked to simultaneously submit their contributions. Fixing player j's contribution,  $s_j$ , player i's utility maximization problem becomes

$$\max_{s_i} [w - s_i]^{0.5} + \left[ m(s_i + s_j) + \alpha \left( s_j - s_j^{ref} \right) \right]^{0.5}$$

And the argument that maximizes this utility function gives us

$$s_{i}^{C}(s_{j}) = \begin{cases} \frac{\alpha s_{j}^{ref} + m^{2}w}{m(1+m)} & \text{if } s_{j} = 0\\ \frac{\alpha s_{j}^{ref} + m^{2}w}{m(1+m)} - \frac{\alpha + m}{m(1+m)}s_{j} & \text{if } s_{j} \in \left(0, \frac{\alpha s_{j}^{ref} + m^{2}w}{\alpha + m(2+m)}\right]\\ 0 & \text{if } s_{j} > \frac{\alpha s_{j}^{ref} + m^{2}w}{\alpha + m(2+m)} \end{cases}$$

Since  $\frac{\alpha s_j^{ref} + m^2 w}{m(1+m)} - \frac{\alpha + m}{m(1+m)} s_j = 0$  at exactly  $s_j = \frac{\alpha s_j^{ref} + m^2 w}{\alpha + m(2+m)}$ . Hence,  $s_i^C(s_j)$  becomes

$$s_{i}^{C}(s_{j}) = \begin{cases} \frac{\alpha s_{j}^{ref} + m^{2}w}{m(1+m)} - \frac{\alpha + m}{m(1+m)}s_{j} \text{ if } s_{j} \in \left(0, \frac{\alpha s_{j}^{ref} + m^{2}w}{\alpha + m(2+m)}\right) \\ 0 \text{ if } s_{j} > \frac{\alpha s_{j}^{ref} + m^{2}w}{\alpha + m(2+m)} \end{cases}$$

## 5.2.10 Proof of Proposition 7

By symmetry, player i and j's best response functions can only cross each other at interior points. Therefore, there must be a unique and interior Nash equilibrium contribution level for every player, which we can obtain by plugging player j's best response function into player i's. In particular,

$$s_i^C = \frac{\alpha s_j^{ref} + m^2 w}{m(1+m)} - \frac{\alpha + m}{m(1+m)} \left(\frac{\alpha s_j^{ref} + m^2 w}{m(1+m)} - \frac{\alpha + m}{m(1+m)} s_i^C\right)$$

Solving for  $s_i^C$ , we have  $s_i^C = \frac{\alpha s_j^{ref} + m^2 w}{\alpha + m(2+m)}$ .

#### 5.2.11 Proof of Corollary 2

Recall that player *i*'s equilibrium contribution in the model where players do not assign value to distances is  $s_i^{NC} = \frac{mw}{2+m}$ . On the other hand, by comparing the equilibrium contribution level when distances are considered,  $s_i^C$ , obtained in the above proposition 7 with respect to  $s_i^{NC}$ ,

$$s_{i}^{C} - s_{i}^{NC} = \frac{\alpha s_{j}^{ref} + m^{2}w}{\alpha + m(2+m)} - \frac{mw}{2+m} = \frac{\alpha \left[s_{j}^{ref} \left(2+m-mw\right)\right]}{\left(2+m\right) \left[\alpha + m\left(2+m\right)\right]}$$

and this difference is positive if and only if  $s_j^{NC} = \frac{mw}{2+m} < s_j^{ref}$ . Hence,  $s_i^C > s_i^{NC}$  if and only if  $D_i^{NC} \equiv \alpha \left( s_j^{NC} - s_j^{ref} \right) < 0$ , which is satisfied for any  $s_j^{ref}$  such that  $s_j^{NC} = \frac{mw}{2+m} < s_j^{ref}$ 

Otherwise, if  $s_j^{NC} = \frac{mw}{2+m} > s_j^{ref}$  holds, then this difference is negative. However, if  $\frac{mw}{2+m} > s_j^{ref}$  and  $\alpha < 0$  are simultaneously satisfied, then this difference is positive, and  $s_i^C > s_i^{NC}$ .

