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Interim Information in Long Term Contracts

Roland Strausz*

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*Roland Strausz, Free University Berlin, Department of Economics, Boltzmannstr. 20, D-14195 Berlin, strausz@fu-berlin.de

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Speaker: Prof. Konrad Stahl, Ph.D. · Department of Economics · University of Mannheim · D-68131 Mannheim,
Phone: +49(0621)1812786 · Fax: +49(0621)1812785

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Roland Strausz*

Free University of Berlin

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Abstract

This paper studies the effectiveness of interim information in reducing inefficiencies in long term relationships. If the interim information is verifiable, it resolves all problems of asymmetric information. Under nonverifiability, the information alleviates the contracting problem only partially and its optimal use depends on the signal's accuracy and timing. Precise and early signals enable the principal to extract all rents and adjust allocations closer to the first best. Imprecise or late signals affect only future allocations and leaves the agent with a rent. Due to a failure of the revelation principle, the optimal contract under non–verifiability is derived by employing the theory of communication equilibrium.

Keywords: long term contracts, repeated adverse selection, verifiability, revelation principle;

JEL Classification No.: D82

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*Free University Berlin, Dept. of Economics, Boltzmannstr. 20, D-14195 Berlin (Germany); Email addresses: *strausz@zedat.fu-berlin.de*. I thank Helmut Bester, Marco Battaglini, and Anette Boom. Financial support by the DFG (German Science Foundation) under SFB/TR 15 is gratefully acknowledged.

1 Introduction

Over the last decades economists have identified pre–contractual asymmetric information as an important source of inefficiency. Most analyses, however, disregard the fact that contracts are typically long term and require repeated interaction during which contracting partners receive additional information. Since such *interim* information may reduce the initial degree of asymmetric information, rational economic agents have an interest in taking it into account when designing their contracts. Real–life examples of these contracts are probation contracts for new employers and accident free discounts in insurance.[1] This paper investigates the role of interim information in reducing inefficiencies.

More specifically, I analyze a standard principal agent setting in which the agent is privately information about his type. During their contractual relationship the principal receives additional information about the agent. I thereby contrast the case where this information is verifiable to one in which it is not. The difference being that the contract can only condition on the information directly if it is verifiable. The paper shows that when the interim information is verifiable, it completely mitigates the problems associated with pre–contractual asymmetric information. That is, independent of the accuracy of the information and its timing, the optimal contract implements first best allocations and the principal extracts all rents.

In contrast, when the principal's interim information is non–verifiable, the effect of interim information depends on its accuracy and timing. When the information has a low accuracy or is received late in the relationship, the principal uses the interim information primarily for rent extraction, but fails to extract all rents. Moreover, the interim information affects only the allocations that occur after the information has been received; earlier allocations remain at their second best levels. In contrast, when the information is more

[1] See Cooper and Hayes (1987) for a concrete analysis of a car insurance model in which accidents provide verifiable interim information about the agent's type.

accurate or received earlier, the principal extracts all rents. In this case, the presence of interim information also affect earlier allocations; they are closer to the first best than without the information.

In addition, the assumption of non–verifiability requires a different analytical approach to the problem. When the information is verifiable, one may solve the contracting problem by using the revelation principle and focus on truthful direct mechanisms. This familiar procedure cannot be followed when the principal's interim information is nonverifiable. In this case, the principal can only be induced to reveal her information so that information revelation becomes a strategic decision to which the principal cannot commit herself contractually.[2]

Faced with a failure of the revelation principle, this paper uses an alternative justification for its focus on truthful direct mechanisms. In particular, it employs the theory of communication equilibrium (e.g. Forges (1986) and Myerson (1986)) to show that, despite the principal's lack of commitment, truthful direct mechanisms are optimal.[3] This literature shows that, when players have private information and imperfect commitment, the introduction of a benevolent mediator, who receives the parties' information and gives subsequent recommendations, restores the revelation principle. In general however such mediators may allow players to attain outcomes which they cannot reach through direct communication.[4] In the current setting however the outcome of the optimal mediated contract may also be implemented by an unmediated one. From this it then follows that the optimal mediated

[2]See Bester and Strausz (2001) for more details about the failure of the revelation principle when a principal has imperfect commitment.

[3]Since interim information exemplifies a setting in which a player receives new information over time, it uses Myerson (1986)'s theory of multistage games which represents the dynamic version of the communication literature.

[4]This has lead to the observation that "mediated contracts" may yield contracting parties strictly more than "unmediated" ones (e.g., Bester and Strausz (2003)). Consequently, Mitusch and Strausz (2001) raise the fundamental question whether the appropriate approach to solving such contracting problems is one with or without a mediator.

and optimal unmediated contract coincide and truthful direct mechanisms are optimal.

