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An AUC-based Permutation Variable Importance Measure for Random Forests

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Abstract

The random forest (RF) method is a commonly used tool for classification with high dimensional data as well as for ranking candidate predictors based on the so-called random forest variable importance measures (VIMs). However the classification performance of RF is known to be suboptimal in case of strongly unbalanced data, i.e. data where response class sizes differ considerably. Suggestions were made to obtain better classification performance based either on sampling procedures or on cost sensitivity analyses. However to our knowledge the performance of the VIMs has not yet been examined in the case of unbalanced response classes. In this paper we explore the performance of the permutation VIM for unbalanced data settings and introduce an alternative permutation VIM based on the area under the curve (AUC) that is expected to be more robust towards class imbalance. We investigated the performance of the standard permutation VIM and of our novel AUC-based permutation VIM for different class imbalance levels using simulated data and real data. The results suggest that the standard permutation VIM loses its ability to discriminate between associated predictors and predictors not associated with the response for increasing class imbalance. It is outperformed by our new AUC-based permutation VIM for unbalanced data settings, while the performance of both VIMs is very similar in the case of balanced classes. The new AUC-based VIM is implemented in the R package party for the unbiased RF variant based on conditional inference trees. The codes implementing our study are available from the companion website: `http://www.ibe.med.uni-muenchen.de/organisation/mitarbeiter/070_drittmittel/janitza/index.html`.

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1 Introduction

In bioinformatics and related fields, such as statistical genomics and genetic epidemiology, data are often high-dimensional, with the number of predictors exceeding the number of observations. For example, in genetic epidemiology data sets usually contain hundreds or thousands of candidate markers whose association with an outcome of interest has to be investigated. From the statistical point of view, one challenge is the high-dimensionality of the data which is also known as the "small n large p" problem. A further challenge is the complex data structure including heterogeneity, correlations and high-order interactions of unknown nature.

The random forest (RF) approach developed by Leo Breiman in 2001 [4] is particularly appropriate to handle such complex data [3]. In bioinformatics, RF is a commonly used tool for classification or regression purposes as well as for ranking candidate predictors through its inbuilt variable importance measures (VIMs). It has been used in many applications involving high-dimensional data. As a nonparametric method RF can deal with nonlinearity, interactions, correlated predictors and heterogeneity, which makes it attractive in genetic epidemiology [5, 7, 20, 23, 28]. However in the context of classification, i.e. when the response to be predicted is a class membership, classification performance of RF has been shown to be suboptimal in case of strongly unbalanced data [21, 19, 17], i. e. when class sizes differ considerably.

In epidemiology, unbalanced data are observed, e.g., in population-based studies where only a small number of subjects develop a certain disease over time, while most subjects remain healthy. Unbalanced data are also common in screening studies, where most of the screened persons are negative, as well as in subclass analyses, e.g., if one wants to differentiate between different subtypes of cancer. Usually some subclasses are more common than other subclasses leading to an imbalance in class sizes. Studies on rare diseases are a further example of unbalanced data settings in medicine. Data can be obtained only from few persons having the specific rare disease, while samples from healthy control persons are much easier to obtain. Of course unbalanced data are also relevant in various other areas of applica-

tion beyond the biomedical field, e.g., the prediction of creditworthiness of a bank's costumers [14], the detection of fraudulent telephone calls [11] or the detection of oil spills in satellite radar images [18], just to name a few examples. Unbalanced data may arise whenever the class memberships are observed after data collection.

Like many other classification methods RF produces classification rules that do not accurately predict the minority class if data are unbalanced. The RF classifier allocates new observations more often to the majority class unless the difference between the classes is large and classes are well separable. For extreme class imbalances, e.g. if the minority class includes only 5% of the observations, it might happen that the RF classifier allocates every observation to the majority class independently of the predictors, yielding a minimal error rate of 5%. Although this error rate of 5% is very small, such a trivial classification is of no practical use.

Some suggestions have been made to yield a useful classification based either on sampling procedures [8, 31, 1, 10] or on cost sensitivity analyses [8]. Sampling procedures create an artificial balance between two or more classes by oversampling the minority class and/or downsampling the majority class. Cost sensitivity analyses attribute a higher cost to the misclassification of an observation from the minority class to impede the trivial systematic classification to the larger class. Both aspects have been widely discussed in the literature with respect to RF's classification performance [30, 29, 8, 31, 15, 16]. Recent simulation studies [19] have shown that the performance of RF classification for unbalanced data depends on (i) the imbalance ratio, (ii) the class overlap and (iii) the sample size.

The impact of class imbalance on the RF VIM, however, has to our knowledge not yet been examined in the literature. In this article we focus on the permutation VIM which is known to be almost unbiased and more reliable than the Gini VIM. The latter has been shown to have a preference for certain types of predictors [27, 24, 22, 2] and therefore its rankings have to be treated with caution. We concentrate on the class imbalance problem for two response classes with respect to the permutation VIM. We investigate the mechanisms of changes in performance for unbalanced data settings and

motivate the use of a new permutation VIM which is not based on the error rate but on the area under the curve (AUC). The AUC can be seen as an accuracy measure putting the same weight on both classes – in contrast to the error rate which essentially gives more weight to the majority class. As such, the AUC is a particularly appropriate prediction accuracy measure in unbalanced data settings [6]. A permutation VIM in which the error rate is replaced by the AUC is therefore a promising alternative to the standard error-rate-based permutation VIM. We performed extensive simulation studies to explore and compare the behaviour of both permutation VIMs for different class imbalance levels, effect sizes and sample sizes.

2 Methods

The RF algorithm is a classification and regression method that combines several individual decision trees to make a final prediction. The final prediction is then the average (for regression) or the majority vote (for classification) of the predictions of all trees in the forest. Each tree is fitted to a random sample of observations (with or without replacement) from the original sample. Observations not used to construct a tree are termed out-of-bag (OOB) observations for that tree. For each split in each tree a randomly drawn subset of predictors is assessed as candidates for splitting and the predictor yielding the best split is finally chosen for the split. In the original version of RF developed by Leo Breiman [4], the selected split is the split with the largest decrease in Gini impurity. In a later version of RF, conditional inference tests are used for selecting the best split in an unbiased way [12]. For each split in a tree, each candidate predictor from the randomly drawn subset is globally tested for its association with the response, yielding a global p-value. The predictor with the smallest p-value is selected, and within this globally selected predictor the best split is finally chosen for the split.

Both forest versions implement so called variable importance measures which can be used to get a ranking of the predictors according to their association with the response. In the following, we briefly introduce the standard error-rate based permutation VIM as well as our novel permutation VIM, which is based on the area under the curve.