#### 5.3 APPENDIX TO CHAPTER 3

#### 5.3.1 Proof of Lemma 1

Both players are asked to simultaneously submit their voluntary contributions to the public good. Fixing subject j's contribution,  $g_j$ , we have that

$$g_i(g_j) = \begin{cases} 1 & \text{if } g_j = 0\\ 1 + \frac{\alpha_i - m}{\alpha_i + m} g_j & \text{if } g_j \in \left(0, \frac{m + \alpha_i}{m - \alpha_i}\right]\\ 0 & \text{if } g_j \in \left(\frac{m + \alpha_i}{m - \alpha_i}, +\infty\right) \end{cases}$$

if  $\alpha_i < m$ . Note that

$$0 \ge 1 + \frac{\alpha_i - m}{\alpha_i + m} g_j \Longleftrightarrow g_j \ge \frac{m + \alpha_i}{m - \alpha_i}$$

and this threshold is positive if  $\alpha_i < m$ , see figure 1(a). In contrast, when  $\alpha_i > m$  this threshold is never binding for any positive  $g_j$ , i.e.,  $g_i$  does not become zero or negative for any positive value of  $g_j$ , see figure 1(b). The corresponding best response function for player i in this case is

$$g_i(g_j) = \begin{cases} 1 \text{ if } g_j = 0\\ 1 + \frac{\alpha_i - m}{\alpha_i + m} g_j \text{ if } g_j > 0 \end{cases}$$

#### 5.3.2 Proof of Lemma 2

Differentiating  $g_i(g_j)$  with respect to  $\alpha_i$ , we have

$$\frac{2mg_j}{\left[\alpha_i\left(g_i-g_j\right)+m\left(g_i+g_j\right)\right]^2}$$

which is strictly positive, for any parameter values.

#### 5.3.3 Proof of Proposition 1

First, take a given player *i*'s best response function,  $g_i(g_j)$ . Then,  $g_i^{Sm} = 1$  only when: (1) the slope of player *j*'s best response function,  $g_j(g_i)$ , is smaller than -1, and (2) the horizontal intercept of player *i*'s best response function,  $g_i(g_j)$ , is higher than 1. Otherwise, both players' best response functions would cross each other in an interior point. The intuition of these two conditions is represented in the following figure.



Figure 28: Both players' best response functions

That is,  $g_i^{Sm} = 1$  if and only if

$$\frac{\alpha_j - m}{\alpha_j + m} \le -1 \iff \alpha_j \le 0$$
, and  $\frac{m + \alpha_i}{m - \alpha_i} \ge 1 \iff \alpha_i > 0$ 

Since  $\alpha_i, \alpha_j \geq 0$ , the above conditions on player *i* and *j*'s concerns about status are  $\alpha_i \geq 0$  and  $\alpha_j = 0$ . Hence,  $g_i^{Sm} = 1$  if and only if  $\alpha_i \geq 0$  and  $\alpha_j = 0$ .

Secondly,  $g_i^{Sm} = 0$  only when the opposite happens. That is, when  $\alpha_i = 0$  and  $\alpha_j > 0$ .

Finally, when none of the above cases is satisfied, i.e., when  $\alpha_i > 0$  and  $\alpha_j > 0$ , then we have an interior solution. Solving for  $g_i$  and  $g_j$  in a system of two equations, we obtain  $g_i^{Sm} = \frac{\alpha_i(\alpha_j + m)}{(\alpha_i + \alpha_j)m}$ , as the interior Nash equilibrium contribution level.

#### Sufficiency

Let us now check that the second order conditions of incentive compatibility are satisfied. Suppose all but player *i* submit a contribution to the public good according to the above equilibrium prediction. I next show that, for any  $\alpha_i$ , contributor *i* maximizes his utility by following  $g_i^{Sm}$ . Let

$$U(g,\alpha_i) = w - g_i + \ln\left[m(g_i + g_j^{Sm}) + \alpha_i\left(g_i - g_j^{Sm}\right)\right]$$

be the utility level of player *i* when contributing *g* to the public good, and having a concern  $\alpha_i$  about status acquisition. We must now show that the derivative  $U_g(g, \alpha_i) \ge 0$  for all  $g < g_i^{Sm}$ , and  $U_g(g, \alpha_i) \le 0$  for all  $g > g_i^{Sm}$ , which imply that  $U(g, \alpha_i)$  is indeed maximized at exactly  $g = g_i^{Sm}$ .

Differentiating  $U(g, \alpha_i)$  with respect to g,

$$U_g\left(g,\alpha_i\right) = -1 + \frac{\alpha_i + m}{\alpha_i \left(g - g_j^{Sm}\right) + m(g + g_j^{Sm})}$$

Let us now suppose that  $g < g_i^{Sm}(\alpha_i)$ , and denote  $\tilde{\alpha}_i$  to be the concern about status for which the equilibrium contribution is exactly g, i.e.,  $g_i^{Sm}(\tilde{\alpha}_i) = g$ . Since  $g_i^{Sm}(\alpha_i)$  is strictly increasing in  $\alpha_i$  (as one can check from the suggested equilibrium contribution  $g_i^{Sm}$ , and confirmed in lemma 4) this implies that  $g_i^{Sm}(\alpha_i) > g_i^{Sm}(\tilde{\alpha}_i)$  if and only  $\alpha_i > \tilde{\alpha}_i$ . Then,  $U_g(g, \tilde{\alpha}_i) \leq U_g(g, \alpha_i)$ . Since by definition,  $g_i^{Sm}(\tilde{\alpha}_i) = g$ , it implies that  $U_g(g, \tilde{\alpha}_i) = 0$ . Hence,  $U_g(g, \alpha_i) \geq 0$  for all  $g < g_i^{Sm}$ . By a similar argument,  $U_g(g, \alpha_i) \leq 0$  for all  $g > g_i^{Sm}$ . Therefore,  $U(g, \alpha_i)$  is maximized at  $g = g_i^{Sm}$ .