Since the interim signal is correlated with the agent's type, it presents a natural example of correlated information. Hence, the paper is related to the literature on the use of correlated information, which demonstrates how correlated information mitigates the problem of private information (e.g. Cremer and McLean 1988 and Riordan and Sappington 1988). The current paper contributes to this literature by emphasizing the role of verifiability.

It is important to note that the literature has studied a different type of interim information. Baron and Besanko (1984) analyze a multi period adverse selection setting in which interim information revelation occurs due to the agent's behavior. Specifically, if the principal offers a revelation mechanism in the first period, the agent's message entails interim information. In the framework of Baron and Besanko the revelation of interim information lies under the full control of the agent and is therefore endogenous. In this case, interim information does not benefit the principal. Indeed, Baron and Besanko show that whenever the principal has full commitment, her optimal policy is to commit not to use interim information. Hence, the current paper differs from the literature on the ratchet effect (e.g., Laffont and Tirole 1988, 1990) in which, due to the principal's limited commitment, the interim information actually hurts the principal and repeated interaction should be avoided. In contrast, this paper explicitly allows the possibility of long term contracts and commitment. The comparison emphasizes that there exists a crucial difference between endogenous and exogenous information.

The rest of the paper is organized as follows. The next section sets up a simple framework of interim information. Section 3 develops two benchmarks against which we may set the use of interim information. Section 4 analyzes interim information when it is verifiable, while Section 5 studies it under non–verifiability. The final section concludes. All formal proofs are relegated to the appendix.

2 The Setup

A principal employs an agent, who is privately informed about the marginal cost of production. With probability α the agent's marginal cost is θ_l and with probability $1-\alpha$ the marginal cost is θ_h, where $\theta_h > \theta_l$. The agent's action a results in a verifiable output $v(a)$. We make the standard assumptions that v is increasing and concave, i.e., $v' > 0$ and $v'' < 0$, and, for technical reasons, $v''' < 0$.

The agent works for the principal for two periods, where we normalize the total length of the relationship to one. The first period of production lasts for a time $\delta \in (0,1)$, after which the principal receives a signal $s \in \{h, l\}$ about the agent's marginal cost. The signal s represents the interim information, while the parameter δ reflects the time at which the principal receives it. The signal is correct with probability $p > 1/2$ so that p represents its accuracy. The second period of production lasts for the remaining time $1-\delta$. Note that the only difference between the two periods is the disclosure of the signal s. Without the signal the model is equivalent to a standard, one-period adverse selection problem.[5]

As the owner of the firm, the principal receives the agent's output and compensates him with a wage w. The principal and agent are risk neutral. Given that the agent receives wages (w_1, w_2) during the two periods and chooses actions (a_1, a_2) the principal's and agent's payoffs are

$$\begin{aligned} V(w_1, a_1, w_2, a_2) &= \delta(v(a_1) - w_1) + (1-\delta)(v(a_2) - w_2), \\ U_i(w_1, a_1, w_2, a_2) &= \delta(w_1 - \theta_i a_1) + (1-\delta)(w_2 - \theta_i a_2), \end{aligned}$$

respectively, where $i \in \{h, l\}$.

Since output is verifiable and invertible, we may interpret an enforceable contract γ as specifying an action a and a wage w for each period, i.e. $\gamma =$

[5] For a formal argument see Laffont and Tirole (1993, p. 103-105) and also Baron and Besanko (1984, p. 290).

$(\gamma_1, \gamma_2) = ((w_1, a_1), (w_2, a_2))$. As is standard, the contract may depend on a message m of the agent about his type. If the principal's interim information s is verifiable, the contract may condition directly on it. In this case, the contract has the form $\gamma_{ms} = (\gamma_1(m), \gamma_2(m, s))$. When the information is unverifiable, the contract can depend only indirectly on the signal, in the sense that the principal may report it by sending some message r. That is, a general contract has the form $\gamma_{mr} = (\gamma_1(m), \gamma_2(m, r))$. The principal offers the contract as a take-it-or-leave-it offer to the agent. The agent's reservation utility is normalized to zero.

3 First and Second Best

This section analyzes two benchmarks to which I will later relate the existence of interim information. The benchmarks show that without the signal s, the model collapses to a static one. This emphasizes that the model allows a straightforward evaluation of the existence of interim information.

First, suppose the principal is fully informed about the agent's type. In this case, the principal can prescribe each type of agent to work efficiently and appropriate the entire surplus. Efficient, *first best action levels*, a_i^{fb}, satisfy

$$v'(a_i^{fb}) = \theta_i,$$

with $i \in \{h, l\}$.[6] Hence, with full information the optimal contract γ is a first best contract that implements in each period the respective first best action levels a_i^{fb} at *first best costs* $\theta_i a_i^{fb}$. Note that the prescribed actions levels are time–invariant and that the two period model is identical to a static one with full information.

