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Abstract

In this paper, we analyze the consequences of bank regulation on the size of the real sector. In particular, we address the question whether exogenous shocks on the return-risk characteristics of the technology and on the equity of the real sector are intensified or damped by a value-at-risk constraint on the credit portfolio of a bank. We consider a one-period model with three risk-averse agents, an investor, a bank, and a firm. The size of the markets for deposits and loans, their prices and the size of the real sector are endogenous. We find that stricter regulation results in higher loan rates, lower deposit rates, and lower activity in the real sector. A negative shock on the return-risk position or on the risk buffer of the real sector reduces the activities in the economy. Surprisingly, the sensitivity of the real sector's activities on negative shocks is smaller for a regulated financial sector than for a non-regulated one. Therefore, in our economy, imperfections in the financial sector do not result in procyclical or acceleration effects.

1 Introduction

The classical Keynesian model and business cycle theory basically assume that the financial structure in the economy has no impact on the real economic outcomes. For about twenty years now, there has been a growth in literature, giving the financial sector of an economy, especially the credit market, a more prominent role in explaining cyclical fluctuations of the real sector. Starting with the work of Bernanke (1983) and Bernanke/Blinder (1988) depressed asset prices and a large number of bank failures are not only reflections of a real downturn, but themselves impact a real depression. Frictions in the credit markets caused by incomplete and asymmetric information influence the borrower-lender relationship and result in agency costs of debt. In a deteriorating real economy these capital costs increase and accelerate the decline of real outcomes. This strand of literature is surveyed in Bernanke/Gertler/Gilchrist (1999).

A second stream of literature discusses the impact of frictions in the financial sector caused by bank regulation. The majority of the work done in this field concentrates

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on the effects of regulation on the individual bank behavior. Koehn/Santomero (1980), Kim/Santomero (1988), and Rochet (1992) show in one-period models that a regulation scheme, as imposed by the Basle Capital Accord, can result in negative incentives in the sense that a regulated bank selects riskier positions than a non-regulated bank.

The discussion of the New Basle Capital Accord (Basle II) has increased the concerns on the consequences of bank regulation. According to this new regulation framework, banks can choose between a standard approach similar to Basle I and an internal rating based approach (IRB approach) to quantify the capital requirements for their credit position. In the long run, it is expected that internal value-at-risk models will be accepted to measure, manage, and control the credit risk of banks. Basle II is, however, subject to an intense discussion. The market risk paper by Danielsson/Zigrand (2002) and the credit risk papers by Eichberger/Summer (2003) and Repullo/Suarez (2003) determine the banking activity under the Basle II regulation scheme. These papers concentrate on an analysis of banking activities and neglect the impact of regulation on the real sector. They also do not allow for the possibility that banks may raise deposits from investors. As the volume of deposits determines to a large extent the volume of granted loans, which in turn affects the real sector, the chain from investors via the bank to the firm in raising funds for the real sector should be taken into account for a discussion on procyclicality or acceleration issues.

Blum/Hellwig (1995) analyze the effect of a Basle I like regulation on the real sector in a classical Keynesian framework. The multiplier of a demand shock in the goods market is larger in an economy with regulated banks than in an unregulated economy. This higher sensitivity is driven by the fact that the firm's investment demand reacts more sensitively to a general demand shock if the capital adequacy requirement is binding compared with the case that it is not binding. Blum and Hellwig's result on the multipliers in the regulated economy are lower than those in an identical, unregulated economy.

On a qualitative level, Danielson/Embrechts/Goodhart/Keating/Muennich/Renault/Shin (2001) point out that the rating of loans on the basis of internal models depends on the state of the business cycle and is therefore procyclical. This procyclicality in internal credit ratings will create procyclicality in capital costs, if banks use similar internal models. As a result, the business downturn goes with a restricted loan granting which amplifies the downturn. The main arguments on the procyclicality of bank regulation are surveyed by Borio/Furfine/Lowe (2001).

To the best of our knowledge, we analyze for the first time the consequences of a regulation of credit portfolios on the volume of granted loans, the loan and deposit rate, and the activity in the real sector in a stochastic setting. This model comprises of a one-period framework with three types of agents: investors, banks, and firms. In this economy with perfect competition the banks collect deposits from the investors and grant loans to the firms. Regulation is introduced by a value-at-risk condition for the unexpected losses of the bank's portfolio. Each agent is equipped with an initial endowment and has a negative exponential utility function. The size of the markets for deposits and loans, their prices and the size of the real sector are endogenous.

We understand our model to be a benchmark model that captures the main effects of banking regulation on the real sector. However, we are well aware of the fact that some stylized features of our model are open to criticism. First, the risky returns of loans and deposits are only partly linked to the returns of the real sector. E.g. deposits bear an exogenous risk like operational risk even if banks invest their funds risk free. Second, the values of loans and deposits at maturity are not capped at zero nor at their face value. These assumptions allow us to derive the demand and supply functions for loans and deposits analytically. If the non-linear characteristics of the loan and deposit contracts at maturity are considered, the demand and supply functions can only be determined numerically. Third, we do not address the question whether regulation of banks is welfare improving for the agents. Therefore, we do not consider any benefits of regulation in our model. In this paper we address exclusively the problem whether in a regulated banking system macroeconomic shocks are amplified relative to a non-regulated financial sector.

Most of the results are obtained numerically. They can be summarized as follows: A stricter regulation results in higher loan rates, lower deposit rates, and lower activity of the real sector. A positive shock on the technology risk has negative effects on the real sector and the loan rate, independent of regulation. An important numerical result is that the sensitivity of the real sector's activity is smaller for a regulated financial sector than for a non-regulated one if the unregulated sector reacts at all. Therefore, in our economy, imperfections in the financial sector do not result in procyclical or acceleration effects.

The paper is organized as follows. Section 2 presents the model framework and introduces the characteristics of the investor, the bank, and the firm. The demand and supply for loans and deposits of these agents is derived in Section 3. Section 4 discusses general conditions and properties in equilibrium. A comparative static analysis of the real sector in terms of various kinds of shocks is accomplished in Section 5. Section 6 concludes.

2 Model Framework

The most simple framework to study the impact of imposing a regulation scheme on banks on the real sector of an economy is a model with three groups of agents: investors, banks, and firms. The agents within each group are assumed to be identical with respect to their preferences, beliefs and endowments. Furthermore, they are assumed to behave competitively. Therefore, they are aggregated to one representative investor I, one bank B, and one firm F.

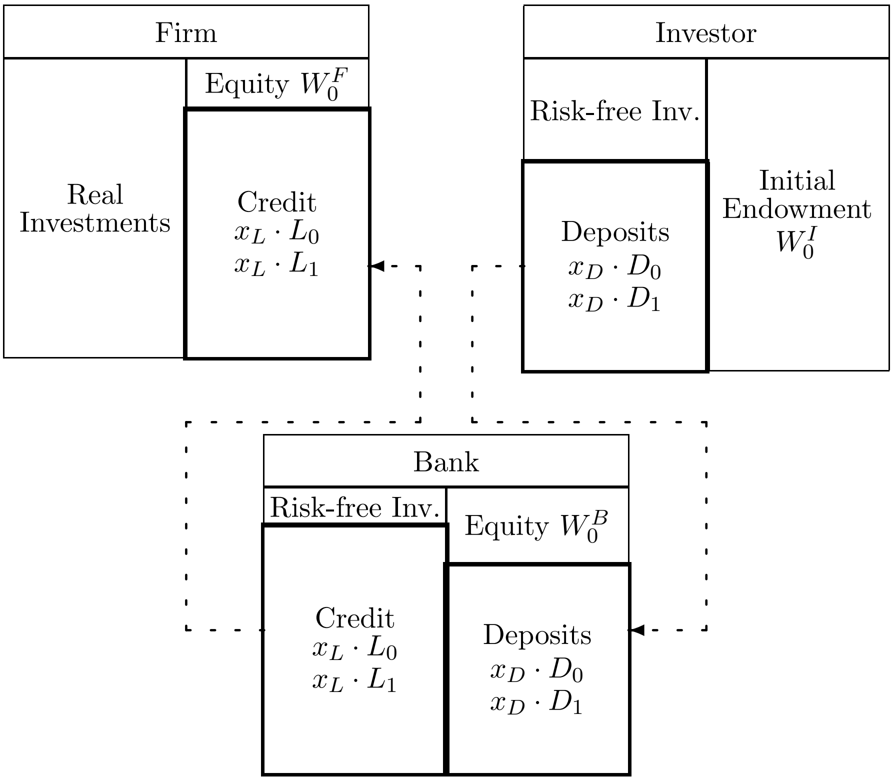
Figure 1 shows the basic structure of the model. For expository purposes, we consider a one-period model in which the agents take positions at time $t = 0$ and hold their position until the planning horizon $t = 1$. The choice set of the agents, their utility functions, the risk sources, and the markets for financial assets will be discussed in detail below.

