# INTERNET QOS MARKET ANALYSIS WITH PEERING AND USAGE-SENSITIVE PRICING: A GAME THEORETIC AND SIMULATION APPROACH 

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# INTERNET QOS MARKET ANALYSIS WITH PEERING AND USAGE-SENSITIVE PRICING: A GAME THEORETIC AND SIMULATION APPROACH 

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One of the major areas for research and investment related to the Internet is the provision of quality of service (QoS). We remain confident that in the not-to-distance future, QoS will be introduced not only in private networks but in the whole Internet. QoS will bring some new features into the Internet market: (1) vertical product differentiation with BE and QoS , (2) usage-sensitive pricing with metering. In this dissertation, the equilibrium outcomes are analyzed when two rural Internet Access Providers (IAPs) interact with several business and technical strategies such as technology (BE or QoS), pricing scheme (flat-rate pricing or two-part tariff), interconnection (transit or peering) and investment in network capacity. To determine the equilibria, we construct a duopoly game model based on Cournot theory. We calibrate this model to data found in real markets. In this model, we study ten cases with a combination of strategic choices of two IAPs. We use two demand functions: one based on uniform distribution and the other based on empirical distribution which comes from the U.S. General Accounting Office (U.S. GAO) survey for Internet usage. We use a two-stage RNG (Random Number Generator) simulation and a linear regression for the latter. If we consider IAPs with the BE and the flat rate pricing as the current Internet, the equilibrium points of each case in this model suggest a progressive market equilibrium path to the future Internet market. Based on the equilibrium analysis of the game model, we conclude that (1) \{QoS, two-part tariff, transit/peering\} or \{QoS, flat-rate pricing, peering\} will be a plausible situation in the future Internet access market, (2) network capacity will still be an important strategy to determine market equilibrium in the future as well as in the current, (3) BE will take a considerable market share in the QoS Internet, and (4) peering arrangements in the QoS Internet will provide a higher social welfare than transit. These implications from the game analysis present an analytical framework for the future Internet policy.

## FOREWORD

When I came to Pittsburgh in August of 1998, I had no idea what telecommunications was. I started basic telecommunication courses like 'Electronic Communications I' and 'Fundamentals of Telecommunications.' Four semesters later, I felt I firmly understood telecommunications. At the fifth semester the Ph.D. qualifying exam made me realize that exhaustive knowledge of telecommunications within four semesters was impossible. Fortunately, the world's best telecommunications program helped me to successfully finish my dissertation. Thank you, telecom faculty!

During my four years and eight months at the University of Pittsburgh, there have been so many people who have inspired and supported. I really appreciate my advisor, Dr. Martin Weiss for his continuous support and wonderful advice. His insightful guidance has allowed me to pursue this dissertation work. I also wish to express special thanks to Dr. Hector Correa. His analytic ability surprised me and spurred me on in my studies. Additionally I would like to thank the dissertation committee members, Drs. Gal-Or, Jedrus, Krishnamurthy, and Professor King for their comments and suggestions.

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## [CHAPTER 1] INTRODUCTION

### 1.1 Research Motivation

After the commercialization of the Internet in 1995, the demand for various Internet services diversified; new real-time and business-critical data applications require improved levels of services, or 'QoS (Quality of Service)' from the network. In the current Internet, most traffic is treated indifferently; there is no discrimination among Internet traffic streams; there is only one class of service to which all traffic belongs; there is no delivery confirmation and no guarantee for timely delivery and there is a possibility for traffic loss. This kind of Internet service is called 'Best Effort (BE).' Compared to BE with no service classification, QoS has various classes: a class for guaranteeing timely delivery, a class for no traffic loss, and a class for delivery confirmation. The relationship between BE and QoS is similar to that of regular mail and priority mail for which users pay a higher price than regular mail. However, guaranteeing QoS in the Internet is not easy. The main reason is that even though Internet Service Providers (ISPs ${ }^{1}$ ), a general term of service providers in the Internet industry, are using the common TCP/IP protocol, they are different: different backbone capacity, different network architecture, different routing protocol, different business model, etc. Furthermore, there is no single entity to coordinate the whole Internet industry. Therefore, end-to-end QoS guarantee is impossible without stronger coordination of multiple ISPs in the Internet industry because any QoS assurances are only as good as the weakest link in the chain between sender and receiver.

In the summer of 2001, large service providers like AT\&T and WorldCom announced that they would provide Internet "Class of Service" (CoS) to their customers. The CoS consists of four classes according to the priority level: Platinum, Gold, Silver, and Bronze. For example, voice or video applications can get the highest priority, while other traffic, such as e-mail or HTTP, can be given the lowest priority, which is the same class as the current Internet. Table 1-1 in the next page describes the characteristic of each class. Since QoS interconnection policies have yet to be established, this CoS capability is limited to traffic that is contained completely in the provider's own network. To address this limitation, BellSouth's Florida Multimedia Internet Exchange (FMIX) announced a plan in 2001 to be the first NAP ${ }^{2}$ (Network Access Point) to support QoS interconnection using MPLS ${ }^{3}$. Some of the challenges (that were not in the

[^0]announcement) will be exactly how class matching between providers will be achieved, and how to disclose needed network information for an end-to-end quality guarantee without compromising the competitive position of the interconnecting parties.
[Table 1-1] QoS Class of Service

| Type of Class |  | Characteristics |
| :---: | :---: | :--- |
| Premium <br> Class | Platinum | Real time application such as voice over IP or video conference |
|  | Gold | Business critical data communications like E-commerce |
|  | Silver | Database applications |
| BE Class | Bronze | Web surfing and E-mail |

Despite these difficulties, we remain confident that, in the not-to-distant future, QoS will be introduced not only in private networks but in the whole Internet. We take the backbone market leaders' movement toward QoS and the emergence of QoS enabled NAP as strong signs of this shift. Other features of this new network include:
(1) Product Diversification Before QoS, there is only one available service level, i.e., BE, in which traffic delay and traffic dropping are possible. With QoS, there are two services in the Internet market: BE and QoS. Since QoS includes BE as its lowest class of service, the new markets will feature vertical product differentiation.
(2) Operational Transition Traditionally, Internet Access Providers (IAPs ${ }^{4}$ ) in the U.S. provided flat rate access plans, and later performed a limited amount of usage metering. Previous research (MacKie-Mason and Varian, 1995) indicates that, in the absence of price-based differentiation, users will choose the highest quality level regardless of traffic type. Thus, it is reasonable to expect a change in pricing and billing practices toward usage-sensitive pricing with metering.
(3) QoS Peering The other traditional characteristic of Internet industry is peering, in which there is no financial payment when ISPs exchange traffic each other. However, after the commercialization of the Internet, peering has gradually evolved with payment. For example, a traffic-ratio based paid peering model is emerging; in which peering is free until traffic asymmetry reaches a certain ratio. At this point, peering get involved with payment. We anticipate the traffic ratio will be tighter in the QoS peering.

[^1](4) Another Balkanization For last several years, the Internet backbone market experienced balkanization according to whether Internet Backbone Providers (IBPs ${ }^{5}$ ) have a nationwide backbone network or not. Another balkanization may happen based on whether ISPs have capability to provide QoS or not. As incumbents with a national backbone network exercise their market power in negotiating with smaller ISPs and new entrants, ISPs armed with QoS capability may discriminate based on their QoS capability.

The QoS Internet with these new features lets IAPs have several options for their strategies besides network capacity. New strategies are chosen for their profit maximization, which is dependent on not only my own strategies but also the other's strategies. With the QoS Internet scenario, we build an Internet QoS Game model through which we show IAPs market equilibrium behaviors. The Internet QoS game analysis will give us answers to several questions, i.e., which is a dominant strategy set in the future Internet market, what is a progressive path of IAP's market behaviors, or do new strategies give us a higher social welfare?

### 1.2 Overview of Internet QoS Game Model

ISPs are competitors and cooperators simultaneously: competitors for market share and cooperators for global connectivity. One ISP's decision has an influence on other ISP's decisions. Thus, they have a strong dependence on each beyond just competitive factors. These characteristics make the Internet provision industry suitable for game theoretic analysis, i.e., each player in the game model is a competitor in a market and there are interactions according to their strategic variables.

In this dissertation, we apply Cournot duopoly theory to our game model, describing two IAPs in a rural area. In Cournot duopoly game model, firms' output level is a strategic variable and its main assumptions are product homogeneity and no entry: there are two firms in the market and each firm produces identical product and the market quantity of two firms determines the market price. In the game model of this dissertation, we use the same assumptions and make an extended duopoly game model, which will be applied to the Internet access market. Each IAP has four strategic choices: technology (BE or QoS), pricing scheme (flat-rate pricing or two-part tariff), investment in network capacity $\left(1 \mathrm{~K}, 2 \mathrm{~K}\right.$, or $3 \mathrm{~K}^{6}$ ), and

[^2]interconnection method (peering or transit). While these four choices are assumed to be long-term strategies, which can not be varied in a short term period, output level is supposed to be a short-term strategy which can be varied within a chosen network capacity. Table 1-2 presents an overall game structure. There are ten unique cases considering interconnection, pricing, and technology strategies. Because of symmetric characteristic, there are duplicates of Cases 2, 3, 4, and 8 . Peering is only possible for two IAPs with the same technology.
[Table 1-2] Internet QoS Game Model

[Table 1-3] Internet QoS Game Model in Detail

|  |  |  |  | Transit |  |  |  |  |  |  |  |  | Peering |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | BE |  |  | QoS |  |  |  |  |  | BE |  |  | QoS |  |  |  |  |  |
|  |  |  |  |  | Flat |  | Flat |  |  | Two-Part |  |  | Flat |  |  | Flat |  |  | Two-Part |  |  |
|  |  |  |  | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |
| Transit | BE | Flat | 1 | Case 1 |  |  | Case2 |  |  | Case3 |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | QoS |  | 1 |  |  |  |  |  |  | Case 5 |  |  | Case 4 |  |  |  |  |  |  |  |  |  |  |  |
|  |  | Flat | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 1 |  |  |  |  |  |  |  |  |  |  | se |  |  |  |  |  |  |  |  |  |  |
|  |  | 2-Part | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Peering | BE | Flat | 1 |  |  |  |  |  |  |  |  |  | Case7 |  |  |  |  |  |  |  |  |
|  |  |  | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | QoS | Flat | 1 |  |  |  |  |  |  |  |  |  |  |  |  | Case9 |  |  | Case8 |  |  |
|  |  |  | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | 2-Part | 1 |  |  |  |  |  |  |  |  |  |  |  |  | Case10 |  |  |  |  |  |
|  |  |  | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Table 1-3 shows the model structure in detail including investment in network capacity. Every case has nine cells ( $3 \times 3$ ), which are characterized by its network capacity ( $1 \mathrm{~K}, 2 \mathrm{~K}$, or 3 K ). There are a total of 90 cells in this model ( $=9$ cells * 10 cases). In each cell, both IAPs try to maximize their profit by choosing an optimal output level, which is determined by interactions between them through the strategic variables.

The following two game trees are equivalent to the above two tables. A game tree is a diagram composed of nodes and branches. Each node in the game tree represents a decision point for one of the players; each branch represents a possible action for a player at that point in the game. (Bierman and Fernandez, 1993, p71) Because this game is a simultaneous game, it makes no difference whoever makes the first move, i.e., IAP1 and IAP2 are interchangeable. This game is processed with two steps: in the first step (figure 1-1), each IAP determines its strategies such as technology, pricing scheme, and interconnection. The game is divided into ten cases by which of these three choices are made. In the second step (figure 1-2), IAP1 and IAP2 in every ten cases have three investment options in their network capacity. After they set all their strategies, they optimize their profit ( $f 1^{*}$ and $f 2^{*}$ ) simultaneously with their output level, i.e., ( $\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{Qos}}$ ) and $\left(\mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{Qos}}\right)^{7}$. By using the technique of backward induction ${ }^{8}$, we can find an optimal investment strategy for each case and an optimal strategy set of the whole Internet QoS game model.

[Figure 1-1] Internet QoS Game Tree

[^3]
[Figure 1-2] Game Tree of Each Case
Case 1 can be considered as the current Internet model and Cases $4 \sim 6$ and Cases $8 \sim 10$, can be interpreted by the future Internet models. The others (Cases 2, 3, and 7) may be viewed as the Internet in transition between the current and future. By analyzing the market equilibrium of current and future Internet, we suggest a progressive path of Internet market equilibrium. In addition, we can calculate social welfare at the equilibrium points of each cell and find whose social welfare is higher.

### 1.3 Research Purpose

The main assumption in this model is that the Internet access market is a duopoly; this is reasonable for many rural areas in the U.S. In reality, there are many IAPs in the access market, most of which are concentrated in urban metropolitan areas. Some of them are large companies with financial power and a high level of technology, but many of them are small-sized, family-operated, rural IAPs. From the universal connectivity point of view the small rural IAPs are very important in providing Internet accessibility to the whole nation. The Telecommunication Act of 1996 expressed that the universal service should be achieved through reducing an information gap between urban and rural areas. However, asymmetric investment in the network infrastructure makes a clear digital divide between the rural and urban areas. According to the 2001 annual ISP survey ${ }^{9}, 80 \%$ of IAPs are classified as relatively small firms. Our model in this dissertation focuses on small IAPs in a rural area. The research purpose of this study is to provide a foundation of policy framework for the QoS connectivity in the future Internet access market. Furthermore, some of the results may also provide guidance for other markets in this industry as well.

[^4]
### 1.4 Research Questions and Hypotheses

In today's dynamic Internet environment, we study how the market behavior of IAPs will be changed in the QoS Internet market. Every IAP has a different set of business and technical strategies: (1) It could choose peering (no financial settlement) or transit (with financial settlement) as interconnection strategies according to its coverage area and its network size; (2) it could choose a pricing strategy flat-rate or usage-sensitive; (3) it could determine its network capacity according to its investment strategy; and (4) it could choose BE or QoS as its technology. All choices would be determined by the interaction with other IAPs in the market.

Based upon the above strategic choices, there are several specific research questions in this dissertation.
(1) What is the market equilibrium behavior of BE-IAPs with a strategy of network capacity?
(2) What is the market equilibrium behavior of QoS-IAPs with different strategies such as capacity planning, output level, and pricing scheme?
(3) What is the market equilibrium behavior between BE-IAP and QoS-IAP with different strategies such as capacity planning and pricing scheme?
(4) What are social welfare values at market equilibrium points of each case and which one is socially desirable?
(5) Is an IAP with usage-sensitive pricing generally profitable than an IAP with flat-rate pricing?

Based upon the above research questions and the game model, we try to test the following five hypotheses:
(H1) An IAP with BE technology has the same profit in market equilibrium as another IAP with the same technology.
(H2) An IAP with QoS technology and usage sensitive QoS pricing has a higher profit in market equilibrium than an IAP with QoS technology and flat-rate QoS pricing.
(H3) An IAP with BE technology and flat-rate pricing has a lower profit in market equilibrium than an IAP with QoS technology and flat-rate QoS pricing.
(H4) An IAP with QoS technology with usage-sensitive QoS pricing has a higher profit in market equilibrium than an IAP with BE technology and flat-rate pricing.
(H5) Both IAPs with QoS technology give a higher social welfare than any other cases.

### 1.5 Research Method

In this dissertation, we make two different assumptions of data distribution for users' willingness-to-pay: (1) uniform distribution and (2) empirical distribution. According to the different assumptions, we use different demand functions. For the demand function of empirical distribution we conduct a two-stage RNG (Random Number Generator) simulation, which will be described in detail in the section 7.1.

Based on the above demand functions and a cost function made by the data ${ }^{10}$ found in the real market, we establish a profit function. Then, we try to find an equilibrium point when both IAPs maximize their profits simultaneously. We use three kinds of softwares to solve this model: (1) CSIM (simulation library), (2) SPSS (statistical package), and (3) Mathematica (math problem-solving package). Figure 1-1 shows an overall research process:

[Figure 1-3] Research Procedure

[^5]
## [CHAPTER 2] INTERNET INDUSTRY

### 2.1 Introduction

The Internet is a system that makes it possible to send and receive information among all the individual and institutional computers associated with it. The Internet industry integrates the equipment, software, and organizational infrastructure required for Internet communications. As a rough approximation it can be said ISPs in the Internet industry are classified into two categories: IBPs that transfer communications in bulk among network exchange points, and IAPs that (1) receive communications from individuals or institutions and transfer them to an IBP's network, and (2) receive communications from IBPs and transfer them to their destination. Generally speaking, the Internet industry has a vertical structure: Upstream IBPs provide an intermediate good and downstream IAPs use this input to sell connectivity to their customers. Therefore, the relationship between IBPs and IAPs is that of wholesalers and retailers.

In reality the Internet is much more complex. The IAPs themselves are networks of users that may directly exchange information among each other. In addition, the IBPs ${ }^{11}$ may provide services directly to users and also may interconnect with other IBPs. In this sense, the Internet is a network of networks that is accessible in many parts of the world. Since the telephone industry is tightly intertwined with the Internet industry, we begin by with an examination of it.

### 2.2 Telephone Industry

The Public Switched Telephone Network (PSTN) was designed and optimized for the transmission of human voice. In the U.S., telephone service is divided into two industries: (1) local telephone service provided by Local Exchange Carriers (LECs) and (2) long distance telephone service provided by Interexchange Carriers (IXCs). This structure creates a vertical hierarchy: The upstream IXCs provide the connection between LECs, and the downstream LECs have direct access to telephone users.

Traditionally, a LEC was a monopoly that served a specific geographic region without competition. Even after deregulation, LECs are still considered by many to be a local monopoly, especially for residential customers. In the U.S., the local telephone services provided under flat-rate billing, that is, a telephone user can originate local calls as many times and as long as he wishes with only monthly flat rate. This type of billing system has been a great influence on the growth of Internet access market.

[^6]The long distance market is now generally considered to be a very competitive market, though it too was a monopoly at one time. Users can use a long distance calling with a pre-selected IXC through their LEC. Any IXC that wishes to handle calls originating in a local service area can build a switching office, called a Point of Presence (POP), there. The function of the POP is to interconnect networks so that now any site where networks interconnect may be referred to as a POP.

### 2.3 Internet Backbone Providers

With some simplification, it can be said that the IBPs receive communications in bulk from POPs or NAPs and distribute them to other POPs or NAPs close to the destination. NAPs are public interconnection points where major providers interconnect their network and consist of a high-speed switch or network of switches to which a number of routers can be connected for the purpose of traffic exchange. The NAPs are similar to major airport hubs; all IAPs and IBPs are gathered at the NAPs to connect each other.

Before the Internet privatization, the NSF (National Science Foundation) was responsible for the operation of the Internet. The NSF backbone ceased operation in late 1994 and was replaced by the four NAPs ${ }^{12}$ (Minoli, 1998, pp27-28). There are probable around 50 major NAPs ${ }^{13}$ world-wide in the Internet, most of which are located in the U.S. (Moulton, 2001, p551). As the Internet continued to grow, the NAPs suffered from congestion because of the enormous traffic loads. Because of the resulting poor performance, private direct interconnections between big IBPs were introduced, called private peering, which will be explained in detail in the section 3.2.

To make the Internet a seamless network, the IBPs have multiple POPs distributed over the whole world. Most frequently they are located in large urban centers. These POPs are connected each other with owned or leased optical carrier lines. Typically, these lines are 622 Mbps (OC-12) or 2.488 Gbps (OC-48) circuits or more, as defined by the SONET ${ }^{14}$ hierarchy. These POPs and optical carrier lines make up the IBP backbone network. The IBP's POPs, are also connected with the POPs of many IAPs. The relationship between an IAP's POP and IBP's POP is the same as that of IAPs and IBPs.

[^7]According to Erickson (2001), the North American backbone market had around $36 \mathrm{IBPs}^{15}$ in the first quarter of 2001. However, these numbers misinterpret the Internet backbone market structure because this market is highly concentrated. There were 11,888 interconnections between backbone and access markets in April 2000. (McCarthy, 2000) Counting by the number of connections to downstream market, $\mathrm{MCI} /$ Worldcom is a dominant player in the backbone market with 3,145 connections and Sprint is the second largest backbone provider with 1,690 connections and AT\&T ( 934 connections) and C\&W (851 connections) are the third and the fourth.

Several of the large IBPs are subsidiaries of large telephone companies such as AT\&T, MCI/WorldCom, Sprint, etc. Since these companies own the infrastructure needed for telephone services, they are very favorably positioned to provide the facilities and equipment required by the IBPs. In addition, due to their size, they are able to offer large volume discount rates or bundling agreements of both telephone and Internet lines for the services they provide. This is possible because the Internet industry is lightly regulated, if at all. In particular, there are no regulations with respect to the tariffs that can be charged for the services provided. From these observations it follows that the large IBPs, supported by the large telephone companies, are in a position to capture large shares of the upstream market.

According to Carlton and Perloff (1999, p247), the most common measure of concentration in an industry is their share of sales by the four largest firms, called a $C 4$ ratio. Generally speaking, if the C 4 ratio is over 60 , the market is considered a tight oligopoly. For the upstream backbone market this ratio ${ }^{16}$ is 73 , which shows high concentration in the market. The entry barrier is also high because there is a large sunk cost for nationwide backbone lines and switching equipment. The number of IBPs for the past three years shows just how high the entry barrier in the backbone market is: 43 (1999), 41 (2000), and $36(2001)^{17}$. The slight reduction for last three years is caused by mergers and acquisitions (M\&A) ${ }^{18}$ and reclassification ${ }^{19}$. According to the number of players, we conclude that the overall backbone market is stable. In addition there are significant economies of scale and the rapid pace of technological change generates a large amount of uncertainty about the future return on investments. It is not easy to enter this market without large investments and high technology.

[^8]The interconnection price is usually determined by the provider's relative strength and level of investment in a particular area (Halabi, 2000, 42p). It is certain that the T1 Internet interconnection price has been decreased continuously in recent years. In 1996, the Internet connectivity for T1 was $\$ 3,000$ per month with $\$ 1,000$ setup fee (Halabi, 1997, 40p). According to Martin (2001), the average price of a T1 connection in 1999 was $\$ 1,729$. In 2000, it was $\$ 1,348$. In 2001, it was $\$ 1,228$. One of reasons for the decreasing T 1 interconnection price is advent of substitute services for T 1 line, such as wireless Internet access technology ${ }^{20}$, digital subscriber line (DSL) technology, and cable-modem technology, which exert a downward pressure on T1 prices.

### 2.4 Internet Access Providers

An IAP's service is public access to the Internet, which includes login authorization, e-mail services, some storage space, and possibly personal web pages. The following diagram illustrates the components of dial-up IAP and its environment. The DS-0 line (Digital Signal level 0$)^{21}$ is a normal telephone local loop. The T1 $1^{22}$ line between CO (Central Office) and IAP's modem pool is dedicated line for the IAP's customer traffic from a particular CO to their data center. Another T1 line is needed to connect to the Internet, which runs from the IAP's router to the IBP's POP.

[Figure 2-1] ISP Business Environment

The IAP's coverage area is usually determined by the existence of an IAP's POP within the local telephone area. IAPs are classified as local, regional, and national according to the scope of their service coverage. The distribution of IAPs is presented in table 2-1.

[^9]Among 307 telephone area codes in U.S., the largest IAP covers 282 area codes and the smallest covers only 1 area code. The IAPs with 1 to 10 area codes constitute $79.81 \%$ of the total number of IAPs. Thus, most IAPs in the downstream market are small, local companies. Some of these small IAPs are subsidiaries or affiliates of CLECs (Competitive Local Exchange Carriers), which are telephone companies established in the 1990s as a result of telephone industry deregulation.
[Table 2-1] Distribution of IAPs by Coverage

| Telephone area codes covered by IAP | Percentage | Type |
| :---: | :---: | :---: |
| 1 | $35.14 \%$ | Local |
| $2-10$ | $44.67 \%$ | Local / Regional |
| $11-24$ | $4.11 \%$ | Regional |
| $25-282$ | $16.08 \%$ | National |

- Source: The $13^{\text {th }}$ edition of the Directory of Internet Service Providers, Boardwatch magazine (www.ispworld.com/isp/introduction.htm) $)^{23}$

AOL-Time Warner is a dominant player in the dial-up access market. According to Fusco (2001), AOLTime Warner had 22.7 million subscribers in the $1^{\text {st }}$ quarter of 2001. Table 2-2 shows the top 10 dial-up IAPs ranked by the number of paying users.
[Table 2-2] Top 10 U.S. Fee-Based IAPs (Dial-up only) ${ }^{24}$

| Rank \& ISP | Paying <br> User | Market <br> Share | Rank \& ISP | Paying <br> User | Market <br> Share |
| :--- | :---: | :---: | :--- | :---: | :---: |
| (1) AOL | 22.7 M | $46 \%$ | (6)Gateway.net | 1.7 M | $3 \%$ |
| (2) MSN | 5.0 M | $10 \%$ | (7)AT\&T WorldNet | 1.3 M | $3 \%$ |
| (3)EarthLink | 4.8 M | $10 \%$ | (8)NetZero+Juno Online | 1.0 M | $2 \%$ |
| (4) Prodigy | 3.1 M | $6 \%$ | (9) Verizon | 0.9 M | $2 \%$ |
| (5)CompuServe | 3.0 M | $6 \%$ | (10) Bell South | 0.8 M | $2 \%$ |

- Source: www.isp-planet.com/research/rankings/usa.html

In the downstream access market the $C 4$ ratio $^{25}$ is 72 , which is also highly concentrated. However, the entry barrier in the downstream market is much lower than in the backbone market. Since subscribers can utilize the PSTN line to connect IAPs' modems, IAPs do not have to invest in access lines to individual subscribers. They can build POPs to link to the PSTN and other IAPs or IBPs. Since T1 lines prices and telecom equipment prices are currently dropping quickly, a large number of small IAPs are possible,

[^10]especially in the less densely populated areas. The number of North American IAPs for the past several years is an evidence of low entry barrier in the downstream market: 1447 (February 1996), 3640 (February 1997), 4470 (February 1998), 5078 (March 1999), 7463 (April 2000), and 7288 (March 2001) ${ }^{26}$.

In summary, considering the market concentration and the entry barriers, IBPs have more market power than IAPs in the Internet industry. Table 2-3 compares the IBP and IAP markets presented above.
[Table 2-3] Comparison of Access and Backbone Markets

|  | No. of companies | Dominant Company | C4 | Entry Barrier |
| :--- | :---: | :---: | :---: | :---: |
| Backbone Market | 36 | MCI/WorldCom $\left(\right.$ UUNET $^{27}$ ) | 73 | Higher |
| Access Market | 7,228 | AOL TimeWarner | 72 | Lower |

- Sources: TeleGeography, Inc., Internet.com Magazine, and Boardwatch Magazine

Most IAPs provide unlimited Internet access with a monthly flat rate. For the major national IAPs, the price ranges generally from $\$ 0$ to $\$ 25$ per month ${ }^{28}$. Some IAPs provide Internet access service with zero monthly subscription fees to their customers ${ }^{29}$; their revenues depend completely on Internet advertising income. According to Zigmont (2000), cost of startup IAP is roughly $\$ 12$ per subscriber including $\$ 7$ for management/maintenance cost plus $\$ 5$ for marketing.

### 2.5 Relationship between Telephone Industry and Internet Industry

Dial-up access using PSTN is the most universal form of Internet access. In the U.S., such a modem call is typically a local call without a per-minute charge. IAP's lines are treated as a business telephone user not as a carrier, so they are not required to pay the measured Common Carrier Line Charge (CCLC) ${ }^{30}$. The switching system in the LEC's CO connects calls between Internet users and IAP's modem pool so the LECs' facilities support dial-up Internet communications. In addition, IBPs and large IAPs often construct their backbone networks by leasing lines from IXCs and LECs. As a result, we can say that telephone industry provide basic infrastructure for the Internet industry.

[^11]The following diagram explains the overall Internet connections from end user to LEC's CO, IAP, IBP, and NAP. In this diagram, the local IAP's POP connected to AT\&T POP is located in the LEC's CO. The AT\&T and Sprint POPs are connected two ways: (1) through the public NAP and (2) through a private peering line. We will explain in detail peering concept in the next chapter.

[Figure 2-2] Relationship among CO, POPs, and NAP

## [CHAPTER 3] INTERNET INTERCONNECTION STRATEGIES: PEERING VS. TRANSIT

In this chapter, we will examine peering and transit as an interconnection strategy, which determines a part of an IAP's cost. The main difference between peering and transit is in the financial rights and obligation that they generate to their customers.

### 3.1 History of Internet Interconnection

To understand the relationship between peering and transit, it is necessary to recall the non-commercial origin of the Internet. During the Internet's early development, there was only one backbone and only one customer, the military, so interconnection was not an issue. In the 1980s, as the Internet was opened to academic and research institutions, the National Science Foundation (NSF) funded the NSFNET as an Internet backbone. Around that time, the Federal Internet Exchange (FIX) served as a first point of interconnection between federal and academic networks. At the time that commercial networks began appearing, general commercial activity on the Internet was restricted by Acceptable Use Policy (AUP), which prevented the commercial networks from exchanging traffic with one another using the NSFNET as the backbone. (Kende, 2000) In the early 1990s, a number of commercial backbone operators including PSINet, UUNET, and CerfNET established the Commercial Internet Exchange (CIX) for the purpose of interconnecting theses backbones and exchanging their end users' traffic. The NSF decided to cease to operate the NSF backbone, which was replaced by four NAPs. (Minoli and Schmidt, 1998, p28) The role of NAPs is similar to that of CIX. The NSF required that any ISP receiving government contracts or receiving money from public universities must connect to all of the NAPs. After the advent of CIX and NAPs, commercial backbones developed a system of interconnection known as peering.

There are 150 Internet exchange points (IXPs) in the worldwide ${ }^{31}$ irrespective of size and most of them are located at U.S. Companies will run public IXPs of various names such as NAP, MAE, and CIX as a service to the industry (Greene, 2000). Those companies price the service so they do not lose money. Their core revenues from running the IXP are gained from (1) infrastructure sold into the IXP, i.e. leased line, and (2) co-location business.

[^12]
### 3.2 Peering

The term "peering" is sometimes used generically to refer to Internet interconnection with no financial settlement ${ }^{32}$, which is known as a "Sender Keeps All (SKA)" or "Bill and Keep." Peering can be divided into several categories: (1) according to its openness, it can be private peering or public peering, (2) according to the numbers of peering partners it can be Bilateral Peering Arrangement (BLPA) or Multilateral Peering Arrangement (MLPA), and (3) according to the market in which it occurs, it can be primary peering in the backbone market or secondary peering in the downstream market.

The original four NAPs were points for public peering. As the Internet traffic grew, the NAPs suffered from congestion. Therefore, direct circuit interconnection between two large IBPs was introduced, so called bilateral private peering, which takes place at a mutually agreed place of interconnection. This private peering is opposed to public peering that takes place at the NAPs. It is estimated that 80 percent of Internet traffic is exchanged via private peering (Kende, 2000).

A peering arrangement is based on equality, that is, ISPs of equal size would peer. The measures of size could be (i) geographic coverage, (ii) network capacity, (iii) traffic volume, (iv) size of customer base, or (v) a position in the market. The ISPs would peer when they perceive equal benefit from peering based on their own subjective terms. (Kende, 2000)

The followings are the characteristics of peering:
(1) Peering partners only exchange traffic that originates with the customer of one ISP and terminates with the customer of the other peered ISP. As part of peering arrangement, an ISP would not act as an intermediary. And it would not accept the traffic of one peering partner for the purpose of transiting this traffic to another peering partner. This characteristic is called a "non-transitive relationship."
(2) Peering partners exchange traffic on a settlement-free basis. The only cost of each partner is its own equipment and the transmission capacity needed for the two peers to meet at each peering point
(3) Peering partners generally meet in a number of geographically dispersed locations. In order to decide where to pass traffic to another, they have adopted what is known as "hot-potato routing," where an ISP will pass traffic to another backbone at the earliest point of exchange.

[^13]According to Block (Cukier, 1999), there are two conditions necessary for the SKA peering, that is, peering with no settlement, to be viable: (1) The traffic flows should be roughly balanced between interconnecting networks; and (2) the cost of terminating traffic should be low in relation to the cost of measuring and billing for traffic. In sum, peering is sustainable under the assumption of mutual benefits and avoidance of costly, unnecessary traffic measuring. Peering partners would make a peering arrangement if they each perceive that they have more benefits than costs from the peering arrangement. Most ISPs in the U.S. historically have not metered traffic flows and accordingly have not erected a pricing mechanism based on usage. Unlimited access with a flat rate is a general form of pricing structure in the Internet industry. Finally, peering makes billing simple: no metering and no financial settlement.

Peering benefits come mainly from the network externality. Network externalities arise when the value or utility that a customer derives from a product or service increases as a function of other customers of the same or compatible products or services; that is, the more users there are, the more valuable the network is. There are two kinds of network externality in the Internet. One is direct network externality: the more E-mail users, the more valuable the Internet is. The other is indirect network externality: the more Internet users there are, the more web contents will be developed, which makes the Internet even more valuable for its users. The ability to provide direct and indirect network externalities to customers provides an almost overpowering incentive for ISPs to cooperate with one another by interconnecting their networks. (Kende, 2000) Another motivation for peering is lower latency because peering needs only one hop to exchange traffic between peering partners.

### 3.3 Transit

Transit is an alternative arrangement between ISPs, in which one pays another to deliver traffic between its customers and the customers of other provider. The relationship of transit arrangement is hierarchical: a provider-customer relationship. Unlike a peering relationship, a transit provider will route traffic from the transit customer to its peering partners. An IBP with many transit customers has a better position when negotiating a peering arrangement with other IBPs. Another difference between peering and transit is existence of a Service Level Agreement (SLA), which describes outage and service objectives, and the financial repercussion for failure to perform. In a peering arrangement, there is no SLA to guarantee rapid resolution of problems. In case of an outage, both peering partners may try to resolve the problem, but it is not mandatory. This is one of the reasons peering agreements with a company short on competent technical staff are broken. In a transit arrangement it is a contract and customers could ask the transit provider to meet the SLA. Many e-commerce companies prefer transit to peering for this reason. Since one minute of outage causes lots of losses to them, rapid recovery is critical to their business.

Furthermore, in the case of transit, there is no threat to quit the relationship while in the case of peering a non-renewal of the peering agreement is a threat. ISPs are not permitted to form transit relationship over public NAPs because these are designed as a neutral meeting place for peering. There is one exception: bypassing the public NAP switching fabric and running a backdoor serial connection between them.

### 3.4 Internet Interconnection Issue - Depeering

In 1996, $\mathrm{AGIS}^{33}$ was the first IBP to unilaterally terminate peering arrangements. After that, a series of IBPs announced that they were ending peering with many of their previous peering partners and were no longer accepting peering arrangements from other networks whose infrastructure would not allow the exchange of a similar traffic level. Instead of peering, they would charge those smaller ISPs for transit. Finally, the large IBPs moved away from public NAPs to a series of private peering or maintained relatively small capacities like T3 in the NAPs and then placed themselves in a new hierarchy, so called top-tier IBPs (Jew and Nicolls, 1999). Most top-tier IBPs are subsidiaries or affiliates of the major facilities-based telecommunication carriers. They are UUNET (Worldcom), C\&W, Genuity, AT\&T WorldNet, and Sprint, the 'so called' Big 5. They don't need transit service from others and they make peering arrangement with each other. Over $80 \%$ US backbone traffic is estimated to pass through their backbones (Weinberg, 2000). The other IBPs make peering arrangements among themselves and simultaneously purchase transit services from the Big 5.

There are two types of cases for which peering is generally refused: (1) Regional IBPs which do not have a national backbone network and (2) content providers or web hosting companies, so called the web farms. The main reason for this refusal is a free-rider issue. Under the hot-potato routing rule (shortest exit routing), someone who does not have a national backbone network must transport its traffic on the others' backbone networks. In addition to that, asymmetric traffic patterns, which occur in file transfer or web surfing, result in increased capacity costs without commensurate revenues.

Some of the Big 5 recently disclosed their policy for peering but some of them still do not. There is a kind of unwritten rule shared by the Big 5 about their peering standard: (i) A coast to coast national backbone with a certain level of bandwidth requirement, (ii) a number of presences in the major exchange points, (iii) 7 days by 24 hours Network Operation Center (NOC) and highly experienced technical staffs, and (iv) a certain level of traffic ratio between inbound and outbound, usually 1:4. We can not tell what the exact requirements for the private peering are since peering agreements are under non-disclosure. Without

[^14]doubt, these requirements could be a significant entry barrier for a new entrant. In Appendix A, peering policies of UUNET and C\&W are summarized.

