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This article presents a reexamination of the Sørensen model. This model derives the pattern of individual careers from structural considerations. If longitudinal data on individual careers are available, Sørensen's model provides two methods to infer the underlying structural parameter. This structural parameter gives a useful measure for unequal career chances. An implementation of these methods, using firm data, shows, however, that they lead to contradictory conclusions; this is shown to be the result of some unrealistic assumptions Sørensen uses in his derivation. Some more realistic assumptions are suggested that produce reasonable results. Finally, it is shown that despite these modifications, the main conclusions of the Sørensen model are preserved. This seems to be promising for future work with this model.

Dynamic Career Models and Inequality Research

A Reexamination of the Sørensen Model

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Theories of social mobility have always been dynamic by their very nature. The methods used in research on social mobility, however, were mostly static. It was not until the mid-1970s that this discrepancy began to vanish. Today a wide variety of dynamic methods is available for social-mobility research (see, e.g., Tuma and Hannan 1984). These methods have had a great impact on mobility research and their applications are now labeled as "life course research" (see, e.g., the volume edited by Mayer and Tuma 1990). The work of Aage Sørensen was certainly one of the more important

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startingpoints for this tradition (besides Nancy Tuma's 1976 work). In Sørensen's (1977, 1979) papers, he presented a mathematical model that showed how social structure shapes individual careers. In addition, this model results in equations that are easily estimable by event-history or panel methods. He showed how it is possible to infer the parameters of the model from these estimates. This close connection between theoretical model and empirical methods was pathbreaking for mobility research. In this respect, Sørensen's model can be seen as a sociological answer to the widely celebrated economic human capital model (cf. Mincer 1974; Becker 1975). The human capital model provides a convincing connection between theory and methods, too. In this model, however, careers are seen as the outcome solely of optimizing behavior of rational actors. The structural context in which careers develop is neglected. Contrary to this, Sørensen presented a model that started only from structural considerations, but finally arrived at conclusions similar to those of the human capital model. For this reason, Sørensen's approach has been widely accepted by sociologists and is today (sometimes under the name "vacancy competition model"; e.g., Sørensen 1983) the standard counterpart to the human capital model in sociological work on mobility processes.

In this article I will present a reexamination of the Sørensen model. For this purpose, the first section of the article gives a concise (and, I think, simpler) restatement of the model, with an emphasis on the two methods that allow for the estimation of the decisive parameter of the model. This parameter, estimated for different groups, informs the researcher (according to Sørensen's model) about the amount of inequality between these groups. In the second section, both methods are implemented by using data from a single firm. It is shown that these methods result in contradictory conclusions in assessing inequality. The third section identifies the reasons for these contradictory results. It is shown that the incompatible findings result from some implicit, unrealistic assumptions in Sørensen's model. In the last section, it is shown that despite the modifications introduced in this article, the main results of the model are still valid. This robustness leads to the conclusion that further work with this model seems to be promising.

THE SØRENSEN MODEL

Sørensen's model (1977, 1979; earlier, related models can be found in Sørensen 1974, 1975) starts with assumptions on the hierarchical structure of a work system and then derives the pattern of individual careers. This enables the researcher to infer the parameter that describes the structural process from data on individual careers. To assess the amount of inequality between the careers of, for example, men and women, then one must assume that men and women are separated into different work systems. On the basis of this assumption it is possible to estimate the structural parameter for every group and to assess how unequal the careers of these groups are. In empirical applications of his model Sørensen always used nationwide survey data, but, in principle, it is possible to apply his model to any work system. In my empirical application I will investigate inequality in an organizational work system. This has the advantage that most terms of the model have a more concrete interpretation than in applications to a national work system. Therefore, the following presentation will be in terms of an organizational system, but one has to recall that the model is capable of describing other work systems as well. Another qualification of the following presentation is that I will focus only on wage careers. Nevertheless, the Sørensen model is also interpretable in terms of status. Thus I will present a somewhat special application of the model. Methodologically, however, even if one special application of a model fails, this casts doubt on the whole approach.

My final aim will be a comparison of the wage careers of different groups in an organization using the Sørensen model. Therefore, the following description of the model refers to each of these groups (for notational convenience I will drop the group index). Sørensen starts with some assumptions:

A1: Wages (Y) for each group are distributed exponentially: $f(Y) = \beta \bullet \exp(-\beta Y)$, where $\beta > 0$. This is an approximation to the conventional wisdom that hierarchies are often shaped like a pyramid. The greater β is, the broader (more equal) is the pyramid [$f(0) = \beta$].
A2: This wage distribution is stable over time.
A3: On each wage level, the exit rate (λ) is the same.

A4: Mobility follows a "restricted vacancy chain," that is, a vacancy (created by an exit) trickles down level by level and is finally filled at the bottom level by a hire from outside. For the employees, this means that promotions are only one level at a time (no jumps), that there are no demotions in the system, and that entries are only at the bottom level.