(a) Investor

The representative investor holds an initial endowment equal to $W_0^I > 0$ at time $t = 0$ which is invested into a portfolio consisting of a risk-free asset and deposits. The amount invested into the risk-free asset and the number of deposits x_D must be non-negative, i.e. short positions are prohibited. Let D_0 be the price of one unit of a deposit, the investor invests

$$x_D \cdot D_0$$

Figure 1: General Relationship between Investor, Bank, and Firm



into deposits and the remaining wealth

$$W_0^I - x_D \cdot D_0 \geq 0 \tag{1}$$

into the risk-free asset. The risk-free asset yields an exogenous fixed return $r \geq 0$ for the period $[0, 1]$. The value D_1 of one deposit unit at time $t = 1$ is assumed to be normally distributed

$$D_1 \sim N\left(\mu_D, \sigma_D^2\right), \qquad \mu_D > 0, \sigma_D > 0,$$

with exogenous expectation μ_D and variance σ_D^2. This assumption means that the risk of deposits is not exclusively determined by the credit risk of loans given to the firms in the real sector. Deposits carry their own risks, e.g. operational risk. In addition, the assumption that the value D_1 of one deposit unit is normally distributed neglects the typical contract structure of deposits. However, as we bind the deposit's risk σ_D to a small fraction of the firm's risk, we believe that this assumption has no major impact on the results. Obviously, we make this assumption to facilitate the analytical derivations of the demand and supply functions.

The investor's objective is to maximize the expected utility $\mathbb{E}\left(U^I\left(W_1^I\right)\right)$ of the terminal wealth

$$W_1^I = x_D \cdot D_1 + \left(W_0^I - x_D \cdot D_0\right) \cdot (1 + r). \tag{2}$$

The utility function $U^I\left(W_1^I\right)$ is assumed to exhibit constant absolute risk aversion (CARA), i.e.

$$U^I\left(W_1^I\right) = -e^{-\lambda^I \cdot W_1^I}, \qquad \lambda^I > 0.$$

A basic determinant for the size of the real sector represents the decision as to how the representative investor distributes his wealth between the riskless asset and deposits. In general, the size of the real sector can vary between zero and the initial endowments of the three groups of agents. In the first case, the investors have a zero demand for deposits, in the second they invest W_0^I fully into deposits. This "breathing" of the real sector with the volume of deposits or with the size of risk-free investments, respectively, is necessary to study the impact of the exogenous parameters on its size. It depends crucially on the assumption that risk-free investments are not channeled back into the banking system where they could be used for loans. Risk-free funds are invested outside our system of three agents and are not available until the end of the period.

(b) Firm

The firm initially has an exogenous equity of $W_0^F > 0$ which is invested together with funds from a loan granted by the bank into a real technology. To strengthen the consequences of bank regulation on the size of the real sector we assume that besides loans, firms have no other possibilities to finance their real investments, e.g. by issuing bonds or stocks. This assumption has the additional advantage that we need not to model the bond and/or the stock market (see also the discussion in Section (e)).

The credit volume is

$$x_L \cdot L_0,$$

where $x_L \geq 0$ denotes the number of credit units and L_0 the value of one credit unit. To reduce the number of different cases, which we have to discuss in Sections 3 and 4, we assume that the firm has no risk-less investment opportunity. As a consequence, the amount invested into the real technology is

$$W_0^F + x_L \cdot L_0 > 0. \tag{3}$$

At the terminal date $t = 1$, the firm obtains a return r_V on the invested capital and the loans are redeemed at L_1. We assume that the firm has unlimited liability. This implies that the terminal equity W_1^F at time $t = 1$ reads

$$W_1^F = \left(W_0^F + x_L \cdot L_0\right) \cdot (1 + r_V) - x_L \cdot L_1. \tag{4}$$

In line with the assumption about the deposit value D_1, we assume that both the return r_V on the technology and the loan redemption L_1 are normally distributed

$$\begin{aligned} r_V &\sim N\left(\mu_V, \sigma_V^2\right), \qquad && \mu_V > 0, \sigma_V > 0, \\ L_1 &\sim N\left(\mu_L, \sigma_L^2\right), \qquad && \mu_L > 0, \sigma_L > 0, \end{aligned}$$

with means μ_V and μ_L and variances σ_V^2 and σ_L^2. The covariance matrix Σ^2 of $(r_V, D_1, L_1)'$ is denoted by

$$\Sigma^2 = \begin{pmatrix} \sigma_V^2 & \rho_{D,V} \cdot \sigma_V \sigma_D & \rho_{L,V} \cdot \sigma_V \sigma_L \\ \rho_{D,V} \cdot \sigma_V \sigma_D & \sigma_D^2 & \rho_{D,L} \cdot \sigma_D \sigma_L \\ \rho_{L,V} \cdot \sigma_V \sigma_L & \rho_{D,L} \cdot \sigma_D \sigma_L & \sigma_L^2 \end{pmatrix}, |\rho_{D,L}| < 1, |\rho_{L,V}| < 1, |\rho_{D,V}| < 1.$$

The firm has — analogous to the investor — a CARA utility function

$$U^F\left(W_1^F\right) = -e^{-\lambda^F \cdot W_1^F}, \qquad \lambda^F > 0,$$

and chooses the credit volume x_L such that the expectation of its terminal utility $\mathbb{E}\left(U^F\left(W_1^F\right)\right)$ is maximized.

We normalize μ_D and μ_L to one and assume that correlations and the expected return $\mu_V - 1$ of the real investment are positive. As the main part of the loan's and deposit's risk stems from the risk σ_V of the technology and is reduced by the firm's and the bank's equity, we assume further that the relations

$$\sigma_V/\mu_V \geq \sigma_L \geq \sigma_D$$

hold.

(c) Bank

The bank is a typical commercial bank which can collect deposits from the investor and grant loans to the firm. It has the properties of a bank for two reasons. First, the firm can raise funds only through bank loans. Therefore, the bank acts as a financial intermediary in its strictest form. Second, the bank is regulated by a value-at-risk constraint on its portfolio. This regulation feature will become apparent below. Thus, the bank is a typical intermediary because the investor cannot invest into claims of the firm directly. This function underlines the effect of a regulation.

The bank raises a non-negative number of deposits with a volume equal to

$$x_D \cdot D_0.$$

The bank invests these funds together with its exogenous initial equity $W_0^B > 0$ into loans $x_L \cdot L_0$ and the remaining part

$$x_C := W_0^B + x_D \cdot D_0 - x_L \cdot L_0 \geq 0$$

into the risk-free asset with exogenous rate of return r. Since the bank also has unlimited liability, the terminal wealth W_1^B of the bank results in:

$$W_1^B = x_C \cdot (1+r) - x_D \cdot D_1 + x_L \cdot L_1 \tag{5}$$

The bank selects a loan and deposit volume such that the expected utility $\mathbb{E}\left(U^B\left(W_1^B\right)\right)$ is maximized. Again, we assume

$$U^B\left(W_1^B\right) = -e^{-\lambda^B \cdot W_1^B}, \qquad \lambda^B > 0.$$

For the trading book of a bank, it is regulatory acceptable to measure and control market risks by an internal model, for example the value-at-risk (VAR) of this book (see e.g. Bank of International Settlement (2003)). According to the third consultative paper of the New Basle Accord, credit risk related to the bank book can be measured and controlled by the standard approach or by a method based on internal ratings (IRB) of individual credits. We extend this internal credit risk model to a VAR-based approach for the *unexpected* losses. In other words, the VAR-approach for credit risk requires that the unexpected loss of terminal wealth given by $\mathbb{E}\left(W_1^B\right) - W_1^B$ exceeds the initial equity $\alpha' \cdot W_0^B$ with probability below or equal to $\overline{p}$:

$$\Pr\left(\mathbb{E}\left(W_1^B\right) - W_1^B \geq \alpha' \cdot W_0^B\right) \leq \overline{p} \tag{6}$$

As the terminal wealth W_1^B is normally distributed, this condition simplifies to

$$SD\left(W_1^B\right) \leq \underbrace{\frac{\alpha'}{N^{-1}\left(1-\overline{p}\right)}}_{=:\alpha} \cdot W_0^B, \tag{7}$$

where $SD\left(W_1^B\right)$ denotes the standard deviation of the terminal wealth of the bank and $N^{-1}(\cdot)$ indicates the inverse of the standard normal distribution function. Therefore, regulation restricts the standard deviation of the bank's wealth through the tightness parameter

$$\alpha = \frac{\alpha'}{N^{-1}\left(1-\overline{p}\right)}$$

and the amount of initial equity W_0^B at the beginning of the planning period. The VAR constraint (6) deserves two comments. First, the assumption that the equity has to cover only negative deviations from the expected wealth at $t = 1$ results in the simple constraint (7) for the standard deviation of the wealth W_1^B. Another possibility would be to relate losses $W_1^B - W_0^B$ to the equity W_0^B. In this case, the expected wealth $\mathbb{E}\left(W_1^B\right)$ would be part of the inequality (7). The qualitative results presented in Section 4 would not change if this version of a VAR-constraint was used. Second, the constraint (6) depends on the bank's total portfolio of investments and deposits. This definition of the risk exposure corresponds with the definition of market risk. It is, however, less suitable to measure credit risk exposure as this risk is related exclusively with the loans. We include deposits into the portfolio to add one degree of freedom to the VAR constraint. Otherwise, a binding constraint would determine the loan volume and the bank has no choice problem. If we extend the model to more than one lending relationship we can exclude the deposits from the risky portfolio.

(d) Risk Sources

Figure 1 suggests that there is only one risk source in our economy. This risk comes from the real sector and determines the risk of the credit and, finally, the risk of the deposits. The real sector risk exposure is reduced by three mechanisms until it is transferred to the deposit investors. First, the equity of the firm and the bank absorbs part of the risk. Second, in the multi-firm-, multi-bank case, there exists, at least in principle, the possibility that individual banks reshape the real sector risk and offer specific risk tranches. Third, the regulation of banks introduces through the credit channel an upper limit to the total risk the real sector can take.