Now assume the agent is privately informed about his type and the signal s is not available. As shown by Baron and Besanko (1984) and Laffont and Tirole (1993, p.103-105) the optimal long term contract is, in this second best

[6]Throughout the paper we assume that the solution of the problem is interior.

world, again time–invariant. The two period model is identical to a standard one period model of adverse selection in which the principal faces the familiar trade–off between efficiency and rent extraction. By the revelation principle an optimal contract is a solution to the following maximization problem:[7]

$$\text{P0: } \max \quad V = \alpha(v(a_l) - w_l) + (1-\alpha)(v(a_h) - w_h)$$

$$\text{s.t.} \quad w_l - \theta_l a_l \geq 0; w_h - \theta_h a_h \geq 0 \tag{1}$$

$$w_l - \theta_l a_l \geq w_h - \theta_l a_h;\ w_h - \theta_h a_h \geq w_l - \theta_h a_l, \tag{2}$$

where (1) represent the participation constraints and (2) the incentive constraints that ensure truthful revelation. Let V^{sb} denote the solution to P0. By standard arguments only the incentive constraint of the efficient type θ_l and the participation constraint of the inefficient type θ_h are binding. The solution to this problem is a second best contract that implements the second best action levels (a_l^{sb}, a_h^{sb}). As is standard, the efficient type receives a strict positive information rent $U_l^{sb} > 0$, while his action is efficient $(a_l^{sb} = a_l^{fb})$. The inefficient type does not receive a rent $U_h^{sb} = 0$, but uses an action, a_h^{sb}, which is below his efficient level, a_h^{fb}, where

$$v'(a_h^{sb}) = \theta_h + \frac{\alpha}{1-\alpha}(\theta_h - \theta_l). \tag{3}$$

4 Verifiable Information

This section studies the optimal use of the interim signal s when it is verifiable. Verifiability implies that a general contract has the form $\gamma_{ms} = (\gamma_1(m), \gamma_2(m,s))$. In this case interim information is extremely powerful: Independent of its accuracy p, and the time δ, at which it is received, it is able to resolve the problem of ex ante asymmetric information completely.

To demonstrate this, the section first derive the optimal contract and, for later references, discusses its properties. By the revelation principle an

[7] To have a non–trivial problem, we assume throughout this paper that the principal wants to employ both agents, which is the case if α is small enough.

optimal contract may be represented by a direct mechanism $\gamma_{\theta s} = (\gamma_{ls}, \gamma_{hs})$ that ensures truthful revelation of the agent's type. The principal's utility from such a direct mechanism is

$$V \equiv \alpha(pV(\gamma_{ll}) + (1-p)V(\gamma_{lh})) + (1-\alpha)(pV(\gamma_{hh}) + (1-p)V(\gamma_{hl})). \quad (4)$$

The direct mechanism $\gamma_{\theta s}$ has to induce the agent to reveal his type truthfully. I.e., the following two incentive compatibility conditions have to hold

$$pU_l(\gamma_{ll}) + (1-p)U_l(\gamma_{lh}) \geq pU_l(\gamma_{hl}) + (1-p)U_l(\gamma_{hh}); \quad (5)$$

$$pU_h(\gamma_{hh}) + (1-p)U_h(\gamma_{hl}) \geq pU_h(\gamma_{lh}) + (1-p)U_h(\gamma_{ll}), \quad (6)$$

where condition (5) ensures that the agent of type l reveals himself truthfully and (6) induces type θ_h to report himself truthfully.

Finally, the contract must ensure acceptance by both types of agents, i.e.,

$$pU_l(\gamma_{ll}) + (1-p)U_l(\gamma_{lh}) \geq 0; \quad (7)$$

$$pU_h(\gamma_{hh}) + (1-p)U_h(\gamma_{hl}) \geq 0. \quad (8)$$

Hence, a solution to the following maximization problem solves the principal's problem:

$$\begin{aligned} \text{P1: } \max \quad & V \\ \text{s.t.} \quad & (5), (6), (7), \text{ and } (8). \end{aligned}$$

Proposition 1 *If the signal s is verifiable, then for any $p > 1/2$ and any $\delta \in (0,1)$ the principal can implement the first best actions at first best costs.*

With a verifiable signal the principal is able to attain her first best payoff. The intuition behind the result is straightforward. As shown in the previous section, the principal's problem without the signal s is to pick a menu that does not give the efficient type θ_l an incentive to claim he is inefficient. With the signal s this problem can be solved costlessly by conditioning second period wages, w_{hs}, on the signal s. Indeed, by setting $w_{hh} > w_{hl}$ the contract that is meant for type θ_h becomes relatively less attractive to type θ_l. The efficient type θ_l would receive the higher wage w_{hh} only with probability $1-p < 1/2$, whereas type θ_h receives it with probability $p > 1/2$. A difference $\Delta w_h \equiv w_{hh} - w_{hl}$ of

$$\Delta \bar{w}_h \equiv \frac{(\theta_h - \theta_l)a_h^{fb}}{(1-\delta)(2p-1)} \tag{9}$$

is sufficient to make the first best allocations incentive compatible. Indeed, from the incentive compatibility constraint (5) it directly follows that the positive wedge of at least $\Delta \bar{w}_h$ is a necessary condition for contracts to attain the first best.

