THREE ESSAYS ON BARGAINING OVER DECISION RIGHTS AND CONTESTS

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Submitted to the Graduate Faculty of
the Department of Economics in partial fulfillment
of the requirements for the degree of

Doctor of Philosophy

University of Pittsburgh

2010

UNIVERSITY OF PITTSBURGH

DEPARTMENT OF ECONOMICS

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This dissertation consists of three chapters where the first two chapters study models of bargaining over decision rights and the third chapter studies a model of contests.

In the first chapter, "Selling Authority," I consider bargaining over decision-making authority in which an informed but self-interested agent makes a price offer to buy decision-rights to an uninformed principal who then decides either to accept or to reject the offer. No matter how large the difference between parties' preferences, there exists a continuum of perfect Bayesian equilibria, each of which yields an *ex-post* efficient action for any realization of the state. In these equilibria, delegation takes place with probability one and no information is transmitted, even though the informed agent's price offers could have been used as a signaling device. However, there exists an infinite sequence of informative equilibria that approximates *ex-post* efficiency in the limit.

The second chapter, "Communication in Bargaining over Decision Right," develops a model of bargaining over decision-rights between an uninformed principal and an informed but self-interested agent. The uninformed principal makes a price offer to the agent who then decides either to accept or to reject the offer. Contrary to the prediction the Coase Theorem provides, actions induced in the unique perfect Bayesian equilibrium do not always satisfy *ex-post* efficiency. Once we introduce explicit communication into the model, however, there exists a truth-telling perfect Bayesian equilibrium, even when the conflict of interest is arbitrarily large. The truth-telling equilibrium outcome is *ex-ante* Pareto superior to that of several dispute-resolution schemes studied in the framework of Crawford and Sobel (1982) and Holmström (1977).

The third chapter, "Contests with a stochastic number of players," studies the Tullock's (1980) n-player contest where each player has an independent probability $0 < p \leq 1$ to participate. A unique symmetric equilibrium is found for any n and p and its properties are analyzed. We show that for a fixed $n > 2$ *the individual equilibrium spending*is non-monotonic whereas *the total equilibrium spending* is monotonically increasing in p and n. We also show that the *ex-post* over-dissipation is a natural feature of the equilibrium.

Keywords: Decision-rights, Bargaining, Cheap Talk, Information Transmission, Contests, Rent-seeking.

TABLE OF CONTENTS

LIST OF FIGURES

PREFACE

The best luck I have ever got in my life is to meet my advisor, Andreas Blume. I am very grateful for his invaluable guidance, encouragement and support. I also thanks other committee members, Oliver Board, Alexander Matros, and Pierre Liang for their help and support. My peers at the University of Pittsburgh have been consistently helpful and willing to contribute comments relative to my papers. Especially helpful were Ernest K. Lai, Jonathan Lafky, and Yeolyong Sung. There are others who has helped me in different ways that I would like to thank—Sourav Bhattacharya, Yeon-koo Che, John Duffy, Maria Goltsman, Tymofiy Mylovanov, and Thomas Rawski. Last but not least, I express my endless love to my wife Jungeun Song and to my son Yujun Lim.

1.0 SELLING AUTHORITY

1.1 INTRODUCTION

In many economic situations, a party, such as a government or firm (principal), initially has full authority to make a decision, but lacks information about the task or project at hand. There is often another, better-informed party (agent) but it lacks the authority to make the decision. Examples of this principal-agent relationship include an international manufacturer who is less informed about specific national market conditions than a domestic distributer, a patentee who is less-experienced in commercializing than a manufacturing company, a policy-maker who knows less about potential impact of a fiscal policy than an economist, a central office who is less informed about a local division than a lower level division manager, and a private investor who is less informed about stock market conditions than a fund manager.

Sometimes the informed agent's interest is so different from the uninformed principal's interest that the informed agent does not want to share useful information with the principal. Crawford and Sobel [27] show that if costless communication, cheap-talk, is allowed between parties whose preferences diverge, then information is transmitted in a strategic way. That is, the informed agent has no incentive to fully reveal his information. Furthermore, when the parties' interests diverge substantially information cannot be transmitted through cheap-talk at all. Although more complicated communication protocols such as communication via a neutral trustworthy mediator (Goltsman, Hörner, Pavlov and Squintani [38]), a biased mediator (Ivanov [53]), an extensive communication (Krishna and Morgan [61]), and communication including noise (Blume, Board, and Kawamura [18]) could facilitate communication between parties, meaningful information is not transmitted when the degree of conflict is sufficiently large.

Organizational theory suggests that delegation of authority might resolve this problem. For example, Milgrom and Roberts [77] point out that comprehensive decision making in a large organization must involve considerable delegation of authority to lower levels of the organization. However, the principal is not always willing to delegate authority because the agent has his own agenda: Dessein [28] shows that the uninformed principal does not prefer to delegate his decision-making authority to the informed agent when the preferences diverge substantially.[1] Although the principal can optimally delegate authority with the restricted set of actions that the agent can take, it is optimal for the principal not to delegate authority when the degree of conflict is large enough (Alonso and Matouschek [3], Holmström [51], Kováč and Mylovanov [56] and Melamud and Shibano [75]).

In this paper, we show that the inefficiency caused by the informational loss can be resolved by efficient reallocation of authority via bargaining with monetary transfers over authority to make a decision, no matter how large the difference between parties' preferences is. This idea is inspired by Coase [22] who asserts that if property rights are well-defined, *voluntary bargaining* between parties results in an efficient outcome under complete information. Since we can think of the authority to make decisions as a well-defined property right, it is natural to investigate how bargaining over the decision-making authority affects the social outcome. One might also expect a socially efficient outcome in our framework. However, the result is not certain in our case because of the presence of an informational asymmetry between the parties. For example, Farrell [30] shows that in the presence of two-sided private information, voluntary negotiation does not lead to the first-best outcome that maximizes joint surplus.

We consider a bargaining game in which an informed but self-interested agent makes a price offer for decision-making authority to an uninformed principal who then decides either to accept or to reject the offer. We demonstrate that this simple bargaining can remove inefficiency by two different ways. First, we find that there exists a continuum of equilibria in which delegation takes place (almost) always and no information is transmitted by the

[1] When conflict of interest is large enough, full delegation cannot lead to ex-post efficient outcome because it is not incentive compatible for the principal. That is, by taking her *ex-ante* ideal action the principal can get higher payoff than that from full delegation.

agent's price offers.[2] Second, we show that there is an equilibrium that approximates full revelation of information in all states of nature. To be more precise, we construct an infinite sequence of informative equilibria with n number of partition intervals in which the agent types in the same interval makes a common price offer to the principal. As n goes to infinity, the length of each partition interval goes to zero and as a result, the principal can take an action arbitrarily close to her ideal action when she retains her decision rights.

Bargaining over authority to make a decision is common in the corporate world between two separately owned companies. When an international manufacturer enters a particular national market, it typically lacks relevant information about local market conditions and has difficulties making decisions on pricing, marketing, advertising, distribution and so on. As a result, it sells an exclusive distributorship to a domestic company who is better-informed but lacks authority to make such decisions. If a license agreement is reached through bargaining, the domestic company pays license fees in return for the exclusive right to make decisions about pricing, marketing, advertising, distribution, and so on in the domestic market.[3] For example, I.B.M., the world's largest computer maker in the 1990's, agreed to allow Mitsubishi to sell an I.B.M. mainframe computer under its own name in Japan in April, 1991.[4] More recently, tobacco industry leader Philip Morris International announced an agreement with Chinese National Tobacco under which Chinese National Tobacco will manufacture Marlboro cigarettes for marketing in China.[5] If international manufacturers cannot find any partners to make a license agreement, then they are able to found their own corporation in the national market and to start their businesses by themselves. For instance, in the mid-1990s, dozens of foreign beer brewers such as Annheuser-Busch, Heineken, South African Breweries (SAB), Carlsberg, Interbrew, San Miguel, Kirin, Lion Nathan and Foster's entered the Chinese market without making any license agreement.[6]

[2]This equilibrium is uninformative in a sense that the agent makes a common price offer regardless of his type.

[3]This license agreement is different from contracting in the adverse selection literature: the international manufacturer does not make a contracting offer which is contingent on every possible decisions or actions. Instead, it usally sells "the right" to make a decision.

[4]Andrew Pollack, "IBM Model to Be Sold By Mitsubishi," *The New York Times* (April 29, 1991), 17.

[5]Nicholas Zamiska and Juliet Ye, "Chinese Cigarettes to Go Global" *The Wall Street Journal*, (January 30, 2008) B4.

[6]After years of failing to break into the market, many of them have recently been cutting back, even selling their new state-of-the-art production facilities to local brewers. See Heracleous [44] for more details.

The situation considered arises frequently in the pharmaceutical industry, especially between an R&D firm with a patent who is less-experienced in commercializing and an experienced manufacturing company. For instance, Animas Corporation, an insulin infusion pump manufacturing company, set up license and development agreements with the Swiss R&D company, Debiotech for intellectual property related to next-generation insulin pumps and micro-needles. In return for the exclusive worldwide license to make, use and sell products utilizing the intellectual property portfolio that includes over 70 issued patents, Animas paid $12 million in cash and issued 400,000 restricted share of Animas common stock for the right.[7]

The efficiently reallocated authority via bargaining allows parties to make a full use of the decision-relevant information and as a result, leads to a Pareto improvement. We compare the equilibrium outcomes of the model with several dispute resolution schemes studied in the literature such as communication (Crawford and Sobel [27]), optimal mediation (Goltsman, Hörner, Pavlov and Squintani [38]), optimal delegation (Alonso and Matouschek [3], Holmström [51], Kováč and Mylovanov [56] and Melamud and Shibano [75]) and optimal compensation contract (Krishna and Morgan [60]). We show that any equilibrium outcomes of this model are Pareto superior to outcomes of those other schemes when the parties' preferences diverge to a substantial degree.

The rest of the paper is organized as follows. In the next section, we discuss the related literature. In section 3, we describe the model. Focusing on the principal's binary decision between accepting and rejecting a price offer, Section 4 provides the full characterization of the set of perfect Bayesian equilibrium outcomes of the model and its properties are analyzed. In section 5, we show that if we extend the principal's strategy space by allowing randomization between accepting and rejecting price offers, there exists an equilibrium with informative price offers that always yields almost efficient outcomes *ex-post*. Section 6 is devoted to analyzing welfare of the model. In section 7, we adopt a stronger equilibrium concept called sequential perfect equilibrium and show that this refinement gives us the unique outcome of the game, which satisfies *ex-post* efficiency. In section 8, we investigate

[7] Rick Baron, "Animas acquires technology for disposable insulin micro-pumps and micro-neddles," http://www.bioalps.org/Bioalps/en/Internet/Documents/1996.pdf

the robustness of the existence of *ex-post* efficient equilibrium against state-dependent biases. We conclude in section 9.

1.2 RELATED LITERATURE

We may divide literature on strategic interactions between an uninformed principal and a self-interested but informed agent into two strands: one on the reallocation of decision rights and the other on the strategic transmission of information.

Holmström [50], [51] initiates works on the reallocation of authority or delegation problem: the uninformed principal's choice from a set of admissible actions from which the agent can take an action. Focusing on the uniform distribution of the state space and particular preferences, Melumad and Shibano [75] provide full characterization of equilibria. With more general distributions of the state space and preferences, Alonso and Matouschek [3] show that the optimal set of admissible actions takes the form of a single interval if the informed party's preferences are sufficiently similar to the uninformed party's. While most papers have restricted attention to deterministic mechanisms, Kováč and Mylovanov [56] study relative performance of both stochastic and deterministic mechanisms and show that stochastic mechanisms perform strictly better than deterministic ones under some circumstances.

Another strand of the literature investigates the strategic information transmission or simply cheap talk between an informed but self-interested agent and an uninformed principal. Crawford and Sobel [27] (hereafter CS) develop a model of strategic communication in which a better-informed agent sends a possibly noisy signal to a principal, who then takes an action that determines the welfare of both. They show that all equilibria in their model have the form of partition equilibria in which there is only a *finite number of actions* chosen in equilibrium and each action is associated with an interval of states. This means that the final outcome of communication may still be inefficient, even though it can improve social welfare by helping parties to transmit information. An important question arises here concerning how to facilitate communication between parties or when the information transmission can be improved. Several papers answer this question by modifying the CS model to allow more extensive communication (Krishna and Morgan [61]) or to consider the possibility of error in communication (Blume, Board and Kawamura [18].) Recently, Goltsman, Hörner, Pavlov and Squintani [38] allow the parties to use any communication protocol including ones that call for a neutral trustworthy mediator and show that information transmission is improved under the optimal mediation rule. Ivanov [53] demonstrates that for any bias in the parties' preferences, there exists a biased mediator that provides the highest expected payoff to the principal as if the players communicated through a neutral mediator. Although there is no explicit communication in our setting, information transmission is an important issue because the informed agent's price offer can be used as a signalling device. We show that meaningful information can be transmitted through bargaining in which the parties are allowed to use monetary transfers, no matter how widely the parties' preferences diverge.

Some recent papers either allow parties to reallocate both information and authority, or consider the principal's choice between communication and delegation. In a setting with a single decision and a single agent, Dessein [28] studies the optimal allocation of decision-making authority when the uninformed party only can commit to an *ex-ante* allocation of decision rights. He assumes that cheap talk takes place whenever the uninformed principal retains some decision rights and shows that complete delegation dominates communication if the conflict of interests is not serious. The same result is obtained in settings with a multi-divisional organization in which there are two agents who have independent private information (Alonso, Dessein and Matouschek [4].) By exploring a setting with multiple, interdependent decisions, Alonso [2] shows that if activities are complementary the uninformed principal can always improve the informativeness of communication by sharing control with the informed agent.

Although we also consider the reallocation of decision-making authority, our paper is significantly different from others in the following ways. First, we allow explicit monetary transfers for parties which are not possible in other papers mentioned above. Second, contrary to most papers on the delegation problem, the informed agent has commitment power in our model so that he makes a price offer for authority to make decisions. This assumption makes the strategic information transmission, which is not an issue in other papers on the delegation problem, play an important role in our paper. Interestingly in our paper, the aspects of both information transmission and delegation appear at the same time, even though we neither allow the parties to communicate via cheap talk nor consider the principal's choice between communication and delegation. This is because the principal may be able to get meaningful information from the price offer for decision-making authority, which is a main determinant for the reallocation of authority.

There are several papers that consider contracting with monetary transfers in the framework of CS. Krishna and Morgan [60] and Bester and Strausz [16] consider the imperfect commitment model in which the uninformed principal is able to commit on the schedule of transfer payment but not on his action rule. Contrary to our model, the uninformed principal always retains decision-making authority in these models since commitment is only on the transfer but not on the allocation of decision-making authority. Under an optimal contract in Krishna and Morgan [60], the principal should never induce the agent to fully reveal what he knows even though this is feasible. Moreover, the principal never pay the agent for imprecise information. Krähmer [57] considers message-contingent delegation in which the principal can commit the allocation of decision rights after observing cheap talk messages from the informed agent and shows that it creates incentives for information revelation. Bester [15] also studies the contracting problem in the setting with monetary transfer when only decision rights are contractible *ex-ante* and focuses on the question of whether a direct and truthful mechanism can implement the same allocation of decision rights as under perfect information. Contrary to all these papers, the informed agent has a bargaining power in our setting so that he makes an offer to the principal.

Our paper shows that reallocation of decision-making authority is important to attain *ex-post* efficient outcomes, whereas most papers mentioned above focus their attention on either informativeness of equilibrium or the principal's welfare. In this sense, our paper is related to the *incomplete contracting* literature. Grossman and Hart [41] and Hart and Moore [46] develop a theory of property rights based on incentives by considering *incomplete contracting* between principal and agent. They assume that parties are not able to make a complete contract that encompasses all contingencies that might arise and argue that it can lead to *inefficient outcomes*. They suggest an optimal allocation of decision-making authority that minimizes the *ex-post* inefficiency. In this framework, Aghion and Tirole [1] explore the determinants of control rights in an alliance between a research unit and a customer firm to develop new technologies when the lack of financial resources makes the research unit eager to form the alliance with a customer.[8] They consider two different cases: when the research unit has bargaining power and when the customer does. When the research unit has the bargaining power, the ownership of the research output will be efficiently allocated. However, when the customer has the bargaining power, an inefficient allocation of the property rights might occur. Instead of assuming the lack of financial resources, however, this paper focuses on the lack of information that the principal faces and shows that voluntary bargaining over decision-making authority yields efficient outcomes *ex-post*.

1.3 THE MODEL

1.3.1 Environment

There are two parties, a principal (P) and an agent (A). The principal who initially has decision-making authority has little information about the state of the world $\theta \in \Theta \equiv [0, 1]$. She has her prior distribution F over $[0, 1]$ which has an absolutely continuous density function $f > 0$. The agent who has different interests from the principal knows the true state of the world θ but does not have decision-making authority. The payoffs for a given allocation of authority depend on an action y taken by the party who has decision-making authority and the state of the world θ. The payoff functions of the parties for a given action being taken are of the form $U^P(y, \theta) = -l(|y - \theta|)$ for the principal and $U^A(y, \theta, b) = -l(|y - (\theta + b)|)$ for the agent.[9] We refer to l as the loss function and assume that $l''(\cdot) > 0$, $l'(0) = 0$ and $l(0) = 0$. This means that the ideal action of the principal is $\overline{y}^P(\theta) = \theta$ and the ideal action of the agent is $\overline{y}^A(\theta, b) = \theta + b$ where $b > 0$ is a parameter that measures how nearly the agent's interest coincides with that of the principal. All of these are common knowledge between parties.

[8] In Aghion and Tirole [1], the research unit initially has decision-making authority. Thus, it is natural to think of the research unit as a principal in our framework.

[9] A special case is a quadratic utility ($U^P(y, \theta) = -(y - \theta)^2$ and $U^A(y, \theta, b) = -(y - \theta - b)^2$) which we are assumed in most examples and applications.

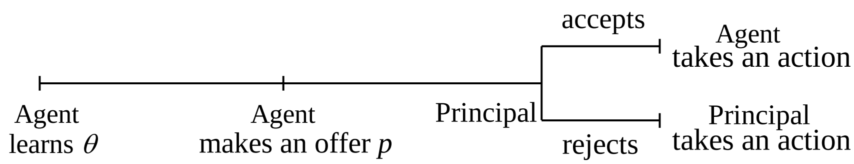

Figure 1: Timing of the game

1.3.2 Bargaining Game

Consider bargaining over the decision-making authority between the informed agent and the uninformed principal. The timing of the game is as follows:

1. The agent privately observes the state of the world $\theta \in \Theta \equiv [0, 1]$.

2. The agent offers to pay a price $p \in \mathbb{R}$ for the authority to take an action.[10]

3. The principal decides whether to accept or reject the offer.

4. If the principal accepts the offer then the agent pays the price offered by himself and takes an action, denoted by y^A. In this case, payoffs become $U^P(y^A, \theta) + p$ and $U^A(y^A, \theta, b) - p$ for the principal and the agent respectively. If the principal rejects the offer, however, she takes an action, denoted by y^P, without transferring the decision-making authority. Then payoffs are $U^P(y^P, \theta)$ and $U^A(y^P, \theta, b)$ for the principal and the agent, respectively.

The solution concept we use here is perfect Bayesian equilibrium. For the agent, a strategy consists of a price offer and an action rule. The price offer $\mu^A : \Theta \to \Delta(\mathbb{R})$ specifies the agent's choice of p when the state is θ. The agent's action rule $y^A : \Theta \times \mathbb{R} \to \mathbb{R}$ specifies the agent's choice of action after the principal's acceptance of p, i.e. $y^A(\theta, p)$ is the action taken by the agent type θ whose offer of p is accepted. For the principal, a strategy consists of a decision rule and an action rule. The decision rule, denoted by $d^P : \mathbb{R} \to \{0, 1\}$, specifies the principal's binary decision between accepting and rejecting for each price offer $p \in \mathbb{R}$ that the principal might receive. That is, $d^P(p) = 0$ implies that the principal accepts the offer p while $d^P(p) = 1$ implies that she rejects the offer. It is important to note that we don't allow the principal to use mixed-strategies in her decision rule and we are focusing on the

[10]We allow p to be negative, which means that the principal pays to the agent.

principal's pure strategy for a while. In section 5, we will investigate equilibria in which the principal is mixing in her decision rule. The action rule $y^P : \mathbb{R} \to \mathbb{R}$ specifies the principal's choice of action for each price offer that the principal might receive. The strategy profile $\{\mu^A, y^A, d^P, y^P\}$ and the principal's posterior belief ρ form a perfect Bayesian equilibrium if:

(**BA1**) for each $\theta \in [0,1]$, $\int_{\mathbb{R}} \mu^A(p|\theta)dp = 1$ and if $p^* \in \mathbb{R}$ is in the support of $\mu^A(\cdot|\theta)$ then p^* solves

$$\max_{p\in\mathbb{R}} d^P(p)U^A(y^P(p),\theta,b) + (1-d^P(p))(U^A(y^A(\theta,p),\theta,b) - p)$$

(**BA2**) for each $p \in \mathbb{R}$, $d^P(p)$ solves

$$\max_{d^P\in\{0,1\}} (1-d^P)(p + \int_0^1 U^P(y^A(\theta,p),\theta)\rho(\theta|p)d\theta) + d^P \int_0^1 U^P(y^P(p),\theta)\rho(\theta|p)d\theta$$

(**BA3**) for each $\theta \in [0,1]$ and each $p \in \mathbb{R}$, $y^A(\theta,p)$ solves

$$\max_{y\in\mathbb{R}} U^A(y,\theta,b)$$

(**BA4**) for each $p \in \mathbb{R}$, $y^P(p)$ solves

$$\max_{y\in\mathbb{R}} \int_0^1 U^P(y,\theta)\rho(\theta|p)d\theta$$

where $\rho(\theta|p)$ is given by Bayes' rule whenever possible.

Notice that the optimal behavior of the informed agent type θ is to take an action $\overline{y}^A(\theta,b) = \theta + b$. After substituting this into (**BA1**), (**BA2**), and (**BA3**), we have the following conditions for the equilibrium.

(**BA1′**) for each $\theta \in [0,1]$, $\int_{\mathbb{R}} \mu^A(p|\theta)dp = 1$ and if $p^* \in \mathbb{R}$ is in the support of $\mu^A(\cdot|\theta)$ then p^* solves

$$\max_{p\in\mathbb{R}} d^P(p)U^A(y^P(p),\theta,b) - p(1-d^P(p))$$

(**BA2′**) for each $p \in \mathbb{R}$, $d^P(p)$ solves

$$\max_{d^P\in\{0,1\}} (1-d^P)(p - l(b)) + d^P \int_0^1 U^P(y^P(p),\theta)\rho(\theta|p)d\theta$$

(**BA3′**) for each $\theta \in [0,1]$ and each $p \in \mathbb{R}$, $y^A(\theta,p) = \theta + b$.

1.4 ANALYSIS

In this section, we identify the set of equilibria of the model. In what follows, we will first categorize equilibrium candidates into several subcategories according to their properties. Then we will show that it is impossible for some of those candidates to be equilibria of the model. Notice that price offers could be used as signaling devices which convey some information to the principal since the informed agent makes an offer in our model. Thus, a price offer could be either *informative* or *uninformative*. Formally, a price offer on the equilibrium path is informative if and only if the posterior belief after observing the price offer is not the same as its prior belief so that the expected value of θ conditional on p is different from $E_P(\theta)$, the prior expectation of θ.

Definition 1. *A price offer $p \in \mathbb{R}$ on the equilibrium path is informative if $E_P(\theta|p) \neq E_P(\theta)$ where $E_P(\theta|p) = \int_\Theta \theta \cdot \rho(\theta|p)d\theta$.*

Moreover, the agent's price offer could be either a *costless* or a *costly* message. Whether it is costless or costly is determined endogenously in equilibrium. To see this explicitly, let us define the set of acceptable prices in equilibrium as

$$\mathcal{P}^\alpha := \{p \in \mathbb{R} | d^P(p) = 0\}$$

and the set of prices offered in equilibrium as

$$\mathcal{P}^o := \{p \in \mathbb{R} | \exists \theta \in [0,1] \quad s.t. \quad \mu^A(p|\theta) > 0\}.$$

A price offer p is *acceptable* if $p \in \mathcal{P}^\alpha$. A price offer p is *unacceptable* if $p \notin \mathcal{P}^\alpha$. Given $\mathcal{P}^\alpha$, if an agent type θ makes an offer $p \in \mathcal{P}^\alpha$ then the principal accepts the offer and the authority to make a decision is transferred to the agent. Then the principal and agent's payoffs are

$$U^P(y^A, \theta) + p \quad and \quad U^A(y^A, \theta, b) - p, \tag{1.1}$$

respectively. In this case, the agent's price offer is a costly message.[11]

[11] This is true only if the agent's price offer is not equal to zero. The price offer $p = 0$ is always costless no matter whether the principal accepts or not.

On the other hand, if the agent type θ makes an offer $p \notin \mathcal{P}^{\alpha}$ then the principal rejects the offer and she retains the authority to make a decision. Then the principal's payoff and the agent's payoff are

$$U^P(y^P, \theta) \quad and \quad U^A(y^P, \theta, b), \tag{1.2}$$

respectively. In this case, the agent's price offer is a costless message because it is not included in the payoffs above.

Therefore, a price offer could be one of the followings: *informative* and *acceptable* (costly) price offer, *informative* but *unacceptable* (costless) price offer, *uninformative* but *acceptable* (costly) price offer, and *uninformative* and *unacceptable* (costless) price offer.

1.4.1 Equilibrium

This section provides the full characterization of equilibrium outcomes of the model. We first focus on the simplest kind of equilibria, in which all agent types make a common price offer which is acceptable. The next proposition shows that there exists a continuum of such equilibria in this model. Let $\sigma = -\int_0^1 U^P(\overline{y}^P, \theta) f(\theta) d\theta > 0$ where $\overline{y}^P = argmax_y \int_0^1 U^P(y, \theta) f(\theta) d\theta$. That is, $-\sigma$ is the principal's expected utility from her ex-ante optimal action $\overline{y}^P$.

Proposition 1. *For any $p^* \in [l(b) - \sigma, l(b)]$, the following strategies and belief form a perfect Bayesian equilibrium.*

i) For all $\theta \in [0,1]$, the agent makes a price offer p^* with probability 1.

ii) $d^P(p) = \begin{cases} 0 & \textit{if } p \geq p^*, \\ 1 & \textit{otherwise.} \end{cases}$

iii) $y^P(p) = \begin{cases} \overline{y}^P & \textit{if } p \geq p^*, \\ 0 & \textit{otherwise.} \end{cases}$

iv) For any $\theta \in [0,1]$ and any $p \in \mathbb{R}$, $y^A(\theta, p) = \theta + b$.

v) For any $p < p^*$, $\rho(\theta|p) = \begin{cases} 0 & \forall \theta \in (0,1], \\ 1 & \textit{if } \theta = 0, \end{cases}$ *and for any $p \geq p^*$, $\rho(\theta|p) = f(\theta)$ $\forall \theta \in [0,1]$.*

Proof. See the appendix. □

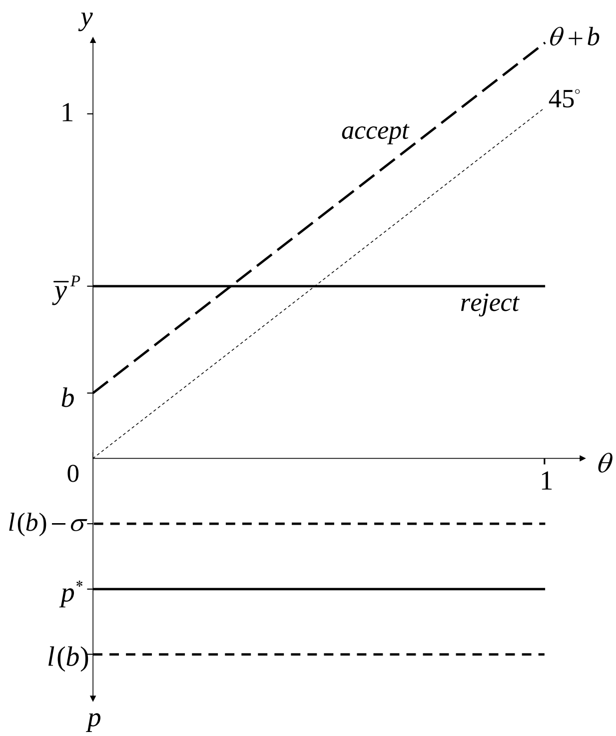

Figure 2: Equilibrium

Figure 2 illustrates the equilibria described in Proposition 1. The horizontal axis measures the state of the world θ. The vertical axis above zero stands for the action taken by players in different states, whereas the vertical axis below zero stands for the price offers made in different states. The dotted line in the upper quadrant represents the agent's action. It is clear that the equilibrium action should be $\theta + b$, which is the ideal action for the agent in each state. The solid line in the upper quadrant represents the principal's action whenever she has the authority. Since the equilibrium price offer is uninformative, the principal's action should be the same as her ex-ante optimal one, $\overline{y}^P$. The solid line in the lower quadrant depicts the price offers made in different states. It is constant in θ because otherwise the price offer conveys some information to the principal. Lastly, the figure shows that the equilibrium price offer must be in-between $l(b) - \sigma$ and $l(b)$.

Observe that in the equilibrium with $p^* = l(b)$ demonstrated in Proposition 1 the agent type 0 is indifferent between making a price offer p^* and making any unacceptable price offer $p < p^*$. Therefore, there exists an equilibrium where agent type 0 reveals itself by making an unacceptable price offer, which results in the principal's action 0, and all the other types make a price offer $l(b)$, which is accepted.

Proposition 2. *The following strategies and belief form a perfect Bayesian equilibrium.*

i) For all $\theta \in (0, 1]$, *the agent makes a price offer* $l(b)$ *with probability 1.*

i) When making a price offer, the agent type $\theta = 0$ *randomizes over* $(-\infty, l(b)]$.

iii) $d^P(p) = \begin{cases} 0 & \text{if } p \geq l(b), \\ 1 & \text{otherwise.} \end{cases}$

iv) $y^P(p) = \begin{cases} \overline{y}^P & \text{if } p \geq l(b), \\ 0 & \text{otherwise.} \end{cases}$

v) For any $\theta \in [0, 1]$ *and any* $p \in \mathbb{R}$, $y^A(\theta, p) = \theta + b$.

vi) For any $p < l(b)$, $\rho(\theta|p) = \begin{cases} 0 & \forall\theta \in (0, 1], \\ 1 & \text{if } \theta = 0, \end{cases}$ *and for any* $p \geq l(b)$, $\rho(\theta|p) = f(\theta)$ $\forall\theta \in [0, 1]$.