# 5.3.4 Proof of Lemma 3

Differentiating  $g_i^{Sm}$  with respect to  $\alpha_i$ , we obtain

$$\frac{\partial g_i^{Sm}}{\partial \alpha_i} = \begin{cases} 0 \text{ if } \alpha_i > 0 \text{ and } \alpha_j = 0\\ \frac{\alpha_j (\alpha_j + m)}{(\alpha_i + \alpha_j)^2 m} \text{ if } \alpha_i > 0 \text{ and } \alpha_j > 0\\ 0 \text{ if } \alpha_i = 0 \text{ and } \alpha_j > 0 \end{cases}$$

which is weakly positive for any parameter values. On the other hand, differentiating  $g_i^{Sm}$  with respect to  $\alpha_j$ , we obtain

$$\frac{\partial g_i^{Sm}}{\partial \alpha_j} = \begin{cases} 0 \text{ if } \alpha_i > 0 \text{ and } \alpha_j = 0\\ \frac{\alpha_i (\alpha_i + m)}{(\alpha_i + \alpha_j)^2 m} \text{ if } \alpha_i > 0 \text{ and } \alpha_j > 0\\ 0 \text{ if } \alpha_i = 0 \text{ and } \alpha_j > 0 \end{cases}$$

which is weakly positive for any parameter values. Finally, differentiating  $g_i^{Sm}$  with respect to  $\alpha_j$ , we obtain

$$\frac{\partial g_i^{Sm}}{\partial \alpha_j} = \begin{cases} 0 \text{ if } \alpha_i > 0 \text{ and } \alpha_j = 0\\ -\frac{\alpha_i \alpha_i}{(\alpha_i + \alpha_j)m^2} \text{ if } \alpha_i > 0 \text{ and } \alpha_j > 0\\ 0 \text{ if } \alpha_i = 0 \text{ and } \alpha_j > 0 \end{cases}$$

which is weakly negative for any parameter values.

# 5.3.5 Proof of Lemma 4

If  $\alpha_i > 0$  and  $\alpha_j = 0$ , then from proposition 1 we know that  $g_i^{Sm} = 1$  and  $g_j^{Sm} = 0$ . Hence,  $G^{Sm} = 1$ . If, on the contrary,  $\alpha_i = 0$  and  $\alpha_j \ge 0$ , then from proposition 1 we also know that  $g_i^{Sm} = 0$  and  $g_j^{Sm} = 1$ . Hence,  $G^{Sm} = 1$  as well. Finally, if  $\alpha_i > 0$  and  $\alpha_j = 0$ , then  $g_i^{Sm} = \frac{\alpha_i(\alpha_j + m)}{(\alpha_i + \alpha_j)m}$ , and similarly for player j, what leads to  $G^{Sm} = 1 + \frac{2\alpha_i\alpha_j}{(\alpha_i + \alpha_j)m}$ .

## 5.3.6 Proof of Proposition 2

Using the second mover's best response function,  $g_j(g_i)$ , from lemma 1, we know that if  $\alpha_j < m$ ,

$$g_j(g_i) = \begin{cases} 1 & \text{if } g_i = 0\\ 1 + \frac{\alpha_j - m}{\alpha_j + m} g_i & \text{if } g_i \in \left(0, \frac{m + \alpha_j}{m - \alpha_j}\right]\\ 0 & \text{if } g_j \in \left(\frac{m + \alpha_j}{m - \alpha_j}, +\infty\right) \end{cases}$$

And if  $\alpha_j > m$ ,

$$g_j(g_i) = \begin{cases} 1 & \text{if } g_i = 0\\ 1 + \frac{\alpha_j - m}{\alpha_j + m} g_i & \text{if } g_i \in (0, +\infty) \end{cases}$$