Corollary 1 *First best actions are implementable only with a wedge Δw_h of at least $\Delta \bar{w}_h$*

A direct comparison with the second best solution shows that the interim information affects both the first and second period action levels of the inefficient type θ_h. That is, even though the signal is received after the first period, the signal influences the first period contract.

The result that the principal is able to implement the first best action level is related to Riordan and Sappington (1988), who show how ex post information may eliminate inefficiencies due to ex ante asymmetric information. Indeed, from the perspective of the first period the interim information may be seen as ex post information. Similarly, Cremer and McLean (1988) show how an auctioneer may exploit the correlation between privately informed bidders to extract the complete surplus. As in Cremer and McLean (1988) the signal s is correlated with the agent's private information and the principal uses a similar scheme to exploit this correlation. The next section shows however that the result depends crucially on the verifiability of the signal.[8]

[8]See Demougin and Garvie (1991), Gary–Bobo and Spiegel (2003), and Kessler et al. (2004) for the use of correlated information when parties have limited liability.

5 Unverifiable Information

This section analyzes intermediate information when it is unverifiable. In this case, the contract γ cannot depend directly on the signal s. Instead, the second period contract can at most depend on a *report* by the principal about her signal s. Intuitively, the principal's report can, in equilibrium, only matter if she reveals her signal truthfully. Hence, it seems natural to look for optimal revelation games that induce a truthful revelation of the agent's *and* principal's information.

Formally, an (unmediated) *revelation game* $\Gamma^u(\gamma)$ is generated by a contract $\gamma = (\gamma_1(m), \gamma_2(m,r))$ and describes the following game: First the agent sends a message $m \in \{h, l\}$ to the principal. The message m determines the first period allocation $\gamma_1(m)$. In the second period the principal sends, after receiving her private signal $s \in \{h, l\}$, a report $r \in \{h, l\}$. The report r together with the agent's message m determines the second period allocation $\gamma_2(m, r)$. We say that an (unmediated) revelation game $\Gamma^u(\gamma)$ is *truthful* if it has a perfect Bayesian equilibrium in which the agent and the principal report their private information truthfully. Moreover, a truthful (unmediated) revelation game $\Gamma^u(\gamma)$ is *acceptable* if it has a truthful equilibrium that yields both types of agents at least their reservation utility of zero.

Whether a game $\Gamma^u(\gamma)$ is truthful and acceptable depends on the underlying contract γ. In a truthful equilibrium of a game $\Gamma^u(\gamma)$, the agent's message is fully revealing. In particular, after receiving a message m, the principal knows that the agent is of type m. Hence, before sending her report r the principal is fully informed about the agent's type. In a truthful revelation game the principal must nevertheless have an incentive to reveal her information truthfully. To the principal the difference in payoffs between reporting $r = l$ and reporting $r = h$ is

$$(v(a_{ml}) - w_{ml}) - (v(a_{mh}) - w_{mh}).$$

This difference is independent of the principal's actual signal s. That is, whenever this difference is positive, the principal has a strict incentive to report $r = l$. If it is negative, she has a strict incentive to report $r = h$. Hence, in a truthful unmediated revelation game $\Gamma^u(\gamma)$ the difference must be zero. I.e., it must hold that

$$v(a_{ll}) - w_{ll} = v(a_{lh}) - w_{lh}. \tag{10}$$

$$v(a_{hl}) - w_{hl} = v(a_{hh}) - w_{hh}. \tag{11}$$

In a truthful revelation game also the agent has to reveal his information. Given that the principal reveals her information truthfully, the earlier conditions (5) and (6) express this requirement. Finally, if the truthful revelation game $\Gamma^u(\gamma)$ is to be acceptable, the contract γ must satisfy equations (7) and (8).

Consequently, we define an *optimal (unmediated) revelation contract* γ^u as a contract which induces an acceptable, truthfully unmediated revelation game $\Gamma^u(\gamma^u)$ that yields the principal the highest payoff. That is, γ^u solves the following problem

$$\begin{array}{lll} \text{P2:} & \max\limits_{\gamma} & V(\gamma) \\ & \text{s.t.} & (5), (6), (7), (8), (10), \text{ and } (11). \end{array}$$

We call an (unmediated) revelation contract γ that satisfies the constraints of program P2 *feasible*. We denote by $V^u \equiv V(\gamma^u)$ the maximum payoff which the principal can achieve in an unmediated revelation game.