Given these assumptions,[1] it is possible to show that the promotion opportunities [p(.)] are equal on each level:

$$p(Y) = \lambda/\beta. \tag{1}$$

Promotion opportunities are defined as the number of vacancies that arrive at a level during a certain time period, divided by the number of workers on that level. As we see from equation 1, opportunities increase with the exit rate and decrease with the width of the hierarchy. This is the first part of the Sørensen model that describes the structural setting in which careers unfold.[2]

The next step is the connection between structure and individual careers. Sørensen accomplishes this by stating that the promotion opportunities on a level must be equal to the sum of the individual promotion rates [r(.)].[3] He assumes that

A5: Promotion rates are not identical over all individuals, but differ according to the time (t) people have spent in the system ("experience" in general work systems, "tenure" in my application).

Hence

$$p(Y) = \int_0^\infty r(Y,t)dt. \tag{2}$$

Because of equation 1, however, both p(.) and r(.) do not depend on the wage level, and therefore an individual rate function is needed that satisfies

$$p = \int_0^\infty r(t)dt. \tag{3}$$

One function that satisfies equation 3 is (there are an infinite number of solutions to 3)

$$r(t) = \exp(-t/p) = \exp(bt), \tag{4}$$

where b is defined as $b = -1/p$. The parameter b is important in the following: It is negative, and the lower its absolute value is, the higher are the promotion opportunities for a group. Equation 4 implies that individual promotion rates decline with increasing tenure because b is negative.

Equation 4 yields the first method to estimate b empirically (Method 1). In this application, t is the current tenure. t is the sum of the tenure at the beginning of the current episode (T) and the present waiting time for a further promotion (d). Sørensen proposes to analyze promotion episodes, whereby one should separate these two time clocks. For each individual of a group, one takes the observed waiting time for a promotion as the dependent variable. Then one estimates an event-history model, where T is introduced as a covariate. The effect of T gives the estimate for b. Method 1 has been applied in the literature several times (e.g., Sørensen and Tuma 1981; Halaby 1982; Sørensen 1984). Sørensen mostly used a Gompertz model of the following form:

$$r(d|x,T) = \exp(\alpha + x'\beta + bT + \gamma d), \tag{5}$$

where x represents additional covariates. I will use this specification too, and apply it to my firm data below.[4]

In the final step, Sørensen derives from the individual promotion rates an expression for the wage profile. A wage profile can be conceived as a sequence of wage jumps. Between two consecutive jumps the wage is constant. Therefore, to describe the profile, an expression is needed for the average height of a jump (ΔY). Sørensen assumes that

> A6: each individual passes during his or her worklife a certain wage span $Y^* - Y(0)$. $Y(0)$ is the starting wage for the workers of a group. Y^* is the maximally obtainable wage for a worker and depends on covariates (z) *and* promotion opportunities: $Y^* = g(z,p)$. Sørensen (1977, p. 973) assumes the following specification: $Y^* = p \bullet (\alpha + z'\beta)$. This means that Y^* increases with increasing promotion opportunities.

Until time t, a worker can expect to experience the following number of jumps $[v(t)]$:[5]

$$v(t) = \int_0^t r(u)du = [\exp(bt) - 1] \bullet 1/b. \tag{6}$$

During his or her whole working life a worker can expect the following number of jumps: $v(\infty) = -1/b$. From this it follows that the average jump height is

$$\Delta Y = \frac{Y* - Y(0)}{v(\infty)} = -b[Y* - Y(0)]. \tag{7}$$

The wage at time t is the starting wage, plus the number of jumps until t, times the average jump height:

$$Y(t) = Y(0) + v(t) \bullet \Delta Y. \tag{8}$$

Using equations 6 and 7, I obtain after rearranging:

$$Y(t) = e^{bt}Y(0) + (1 - e^{bt})Y*. \tag{9}$$

This is the final expression for the wage profile of a group. This profile shows the well-known concave shape that is also predicted by human capital models.

Equation 9 yields the second possibility for estimating b (Method 2). One needs information on the wages of workers for several time points (from a panel, for instance). To obtain an estimable equation, equation 9 is rewritten as a difference equation:

$$Y_t = e^b Y_{t-1} + (1 - e^b)Y*. \tag{10}$$

Now introduce the covariates in Y* and add an error term to get

$$Y_t = a_1 Y_{t-1} + a_0 + z'a + u_t, \tag{11}$$

where $a_1 = \exp(b)$, $a_0 = [1 - \exp(b)]\alpha p$, $a = [1 - \exp(b)]\beta p$. This is a simple regression equation with a lagged wage variable. One has to construct a data set where each wage information (besides the first one) is a record. Then ordinary least square (OLS) is applied and $\hat{b} = \ln \hat{a}_1$. This second method to obtain an estimate of b has been used frequently in the literature (Sørensen 1975, 1979; Rosenfeld 1980; Spenner, Otto, and Call 1982; Sørensen and Blossfeld 1989). However, to the best of my knowledge, no one has used both methods with the same data set. If Sørensen's reasoning is correct, both methods should give compatible estimates of b. In the next section I will use both methods and compare the results.