Proof. Since the proof is obvious, I skip it. □

We say that an equilibrium price offer is non-degenerately informative if there is an-

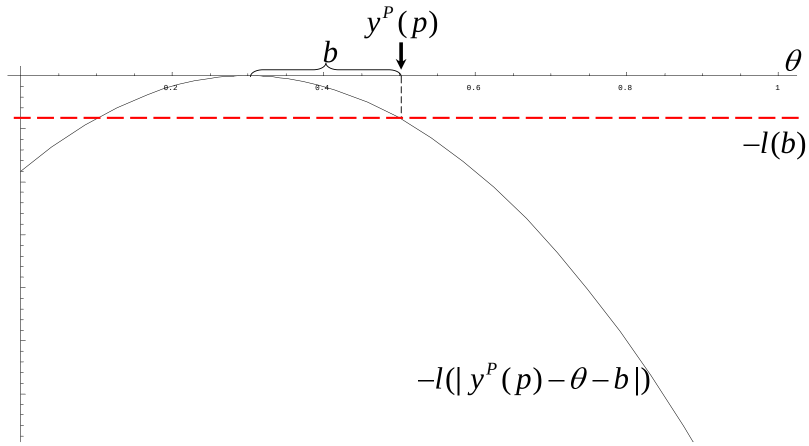

Figure 3: Nonexistence of equilibria with a non-degenerately informative offer

other informative price offer on the equilibrium path. In the equilibrium above, there is an informative price offer but not non-degenerately informative price offer. In the remainder of the paper, we simply say that an equilibrium is informative if and only if there exists a non-degenerately informative price offer on the equilibrium path. Is there any equilibrium with a non-degenerately informative price offer? It is interesting to notice that if all agent types make unacceptable offers in equilibrium, then price offers play exactly the same role as cheap talk messages so that outcome should be the same as one of the equilibria of CS model.[12] That is, unacceptable price offers could be informative non-degenerately. However, not only is there no such equilibrium in this model but also an unacceptable price offer made by a positive measure of agent types can never be part of an equilibrium.

Lemma 1. *There is no equilibrium in which a positive measure of agent types makes an unacceptable offer.*

Proof. See the appendix. □

Why does a positive measure of agent types not make an unacceptable offer in equilibrium? Notice that an unacceptable price offer in our model plays exactly the same role

[12]In this case, the message space is $\mathbb{R}\backslash\mathcal{P}^\alpha$.

as a costless message in CS model[13] so that if there is an unacceptable price offer on the equilibrium path then the set of agent types who make the offer should be an interval. As a result, the unacceptable price offer could not convey precise information without noise to the principal. However, as shown in Figure **??**, some agent types in the interval strictly prefer to take an action by themselves after obtaining decision-making authority rather than allowing the principal to make a decision based on imprecise information. In this figure, the solid curve depicts the agent's expected payoff from making the unacceptable offer p that induces the principal's action $y^P(p)$, whereas the dotted line depicts the agent's expected payoff from deviating to the acceptable offer $l(b)$. Notice that the principal accepts any price offer greater than $l(b)$ because accepting the offer always gives her nonnegative expected payoff, while rejecting it could give her at most zero expected payoff. This implies that each agent type in the interval should get at least $-l(b)$ from making the unacceptable offer and does not have an incentive to make an acceptable offer. This is impossible because in any given interval, $y^P(p)$ is in the interior of this interval so that the agent types on the right of $y^P(p)$ are strictly better off by deviating to making the acceptable offer $l(b)$.

This result tells us that there remain only two possibilities: a price offer can be either an *informative and acceptable* offer or an *uninformative but acceptable* offer. The next lemma tells even more about the equilibrium price offer and allows us to eliminate the possibility for having an informative price offer made by a positive measure of agent types.

Lemma 2. *There is no equilibrium in which a positive measure of agent types makes an informative price offer.*

Proof. See the appendix. □

Where does the nonexistence of equilibrium with a non-degenerately informative price offer come from? In our model two different price offers on the equilibrium path cannot be accepted at the same time, because otherwise the agent type who makes a higher price offer has an incentive to make the lower offer. Thus, if an equilibrium is informative, there

[13] In CS, every equilibrium should be an interval partitional, i.e. for every message on the equilibrium path, the set of sender types who send the message is a convex interval. This implies that a cheap talk message sent by the informed sender conveys imprecise information to the receiver in equilibrium.

should be at least one unacceptable price offer on the equilibrium path, which leads to a contradiction.

Lemma 1 and 2 result in the following proposition.

Proposition 3. *All equilibria are outcome equivalent to equilibria demonstrated in Proposition 1 and 2.*

Proof. See the appendix. □

1.4.2 *Ex-post* Efficiency

In this section, we demonstrate ex-post efficiency of the equilibrium outcome in this model. Although the model of informed agent and uninformed principal has been extensively studied in the literature on communication (Crawford and Sobel [27]) and optimal delegation (Holmström [50]), most of the schemes considered do not completely resolve *ex-post* inefficiency caused by the tension between access to information and authority to make a decision. We demonstrate *ex-post* inefficiency of these schemes in the following example and show that our model leads to a Pareto efficient outcome.

Example: Suppose that F is uniform and utilities are quadratic. Let $b = 1/5$ and the realized state of the world $\theta = 7/8$. As you can see in Figure **??**, if an action y is not in $[\theta, \theta + b] = [7/8, 43/40]$, then there exists another action y' such that both parties strictly prefer y' to y. Suppose that parties communicate via cheap talk. In the most informative equilibrium, only two actions $y_1 = \frac{1}{20}$ and $y_2 = \frac{11}{20}$ are induced.[14] Since $y_1 < \theta$ and $y_2 < \theta$, both actions are inefficient *ex-post*. Alternatively, suppose that the principal optimally proposes the set of admissible actions that the agent can take. In the optimal delegation, the proposed set is $[0, 1 - b] = [0, 4/5]$.[15] As the result, the agent cannot take any action $y \in [7/8, 43/40]$.

[14] See the leading example of Crawford and Sobel [27].

[15] See Holmström [50][51], Melumad and Shibano [75], Alonso and Matouschek [3] and Kováč and Mylovanov [56].

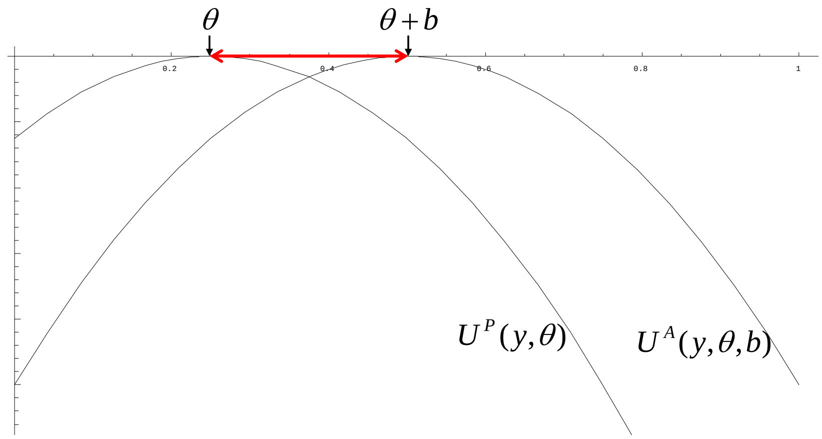

Figure 4: Ex-post Efficiency

Our focus is on whether an ex-post efficient action $y \in [\theta, \theta + b]$ can be taken for any realization of the state of the world θ in an equilibrium of our model. From the previous analysis, we know that parties come to an agreement for (almost) all realization of the state of the world so that the informed agent takes his ex-post ideal action $\theta + b$. Therefore, a socially efficient outcome is always attained in our model. To be more formal, define *ex-post* efficiency as follows. An action is said to be *efficient ex-post* if and only if there is no other feasible action that makes some individual better off without making other individuals worse off after the true state of the world θ is publicly known.

Definition 2. *An action $y \in \mathbb{R}$ is efficient ex-post at θ if there is no other action $z \in \mathbb{R}$ such that*

$$U^P(z, \theta) \geq U^P(y, \theta) \quad \textit{and} \quad U^A(z, \theta, b) \geq U^A(y, \theta, b) \tag{1.3}$$

with at least one strict inequality.

Now, we are ready to state the main result of this section. It is important to notice that in any perfect Bayesian equilibrium of this model the informed agent makes a decision after buying authority from the principal. The agent is able to use her private information fully so that a socially desirable outcome is attained. The following proposition summarizes this result.

Proposition 4. *In any perfect Bayesian equilibrium of the model an ex-post efficient action is taken for all* $\theta \in [0,1]$.

Proof. Since an action $y = (\theta + b)$ is a unique maximizer of $U^A(y, \theta, b)$, any other actions make the agent worse off. Similarly, an action $y = \theta$ is a unique maximizer of $U^P(y, \theta)$, any other actions make the principal worse off. Notice that the action taken by the party in control in any equilibrium is either θ or $\theta + b$. This completes the proof. □

One might be tempted to argue that the concept of *ex-post* efficiency should be defined over the space of actions and transfers. However, in this paper, it is assumed that decision-right is transferable and contractible but not the decision or action itself. The principal can commit only on the *ex-ante* allocation of decision rights and corresponding monetary transfers. Once the *ex-ante* allocation of decision rights is determined and monetary transfers are made according to the agreement, the monetary transfers have no effect on the incentives of a decision-maker at all and what action can be taken is only an informational issue. Therefore, it is reasonable to discuss efficiency of actions separately from monetary transfers, especially when actions are not contractible.

1.5 INFORMATIVE EQUILIBRIA

Several recent papers provide theoretical evidence showing that uncertainty could improve the informativeness of communication (Blume, Board and Kawamura [18], Krishna and Morgan [61], and Goltsman, Hörner, Pavlov and Squintani [38].) How then does uncertainty affect the informativeness of equilibria in our model? Instead of introducing an exogenous source of uncertainty, we consider the strategic uncertainty resulting from the players' randomization by allowing the principal to use mixed strategies in her decision rule and investigate the effect of the strategic uncertainty on the informativeness of equilibria. In what follows, we first investigate some general properties of informative equilibria and show that there exist both monotonic[16] and non-monotonic equilibria with informative price of-

[16] The equilibrium action taken by the principal is a nondecreasing function of the state.

fers. Next, we study the *ex-post* efficiency of these equilibria and see that there exists an informative equilibrium that yields an outcome arbitrarily close to the *ex-post* efficient one.

Let $d^P : \mathbb{R} \to [0, 1]$ be the principal's decision rule. Precisely, $d^P(\cdot)$ specifies the *rejection probability* for each price offer $p \in \mathbb{R}$ that the principal might receive. The first question that arises is whether there is an informative equilibrium in this extended strategy space. The following proposition tells us that there is no fully separating equilibrium, even though we allow mixing in the principal's decision rule. Intuitively, if the agent type θ makes an informative price offer that fully reveals her private information, then the principal has a strong incentive to take her optimal action θ by herself after rejecting the offer. Therefore, it is profitable for the agent type $\theta - b$ to imitate the agent type θ.

Proposition 5. *There is no fully separating equilibrium.*

Proof. See the appendix. □

The nonexistence of a fully separating equilibrium does not imply that there is no equilibrium with informative price offers. There indeed exist equilibria with informative price offers in this model. Before we start looking at some examples of informative equilibria in the next section, let us explain why we suddenly have such equilibria once we remove the restriction on the principal's decision rule. Recall that two different acceptable price offers cannot be made by agent types in equilibrium if we prohibit the principal from mixing her strategies because otherwise, making the lower price offer is profitable for the agent types who are making higher but still acceptable offers. This prevents us from having an equilibrium with informative price offers. It turns out that two (or even more) price offers can be accepted *with positive probability* in equilibrium of the extended model. To see details, suppose that there are two (high and low) price offers that will be accepted by the principal with some positive probabilities. It is clear that all agent types get higher payoff from making the low-price offer if it is accepted. However, some agent types might still prefer to make the high-price offer because they infer that an action induced by the offer is much more attractive to them in case of rejection that takes place with positive probability. We will discuss more on this by looking at some examples of informative equilibria in the next session.

1.5.1 Properties and Examples

What do equilibria with informative price offers look like? Recall that, by Lemma 2, there is no (non-degenerately) informative equilibrium when the principal uses pure strategies. This implies that for equilibria to be informative it is necessary for the principal to use a mixed strategy in her decision rule. Thus, on the equilibrium path, agent types make some offers that the principal is indifferent between accepting and rejecting.

Lemma 3. *In any (non-degenerately) informative equilibrium, agent types make price offers that the principal is indifferent between accepting and rejecting.*

To get a clear picture of the informative equilibria, we first focus on the monotonic equilibrium in which the equilibrium action taken by the principal is an increasing function of the state, which is often reported in the cheap talk literature. Let $\Theta(N) \equiv (\Theta_0(N), ..., \Theta_N(N))$ denote a partition of $[0,1]$ with N steps and dividing points between steps $\theta_0(N)$,...,$\theta_N(N)$, where $0 = \theta_0(N) < \theta_1(N) < \cdots < \theta_{N+1}(N) = 1$. Whenever it can be done without loss of clarity in what follows, we shall write θ or θ_n instead of $\theta(N)$ or $\theta_n(N)$. Define, for all $\underline{\theta}, \overline{\theta} \in [0,1]$ with $\underline{\theta} \leq \overline{\theta}$,

$$y(\underline{\theta}, \overline{\theta}) = \begin{cases} argmax \int_{\underline{\theta}}^{\overline{\theta}} U^P(y,\theta) f(\theta) & if\ \underline{\theta} < \overline{\theta}, \\ \overline{\theta} & if\ \underline{\theta} = \overline{\theta} \end{cases} \tag{1.4}$$

Then in any monotonic informative equilibria the following indifference conditions are necessary.

Proposition 6. *In any N-step monotonic informative equilibria,*

$$p_n - l(b) = \int_{\theta_{n-1}}^{\theta_n} U^P\left(y(\theta_{n-1}, \theta_n), \theta\right) \rho(\theta|p_n) d\theta, \qquad (ID-P)$$

$$U^A\left(y(\theta_{n-1}, \theta_n), \theta_n, b\right) d_n - p_n(1-d_n) = U^A\left(y(\theta_n, \theta_{n+1}), \theta_n, b\right) d_{n+1} - p_{n+1}(1-d_{n+1}), \quad (ID-A)$$

and

$$U^A\left(y(\theta_{n-1}, \theta_n), \theta_n, b\right) d_n - p_n(1-d_n) \geq -l(b), \quad \forall \theta \in \Theta_n,$$

for all $n = 1, 2, ...N-1$.

$(ID-P)$ is the indifference condition of the principal for equilibrium price offers and $(ID-A)$ is the indifference condition of the critical agent types. The third condition tells us that equilibrium payoffs of agent types are at least $-l(b)$ so that there are no agent types who have incentives to deviate to off-the-equilibrium-path price offers. The following example gives us one of the simplest two-step equilibrium. In this example, we take a uniform distribution and quadratic utilities for simplicity.

Example 1 (Monotonic equilibrium)**.** *Consider the following strategy profile.*

- *The agent types in* $[0, \theta_1]$ *makes a price offer* $p_1 = b^2 - \frac{\theta_1^2}{12}$.
- *The agent types in* $(\theta_1, 1]$ *makes a price offer* $p_2 = b^2 - \frac{(1-\theta_1)^2}{12}$.
- *The principal rejects the price offers* p_1 *with probability* $d^P(p_1) \in (0, 1)$ *but accepts* p_2 *with probability 1.*
- *The principal accepts any price offer* $p \geq b^2$ *with probability 1 but rejects any* $p < b^2$ *which is different from* p_1 *and* p_2 *with probability 1.*
- *The principal takes an action* $y = 0$ *whenever she rejects a price offer* $p < b^2$ *which is different from* p_1 *or* p_2*. The principal takes an action* $y = \frac{1}{2}$ *whenever she rejects a price offer* $p \geq b^2$.
- *The principal takes an action* $y_1 = \frac{\theta_1}{2}$ *if she rejects* p_1 *and takes an action* $y_2 = \frac{1+\theta_1}{2}$ *if she rejects* p_2.
- *The agent type* θ *takes an action* $\theta + b$ *whenever she has decision-making authority.*

It is interesting to see that, for any $b > 0$, we can find proper beliefs and $d^P(p_1) \in (0, 1)$ that form a perfect Bayesian equilibrium together with the strategy profile specified above. The strategy profile satisfies all three necessary conditions in Proposition 6. Figure 5 illustrates this equilibrium. All missing calculations can be found in the appendix.

In CS model, there exist indifference conditions similar to $(ID - A)$ for critical agent types which are both necessary and sufficient. Unlike CS, however, the above conditions are not sufficient for equilibria in our model because the agent's interim utility function does not satisfy the single-crossing property, the sorting condition crucially used by CS to get a full characterization of equilibria. This may give us another type of equilibria called non-monotonic equilibria in which equilibrium action taken by the principal is not an increasing

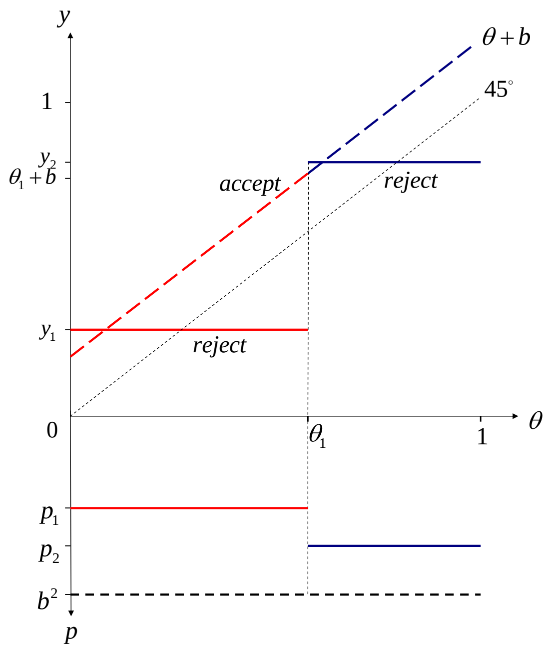

Figure 5: 2-step Monotonic Equilibrium

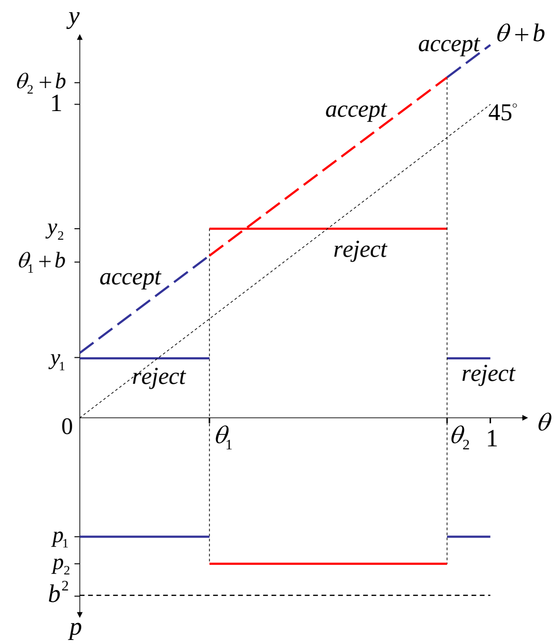

Figure 6: 3-step Non-monotonic Equilibrium

function of the state. In a setting with a multi-stage communication, Krishna and Morgan [61] report the existence of such non-monotonic equilibria. Similarly, there are non-monotonic equilibria in our model. Again, for simplicity, we take a uniform distribution and quadratic utilities in the following example.

Example 2 (Non-monotonic equilibrium)**.** *Consider the following strategy profile as a simple candidate for the (three-step) non-monotonic informative equilibrium. Specifically, suppose that* $b = 1$*,* $\theta_1 = \frac{49}{100}$*, and* $\theta_2 = \frac{99}{100}$*.*

- *The agent types in* $[0, \theta_1] \cup [\theta_2, 1]$ *make a price offer* p_1 *that the principal is indifferent between accepting and rejecting.*
- *The agent types in* (θ_1, θ_2) *make a price offer* p_2 *that the principal is indifferent between accepting and rejecting.*
- *The principal rejects the price offers* p_1 *and* p_2 *with probability* $d^P(p_1)$ *and* $d^P(p_2)$ *respectively.*
- *The principal accepts any price offer* $p \geq b^2$ *with probability 1 but rejects any* $p < b^2$ *which*

is different from p_1 and p_2 with probability 1.

- *The principal takes an action $y = 0$ whenever she rejects a price offer $p < b^2$ which is different from p_1 or p_2. The principal takes an action $y = \frac{1}{2}$ whenever she rejects a price offer $p \geq b^2$.*
- *The principal takes an action $y_1 = \frac{1+\theta_1^2-\theta_2^2}{2(1+\theta_1-\theta_2)}$ if she rejects p_1 and takes an action $y_2 = \frac{\theta_1+\theta_2}{2}$ if she rejects p_2.*
- *The agent type θ takes an action $\theta + b$ whenever she has decision-making authority.*

Figure 6 illustrates this equilibrium. Again, all missing calculations can be found in the appendix.

1.5.2 No Upper Bound in N: Uniform Quadratic Example

What is the full characterization of the equilibrium with informative price offers in this model? While we cannot offer a complete answer, this section demonstrates the existence of the infinite sequence of informative equilibria with N partition elements that converges to an *ex-post* efficient equilibrium as N tends to infinity, by focusing on uniform prior and quadratic utilities.[17]

Let us construct a monotonic equilibrium with N interval partitions. Notice that a price offer in equilibrium depends solely on the conditional variance which is determined by the length of the interval in the case of a uniform distribution due to the indifference condition $(ID - P)$. Thus, no two interval partitions can have the same length. Finally, we have the following necessary conditions for monotonic informative equilibria:

1) The lengths of intervals are different from each other.

2) The agent types in each interval make a price offer which the principal is indifferent between accepting and rejecting.

3) The principal randomizes in her decision rule for any price offer on the equilibrium path.

4) The critical agent type θ_n is indifferent between making price offer p_n and p_{n+1}, for $n = 1, 2, ...N - 1$.

Figure 7 illustrates this monotonic informative equilibrium in which the lengths of inter-

[17] For simplicity we have this assumptions, but the result is clearly not driven by these assumptions.

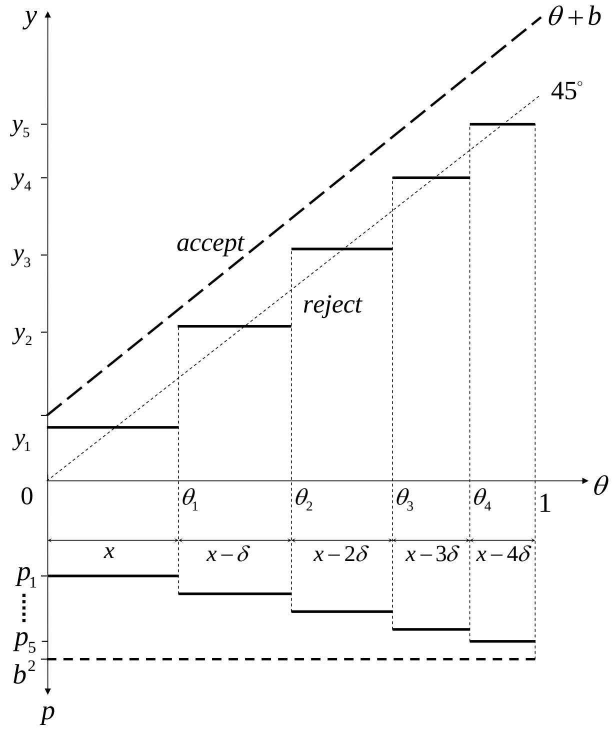

Figure 7: Construction of 5-step Equilibrium

vals are strictly decreasing in n for $N = 5$. Let x and δ denote the length of the first interval and the difference between the lengths of any two adjacent intervals, respectively. Then by the principal's indifference condition, the price offer p_n made by the agent types in the n_{th} interval is strictly increasing in n. Notice that any p_n is less than b^2. By sequential rationality, an agent type takes an action $\theta + b$ whenever she has the authority. If the principal rejects an offer p_n, she then takes the action which is a mid-point of the n_{th} interval. The principal randomizes between accepting and rejecting for any p_n. The following proposition demonstrates the existence of such informative equilibria for any positive integer N and any positive b.

Proposition 7. *Suppose that F is uniform and utilities are quadratic. For any $b > 0$ and any positive integer N, there exist $\delta \in (0, 1)$ and $x \in (0, 1)$ such that the following strategy profile forms a perfect Bayesian equilibrium;*

i) For all $\theta \in [\theta_{n-1}, \theta_n]$ the agent makes a price offer p_n with probability 1, for all $n = 1, \ldots\ldots, N$.

ii) $$d^P(p) = \begin{cases} 0 & \text{if } p \geq b^2, \\ d^* = \frac{\delta}{4(3b+\delta)} & \text{if } p = p_n \text{ for all } n = 1, \ldots., N, \\ 1 & \text{if } p < b^2 \text{ and } p \neq p_n. \end{cases}$$

iii) $$y^P(p) = \begin{cases} \frac{1}{2} & \text{if } p \geq b^2, \\ y_n & \text{if } p = p_n \text{ for all } n = 1, \ldots., N, \\ 0 & \text{if } p < b^2 \text{ and } p \neq p_n. \end{cases}$$

iv) For any $\theta \in [0, 1]$ and any $p \in \mathbb{R}$, $y^A(\theta, p) = \theta + b$.

v) $$\rho(\theta|p) = \begin{cases} 0 & \text{if } \theta \in (0, 1], \\ 1 & \text{if } \theta = 0. \end{cases} \quad \text{for any } p < b^2 \text{ and } p \neq p_n, \text{ and}$$

$$\rho(\theta|p_n) = \begin{cases} \frac{1}{\theta_n - \theta_{n-1}} & \text{if } \theta \in (\theta_{n-1}, \theta_n], \\ 0 & \text{otherwise.} \end{cases} \quad \text{for all } n = 1, \ldots., N, \text{ and } \rho(\theta|p) = 1 \text{ for any } p \geq b^2,$$

where

(a) $\theta_n = nx - \frac{n(n-1)\delta}{2}$ for all $n = 1, \ldots\ldots, N$,

(b) $\theta_0 = 0$ and $\theta_N = 1$

(c) $p_n = b^2 - \frac{(x-(n-1)\delta)^2}{12}$ for all $n = 1, \ldots\ldots, N$,

(d) $y_n = \frac{(2n-1)x}{2} - \frac{(n-1)^2\delta}{2}$ for all $n = 1, \ldots\ldots, N$.

Proof. See the appendix. □

It is important to note that for any $b > 0$ there is no upper bound for N in this proposition. This means that meaningful information can be transmitted without limit, even though the agent's bias is so great that communication would not be possible via cheap talk. Surprisingly, this remains true even if the divergence of interests is sufficiently large that no meaningful communication occurs in an equilibrium that allows parties to use any communication protocol including the neutral trustworthy mediator studied by Goltsman, Hörner, Pavlov and Squintani [38]. Our model, however, allows the parties to use monetary transfers excluded in Goltsman, Hörner, Pavlov and Squintani [38]. Krishna and Morgan [60] show that if parties can use monetary transfers, then full revelation of agent's private information can be induced by some contract, no matter how largely the parties' preferences diverge. Therefore, our result is consistent with that of contracting with monetary transfers.

Observe that these equilibria do not need to yield an *ex-post* efficient outcome. Any equilibrium with informative price offers, including the equilibria demonstrated in Proposition 7, contains the principal's mixing in her decision rule. Since there is no fully separating equilibrium, an action taken by the principal after the rejection of any equilibrium price offer is based on imprecise information. As a result, it has to yield an *ex-post* inefficient outcome in some states. However, it is always possible to achieve an outcome *arbitrarily close to* the *ex-post* efficient one in some equilibria. By construction of the equilibrium in Proposition 7, the length of each interval decreases as the number of partitions, N, increases. This implies that increasing N gives more precise information to the principal and allows her to choose an action very close to her (*ex-post*) optimal one, even though *ex-post* optimality is not guaranteed.

1.6 WELFARE

In this section, we assume that utilities are quadratic in order to get not only more precise welfare analysis of our model but also clear welfare comparisons to other models that studied quadratic utility functions. That is,

$$U^P(y,\theta) = -(y-\theta)^2 \quad and \quad U^A(y,\theta,b) = -(y-\theta-b)^2. \tag{1.5}$$

1.6.1 No *Ex-Ante* Pareto Ranking

Facing multiplicity of perfect Bayesian equilibria, what criterion can we use to get a unique prediction of the game? CS use *ex-ante* Pareto ranking to select the most informative equilibrium. We might also use the same argument to select one equilibrium if there is such a Pareto ranking among equilibria in this model. However, there is no Pareto ranking in our model.

Let $p(\theta)$ be an equilibrium price offer made by the agent type θ. Define the *ex-ante* expected payoff of the principal as

$$EU^P = \int_0^1 \{-(y^P(p(\theta)) - \theta)^2 d^P(p(\theta)) + (1 - d^P(p(\theta)))(p(\theta) - b^2)\} f(\theta) d\theta, \tag{1.6}$$

and the *ex-ante* expected payoff of the agent as

$$\begin{aligned} EU^A &= \int_0^1 \{-(y^P(p(\theta)) - \theta - b)^2 d^P(p(\theta)) - (1 - d^P(p(\theta)))p(\theta)\} f(\theta) d\theta \\ &= \int_0^1 \{-(y^P(p(\theta)) - \theta)^2 d^P(p(\theta)) - (1 - d^P(p(\theta)))p(\theta)\} f(\theta) d\theta - b^2 \int_0^1 d(p(\theta)) f(\theta) d\theta \\ &= EU^P - 2\int_0^1 (1 - d^P(p(\theta)))p(\theta) f(\theta) d\theta + b^2 \int_0^1 (1 - 2d^P(p(\theta))) f(\theta) d\theta. \end{aligned} \tag{1.7}$$

While equilibrium actions taken by the principal and the bias between parties are the only determinant of parties' expected payoffs in CS, price offers and corresponding rejection probabilities play an additional important role to determine parties' payoffs in (1.7) so that we cannot have any Pareto ranking between equilibria in this model.

By $(ID-P)$, the following holds in any informative equilibrium:

$$\int_0^1 -(y^P(p(\theta)) - \theta)^2 d^P(p(\theta)) f(\theta) d\theta = \int_0^1 (p(\theta) - b^2) d^P(p(\theta)) f(\theta) d\theta. \tag{1.8}$$

After substituting this into (1.6) and (1.7), we have

$$EU^P = \int_0^1 p(\theta) f(\theta) d\theta - b^2 \ and \quad EU^A = -\int_0^1 \{(1 - 2d^P(p(\theta)))p(\theta) + 2d^P(p(\theta))b^2\} f(\theta) d\theta. \tag{1.9}$$

Notice that by $(ID-P)$, $p(\theta) \in (b^2 - \sigma, b^2)$ for all $\theta \in [0,1]$. This gives the following result.

Proposition 8. *Suppose that utility functions satisfy (1.5). In any informative equilibria,* $EU^P \in (-\sigma, 0)$ *and* $EU^A \in (-b^2, \sigma - b^2)$.

Proof. See the appendix. □

This result tells us that although there is no general *ex-ante* Pareto ranking among equilibria, both uninformative and informative equilibria should be in between two extreme pure-strategy equilibria- $E1$ (the perfect Bayesian equilibrium with $p^* = b^2 - \sigma$) and $E2$ (the perfect Bayesian equilibrium with $p^* = b^2$) in terms of ex-ante payoffs. This means that $E1(E2)$ is the best(worst) equilibrium for the agent and at the same time the worst(best) equilibrium for the principal, in terms of the *ex-ante* payoffs. We use this result to get a clear welfare comparison between our model and other models studied in the literature on Crawford and Sobel [27] and Holmström [50][51].