In our preliminary model, we specify the risk transfer from the real sector to the deposit investors by the following assumptions:

- Real risks are reshaped only by the equity of the firm and the bank. An additional reshaping of risks by tranching is not considered.
- The credit value L_1 per unit and the deposit value D_1 per unit at the repayment date are normally distributed.

As discussed in the introduction, this assumption destroys the typical put-option characteristic of credits and deposits. Particularly, it has the obvious unrealistic consequence that the bank or the investor can lose more than its initial wealth. The well-known advantage of this assumption is that we can characterize the optimal behavior of the agents in a mean-variance framework. Therefore, the results of this preliminary model should be considered as an 'easily computable' benchmark for more general versions in which the risk transfer, the option feature of credits and deposits, and the diversification or tranching possibilities of banks are considered explicitly.

On the other hand, the assumption of normally distributed, not perfectly correlated risks of the loan and the deposit adds additional risk factors to the real sector risk. On the firm and bank level these additional risks can be a consequence of market- and operational risk factors. In the comparative static analysis we will relate the volatilities σ_D and σ_L of the deposit and loan value D_1, L_1 to the volatility σ_V of the real-sector risk.

(e) Markets

In our model, we consider a market for credits and deposits only. The prices D_0 and L_0 for one unit of deposit or credit are characterized by a standard Walrasian equilibrium.

We do not consider a market for stocks issued by the firm or the bank for two reasons. First, the possibility to increase the firm's or bank's equity would require the modelling of the stock market and increase the complexity of the model. Second, and more importantly, in the intended dynamic extension of the model in a dynamically complete market setting, the investor could replicate the loan position of the bank. As a consequence the bank would be superfluous in the model.

The assumption that the bank and the firm cannot issue new equity implies that in the multi-period extension of the model the equity of the firms and banks is determined by the exogenous initial equity and the aggregated losses and profits over time.

3 Demand and Supply of Deposits and Loans

Since the terminal wealth W_1^i, $i = I, B, F$ of each agent is normally distributed and all three agents have negative exponential utility functions, the maximization of the expected utility is equivalent to the maximization of the following mean-variance objective functions OF^i:

$$OF^i := \max \mathbb{E}\left(W_1^i\right) - \frac{1}{2}\lambda^i \cdot \mathbb{V}\left(W_1^i\right), \, i = I, B, F,$$

where $\mathbb{V}(W_1^i)$ is the variance of terminal wealth of agent i. We use this objective function to determine the demand and supply of deposits and loans.

(a) Demand of Deposits

The investor determines his optimal demand $x_D^I(D_0)$ of deposits by solving OF^I with an expected value and variance of his final wealth of

$$\begin{aligned}\mathbb{E}\left(W_1^I\right) &= x_D + \left(W_0^I - x_D \cdot D_0\right) \cdot (1+r), \\ \mathbb{V}\left(W_1^I\right) &= x_D^2 \cdot \sigma_D^2.\end{aligned}$$

The first order condition of the **unconstrained** problem provides us with the solution

$$x_D = \frac{1 - D_0 \cdot (1+r)}{\lambda^I \cdot \sigma_D^2}.$$

As a short sale of the risk-free instrument is excluded by (1), his demand is limited by $x_D^I(D_0) \leq \frac{W_0^I}{D_0}$. In the case $x_D > \frac{W_0^I}{D_0}$, we can easily see that the expected utility of the investor is maximized for $x_D^I(D_0) = \frac{W_0^I}{D_0}$. Thus, we obtain the demand function of deposits:

$$x_D^I(D_0) = \min\left(\max\left(\frac{1 - D_0 \cdot (1+r)}{\lambda^I \cdot \sigma_D^2}, 0\right), \frac{W_0^I}{D_0}\right) \tag{8}$$

The demand of deposits is positive if and only if $D_0 < \frac{1}{1+r}$ holds, and $x_D^I(D_0)$ is strictly monotonic decreasing in D_0 as long as $x_D^I(D_0)$ is positive. The deposit volume $x_D^I(D_0) \cdot D_0$ is concave and increases first and then declines in D_0 on the interval $[0, 1/(1+r)]$.

(b) Demand of Loans

The demand of loans $x_L^F(L_0)$ is determined by the firm. The expected value and the variance of its terminal wealth follow from (4):

$$\begin{aligned}
\mathbb{E}\left(W_1^F\right) &= \left(W_0^F + x_L \cdot L_0\right) \cdot \mu_V - x_L, \\
\mathbb{V}\left(W_1^F\right) &= \left(W_0^F + x_L \cdot L_0\right)^2 \cdot \sigma_V^2 + x_L^2 \cdot \sigma_L^2 \\
&\quad - 2 \cdot \left(W_0^F + x_L \cdot L_0\right) \cdot x_L \cdot \rho_{L,V} \cdot \sigma_V \sigma_L
\end{aligned}$$

Again, the **unconstrained** demand x_L of loans follows from the first order condition and is given by:

$$x_L = \frac{\mu_V \cdot L_0 - 1 + \lambda^F W_0^F \cdot (\rho_{L,V}\sigma_L\sigma_V - \sigma_V^2 \cdot L_0)}{\lambda^F \cdot (\sigma_V^2 L_0^2 - 2\rho_{L,V}\sigma_L\sigma_V \cdot L_0 + \sigma_L^2)}$$

If L_0 is sufficiently low, the firm wants to take a short position in loans $x_L < 0$ which is excluded. In this case, $x_L^F(L_0) = 0$ is optimal and the demand function $x_L^F(L_0)$ reads

$$x_L^F(L_0) = \max\left(\frac{\left(\mu_V - \lambda^F W_0^F \sigma_V^2\right) \cdot L_0 - 1 + \lambda^F W_0^F \cdot \rho_{L,V}\sigma_L\sigma_V}{\lambda^F \cdot (\sigma_V^2 L_0^2 - 2\rho_{L,V}\sigma_L\sigma_V \cdot L_0 + \sigma_L^2)}, 0\right).$$

For the discussion of the demand function $x_L^F(L_0)$ we first note that the denominator is always positive. The qualitative behavior of the demand for loans depends crucially on the expression $\mu_V - \lambda^F W_0^F \sigma_V^2$. This term is equal to the marginal expected utility $\frac{\partial \mathbb{E}\left(U^F\left(W_1^F\right)\right)}{\partial W_1^F}$ if the firm is restricted to equity financing. If this marginal utility at the firm's exogenous amount of equity W_0^F is non-positive, a larger equity volume would not increase the firm's expected utility. As this case is not interesting, we exclude it by the assumption

$$\mu_V - \lambda^F W_0^F \sigma_V^2 > 0 \tag{9}$$

In this case the demand $x_L^F(L_0)$ is positive if the loan value L_0 per unit is above the critical value

$$\overline{L}_0 = \frac{1 - \lambda^F \cdot W_0^F \cdot \rho_{L,V} \cdot \sigma_L\sigma_V}{\mu_V - \lambda^F \cdot W_0^F \cdot \sigma_V^2} \tag{10}$$

It can be further shown that $x_L^F(L_0)$ for $L_0 \geq \overline{L}_0$ is first strictly increasing in L_0 until a unique maximum $\widehat{L}_0$. After $\widehat{L}_0$ it strictly decreases in L_0 and converges to zero if L_0 goes to infinity. The demand for the loan volume $x_L^F(L_0) \cdot L_0$ is strictly monotonic increasing for $L_0 \geq \overline{L}_0$ and converges to $\frac{\mu_V}{\lambda^F \sigma_V^2} - W_0^F$. The first term in this difference is the optimal real investment of the firm if it took no credit and could adapt its equity to the investment volume.

It follows from the assumptions $\mu_V > 1$, $\sigma_V/\mu_V \geq \sigma_L$, and $\rho_{L,V} > 0$ that the critical value $\overline{L}_0$ is positive and below one.

(c) Supply of Deposits and Loans of the Unregulated Bank

The bank simultaneously supplies deposits and loans. Due to the non-negativity restrictions for the choice variables, we have to discuss five different cases for the optimal bank decision. They differ by the structure of the bank's balance sheet. In addition to the equity the balance sheet can consist of

(i) loans, deposits, and the risk-free asset,

(ii) loans and deposits only,

(iii) loans and the risk-free asset,

(iv) deposits and the risk-free asset, or

(v) the risk-free asset only.

First, we consider an unregulated bank and discuss the structure of the non sign-restricted solution and the feasible optimal holdings in sequence.

Case (i)

In general case (i) will hold if the choice variables of the bank are not restricted in sign. Thus, the expected value and the variance of the unregulated bank's wealth read:

$$\mathbb{E}\left(W_1^B\right) = x_L - x_D + x_C \cdot (1+r),$$
$$\mathbb{V}\left(W_1^B\right) = x_D^2 \cdot \sigma_D^2 + x_L^2 \cdot \sigma_L^2 - 2 \cdot x_D \cdot x_L \cdot \rho_{D,L} \cdot \sigma_D \sigma_L$$

The expected utility of its terminal wealth is a strictly concave function. From the first order conditions, we obtain the **unrestricted** solution

$$x_D = \frac{(1 - L_0 \cdot (1+r)) \cdot \rho_{D,L} \cdot \sigma_D \sigma_L - (1 - D_0 \cdot (1+r)) \cdot \sigma_L^2}{\lambda^B \cdot \left(1 - \rho_{D,L}^2\right) \cdot \sigma_D^2 \sigma_L^2}, \tag{11}$$
$$x_L = \frac{(1 - L_0 \cdot (1+r)) \cdot \sigma_D^2 - (1 - D_0 \cdot (1+r)) \cdot \rho_{D,L} \cdot \sigma_D \sigma_L}{\lambda^B \cdot \left(1 - \rho_{D,L}^2\right) \cdot \sigma_D^2 \sigma_L^2}.$$

If this solution provides non-negative volumes for deposits $x_D \geq 0$, loans $x_L \geq 0$, and also non-negative holdings in the risk-free asset, $x_C \geq 0$, the units x_D and x_L represent the supply of the bank. Otherwise, the bank's optimal balance sheet has a different structure.