AN EMPIRICAL APPLICATION OF THE MODEL

In this section I will apply Methods 1 and 2 to a particular data set. My aim is to estimate b for four different groups of the organizational work force: German men, German women, foreign men, and foreign women. With U.S. data, the criterion "nationality" is usually substituted by "race." In the German labor force, foreign workers have a similar (disadvantaged) position in the labor market. Therefore, I will use this criterion. Nevertheless, it would be possible to use any other criterion for which one suspects that career patterns are unequal.

THE DATA

As empirical data I use personnel records of blue-collar workers of a large West German mechanical engineering company (pseudonym: "South-Factory"). These data include all blue-collar workers who had an employment contract with South-Factory in the period of 1976-1984, either over the whole time period or only for a certain interval (about 7,500 workers). The data have the character of panel data (more specifically: nine-wave panel data) because there is information about all workers at the end of each year (if the worker was employed in the particular year). Besides some sociodemographic variables (sex, nationality, and age), there are the exact date of entry into the firm, the date of exit, if there was an exit before December 1984, the occupational group to which each worker belonged at the end of each year, and the respective wage group at the end of each year. The wage groups ranged from 2-10 (for more details about the data, see Brüderl 1991). The data contain no information on education and social origin. However, by looking at the blue-collar workers of a single firm only, the amount of unobserved heterogeneity introduced by these missing variables can be assumed to be not very high.

In the following I will not use the information for all these workers. Instead, I will analyze only the wage careers of those workers who entered South-Factory in the observation period. This is done because the careers of the workers who were already in the firm, when the observation period started in 1976, are left censored. Including them could introduce potential bias in the analyses (for instance, mixing up cohort with age effects). There were 3,362 new entries from 1976-1984.

To apply both methods, two different data sets have to be constructed. First, for using Method 2, one has to pool all the wage information (I assigned monthly deutsche mark [DM] wages to the wage groups by using the wage table of 1984). In the data set, there is for each worker, depending on the length of observation, at least one and at most nine pieces of information. Every wage information (besides the first one) forms an observation for the dependent variable. There are 10,608 wage observations. As covariates I use the lagged wage, if payment was per hour or piece-rate, and the current experience (computed as age-16 because there was no information on education).

To apply Method 1, the waiting time for a wage change has to be known. Based on the yearly wage information, one can determine how long a worker received the same wage; this is the waiting time. If two consecutive wages differed, it was assumed that the change occurred in the middle of the year. For most workers, more than one episode was observed. For some there were up to five episodes. Next, all these episodes have to be pooled. If an episode ended with a wage increase, an event is recorded. Episodes are censored if there was a wage decrease (very seldom), an exit from the firm, or the observation period ended. There were 6,875 episodes and 3,301 events. As covariates I use experience, the form of payment, the monthly wage, an additional dummy for the wage groups 7-10, and the tenure (in months). All these variables are measured at the beginning of the episode.

THE RESULTS

To implement Method 1, I estimate equation 5 for each of our four groups (by Maximum-Likelihood, using Trond Petersen's 1986 BMDP3R subprogram). The results are given in Table 1. It can be seen

TABLE 1: Results of the Gompertz Model (Method 1)

	German Men	*German Women*	*Foreign Men*	*Foreign Women*
Constant	–0.95	–5.71	6.06	–8.69
Experience	–0.014	–0.017	–0.023	–0.025
(in years)	(5.17)	(2.12)	(5.89)	(1.28)
Hourly paid	0.18	0.33	–0.72	–0.88
(1 = yes)	(3.77)	(0.79)	(9.61)	(2.37)
Monthly wage	–0.0014	0.0010	–0.0054	0.0031
(in DM)	(4.01)	(0.65)	(7.59)	(0.71)
Wage group 7-10	0.11	–0.10	1.43	–0.12
(1 = yes)	(1.26)	(0.10)	(6.51)	(0.01)
Firm tenure (T)	–0.0225	–0.0016	–0.0275	–0.0239
(in months)	(12.02)	(0.24)	(9.44)	(1.42)
Waiting time (d)	0.0085	0.0011	0.0043	0.0070
(in months)	(5.99)	(0.17)	(2.19)	(0.72)
Chi-square (*df*)	376 (6)	11.7 (6)	339 (6)	13.0 (6)
N	4,154	303	2,303	115