2.1 Random forest variable importance measures

The two standard VIMs for feature selection with RF are the Gini VIM and the permutation VIM. Roughly speaking the Gini VIM of a predictor of interest is the sum over the forest of the decreases of Gini impurity generated by this predictor whenever it was selected for splitting, scaled by the number of trees. This measure has been shown to prefer certain types of predictors [27, 24, 22, 2]. The resulting predictor ranking should therefore be treated with caution. That is why in this paper we focus on the permutation VIM that gives essentially unbiased error rate rankings of the predictors.

Error-rate-based permutation VIM

From now on, we denote the standard permutation VIM as "error-rate-based permutation VIM", since it is based on the OOB error rate, as outlined below. More precisely, it measures the difference between the OOB error rate after and before permuting the values of the predictor of interest. The error-rate-based permutation variable importance (VI) for predictor j is defined by

$$VI_j^{(ER)} = \frac{1}{ntree} \sum_{t=1}^{ntree} (ER_{t\tilde{j}} - ER_{tj}) \tag{1}$$

where

- $ntree$ denotes the number of trees in the forest,
- ER_{tj} denotes the mean error rate over all OOB observations in tree t before permuting predictor j,
- $ER_{t\tilde{j}}$ denotes the mean error rate over all OOB observations in tree t after randomly permuting predictor j.

The idea underlying this VIM is the following: If the predictor is not associated with the response, the permutation of its values has no influence on the classification, and thus also no influence on the error rate. The error rate of the forest is not substantially affected by the permutation and the VI of the predictor takes a value close to zero, indicating no association between the predictor and the response. In contrast, if response and predictor are associated, the permutation of the predictor values destroys this

association. "Knocking out" this predictor by permuting its values results in a worse classification leading to an increased error rate. The difference in error rates before and after randomly permuting the predictor thus takes a positive value reflecting the high importance of this predictor.

A novel AUC-based permutation VIM

Our new AUC-based permutation VIM is closely related to the error-rate-based permutation VIM. They only differ with respect to the prediction accuracy measure: In a nutshell, the error rate of a tree involved in (1) is replaced by the area under the curve (AUC) [26]. We define the AUC-based permutation VI for predictor j as:

$$VI_j^{(AUC)} = \frac{1}{ntree^*} \sum_{t=1}^{ntree^*} (AUC_{tj} - AUC_{t\tilde{j}}) \tag{2}$$

where

- $ntree^*$ denotes the number of trees in the forest whose OOB observations include observations from both classes,
- AUC_{tj} denotes the area under the curve computed from the OOB observations in tree t before permuting predictor j,
- $AUC_{t\tilde{j}}$ denotes the area under the curve computed from the OOB observations in tree t after randomly permuting predictor j.

Instead of computing the error rate for each tree after and before permuting a predictor, the AUC is computed. The AUC for a tree is based on the so-called class probabilities, i.e. the estimated probability of each observation to belong to the class $Y = 0$ or $Y = 1$, respectively. The class probabilities of an observation are determined by the relative amount of training observations belonging to the corresponding class in the terminal node in which an observation falls into. If one considers an OOB observation with $Y = 0$ and an OOB observation with $Y = 1$, a "good tree" is expected to assign a larger class probability for class $Y = 1$ to the observation truly belonging to class $Y = 1$ than to the observation belonging to class $Y = 0$. The AUC for a tree corresponds to the proportion of pairs for which this is the case. It can be seen as an estimator of the probability that a randomly chosen

observation from class $Y = 1$ receives a higher class probability for class $Y = 1$ than a randomly chosen observation from class $Y = 0$.
Note that with the use of the AUC, the information contained in the class probabilities returned by a tree are adequately exploited. This is not the case for the error rate that requires a dichotomization of class probabilities. From a practical point of view, the AUC is computed by making use of its equivalence with the Mann-Whitney-U statistic. The Mann-Whitney-U statistic is solely based on the rankings of two independent samples. AUC values of 1 correspond to a perfect tree classifier, since a perfect classifier would attribute each observation from one class a higher probability to belong to this class than any observation from the other class. AUC values of 0.5 correspond to a useless tree classifier that randomly allocates class probabilities to the observations. In this case in about half the cases a randomly drawn observation from one class receives a higher probability of belonging to that class than a randomly drawn observation from the other class.

The novel AUC-based permutation VIM is implemented in the package **party** for the unbiased RF variant based on conditional inference trees. Note that the discrepancy in performance between the standard permutation VIM and the AUC-based permutation VIM is transferable to the original version of RF since the VI ranking mechanism is completely independent from the construction of the trees.

2.2 Comparison studies

The behavior of the two introduced permutation VIMs is expected to be different in the presence of unbalanced data. The AUC is a prediction accuracy measure which puts the same weight on both classes independently of their sizes [6]. The error rate, in contrast, gives essentially more weight to the majority class because it does not take class affiliations into account and regards all misclassifications equally important. In the results section we try to explain the consequences for the performance of the permutation VIMs for unbalanced data settings and provide evidence for our supposition. We performed studies on simulated and on real data to explore and contrast the performance of both permutation VIMs. Using simulated data we aim to see whether total sample size and effect size play a role for the class imbalance problem. We explored this by varying the total number of

observations and by simulating predictors with different effect sizes. Furthermore we conducted analyses based on real data to provide additional evidence based on realistic data structures which usually incorporate complex interdependencies.

If the data are unbalanced, the depth of the trees in a RF, that is determined by tree pruning (for classical RF) or early stopping (for the unbiased RF variant), is expected to affect the AUC-based VIM and the error-rate-based VIM in different ways. This results from the fact that with the use of the AUC-based VIM the information on class probabilities given by a tree are preserved, while the commonly used error-rate-based VIM requires a dichotomization of the class probabilities. We later illustrate that if trees are not pruned or stopped early, preserving the class probabilities is of advantage for a permutation VIM in unbalanced data settings. We conducted simulation studies with different stopping criteria to provide evidence that the AUC-based permutation VIM is unaffected while the error-rate-based permutation VIM is impaired by tree pruning or early stopping.

Our comparison studies on simulated and on real data were conducted using the unbiased RF variant based on conditional inference trees. The implementation of this unbiased RF variant is available in the R system for statistical computing via the package **party** [13].