PSINet, which was one of the large IBPs, used a peering standard called "open peering policy", that was different from others. It would peer with any ISP including local, regional, and national except for companies whose primary business was web hosting or content collection. Some of the Big 5 did not want to peer with PSINet, because some of PSINet's private peering partners are transit customers of the Big 5. Whenever the Big 5 upgrade their networks, they upgrade their peering policy. From the tier- 2 IBP's point of view, peering requirements are getting tougher and tougher. Nobody can enter into the top tier group without their approvals. This kind of cartel-like behavior has been an important issue in the Internet industry for several years.

After being refused peering in the backbone market, IAPs in the downstream access market, usually operating in a limited geographic region, tried to peer among themselves. Cremer and Tirole (2000) in their paper call this kind of peering "local secondary peering." This is a major factor in proliferation of local and regional Internet exchange points. These smaller exchange points (compared to NAPs) referred to as Metropolitan Exchange Points (MXPs) (OECD, 1998).

Most of Internet exchange points, or POPs of major IBPs are located near the metropolitan areas, which are far from the rural areas. The local IAPs in a rural area have to lease long expensive lines to reach an interconnection point. This long distance from private or public peering points is additional obstacle to overcome for the rural IAPs.

The Pittsburgh Internet Exchange ( PITX $^{34}$ ) is an example of local peering arrangement. Without this local peering, all network traffic passing from one Pittsburgh network to another has to be sent through Washington, D.C., Chicago, or New York City. The sending and receiving networks pay an unnecessary cost for this inefficient handling of data that should remain local. Participants in this local exchange point reduce their costs and improve performance and reliability for their local Internet traffic with the equal basis of cost recovery. However, the local peering is confined to only local traffic. Outbound traffic to other areas still has to depend on the IBP's transit service.

[^15]GigaPoP (Gigabit Point of Presence) is another future Internet exchange model, which is being realized by the Internet 2 project $^{35}$. The difference between NAP and GigaPoP is while a NAP is a neutral, Layer 2 meet point, a GigaPoP is a value-added, layer $2 / 3$ meet point. (Minoli and Schmidt, 1999) GigaPoP serves a peering point as well as a regional aggregation point. The consolidation of regional traffic allows members of GigaPoP to save cost by economies of scale and it gives them a market power to negotiate with other networks.

### 3.5 Peering Decision-Making Process

An interconnection strategy may be different according to its priority. If expense of interconnection is the number one issue, ISPs will try to find as many peering partners as they can and try to choose minimum combination costs among them. Or if performance is the top priority, they may prefer private peering or transit to public peering. All of interconnection decisions should start from the analysis of their own traffic. Then the ISP tries to find the available options and negotiates with their interconnection partners for interconnection methodology, interconnection line capacity, interconnection settlement, etc. This process will be explained below in detail. (Norton, 1999)

## Phase I: Identification of ISP's Traffic

The costs of peering and transit vary according to the distance of the ISPs' POP and interconnection point. Generally, the cost of transit is expensive than that of peering. Before deciding on a transit or peering arrangement, the ISP may systematically sample inbound and outbound traffic flows and then map these flows to the originating Autonomous System $\left(\mathrm{AS}^{36}\right)$. Calculations are made to determine where to reduce the load on the expensive transit paths.

## Phase II: Finding Potential Interconnection Partners

Based on the traffic map and its analysis, ISPs try to find interconnection partners. Because peering policies are often exposed only under Non-Disclosure Agreements (NDA), it is not easy to know them in advance of negotiations. It is reasonable for an ISP to find its peering partners in its own level of Internet industry hierarchy. If a top-tier IBPs makes a peering arrangement with a second tier IBPs, then the latter could be formers' competitor. Therefore, higher tier ISPs prefer selling transit service to lower tier ISPs and have an incentive to reduce the number of their own competitors. Many ISPs except for top tier IBPs

[^16]have adopted a hybrid approach to interconnection, peering with a number of ISPs and paying for transit from one or more IBPs in order to have access to the transit provider as well as the peering partners of the transit provider.

## Phase III: Implementing Interconnection Methodology

Since peering is seen as being of mutual benefit, both parties explore the interconnection methods that will most effectively exchange traffic. Both parties decide (1) how many interconnection points they have, (2) where to locate the interconnection points, (3) how they interconnect, direct circuit or exchange-based interconnection, (4) what line capacity they will use, (5) what kind of peering, settlement-free or ratio-based peering, etc.

Table 3-1 illustrates comparison of per Mega-bit cost of transit, private peering, and public peering. If we compare cost per Mbps shipped $\left(\mathrm{CPMS}^{37}\right)$ per month of OC3 capacity, the order from the cheapest is public peering $(\$ 30)$, private peering (\$64~\$129), and transit (\$464).
[Table 3-1] Per M-bit Cost Comparison

| Interconnection Type | Capacity | Cost / Capacity | Per M bit Cost |
| :---: | :---: | ---: | ---: |
| Transit | DS3 | $\$ 26,000 / 45 \mathrm{M}$ | $\$ 578 / \mathrm{M}$ |
|  | OC3 | $\$ 72,000 / 155 \mathrm{M}$ | $\$ 464 / \mathrm{M}$ |
| Private Peering | OC3 | $(\$ 10,000 \sim \$ 20,000) / 155 \mathrm{M} / 2$ | $\$ 64 / \mathrm{M} \sim \$ 129 / \mathrm{M}$ |
|  | OC12 | $(\$ 20,000 \sim \$ 30,000) / 622 \mathrm{M} / 2$ | $\$ 32 / \mathrm{M} \sim \$ 48 / \mathrm{M}$ |
| Public Peering | DS3 | $\$ 3,900 / 45 \mathrm{M}$ | $\$ 87 / \mathrm{M}$ |
|  | OC3 | $\$ 4,700 / 155 \mathrm{M}$ | $\$ 30 / \mathrm{M}$ |

- Source: AT\&T (Transit), Norton (2000, Private Peering), and Chicago NAP (Public Peering)


### 3.6 New Approaches

To overcome issues of free-rider and asymmetric traffic pattern under the current peering arrangement, new approaches are introduced: (1) Best Exit Routing and (2) Traffic Ratio-Based Peering. The peering burden upon the ISPs' networks is aggravated by the hot potato routing. The only solution to overcome this scenario is "best exit routing," which involves imposing responsibility on the web farm to carry the traffic to an exit point closest to the location of the IBP's customers. To overcome the current free peering problem, a traffic-ratio based paid peering model is emerging. In this approach, peering is free until traffic asymmetry reaches a certain ratio, i.e., $4: 1$. At this point, the net source of traffic will pay the net sink of traffic a fee based upon traffic flow above this ratio. (Norton, 1999)

[^17]
## [CHAPTER 4] QOS AND ITS TECHNOLOGY

In this chapter, we introduce concepts of QoS, MPLS, and Diffserv, which will be a basis in the network modeling part of this dissertation. At the end, we explain the general requirements for QoS peering.

### 4.1 Quality of Service

The Internet was historically built on the simple concept of BE: a datagram with source and destination addresses traverses different interconnected networks. Routing decisions are made independently at each node, and no guarantees are made for the reliable, timely delivery of data.

As communications services move to the Internet as a basic infrastructure, whether carriers have an ability to provide QoS or not is an emerging business issue. For example, with a growth of E-commerce and real time applications such as VoIP (Voice over IP) or video-conferencing, the customer demand for guaranteed service is rapidly increasing. Further, there are diverse Internet applications, each of which may have different quality requirements.

The term QoS was used in the Open System Interconnection (OSI) reference model ${ }^{38}$. The fundamental idea of QoS is 'to enable customers to ask ISPs to establish different levels of service pertaining to the customers' traffic throughput, traffic loss, and response time and to charge the customer for the actual level of service provided.' (Black, 2001, p12) The traffic requirements can be made up of four categories: (1) bandwidth, (2) delay, (3) delay jitter, and (4) traffic loss. The provision of bandwidth for an application means that the network has sufficient capacity to support the application's throughput requirements, measured in packets per second. Delay can be measured in round-trip-time (RTT), which is the time it takes to send a packet to a destination node and receive a reply from that node. Jitter is the variation in the delay between the arrivals of packets at the receiver. Traffic loss is the percentage of traffic discarded. Failure to meet the requirements of above four parameters can mean unsatisfactory service. (Black, 2000, pp.2-3)

This limitation of traditional BE service has not been a serious problem for traditional Internet applications such as e-mail and file transfer. Internet traffic has increased as the number of users and applications has increased. Internet traffic has not only increased, but it has changed in character. Today's Internet applications such as video-conferencing need high bandwidth and low latency. Especially, E-

[^18]commerce business traffic needs high reliability without even small traffic loss. Therefore, we need QoS, which can provide consistent, reliable, and timely data delivery.

There have been several approaches for the Internet Engineering Task Force (IETF) ${ }^{39}$ to provide QoS: Integrated Services (IntServ), Differentiated Service (DiffServ), and Multi-Protocol Label Switching (MPLS). The IntServ architecture is based on per-flow resource reservation. To receive resource assurance, an application must make a reservation before it can transmit traffic onto the network. Once the reservation is established, the application can start to send traffic over the path for which it has exclusive use of the resource. The IntServ model was the first attempt to enhance the Internet with QoS capabilities. (Wang, 2001) By the late 1990s, ISPs found that IntServ had a scalability problem. To find more scalable way to provide better than a BE service, DiffServ and MPLS were developed. However, none of above three approaches can provide a full QoS service throughout the whole Internet. The next generation Internet must integrate many of the qualities and attributes of switched networks. One of the recently emerging models is a DiffServ-MPLS model: MPLS QoS in the backbone network and DiffServ QoS in the access network. MPLS technology was developed in direct response to improve quality in carrier networks and DiffServ is the leading technology for QoS in the enterprise networks. Therefore MPLS can be used with DiffServ to address network layer QoS requirements for specific IP applications. We will see more detail for MPLS, DiffServ, and the MPLS-DiffServ model in the following sections.

### 4.2 Multi-Protocol Label Switching ${ }^{40}$

MPLS is a label swapping (mapping) and forwarding technology. The idea of MPLS is to improve the performance of network layer routing and the scalability of the network layer. The MPLS uses a concept called Functional (Forwarding) Equivalence Class (FEC). An FEC is associated with a prefix of IP address (network and sub-network part) and a class of datagrams such as an Internet destination port number, or values in the IP type of service (TOS), or IP protocol ID fields. Based on the FEC, a label is then negotiated between neighbor MPLS enabled routers from ingress to egress of a routing domain. (Black, 2001, p15)

According to Black (2001, pp.6-7), the basic idea of MPLS is similar to zip code in the postal system. A piece of mail is forwarded using the zip code to relay the mail. After the mail reaches its destination ZIP area, the address is used to forward the mail to its intended reader. The same idea holds for MPLS. An IP

[^19]packet is sent to an MPLS-enabled router, called an ingress Label Switching Router (LSR), for delivery to a destination IP address. The router appends a label to the packet. Therefore, the label, instead of the IP address, is used in the network to forward the traffic. Once the traffic has reached the end of the label path, i.e., an egress LSR, the IP address is used to make the final delivery to the end user. Figure 4-1 shows MPLS network components and a format of the MPLS label.

[Figure 4-1] MPLS Network and Label Format

The MPLS header is 32 bits long and is created at the ingress LSR. It must be behind any layer-2 headers and in front of a layer-3 header. Table 4-1 explains MPLS header.
[Table 4-1] MPLS Header

| Label | Label Value | 20 bits | MPLS label |
| :---: | :---: | :---: | :--- |
| Exp | Experimental Use | 3 bits | QoS classification with DiffServ |
| $\mathbf{S}$ | Stack | 1 bit | Used to stack multiple labels (Nested Multiple <br> MPLS structure) |
| TTL | Time to Live | 8 bits | laces a limit on how many hops the MPLS <br> packet can traverse |

The LSR creates a local binding for a particular FEC by arbitrarily selecting a label from a pool of free labels in the label information base and updating its LFIB (Label Forwarding Information Base). The LFIB includes incoming label, outgoing label, next hop, and outgoing interface. The LFIB incoming label is set to the label selected from the label pool. The next hop is set to the layer-3 address of the next hop associated with the FEC, and the outgoing interface is set to the egress interface used to reach the next hop. (Alwayn, 2002, p60) The route taken by an MPLS-labeled packet is called the Label Switched Path (LSP).

The packet is encapsulated into an outer packet, and the header of the outer packet has the label number. This idea is called a label switching tunnel, which means the inner packet is not examined by the internal LSRs within the network. Their only concern is the processing of the outer packet header's label and handling the packet. At the egress LSR, the packet is decapsulated, and the destination IP address, along with the other identifier, is used to determine how the packet is treated at the receiving node. (Black, 2001, p26)

### 4.3 Differentiated Service ${ }^{41}$

DiffServ (DS) is a unique layer-3 QoS technology and it can address a wide range of QoS requirements for a connectionless IP network. The key ideas of DS are: (1) to classify traffic at the boundaries of a network and (2) to condition this traffic at the boundaries. DS classifies packets according to their common characteristics. The collected packets are called Behavioral Aggregates (BAs). The traffic is assigned a value, a differentiated services code-point (DSCP), which uses the first 6 bits of the IPv4 type of service (TOS) field. After the packets have been classified at the boundary of the network, they are forwarded through the network based on the DSCP. The forwarding is performed on a per-hop basis, which is called Per-Hop Behavior (PHB). (Black, 2001, p158) The following table shows a matching between TOS field and DSCP field.
[Table 4-2] TOS and DSCP

| Bit position | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| TOS (IPv4) |  | Prece |  | D | T | R | C | MBZ |
| DS-Field | DSCP |  |  |  |  |  | CU (Currently Unused) |  |
|  | Classification |  |  | Drop-Priority |  |  |  |  |

* D: Delay, T: Throughput, R: Reliability, and C: Cost

[^20]The IPv4 type of service (TOS) field can be used to identify several QoS functions provided for an Internet application. Transit delay, throughput, reliability, and cost can be requested with this field. The DSCP is 6 bits in length. The remaining two bits of the TOS field are currently unused (CU). The leftmost bit signifies bit 0 of the field, and the right-most bit signifies bit 5 . The entire 6 bits of the DSCP are used by DS nodes as an index into a table to select a specific packet handling mechanism. The first 3 bits in the DSCP are used for classification and the last 3 bits in the DSCP are used for drop-priority.

DS domain is a network or a collection of networks operating under an administration with a common provisioning policy. DS domain consists of a contiguous set of nodes that are DS-compliant and DS domain also operates with common PHB definition. The PHB defines how a collection of packets with the same DSCP is treated. The PHB provides a hop-by-hop resource allocation and allows the support of differentiated services. A PHB group is a set of one or more PHBs. The PHB group allows a set of related forwarding behavior to be specified together. Once past the ingress node and inside the DS domain, the internal nodes continue to forward packets based on the DSCP. Their job is to map the DSCP value to a supported PHB. (Black, 2000, pp256-7) Figure 4-2 summarizes the DS network explained above.

[Figure 4-2] DS Network

The packets that are presented to a specific traffic conditioner constitute a Traffic Profile (TP). Figure 4-3 shows a logical view of the relationships of the key DS functions for DS packet classification and traffic conditioning operations. A traffic stream is selected by a classifier and sent to a traffic conditioner. DS uses the term Traffic Conditioning Block (TCB) to describe the overall conditioning operations. If appropriate, a meter is used to measure the traffic against a traffic profile. The result of the metering
procedure may be used to mark, shape, or drop the traffic based on the packet being "in-profile" or "out-of-profile." (Black, 2000, p260)

[Figure 4-3] Traffic Conditioning Block

Multiple DS domains constitute a DS region. Theses regions support differentiated services along paths in the domains that make up the DS region. One advantage to defining a DS region is that DS allows the DS domains in the same DS region to support different PHB groups. For example, a DS domain on a college campus may have different PHB group than a DS domain in a corporate network. These two peering DS domains must be able to inter-work in a predictable and structured manner. They must define a peering requirement that establishes a TCA for the traffic that flows between them. (Black, 2000, p258) Figure 44 shows the relationship between DS Domain and DS Region.

[Figure 4-4] DS Region and DS Domain

There are three classes of PHBs: (1) Default (DE), (2) Expedited Forwarding (EF), and (3) Assured Forwarding (AF). The default PHB is the conventional best-effort forwarding operation. A default PHB should be given some bandwidth. The default code-point for the default PHB is ' 000000 '. The code point for the expedited forwarding (EF) is '101 110.' The DS traffic conditioning block must treat the EF PHB as the highest priority of all traffic. Assured Forwarding (AF) enables a DS domain to support different levels of forwarding assurances for IP traffic. Four AF classes are defined in relation to three droppreferences. In case of congestion, the drop preference for the packet determines its relative importance within the AF class. Even if PHBs are implemented on both sides of a DS boundary, the DSCP still may be re-marked. Table 4-3 shows the DSCP values of AF PHB.
[Table 4-3] DSCP of AF PHB

| Drop Preference | Class 1 (AF1) | Class 2 (AF2) | Class 3 (AF3) | Class 4 (AF4) |
| :---: | :---: | :---: | :---: | :---: |
| Low | 001010 | 010010 | 011010 | 100010 |
| Medium | 001100 | 010100 | 011100 | 100100 |
| High | 001110 | 010110 | 011110 | 100110 |

### 4.4 Network Model: DiffServ-MPLS-DiffServ ${ }^{42}$

The IETF (Internet Engineering Task Force) has been considering how to integrate DiffServ and MPLS. Before I explain the network model, it would be better to compare the two protocols. Although MPLS is connection-oriented and DS is connectionless, there is a similarity in marking priority at boundaries in each network. Table 4-4 summarizes comparison of DS and MPLS (Stephenson, 1999).
[Table 4-4] Comparison between DiffServ and MPLS

|  | DiffServ | MPLS |
| :---: | :---: | :---: |
| Position | Layer-3 | Between Layer-2 and Layer-3 |
| Operation | Take IP TOS field and rename it DS <br> field and use it to carry information <br> about packet service requirement | Layer-3 traffic can be mapped to label <br> containing specific routing information to <br> each IP packet and allow routers to assign <br> explicit path to various classes of traffic |
| Decision Point | DS Ingress Router | Ingress LSR |
| Upgrade | Software Upgrade | New Investment for MPLS capable router <br> but compatible with Layer-2 switch |
| Type of Network | Enterprise Network | Core of Carrier Network |

[^21]AT\&T and Worldcom already started offering Class of Service (CoS) based on the MPLS. The other large IBPs have been upgrading their backbone network for the MPLS. According to the above table, even though MPLS technology is compatible with existing layer-2 backbone network technologies such as ATM (Asynchronous Transfer Mode) or FR (Frame Relay), a new investment in MPLS capable routers is needed. However, in the case of DS, if the current network routers are distribution layer routers ${ }^{43}$, upgrading an NOS (Network Operating System) makes it possible to use DS. Therefore, the MPLS core network and the DS access network model is a plausible combination. In this dissertation, we assume that IAP1 and IAP2 have a choice of DS and that IBP is equipped with MPLS.

How can the integration between MPLS and DS work? The boundary LSRs of MPLS network must have an ability to recognize DS packets. There are two requirements for the MPLS ingress LSR. It has to (1) be configured to support both MPLS and DS operations and (2) match DS PHB to MPLS flow.

MPLS header has a 3-bit field called "Exp", which can provide MPLS for up to eight DS classes $\left(=2^{3}\right)$. However, the length of DSCP is 6 bits, which supports up to 64 classes of service $\left(=2^{6}\right)$. In this case, there are two methods for matching classes between MPLS and DS according to the number of supporting classes: (1) Exp-LSP ${ }^{44}$ and (2) Label-LSP ${ }^{45}$.

### 4.4.1 Exp-LSP

By using the Exp field of MPLS header, a single LSP can be used to support up to eight BAs, which means that the PHB of the LSR is inferred from the Exp field. The first 3 bits of the DSCP field are copied into the MPLS Exp field at the ingress LSR. Each LSR along the LSP maps Exp bits to a PHB. Figure 4-5 explains how Exp-LSP can work in matching between MPLS and DS.

For example, in figure 4-6 there are only two classes of services (BE and Premium) and we could use Exp value 000 for BE class and 001 for Premium class. LSRs would be configured to provide default behavior to the 000 packets and EF PHB to the 001 packets. The MPLS label tells LSRs where to forward a packet while the Exp bits tell them what PHB to treat the packet with. In the following figure, a single Exp-LSP carries packets of two different PHBs. The PHB to be applied to packets is determined by examining the Exp bits: ‘ 000 ' for BE class and ' 001 ' for Premium class. (Davie and Rekhter, 2001)

[^22]
[Figure 4-5] Exp-LSP

[Figure 4-6] Exp-LSP Example

### 4.4.2 Label-LSP

If more than eight PHBs are needed in the MPLS network, Label LSPs are used. In the Label-LSP case, the PHB of LSR is inferred from the label. The label-to-PHB mapping is signaled. In the case of DE or EF , one PHB can be mapped onto one Label-LSP. However, in the AF case, AF packets of the same class, for example AF11, AF12, and AF13, can be aggregated into a FEC, which can be assigned into an LSP. We call a group of the same AF class PHB Scheduling Class (PSC). The drop precedence is encoded into the Exp field in the MPLS header. In the case of ATM or FR, where the MPLS header is not used, the PHB is carried in the VCI (Virtual Channel Identifier) field or DLCI (Data Link Connection Identifier) field and the drop precedence is encoded into the CLP (Cell Loss Priority) bit or DE (Discard Eligibility) bit. Figure 4-7 explains how Label-LSP can work in matching between MPLS and DS.

For example, there are 14 classes of services (EF, $\mathrm{AFxy}^{46}$, and DE). In figure 4-8, two Label-LSPs are established even with a same LSP: One for DF and the other for AF1 (AF11, AF12, AF13). The packets

[^23]with ' 000000 ' DSCP will take the lower Label-LSP for the DE PHB and the packets with ' 001 010', '001 100', and '001 110' DSCPs will take the upper Label-LSP for the AF1 PSC. For DF or EF traffic, a single LSP will carry packets of single PHB, and for AF traffic, an LSP will carry of the same AFx class with different drop preferences. In the Exp-LSP, a single LSP is enough to carry up to 8 classes but in the Label-LSP multiple LSPs are needed to carry various kinds of traffic classes. (Davie and Rekhter, 2001)

[Figure 4-7] Label-LSP

[Figure 4-8] Label-LSP

### 4.5 QoS Enabled NAP and QoS Peering

Even a large IBP can not cover all users everywhere, so a global company with many branches worldwide needs more than one IBP's connectivity. For example, if a branch in Europe has a service from UUNET
and another branch in Asia has a service from AT\&T WorldNet, UUNet and AT\&T WorldNet need to connect each other for the end-to-end integrated QoS service for their common customer. However, they have a reluctance to connect directly with each other since they have to show their MPLS network information to the other for the end-to-end QoS connection. This is critical business information, i.e., location of major interconnecting point, network capacity, and detailed routing technology. Therefore, an MPLS-enabled neutral NAP is inevitable for QoS peering. The router in the MPLS-enabled NAP has to connect two MPLS domains.

Traditional NAPs with an architecture based on LAN or ATM are under the control of big telecom carriers. For the QoS peering, many ISPs and IBPs want a neutral NAP, which does not belong to any carrier. In the QoS Internet, in addition to the BE peering requirements such as network interface capacity and routing power, peering ISPs have to consider (1) that they have the same CoS structure, (2) that they treat traffic of same class fairly, and (3) that they agree on traffic ratio, settlement rate, traffic loss penalty rate, and monitoring method. Each QoS peering ISP wants its traffic treated equally in the other's network as it treats its traffic in its own network. Therefore, each QoS peering ISP has to have an agreement of the following areas such as admission control, congestion control, and dropping discipline.
(1) Admission Control at the peering point: At the boundary router, traffic will be metered according to its class. If more traffic enters than its traffic profile, traffic rate management is needed. QoS peering ISPs should have an agreement on whether they will use traffic policing ${ }^{47}$ or traffic shaping ${ }^{48}$ for the bursting traffic. At the boundary router, token bucket will be used as a means to determine whether a packet is compliant or noncompliant. When metering traffic by a token bucket, parameters such as token arriving rate or conformed burst size must be agreed to in advance between two QoS peering ISPs, which will decide dropping policy for exceeding traffic against conformed burst point.
(2) Congestion Control at each network domain: After one ISP's QoS traffic is admitted into the other's network, the two ISPs have to agree on the treatment of traffic during network congestion. It is related to scheduling, queue management, and bandwidth allocation. If one peering ISP uses $\mathrm{WFQ}^{49}$ (Weighted Fair Queuing) as a scheduling discipline and the other uses strict priority

[^24]queuing ${ }^{50}$, the traffic of the same class would be treated differently in each network. QoS peering ISPs have to consider how to calibrate a weight to be served equally. For example, delay-sensitive traffic like VoIP, might be served by strict priority queuing and the other class of traffic might be served by the WFQ with a ratio of 5:3:2.
(3) Packet Dropping Policy: When the congestion happens, what kind of packet dropping approach will be used, tail-dropping or random early dropping (RED)? If both ISPs use the RED, which algorithm will be used, average queue size computation or packet drop probability? If the platinum queue is overflowing, dropping the excess traffic or transferring it to a lower level queue should be agreed in advance.
(4) Settlement Rate and Packet Loss Penalty Rate: In the premium classes (platinum, gold, and silver), it is possible to use traffic ratio based peering: The higher the class, the tighter the traffic ratio, i.e., 1:1 for platinum class, 1:2 for gold class, and 1:3 for silver class. Different settlement rates will be applied to the different classes: The higher the class, the higher the settlement rate, because of higher value of traffic in the higher class. The other important ratio in the QoS peering is the packet-dropping ratio. Even in the platinum class, there is a possibility of packet loss. If it faces congestion, overflow traffic might be transferred to lower level of classes or just dropped. In the case of real time application or business critical data, packet dropping can cause a serious problem for QoS. If a QoS peering partner cannot accommodate the other's traffic in profile, the former should pay the penalty for the dropped traffic. In this case, the higher the class, the higher the packet-dropping penalty.

[^25]
## [CHAPTER 5] BACKGROUND OF INTERNET QOS GAME MODEL

### 5.1 Model Components

There are two well-known models in the game theory: Cournot and Bertrand. In the Cournot model firms' output level is a strategic variable and in the Bertand model price of a product is a strategic variable. The main assumption of the above two models is homogeneity of the product and no entry. According to Carlton and Perloff (1999, pp155-7), game theory analyzes the interactions between rational, decisionmaking individuals who may not be able to predict fully the outcomes of their decisions. A duopoly game has three common elements: (1) there are two firms, (2) each firm attempts to maximize its profit, and (3) each firm is aware that other firm's actions can affect its profit. Game theory uses the concept of 'Nash equilibrium,' which is a set of strategies if, holding the strategies of all other firms constant, no firm can obtain a higher payoff (profit) by choosing a different strategy.

In this dissertation, we choose the Cournot duopoly model with an assumption that each IAP produces a homogeneous Internet access service with limited capacity. The sum of their products equals the market output, which determines the market price. In our model, beside output level, which is assumed to be a short-term variable, there are four long-term strategic variables: quality, network capacity, pricing, and interconnection. Once each IAP determines its long term decisions, it is assumed to maximize its profit simultaneously. In this model, we use a best-response function (or best-reaction function), which is a best action by a firm given its beliefs about the action its rival takes. (Carlton and Perloff, p159) The Nash equilibrium lies in the intersection of both firm's best response functions.

The basic model components are: (1) two IAPs (IAP1 and IAP2) in a rural area Internet access market without the possibility of new entrants, and (2) one IBP who can connect both of IAPs. The two IAPs can independently buy a transit service from the IBP for the Internet access, or they can peer with each other. Our concern is the Internet access market even if the IBP can influence the profits of the two IAPs. Figure 5-1 shows the basic model components.

[Figure 5-1] Basic Model Components

This duopoly and no entry scenario is a reasonable one to analyze even in a highly competitive Internet industry. According to Greenstein (1999), 2069 counties ( $66 \%$ ) out of 3139 in the U.S. had two or fewer IAPs in the fall of $1998 ; 87 \%$ of these 2069 counties are rural. While national IAPs usually concentrate in major urban areas and moderate density suburban areas, low-density rural areas are usually served by local providers. In these low-density areas, the national IAPs do not have local POPs ${ }^{51}$ (Points of Presences) so their customers would be forced to use measured service via a toll free number ${ }^{52}$. Thus, users of national IAPs have to pay a usage-based data communication fee as well as the IAP's subscription fee. An important competitive advantage of local IAPs is a lower access cost for the local population. In addition to the cost advantage against the national IAPs, the rural market is too small to sustain additional IAP beyond existing two IAPs. Therefore, we use 'no entry' assumption in the model even though the entry barrier in the access market is relatively low.

These two IAPs can produce BE or QoS . We assume that there is only one class in BE and there are two classes in QoS . The lower class of QoS is the same as BE and a higher class of QoS is a guaranteed service. Therefore, all QoS users are BE users in our model and the number of the QoS users as well as the BE users can influence a price of BE . The service with same quality is assumed to be homogeneous, i.e. there is no quality difference between QoS services from IAP1 and IAP2 and it is assumed that users' Internet connection time after introducing QoS will not change.

### 5.2 Internet Access Technology

The Internet access technologies are roughly divided into two categories: narrow band access using dialup modem technology and broadband access such as cable-modem, DSL, and wireless technology. Broadband access technologies will eventually replace the dial-up modem narrowband access technology in the future Internet access market. However, dial-up narrowband access is still dominant in the U.S. Internet access market and will be for the foreseeable future especially in rural areas. According to several research studies of Internet access technology's market penetration in Appendix B (U.S. GAO 2001, McClure 2001, Bright 2000, NTIA \& RUS 2000, and Grassman 2001), the penetration of broadband technology is $10 \%$ of households (or subscribers) in the U.S. In a recent paper (Strover, 2002), the prospects for near-term broadband services in rural region are dim and existing policy approaches appear insufficient to achieve the goal of widespread rural deployment. Therefore, we assume that subscribers in the rural area generally use dial-up modems to access their IAP, so our analysis will be restricted to 56

[^26]Kbps dial-up modem technology. Thus, the QoS services that subscribers might use are most likely audiooriented, such as Internet radio, VoIP or audio conferencing, and possibly also low bit rate video.

### 5.3 Pricing Scheme

The main pricing scheme in the current Internet access market is flat rate pricing, for example, unlimited Internet access for a fixed amount of money per month. A consumer can connect to the Internet as much as he wants, 24 hours a day and 7 days a week. The reasons to use flat rate pricing are: (1) IAPs do not have to meter their customer's traffic for billing, (2) customers prefer flat rating pricing to usage-sensitive pricing, and (3) flat-rate pricing encourages Internet usage because users do not worry about additional cost for usage. However, flat rate pricing leads to a 'tragedy of commons' phenomenon, i.e., the congestion of the Internet. After introducing QoS, it may be possible or even necessary to introduce usage-sensitive pricing, because the value of QoS traffic is higher than that of BE traffic. To reduce customers' resistance to pure usage-sensitive pricing, flat rate pricing will remain for BE and a new pricing scheme will be introduced for the new product $(\mathrm{QoS})$.

Generally speaking, when firms do not exactly know consumers' willingness-to-pay, a common approach is to use two-part tariff, which charges a lump-sum fee for the right to purchase plus a per-unit charge for each unit consumed. In the Internet industry, the two-part tariff is not fully efficient because the added fixed charges may deter some users who at marginal would be willing to join the network and consume. (Cawley, 1997) One QoS pricing scheme in the model is the two-part tariff: Flat-rate for the BE class and per-hour charge for the premium class. The fixed part lump-sum fee is the right to use the lowest class of the QoS service ( BE ) and the variable part is for the consumption of the premium classes of the QoS service. Someone who only wants to use a BE service pays only fixed part lump-sum fee. We assume that each IAP uses flat-rate pricing for its BE service and flat-rate or a two-part tariff for its QoS service.

### 5.4 Product Differentiation Theory

IAPs usually wish to differentiate themselves from other IAPs because product differentiation gives them market power. However, the current Internet, which can not guarantee any quality level, offers few opportunities for product differentiation except for increasing capacity to avoid congestion. This method, which we call 'over-engineering,' also has limitations because of the exponential increase of Internet traffic, Internet users, and IAPs.

According to Carlton and Perloff (1999, p79), product differentiation is defined as related products that have varying characteristics so that consumers do not view them as perfect substitutes. For example,

Apple computers (Mac) are not perfect substitutes for IBM compatible computers. This kind of product differentiation between Apple and IBM is called 'horizontal product differentiation.' The other product differentiation concept is 'vertical product differentiation.' In a vertically differentiated product space, all consumers agree on the most preferred mix of characteristics and, more generally, over the preference ordering. (Tirole, 1988, p96) For example, Internet users agree that the quality of QoS service is superior to that of BE service.

Professor Gal-Or ${ }^{53}$ has studied quality differentiation and its strategic usage. (Gal-Or, 1983, 1985, 1987) We apply her theory as a foundation of the QoS game theoretic modeling. There are several assumptions for the theory: (1) customers are uniformly distributed according to a single taste parameter, and the taste parameter determines user's utility and marginal utility, (2) products are differentiated on the basis of quality ${ }^{54}$, (3) each firm can choose levels of quality and quantity ${ }^{55}$, and (4) each consumer is assumed to purchase only one unit of the product if he expects a nonnegative payoff from the purchase.

The preference of a consumer of type X , when purchasing a product of quality $M(\geq 0)$, is represented by his willingness-to-pay function $U(M, X)$, where $U_{M}>0, U_{X}>0, U_{M X}>0, U_{M M} \leqq 0, U_{X X} \leqq 0$. The taste parameter $(X)$ of a consumer is assumed to be uniformly distributed over the compact interval $[0, X]$. The above assumptions imply that utility is gained by purchasing a product of higher quality $\left(U_{M}>0\right)$, that consumers of a higher taste parameter value the product more highly $\left(U_{X}>0\right)$, that those who value the product more highly gain more by buying products of higher quality ( $U_{M X}>0$ ), and that the utility derived from the product less than proportionately with the type of the consumer $\left(U_{X X} \leqq 0\right)$ and with the quality purchased $\left(U_{M M} \leqq 0\right)$. Under the above assumptions, each firm decides how much to produce at each quality level. At the Nash equilibrium: (1) this decision is the best response to the output decisions of all the other firms, given the optimization problem solved by consumers; (2) the price of each quality level is determined so the supply of products of quality equals the demand for it. (Gal-Or, 1983)

In the game model of this dissertation, we follow the same assumptions as described above: (1) Users are uniformly distributed according to the preference of QoS , (2) services are differentiated on the basis of ability to provide QoS, (3) each IAP can choose two quality levels for their service: BE (low quality) or

[^27]QoS (high quality) and various quantity levels between 0 and its maximum capacity ( 999,1999 , or 2999), and (4) each user is assumed to purchase only one of services, BE or QoS.

### 5.5 Demand Function

There are two demands in the Internet access market: (1) access demand and (2) usage demand. The demand for access means how much the subscriber would pay for the right to be connected to an IAP's network and the demand for usage means how many hours he would connect at whatever price is charged for usage. (Wenders, 1987, pp46-7).

The following two demands equations can be expressed as the access demand for the number of users $(Q)$ in the $[\mathrm{Eq}-1]$, and the usage demand for the number of QoS usage hours $\left(h_{Q o S}\right)$ in the [Eq-2]:

$$
\begin{align*}
& \mathrm{Q}=\mathrm{A}_{0}-\mathrm{A}_{1} * \mathrm{~F}-\mathrm{A}_{2}{ }^{*} \mathrm{r} \\
& \text {.Eq-1] } \\
& h_{\text {QoS }}=B_{0}-B_{1}{ }^{*} \mathrm{r} \tag{Eq-2}
\end{align*}
$$

where $Q$ is the number of users ( BE and QoS ), $h_{Q o S}$ is the number of QoS connection hours, $F$ is the fixed access price, and the $r$ is the hourly usage rate for QoS connection. The relationships between quantities $\left(Q, h_{Q o s}\right)$ and price factors $(F, r)$ are assumed to be negative, i.e., increasing prices means decreasing quantities.