Case (ii)

If the bank optimizes its portfolio under the constraint $x_C = 0$, the supply of loans and deposits have to satisfy the following budget constraint

$$W_0^B + x_D \cdot D_0 = x_L \cdot L_0.$$

In this case the expectation and the variance of the bank's terminal wealth are

$$\mathbb{E}\left(W_1^B\right) = x_L - x_D,$$
$$\mathbb{V}\left(W_1^B\right) = x_D^2 \cdot \sigma_D^2 + x_L^2 \cdot \sigma_L^2 - 2 \cdot x_D \cdot x_L \cdot \rho_{D,L} \cdot \sigma_D \sigma_L.$$

Substituting x_D by means of the budget constraint, we obtain the unrestricted optimal number of deposits x_D and loans x_L

$$x_D = \frac{x_L \cdot L_0 - W_0^B}{D_0}, \tag{12}$$
$$x_L = \frac{D_0^2 - D_0 L_0 + \lambda^B W_0^B \left(\sigma_D^2 L_0 - D_0 \cdot \rho_{D,L} \cdot \sigma_D \sigma_L\right)}{\lambda^B \cdot \left(\sigma_D^2 L_0^2 - 2 \cdot \rho_{D,L} \cdot \sigma_D \sigma_L D_0 L_0 + \sigma_L^2 D_0^2\right)}.$$

If x_D and x_L are non-negative, this solution is a feasible candidate for the optimal supply of deposits and loans. Note that x_D and x_L are non-linear in L_0 and D_0.

Case (iii)

If the balance sheet of the bank is restricted to the risk-free asset and loans, we obtain the following solution for the supply of loans:

$$x_L^B = \min\left(\max\left(\frac{1 - L_0 \cdot (1+r)}{\lambda^B \cdot \sigma_L^2}, 0\right), \frac{W_0^B}{L_0}\right) \tag{13}$$

This solution is determined analogously to the optimal investment of the investor.

Case (iv)

If the bank collects deposits and holds their total funds in the risk-free asset without granting loans to the firm, the optimal number of deposits amounts to

$$x_D^B = \max\left(\frac{D_0 \cdot (1+r) - 1}{\lambda^B \cdot \sigma_D^2}, 0\right). \tag{14}$$

We will later illustrate the fact that case (iv) cannot represent an equilibrium.

Case (v)

In case (v), the bank invests all of its equity in the risk-free asset; i.e. $x_D = x_L = 0$. This portfolio cannot be optimal if x_L is positive in case (iii).

Summary of Cases (i) – (v)

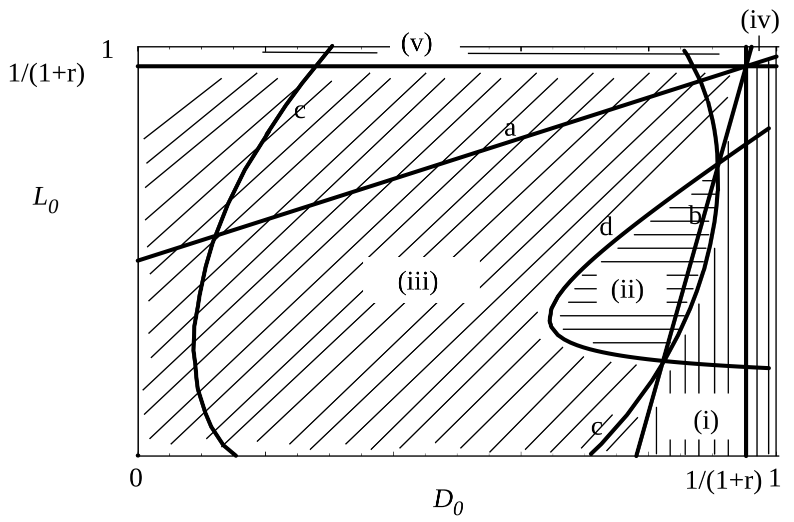
The optimal supply of deposits and loans can now be determined as follows: If the general case (i) results in non-negative amounts for deposits, loans, and the risk-free asset, the optimal supply $x_D^B(D_0, L_0)$, $x_L^B(D_0, L_0)$ is represented by (11).

If (11) does not satisfy the non-negativity constraints, the same analysis is done for case (ii). Given that (12) results in a non-negative solution this will represent the optimal portfolio of the bank if in addition the following relationship for the expected gross returns for the risk-free asset, the deposits and the loans holds:

$$1 + r < \frac{1}{D_0} < \frac{1}{L_0} \tag{15}$$

Figure 2: Bank's Optimal Portfolio Decision as a Function of current Values D_0 and L_0

The parameter values used are: $r = 0.05$, $\mu_V = 1.2$, $\sigma_V = 0.3$, $\sigma_L = 0.06$, $\sigma_D = 0.036$, $\rho_{L,V} = 0.3$, $\rho_{D,L} = 0.3$, $W_0^B = 100$, *and* $\lambda^B = 0.5$.



If this condition is violated, the optimal portfolio is attained by evaluating the objective function for the three portfolios given by (12), (13), and (14).

Figure 2 summarizes these findings for the unregulated bank. The area of case (i) is formed by two linear restrictions a and b and and an ellipse c. Lines a and b are the result of the non-negativity condition of x_L and x_D in (11), while the ellipse controls for non-negative holdings $x_C \geq 0$ in the risk-free asset. For all feasible parameter sets, we can show that both linear functions intersect at the point $(1/(1+r), 1/(1+r))$ and have a positive slope. The slope of line b is always higher than that of a. If the ellipse c and line b intersect at all, such as in Figure 2, each intersection has the property $L_0 \leq 1/(1+r)$.

If the risk aversion of the bank is sufficiently large, the ellipse does not intersect with b. In this case, the non-negativity condition for the risk free asset is satisfied for every pair of prices (D_0, L_0) below a and b, i. e. this condition does not become binding in case (i).

Line d separates the price pairs (D_0, L_0) for which x_D is non-negative in case (ii). All price pairs (D_0, L_0) below curve d and on the left of the ellipse c result in portfolios of the structure (ii). If the risk aversion λ_B of the bank increases, the region where the bank desires to have deposits and loans without risk-free assets in its balance sheet shrinks and can become void.

The optimal bank portfolio has structure (iii) if case (i) or (ii) are not optimal and L_0 is below $1/(1+r)$, i.e. the expected returns of the risky loans exceed the risk-free rate r. Accordingly, the bank decides upon the optimal portfolio relating to case (iv) if cases (i) to (iii) are not optimal and D_0 is above $1/(1+r)$. Otherwise, the bank holds its whole equity in the risk-free asset.

We can see in Figure 2, that for a fixed L_0 below $1/(1+r)$ and a sufficiently high D_0 case (i) is optimal. In this case, the bank can raise funds at relatively 'cheap' expected costs and invests said deposits in loans. In order to control the risk of the bank's portfolio, the bank also has holdings in the risk-free asset.

When D_0 declines for a fixed L_0, the bank decreases its risk-free investment to zero. In the unconstrained case, the bank would like to have a short position in the risk-free asset for price pairs (D_0, L_0) in the area (ii). The short-sale restriction forces the bank to take additional deposits instead.

If the price of a deposit unit declines further, a transition from the case (ii) to the case (iii) occurs. For low D_0 (relative to L_0), the deposits are expensive and therefore the bank holds its wealth W_0^B in loans and the risk-free asset only.

If the loan price is above $1/(1+r)$ three cases can occur. For high prices D_0, it is optimal for the bank to have deposits, loans, and the risk-free asset according to case (i). If D_0 declines, a transition from case (i) to case (iv) occurs. The frontier is given by the condition that x_L in case (i) is zero (line a). If D_0 declines further, it is not optimal for the bank to take neither loans nor deposits (case (v)).

As a general result, we obtain that the bank simultaneously supplies positive amounts of deposits and loans, if $D_0 \geq 1/(1+r)$ and $L_0 < 1/(1+r)$ hold. In addition, the deposit supply of the bank is strictly positive even for $D_0 = 1/(1+r)$ and $L_0 < 1/(1+r)$. Figure 2 verifies this result. Therefore, case (i) or case (ii) can be optimal but not case (iii). These properties are the core for the analysis of the equilibria in Section 4.