NOTE: *t* values in parentheses. Chi-square test against the model without covariates. The dependent variable is the rate to get a wage increase. All independent variables are measured at the beginning of an episode. "Experience" is age-16 (because education is unknown). "Tenure" measures the time a worker spent in the firm at the beginning of an episode. "Waiting time" captures the duration dependence. Reference group: piece-rate payment, wage group 2-6.

that experience lowers the promotion rate. The same is (in two groups) true for the wage. That is, the higher in the hierarchy a person is, the longer it will take until he gets a further promotion. This contradicts Sørensen's assumption that promotion rates are independent of the wage level.[6] In addition, the promotion rate is increasing with the waiting time. This, however, results from using the monotonic Gompertz model. Elsewhere (Brüderl 1991), I show that the promotion rates are actually inverse U-shaped. Most interesting for this purpose, however, are the estimated effects of "tenure." They give the estimates of b. As one would expect according to the Sørensen model, these are all negative. The lowest absolute value is observed for German women, followed by German men, foreign women, and foreign men. One would conclude that German women have the best promotion opportunities.

Now apply Method 2. One has to estimate four different regressions with the pooled wage data. The OLS estimates of equation 11, however, would be biased because Y_{t-1} is correlated with the error term u_t. This is the consequence of unobserved heterogeneity. Generally, the quality of a worker-job match is not measurable, so quality is part of the error term. Further, if quality and wages are positively correlated, the error term and the lagged wage variable will be correlated. Therefore, one has to apply certain correction methods (Rosenfeld and Nielsen 1984 give a comprehensive discussion of these points). I decided to use a method proposed by Abraham and Farber (1987). This method tries to capture the unobserved quality of a worker-job match by using a proxy variable: the expected tenure of a worker-job match. The reasoning behind this is that high-quality matches will endure longer. To implement the Abraham-Farber method one has to estimate a firm tenure model (by event-history methods), and then one calculates from these estimates the expected tenure for each censored worker (for workers with an exit one knows the exact tenure). Then this additional variable (and its square) is added to equation 11 (for more details on this procedure, see Brüderl 1991). Estimation can now be done by OLS because the unobserved worker-job match quality is captured by the new variable. The results are given in Table 2. Experience lowers the wage in most cases. The same is true for expected tenure. But if one expects to be very long in the firm, then one will receive considerable wage premiums. This corroborates Abraham and Farber's reasoning concerning their correction for unobserved heterogeneity. (As the Durbin-Watson test statistics show, there is no need for additional corrections for autocorrelation.) But the primary interest is in the effects of the lagged wage variables. The logarithm of these estimates is my estimate for b. In the second panel of Table 2 these estimates are given; again, all are negative. However, now foreign women have the lowest absolute value.[7]

To make the comparison easier, Table 3 summarizes the estimates of b. Note that Sørensen's two estimation methods not only give very different values for b, but even worse, the rankings of the career chances of the groups are contradictory. From Method 1, one would conclude that German women face the best opportunities; from Method 2, one would infer that foreign women (who have very bad career chances according to Method 1) have the best opportunities.[8]

TABLE 2: Pooled Regression Results (Method 2)

	German Men	*German Women*	*Foreign Men*	*Foreign Women*
Constant	184	152	135	140
Experience (EXP)	–0.47	–1.35	0.28	–1.61
(in years)	(2.14)	(3.36)	(1.33)	(3.68)
$EXP^2/100$	1.17	3.66	–1.13	4.52
	(1.79)	(3.17)	(1.78)	(4.02)
Hourly paid	9.69	7.08	–7.03	–7.56
(1 = yes)	(8.28)	(1.72)	(6.56)	(3.13)
Expected tenure (D)	–2.35	–2.78	–1.93	–4.03
(in years)	(10.48)	(4.47)	(8.81)	(4.50)
$D^2/100$	12.78	17.17	9.35	17.83
	(18.50)	(7.16)	(12.61)	(4.89)
Previous wage	0.908	0.915	0.932	0.937
(Y_{t-1}, in DM)	(178.1)	(50.5)	(141.8)	(35.29)
$\overline{Y}(1)$	1,757	1,672	1,692	1,651
$\overline{Y}^*$	2,066	1,814	1,930	1,756
$\hat{b}$	–0.097	–0.089	–0.070	–0.065
N	5,793	497	4,053	265
R^2	0.89	0.88	0.86	0.90
Durbin-Watson test	2.03	2.06	1.99	2.12

NOTE: *t* values in parentheses. $\overline{Y}(1)$ is the observed average wage at the first observation, $\overline{Y}^*$ is the maximum wage predicted for each group by the model. b is the parameter for this model. The dependent variable is the monthly wage in German marks (Y_t). "Experience" is age at the time of the observation minus 16. Reference group: piece-rate payment. "Expected tenure" is calculated as described by Abraham and Farber (1987).