Although our focus on truthful revelation games may seem natural, it is unclear whether they are indeed optimal. To ensure this one must show the solution to problem P2 is also optimal with respect to mechanisms that do not induce truthful revelation. As in Section 4 this normally follows from the revelation principle. Yet, because the principal has imperfect commitment concerning her reporting behavior, the principle does not apply.[9]

[9]For instance, a game in which the agent only takes an action after the principal

However, we may use an indirect argument to show that, despite imperfect commitment, truthful revelation games are nevertheless optimal. The first step in showing this is to recognize that the setting with unverifiable information is an example of a multi–stage game as defined in Myerson (1986). Following Myerson (1986) we may use the concept of communication equilibrium and solve a relaxed version of the principal's contracting problem. Effectively, this allows the principal to use a mediator for contract execution, where a mediator is a device that receives signals (information) from players and gives players recommendations about their play.[10] It is well known that in general mediators may be strictly beneficial to a principal with imperfect commitment, in the sense that the principal is unable to implement the optimal mediated contract without an explicit mediator (e.g. Bester and Strausz (2003) and Mitusch and Strausz (2001)). However, in the specific setup this does not occur and the principal may implement the optimal mediated contract by an unmediated revelation contract.

The advantage of introducing a mediator in the principal's problem is that it restores the revelation principle. Following Myerson (1986 and 1991, p. 296–298)), one may show that any equilibrium outcome of any game between the principal and the agent is an equilibrium outcome of some incentive compatible, direct mechanism involving a mediator, who induces the players to report their types truthfully. This revelation principle for Bayesian games implies that, we may, without loss of generality, restrict attention to the following type of revelation game:

In the first period $t = 1$ the agent sends a message m about his type to the mediator. The mediator then executes the first period contract in that he

reports his information may have an equilibrium outcome in which the agent's action is type–dependent. In such an equilibrium the principal's revelation strategy will depend on her beliefs. Consequently, the equilibrium outcome of such a game cannot be achieved by a truthful revelation game.

[10]Since the agent's action is contractible, the mediator will prescribe rather than recommend the action to the agent.

implements the contractual first period allocation $\gamma_1(m)$. Without observing either the agent's message nor the prescribed allocation $\gamma_1(m)$, the principal reports his signal $s \in \{h, l\}$ to the mediator. Subsequently, the mediator implements the second period allocation $\gamma_2(m, r)$.

Effectively, the difference between an unmediated and a mediated revelation game is whether the principal actually observes the message and/or the first period allocation before sending her own report. Therefore, in an unmediated revelation game the principal learns from the agent's behavior, whereas in a mediated revelation game she does not.

Formally, a *mediated revelation game* $\Gamma^m(\gamma)$ is a Bayesian game between the principal and the agent that is generated by a contract $\gamma = (\gamma_1(m), \gamma_2(m, r))$. It has the following structure. The agent has two possible actions $m \in \{h, l\}$ and may be of two types, i.e. the type space is $T_a = \{h, l\}$. Similarly, also the principal has two actions $r \in \{h, l\}$ and may also be of two types, i.e. $T_p = \{h, l\}$. The agent's type–dependent payoff functions are

$$u_h(m, r) = U_h(w_m, a_m, w_{mr}, a_{mr}) \text{ and } u_l(m, r) = U_l(w_m, a_m, w_{mr}, a_{mr});$$

while the principal's payoff functions are

$$v_h(m, r) = v_l(m, r) = V(w_m, a_m, w_{mr}, a_{mr}).$$

Note that, the principal's payoff is actually type–independent.

To complete the description of the Bayesian game $\Gamma^m(\gamma)$, we must specify the ex ante beliefs of the players. Since the principal's signal s is correct with probability p, the Bayes' consistent belief of an agent of type i that the principal is of type i is $p(i|i) = p$. The belief of a principal of type h that the agent is of type l follows from Bayes' rule as

$$\nu_h = \frac{\nu(1-p)}{\nu(1-p) + (1-\nu)p}.$$

Likewise, the belief of a principal of type l that the agent is of type l is

$$\nu_l = \frac{\nu p}{\nu p + (1-\nu)(1-p)}.$$

The beliefs (p, ν) completes the description of the Bayesian game $\Gamma^m(\gamma)$.

Continuing previous terminology, a mediated revelation game $\Gamma^m(\gamma)$ is *truthful*, if it has an equilibrium in which the agent's message and the principal's report are both truthful. In a truthful equilibrium of a truthful mediated revelation game $\Gamma^m(\gamma)$, the equilibrium outcome coincides with γ and the principal's ex ante equilibrium payoff is $V(\gamma)$.