2.2.1 Simulated data

The considered simulation design represents a scenario where the predictors associated with the response variable Y (binary) are to be identified from a set of continuous predictors. We performed simulations for varying imbalance levels: 50% corresponding to a completely balanced sample, 40%, 30%, 20%, 10%, 5% and 1% corresponding to different imbalance levels from slight to very extreme class imbalances. The simulation setting comprises both predictors not associated with the response and associated predictors with three different levels of effect sizes. Table 1 presents the data setting used throughout this simulation. The first five predictors $X_1, \ldots, X_5$ differ strongly between classes with mean $\mu_1 = 1$ in one class and mean $\mu_2 = 0$ in the other class. The predictors $X_6, \ldots, X_{10}$ have a moderate mean difference between the two classes with $\mu_1 = 0.75$ and $\mu_2 = 0$. For $X_{11}, \ldots, X_{15}$ there

is only a small difference between the classes with $\mu_1 = 0.5$ and $\mu_2 = 0$. We simulated 50 additional predictors following a standard normal distribution with no association to the response variable (termed noise predictors).

Predictors	Distribution in class 1	Distribution in class 2	Effect size
$X_1, \ldots, X_5$	$N(1.00, 1)$	$N(0, 1)$	strong effect
$X_6, \ldots, X_{10}$	$N(0.75, 1)$	$N(0, 1)$	moderate effect
$X_{11}, \ldots, X_{15}$	$N(0.50, 1)$	$N(0, 1)$	weak effect
$X_{16}, \ldots, X_{65}$	$N(0, 1)$	$N(0, 1)$	no effect

Table 1: Distribution of predictors in class 1 and class 2.

We performed analyses with varying sample sizes and report the results for total sample sizes of $n = 100$, $n = 500$ and $n = 1000$. For each parameter combination, i.e. imbalance level and sample size, we simulated 100 datasets and computed AUC-based and error-rate-based permutation VIs for each dataset. Note that for a sample size of $n = 100$ an imbalance of 1% is not meaningful since there is only one observation in the minority class.

Forest and tree parameters were held fixed. The parameter `ntree` denoting the number of trees in a forest was set to 1000, the parameter for the number of candidate splits `mtry` was set to the default value of 5. We used subsampling instead of bootstrap sampling for constructing the trees, i.e. setting the parameter `replace` to `FALSE` [27]. Conditional inference trees were grown to maximal possible depth, i.e. setting the parameters `minsplit`, `minbucket` and `mincriterion` in the `cforest` function to zero.

In additional analyses we examined the influence of early stopping on the performance of both VIMs for unbalanced data settings. We explored this by inspecting different values for early stopping criteria in conditional inference trees such as `minsplit`, `minbucket` and `mincriterion`. The parameter `minsplit` denotes the minimal number of observations a node should contain in order to be split. The parameter `minbucket` denotes the minimal number of observations in a node and `mincriterion` corresponds to the significance level threshold for a node in order to be split. The same simulation setting described above and in Table 1 (i.e. effect sizes, number of predictors and

data generating process) was used to explore the influence of early stopping. We simulated 1000 datasets for each parameter setting to get stable results which allow for the detection of even slight differences in performance.

2.2.2 Real data

We also investigated the performance of the error-rate-based and the AUC-based permutation VIM on real data including complex dependencies (e.g. correlations) and predictors of different scales. The dataset is about RNA editing in land plants [9]. RNA editing is the modification of the RNA sequence from the corresponding DNA template. It occurs e.g. in plant mitochondria where some cytidines are converted to uridines before translation (abbreviated with C-to-U conversion in the following). The dataset comprises a total of 43 predictors: 41 categorical predictors (40 nucleotides at positions -20 to 20 relative to the edited site and one predictor describing the codon position) and two continuous predictors (one for the estimated folding energy and one predictor describing the difference in estimated folding energy between pre-edited and edited sequences). It includes 2694 observations, where exactly one half has an edited site and the other half has a non-edited site. The data are publicly available from the journal's homepage. After excluding observations with missing values, a total of 2613 observations were left, where 1307 had a non-edited site and 1306 observations had an edited site. We used this balanced dataset to explore the performance of ER- and AUC-based permutation VIM for varying class imbalances – but now with realistic dependencies and predictors of different scales. For this purpose, we artificially created different imbalance levels by drawing random subsets from the class with edited sites.

Application of the standard permutation VIM to the data using the 2613 observations without missing values gave VIs greater than zero for all 43 predictors for different random seeds (i.e. different starting values for the random permutation), indicating that all predictors seem to have at least a small predictive power (data not shown). We generated and added additional predictors without any effect (termed noise predictors in the following) in order to evaluate the performance of error-rate-based and AUC-based permutation VIMs. Provided that there is a higher association between the response and any of the original predictors than between the response and

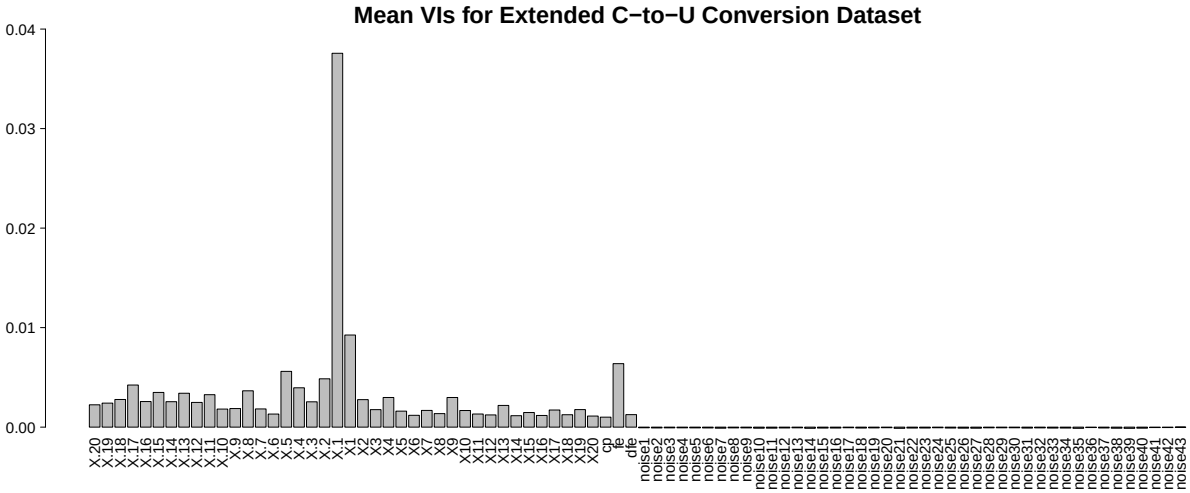

Figure 1: Mean VIs for the 43 original predictors and 43 noise predictors from the balanced modified C-to-U conversion dataset are shown. Mean VIs were obtained by averaging the VIs (by commonly used error-rate-based permutation VIM) over 100 extended versions of the C-to-U conversion dataset.