This leads to the conclusion that there must be something wrong with Sørensen's arguments as presented above. I will investigate the reasons for this in the next section.

But before this is done, I will show the actual situation in South-Factory. The actual situation can best be figured out by drawing the observed and estimated wage profiles for the four groups. The observed wage profile is obtained by calculating the average wage for each year. The estimated profile is obtained by using the estimates from Table 2 and inserting them into equation 9. First, for each group, the average maximum wage must be calculated by $\hat{\overline{Y}}^* = \widehat{\alpha p} + \overline{z}'\widehat{\beta p}$

TABLE 3: Comparison of the Estimates of b (and p)

	German Men	*German Women*	*Foreign Men*	*Foreign Women*
b from Method 1	–0.0225	–0.0016	–0.0275	–0.0239
Rank	2	1	4	3
b from Method 2	–0.097	–0.089	–0.070	–0.065
Rank	4	3	2	1
p from modified Method 1	0.034	0.017	0.029	0.012
Rank	1	3	2	4
p from modified Method 2	0.092	0.085	0.068	0.063
Rank	1	2	3	4

NOTE: Results in the first panel are from Table 1, results in the second panel are from Table 2, results in the third panel are from estimates obtained with equation 4′ (not shown here), and results in the fourth panel are from Table 2, using the modified Method 2 as described in the text.

(look at Table 2 for these estimates). Then the estimated wage profile is obtained by

$$\hat{Y}(t) = e^{\hat{b}(t-1)}\overline{Y}(1) + (1 - e^{\hat{b}(t-1)})\hat{Y}*.$$

$\overline{Y}(1)$ is used, because the starting wage is unknown; only the wage at the end of the first year can be determined. Figure 1 gives these profiles. The solid lines are the estimated profiles and the dashed lines are the observed profiles. The first remarkable thing is that the estimated and observed profiles are very close. Equation 9 seems to fit the data reasonably well. If the wage profiles are compared among groups, one can at least conclude that German men (the highest profile) are the most privileged group: They get the highest wages, they have the greatest wage span (look at Panel 2 in Table 2), and they have the steepest profiles. Any reasonable method that is aimed at determining inequality should conclude that German men are the most privileged group and have the highest opportunities.[9] However, none of the two methods proposed by Sørensen do so. Thus one can conclude that these two methods not only lead to contradictory results but also that none gives the correct result.

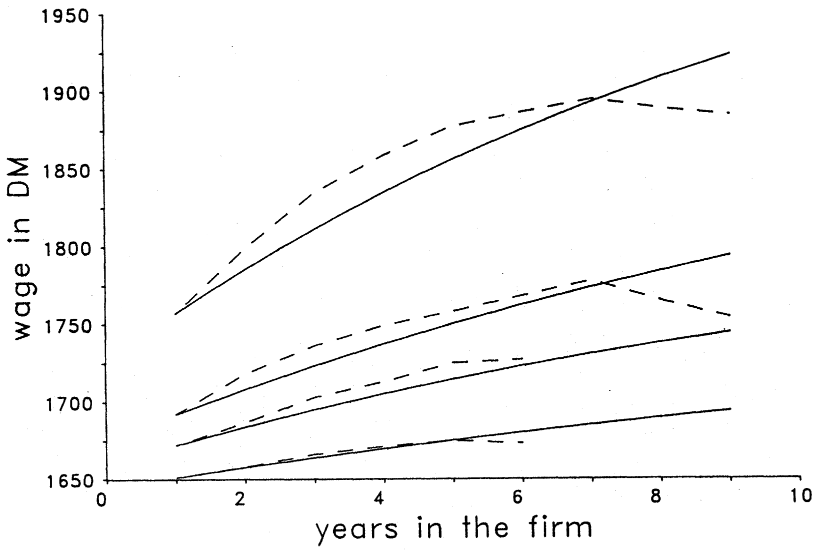

Figure 1: Observed and Estimated Wage Profiles

NOTE: Solid lines are the estimated wage profiles, and the dashed lines are the observed wage profiles (observed profiles are plotted only for $N \geq 10$, therefore the profiles for women end after 6 years). From top to bottom: German men, foreign men, German women, foreign women.

PROBLEMS OF THE SØRENSEN MODEL

In this section I will investigate the reasons why the two methods put forward by the Sørensen model apparently failed. The following reasoning will show that this is to be expected in most applications: It is not a consequence of my special application.