There is no usage demand under flat-rate pricing. Only access demand equation exists with $r=0$ in [Eq1]. In the two-part tariff, there exist both demand functions. Table 5-1 shows values of coefficients we use in the game model. $\mathrm{A}_{0}=5,000$ means that when $\mathrm{A}_{1}=\mathrm{A}_{2}=0$, the maximum number of users in the market is $5,000 . \mathrm{A}_{1}=100$ means that one dollar increase in the fixed price causes 100 customers to stop using the service. $\mathrm{A}_{2}=500$ means that one dollar increase in the QoS usage rate $(r)$ causes 500 customers to stop using the service. $\mathrm{B}_{1}=33.3$ implies one dollar increase of the QoS usage rate ( $r$ ) causes 33.3 QoS hour reduction. $\mathrm{B}_{0}=100$ means that the maximum number of QoS Internet connection hours is 100 hours per month. The detailed explanation for the demand functions of each pricing scheme will be provided below.
[Table 5-1] Values of Coefficients

| Coefficients | $\mathbf{A}_{\mathbf{0}}$ | $\mathbf{A}_{\mathbf{1}}$ | $\mathbf{A}_{\mathbf{2}}$ | $\mathbf{B}_{\mathbf{0}}$ | $\mathbf{B}_{\mathbf{1}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Values | 5,000 | 100 | 500 | 100 | 33.3 |

### 5.5.1 Demand Function for Flat-Rate Pricing

We assume that the potential number of Internet user is 5,000, a number derived from demographic data. According to Greenstein (1999), in 1996 the IAPs in rural counties with under 50,000 population were
overwhelmingly local or regional, and in the fall of 1998 the equivalent figure was 30,000 . Extrapolating from this trend, we assume that the population of $2001^{56}$ in our model market is 25,000 . On average, $20 \%$ of the population would subscribe to dialup service. The estimated number of dial-up users was 60 million ${ }^{57}$ in 2001, compared with the U.S. population of 300 million. Applying this percentage (20\%) to our model market, we compute the number of potential users to be 5,000 .

Each user is assumed to purchase exclusively one service, i.e., BE or QoS , from one IAP. Because BE is a lowest class of QoS, a QoS user does not have to buy BE and QoS simultaneously. Furthermore, the number of Internet connection hours is assumed to be constant after introducing QoS. Under these consumer behavior assumptions, we will use the same willingness-to-pay function as Gal-Or did (1983, 1985):

$$
U(X, M)=M^{*}(a+b X), a>0, b>0,
$$

where $M=$ level of quality, $X=$ taste factor (i.e., preference to QoS service), and $U_{M}>0, U_{X}>0, U_{M X}>0$, $U_{M M} \leqq 0, U_{X X} \leqq 0$.

The user payoff ( $V$ ) is determined by the difference between the user's willingness-to-pay from the consumption of a product and the price of that product. For example, the payoff for a consumption of BE is $V_{B E}=U\left(X, M_{B E}\right)-P_{B E}$, and the payoff for a consumption of QoS is $V_{Q o S}=U\left(X, M_{Q o S}\right)-P_{Q o s}$. The user will choose his consumption according to the positive value of his payoff, i.e., if $V_{B E}>V_{Q O S}$, then he would choose BE, otherwise he would choose QoS. We can sort the users according to their preference to QoS, i.e., begin with the user with the least QoS preference to the highest preference user. Figure 5-2 shows the ordering of users.

[Figure 5-2] QoS Preference Line-Up

[^28]In the above figure, there are two special users: $X^{*}$ and $X^{* *} . X^{*}$ is a marginal user whose payoff is the same when he consumes QoS or BE. $\mathrm{X}^{* *}$ is a marginal user whose payoff is the same when he consume BE or nothing. The order of $\mathrm{X}^{*}$ in the line equals (5000- $\left.Q_{Q o S}\right)$ and that of $\mathrm{X}^{* *}$ equals $\left(5000-Q_{Q o s}-Q_{B E}\right)$. The following equation expresses the above two users' payoff with $\left(\mathrm{X}^{*}=5000-\mathrm{Q}_{\mathrm{Qos}}\right)$ and $\left(\mathrm{X}^{* *}=5000-\right.$ $\left.\mathrm{Q}_{\mathrm{Qos}}-\mathrm{Q}_{\mathrm{BE}}\right)$.
(1) For the marginal user $\mathrm{X}^{* *}$,
$($ Payoff of not purchasing $)=($ Payoff of BE consumption $)$

$$
\begin{align*}
0 & =\left(\text { Willingness-to-pay of BE at } \mathrm{X}^{* *}\right)-(\text { Price of } \mathrm{BE}) \\
0 & =\left[\mathrm{a}+\mathrm{b}^{*}\left(5000-\mathrm{Q}_{\mathrm{QoS}}-\mathrm{Q}_{\mathrm{BE}}\right)\right]^{*} \mathrm{M}_{\mathrm{BE}}-\mathrm{P}_{\mathrm{BE}} \\
\mathbf{P}_{\mathbf{B E}} & =\left[\mathbf{a}+\mathbf{b}^{*}\left(\mathbf{5 0 0 0}-\mathbf{Q}_{\mathrm{QoS}^{-}} \mathbf{Q}_{\mathbf{B E}}\right)\right]^{*} \mathbf{M}_{\mathbf{B E}} \ldots \ldots \ldots \ldots \ldots \ldots . \tag{Eq-3}
\end{align*}
$$

(2) For the marginal user $X^{*}$,
$($ Payoff of BE consumption $)=($ Payoff of QoS consumption $)$
(Willingness-to-pay of BE at $\mathrm{X}^{*}$ ) - $\mathrm{P}_{\mathrm{BE}}=\left(\right.$ Willingness-to-pay of QoS at $\mathrm{X}^{*}$ ) $-\mathrm{P}_{\mathrm{QoS}}$

$$
\begin{align*}
{\left[\mathrm{a}+\mathrm{b}^{*}\left(5000-\mathrm{Q}_{\mathrm{QoS}}\right)\right]^{*} \mathrm{M}_{\mathrm{BE}}-\mathrm{P}_{\mathrm{BE}} } & =\left[\mathrm{a}+\mathrm{b}^{*}\left(5000-\mathrm{Q}_{\mathrm{QoS}}\right)\right]^{*} \mathrm{M}_{\mathrm{QoS}}-\mathrm{P}_{\mathrm{QoS}} \\
\mathbf{P}_{\mathrm{QoS}} & =\mathbf{P}_{\mathbf{B E}}+\left[\mathbf{a}+\mathbf{b}^{*}\left(\mathbf{5 0 0 0}-\mathbf{Q}_{\mathrm{Qos}}\right)\right] *\left(\mathbf{M}_{\mathbf{Q o S}}-\mathbf{M}_{\mathbf{B E}}\right) . \tag{Eq-4}
\end{align*}
$$

In the above two equations, we have to be careful to understand total quantity $\left(=\mathrm{Q}_{\mathrm{QoS}}+\mathrm{Q}_{\mathrm{BE}}\right)$, which means the quantity demanded by the users who can use BE, because QoS users can consume BE as one of QoS classes.

We can say that [Eq-3] is an inverse demand function for the BE service and [Eq-4] is an inverse demand function for the QoS service. We assume the number of classes that each IAP can produce with its chosen technology could be a value of $M$, i.e., a value of quality level. For example, the quality level of BE, i.e., $M_{B E}=$ ' 1 ' and the quality level of QoS, i.e., $M_{Q o S}=$ ' 2 ' because the QoS includes the BE as a subset of classes. We can put $M_{B E}=' 1$ ' and $M_{Q o S}=$ ' 2 ' into [Eq-3] and [Eq-4], which can be rewritten as:

$$
\begin{aligned}
& \mathrm{P}_{\mathrm{BE}} \quad=\left[\mathrm{a}+\mathrm{b}^{*}\left(5000-\mathrm{Q}_{\mathrm{QoS}}-\mathrm{Q}_{\mathrm{BE}}\right)\right]^{*} 1 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . .[\mathrm{Fq}-5] \\
& \mathrm{P}_{\mathrm{QoS}}=\mathrm{P}_{\mathrm{BE}}+\left[\mathrm{a}+\mathrm{b}^{*}\left(5000-\mathrm{Q}_{\mathrm{QoS}}\right)\right]^{*}(2-1) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots .[\mathrm{Fq}-6]
\end{aligned}
$$

The current price range of dial-up access service is expected as $\$ 0$ to $\$ 25$ per month ${ }^{58}$. If we assume that all potential customers will buy BE access service with $\$ 0$ per month, the number of customers will be 5,000, i.e., $\left(P_{B E}, Q\right)=(\$ 0,5000$ users $)$, becasue $\left(Q_{Q o S}+Q_{B E}\right)$ in the $[\mathrm{Eq}-5]$ means that the number of users

[^29]who can consume a BE service or a BE class as a part of QoS . If we put this pair of numbers into equation [Eq-5], we can easily get a value of parameter ' $a$ ' $(=0)$. If we assume the price of DSL service is a ceiling price for dial-up access, $\$ 50$ is the maximum price for dial-up access. Over $\$ 50$ per month, a rational user will not buy the dial-up access. Instead he will buy DSL service because it is a superior service. Then, we can put $\left(P_{B E}, Q\right)=(\$ 50,0$ user) into the $[\mathrm{Eq}-5]$ and we get a value of parameter $b(=0.01)$. We can rewrite the equation $[\mathrm{Eq}-5]$ and $[\mathrm{Eq}-6]$ with $a=0$ and $b=0.01$.
where $\mathrm{Q}_{\mathrm{BE}}=\mathrm{q} 1_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{BE}}$ and $\mathrm{Q}_{\mathrm{Qos}}=\mathrm{q} 1_{\mathrm{Qos}}+\mathrm{q} 2_{\mathrm{QoS}}$.

From the [Eq-7], the BE demand function expresses that $\$ 1$ increasing for local Internet access results in 100 users leaving. This value is rather price sensitive, which conflicts a general idea of insensitivity of Internet access service. Usually, the demand for Internet access service is considered to be insensitive to price. The reasons for price-insensitivity are:
(1) The access demand is considered less sensitive than the usage demand. Under flat-rate pricing, user's price includes both usage and access prices. But precisely speaking, because the quantity that users can consume is unbounded, there is no usage demand and the usage price is zero. (Wenders, 1987, p46). Therefore, Internet access service itself is insensitive to the price.
(2) Network externality effect. A potential user has a tendency to subscribe to the same IAP of his friends and family for better and reliable communications between them. Even if monthly price is higher than he is willing to pay, he prefers to choose the IAP with as many acquaintances as possible.
(3) Customer lock-in effect. If someone wants to change his current IAP, he has to tolerate the inconvenience of notifying his correspondents of his new e-mail addresses. That is the same as the local number portability ${ }^{59}$ issue in the telephone industry, though services such as hotmail, yahoo and others minimize this effect by offering addresses that are not associated with IAPs.

However, there are two supporting ideas for our assumption that the demand function that is sensitive to price:
(1) According to MacKie-Mason and Varian (1995), there are two types of users. One has a very high value for the service, but only wants to use a little of it, i.e. ASCII e-mail. The other user has a

[^30]low willingness-to-pay for the service but wants to consume a very large amount of it, i.e, teenager's downloading MTV videos. The demand function for high value users is insensitive to the price but the demand function for low value users is sensitive to the price because they are marginal users. With the advent of broadband Internet access service, the high value users for dial-up Internet access have been moving to broadband. Therefore, the fraction of low value users in the dial-up Internet access market is increasing, which means the demand function is becoming more sensitive to the price.
(2) The USGAO (2001, pp26-27) reports that the largest percentage of users (35\%) indicates that price is a basis for their choice of dial-up IAP. But among the broadband users, the most common reason is that they selected their IAP because it was the company that provided the features and applications of most interest to them ( $23 \%$ ), which means price is not a top concern of those who chose a broadband IAP. Thus, we stand by our assumption that dial-up users in our model are relatively price-sensitive.

We can interpret the above two equations [Eq-7] and [Eq-8] as follows:
(1) Under the BE demand function [Eq-7], we can derive total quantity demanded $(Q)$ in the market. The total demand means the quantity consumed for BE and QoS , which can be interpreted as the demand by users who want to consume the BE class of service.
(2) We can derive the price of $\mathrm{BE}\left(P_{B E}\right)$, which corresponds to the total quantity demanded ( $Q=Q_{Q O S}$ $+Q_{B E}$ ) in the [Eq-7].
(3) We can put this price $\left(P_{B E}\right)$ into the QoS demand function $[\mathrm{Eq}-8]$ and find QoS quantity $\left(Q_{Q o S}\right)$ and QoS price $\left(P_{Q O S}\right)$.
(4) BE quantity ( $Q_{B E}$ ) is residual demand after subtracting QoS demand ( $Q_{Q_{O} S}$ ) from total quantity $(Q)$.

Figure 5-3 explains the process of determination of $Q_{Q O S}$ and $Q_{B E}$.

[Figure 5-3] The process of determination of $Q_{Q o s}$ and $Q_{B E}$

### 5.5.2 Demand Function for Two-Part Tariff

In this section we try to explain the coefficient value of $\mathrm{A}_{2}, \mathrm{~B}_{0}$, and $\mathrm{B}_{1}$ in the equation of $[\mathrm{Eq}-1]$ and $[\mathrm{Eq}-$ 2], which are essential to the two-part tariff. For the value of $\mathrm{A}_{2}$, we don't have any data because the QoS service does not exist at present. The assumptions for $\mathrm{A}_{2}$ are (1) QoS hourly rate ( $r$ ) also influences a quantity of access market and (2) its relationship between $r$ and $Q$ in the [Eq-1] is also negative. We tested several values of $\mathrm{A}_{2}$ such as ' 0.1 ', ' 0.5 ', ' 1 ', ' 5 ', ' 10 ', and ' 50 ' and finally $\mathrm{A}_{2}=5$ chose considering a reasonable range of other variables like $\mathrm{Q}_{\mathrm{BE}}, \mathrm{Q}_{\mathrm{Qos}}$, and profits of IAP1 and IAP2.

To estimate the usage demand function of [Eq-2], we use the long distance telephone price as a proxy for VoIP tariffs, which we anticipate to be a major application of the QoS service in the rural dialup market. We assume that 5 cents per minute, i.e., $\$ 3$ per hour, is a typical competitive long distance rate in the United States. From the USGAO report on Internet usage (2001), we can derive an estimate for an average number of connection hours ${ }^{60}$ as shown in table 5-2.
[Table 5-2] Internet Usage

| Answer | $\sim 1 \mathrm{hr}$ | $\sim 4 \mathrm{hrs}$ | $\sim 10 \mathrm{hrs}$ | $\sim 15 \mathrm{hrs}$ | $\sim 25 \mathrm{hrs}$ | $\sim 40 \mathrm{hrs}$ | $\sim 60 \mathrm{hrs}$ | $60 \mathrm{hrs} \sim$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Percent | 0.0 | 6.3 | 12.1 | 19.4 | 29.3 | 19.8 | 6.3 | 6.9 |

[^31]Based on the above table, we can use 100 hours ${ }^{61}$ per month as the typical monthly Internet connection time. We assume that after introducing QoS the 100 hours would be divided into BE connection hours and QoS connection hours. Total number of connection hours is assumed to be constant. Users of flat rate QoS pricing will use $100 \%$ QoS connection but users of two-part tariff will not make a QoS connect all the time. Our assumptions for the relationship between hourly rate ' $r$ ' and QoS connection hours ' $h_{Q o s}(r)$ ' are:
(1) The higher the rate $r$, the lower the QoS usage percentage,
(2) A range of rate ' $r$ ' is from $\$ 0$ to $\$ 3$ per hour.
(3) For example, if $r=\$ 0$, all users use $100 \%$ QoS connection. If $r=\$ 3$, no one uses QoS connection. Instead users will use a long distance telephone service. In the absence of market data for the QoS, we assume linearity between these endpoints, resulting in table 5-3.
[Table 5-3] Relationship between Usage Rate ( $r$ ) and QoS Usage ( $h_{Q o s}$ )

| $\boldsymbol{r}$ | $\$ 0$ | $\$ 1$ | $\$ 2$ | $\$ 3$ |
| :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{h}_{\text {OOS }}$ | 100 | 67 | 33 | 0 |
| $\boldsymbol{h}_{\text {BE }}$ | 0 | 33 | 67 | 100 |

According to the above assumptions, we can rewrite [Eq-1] and [Eq-2] into the following [Eq-9] and [Eq10]:

$$
\begin{align*}
\mathrm{F} & =50-0.01 * \mathrm{Q}-5 * \mathrm{r} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots  \tag{Eq-9}\\
\mathrm{r} & =3-(1 / 33.3)^{*} \mathrm{~h}_{\mathrm{QoS}} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots
\end{align*}
$$

In our assumption, the fixed part of two-part tariff is the price of BE service, so [Eq-9] can be rewritten as

$$
\begin{equation*}
\mathrm{P}_{\mathrm{BE}}=50-0.01 * \mathrm{Q}-5 * \mathrm{r} \tag{Eq-11}
\end{equation*}
$$

$\qquad$

These equations imply that the QoS usage rate $(r)$ as well as the number of users $(Q)$ can influence the price of BE service. The demand function of the [Eq-11] will be used when the two-part tariff is involved in the game model, i.e., Cases $3,4,6,8$, and 10 .

### 5.6 Revenue Function

The revenue function of each IAP is assumed to have generally two components: (1) the revenue obtained from the subscribers and (2) those received from the advertisers that present their announcements in the IAP's web pages. In our analysis these two revenues will be calculated on a monthly basis.

[^32]The subscription revenue is directly related with each IAP' price structure and quantity demanded in the market situation. If an IAP produces QoS with a flat-rate, its revenue is sum of BE revenue ( $=Q_{B E} * P_{B E}$ ) and QoS revenue ( $=Q_{Q o s} * P_{Q O S}$ ), otherwise, BE revenue alone. If the IAP chooses a usage-sensitive price structure for QoS , then QoS subscription revenue will have a different form, $\left(=Q_{Q o s} * P_{B E}+Q_{Q o s} * h_{Q o s} * r\right)$. This means that all QoS users have to pay $P_{B E}$ for their connection and right to use BE class, and they have also to pay additional QoS usage.

The advertising revenue is similar to that of newspaper and broadcasting industries. In reality, the number of hits on a specific advertisement in web pages determines payment for that advertisement, but in our model we simply assume that the monthly advertising revenue per subscriber is constant. According to AOL's 2000 annual report ${ }^{62}$, the advertising revenue was $\$ 2,000$ million. If we assume that the number of AOL's subscribers in 2000 is 20 million $^{63}$, the average monthly advertising revenue was approximately $\$ 8$ per subscriber. However, the rural IAPs will probably not earn the same per subscriber in advertising revenue as AOL. Because of their small size and scope, the IAPs with local coverage are not as attractive for advertisers as AOL with national coverage and large customer base. With the absence of rural IAP's advertising rate, we will use $\$ 8$ per subscriber as an advertising rate. From this point of view, the number of subscribers is an important factor for the advertising revenue of IAPs. This type of revenue can justify increase of capacity with a lower price of Internet access service to acquire more subscribers. In an extreme case, subscription price per month may be reduced to zero and only the advertising revenue can be a source of income for an IAP.

### 5.7 Cost Function

The cost structure of the Internet industry is characterized by large, up-front sunk costs and near zero, short run marginal traffic cost. It is well known that with congestion-free network the cost to carry or process an additional minute of Internet traffic approaches zero, because the incremental cost is near zero. (Frieden, 1998) In our model, the measuring unit of cost is not traffic but a subscriber, i.e. the cost is calculated by the number of subscribers. The basic assumptions of the cost structure in our model are (1) large, up-front, irreversible sunk cost and (2) low constant marginal cost for additional subscriber. Under these assumptions, the duopolists must cover the following three types of costs to be able to provide their services: capital ( $\mathrm{c}_{\mathrm{c}}$ ), interconnection (transit: $\mathrm{c}_{\mathrm{t}}$ or peering: $\mathrm{c}_{\mathrm{p}}$ ), and operation costs ( $\mathrm{c}_{\mathrm{o}}$ ). Capital and

[^33]interconnection costs are evaluated in similar ways, and are characterized simultaneously. Operation costs are treated differently and will be discussed below.

### 5.7.1 Cost of BE-IAP

The operation costs are related with the number of subscribers. The operatinon cost includes the set-up cost for network connectivity such as login account, allocation of storage, user registration, etc. and maintenance costs for a single user of the network. These types of costs increase as the number of users increase. The operation cost for the BE service is assumed to be linear function.

The capital costs are consist predominantly of the equipment that an IAP needs to provide its services, that is, mail-server, access layer router, and modem pool. The transit costs are payments by an IAP to an IBP for the right to use the IBP's facilities to transmit the communications of the IAP's subscribers. Although the price of bandwidth is decreasing substantially and the demand for T3 lines ${ }^{64}$ and optical links are increasing, a T1 line still dominates the market ${ }^{65}$ especially for rural IAPs. We assume that the IBP sells only T1 connections to the IAPs, which is reasonable given that these are small IAPs serving a rural area.

It is assumed here that these two types of costs (capital and interconnection) increase in equal steps. This means that an IAP to provide services to 0 to $n-1$ subscriber(s) must purchase equipment worth $\$ c_{c}$ and must pay $\$ c_{t}$ to an IBP for transit capabilities. For $n$ to $2 * n-1$ subscribers the cost increases to $2 *\left(c_{c}+c_{t}\right)$, and so on. When the number of subscribers of a duopolist ranges between $k^{*} n$ and $(k+1) * n-1$, the costs of the duopolist are $(k+1) *\left(c_{c}+c_{t}\right)$.

We assume that the value $n$ is $1,000 \mathrm{BE}$ subscribers. The calculation of $1,000 \mathrm{BE}$ subscribers per one set of equipment and one T 1 line is made under the following assumptions:
(1) The capacity between CO and IAP modem pool is determined by a concentration ratio of 1:10 (the number of modems to the number of subscribers). That means 100 modems are enough to support 1,000 subscribers.
(2) The capacity between the IAP and the IBP is determined by several factors. We already assume T1 line ( $=1.544 \mathrm{Mbps}$ ) and 56 Kbps modems as a basic connection, and we add 1:6 bandwidthratio to this assumptions. The bandwidth-ratio occurs because a user does not consume 56 Kbps

[^34]for the duration of the connection. Therefore, $162(=1.544 \mathrm{Mbps} / 56 \mathrm{Kbps} * 6)$ users can access the Internet simultaneously at one time.
(3) Peak load time is used to calculate the Internet traffic capacity; the standard duration of peaks in the industry is assumed to be 4 hours a day. If we assume the average holding time per user is 30 minutes, the number of users using the Internet during 4 peak hours is 1,296 users $(=162 *(4 \mathrm{hrs} * 60 \mathrm{~min} / 30 \mathrm{~min}))$. From a network-engineering point of view, IAPs try to plan to have at least $20 \%$ excess capacity at their peak times, and therefore $1,000 \mathrm{BE}$ subscribers per T1 line is reasonable number.
(4) It is assumed that the value of the equipment ${ }^{66}\left(c_{c}\right)$ is $\$ 9,000$, which comes from the retail price of an access router, a server, and 100 modems in 2001. The value of transit $\operatorname{cost}^{67}\left(c_{t}\right)$ is assumed to be $\$ 1,000$. The operation cost $\left(c_{o}\right)$ per subscriber is assumed to be $\$ 1$. In summary, the IAP will spend $\$ 10,000$ of capital and transit costs for the first $(1,000-1)$ subscribers before it start its business and it will spend $\$ 1$ for every subscriber. When the IAP has a plan for network capacity to support up to $(2,000-1)$ users, the IAP will spend $\$ 20,000(=2 * \$ 10,000)$ for the costs of capital and interconnection. Therefore, $\$ 10,000$ can be viewed as a lump sum cost which is independent of number of customers within the network capacity.

### 5.7.2 Cost of QoS-IAP

The operation cost for QoS is assumed to be exponential. Comparing the BE , there might be a lots of operational works for the QoS-IAP such as (1) classification of traffic, (2) monitoring QoS measurement like delay, jitter, packet loss rate, and (3) complicated billing by multi-dimensions such as time, class, destination. We assume that the complexity of QoS operation will increase exponentially as the number of subscribers increases. There is a difference in operation cost with different QoS pricing: $\left(\mathrm{q} 1_{\mathrm{Qos}}\right)^{\wedge} 1.5$ is used for the QoS-IAP with the flat-rate pricing and ( $\mathrm{q} 2 \mathrm{Qos}^{\mathrm{s}}{ }^{\wedge} 1.6$ is used for the QoS-IAP with the two-part tariff. Since the usage-sensitive pricing has more complicated (and therefore more costly) measurement and billing functions than the flat-rate pricing, we assign powers of 1.6 for the two-part tariff and 1.5 for the flat-rate pricing for the operation cost functions. These numbers are chosen after testing several others considering a reasonable range of variables and profits.

For capital cost of the QoS-IAP, since currently ${ }^{68} \mathrm{QoS}$ is not supported by an access layer router, a distribution layer router with DiffServ capability $(\$ 10,000)$ is needed for QoS service and the distribution

[^35]layer router has an enough capability to support 5,000 subscribers. The QoS-IAP also needs server ( $\$ 3000$ ) and 100-modem pool (\$3000) by every 1,000 subscribers. However, the QoS-IAP does not worry about the capital cost for the BE service because QoS equipment produces BE as well as QoS. As a transit cost of QoS-IAP, two T1 lines are assumed to support 1,000 QoS subscribers for guaranteed service, which costs $\$ 2,000$. The reason to use twice numbers of T1 lines for QoS service comes from the previous assumption that the quality ratio between BE and QoS is $1 / 2$, i.e., $M_{B E}=1$ and $M_{Q o S}=2$.

### 5.7.3 Cost of Peering

In the case of peering, we replace the transit cost for peering cost. For BE-IAPs' peering, they need at least one distribution layer router $(\$ 10,000)$ to connect both access routers, which will be taken charge of equally ( $\$ 5,000$ per IAP) and they need T1 lines to connect each other. The number of T1 lines in the peering arrangement is determined by maximum network capacity between the two IAPs, i.e., Max \{kl, $k 2\}$ because a peering IAP has to take care of the other's traffic as well as its own traffic. For example, if the IAP1 has 2K network capacity and the IAP2 has 1K network capacity, both IAPs need two T1 lines to support both customers' traffic. The leased T1 price is assumed to be $\$ 150$ in the local market ${ }^{69}$.

Figure 5-4 shows the BE-peering structure.

[Figure 5-4] BE-Peering

For QoS-IAPs' peering cost, since they already have distribution-layer routers, they only need T1 lines to support both IAP's customers' traffic. For guaranteeing service, they need two T1 lines per 1,000 users, which costs $2^{*} \operatorname{Max}\{k 1, k 2\}^{*} \$ 150$. Figure $5-5$ shows the QoS-peering structure.

[Figure 5-5] QoS-Peering

[^36]Table 5-4 summarizes cost functions of the IAP1 used in the game model. The same cost function is applied to the IAP2.
[Table 5-4] Cost Functions

| Category |  | Parameter | BE-IAP | QoS-IAP (flat-rate pricing) | QoS-IAP (two-part tariff) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Capital |  | $\mathrm{c}_{\mathrm{c}} / 1,000$ | k1*9,000 | 10,000+kI*6,000 | 10,000+kI*6,000 |
| Inter-connection | Transit | $\begin{gathered} \hline \mathrm{c}_{\mathrm{t}} / 1,000 \\ \text { subscribers } \end{gathered}$ | k1*1,000 | k1*\$2,000 | k1*2,000 |
|  | Peering | $\begin{gathered} \hline \mathrm{c}_{\mathrm{p}} / 1,000 \\ \text { subscribers } \end{gathered}$ | $\operatorname{Max}\{k 1, k 2\}^{* 150}$ | 2*Max $\{k 1, k 2\} * 150$ | 2*Max $\{k 1, k 2\}{ }^{*} 150$ |
| Operation |  | $\mathrm{c}_{0} /$ subscriber | \$1*q1 $1_{\text {BE }}$ | \$1* $\mathrm{q}_{1 \mathrm{BE}}+\left(\mathrm{q} 1_{\mathrm{QoS}}\right)^{\wedge} 1.5$ | \$1* $1^{\text {BE }}+\left(\mathrm{q} 1_{\mathrm{QoS}}\right)^{\wedge} 1.6$ |
| Total |  | Transit | \$1*q1 $1_{\mathrm{BE}}+k 1^{*} 10,000$ | $\begin{gathered} \$ 1 *{ }^{*} 1_{\mathrm{BE}}+\left(\mathrm{q} 1_{\mathrm{QOS}}\right)^{\wedge} 1.5 \\ +10,000+k 1 * 8000 \end{gathered}$ | $\begin{gathered} \$ 1 * \mathrm{q}_{\mathrm{BE}}+\left(\mathrm{q} 1_{\mathrm{OoS}}\right)^{\wedge} 1.6 \\ +10,000+\mathrm{n} 1 * 8000 \end{gathered}$ |
|  |  | Peering | $\begin{gathered} \$ 1 * \mathrm{q}_{\mathrm{BE}}+5,000 \\ +k 1 * 6,000 \\ +\operatorname{Max}\{k l, k 2\} * 150 \\ \hline \end{gathered}$ | $\begin{gathered} \hline \$ 1 *{ }^{*} 1_{\mathrm{BE}}+\left(\mathrm{q} 1_{\mathrm{Qos}}\right)^{\wedge} 1.5 \\ +10,000+k 1 * 6000+ \\ 2 * \operatorname{Max}\{k 1, k 2\}^{*} 150 \end{gathered}$ | $\begin{gathered} \hline \$ 1 * \mathrm{q}_{\mathrm{BE}}+\left(\mathrm{q} 1_{\mathrm{os}}\right)^{\wedge} 1.6 \\ +10,000+k 1 * 600+ \\ 2 * \operatorname{Max}\{k 1, k 2\} * 150 \end{gathered}$ |

## [CHAPTER 6] ANALYSIS OF UNIFORM DISTRIBUTION

According to the demand function based on the uniform distribution, we make 10 cases with a combination of three strategies, (1) technology choice, (2) pricing scheme, and (3) interconnection method. Each IAP has three investment options for its network capacity, $1 \mathrm{~K}, 2 \mathrm{~K}$, or 3 K . The first 6 cases, i.e., Case $1 \sim$ Case 6 , are related to transit interconnection and the remaining 4 cases, i.e., Case $7 \sim$ Case 10, are related to peering interconnection. We analyze Case 1 in detail as a sample case. The rest of the cases are explained focusing on the results since all use a similar methodology. In the end, we present overall analysis of this game model and social welfare analysis.

### 6.1 Analysis of Case 1

In this case, both IAPs choose (1) BE service, (2) transit, and (3) flat-rate pricing. The profit functions of two IAPs are:

- $\mathrm{f} 1\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}\right]=\mathrm{q} 1_{\mathrm{BE}} *\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}\right]+8\right)-\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{k} 1 * 10000\right)$, if $0 \leqq \mathrm{q} 1_{\mathrm{BE}}<\mathrm{k} 1^{*} 1000$,
- $\mathrm{f} 2\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}\right]=\mathrm{q} 2_{\mathrm{BE}} *\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}\right]+8\right)-\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{k} 2 * 10000\right)$, if $0 \leqq \mathrm{q} 2_{\mathrm{BE}}<\mathrm{k} 2^{*} 1000$, where $\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}\right]=50-0.01 *\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{BE}}\right)$, and $\mathrm{k} 1, \mathrm{k} 2=1,2$, or 3 .

We assumed that each IAP maximizes profit for any number of subscribers that the other IAP is able to serve. This means that the maximum of the functions in $f 1\left[q 1_{B E}, q 2_{B E}\right]$ and $f 2\left[q 1_{B E}, q 2_{B E}\right]$ have to be obtained with respect to $q 1_{B E}$ and $q 2_{B E}$. Since the profit functions are different according to the investment level, the maximization has to be obtained within each network capacity. This is done using the KuhnTucker conditions for maximization with inequality constraints.

Using the first order conditions for the maximization of each IAP's profit function with the assumption that the number of subscribers of the other IAP is fixed, we obtain a system of two equations and two unknowns. The maximization values of $q 1_{B E}$ and $q 2_{B E}$ are obtained solving this system of equations.

If we solve the above the two profit functions simultaneously without constraints, the equilibrium quantities are:

$$
q 1 *_{B E}=q 2 *_{B E}=1,900 \text { subscribers }
$$

with reaction functions of $q 1_{B E}=-50 *\left(-57+q 2_{B E} / 100\right)$ and $q 2_{B E}=-50 *\left(-57+q 1_{B E} / 100\right)$. The following graph shows two reaction functions: (1) $R^{l}\left[q 2_{B E}\right]=2850-0.5^{*} q 2_{B E}$, and (2) $R^{2}\left[q 1_{B E}\right]=2850-0.5^{*} q 1_{B E}$.

[Figure 6-1] Reaction Functions
These two reaction functions satisfy the stability condition of the Cournot model, which is $\left|\partial q_{i} / \partial q_{j}\right|<1$. In our reaction functions this value is $\left|\partial q 1_{B E} / \partial q 2_{B E}\right|=0.5$. Therefore, the Nash equilibrium quantity exists in the Case 1. The intersection point of two reaction functions is an equilibrium, $\left(q 1 *_{B E}, q 2 *_{B E}\right)=$ ( 1900,1900 ). However, that equilibrium is only meaningful when $k l \geqq 2$ and $k 2 \geqq 2$, because 1,900 is only available when the network capacity is up to 1999 subscribers or higher.