Regarding player i, we know that he inserts the above best response function of the follower into his utility function,

$$U_{i} = w - g_{i} + \ln \left[ m(g_{i} + g_{j}(g_{i})) + \alpha_{i} \left( g_{i} - g_{j}(g_{i}) \right) \right]$$

which is maximized at

$$g_i^{Seq} = \begin{cases} 0 \text{ if } \alpha_i \in [0, \bar{\alpha}_i] \\ \frac{\alpha_i \alpha_j + 3\alpha_i m + \alpha_j m - m^2}{2m(\alpha_i + \alpha_j)} \text{ if } \alpha_i \in (\bar{\alpha}_i, +\infty) \end{cases}$$

where  $\bar{\alpha}_i = \frac{m(m-\alpha_j)}{3m+\alpha_j}$ 

Given the above contribution of the first donor and  $g_j(g_i)$  specified above, player j submits an equilibrium contribution of

$$g_j^{Seq} = \begin{cases} 1 & \text{if } \alpha_i \in [0, \bar{\alpha}_i) \\ \frac{1}{2} \left( \frac{\alpha_i \alpha_j}{(\alpha_i + \alpha_j)m} + \frac{m}{\alpha_i + \alpha_j} + \frac{4\alpha_j}{\alpha_j + m} - 1 \right) & \text{if } \alpha_i \in [\bar{\alpha}_i, \hat{\alpha}_i) \\ 0 & \text{if } \alpha_i \in [\hat{\alpha}_i, +\infty) \end{cases}$$

if  $\alpha_j < m$ . Clearly, note that when player j's best response function is negative, i.e.,  $\alpha_j < m$ , player j submits no positive contribution if  $1 - \frac{\alpha_i - m}{\alpha_i + m} g_j \ge \frac{m + \alpha_j}{m - \alpha_j}$ , or in equilibrium, when  $\alpha_i \ge \hat{\alpha}_i$ , where  $\hat{\alpha}_i = \frac{m(3\alpha_j^2 + m^2)}{-\alpha_j^2 - 4\alpha_j m + m^2}$ 

On the other hand, if player j's best response function is positive,  $\alpha_j > m$ , player j submits

$$g_j^{Seq} = \begin{cases} 1 & \text{if } \alpha_i \in [0, \bar{\alpha}_i) \\ \frac{1}{2} \left( \frac{\alpha_i \alpha_j}{(\alpha_i + \alpha_j)m} + \frac{m}{\alpha_i + \alpha_j} + \frac{4\alpha_j}{\alpha_j + m} - 1 \right) & \text{if } \alpha_i \in [\bar{\alpha}_i, +\infty) \end{cases}$$

Clearly, the above two expressions for  $g_{j}^{Seq}$  can be simplified to

$$g_{j}^{Seq} = \begin{cases} 1 & \text{if } \alpha_{i} \in [0, \bar{\alpha}_{i}); \\ \frac{1}{2} \left( \frac{\alpha_{i} \alpha_{j}}{(\alpha_{i} + \alpha_{j})m} + \frac{m}{\alpha_{i} + \alpha_{j}} + \frac{4\alpha_{j}}{\alpha_{j} + m} - 1 \right) & \text{if } \alpha_{j} < m \text{ and } \alpha_{i} \in [\bar{\alpha}_{i}, \hat{\alpha}_{i}), \\ & \text{or if } \alpha_{j} > m \text{ and } \alpha_{i} \in [\bar{\alpha}_{i}, +\infty); \\ & 0 & \text{if } \alpha_{j} < m \text{ and } \alpha_{i} \in [\hat{\alpha}_{i}, +\infty) \end{cases}$$

## 5.3.7 Proof of Corollary 1

First result: From proposition 2, we know that player *i* submits strictly positive contributions if and only if  $\alpha_i > \frac{m(m-\alpha_j)}{3m+\alpha_j}$ . Then, if  $\alpha_i = 0$ , the former condition can only be satisfied if

$$0 > \frac{m\left(m - \alpha_j\right)}{3m + \alpha_j} \Longleftrightarrow \alpha_j > m$$

Second result: Since  $\bar{\alpha} = \frac{m(m-\alpha_j)}{3m+\alpha_j} < m$ , for any  $\alpha_j \ge 0$ , then if  $m < \alpha_i$  we must have  $\bar{\alpha} < m < \alpha_i$  for any  $\alpha_j \ge 0$ . Therefore,  $\bar{\alpha} < \alpha_i$ , and player *i* submits a strictly positive contribution for any concern about status player *j* may have,  $\alpha_j \ge 0$ .

## 5.3.8 Proof of Lemma 5

Differentiating  $g_i^{Seq}$  with respect to  $\alpha_i$ , we obtain

$$\frac{\partial g_i^{Seq}}{\partial \alpha_i} = \begin{cases} 0 \text{ if } \alpha_i \in [0, \bar{\alpha}_i] \\ \frac{(\alpha_j + m)^2}{2(\alpha_i + \alpha_j)^2 m} \text{ if } \alpha_i > \bar{\alpha}_i \end{cases}$$

which is weakly positive for any parameter values. On the other hand, differentiating  $g_i^{Seq}$  with respect to  $\alpha_j$ , we obtain  $\frac{(\alpha_i - m)^2}{2(\alpha_i + \alpha_j)^2 m}$ 

$$\frac{\partial g_i^{Seq}}{\partial \alpha_j} = \begin{cases} 0 \text{ if } \alpha_i \in [0, \bar{\alpha}_i] \\ \frac{(\alpha_i - m)^2}{2(\alpha_i + \alpha_j)^2 m} \text{ if } \alpha_i > \bar{\alpha}_i \end{cases}$$

which is weakly positive for any parameter values.