1.6.2 Benefit from Trading Decision-making Authority

In this section, we demonstrate the benefit from trade of decision-making authority by comparing the equilibrium outcomes of our model to those of several dispute resolution processes studied in the same framework: communication (Crawford and Sobel [27]), optimal mediation (Goltsman, Hörner, Pavlov, and Squintani [38]), optimal delegation (Holmström [50][51], Melumad and Shibano [75], Alonso and Matouschek [3] and Kováč and Mylovanov [56]) and optimal compensation contract (Krishna and Morgan [60]). To get more clear comparative results we focus on the uniform distribution in this section. It will be shown that there exist perfect Bayesian equilibria of this model Pareto superior to the equilibrium outcomes of all of these schemes especially when the parties' preferences are misaligned to a substantially large degree.

CS consider a situation in which the principal has no commitment power at all and sends cheap-talk messages to the agent. It is shown that all equilibria in their model are interval partitional so that there is only a finite number of actions chosen in equilibrium, each associated with an interval of states. With uniform quadratic assumption, they show that the number of distinct equilibrium outcome, denoted by $N_{CS}(b)$, is

$$N_{CS}(b) = \Big\langle -\frac{1}{2} + \frac{1}{2}\sqrt{1+\frac{2}{b}}\Big\rangle \tag{1.10}$$

where $\langle z \rangle$ denotes the smallest integer greater than or equal to z. Moreover, there is a Pareto ranking among $N_{CS}(b)$ equilibria so that, for any $b > 0$, the number of elements of the partition associated with the *Pareto dominant* equilibrium, which we will call the *best* equilibrium, is $N_{CS}(b)$. The expected payoff of the principal in this *best* equilibrium is

$$EU^P_{CS}(b) = -\frac{1}{12N_{CS}(b)^2} - \frac{b^2(N_{CS}(b)^2-1)}{3} \tag{1.11}$$

while the *ex-ante* expected payoff for the informed agent is

$$EU^A_{CS}(b) = EU^P_{CS}(b) - b^2. \tag{1.12}$$

Recently, Goltsman, Hörner, Pavlov, and Squintani [38] allow the parties to use any communication protocol, including the ones that call for a neutral trustworthy mediator. According to the optimal mediation rule, the parties' expected payoffs are

$$EU^P_{mediation}(b) = -\frac{b(1-b)}{3} \quad \textit{and} \quad EU^A_{mediation}(b) = EU^P_{mediation}(b) - b^2. \tag{1.13}$$

Holmström [50][51], Melumad and Shibano [75], Alonso and Matouschek [3] and Kováč and Mylovanov [56] study the principal's optimal choice of the set of admissible actions that the agent can take and show that under the optimal delegation scheme, the principal restricts project choices of the agent to be from 0 up to a maximum of $1-b$. Under this scheme, the parties' expected payoffs are

$$EU^P_{delegation}(b) = -\frac{b^2(3-4b)}{3} \quad \textit{and} \quad EU^A_{delegation}(b) = -\frac{8b^3}{3}. \tag{1.14}$$

In these papers, the principal also has imperfect commitment power so that she can only commit on the *ex-ante* allocation of decision rights. Moreover, the monetary transfer is impossible.

Krishna and Morgan [60] consider the situation in which the principal can commit to pay the agent for his advice but retains decision-making authority. They fully characterize the optimal compensation contract: the optimal compensation contract involves separation

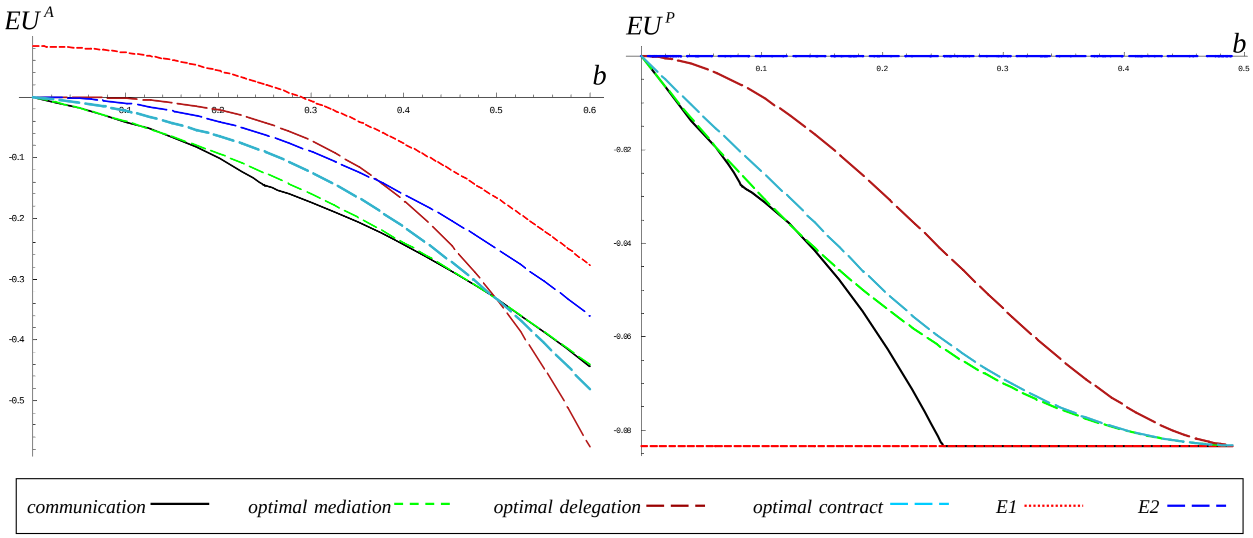

Figure 8: (a) Agent's expected payoff (b) Principal's expected payoff

in low states and a finite number of pooling intervals in high state and the principal never pays for imprecise information. In this optimal compensation contract, the expected payoffs for the principal and the agent are

$$EU^P_{contract}(b) = -\int_0^{a_0} (2b(a_0-\theta)+t_0)d\theta - \frac{1}{12}\sum_{i=1}^{K}\left(\frac{1}{K}-\frac{a_0}{K}-2b(K-2i+1)\right)^3 \tag{1.15}$$

and

$$EU^A_{contract}(b) = EU^P_{contract}(b) - b^2 + 2\int_0^{a_0}(2b(a_0-\theta)+t_0)d\theta \tag{1.16}$$

where

$$K = \left\langle -\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{3}{2b}}\right\rangle,$$

$$a_0 = \frac{3}{4}-\frac{1}{4}\sqrt{4+\frac{1}{3}(3-8bK(K-1))(8bK(K+1)-3)} \quad and$$

$$t_0 = \frac{(1-a_0-2K(K-1)b)(2bK(K+1)-(1-a_0))}{4K^2}.$$

In our model, the principal has stronger commitment power so that she can commit on the allocation of decision rights that corresponds to the agent's monetary transfers. In equilibrium, the agent takes an action $\theta+b$ after paying the price $p^* \in [b^2-\frac{1}{12}, b^2]$ of authority to the principal.[18] As a result, the equilibrium payoffs are

$$EU^A(p^*, b) = \int_0^1 -((\theta+b)-\theta-b)^2 f(\theta)d\theta - p^* = -p^* \tag{1.17}$$

$$EU^P(p^*, b) = \int_0^1 -((\theta+b)-\theta)^2 f(\theta)d\theta + p^* = p^* - b^2 \tag{1.18}$$

for the agent and the principal, respectively.

The (*ex-ante*) payoff comparison between equilibrium outcomes of these schemes and equilibria of our model is shown in Figure 8. $E1$ and $E2$ represent the perfect Bayesian equilibria with $p^* = b^2 - \frac{1}{12}$ and $p^* = b^2$, respectively. For any $b > 0$ and any $p^* \in [b^2 - \frac{1}{12}, b^2]$, the informed agent's expected payoff in our model is strictly greater then the equilibrium payoff of CS and optimal mediation. Although optimal delegation yields higher expected payoff than some equilibrium of this model for the agent for small b, there always exists a continuum of equilibria that gives strictly higher payoff to the agent than all other schemes compared. Furthermore, for any $b > 0$, the set of equilibria that give the principal strictly higher payoff than the equilibrium of any other schemes is nonempty. It is interesting to note that this set becomes larger as b increases. When $b > \frac{1}{2}$, all equilibria in our model are Pareto superior to all other schemes considered. This result is summarized in the following proposition. Detailed proofs are omitted since Figure 8 demonstrates a clear welfare comparison.

Proposition 9. *Suppose that F is uniform and utilities are quadratic. For any $b > 0$, there exists a perfect Bayesian equilibrium in this model which is ex-ante Pareto superior to equilibria of several other dispute resolution schemes such as communication, optimal mediation, optimal delegation and optimal contract. Moreover, for sufficiently large b, all equilibria in this model are ex-ante Pareto superior to equilibria of these schemes.*

This welfare result does not imply that bargaining mechanism is superior to all other schemes considered in the literature. The higher ex-ante utilities comes from different assumptions on the principal's commitment power. Unlike the most papers in the literature,

[18] Note that $\frac{1}{12}$ is a variance of the random variable distributed uniformly over $[0, 1]$.

this model assumes not only the principal can commit on the ex-ante allocation of decision right but also monetary transfer is available. The welfare comparison shows the benefit of using monetary transfer to trade decision rights in our environment.

1.7 REFINEMENT

From the previous sections, we know that there is a continuum of pure strategy equilibria, each of which has a different price offer on the equilibrium path and (at least) infinitely many mixed strategy equilibria. Therefore, it seems necessary to reduce the set of equilibrium outcomes to understand the model completely. In this section, we apply a stronger equilibrium concept called *perfect sequential equilibrium* developed by Grossman and Perry [43] to refine equilibria. We will first show that there is a unique pure-strategy perfect sequential equilibrium for any $b > 0$ in our model. Next, we will extensively apply the refinement to the mixed strategy equilibria and show that there is no mixed strategy perfect sequential equilibrium when $b < l^{-1}(\sigma)$.

Where does the multiplicity of equilibria come from? In any equilibrium of this game, there exists a continuum of out-of-equilibrium path price offers. If the price offer is one to which the equilibrium assigns positive probability, a posterior distribution of an agent's type can be computed using Bayes' rule. However, Bayes' rule does not determine the posterior distribution over type after observing the price offer to which the equilibrium assigns probability 0. Notice that the principal's choice of action after rejection of out-of-equilibrium price offer depends solely on the principal's posterior belief that we are completely free to choose. Therefore, without additional restriction on beliefs off the equilibrium path, any action can be induced as a best response to some beliefs off the equilibrium path and this leads to multiplicity of perfect Bayesian equilibrium.

To see this more precisely, compare the following extreme perfect Bayesian equilibria from proposition 1.

E1: the perfect Bayesian equilibrium with $p^* = l(b) - \sigma$,

E2: the perfect Bayesian equilibrium with $p^* = l(b)$

Notice that the price offer in *E1* is the least costly and most attractive to the agent. Thus, it seems reasonable that the agent makes this price offer to minimize the loss from the payment. This is also true in any equilibrium including *E2*. However, this could not happen in *E2* (and also any other equilibria except *E1*) because the price offer $l(b) - \sigma$ would be rejected in the equilibrium. Interestingly, the reason why the principal rejects the offer is that she certainly believes the state of the world is $\theta = 0$ if she accidently observes the offer. This belief gives the principal higher expected payoff from the rejection than from the acceptance, even though the price offer is the least costly. However, there is no reason why the principal has such an extreme belief off the equilibrium path. Thus, *E2*, as well as any other equilibria except *E1*, might not be supported by some different belief.

How could we eliminate some perfect Bayesian equilibria which contain unreasonable behaviors? In other words, how can we find a reasonable restriction on the belief off the equilibrium path? We adopt a stronger refinement called *perfect sequential equilibria* introduced by Grossman and Perry [43]. This refinement is closely related to the concept of *neologism-proof equilibria* developed by Farrell [31].[19] The refinement involves a *consistent interpretation* of a deviation from a perfect Bayesian equilibrium.[20] Formally, an interpretation of a deviation is a hypothesized (nonempty) subset of the type space by the principal who observes the deviation that members of the specified subset are responsible for the deviation. For a given interpretation, the principal's posterior belief is her prior belief renormalized over the interpretation. Then, sequential rationality determines whether the principal accepts or rejects the deviation. Each agent type θ in $[0, 1]$ can compute its payoff from offering the deviation and can compare this with its payoff in the perfect Bayesian equilibrium. A *consistent interpretation* is a fixed point of the map described above.

Definition 3 (**Gertner, Gibbons and Scharfstein**). *An interpretation of a deviation is consistent if the set of agent types who strictly prefer its payoff from offering the deviation to its equilibrium payoff is equivalent to the interpretation.*

[19] However, *neologism-proof* differs from *perfect sequential equilibria* because it imposes stronger requirement to an equilibrium. Precisely, an equilibrium is *neologism-proof* only if it is supported by all, rather than one, credible updating rules for interpreting deviations off the equilibrium path.

[20] A *consistent interpretation* of a deviation is used first by Gertner *et al* [37]. It is equivalent to the *credibility of neologism* in Farrell [31] and *consistent belief* derived by credible updating rule in Grossman and Perry [43].

It seems reasonable to take the associated posterior belief for the principal if a deviation has a consistent interpretation. By construction, this belief destroys the equilibrium by motivating agent types in the interpretation to deviate. This argument gives us the following additional restriction on the perfect Bayesian equilibrium.

(**BA5**) There is no deviation with a consistent interpretation.

An equilibrium that satisfies condition (**BA1**)∼(**BA5**) is called as a *perfect sequential equilibrium.*

Let us go back to the previous comparison to see if there is a perfect sequential equilibrium. Does the deviation $\widehat{p} = p^* - \varepsilon > l(b) - \sigma$ (with arbitrarily small $\varepsilon > 0$) have a consistent interpretation in equilibrium $E2$? Observe that this deviation might be beneficial for all agent types in $[0, 1]$ if they believe that the deviation is acceptable. So, assume that the principal who observes the deviation interprets the deviation as an offer from all agent types in $[0, 1]$. The principal's posterior belief is the same as her prior so that her optimal action is μ once the offer is rejected. Then the principal expects to get $-\sigma$ from rejecting the offer. However, accepting the offer gives the principal $\widehat{p} - l(b) > -\sigma$ so that it is optimal for the principal to accept the offer. Under the deviation being accepted, all agent types become strictly better off by deviating to the offer. As the result, the deviation $\widehat{p}$ in equilibrium $E2$ has a consistent interpretation $[0, 1]$. Notice that this is true for any perfect Bayesian equilibria except $E1$. Therefore, any perfect Bayesian equilibrium with the price offer $p^* \in (l(b) - \sigma, l(b)]$ does not satisfy the condition (**BA5**). It remains to show that the set of perfect sequential equilibria is nonempty.[21] The following proposition shows that the perfect Bayesian equilibrium with the price offer $p^* = l(b) - \sigma$ demonstrated in proposition 1 is indeed a perfect sequential equilibrium.

Proposition 10. *There is a unique pure-strategy perfect sequential equilibrium in this model. In the perfect sequential equilibrium, all agent types make a common price offer $l(b) - \sigma$ and the principal accepts the offer.*

[21] It is possible that the set of perfect sequential equilibria is empty. See Grossman and Perry [43]. Neologism-proof equilibria are also suffering from the same problem. See Farrell [31].

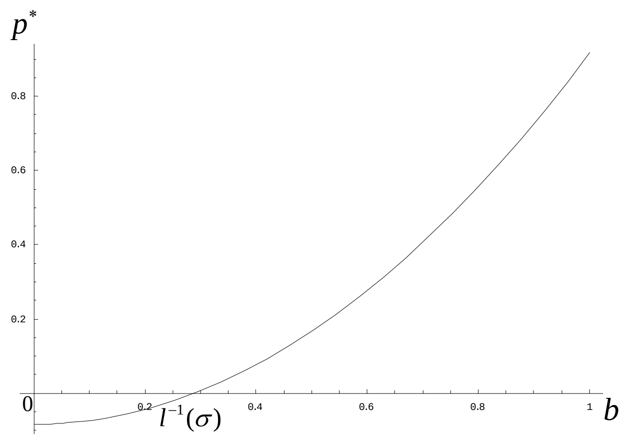

Figure 9: Equilibrium payment

Proof. See the appendix. □

Remark: The above result points to an interesting relationship between the equilibrium price offer and the parameter b, which measures the divergence of preferences. Notice that if $b > l^{-1}(\sigma)$, then the equilibrium price offer is positive. (See Figure 9.) This means that the agent pays some amount of money to the principal for the authority. In reality, such payment is often called a *license fee*. Intuitively, when b is large, using the agent's private information by means of selling authority becomes less valuable or sometimes even harmful for the principal because it allows the agent to choose her optimal action, which is distant from the principal's optimal one. Thus, the agent has to give the principal monetary compensation in order to get authority. This explains the positive price offer when b is large. When $b < l^{-1}(\sigma)$, however, the equilibrium price offer is negative. This means that the principal pays the agent, as we frequently observe in interfirm relationships. Examples include *consulting fees* in projects conducted through the outside consultation by other firms. Intuitively, small b increases the principal's relative value of information against the authority because the agent's private information allows her to choose an action similar to the principal's optimal action. Therefore, the principal is willing to pay some amount of money to the informed agent for giving her the authority to make a decision that fully reflects the agent's private information.

We can apply the same logic to refine outcomes of informative perfect Bayesian equilibria. Take an arbitrary informative equilibrium of the model and consider the deviation $\widehat{p} = l(b) - \sigma$. Notice that an agent type's interim payoff is a convex combination of the price he would pay and some negative utility driven by some action taken by the principal. Since the price offer made by the principal in any equilibrium should be less than $l(b) - \sigma$, any agent type in $[0, 1]$ prefers to make the price offer $\widehat{p}$ if $\widehat{p} > 0$. Together with the fact that $\widehat{p}$ is one of the acceptable common price offer of uninformative equilibria, this results in the following proposition.

Proposition 11. *There is no informative perfect sequential equilibrium if $b < l^{-1}(\sigma)$.*

Proof. See the appendix. □

1.8 STATE-DEPENDENT BIASES

In this section, we investigate the robustness of the existence of the equilibrium in which an *ex-post* efficient action is taken for any realization of the state under state-dependent biases: we find the necessary and sufficient condition for such equilibria to exist. The state-dependent biases are studied by several papers in the literature, for example, Alonso and Matouschek [3], Melumad and Shibano [75] and Gordon [39].

Let $b(\theta)$ be the state-dependent bias, a continuous function from type space to real number. Since the type space is compact and the function is continuous, $b(\theta)$ has its maximum and minimum. Recall that if there exists any *ex-post* efficient equilibrium then it should be pooling equilibria. This allows us to start with the following strategy of the agent: all agent types make a common price offer p^*. Since the equilibrium has to be efficient *ex-post*, the principal should accept the price offer with probability 1. This gives the following necessary condition:

$$-\sigma \leq p^* - \int_0^1 l(|b(\theta)|) \cdot f(\theta) d\theta = p^* - \overline{b}, \tag{1.19}$$

where $\overline{b} = \int_0^1 l(|b(\theta)|) \cdot f(\theta) d\theta$.

Consider the principal's strategy as conservative as possible that includes the worst action for agent types who deviate from the equilibrium behavior: the principal accepts any price offer $p \geq p^*$ and rejects any price offer $p < p^*$. After rejecting $p < p^*$, she takes an action y^* where

$$y^* = arg \min_{y \in [0,1]} \left(\max_{\theta \in [0,1]} U^A(y, \theta, b(\theta)) \right). \tag{1.20}$$

Notice that such a y^* exists since the set of state is compact and utilities are continuous. Under the principal's strategy specified above, the following is sufficient for any agent type in $[0, 1]$ not to have an incentive to deviate to any price offer $p < p^*$:

$$\min_{y \in [0,1]} \left(\max_{\theta \in [0,1]} U^A(y, \theta, b(\theta)) \right) \leq -p^*. \tag{1.21}$$

An *ex-post* efficient equilibrium exists if and only if there exists p^* satisfying (1.19) and (1.21).

Proposition 12. *There exists an equilibrium in which an ex-post efficient action is taken for any realization of the state if and only if*

$$\min_{y \in [0,1]} \left(\max_{\theta \in [0,1]} U^A(y, \theta, b(\theta)) \right) \leq \sigma - \bar{b}. \tag{1.22}$$

Observe that the left-hand side of equation (1.22) is at most 0. This means that if $\bar{b}$ is small enough then there always exists an equilibrium in which an *ex-post* efficient action is taken for any realization of the state.

1.9 CONCLUSION

This paper considers bargaining over the authority to make a decision between an uninformed principal and a privately informed but self-interested agent. In spite of private information, bargaining between parties results in *ex-post* efficient outcomes. This explains why we frequently observe delegation in both interfirm and intrafirm relationships.

Meaningful information can always be transmitted in equilibrium, however, if we allow mixing in the principal's decision rule. Although equilibria with informative price offers may not yield *ex-post* efficient outcomes, there exists an informative equilibrium that yields an outcome arbitrarily close to the *ex-post* efficient one.

Some extensions of this model might be interesting. We may investigate bargaining models with different procedures such as, multi-stage bargaining with alternative offers. Moreover, it seems natural that bargainers can communicate through cheap-talk in bargaining procedures. So, it might be worthwhile to consider bargaining either before or after communication. More general contracting offers by informed agents in the same environment is worth investigating. We leave them for future research.

2.0 COMMUNICATION IN BARGAINING OVER DECISION RIGHTS

2.1 INTRODUCTION

In many economic situations a party, such as a government or firm (principal), initially has full authority to make a decision but lacks information about the task or project at hand. There is often another, better-informed party (agent), but its interest may be so different from that of the principal that it may not be willing to share useful information with the uninformed principal.[1] This creates an incentive for the principal to delegate his or her decision-rights to the agent in order to make full use of the agent's private information. When the principal delegates decision-making authority to the agent, it may be beneficial for the principal to make some restriction on the set from which the agent can choose an action. The principal's optimal choice of the restricted set of actions is extensively studied in the literature of optimal delegation (Holmström [**?**, **?**], Goltsman, Hörner, Pavlov, and Squintani [38], Alonso and Matouschek [3], Kováč and Mylovanov [56] and Melumad and Shibano [75]).

It is remarkable that most of papers in the delegation literature focus on settings without monetary transfers.[2] Although there are many settings in which the use of monetary transfers is limited or ruled out, sometimes it is more natural to assume that nothing prevents parties from using financial incentives. In practice, the principal can and do use contracts or bargaining mechanisms that include financial incentives in order to transfer decision-rights.[3]

[1]See Crawford and Sobel [27]. For more general communication mechanisms, see Goltsman, Hörner, Pavlov, and Squintani [38].

[2]There are few exceptions in the literature on optimal delegation. See, for example, Krähmer [57]. Recently, Ambrus and Egorov [5] investigate the effect of money burning on the optimal delegation structure.

[3]In the framework of incomplete contracts (Grossman and Hart [41] and Hart and Moore [46]), Baker, Gibbons and Murphy [10] model the allocation of decision-rights via contracts. They assume that decisions

For example:

- Workers and managements typically negotiate the right and responsibility to choose how workers spend their time in workplace or how workers participate in the firm's managemental decision-making process. There is a strong empirical evidence that shows a positive relation between degree of delegation and wage levels, controlling for a variety of worker and firm characteristics (Caroli and Van Reenen [19], Black, Lynch and Krivelyova [17], Bauer and Brender [12]). Managements rarely have good information about the ease with which workers could increase their personal productivity. In some settings, workers may also have superior information about changes in workplace organization, job descriptions, or work flows that would increase firm productivity. Managements also may not have very good information about worker preferences, such as the trade-offs workers would be willing to make between such matters as safety, work rates, wages, job security, and the like (Bainbridge [9]).
- When launching into a new business partnership, auto manufacturers and their dealers negotiate the right to determine the size and qualification of the sales force, or the right to set prices. Auto manufacturers may not have very good information about consumer preferences so that they get some difficulties to determine price, advertising strategy and so on. Arruñada, Garicano, and Vasquez [7] empirically analyze the allocation of rights and monetary incentives in automobile franchise contracts. Similarly, when an international manufacturer enters a particular national market, it typically lacks relevant information about local market conditions and has difficulties making decisions on pricing, marketing, advertising, distribution and so on. As a result, it sells an exclusive distributorship to a domestic company who is better-informed but lacks authority to make such decisions. If a license agreement is reached through bargaining, the domestic company pays license fees in return for the exclusive right to make decisions about pricing, marketing, advertising, distribution, and so on in the domestic market.[4]

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are not contractible ex post, the parties cannot negotiate over the decision after the state is revealed. Instead, the party in control simply takes her self-interested decision.

[4]For example, I.B.M., the world's largest computer maker in the 1990's, agreed to allow Mitsubishi to sell an I.B.M. mainframe computer under its own name in Japan in April, 1991. See Andrew Pollack, "IBM Model to Be Sold By Mitsubishi," *The New York Times* (April 29, 1991), 17. More recently, tobacco industry leader Philip Morris International announced an agreement with Chinese National Tobacco under

- It is typical that venture capitalists face substantial uncertainty when financing new ventures. (Kaplan and Strömberg [54] and Dessein [29]). Due to this uncertainty, biotechnology companies often *sell* its patent or legal rights to manufacture the final product to pharmaceutical firms who are better-informed through a license and development agreements.[5]

All these examples share the following commonalities: (i) there is a principal who has to make a decision but lacks decision-relevant information or knowledge; (ii) there is another party who is better-informed or more-experienced but it has its own agenda; (iii) these two parties negotiate the allocation of decision-rights by using various monetary incentives schemes; (iv) the final decision made by the party in control determines both parties' welfare. In order to capture this situation, we follow the framework of Crawford and Sobel [27] and Holmström [**?**, **?**]: there are two parties, uninformed principal (P) and informed but self-interested agent (A), with one dimensional decision-making that affects welfare of both under one dimensional uncertainty.

The novel feature of this model is as follows: we investigate reallocation of decision-rights in settings with monetary transfers. In particular, we consider an uninformed principal's optimal choice of a price offer for decision-rights when an informed agent decides either to accept or to reject the offer. If the price offer is accepted then the agent pays the principal the price for decision-rights and makes a decision. Otherwise, the principal retains decision-rights without making any monetary transfers. We call this game *bargaining over decision-rights*.[6] Our main finding is as follows: it is optimal for the uninformed principal to use the price offer as a device for screening some agent types out. In equilibrium, the principal makes a price offer that is accepted by some agent types but not by all agent types. It means

which Chinese National Tobacco will manufacture Marlboro cigarettes for marketing in China. See Nicholas Zamiska and Juliet Ye, "Chinese Cigarettes to Go Global" *The Wall Street Journal*, (January 30, 2008) B4.

[5]For instance, *Animas Corporation*, an insulin infusion pump manufacturing company, set up a license and development agreements with the Swiss R&D company, *Debiotech*, for intellectual property related to next-generation insulin pumps and micro-needles. In return for the exclusive worldwide license to make, use, and sell products utilizing the intellectual property portfolio that includes over 70 issued patents, *Animas* paid $12 million in cash and issued 400,000 restricted shares of Animas common stock. See Rick Baron, "Animas acquires technology for disposable insulin micro-pumps and micro-neddles," http://www.bioalps.org/Bioalps/en/Internet/Documents/1996.pdf. Also see Lerner and Merges [64] and Higgins [47] for some empirical evidences.

[6]A companion model of bargaining is considered by Lim [65] by focusing on the situation where the informed agent has bargaining power so that he makes a take-it-or-leave-it price offer.

that the principal sometimes retains decision-rights while lacking precise information. As a result, actions taken by the principal without precise information may be inefficient *ex-post* for some realization of the state. That is, sometimes there exists an action that makes the principal better off without making the agent worse off or vice versa, once the true state of the world becomes public.

This result seems to be contrary to Coase [22] who asserts that if the market outcome is inefficient and there are no transaction costs, then the parties concerned will negotiate their way to efficiency. The main obstacle to the efficient bargaining seems to be bargaining costs due to incomplete or asymmetric information. Farrell [30] shows that in the presence of private or incomplete information, voluntary negotiation could not lead to the first-best outcome that maximizes joint surplus. The important issue is how to interpret "no transaction costs" in the presence of private information. In the basic model, we assume that bargaining is *tacit* in the sense that parties can communicate only by making a price offer that directly affects their payoffs. As pointed out by Crawford [26], real bargaining, by contrast, is usually *explicit*, in that parties can furthermore communicate by sending non-binding messages with no direct effects on their payoffs. Thus, it is natural to interpret the absence of transaction costs in bargaining under asymmetric information as the absence of communication costs: people freely get together and communicate with each other without any costs. Therefore, it is impetuous to conclude that Coase's assertion is unwarranted in our environment without investigating the impact of communication into the tacit bargaining carefully.

There are several theoretical evidences showing that such cheap talk messages play an important role in coordinating bargainers' expectations so that they can reach agreement and in determining how they share the resulting surplus (Farrell and Gibbons [34] and Matthews [72]). Farrell and Gibbons [34] study a two-stage bargaining game in which talk may be followed by the sealed-bid double auction studied by Chatterjee and Samuelson [20], a well-known model of bargaining under incomplete information. They show that talk can matter in the sense that the cheap talk equilibrium features bargaining outcomes that could not be an equilibrium behavior in the absence of talk. Matthews [72] considers a specific bargaining situation with a veto-threat and shows there exists an equilibrium in which an informed party (proposer) tells the other (chooser) which of two sets contains his type. This equilibrium behavior is not a part of equilibrium behavior in the absence of talk. These results intimate that communication may resolve inefficiency caused by the presence of incomplete or asymmetric information in our model.

How then does introducing talk into the tacit bargaining affect the behaviors of the parties? To answer this question, we devote the second half of our paper to bargaining over decision-making rights with explicit communication. Specifically, we assume the informed agent can send a cheap talk message before bargaining begins.[7] Once we allow parties to communicate via cheap talk before bargaining, there exists a truth-telling equilibrium. The existence of the truth-telling equilibrium is surprising because neither the tacit bargaining nor communication via cheap talk alone allows parties to make full use of the agent's private information to make a decision. In this equilibrium, the principal uses the following trigger type of strategy: "*Tell me the truth and prove your honesty by accepting my price offer. I will make a specific message-independent price offer that must be accepted by a truthful agent. Thus, I consider the rejection of my price offer as evidence for your dishonesty so that I will punish you by taking an action that makes you much worse than telling the truth.*" The *threat* action compels the agent to report the true state in the cheap talk stage and to accept the equilibrium-path price offer from the principal in the bargaining stage. It turns out that the threat action coincides with the unique agent type who rejects the price offer on the equilibrium path with positive probability. Consequently, taking the threat action becomes rational for the principal, and more importantly, to be credible to the agent. In this truth-telling equilibrium, induced actions always satisfy *ex-post* efficiency.

We apply two standard cheap-talk refinements, *neologism-proofness* (Farrell [31]) and NITS (Chen, Kartik and Sobel [21]), and show that the existence of the truth-telling equilibrium is robust against those refinements: no matter how large the difference between parties' preferences is, the equilibrium is neologism proof in Farrell [31]'s sense. Moreover, it is the unique neologism-proof equilibrium under some parameter value. Imposing NITS (no incentive to separate), the criterion proposed by Chen, Kartik and Sobel [21] to refine equilibria in cheap-talk games (Crawford and Sobel [27]) leads to the same result. That

[7] However, our main result, the existence of the truth-telling equilibrium, does not depend on the exact timing of the game.

is, not only the truth-telling equilibrium *always* satisfies NITS, but also it is the unique equilibrium satisfying NITS under some parameter value. We also consider the notion of sequential perfect equilibrium or extensive-form trembling-hand perfection and show that the truth-telling equilibrium is robust against those refinements too.