We use Lagrangian function to maximize the two profit functions simultaneously with inequality constraints. For example, when $k l=k 2=1$, i.e., network capacity of [ $1 \mathrm{~K}, 1 \mathrm{~K}$ ], the inequalities, $\left(0 \leqq q 1_{B E}\right.$ $<1,000)$ and $\left(0 \leqq q 2_{B E}<1,000\right)$ are the same as $\left(0 \leqq q 1_{B E} \leqq 999\right)$ and $\left(0 \leqq q 2_{B E} \leqq 999\right)$ because $q 1_{B E}$ and $q 2_{B E}$ must be an integer. We use slack (dummy) variables, $n 1^{2}$ and $n 2^{2}$ and Lagrangian multipliers, $r_{l}$ and $r_{2}$, for the purpose of transforming the inequality conditions into equation form and satisfying nonnegativity condition. The followings are input equations of the Mathematica to solve the Case 1 with network capacity of $[1 \mathrm{~K}, 1 \mathrm{~K}]$.
[1] pbe[q1be_, q2be] = 50-0.01*(q1be+q2be) /* Inverse BE demand function */
[2] $f 1[q 1 b e, q 2 b e]=q 1 b e *(p b e[q 1 b e, q 2 b e]+8)-(q 1 b e+1 * 1000)$
/* Profit function of IAP1 with $k l=1^{* /}$
[3] $f 2[q 1 b e, q 2 b e]=q 2 b e *(p b e[q 1 b e, q 2 b e]+8)-(q 2 b e+1 * 1000)$
/* Profit function of IAP2 with $k 2=1$ */
[4] $c 1[q 1 b e]=q 1 b e+n 1^{\wedge} 2-999$
/* Capacity constraint of IAP1 and non-negative constraint of qlbe*/
[5] $c 2[q 2 b e]=q 2 b e+n 2^{\wedge} 2-999$
/* Capacity constraint of IAP2 and non-negative constraint of q2be*/
[6] g1[q1be_,q2be_r1_r2_,n1_,n2]]=Simplify[Expand[f1[q1be,q2be]+r1*c1[q1be]]]
/* Lagrangian function of IAP1, r1: Largrangian multiplier */
[7] $g 2\left[q 1 b e, q 2 b e \_r 1 \_, r 2 \_, n 1 \_, n 2\right]=$ Simplify[Expand[f2[q1be,q2be]+r2*c2[q1be]]]
/* Lagrangian function of IAP2, r2: Largrangian multiplier */
[8] glq1be[q1be_,q2be_,r1_r2_,n1_,n2] =
Simplify[Expand[D[g1[q1be,q2be,r1,r2,n1,n2],q1be]]]
[9] g1r1[q1be_,q2be_r1_,r2_,n1_n2] = Simplify[Expand[D[g1[q1be,q2be,r1,r2,n1,n2],r1]]]
[10] g1n1[q1be_,q2be_r1_,r2_,n1_,n2]] = Simplify[Expand[D[g1[q1be,q2be,r1,r2,n1,n2],n1]]]
/* First order conditions of IAP1 with respect to qlbe, rl, nl */
[11] gln1v1[q1be_q2be_,r1_,r2_,v1_,v2]] = gln1[q1be,q2be,r1,r2,(999-q1be),n2]
/* To reduce the number of variables, the variable $v 1$ (=999-qlbe) is used */
[12] g2q2be[q1be_, q2be_r1_,r2_,n1_,n2] =
Simplify[Expand[D[g2[q1be,q2be,r1,r2,n1,n2],q2be]]]
[13] g2r2[q1be_q2be_r1_,r2_,n1_n2] = Simplify[Expand[D[g2[q1be,q2be,r1,r2,n1,n2],r2]]]
[14] g2n2[q1be_,q2be_r1_,r2_,n1_,n2]] = Simplify[Expand[D[g2[q1be,q2be,r1,r2,n1,n2],n2]]]
/* First order conditions of IAP2 with respect to q2be, r2, n2 */
[15] g2n2v2[q1be_q2be_r1_,r2_,v1_,v2] = g2n2[q1be,q2be,r1,r2,n1,(999-q2be)]
/* To reduce the number of variable, the variable $v 2$ (=999-q2be) is used */
[16] sol $=$ Simplify [Expand[Flatten[Nsolve[\{g1q1be[q1be,q2be,r1,r2,n1,n2] $==0$,
$g \ln 1 v 1[q 1 b e, q 2 b e, r 1, r 2, v 1, v 2]==0, g 2 q 2 b e[q 1 b e, q 2 b e, r 1, r 2, n 1, n 2]==0$,
$g 2 n 2 v 2[q 1 b e, q 2 b e, r 1, r 2, v 1, v 2]==0\},\{q 1 b e, q 2 b e, r 1, r 2\}]]]]$
/* Solve four first-order conditions simultaneously with respect to qlbe, q2be, r1, r2 */

From the input equation [16], we can get an equilibrium solution: $\left(q 1^{*}{ }_{B E}, q 2^{*}{ }_{B E}, r 1^{*}, r 2^{*}\right)=(999,999$, -$27.03,-27.03$ ). The profit of each $\operatorname{IAP}\left(=f 1^{*}, f 2^{*}\right)$ is $\$ 26,983$ and the equilibrium price $\left(=p^{*}{ }_{B E}\right)$ is $\$ 30.02$. With a change in the capacity constraint, i.e., $[k 1 \mathrm{~K}, k 2 \mathrm{~K}]$, and its profit function, we get the following nine equilibria in Case 1.
[Table 6-1] Equilibrium Points of Case 1

| Case $\mathbf{1}$ | $\mathbf{q 1}{ }^{\text {BE }}$ | $\mathbf{q 2}^{*}{ }_{\text {BE }}$ | $\mathbf{f 1}^{*}$ | $\mathbf{f} \mathbf{2}^{*}$ | $\mathbf{P}^{*}{ }_{\mathbf{B E}}$ |
| :---: | ---: | ---: | ---: | ---: | ---: |
| $[1 \mathrm{~K}, 1 \mathrm{~K}]$ | 999 | 999 | 26983 | 26983 | 30.02 |
| $[1 \mathrm{~K}, 2 \mathrm{~K}]$ | 999 | 1999 | 16993 | 34013 | 20.02 |
| $[1 \mathrm{~K}, 3 \mathrm{~K}]$ | 999 | 2350 | 13486 | 25249 | 16.51 |
| $[2 \mathrm{~K}, 1 \mathrm{~K}]$ | 1999 | 999 | 34013 | 16993 | 20.02 |
| $[2 \mathrm{~K}, 2 \mathrm{~K}]$ | 1900 | 1900 | 16100 | 16100 | 12.00 |
| $[2 \mathrm{~K}, 3 \mathrm{~K}]$ | 1900 | 1900 | 16100 | 6100 | 12.00 |
| $[3 \mathrm{~K}, 1 \mathrm{~K}]$ | 2350 | 999 | 25249 | 13486 | 16.51 |
| $[3 \mathrm{~K}, 2 \mathrm{~K}]$ | 1900 | 1900 | 6100 | 16100 | 12.00 |
| $[3 \mathrm{~K}, 3 \mathrm{~K}]$ | 1900 | 1900 | 6100 | 6100 | 12.00 |

The following table is a payoff matrix of Case 1 , which is based on the above table. Table 6-2 is symmetric along the diagonal where both IAPs' profits are the same.
[Table 6-2] Payoff Matrix for Case 1

| IAP1 IAP2 | 1K | 2K | 3K | Optimal k2* |
| :---: | :---: | :---: | :---: | :---: |
| 1K | [26983, 26983] | [16993, 34013] | [13486, 25249] | 2K |
| 2K | [34013, 16993] | [16100, 16100] | [16100, 6100] | 1K |
| 3K | [25249, 13486] | [6100, 16100] | [6100, 6100] | 2K |
| Optimal k1* | 2 K | 1 K | 2 K |  |

We try to find an equilibrium strategy of network capacity among the above nine cells, i.e., an optimal $k l^{*}$ and $k 2 *$. There are two equilibria in the matrix, $[1 \mathrm{~K}, 2 \mathrm{~K}]$ and $[2 \mathrm{~K}, 1 \mathrm{~K}]$. We only consider four shaded cells instead of all nine cells, because 3 K is always inferior to 2 K .

- If IAP1 chooses 1 K , IAP2's optimal strategy is 2 K , because $(f 2[999,999]=\$ 26,983)<$ $(f 2[999,1999]=\$ 34,013)$.
- If IAP1 chooses 2 K , IAP2's optimal strategy is 1 K because $(f 2[1999,999]=\$ 16,993)>$ (f2[1900,1900] = \$16,100).
The same logic can be applied to the IAP2.
- If IAP2 chooses 1K, IAP1's optimal strategy is 2K because (f1[999,999] $=\$ 26,983)<$ $(f 1[1999,999]=\$ 34,013)$.
- If IAP2 chooses 2 K , IAP1's optimal strategy is 1 K because $(f 1[999,1999]=\$ 16,9893)>$ $(f 1[1900,1900]=\$ 16,100)$.
At these equilibrium points, either IAP cannot produce higher profit to change its own strategy. In this case, being a larger network capacity $(2 K)$ earlier than the other $(1 K)$ will pay off.


### 6.2 Analysis of Case 2

In this case, IAP1 chooses BE and IAP2 chooses QoS as its technology. Both IAPs' interconnection strategies are transit and they use flat-rate pricing for their services. In this case, IAP2 is a monopoly provider of QoS service. The profit functions of the two IAPs are:

- $\mathrm{f} 1\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{Qos}}\right]=\mathrm{q} 1_{\mathrm{BE}} *\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{Qos}}\right]+8\right)-\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{k} 1^{*} 10000\right)$, if $0 \leqq \mathrm{q} 1_{\mathrm{BE}}<$ k1*1000,
- $\mathrm{f} 2\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{Qos}}\right]=\mathrm{q} 2_{\mathrm{BE}} *\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{Qos}}\right]+8\right)+\mathrm{q} 2_{\mathrm{Qos}} *\left(\mathrm{p}_{\mathrm{Qos}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{Qos}}\right]+8\right)-$ $\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{Qos}}{ }^{\wedge} 1.5+10000+\mathrm{k} 2 * 8000\right)$, if $0 \leqq\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{Qos}}\right)<\mathrm{k} 2 * 1000$,
where $\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{Qos}}\right]=50-0.01^{*}\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{QoS}}\right)$,
$\mathrm{p}_{\mathrm{Qos}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{QoS}}\right]=\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{Qos}}\right]+\left(50-0.01 * \mathrm{q} 2_{\mathrm{Qos}}\right)$, and $\mathrm{k} 1, \mathrm{k} 2=1,2$, or 3 .

With adding a new variable ( $q_{2} Q_{Q o s}$ ), we solve the above equations by the same approach as we did in the Case 1. The following table displays quantities, prices, and profits at the equilibrium point of each investment in the network capacity.
[Table 6-3] Equilibrium Points of Case 2

| Case 2 | $\mathrm{q} 1{ }^{\text {BE }}$ | 2* ${ }_{\text {bE }}$ | Q2* ${ }_{\text {os }}$ | Q1 | Q2 | f1* | f2* | P** ${ }_{\text {BE }}$ | P* ${ }_{\text {O }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [1K,1K] | 999 | 354 | 645 | 999 | 999 | 26983 | 31337 | 30.02 | 73.57 |
| [1K,2K] | 999 | 1354 | 645 | 999 | 1999 | 16993 | 40367 | 20.02 | 63.57 |
| [1K,3K] | 999 | 1705 | 645 | 999 | 2350 | 13487 | 33602 | 16.51 | 60.06 |
| [2K,1K] | 1999 | 354 | 645 | 1999 | 999 | 52013 | 21347 | 20.02 | 63.57 |
| [2K,2K] | 1900 | 1255 | 645 | 1900 | 1900 | 16100 | 22454 | 12.00 | 55.55 |
| [2K,3K] | 1900 | 1255 | 645 | 1900 | 1900 | 16100 | 14454 | 12.00 | 55.55 |
| [3K,1K] | 2350 | 354 | 645 | 2350 | 999 | 25249 | 17840 | 16.51 | 60.06 |
| [3K,2K] | 1900 | 1255 | 645 | 1900 | 1900 | 6100 | 22454 | 12 | 55.55 |
| [3K,3K] | 1900 | 1255 | 645 | 1900 | 1900 | 6100 | 14454 | 12 | 55.55 |

$$
* Q 1=q 1_{B E} \text { and } Q 2=q 2_{B E}+q 2_{Q o S}
$$

The following table is a payoff matrix based on the above table. The equilibrium network capacity is $[1 \mathrm{~K}$, 2K], where IAP2 has larger network capacity and a higher profit than IAP1. This result implies that a QoS-IAP will have a better market position than a BE-IAP when they exist in the same market and upgrading to QoS technology earlier than the other will pay off.
[Table 6-4] Payoff Matrix for Case 2

| IAP1 | IAP2 | $\mathbf{1 K}$ | $\mathbf{2 K}$ |
| :---: | :---: | :---: | :---: |
| $\mathbf{1 K}$ | $[26983,31337][\mathbf{1 6 9 9 3}, \mathbf{4 0 3 6 7 ]}[13487,33602]$ | $\mathbf{3 K}$ | Optimal $\boldsymbol{k}^{* 2}$ |
| $\mathbf{2 K}$ | $[52013,21347][16100,22454][16100,14454]$ | 2 K |  |
| $\mathbf{3 K}$ | $[25249,17840][6100,22454][6100,14454]$ | 2 K |  |
| Optimal $\boldsymbol{k 1 *}$ | 2 K | 1 K | 2 K |

The following graph shows the IAP2's pricing structure at the network capacity of $[1 \mathrm{~K}, 2 \mathrm{~K}]$.

[Figure 6-2] Pricing Structure of QoS-IAP in Case 2

### 6.3 Analysis of Case 3

In this case, every strategy is the same as Case 2 except for the pricing scheme of IAP2, i.e., two-part tariff. The profit function of IAP1 is the same as in Case 2. The profit functions of the two IAPs are:

- $\mathrm{f} 1\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{QoS}}\right]=\mathrm{q} 1_{\mathrm{BE}} *\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{QoS}}, \mathrm{r}\right]+8\right)-\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{k} 1^{*} 10000\right)$, if $0 \leqq \mathrm{q} 1_{\mathrm{BE}}<$ k1*1000,
- $\mathrm{f} 2\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{QoS}}, \mathrm{r}\right]=\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{Qos}}\right)^{*}\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{QoS}}, \mathrm{r}\right]+8\right)+\mathrm{h}_{\mathrm{QoS}}[\mathrm{r}]^{*} \mathrm{r}^{*} \mathrm{q} 2_{\mathrm{QoS}}-\left(\mathrm{q} 2_{\mathrm{BE}}+\right.$ $\left.\mathrm{q} 2_{\mathrm{Qos}}{ }^{\wedge} 1.6+10000+\mathrm{k} 2 * 8000\right)$, if $0 \leqq\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{Qos}}\right)<\mathrm{k} 2 * 1000$,
where $\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{Qos}}, \mathrm{r}\right]=50-0.01 *\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{Qos}}\right)-5^{*} \mathrm{r}$, $\mathrm{h}_{\mathrm{Qo}}[\mathrm{r}]=100-33.3^{*} \mathrm{r}$, and $\mathrm{k} 1, \mathrm{k} 2=1,2$, or 3 .

In this case, there are four variables, $q 1_{B E}, q 2_{B E}, q 2_{Q o S}$, and $r$. We use the same approach as we did in the previous case, but the solution in this case is imaginary. As the next best method, we try to solve this case with three variables $\left(q 1_{B E}, q 2_{B E}\right.$, and $\left.q 2_{Q O S}\right)$ with a constant value of $r$. We modify the above profit functions into the followings:

- $\mathrm{f} 1\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{QoS}}\right]=\mathrm{q} 1_{\mathrm{BE}}{ }^{*}\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{Qos}}\right]+8\right)-\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{k} 1^{*} 10000\right)$, if $0 \leqq \mathrm{q} 1_{\mathrm{BE}}<$ k1*1000,
- $\mathrm{f} 2\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{Qos}}\right]=\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{Qos}}\right)^{*}\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{Qos}}\right]+8\right)+\mathrm{h}_{\mathrm{Qos}}{ }^{*} \mathrm{r}^{*} \mathrm{q}_{\mathrm{QoS}}-\left(\mathrm{q} 2_{\mathrm{BE}}{ }^{+}\right.$ $\left.\mathrm{q} 2_{\mathrm{Qos}}{ }^{\wedge} 1.6+10000+\mathrm{k} 2 * 8000\right)$, if $0 \leqq\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{Qos}}\right)<\mathrm{k} 2 * 1000$,
where $\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{Qos}}\right]=50-0.01^{*}\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{Qos}}\right)-5^{*} \mathrm{r}$,
$h_{\mathrm{Qos}}=100-33.3^{*} \mathrm{r}, \mathrm{k} 1, \mathrm{k} 2=1,2$, or 3 , and $\mathrm{r}=1.0,1.1,1.2, \ldots$, or 2.0 .

We maximize the above two equations 11 times, each time with different pair of $\left(r, h_{Q o s}\right)$, i.e., $\{(\$ 1.0,67)$, (\$1.1, 63), (\$1.2, 60), (\$1.3, 57), (\$1.4, 53), (\$1.5, 50), (\$1.6, 47), (\$1.7, 43), (\$1.8, 40), (\$1.9, 37), (\$2.0, 33) \}. Experience gives us an optimal range of ' $r$ ', which lies between 1.0 and 2.0 . We use an integer number of QoS connection hours based on the equation of $h_{Q o S}=100-33.3^{*} r$. For example, at first, put $r=$ $\$ 1.0$ /hour and $h_{\text {QoS }}=67$ hours into the above equations and solve them simultaneously. And next, put $r=$ $\$ 1.1$ /hour and $h_{Q o S}=63$ hours into the above equations and solve them simultaneously. Do the same thing until $r=2.0$ /hour and $h_{Q O S}=33$ hours. And compare the eleven equilibrium outputs and find a maximum profit of IAP2 among them. For example, in the following table, there are eleven equilibrium points in the network capacity of $[1 \mathrm{~K}, 1 \mathrm{~K}]$. Among them, $r=\$ 1.3 /$ hour and $h_{Q O S}=57$ hours give the maximum profit to the IAP2. We use this modified method when at least one of IAPs chooses the two-part tariff, i.e., Case 3, $4,6,8$, and 10 .
[Table 6-5] Equilibrium Quantities, Prices, and profits with a change of $r$

| r | q1* ${ }_{\text {BE }}$ | q2* ${ }_{\text {BE }}$ | q2* ${ }_{\text {OoS }}$ | Q1 | Q2 | f1* | f2* | P* ${ }^{\text {BE }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.0 | 999 | 481 | 518 | 999 | 999 | 21988 | 27186 | 25.02 |
| 1.1 | 999 | 452 | 547 | 999 | 999 | 21489 | 27911 | 24.52 |
| 1.2 | 999 | 416 | 583 | 999 | 999 | 20989 | 28937 | 24.02 |
| 1.3 | 999 | 388 | 611 | 999 | 999 | 20490 | 29690 | 23.52 |
| 1.4 | 999 | 387 | 612 | 999 | 999 | 19990 | 29252 | 23.02 |
| 1.5 | 999 | 376 | 623 | 999 | 999 | 19491 | 29246 | 22.52 |
| 1.6 | 999 | 373 | 626 | 999 | 999 | 18991 | 28871 | 22.02 |
| 1.7 | 999 | 402 | 597 | 999 | 999 | 18492 | 27088 | 21.52 |
| 1.8 | 999 | 416 | 583 | 999 | 999 | 17992 | 25940 | 21.02 |
| 1.9 | 999 | 439 | 560 | 999 | 999 | 17493 | 24469 | 20.52 |
| 2.0 | 999 | 494 | 505 | 999 | 999 | 1693 | 21680 | 20.02 |

The following table shows IAP2's best $r$ value of each investment level in Case 3. Except for the network capacity of $[3 \mathrm{~K}, 1 \mathrm{~K}]$ the best r values of all other network capacities appear to be $\$ 1.3$ /hour.
[Table 6-6] Best rof Case 3

| $\mathbf{I A P 1}^{\mathbf{1 A P}}{ }^{\mathbf{1 A P 2}}$ | $\mathbf{1 K}$ | $\mathbf{2 K}$ | $\mathbf{3 K}$ |
| :---: | :---: | :---: | :---: |
| $\mathbf{1 K}$ | 1.3 | 1.3 | 1.3 |
| $\mathbf{2 K}$ | 1.3 | 1.3 | 1.3 |
| $\mathbf{3 K}$ | 1.5 | 1.3 | 1.3 |

According to the above best $r$ values, the following table is quantities, prices, and profits at the equilibrium point of each investment on the network capacity.
[Table 6-7] Equilibrium Points of Case 3

| Case 3 | $\mathbf{r}$ | $\mathbf{q 1 *}{ }_{\mathbf{B E}}$ | $\mathbf{q 2}{ }^{*}{ }_{\mathbf{B E}}$ | $\mathbf{q 2}{ }^{*}{ }_{\mathbf{o} \mathbf{o s}}$ | $\mathbf{Q 1}$ | $\mathbf{Q 2}$ | $\mathbf{f 1 *}$ | $\mathbf{f 2 *}$ | $\mathbf{P}^{*}{ }_{\mathbf{B E}}$ |
| :---: | :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $[1 \mathrm{~K}, 1 \mathrm{~K}]$ | 1.3 | 999 | 388 | 611 | 999 | 999 | 20490 | 29690 | 23.52 |
| $[1 \mathrm{~K}, 2 \mathrm{~K}]$ | 1.3 | 999 | 1388 | 611 | 999 | 1999 | 10500 | 32220 | 13.52 |
| $[1 \mathrm{~K}, 3 \mathrm{~K}]$ | 1.3 | 999 | 1415 | 611 | 999 | 2026 | 10230 | 24227 | 13.25 |
| $[2 \mathrm{~K}, 1 \mathrm{~K}]$ | 1.3 | 1999 | 388 | 611 | 1999 | 999 | 21020 | 19700 | 13.52 |
| $[2 \mathrm{~K}, 2 \mathrm{~K}]$ | 1.3 | 1683 | 1073 | 611 | 1683 | 1684 | 8325 | 19542 | 9.83 |
| $[2 \mathrm{~K}, 3 \mathrm{~K}]$ | 1.3 | 1683 | 1073 | 611 | 1683 | 1684 | 8325 | 11542 | 9.83 |
| $[3 \mathrm{~K}, 1 \mathrm{~K}]$ | 1.5 | 1976 | 376 | 623 | 1976 | 999 | 9026 | 19486 | 12.75 |
| $[3 \mathrm{~K}, 2 \mathrm{~K}]$ | 1.3 | 1683 | 1073 | 611 | 1683 | 1684 | -1675 | 19542 | 9.83 |
| $[3 \mathrm{~K}, 3 \mathrm{~K}]$ | 1.3 | 1683 | 1073 | 611 | 1683 | 1684 | -1675 | 11542 | 9.83 |

[^37]The following table is a payoff matrix based on the above table. There are two equilibrium points in this case: $[1 \mathrm{~K}, 2 \mathrm{~K}]$ and $[2 \mathrm{~K}, 1 \mathrm{~K}]$. This result implies that even a BE-IAP can have a better market position when it chooses larger network capacity than a QoS-IAP with two-part tariff. If we compare tables 6-3 and 6-7, equilibrium profits in Case 2 are higher than those in Case 3 at the network capacity of [ $1 \mathrm{~K}, 2 \mathrm{~K}$ ]. In conclusion, when competing with a BE-IAP, flat-rate pricing is the superior QoS pricing scheme to the two-part tariff.
[Table 6-8] Payoff Matrix for Case 3

| IAP1 | IAP2 | $\mathbf{1 K}$ | $\mathbf{2 K}$ | $\mathbf{3 K}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 K}$ | $[20490,29690]$ | $[\mathbf{1 0 5 0 0 , 3 2 2 2 0}]$ | $[10230,24227]$ | Optimal $\boldsymbol{k 2 *}$ |
| 2K | $[\mathbf{2 1 0 2 0 , 1 9 7 0 0}]$ | $[8325,19542]$ | $[8325,11542]$ | 1 K |
| $\mathbf{3 K}$ | $[9026,19486]$ | $[-1675,19542]$ | $[-1675,11542]$ | 2 K |
| Optimal $\boldsymbol{k 1 *}$ | 2 K | 1 K | 1 K |  |

The following figure shows the IAP2's pricing structure at the network capacity of $[1 \mathrm{~K}, 2 \mathrm{~K}]$.

[Figure 6-3] Pricing Structure of QoS-IAP in the Case 3

### 6.4 Analysis of Case 4

In this case, both IAPs choose QoS with different pricing schemes: IAP1 with flat-rate pricing and IAP2 with two-part tariff. Both IAPs choose transit for their interconnection. The following are the modified profit functions of the two IAPs with constant $r$.

- $\mathrm{f} 1\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{Qos}}, \mathrm{q} 2_{\mathrm{Qos}}\right]=\mathrm{q} 1_{\mathrm{BE}} *\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]+8\right)+\mathrm{q} 1_{\mathrm{Qos}} *\left(\mathrm{p}_{\mathrm{Qos}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}\right.\right.$, $\left.\left.\mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]+8\right)-\left(\mathrm{q}_{\mathrm{BE}}+\mathrm{q} 1_{\mathrm{Qos}} \wedge 1.5+10000+\mathrm{k} 1^{*} 8000\right)$, if $0 \leqq\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{q} 1_{\mathrm{Qos}}\right)<\mathrm{k} 1^{*} 1000$,
- $\mathrm{f} 2\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]=\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{QoS}}\right) *\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QOS}}, \mathrm{q} 2_{\mathrm{QoS}}\right]+8\right)+\mathrm{h}_{\mathrm{Qos}} * \mathrm{r}^{*} \mathrm{q}_{\mathrm{QoS}}-$ $\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{Qos}} \wedge 1.6+10000+\mathrm{k} 2 * 8000\right)$, if $0 \leqq\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{QoS}}\right)<\mathrm{k} 2 * 1000$,
where $\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]=50-0.01^{*}\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 1_{\mathrm{Qos}}+\mathrm{q} 2_{\mathrm{QoS}}\right)-5{ }^{2}$,
$\mathrm{p}_{\mathrm{Qos}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]=\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{QoS}}\right]+50-0.01^{*}\left(\mathrm{q} 1_{\mathrm{QoS}}+\mathrm{q} 2_{\mathrm{QoS}}\right)$,
$\mathrm{h}_{\mathrm{QoS}}=100-33.3^{*} \mathrm{r}, \mathrm{k} 1, \mathrm{k} 2=1,2$, or 3 , and $\mathrm{r}=1.0,1.1,1.2, \ldots$, or 2.0 .

The following table is quantities, prices, and profits at the equilibrium of each investment in network capacity. QoS market demand ( $=\mathrm{q} 1{ }^{*}{ }_{\mathrm{Qos}}{ }^{+} \mathrm{q}^{2}{ }_{\mathrm{Qos}}$ ) is relatively stable regardless of network capacity and QoS-IAP with tow-part tariff has a more market share than QoS-IAP with flat-rate. ( $\mathrm{q} 1 *{ }_{\mathrm{oos}}<\mathrm{q} 2 *$ Qos)
[Table 6-9] Equilibrium Points of Case 4

| Case 4 | r | q1* ${ }_{\text {be }}$ | q2* ${ }_{\text {be }}$ | q1* oos | q2* ${ }_{0 \text { os }}$ | Q1 | Q2 | f1* | f2* | P** ${ }_{\text {be }}$ | P |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [1K,1K] | 1.3 | 474 | 388 | 525 | 611 | 999 | 999 | 21272 | 29690 | 23.53 | 62.16 |
| [1K,2K] | 1.3 | 474 | 1388 | 525 | 611 | 999 | 1999 | 11281 | 32220 | 13.52 | 52.16 |
| [1K,3K] | 1.3 | 474 | 1415 | 525 | 611 | 999 | 2026 | 11012 | 24227 | 13.25 | 51.89 |
| [2K,1K] | 1.3 | 1474 | 388 | 525 | 611 | 1999 | 999 | 23801 | 19700 | 13.52 | 52.16 |
| [2K,2K] | 1.3 | 1158 | 1073 | 525 | 611 | 1683 | 1684 | 11107 | 19542 | 9.83 | 48.47 |
| [2K,3K] | 1.3 | 1158 | 1073 | 525 | 611 | 1683 | 1757 | 11107 | 11542 | 9.83 | 48.47 |
| [3K,1K] | 1.5 | 1452 | 376 | 523 | 623 | 1975 | 999 | 13745 | 19496 | 12.76 | 51.30 |
| [3K,2K] | 1.3 | 1158 | 1073 | 525 | 611 | 1683 | 1757 | 3107 | 19542 | 9.83 | 48.47 |
| [3K,3K] | 1.3 | 1158 | 1073 | 525 | 611 | 1683 | 1757 | 3107 | 11542 | 9.83 | 48.47 |

*Q1 $=q 1_{B E}+q 2_{B E}$ and $Q 2=q 2_{B E}+q 2_{Q O S}$

The following table is a payoff matrix based on the above table. In this case, there are also two equilibrium points: [ $1 \mathrm{~K}, 2 \mathrm{~K}$ ] where QoS-IAP with a two-part tariff has a higher profit and $[2 \mathrm{~K}, 1 \mathrm{~K}$ ] where QoS-IAP with flat-rate has a higher profit. This result implies that larger QoS-IAP will be a better market position regardless of pricing scheme.
[Table 6-10] Payoff Matrix for Case 4

| $\mathbf{I A P 1}^{\text {IAP2 }}$ | $\mathbf{1 K}$ | $\mathbf{2 K}$ | $\mathbf{3 K}$ | Optimal $\mathbf{k 2}^{*}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 K}$ | $[21272,29690]$ | $[\mathbf{1 1 2 8 1 , 3 2 2 2 0 ]}$ | $[11012,24227]$ | 2 K |
| $\mathbf{2 K}$ | $[\mathbf{2 3 8 0 1 , 1 9 7 0 0 ]}$ | $[11107,19542]$ | $[11107,11542]$ | 1 K |
| $\mathbf{3 K}$ | $[13745,19496]$ | $[3107,19542]$ | $[3107,11542]$ | 2 K |
| Optimal $\boldsymbol{k 1}^{*}$ | 2 K | 1 K | 2 K |  |

### 6.5 Analysis of Case 5

In this case, both IAPs choose QoS with flat-rate pricing. The profit functions of the two IAPs are:

- $\mathrm{f} 1\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{QoS}}\right]=\mathrm{q} 1_{\mathrm{BE}} *\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]+8\right)+\mathrm{q} 1_{\mathrm{Qos}} *\left(\mathrm{p}_{\mathrm{QoS}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}\right.\right.$, $\left.\left.\mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]+8\right)-\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{q} 1_{\mathrm{Qos}} \wedge^{\wedge} 1.5+10000+\mathrm{k} 1 * 8000\right)$, if $0 \leqq\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{q} 1_{\mathrm{QoS}}\right)<\mathrm{k} 1^{*} 1000$,
- $\mathrm{f} 2\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{QoS}}\right]=\mathrm{q} 2_{\mathrm{BE}} *\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]+8\right)+\mathrm{q} 2_{\mathrm{Qos}} *\left(\mathrm{p}_{\mathrm{QoS}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}\right.\right.$, $\left.\left.\mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]+8\right)-\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{Qos}} \wedge 1.5+10000+\mathrm{k} 1^{*} 8000\right)$, if $0 \leqq\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{QoS}}\right)<\mathrm{k} 2 * 1000$,
where $\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{Qos}}, \mathrm{q} 2_{\mathrm{QoS}}\right]=50-0.01 *\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 1_{\mathrm{Qos}}+\mathrm{q} 2_{\mathrm{QoS}}\right)$,
$\mathrm{p}_{\mathrm{Qos}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]=\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{QoS}}\right]+50-0.01 *\left(\mathrm{q} 1_{\mathrm{Qos}}+\mathrm{q} 2_{\mathrm{Qos}}\right)$,
and $\mathrm{k} 1, \mathrm{k} 2=1,2$, or 3 .

The following table is quantities, prices, and profits at the equilibrium points of each investment in the network capacity. QoS market share of both IAPs are equal and constant regardless of network capacity. Therefore, market share of BE service determines the overall profit of both IAPs.
[Table 6-11] Equilibrium Points of Case 5

| Case 5 | q1* ${ }_{\text {BE }}$ | q2* ${ }_{\text {BE }}$ | q1* ${ }^{\text {os }}$ | q2* ${ }^{\text {oS }}$ | Q1 | Q2 | f1* | f2* | p* ${ }_{\text {BE }}$ | p * Oos |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [1K, 1K] | 460 | 460 | 539 | 539 | 999 | 999 | 28148 | 28148 | 30.02 | 69.24 |
| [1K, 2 K ] | 460 | 1460 | 539 | 539 | 999 | 1999 | 18158 | 37178 | 20.02 | 59.24 |
| [1K,3K] | 460 | 1811 | 539 | 539 | 999 | 2350 | 14651 | 30414 | 16.51 | 55.73 |
| [2K, 1 K ] | 1460 | 460 | 539 | 539 | 1999 | 999 | 37178 | 18158 | 20.02 | 59.24 |
| [2K,2K] | 1361 | 1361 | 539 | 539 | 1900 | 1900 | 19265 | 19265 | 12.00 | 51.22 |
| [2K, 3 K ] | 1361 | 1361 | 539 | 539 | 1900 | 1900 | 19265 | 11265 | 12.00 | 51.22 |
| [3K,1K] | 1811 | 460 | 539 | 539 | 2350 | 999 | 30414 | 14651 | 16.51 | 55.74 |
| [3K,2K] | 1361 | 1361 | 539 | 539 | 1900 | 1900 | 11265 | 19265 | 12.00 | 51.22 |
| [3K,3K] | 1361 | 1361 | 539 | 539 | 1900 | 1900 | 11265 | 11265 | 12.00 | 51.22 |

The following table is a payoff matrix based on the above table. Both IAPs' dominant strategy is 2 K , therefore there is one equilibrium, $[2 \mathrm{~K}, 2 \mathrm{~K}]$. At this equilibrium, every strategy is symmetric between two IAPs, i.e., the same strategy set of two IAPs, $\{$ QoS, flat-rate, 2 K , transit $\}$. So, there is a possibility of collusion between the two identical providers, i.e., to reduce the network capacity from 2 K to 1 K , and to get a higher profit than at the equilibrium point. $(f 1[1 \mathrm{~K}, 1 \mathrm{~K}]=f 2[1 \mathrm{~K}, 1 \mathrm{~K}]=28,148>f 1[2 \mathrm{~K}, 2 \mathrm{~K}]=f 2[2 \mathrm{~K}$, $2 \mathrm{~K}]=19,265)$. In the Cournot model, market quantity determines the market price of a product. Reducing market quantity means (1) increasing the market price and (2) reducing cost. Even though output of the product decreases, profit would rise because of the increased price and the decreased cost.
[Table 6-12] Payoff Matrix for Case 5

| IAP1 IAP2 | 1K | 2K | 3K | Optimal k2* |
| :---: | :---: | :---: | :---: | :---: |
| 1K | [28148, 28148] | [18185, 37178] | [14651, 30414] | 2K |
| 2K | [37178, 18158] | [19265, 19265] | [19265, 11265] | 2K |
| 3K | [30414, 14651] | [11265, 19265] | [11265, 11265] | 2K |
| Optimal k1* | 2K | 2K | 2K |  |

### 6.6 Analysis of Case $\mathbf{6}$

In this case, both IAPs choose the QoS with two-part tariff. The profit functions of two IAPs are:

- $\mathrm{f} 1\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{QoS}}\right]=\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{q} 1_{\mathrm{Qos}}\right)^{*}\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]+8\right)+\mathrm{h}_{\mathrm{Qos}} *{ }^{*} \mathrm{r}^{*} 1_{\mathrm{QoS}}-$ $\left(\mathrm{q}_{1} \mathrm{BE}^{+}+1_{\mathrm{Qos}}{ }^{\wedge} 1.6+10000+\mathrm{k} 1 * 8000\right)$, if $0 \leqq\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{q} 1_{\mathrm{QoS}}\right)<\mathrm{k} 1 * 1000$,
- $\mathrm{f} 2\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{QoS}}\right]=\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{QoS}}\right)^{*}\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{QoS}}\right]+8\right)+\mathrm{h}_{\mathrm{Qos}} * \mathrm{r}^{*} \mathrm{q}_{\mathrm{QoS}}-$ $\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{Qos}} \wedge 1.6+10000+\mathrm{k} 2 * 8000\right)$, if $0 \leqq\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{QoS}}\right)<\mathrm{k} 2 * 1000$,
where $\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]=50-0.01 *\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 1_{\mathrm{Qos}}+\mathrm{q} 2_{\mathrm{Qos}}\right)-5 * \mathrm{r}$,
$h_{\mathrm{Qos}}=100-33.3^{*} \mathrm{r}, \mathrm{k} 1, \mathrm{k} 2=1,2$, or 3 , and $\mathrm{r}=1.0,1.1,1.2, \ldots$, or 2.0 .

The following table is quantities, prices, and profits at the equilibrium point of each investment in the network capacity. In this case, QoS market share of both IAPs are equal and BE market shares determine the overall profits of both IAPs.
[Table 6-13] Equilibrium Points of Case 6

| Case 6 | $\mathbf{r}$ | $\mathbf{q 1 *}{ }_{\mathbf{B E}}$ | $\mathbf{q 2}{ }^{*}{ }_{\mathbf{B E}}$ | $\mathbf{q 1 *}{ }_{\mathbf{O} \mathbf{O S}}$ | $\mathbf{q 2} *_{\mathbf{O} \mathbf{o S}}$ | $\mathbf{Q 1}$ | $\mathbf{Q 2}$ | $\mathbf{f 1 *}$ | $\mathbf{f 2}{ }^{*}$ | $\mathbf{P}^{*}{ }_{\mathbf{B E}}$ |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $[1 \mathrm{~K}, 1 \mathrm{~K}]$ | 1.3 | 388 | 388 | 611 | 611 | 999 | 999 | 29690 | 29690 | 23.53 |
| $[1 \mathrm{~K}, 2 \mathrm{~K}]$ | 1.3 | 388 | 1388 | 611 | 611 | 999 | 1999 | 19700 | 32220 | 13.52 |
| $[1 \mathrm{~K}, 3 \mathrm{~K}]$ | 1.3 | 388 | 1415 | 611 | 611 | 999 | 2026 | 19430 | 24227 | 13.25 |
| $[2 \mathrm{~K}, 1 \mathrm{~K}]$ | 1.3 | 1388 | 388 | 611 | 611 | 1999 | 999 | 32220 | 19700 | 13.52 |
| $[2 \mathrm{~K}, 2 \mathrm{~K}]$ | 1.3 | 1073 | 1073 | 611 | 611 | 1684 | 1684 | 19525 | 19525 | 9.82 |
| $[2 \mathrm{~K}, 3 \mathrm{~K}]$ | 1.3 | 1073 | 1073 | 611 | 611 | 1684 | 1684 | 19525 | 11525 | 9.82 |
| $[3 \mathrm{~K}, 1 \mathrm{~K}]$ | 1.5 | 1353 | 376 | 623 | 623 | 1976 | 999 | 22782 | 19486 | 12.75 |
| $[3 \mathrm{~K}, 2 \mathrm{~K}]$ | 1.3 | 1073 | 1073 | 611 | 611 | 1684 | 1684 | 11525 | 19525 | 9.82 |
| $[3 \mathrm{~K}, 3 \mathrm{~K}]$ | 1.3 | 1073 | 1073 | 611 | 611 | 1684 | 1684 | 11525 | 11525 | 9.82 |

${ }^{*} Q 1=q 1_{B E}+q 2_{B E}$ and $Q 2=q 2_{B E}+q 2_{Q O S}$

The following table is a payoff matrix based on the above table. There are two equilibra in this case: [1K, $2 \mathrm{~K}]$ and $[2 \mathrm{~K}, 1 \mathrm{~K}]$. It concludes that when both IAPs produce QoS services with a two-part tariff, large QoS-IAP will pay off.
[Table 6-14] Payoff Matrix for Case 6

| IAP1 | IAP2 | $\mathbf{1 K}$ | 2K | 3K |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 K}$ | $[29690,29690]$ | $[\mathbf{1 9 7 0 0 , 3 2 2 2 0}]$ | $[19430,24227]$ | Optimal $\boldsymbol{k 2 *}$ |
| 2K | $[\mathbf{3 2 2 2 0 , 1 9 7 0 0 ]}$ | $[19525,19525]$ | $[19525,11525]$ | 1 K |
| $\mathbf{3 K}$ | $[22782,19486]$ | $[11525,19525]$ | $[11525,11525]$ | 2 K |
| Optimal $\boldsymbol{k} \mathbf{1}^{*}$ | 2 K | 1 K | 1 K |  |

### 6.7 Analysis of Case 7

Cases 7, 8, 9, and 10 are related to the peering of the two IAPs. In Case 7 both IAPs are BE-IAPs and in the other cases they are QoS-IAPs. The payoff functions of the Case 7 are:

- $\mathrm{fl}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}\right]=\mathrm{q} 1_{\mathrm{BE}} *\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}\right]+8\right)-\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{k} 1^{*} 9000\right)-\left(0.5^{*} 10000+\max \{\mathrm{k} 1, \mathrm{k} 2\}^{*} 150\right)$, if $0 \leqq \mathrm{q} 1_{\mathrm{BE}}<\mathrm{k} 1 * 1000$,
- $\mathrm{f} 2\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}\right]=\mathrm{q} 2_{\mathrm{BE}} *\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}\right]+8\right)-\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{k} 2^{*} 9000\right)-(0.5 * 10000+\max \{\mathrm{k} 1, \mathrm{k} 2\} * 150)$, if $0 \leqq \mathrm{q} 2_{\mathrm{BE}}<\mathrm{k} 2 * 1000$,
where $\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}\right]=50-0.01 *\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{BE}}\right)$, and $\mathrm{k} 1, \mathrm{k} 2=1,2$, or 3 .