any of the simulated noise predictors, a well performing VIM would attribute a higher VI to original predictors than to simulated noise predictors. The noise predictors were generated by randomly permuting the values of the original predictors. Each original predictor was permuted once, resulting in a total of 43 noise predictors. The whole process consisting of (1) creating 43 noise predictors, (2) merging them to the original dataset, (3) randomly subsampling to create an unbalanced dataset and (4) computing the error-rate-based and AUC-based permutation VIs, was repeated 100 times for each imbalance level to get stable results for the VIM performance. To check the assumption that there is a higher association between the response and any of the original predictors than between the response and any of the simulated predictors, we computed the mean VI over 100 completely balanced datasets that had been extended by noise predictors. Figure 1 shows that all mean VIs of the original predictors are higher than any mean VI of a simulated noise predictor and hence confirms our first impression.

2.2.3 Performance evaluation criteria

VIMs give a ranking of the predictors according to their association with the response. To evaluate the quality of the rankings by the permutation

VIMs the AUC was used as performance measure. The AUC was computed to assess the ability of a VIM to differentiate between associated predictors and predictors not associated with the response. AUC values of 1 mean that each associated predictor receives a higher VI than any noise predictor, thus indicating a perfect discrimination. AUC values of 0.5 mean that a randomly drawn associated predictor receives a higher VI than a randomly drawn noise predictor in only half of the cases, indicating no discriminative ability.

For our comparison studies we defined the two classes which are to be differentiated by a VIM in the following way. In the first instance of our studies on simulated data, all predictors which are associated with the response formed one class and noise predictors built the other class. In more detailed subsequent analyses we then explored the ability of the VIMs to discriminate between predictors with the same effect size and predictors without an effect. For this analysis one class comprised the noise predictors while the other class comprised only predictors with the same effect. For the studies on real data it was not possible to conduct such detailed analyses because the true ordering of the predictors according to their association with the response is not known. Hence in the analysis on real data we restricted our analysis to the discrimination between original predictors forming one class and simulated noise predictors forming the other class.

3 Results and Discussion

Why may the error-rate-based permutation VIM fail in case of class imbalance?

The prioritisation of the majority class in unbalanced data settings is well known in the context of RF classification and can easily be seen from trees constructed on unbalanced data. Trees trained on unbalanced data more often predict the majority class, which leads to the minimization of the overall error rate. But how does this affect the performance of the permutation VIMs? And why is the AUC-based permutation VIM expected to be more robust towards class imbalance than the commonly used error-rate-based permutation VIM?

To answer these questions we consider an extremely unbalanced data setting and illustrate what happens in a tree when permuting the values of an associated predictor. We will first have a look at observations from the majority class. For this class nearly all observations are correctly classified by a tree which has been trained on extremely unbalanced data. If we now permute the values of an associated predictor, this does generally not result in a classification into the minority class since a classification into the minority class is an unlikely event – even for an observation from this class. A very specific data pattern is required for an observation to be classified into the minority class. It is unlikely that a random permutation of an associated predictor results in such a specific data pattern just by chance. Thus, for the majority class we expect hardly any observation to be incorrectly classified to the minority class after the permutation of an associated predictor. Thus the error rate does not considerably increase after the permutation of an associated predictor, finally leading to a rather low contribution to the VI.

Now let us consider the classifications by a tree for observations from the minority class. For an extreme class imbalance most of the observations from the minority class are falsely classified to the majority class due to the above described focus on the majority class. It might be the case that some observations from the minority class are correctly classified by the tree because these observations have that specific pattern of predictor values which is required for an observation to be classified into the minority class. It is likely that a permutation of the values of an associated predictor might then destroy that specific pattern so that after the permutation, these observations are not identified anymore to be in the minority class. Thus a misclassification due to the elimination of an associated predictor is much more likely to appear in observations from the minority class than in observations from the majority class. Note that only a small number of observations from the minority class are affected since most of the observations from the minority class are classified into the majority class anyway (before as well as after the permutation). The change in error rates is thus expected to be rather small – albeit it is more pronounced than the change in error rates in the majority class.

Note that the error-rate-based permutation VIM does not take class affilia-

tions into account. Thus the change in error rates is actually not computed separately for each class. However if class proportions are equal in all OOB samples, the actual VI of a predictor can be derived from a weighted average of class specific differences in error rates. The weights correspond to the proportion of observations from the respective class in the OOB samples. Class frequencies are in general not equal in all OOB samples (unless one explicitly specifies it in the RF algorithm) but it illustrates the fact that for unbalanced data settings the VI is mainly driven by the change in error rates derived from observations from the majority class. Since the change in error rates in the majority class is expected to be much smaller compared to the change in error rates in the minority class, the computed VIs are rather low. This results in low VIs even for associated predictors and in a poor differentiation of associated predictors and predictors not associated with the response.

Class specific VIs

This theory is supported by computing class specific VIs (corresponding to mean changes in error rates computed only from observations belonging to the same class). Computing class specific VIs was done using the R package `randomForest` implementing the standard RF algorithm. The `importance` function of this package provides permutation VIs computed separately for each class (besides the VIs by the standard permutation VIM and by the Gini VIM). The class specific VIs for a total sample size of $n = 500$ and an imbalance level of 5% are shown in Figure 2, where predictors X_1 to X_{15} have an effect while the remaining 50 predictors do not have an effect, corresponding to the simulation setting previously described in Table 1 in the context of the comparison study (for simplicity, we use the same setting as in the comparison study, although the addressed problem is here a different one). Different sample sizes and imbalance levels give similar results (thus not shown). They confirm our argumentation that the change in the error rates computed from OOB observations from the majority class is smaller than the change in error rates computed from OOB observations from the minority class. This results in an underestimation of the actual permutation VI due to a much higher weighting of the majority class in the computation of the VI (see concordance of VIs in middle and lower panel of Figure 2). The discrepancy between the VIs computed from observations of the