Whether a game $\Gamma^m(\gamma)$ is truthful depends on its generating contract γ. In particular, the contract γ must satisfy constraints (5) and (6) if the agent is to reveal his type truthfully. Similarly, a principal of type h reveals her type truthfully if

$$\nu_h V(\gamma_{lh}) + (1 - \nu_h) V(\gamma_{hh}) \geq \nu_h V(\gamma_{ll}) + (1 - \nu_h) V(\gamma_{hl}). \tag{12}$$

Likewise, a principal of type l reveals her type truthfully if

$$\nu_l V(\gamma_{ll}) + (1 - \nu_l) V(\gamma_{hl}) \geq \nu_l V(\gamma_{lh}) + (1 - \nu_l) V(\gamma_{hh}). \tag{13}$$

An important observation is that as a system the constraints (12) and (13) simplify to

$$v(a_{ll}) - w_{ll} \geq v(a_{lh}) - w_{lh}. \tag{14}$$

$$v(a_{hl}) - w_{hl} \geq v(a_{hh}) - w_{hh}. \tag{15}$$

Finally, a truthful mediated revelation game $\Gamma^m(\gamma)$ is *acceptable*, if it yields the agent at least his reservation utility of zero. That is, for a contract γ the inequalities (7) and (8) must be satisfied.

By the revelation principal for Bayesian games we may find an *optimal mediated contract* γ^m by solving the following maximization program:

$$\text{P3:} \quad \max_{\gamma} \quad V(\gamma)$$
$$\text{s.t.} \quad (5), (6), (7), (8), (14), (15).$$

Let $V^m \equiv V(\gamma^m)$ denote the maximum of program P3. Note that the revelation principle guarantees that V^m represents the maximum payoff that the principal can attain in any mediated contracting game. Hence, V^m denotes the principal's maximum payoff of any mediated contract.

Program P2 and P3 differ only in the principal's incentive constraints. More specifically, program P2 obtains from program P3 by the additional requirement that constraints (14) and (15) are satisfied in equality. Hence, program P2 is more constrained than program P3. This confirms that $V^m \geq V^u$. Yet, if a solution γ^m of program P3 exists such that constraints (14) and (15) are satisfied in equality then γ^m is also feasible in the unmediated revelation game. It follows that unmediated revelation games are optimal despite the failure of the revelation principle for such games. Moreover, γ^m represents an optimal contract of the original (unmediated) contracting problem.

As a first step, the following lemma shows that any solution to program P3 satisfies condition (15) in equality. The reason is that if condition (15) does not restrict the principal's problem, she could implement the first best, which, by Corollary 1, requires wages that violate the constraint.

Lemma 1 *For any solution γ^m to program P3 the constraint (15) is binding.*

In contrast, condition (14) does not restrict the problem, but we may show that it can be satisfied costlessly through an appropriate distribution of the wages w_{ll} and w_{lh}. The intuition for this is that because the incentive constraint of type h does not bind, one may freely distribute type l's wage between w_{ll} and w_{lh} to satisfy condition (14) in equality without affecting incentives and payoffs. Hence, we may find a solution γ^m to P3 that is also feasible for problem P2 and therefore represents an optimal (unmediated) contract. The following proposition characterizes the solution in relation to p and δ.

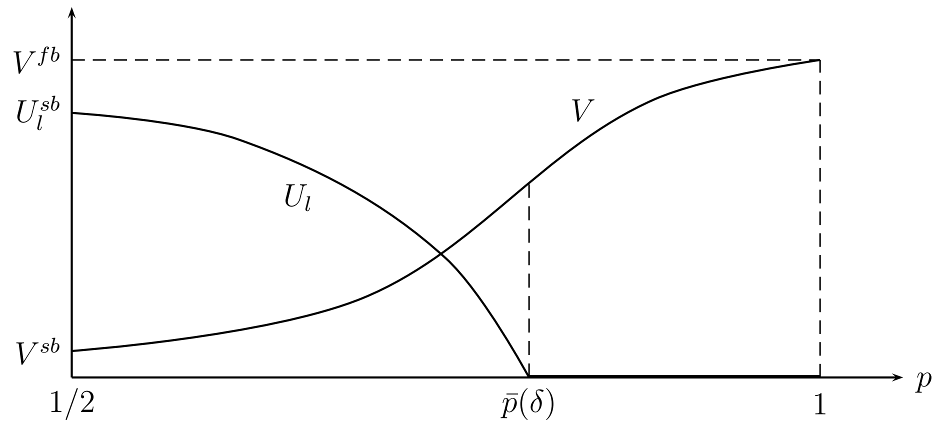

Figure 1: Utilities with nonverifiability

Proposition 2 *There exists a* $\bar{p}(\delta) \in (1/2, 1/(1+\alpha))$ *such that for all* $p < \bar{p}(\delta)$ *the optimal contract leaves the efficient agent a positive rent and prescribes action levels* $a_{hl} < a_h = a_h^{sb} < a_{hh}$. *For* $p > \bar{p}(\delta)$ *the optimal contract extracts all rents and the first period action level,* a_h, *is more efficient than the 2nd best* a_h^{sb}. *For* $p = 1/2$ *the solution coincides with the second best. For* $p = 1$ *the solution yields the first best. Moreover,* $\bar{p}'(\delta) > 0$.