The following table shows quantities, price, and profits at the equilibrium point of each investment in the network capacity.
[Table 6-15] Equilibrium Points of Case 7

| Case 7 | $\mathbf{q} \mathbf{1}^{*}{ }_{\text {BE }}$ | $\mathbf{q 2}{ }^{*}{ }_{\text {BE }}$ | $\mathbf{f 1 *}$ | $\mathbf{f 2}{ }^{*}$ | $\mathbf{P}^{*}{ }_{\text {BE }}$ |
| :---: | ---: | ---: | ---: | ---: | ---: |
| $[1 \mathrm{~K}, 1 \mathrm{~K}]$ | 999 | 999 | 22833 | 22833 | 30.02 |
| $[1 \mathrm{~K}, 2 \mathrm{~K}]$ | 999 | 1999 | 12693 | 30713 | 20.02 |
| $[1 \mathrm{~K}, 3 \mathrm{~K}]$ | 999 | 2350 | 9036 | 22799 | 16.51 |
| $[2 \mathrm{~K}, 1 \mathrm{~K}]$ | 1999 | 999 | 30713 | 12693 | 20.02 |
| $[2 \mathrm{~K}, 2 \mathrm{~K}]$ | 1900 | 1900 | 12800 | 12800 | 12.00 |
| $[2 \mathrm{~K}, 3 \mathrm{~K}]$ | 1900 | 1900 | 12650 | 3650 | 12.00 |
| $[3 \mathrm{~K}, 1 \mathrm{~K}]$ | 2350 | 999 | 22799 | 9036 | 16.51 |
| $[3 \mathrm{~K}, 2 \mathrm{~K}]$ | 1900 | 1900 | 3650 | 12650 | 12.00 |
| $[3 \mathrm{~K}, 3 \mathrm{~K}]$ | 1900 | 1900 | 3650 | 3650 | 12.00 |

The following is a payoff matrix based on the above table. 2 K is a dominant strategy of both IAPs so, there is one equilibrium, $[2 \mathrm{~K}, 2 \mathrm{~K}]$. At the equilibrium, strategy sets of two IAPs’ are identical, i.e., $\{\mathrm{BE}$, flat-rate, 2 K , peering\} and they have to cooperate closely under the peering arrangement. There is a strong possibility for them to collude by cutting their investment level from 2 K to 1 K ; then they can obtain a higher profit than at their equilibrium point. At the point of $[1 \mathrm{~K}, 1 \mathrm{~K}]$, the profit of each IAP is the same as a half of monopoly profit, which means both IAP can produce a half of monopoly quantity ${ }^{70}$ and divide the monopoly profits equally.
[Table 6-16] Payoff Matrix for Case 7

| IAP1 $\mathbf{\text { IAP2 }}$ | $\mathbf{1 K}$ | $\mathbf{2 K}$ | $\mathbf{3 K}$ | Optimal $\boldsymbol{k 2 *}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 K}$ | $[22833,22833]$ | $[12693,30713]$ | $[9036,22799]$ | 2 K |
| $\mathbf{2 K}$ | $[30713,12693]$ | $[\mathbf{1 2 8 0 0 , 1 2 8 0 0 ]}$ | $[12650,3650]$ | 2 K |
| $\mathbf{3 K}$ | $[22799,9036]$ | $[3650,12650]$ | $[3650,3650]$ | 2 K |
| Optimal $\boldsymbol{k} \mathbf{1 *}^{*}$ | 2 K | 2 K | 2 K |  |

In Case 1 and Case 7, both IAP provide BE service to the market. If we compare tables $6-1$ and $6-15$, equilibrium quantities and prices at each investment level are the same in the both cases. The only difference between the two tables is the equilibrium profit of each investment level because of different cost structure; the equilibrium profits in Case 7 are lower than those in Case 1. In conclusion, because of relatively heavy capital cost, BE-IAPs in Case 1 do not have a motivation to make a peering arrangement to move to the Case 7 .

[^38]
### 6.8 Analysis of Case 8

This case is the same as the Case 4 except for peering instead of transit, i.e., both choose QoS with difference pricing scheme and decide to make QoS peering arrangement. The followings are the profit functions of both IAPs:

- $\mathrm{f} 1\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{QoS}}\right]=\mathrm{q} 1_{\mathrm{BE}} *\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]+8\right)+\mathrm{q} 1_{\mathrm{Qos}} *\left(\mathrm{p}_{\mathrm{QoS}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}\right.\right.$, $\left.\left.\mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{QoS}}\right]+8\right)-\left(\mathrm{q}_{\mathrm{BE}}+\mathrm{q} 1_{\mathrm{Qos}}{ }^{\wedge} 1.5+10000+\mathrm{k} 1 * 6000\right)-\left(\max \{\mathrm{k} 1, \mathrm{k} 2\}^{*} 150 * 2\right)$, if $0 \leqq\left(\mathrm{q} 1_{\mathrm{BE}}+\right.$ $\left.\mathrm{q} 1_{\mathrm{Qos}}\right)<\mathrm{k} 1 * 1000$,
- $\mathrm{f} 2\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]=\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{QoS}}\right)^{*}\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{QoS}}\right]+8\right)+\mathrm{h}_{\mathrm{Qos}} * \mathrm{r}^{*} \mathrm{q}_{\mathrm{QoS}}-$ $\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{Qos}} \wedge 1.6+10000+\mathrm{k} 2 * 6000\right)-(\max \{\mathrm{k} 1, \mathrm{k} 2\} * 150 * 2)$, if $0 \leqq\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{QoS}}\right)<\mathrm{k} 2 * 1000$,
where $\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]=50-0.01 *\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 1_{\mathrm{QoS}}+\mathrm{q} 2_{\mathrm{QoS}}\right)-5 * \mathrm{r}$,
$\mathrm{p}_{\mathrm{QOS}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]=\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{QoS}}\right]+50-0.01 *\left(\mathrm{q} 1_{\mathrm{QoS}}+\mathrm{q} 2_{\mathrm{QOS}}\right)$,
$h_{\mathrm{Qos}}=100-33.3^{*} \mathrm{r}, \mathrm{k} 1, \mathrm{k} 2=1,2$, or 3 , and $\mathrm{r}=1.0,1.1,1.2, \ldots$, or 2.0 .

The following table shows quantities, prices, and profits at the equilibrium point of each investment in the network capacity.
[Table 6-17] Equilibrium Points of Case 8

| Case 8 | $r$ | q1* ${ }_{\text {BE }}$ | q2 ${ }^{\text {* }}$ * | q1* ${ }_{\text {Oos }}$ | q2* ${ }_{\text {os }}$ | Q1 | Q2 | f1* | f2* | p* ${ }_{\text {BE }}$ | $\mathrm{p}^{*}{ }_{\text {OOS }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [1K, 1K] | 1.3 | 474 | 388 | 525 | 611 | 999 | 999 | 22971 | 31390 | 23.52 | 62.16 |
| [1K, 2K] | 1.3 | 474 | 1388 | 525 | 611 | 999 | 1999 | 12681 | 35620 | 13.52 | 52.16 |
| [1K, 3K] | 1.3 | 474 | 1415 | 525 | 611 | 999 | 2026 | 12111 | 29327 | 13.25 | 51.89 |
| [2K, 1K] | 1.3 | 1474 | 388 | 525 | 611 | 1999 | 999 | 27201 | 21100 | 13.52 | 52.16 |
| [2K, 2K] | 1.3 | 1158 | 1073 | 525 | 611 | 1683 | 1684 | 14507 | 22942 | 9.83 | 48.47 |
| [2K, 3K] | 1.3 | 1158 | 1073 | 525 | 611 | 1683 | 1684 | 14207 | 16642 | 9.83 | 48.47 |
| [3K, 1K] | 1.5 | 1452 | 376 | 523 | 623 | 1975 | 999 | 18845 | 20596 | 12.76 | 51.3 |
| [3K, 2K] | 1.3 | 1158 | 1073 | 525 | 611 | 1683 | 1684 | 8207 | 22642 | 9.83 | 48.47 |
| [3K, 3K] | 1.3 | 1158 | 1073 | 525 | 611 | 1683 | 1684 | 8207 | 16642 | 9.83 | 48.47 |

*Q1 $=q 1_{B E}+q 2_{B E}$ and $Q 2=q 2_{B E}+q 2_{Q O S}$

The following table is a payoff matrix based on the above table. [2K, 2 K ] is an equilibrium strategy of network capacity. At this point, even though they make an investment of the same network capacity, QoS-IAP with two-part tariff has a higher profit than QoS-IAP with flat-rate pricing.
[Table 6-18] Payoff Matrix for Case 8

| IAP1 IAP2 | $\mathbf{1 K}$ | $\mathbf{2 K}$ | $\mathbf{3 K}$ | Optimal $\boldsymbol{k 2 *}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 K}$ | $[22971,31390]$ | $[12681,35620]$ | $[12111,29327]$ | 2 K |
| $\mathbf{2 K}$ | $[27201,21100]$ | $[\mathbf{1 4 5 0 7 , 2 2 9 4 2}]$ | $[14207,11642]$ | 2 K |
| $\mathbf{3 K}$ | $[18845,20596]$ | $[8207,22642]$ | $[8207,16642]$ | 2 K |
| Optimal $\boldsymbol{k} \mathbf{1}^{*}$ | 2 K | 2 K | 2 K |  |

### 6.9 Analysis of Case 9

In this case, every strategy is the same as the Case 5 except for peering instead of transit. Both IAP choose the QoS with the flat-rate pricing. The profit functions are:

- $\mathrm{f} 1\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]=\mathrm{q} 1_{\mathrm{BE}} *\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]+8\right)+\mathrm{q} 1_{\mathrm{Qos}} *\left(\mathrm{p}_{\mathrm{Qos}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}\right.\right.$, $\left.\left.\mathrm{q} 1_{\mathrm{Qos}}, \mathrm{q} 2_{\mathrm{Qos}}\right]+8\right)-\left(\mathrm{q}_{\mathrm{BE}^{+}} \mathrm{q}_{\mathrm{Q}_{\mathrm{Qos}}} 1.5+10000+\mathrm{k} 1^{*} 6000\right)-\max \{\mathrm{k} 1, \mathrm{k} 2\}^{*} 150 * 2$, if $0 \leqq\left(\mathrm{q} 1_{\mathrm{BE}}+\right.$ $\left.\mathrm{q} 1_{\mathrm{Qos}}\right)<\mathrm{k} 1^{*} 1000$,
- $\quad \mathrm{f} 2\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{QoS}}\right]=\mathrm{q} 2_{\mathrm{BE}} *\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]+8\right)+\mathrm{q} 2_{\mathrm{Qos}} *\left(\mathrm{p}_{\mathrm{QoS}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}\right.\right.$, $\left.\left.\mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]+8\right)-\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{Qos}} \wedge^{\wedge} 1.5+10000+\mathrm{k} 1^{*} 6000\right)-\max \{\mathrm{k} 1, \mathrm{k} 2\}^{*} 150 * 2$, if $0 \leqq\left(\mathrm{q} 2_{\mathrm{BE}}+\right.$ $\left.\mathrm{q} 2_{\mathrm{Qos}}\right)<\mathrm{k} 2 * 1000$,
where $\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]=50-0.01 *\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 1_{\mathrm{Qos}}+\mathrm{q} 2_{\mathrm{Qos}}\right)$,
$\mathrm{p}_{\mathrm{Qos}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]=\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{QoS}}\right]+50-0.01 *\left(\mathrm{q} 1_{\mathrm{Qos}}+\mathrm{q} 2_{\mathrm{QOS}}\right)$, and $\mathrm{k} 1, \mathrm{k} 2=1,2$, or 3 .

The following table shows quantities, prices, and profits at the equilibrium point of each investment in the network capacity. QoS market share of both IAPs are same and constant regardless of network capacity, so the market shares of BE service determine profits of both IAPs. Compared to table 6-11, every thing is same except for equilibrium profits, which is caused by the different cost structures between peering and transit. If we take a careful look at the equilibrium tables of cases $1,5,7$, and 9 , where both IAPs choose flat-rate, since QoS users are also BE users total market shares of each IAP (Q1, Q2) is always same.
[Table 6-19] Equilibrium Points of Case 9

| Case 9 | q1* ${ }_{\text {be }}$ | q2* ${ }_{\text {bE }}$ | q1* ${ }_{\text {ob }}$ | q2* ${ }_{\text {ob }}$ | Q1 | Q2 | f1* | f2* | P** ${ }_{\text {BE }}$ | P** ${ }_{\text {os }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [1K,1K] | 460 | 460 | 539 | 539 | 999 | 999 | 29848 | 29848 | 30.02 | 69.24 |
| [1K,2K] | 460 | 1460 | 539 | 539 | 999 | 1999 | 19558 | 40578 | 20.02 | 59.24 |
| [1K,3K] | 460 | 1811 | 539 | 539 | 999 | 2350 | 15751 | 35514 | 16.51 | 55.73 |
| [2K, 1K] | 1460 | 460 | 539 | 539 | 1999 | 999 | 40578 | 19558 | 20.02 | 59.24 |
| [2K, 2 K ] | 1361 | 1361 | 539 | 539 | 1900 | 1900 | 22665 | 22665 | 12.00 | 51.22 |
| [2K, 3 K$]$ | 1361 | 1361 | 539 | 539 | 1900 | 1900 | 22365 | 16365 | 12.00 | 51.22 |
| [3K, 1K] | 1811 | 460 | 539 | 539 | 2350 | 999 | 35514 | 15751 | 16.51 | 55.73 |
| [3K,2K] | 1361 | 1361 | 539 | 539 | 1900 | 1900 | 16365 | 22365 | 12.00 | 51.22 |
| [3K,3K] | 1361 | 1361 | 539 | 539 | 1900 | 1900 | 16365 | 16365 | 12.00 | 51.22 |

$* Q 1=q 1_{B E}+q 2_{B E}$ and $Q 2=q 2_{B E}+q 2_{Q O S}$

The following is a payoff matrix based on the above table. 2 K is a dominant strategy of both IAPs and [ $2 \mathrm{~K}, 2 \mathrm{~K}$ ] is an equilibrium strategy of this case, where both IAPs have an identical strategy set.
[Table 6-20] Payoff Matrix for Case 9

| IAP1 $\mathbf{I A P 2}$ | $\mathbf{1 K}$ | $\mathbf{2 K}$ | $\mathbf{3 K}$ | Optimal $\boldsymbol{k 2}^{*}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 K}$ | $[29848,29848]$ | $[19558,40578]$ | $[15751,35514]$ | 2 K |
| $\mathbf{2 K}$ | $[40578,19558]$ | $[22665,22665]$ | $[22365,16365]$ | 2 K |
| $\mathbf{3 K}$ | $[35514,15751]$ | $[16365,22365]$ | $[16365,16365]$ | 2 K |
| Optimal $\boldsymbol{k} \mathbf{1}^{*}$ | 2 K | 2 K | 2 K |  |

### 6.10 Analysis of Case 10

This case is the same as the Case 6 except for peering instead of transit. Both IAPs choose QoS with twopart tariff. The profit functions are:

- $\mathrm{f} 1\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{QoS}}\right]=\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{q} 1_{\mathrm{QoS}}\right) *\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{QoS}}\right]+8\right)+\mathrm{h}_{\mathrm{QoS}} * \mathrm{r}^{*} \mathrm{q} 1_{\mathrm{QoS}}-$ $\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{q} 1_{\mathrm{Qos}} \wedge^{\wedge} 1.6+10000+\mathrm{k} 1 * 6000\right)-\max \{\mathrm{k} 1, \mathrm{k} 2\}^{*} 150 * 2$, if $0 \leqq\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{q} 1_{\mathrm{QoS}}\right)<\mathrm{k} 1 * 1000$,
- $\quad \mathrm{f} 2\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{QoS}}\right]=\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{QoS}}\right)^{*}\left(\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{QoS}}\right]+8\right)+\mathrm{h}_{\mathrm{QoS}} * \mathrm{r}^{*} \mathrm{q} 2_{\mathrm{QoS}}-$ $\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{Qos}} \wedge 1.6+10000+\mathrm{k} 1 * 6000\right)-\max \{\mathrm{k} 1, \mathrm{k} 2\} * 150 * 2$, if $0 \leqq\left(\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{Qos}}\right)<\mathrm{k} 2 * 1000$,
where $\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{QoS}}\right]=50-0.01 *\left(\mathrm{q} 1_{\mathrm{BE}}+\mathrm{q} 2_{\mathrm{BE}}+\mathrm{q} 1_{\mathrm{QoS}}+\mathrm{q} 2_{\mathrm{QoS}}\right)-5 * \mathrm{r}$,
$\mathrm{p}_{\mathrm{QoS}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{QoS}}, \mathrm{q} 2_{\mathrm{Qos}}\right]=\mathrm{p}_{\mathrm{BE}}\left[\mathrm{q} 1_{\mathrm{BE}}, \mathrm{q} 2_{\mathrm{BE}}, \mathrm{q} 1_{\mathrm{Qos}}, \mathrm{q} 2_{\mathrm{Qos}}\right]+50-0.01^{*}\left(\mathrm{q} 1_{\mathrm{QoS}}+\mathrm{q} 2_{\mathrm{QoS}}\right)$, $\mathrm{h}_{\mathrm{QoS}}=100-33.3 * \mathrm{r}, \mathrm{k} 1, \mathrm{k} 2=1,2$, or 3 , and $\mathrm{r}=1.0,1.1,1.2, \ldots$, or 2.0. .

The following table is quantities, prices, and profits at the equilibrium point of each investment in the network capacity. Comparing Case 6 and Case 10, where both QoS-IAPs choose a two-part tariff, everything is same except for the equilibrium profits.
[Table 6-21] Equilibrium Points of Case 10

| Case 10 | r | q1* ${ }_{\text {BE }}$ | q2* ${ }_{\text {BE }}$ | $\mathrm{q} 1{ }^{*}{ }_{\text {os }}$ | q2* ${ }_{00 \mathrm{~S}}$ | Q1 | Q2 | f1* | f2* | p* ${ }_{\text {BE }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [1K, 1K] | 1.3 | 388 | 388 | 611 | 611 | 999 | 999 | 31390 | 31390 | 23.52 |
| [1K, 2K] | 1.3 | 388 | 1388 | 611 | 611 | 999 | 1999 | 21100 | 35620 | 13.52 |
| [1K, 3K] | 1.3 | 388 | 1415 | 611 | 611 | 999 | 2026 | 20530 | 29327 | 13.25 |
| [2K, 1K] | 1.3 | 1388 | 388 | 611 | 611 | 1999 | 999 | 35620 | 21100 | 13.52 |
| [2K, 2K] | 1.3 | 1073 | 1073 | 611 | 611 | 1684 | 1684 | 22925 | 22925 | 9.82 |
| [2K, 3K] | 1.3 | 1073 | 1073 | 611 | 611 | 1684 | 1684 | 22625 | 16625 | 9.82 |
| [3K, 1K] | 1.5 | 1353 | 376 | 623 | 623 | 1976 | 999 | 27882 | 20586 | 12.75 |
| [3K, 2K] | 1.3 | 1073 | 1073 | 611 | 611 | 1684 | 1684 | 16625 | 22625 | 9.82 |
| [3K, 3K] | 1.3 | 1073 | 1073 | 611 | 611 | 1684 | 1684 | 16625 | 16625 | 9.82 |

The following is a payoff matrix based on the above table. 2 K is a dominant strategy for both IAPs and $[2 \mathrm{~K}, 2 \mathrm{~K}]$ is an equilibrium strategy of this case, where both IAPs have an identical strategy set.
[Table 6-22] Payoff Matrix for Case 10

| IAP1 $\mathbf{I A P 2}$ | $\mathbf{1 K}$ | $\mathbf{2 K}$ | $\mathbf{3 K}$ | Optimal $\boldsymbol{k 2 *}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 K}$ | $[31390,31390]$ | $[21100,35620]$ | $[20530,29327]$ | 2 K |
| $\mathbf{2 K}$ | $[35620,21100]$ | $[22925,22925]$ | $[22625,16625]$ | 2 K |
| $\mathbf{3 K}$ | $[27882,20586]$ | $[16625,22625]$ | $[16625,16625]$ | 2 K |
| Optimal $\boldsymbol{k} \mathbf{1}^{*}$ | 2 K | 2 K | 2 K |  |

All peering-QoS cases (Cases 8,9 , and 10 ) have an equilibrium at $[2 \mathrm{~K}, 2 \mathrm{~K}]$. The profits at $[1 \mathrm{~K}, 1 \mathrm{~K}]$ in all three cases are higher than those at $[2 \mathrm{~K}, 2 \mathrm{~K}]$. So, all these three cases provide an incentive for the IAPs to reduce their investment in network capacity from 2 K to 1 K .

If we compare the transit-QoS cases (Cases 4,5 , and 6 ) with the peering-QoS cases (Cases 8,9 , and 10), the former has the same strategies as the latter except for the interconnection strategy. In detail, if we compare tables (1) 6-9 (Case 4) and 6-17 (Case 8), (2) 6-11 (Case 5) and 6-19 (Case 9), and (3) 6-13 (Case 6) and 6-21 (Case 10), the equilibrium quantities and prices are the same. But the equilibrium profits in the peering cases are higher than those in the transit cases. So, QoS-IAPs would have an incentive to peer with each other. In conclusion, once IAPs upgrade their technology from BE to QoS , it would be better for them to make a peering arrangement. The following table compares equilibrium profits of QoS-IAPs at the network capacity of [ $1 \mathrm{~K}, 2 \mathrm{~K}]$, $[2 \mathrm{~K}, 1 \mathrm{~K}]$, and $[2 \mathrm{~K}, 2 \mathrm{~K}]$ between transit and peering cases.
[Table 6-23] Comparison of QoS Equilibrium Profits between Transit and Peering

| Transit | Case 4 | Case 5 | Case 6 |
| :---: | :---: | :---: | :---: |
| $[1 \mathrm{~K}, 2 \mathrm{~K}]$ | $[11281,32220]$ | $[18158,37178]$ | $[19700,32220]$ |
| $[2 \mathrm{~K}, 1 \mathrm{~K}]$ | $[23801,19700]$ | $[37178,18158]$ | $[32220,19700]$ |
| $[2 \mathrm{~K}, 2 \mathrm{~K}]$ | $[11107,19542]$ | $[19265,19265]$ | $[19525,19525]$ |
| Peering | Case 8 | Case 9 | Case 10 |
| $[1 \mathrm{~K}, 2 \mathrm{~K}]$ | $[12681,35620]$ | $[19558,40578]$ | $[21100,35620]$ |
| $[2 \mathrm{~K}, 1 \mathrm{~K}]$ | $[27201,21100]$ | $[40578,19558]$ | $[35620,21100]$ |
| $[2 \mathrm{~K}, 2 \mathrm{~K}]$ | $[14507,22942]$ | $[22665,22665]$ | $[22925,22925]$ |

At their equilibrium network capacity, all peering QoS-IAPs provide $4 \mathrm{~K}(=2 \mathrm{~K}+2 \mathrm{~K})$ network capacity to the market but except for the Case 5 the other transit QoS-IAPs provide $3 \mathrm{~K}(=1 \mathrm{~K}+2 \mathrm{~K})$ to the market. In conclusion, QoS-peering gives more capacity to the market at a lower price, which is desirable to the customers in the market.

### 6.11 Progressive Market Equilibrium Paths

If we combine equilibrium points of all ten cases, we can suggest a progressive path of IAPs' market equilibrium. We can classify the above ten cases into three categories: (1) current BE Internet: Case 1, (2) Internet in transit: Cases 2, 3, and 7, and (3) future QoS Internet: Cases 4, 5, 6, 8, 9, and 10.
[Table 6-24] Current, Transitive, and Future Internet Market Classification

| Current Internet Market | Internet Market in Transition | Future QoS Internet Market |
| :---: | :---: | :---: |
| Case 1 | Case 2, Case 3, and Case 7 | Case 4~6 and Case 8~10 |
| BE Internet | BE-Peering or Co-existence of BE and QoS | QoS Internet and QoS peering |

From the equilibrium points in Case 1, we can find a progressive path of market equilibrium over time when the technology changes from BE to QoS and the interconnection method changes from transit to peering. The decision making rules of transition from one equilibrium to another, when a new strategy is introduced, are (1) whether the profit of new equilibrium point is higher than the previous one and (2) increasing network capacity is possible but decreasing is impossible because network capacity is assumed to be classified into the irreversible sunk cost. This transition is also based on the assumption that the other's strategy set does not change. The following tables show two progressive paths:
[Table 6-25a] Progressive Market Equilibrium Paths

|  | Period 1 | Period 2 | Period 3 | Period 4 |
| :---: | :---: | :---: | :---: | :---: |
| Path I | Case 1 $[1 \mathrm{~K}, 2 \mathrm{~K}]$ | Case 2 $2 \mathrm{~K}, 2 \mathrm{~K}]$ | Case 5 $[2 \mathrm{~K}, 2 \mathrm{~K}]$ | Case 9 $[2 \mathrm{~K}, 2 \mathrm{~K}]$ <br>  <br>  <br> $[16993,34013]$ |
|  | $[19265,19265]$ |  |  |  |
| $[22665,22665]$ |  |  |  |  |
| Path II | Case 1 $[2 \mathrm{~K}, 1 \mathrm{~K}]$ | Case 2 $[2 \mathrm{~K}, 1 \mathrm{~K}]$ | Case $6[2 \mathrm{~K}, 1 \mathrm{~K}]$ |  |
|  | $[34013,16993]$ | $[21020,19700]$ | $[32220,19700]$ |  |

[Table 6-25b] Progressive Market Equilibrium Paths


At the equilibrium point of $[1 \mathrm{~K}, 2 \mathrm{~K}]$ in Case 1 , there are three possibilities to move to the next phase: (1) one IAP chooses QoS with a flat-rate pricing (Case 2), (2) one IAP chooses QoS with a two-part tariff (Case 3), and (3) both IAPs make a BE peering arrangement (Case 7). From IAP1's point of view, all three options give it the same or lower profit than just staying at the current equilibrium point. However, if IAP2 changes its technology from BE to QoS with the same flat-rate pricing, its profit would be higher than the current $(\$ 34,013 \rightarrow \$ 40,367)$. So, $[1 \mathrm{~K}, 2 \mathrm{~K}]$ in Case 2 is the second period equilibrium. Since at this point IAP2's profit is the highest among all equilibrium points in transit cases, there is no motivation for IAP2 to change its strategy set. However, even though IAP1 is not the first mover to QoS, if it changes its strategy set like $\{(\mathrm{BE} \rightarrow \mathrm{QoS})$, flat-rate, $(1 \mathrm{~K} \rightarrow 2 \mathrm{~K})$, transit $\}$, it will pay off $(\$ 16,993 \rightarrow$ $\$ 19,265$ ). The third period equilibrium point is [2K, 2 K ] in Case 5. At this point, there is an incentive to make a QoS peering arrangement between the two IAPs because of higher profits ( $\$ 19,265 \rightarrow \$ 22,665$ ). So, $[2 \mathrm{~K}, 2 \mathrm{~K}]$ in Case 9 is a final destination of Path I.

At the equilibrium point of $[2 \mathrm{~K}, 1 \mathrm{~K}]$ in Case 1, IAP2 wants to change its strategy set like $\{\mathrm{BE} \rightarrow \mathrm{QoS}$, flat-rate $\rightarrow$ two-part, 1 K , transit $\}$, which gives higher profit $(\$ 16,993 \rightarrow \$ 19,700)$. So, $[2 \mathrm{~K}, 1 \mathrm{~K}]$ in Case 3 is the second period equilibrium. At this point, IAP1's profit is reduced, so it also wants to change its strategy set like $\{\mathrm{BE} \rightarrow \mathrm{QoS}$, flat-rate $\rightarrow$ two-part, 2 K , transit $\}$, which also gives higher profit $(\$ 21,020 \rightarrow$ $\$ 32,220$ ). Therefore, the third period equilibrium is $[2 \mathrm{~K}, 1 \mathrm{~K}]$ in Case $\mathrm{VI}^{71}$. However, in Path II, due to its profit reduction ${ }^{72}$ and inequality of network capacity, IAP2 does not want to make a QoS peering arrangement. So, $[2 \mathrm{~K} 1 \mathrm{~K}]$ in Case 6 is a final destination of Path II. The following graph shows changes of profits along the two progressive paths.

[Figure 6-4] Equilibrium Profits of Progressive Paths

[^39]
### 6.12 Overall Analysis of the Game Model

In this section, we gather all the payoff matrixes of 10 cases and make them into one overall matrix. We try to find global equilibrium points in the uniform distribution. The following tables show the optimal response strategy set of IAP1 and IAP2.
[Table 6-26] IAP1's Optimal Strategy Set

| IAP2's <br> Strategy | Transit-BE-Flat |  |  | Transit-QoS-Flat |  |  | Transit-QoS-2P |  |  | Peering-BE-Flat |  |  | Peering-QoS-Flat |  |  | Peering-QoS-2P |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1K | 2K | 3K | 1K | 2K | 3K | 1K | 2K | 3K | 1K | 2K | 3K | 1K | 2K | 3K | 1K | 2K | 3K |
| IAP1's | Tran | Tran | Tran | Tran | Tran | Tran | Tran | Tran | Tran | Peer | Peer | Peer | Peer | Peer | Peer | Peer | Peer | Peer |
| Optimal | QoS | QoS | QoS | BE | QoS | QoS | QoS | QoS | QoS | BE | BE | BE | QoS | QoS | QoS | QoS | QoS | QoS |
| Strategy | Flat | Flat | Flat | Flat | 2 P | Flat | 2P | 2P | 2 P | Flat | Flat | Flat | Flat | Flat | Flat | 2P | 2P | 2P |
| Set | 2K | 2K | 2 K | 2K | 1 K | 2K | 2 K | 1 K | 2 K | 2K | 2K | 2K | 2K | 2K | 2K | 2 K | 2 K | 2 K |

[Table 6-27] IAP2's Optimal Strategy Set

| IAP1's <br> Strategy | Transit-BE-Flat |  |  | Transit-QoS-Flat |  |  | Transit-QoS-2P |  |  | Peering-BE-Flat |  |  | Peering-QoS-Flat |  |  | Peering-QoS-2P |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1K | 2K | 3K | 1K | 2K | 3K | 1K | 2K | 3K | 1K | 2K | 3K | 1 K | 2K | 3K | 1 K | 2K | 3K |
| IAP2's | Tran | Tran | Tran | Tran | Tran | Tran | Tran | Tran | Tran | Peer | Peer | Peer | Peer | Peer | Peer | Peer | Peer | Peer |
| Optimal | QoS | QoS | QoS | BE | QoS | QoS | BE | QoS | QoS | BE | BE | BE | QoS | QoS | QoS | QoS | QoS | QoS |
| Strategy | Flat | Flat | Flat | Flat | 2 P | 2P | Flat | 2P | 2P | Flat | Flat | Flat | Flat | Flat | Flat | 2 P | 2P | 2P |
| Set | 2K | 2K | 2K | 2K | 1 K | 2K | 1 K | 1 K | 2 K | 2K | 2K | 2K | 2K | 2K | 2K | 2K | 2 K | 2 K |

There are four shaded strategy sets in the above tables, which means there are four global equilibrium points in this game model. These four global equilibrium points are also equilibrium points in Cases 6,7 , 9 , and 10. Table $6-28$ shows the global equilibrium points (shaded cells) among the local equilibrium points in each case. The cells with a letter ' $\alpha$ ', ' $\beta$ ', ' $\chi$ ', and ' $\delta$ ' in table 6-29 are global equilibrium points considering all strategies. In the peering cases, there are three global equilibrium points and all of them are on a diagonal line. There is only one equilibrium point in the transit case: $[2 \mathrm{~K}, 1 \mathrm{~K}]$ of the Case 6 . The reason for more global equilibrium points in the peering case is that all transit cases have 9 options to be considered in determining an optimal strategy set but in the peering the number of options are three or six. For example, both BE-IAPs with peering have three options: $\{\mathrm{BE}$, flat-rate, peering, $(1 \mathrm{~K} / 2 \mathrm{~K} / 3 \mathrm{~K})\}$, the same options in Case 7. So, the local equilibrium in Case 7 is always the global equilibrium.
[Table 6-28] Equilibrium Profits

|  | $[\mathbf{1 K}, \mathbf{2 K}]$ | $[\mathbf{2 K}, \mathbf{1 K}]$ | $[\mathbf{2 K}, \mathbf{2 K}]$ |
| :---: | :---: | :---: | :---: |
| Case 1 | $[16993,34013]$ | $[34013,16993]$ |  |
| Case 2 | $[16993,40367]$ |  |  |
| Case 3 | $[10500,32220]$ | $[21020,19700]$ |  |
| Case 4 | $[11281,32220]$ | $[23801,19700]$ |  |
| Case 5 |  |  | $[19265,19265]$ |
| Case 6 | $[19700,32220]$ | $\alpha:[\mathbf{3 2 2 2 0}, \mathbf{1 9 7 0 0 ]}$ |  |
| Case 7 |  |  | $\beta:[\mathbf{1 2 8 0 0}, \mathbf{1 2 8 0 0 ]}$ |
| Case 8 |  |  | $[14507,22942]$ |
| Case 9 |  |  | $\chi:[22665, \mathbf{2 2 6 6 5}]$ |
| Case 10 |  |  | $\delta:[\mathbf{2 2 9 2 5}, \mathbf{2 2 9 2 5}]$ |

[Table 6-29] Equilibrium Points in Uniform Distribution

| Uniform Distribution |  |  |  | Transit |  |  |  |  |  |  |  |  | Peering |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | BE |  |  | QoS |  |  |  |  |  | $\begin{gathered} \hline \text { BE } \\ \hline \text { Flat } \end{gathered}$ |  |  | QoS |  |  |  |  |  |
|  |  |  |  | Flat |  |  | Flat |  |  | Two-Part |  |  |  |  |  | Flat |  |  | Two-Part |  |  |
|  |  |  |  | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |
|  |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | BE | Flat | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Transit |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | Flat | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | QoS |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | 2-Part | 2 |  |  |  |  |  |  | $\alpha$ |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Peering | BE | Flat | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 2 |  |  |  |  |  |  |  |  |  |  | $\beta$ |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | QoS | Flat | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 2 |  |  |  |  |  |  |  |  |  |  | $\chi$ |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | 2-Part | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  | $\delta$ |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

### 6.13 Social Welfare Analysis

According to Carlton and Perloff (1999, p71-72), one common measure of welfare from a market is the sum of consumer surplus (CS) and producer surplus (PS). This measure of welfare is the value that consumers and producers would be willing to pay and to produce the equilibrium quantity of output at the equilibrium price. The CS is defined as the amount above price paid that a consumer would willingly spend, if necessary, to consume the units purchased. The PS is defined as revenues minus variable costs, or equivalently, profits plus the fixed costs. (Varian, 1999, p382) The variable cost is dependent on output while the fixed cost is independent on the output. In our model, the interconnection and capital costs are not dependent on the number of subscribers. Therefore, in the short-run we can assume the interconnection and capital costs are fixed and the operation cost is variable.