We also show that the truth-telling equilibrium outcome of the explicit bargaining is *ex-ante* Pareto superior to that of several other protocols studied in the literature, such as communication (Crawford and Sobel [27]), optimal mediation (Goltsman, Hörner, Pavlov, and Squintani [38]), optimal delegation (Holmström [50][51], Alonso and Matouschek [3], Kováč Mylovanov [56] and Melumad and Shibano [75]) and optimal compensation contract (Krishna and Morgan [60]) if parties' interests are substantially misaligned. This might explain why bargaining over decision-rights often takes place between two separately owned companies whose interests diverge widely.

We extend the original model with unidimensional state space to multidimensional state space case. Interestingly, the main result, the existence and characterization of fully revealing equilibria, still holds with multidimensional state space if the space is compact. The result shows that compactness, as opposed to the dimensionality, of state space plays an important role for fully revealing equilibria to exist. Contrary to the main finding of the multidimensional cheap talk literature, compactness of state space is a sufficient condition for the existence of fully revealing equilibria.[8]

The rest of the paper is organized as follows. The next section describes the environment. In section 3, we setup the basic model of bargaining over decision-making authority and show that there is no equilibrium in which an *ex-post* efficient action is taken for any realization of the state. Full characterization of equilibria is provided assuming the parties' prior beliefs are *uniform*. In section 4, we extend the basic model and allow parties to communicate before bargaining by sending cheap talk messages. We show that there exists a truth-telling perfect Bayesian equilibrium in which actions induced are efficient *ex-post*. The robustness of the truth-telling equilibrium is also discussed. We devote section 5 to welfare comparisons. We discuss two extensions of the basic model in section 6. First, we consider the model

[8]For example, unboundedness of state space is a sufficient condition for fully revealing equilibria to exist in Ambrus and Takahashi [6].

with multidimensional state space and investigate the existence of truth-telling equilibria. Second, we follow a mechanism design approach and show that under certain conditions the explicit bargaining is an optimal bargaining mechanism that maximizes a joint surplus of parties. We conclude in section 7.

2.2 BASIC MODEL

2.2.1 Environment

There are two parties, a principal (P) and an agent (A). The principal who initially has decision-making authority has little information about the state of the world $\theta \in \Theta \equiv [0,1]$. She has a prior distribution F over $[0,1]$ with an absolutely continuous density function $f > 0$. The agent who has different interests from the principal knows the true state of the world θ but does not have decision-making authority. The payoffs for a given allocation of authority depend on an action y taken by the party who has decision-making authority and the state of the world θ. The payoff functions of the parties are of the form $U^P(y,\theta) = -l(|y-\theta|)$ for the principal and $U^A(y,\theta,b) = -l(|y-(\theta+b)|)$ for the agent.[9] We refer to l as the loss function and assume that $l''(\cdot) > 0$, $l'(0) = 0$ and $l(0) = 0$. This means that the ideal action of the principal is $\overline{y}^P(\theta) = \theta$ and the ideal action of the agent is $\overline{y}^A(\theta,b) = \theta + b$ where $b > 0$ is a parameter that measures how nearly the agent's interest coincides with that of the principal. All of these are common knowledge between parties.

2.2.2 *Ex-post* Efficient Actions

When utility functions are quasi-linear there exists unique *ex-post* efficient action in between two ideal actions, which maximizes a joint surplus of parties. For example, if both U^A and U^P are quadratic, the ex-post efficient action is the mid-point of θ and $\theta+b$. It is well known that property rights and voluntary private negotiation are not able to achieve this first-best

[9] A special case is a quadratic utility ($U^P(y,\theta) = -(y-\theta)^2$ and $U^A(y,\theta,b) = -(y-\theta-b)^2$) which we are assumed in most examples and applications. Similar utility functions are assumed in many other paper, for example, Dessein [28]. To see more papers assuming quadratic utilities, see Kováč and Mylovanov [56].

efficient outcomes in the presence of important private information.[10] In this section, we shall argue for some kind of second-best comparison as against comparing things with first-best efficiency.

Define a (second-best) *ex-post* efficient action as follows. An action is said to be (second-best) *efficient ex-post* if and only if there is no other feasible action that makes some individual better off without making other individuals worse off after the true state of the world θ is publicly known.

Definition 4. *An action $y \in \mathbb{R}$ is efficient ex-post at θ if there is no other action $z \in \mathbb{R}$ such that*

$$U^P(z,\theta) \geq U^P(y,\theta) \quad \textit{and} \quad U^A(z,\theta,b) \geq U^A(y,\theta,b) \tag{2.1}$$

with at least one strict inequality.

In our environment, an action y is (second-best) efficient *ex-post* if and only if $y \in [\theta, \theta+b]$ when the realization of the state is θ, as one can see in Figure 10. Notice that most mechanisms considered in the literature on strategic information transmission (Crawford and Sobel [27]) and optimal delegation (Holmstrom [50][51]) lead to efficient actions for some states of the world but not all. The following example demonstrates *ex-post* inefficiency of actions in cheap talk and optimal delegation.

Example 1. Suppose that F is uniform and utilities are quadratic. Let $b = 1/5$ and the realized state of the world $\theta = 7/8$. As you can see in Figure 10, if an action y is not in $[\theta, \theta + b] = [7/8, 43/40]$, then there exists another action y' such that both parties strictly prefer y' to y. Suppose that parties communicate via cheap talk. In the most informative equilibrium, only two actions, $y_1 = \frac{1}{20}$ and $y_2 = \frac{11}{20}$, are induced.[11] Since $y_1 < \theta$ and $y_2 < \theta$, both actions are inefficient *ex-post*. Alternatively, suppose that the principal optimally proposes the set of admissible actions that the agent can take. In the optimal delegation, the proposed set is $[0, 1-b] = [0, 4/5]$.[12] As the result, the agent cannot take any action $y \in [7/8, 43/40]$.

[10] See, for example, Myerson and Satterthwaite [75].

[11] See the leading example of Crawford and Sobel [27].

[12] See Holmström [50][51], Melumad and Shibano [75], Alonso and Matouschek [3] and Kováč and Mylovanov [56].

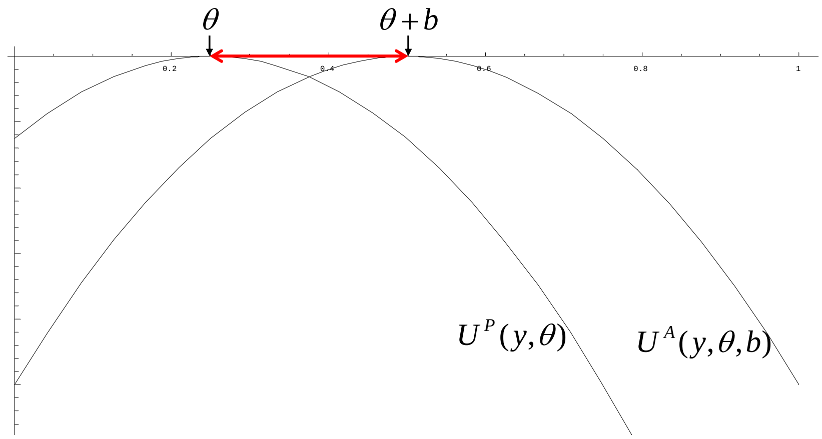


Figure 10: Ex-post Efficient Actions

2.3 BENCHMARK: TACIT BARGAINING

Consider bargaining over decision-making authority between the informed agent and the uninformed principal. The timing of the game is as follows:

1. The agent privately observes the state of the world $\theta \in \Theta \equiv [0, 1]$.

2. The principal makes an offer $p \in \mathbb{R}$ for the authority to take an action.[13]

3. The agent decides whether to reject or accept the offer.

4. If the agent accepts the offer then he pays the price to the principal and takes an action, denoted by y^A. In this case, payoffs become $U^P(y^A, \theta) + p$ and $U^A(y^A, \theta, b) - p$ for the principal and the agent respectively. If the agent rejects the offer, however, the principal takes an action, denoted by y^P, without transferring the decision-making authority. Then payoffs are $U^P(y^P, \theta)$ and $U^A(y^P, \theta, b)$ for the principal and the agent, respectively.

The equilibrium concept we use is perfect Bayesian equilibrium. For the principal, a strategy consists of a price offer p^* and an action rule y^P. The action rule, denoted by $y^P : \mathbb{R} \to \mathbb{R}$ specifies the principal's action after the rejection of each price offer $p \in \mathbb{R}$ that the he might make. Since the utility function is strictly concave in y, the principal will never use mixed strategies in equilibrium. For the agent, a strategy consists of a decision

[13] We allow p to be negative, which means that the principal pays $|p|$ to the agent.

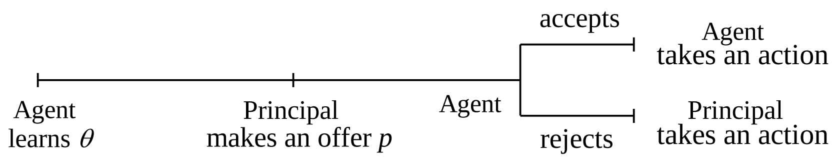


Figure 11: Timing of the game

rule and an action rule. The decision rule, denoted by $d^A : \Theta \times \mathbb{R} \to [0, 1]$, specifies the probability of rejection for each price offer $p \in \mathbb{R}$ that the agent might receive. The action rule $y^A : \Theta \times \mathbb{R} \to \mathbb{R}$, specifies the agent's choice of action after he accepts the principal's price offer p. The strategy profile $\{(p^*, y^P), (d^A, y^A)\}$ forms a perfect Bayesian equilibrium if:

(**B1**) p^* solves

$$\max_{p\in\mathbb{R}} \int_0^1 \{d^A(\theta, p)U^P(y^P(p), \theta) + (1 - d^A(\theta, p))(p - l(b))\}f(\theta)d\theta$$

(**B2**) for each $p \in \mathbb{R}$ and each $\theta \in [0, 1]$, $d^A(\theta, p)$ solves

$$\max_{d^A\in[0,1]} (1 - d^A)(-p) + d^A \cdot U^A(y^P(p), \theta, b)$$

(**B3**) for each $\theta \in [0, 1]$ and $p \in \mathbb{R}$, $y^A(\theta, p) = \overline{y}^A(\theta) = \theta + b$

(**B4**) for each $p \in \mathbb{R}$, $y^P(p)$ solves

$$\max_{y\in\mathbb{R}} \int_0^1 U^P(y, \theta)\rho(\theta|p)d\theta$$

where $\rho(\theta|p)$ is the principal's updated belief after observing the agent's rejection of p, which is given by Bayes' rule whenever possible.

2.3.1 Equilibrium

2.3.1.1 Example: uniform distribution In this section, we illustrate the main idea behind our analysis while assuming that f is *uniform* over $[0, 1]$. This setting, together with quadratic utilities, is a leading example of Crawford and Sobel [27] and has been widely used in the literature on strategic information transmission and optimal delegation. We will extend our result to more general distributions in the next subsection. In what follows we first focus on the agent's decision whether to accept a given price offer or not. We will show that the agent's decision rule satisfies an interesting property called *monotonicity*. Next, with the full characterization of the agent's decision rule we show there exists a unique price offer that maximizes the principal's expected utility.

For an arbitrary $p \in \mathbb{R}$, define the set of agent types who accept p with probability one as

$$\Theta(p) = \{\theta \in [0, 1] | d(\theta, p) = 0\}.$$

Define the set of agent types who reject the offer p with probability one as

$$\Theta^{-1}(p) = \{\theta \in [0, 1] | d(\theta, p) = 1\}.$$

Lemma 4 (Monotonicity)**.** *For any price offer, if there is an agent type θ who accepts the offer with positive probability then all agent types higher than θ have to accept it with probability one.*

Proof. See the appendix. □

To see the intuition of Lemma 4, consider the decision problem of agent type $\theta \in [0, 1]$ who observes a price offer p. Define the agent type θ's willingness to pay as follows:

$$W(\theta, p, y^P(p)) = U^A(\overline{y}^A(\theta), \theta, b) - U^A(y^P(p), \theta, b) = l(|y^P(p) - (\theta + b)|), \qquad (2.2)$$

where $y^P(p)$ is the action taken by the principal after the price offer p is rejected and $\overline{y}^A(\theta) = \theta + b$ is the agent type θ's optimal action. The agent type θ accepts the offer only if the gain of getting decision-making authority (or willingness to pay for authority) is at least as big as the loss of it, that is,

$$W(\theta, p, y^P(p)) \geq p.$$

It is not possible that $y^P(p)$ is on the right of θ, because otherwise, the quasi-concavity of U^A implies that the set of agent types who reject the offer should be on the right of θ and the mid-point of the set should be $y^P(p) - b$ but not $y^P(p)$, which is contradicted by **(B4)**. It means that $y^P(p)$ is on the left of θ, and as a result, the agent type $\theta' > \theta$ whose most preferred action is higher than that of the agent type θ is willing to pay more to get authority to make a decision, that is,

$$\frac{\partial W(\theta, p, y^P(p))}{\partial \theta} \geq 0.$$

This means that agent type θ accepts any price offer which is accepted by agent type $\theta' < \theta$.

Lemma 4 implies that for any $p \in \mathbb{R}$, both $\Theta(p)$ and $\Theta^{-1}(p)$ are convex if they are non-empty. Further, $\Theta(p)$ cannot be to the left of $\Theta^{-1}(p)$. These guarantee that for any $p \in \mathbb{R}$ there is at most one agent type who is indifferent between accepting and rejecting the offer. Let $\theta_p \in [0, 1]$ denote the agent type if it exists. Then we can write that $\Theta(p) = (\theta_p, 1]$ and $\Theta^{-1}(p) = [0, \theta_p)$. From the indifference condition at θ_p we have

$$p = l(|y^P(p) - \theta_p - b|), \tag{2.3}$$

where $y^P(p) = arg\max_y \int_0^{\theta_p} -l(|y - \theta - b|) \cdot \frac{1}{\theta_p} d\theta = \frac{\theta_p}{2}$. Thus, we have

$$p = l(\frac{\theta_p}{2} + b) \quad or \quad \theta_p = 2(l^{-1}(p) - b). \tag{2.4}$$

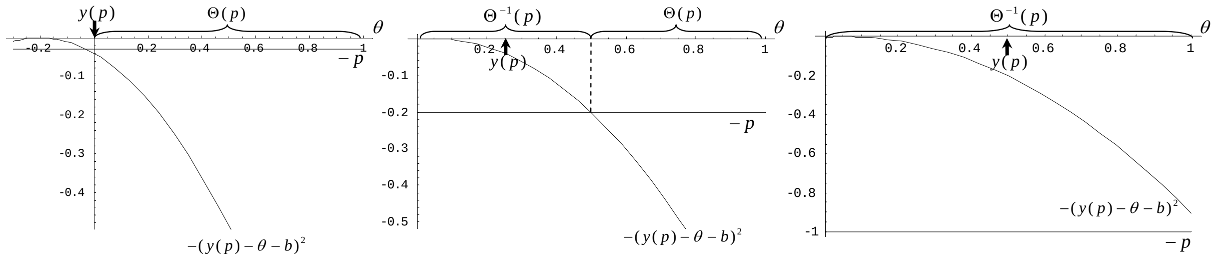
Since $\theta_p \in [0, 1]$, we have the following corollary.

Corollary 1.

$$\Theta(p) = \begin{cases} [0, 1] & \text{if } p < l(b), \\ (2(l^{-1}(p) - b), 1] & \text{if } l(b) \leq p \leq l(b + \frac{1}{2}), \\ \emptyset & \text{if } p > l(b + \frac{1}{2}) \end{cases}$$

In words, all agent types in $[0, 1]$ accept a low price offer ($p < l(b)$) with probability one, and once the price offer becomes greater than $l(b)$ then low agent types start rejecting it. As p increases, the set $\Theta(p)$ becomes smaller and finally all agent types in $[0, 1]$ reject a high price offer ($p > l(b + \frac{1}{2})$) with probability one.

In Figure 12, we see the clear trade-off between higher price and more rejections that the principal faces. Although a higher price offer gives a higher payoff to the principal if



(*a*) low *p* (*b*) moderate *p* (*c*) high *p*

Figure 12: The agent's decision rule

accepted, it does not seem to be optimal for the principal to make a very high offer because it cannot be accepted (Figure 12(a)). Similarly, making a price offer that could be accepted by all agent types does not seem to be optimal either because it is very low (Figure 12(c)). These suggest that the principal's optimal price offer should lie halfway between two extremes (Figure 12(b)). To confirm this idea, consider the principal's optimal price offer as a best response to the agent's strategy. The principal chooses p^* to solve

$$\max_{p\in\mathbb{R}} EU^P = \int_0^{\theta_p} -l(|y^P(p)-\theta|)d\theta + (1-\theta_p)(p-l(b)) \quad (2.5)$$
$$\text{s.t. } y^P(p) = \frac{\theta_p}{2} \text{ and } \theta_p = 2(l^{-1}(p)-b).$$

Notice that there exists a unique interior solution of this maximization problem because of the strict concavity of EU^P in p. From the first order condition, we get

$$p^* = l(\frac{1}{4}+b) \quad \textit{and} \quad \theta_{p^*} = \frac{1}{2}. \quad (2.6)$$

This implies that it is optimal for the principal to make the price offer that is acceptable for some agents of high type but not for the remaining agents of low type. This result is summarized in the following proposition.

Proposition 13. *In equilibrium, the principal makes a price offer* $p^* = l(\frac{1}{4}+b)$*. The agent type* $\theta \in [0, \frac{1}{2})$ *rejects the offer with probability one and* $\theta \in (\frac{1}{2}, 1]$ *accepts the offer with probability one. As a result, ex-post efficiency does not hold.*

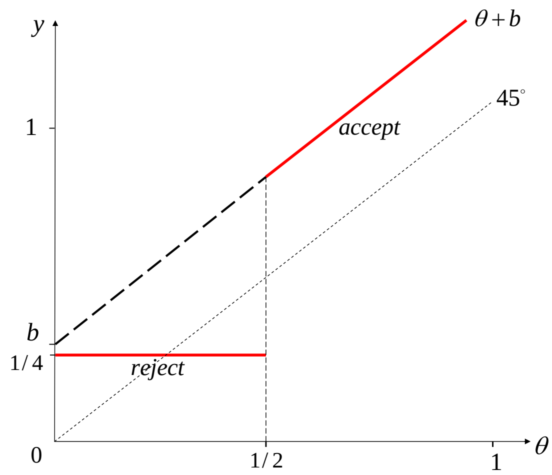


Figure 13: Equilibrium Outcome

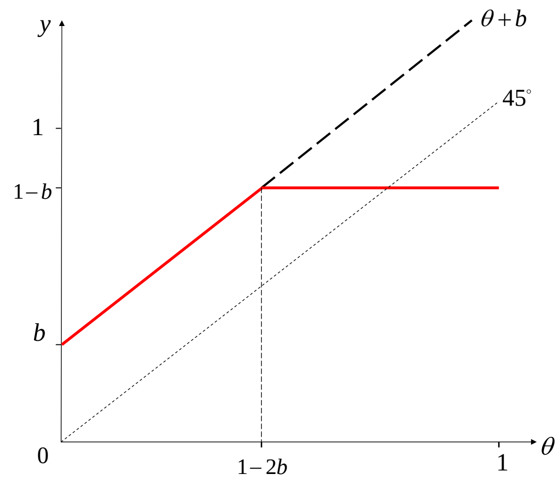


Figure 14: Optimal Delegation

It is interesting to compare this result with the outcome of optimal delegation studied by Holmström [50][51], Melumad and Shibano [75], Alonso and Matouschek [3], Goltsman *et al.* [38] and Kováč and Mylovanov [56]. They studied the uninformed principal's optimal choice of the set of admissible actions that the informed agent can take. According to the optimal delegation rule in the model with a uniform prior and quadratic utility functions, the informed agent can enforce any decision he likes, as long as it does not exceed $1-b$ (see Figure 14). The intuition is as follows: since the informed party's most preferred action is always higher than that of the principal, it pays to impose an upper bound on the allowable actions. In case of the low state, on the other hand, the best way to make use of the informed agent's information is to grant complete freedom of choice of the action to the informed agent.

Although the outcome of the equilibrium in this model in which the agent obtains complete freedom of choice of the action only in case of the high state looks exactly opposite of that of optimal delegation, the underlying intuition is exactly the same as that of optimal delegation. Recall that the type θ agent's willingness to pay is strictly increasing in the distance between $y^P(p)$ and $(\theta + b)$ where $y^P(p)$ is determined by Bayes' rule. This means that the principal can maximize the willingness to pay by choosing $y^P(p)$ and the agent's most preferred action $(\theta + b)$ as distant as possible. Since $b > 0$, the principal can maximize this distance by making the price offer that the agent rejects in case of a low state and accepts in case of high state so that $y^P(p)$ is low and $(\theta + b)$ is high. Clearly, we should get exactly the opposite outcome in which the informed agent gets a complete freedom to choose the action in case of low state if the agent's most preferred action is always lower than that of the principal.

2.3.1.2 General Case

In this section, we extend the analysis in the previous section to more general distributions. Recall that in the previous section the monotonicity of the agent's decision rule allows us to have a unique optimal price offer for the principal. The following regularity condition on the parties' prior belief f is necessary for us to have the same monotonicity of the agent's decision rule and as a result, ensures that all results we got in the previous section are preserved.

Condition 1. *For a given value of* $b > 0$*,*

$$y(\underline{\theta}, \overline{\theta}) - b < \frac{\underline{\theta} + \overline{\theta}}{2} \tag{2.7}$$

for any $\underline{\theta}$ *and* $\overline{\theta}$ *with* $0 \leq \underline{\theta} \leq \overline{\theta} \leq 1$*, where*

$$y(\underline{\theta}, \overline{\theta}) = \begin{cases} argmax \int_{\underline{\theta}}^{\overline{\theta}} U^P(y,\theta) f(\theta) d\theta & \text{if } \underline{\theta} < \overline{\theta}, \\ \overline{\theta} & \text{if } \underline{\theta} = \overline{\theta}. \end{cases}$$

In words, this condition implies that for any interval subset of Θ, an action that maximizes the expected payoff for a principal who believes that agent type is in the interval is not lopsided too much toward the right of the interval. Any prior f satisfies this regularity condition if $b \geq 1/2$. Moreover, this condition holds for any $b > 0$ if f is non-increasing in θ. In particular, it is satisfied in the setting with *uniform* distribution considered in the previous subsection.

Under this regularity condition, the agent types' decision rule satisfies the *monotonicity*.[14] As we already saw in the example with uniform distribution, the monotonicity makes it optimal for the principal to use her price offer as a screening device. This result is summarized in the following proposition.

[14] In the proof of Lemma 4, we first show that under condition 1 the agent's decision rule satisfies the monotonicity. The proof is completed by pointing the fact out that the condition 1 holds with the uniform distribution.

Proposition 14. *Under Condition 1, there exists a unique perfect Bayesian equilibrium. In the equilibrium, the principal makes a price offer accepted by positive measure of agent types but not all.*

Proof. See the appendix. □

2.4 EXPLICIT BARGAINING

In this section, we explore how introducing explicit communication into the basic model affects its outcomes. The timing of the game is as follows:

1. The agent privately observes the state of the world $\theta \in \Theta \equiv [0, 1]$.
2. The agent sends a message $m \in M$ to the principal.
3. After observing the message from the agent, the principal makes a price offer $p \in \mathbb{R}$ for authority to take an action.
4. The agent decides whether to accept or reject the offer.
5. If the agent accepts the offer then he pays the price offered by the principal and takes an action, denoted by y^A. In this case, payoffs become $U^P(y^A, \theta) + p$ and $U^A(y^A, \theta, b) - p$ for the principal and the agent respectively. If the agent rejects the offer, however, the principal takes an action, denoted by y^P, without transferring the decision-making authority. Then payoffs are $U^P(y^P, \theta)$ and $U^A(y^P, \theta, b)$ for the principal and the agent, respectively.

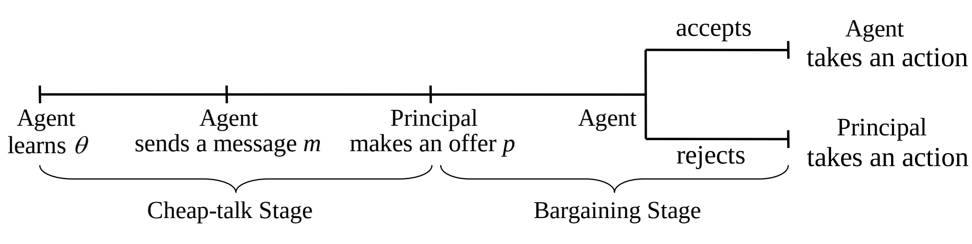


Figure 15: Communication before Bargaining

Again, the equilibrium concept we use is perfect Bayesian equilibrium. For the agent, a strategy consists of a message rule, a decision rule and an action rule. The message rule

$\mu : \Theta \to \Delta(M)$ specifies the choice of message for each type $\theta \in \Theta$. The decision rule, denoted by $d : \Theta \times M \times \mathbb{R} \to [0,1]$, specifies the probability of rejection for each price offer $p \in \mathbb{R}$ that the agent who sent the message m might receive. The action rule $y^A : \Theta \times M \times \mathbb{R} \to \mathbb{R}$, specifies the action taken by the agent type θ who sent a message m and accepted the principal's price offer p. For the principal, a strategy consists of a price rule and an action rule. The price rule $p^* : M \to \mathbb{R}$ specifies the principal's choice of price offer for each message $m \in M$ that the principal might receive. The action rule, denoted by $y^P : M \times \mathbb{R} \to \mathbb{R}$ specifies the action taken by the principal who observed a message m and her price offer p was rejected. The strategy profile $\{(\mu, d, y^A), (p^*, y^P)\}$ and the principal's posterior beliefs ρ_1 and ρ_2 form a perfect Bayesian equilibrium if:

(**CB1**) for each $\theta \in [0,1]$, $\int_M \mu(m|\theta)dm = 1$ and if $m^* \in M$ is in the support of $\mu(\cdot|\theta)$ then m^* solves

$$\max_{m \in M} d(\theta, m, p^*(m))U^A(y^P(m, p^*(m)), \theta, b) - p^*(m)(1 - d(\theta, m, p^*(m)))$$

(**CB2**) for each $m \in M$, $p^*(m)$ solves

$$\max_{p \in \mathbb{R}} \int_0^1 \{d(\theta, m, p)U^P(y^P(m, p), \theta) + (1 - d(\theta, m, p))(p - l(b))\}\rho_1(\theta|m)d\theta$$

(**CB3**) for each $\theta \in [0,1]$, $m \in M$, and $p \in \mathbb{R}$, $d(\theta, m, p)$ solves

$$\max_{d \in [0,1]} (1 - d)(-p) + d \cdot U^A(y^P(m, p), \theta, b)$$

(**CB4**) for each $\theta \in [0,1]$, $m \in M$, and $p \in \mathbb{R}$, $y^A(\theta, m, p) = \theta + b$

(**CB5**) for each $m \in M$ and $p \in \mathbb{R}$, $y^P(m, p)$ solves

$$\max_{y \in \mathbb{R}} \int_0^1 U^P(y, \theta)\rho_2(\theta|m, p)d\theta$$

(**CB6**)

$$\rho_1(\theta|m) = \frac{\mu(m|\theta)}{\int_0^1 \mu(m|\theta')d\theta'} \qquad \text{and} \qquad \rho_2(\theta|m, p) = \frac{d(\theta, m, p)\rho_1(\theta|m)}{\int_0^1 d(\theta', m, p)\rho_1(\theta'|m)d\theta}$$

where $\rho_1(\theta|m)$ is the principal's updated belief after observing the message m from the agent and $\rho_2(\theta|m, p)$ is the updated belief of the principal receiving the message m and observing the rejection of the price offer p.

2.4.1 Truth-telling Equilibrium

Is it possible that communication before bargaining is informative and as a result, improves efficiency of bargaining? Surprisingly, there exists a truth-telling perfect Bayesian equilibrium once we allow parties to communicate before bargaining.

Consider the following strategy profile: the principal makes a price offer $l(b)$ regardless of the message he observed and takes an action $y = \overline{y}^P(0) = 0$ if the offer is rejected. The agent fully reveals his private information by sending a truth-telling message in the cheap talk stage and accepts any offer less than or equal to $l(b)$ with probability one but rejects any other offer with probability one. If the principal makes any price offer $p \neq l(b)$ and it is rejected, then the principal takes an action $y = \theta$. It is easy to see that the agent types' strategy is a best response to the principal's strategy. First, no agent type has an incentive to deviate in cheap talk stage because the principal's price offer is message-independent. Second, no agent type has an incentive to reject the offer in the bargaining stage because for all $\theta \in [0, 1]$

$$\underbrace{-l(b)}_{\text{from accepting } l(b)} \geq \underbrace{-l(|0 - \theta - b|)}_{\text{from rejecting } l(b)} = -l(\theta + b).$$

Given the agent's strategy specified above, the principal's best response is to make an offer $l(b)$, the highest price offer accepted by agent types who tells the truth in the cheap talk stage. The principal does not have an incentive to make any offer $p < l(b)$ because such a price offer will be accepted by all agent types and gives a strictly less payoff to the principal than making the price offer $l(b)$. The principal does not have a strict incentive to make any offer $p > l(b)$ because such a price offer will be rejected by all agent types and the action taken by the principal after $p > l(b)$ is rejected will give him exactly the same payoff as he could get by making the offer $p = l(b)$. After the price offer $l(b)$ is rejected, the principal believes that the true state of the world is $\theta = 0$ with probability one. This is a reasonable belief in the sense that the agent with type $\theta = 0$ is the only type who is indifferent between accepting and rejecting the offer $l(b)$, and all other agent types strictly prefer accepting the offer. We will discuss this issue more carefully in section 2.4.2.3.

Proposition 15 (Truth-telling equilibrium)**.** *For any $b > 0$, there exists a perfect Bayesian equilibrium in which the informed agent fully reveals his private information by sending* *truth-telling messages in cheap talk stage.*

Proof. The proof is constructive. Consider the following strategies and belief:

i) The agent type θ fully reveals his private information by sending a message θ.

ii) For any $m \in M$, the principal makes the price offer $l(b)$.

iii) For any $\theta \in [0,1]$, the agent accepts the offer p with probability one if $p \leq l(b)$ but rejects p with probability one if $p > l(b)$, regardless of the message he sent.

iv) If a price offer $p = l(b)$ is rejected then the principal takes an action $y = 0$ regardless of the message she received. If a price offer $p \neq l(b)$ is rejected then the principal who received a message m takes an action $y = m$.

v) For any $m \in M$, $\rho_1(\theta|m) = \begin{cases} 0 & \forall \theta \in [0,1] \setminus m, \\ 1 & \text{if } \theta = m. \end{cases}$

vi) For any $m \in M$ and any $p = l(b)$, $\rho_2(\theta|m,p) = \begin{cases} 0 & \forall \theta \in (0,1], \\ 1 & \text{if } \theta = 0. \end{cases}$

vii) For any $m \in M$ and any $p \neq l(b)$, $\rho_2(\theta|m,p) = \begin{cases} 0 & \forall \theta \in [0,1] \setminus m, \\ 1 & \text{if } \theta = m. \end{cases}$

First, consider the agent's incentive. Under the principal's strategy and beliefs above, the agent has no incentive to deviate in his message rule because the principal makes the message-independent price offer $l(b)$. For any $m \in M$, any agent type $\theta \in [0,1]$ accepts an offer p with probability one if $p \leq l(b)$ since he gets $-p$ which is greater than or equal to $-l(b)$ from accepting the offer, but the expected payoff of the agent type θ from rejecting the offer is

$$-l(|0 - \theta - b|) = -l(\theta + b) \leq -l(b), \quad \forall \theta \in [0,1].$$

Any agent type $\theta \in [0,1]$ who reveals his private information fully rejects an offer p with probability one if $p > l(b)$ since he gets $-p$ which is less than $-l(b)$ from accepting the offer, but the expected payoff of the agent type θ from rejecting the offer is $-l(b)$.