(d) Supply of Deposits and Loans of the Regulated Bank

The regulated bank differs from the unregulated one by the additional VAR-constraint (7) that relates the standard deviation of the terminal wealth to the regulation parameter α and the initial wealth of the bank:

$$SD\left(W_1^B\right) := \sqrt{x_D^2 \cdot \sigma_D^2 + x_L^2 \cdot \sigma_L^2 - 2 \cdot x_D x_L \cdot \rho_{D,L} \cdot \sigma_D \sigma_L} \leq \alpha \cdot W_0^B \tag{16}$$

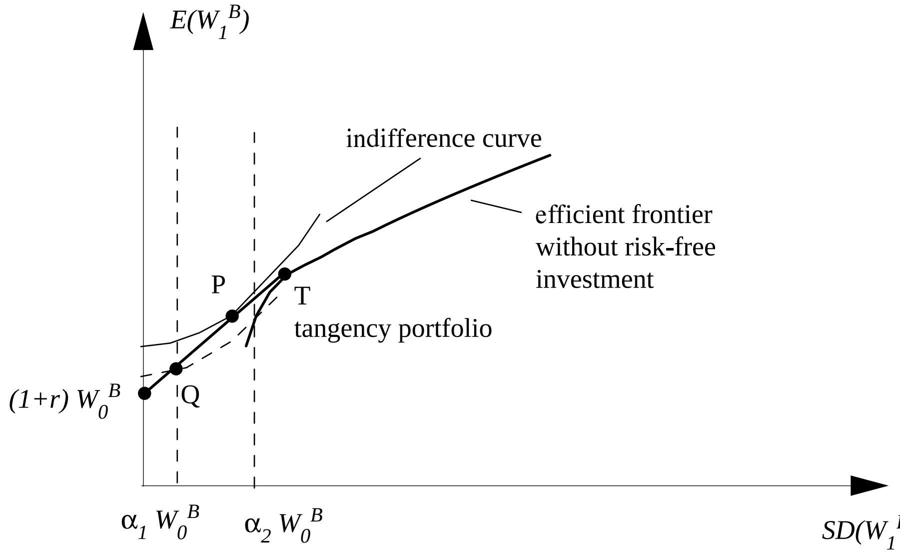
Analogously to the unregulated bank, we have to analyze the cases (i) to (v). Contrary to the analysis above we now discuss each case conditional on the assumption that this case represents the optimal portfolio of the unregulated bank.

Case (i)

In the first case, the bank grants loans, takes deposits, and adjusts the risk-free holdings such that the regulatory condition is satisfied. To illustrate this result it is helpful to characterize graphically the bank's efficient portfolios in the well-known expected-wealth-standard-deviation-of-wealth diagram (see Figure 3).

If the unrestricted solution (11) for the unregulated bank satisfies the non-negativity constraints, then its E-SD-combination P lies on the tangent line through the points $\left(0, (1+r)\, W_0^B\right)$ and T (the tangency portfolio T exists and is characterized explicitly below). If, in addition, the VAR-constraint is not binding ($\alpha = \alpha_2$), then this portfolio is obviously also optimal for the regulated bank. If the VAR-constraint is binding ($\alpha = \alpha_1$), the portfolio on the tangency line with $SD\left(W_1^B\right) = \alpha \cdot W_0^B$ satisfies the non-negativity constraints. It differs from the optimal portfolio of the non-regulated bank by a larger investment in the risk-free asset. This portfolio is optimal for the regulated bank as all expected-wealth-standard-deviation-of-wealth efficient portfolios lie on this line.

Figure 3: Expected Wealth–Standard Deviation of Wealth Diagram



The unrestricted tangency portfolio $\left(x_D^T, x_L^T\right)$ exists and has the analytical representation:

$$x_D^T = W_0^B \sigma_L^2 \frac{A}{C},$$
$$x_L^T = W_0^B \sigma_D^2 \frac{B}{C},$$

where

$$\begin{aligned} A &:= 1 - D_0 \cdot (1+r) - (1 - L_0 \cdot (1+r)) \frac{\sigma_D}{\sigma_L} \rho_{D,L} \\ B &:= L_0 \cdot (1+r) - 1 + (1 - D_0 \cdot (1+r)) \frac{\sigma_L}{\sigma_D} \rho_{D,L}, \\ C &:= (L_0 \cdot (1+r) - 1) L_0 \sigma_D^2 - (1 - D_0 \cdot (1+r)) D_0 \sigma_L^2 \\ &\quad + \sigma_D \sigma_L \cdot \rho_{D,L} \cdot (D_0 + L_0 - 2 L_0 D_0 \cdot (1+r)) . \end{aligned} \tag{17}$$

If case (i) is assumed to be optimal for the unregulated bank, x_D^T and x_L^T are non-negative and the optimal supply of deposits and loans by the regulated bank are

$$\begin{aligned} x_D^{reg}(D_0, L_0) &= \frac{\alpha \cdot W_0^B}{SD^T} x_D^T, \\ x_L^{reg}(D_0, L_0) &= \frac{\alpha \cdot W_0^B}{SD^T} x_L^T, \end{aligned} \tag{18}$$

where SD^T stands for the standard deviation of the tangency portfolio.

Compared to the optimal portfolio of the unregulated bank, regulation in case (i) has the obvious effect that a larger amount of the equity funds and the deposits is invested into the risk-free asset at the expense of loans given to the firm.

It is notable that the adverse risk incentive as discussed by Kim/Santomero (1988) and Rochet (1992) does not occur. If case (i) is optimal for the unregulated bank, regulation always results in a non-negative transfer of risky assets into the risk-free asset as the regulator intends. It is also notable that the structure of the optimal solution is not changed by an effective VAR-constraint in case (i).

Case (ii)

If it is optimal for the non-regulated bank to hold deposits and loans only, we obtain the solution (12) in the case that the VAR-constraint is not binding. Otherwise the budget restriction and

$$x_D^2\sigma_D^2 + x_L^2\sigma_L^2 - 2x_D x_L \rho_{D,L}\sigma_D\sigma_L = \left(\alpha \cdot W_0^B\right)^2 \tag{19}$$

determine x_D and x_L. It can be shown that there exist two solutions of which only the following is efficient:

$$x_D^{reg} = \frac{x_L^{reg} L_0 - W_0^B}{D_0}$$

$$x_L^{reg} = W_0^B \frac{\sigma_D \cdot (L_0\sigma_D - D_0\sigma_L \cdot \rho_{D,L}) - Sign\left(\frac{1}{D_0} - \frac{1}{L_0}\right) \cdot D_0 \cdot \sqrt{A'}}{B'},$$

The parameters A' and B' depend on the current values D_0, L_0 of deposits and loans and on the variances and covariance of D_1, L_1 only.

In case (ii), the optimal portfolio of the unregulated bank can be represented graphically in Figure 3 by an E-SD-combination on the non-linear part of the efficient frontier. If the tangency portfolio satisfies the VAR-constraint (19), then case (ii) also characterizes the optimal balance-sheet structure for the regulated bank. If the tangency portfolio violates the VAR-restriction, the regulated bank will also invest into the risk-free asset and the optimal balance sheet will exhibit the same structure as in case (i). Again, regulation results in a simultaneous reduction of loans and deposits of the same size. Again, contrary to the result by Kim/Santomero (1998) and Rochet (1992), regulation has the desired impact on the risk exposure of the bank.

Case (iii)

If the unregulated bank grants loans and holds risk-free assets without taking deposits, then a binding VAR condition results in

$$x_L^{reg} = \alpha \cdot \frac{W_0^B}{\sigma_L}.$$

Case (iv)

If it was optimal for the unregulated bank to collect deposits and to hold their total funds in the risk-free asset, and if the regulation constraint was binding, (14) would reduce to

$$x_D^{reg} = \alpha \cdot \frac{W_0^B}{\sigma_D}. \tag{20}$$

Case (v)

As the bank has only equity and the risk-free assets, the VAR constraint has no effect.

Summing up, the discussion above shows that if one of the cases (i), (iii), (iv), or (v) is optimal for the unregulated bank, the same case is also optimal for the VAR-regulated bank. Only in case (ii), a stricter regulation can change the optimal structure to case (i). Due to the risk appetite of the bank, case (ii) is preferable compared to case (i) without regulation. If regulation becomes binding, the bank can be forced to reduce the volume of loans and deposits and to invest in the risk-free asset.

We note that the general and important result that the bank always supplies a positive amount of deposits and loans, if $D_0 \geq 1/(1+r)$ and $L_0 < 1/(1+r)$ hold, is still valid under regulation as this is the case for the unregulated banking system.

4 Equilibrium

We define an equilibrium for the non-regulated financial sector in the standard way as a vector $\left(D_0^*, L_0^*, x_D^B, x_L^B, x_C^B, x_D^I, x_L^F\right)$ that satisfies the following conditions:

- $x_i^B\left(D_0^*, L_0^*\right)$, $i \in \{D, L, C\}$, $x_D^I\left(D_0^*\right)$, and $x_L^F\left(L_0^*\right)$ are optimal choices of the bank, the investor, and the firm.
- The deposit and loan market clear for the prices D_0^* and L_0^*:

$$\begin{aligned} x_D^I\left(D_0^*\right) &= x_D^B\left(D_0^*, L_0^*\right) \\ x_L^F\left(L_0^*\right) &= x_L^B\left(D_0^*, L_0^*\right) \end{aligned} \tag{21}$$

If the bank is regulated x_D^B and x_L^B in (21) have to be substituted by $x_D^{reg}\left(D_0^*, L_0^*\right)$ and $x_L^{reg}\left(D_0^*, L_0^*\right)$.