For an explanation of calculating a social welfare, we take the equilibrium point at [ $2 \mathrm{~K}, 2 \mathrm{~K}$ ] in Case 4 (QoS-IAP1 with the flat-rate pricing and QoS-IAP2 with the two-part tariff), which is one of the complicated cases in the model. The followings are information used for the social welfare.
[Table 6-30] Information for Social Welfare

| $\underline{q 1 *}{ }_{\text {be }}$ | q1* ${ }_{\text {oos }}$ | P* ${ }_{B E}$ | f1* | $\underline{q} 1 *_{B E}+\boldsymbol{q} 2 *_{B E}$ | q1* ${ }_{B E}+q 1 *_{o o S}$ | FC1 | $\boldsymbol{h}_{\text {OoS }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1,158 | 525 | \$9.83 | \$11,107 | 2,231 | 1,683 | \$26,000 | 57 hrs |
| q2 ${ }_{\text {* }}{ }_{\text {B }}$ | q2* ${ }_{\text {o }}$ S | ${ }^{*}{ }^{*}{ }_{\text {oos }}$ | $\mathrm{f}^{*}$ | q1* ${ }_{0,5}+q 2 *{ }_{0}{ }_{\text {os }}$ | q2* ${ }_{B E}+q 2 *_{o o S}$ | FC2 | $r$ |
| 1,073 | 611 | \$48.47 | \$19,542 | 1,136 | 1,684 | \$26,000 | \$1.3 |

*FC: Fixed Cost

The CS in this case consists of (1) CS from BE and (2) CS from QoS.

## (1) CS from BE

The following equation is the BE demand function at $[2 \mathrm{~K}, 2 \mathrm{~K}]$ in Case 4.

- $\mathrm{P}_{\mathrm{BE}}=50-0.01 * \mathrm{Q}_{\mathrm{BE}}-5^{*} \mathrm{r}=43.5-0.01 * \mathrm{Q}_{\mathrm{BE}}$

We need two prices for the calculation of CS from the BE consumption because QoS users have an influence on determining the BE price. The actual number of BE users are 3,367 (= $\left.q 1{ }^{*}{ }_{B E}+q 2{ }^{*}{ }_{B E}+q 1{ }^{*}{ }_{Q O S}+q 2{ }^{*}{ }_{Q o S}\right)$ including $2,231 \mathrm{BE}$ only users. So, we need the following two prices: $P^{*}{ }_{B E}\left(Q_{B E}+Q_{Q o s}=3367\right)=\$ 9.83$, and $P_{B E}\left(Q_{B E}=2231\right)=\$ 21.19$. The shaded area in figure 6-3 shows the CS from the BE consumption. The triangle is calculated by ( $\$ 43.5-$ $\$ 21.19$ )*2231*0.5 and the rectangle part is calculated by (\$21.19-\$9.83)*2231. The sum of triangle and rectangle is the CS from the BE consumption, which equals $\$ 50,231$.

[Figure 6-5] Consumer Surplus for BE
(2) CS from QoS

The following is the QoS demand function at $[2 \mathrm{~K}, 2 \mathrm{~K}]$ in Case 4.

- $\mathrm{P}_{\mathrm{Qos}}=\mathrm{P} *{ }_{\text {BE }}+50-0.01 * \mathrm{Q}_{\mathrm{QoS}}=9.83+50-0.01 * \mathrm{Q}_{\mathrm{Qos}}=59.83-0.01 * \mathrm{Q}_{\mathrm{QoS}}$

In the case of QoS CS, we need to follow the definition of CS. Because an actual payment of the two-part tariff based on the usage hours can not be expressed in the two-dimension graph with fixed part of price $(F)$ and quantity $(Q)$, we calculate separately the total benefit and total payment from the QoS consumption, and then we calculate net benefit, i.e., (total benefit - total payment), as the CS from the QoS consumption. The total number of QoS users is 1,136 ( $=q 1{ }^{*}{ }_{Q o S}$ $\left.+q 2^{*}{ }_{Q o S}\right)$. The shaded area of the following figure shows the total benefit from the QoS
consumption, which is calculated by $\left\{(\$ 59.83-\$ 48.47)^{*} 1136 * 0.5+\$ 48.47 * 1136\right\}=\$ 61,514$. The total payment consists of (1) payment from the customer whose pricing is the flat-rate pricing (= $\left.q 1_{Q o S} * p_{Q O S}=525 * \$ 48.47=\$ 25,447\right)$ and (2) payment from the customers whose pricing is the two-part tariff $\left(=q 2_{\text {Qos }} *\left(r * h+p_{B E}\right)=611 *(\$ 1.3 * 57+\$ 9.83)=\$ 51,281\right)$. Therefore, the CS from the QoS is $-\$ 15,214(=61,514-(25,447+51,281))$. The total CS from both BE and QoS is $\$ 35,017$ ( $=\$ 50,231-\$ 15,214)$.

[Figure 6-6] Consumer Benefit for QoS

The PS can be calculated by sum of profits $\left(=30,649=f 1^{*}+f 2^{*}\right)$ and sum of fixed costs $(=52,000=$ $F C 1+F C 2)$. In this case, the PS is $\$ 82,649(=\$ 30,649+\$ 52,000)$. Finally, the social welfare is $\$ 117,666$ $(=\mathrm{CS}+\mathrm{PC}=\$ 35,017+\$ 82,649)$. The following table is a summary of these calculations. The detailed numbers of social welfare of all cases are provided in Appendix F.
[Table 6-31] Social Welfare at [2K, 2K] of Case 4

| Uniform Case 4 | CS |  |  | PS |  |  | SW |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | BE | QoS | Total | Profits | Fixed Cost | Total |  |
| [2K, 2K] | 50,231 | -15,214 | 35,017 | 30,649 | 52,000 | 82,624 | 117,666 |

The following table shows points of the highest social welfare in the model, which is based on table F-1. There are 8 points whose social welfare values are $\$ 162$, 730 : $[2 \mathrm{~K}, 2 \mathrm{~K}],[2 \mathrm{~K}, 3 \mathrm{~K}],[3 \mathrm{~K}, 2 \mathrm{~K}]$, and $[3 \mathrm{~K}$, $3 \mathrm{~K}]$ of Case 5 and Case 9, which are higher than any other. This means that the strategy set of $\{\mathrm{QoS}$, flatrate, transit/peering, $2 \mathrm{~K} / 3 \mathrm{~K}\}$ is a socially desirable one. These points (Case 5 and 9 ) are different from the destination points of market equilibrium progressive paths (Case 6 and 9 ) and global equilibrium points of this model (Case 6, 7, 9, and 10), which suggests that market equilibrium does not mean socially desirable points.
[Table 6-32] The Highest SW Points

| Uniform Distribution |  |  |  | Transit |  |  |  |  |  |  |  |  | Peering |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | QoS |  |  |  |  |  | $\frac{\text { BE }}{\text { Flat }}$ |  |  | QoS |  |  |  |  |  |
|  |  |  |  | Flat |  |  | Flat |  |  | Two-Part |  |  |  |  |  | Flat |  |  | Two-Part |  |  |
|  |  |  |  | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |
|  | $\begin{aligned} & \mathrm{B} \\ & \mathrm{E} \end{aligned}$ | F | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | $\begin{aligned} & \mathrm{Q} \\ & \mathrm{o} \\ & \mathrm{~S} \end{aligned}$ | F | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | $\begin{aligned} & 2 \\ & \mathrm{P} \end{aligned}$ | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & \mathrm{P} \\ & \mathrm{e} \\ & \mathrm{e} \\ & \mathrm{r} \\ & \mathrm{i} \\ & \mathrm{n} \\ & \mathrm{~g} \end{aligned}$ | $\begin{aligned} & \mathrm{B} \\ & \mathrm{E} \end{aligned}$ | F | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | F | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | Q |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 0 |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | S | 2 | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

In section 6.12, we found multiple global equilibrium points ( $\alpha, \beta, \chi$, and $\delta$ ). An Internet policy maker, i.e., a government agency, has a responsibility to induce the Internet into a socially desirable point among them. According to the progressive market equilibrium paths, $\alpha$ (Case 6) and $\chi$ (Case 9) are plausible in the future $\operatorname{QoS}$ Internet. Through social welfare analysis we realize that $\chi$ is more desirable than $\alpha$. The differences between $\chi$ and $\alpha$ are interconnection strategy and pricing scheme: $\alpha$ (Case 6) has transit interconnection strategy and two-part tariff and $\chi$ (Case 9) has peering interconnection strategy and flatrate pricing. Therefore, in the QoS Internet the government agency would have to encourage flat-rate pricing and peering: it can require IAPs to offer a flat-rate option to its customers, which has been the case in the U.S. telephone industry and it can drive the industry to make a peering arrangement. Currently, peering has been achieved among big IAPs and IBPs, but it is not popular between local small IAPs. It is technically possible for QoS-IAPs to peer each other. However, it is also possible for them to remark and degrade the other's traffic in their own network. In advance of the QoS-peering, each QoSIAP needs to trust that their traffic will be treated fairly and equally. From this perspective, one of the government's roles is to help an ease of peering without distrust by establishing regulations and standards.

## [CHAPTER 7] SIMULATION AND ANALYSIS OF EMPIRICAL DISTRIBUTION

In the previous chapter we use the theoretical uniform distribution for the customers' willingness-to-pay. In this chapter, instead of the uniform distribution, we use an empirical distribution from a survey of Internet usage, on which we draw an estimated demand function by simulation and linear regression. We expect the empirical distribution to make the game model close to the real market situation.

### 7.1 Demand Function Based on the Empirical Distribution

In the U.S. GAO report (2001), there is a survey question: "About how much do you pay per month to access the Internet from your home?" Although this question does not provide an exact willingness-topay for Internet access but we can use this answer as a proxy for customer's demand. The following is the distribution from this question.
[Table 7-1] Distribution of Household Expenditure for Internet Access (per Month)

| Sub-Range | $\mathbf{\$ 0}$ | $\sim \mathbf{~ \$ 5}$ | $\mathbf{\sim 1 0}$ | $\boldsymbol{\sim 1 5}$ | $\boldsymbol{\sim} \mathbf{\$ 2 0}$ | $\sim \mathbf{\$ 3 0}$ | $\sim \mathbf{\$ 4 0}$ | $\sim \mathbf{\$ 5 0}$ | $\mathbf{\$ 5 0} \sim$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 8.9 | 1.4 | 3.8 | 8.3 | 21.0 | 31.7 | 11.1 | 8.7 | 5.1 |

* Source: GAO-01-345, Characteristics and Choices of Internet Users, p46.

We assume price range for BE is from $\$ 0$ to $\$ 50$, which is the same as in the uniform distribution and the data within a sub-range is uniformly distributed. We modified the above table into an equal sub-range: [ $\$ 0-\$ 10],[\$ 10-\$ 20],[\$ 20-\$ 30]$, $[\$ 30-\$ 40]$, and $[\$ 40-\$ 50]$. The following is a modified piecewise uniform distribution table according to the above assumptions.
[Table 7-2] Piecewise Uniform Distribution

| Sub-Range | $\mathbf{\$ 0} \mathbf{- \$ 1 0}$ | $\mathbf{\$ 1 0 - \$ 2 0}$ | $\mathbf{\$ 2 0 - \$ 3 0}$ | $\mathbf{\$ 3 0} \mathbf{- \$ 4 0}$ | $\mathbf{\$ 4 0 - \$ 5 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\%$ | 14.1 | 29.3 | 31.7 | 11.1 | 13.8 |

We use a two-stage RNG: (1) first, we use the above empirical distribution table (table 7-1) to match a sub-range and (2) then we use the piecewise uniform distribution (table 7-2) for a specific value. For example, if a random number $(\alpha)$ generated from the empirical distribution is between 0.0 and 0.141 (the $1^{\text {st }}$ sub-range), then this customer's willingness-to-pay ( $\beta$ ) is determined by a function of uniform [ $\$ 0$, $\$ 10]$. Table 7-3 shows this two-stage RNG method.
[Table 7-3] Two-Stage RNG

| $\alpha=$ RNG-1 | Sub-Range | $\beta=$ RNG-2 |
| :---: | :---: | :---: |
| $0.0000<\alpha<0.1410$ | $\$ 0 \sim \$ 10$ | $\beta=$ Uniform $[\$ 0, \$ 10]$ |
| $0.1411<\alpha<0.4340$ | $\$ 10 \sim \$ 20$ | $\beta=$ Uniform $[\$ 10, \$ 20]$ |
| $0.4341<\alpha<0.7510$ | $\$ 20 \sim \$ 30$ | $\beta=$ Uniform $[\$ 20, \$ 30]$ |
| $0.7511<\alpha<0.8620$ | $\$ 30 \sim \$ 40$ | $\beta=$ Uniform $[\$ 30, \$ 40]$ |
| $0.8621<\alpha<1.0000$ | $\$ 40 \sim \$ 50$ | $\beta=$ Uniform $[\$ 40, \$ 50]$ |

Based on these assumptions and methods, we conducted 30 times simulations and linear regressions, the results of which are presented in Appendix C. The estimated demand function for BE is presented in figure 7-1. The kinked line represents a demand curve from the simulation and the straight line represents a demand curve from the linear regression, which is " $P=44.55-0.00855^{*} Q$." We estimate a QoS demand function by applying the product differentiation theory as we did in the uniform distribution case.

[Figure 7-1] Demand Curves for Empirical Distribution

Based on this empirically estimated demand function, we analyze the 10 cases and the overall model as we did in the uniform distribution. We do not explain each case of empirical distribution in detail as we did in the uniform distribution. The detailed equilibrium quantities, prices, and profits at the equilibrium points of each case are presented in Appendix D and the payoff matrixes of 10 cases are provided in

Appendix E. The following table shows the basic demand functions of uniform and empirical distributions.
[Table 7-4] Demand Function Comparison

|  | Uniform | Empirical |
| :---: | :---: | :---: |
| $\mathbf{B E}$ | $\mathrm{P}_{\mathrm{BE}}=50-0.01 * \mathrm{Q}_{\mathrm{BE}}$ | $\mathrm{P}_{\mathrm{BE}}=44.55-0.00855 * \mathrm{Q}_{\mathrm{BE}}$ |
| $\mathbf{Q o S}$ | $\mathrm{P}_{\mathrm{Q} \text { OS }}=\mathrm{P}_{\mathrm{BE}}+50-0.01 * \mathrm{Q}_{\mathrm{QoS}}$ | $\mathrm{P}_{\mathrm{QoS}}=\mathrm{P}_{\mathrm{BE}}+44.55-0.00855 * \mathrm{Q}_{\mathrm{QoS}}$ |

If we compare two demand functions, the demand function of uniform distribution has a higher intercept $(50>44.55)$ and a steeper slope $(0.01>0.00855)$ than that of empirical distribution. For the most of parts, the demand curve of the uniform distribution is above the demand curve of the empirical distribution.

### 7.2 Progressive Market Equilibrium Paths

The empirical distribution has the same equilibrium network capacity as in the uniform distribution except for Case $7^{73}$. We try to find progressive market equilibrium paths from the equilibrium points of Case 1 as we did in the uniform distribution. There are identical, two progressive market equilibrium paths as in the uniform distribution. Tables $7-5 \mathrm{a}$ and $7-5 \mathrm{~b}$ show these two paths. In conclusion, there is no significant difference in the results of equilibrium analyses between two distributions, thus uniform distribution assumption is reasonable in this game model.
[Table 7-5a] Progressive Market Equilibrium Paths

| Empirical Distribution |  |  |  | Transit |  |  |  |  |  |  |  |  | Peering |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | BE |  |  | QoS |  |  |  |  |  | BE |  |  | QoS |  |  |  |  |  |
|  |  |  |  |  | Flat |  |  | Flat |  |  | o-P |  |  | Flat |  |  | Flat |  |  | o-P |  |
|  |  |  |  | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |
|  |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | BE | Flat | 2 |  |  |  |  | - |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  | ! |  |  |  |  |  |  |  |  |  |  |  |
| Transit |  |  | 1 |  |  |  |  | $\downarrow$ |  | ! |  |  |  |  |  |  |  |  |  |  |  |
|  |  | Flat | 2 |  |  |  |  |  |  | $\vdots$ |  |  |  |  |  |  |  |  |  |  |  |
|  | QoS |  | 3 |  |  |  |  |  |  | ! |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | 2-Part | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Peering | BE | Flat | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | QoS | Flat | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | 2-Part | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

[^40][Table 7-5b] Progressive Market Equilibrium Paths

|  | Period 1 | Period 2 | Period 3 | Period 4 |
| :---: | :---: | :---: | :---: | :---: |
| Path I | Case 1 [1K, 2K] | Case 2[1K, 2K] | Case 5 $[2 \mathrm{~K}, 2 \mathrm{~K}]$ | Case 9 $[2 \mathrm{~K}, 2 \mathrm{~K}]$ |
|  | $[15891,31808]$ | $[15891,35385]$ | $[16101,16101]$ | $[19501,19501]$ |
| Path II | Case 1 [2K, 1K] | Case 3 $[2 \mathrm{~K}, 1 \mathrm{~K}]$ | Case 6 $[2 \mathrm{~K}, 1 \mathrm{~K}]$ |  |
|  | $[31808,15891]$ | $[18815,18598]$ | $[30015,18598]$ |  |

### 7.3 Overall Analysis of the Game Model

In this section, we make all 10 payoff matrixes into one combined matrix and then try to find global equilibria in the game model. The following two tables show optimal strategy sets of IAP1 and IAP2 in the empirical distribution.
[Table 7-6] IAP1's Optimal Strategy Set

| IAP2's Strategy | Transit-BE-Flat |  |  | Transit-QoS-Flat |  |  | Transit-QoS-2P |  |  | Peering-BE-Flat |  |  | Peering-QoS-Flat |  |  | Peering-QoS-2P |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1K | 2K | 3 K | 1K | 2K | 3 K | 1 K | 2K | 3 K | 1K | 2K | 3K | 1 K | 2K | 3K | 1K | 2K | 3K |
| IAP1's | Tran | Tran | Tran | Tran | Tran | Tran | Tran | Tran | Tran | Peer | Peer | Peer | Peer | Peer | Peer | Peer | Pee | Peer |
| Optimal | QoS | QoS | Qos | QoS | QoS | Qo | Qos | Qos | QoS | BE | BE | BE | Qos | Qos | Qos | Qos | Qos | oS |
| Strategy | Fla | Flat | Fla | Flat | Flat | Fl | 2 P | 2 P | 2 P | Flat | Fl | Flat | Flat | Flat | Flat | 2 P | 2 P | 2 P |
| Set | 2K | 2 K | 2 K | 2 K | 2 K | 2 K | 2K | 1 K | 2 K | 2 K | 1 K | 2K | 2 K | 2K | 2K | 2 K | 2 K | 2 K |

[Table 7-7] IAP2's Optimal Strategy Set

| IAP1's <br> Strategy | Transit-BE-Flat |  |  | Transit-QoS-Flat |  |  | Transit-QoS-2P |  |  | Peering-BE-Flat |  |  | Peering-QoS-Flat |  |  | Peering-QoS-2P |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1K | 2K | 3K | 1K | 2K | 3K | 1K | 2K | 3K | 1K | 2K | 3K | 1K | 2K | 3K | 1K | 2K | 3K |
| IAP2's | Tran | Tran | Tran | Tran | Tran | Tran | Tran | Tran | Tran | Peer | Peer | Peer | Peer | Peer | Peer | Peer | Peer | Peer |
| Optimal | QoS | QoS | QoS | QoS | QoS | QoS | QoS | QoS | QoS | BE | BE | BE | QoS | QoS | QoS | QoS | QoS | QoS |
| Strategy | Flat | 2P | Flat | Flat | 2 P | 2 P | 2P | 2P | 2 P | Flat | Flat | Flat | Flat | 2P | 2P | 2 P | 2P | 2P |
| Set | 2K | 1 K | 2 K | 2K | 1 K | 2K | 2 K | 1 K | 2 K | 2K | 1K | 2 K | 2K | 2 K | 2 K | 2K | 2 K | 2 K |

There are four global equilibrium points, $\varepsilon$, two $\phi$ s, and $\gamma$ in table $7-9$, and two of them, $\varepsilon([2 \mathrm{~K}, 1 \mathrm{~K}]$ of Case 6 ) and $\gamma([2 \mathrm{~K}, 2 \mathrm{~K}]$ of Case 10 ), are the same positions as $\alpha$ and $\delta$ in the uniform distribution. These four global equilibria are also the local equilibrium points in Cases 6, 7, and 10. While in the result of uniform distribution QoS-IAPs could choose flat-rate or a two-part tariff in their global equilibrium points, in the empirical distribution QoS-IAPs could not choose flat-rate. Table 7-8 shows a collection of local equilibrium points in the empirical distribution and the shaded cells are the global equilibrium points.
[Table 7-8] Equilibrium Profits

|  | $[\mathbf{1 K}, \mathbf{2 K}]$ | $[\mathbf{2 K}, \mathbf{1 K}]$ | $[\mathbf{2 K}, \mathbf{2 K}]$ |
| :--- | :---: | :---: | :---: |
| Case 1 | $[15891,31808]$ | $[31808,15891]$ |  |
| Case 2 | $[15891,35385]$ |  |  |
| Case 3 | $[9730,30782]$ | $[18815,18598]$ |  |
| Case 4 | $[8273,30015]$ | $[19690,18598]$ |  |
| Case 5 |  |  | $[16101,16101]$ |
| Case 6 | $[18598,30015]$ | $\varepsilon:[\mathbf{3 0 0 1 5 , 1 8 5 9 8}]$ |  |
| Case 7 | $\phi:[\mathbf{1 1 5 9 1 , 2 8 5 0 8 ]}$ | $\phi:[\mathbf{2 8 5 0 8 , 1 1 5 9 1 ]}$ |  |
| Case 8 |  |  | $[10640,20965]$ |
| Case 9 |  |  | $[19501,19501]$ |
| Case 10 |  |  | $\gamma:[\mathbf{2 0 9 6 5}, \mathbf{2 0 9 6 5}]$ |

[Table 7-9] Equilibrium Points in Empirical Distribution

| Empirical Distribution |  |  |  | Transit |  |  |  |  |  |  |  |  | Peering |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | BE |  |  | QoS |  |  |  |  |  | BE |  |  | QoS |  |  |  |  |  |
|  |  |  |  | Flat |  |  | Flat |  |  | Two-Part |  |  | Flat |  |  | Flat |  |  | Two-Part |  |  |
|  |  |  |  | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |
| Transit | BE | Flat | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | QoS | Flat | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | 2-Part | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 2 |  |  |  |  |  |  | $\varepsilon$ |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Peering | BE | Flat | 1 |  |  |  |  |  |  |  |  |  |  | $\phi$ |  |  |  |  |  |  |  |
|  |  |  | 2 |  |  |  |  |  |  |  |  |  | $\phi$ |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | QoS | Flat | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | 2-Part | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\gamma$ |  |
|  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

### 7.4 Social Welfare Analysis

We calculate social welfares of all 90 equilibrium points ( $=9$ cells * 10 cases) in the empirical distribution as we did in the uniform distribution. The detailed numbers are presented at table F-2 in Appendix F. The following table shows the highest social welfare in the empirical distribution. This point is $[1 \mathrm{~K}, 3 \mathrm{~K}]$ in Case $2(=\$ 147,936)$, which is not a local equilibrium in Case 2 and which is different from the highest social welfare points in the uniform distribution. When we compare social welfares of [ 2 K , 2 K ] in Case 5 (one of the highest social welfare points in the uniform distribution) between table F-1 (uniform) and table F-2 (empirical) in Appendix F, there is a big difference in the BE consumer surplus ( 66,390 vs. 39,145 ). But there is a small difference of BE consumer surplus at [ $1 \mathrm{~K}, 3 \mathrm{~K}$ ] in Case 2 (the highest social welfare in the empirical distribution) in both distributions ( 53,999 vs. 51,400 ). The reason for different points of highest social welfares in the two distributions mainly comes from the level of
difference in the BE consumer surplus by using different estimated demand functions. However, if we consider only local equilibrium of each case, the highest social welfares are [ $2 \mathrm{~K}, 2 \mathrm{~K}$ ] of Case 5 and Case $9(=\$ 127,386)$, which are the same points as in the uniform distribution.
[Table 7-10] Highest Social Welfare Points


If we try to compare the social welfares at the global equilibrium points from the two distributions, social welfares of uniform distribution are higher than those of empirical distribution. The reason for higher social welfare in the uniform distribution comes from the fact that the demand functions of empirical distribution are below those of uniform distribution, which causes lower consumer surplus, lower profits, and finally lower social welfare. The following two tables display the social welfare values of the global equilibrium points in both distributions. In the both distributions, social welfares of peering equilibrium points ( $\beta, \chi, \delta, \phi$, and $\gamma$ ) are higher than those of transit equilibrium points ( $\alpha$ and $\varepsilon$ ), which means that peering is socially more desirable than transit. But comparing the combined profit of both IAPs at these global equilibrium points, the profit of transit is higher than that of peering, which is another obstacle to peer each other.
[Table 7-11] Social Welfare of Global Equilibrium in the Uniform Distribution

| Uniform Distribution |  | CS |  |  | PS |  |  | SW |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | BE | QoS | Total | Profits | Fixed Cost | Total |  |
| $\alpha$ | Case $6[2 \mathrm{~K}, 1 \mathrm{~K}]$ | 38806 | -14725 | 24081 | 51920 | 44000 | 95920 | 120,001 |
| $\beta$ | Case 7 [2K,2K] | 72200 | N/A | 72200 | 25600 | 46600 | 72200 | 144,400 |
| $\chi$ | Case 9 [2K,2K] | 66390 | 5810 | 72200 | 45330 | 45200 | 90530 | 162,730 |
| $\delta$ | Case 10 [2K, 2 K ] | 49251 | -14725 | 34526 | 45850 | 45200 | 91050 | 125,576 |

[Table 7-12] Social Welfare of Global Equilibrium in the Empirical Distribution

| Empirical <br> Distribution |  |  | CS |  |  | PS |  |  |
| :---: | :---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
|  | BE | QoS | Total | Profits | Fixed Cost | Total |  |  |
| $\boldsymbol{\varepsilon}$ | Case 6 [2K,1K] | 32040 | -18055 | 13985 | 48613 | 44000 | 92613 | 106,598 |
| $\phi$ | Case 7 <br> $[1 \mathrm{~K}, 2 \mathrm{~K}],[2 \mathrm{~K}, 1 \mathrm{~K}]$ | 38424 | N/A | 38424 | 40099 | 37600 | 77699 | $\mathbf{1 1 6 , 1 2 3}$ |
| $\gamma$ | Case 10 [2K,2K] | 46405 | -18055 | 28350 | 41930 | 45200 | 87130 | 115,480 |

## [CHAPTER 8] CONCLUSION

### 8.1 Answers for Research Hypotheses

We have five research hypotheses in the section 1.4. Based on the 10 equilibrium results of each distribution, we can accept or reject these five hypotheses.
(H1) An IAP with BE technology has the same profit in market equilibrium as another IAP with the same technology.

- This hypothesis is related with Case 1 and Case 7. The following table shows the equilibrium profits in Case 1 and Case 7.
[Table 8-1] Equilibrium Profits of Case 1 and Case 7

|  | Uniform Distribution | H1 | Empirical Distribution | H1 |
| :--- | :---: | :---: | :---: | :---: |
| Case 1 | $(1 \mathrm{~K}, 2 \mathrm{~K})=[16993,34013]$ | F | $(1 \mathrm{~K}, 2 \mathrm{~K})=[15891,31808]$ | F |
|  | $(2 \mathrm{~K}, 1 \mathrm{~K})=[34013,16993]$ |  | $(2 \mathrm{~K}, 1 \mathrm{~K})=[31808,15891]$ |  |
| Case 7 | $(2 \mathrm{~K}, 2 \mathrm{~K})=[12800,12800]$ | T | $(1 \mathrm{~K}, 2 \mathrm{~K})=[11591,28508]$ <br> $(2 \mathrm{~K}, 1 \mathrm{~K})=[28508,11591]$ | F |

- According to the above table, when the network capacities of both IAPs are different, i.e., ( 1 K , $2 \mathrm{~K})$ or $(2 \mathrm{~K}, 1 \mathrm{~K})$, a profit of the larger IAP is higher than that of the smaller IAP. The H1 is only true when the capacities of both IAPs are the same.
- Therefore, we reject H1. This implies that the same technology does not guarantee the same profit.
- However, in Case 1 and Case 7, every strategy of both IAPs is the same except for the asymmetric investment in network capacity. If we gradually reduce the number of users for one set of equipment and T1 lines from 1000 to $500,300,200$, and 100 , the network capacity of equilibrium asymptotically converge equally, which leads to the symmetric equilibrium strategies of both IAPs and the equal equilibrium profits. This means that the asymmetric outcome is an artifact of the lumpiness of investment and the relatively small market size of the rural market.
(H2) An IAP with QoS technology and usage sensitive QoS pricing has a higher profit in market equilibrium than an IAP with QoS technology and flat rate QoS pricing.
- This hypothesis is related with Case 4 and Case 8 , where the IAP1 chooses the flat rate pricing and the IAP2 chooses the two-part tariff. The following table shows the equilibrium profits in Case 4 and Case 8.
[Table 8-2] Equilibrium Profits of Case 4 and Case 8

|  | Uniform Distribution | H2 | Empirical Distribution | H2 |
| :--- | :---: | :---: | :---: | :---: |
| Case 4 | $(1 \mathrm{~K}, 2 \mathrm{~K})=[11281,3220]$ <br> $(1 \mathrm{~K}, 2 \mathrm{~K})=[23801,19700]$ | F | $(1 \mathrm{~K}, 2 \mathrm{~K})=[8273,30015]$ |  |
| $(1 \mathrm{~K}, 2 \mathrm{~K})=[19690,18598]$ |  |  |  |  | F.

- According to the above table, when the network capacities of both IAPs are the same, the IAP with the two-part tariff has a higher profit than the IAP with the flat-rate pricing. But when the network capacities are different, the larger has a higher profit than the smaller regardless of pricing scheme. Therefore we reject H2.
(H3) An IAP with BE technology and flat-rate pricing has a lower profit in market equilibrium than an IAP with QoS technology and flat-rate QoS pricing.
- This hypothesis is related with Case 2. The following table shows the equilibrium profits of the Case 2.
[Table 8-3] Equilibrium Profits of Case 2

|  | Uniform Distribution | H3 | Empirical Distribution | H3 |
| :---: | :---: | :---: | :---: | :---: |
| Case 2 | $(1 \mathrm{~K}, 2 \mathrm{~K})=[16993,40367]$ | T | $(1 \mathrm{~K}, 2 \mathrm{~K})=[15891,35385]$ | T |

- In the Case 2, QoS-IAP is larger and it has a higher profit in the both distributions. Therefore we accept H3.
(H4) An IAP with QoS technology with usage-sensitive QoS pricing has a higher profit in market equilibrium than an IAP with BE technology and flat-rate pricing.
- This hypothesis is related with Case 3. The following table shows the equilibrium profits.
[Table 8-4] Equilibrium Profits of Case 3

|  | Uniform Distribution | H4 | Empirical Distribution | H4 |
| :---: | :---: | :---: | :--- | :---: |
| Case 3 | $[1 \mathrm{~K}, 2 \mathrm{~K}]=(10500,32220)$ | F | $[1 \mathrm{~K}, 2 \mathrm{~K}]=(9730,30782)$ | F |
|  | $[2 \mathrm{~K}, 1 \mathrm{~K}]=(21020,19700)$ |  | $[2 \mathrm{~K}, 1 \mathrm{~K}]=(18815,18598)$ |  |

- Even a BE-IAP, if it has a larger network capacity at equilibrium, has a higher profit than a QoSIAP with the two-part tariff. Therefore, we reject H 4 .
(H5) Both IAPs with QoS technology give a higher social welfare than any other cases.
- According to tables 7-11 and 7-12 (social welfare of global equilibrium in both distributions), the points $\beta$ and $\phi$ are related to BE technology and the other points $\alpha, \chi, \delta, \varepsilon$, and $\gamma$, are related to QoS technology. The social welfares of $\beta$ and $\phi$ are the second highest in each distribution respectively.
- Thus, the social welfare values of QoS technology are not always higher than those of BE technology. Therefore we reject H5, which means introducing QoS technology does not always means higher social welfare.

Through testing the five research hypotheses, we conclude that there is no single dominant strategy to win in the Internet access market. Technology strategy alone or pricing strategy alone is not enough to dominate in the market. The winning strategy set is a combination of multiple strategies and it is different according to the other's strategy set. The result of H 5 gives an implication to non-rural networks: Introducing new technology does not always guarantee social welfare improvement. If cost of new technology is larger than benefit or if demand of new technology is not enough to cover the cost, it would make social welfare worse than before.

### 8.2 Further Study

There are some limitations in the QoS Internet game model. The followings should be considered in the future study:
(1) IAPs' access technology is confined to the narrowband Internet service. We need to expand our research to cover broadband service. If we introduce broadband technologies such as DSL, cablemodem, or wireless technologies, we need to modify the cost structure of the game. Furthermore, whether IAPs start as a broadband provider or migrate from a narrowband provider, their cost structure also differs.
(2) The assigning mechanism of the quality level is arbitrary. We assume the number of classes that a service can offer is the quality level of that service. ( $M_{B E}=1$ and $M_{Q O S}=2$ ) We need more logical method to assign the quality level of each service or we introduce the quality level $(M)$ as a control variable in the profit function.
(3) We only consider the Cournot game model. When a product is differentiated with various qualities, the Bertrand price competition may be the appropriate model to apply.
(4) Price of two-part tariff QoS-IAP $\left(=\left(P_{B E}+r^{*} h_{Q o S}\right) /\right.$ user $)$ is relatively high comparing that of flat rate QoS-IAP ( $=P_{Q o S} /$ user). Considering the current practice in the industry, we should consider
to give a certain mount of allowance (i.e., free QoS usage for the first 10 hours) to the two-part tariff QoS users.
(5) We need to reconsider cost of QoS-IAP with the flat-rate pricing. QoS users with the flat-rate pricing will generate much more QoS traffic than QoS users with the two-part tariff. Therefore, QoS-IAP with the flat-rate pricing costs higher than QoS-IAP with the two-part tariff when they support the same number of QoS users.
(6) In the local peering, even peering IAPs are usually dependent on an IBP for their outgoing traffic. So, it is reasonable for rural local IAPs to have a mixed interconnection strategy, i.e., local peering with local IAPs and transit with one or two IBPs.

### 8.3 Conclusion

The Internet has become an important social and business tool. The market has been quite dynamic since it was privatized. The Internet will evolve from our current situation but we don't know what the future Internet will be. The one thing we do know is QoS will be introduced in the near future. By studying rural markets where the market structure is simpler, we are able to construct reasonable economic model for QoS Internet. In this dissertation, we show that the unique cost and revenue structure of the Internet access market has a significant influence on the equilibrium results.

Internet technology has been developed and it impacts on the Internet market. The current Internet has flourished in the last three decades on the basis of BE technology and flat-rate pricing but it also has several problems in the areas of congestion, over-subscription, and interconnection. What will the future QoS Internet market be? The future IAPs might have more options for their strategic choices. Considering analyses in the dissertation, we can draw what the future Internet might be as follows:
(1) QoS technology will be introduced with \{two-part tariff, transit\}, \{two-part tariff, peering\}, or \{flat-rate, peering\},
(2) Network capacity is an important strategy to decide the market equilibrium in the future as well as in the current,
(3) A peering arrangement will provide higher social welfare than transit will in the QoS Internet, and
(4) A BE service will survive in the QoS Internet and will contribute a considerable portion to the IAPs' revenue.