Second, consider the principal's incentive. Under the agent's strategy and beliefs above, the principal's optimal behavior after observing a truthful message (or a message θ) is to make the price offer $l(b)$, because any offer less than $l(b)$ will be accepted by all types of agent with probability one and give her the expected payoff strictly less than 0, the principal's expected payoff from making the offer $l(b)$ and any offer greater than $l(b)$ will be rejected with probability one and induces the principal's action θ which gives the principal the expected payoff 0.

By the construction, no price offer is rejected with positive probability on the equilibrium path so that we cannot use Bayes' rule to determine beliefs that the principal has after price offers are rejected. The principal's action rule specified above is sequentially rational under the beliefs we take. This completes our proof. □

In this truth-telling equilibrium, the informed agent accepts the equilibrium price offer $l(b)$ with probability one so that the final outcome is always efficient *ex-post*. This is surprising because neither the tacit bargaining nor communication via cheap talk alone allow parties to make full use of the agent's private information to make a decision. This result is similar to Farrell and Gibbons [32] in the sense that not only information conveyed by cheap talk in equilibrium, but the equilibrium outcomes differ from any that could occur in an equilibrium without talk.

What is the role of communication in this equilibrium? In fact, the principal completely ignores the messages she got from the agent in the cheap-talk stage on the equilibrium path. Nonetheless, the role of communication is clear in this equilibrium. The principal uses her information she got from communication when any off-the-equilibrium-path price offer $p \neq l(b)$ is rejected, and takes an action $y = \theta$. Hence, the agent types who send truthful messages in the cheap-talk stage would not want to accept any price offer $p > l(b)$. This leads the principal not to make any price offer $p > l(b)$ in this equilibrium. As a result, making a price offer $l(b)$ is optimal for the principal. Recall that without communication, making the price offer $l(b)$ is not optimal. Since almost all of agent types still accept a price offer $l(b) + \varepsilon$ with an arbitrarily small $\varepsilon > 0$, the principal has an incentive to make an offer $l(b) + \varepsilon$ in the case without communication.

It is remarkable to see that the existence of this equilibrium is robust against the exact timing of the game. To be more precise, consider the game in which bargaining comes first and communication comes next under the contingency that an agreement is not reached. Then there exists the following perfect Bayesian equilibrium in this game which is outcome equivalent to the truth-telling equilibrium in the original model. The construction of the equilibrium is almost the same as before: the principal makes a price offer $l(b)$ and takes an action $y = 0$ regardless of the message she received from the agent if the offer is rejected. All agent types in Θ always accept the offer $l(b)$ with probability one and fully reveal their private information by sending truth-telling messages off the equilibrium path (i.e. when any offer is rejected.) Since it is straightforward to see that this strategy profile satisfies the mutual best response under some belief derived by Bayes' rule, I skip the detailed proof.

2.4.2 Robustness of the Truth-telling Equilibrium

In this section, we apply two equilibrium refinements for cheap-talk models- neologism-proofness developed by Farrell [31] and NITS (no incentive to separate) developed by Chen, Kartik, and Sobel [21]. We show that the truth-telling equilibrium satisfies both neologism-proofness and NITS condition for any $b > 0$ whereas the babbling equilibrium does not satisfy either of them for some parameter value of b. Moreover, we discuss robustness of the truth-telling equilibrium against support restriction, the assumption used by several papers such as Grossman and Perry [43][42], Harrington [45], Kreps and Wilson [58] and Rubinstein [85]. The extensive-form trembling hand perfection by Selten [86] and sequential equilibria by Kreps and Wilson [59] will be discussed.

2.4.2.1 Neologism-proofness There are several papers (see Gertner, Gibbons, and Scharfstein [37], Farrell and Gibbons [32][33], and Matthews [72]) that use neologism proofness (Farrell [31]) to refine equilibrium outcomes of cheap talk games, to refine their equilibrium outcomes. In what follows, we show that the truth-telling perfect Bayesian equilibrium is always neologism proof and for some parameter values, it is a unique neologism-proof equilibrium.

According to Farrell [31], assume that for every non-empty subset X of Θ, and for every perfect Bayesian equilibrium of the game, there exists a message $m(X)$ that is unused in the equilibrium and whose literal meaning is that $\theta \in X$. If the principal observes the message $m(X)$, then she hypothesizes that some members of specified subset X are responsible for the message and makes a price offer that is a best response under the posterior belief derived by Bayes' rule from her prior. For example, by Lemma 4 and the tacit bargaining analysis, for any convex $X \subseteq \Theta$ it is the principal's best response against getting the message $m(X)$ to make a price offer which is rejected by agent types in the low half of X and accepted by remaining agent types in X. The parties' behaviors in the remainder of the game satisfy sequential rationality and the final payoffs for the agent types from sending the message $m(X)$ are determined. Let $P(X)$ denote the set of all agent types that strictly prefer their payoffs from sending the message $m(X)$ to their equilibrium payoffs. We say that a subset X is *self-signaling* if $P(X) = X$. The neologism $m(X)$ is credible if X is self-signaling. If there is a credible neologism available in an equilibrium, we say that such an equilibrium is not *neologism proof*.

The notion of neologism-proofness has a refining power in our model. To see this, suppose that parties' prior belief is *uniform* over $[0, 1]$ and utilities are quadratic. Notice that in the babbling equilibrium, it might be the case that some agent types prefer to reveal their types because either the unique action induced in equilibrium is too small ($\frac{1}{4}$) for them or the amount of money they should pay to the principal ($p = (\frac{1}{4} + b)^2$) is too high (See Figure 16). We therefore investigate if there is a self-signaling subset of the form $X = [\widetilde{\theta}, 1]$ with $\widetilde{\theta} > 0$. Suppose that $X = [\widetilde{\theta}, 1]$ send a neologism to the principal. Then, by Lemma 4 and analysis on the tacit bargaining, the principal's optimal response is to make a price offer, p', which is rejected by $[\widetilde{\theta}, \frac{\widetilde{\theta}+1}{2}]$ but accepted by $(\frac{\widetilde{\theta}+1}{2}, 1]$. From the indifference condition at $\frac{\widetilde{\theta}+1}{2}$, we have $p' = (\frac{1-\widetilde{\theta}}{4} + b)^2$. For X to be self-signaling, it is necessary and, for $\widetilde{\theta} \in (0, 1)$, sufficient that i) the agent type $\widetilde{\theta}$ is indifferent between sending the neologism inducing the action $\frac{3\widetilde{\theta}+1}{4}$ and sending his equilibrium message inducing the action $\frac{1}{4}$ and ii) the agent type 0 does not want to deviate to the neologism $m(X)$. This requires that

$$\frac{1}{4} - \widetilde{\theta} - b = -\frac{3\widetilde{\theta} + 1}{4} + \widetilde{\theta} + b \tag{2.8}$$

and

$$-(\frac{1}{4} - b)^2 \geq -p' = -(\frac{1 - \widetilde{\theta}}{4} + b)^2. \tag{2.9}$$

If the equation (2.8) gives a value of $\widetilde{\theta}$ in the range of $(0, 1)$ and the value of $\widetilde{\theta}$ satisfies the inequality (2.9), then we have constructed a self-signaling subset X. It is immediately clear

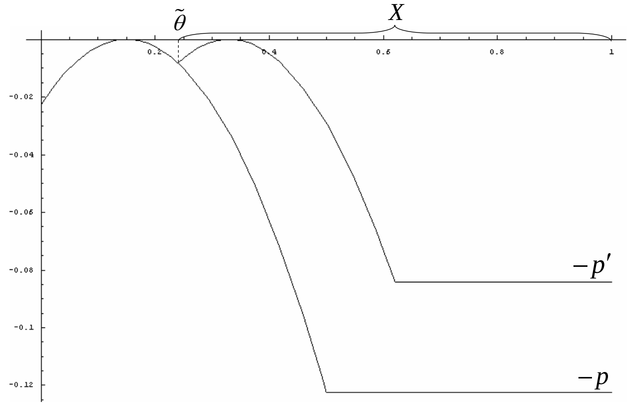


Figure 16: Neologism-proofness

that $\widetilde{\theta}$ satisfies both conditions if and only if $\frac{1}{24} \leq b < \frac{1}{4}$. Therefore, if $\frac{1}{24} \leq b < \frac{1}{4}$, then any babbling equilibrium is not neologism proof.

It is well-known that a neologism-proof equilibrium may select only a pooling equilibrium (Gertner, Gibbons, and Scharfstein [37]). Moreover, there might be no neologism-proof equilibrium in some models (Matthews [72]). However, truth-telling is always neologism proof in our model.

Proposition 16. *For any $b > 0$, the truth-telling perfect Bayesian equilibrium is neologism proof.*

Proof. See the appendix. □

The proof in the appendix shows that for any non-empty subset X of Θ there must be an agent type $\theta \in X$ such that sending the neologism $m(X)$ generates a payoff less than $-l(b)$, the payoff from the truth-telling equilibrium.

2.4.2.2 NITS (No Incentive To Separate) Chen, Kartik, and Sobel [21] pose a criterion to select equilibria in Crawford and Sobel [27] cheap-talk games: NITS, for *no incentive to separate*. An equilibrium satisfies NITS if the agent of the lowest type weakly prefers the equilibrium outcome to credibly revealing his type. They show that equilibria satisfying NITS always exist in Crawford and Sobel [27], and the most informative equilibrium outcome is the unique equilibrium satisfying NITS under the monotonicity condition M in Crawford and Sobel [27]. In this section, we apply NITS to our model and show that the criterion is selective; under some value of b the babbling equilibrium does not survive, while the truth-telling equilibrium does survive for any $b > 0$.

Suppose that parties' prior belief is *uniform* over $[0, 1]$ and utilities are quadratic. Notice that in the babbling equilibrium, the agent of the lowest type gets $-(\frac{1}{4} - 0 - b)^2$. Thus, the babbling equilibrium does not satisfy NITS if and only if

$$-(\frac{1}{4} - b)^2 < -b^2,$$

which is equivalent to $b < \frac{1}{8}$.

It is straightforward that NITS holds in the truth-telling equilibrium, in which all agent types reveal their types fully.

Proposition 17. *For any $b > 0$, the truth-telling perfect Bayesian equilibrium satisfies NITS.*

2.4.2.3 Support Restriction and Perfection One might be tempted to argue that the truth-telling equilibrium is not very reasonable because it does not satisfy "support restriction", the assumption used by several papers such as Grossman and Perry [43][42], Harrington [45], Kreps and Wilson [58] and Rubinstein [85]. This restriction requires that the support of beliefs at an information set should be contained in the supports of beliefs at preceding information sets.

In our truth-telling equilibrium the principal has probability one beliefs after getting messages in the cheap-talk stage and switches away from these beliefs to the new belief that assigns probability one to the type $\theta = 0$ after observing "rejection" of the equilibrium price offer which takes place off-the-equilibrium path, and therefore the equilibrium violates the support restriction. There are two responses to the support restriction. First, it has been shown, however, that not only the support restriction may be based on a wrong interpretation of the concept of a belief in some games but also violations of the support restriction may represent a sensible reasoning process which supports interesting equilibrium behaviors by Madrigal, Tan and Werlang [69] and Nöldeke and van Damme [82]:

> ... violations of the support restriction may very well reflect the fact that once a deviation from equilibrium behavior has been observed, a reassessment of all previous beliefs - which were based on the assumption that equilibrium strategies are followed - is called for. In this light such "switching beliefs" is not an unfortunate problem, which cannot be avoided in some cases, but actually is a natural consequence of observing a deviation. (Nöldeke and van Damme [82], p. 9.)

Second, there exists another truth-telling equilibrium in which there is no such a problem off the equilibrium path. Recall that in the previous equilibrium the agent type $\theta = 0$ is indifferent between accepting and rejecting the equilibrium-path price offer $p = l(b)$. Now, consider the strategy profile that has only one difference from the previous construction: on the equilibrium path, the principal makes a message-independent price offer $l(b)$ and all agent types (0,1] accept it with probability 1 but the agent type 0 rejects it with probability 1.[15] Except this, all of the strategy profile are exactly the same as before. It is straightforward that this strategy profile forms a perfect Bayesian equilibrium. Importantly, "rejection" of $p = l(b)$ is not an off-the-equilibrium-path event anymore in this construction. That is, there is no switching away from beliefs so that this equilibrium satisfies the support restriction.

One may also wonder if the truth-telling equilibrium is robust against the extensive-form trembling-hand perfection by Selten [86] or sequential equilibria by Kreps and Wilson [59]. While the definition of trembling-hand perfect equilibria or sequential equilibria does not directly apply to our game[16], the truth-telling equilibrium does not violate any possible implementation of sequential equilibria or trembling-hand perfect equilibria. To see this, suppose that some trembles are allowed in the bargaining stage so that the principal makes some rejected offers with a positive probability. As a result, beliefs after every history can be derived by Bayes' rule. Then, regardless of the trembles made, the principal facing the price offer by trembles sticks to the belief he updated through the cheap-talk stage and thus taking an action $y = \theta$ is optimal.

[15] It is not necessary in this construction that the agent type 0 rejects $l(b)$ with probability 1. If the agent type $\theta = 0$ accepts $p = l(b)$ with a positive probability, then we have an equilibrium in which "rejection" of $p = l(b)$ is not an off-the-equilibrium-path event anymore.

[16] For more details, see Simon and Stinchcombe [87]

2.5 WELFARE COMPARISON

2.5.1 Comparisons to Other Schemes

In this section, we demonstrate the benefit from trade of decision-rights by comparing the equilibrium outcomes of our model to those of several dispute resolution schemes studied in the framework of Crawford and Sobel [27]: communication (Crawford and Sobel [27]), optimal mediation (Goltsman, Hörner, Pavlov, and Squintani [38]), optimal delegation (Holmström [50][51], Melumad and Shibano [75], Alonso and Matouschek [3] and Kováč and Mylovanov [56]) and optimal compensation contract (Krishna and Morgan [60]). For the comparison, we assume that f is *uniform* and utility functions are *quadratic* as follows:

$$U^P(y,\theta) = -(y-\theta)^2 \quad \textit{and} \quad U^A(y,\theta,b) = -(y-\theta-b)^2.$$

Crawford and Sobel [27] consider a situation in which the principal has no commitment power at all and sends cheap-talk messages to the agent. It is shown that all equilibria in their model are interval partitional so that there is only a finite number of actions chosen in equilibrium, each associated with an interval of states. With uniform quadratic assumption, they show that the number of distinct equilibrium outcomes, denoted by $N_{CS}(b)$, is

$$N_{CS}(b) = \left\langle -\frac{1}{2} + \frac{1}{2}\sqrt{1+\frac{2}{b}} \right\rangle \tag{2.10}$$

where $\langle z \rangle$ denotes the smallest integer greater than or equal to z. Moreover, there is a Pareto ranking among $N_{CS}(b)$ equilibria so that, for any $b > 0$, the number of elements of the partition associated with the *Pareto dominant* equilibrium, which we will call the *best* equilibrium, is $N_{CS}(b)$. The expected payoff of the principal in this *best* equilibrium is

$$EU^P_{CS}(b) = -\frac{1}{12N_{CS}(b)^2} - \frac{b^2(N_{CS}(b)^2-1)}{3} \tag{2.11}$$

while the *ex-ante* expected payoff for the informed agent is

$$EU^A_{CS}(b) = EU^P_{CS}(b) - b^2. \tag{2.12}$$

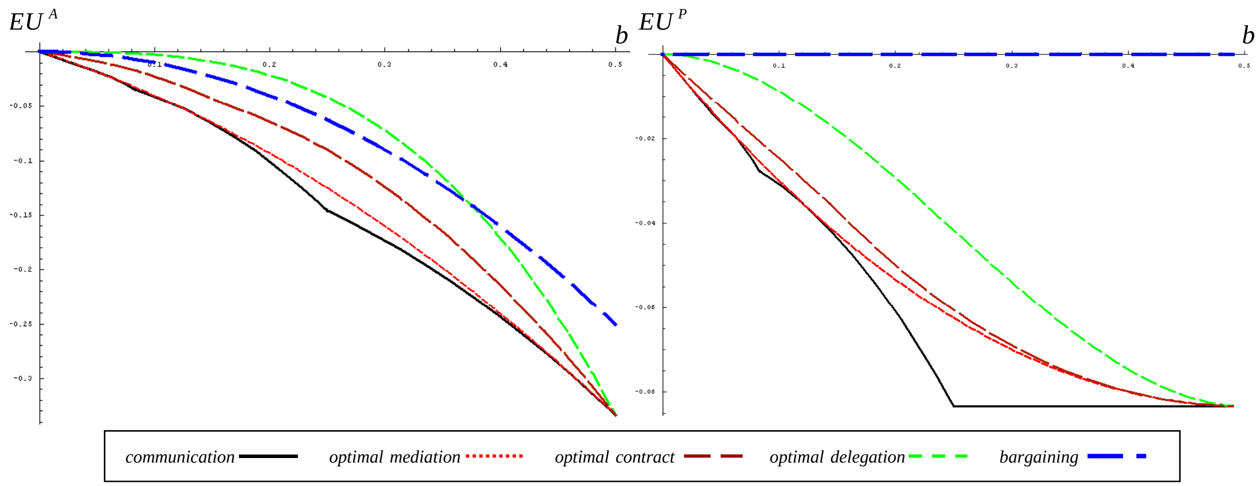


Figure 17: (a) Agent's expected payoff (b) Principal's expected payoff

Recently, Goltsman, Hörner, Pavlov, and Squintani [38] allow the parties to use any communication protocol, including the ones that call for a neutral trustworthy mediator. According to the optimal mediation rule, the parties' expected payoffs are

$$EU^P_{mediation}(b) = -\frac{b(1-b)}{3} \quad \text{and} \quad EU^A_{mediation}(b) = EU^P_{mediation}(b) - b^2. \tag{2.13}$$

Holmström [50][51], Melumad and Shibano [75], Alonso and Matouschek [3] and Kováč and Mylovanov [56] study the principal's optimal choice of the set of admissible actions that the agent can take and show that under the optimal delegation scheme, the principal restricts project choices of the agent to be from 0 up to a maximum of $1-b$. Under this scheme, the parties' expected payoffs are

$$EU^P_{delegation}(b) = -\frac{b^2(3-4b)}{3} \quad \text{and} \quad EU^A_{delegation}(b) = -\frac{8b^3}{3}. \tag{2.14}$$

In these papers, the principal also has imperfect commitment power so that she can only commit on the *ex-ante* allocation of decision-rights. Moreover, the monetary transfer is impossible.

Krishna and Morgan [60] consider the situation in which the principal can commit to pay the agent for his advice but retains decision-making authority. They fully characterize the optimal compensation contract: the optimal compensation contract involves separation in low states and a finite number of pooling intervals in high state, and the principal never pays for imprecise information. In this optimal compensation contract, the expected payoffs for the principal and the agent are

$$EU^P_{contract}(b) = -\int_0^{a_0}(2b(a_0-\theta)+t_0)d\theta - \frac{1}{12}\sum_{i=1}^{K}\left(\frac{1}{K}-\frac{a_0}{K}-2b(K-2i+1)\right)^3 \quad (2.15)$$

and

$$EU^A_{contract}(b) = EU^P_{contract}(b) - b^2 + 2\int_0^{a_0}(2b(a_0-\theta)+t_0)d\theta \quad (2.16)$$

where

$$K = \left\langle -\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{3}{2b}}\right\rangle,$$

$$a_0 = \frac{3}{4}-\frac{1}{4}\sqrt{4+\frac{1}{3}(3-8bK(K-1))(8bK(K+1)-3)} \quad \textit{and}$$

$$t_0 = \frac{(1-a_0-2K(K-1)b)(2bK(K+1)-(1-a_0))}{4K^2}.$$

Figure 17 illustrates the comparison. As the figure shows, the truth-telling equilibrium outcome of explicit bargaining is *ex-ante* Pareto superior to communication, optimal mediation rule and optimal contract for any $b > 0$. Furthermore, it is *ex-ante* Pareto superior to all other schemes (including optimal delegation) when $b > .375$. This might explain why bargaining over decision-rights often takes place between two separately owned companies whose interests diverge widely.

It is impertinent to interpret this welfare comparison as a result showing that bargaining mechanism we considered is superior to all other schemes considered in the literature. It is more appropriate to say that the higher *ex-ante* utilities for both parties comes from different assumptions on the principal's commitment power rather than from the superiority of the mechanism. Unlike the most papers in the literature on communication and optimal delegation, this model assumes not only the principal can commit on the *ex-ante* allocation of decision-rights but also monetary transfer is available. The welfare comparison shows the benefit of using monetary transfer to trade decision-rights in our environment though. The use of monetary incentives allows parties to make full use of the agent's private information when making a decision, and as a result, the truth-telling outcome of explicit bargaining is *ex-ante* Pareto superior to the outcomes of several dispute-resolution schemes studied in the literature.

2.5.2 Comparisons to Bargaining with Agents Making Offers

Lim [65] considers bargaining over decision-making authority in which the informed agent makes a price offer (*A-offer Bargaining*) and shows that there are continuum of perfect Bayesian equilibria, each of which yields an *ex-post* efficient outcome. Although there is no general Pareto ranking among equilibria, Lim [65] shows that there exists the principal optimal equilibrium and the agent optimal equilibrium. Moreover, the principal optimal equilibrium gives the lowest payoff to the agent among all equilibria of the model and vice versa. By using the refinement of perfect sequential equilibrium (Grossman and Perry [43]), Lim [65] gets a unique equilibrium outcome, which coincides with the agent optimal equilibrium.

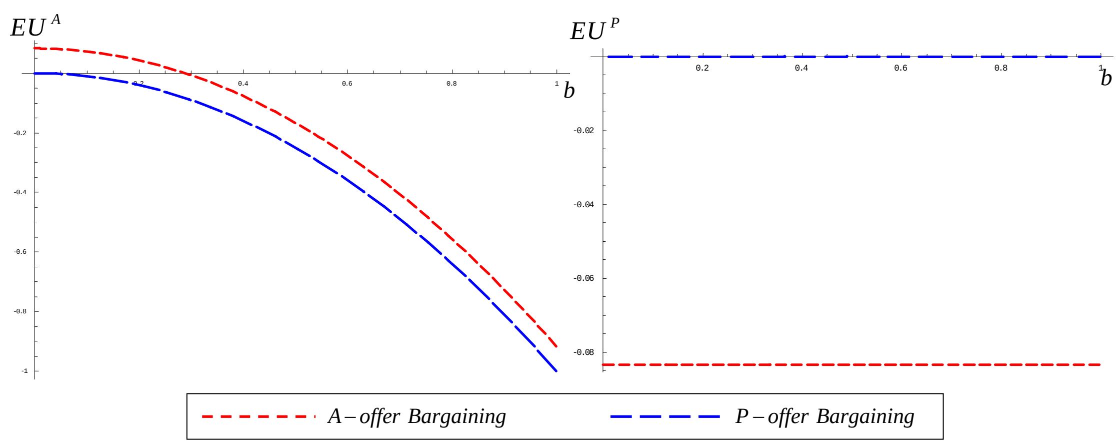


Figure 18: (a) Agent's expected payoff (b) Principal's expected payoff

Since the uninformed principal makes a price offer in our model (*P-offer Bargaining*), the equilibrium outcomes are quite different from those of *A-offer Bargaining*. Interestingly, the truth-telling equilibrium outcome of *P-offer Bargaining* coincides with the outcome of the principal optimal equilibrium of *A-offer Bargaining*. Figure **??** illustrates this result. It tells us that although bargaining over decision-rights can lead to a Pareto-efficient outcome regardless of who has bargaining power, allocation of initial bargaining power plays an important role in determining how they share the resulting surplus.

2.6 DISCUSSION AND EXTENSIONS

2.6.1 Multidimensional State Space

In this section, we extend our model to the case where the state space is multidimensional. We show that the compactness, as opposed to dimensionality, of the state space plays a crucial role for the existence of the truth-telling equilibrium. Suppose that the state space Θ is a compact subset of $\mathbb{R}^d$. Similarly, the action space Y is a subset of $\mathbb{R}^d$. $b \in \mathbb{R}^d$ is a bias of the agent. For state θ and action y, the principal's utility is $-\sum_{j=1}^{d} l(|y_j - \theta_j|)$ and the agent's utility is $-\sum_{j=1}^{d} l(|y_j - \theta_j - b_j|)$ where y_j, θ_j, and b_j are jth coordinate of y, θ, and b, respectively.

It is straightforward to see that the construction of truth-telling equilibrium from the previous section can be extended to the current model with multidimensional state space, if the state space is compact. Let $\underline{W}_j$ and $\overline{W}_j$ denote minimum and maximum of j's coordinate of Θ, respectively. Define $\theta^* = (\theta_1^*, \theta_2^*, ..., \theta_d^*)$ such that $\theta_j^* = \underline{W}_j$ if $b_j \geq 0$ and $\theta_j^* = \overline{W}_j$ otherwise. The following strategy profile constitutes a perfect Bayesian equilibrium: (i) All agent types fully reveal their types in the cheap talk stage. (ii) All agent types but θ^* accept any offer $p \leq \sum_{j=1}^{d} l(|b_j|)$ with probability one and otherwise, reject with probability one. (iii) The agent type θ^* rejects the offer $p = \sum_{j=1}^{d} l(|b_j|)$ with positive probability and behaves in the same way as other agent types for any other price offers. (iv) The principal makes a price offer $\sum_{j=1}^{d} l(|b_j|)$ regardless of the message he received from the agent. (v) If the offer $p = \sum_{j=1}^{d} l(|b_j|)$ is rejected, the principal believes that the type is θ^* with probability one and takes an action $y = \overline{y}^P(\theta^*) = \theta^*$. (vi) If the offer $p \neq \sum_{j=1}^{d} l(|b_j|)$ is rejected, the principal believes messages from agent types and takes an action $y = \theta$. (vii) If any offer is accepted, the agent type θ takes his ideal action $\theta + b$.

This equilibrium is graphically illustrated in Figure 19. In this figure, the solid circle

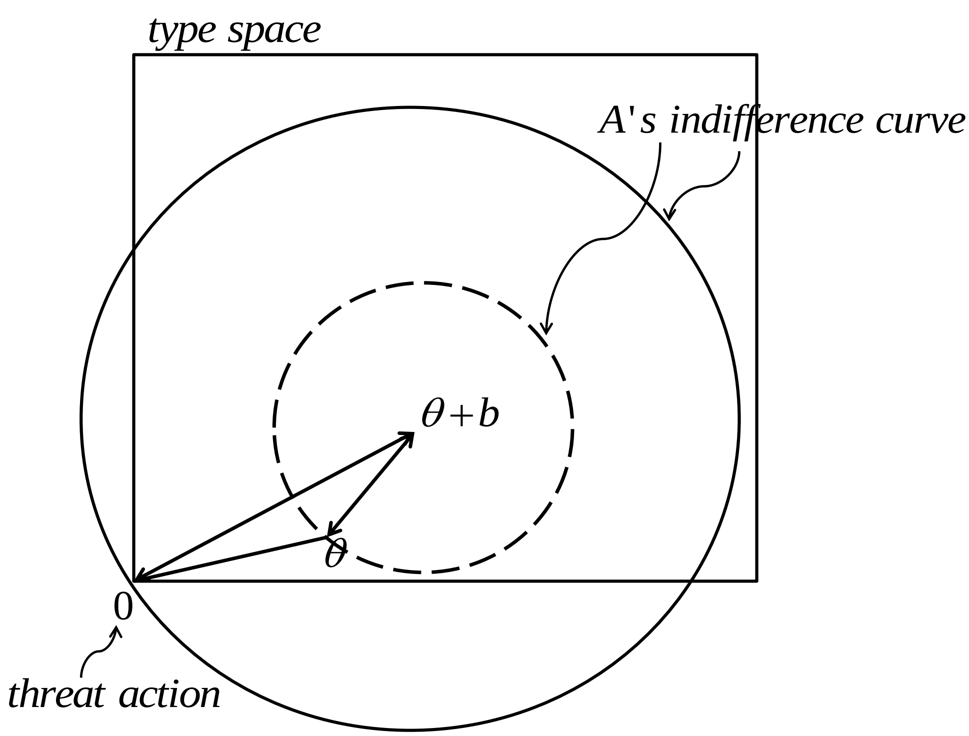


Figure 19: Constructing a truth-telling equilibrium

represents the indifference curve of the agent at the threat action θ^* (in this example, it is 0) that would be taken by the principal in case that the equilibrium price offer $\sum_{j=1}^{d} l(|b_j|)$ is rejected. The distance between θ and $\theta+b$ represents the equilibrium price offer $\sum_{j=1}^{d} l(|b_j|)$. As you can see in this figure, any agent type $\theta \neq 0$ has to accept the equilibrium price offer since the distance between θ and $\theta + b$ is shorter than the radius of the circle with the origin of $\theta + b$. Remarkably, those two coincide with each other only if $\theta = \theta^* = 0$, which means that the agent type θ^* is indifferent between accepting and rejecting $\sum_{j=1}^{d} l(|b_j|)$. Taking the action θ^* turns out to be rational for the principal (and as a result credible to the agent) under the belief that the agent type θ^* is the only type who randomizes between accepting and rejecting the price offer. Again, truth-telling is optimal for the agent since the principal makes a message-independent price offer $\sum_{j=1}^{d} l(|b_j|)$. The dotted circle represents the indifference curve of the agent at the action $y = \theta$. Truth-telling guarantees the agent the utility level in the dotted indifference curve so that truthful agent will always reject a price offer higher than $\sum_{j=1}^{d} l(|b_j|)$. This makes it optimal for the principal to make such an offer $\sum_{j=1}^{d} l(|b_j|)$ regardless of messages.

Figure 19 also highlights the importance of the compactness of the state space. Especially, the construction of the credible threat action heavily relies upon compactness of the state space. If each of the unidimensional subspaces of the entire state space is neither bounded below nor bounded above, then it is impossible to construct any credible threat action or equivalently an agent type who is indifferent between accepting and rejecting some message-independent price offer. Indeed, compactness of the state space is a sufficient condition for truth-telling equilibrium to exist. This result is summarized in the following proposition.