Let me start with the *problems of Method 2*. Obviously, I did not succeed in deriving correct estimates of the promotion opportunities from equation 11. This method shows a picture of inequality that is absolutely contrary to what can be seen in Figure 1. This leads to the presumption that there is something wrong with the derivation of equation 11. The problem is, I think, an implicit assumption that Sørensen makes on his way to equation 11 that is not met in reality. He assumes that Y* depends on the promotion opportunities. This is realistic, as seen in Figure 1. Groups with better promotion chances have higher maximum wages (ceteris paribus). This assumption has the consequence that these groups have greater wage spans. However, Sørensen assumes in addition (implicitly) that the starting wages do not depend on promotion opportunities. That is, all groups have identical starting wages. This is unrealistic, as a look at Figure 1 shows (see also Rosenfeld's [1980] results). If starting wages are correlated with opportunities, then Method 2 fails.

To see this, define a positive function s(b) that increases in b. Multiplying Y(0) with s(b) captures this correlation. Now Y* depends on b, too. To make things simple, assume that this dependence is also captured by s(b). Therefore, the wage span is now: s(b)[Y* – Y(0)]. This implies that the wage span increases with promotion opportunities, an assumption that is not unrealistic as a look at Figure 1 shows. If one uses this expression for the wage span, equation 7 has to be modified:

$$\Delta Y = -b \bullet s(b) \bullet [Y^* - Y(0)]. \tag{7'}$$

If one does the further calculations with equation 7′ instead of equation 7, one finally arrives at

$$Y(t) = (1 - s(b) + s(b)e^{bt})Y(0) + s(b)(1 - e^{bt})Y^*. \tag{9'}$$

Equation 9′ shows that, in general, it is not possible to infer an estimate for the promotion opportunities (that is, b) from a regression like equation 11 in a simple way [a_1 is now 1 – s(b) + s(b)exp(b)]. Generally, b is no longer identifiable from a_1 unless some restrictive assumptions on s(b) are made. The problem with such assumptions is that one never knows whether they are met in this particular application. If they are not met, conclusions will be false.

As noted above, German men (the group that actually has the highest promotion opportunities) have the lowest a_1 (see Table 2). That is, in this case one can observe that a_1 decreases with increasing promotion opportunities (increasing b). To see under which circumstances this might happen, take the derivative of a_1 with respect to b. This derivative is negative if $s' > (s' + s)\exp(b)$ is fulfilled. In a realistic setting,[10] p will be, say, 0.1. Thus b = –10. Then this condition will be satisfied in most cases. Therefore, if the promotion opportunities are not extremely high, one can observe that a_1 decreases with better promotion opportunities. If these a_1s are interpreted in the way Sørensen proposed, the groups will be ranked in a way that is contrary to what is actually going on. This obviously happened in my application.

Now I will propose a concrete assumption on s(b) that leads to more reasonable results (modified Method 2). Assume that $s(b) = -1/b = p$. This assumption ensures that with increasing opportunities, starting wages, maximum wages, and the wage span are higher. Using the fact that exp(b) will be almost zero, $p \approx 1 - a_1$. On the basis of this formula, the result is that German men have the highest promotion opportunities (look at Panel 4 in Table 3). This makes sense. German women rank second, which is questionable in light of Figure 1. Thus this modified Method 2 leads only to partially convincing results. It is, nevertheless, a great improvement compared with the original method. The estimated promotion opportunities now have reasonable values and the ranking is at least correct in the extremes. In spite of all this, remember that the conclusions one gets from this method depend in a decisive way on the assumption about the form of s(b).

This discussion shows that it is problematic to infer from a regression like equation 11 the underlying promotion opportunities. How then, can one interpret such regressions alternatively? I am not the first to note that Sørensen's interpretation leads to problems. Nielsen and Rosenfeld (1981; Rosenfeld and Nielsen 1984) also argued that one should not follow Sørensen in this point. However, they did not point out the problems in the model, as I did above, but arrived at this conclusion by using an alternative derivation of equation 9. Equation 9 is the solution to a particular differential equation: the so-called *partial-adjustment model* (see Coleman 1968; Tuma and Hannan 1984, chap. 11):

$$\frac{dY(t)}{dt} = -b[Y^* - Y(t)].$$

Here b is interpretable as an adjustment parameter: the higher the absolute value of b, the faster the gap between Y^* and $Y(t)$ closes. Estimates of the adjustment parameter are obtainable by taking the logarithm of the lagged wage effect. This is identical to what Method 2 proposes, but the interpretation of b is totally different: A higher absolute value of the adjustment parameter b now (ceteris paribus) points toward an advantageous position in the labor market because people approach their maximum wages much faster. This is contrary to what Sørensen proposed. However, it accords to my modified Method 2. To see this, compare Panels 2 and 4 in Table 3. Panel 4 gives the ranking obtained from modified Method 2. Panel 2 gives an identical ranking, if interpreted in accordance with the partial adjustment model (the higher the absolute value of b, the better the rank). Thus my modification resolves the discordance between the Sørensen model and the partial-adjustment model. Both models now give the same ordering of the groups. As Nielsen and Rosenfeld point out, however, b alone does not suffice to assess inequality: One should also take account of the maximum wage level and the wage span. This can best be done by plotting the estimated wage profiles, as I did in Figure 1.