## 5.3.9 Proof of Lemma 6

We must find the set of parameter values under which player i's contribution is above that of player j in this sequential game. Specifically, manipulating both expressions, we find that

$$g_i^{Seq} \ge g_j^{Seq} \iff \alpha_i \ge \frac{\alpha_j^2 + m^2}{2m}$$

#### 5.3.10 Proof of Lemma 7

Differentiating  $g_j(g_i)$  with respect to  $g_i$ , we have  $\frac{\alpha_j - m}{\alpha_j + m}$ , which is positive if  $\alpha_j > m$ . Otherwise, if  $\alpha_j < m$ ,  $g_j(g_i)$  decreases in  $g_i$ . On the other hand, differentiating  $U_i$  with respect to  $g_j$  we obtain

$$\frac{-\alpha_i + m}{\alpha_i \left(g_i - g_j\right) + m \left(g_i + g_j\right)}$$

which is negative if  $\alpha_i > m$ . Otherwise, if  $\alpha_i < m$ ,  $U_i$  increases in  $g_j$ .

# 5.3.11 Proof of Lemma 8

When  $\alpha_i < \bar{\alpha}_i$ , we know from proposition 2 that player *i* does not contribute, but player *j* responds submitting a contribution of  $g_j^{Seq} = 1$ . This is valid both when  $\alpha_j < m$  and when  $\alpha_j < m$ . Then,  $G^{Seq} = 1$ .

In contrast, when  $\alpha_i \in [\bar{\alpha}_i \hat{\alpha}_i)$  and  $\alpha_j < m$  (or when  $\alpha_i \in [\bar{\alpha}_i + \infty)$  and  $\alpha_j > m$ ) from proposition 2 we know that player *i* submits  $g_i^{Seq} = \frac{\alpha_i \alpha_j + 3\alpha_i m + \alpha_j m - m^2}{2m(\alpha_i + \alpha_j)}$  while player *j* responds by submitting

$$g_j^{Seq} = \frac{1}{2} \left( \frac{\alpha_i \alpha_j}{(\alpha_i + \alpha_j)m} + \frac{m}{\alpha_i + \alpha_j} + \frac{4\alpha_j}{\alpha_j + m} - 1 \right)$$

Then, the total contributions when  $\alpha_i > \bar{\alpha}_i$  adds up to  $G^{Seq} = \frac{2\alpha_j}{\alpha_j + m} + \frac{\alpha_i(\alpha_j + m)}{(\alpha_i + \alpha_j)m}$ 

Finally, if  $\alpha_i \in [\bar{\alpha}_i + \infty)$  and  $\alpha_j < m$ , from proposition 2 we know that player *i* submits  $g_i^{Seq} = \frac{\alpha_i \alpha_j + 3\alpha_i m + \alpha_j m - m^2}{2m(\alpha_i + \alpha_j)}$  and player *j* does not submit any positive contribution (since his best response function is positively sloped and, for these parameter values, it crosses the  $g_i$ -axis), what implies  $G^{Seq} = \frac{\alpha_i \alpha_j + 3\alpha_i m + \alpha_j m - m^2}{2m(\alpha_i + \alpha_j)}$ .

# 5.3.12 Proof of Lemma 9

Regarding player i, the difference between his equilibrium contribution in the simultaneous and sequential game is

$$\frac{\left(\alpha_{i}-m\right)\left(\alpha_{j}-m\right)}{2\left(\alpha_{i}+\alpha_{j}\right)m}$$

which is positive if either  $\alpha_i > m$  and  $\alpha_j > m$ , or if  $\alpha_i < m$  and  $\alpha_j < m$ . Hence, if  $\alpha_i > m$ and  $\alpha_j > m$  (or if  $\alpha_i < m$  and  $\alpha_j < m$ ), then  $g_i^{Sm} > g_i^{Seq}$ . Regarding player j, the difference between his equilibrium contribution in the simultaneous and sequential game is

$$\frac{(\alpha_i - m) (\alpha_j - m)^2}{2 (\alpha_i + \alpha_j) m (\alpha_j - m)}$$

which is positive if and only if  $\alpha_i > m$ . Hence, if  $\alpha_i > m$ ,  $g_j^{Sm} > g_j^{Seq}$ .