Therefore, new technology (QoS) and larger capacity will pay off in the future Internet market. Even if you are not the first mover toward QoS, a second mover will pay off.

The analysis in this dissertation is based on the rural area. Is it possible to apply this analysis to urban areas? There are more IAPs in the urban area and competition is fiercer than in the rural area. But what we have learned from this dissertation can serve as a guideline to the urban Internet access market, too.

What is a good strategy for a start-up IAP in the rural area? It depends on how much a new IAP has financial power and how much demand exists in that area. Without adequate fianacing, the start-up rural IAP has a limited choice of strategies, i.e., BE technology and 1 K network capacity. According to table 22 in the section 2.4, the average number paying dial-up users ${ }^{74}$ per local IAP (without top 10 dial-up IAPs) is roughly 800 . That means those local IAPs may not have large enough subscriber base to accumulate money to invest in the network capacity and upgrading to QoS. The 1 K network capacity is enough to support 800 subscribers, thus we can say that $[1 \mathrm{~K}, 1 \mathrm{~K}]$ in Case 1 might be a real situation in the rural Internet access market. At the equilibrium of $[1 \mathrm{~K}, 1 \mathrm{~K}]$ in Case 1 , in all 90 equilibrium points, the sum of profits of IAP1 and IAP2 is relatively very high but the social welfare value of this point is relatively very low. Therefore, there is no incentive for additional investment to expand their business for more network capacity or upgrading their technology to QoS: minimum investment but relatively high profits.

A policy maker's goal for the Internet industry is continuing growth and innovation. To achieve this goal, they are trying to encourage competition and to give incentives for ongoing investment and deploying new technology, which will give benefit to consumers in the Internet market. Therefore, it is a role of Internet policy makers to induce the rural IAPs to move into the socially desirable equilibrium point in the future without market distortion.

[^41]
## APPENDIX(CES)

## APPENDIX A

## A Summary of Peering Policies of UUNET and C\&W

[Table A-1] Example of Peering Policy

| Requirement | UUNET $^{\mathbf{7 5}}$ | C\&W $^{\mathbf{7 6}}$ |
| :---: | :--- | :--- |
| Geographic Scope | 50\% of the geographic region of <br> UUNET, 15 states in U.S., 8 <br> countries in Europe, and 2 countries <br> in Aisa-Pacific Region | Backbone node at the 9 geographic <br> regions in which C\&W also has <br> nodes (3 in west coast, 2 in east <br> coast, 4 in middle) |
| Number of meeting <br> place in the U.S. | At least 4, one in East Coast, one in <br> West Coast, and 2 in Midwest | At least 4, East Coast, West Coast, <br> Midwest, and South |
| Backbone Capacity | Nationwide redundant OC-12 <br> Traffic Exchange <br> Ratio | Nationally deployed redundant OC- <br> 48 |
| Traffic Volume | At least 150 Mbps | At least 45 Mbps |
| NOC | 24 hours x 7 days | 24 hours x 7 days |
| Measurement | All traffic or sample, Peak <br> Utilization | Measuring in each direction <br> (inbound and outbound) |
| ETC | Mutual Non-Disclosure Agreement <br> Shortest Exit Routing | Providing Network information <br> (map, topology, capacity). No <br> Transit customer of C\&W |

[^42]
## APPENDIX B

## Market Research for Broadband Internet Access Technology

(1) According to the USGAO (United States General Accounting Office) report (2001), the conventional telephone line was still the most common method of transport to the Internet at 2000. The following is the distribution table for Internet access method from that report.

## [Table B-1] Distribution of Internet Access Methods

| Access Methods | Dial-up | DSL | Cable Modem | Wireless |
| :---: | :---: | :---: | :---: | :---: |
| Percent | $87.5 \%$ | $3.2 \%$ | $8.9 \%$ | $0.4 \%$ |

- Source: GAO-01-345, Characteristics and Choices of Internet Users
(2) According to the Strategies Group, with an annual growth rate of $230 \%$, the broadband market will surpass 36 million US household by 2005 (McClure, 2001), which means that until 2005 dial-up access technology will be dominant in the market.
(3) A Department of Commerce study shows that $7.3 \%$ of rural subscribers use broadband compared to $11.8 \%$ of urban subscribers (Bright, 2000), which means that rural areas have a higher dependency on the dial-up Internet access than urban areas.
(4) According to the report from the joint team of NTIA (National Telecommunications and Information Administration) and RUS (Rural Utility Services) (2000), many broadband companies concentrate on large cities and well populated towns, and rural towns (under 25,000 population) do not have a broadband service. The following tables, which come from the above report, show the deployment of cable modem service and RBOC's DSL service across cities and towns of various sizes. Our concerned town size in this paper is below 25,000 populations.
[Table B-2] Broadband Cable Access by Town Size
$\left.\begin{array}{|l|l|l|l|l|l|l|}\hline \text { Town Size } \\ \text { (Urban) }\end{array} \begin{array}{l}\text { Over } \\ \mathbf{1 M}\end{array} \quad \begin{array}{l}\mathbf{5 0 0 K} \\ \sim \mathbf{\sim M}\end{array}\right)$
* Source: Cable Modem Deployment Update, CED Magazine (March 2000)
[Table B-3] RBOC Provided DSL by Town Size

| Town Size (Urban) | $\begin{aligned} & \hline \text { Over } \\ & 1 M \\ & \hline \end{aligned}$ | $\begin{gathered} \hline 500 \mathrm{~K} \\ \sim 1 M \\ \hline \end{gathered}$ | $\begin{aligned} & \hline 250 \mathrm{~K} \\ & \sim 500 \mathrm{~K} \end{aligned}$ | $\begin{aligned} & \hline 100 \mathrm{~K} \\ & \sim 250 \mathrm{~K} \end{aligned}$ | $\begin{array}{\|l\|} \hline 50 \mathrm{~K} \\ \sim 100 \mathrm{~K} \\ \hline \end{array}$ | $\begin{aligned} & \hline 25 K \\ & \sim 50 K \\ & \hline \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \% of Town Served | $100 \% \text { of }$ $8 \text { towns }$ | $73.3 \% \text { of }$ $15 \text { towns }$ | $87.8 \% \text { of }$ <br> 41 towns | $56.6 \%$ of 136 towns | $\begin{array}{\|l} \hline 32.1 \% \text { of } \\ 355 \\ \text { towns } \\ \hline \end{array}$ | $\begin{aligned} & 17.0 \% \text { of } \\ & 742 \text { towns } \end{aligned}$ |
| Town Size <br> (Rural) | $\begin{aligned} & \hline 10 K \\ & \sim 25 K \end{aligned}$ | $\begin{aligned} & \hline 5 K \\ & \sim 10 K \end{aligned}$ | $\begin{aligned} & \hline 2.5 K \\ & \sim 5 K \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 1 K \\ & \sim 2.5 K \end{aligned}$ | Under 1K |  |
| \% of Town <br> Served | $\begin{aligned} & \hline 4.6 \% \text { of } \\ & 1852 \\ & \text { towns } \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 1.4 \% \text { of } \\ & 2336 \\ & \text { towns } \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 0.6 \% \text { of } \\ & 3022 \end{aligned}$ towns | $\begin{aligned} & \hline 0.1 \% \text { of } \\ & 4936 \\ & \text { towns } \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.0 \% \text { of } \\ & 9993 \end{aligned}$ towns |  |

* Source: Public Data from RBOCs (March 2000)
(5) By the end of 2001, only about one in every ten U.S. households will have broadband -mainly provided by either DSL technology or by cable. (Grassman, 2001)


## APPENDIX C

## Estimated BE Demand Curve by Simulation and Regression

[Table C-1] Linear Regression Result by 30 Time Simulations

| Simulation No. | BE Demand |  |  |
| :---: | :---: | :---: | :---: |
|  | Constant | Slope | $\mathbf{R}^{2}$ |
| 1 | 45.010 | . 00867 | . 974 |
| 2 | 44.020 | . 00847 | . 969 |
| 3 | 44.449 | . 00849 | . 970 |
| 4 | 44.992 | . 00859 | . 972 |
| 5 | 44.662 | . 00855 | . 972 |
| 6 | 44.635 | . 00859 | . 971 |
| 7 | 44.389 | . 00850 | . 971 |
| 8 | 44.522 | . 00854 | . 971 |
| 9 | 44.460 | . 00853 | . 971 |
| 10 | 44.716 | . 00860 | . 972 |
| 11 | 44.581 | . 00864 | . 974 |
| 12 | 44.854 | . 00853 | . 970 |
| 13 | 44.948 | . 00864 | . 974 |
| 14 | 44.147 | . 00846 | . 968 |
| 15 | 44.158 | . 00840 | . 967 |
| 16 | 44.350 | . 00856 | . 968 |
| 17 | 44.646 | . 00862 | . 971 |
| 18 | 44.190 | . 00846 | . 969 |
| 19 | 44.284 | . 00845 | . 970 |
| 20 | 44.143 | . 00851 | . 969 |
| 21 | 44.248 | . 00851 | . 969 |
| 22 | 44.507 | . 00856 | . 971 |
| 23 | 44.210 | . 00845 | . 967 |
| 24 | 44.827 | . 00860 | . 971 |
| 25 | 44.612 | . 00849 | . 970 |
| 26 | 44.714 | . 00860 | . 971 |
| 27 | 44.851 | . 00866 | . 976 |
| 28 | 44.943 | . 00863 | . 974 |
| 29 | 44.640 | . 00860 | . 969 |
| 30 | 44.794 | . 00870 | . 971 |
| Mean | 44.550 | . 00855 |  |
| 95\% CI for Mean | $\begin{aligned} & \mathrm{LB}=44.443 \\ & \mathrm{UB}=44.667 \end{aligned}$ | $\begin{aligned} & \mathrm{LB}=.00852 \\ & \mathrm{UB}=.00858 \end{aligned}$ |  |

## APPENDIX D

## Equilibrium Point of Empirical Distribution

[Table D-1] Equilibrium Quantities, Prices, and Profits of Case 1

| Case 1 | $\mathbf{q 1 *}{ }_{\mathbf{B E}}$ | $\mathbf{q 2}^{2}{ }_{\mathbf{B E}}$ | $\mathbf{f 1 *}$ | $\mathbf{f 2}^{*}$ | $\mathbf{P}^{*}{ }_{\mathbf{B E}}$ |
| :---: | ---: | ---: | ---: | ---: | ---: |
| $[1 \mathrm{~K}, 1 \mathrm{~K}]$ | 999 | 999 | 24433 | 24433 | 27.47 |
| $[1 \mathrm{~K}, 2 \mathrm{~K}]$ | 999 | 1999 | 15891 | 31808 | 18.92 |
| $[1 \mathrm{~K}, 3 \mathrm{~K}]$ | 999 | 2515 | 11484 | 24086 | 14.51 |
| $[2 \mathrm{~K}, 1 \mathrm{~K}]$ | 1999 | 999 | 31808 | 15891 | 18.92 |
| $[2 \mathrm{~K}, 2 \mathrm{~K}]$ | 1999 | 1999 | 14717 | 14717 | 10.37 |
| $[2 \mathrm{~K}, 3 \mathrm{~K}]$ | 1999 | 2015 | 14443 | 4719 | 10.23 |
| $[3 \mathrm{~K}, 1 \mathrm{~K}]$ | 2515 | 999 | 24086 | 11484 | 14.51 |
| $[3 \mathrm{~K}, 2 \mathrm{~K}]$ | 2015 | 1999 | 4719 | 14443 | 10.23 |
| $[3 \mathrm{~K}, 3 \mathrm{~K}]$ | 2010 | 2010 | 4530 | 4530 | 10.18 |

[Table D-2] Equilibrium Quantities, Prices, and Profits of Case 2

| Case 2 | q1* ${ }_{\text {be }}$ | q2* ${ }_{\text {be }}$ | 92* ${ }_{\text {os }}$ | Q1 | Q2 | f1* | f2* | P* ${ }_{\text {bE }}$ | P* ${ }^{\text {os }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [1K,1K] | 999 | 429 | 570 | 999 | 999 | 24433 | 26010 | 27.47 | 67.14 |
| [1K,2K] | 999 | 1429 | 570 | 999 | 1999 | 15891 | 35385 | 18.92 | 58.59 |
| [1K,3K] | 999 | 1945 | 570 | 999 | 2515 | 11484 | 29663 | 14.51 | 54.18 |
| [2K,1K] | 1999 | 429 | 570 | 1999 | 999 | 31808 | 17468 | 18.92 | 58.59 |
| [2K, 2 K ] | 1999 | 1429 | 570 | 1999 | 1999 | 14717 | 18294 | 10.37 | 50.04 |
| [2K,3K] | 1999 | 1445 | 570 | 1999 | 2015 | 14443 | 10296 | 10.23 | 49.91 |
| [3K,1K] | 2515 | 429 | 570 | 2515 | 999 | 24086 | 13061 | 14.51 | 54.18 |
| [3K,2K] | 2015 | 1429 | 570 | 2015 | 1999 | 4719 | 18020 | 10.23 | 49.91 |
| [3K,3K] | 2010 | 1440 | 570 | 2010 | 2010 | 4530 | 10107 | 10.18 | 49.86 |

[Table D-3] Equilibrium Quantities, Prices, and Profits of Case 3

| Case 3 | r | q1* ${ }_{\text {bE }}$ | q2* ${ }_{\text {bE }}$ | 92* ${ }_{\text {os }}$ | Q1 | Q2 | f1* | f2* | p* ${ }_{\text {be }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [1K, 1K] | 1.3 | 999 | 388 | 611 | 999 | 999 | 17939 | 27140 | 20.97 |
| [1K, 2K] | 1.5 | 999 | 1353 | 623 | 999 | 1976 | 9730 | 30782 | 12.75 |
| [1K, 3K] | 1.3 | 999 | 1524 | 611 | 999 | 2135 | 8236 | 22173 | 11.25 |
| [2K, 1K] | 1.3 | 1999 | 388 | 611 | 1999 | 999 | 18815 | 18598 | 12.42 |
| [2K, 2K] | 1.3 | 1756 | 1146 | 611 | 1756 | 1757 | 6364 | 17580 | 8.01 |
| [2K, 3K] | 1.3 | 1756 | 1146 | 611 | 1756 | 1757 | 6364 | 9580 | 8.01 |
| [3K, 1K] | 1.5 | 2077 | 376 | 623 | 2077 | 999 | 6867 | 17488 | 10.75 |
| [3K, 2K] | 1.3 | 1756 | 1146 | 611 | 1756 | 1757 | -3636 | 17580 | 8.01 |
| [3K, 3K] | 1.3 | 1756 | 1146 | 611 | 1756 | 1757 | -3636 | 9580 | 8.01 |

[Table D-4] Equilibrium Quantities, Prices, and Profits of Case 4

| Case 4 | r | q1* ${ }_{\text {bE }}$ | q2* ${ }^{\text {bE }}$ | q1* ${ }_{\text {os }}$ | q2 * ${ }_{\text {oos }}$ | Q1 | Q2 | f1* | f2* | $\mathbf{p}^{*}$ вE | $\mathbf{p}^{*}{ }_{\text {Oos }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [1K, 1K] | 1.3 | 533 | 388 | 466 | 611 | 999 | 999 | 16815 | 27240 | 20.97 | 56.31 |
| [1K, 2K] | 1.3 | 533 | 1388 | 466 | 611 | 999 | 1999 | 8273 | 30015 | 12.42 | 47.76 |
| [1K, 3K] | 1.3 | 533 | 1524 | 466 | 611 | 999 | 2135 | 7112 | 22173 | 11.25 | 46.6 |
| [2K, 1K] | 1.3 | 1533 | 388 | 466 | 611 | 1999 | 999 | 19690 | 18598 | 12.42 | 47.76 |
| [2K, 2K] | 1.3 | 1291 | 1146 | 466 | 611 | 1757 | 1757 | 7240 | 9565 | 8.01 | 43.35 |
| [2K, 3K] | 1.3 | 1291 | 1146 | 466 | 611 | 1757 | 1757 | 7240 | 9565 | 8.01 | 43.35 |
| [3K, 1K] | 1.5 | 1613 | 376 | 464 | 623 | 2077 | 999 | 9695 | 17488 | 10.75 | 46.01 |
| [3K, 2K] | 1.3 | 1291 | 1146 | 466 | 611 | 1757 | 1757 | -760 | 17565 | 8.01 | 43.35 |
| [3K, 3K] | 1.3 | 1291 | 1146 | 466 | 611 | 1757 | 1757 | -760 | 9565 | 8.01 | 43.35 |

[Table D-5] Equilibrium Quantities, Prices, and Profits of Case 5

| Case 5 | q1* ${ }_{\text {bE }}$ | q2* ${ }^{\text {BE }}$ | q1* ${ }_{\text {os }}$ | q2* ${ }_{0} \mathrm{~S}$ | Q1 | Q2 | f1* | f2* | P* ${ }_{\text {be }}$ | $\mathbf{P}^{*}$ Qos |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [1K,1K] | 513 | 513 | 486 | 486 | 999 | 999 | 23817 | 23817 | 27.47 | 63.71 |
| [1K, 2 K ] | 513 | 1513 | 486 | 486 | 999 | 1999 | 15276 | 33193 | 18.92 | 55.16 |
| [1K,3K] | 513 | 2029 | 486 | 486 | 999 | 2515 | 10868 | 27470 | 14.51 | 50.74 |
| [2K,1K] | 1513 | 513 | 486 | 486 | 1999 | 999 | 33193 | 15276 | 18.92 | 55.16 |
| [2K, 2 K ] | 1513 | 1513 | 486 | 486 | 1999 | 1999 | 16101 | 16101 | 10.37 | 46.61 |
| [2K,3K] | 1513 | 1529 | 486 | 486 | 1999 | 2015 | 15828 | 8103 | 10.23 | 46.47 |
| [3K,1K] | 2029 | 513 | 486 | 486 | 2515 | 999 | 27470 | 10868 | 14.51 | 50.74 |
| [3K, 2 K ] | 1529 | 1513 | 486 | 486 | 2015 | 1999 | 8103 | 7139 | 18.78 | 55.02 |
| [3K,3K] | 1523 | 1523 | 486 | 486 | 2009 | 2009 | 7931 | 7931 | 10.20 | 46.44 |

[Table D-6] Equilibrium Quantities, Prices, and Profits of Case 6

| Case 6 | r | q1* ${ }_{\text {bE }}$ | q2* ${ }^{\text {BE }}$ | q1* ${ }_{\text {oos }}$ | q2* ${ }_{\text {oos }}$ | Q1 | Q2 | f1* | f2* |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [1K, 1K] | 1.3 | 388 | 388 | 611 | 611 | 999 | 999 | 27140 | 27140 | 20.97 |
| [1K, 2K] | 1.3 | 388 | 1388 | 611 | 611 | 999 | 1999 | 18598 | 30015 | 12.42 |
| [1K, 3K] | 1.3 | 388 | 1524 | 611 | 611 | 999 | 2135 | 17436 | 22173 | 11.25 |
| [2K, 1K] | 1.3 | 1388 | 388 | 611 | 611 | 1999 | 999 | 30015 | 18598 | 12.42 |
| [2K, 2K] | 1.3 | 1146 | 1146 | 611 | 611 | 1757 | 1757 | 17565 | 17565 | 8.01 |
| [2K, 3K] | 1.3 | 1146 | 1146 | 611 | 611 | 1757 | 1757 | 17565 | 9565 | 8.01 |
| [3K, 1K] | 1.5 | 1454 | 376 | 623 | 623 | 2077 | 999 | 20623 | 17488 | 10.75 |
| [3K, 2K] | 1.3 | 1146 | 1146 | 611 | 611 | 1757 | 1757 | 9565 | 17565 | 8.01 |
| [3K, 3K] | 1.3 | 1146 | 1146 | 611 | 611 | 1757 | 1757 | 9565 | 9565 | 8.01 |

[Table D-7] Equilibrium Quantities, Prices, and Profits of Case 7

| Case 7 | $\mathbf{q 1}^{*}{ }_{\text {BE }}$ | $\mathbf{q 2}^{*}{ }_{\text {BE }}$ | $\mathbf{f 1 *}$ | $\mathbf{\text { f2* }}$ | P* ${ }_{\text {BE }}$ |
| :---: | ---: | ---: | ---: | ---: | ---: |
| $[1 \mathrm{~K}, 1 \mathrm{~K}]$ | 999 | 999 | 20283 | 20283 | 27.47 |
| $[1 \mathrm{~K}, 2 \mathrm{~K}]$ | 999 | 1999 | 11591 | 28508 | 18.92 |
| $[1 \mathrm{~K}, 3 \mathrm{~K}]$ | 999 | 2515 | 7034 | 21636 | 14.51 |
| $[2 \mathrm{~K}, 1 \mathrm{~K}]$ | 1999 | 999 | 28508 | 11591 | 18.92 |
| $[2 \mathrm{~K}, 2 \mathrm{~K}]$ | 1999 | 1999 | 11417 | 11417 | 10.37 |
| $[2 \mathrm{~K}, 3 \mathrm{~K}]$ | 1999 | 2015 | 10993 | 2269 | 10.23 |
| $[3 \mathrm{~K}, 1 \mathrm{~K}]$ | 2515 | 999 | 21636 | 7034 | 14.51 |
| $[3 \mathrm{~K}, 2 \mathrm{~K}]$ | 2015 | 1999 | 2269 | 10993 | 10.23 |
| $[3 \mathrm{~K}, 3 \mathrm{~K}]$ | 2010 | 2010 | 2080 | 2080 | 10.18 |

[Table D-8] Equilibrium Quantities, Prices, and Profits of Case 8

| Case 8 | r | q1* ${ }_{\text {bE }}$ | q2* ${ }_{\text {bE }}$ | q1* ${ }_{\text {OSS }}$ 92* ${ }_{\text {os }}$ |  | Q1 | Q2 | f1* | f2* | p* ${ }_{\text {be }}$ | $\mathrm{p}^{*}{ }_{\text {o }}$ S |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [1K, 1K] | 1.3 | 533 | 388 | 466 | 611 | 999 | 999 | 18515 | 28840 | 20.97 | 56.31 |
| [1K, 2K] | 1.3 | 533 | 1388 | 466 | 611 | 999 | 1999 | 9673 | 33415 | 12.42 | 47.76 |
| [1K, 3K] | 1.3 | 533 | 1524 | 466 | 611 | 999 | 2135 | 8212 | 27273 | 11.25 | 46.60 |
| [2K, 1K] | 1.3 | 1533 | 388 | 466 | 611 | 1999 | 999 | 23090 | 19998 | 12.42 | 47.76 |
| [ $2 \mathrm{~K}, 2 \mathrm{~K}$ ] | 1.3 | 1291 | 1146 | 466 | 611 | 1757 | 1757 | 10640 | 20965 | 8.01 | 43.35 |
| [2K, 3K] | 1.3 | 1291 | 1146 | 466 | 611 | 1757 | 1757 | 10340 | 14665 | 8.01 | 43.35 |
| [3K, 1K] | 1.5 | 1613 | 376 | 464 | 623 | 2077 | 999 | 14795 | 18588 | 10.75 | 46.01 |
| [3K, 2K] | 1.3 | 1291 | 1146 | 466 | 611 | 1757 | 1757 | 4340 | 20665 | 8.01 | 43.35 |
| [3K, 3K] | 1.3 | 1291 | 1146 | 466 | 611 | 1757 | 1757 | 4340 | 14665 | 8.01 | 43.35 |

[Table D-9] Equilibrium Quantities, Prices, and Profits of Case 9

| Case 9 | q1 ${ }^{\text {BE }}$ |  |  |  | Q1 | Q2 | f1* | f2* | p * $_{\text {BE }}$ | P* 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [1K,1K] | 513 | 513 | 486 | 486 | 999 | 999 | 25517 | 25517 | 27.47 | 63.7 |
| [1K,2K] | 513 | 1513 | 486 | 486 | 999 | 1999 | 16676 | 36593 | 18.92 | 55.1 |
| [1K,3K] | 513 | 2029 | 486 | 486 | 999 | 2515 | 11968 | 32570 | 14.51 | 50.7 |
| [2K,1K] | 1513 | 513 | 486 | 486 | 1999 | 999 | 36593 | 16676 | 18.92 | 55.1 |
| [2K,2K] | 1513 | 1513 | 486 | 486 | 1999 | 1999 | 19501 | 19501 | 10.37 | 46.6 |
| [2K,3K] | 1513 | 1529 | 486 | 486 | 1999 | 2015 | 18928 | 13203 | 10.23 | 46.4 |
| [3K,1K] | 2029 | 513 | 486 | 486 | 2515 | 999 | 32570 | 11968 | 14.51 | 50.7 |
| [3K,2K] | 1529 | 1513 | 486 | 486 | 2015 | 1999 | 13203 | 18928 | 10.23 | 46.4 |
| [3K,3K] | 1523 | 1523 | 486 | 486 | 2009 | 2009 | 13031 | 13031 | 10.20 | 46. |

[Table D-10] Equilibrium Quantities, Prices, and Profits of Case 10

| Case 10 | r | q1* ${ }_{\text {be }}$ | q2* ${ }^{\text {bE }}$ | q1* ${ }^{\text {os }}$ | q2* oos | Q1 | Q2 | f1* | f2* | p* ${ }^{\text {bE }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [1K, 1K] | 1.3 | 388 | 388 | 611 | 611 | 999 | 999 | 28840 | 28840 | 20.97 |
| [1K, 2K] | 1.3 | 388 | 1388 | 611 | 611 | 999 | 1999 | 19998 | 33415 | 12.42 |
| [1K, 3K] | 1.3 | 388 | 1524 | 611 | 611 | 999 | 2135 | 18536 | 27273 | 11.25 |
| [2K, 1K] | 1.3 | 1388 | 388 | 611 | 611 | 1999 | 999 | 33415 | 19998 | 12.42 |
| [2K, 2K] | 1.3 | 1146 | 1146 | 611 | 611 | 1757 | 1757 | 20965 | 20965 | 8.01 |
| [2K, 3K] | 1.3 | 1146 | 1146 | 611 | 611 | 1757 | 1757 | 20665 | 14665 | 8.01 |
| [3K, 1K] | 1.5 | 1454 | 376 | 623 | 623 | 2077 | 999 | 25723 | 18588 | 10.75 |
| [3K, 2K] | 1.3 | 1146 | 1146 | 611 | 611 | 1757 | 1757 | 14665 | 20665 | 8.01 |
| [3K, 3K] | 1.3 | 1146 | 1146 | 611 | 611 | 1757 | 1757 | 14665 | 14665 | 8.0 |

## APPENDIX E

## Payoff Matrixes of Empirical Distribution

[Table E-1] Payoff Matrix of Case 1

| IAP1 $\mathbf{I A P 2}$ | $\mathbf{1 K}$ | 2K | 3K | Optimal $\boldsymbol{k 2 *}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 K}$ | $[24433,24433]$ | $[\mathbf{1 5 8 9 1}, \mathbf{3 1 8 0 8}]$ | $[11484,24086]$ | 2 K |
| $\mathbf{2 K}$ | $[31808, \mathbf{1 5 8 9 1}]$ | $[14717,14717]$ | $[14443,4719]$ | 1 K |
| $\mathbf{3 K}$ | $[24086,11484]$ | $[4719,14443]$ | $[4530,4530]$ | 2 K |
| Optimal $\boldsymbol{k} \mathbf{1}^{*}$ | 2 K | 1 K | 1 K |  |

[Table E-2] Payoff Matrix of Case 2

| IAP1 $\mathbf{\text { IAP2 }}$ | $\mathbf{1 K}$ | $\mathbf{2 K}$ | 3K | Optimal $\boldsymbol{k 2 *}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 K}$ | $[24433,26010]$ | $[\mathbf{1 5 8 9 1}, \mathbf{3 5 3 8 5 ]}$ | $[11484,29663]$ | 2 K |
| $\mathbf{2 K}$ | $[31808,17468]$ | $[14717,18294]$ | $[14443,10296]$ | 2 K |
| $\mathbf{3 K}$ | $[24086,13061]$ | $[4719,18020]$ | $[4530,10107]$ | 2 K |
| Optimal $\boldsymbol{k 1 ^ { * }}$ | 2 K | 1 K | 2 K |  |

[Table E-3] Payoff Matrix of Case 3

| IAP1 $\mathbf{\text { IAP2 }}$ | $\mathbf{1 K}$ | $\mathbf{2 K}$ | $\mathbf{3 K}$ | Optimal $\boldsymbol{k 2 *}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 K}$ | $[17939,27140]$ | $[\mathbf{9 7 3 0 , 3 0 7 8 2}]$ | $[8236,22173]$ | 2 K |
| $\mathbf{2 K}$ | $[\mathbf{1 8 8 1 5 , 1 8 5 9 8}]$ | $[6364,17580]$ | $[6364,9580]$ | 1 K |
| $\mathbf{3 K}$ | $[6867,17488]$ | $[-3636,17580]$ | $[-3636,9580]$ | 2 K |
| Optimal $\boldsymbol{k 1 *}$ | 2 K | 1 K | 1 K |  |

[Table E-4] Payoff Matrix of Case 4

| ${ }^{\text {IAP1 }}{ }^{\text {IAP2 }}$ | $\mathbf{1 K}$ | $\mathbf{2 K}$ | $\mathbf{3 K}$ | Optimal $\mathbf{k 2 *}^{*}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 K}$ | $[16815,27240]$ | $[8273,30015]$ | $[7112,22173]$ | 2 K |
| $\mathbf{2 K}$ | $[19690,18598]$ | $[7240,17565]$ | $[7240,9565]$ | 1 K |
| $\mathbf{3 K}$ | $[9695,17488]$ | $[-760,17565]$ | $[-760,9565]$ | 2 K |
| Optimal $\boldsymbol{k 1}^{*}$ | 2 K | 1 K | 1 K |  |

[Table E-5] Payoff Matrix of Case 5

| IAP1 $\mathbf{I A P 2}^{2}$ | $\mathbf{1 K}$ | $\mathbf{2 K}$ | $\mathbf{3 K}$ | Optimal $\boldsymbol{k 2}^{*}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 K}$ | $[23817,23817]$ | $[15276,33193]$ | $[10868,27470]$ | 2 K |
| $\mathbf{2 K}$ | $[33193,15276]$ | $[\mathbf{1 6 1 0 1 , 1 6 1 0 1 ]}$ | $[15828,8103]$ | 2 K |
| $\mathbf{3 K}$ | $[27470,10868]$ | $[8103,7139]$ | $[7931,7931]$ | 1 K |
| Optimal $\boldsymbol{k 1}^{*}$ | 2 K | 2 K | 2 K |  |

[Table E-6] Payoff Matrix of Case 6

| IAP1 $\mathbf{I A P 2}$ | $\mathbf{1 K}$ | $\mathbf{2 K}$ | $\mathbf{3 K}$ | Optimal $\boldsymbol{k 2 *}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 K}$ | $[27140,27140]$ | $[\mathbf{1 8 5 9 8 , 3 0 0 1 5 ]}$ | $[17436,22173]$ | 2 K |
| $\mathbf{2 K}$ | $[\mathbf{3 0 0 1 5 , 1 8 5 9 8 ]}$ | $[17565,17565]$ | $[17565,9565]$ | 1 K |
| $\mathbf{3 K}$ | $[20623,17488]$ | $[9565,17565]$ | $[9565,9565]$ | 2 K |
| Optimal $\boldsymbol{k 1 *}$ | 2 K | 1 K | 1 K |  |

[Table E-7] Payoff Matrix of Case 7

| IAP1 $\mathbf{\text { IAP2 }}$ | $\mathbf{1 K}$ | $\mathbf{2 K}$ | $\mathbf{3 K}$ | Optimal $\boldsymbol{k 2 *}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 K}$ | $[20283,20283]$ | $[\mathbf{1 1 5 9 1 , 2 8 5 0 8 ]}$ | $[7034,21636]$ | 2 K |
| $\mathbf{2 K}$ | $[\mathbf{2 8 5 0 8 , 1 1 5 9 1 ]}$ | $[11417,11417]$ | $[10993,2269]$ | 1 K |
| $\mathbf{3 K}$ | $[21636,7034]$ | $[2269,10993]$ | $[2080,2080]$ | 2 K |
| Optimal $\boldsymbol{k 1 *}$ | 2 K | 1 K | 2 K |  |

[Table E-8] Payoff Matrix of Case 8

| IAP1 IAP2 | 1K | 2K | 3K | Optimal k2* |
| :---: | :---: | :---: | :---: | :---: |
| 1K | [18515,28840] | [9673,33415] | [8212,27273] | 2K |
| 2K | [23090,19998] | [10640,20965] | [10340,14665] | 2K |
| 3K | [14795,18588] | [4340,20665] | [4340,14665] | 2K |
| Optimal $k 1^{*}$ | 2 K | 2K | 2 K |  |

[Table E-9] Payoff Matrix of Case 9

| IAP1 $\mathbf{\text { IAP2 }}$ | $\mathbf{1 K}$ | $\mathbf{2 K}$ | $\mathbf{3 K}$ | Optimal $\boldsymbol{k 2 *}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 K}$ | $[25517,25517]$ | $[[16676,36593]$ | $[11968,32570]$ | 2 K |
| $\mathbf{2 K}$ | $[36593,16676]$ | $[19501,19501]$ | $[18928,13203]$ | 2 K |
| $\mathbf{3 K}$ | $[32570,11968]$ | $[13203,18928]$ | $[13031,13031]$ | 2 K |
| Optimal $\boldsymbol{k 1 *}$ | 2 K | 2 K | 2 K |  |

[Table E-10] Payoff Matrix of Case 10

| IAP1 $\mathbf{\text { IAP2 }}$ | $\mathbf{1 K}$ | $\mathbf{2 K}$ | $\mathbf{3 K}$ | Optimal $\boldsymbol{k} \mathbf{2}^{*}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 K}$ | $[28840,28840]$ | $[19998,33415]$ | $[18536,27273]$ | 2 K |
| $\mathbf{2 K}$ | $[33415,19998]$ | $[\mathbf{2 0 9 6 5 , 2 0 9 6 5 ]}$ | $[20665,14665]$ | 2 K |
| $\mathbf{3 K}$ | $[25723,18588]$ | $[14665,20665]$ | $[14665,14665]$ | 2 K |
| Optimal $\boldsymbol{k 1 ^ { * }}$ | 2 K | 2 K | 2 K |  |