Now turn to the *problems of Method 1*. The problem lies in equation 3. On the left-hand side is the average promotion rate for a group. On the right-hand side is the integral of the rates. What does this integral represent? The correct answer is given in equation 6: Here Sørensen interprets this integral as the number of jumps one can expect during one's worklife. Thus equation 3 cannot be correct. On the right-hand side, an average rate is also needed. Hence the expectation of the rate on the right side of equation 3 should be used. An improved version of equation 3 is

$$p = \int_0^\infty r(t)h(t)dt, \tag{3'}$$

where h(.) is the density of t. The solution of this integral equation will no longer be equation 4. That will be the case only if one assumes h(t) = 1, as Sørensen does. This, however, is not reasonable. In a system with restricted vacancy chains, the young workers enter at the bottom and the old ones are at the top. Thus it seems advisable to introduce a nonconstant density h(t). Again, the problem is that normally there are no indications on the functional form of h(t).

Nevertheless, I will now present a modification that seems more reasonable (modified Method 1). Assume an exponential tenure distribution: $h(t) = \gamma\exp(-\gamma t)$, with $\gamma > 0$.[11] Then a solution to the integral equation 3′ would be

$$r(t) = \frac{1}{\gamma}\exp[(b + \gamma)t] = \exp[-\ln\gamma + (b + \gamma)t]. \tag{4'}$$

This is still a monotonic declining function of t if $|b| > |\gamma|$. Equation 4′ can be estimated with my data. I estimated a constant rate model with t as the sole covariate (t is time-dependent, so I used the method of episode-splitting; results not shown here). Then an estimate of b is obtained by $\hat{b} = \hat{c} - \exp(-\hat{\alpha})$, where $\hat{c}$ is the effect of t and $\hat{\alpha}$ is the constant of this model. Then the estimates of the promotion opportunities are derived by $\hat{p} = -1/\hat{b}$. These are given in Panel 3 of Table 3. This modified Method 1 gives a ranking that is in accordance with what is known from Figure 1. Therefore, this modification finally shows how it is possible to arrive at reasonable conclusions from the Sørensen model. I think the modified Method 1 could prove valuable in future research. Nevertheless, one should always remember that the conclusions one draws depend on the assumption about h(t).[12]

CONCLUSION

The Sørensen model derives the pattern of individual careers from structural considerations. It provides two methods that allow inferring the underlying structural parameter from individual careers. An empirical application of these methods has shown that they lead to contradictory and incorrect results. I pointed out that the reasons for this are some very special and unrealistic assumptions. In most realistic settings these assumptions will not hold, and therefore the methods are not appropriate for inferring the underlying structural parameter. Thus mobility researchers should not use these methods to draw conclusions on the underlying structure. Instead, I proposed some modifications that produce reasonable results. These modified methods might be useful for future research on inequality.

Nevertheless, I think Sørensen's model is a fruitful approach for conceptualizing mobility processes. It shows how structure shapes individual careers. This is a viewpoint that must be very attractive to sociologists. The discussion above showed that there are some problems with the particular formalization that Sørensen presented. In spite of all this, however, even the modifications I introduced here do not alter the main conclusion of the model: The structural restrictions alone, given by equation 1, are sufficient to ensure concave individual wage profiles. To see this, take the first and second derivatives of equation 8 with respect to t:[13]

$$Y' = r(t) \bullet \Delta Y$$
$$Y'' = r' \bullet \Delta Y.$$

The necessary and sufficient conditions for concavity are $Y' > 0$, and $Y'' < 0$. The first condition is always met. The second is true if the individual rates are monotonically declining. Therefore, even with the modifications I proposed above, it is still the case that the main conclusion of the Sørensen model is true (equation 4′ is still a monotonically declining rate if $|b| > |\gamma|$, a condition that usually will be met in real work systems).[14] I think it would be very rewarding to try further generalizations of the model. It seems to be especially promising to modify assumptions in the structural setting, with which Sørensen started his formalization. But I leave this task to future research.

Concerning empirical research on inequality, however, the arguments put forward in this article show how strongly the conclusions one draws from the Sørensen model depend on the underlying assumptions. If one is willing to make these assumptions, the modified methods proposed above can be used to assess inequality. If one does not want to make these assumptions, the general methodology still can be used (now, however, lacking the theoretical foundation). Both methods have been applied widely in mobility research. Event-history models for the waiting time until a wage (or status) change occurs give valuable information on the timing of mobility processes. In addition, it is easy to incorporate structural aspects in such models, as shown in other papers (see Brüderl, Diekmann, and Preisendörfer forthcoming; Brüderl, Preisendörfer, and Ziegler 1991). With these models it is possible to show, for instance, how promotion rates differ between men and women. Regressions like equation 11 also have been applied often (which Rosenfeld and Nielsen 1984 review). The interpretation of b that follows from the partial adjustment framework gives valuable hints on the inequality of careers.