# 5.3.13 Proof of Proposition 3

Applying proposition 1 of Romano and Yildirim (2001), we know that whenever  $1 + \frac{\partial g_j(g_i)}{\partial g_i} > 0$ , the sign of

$$\frac{\partial U_i}{\partial g_j} \frac{\partial g_j(g_i)}{\partial g_i}$$
 and  $G^{Seq} - G^{Sm}$ 

coincide. Let us then first find  $1 + \frac{\partial g_j(g_i)}{\partial g_i}$ . In particular,  $1 + \frac{\partial g_j(g_i)}{\partial g_i} = 1 + \frac{\alpha_j - m}{\alpha_j + m}$  which is positive for any  $\alpha_j > 0$ . On the other hand, from corollary 1, we know that for any  $i, j = \{1, 2\}$  where  $j \neq i$ 

$$\frac{\partial U_i}{\partial g_j} \frac{\partial g_j(g_i)}{\partial g_i} = \begin{cases} > 0 \text{ if } \alpha_i < m \text{ and } \alpha_j > m, \\ < 0 \quad \text{otherwise} \end{cases}$$

Therefore, if  $\alpha_i < m$  and  $\alpha_j > m$ , for all  $i, j = \{1, 2\}$  and  $j \neq i$ , then

$$\frac{\partial U_i}{\partial g_j} \frac{\partial g_j(g_i)}{\partial g_i} > 0 \text{ and } G^{Seq} > G^{Sm}$$

and if  $\alpha_i > m$  and  $\alpha_j > m$  (or if  $\alpha_i < m$  and  $\alpha_j < m$ ), then

$$\frac{\partial U_i}{\partial g_j} \frac{\partial g_j(g_i)}{\partial g_i} < 0 \text{ and } G^{Seq} < G^{Sm}$$

#### 5.3.14 Proof of Lemma 10

When seniority in status is considered, player i's utility maximization problem becomes

$$\max_{g_i} w - g_i + \ln \left[ m(g_i + g_j) + \alpha_i \left( S_i + g_i - g_j \right) \right]$$

Differentiating with respect to  $g_i$ , setting it equal to zero, and solving for  $g_i$ , we obtain  $1 - \frac{\alpha_i S_i}{\alpha_i + m} + \frac{\alpha_i - m}{\alpha_i - m} g_j$ . Hence, player *i*'s best response function is

$$g_i(g_j) = \begin{cases} 1 - \frac{\alpha_i S_i}{\alpha_i + m} \text{ if } g_j = 0\\ 1 - \frac{\alpha_i S_i}{\alpha_i + m} + \frac{\alpha_i - m}{\alpha_i - m} g_j \text{ if } g_j \in [0, \frac{\alpha_i S_i - \alpha_i - m}{\alpha_i - m}]\\ 0 \text{ if } g_j > \frac{\alpha_i S_i - \alpha_i - m}{\alpha_i - m} \end{cases}$$

where  $1 - \frac{\alpha_i S_i}{\alpha_i + m} + \frac{\alpha_i - m}{\alpha_i - m} g_j = 0$  at exactly  $g_j = \frac{\alpha_i S_i - \alpha_i - m}{\alpha_i - m}$ . In particular, note that  $\frac{\alpha_i S_i - \alpha_i - m}{\alpha_i - m} > 0$  if and only if  $S_i > 1$ .

## 5.3.15 Proof of Proposition 4

First, take a given player *i*'s best response function,  $g_i(g_j)$ . Then,  $g_i^{Sm,Sen} = 1 - \frac{\alpha_i S_i}{\alpha_i + m}$  only when: (1) the slope of player *j*'s best response function,  $g_j(g_i)$ , is smaller than -1, and (2) the horizontal intercept of player *i*'s best response function,  $g_i(g_j)$ , is higher than  $1 - \frac{\alpha_j S_j}{\alpha_j + m}$ . Otherwise, both players' best response functions would cross each other in an interior point. Therefore,  $g_i^{Sm,Sen} = 1 - \frac{\alpha_i S_i}{\alpha_i + m}$  if and only if

$$\begin{array}{ll} \displaystyle \frac{\alpha_j - m}{\alpha_j + m} &\leq & -1 \Longleftrightarrow \alpha_j \leq 0, \text{ and} \\ \\ \displaystyle \frac{\alpha_i S_i - \alpha_i - m}{\alpha_i - m} &\geq & 1 - \frac{\alpha_j S_j}{\alpha_j + m} \Longleftrightarrow \alpha_i \geq \frac{\alpha_j S_j m}{\alpha_j \left(S_i + S_j - 2\right) + \left(S_i - 2\right) m} \end{array}$$