Proposition 18. *Suppose that $\Theta \subset \mathbb{R}^d$ is compact. For any $b \in \mathbb{R}^d$, there exists a truth-telling equilibrium.*

Proof. It suffices to show that there exists $z \in \Theta$ such that $-l(|b|) > -l(|z - \theta - b|)$ for all $\theta \in \Theta \setminus \{z\}$. Since $l'(\cdot) < 0$, the condition is equivalent to $|b| < |z - \theta - b|$ or

$$b \cdot b < b \cdot b + 2b \cdot (\theta - z) + (\theta - z) \cdot (\theta - z), \quad \forall \theta \in \Theta \setminus \{z\}.$$

Consider a function $b \cdot \theta$. Notice that it is a continuous function on the compact set Θ and therefore achieves a minimum on Θ. Set z to be a minimizer of $b \cdot \theta$ on Θ. Then 1) $b \cdot \theta \geq b \cdot z$ for all $\theta \in \Theta$ and 2) $(\theta - z) \cdot (\theta - z) > 0$ for all $\theta \in \Theta \setminus \{z\}$. This completes the proof. $\square$

It is interesting to compare this result to the main finding of multi-dimensional cheap talk literature. Battaglini [11] shows that full revelation of information in all states of nature is generically possible if there is more than one sender and the state space is multidimensional. Ambrus and Takahashi [6] further investigate the existence of fully revealing equilibrium and find that boundedness, as opposed to dimensionality, of the state space plays an important role in determining the qualitative feature of a cheap talk model. They show that if the state space is bounded and biases are large enough then there is no fully revealing perfect Bayesian equilibrium. It is worth emphasizing that although the boundedness of state space also plays an important role for fully revealing equilibria to exist in our model, the role is not the same as in Ambrus and Takahashi [6]. If the state space is compact, then there exists a fully revealing equilibrium even when conflict of interest is arbitrarily large. Unlike Battaglini [11] and Ambrus and Takahashi [6], multiple agents are not necessary even though the construction does not rely on the fact that there is only one agent. The direction of biases does not matter for this result.

2.6.2 Optimal Bargaining Mechanism

One important observation is that bargaining over decision-rights that we have considered is not the only bargaining mechanism for decision-rights in our environment. For example, after the principal's price offer is rejected, the principal may want to make another price offer instead of taking action by herself. It may be possible that the agent makes a price offer to buy decision-rights after he rejects a price offer from the principal in the first round. By a bargaining mechanism, we mean any kind of scheme by which principal and agent make offers directly, indirectly, once, repeatedly, sequentially, simultaneously, alternatively and so on. Accordingly, we have *infinitely many* alternatives as a bargaining mechanism for decision-rights. In this section, we follow a mechanism design approach and show that the explicit bargaining we have considered is an optimal mechanism among all feasible mechanisms in the sense that it achieves the upper bound of the *ex-ante* social welfare. Although the analysis in this section is restricted to the quadratic utility functions, I believe that one can get a similar result for a broader class of utility functions.[17]

A *bargaining mechanism* is one in which the informed agent send a message to a mediator who then credibly commits to the final allocation of decision-rights and the monetary transfers. We restrict our attention to a *direct bargaining mechanism* in which the informed agent reports the true state of the world to a mediator who then determines the final allocation of decision-rights and the monetary transfers. This means that a direct mechanism is characterized by two outcome functions, denoted by $x(\cdot)$ and $p(\cdot)$, where $x(\theta)$ is the probability that the decision-right is transferred to the agent and $p(\theta)$ is the expected payment from the agent to the principal if θ is the reported state of the world from the agent.

Bester and Strausz [16] shows that the standard revelation principle by Myerson [70] may fail if the mechanism designer is not able to credibly commit to the outcome of the mechanism. In our case, however, it is straightforward to see that the standard argument for the revelation principle holds. Consider an indirect mechanism (x, p) where the agent follows a message rule $m : \Theta \rightarrow M$ in an equilibrium of the mechanism where M is a Borel-measurable message space. In this mechanism, 1) the agent takes an action $\theta + b$ and $p(m(\theta))$ is transferred from the agent to the principal with probability $x(m(\theta))$ and 2) the principal takes an action based on the information she updated from observing $p(m(\theta))$ and $x(m(\theta))$ with probability $1 - x(m(\theta))$. Define a direct mechanism $(x', p') \equiv (x \circ m, p \circ m)$. The outcome of this mechanism is that 1) the agent takes an action $\theta + b$ and $p'(\theta)$ is transferred from the agent to the principal with probability $x'(\theta)$ and 2) the principal takes an action based on the information she updated from observing $p'(\theta)$ and $x'(\theta)$ with probability $1 - x'(\theta)$. By definition of the direct mechanism, $p(m(\theta)) = p'(\theta)$ and $x(m(\theta)) = x'(\theta)$ for any state of the world $\theta \in \Theta$. Furthermore, this implies that for any state of the world the actions taken by the principal under two mechanisms are exactly the same since the same information is transmitted from the agent to the principal through the mechanisms. Therefore, these two mechanisms are outcome equivalent. By invoking this version of revelation principal, we restrict our attention to a direct mechanism without loss of generality.

[17] However, the result does not depend on the distribution function.

Our goal is to find a mechanism that maximizes a social welfare which is defined as the sum of expected payoff of the principal and of the agent. By focusing on this mechanism, we are able to see if a bargaining mechanism can achieve the first-best efficient outcome in our environment. Recall that in the quasi-linear environment there exists a unique first-best outcome that maximizes the joint surplus of parties. In case of quadratic utility, it is the midpoint of θ and $\theta + b$. At the first-best outcome, the social welfare is $-\frac{b^2}{4}$. Does any bargaining mechanism achieve this first-best outcome? Otherwise, what is the efficiency bound?

Given a mechanism (p, x), define $\mathcal{U}$ as a social welfare. Then

$$\begin{aligned}
\mathcal{U} &= EU^A + EU^P \\
&= \int_\Theta \{x(\theta) \cdot U^A(\overline{y}^A(\theta), \theta, b) + (1 - x(\theta)) \cdot U^A(\overline{y}^P(p(\theta)), \theta, b) - p(\theta)\} f(\theta) d\theta \\
&\quad + \int_\Theta \{x(\theta) \cdot U^P(\overline{y}^A(\theta), \theta) + (1 - x(\theta)) \cdot U^P(\overline{y}^P(p(\theta)), \theta) + p(\theta)\} f(\theta) d\theta \\
&= \int_\Theta \{-x(\theta) \cdot l(b) + (1 - x(\theta)) \cdot [U^P(\overline{y}^P(p(\theta)), \theta) + U^A(\overline{y}^P(p(\theta)), \theta, b)]\} f(\theta) d\theta
\end{aligned} \tag{2.17}$$

We say that a mechanism (p, x) is optimal if it maximizes the social welfare. That is, an optimal mechanism is a solution for the following optimization problem:

$$\max_{p(\cdot), x(\cdot)} \mathcal{U}$$

subject to the incentive compatibility constraint.

Equation (2.17) shows that an optimal mechanism should assign decision-rights to the principal if and only if the sum of interim utilities resulting from the action $\overline{y}^P(p(\theta))$ is greater than $-l(b)$ where $\overline{y}^P(p(\theta))$ is an action taken by the principal who updates her belief after observing $p(\theta)$. Thus, if a mechanism achieves a social welfare $\mathcal{U} > l(b)$, there exists a nonempty set of agent types, denoted by S, such that

$$\forall \theta \in S \quad x(\theta) = \overline{x} \quad \textit{and} \quad p(\theta) = \overline{p}, \tag{2.18}$$

and

$$\int_S [U^P(\overline{y}^P(\overline{p}), \theta) + U^A(\overline{y}^P(\overline{p}), \theta, b)] f(\theta) d\theta > -l(b) \tag{2.19}$$

where

$$
\begin{aligned}
\overline{y}^P(\overline{p}) &= argmax_y \int_\Theta U^P(y,\theta)\frac{q(\overline{p}|\theta)f(\theta)}{\int_\Theta q(\overline{p}|\theta')f(\theta')d\theta'}d\theta \\
&= argmax_y \int_S U^P(y,\theta)f(\theta)d\theta.
\end{aligned}
$$

Let $\overline{\mathcal{U}}$ denote the upper bound of the social welfare. The next proposition shows that when the utility function is quadratic the upper bound of the social welfare is $-l(b) = -b^2$ in any bargaining mechanism.

Proposition 19. *Suppose that the utility function is quadratic. Then* $\overline{\mathcal{U}} = -l(b) = -b^2$.

Proof. See the appendix. □

The intuition of this result is straightforward. A bargaining mechanism determines the final allocation of decision-rights but has no effect on the incentive in the decision-making stage. That is, the final decision depends only on the decision-making party's own interest and private information the party possess. It is well-known from the cheap-talk literature that more precise information is always beneficial *ex-ante* not only to the principal but also to the agent. Therefore, the social welfare cannot be higher than $-l(b)$, the social welfare that results from the most informative decision-making. Recall that the explicit bargaining we have considered in the previous sections leads to the social welfare $-l(b)$ in the truth-telling equilibrium. This leads to the following corollary.

Corollary 2. *When the utility function is quadratic, the explicit bargaining is an optimal mechanism.*

Notice that the efficiency of bargaining mechanisms is bounded away from the first-best efficiency. Therefore, one can interpret this result as theoretical supports reinforcing the previous finding that property rights and voluntary private negotiation are not able to achieve this first-best efficient outcomes when information is asymmetrically distributed.

2.7 CONCLUSION

This paper studies bargaining over decision-making rights between an informed but self-interested agent and an uninformed principal in which the uninformed principal makes a price offer to the agent who then decides either to accept or to reject it. We show that the unique perfect Bayesian equilibrium outcome does not satisfy *ex-post* efficiency. Once we introduce explicit communication into the model, however, there exists a truth-telling perfect Bayesian equilibrium, which is not only efficient *ex-post* but also neologism proof. Moreover, it is the unique neologism-proof equilibrium if parties' preferences are sufficiently similar.

We compare the equilibrium outcome of our model to that of some dispute resolution schemes studied in the framework of Crawford and Sobel [27] and and Holmström [50] and show that it is *ex-ante* Pareto superior to all other schemes when the parties' interests diverge substantially. This might explain why bargaining over decision-rights often takes place between two separately owned companies whose interests diverge widely. Although bargaining over decision-rights can lead to a Pareto-efficient outcome regardless of who has bargaining power, allocation of initial bargaining power plays an important role in determining how they share the resulting surplus.

3.0 CONTESTS WITH A STOCHASTIC NUMBER OF PLAYERS

3.1 INTRODUCTION

Contest theory, which started from the famous Tullock's [89] paper,[1] is used to consider a fixed number of players.[2] However, sometimes players do not know the actual number of players participating in the contest. For example, in many rent-seeking contests an individual lobbyist does not know how many other lobbyists are competing for the rent when she exerts her effort.[3] In this paper, we consider Tullock's n-player contests where each player has an independent probability $0 < p \leq 1$ of participation. It means that the actual number of players in the contest can vary between 0 and n.

We show that such contests have a unique symmetric equilibrium which can be described in the closed form. Our main interest is to analyze properties of this equilibrium and compare the individual and the total equilibrium spending in the standard Tullock's and our cases. Note that the standard Tullock's model is a particular case of our model where the participation probability is one, $p = 1$. The individual equilibrium spending, $X^*(r, V, n, p)$, depends on four parameters: the marginal return, the prize value, the number of potential players, and the probability of participation. It turns out that the individual equilibrium spending is strictly increasing in the marginal return and the value of the prize for any fixed n and p. Therefore, the total equilibrium spending is also strictly increasing in the same

[1]Models similar to Tullock's have been studied in the literature on advertising and rivalry for market shares. Friedman's [36] paper is probably one of the first in the literature.

[2]Surveys of the contest literature can be found in Nitzan [81], Szymanski [88], Congleton, et al. [23], and Konrad [55].

[3]The same situation often takes place in R&D races when each firm does not know the actual number of R&D race competitors. Another example is a usual lottery where one player wins a unique main prize. Typically, when players buy lottery tickets they do not know the actual number of players.

parameters. These observations are of course consistent with the standard Tullock's model, $p = 1$.

We show that the individual equilibrium spending as a function of the participation probability p is *single-peaked*. Moreover, for any number of potential players n, a unique positive probability which maximizes the individual equilibrium spending is always below one. It means that each active player spends more under some uncertainty about the number of players than under certainty, in the Tullock's case. There are several reasons why it is important to know about the individual equilibrium spending. Sometimes, the contest outcome depends solely on the winner's individual spending. In many R&D races, for instance, losing firms can not have patents or rights to produce so their effort and investments are socially wasteful. In this case, the total spending is not important but the individual spending made by the winning firm is. Our analysis suggests that it can be beneficial for the society to have some degree of uncertainty in this case.

We also demonstrate that the *single-crossing* property[4] holds for the individual equilibrium spending: for a fixed marginal return and a prize value, *any* two individual equilibrium spending curves (as functions of participation probability p) with different potential numbers of players cross only once. This property is surprising and important. First, the single-crossing property helps to identify the unique probability which maximizes the individual equilibrium spending. We prove that two *adjacent* individual equilibrium spending curves cross at the *peaked point* of the curve with a higher number of potential players. Second, based on the single-crossing property, we show that the interval of participation probabilities is divided into two parts. If $p > 0.9$, then the standard contest result that the individual equilibrium spending is decreasing in the number of participating players holds. However, if $0 < p < 0.9$, then the above result does not hold.

It turns out that the comparative statics is much simpler for the total equilibrium spending than for the individual equilibrium spending. Even though the individual equilibrium spending is strictly increasing in only two out of four parameters, the total equilibrium spending is strictly increasing in *all* four parameters. In particular, unlike the individual

[4] This property is different from the Spence's single crossing condition that has an important role in signaling, contract theory, and mechanism design.

equilibrium spending, the *ex-ante* total equilibrium spending (as a function of probability p) is maximized under certainty, $p = 1$.

Our model provides another possible answer for a long-standing question about over-dissipation in the equilibrium. Since Krueger [62], Posner [83], and Tullock [89], there have been continuous endeavors including Corcoran [24], Corcoran and Karels [25], Higgins, et al. [48], Michaels [76], and Leininger and Yang [63] to explain over-dissipation within theoretical framework. Now, it is a well-known result that ex-ante over-dissipation is not consistent with an equilibrium behavior.[5] The ex-ante over-dissipation *never* takes place in our model too. Since the total equilibrium spending is monotonically increasing in p, the highest expected (ex-ante) total spending is achieved in the Tullock's certain case, $p = 1$.

Hillman and Samet [49] and Baye, et al. [13][14] show that ex-post over-dissipation can take place as a particular realization of a *mixed-strategy* equilibrium. We demonstrate that ex-post over-dissipation is a natural feature of the *pure-strategy* equilibrium if the number of players is stochastic. The intuition for this observation is straightforward. If the participation probability is below one, $p < 1$, the *actual* number of players in the contest can be different from the expected number of players. We demonstrate that if $n > 3$ and the actual number of players is "much higher" than the expected number of players, the ex-post over-dissipation occurs. However, the expected number of players always coincides with the actual number of players if $p = 1$ and, therefore, the ex-post over-dissipation *never* takes place in the certain world (Tullock's model).

Games with uncertain or stochastic number of players are a natural extension of games with a fixed number of players. In the auction theory, McAfee and McMillan [74] are the first to study the model with a stochastic number of bidders. They show that the standard auction theory is sensitive to the assumption that a number of players is common knowledge. Levin and Smith [67] consider auctions with stochastic entrants resulting from endogenous entry. They extend the revenue-equivalence and ranking theorems and show that the seller and society can benefit from policies that reduce market thickness. Levin and Ozdenoren [66] investigate bidders' and seller's responses to ambiguity about a number of bidders in the

[5] Recently, Baharad and Nitzan [8] have shown that over-dissipation is possible in equilibrium, when the contestants distort their winning probabilities.

standard auctions with independent private valuations. They show that the general revenue equivalence breaks down under ambiguity about a number of bidders. Myerson [79] develops a mathematical framework to analyze games with population uncertainty and shows special properties of the Poisson games.[6]

Myerson and Wärneryd [80] and Münster [78] are the first to consider contests with a stochastic number of players. Myerson and Wärneryd [80] analyze a model with infinitely many potential players where the number of players is a random variable. They show that if it is known for certain that there will be at least one participant, then the total equilibrium spending is strictly lower in a contest with population uncertainty than in a non-uncertain contest with the same expected number of players. In our model, the number of players is a random variable which follows the *binomial* distribution. Even though we do not require that it has to be at least one participant in the contest, we are also able to demonstrate the same result as in Myerson and Wärneryd [80]. Münster [78] considers a model similar to ours. However, he focuses on the players' risk attitude and shows that equilibrium rent seeking efforts are lower under risk aversion if and only if the expected fraction of active contestants is low.

The rest of the paper is organized as follows. In the next section, we describe the model and present a unique symmetric equilibrium. We focus on the properties of the individual equilibrium spending in Section 3. Section 4 is devoted to the properties of the total equilibrium spending. We conclude in Section 5. All proofs are in the Appendix.

3.2 THE MODEL

Consider a contest with N potential risk neutral players. We assume that each potential player participates (becomes active) in the contest with independent probability $p \in (0, 1]$. All active players compete for a single prize of value V. The timing of the game is as follows. First, the nature chooses active players. Then, without knowing the actual number of participants, each active player i makes an expenditure, denoted by $X_i \geq 0$. The winner

[6] In the Poisson game, the number of players is a random variable which follows Poisson distribution.

of the contest is determined through the following contest success function

$$P_i(X_i;\mathbf{M}) = \begin{cases} \frac{X_i^r}{X_i^r+\sum_{j\in\mathbf{M}} X_j^r}, & \text{if} \quad X_i > 0, \\ 0, & \text{if} \quad X_i = 0, \end{cases} \tag{3.1}$$

where $\mathbf{M}$ is the set of active players in the contest.

If player i participates in the contest, she maximizes the following objective function

$$\max_{X_i} V \cdot \left(\sum_{\mathbf{M}\in\mathcal{P}^{\mathbf{N}_i}} p^{|\mathbf{M}|}(1-p)^{|\mathbf{N}_i\setminus\mathbf{M}|} P_i(X_i;\mathbf{M}) \right) - X_i, \tag{3.2}$$

where $\mathbf{N}_i$ is the set of player i's possible opponents, $\mathcal{P}^{\mathbf{N}_i}$ is the set of all subsets of $\mathbf{N}_i$ and $|\mathbf{M}|$ denote the cardinality of the set $\mathbf{M}$.

The first order condition for player i is

$$V \cdot \left(\sum_{\mathbf{M}\in\mathcal{P}^{\mathbf{N}_i}} p^{|\mathbf{M}|}(1-p)^{|\mathbf{N}_i\setminus\mathbf{M}|} \frac{rX_i^{r-1}\sum_{j\in\mathbf{M}} X_j^r}{(X_i^r+\sum_{j\in\mathbf{M}} X_j^r)^2} \right) - 1 = 0. \tag{3.3}$$

We focus on a symmetric equilibrium where $X_1 = ... = X_n = X^*$. Then the first order condition (3.3) becomes

$$X^* = rV \cdot \left(\sum_{\mathbf{M}\in\mathcal{P}^{\mathbf{N}_i}} p^{|\mathbf{M}|}(1-p)^{|\mathbf{N}_i\setminus\mathbf{M}|} \frac{|\mathbf{M}|}{(|\mathbf{M}|+1)^2} \right). \tag{3.4}$$

Note that there are $C_{|\mathbf{M}|}^{n-1}$ different ways to make a set which has exactly $|\mathbf{M}|$ elements from the set $\mathbf{N}_i$. Using this fact, we can rewrite the individual spending (3.4) as follows

$$X^* = rV \cdot \left(C_{n-1}^{n-1} p^{n-1} \frac{(n-1)}{n^2} + ... + C_1^{n-1} p^1 (1-p)^{n-2} \frac{1}{2^2} \right),$$

where $C_j^{n-1} = \frac{(n-1)!}{j!(n-j-1)!}$.

Now we can state the main result of this section.

Theorem 1. *Suppose that* $0 < r \leq \frac{n+1}{n}$. *Then, there exists a unique symmetric pure-strategy equilibrium where each player expenditure is*

$$X^*(r,V,n,p) = rV \cdot \left(\sum_{i=1}^{n-1} C_i^{n-1} p^i (1-p)^{n-i-1} \frac{i}{(i+1)^2} \right). \tag{3.5}$$

Note that the original Tullock's model is a particular case of our model where all players participate in the contest with probability 1. Thus, we get the following corollary.

Corollary 3 (Tullock's individual spending)**.** *If* $0 < r \leq \frac{n+1}{n}$ *and* $p = 1$*, then*

$$X^*(r, V, n, 1) = \frac{(n-1)}{n^2} rV. \tag{3.6}$$

3.3 INDIVIDUAL SPENDING

In this section, we examine the impact of each parameter on the individual equilibrium spending, $X^*(r, V, n, p)$. First, we consider a marginal return and a prize value. It is straightforward to see from (3.5) that the individual equilibrium spending is strictly increasing in both r and V. This means that Tullock's ($p = 1$) finding that the individual equilibrium spending is a strictly increasing function of a marginal return and a prize value is robust to introducing a stochastic number of players.

Let us move our attention to the remaining parameters, the number of players and the participation probability. Note that if there are only two potential players, then the individual equilibrium spending is monotonic (strictly increasing) in the probability of participation, p, because

$$X^*(r, V, 2, p) = \frac{rV}{4} p. \tag{3.7}$$

However, our next example demonstrates that the individual equilibrium spending is not monotonic in the probability of participation for $n \geq 3$.

Example 3. *Suppose that* $n = 3$ *and* $r = V = 1$*. Then*

$$X^*(1, 1, 3, 0.8) = 0.222 < X^*(1, 1, 3, 0.9) = 0.225 > X^*(1, 1, 3, 1) = 0.222.$$

This example shows that the individual equilibrium spending can be higher if there is some degree of uncertainty ($p < 1$) than under certainty ($p = 1$). The following theorem demonstrates that this observation is true in general: the individual equilibrium spending is single-peaked in p.

Theorem 2 (Single-Peak Property)**.** *For any $n \geq 3$, there exists a unique $p^*(n) \in (0,1)$ that maximizes the individual equilibrium spending. Moreover, for given r, V, and n, $X^*(r,V,n,p)$ is single-peaked in p.*

In order to see the intuition behind the single-peaked property, let us take a derivative in equation (3.5) with respect to p

$$\frac{1}{rV}\frac{\partial X^*(r,V,n,p)}{\partial p} = \underbrace{\sum_{i=1}^{n-1} C_i^{n-1} p^{i-1}(1-p)^{n-i-1}\left(\frac{i}{i+1}\right)^2}_{\text{Positive effect}} - \underbrace{\sum_{i=1}^{n-2} C_i^{n-1} p^{i}(1-p)^{n-i-2}\frac{(n-i-1)i}{(i+1)^2}}_{\text{Negative effect}} . \tag{3.8}$$

There are two competing effects in (3.8) on the individual equilibrium spending: positive and negative. The positive effect: a player wants to increase her spending, because she faces just a few opponents and she wants to increase her chance to win in this case. The negative effect: a player wants to decrease her spending, because she faces a lot opponents and her chance to win is small, so she does not want to waste a lot of resources in this case. When p is small (big), the expected number of players is small (big), therefore, the positive (negative) effect dominates the negative (positive) effect and the individual equilibrium spending curve increases (decreases). At the peak point the positive and negative effects cancel each other out. The following identity demonstrates the relationship between two adjacent individual spending curves and helps to characterize graphically the peak points.

Theorem 3. *Suppose that $n \geq 3$, then*

$$(n-1)(X^*(r,V,n,p) - X^*(r,V,n-1,p)) \equiv p \cdot \frac{\partial X^*(r,V,n,p)}{\partial p}. \tag{3.9}$$

Since the right-hand side of (3.9) can be equal to zero only at $p^*(n)$ from the single peak property, the individual equilibrium spending curves $X^*(r,V,n,p)$ and $X^*(r,V,n-1,p)$ can only intersect once if $p \in (0,1)$. Moreover, the intersection point must be the peak point of the curve $X^*(r,V,n,p)$. This gives the following corollary.

Corollary 4. *For all $n \geq 2$, curve $X^*(r,V,n,p)$ intersects curve $X^*(r,V,n+1,p)$ at the peak point of $X^*(r,V,n+1,p)$.*

What is the participation probability $p^*(n)$ which maximizes the individual equilibrium spending for given r, V, and n? Although it is hard to obtain an analytical description of $p^*(n)$ for all $n \geq 2$, we can investigate properties of $p^*(n)$. In fact, $p^*(n)$ and $X^*(r,V,n,p^*(n))$ decrease as n increases. These results are derived directly from the single peak property and identity (3.9).

Theorem 4. *Suppose that r and V are given. Then,*
i) $p^*(n)$ strictly decreases as n increases;
ii) $X^*(r,V,n,p^*(n))$ strictly decreases as n increases.

Theorem 4 shows that the point $p^*(n)$ where the positive and negative effects cancel each other out "moves to the left" as the number of potential players increases. In other words, the negative effects becomes stronger as the number of potential players increases. This observation is intuitive, because more potential players means more expected players, therefore, the negative effects dominates for smaller participation probabilities.

Let us move our attention to the relationship between individual equilibrium spending curves with different potential number of players. In the original Tullock's [89] model, the individual equilibrium spending is a strictly decreasing function of the number of players. However, the following example shows that this is not the case under uncertainty, $p = 0.8$.

Example 4. *Suppose that $p = 0.8$ and $r = V = 1$. Then*

$$X^*(1,1,2,0.8) = 0.2 < X^*(1,1,3,0.8) = 0.2222 > X^*(1,1,4,0.8) = 0.2053.$$ [7]

It is possible to describe all participation probabilities where the individual equilibrium spending is decreasing in the number of participating players. First, we show that *any* two individual equilibrium spending curves with different number of potential players cross only once on the interval $p \in (0,1]$. The following theorem states it formally.

Theorem 5 (Single-Crossing Property)**.** *For any $2 \leq m < n$, there exists a unique $p(m,n) \in (0,1)$ such that*

$$X^*(r,V,m,p(m,n)) = X^*(r,V,n,p(m,n)). \tag{3.10}$$

[7] Münster (2006) also points this possibility out.

Moreover,

$$X^*(r, V, m, p) < X^*(r, V, n, p), \quad \textit{if} \quad 0 < p < p(m, n)$$

and

$$X^*(r, V, m, p) > X^*(r, V, n, p), \quad \textit{if} \quad p(m, n) < p < 1.$$

There are two competing effects on the individual equilibrium spending again. Both effects are stronger if there are more potential players. Therefore, before two individual equilibrium spending curves with different number of potential players cross, the curve with more potential players is always above the curve with less potential players: the positive effect is stronger. The situation is completely opposite after the curves cross, because the negative effect is stronger now.

Theorem 5 demonstrates that the individual equilibrium spending curves do not intersect on the right of $p^*(2)$. Since $p^*(2) = 0.9$, the following corollary describes all participation probabilities where the individual equilibrium spending is decreasing in the number of participating players

Corollary 5. *Suppose that* r *and* V *are given and* $0.9 < p \leq 1$. *Then, the individual equilibrium spending,* $X^*(r, V, n, p)$*, is monotonic in* n.

All results of this section can be seen on Figure 1.

3.4 TOTAL SPENDING

In this section, we examine the impact of each parameter on the the total equilibrium spending. Even though in the previous section we show that the individual equilibrium spending is not monotonic in p and n, here, we demonstrate that the total equilibrium spending is monotonic in all parameters. Then, we show that ex-post over-dissipation can occur. Finally, we discuss properties of the total equilibrium spending if the expected number of players if fixed.

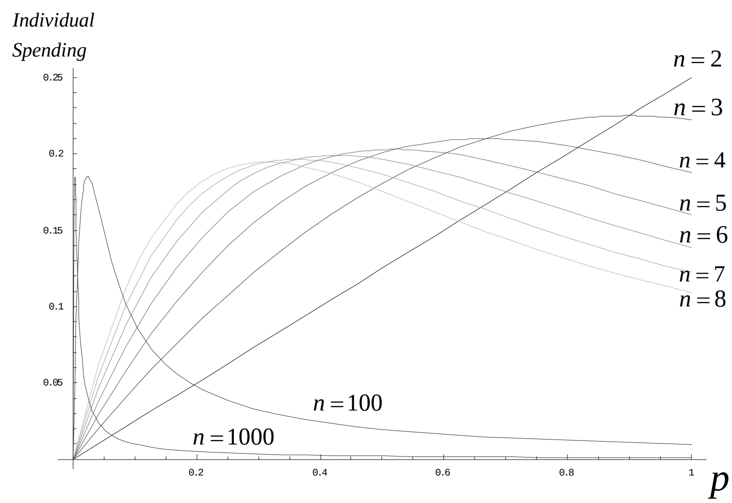


Figure 20: Equilibrium individual spending when $rV = 1$.

3.4.1 Properties

Let N^* and T^* denote the *ex-ante* expected number of players and the *ex-ante* total equilibrium spending, respectively. Then

$$N^* = C_n^n p^n n + C_{n-1}^n p^{n-1}(1-p)(n-1) + ... + C_1^n p(1-p)^{n-1} 1 = np. \tag{3.11}$$

Note that the expected number of players is the same as an expectation of the random variable which follows a binomial distribution $B(n,p)$ where the probability of success is p and the number of trials is n.

The expected total equilibrium spending is

$$T^*(r, V, n, p) = rV \cdot \left(\sum_{i=1}^{n} C_i^n p^i (1-p)^{n-i} \left(1 - \frac{1}{i}\right) \right). \tag{3.12}$$

From equation (3.12), it is straightforward to see that the total spending increases as either V or r increases.

Consider now how a participation probability and the number of potential players effect the total equilibrium spending. From Theorem 2 (Theorem 5), it is clear that for a small

.

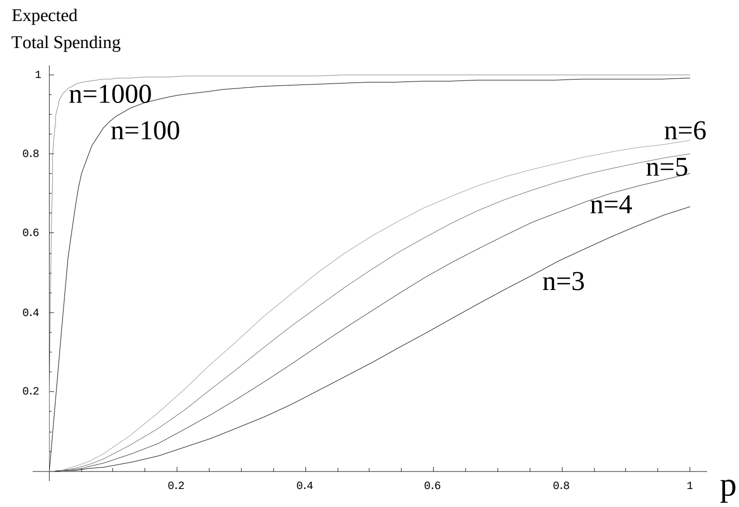


Figure 21: Expected total equilibrium spending. $rV = 1$.

$p < p^*(n)$ ($0 < p < p(m,n)$, $m < n$) the expected number of players and the individual equilibrium spending will both increase if p (n) increases. Therefore, the total equilibrium spending will increase too. However, if $p \geq p^*(n)$ ($p(m,n) < p < 1$), then an increase of p (n) has two different effects on the expected total equilibrium spending. First, as we can see in equation (3.11), it always increases the expected number of players. Second, it decreases the individual equilibrium spending. The following theorem shows that the first effect is *always* stronger.

Theorem 6. *Suppose that r and V are given. Then,*
i) for any $n \geq 2$, the expected total equilibrium spending increases as p increases;
ii) for any $p \in (0,1]$, the expected total equilibrium spending increases as n increases.

Theorem 6 shows that the expected total equilibrium spending is monotonic in the participation probability and in the number of potential players. Figure 21 illustrates Theorem 6.

3.4.2 Over-dissipation

In this subsection, we show that ex-post over-dissipation can be a natural feature of the *pure-strategy* equilibrium. It happens if the actual number of players in the contest is "much higher" than the expected number of players.