Both methods, however, give only partial insights. Careers consist of two components: waiting times and jumps. The Sørensen model shows that for a full assessment of inequality one needs to analyze both of these components. With event-history models researchers investigate only the timing aspect of careers. The partial-adjustment model gives only a description of the resultant of these two components but does not allow for a separation of them. For instance, steep wage profiles can be the result of long waiting times and high wage increases, short waiting times and low increases, or short waiting times and high increases. Thus the conclusion that profiles of one group are steeper than the profiles of another group tells nothing about the process that generated these profiles. A method is needed for investigating both components simultaneously. Such a method has already been proposed by Trond Petersen (1988, 1990). This method conceives careers as being created by two processes: The first process determines when somebody will have a jump, the second gives the height of the jump. Petersen shows that under certain assumptions it is possible to separate both processes and analyze them separately. I demonstrated elsewhere the fruitfulness of this approach using data from South-Factory (Brüderl 1990). For example, it is possible to see with this method that the steep careers of German men result from both higher promotion chances (i.e., shorter waiting times) and higher wage increases. So this group profits from both processes. Contrary to this, the higher in the hierarchy a person is placed, the longer he or she has to wait for a promotion. If a promotion occurs, however, the resulting wage jump is higher than on lower levels. This demonstrates that both processes will not always operate in the same direction. The Petersen method is able to detect this. A refined Sørensen model and the Petersen method (perhaps combined in a new model) seem to be very promising for future mobility research.

NOTES

1. One could criticize these assumptions as being unrealistic. However, this will not be the point of this article. In model building it is always advisable to start from simple assumptions.

2. Up to this point, the Sørensen model is nothing but a continuous reformulation of a special case of Bartholomew's (1967, 163f) renewal model. There are several other variants of Bartholomew's

model in the literature. For instance, Stewman and Konda's (1983, 649) formula for the promotion opportunities is also a special case of the renewal model (cf. Brüderl 1991, chap. 2). Therefore, it is no surprise that equation 1 is a special case of the Stewman-Konda model, as Skvoretz (1984) has shown.

3. This is questionable, as will be shown below.

4. Nevertheless, equation 5 is an ad hoc augmentation of equation 4. In particular, I believe that the introduction of a constant produces some of the problems with Method 1. Later on, I will present a modification where the constant follows from the theoretical model.

5. Sørensen does not justify equation 6. However, it can be easily seen that the promotion process as described here is a time-dependent Poisson process. Therefore, the number of events until t is a Poisson variate with mean $\int_0^t r(u)du$ (cf. Lancaster 1990, 88).

6. One reviewer argued that the inclusion of the wage level is not in accordance with the Sørensen model. This is correct (see our arguments in connection with equation 3). However, including the wage level in the rate models allows for a test of the assumptions. In addition, the qualitative results obtained from Table 1 do not change if one drops the wage level from the equations.

7. Again, a reviewer argued that the inclusion of the time-related variables EXP and D is not what Sørensen intended. The expected tenure, however, is used in these equations to correct for a potential bias, and one should not drop this variable. Results are almost the same if EXP and EXP^2 are dropped from the equations.

8. A test for equality of the four coefficients in the first two panels of Table 3 led to the following results (assuming independently, normally distributed estimators): The null hypothesis of equal bs can be rejected on the 5% level for both panels. Thus the contradictory rankings cannot be explained simply by random variation.

9. Rosenfeld (1980), who also uses model (9), gets a remarkably similar picture with her U.S. data: White men have a profile that is much higher and steeper than the profiles of the other groups.

10. The estimates of b in Table 3 would result in ps that are all greater than 10. These numerical estimates alone indicate that there must be something wrong with the two methods.

11. This tenure distribution still does not depend on Y. In a more realistic model, h(.) should be made dependent on Y. This would introduce wage dependence of r(t), too.

12. If one accepts this modification of Method 1, then Method 2 is no longer viable on its own. This can be seen by starting from equation 4′ and going through the calculations that lead to equation 11. Then $a_1 = \exp(b + \gamma)$. Thus b can no longer be identified from a regression with a lagged wage variable.

13. I owe this idea to Rudolf Schüßler (unpublished).

14. Because there are an infinite number of solutions for equation 3′, the choice of rate function equation 4′ is actually an assumption of the model. There are nonmonotonic rate functions that satisfy equation 3′. An inverse U-shaped promotion rate, for instance, would produce a wage profile that is convex first and concave later on. Thus the main conclusion of the Sørensen model follows from an empirical testable assumption.

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