We discuss subsequently important properties of the equilibria for the non-regulated and the regulated bank. In the proof of these propositions, we make use of the general result that the bank supplies a positive amount of deposits, if either $D_0 > 1/(1+r)$ or $D_0 = 1/(1+r)$ together with $L_0 < 1/\left(1+r\right)$ hold. Accordingly, the bank supplies a positive amount of loans for $L_0 < 1/(1+r)$. These properties hold for an unregulated bank as well as for a regulated bank if $\alpha > 0$. Generally, we presume $\alpha > 0$. The case $\alpha = 0$ is uninteresting as the bank cannot take any risky positions.

An important determinant for the discussion of equilibria is the critical value $\overline{L}_0$ characterized by (10). Depending on the size of $\overline{L}_0$ relative to $1/\left(1+r\right)$ the equilibria are characterized differently.

Result 1 *For $\overline{L}_0 < \frac{1}{1+r}$, the following relations must hold in equilibrium:*

$$\begin{aligned} D_0^* &< \frac{1}{1+r}, \\ \overline{L}_0 < L_0^* &< \frac{1}{1+r}. \end{aligned}$$

The properties $D_0^* \leq 1/(1+r)$ and $L_0^* \leq 1/(1+r)$ are immediate consequences of the fact that otherwise the investor has a zero demand for deposits whereas the bank supplies deposits in a positive amount, $x_D^B > 0$ or $x_D^{reg} > 0$. Analogously, $\overline{L}_0 < L_0^* \leq \frac{1}{1+r}$ must hold for the firm to have a positive demand for loans.

If $L_0^* = 1/(1+r)$ holds, the demand x_L^F of the firm is positive, while for the relevant region $D_0^* \leq \frac{1}{1+r}$ the supply x_L^B and x_L^{reg} of the bank is zero. Therefore, the price $L_0^* = 1/(1+r)$ cannot represent an equilibrium.

For $D_0^* = 1/(1+r)$ and $L_0^* < 1/(1+r)$, $x_D^I = 0$ holds but x_D^B and x_D^{reg} are positive. Hence, the equilibrium price D_0^* of a deposit must also be below $1/(1+r)$. As a consequence of deposit and loan prices below $1/(1+r)$ and $\overline{L}_0 < 1/(1+r)$, the number $x_D^I(D_0^*)$ and $x_L^F(L_0^*)$ of deposits and loans in equilibrium are strictly positive.

Result 2 *For $\overline{L}_0 \geq \frac{1}{1+r}$, the equilibrium prices of deposits and loans are:*

$$D_0^* = \frac{1}{1+r},$$
$$L_0^* \in \left[\frac{1}{1+r}, \overline{L}_0\right].$$

These prices result in zero supply and demand on both markets and represent equilibria. For other prices, the markets do not clear. For $L_0 > \overline{L}_0$, the firm has a positive demand in loans but the bank does not supply loans and for prices $L_0 < 1/(1+r)$, it is vice versa. Accordingly, if D_0 differs from $1/(1+r)$, either only the bank $(D_0 > 1/(1+r))$ or only the investor $(D_0 < 1/(1+r))$ are active in the deposit market.

These two results have important implications. In equilibrium, the volume of loans and deposits is either strictly positive on **both** markets or the loan market and the deposit market break down. Therefore, cases (iii) and (iv), in which the bank is either active in the deposit or in the loan market, are not possible in equilibrium.

5 Comparative Static Analysis

In this section we analyze

- the expected loan rate $r_L := 1/L_0^* - 1$ and
- the loan volume $LV = x_L^F(L_0^*) \cdot L_0^*$

in the case of a non-regulated and a regulated bank. In the core of our comparative static analysis lies the variation of the regulation tightness parameter α. In addition, we consider changes of the expected value μ_V and the standard deviation σ_V of the return r_V and the initial equity W_0^F of the firm.

The parameters of the base case are fixed as follows:

Risk-free rate r: 5%.

Expected value of the firm's gross return μ_V: 120%.

Volatility of the firm's return σ_V: 30%.

Volatility of a loan unit σ_L: 6%.

Volatility of a deposit unit σ_D: 3.6%.

Correlation between the firm's return and the loan rate $\rho_{L,V}$: 0.3.

Correlation between the loan rate and deposit rate $\rho_{D,L}$: 0.3.

Regulation tightness α: 0.4.

Initial wealth of the investor W_0^I: 2000.

Initial equity of the bank W_0^B: 100.

Initial equity of the firm W_0^F: 600.

Risk aversion of the investor λ^I: $2/W_0^I = 0.001$.

Risk aversion of the bank λ^B: $0.2/W_0^B = 0.002$.

Risk aversion of the firm λ^F: $0.5/W_0^F = 0.0008$.

With these choices of parameter values, we try to describe a typical economy. The relation between the initial wealth of the investor and the equity of the firm basically captures the relation between the volume of the fixed income market and the stock market in Germany. The equity of the banking sector is supposed to be smaller than that of the other two sectors. Investors have a risk aversion with $\lambda^I \cdot W_0^I$ equal to two. The firm, whose primary goal is not to hedge risks, has a lower risk aversion. The relation between the volatility σ_V of the real technology and the volatilities of loans σ_L and deposits σ_D is described in Subsection 5.2, the choice of the regulation tightness parameter in Subsection 5.1.

As we consider a static model, we are not able to analyze the consequences of regulation in a business cycle model. Instead, we approximate the effects of external shocks by the relative sensitivity:

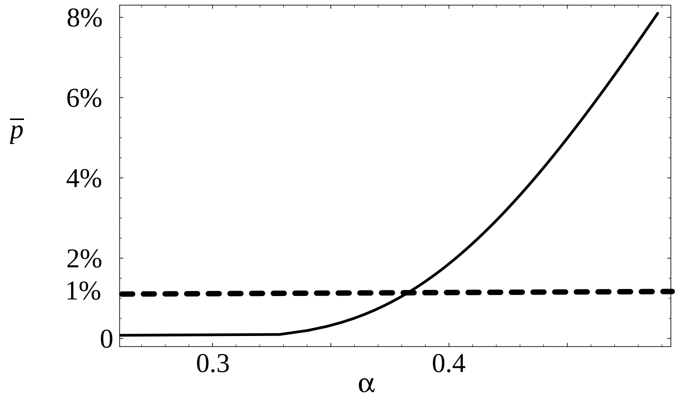
$$\frac{\partial\,(\text{endog. variable})^{reg}}{\partial\text{exog. variable}} \Big/ \frac{\partial\text{endog. variable}}{\partial\text{exog. variable}}$$

The superscript "reg" indicates the case of a VAR-regulated bank. Of special interest is how the credit volume $LV = x_L^F\,(L_0^*) \cdot L_0^*$, that determines together with the exogenous initial equity W_0^F the size of the real sector, reacts to a change of the investment risk σ_V, the regulation tightness α, and the firm's equity W_0^F. This last exogenous variable can be understood as a state variable that characterizes the current state of the economy in a business cycle. A relatively small value of W_0^F can be a consequence of previous losses that reduced the equity or as a bad economic outlook. If this view is accepted, a small value W_0^F characterizes a depressed economy and a relatively high value a prosperous one.

For these values of the exogenous variables the equilibrium in the non-regulated case is characterized by the following values: The deposit volume $x_D^I\,(D_0^*) \cdot D_0^*$ amounts to 1279.3 units and the expected deposit rate to 5.2%. The loan volume is 1379.3 units with an expected loan rate of 6.3%. The bank does not have the risk-free asset on its balance sheet. Therefore, in equilibrium, the optimal portfolio of the non-regulated bank is characterized by case (ii). The investor holds 36% of his wealth in the risk-free assets and

Figure 4: $\overline{p}$—α Diagram

This figure shows the probability $\overline{p}$ that the bank's unexpected loss exceeds one third of its equity as a function of the regulation tightness parameter α.



64% in deposits. The firm invests a total of 1979.3 units into its production technology.

5.1 Response to Regulation Tightness α

The parameter α in the VAR-constraint

$$\sqrt{x_D^2\sigma_D^2 + x_L^2\sigma_L^2 - 2x_D x_L \rho_{D,L}\sigma_D\sigma_L} \leq \alpha \cdot W_0^B$$

describes the regulation tightness. The tightness increases with a decreasing value of α. For $\alpha \geq 0.87$ the VAR-constraint is not binding in equilibrium. If we select $\alpha' = 1/3$ (this parameter choice corresponds with the regulation of market risk), the non-regulated bank experiences unexpected losses that exceed one third of its equity with a probability of 35%. For this value of α' the relationship is shown in Figure 4. $\overline{p} = 1\%$ is related with an α-value of about 0.4. For the non-regulated bank the probability of losses larger than $1/3 \cdot W_0^B$ is 35%.

Figure 5 shows the size of the real sector $W_0^F + LV = x_L^F(L_0^*) \cdot L_0^* + W_0^F$, the deposit volume $DV = x_D^I(D_0^*) \cdot D_0^*$, the expected deposit rate r_D, and the expected loan rate r_L in equilibrium for a varying regulation tightness.