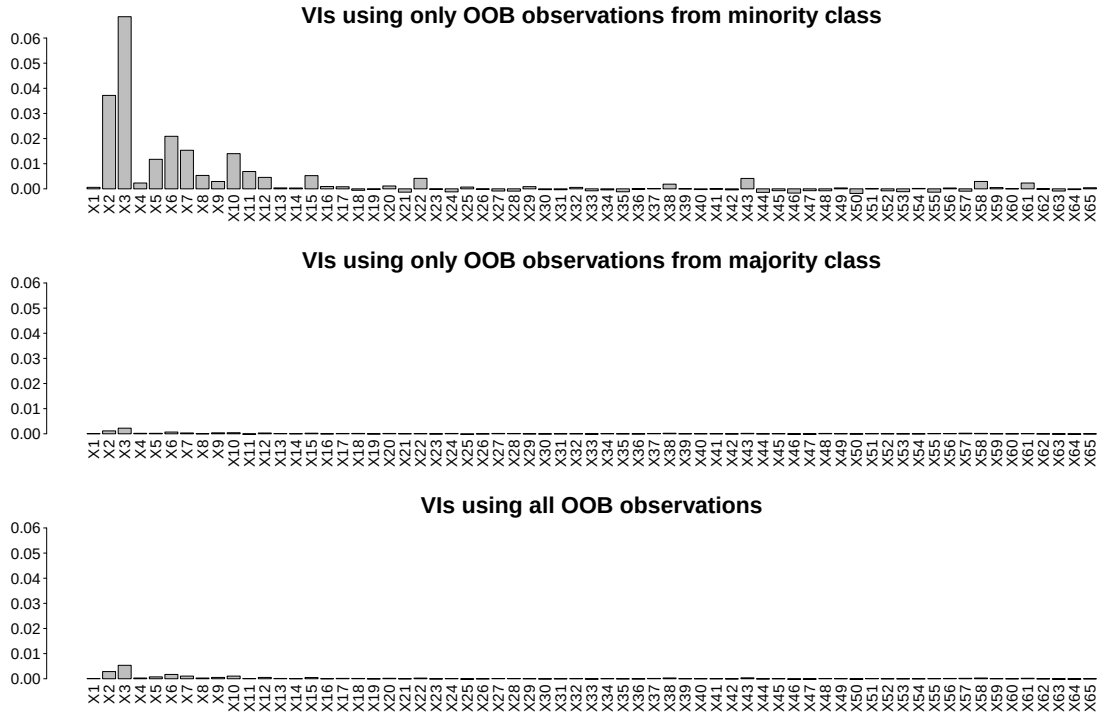

Figure 2: VIs computed only from OOB observations of the minority class (top), from OOB observations of the majority class (middle) and from all OOB observations (bottom). The first 15 predictors are associated with the response while the remaining predictors are noise predictors. VIs are shown for a total sample size of $n = 500$ and an imbalance level of 5%.

minority class and VIs computed from observations of the majority class depends on the class imbalance and is more pronounced for more extreme class imbalances.

This motivates the use of an alternative accuracy measure which better incorporates the minority class. While the error rate gives the same weight to all observations, therefore focusing more on the majority class, the AUC is a measure which does not prefer one class over the other but instead puts exactly the same weight on both classes. Therefore the AUC-based permutation VIM is expected to detect changes in tree predictions for observations from the minority class, which might not be grasped by the error-rate-based permutation VIM due to a much higher weighting of the majority class. The VIs for associated predictors obtained by the AUC-based permutation VIM are thus expected to be comparatively higher than the VIs obtained by the error-rate-based permutation VIM. This would result in a better differentiation of associated and noise predictors by the AUC-based permutation VIM. These conjectures are assessed in the comparison study presented in the next section [1].

3.1 Comparison study with simulated data

The performance of the error-rate-based and AUC-based VIMs as measured by the AUC is shown in Figure 3 for the three different total sample sizes with $n = 100$ (left panel), $n = 500$ (middle panel) and $n = 1000$ observations (right panel) and different class imbalance levels. Filled boxes correspond to the AUC-based permutation VIM and unfilled boxes correspond to the error-rate-based permutation VIM. Figure 3 shows that the performance of both VIMs decreases with an increasing class imbalance for all sample sizes. Note that the decrease in performance for both VIMs is not solely attributable to the imbalance ratio per se but also to the reduced number of observations in the minority class with an increasing class imbalance. This is induced by the simulation setting since we held the total number of observations fixed and varied the number of observations in both classes to create different class imbalances. If there are only few observations in one

[1] An additional performance comparison between the AUC-based permutation VIM and the error-rate-based permutation VIM based only on observations from the minority class is documented in the supplementary material.

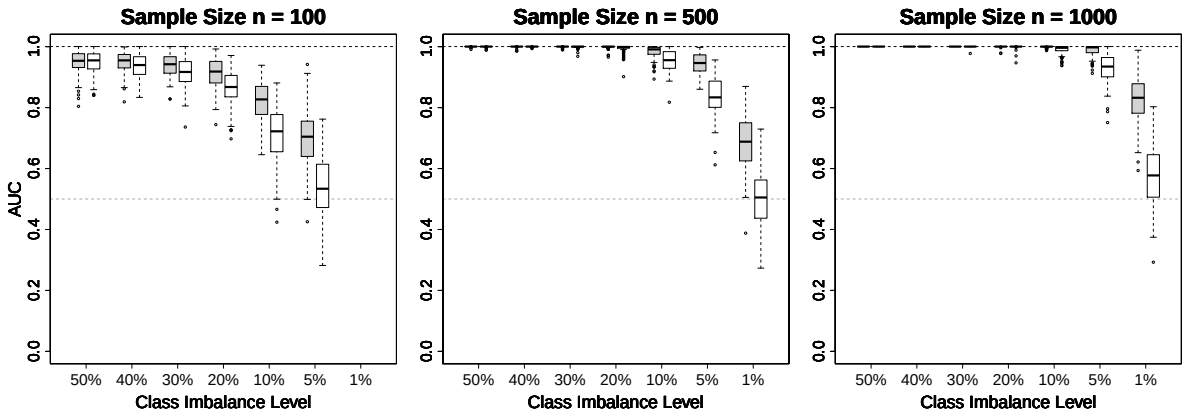

Figure 3: Distribution of AUC-values for 100 simulated datasets for AUC-based (filled) and error-rate-based (unfilled) permutation VIMs for different class imbalances. The AUC is used to assess the ability of a VIM to discriminate between predictors with an effect and predictors without an effect. Distributions are shown for total sample sizes of $n = 100$ (left panel), $n = 500$ (middle panel) and $n = 1000$ (right panel).

class then the tree predictions are less accurate. However the performance of the AUC-based permutation VIM decreases less dramatically than the performance of the error-rate-based permutation VIM. The discrepancy in performances between the VIMs increases with increasing imbalance level and is maximal for the most extreme class imbalance. While for a sample size of $n = 500$ the error-rate-based permutation VIM is no longer able to discriminate between associated and noise predictors (AUC values randomly vary around 0.5) for the most extreme class imbalance of 1%, the AUC-based permutation VIM still is, showing that it can be used to identify associated predictors even if the minority class comprises only few observations. It can be ruled out that the better performance of the AUC-based permutation VIM is due to chance since the distributions of AUC values significantly differ. Furthermore this difference in performances between both VIMs becomes even larger for larger sample sizes.