Denote by $RT^*(r, V, n, p; k)$ the *actual* total spending, where k is the *actual* number of players. Then in the symmetric equilibrium,

$$RT^*(r, V, n, p; k) = k \cdot X^*(r, V, n, p).$$

The *ex-post* over-dissipation takes place if and only if the actual total spending exceeds the prize value, V. That is,

$$RT^*(r, V, n, p; k) = k \cdot X^*(r, V, n, p) > V. \tag{3.13}$$

From (3.13) and (3.5), it follows that ex-post over dissipation occurs if

$$n \geq k > k^* := \frac{1}{r \cdot \left(\sum_{i=1}^{n-1} C_i^{n-1} p^i \left(1-p\right)^{n-i-1} \frac{i}{(i+1)^2} \right)}. \tag{3.14}$$

It is clear from (3.14) that ex-post over-dissipation never happens if the critical number k^* is greater than the number of potential players $k^* \geq n$. This happens for small numbers of potential players, $n = 2, 3$. For example, if $n = 3$, we get

$$V > 0.9V = RT^*(4/3, V, 3, 0.9; 3) \geq RT^*(r, V, 3, p; k), \quad \text{for all } r \in (0, 4/3] \text{ and } p \in (0, 1].$$

The smallest number of potential players when ex-post over-dissipation is possible is 4:

$$RT^*(5/4, V, 4, 2/3; 4) \approx 1.049V > V.$$

Figures 22a and 22b illustrate the ex-post under-dissipation in the case of $n = 3$ and the ex-post over-dissipation in the case of $n = 10$. We can see on Figures 22b that ex-post over-dissipation can occur if $k \geq 6$. Moreover, the range of p where ex-post over-dissipation can occur becomes wider as k increases.

It is important to note that ex-post over-dissipation *never* takes place in the certain world, $p = 1$, because the actual number of players is the same as the expected number of

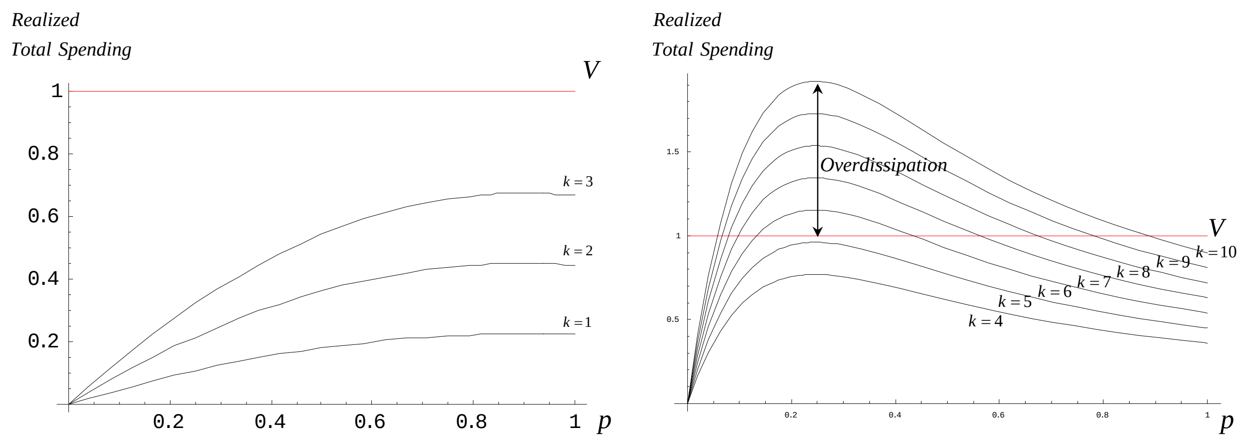


Figure 22: (a) realized total spending when $n = 3$ (b) realized total spending when $n = 10$

players in this case. Therefore, some uncertainty must be present, $p < 1$, in order to have ex-post over-dissipation. This intuition is similar to the idea behind Baye, et al. [13][14] who show that ex-post over-dissipation can take place as a particular realization of a *mixed-strategy* equilibrium.

We summarize this subsection in the following theorem.

Theorem 7. *Ex-post over-dissipation takes place if and only if condition* (3.14) *holds. Moreover, if $n = 2$ or 3, the ex-post over-dissipation never occurs; for any $n \geq 4$ there exist $p \in (0, 1)$ and $r \in (0, \frac{n+1}{n}]$ such that ex-post over-dissipation is possible.*

The possibility of ex-post over-dissipation is reminiscent to Proposition 3 in Levin and Smith [67]. They study auctions with stochastic entrants and show that in the common-value auction "without the entry fee, entry would be excessive from social and private points of view."

3.4.3 The same expected number of players

In this subsection, we fix the expected number of players, $\overline{n} = np$, and compare the total equilibrium spending across the total-equilibrium-spending curves in Figure 2. In particular, we are interested to compare the total equilibrium spending in the certain (if $p = 1, n = \overline{n}$), $T^*(r, V, \overline{n}, 1)$, and in uncertain cases (if $0 < p = \frac{\overline{n}}{n} < 1, n > \overline{n}$), $T^*(r, V, n, \frac{\overline{n}}{n})$. Note that the expected number of players is the same in both cases.

From equations (3.6) and (3.12), we obtain

$$T^*(r, V, \overline{n}, 1) = rV \frac{(\overline{n} - 1)}{\overline{n}}$$

and

$$T^*(r, V, n, \frac{\overline{n}}{n}) = rV \cdot \left(\sum_{i=1}^{n} C_i^n \left(\frac{\overline{n}}{n} \right)^i \left(1 - \frac{\overline{n}}{n} \right)^{n-i} \left(\frac{i-1}{i} \right) \right). \tag{3.15}$$

The following theorem presents the main result of this subsection that the total equilibrium spending is strictly lower in any uncertain case in comparison with the certain case.

Theorem 8. *Suppose that $\overline{n} \geq 2$. Then*

$$T^*(r, V, \overline{n}, 1) > T^*(r, V, n, \frac{\overline{n}}{n}),$$

for any $n > \overline{n}$.

Theorem 8 shows that uncertainty about the actual number of players induces lower ex-ante under-dissipation. This result is consistent with Myerson and Wärneryd's [80] finding for another distribution of potential players. They consider a contest with infinitely many potential players where the number of players is a random variable with expectation $\mu > 1$ and show that the total equilibrium spending is strictly lower in such a contest than in a contest where the number of players is known with certainty to be μ.

3.5 CONCLUSION

We consider Tullock's n-player contest where each player has an independent probability to participate. We find a unique symmetric equilibrium and analyze its properties.

There are several natural extensions of our paper. It will be interesting to test our theory in the experimental lab. In particular, our predictions about single-peak and single-crossing properties of the individual equilibrium spending and the monotonicity of the total equilibrium spending should be checked.

Finally, the actual number of players in our contest follows the Binomial distribution, $B(n, p)$. This assumption captures the main source of the population uncertainty that results from the uncertainty each individual faces. Moreover, it is a well-know result that the *binomial* distribution converges towards the *Poisson* distribution with parameter $\lambda = np$ as n goes to infinity while the product np remains fixed.[8] In this sense, we analyze a finite version of the "*Poisson* contest."

[8] See, for example, Feller [35].

APPENDIX A

PROOFS AND CALCULATIONS FOR CHAPTER 1

Proof of Proposition 1. First, consider the principal's incentives on the equilibrium path. After observing p^*, the principal updates her belief by Bayes' rule, i.e.

$$\rho(\theta|p^*) = \frac{\mu^A(p^*|\theta)f(\theta)}{\int_0^1 \mu^A(p^*|\theta')f(\theta')d\theta'} = f(\theta), \quad \forall \theta \in [0,1]$$

Under this belief, the principal's optimal action after observing p^* is

$$y^P(p^*) = arg \max_y \int_0^1 U^P(y,\theta)f(\theta)d\theta = \overline{y}^P$$

Then the principal would get the expected payoff $\int_0^1 U^P(y^P(p^*),\theta)d\theta = -\sigma$ from rejecting p^* whereas she would get $p^* - l(b)$ from accepting p^*. Since $p^* \geq l(b) - \sigma$, the principal has no incentive to reject p^*.

Second, consider the principal's incentives off the equilibrium path. Notice that we are completely free to take any out-of-equilibrium path belief. Let us take the following out-of-equilibrium-path belief for any $p < p^*$:

$$\rho(\theta|p) = \begin{cases} 0 & \forall \theta \in (0,1], \\ 1 & if\, \theta = 0. \end{cases}$$

In words, this implies that the principal is so certain and confident that she believes the true state of the world is $\theta = 0$ for sure whenever she observes any $p < p^*$. Under the belief we take, $y^P(p) = 0$ for any $p < p^*$. Then the principal would get 0 in expectation from rejecting any $p < p^*$ while she would get $p - l(b)$ from accepting it in expectation. Since $p < p^* \leq l(b)$, the principal rejects the offer p.

Let us take the following out-of-equilibrium-path belief for any $p > p^*$:

$$\rho(\theta|p) = f(\theta)$$

Under the belief we take, $y^P(p) = \mu$ for any $p > p^*$. Then the principal would get $-\sigma$ in expectation from rejecting any $p > p^*$ while she would get $p - l(b)$ from accepting it in expectation. Since $p^* \geq l(b) - \sigma$, the principal accepts the offer p.

Third, consider the agent's incentives. Given the principal's strategy and belief above, if the agent type θ makes an acceptable offer $p > p^*$ then her expected utility is $-p < -p^*$. If the agent type θ makes an unacceptable offer $p < p^*$, then she gets $-l(|\theta+b|) < -l(b) \leq -p^*$. If the agent type θ offers p^* then she gets $-p^*$ because the principal would accept this offer and the agent chooses her optimal action $y^A(\theta, p^*) = \theta + b$. Notice that $p^* \leq l(b)$. Since any agent type in $[0, 1]$ would get a payoff less than $-p^*$ by offering any $p \neq p^*$, offering p^* with probability 1 is optimal for all agent types in $[0, 1]$. This completes the proof.

Proof of Lemma 1. The proof of Lemma **??** consists of three claims. Let $\overline{P} \equiv \{p|d^P(p) = 1 \quad and \quad y^P(p) = y\}$. We say that principal's action y is induced by an agent type θ if $\int_{\overline{P}} \mu(p|\theta)dp > 0$.

Claim 1. *In any equilibrium, for every action y, the set of agent types who induce principal's action y is an interval. If this interval has a nonempty interior, then all types in the interior induce only action y.*

Proof. To get a contradiction, suppose that agent types θ_1 and θ_2 with $\theta_1 < \theta_2$ both induce the principal's action y, but an agent type $\theta_3 \in (\theta_1, \theta_2)$ induces the principal's action $y_1 > y$. Then since $U^A_{12}(y, \theta, b) > 0$, agent type θ_2 strictly prefer y_1 to y. Thus, types $\theta_3 \in (\theta_1, \theta_2)$ never induce an action $y_1 > y$. Similarly, types $\theta_3 \in (\theta_1, \theta_2)$ never induce an action $y_2 < y$, because otherwise, by single crossing property agent type θ_1 strictly prefer y_2 to y. Hence agent types in the interval (θ_1, θ_2) have no incentive to induce actions different from y.

Next, let us show that the agent type θ_3 has no incentive to make an acceptable offer. To see this, suppose that the agent type $\theta_3 \in (\theta_1, \theta_2)$ makes an acceptable price offer, denoted by ω. Then revealed preference yields $U^A(y, \theta_1, b) \geq -\omega$ and $U^A(y, \theta_2, b) \geq -\omega$. However, by the concavity of U^A in θ, $U^A(y, \theta_3, b) > -\omega$. That is, the agent type θ_3 strictly prefers to induce the action y rather than make the acceptable price offer ω. This is a contradiction. Thus, agent types in the interval (θ_1, θ_2) have no incentive to make an acceptable offer. This completes the proof. □

Claim 2. *In any equilibrium, for any principal's action y, y is in the interior of the set of agent types who induce the action.*

Proof. Let y denote the action taken by the principal after rejecting p. By Lemma 1, the set of agent types who induces the action y is an interval. $f > 0$ and the concavity of U^P in y complete the proof. □

Claim 3. *There is no equilibrium in which a positive measure of agent types makes an unacceptable offer.*

Proof. To get a contradiction, suppose that an unacceptable price offer p is made by a positive measure of agent types in equilibrium. Let y denote the action taken by the principal after rejecting p. By Claim 1 and 2, the set of agent types who induces the action y can be denoted by $[x, z]$ where $0 \leq x < y < z \leq 1$. By the concavity of U^A in θ, we have

$$U^A(y, \theta, b) < U^A(y, y, b) = -l(b), \quad \forall \theta \in (y, z] \tag{A.1}$$

However, by the sequential rationality (**BA2′**) the principal should accept any price offer $p > l(b)$ because she expects to get at most 0 from rejecting it while she expects to get $p - l(b) > 0$ from allowing the agent to take an action by accepting it. This implies that we are always able to find $\varepsilon > 0$ such that the acceptable price offer $l(b) + \varepsilon$ is profitable for these agent types. This leads to a contradiction. □

Proof of Lemma 2. I claim that $\mathcal{P}^\alpha \cap \mathcal{P}^o$ is a singleton.

Proof. To get a contradiction, suppose that $p', p'' \in \mathcal{P}^\alpha \cap \mathcal{P}^o$ where $p' > p''$. Then there exist $\theta', \theta'' \in [0,1]$ (possibly $\theta' = \theta''$) such that $\mu^A(p'|\theta') > 0$ and $\mu^A(p''|\theta'') > 0$. Since $p', p'' \in \mathcal{P}^\alpha$, $d^P(p') = 0$ and $d^P(p'') = 0$. Then from **(BA1′)**, the agent type θ' gets $-p'$ if she makes the offer p' while she gets $-p''$ if she makes the offer p''. Since $p' > p''$, making the offer p'' is profitable for the agent type θ'. This is a contradiction. This implies that two different prices cannot be accepted in equilibrium. □

Proof of Proposition 3. From Lemma 1 and 2, the set of agent types making an unacceptable price offer is either a singleton or an empty set. Suppose that it is a singleton. Let $\widehat{\theta}$ denote the agent type making the unacceptable price offer. Let p^* denote an acceptable price offer in the equilibrium. Since the agent type $\widehat{\theta}$ reveals a weak preference for making the unacceptable price offer over making the acceptable price offer, we have

$$p^* \geq l(b). \tag{A.2}$$

Moreover, any agent types except $\widehat{\theta}$ reveal a weak preference for making the acceptable price offer over making the unacceptable price offer, we have

$$-l(\widehat{\theta} - \theta - b) \leq -p^*, \quad \forall \theta \in \Theta \setminus \{\widehat{\theta}\}. \tag{A.3}$$

By (A.2) and (A.3), we have

$$\widehat{\theta} = 0 \quad \textit{and} \quad l(b) = p^*.$$

This implies that any equilibria with informative price offer have to be outcome equivalent to the equilibrium demonstrated in Proposition 2.

Now, suppose that the set of agent types making an unacceptable price offer is empty. Suppose that all agent types in $[0,1]$ make an acceptable price offer $p < l(b) - \sigma$. Notice that the principal would get $p - l(b) < -\sigma$ from accepting the offer. However, rejecting the offer is profitable for the principal because she can get the expected payoff $-\sigma$ by choosing $y^P(p) = \overline{y}^P$. This is a contradiction. Suppose that all agent types in $[0,1]$ make an acceptable price offer $p > l(b)$. Suppose that the principal would take an action $y \in [0,1]$ if the equilibrium price offer p is rejected. Since all agent types in $[0,1]$ reveal a weak preference for making an acceptable price offer over making an unacceptable price offer, we have $\forall y \in [0,1]$,

$$-l(y-\theta-b) \leq -p, \quad \forall \theta \in [0,1].$$

This implies that $p \leq l(b)$ which leads to a contradiction. Therefore, any equilibria with an uninformative price offer have to be outcome equivalent to one of the equilibrium demonstrated in Proposition 1.

Proof of Proposition 5. Suppose that there is a separating equilibrium. Then the agent type θ induces either the principal's action $y^P = \theta$ after the rejection, the agent's action $y^A = \theta + b$ after the acceptance, or both with positive probability. Since two different price offers cannot be accepted with probability 1 in equilibrium, there is at most one agent type who makes an acceptable price offer. Moreover, the principal randomizes accepting and rejecting only if she is indifferent between them, i.e. $p - l(b) = 0$. Hence, there is at most one agent type who makes the price offer of which the principal randomizes accepting and rejecting. This implies that there are at most two agent types who make the offer accepted with positive probability. Thus, almost all agent types in $[0,1]$ make a price offer rejected with probability 1 in this separating equilibrium. Therefore, we can choose one agent type, denoted by θ_1, who makes an offer rejected with probability 1 such that there exists an agent type $\theta_2 \in (\theta_1, \theta_1 + b]$ who also makes an offer rejected with probability 1 for any $b > 0$. Since $-l(|\theta_1 - \theta_1 - b|) = -l(b) < -l(|\theta_2 - \theta_1 - b|)$, the agent type θ_1 has an incentive to pretend to be an agent type θ_2. This is a contradiction.

Calculations for Example 1. In order to verify that the strategy forms a perfect Bayesian equilibrium with some belief, let us consider the principal's incentive on the equilibrium path. After observing p_1, the principal updates her belief by Bayes' rule, i.e.

$$\rho(\theta|p_1) = \frac{\mu^A(p_1|\theta)}{\int_0^1 \mu^A(p_1|\theta')d\theta'} = \begin{cases} \frac{1}{\theta_1} & \textit{if } \theta \in (0, \theta_1], \\ 0 & \textit{otherwise.} \end{cases}$$

Under this belief, the principal's optimal action after observing p_1 is

$$y_1 = arg\max_y \int_0^1 -(y-\theta)^2 \cdot \rho(\theta|p_1)d\theta = \frac{\theta_1}{2}$$

Notice that the expected payoff from accepting p_1 is exactly the same as the expected payoff from rejecting p_1 because

$$\int_0^1 -(y_1-\theta)^2 \cdot \rho(\theta|p_1)d\theta = -\frac{(\theta_1)^2}{12} = p_1 - b^2.$$

Hence, the principal is indifferent between accepting and rejecting p_1. Therefore, any $d^P(p_1) \in [0,1]$ is sequentially rational. Applying the same logic to p_2 allows us to conclude that the principal is indifferent between accepting and rejecting p_2 so that rejecting p_2 with probability 1 is also sequentially rational.

Second, let us look at the principal's incentive off the equilibrium under the following beliefs;

$$\rho(\theta|p) = \begin{cases} 0 & if\, \theta \in (0,1], \\ 1 & if\, \theta = 0 \end{cases} \qquad for\ any\ p < b^2\ and\ p \neq p_1, p_2,$$

and

$$\rho(\theta|p) = 1 \qquad for\ any\ p \geq b^2.$$

Suppose that the out-of-equilibrium price offer is $p < b^2$. Then the principal's action, which is sequentially rational under the belief we take, is $y^P(p) = 0$. Then the principal's expected payoff from rejecting any $p < b^2$ is 0 while she would expect to get $p - b^2 < 0$ from accepting the offer. Therefore, the principal rejects the offer whenever $p < b^2$. On the other hand, suppose that the out-of-equilibrium price offer is $p \geq b^2$. Then the principal's action, which is sequentially rational under the belief we take, is $\frac{1}{2}$. Then the principal would expect to get $-\frac{1}{12}$ from rejecting any $p \geq b^2$ while her expected payoff from accepting the offers is $p - b^2 \geq 0$. Thus, the principal accepts the price offer whenever $p \geq b^2$.

Third, consider the agent's incentives. Let us define the agent type θ's expected utility from making a price offer p_i as follows;

$$EU(\theta, p_i) = -(y_i - \theta - b)^2 \cdot d^P(p_i) - (1 - d^P(p_i)) \cdot p_i. \tag{A.4}$$

We will show that any agent of type $\theta \in (\theta_{n-1}, \theta_n)$ has no incentive to make an offer p_m where $m \neq n$. By the strict concavity of $EU(\theta, p_1)$ on θ and the linearity of $EU(\theta, p_2)$ on θ, it is sufficient to show that the boundary type θ_1 is indifferent between making the offers p_1 and p_2. Formally, we need to find $\theta_1 \in (0,1)$ such that

$$EU(\theta_1, p_1) = EU(\theta_1, p_2). \tag{A.5}$$

Plugging equation (**??**) into (**??**) gives the following;

$$\left(\frac{\theta_1}{2} + b\right)^2 \cdot d^P(p_1) + \left(b^2 - \frac{\theta_1^2}{12}\right) \cdot (1 - d^P(p_1)) = b^2 - \frac{(1-\theta_1)^2}{12}. \tag{A.6}$$

After some rearrangement, we have

$$4d^P(p_1)\theta_1^2 + 2(6bd^P(p_1) - 1)\theta_1 + 1 = 0. \tag{A.7}$$

Since $d^P(p_1) \neq 0$, the above equation can be treated as a polynomial in θ_1. Therefore,

$$\theta_1 = \frac{(1 - 6bd^P(p_1)) \pm \sqrt{(1 - 6bd^P(p_1))^2 - 4d^P(p_1)}}{4d^P(p_1)}. \tag{A.8}$$

To complete our discussion on the existence of monotonic informative equilibrium, it is necessary to show that there exist $d^P(p_1) \in (0,1)$ and $\theta_1 \in (0,1)$ that satisfy the equation (**??**). It is easy to verify that equation (A.8) is satisfied by the following parameters:

$$\theta_1 = 0.6 \quad and \quad d^P(p_1) = \frac{5}{36(1+5b)}.$$

Since $0 < \frac{5}{36(1+5b)} < 1$ for all $b > 0$, we conclude that for any $b > 0$, there exists a (two-step) monotonic perfect Bayesian equilibrium with informative price offers.

Calculations for Example 2. Let us look at the principal's incentive. Under the strategy profile above, if the principal observes the price offer p_1 she then updates her belief by Bayes' rule, i.e.

$$\rho(\theta|p_1) = \begin{cases} \frac{1}{1+\theta_1-\theta_2} & if\ \theta \in [0, \theta_1] \cup [\theta_2, 1], \\ 0 & if\ \theta \in (\theta_1, \theta_2). \end{cases}$$

The principal's optimal action y_1 after rejecting p_1 is the following;

$$y_1 = arg\max_y \int_0^1 -(y-\theta)^2 \rho(\theta|p_1)d\theta = \frac{1+\theta_1^2-\theta_2^2}{2(1+\theta_1-\theta_2)} = \frac{13}{50} \tag{A.9}$$

If the principal observes the price offer p_2 then she again updates her belief by Bayes' rule, i.e.

$$\rho(\theta|p_2) = \begin{cases} \frac{1}{\theta_2-\theta_1} & if\ \theta \in (\theta_1, \theta_2), \\ 0 & otherwise. \end{cases}$$

The principal's optimal action y_2 after rejecting p_2 is the following;

$$y_2 = arg\max_y \int_0^1 -(y-\theta)^2 \rho(\theta|p_2)d\theta = \frac{\theta_1+\theta_2}{2} = \frac{37}{50} \tag{A.10}$$

Thus, the principal's actions induced by the price offers on the equilibrium path are sequentially rational under the posterior belief derived from Bayes' rule. It is easy to see that the principal's out-of-equilibrium strategy is sequentially rational if we have the same belief as in the previous example.

Since the principal should be indifferent between accepting and rejecting p_1, we have

$$p_1 = b^2 + \int_0^1 -(y_1-\theta)^2 \rho(\theta|p_1)d\theta = b^2 + \frac{(y_1-\theta_1)^3 - y_1^3 + (y_1-1)^3 - (y_1-\theta_2)^3}{3(1+\theta_1-\theta_2)} = \frac{29081}{30000} \tag{A.11}$$

Similarly, we have

$$p_2 = b^2 + \int_0^1 -(y_2-\theta)^2 \rho(\theta|p_2)d\theta = b^2 - \frac{(\theta_1-\theta_2)^2}{12} = \frac{47}{48} \tag{A.12}$$

To see that the agent's price offers are incentive compatible, it is sufficient to show that the boundary types θ_1 and θ_2 are indifferent between making the offer p_1 and p_2. From these arbitrage conditions, we get

$$EU(\theta_1, p_1) = EU(\theta_1, p_2) \tag{A.13}$$

and

$$EU(\theta_2, p_1) = EU(\theta_2, p_2), \tag{A.14}$$

where the agent type θ's expected payoff from making the offer p_i is

$$EU(\theta, p_i) = -(y_i-\theta-b)^2 d^P(p_i) - (1-d^P(p_i))p_i.$$

From (A.9), (A.10), (A.11), (A.12), (A.13), and (A.14), we get

$$d^P(p_1) = 0.00844682 \quad and \quad d^P(p_2) = 0.0125013.$$

It remains to show that no agent type has an incentive to deviate to offers off the equilibrium path. This is equivalent to showing that all agent types who make the offer p_i get higher expected payoff than $-b^2$ for all $i = 1, 2$. Formally, we need to show that

$$EU(\theta, p_1) > -b^2, \quad \forall\theta \in [0, \theta_1] \quad and \quad EU(\theta, p_2) > -b^2, \quad \forall\theta \in (\theta_1, 1] \tag{A.15}$$

Notice that two indifference conditions and the convexity of utility function on θ guarantee that for any $\theta \in [0, \theta_1] \cap [\theta_2, 1]$, $EU(\theta, p_1) > EU(1, p_1)$ and for any $\theta \in [\theta_1, \theta_2]$, $EU(\theta, p_2) > EU(1, p_1)$. Since we have $EU(1, p_1) = -0.986752 > -1 = -b^2$, there is no agent type who wants to deviate to any other price offers. This completes our discussion on the existence of non-monotonic equilibria.

Proof of Proposition 7. First, consider the principal's incentives on the equilibrium path. After observing p_n, the principal updates her belief using Bayes rule, i.e.

$$\rho(\theta|p_n) = \frac{\mu^A(p_n|\theta)}{\int_0^1 \mu^A(p_n|\theta')d\theta'} = \begin{cases} \frac{1}{\theta_n - \theta_{n-1}} & if\ \theta \in (\theta_{n-1}, \theta_n], \\ 0 & otherwise. \end{cases}$$

Under this belief, the principal's optimal action after observing p_n is

$$y^P(p_n) = arg\max_y \int_0^1 -(y-\theta)^2 \cdot \rho(\theta|p_n)d\theta = \frac{\theta_{n-1} + \theta n}{2}$$

Rewriting this by using (a) gives us the following:

$$y^P(p_n) = \frac{(2n-1)x}{2} - \frac{(n-1)^2\delta}{2}.$$

Notice that the expected payoff from accepting p_n is exactly the same as the expected payoff from rejecting p_n because

$$\int_0^1 -(y^P(p_n) - \theta)^2 \cdot \rho(\theta|p_n)d\theta = -\frac{(x-(n-1)\delta)^2}{12} = p_n - b^2$$

Hence, the principal is indifferent between accepting and rejecting p_n for any $n = 1, ...N$.

Second, consider the principal's incentives off the equilibrium path. Suppose that the out-of-equilibrium price offer $p < b^2$. Then the principal's action which is sequentially rational under the belief we take is $y^P(p) = 0$. Then the principal would get 0 in expectation from rejecting any p while she would get $p - b^2 < 0$ from accepting it in expectation. Therefore, the principal rejects the offer whenever $p < b^2$. On the other hand, suppose that the out-of-equilibrium price offer $p > b^2$. Then the principal's action which is sequentially rational under the belief we take is $y^P(p) = \frac{1}{2}$. Then the principal would get $-\frac{1}{12}$ in expectation from rejecting any $p > b^2$ while she would get $p - b^2$ from accepting it in expectation. Thus, the principal accepts the price offer whenever $p > b^2$.

Third, consider the agent's incentives. We will show that we can choose $\delta \in (0, 1)$ and $x \in (0, 1)$ such that the agent's price offers are incentive compatible. As a first step, we will show that any agent type $\theta \in (\theta_{n-1}, \theta_n)$ has no incentive to make an offer p_m where $m \neq n$. As a second step, we will show that any agent type $\theta \in [0, 1]$ has no incentive to make an offer off the equilibrium path.

Given y_n, p_n, d^* and θ_n, let us define the agent type θ's expected utility from making a price offer p_n as follows;

$$EU(\theta, p_n) = -(y_n - \theta - b)^2 \cdot d^* - (1 - d^*) \cdot p_n. \tag{A.16}$$

Then, it is easy to see that

$$EU(\theta_n, p_n) = \frac{b(-12b^2 + x^2 + n(n-1)\delta^2 + (x(1-2n) - 4b)\delta)}{4(3b + \delta)} = EU(\theta_n, p_{n+1}) \tag{A.17}$$

Therefore, the following arbitrage condition hold for all $n = 1, ..., N - 1$.

$$EU(\theta_n, p_n) = EU(\theta_n, p_{n+1}). \tag{A.18}$$

We need to show that for any $\theta \in (\theta_{n-1}, \theta_n)$,

$$EU(\theta, p_n) \geq EU(\theta, p_m), \quad \forall m \neq n.$$

By the arbitrage condition and the strict concavity of $EU(\theta, p)$ on θ, this is the same as showing that

$$EU(\theta_n, p_n) - EU(\theta_n, p_m) \geq 0, \quad \forall m \neq n. \tag{A.19}$$

From equation (**??**), we have

$$EU(\theta_n, p_n) - EU(\theta_n, p_m) = -(y_n - \theta_n - b)^2 d^* - (1 - d^*)p_n + (y_m - \theta_n - b)^2 d^* + (1 - d^*)p_m$$

$$= \frac{\delta}{16(3b + \delta)}(m - n)(m - n - 1)(-2x + (m + n - 2)\delta)(-2x + (m + n - 1)\delta). \quad \text{(A.20)}$$

Notice that the length of each interval should be positive, i.e.

$$x - (n - 1)\delta > 0. \quad \text{(A.21)}$$

Then,

$$-2x + (m + n - 2)\delta < (m - n)\delta, \ \ and \ -2x + (m + n - 1)\delta < (m - n + 1)\delta. \quad \text{(A.22)}$$

Thus, if $m \leq n - 1$ then

$$EU(\theta_n, p_n) - EU(\theta_n, p_m) > \frac{\delta^3}{16(3b + \delta)}(m - n)^2((m - n)^2 - 1) \geq 0. \quad \text{(A.23)}$$

Suppose that $m \geq n + 1$. Then the choice of $\delta \in (0, \frac{2x}{m+n-1})$ gives us

$$(-2x + (m + n - 2)\delta) < 0 \ and \ (-2x + (m + n - 1)\delta) < 0, \quad \text{(A.24)}$$

so that condition (A.19) holds. Choose $\delta = \delta_1^* \in (0, \frac{x}{N-1})$. Then, for all $n = 1, ...N$,

$$EU(\theta_n, p_n) \geq EU(\theta_n, p_m), \quad for\ all\ m \neq n \quad \text{(A.25)}$$

It remains to show that for all $n = 1, ..., N$, $EU(\theta, p_n) > -b^2$ for any agent type $\theta \in (\theta_{n-1}, \theta_n)$. Since $EU(\theta_n, p_n) < EU(\theta, p_n)$ for all $\theta \in (\theta_{n-1}, \theta_n)$, it is sufficient to show that $EU(\theta_n, p_n) > -b^2$. From equation (A.16), we have

$$EU(\theta_n, p_n) + b^2 = \frac{b(x^2 + n(n - 1)\delta^2 + n\delta(1 - 2n))}{4(3b + \delta)}. \quad \text{(A.26)}$$

Therefore, if we choose $\delta_n \in (0, \frac{x^2}{n(2n-1)})$, then $EU(\theta_n, p_n) + b^2 > 0$. Since N is finite, we can choose $\delta_2^* = \min\{\delta_1, \delta_2, ..., \delta_N\}$. Take $\delta = \min\{\delta_1^*, \delta_2^*\}$.