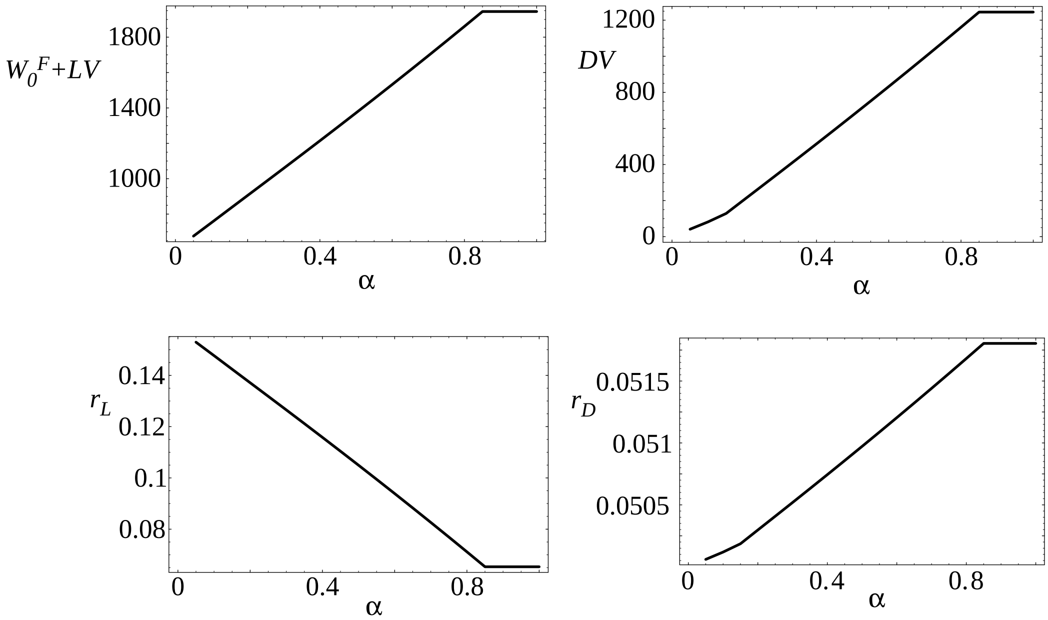
For values of α larger than 0.87 these four endogenous variables are not affected by α as the constraint is not binding. The maximum demand for loans by the firm equals $\mu_V / \left(\lambda^F \sigma_V^2\right) - W_0^F = 15,400$. The critical price $\overline{L}_0$ below which the demand for loans is zero amounts to 0.863 or to an expected loan rate of 15.9%.

Figure 5 shows that increasing regulation tightness forces the bank to reduce the size of loans LV and deposits DV. As the size of the real sector differs only by the fixed equity W_0^F of the firm from the loan volume, an increasing regulatory tightness results in a shrinking real sector.

If α is below 0.15, the bank holds also the risk-free asset. This holding increases

Figure 5: Impact of Regulation Tightness α

The parameter values used are: $r = 0.05$, $\mu_V = 1.2$, $\sigma_V = 0.3$, $\sigma_L = 0.06$, $\sigma_D = 0.036$, $\rho_{L,V} = 0.3$, $\rho_{D,L} = 0.3$, $W_0^I = 2000$, $W_0^B = 100$, $W_0^F = 600$, $\lambda^I = 0.001$, $\lambda^B = 0.002$, *and* $\lambda^F = 0.0008$.



monotonously if α decreases but it is always smaller than the bank's equity as long as α is positive. For $\alpha = 0$, the bank's optimal portfolio consists of the risk-free asset only.

The deposit rate r_D decreases (D_0^* increases) slightly with increasing regulation tightness and converges to the risk-free rate $r = 5\%$. On the contrary, the loan rate r_L increases (L_0^* decreases) if the regulation tightness increases. This behavior of the two rates follows immediately from the demand functions of the investor and the firm together with the fact that for $\overline{L}_0 < \frac{1}{1+r} = 0.952 < \widehat{L} = 1.69$ the firm's loan demand increases in L_0.

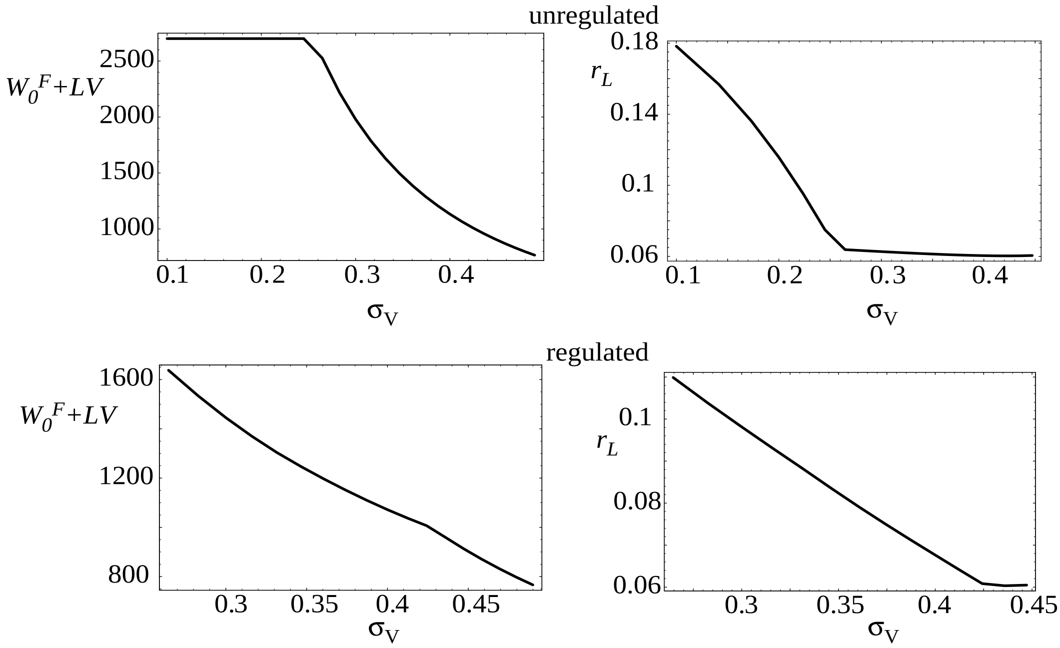
5.2 Response to Risk Shocks

In a model in which the production risk σ_V determines completely the loan and deposit risk, only σ_V can be varied. For our analysis of a risk shock, we link the risk of a loan σ_L to the production risk by considering the volatility $\sigma_L(\sigma_V, L_0)$ which results from an otherwise identical firm but whose liability is limited by the nominal amount $x_L^F(L_0)$ of debt. The variance $\sigma_L^2(\sigma_V, L_0)$ of a loan contract with this repayment characteristic and with production risk σ_V is given by

$$\sigma_L^2\left(\sigma_V^2, L_0\right) = \frac{\int_{-\infty}^{\infty}\left(\min\left(x_L^F(L_0), \left(x_L^F(L_0)\, L_0 + W_0^F\right)\cdot y\right) - EW_L\right)^2 \cdot n\left(y; \mu_V, \sigma_V\right) dy}{x_L^F(L_0)^2},$$

Figure 6: Impact of Risk Shock σ_V

The parameter values used are: $r = 0.05$, $\mu_V = 1.2$, $\rho_{L,V} = 0.3$, $\rho_{D,L} = 0.3$, $\alpha = 0.4$, $W_0^I = 2000$, $W_0^B = 100$, $W_0^F = 600$, $\lambda^I = 0.001$, $\lambda^B = 0.002$, *and* $\lambda^F = 0.0008$.



where EW_L denotes the expectation

$$EW_L = \int_{-\infty}^{\infty} \min\left(x_L^F(L_0), \left(x_L^F(L_0)L_0 + W_0^F\right)\cdot y\right)\cdot n(y;\mu_V,\sigma_V)\,dy$$

and $n(y;\mu,\sigma^2)$ stands for a normal density function with expectation μ and variance σ^2. Similarly, the variance $\sigma_D^2(\sigma_V, L_0, D_0)$ results from the fact that the bank's liability is limited by nominal value $x_D^I(D_0)$ of the deposits (we assume that the bank holds no risk-free asset):

$$\sigma_D^2\left(\sigma_V^2, L_0, D_0\right) = \frac{\int_{-\infty}^{\infty}\left(\min\left(x_D^I(D_0), x_L^B(D_0,L_0)\cdot y\right) - EW_D\right)^2\cdot n\left(y;\frac{1}{L_0},\sigma_L(\sigma_V,L_0)\right)dy}{x_D^I(D_0)^2}$$

with

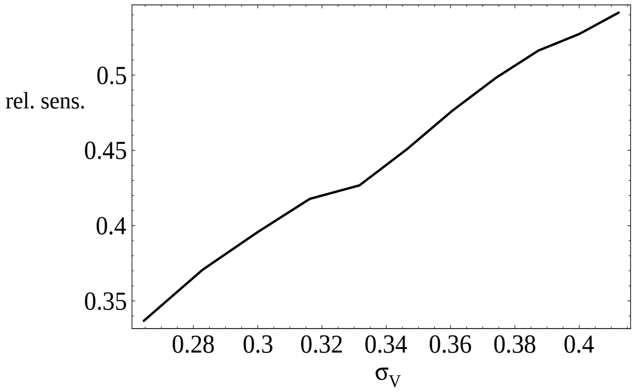
$$EW_D = \int_{-\infty}^{\infty}\min\left(x_D^I(D_0), x_L^B(D_0,L_0)\cdot y\right)\cdot n\left(y;\frac{1}{L_0},\sigma_L(\sigma_V,L_0)\right)dy.$$

As a result of these relations, a higher volatility σ_V of the technology is carried forward to a volatility σ_L of loans and deposits σ_D.