In a nutshell, in this first simulation the AUC-based permutation VIM performed better in case of class imbalance. The following simulations focus on the influence of sample size and effect size on the respective performance of both permutation VIMs in unbalanced data settings.

Influence of sample size

In Figure 3, the performance of both VIMs improves with an increased total sample size for a fixed imbalance level since an increase in the sample size results in more accurate tree predictions. The right panel of Figure 3 shows that both permutation VIMs are hardly affected by class imbalances up to 10% when the sample size is rather large ($n = 1000$). If the sample size is smaller ($n = 100$), however, the performance of the VIMs is considerably decreased for a 10% imbalance level. A decrease in performance for a 10% imbalance level is also observed for a sample size of $n = 500$, especially for error-rate-based permutation VIM. In a nutshell, class imbalance seems to be more problematic for the permutation VIMs if the total sample size is small.

Influence of effect size

In the following simulation we explored the ability of the permutation VIMs to identify predictors with different effect sizes in presence of unbalanced data. The AUC was again used as an evaluation criterion to compare the ability of the AUC-based and error-rate-based permutation VIMs to discriminate between associated and non-associated predictors. Here the evaluation was done for each effect size separately meaning that one class comprised all the noise predictors while the other class comprised only predictors with the considered effect size (either strong, moderate or weak). Figure 4 shows the results for the setting with $n = 100$. The results for other sample sizes are given in the supplementary material. The left panel of Figure 4 shows the performance of both permutation VIMs according to their ability to discriminate between predictors with weak effects and predictors without an effect. The middle panel corresponds to the AUC values for predictors with a moderate effect versus noise predictors and the right panel corresponds to the AUC values for predictors with a strong effect versus noise predictors. Unsurprisingly, for both permutation VIMs predictors having only a weak effect are less discriminable from noise predictors than predictors with stronger effects. For imbalances up to 20% both VIMs identify nearly all predictors with a strong effect. Obviously there are unbalanced data settings where the standard permutation VIM still perfectly separates between noise predictors and predictors with pronounced effects. We conclude that class imbalance

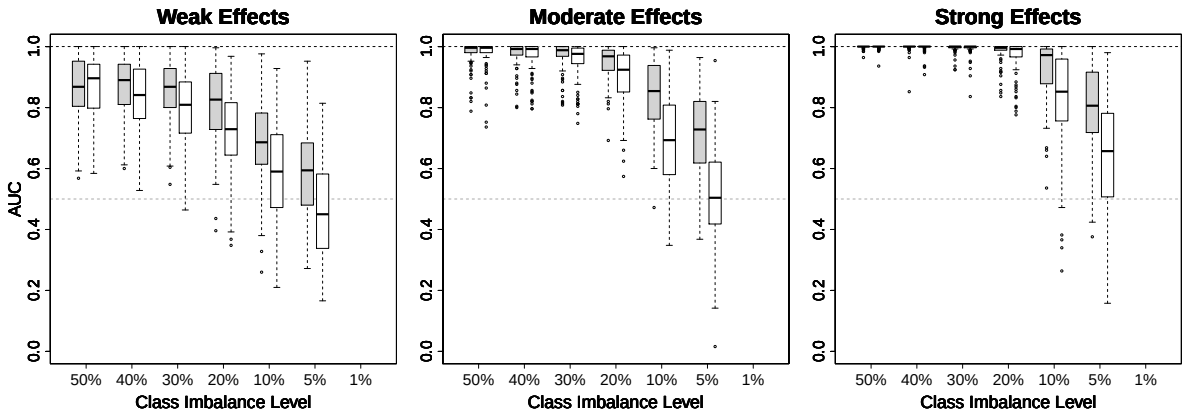

Figure 4: Distribution of AUC-values for 100 simulated datasets for AUC-based (filled) and error-rate-based (unfilled) permutation VIMs for different class imbalances. The AUC is used to assess the ability of a VIM to discriminate between noise predictors and predictors with a weak (left panel), moderate (middle panel) and strong (right panel) effect. Distributions are shown for a total sample size of $n = 100$.

is more problematic if predictors with weak effects are to be identified while it plays a minor role if the classes are well separable.

Influence of early stopping a tree

Early stopping in the presence of unbalanced data is expected to further aggravate the performance of the standard permutation VIM. In order to predict a class, observations from that class have to outnumber observations of the other class in a terminal node. Terminal nodes of early pruned trees, i.e. trees that were not grown to maximal depth, contain more observations than terminal nodes of trees which were grown to maximal depth. It is then more difficult for the minority class to outnumber the majority class in terminal nodes. The earlier a tree is stopped, the higher the number of observations in terminal nodes and the more difficult it gets for the minority class to outnumber the majority class. This directly affects the performance of the error-rate-based permutation VIM: The number of terminal nodes which predict the minority class decreases and a classification into the minority class becomes even rarer. If all terminal nodes in a tree predict the majority class, the error rate of that tree is fixed and not affected by permutations of the predictors. Thus VIs of all predictors are zero for that tree

and the VIM loses its discriminative ability. For extreme early stopping it can even happen that all trees in the forest predict the majority class which leads to a VI of zero for all predictors.

In contrast, the AUC-based permutation VIM is unaffected by early stopping since it is based on the class probabilities given by a terminal node. Even if the majority class outnumbers the minority class in all terminal nodes there can still be a change in AUCs before and after the permutation of a predictor. We provide evidence for this by the results of our simulation studies which are presented in the following.