## APPENDIX F

## Welfare of the Internet QoS Game Model

[Table F-1] Social Welfare of Uniform Distribution

| Uniform | Dist. | CS |  |  | PS |  |  | SW |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CASE | Capacity | BE | QoS | Total | Profits | Fixed | Total |  |
| 1(Transit) | [1K,1K] | 19960 | 0 | 19960 | 53966 | 20000 | 73966 | 93926 |
| (BE-BE) | [1K,2K] | 44940 | 0 | 44940 | 51006 | 30000 | 81006 | 125946 |
| (F-F) | [1K,3K] | 56079 | 0 | 56079 | 38735 | 40000 | 78735 | 134814 |
|  | [2K,1K] | 44940 | 0 | 44940 | 51006 | 30000 | 81006 | 125946 |
|  | [2K,2K] | 72200 | 0 | 72200 | 32200 | 40000 | 72200 | 144400 |
|  | [2K,3K] | 72200 | 0 | 72200 | 22200 | 50000 | 72200 | 144400 |
|  | [3K,1K] | 56079 | 0 | 56079 | 38735 | 40000 | 78735 | 134814 |
|  | [3K,2K] | 72200 | 0 | 72200 | 22200 | 50000 | 72200 | 144400 |
|  | [3K,3K] | 72200 | 0 | 72200 | 12200 | 60000 | 72200 | 144400 |
| 2(Transit) | [1K,1K] | 17880 | 2080 | 19960 | 58320 | 28000 | 86320 | 106280 |
| (BE-QoS) | [1K,2K] | 42860 | 2080 | 44940 | 57360 | 36000 | 93360 | 138300 |
| (F-F) | [1K,3K] | 53999 | 2080 | 56079 | 47089 | 54000 | 101089 | 157168 |
|  | [2K,1K] | 42860 | 2080 | 44940 | 73360 | 38000 | 111360 | 156300 |
|  | [2K,2K] | 70120 | 2080 | 72200 | 38554 | 46000 | 84554 | 156754 |
|  | [2K,3K] | 70120 | 2080 | 72200 | 30554 | 54000 | 84554 | 156754 |
|  | [3K,1K] | 53999 | 2080 | 56079 | 43089 | 48000 | 91089 | 147168 |
|  | [3K,2K] | 70120 | 2080 | 72200 | 28554 | 56000 | 84554 | 156754 |
|  | [3K,3K] | 70120 | 2080 | 72200 | 20554 | 64000 | 84554 | 156754 |
| 3(Transit) | [1K,1K] | 18093 | -16592 | 1502 | 50180 | 28000 | 78180 | 79682 |
| (BE-QoS) | [1K,2K] | 43073 | -16592 | 26482 | 42720 | 36000 | 78720 | 105202 |
| (F-2P) | [1K,3K] | 43887 | -16592 | 27295 | 34457 | 54000 | 88457 | 115752 |
|  | [2K,1K] | 43073 | -16592 | 26482 | 40720 | 38000 | 78720 | 105202 |
|  | [2K,2K] | 54817 | -16592 | 38225 | 27867 | 46000 | 73867 | 112092 |
|  | [2K,3K] | 54817 | -16592 | 38225 | 19867 | 54000 | 73867 | 112092 |
|  | [3K,1K] | 42312 | -17516 | 24797 | 28512 | 48000 | 76512 | 101309 |
|  | [3K,2K] | 54817 | -16592 | 38225 | 17867 | 56000 | 73867 | 112092 |
|  | [3K,3K] | 54817 | -16592 | 38225 | 9867 | 64000 | 73867 | 112092 |
| 4(Transit) | [1K,1K] | 13508 | -15214 | -1706 | 50962 | 36000 | 86962 | 85256 |
| (QoS-QoS) | [1K,2K] | 38488 | -15214 | 23274 | 43501 | 44000 | 87501 | 110775 |
| (F-2P) | [1K,3K] | 39301 | -15214 | 24087 | 35239 | 52000 | 87239 | 111326 |
|  | [2K,1K] | 38488 | -15214 | 23274 | 43501 | 44000 | 87501 | 110775 |
|  | [2K,2K] | 50231 | -15214 | 35017 | 30649 | 52000 | 82649 | 117666 |
|  | [2K,3K] | 50231 | -15214 | 35017 | 22649 | 60000 | 82649 | 117666 |
|  | [3K,1K] | 37657 | -16148 | 21509 | 33241 | 52000 | 85241 | 106750 |


|  | $\left\|\begin{array}{l} {[3 \mathrm{~K}, 2 \mathrm{~K}]} \\ {[3 \mathrm{~K}, 3 \mathrm{~K}]} \end{array}\right\|$ | 50231 50231 | -15214 -15214 | 35017 35017 | 22649 14649 | 60000 68000 | 82649 82649 | 117666 117666 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \hline 5 \text { (Transit) } \\ \text { (QoS-QoS) } \\ (\text { F-F }) \end{gathered}$ | [1K,1K] | 14150 | 5810 | 19960 | 56296 | 36000 | 92296 | 112256 |
|  | [1K,2K] | 39130 | 5810 | 44940 | 55363 | 44000 | 99363 | 144303 |
|  | [1K,3K] | 50269 | 5810 | 56079 | 45065 | 52000 | 97065 | 153144 |
|  | [2K,1K] | 39130 | 5810 | 44940 | 55336 | 44000 | 99336 | 144276 |
|  | [2K,2K] | 66390 | 5810 | 72200 | 38530 | 52000 | 90530 | 162730 |
|  | [2K,3K] | 66390 | 5810 | 72200 | 30530 | 60000 | 90530 | 162730 |
|  | [3K,1K] | 50269 | 5810 | 56079 | 45065 | 52000 | 97065 | 153144 |
|  | [3K,2K] | 66390 | 5810 | 72200 | 30530 | 60000 | 90530 | 162730 |
|  | [3K,3K] | 66390 | 5810 | 72200 | 22530 | 68000 | 90530 | 162730 |
| $\begin{array}{\|c\|} \hline \text { 6(Transit) } \\ \text { (QoS-QoS) } \\ (2 P-2 P) \end{array}$ | [1K,1K] | 13076 | -14725 | -1650 | 59380 | 36000 | 95380 | 93731 |
|  | [1K,2K] | 38806 | -14725 | 24081 | 51920 | 44000 | 95920 | 120001 |
|  | [1K,3K] | 39639 | -14725 | 24914 | 43657 | 52000 | 95657 | 120571 |
|  | [2K,1K] | 38806 | -14725 | 24081 | 51920 | 44000 | 95920 | 120001 |
|  | [2K,2K] | 50860 | -14725 | 36135 | 39050 | 52000 | 91050 | 127185 |
|  | [2K,3K] | 50860 | -14725 | 36135 | 31050 | 60000 | 91050 | 127185 |
|  | [3K,1K] | 38652 | -15575 | 23077 | 42268 | 52000 | 94268 | 117345 |
|  | [3K,2K] | 50860 | -14725 | 36135 | 31050 | 60000 | 91050 | 127185 |
|  | [3K,3K] | 50860 | -14725 | 36135 | 23050 | 68000 | 91050 | 127185 |
| $\begin{gathered} \hline \text { 7(Peering) } \\ \text { (BE-BE) } \\ \text { (F-F) } \end{gathered}$ | [1K,1K] | 19960 | 0 | 19960 | 45666 | 28300 | 73966 | 93926 |
|  | [1K,2K] | 44940 | 0 | 44940 | 43406 | 37600 | 81006 | 125946 |
|  | [1K,3K] | 56079 | 0 | 56079 | 31835 | 46900 | 78735 | 134814 |
|  | [2K,1K] | 44940 | 0 | 44940 | 43406 | 37600 | 81006 | 125946 |
|  | [2K,2K] | 72200 | 0 | 72200 | 25600 | 46600 | 72200 | 144400 |
|  | [2K,3K] | 72200 | 0 | 72200 | 16300 | 55900 | 72200 | 144400 |
|  | [3K,1K] | 56079 | 0 | 56079 | 31835 | 46900 | 78735 | 134814 |
|  | [3K,2K] | 72200 | 0 | 72200 | 16300 | 55900 | 72200 | 144400 |
|  | [3K,3K] | 72200 | 0 | 72200 | 7300 | 64900 | 72200 | 144400 |
| $\begin{array}{\|c} \hline \text { 8(Peering) } \\ \text { (QoS-QoS) } \\ \text { (F-2P) } \end{array}$ | [1K,1K] | 13508 | -15214 | -1706 | 54361 | 32600 | 86961 | 85255 |
|  | [1K,2K] | 38488 | -15214 | 23274 | 48301 | 39200 | 87501 | 110775 |
|  | [1K,3K] | 39301 | -15214 | 24087 | 41438 | 45800 | 87238 | 111325 |
|  | [2K,1K] | 38488 | -15214 | 23274 | 48301 | 39200 | 87501 | 110775 |
|  | [2K,2K] | 50231 | -15214 | 35017 | 37449 | 45200 | 82649 | 117666 |
|  | [2K,3K] | 50231 | -15214 | 35017 | 30849 | 51800 | 82649 | 117666 |
|  | [3K,1K] | 37657 | -16148 | 21509 | 39441 | 45800 | 85241 | 106750 |
|  | [3K,2K] | 50231 | -15214 | 35017 | 30849 | 51800 | 82649 | 117666 |
|  | [3K,3K] | 50231 | -15214 | 35017 | 24849 | 57800 | 82649 | 117666 |
| $\begin{array}{\|c} \hline 9 \text { (Peering) } \\ \text { (QoS-QoS) } \\ \text { (F-F) } \end{array}$ | [1K,1K] | 14150 | 5810 | 19960 | 59696 | 32600 | 92296 | 112256 |
|  | [1K,2K] | 39130 | 5810 | 44940 | 60136 | 39200 | 99336 | 144276 |
|  | [1K,3K] | 50269 | 5810 | 56079 | 51265 | 45800 | 97065 | 153144 |
|  | [2K,1K] | 39130 | 5810 | 44940 | 60136 | 39200 | 99336 | 144276 |
|  | [2K,2K] | 66390 | 5810 | 72200 | 45330 | 45200 | 90530 | 162730 |
|  | [2K,3K] | 66390 | 5810 | 72200 | 38730 | 51800 | 90530 | 162730 |
|  | [3K,1K] | 50269 | 5810 | 56079 | 51265 | 45800 | 97065 | 153144 |
|  | [3K,2K] | 66390 | 5810 | 72200 | 38730 | 51800 | 90530 | 162730 |
|  | [3K,3K] | 66390 | 5810 | 72200 | 32730 | 57800 | 90530 | 162730 |


| $\mathbf{1 0 ( P e e r i n g )}$ | $[1 \mathrm{~K}, 1 \mathrm{~K}]$ | 12494 | -14725 | -2232 | 62780 | 32600 | 95380 | 93149 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | ---: |
| $\mathbf{( Q o S - Q o S})$ | $[1 \mathrm{~K}, 2 \mathrm{~K}]$ | 37474 | -14725 | 22749 | 56720 | 39200 | 95920 | 118669 |
| $\mathbf{( 2 P - 2 P )}$ | $[1 \mathrm{~K}, 3 \mathrm{~K}]$ | 38287 | -14725 | 23562 | 49857 | 45800 | 95657 | 119219 |
|  | $[2 \mathrm{~K}, 1 \mathrm{~K}]$ | 37474 | -14725 | 22749 | 56720 | 39200 | 95920 | 118669 |
|  | $[2 \mathrm{~K}, 2 \mathrm{~K}]$ | 49251 | -14725 | 34526 | 45850 | 45200 | 91050 | 125576 |
|  | $[2 \mathrm{~K}, 3 \mathrm{~K}]$ | 49251 | -14725 | 34526 | 39250 | 51800 | 91050 | 125576 |
|  | $[3 \mathrm{~K}, 1 \mathrm{~K}]$ | 36491 | -15575 | 20916 | 48468 | 45800 | 94268 | 115184 |
|  | $[3 \mathrm{~K}, 2 \mathrm{~K}]$ | 49251 | -14725 | 34526 | 39250 | 51800 | 91050 | 125576 |
|  | $[3 \mathrm{~K}, 3 \mathrm{~K}]$ | 49251 | -14725 | 34526 | 33250 | 57800 | 91050 | 125576 |

[Table F-2] Social Welfare of Empirical Distribution

| Empirical | Dist. | CS |  |  | PS |  |  | SW |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CASE | Capacity | BE | QoS | Total | Profits | Fixed | Total |  |
| 1(Transit) | [1K, 1K] | 17066 | 0 | 17066 | 48866 | 20000 | 68866 | 85932 |
| (BE-BE) | [1K,2K] | 38424 | 0 | 38424 | 47699 | 30000 | 77699 | 116123 |
| (F-F) | [1K, 3 K$]$ | 52789 | 0 | 52789 | 35570 | 40000 | 75570 | 128359 |
|  | [2K, 1K] | 38424 | 0 | 38424 | 47699 | 30000 | 77699 | 116123 |
|  | [2K, 2 K ] | 68332 | 0 | 68332 | 29434 | 40000 | 69434 | 137766 |
|  | [2K, 3 K$]$ | 68880 | 0 | 68880 | 19162 | 50000 | 69162 | 138042 |
|  | [3K, 1K] | 52789 | 0 | 52789 | 35570 | 40000 | 75570 | 128359 |
|  | [3K,2K] | 68880 | 0 | 68880 | 19162 | 50000 | 69162 | 138042 |
|  | [3K,3K] | 69086 | 0 | 69086 | 9060 | 60000 | 69060 | 138146 |
| 2(Transit) | [1K,1K] | 15677 | 1389 | 17066 | 50443 | 28000 | 78443 | 95509 |
| (BE-QoS) | [1K,2K] | 37035 | 1389 | 38424 | 51276 | 36000 | 87276 | 125700 |
| (F-F) | [1K,3K] | 51400 | 1389 | 52789 | 41147 | 54000 | 95147 | 147936 |
|  | [2K, 1 K$]$ | 37035 | 1389 | 38424 | 49276 | 38000 | 87276 | 125700 |
|  | [2K, 2 K ] | 66943 | 1389 | 68332 | 33011 | 46000 | 79011 | 147343 |
|  | [2K, 3 K$]$ | 67491 | 1389 | 68880 | 24739 | 54000 | 78739 | 147619 |
|  | [3K, 1K] | 51400 | 1389 | 52789 | 37147 | 48000 | 85147 | 137936 |
|  | [3K,2K] | 67491 | 1389 | 68880 | 22739 | 56000 | 78739 | 147619 |
|  | [3K,3K] | 67697 | 1389 | 69086 | 14637 | 64000 | 78637 | 147723 |
| 3(Transit) | $[1 \mathrm{~K}, 1 \mathrm{~K}]$ | 15470 | -19651 | -4181 | 45079 | 28000 | 73079 | 68898 |
| (BE-QoS) | [1K,2K] | 36177 | -20630 | 15548 | 40512 | 36000 | 76512 | 92060 |
| (F-2P) | [1K, 3 K$]$ | 40393 | -19651 | 20742 | 30409 | 54000 | 84409 | 105151 |
|  | [2K, 1 K ] | 36828 | -19651 | 17177 | 37413 | 38000 | 75413 | 92590 |
|  | [2K, 2 K ] | 51163 | -19651 | 31512 | 23944 | 46000 | 69944 | 101456 |
|  | [2K, 3 K$]$ | 51163 | -19651 | 31512 | 15944 | 54000 | 69944 | 101456 |
|  | [3K, 1K] | 38790 | -20630 | 18160 | 24355 | 48000 | 72355 | 90515 |
|  | [3K,2K] | 51163 | -19651 | 31512 | 13944 | 56000 | 69944 | 101456 |
|  | [3K,3K] | 51163 | -19651 | 31512 | 5944 | 64000 | 69944 | 101456 |
| 4(Transit) | [1K,1K] | 12107 | -18723 | -6616 | 44055 | 36000 | 80055 | 73439 |
| (QoS-QoS) | [1K,2K] | 33465 | -18723 | 14742 | 38288 | 44000 | 82288 | 97030 |
| (F-2P) | [1K, 3 K$]$ | 37030 | -18723 | 18308 | 29285 | 52000 | 81285 | 99593 |
|  | [2K, 1 K ] | 33465 | -18723 | 14742 | 38288 | 44000 | 82288 | 97030 |
|  | [2K, 2 K ] | 47830 | -18723 | 29107 | 16805 | 52000 | 68805 | 97912 |


|  | $\begin{aligned} & {[2 \mathrm{~K}, 3 \mathrm{~K}]} \\ & {[3 \mathrm{~K}, 1 \mathrm{~K}]} \\ & {[3 \mathrm{~K}, 2 \mathrm{~K}]} \\ & {[3 \mathrm{~K}, 3 \mathrm{~K}]} \end{aligned}$ | $\begin{aligned} & 47830 \\ & 35398 \\ & 47830 \\ & 47830 \end{aligned}$ | $\begin{aligned} & -18723 \\ & -19709 \\ & -18723 \\ & -18723 \\ & \hline \end{aligned}$ | $\begin{aligned} & 29107 \\ & 15689 \\ & 29107 \\ & 29107 \\ & \hline \end{aligned}$ | $\begin{array}{r} 16805 \\ 27183 \\ 16805 \\ 8805 \\ \hline \end{array}$ | $\begin{aligned} & 60000 \\ & 52000 \\ & 60000 \\ & 68000 \\ & \hline \end{aligned}$ | $\begin{aligned} & 76805 \\ & 79183 \\ & 76805 \\ & 76805 \\ & \hline \end{aligned}$ | $\begin{array}{r} 105912 \\ 94872 \\ 105912 \\ 105912 \\ \hline \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5(Transit) | [1K, 1 K ] | 4500 | 4039 | 8539 | 47634 | 36000 | 83634 | 92173 |
| (QoS-QoS) | [1K, 2 K ] | 17547 | 4039 | 21586 | 48469 | 44000 | 92469 | 114055 |
| (F-F) | [1K,3K] | 27624 | 4039 | 31663 | 38338 | 52000 | 90338 | 122001 |
|  | [2K, 1 K ] | 17547 | 4039 | 21586 | 48469 | 44000 | 92469 | 114055 |
|  | [2K, 2 K ] | 39145 | 4039 | 43184 | 32202 | 52000 | 84202 | 127386 |
|  | [2K,3K] | 39560 | 4039 | 43599 | 23931 | 60000 | 83931 | 127530 |
|  | [3K,1K] | 27624 | 4039 | 31663 | 38338 | 52000 | 90338 | 122001 |
|  | [3K,2K] | 39560 | 4039 | 43599 | 15242 | 60000 | 75242 | 118841 |
|  | [3K,3K] | 39664 | 4039 | 43703 | 15862 | 68000 | 83862 | 127565 |
| $\begin{array}{\|c} \text { 6(Transit) } \\ \text { (QoS-QoS) } \\ (2 \mathrm{P}-2 \mathrm{P}) \end{array}$ | [1K, 1 K ] | 10682 | -18055 | -7373 | 54280 | 36000 | 90280 | 82907 |
|  | [1K, 2 K ] | 32040 | -18055 | 13985 | 48613 | 44000 | 92613 | 106598 |
|  | [1K,3K] | 35605 | -18055 | 17550 | 39609 | 52000 | 91609 | 109159 |
|  | [2K, 1 K ] | 32040 | -18055 | 13985 | 48613 | 44000 | 92613 | 106598 |
|  | [2K, 2 K ] | 46405 | -18055 | 28350 | 35130 | 52000 | 87130 | 115480 |
|  | [2K, 3 K ] | 46405 | -18055 | 28350 | 27130 | 60000 | 87130 | 115480 |
|  | [3K,1K] | 33812 | -18970 | 14842 | 38111 | 52000 | 90111 | 104953 |
|  | [3K,2K] | 46405 | -18055 | 28350 | 27130 | 60000 | 87130 | 115480 |
|  | [3K,3K] | 46405 | -18055 | 28350 | 19130 | 68000 | 87130 | 115480 |
| $\begin{array}{\|c} \text { 7(Peering) } \\ \text { (BE-BE) } \\ \text { (F-F) } \end{array}$ | [1K, 1 K ] | 17066 | 0 | 17066 | 40566 | 28300 | 68866 | 85932 |
|  | [1K, 2 K ] | 38424 | 0 | 38424 | 40099 | 37600 | 77699 | 116123 |
|  | [1K,3K] | 52789 | 0 | 52789 | 28670 | 46900 | 75570 | 128359 |
|  | [2K, 1 K ] | 38424 | 0 | 38424 | 40099 | 37600 | 77699 | 116123 |
|  | [2K, 2 K ] | 68332 | 0 | 68332 | 22834 | 46600 | 69434 | 137766 |
|  | [2K, 3 K ] | 68880 | 0 | 68880 | 13262 | 55900 | 69162 | 138042 |
|  | [3K,1K] | 52789 | 0 | 52789 | 28670 | 46900 | 75570 | 128359 |
|  | [3K,2K] | 68880 | 0 | 68880 | 13262 | 55900 | 69162 | 138042 |
|  | [3K,3K] | 69086 | 0 | 69086 | 4160 | 64900 | 69060 | 138146 |
| $\begin{array}{\|c} \text { 8(Peering) } \\ \text { (QoS-QoS) } \\ \text { (F-2P) } \end{array}$ | [1K, 1 K$]$ | 12107 | -18723 | -6616 | 47355 | 32600 | 79955 | 73339 |
|  | [1K, 2 K ] | 33465 | -18723 | 14742 | 43088 | 39200 | 82288 | 97030 |
|  | [1K,3K] | 37030 | -18723 | 18308 | 35485 | 45800 | 81285 | 99593 |
|  | [2K, 1 K ] | 33465 | -18723 | 14742 | 43088 | 39200 | 82288 | 97030 |
|  | [2K, 2 K ] | 47830 | -18723 | 29107 | 31605 | 45200 | 76805 | 105912 |
|  | [2K, 3 K ] | 47830 | -18723 | 29107 | 25005 | 51800 | 76805 | 105912 |
|  | [3K,1K] | 35398 | -19709 | 15689 | 33383 | 45800 | 79183 | 94872 |
|  | [3K,2K] | 47830 | -18723 | 29107 | 25005 | 51800 | 76805 | 105912 |
|  | [3K,3K] | 47830 | -18723 | 29107 | 19005 | 57800 | 76805 | 105912 |
| $\begin{gathered} \hline 9 \text { (Peering) } \\ \text { (QoS-QoS) } \\ \text { (F-F) } \end{gathered}$ | [1K, 1 K ] | 4500 | 4039 | 8539 | 51034 | 32600 | 83634 | 92173 |
|  | [1K, 2 K ] | 17547 | 4039 | 21586 | 53269 | 39200 | 92469 | 114055 |
|  | [1K,3K] | 27624 | 4039 | 31663 | 44538 | 45800 | 90338 | 122001 |
|  | [2K, 1 K ] | 17547 | 4039 | 21586 | 53269 | 39200 | 92469 | 114055 |
|  | [2K, 2 K ] | 39145 | 4039 | 43184 | 39002 | 45200 | 84202 | 127386 |
|  | [2K, 3 K ] | 39560 | 4039 | 43599 | 32131 | 51800 | 83931 | 127530 |
|  | [3K, 1K] | 27624 | 4039 | 31663 | 44538 | 45800 | 90338 | 122001 |


|  | $[3 \mathrm{~K}, 2 \mathrm{~K}]$ | 39560 | 4039 | 43599 | 32131 | 51800 | 83931 | 127530 |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | $[3 \mathrm{~K}, 3 \mathrm{~K}]$ | 39664 | 4039 | 43703 | 26062 | 57800 | 83862 | 127565 |
| $\mathbf{1 0 ( P \text { Peering } )}$ | $[1 \mathrm{~K}, 1 \mathrm{~K}]$ | 10682 | -18055 | -7373 | 57680 | 32600 | 90280 | 82907 |
| $\mathbf{( Q o S - Q o S})$ | $[1 \mathrm{~K}, 2 \mathrm{~K}]$ | 32040 | -18055 | 13985 | 53413 | 39200 | 92613 | 106598 |
| $\mathbf{( 2 P - 2 P )}$ | $[1 \mathrm{~K}, 3 \mathrm{~K}]$ | 35605 | -18055 | 17550 | 45809 | 45800 | 91609 | 109159 |
|  | $[2 \mathrm{~K}, 1 \mathrm{~K}]$ | 32040 | -18055 | 13985 | 53413 | 39200 | 92613 | 106598 |
|  | $[2 \mathrm{~K}, 2 \mathrm{~K}]$ | 46405 | -18055 | 28350 | 41930 | 45200 | 87130 | 115480 |
|  | $[2 \mathrm{~K}, 3 \mathrm{~K}]$ | 46405 | -18055 | 28350 | 35330 | 51800 | 87130 | 115480 |
|  | $[3 \mathrm{~K}, 1 \mathrm{~K}]$ | 33812 | -18970 | 14842 | 44311 | 45800 | 90111 | 104953 |
|  | $[3 \mathrm{~K}, 2 \mathrm{~K}]$ | 46405 | -18055 | 28350 | 35330 | 51800 | 87130 | 115480 |
|  | $[3 \mathrm{~K}, 3 \mathrm{~K}]$ | 46405 | -18055 | 28350 | 29330 | 57800 | 87130 | 115480 |

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[^0]:    ${ }^{1}$ The term ISP in this dissertation includes IAP (Internet Access Provider) in the Internet access market and IBP (Internet Backbone Provider) in the Internet backbone market. See [Chapter 2] section 2.1. A company that provides access to the Internet. For a monthly fee, the service provider gives you a software package, username, password, and access phone number. Equipped with a modem, you can then log on to the Internet and browse the World Wide Web and Usenet, and send and receive e-mail. (source: www.webopedia.com/TERM/I/.ISP.html)
    ${ }^{2}$ A NAP is where Internet interconnection among different providers occurs.
    ${ }^{3}$ Multi-Protocol Label Switching, see [Chapter 4] section 4.2.

[^1]:    ${ }^{4}$ A company that provides access to the Internet IAPs generally provide dial-up access through a modem and PPP connection, though companies that offer Internet access with other devices, such as cable modems or wireless connections, could also be considered IAPs. The term IAPs and ISPs are often used interchangeably, though some people consider IAPs to be a subset of ISPs. (Source: www.webopedia.com/TERM/I/IAP.html) See [Chapter 2] section 2.4.

[^2]:    ${ }^{5}$ A backbone provider supplies access to high-speed transmission lines that connect users to the Internet. These lines comprise the backbone of the Internet. An IBP supplies the ISPs with access to the lines sch as T1 or T3 lines, that connect ISPs to each other. (Source: www.webopedia.com/TERM/b/backbone_provider.html). See [Chapter 2] section 2.3
    ${ }^{6}$ "K" means 1,000 . For example $1 \mathrm{~K}(2 \mathrm{~K}, 3 \mathrm{~K})$ means an IAP can support up to $999(1999,2999)$ users. Because we assume 5,000 potential users in the market, network capacity for 3,999 or 4,999 users means negative profit to the IAPs.

[^3]:    ${ }^{7}$ Output is defined in the model as a number of users of BE and QoS , i.e., $\mathrm{q} 1_{\mathrm{BE}} / \mathrm{q} 1_{\mathrm{QoS}}$ is a number of $\mathrm{BE} / \mathrm{QoS}$ users subscribed to the IAP1.
    ${ }^{8}$ This idea is to begin with each of the final nodes in the game tree and determine the optimal decision to make at each of these points. For example, at first, find the maximum $f 2 *$ of each node of IAP2 and then find the maximum $f 1$ * at the node of IAP1 in the above game model.

[^4]:    ${ }^{9}$ The $13^{\text {th }}$ edition of the Directory of Internet Service Providers, Boardwatch magazine (www.ispworld.com/isp/introduction.htm)

[^5]:    ${ }^{10}$ Most data in this dissertation are based on the year 2001.

[^6]:    ${ }^{11}$ IBPs like AT\&T WorldNet, Broadwing, CAIS, Epoch, Netaxs, Savvis Communications, XO also support dial-up access customers in the downstream market. (http://www.boardwatch.com/isp/bb/n_america.htm)

[^7]:    ${ }^{12}$ Four NAPs are Chicago NAP (Ameritech), San Francisco NAP (Pacific Bell), New York NAP (Sprint), and Washington D.C. NAP (Metropolitan Fiber Systems).
    ${ }^{13}$ Sometimes NAPs are known by names such as Commercial Internet Exchange (CIX), Federal Internet Exchange (FIX), and Metropolitan Area Exchange (MAE).
    ${ }^{14}$ SONET stands for Synchronous Optical Networking. The capacity of OCx is based on that of OC1 ( 51.84 Mbps ). For example, the capacity of OC3 is 3 times of OC1. (155.52 Mbps $=3 * 51.84 \mathrm{Mbps}$ )

[^8]:    ${ }^{15}$ These numbers are based on North America region.
    ${ }^{16}$ Source: TeleGeography, Inc. The calculation is based on the data of 1999 U.S. backbone revenue (Worldcom $38 \%$, Genuity $15 \%$, AT\&T $11 \%$, Sprint 9\%)
    ${ }^{17}$ The Boardwatch magazine's annual survey for ISPs.
    ${ }^{18}$ AOL and Time Warner, GTE and Bell Atlantic, Concentric and NextLink, Qwest and US West, etc
    ${ }^{19}$ The backbone section was divided into backbone provider and data center provider from 2001 survey.

[^9]:    ${ }^{20}$ LMDS (Local Multiple Distribution System) and satellite based Internet service
    ${ }^{21}$ DS0 is a basic digital signal converted from analog voice ( 64 Kbps ).
    ${ }^{22} \mathrm{~T} 1$ line has 1.544 Mbps capacity, which is the same as 24 DS 0 ( 64 Kbps ) telephone lines.

[^10]:    ${ }^{23}$ Total number of registered IAPs to this survey is 7,288 at March, 2001
    ${ }^{24}$ Total number of customers of paid dial-up IAPs is 49.6 M at the first quarter of 2001 according to the Telecommunications Report International Inc. We calculated market share of each companies. The rest of IAPs' market share except top 10 IAPs is expected as $11 \%$.
    ${ }^{25}$ Source: www.isp-planet.com/research/rankings/usa.html

[^11]:    ${ }^{26}$ Source: www.ispworld.com/isp/images/NA_ISPs_chart.gif. At the time of July 2001, the number of ISPs is over 8,000.
    ${ }^{27}$ UUNET is a subsidiary of the MCI/WorldCom
    ${ }^{28}$ Telecommunications International Inc.'s Quarterly Online Census at March 31, 2000
    ${ }^{29}$ AltaVista, FreeInternet.com, NetZero, FreeLane, Source: www.ispworld.com/introduction.htm.
    ${ }^{30}$ LECs have traditionally been charging IXCs $\$ .03 /$ minute at each end for originating and terminating calls. This CCLC recovers part of the cost of the local loop.

[^12]:    ${ }^{31}$ Source: http://www.telegeography.com

[^13]:    ${ }^{32}$ Settlment can be thought of as payments or financial transfers between ISPs in return for interconnection and interoperability (Cawley, 1997)

[^14]:    ${ }^{33}$ On May 19, 2000, Telia International Inc., substantially purchased all the operating assets of AGIS.

[^15]:    ${ }^{34}$ Source: http://www.pitx.net/about.html

[^16]:    ${ }^{35}$ Internet2 is a cooperative project involving more than 120 top U.S. universities, along with invited hightechnology corporations. One of the primary goal s of Internet2 is to develop the next generation of computer network applications and the underlying broadband infrastructure to facilitate the research and education missions of universities. (Minoli and Schmidt, 1999)
    ${ }^{36} \mathrm{AS}$ is defined as a collection of networks that are under the administrative control of a single organization and that share a common routing strategy.

[^17]:    ${ }^{37}$ The cost per Mbit/s shipped figure is often used as a benchmark to determine interconnection strategy.

[^18]:    ${ }^{38}$ OSI reference model was developed by the International Standards Organization (ISO) as a model for a computer communications architecture. It consists of seven layers: (1) physical layer, (2) data link layer, (3) network layer, (4) transport layer, (5) session layer, (6) presentation layer, and (7) application layer.

[^19]:    ${ }^{39}$ IETF is one of the organizations under the Internet Society, which is responsible for protocol engineering and development.
    ${ }^{40}$ Most of this section comes from the paper, "Rosen, E., et. al., "Multiprotocol Label Switching Architecture," RFC3031, Jan. 2001.

[^20]:    ${ }^{41}$ Most of this section comes from (1) Black, S. et. al., "An Architecture for Differentiated Services," RFC2475, Dec. 1998 and (2) Nichols, K. et. al., "Definition of the Differentiated Services Field (DS Field) in the IPv4 and Ipv6 headers," RFC 2474, Dec. 1998.

[^21]:    ${ }^{42}$ Most of this part comes from the paper: Le Faucheur, F., et al., "MPLS Support of Differentiated Services," draft-ietf-mpls-diff-ext-09.txt, April, 2001.

[^22]:    ${ }^{43}$ There is three-layer router hierarchy: access layer, distribution layer, and core layer. Access layer routers have a function of access server connecting remote users to internetworks. Distribution layer routers are used to separate slow-speed local traffic from the high-speed backbone. Core layer is the backbone layer.
    ${ }^{44}$ Exp-Inferred-PSC LSPs in the original draft
    ${ }^{45}$ Label-Only-Inferred-PSC LSPs in the original draft

[^23]:    ${ }^{46} \mathrm{x}=$ class $1 \sim 4$ and $\mathrm{y}=$ drop preference $1 \sim 3$, i.e., AF11, AF12, AF13, AF21, AF22, AF23, AF31, AF32, AF33, AF41, AF42, AF43.

[^24]:    ${ }^{47}$ Traffic Policing is provided by Committeed Access Rate (CAR), which doesn't buffer or smooth traffic and might drop packets when the allowed bursting capability is exceeded. (Vegesna, 2001)
    ${ }^{48}$ Traffic Shaping is a mechanism to smooth the traffic flow on an interface to avoid congestion on the link and to meet a service provider's requirements. (Vegesna, 2001)
    ${ }^{49}$ In WFQ, each flow or traffic class is assigned a weight, and the rate at which a flow or a traffic class is serviced is proportional to its assigned weight. (Vegesna, 2001)

[^25]:    ${ }^{50}$ In the Priority Queuing, packets on the highest priority queue are transmitted first. No packets in the lower priority queue are served if packets in the higher priority queue are waiting for service. (Vegesna, 2001)

[^26]:    ${ }^{51}$ Any site where networks interconnect is may be referred to as a POP.
    ${ }^{52}$ For example, AOL's usage price for $1-800$ number ( 28.8 Kbps ) is 10 cents / minute.

[^27]:    ${ }^{53}$ Esther Gal-Or, the Katz School of Business, University of Pittsburgh.
    ${ }^{54}$ She restricts quality to a single dimension, which implies that any two consumers will agree on which of two products they prefer.
    ${ }^{55}$ She considers output and not price as part of the strategy of a firm so as to extend the Cournot equilibria concept from market of homogeneous products to markets of differentiated products.

[^28]:    ${ }^{56}$ Most of the data in this dissertation come from the year 2001.
    ${ }^{57}$ Including free users ( 10 millions) and paid users ( 49.6 millions) at the first quarter of 2001, Source: Telecommunications Report International Inc.

[^29]:    ${ }^{58}$ Telecommunications International Inc.'s Quarterly Online Census at March 31, 2000.

[^30]:    ${ }^{59}$ With the advent of local competition, telephone subscribers do not have to change their telephone number when they move from one local telephone company to another (2001,Moulton, p101).

[^31]:    ${ }^{60}$ One of the survey questions was "On average, how many hours per week do you and all your members of your household spend on the Internet from your home?"

[^32]:    ${ }^{61}\{(2 \mathrm{hrs} * 0.063)+(7 \mathrm{hrs} * 0.121)+(12.5 \mathrm{hrs} * 0.194)+(20 \mathrm{hrs} * 0.293)+(32.5 \mathrm{hrs} * 0.198)+(50 \mathrm{hrs} * 0.063)+(75$ hrs*0.069) $\} * 4.3=103.2774$ hours per month.

[^33]:    ${ }^{62}$ We chose this year instead of 2001 because it preceded the merger with Time Warner, which could obscure the reporting of these revenues.
    ${ }^{63}$ This number is estimated from the fact that the number of subscriber at first quarter of 2001 is 22.7 millions.

[^34]:    ${ }^{64}$ The bandwidth of T3 line is 45.736 Mbps , which equals to that of 28 T 1 lines.
    ${ }^{65}$ At year 2000, T1:1.2 million, T-3:58,000, Ocx:14,000, Source: Gartner/Dataquest

[^35]:    ${ }^{66}$ A low-end access router $(\$ 3,000)+$ a low-end mail server with software $(\$ 3,000)+100$ Modems $(\$ 3000)$
    ${ }^{67}$ The average price of T 1 transit service is $\$ 1,288$ per month (www.ispworld.com/isp). The big IBP's price of T1 is close to $\$ 2,000$ per month and small provider's T1 price is less than $\$ 1,000$ per month.
    ${ }^{68}$ At the time of data acquisition (2001)

[^36]:    ${ }^{69}$ The T1 price is based on mileage from the CO or POP. For example, Bell South charges $\$ 80 /$ month for the first half mile, and $\$ 30 /$ month for each additional half mile. Advent of business DSL service makes the price of T1 down to $\$ 150 /$ mont flat rate in the local market. (www.teleconnect.com/article/TCM20000509S0037)

[^37]:    * $Q 1=q 1_{B E}$ and $Q 2=q 2_{B E}+q 2_{Q o S}$

[^38]:    ${ }^{70}$ A monopoly IAP can produce 1,999 with 2 K of network capacity.

[^39]:    ${ }^{71}$ Because this payoff matrix is symmetric, another equilibrium point in Case VI, i.e., $[1 \mathrm{~K}, 2 \mathrm{~K}]$ also can be a third period equilibrium.
    ${ }^{72}$ [32220,19700] at [2K, 1 K$]$ in Case 6 and [22925,22925] at [2K,2K] in Case 10.

[^40]:    ${ }^{73}$ Case 7 's equilibrium network capacity: $[2 \mathrm{~K}, 2 \mathrm{~K}]$ in the uniform distribution and $[1 \mathrm{~K}, 2 \mathrm{~K}]$ and $[2 \mathrm{~K}, 1 \mathrm{~K}]$ in the empirical distribution.

[^41]:    ${ }^{74}$ \{The number of paying dial-up users - Sum of top 10 dial-up IAP users \} / \{Number of IAPs in the downstream market $\}=(49.6$ millions -44.3 millions $) / 7000=800$ subscribers $/$ IAP

[^42]:    ${ }^{75}$ www2.uu.net/peering
    ${ }^{76}$ www.cw.com