Figure 6 shows the results of a variation of σ_V. The volume of the technology $W_0^F + LV$ in the unregulated banking system is not affected by the risk of the technology σ_V below 0.24 but then strictly decreases with σ_V. In the case of a regulated banking system, the

Figure 7: Relative Sensitivity $\frac{\partial\left(W_0^F+LV\right)^{reg}}{\partial\sigma_V} / \frac{\partial\left(W_0^F+LV\right)}{\partial\sigma_V}$

The parameter values used are: $r = 0.05$, $\mu_V = 1.2$, $\rho_{L,V} = 0.3$, $\rho_{D,L} = 0.3$, $\alpha = 0.4$, $W_0^I = 2000$, $W_0^B = 100$, $W_0^F = 600$, $\lambda^I = 0.001$, $\lambda^B = 0.002$, *and* $\lambda^F = 0.0008$.



real sector $W_0^F + LV$ is also constant for $\sigma_V \leq 0.15$ as for these volatilities the regulation is not binding.[1] For a binding VAR constraint, $W_0^F + LV$ declines with σ_V. If the volatility is above 0.42, regulation is again not binding and the decline of the real sector $W_0^F + LV$ is identical to the case without regulation. The result that the real sector cannot increase with σ_V is intuitive. When the risk σ_V in the economy increases, the positions held by the firm, the bank, and the investor become more risky. Due to the risk aversion of the agents, they are less willing to take these positions and therefore the investor invests a lower amount in deposits such that the firm has a lower loan volume in equilibrium.

Since the regulated system coincides with the unregulated system for $\sigma_V < 0.15$ and $\sigma_V > 0.42$, the relative sensitivity $\frac{\partial\left(W_0^F+LV\right)^{reg}}{\partial\sigma_V} / \frac{\partial\left(W_0^F+LV\right)}{\partial\sigma_V}$ is equal to one for these volatilities. In the region $0.15 < \sigma_V < 0.24$, the unregulated real sector is not affected such that the relative sensitivity is not defined. Since the real sector decreases under regulation rather than without regulation, we can speak from a procyclical effect for those volatilities. For higher volatilities for which a regulation is binding, $0.24 < \sigma_V < 0.42$, the decrease of the real sector under regulation is less pronounced than without regulation. Hence, the relative sensitivity is below one and regulation has a dampening effect on the real sector as shown in Figure 7.

5.3 Response to Equity Shocks

The volume W_0^F of the firms equity — as discussed above — can be understood as a state variable that characterizes the current state of the economy in a business cycle. Low values of W_0^F are attributed to a depressed economy and vice versa.

An increase of the firm's equity W_0^F results in a decrease of the firm's optimal demand

[1]For those volatilities with a binding regulation where the real sector in the unregulated system is independent of σ_V, we were not able to obtain equilibria numerically.

Figure 8: Impact of Equity Shock

The parameter values used are: $r = 0.05$, $\mu_V = 1.2$, $\sigma_V = 0.3$, $\sigma_L = 0.06$, $\sigma_D = 0.036$, $\rho_{L,V} = 0.3$, $\rho_{D,L} = 0.3$, $\alpha = 0.4$, $W_0^I = 2000$, $W_0^B = 100$, $\lambda^I = 0.001$, $\lambda^B = 0.002$, *and* $\lambda^F = 0.0008$.

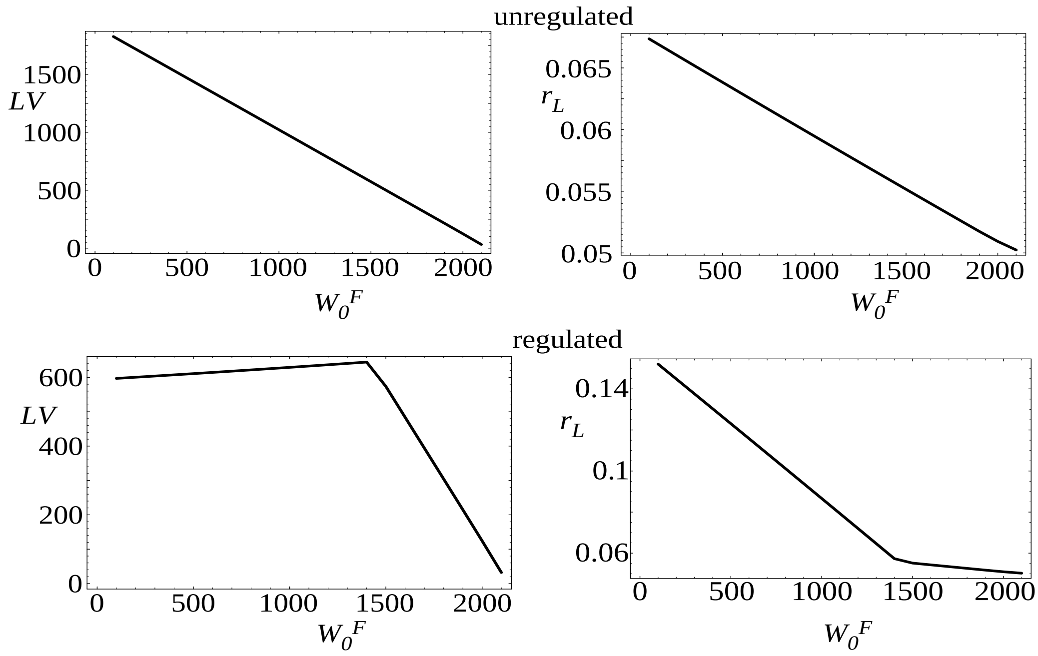
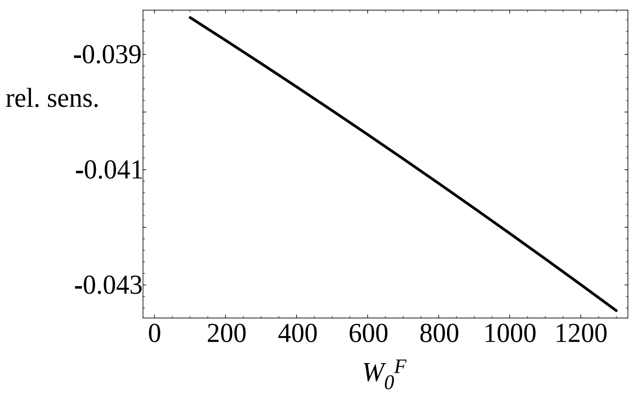


Figure 9: Relative Sensitivity $\frac{\partial (LV)^{reg}}{\partial W_0^F} / \frac{\partial LV}{\partial W_0^F}$

The parameter values used are: $r = 0.05$, $\mu_V = 1.2$, $\sigma_V = 0.3$, $\sigma_L = 0.06$, $\sigma_D = 0.036$, $\rho_{L,V} = 0.3$, $\rho_{D,L} = 0.3$, $\alpha = 0.4$, $W_0^I = 2000$, $W_0^B = 100$, $\lambda^I = 0.001$, $\lambda^B = 0.002$, *and* $\lambda^F = 0.0008$.



$x_L^F(L_0)$ for loans as long as this demand is positive. This property of x_L^F follows immediately from the representation of x_L^F in Section 3 and the fact that the inequality $\sigma_V > \rho_{L,V} \cdot \sigma_L$ holds. Therefore, in the unregulated economy the loan volume and the expected loan rate decrease strictly with the size of the equity, as shown in Figure 8. For equity values larger than $2,100$ units the firm's demand for loans is zero.

In the unregulated economy the VAR-constraint is binding for equity values below $1,400$ units. For values above this critical size the loan volume behaves as in the unregulated economy. Below this critical value, the loan volume increases slightly with the firm's equity. The reason for this surprising behavior is the fact that the nominal volume of loans and deposits is almost unaffected by an increase of W_0^F. But L_0 (r_L) increases (decreases) with the firms's equity as the firm's demand function is shifted downward.

As a consequence, the relative sensitivity of the loan volume LV, which is shown in Figure 9, is negative when regulation is binding. However, the sensitivity is close to zero as the effect of the initial wealth W_0^F on the loan volume LV in the regulated economy is relatively small.

6 Summary and Conclusions

In this paper we analyze the effect of a Value-at-Risk on the banks choice variables on the size of the real sector and on the loan and deposit rates in equilibrium. We consider as a starting point the most simple model to describe the relations between the real and the financial sector in an economy. This model is characterized by one period, agents with negative exponential utility and normally distributed risk factors. Our findings can be summarized as follows:

A stricter regulation results in higher loan rates, lower deposit rates, and in a lower activity of the real sector. A positive shock on the technology risk has negative effects on the real sector and the loan rate in the regulated and unregulated economy. Our most important and surprising finding is that contrary to the literature, regulation does not accelerate but dampens the effects of exogenous risk shocks on the size of the real sector if the unregulated economy reacts to this shock at all. This dampening effect of bank regulation holds also for productivity shocks, a result that is not shown in this paper. The basic reason for this unexpected result is that in the unregulated economy the firm's equilibrium demand for loans is fully affected by exogenous shocks. In the regulated system a positive risk shock reduces the equilibrium volume of loans through the VAR-constraint.

Since we consider this approach as a first benchmark model, there are many possibilities to generalize it. For example, we can study effects from risk diversification by introducing further firms. Analogously, we can allow for multiple banks in the economy to investigate effects from competition among banks. Moreover, we can replace the unlimited liable agents by agents with limited liability. This case explicitly takes into account the option characteristics of loans and deposits. Then, only the risk from the real sector matters and it is partly transferred to the bank and the investor via loans and deposits.

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