The effect of early stopping was examined by varying the values for three different parameters controlling the size of a tree. Figures 5, 6 and 7 show the results for the sample size $n = 100$ and an imbalance level of 10% contrasted to the results for a completely balanced dataset. Figure 5 shows the performance of both permutation VIMs for varying values of the minimal number of observations in a node (`minbucket`). Figure 6 shows the results for varying values of the minimal number of observations in a node that are required for a node to be split (`minsplit`). While the AUC-based permutation VIM is not at all affected by early stopping with these two parameters for both the balanced and unbalanced data setting, the error-rate-based permutation VIM is obviously sensitive to early stopping in case of unbalanced data. If data is unbalanced the performance of the error-rate-based permutation VIM is stable for small values of `minsplit` and `minbucket` while it is clearly decreased for larger values. Since its performance is stable for the balanced data setting [2], we attribute this reduction in performance for the unbalanced setting to the above described mechanism resulting from class imbalance.
In contrast, for different `mincriterion` values corresponding to different significance level thresholds for a node in order to be split, no systematic difference w.r.t. performance reduction for higher values of `mincriterion` can be observed for the unbalanced data setting compared to the balanced data setting (Figure 7). Increased values of `mincriterion` seem to have no apparent different effect on the VIM performance for unbalanced data

[2] No RF is grown for too extreme values of `minbucket`, resulting in VIs of zero for all predictors. The corresponding AUC takes value of 0.5.

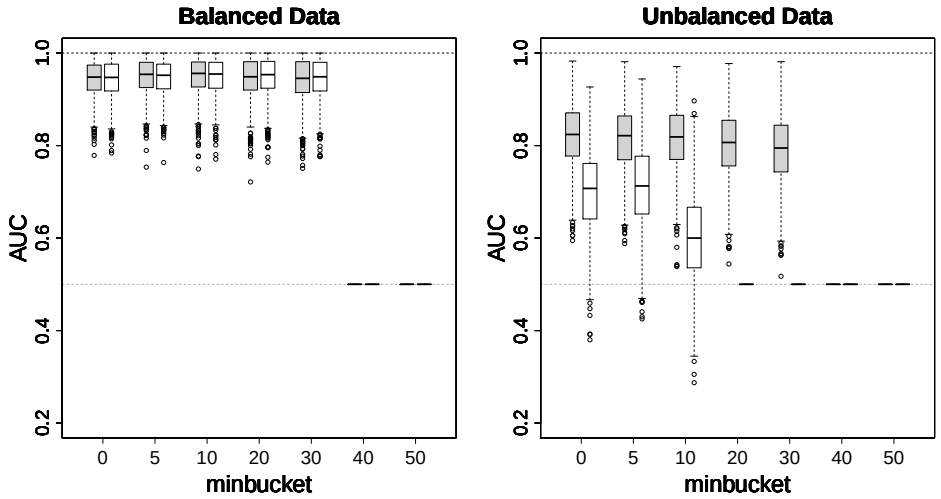

Figure 5: Distribution of AUC-values for 1000 simulated datasets for AUC-based (filled) and error-rate-based (unfilled) permutation VIMs for different values of `minbucket`. High values for `minbucket` correspond to trees stopped earlier. Distributions are shown for a total sample size of $n = 100$ in presence of complete balance (left panel) and an imbalance level of 10% (right panel).

settings compared to balanced data settings and early stopping with this parameter aggravates the performance for both unbalanced and balanced data settings as well.

The simulation studies support our hypothesis that early stopping a tree (at least with `minsplit` and `minbucket`) impairs the performance of the commonly used error-rate-based permutation VIM for unbalanced data settings while the AUC-based permutation VIM is obviously robust towards early stopping.

3.2 Comparison study with real data

Figure 8 shows the distribution of AUC values for 100 modified C-to-U conversion datasets for varying imbalance levels. For the balanced dataset and for slight class imbalances up to 40% both VIMs have a perfect discriminative ability since all associated predictors receive a higher VI than any noise predictor. Overall the performance of both VIMs decreases with an increasing class imbalance. Note that the decreasing performance for increasing class imbalances might be partly attributable to the reduced total sample size as the imbalance was created by randomly subsampling obser-

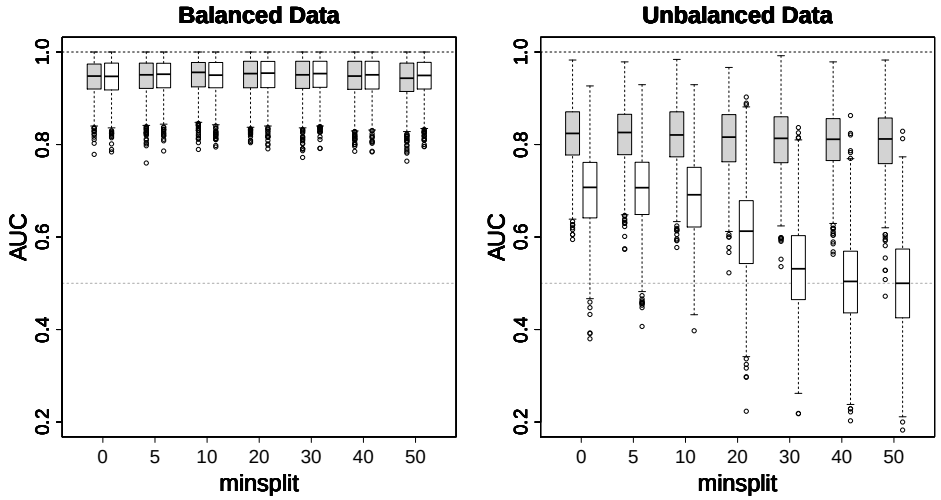

Figure 6: Distribution of AUC-values for 1000 simulated datasets for AUC-based (filled) and error-rate-based (unfilled) permutation VIMs for different values of `minsplit`. High values for `minsplit` correspond to trees stopped earlier. Distributions are shown for a total sample size of $n = 100$ in presence of complete balance (left panel) and an imbalance level of 10% (right panel).

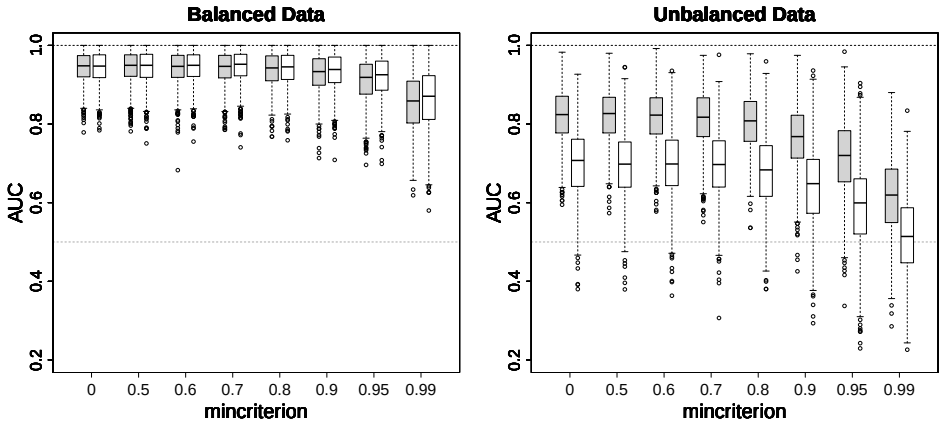

Figure 7: Distribution of AUC-values for 1000 simulated datasets for AUC-based (filled) and error-rate-based (unfilled) permutation VIMs for different values of `mincriterion`. High values for `mincriterion` correspond to trees stopped earlier. Distributions are shown for a total sample size of $n = 100$ in presence of complete balance (left panel) and an imbalance level of 10% (right panel).