Since  $\alpha_i, \alpha_j \geq 0$ , the above conditions on player *i* and *j*'s concerns about status are  $\alpha_i \geq \frac{\alpha_j S_j m}{\alpha_j (S_i + S_j - 2) + (S_i - 2)m}$  and  $\alpha_j = 0$ . Secondly,  $g_i^{Sm,Sen} = 0$  when the opposite happens. That is, when  $\alpha_i = 0$  and  $\alpha_j \geq \frac{\alpha_i S_i m}{\alpha_i (S_j + S_i - 2) + (S_j - 2)m}$ . Finally, when both  $\alpha_i \geq \frac{\alpha_j S_j m}{\alpha_j (S_i + S_j - 2) + (S_j - 2)m}$  and  $\alpha_j \geq \frac{\alpha_i S_i m}{\alpha_i (S_j + S_i - 2) + (S_j - 2)m}$ , we have an interior solution. Solving for  $g_i$  and  $g_j$  in a system of two equations, we obtain

$$g_i^{Sm,Sen} = \frac{\alpha_j S_j m - \alpha_i \left[\alpha_j \left(S_i + S_j - 2\right) + m \left(S_i - 2\right)\right]}{2 \left(\alpha_i + \alpha_j\right) m}$$

Therefore,

$$g_i^{Sm,Sen} = \begin{cases} 1 - \frac{\alpha_i S_i}{\alpha_i + m} \text{ if } \alpha_i \ge \tilde{\alpha}_i \text{ and } \alpha_j \ge 0\\ \frac{\alpha_j S_j m - \alpha_i [\alpha_j (S_i + S_j - 2) + m(S_i - 2)]}{2(\alpha_i + \alpha_j)m} \text{ if } \alpha_i \ge \tilde{\alpha}_i \text{ and } \alpha_j \ge \tilde{\alpha}_j\\ 0 \text{ if } \alpha_i \ge 0 \text{ and } \alpha_j \ge \tilde{\alpha}_j \end{cases}$$

where  $\tilde{\alpha}_i = \frac{\alpha_j S_j m}{\alpha_j (S_i + S_j - 2) + (S_i - 2)m}$  and  $\tilde{\alpha}_j = \frac{\alpha_i S_i m}{\alpha_i (S_j + S_i - 2) + (S_j - 2)m}$ .

# 5.3.16 Proof of Lemma 11

Differentiating  $g_i^{Sm,Sen}$  with respect to  $S_i$ ,

$$\frac{\partial g_i^{Sm,Sen}}{\partial S_i} = -\frac{\alpha_i \left(\alpha_j + m\right)}{2 \left(\alpha_i + \alpha_j\right) m}$$

which is negative for all parameter values. Similarly, differentiating  $g_i^{Sm,Sen}$  with respect to  $S_j$ , we have

$$\frac{\partial g_i^{Sm,Sen}}{\partial S_j} = \frac{\alpha_j \left(m - \alpha_i\right)}{2 \left(\alpha_i + \alpha_j\right) m}$$

which is negative if and only if  $m < \alpha_i$ .

# 5.3.17 Proof of Proposition 5

Using the second mover's best response function,  $g_j(g_i)$ , from lemma 10, we know that

$$g_i(g_j) = \begin{cases} 1 - \frac{\alpha_j S_j}{\alpha_j + m} \text{ if } g_i = 0\\ 1 - \frac{\alpha_j S_j}{\alpha_j + m} + \frac{\alpha_j - m}{\alpha_j - m} g_j \text{ if } g_i \in [0, \frac{\alpha_j S_j - \alpha_j - m}{\alpha_j - m}]\\ 0 \text{ if } g_i > \frac{\alpha_j S_j - \alpha_j - m}{\alpha_j - m} \end{cases}$$

Regarding player i, we know that he inserts the above best response function into his utility function,

$$U_{i} = w - g_{i} + \ln \left[ m(g_{i} + g_{j}(g_{i})) + \alpha_{i} \left( S_{i} + g_{i} - g_{j}(g_{i}) \right) \right]$$

and differentiating with respect to  $g_i$ , and solving for  $g_i$  we obtain the following optimal contribution

$$g_i^{Seq,Sen} = \begin{cases} 0 \text{ if } \alpha_i \in [0, \alpha_i^A] \\ \frac{(\alpha_j + \alpha_j S_j - m)m - \alpha_i [\alpha_j (S_i + S_j - 1) + (S_i - 3)m]}{2(\alpha_j + \alpha_j)m} \text{ if } \alpha_i > \alpha_i^A \end{cases}$$

where  $\alpha_i^A = \frac{(\alpha_j + \alpha_j S_j - m)m}{\alpha_j (S_i + S_j - 1) + (S_i - 3)m}$ . Given the above contribution of the first mover, we can now use  $g_j(g_i)$  to find player j's equilibrium contribution.