The proposition shows that, despite unverifiability, the principal benefits from interim information by adjusting the second period allocations a_{hh} and a_{hl}. Surprisingly, the principal adjusts the first period allocation a_h only if the signal is informative enough. These results may be explained by reconsidering the fundamental trade–off between efficiency and rent extraction that is responsible for the distortion in the original adverse selection model. If the signal reveals little information, the principal must leave an information rent to the efficient type. Hence, the principal still faces the standard trade–off between efficiency and rent extraction. With respect to the first period, however, the interim information does not change this trade–off and the allocation a_h remains, therefore, at its second best level.[11] In contrast,

[11] A marginal change in a_h raises efficiency by $v'(a_h) - \theta_h$, but requires an increase in the information rent of $\theta_h - \theta_l$. Evaluating changes by their respective probabilities,

the trade–offs concerning the second period allocations a_{hh} and a_{hl} do depend on the accuracy of the signal. Indeed, since an inefficient type receives the allocation a_{hh} more often than a (lying) efficient agent would, the trade–off with respect to a_{hh} changes in favor of the efficiency effect. Hence, as p rises, the efficiency argument gains in importance relative to the rent extraction argument and, as a result, the principal adjusts the second period allocation a_{hh} closer to the first best. For the allocation a_{hl} the contrary happens. Here the rent extraction problem intensifies relatively to the efficiency effect, and leads the principal to distort the allocation further away from the first best. As of $\bar{p}(\delta)$ the signal is informative enough to extract all information rents. As a consequence, there is no longer a trade–off between efficiency and rent extraction. To see that this implies that the principal now also adjusts a_h, suppose that the principal did choose allocations on the basis of the previous trade–off. In this case, the signal is so informative that the corresponding contract violates the efficient type's individual rationality. Hence, if the principal wants to implement these allocation, she must raise the efficient type's wages. This, however, results in a slack incentive compatibility constraint. But since the need for incentive compatibility actually causes the distortion on type θ_h's allocations, its slackness allows the principal to adjust them closer to the first best.

Figure 1 illustrates how the utility of the principal and the efficient type θ_l depend on the accuracy of the signal s. For $p = 1/2$ the signal is uninformative and the principal's utility coincides with the second best. As p increases, the signal s becomes more informative and relaxes the agent's incentive constraint. In the range of $1/2$ to $\hat{p}$ the principal's uses the signal to reduce the information rent to the agent. At $p = \bar{p}(\delta)$ the information rent is completely extracted and the individual rationality constraint of the efficient agent becomes binding. From $\bar{p}(\delta)$ onwards, the principal reduces the

the principal net gain from a change in a_h is $(1-\alpha)\delta(v'(a_h) - \theta_h) - \delta\alpha(\theta_h - \theta_l)$. It is independent of p and, at the optimum, must be zero, leading to the first order condition (3).

allocative distortions. At $p = 1$ all distortions disappear and the principal achieves the first best.

6 Conclusion and Extensions

This paper studied the use of interim information. The occurrence of such information is a natural characteristic of most long term contractual relationship. In fact, it may be the very reason why economic agents enter into long term contractual relationships rather than transact on anonymous spot markets in which such information cannot be used. Interim information mitigates the adverse selection problem. Hence, in repeated contractual relationships that start under asymmetric information the adverse selection problem may be less problematic than the standard static model suggests.

When interim information is unverifiable, the principal may look for ways to circumvent the non–verifiability. One approach is to delegate the signal gathering to a third party who reports the signal to the principal. If the third party's report is truthful and verifiable, the signal itself becomes verifiable. Hence, non–verifiability creates incentives to delegate. However, delegation may involve additional costs. First, the third party must be given incentives to report his signal truthfully. A problem that is possibly exacerbated by collusive pressures between the principal and the third party.[12] Second, in general the employment of a third party will be costly in itself. The principal's decision whether to delegate will therefore explicitly depend on a comparison between the solution with delegation to the one without delegation as derived here.

[12] See Strausz (1997) for an approach to model these pressures explicitly.

Appendix

Proof of Proposition 1: Consider the contract γ^* with $a_h = a_{hh} = a_{hl} = a_h^{fb}$, $a_l = a_{lh} = a_{ll} = a_l^{fb}$, and wages $w_{hh} = \frac{a_h^{fb}(p\theta_h-(1-p)\theta_l)}{(1-\delta)(2p-1)}$, $w_{hl} = \frac{a_h^{fb}(p\theta_l-(1-p)\theta_h)}{(1-\delta)(2p-1)}$, $w_h = w_l = 0$, $w_{ll} = w_{lh} = \theta_l a_l^{fb}/(1-\delta)$. The contract γ^* gives each type of agent an incentive to report his type truthfully. Moreover, each type receives his reservation utility such that the contract implements the first best at first best costs. Q.E.D.