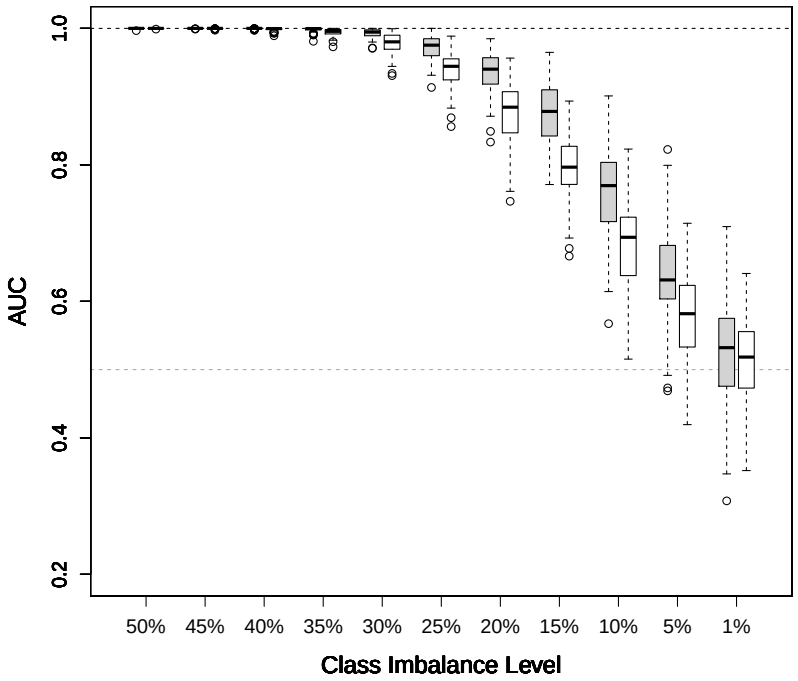

Figure 8: Distribution of AUC-values for AUC-based (filled) and error-rate-based (unfilled) permutation VIMs for different class imbalances derived from 100 modified datasets from C-to-U conversion data. The AUC is used to assess the ability of a VIM to discriminate between associated predictors and predictors not associated with the response.

vations from the class with the edited sites. When comparing both VIMs the AUC-based permutation VIM significantly outperformed the standard permutation VIM. For an imbalance of 30% the AUC-based permutation VIM clearly identified more associated predictors than the error-rate-based permutation VIM. The superiority of the AUC-based permutation VIM over the standard permutation VIM increased with an increasing class imbalance. For imbalances between 15% and 5% the discrepancy between the performance of AUC-based and standard permutation VIM was maximal.

Overall, this study on real data impressively shows that the AUC-based permutation VIM also works for complex real data and outperforms the standard permutation VIM in almost all class imbalance settings.

4 Conclusions

The problem of unbalanced data has been widely discussed in the literature for diverse classifiers including random forests. Many approaches have been developed to improve the predictive ability of RF classifiers for unbalanced data settings. However less attention has been paid to the behaviour of random forests' variable importance measures for unbalanced data. In this paper we explored the performance of the permutation VIM for different class imbalances and proposed an alternative permutation VIM which is based on the AUC.

Our studies on simulated as well as on real data show that the commonly used error-rate-based permutation VIM loses its ability to discriminate between associated predictors and predictors not associated with the response for increasing class imbalances. This is particularly crucial for small sample sizes and if predictors with weak effects are to be detected. Early stopping trees even aggravates the performance of the error-rate-based permutation VIM. The decreasing performance of the standard permutation VIM results from two sources: the class imbalance on the training data level leading to trees more often predicting the majority class and the class imbalance at the OOB data level leading to blurred VIs due to a much higher weighting of error rate differences in the majority class. A higher weighting of the majority class in the VI calculation is problematic because the difference in error rates is shown to be less pronounced in the majority class than in the minority class. Note that in some cases it might be interesting to assess the increase in error rate obtained when a certain predictor is removed. In this case the error-rate-based permutation VIM can be considered. If the goal is to rank the predictors according to their discrimination power, however, the AUC-based permutation VIM should be preferred.
The problem of imbalance at the OOB data level is directly addressed with the use of a novel AUC-based permutation VIM. This VIM puts the same weight on both classes by measuring the difference in AUCs instead of the difference in error rates. It is thus able to detect changes in tree predictions when permuting associated predictors which might not be grasped by the standard permutation VIM. In contrast, the imbalance on training data level is not addressed by the AUC-based permutation VIM, meaning that the

structure of a tree remains untouched. On the one hand this is a drawback since class predictions before and after permuting a predictor are similar even if the respective predictor is associated with the response, resulting in a reduced change in the AUCs. On the other hand preserving the tree structure can be regarded as an advantage since a change in tree structure might open space for new unexpected behaviours. It is a major advantage of our novel AUC-based permutation VIM that it is based on exactly the same principle and differs from the standard permutation VIM only with respect to the accuracy measurement. It is thus expected to share the advantages of the standard permutation VIM and its properties and behaviours discovered in recent years (e.g. its behaviour in presence of correlated predictors [25] and in presence of predictors with different scales [27] and category sizes in the predictors [22, 2]).

Our studies on simulated as well as on real data show that the AUC-based permutation VIM outperforms the commonly used error-rate-based permutation VIM as well as the error-rate-based permutation VIM computed only using observations from the minority class in case of unbalanced data settings (the comparison to the class specific VIM is shown in the supplementary material). The difference in performance between our novel AUC-based permutation VIM and the standard permutation VIM can be substantial, especially for extremely unbalanced data settings. But even for slight class imbalances the AUC-based permutation VIM has shown to be superior to the standard permutation VIM. We conclude from our studies that the AUC-based permutation VIM should be preferred to the standard permutation VIM whenever two response classes have different class sizes and the aim is to identify relevant predictors.

Supplementary material

Additional files referenced in Section 3 are available under `http://www.ibe.med.uni-muenchen.de/organisation/mitarbeiter/070_drittmittel/janitza/index.html`.

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