Notice that from the condition (a) and (b) we have $\delta = \frac{2x}{N-1} - \frac{2}{(N-1)N}$. To complete this proof, we need to show that there exists $x \in (0, 1)$ such that $\delta \in (0, \min\{\delta_1^*, \delta_2^*\}] =$

$(0, \min\{\frac{x}{N-1}, \frac{x^2}{N(2N-1)}\}]$. Take $x = \frac{1}{N} + \varepsilon$ with arbitrarily small $\varepsilon > 0$. Then $\delta = \frac{\varepsilon}{N-1}$. Since both δ_1^* and δ_2^* are strictly increasing in x, $\min\{\delta_1^*, \delta_2^*\} > \min\{\frac{1}{N(N-1)}, \frac{1}{N^3(2N-1)}\} = \frac{1}{N^3(2N-1)}$. It is straight forward to see that with an arbitrarily small $\varepsilon > 0$ (or more precisely when $0 < \varepsilon < \frac{N-1}{N^3(2N-1)}$), $\delta = \frac{\varepsilon}{N-1} < \frac{1}{N^3(2N-1)}$ so that $\delta \in (0, \min\{\delta_1^*, \delta_2^*\}]$. This completes the proof.

Proof of Proposition 8. The first result is directly from the previous discussion. It remains to show that $EU^A \in (-b^2, \sigma - b^2)$. Before I show this, I will first show that $d^P(p(\theta)) \in [0, \frac{1}{2}]$ for all $\theta \in [0,1]$ and use this to prove the result. Take an arbitrary price offer p_0 on the equilibrium path and let Θ_0 be the set of all agent types who make the offer p_0. Let σ_0 denote the conditional variance of θ, i.e. $\sigma_0 = Var(\theta|\theta \in \Theta_0)$. Then by $(ID-P)$, $p_0 = b^2 - \sigma_0$. Moreover, in order for any agent types in Θ_0 not to have an incentive to deviate to off the equilibrium price offers, it is necessary that

$$-d^P(p_0) \cdot (y^P(p_0) - \theta - b)^2 - (1 - d^P(p_0))(b^2 - \sigma_0) \geq -b^2. \tag{A.27}$$

Take integral with respect to θ in both sides. After some rearrangements, we get

$$d^P(p_0)(-2\sigma_0^2) + \sigma_0^2 \geq 0 \;\; or \quad d^P(p_0) \leq \frac{1}{2} \tag{A.28}$$

This is true for any equilibrium price offers so that we have $d^P(p(\theta)) \in [0, \frac{1}{2}]$ as was to be shown.

Next, to get the upper bound of EU^A, let us rearrange (1.9).

$$EU^A = -\int_0^1 p(\theta) f(\theta) d\theta + 2\int_0^1 d^P(p(\theta))\{p(\theta) - b^2\} f(\theta) d\theta.$$

Since $p(\theta) - b^2 \in (-\sigma, 0)$, $p(\theta) \in (b^2 - \sigma, b^2)$, and $d^P(p(\theta)) \in [0, \frac{1}{2}]$ for all $\theta \in [0,1]$, we have

$$-\sigma - b^2 < EU^A < \sigma - b^2. \tag{A.29}$$

In order for the agent type not to have an incentive to deviate to off-the-equilibrium-path price offers,

$$-(y^P(p(\theta)) - \theta - b)^2 d^P(p(\theta)) - (1 - d^P(p(\theta)))p(\theta) \geq -b^2, \quad \forall \theta \in [0,1].$$

The non-existence of fully separating equilibrium gives

$$-(y^P(p(\theta)) - \theta - b)^2 d^P(p(\theta)) - (1 - d^P(p(\theta)))p(\theta) > -b^2, \quad \textit{for some } \theta \in [0,1].$$

Taking integral with respect to θ in both sides gives

$$EU^A > -b^2. \tag{A.30}$$

From (A.29) and (A.30), we have $EU^A \in (-b^2, \sigma - b^2)$. This completes our proof.

Proof of Proposition 10. First, we claim that the perfect Bayesian equilibrium with $p^* = l(b) - \sigma$ demonstrated in proposition 1 satisfies the condition (**BA5**). To see this, suppose that the principal observes the deviation $p < p^*$ and hypothesizes that a subset Θ' of Θ is responsible for the deviation. Let $y^P(\Theta')$ be the principal's optimal action under the posterior belief that is the prior belief renormalized over Θ'. Then the principal's expected payoff from rejecting the deviation is $\int_{\Theta'} U^P(y^P(\Theta'), \theta) \cdot \rho(\theta|p) d\theta \geq -\sigma$. Since the principal's expected payoff from accepting the deviation is $p - l(b) < -\sigma$, the principal rejects the deviation. Then the agent type θ gets $U^A(y^P(\Theta'), \theta, b)$ from offering the deviation while she gets $-p^* = \sigma - l(b)$ in equilibrium. In order for Θ' to be a consistent interpretation for the deviation p, we need the following condition:

$$\sigma - l(b) < U^A(y^P(\Theta'), \theta, b), \quad \forall \theta \in \Theta'. \tag{A.31}$$

After rearranging the righthand side of the equation , we get

$$\sigma < l(b) - l(|\theta - y^P(\Theta') + b|), \quad \forall \theta \in \Theta'. \tag{A.32}$$

Since $y^P(\Theta') \in C(\Theta')$, where $C(\Theta')$ is the convex hull of Θ', there exists $\theta \in \Theta'$ such that $\theta > y^P(\Theta')$. Then at $\theta > y^P(\Theta')$, $l(b) < l(|\theta - y^P(\Theta') + b|)$ so that we have

$$\sigma < 0,$$

which leads to a contradiction. Therefore, any deviation $p < p^*$ cannot have a consistent interpretation.

Similarly, suppose that the principal observes the deviation $p > p^*$ and hypothesize that a subset Θ' of Θ is responsible for the deviation. If the sequential rationality determines that the principal accepts the deviation, then the agent type θ gets $-p < -p^*$ from offering the deviation. Therefore, there is no agent type who wants to offer the deviation. If the sequential rationality determines that the principal rejects the deviation, then the agent type θ gets $-l(|y^P(\Theta') - \theta - b|)$ from offering the deviation while she gets $-p^* = \sigma - l(b)$ in equilibrium. Then, we get the condition (A.31) again for Θ' to be a consistent interpretation for the deviation p. This is a contradiction. Therefore, any deviation $p > p^*$ of the perfect Bayesian equilibrium cannot have a consistent deviation.

Second, I claim that in any perfect Bayesian equilibrium with $p^* \neq l(b) - \sigma$, a deviation $\widehat{p} = p^* - \varepsilon$ (with arbitrarily small $\varepsilon > 0$ so that $\widehat{p} > l(b) - \sigma$) has a consistent interpretation $\Theta = [0, 1]$. To prove this, suppose that the principal observes the deviation $\widehat{p}$ and hypothesizes that all agents types are responsible for the deviation. Then the principal's posterior belief is the same as the prior belief. Given this belief, the principal optimally chooses an action μ if he rejects the deviation and his expected payoff is $-\sigma$. If he accepts the deviation, then he also gets the expected payoff $(p^* - \varepsilon) - l(b) > -\sigma$. Thus, it is optimal for the principal to accept the deviation. Then the agent type θ gets $-\widehat{p} = \sigma - l(b)$ from the deviation while he gets $-p^* < -\widehat{p}$. Therefore, all agent types in $[0, 1]$ strictly prefer their payoffs from offering the deviation to their equilibrium payoff. This completes the proof.

Proof of Proposition 11. Suppose that $b < l^{-1}(\sigma)$. Take any informative perfect Bayesian equilibrium and consider a deviation $\widehat{p} = 0$. Suppose that the principal hypothetically assumes that $\widehat{p}$ is from $[0, 1]$. Then the principal accepts the offer because accepting the offer gives $-l(b)$ while rejecting it gives $-\sigma$. Notice that, for all $\theta \in [0, 1]$, the equilibrium payoff of the agent type θ is strictly less than 0. Thus, all agent types in $[0, 1]$ strictly prefer making $\widehat{p}$ to making equilibrium price offers. This implies that the deviation $\widehat{p}$ has consistent interpretation $[0, 1]$, which destroys the original equilibrium.

APPENDIX B

PROOFS FOR CHAPTER 2

Proof of Lemma 4. First, any price offer $p < 0$ is accepted by all agent types because for any $\theta \in [0,1]$, $U^A(y,\theta,b) < -p$ for any action $y \in \mathbb{R}$. Thus, in the remainder of the proof, take $p \geq 0$. Let $\overline{y}$ denote the principal's action induced by the offer p. Suppose that an agent type $\overline{\theta} \in [0,1]$ accepts the price offer p with positive probability in equilibrium. Then we have $U^A(\overline{y},\overline{\theta},b) \leq -p$. By continuity, there exists $\theta_p \in [0,1]$ such that $U^A(\overline{y},\theta_p,b) = -p$. (Otherwise, we have $U^A(\overline{y},\theta,b) < -p$ for any $\theta \in [0,1]$ so that all agent types accept the offer, which means the proof is done.) Now, suppose that $\theta_p > \overline{\theta}$. Then by quasi-concavity of U^A, the set of agent types who reject p with probability one is $(\theta_p,\theta']$ with $\theta_p < \theta' \leq 1$. Since the agent type θ' rejects the offer, we have $U^A(\overline{y},\theta',b) \geq -p$. However, by **(B4)** and Bayes' rule, we have $\overline{y} = y(\theta_p,\theta')$ and by Condition 1, $U^A(\overline{y},\theta',b) < U^A(\overline{y},\theta_p,b) = -p$, which leads to a contradiction. Thus, we have $\theta_p \leq \overline{\theta}$. Then, by the strict-concavity of U^A, we get $U^A(\overline{y},\theta,b) < -p$ for all $\theta > \theta_p$. This implies that under Condition 1, the monotonicity holds. We complete our proof by pointing out that the uniform prior satisfies Condition 1.

Proof of Proposition 14. Lemma 4 implies that for any $p \in \mathbb{R}$, both $\Theta(p)$ and $\Theta^{-1}(p)$ are convex if they are non-empty. Further, $\Theta(p)$ cannot be to the left of $\Theta^{-1}(p)$. These guarantee that for any $p \in \mathbb{R}$ there is at most one agent type who is indifferent between accepting and rejecting the offer. Let $\theta_p \in [0,1]$ denote the agent type if it exists. Then we can write that $\Theta(p) = (\theta_p, 1]$ and $\Theta^{-1}(p) = [0,\theta_p)$. From the indifference condition at θ_p we have

$$p = l(|y^P(p) - \theta_p - b|), \tag{B.1}$$

where

$$y^P(p) = arg\max_y \int_0^{\theta_p} -l(|y-\theta|)\cdot\frac{f(\theta)}{F(\theta_p)}d\theta = y(0,\theta_p). \tag{B.2}$$

Notice that $y(0,\theta_p) < \theta_p$. Then from (**??**), we have

$$\theta_p = y(0,\theta_p) + l^{-1}(p) - b. \tag{B.3}$$

Then the principal chooses p^* to solve

$$\begin{aligned}\max_{p\in\mathbb{R}} EU^P &= \int_0^{\theta_p} -l(|y^P(p)-\theta|)d\theta + (1-\theta_p)(p - l(b)) \\ &\quad \text{s.t. (??).}\end{aligned} \tag{B.4}$$

Since, from (B.2), $\frac{\partial y^P(p)}{\partial\theta_p} = \frac{\partial y(0,\theta_p)}{\partial\theta_p}$ and f has a full support, $0 < \frac{\partial y^P(p)}{\partial\theta_p} < 1$. From (B.3), we have

$$\frac{\partial\theta_p}{\partial p} = \frac{\partial y^P(p)}{\partial\theta_p}\cdot\frac{\partial\theta_p}{\partial p} + \frac{\partial l^{-1}(p)}{\partial p}.$$

After some rearrangement, we get

$$\frac{\partial\theta_p}{\partial p} = \frac{1}{1-\frac{\partial y^P(p)}{\partial\theta_p}}\cdot\frac{\partial l^{-1}(p)}{\partial p}.$$

Since $\frac{\partial l^{-1}(p)}{\partial p} \geq 0$, we have $\frac{\partial\theta_p}{\partial p} \geq 0$. This, together with $0 < \frac{\partial y^P(p)}{\partial\theta_p} < 1$, implies that $\frac{\partial y^P(p)}{\partial p} > 0$. Taking a derivative in (**??**) w.r.t. p yields

$$\frac{\partial EU^P}{\partial p} = \frac{\partial\theta_p}{\partial p}\cdot(-l(|y(0,\theta_p)-\theta_p|) - p + l(b)) + (1-\theta_p). \tag{B.5}$$

At $\theta_p = 0$, we have

$$\frac{\partial EU^P}{\partial p}\Big|_{\theta_p=0} = \frac{\partial\theta_p}{\partial p}\Big|_{\theta_p=0}\cdot(l(b)-l(b)) + (1-0) = 1 > 0. \tag{B.6}$$

At $\theta_p = 1$, we have

$$\frac{\partial EU^P}{\partial p}\Big|_{\theta_p=1} = \frac{\partial\theta_p}{\partial p}\Big|_{\theta_p=1}\cdot(-l(|y(0,1)-1|) - l(|1-y(0,1)+b|) + l(b)) < 0. \tag{B.7}$$

Taking a derivative in (B.5) w.r.t. p yields

$$\begin{aligned}\frac{\partial^2 EU^P}{\partial p^2} &= \frac{\partial^2 \theta_p}{\partial p^2}\cdot(-l(|y(0,\theta_p)-\theta_p|)-p+l(b)) \\ &+\frac{\partial \theta_p}{\partial p}\cdot(-l'(|y(0,\theta_p)-\theta_p|)\cdot(-\frac{\partial y(0,\theta_p)}{\partial p}+\frac{\partial \theta_p}{\partial p})-1)-\frac{\partial \theta_p}{\partial p}. \end{aligned} \tag{B.8}$$

It is routine to verify that

$$\frac{\partial^2 EU^P}{\partial p^2}<0 \quad \textit{if } \theta_p\in[0,1].$$

Therefore, by continuity, the principal's optimal price offer p^* is unique and $\theta_{p^*}\in[0,1]$. This completes our proof.

Proof of Proposition 16. Note that for any $\theta\in\Theta$, the agent's payoff in the truth-telling equilibrium is $-l(b)$. Any singleton subset of Θ could not be self-signaling because sending a neologism message by himself reveals his true type to the principal so that the agent type in the set could not get more than $-l(b)$. Thus, suppose that an arbitrary non-singleton subset $\widehat{\Theta}$ of Θ sends a neologism message $\widehat{m}$ to the principal. Let $\widehat{p}$ denote the price offer induced by $\widehat{\Theta}$. Then $\widehat{p}\geq l(b)$ because, otherwise, all agent types in Θ would be strictly better off by accepting the offer $\widehat{p}$ which implies $\widehat{\Theta}=\Theta$. However, by Lemma 4, making the price offer $\widehat{p}<l(b)$ is never optimal for the principal who believes that $\widehat{\Theta}=\Theta$. Thus, the price offer induced by $\widehat{\Theta}$ is greater than or equal to $l(b)$. In order for the neologism $\widehat{m}$ to be credible, all agent types in $\widehat{\Theta}$ should reject $\widehat{p}$ since accepting $\widehat{p}$ gives them at most $-l(b)$. However, rejecting $\widehat{p}$ cannot give all agent types in $\widehat{\Theta}$ higher payoff than $-l(b)$ either because the principal's action induced by $\widehat{\Theta}$, denoted by $y^P(\widehat{p})$, is always in the interior of $C(\widehat{\Theta})$, convex hull of $\widehat{\Theta}$, and as a result, there always exist some agent types to the right of $y^P(\widehat{p})$ who get strictly less payoff than $-l(b)$. Therefore, $\widehat{m}$ cannot be a credible neologism.

Proof of Proposition 19. It suffices to show that for an arbitrary subset S of Θ such that for any $\theta\in S$ $x(\theta)=\overline{x}$ and $p(\theta)=\overline{p}$,

$$\int_S [U^P(\overline{y}^P(\overline{p}),\theta)+U^A(\overline{y}^P(\overline{p}),\theta,b)]f(\theta)d\theta\leq -l(b)=-b^2. \tag{B.9}$$

It is well-known from cheap-talk literature that

$$\int_S U^A(\overline{y}^P(\overline{p}), \theta, b) f(\theta) d\theta = \int_S U^P(\overline{y}^P(\overline{p}), \theta) f(\theta) d\theta - b^2.$$

Therefore, we get

$$\int_S [U^P(\overline{y}^P(\overline{p}), \theta) + U^A(\overline{y}^P(\overline{p}), \theta, b)] f(\theta) d\theta = \int_S 2U^P(\overline{y}^P(\overline{p}), \theta) f(\theta) d\theta - l(b) \leq -l(b) = -b^2.$$

APPENDIX C

PROOFS FOR CHAPTER 3

Proof of Theorem 1. In order to verify that (3.5) is indeed an equilibrium, we have to check the second order condition and confirm that individual spending (3.5) leads to a non-negative expected payoff for each player.

In the symmetric equilibrium, player i's expected payoff is

$$V \cdot \left(\sum_{i=0}^{n-1} C_i^{n-1} p^i \left(1-p\right)^{n-i-1} \frac{(1-r)i+1}{(i+1)^2} \right) \geq 0. \tag{C.1}$$

It means that each player prefers to spend (3.5) instead of nothing, 0, given that each other player also spends (3.5). Condition (C.1) is equivalent to

$$0 < r \leq \frac{\sum_{i=0}^{n-1} C_i^{n-1} p^i \left(1-p\right)^{n-i-1} \frac{1}{(i+1)}}{\sum_{i=0}^{n-1} C_i^{n-1} p^i \left(1-p\right)^{n-i-1} \frac{i}{(i+1)^2}}. \tag{C.2}$$

The second order condition, from equation (3.3), is

$$V \cdot \left(\sum_{\mathbf{M} \in \mathcal{P}^{\mathbf{N}_i}} p^{|\mathbf{M}|} (1-p)^{|\mathbf{N}_i \setminus \mathbf{M}|} \frac{r \sum_{j \in \mathbf{M}} X_j^r}{(X_i^r + \sum_{j \in \mathbf{M}} X_j^r)^3} \Big((r-1) X_i^{r-2} (X_i^r + \sum_{j \in \mathbf{M}} X_j^r) - 2r X_i^{2r-2} \Big) \right) \leq 0.$$

In the symmetric equilibrium, this condition becomes

$$0 < r \leq \frac{\sum_{i=0}^{n-1} C_i^{n-1} p^i (1-p)^{n-i-1} \frac{i}{(i+1)^2}}{\sum_{i=0}^{n-1} C_i^{n-1} p^i (1-p)^{n-i-1} \frac{i(i-1)}{(i+1)^3}}. \tag{C.3}$$

Note that if $0 < r \leq \frac{n+1}{n}$, then inequalities (C.2) and (C.3) always hold.

Proof of Theorem 2. Fix $n \geq 3$. Then from (3.5) and (3.8)

$$\frac{1}{rV} \cdot \frac{\partial X^*(r,V,n,p)}{\partial p} = \frac{(n-1)(1-p)^{n-2}}{4} + \sum_{i=1}^{n-2} \frac{(n-1)!}{i!(n-i-2)!} p^i (1-p)^{n-i-2} \left(\frac{i+1}{(i+2)^2} - \frac{i}{(i+1)^2} \right)$$

$$= \frac{1}{rV} \cdot \frac{\partial X^*(r,V,n,p)}{\partial p} = (n-1)(1-p)^{n-2} \left(\frac{1}{4} - G(p,n) \right), \tag{C.4}$$

where

$$G(p,n) = \sum_{i=1}^{n-2} C_i^{n-2} \left(\frac{p}{1-p} \right)^i \left(\frac{i^2+i-1}{(i+1)^2(i+2)^2} \right). \tag{C.5}$$

Note that

$$\frac{1}{rV} \cdot \frac{\partial X^*(r,V,n,0)}{\partial p} = \frac{(n-1)}{4} > 0$$

and since $n \geq 3$

$$\frac{1}{rV} \cdot \frac{\partial X^*(r,V,n,1)}{\partial p} = \frac{-n^2+3n-1}{n^2(n-1)} = \frac{-n(n-3)-1}{n^2(n-1)} < 0.$$

Since $\frac{1}{rV} \cdot \frac{\partial X^*(r,V,n,p)}{\partial p}$ is a continuous function of p on the interval $[0,1]$, there must exist an interior $p^*(n) \in (0,1)$ such that $\frac{1}{rV} \cdot \frac{\partial X^*(r,V,n,p^*(n))}{\partial p} = 0$. Now, we shall show that $p^*(n)$ is unique.

Note that $G(p,n) > 0$ for all $p \in [0,1]$ and function $G(p,n)$ is strictly increasing in p since

$$\frac{\partial}{\partial p} G(p,n) = \sum_{i=1}^{n-2} C_i^{n-2} \left(\frac{p}{1-p} \right)^{i-1} \frac{i}{(1-p)^2} \left(\frac{i^2+i-1}{(i+1)^2(i+2)^2} \right) > 0. \tag{C.6}$$

It means that $G(p,n)$ is equal to $1/4$ only at the unique point $p^*(n)$. Therefore, there exists a unique $p^*(n)$ such that $\frac{1}{rV} \cdot \frac{\partial X^*(r,V,n,p^*(n))}{\partial p} = 0$. Finally, from equations (C.4) and (C.6) it is clear that if $p < p^*(n)$ $(p > p^*(n))$, then $\frac{\partial X^*(r,V,n,p)}{\partial p} > 0 (< 0)$. This completes the proof.

Proof of Theorem 3. From equation (3.5), we have

$$\frac{X^*(r,V,n,p) - X^*(r,V,n-1,p)}{rV} =$$

$$= p^{n-1} \frac{(n-1)}{n^2} + \sum_{i=1}^{n-2} C_i^{n-1} p^i (1-p)^{n-i-1} \frac{i}{(i+1)^2} - \sum_{i=1}^{n-2} C_i^{n-2} p^i (1-p)^{n-i-2} \frac{i}{(i+1)^2}$$

$$
\begin{aligned}
&= p^{n-1}\frac{(n-1)}{n^2} + \sum_{i=1}^{n-2}(C_i^{n-2}+C_{i-1}^{n-2})p^i\,(1-p)^{n-i-1}\,\frac{i}{(i+1)^2} - \sum_{i=1}^{n-2}C_i^{n-2}p^i\,(1-p)^{n-i-2}\,\frac{i}{(i+1)^2} \\
&= \frac{p}{(n-1)}\left(\sum_{i=1}^{n-1}C_i^{n-1}p^{i-1}\,(1-p)^{n-i-1}\,\frac{i^2}{(i+1)^2} - \sum_{i=1}^{n-2}C_i^{n-1}p^i\,(1-p)^{n-i-2}\,\frac{i(n-i-1)}{(i+1)^2}\right) \\
&= \frac{p}{(n-1)}\cdot\frac{1}{rV}\frac{\partial X^*(p,n)}{\partial p}. && \text{(C.7)}
\end{aligned}
$$

Proof of Theorem 4. First, we shall show that $p^*(n)$ strictly decreases as n increases. From equation (C.4), the unique optimizer $p^*(n)$ must satisfy

$$G(p^*(n),n) = \frac{1}{4}. \tag{C.8}$$

Since

$$G(p,n+1) - G(p,n) =$$

$$\left(\frac{p}{1-p}\right)^{n-1}\left(\frac{n^2-n-1}{n^2(n+1)^2}\right) + \sum_{i=1}^{n-2}\frac{i}{n-i-1}C_i^{n-2}\left(\frac{p}{1-p}\right)^i\left(\frac{i^2+i-1}{(i+1)^2(i+2)^2}\right) > 0,$$

for any $p \in (0,1)$ and $\frac{\partial G(p^*,n)}{\partial p^*} > 0$, we get

$$p^*(n) > p^*(n+1), \quad \text{for} \quad n \geq 2. \tag{C.9}$$

Second, we shall show that function $X^*(r,V,n,p^*(n))$ strictly decreases as n increases. From Theorem 3, we have

$$X^*(r,V,n,p^*(n)) = X^*(r,V,n-1,p^*(n)). \tag{C.10}$$

From (C.9), it follows that $p^*(n) < p^*(n-1)$. By the definition of $p^*(n-1)$ and its uniqueness, we get

$$X^*(r,V,n-1,p^*(n-1)) > X^*(r,V,n-1,p^*(n)). \tag{C.11}$$

Therefore, from (C.10) and (C.11)

$$X^*(r,V,n-1,p^*(n-1)) > X^*(r,V,n,p^*(n)).$$

Proof of Theorem 5. First, we show that two curves $X^*(r, V, m, p)$ and $X^*(r, V, n, p)$ intersect. Note that $G(0, n) = G(0, m) = 0$. Therefore, from (C.4), we get

$$\frac{\partial X^*(r, V, n, 0)}{\partial p} > \frac{\partial X^*(r, V, m, 0)}{\partial p} > 0. \tag{C.12}$$

From (3.5) and (3.6), we have

$$X^*(r, V, n, 0) = 0 = X^*(r, V, m, 0) \tag{C.13}$$

and

$$X^*(r, V, n, 1) = \frac{(n-1)}{n^2} rV < \frac{(m-1)}{m^2} rV = X^*(r, V, m, 1). \tag{C.14}$$

Continuity of $X^*(r, V, n, p)$ in p together with (C.12), (C.13), and (C.14) provide the existence of an interior solution of equation (3.10). Now, we demonstrate that this interior solution is unique.

From Theorem 3, if $m = n - 1$, then there exists a unique interior solution of equation (3.10), $p(n-1, n) = p^*(n)$. It means that for all $n > m \geq 2$ the following equations have a unique interior solution

$$X^*(r, V, n, p^*(n)) = X^*(r, V, n-1, p^*(n))$$

and

$$X^*(r, V, m, p^*(m+1)) = X^*(r, V, m+1, p^*(m+1)).$$

Moreover, from Theorem 4

$$X^*(r, V, n, p) > X^*(r, V, n-1, p), \quad \textit{if} \quad 0 < p < p^*(n)$$

and

$$X^*(r, V, n, p) < X^*(r, V, n-1, p), \quad \textit{if} \quad p > p^*(n).$$

Analogously,

$$X^*(r, V, m, p) < X^*(r, V, m+1, p), \quad \textit{if} \quad 0 < p < p^*(m)$$

and

$$X^*(r,V,m,p) > X^*(r,V,m+1,p), \quad \text{if} \quad p > p^*(m).$$

Since $n > m$, Theorem 4 gives $p^*(n) < p^*(m)$. Therefore,

$$X^*(r,V,n,p) > X^*(r,V,m,p), \quad \text{if} \quad 0 < p < p^*(n)$$

and

$$X^*(r,V,n,p) < X^*(r,V,m,p), \quad \text{if} \quad p > p^*(m).$$

Hence, $X^*(r,V,m,p)$ and $X^*(r,V,n,p)$ can cross only on the interval $[p^*(n), p^*(m)]$. Note that function $X^*(r,V,m,p)$ is strictly increasing and function $X^*(r,V,n,p)$ is strictly decreasing on the interval $p \in [p^*(n), p^*(m)]$. Therefore, if there exists a solution of equation (3.10), it must be unique.

Proof of Theorem 6. First, we prove part i). Equation (3.12) yields

$$\frac{1}{rVn} \cdot \frac{\partial T^*(r,V,n,p)}{\partial p} = \sum_{i=1}^{n-1} C_i^{n-1} p^i (1-p)^{n-i-1} \frac{i}{(i+1)} - \sum_{i=1}^{n-2} C_i^{n-1} p^{i+1} (1-p)^{n-i-2} \frac{(n-i-1)i}{(i+1)^2}$$

$$= \frac{(n-1)}{2} p(1-p)^{n-2} + \sum_{i=1}^{n-2} \left(C_{i+1}^{n-1} p^{i+1} (1-p)^{n-i-2} \frac{(i+1)}{(i+2)} - C_i^{n-1} p^{i+1} (1-p)^{n-i-2} \frac{(n-i-1)i}{(i+1)^2} \right)$$

$$= \frac{(n-1)}{2} p(1-p)^{n-2} + \sum_{i=1}^{n-2} p^{i+1} (1-p)^{n-i-2} \frac{(n-1)!}{(i+1)!(n-i-2)!} \left(\frac{(i+1)}{(i+2)} - \frac{i}{(i+1)} \right)$$

$$= \left(\sum_{i=0}^{n-2} C_{i+1}^{n-1} p^{i+1} (1-p)^{n-i-2} \frac{1}{(i+1)(i+2)} \right) > 0.$$

We prove part ii) now. From equation (3.12), we have

$$\frac{1}{rV} \left(T^*(r,V,k+1,p) - T^*(r,V,k,p) \right) =$$

$$p^{k+1} \frac{k}{(k+1)} + \sum_{i=1}^{k} C_i^{k+1} p^i (1-p)^{k-i+1} \frac{(i-1)}{i} - \sum_{i=1}^{k} C_i^k p^i (1-p)^{k-i} \frac{(i-1)}{i} =$$

$$\sum_{i=1}^{k+1} C_{i-1}^k p^i (1-p)^{k-i+1} \frac{(i-1)}{i} - \sum_{i=1}^{k} C_i^k p^{i+1} (1-p)^{k-i} \frac{(i-1)}{i} =$$

$$\sum_{i=2}^{k+1} C_{i-1}^{k} p^i (1-p)^{k-i+1} \frac{1}{i(i-1)} > 0.$$

Proof of Theorem 8. From equation (3.12), we get

$$T^*(r, V, n, p) = rV \cdot \left(1 - (1-p)^n - \sum_{i=1}^{n} C_i^n p^i (1-p)^{n-i} \frac{1}{i}\right). \tag{C.15}$$

Note that the function $\frac{1}{i}$ is strictly convex. By the Jensen's inequality, we have

$$\sum_{i=1}^{n} C_i^n p^i (1-p)^{n-i} \frac{1}{i} > \frac{(\sum_{i=1}^{n} C_i^n p^i (1-p)^{n-i})^2}{\sum_{i=1}^{n} C_i^n p^i (1-p)^{n-i} i} =$$

$$= \frac{(\sum_{i=1}^{n} C_i^n p^i (1-p)^{n-i})^2}{\overline{n}} = \frac{(1-(1-p)^n)^2}{\overline{n}}. \tag{C.16}$$

Combining inequality (C.16) with equation (C.15), we obtain

$$T^*(r, V, n, p) < rV \cdot \left(1 - (1-p)^n - \frac{(1-(1-p)^n)^2}{\overline{n}}\right) =$$

$$rV \frac{(\overline{n}-1)}{\overline{n}} + rV \left(\frac{2(1-p)^n}{\overline{n}} - (1-p)^n - \frac{(1-p)^{2n}}{\overline{n}}\right) < T^*(r, V, \overline{n}, 1) - \frac{rV(1-p)^{2n}}{\overline{n}}.$$

The last inequality comes from the fact that $\overline{n} \geq 2$. This completes the proof.

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