$$g_{j}^{Seq,Sen} = \begin{cases} 1 - \frac{\alpha_{j}S_{j}}{\alpha_{j}+m} \text{ if } \alpha_{i} \in [0, \alpha_{i}^{A}] \\ \frac{1}{2} \left[ \frac{m}{\alpha_{i}+\alpha_{j}} + \frac{4\alpha_{j}}{\alpha_{j}+m} - \frac{\alpha_{i}+\alpha_{j}-\alpha_{i}S_{i}+\alpha_{j}S_{j}}{(\alpha_{i}+\alpha_{j})m} - \frac{\alpha_{i}\alpha_{j}(S_{i}+S_{j}-4)}{\alpha_{i}+\alpha_{j}} \right] \text{ if } \alpha_{i} \in \left[\alpha_{i}^{A}, \alpha_{i}^{B}\right] \\ 0 \text{ if } \alpha_{i} > \alpha_{i}^{B} \end{cases}$$

where  $\alpha_i^A = \frac{(\alpha_j + \alpha_j S_j - m)m}{\alpha_j (S_i + S_j - 1) + (S_i - 3)m}$  and  $\alpha_i^B = \frac{m(m^2 - (S_j - 3)\alpha_j^2 - \alpha_j S_j m)}{\alpha_j^2 (S_i + S_j - 1) + m\alpha_j (S_j - 4) - m^2 (S_i - 1)}$ .

## 5.3.18 Proof of Lemma 12

Differentiating  $g_i^{Seq,Sen}$  and  $g_j^{Seq,Sen}$  with respect to  $S_i$  and  $S_j$ , respectively, we obtain

$$\frac{\partial g_i^{Sm,Sen}}{\partial S_i} = -\frac{\alpha_i \left(\alpha_j + m\right)}{2 \left(\alpha_i + \alpha_j\right) m}, \text{ and } \frac{\partial g_j^{Sm,Sen}}{\partial S_j} = -\frac{\alpha_j \left(\alpha_i + m\right)}{2 \left(\alpha_i + \alpha_j\right) m}$$

which are negative for all parameter values. Similarly, differentiating  $g_i^{Seq,Sen}$  and  $g_j^{Seq,Sen}$  with respect to  $S_j$  and  $S_i$ , respectively, we have

$$\frac{\partial g_i^{Sm,Sen}}{\partial S_j} = \frac{\alpha_j \left(m - \alpha_i\right)}{2\left(\alpha_i + \alpha_j\right)m}, \text{ and } \frac{\partial g_j^{Sm,Sen}}{\partial S_i} = \frac{\alpha_i \left(m - \alpha_j\right)}{2\left(\alpha_i + \alpha_j\right)m}$$

which are negative if and only if  $m < \alpha_i$  and  $m < \alpha_j$  respectively.

# 5.3.19 Proof of Proposition 6

Applying Romano and Yildirim (2001), we know that whenever  $1 + \frac{\partial g_j(g_i)}{\partial g_i} > 0$ , the sign of

$$\frac{\partial U_i}{\partial g_j} \frac{\partial g_j(g_i)}{\partial g_i}$$
 and  $G^{Seq} - G^{Sm}$ 

coincide. Let us then first find  $1 + \frac{\partial g_j(g_i)}{\partial g_i}$ . In particular,  $1 + \frac{\partial g_j(g_i)}{\partial g_i} = 1 + \frac{\alpha_j - m}{\alpha_j + m}$  which is positive for any  $\alpha_j > 0$ . On the other hand,

$$\frac{\partial U_i}{\partial g_j} = \frac{-\alpha_i + m}{\alpha_i \left(S_i + g_i - g_j\right) + m \left(g_i + g_j\right)}$$

which is negative if and only if  $\alpha_i > m$ . Then, from corollary 1, we know that for any  $j \neq i$ 

$$\frac{\partial U_i}{\partial g_j} \frac{\partial g_j(g_i)}{\partial g_i} = \begin{cases} > 0 \text{ if } \alpha_i < m \text{ and } \alpha_j > m, \\ < 0 \text{ otherwise} \end{cases}$$

Therefore, if  $\alpha_i < m$  and  $\alpha_j > m$ , for all  $j \neq i$ , then

$$\frac{\partial U_i}{\partial g_j} \frac{\partial g_j(g_i)}{\partial g_i} > 0 \text{ and } G^{Seq} > G^{Sm}$$

and if  $\alpha_i > m$  and  $\alpha_j > m$  (or if  $\alpha_i < m$  and  $\alpha_j < m$ ), then

$$\frac{\partial U_i}{\partial g_j} \frac{\partial g_j(g_i)}{\partial g_i} < 0 \text{ and } G^{Seq} < G^{Sm}$$

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