Proof of Corollary 1: In the first best it holds, by definition, that $a_h = a_{hh} = a_{hl} = a_h^{fb}$, $a_l = a_{ll} = a_{lh} = a_l^{fb}$ and that constraints (7) and (8) bind. Incentive compatibility constraint (5) may therefore be rewritten as

$$(1-\delta)[pw_{hl} + (1-p)w_{hh}] \leq \theta_l a_h^{fb}.$$

Moreover, a binding (8) implies $(1-\delta)[pw_{hh}+(1-p)w_{hl}] = \theta_h a_h^{fb}$. Subtracting this equation from the previous inequality and a rearrangement of terms yields

$$(1-\delta)(2p-1)(w_{hh} - w_{hl}) \geq (\theta_h - \theta_l)a_h^{fb}.$$

Q.E.D.

Proof of Lemma 1: First note that the contract γ^* defined in the proof of Proposition 1 satisfies all the constraints of P3 apart from (15). Hence, if (15) does not restrict the optimum, the principal may obtain her first best payoff. Since the principal cannot obtain more than her first best payoff, the contract γ^* must be a solution to P3. Moreover, since any contract that yields first best payoffs and satisfies (7) and (8) must implement the first best actions, any solution to the problem P3 without the constraint (15) exhibits $a_h = a_{hl} = a_{hh} = a_h^{fb}$. But according to Corollary 1 any implementation of the first best actions requires $w_{hh} - w_{hl} \geq \Delta\bar{w}_h > 0$. Yet, this together with $a_{hl} = a_{hh}$ violates (15). Hence, (15) restricts the optimum. Q.E.D.

Proof of Proposition 2: I solve problem P3 and show that its solution is also feasible in P2. For $p = 1/2$ the constraints (5)-(8) represent standard incentive and individual rationality constraints. In this case, (5) and (8) imply (7). Our approach is therefore first to solve the relaxed problem

$$P3' : \max_{\gamma} V \text{ s.t. } (5), (8), \text{ and } (15).$$

I show that for p close enough to $1/2$ the solution satisfies the remaining constraints (6), (7), (8), and (14), but that for p close to $1/(1-\alpha)$ the solution violates (7). In the relaxed problem P3', the three constraints are all binding. First, (8) binds at the optimum, because otherwise one may lower all wages by at least some $\varepsilon > 0$. By Lemma 1 also the constraint (15) binds at the optimum. Finally, constraint (5) binds at the optimum, since otherwise one can implement the first best.

Solving for the constraints (5), (8), and (15) yields

$$w_{hh} = a_{hl}\theta_h + \delta a_h\theta_h/(1-\delta) + (a_{hh} - a_{hl})p\theta_h + (1-p)(v(a_{hh}) - v(a_{hl})) \quad (16)$$

$$w_{hl} = a_{hl}\theta_h + \delta a_h\theta_h/(1-\delta) + (a_{hh} - a_{hl})p\theta_h - p(v(a_{hh}) - v(a_{hl})) \quad (17)$$

$$\begin{aligned} w_{ll} =& \frac{a_{hl}\theta_h + (a_{lh} - a_{hh})\theta_l - w_{lh} + (1-2p)(v(a_{hh}) - v(a_{hl}))}{p} \\ &+ \frac{\delta(a_h\theta_h + (a_l - a_h)\theta_l)}{(1-\delta)p} + (a_{hh} - a_{hl})(\theta_h + \theta_l) + (a_{ll} - a_{lh})\theta_l + w_{lh} \quad (18) \end{aligned}$$

Substitution and a rearrangement of terms yields

$$\begin{aligned} V(p) \equiv\ & \alpha \{\delta[v(a_l) - a_l\theta_l - a_h(\theta_h - \theta_l)] + (1-\delta)[p(v(a_{ll}) - a_{ll}\theta_l) \\ & + (1-p)(v(a_{lh}) - a_{lh}\theta_l) + (2p-1)(v(a_{hh}) - v(a_{hl})) \\ & + ((1-p)\theta_l - p\theta_h)a_{hh} + (p\theta_l - (1-p)\theta_h)a_{hl}]\} \\ & + (1-\alpha)\{\delta[v(a_h) - a_h\theta_h] \\ & + (1-\delta)[p(v(a_{hh}) - a_{hh}\theta_h) + (1-p)(v(a_{hl}) - a_{hl}\theta_h)]\}. \end{aligned}$$

Since this expression is independent of w_{lh} we may, without affecting payoffs or any other constraints, choose w_{ll} and w_{lh} such that both (14) and (18) are satisfied in equality.

The first order conditions with respect to $a_{l\cdot}$ is $v'(a_{l\cdot}) = \theta_l$ and implies the first best action level a_l^{fb}. The remaining first order conditions are

$$(1-\alpha)[v'(a_h) - \theta_h] = \alpha(\theta_h - \theta_l) \quad (19)$$

$$\begin{aligned}(1-(1+\alpha)p)[v'(a_{hl})-\theta_h] &= \alpha p(\theta_h-\theta_l) & (20)\\ (p-\alpha(1-p))[v'(a_{hh})-\theta_h] &= \alpha(1-p)(\theta_h-\theta_l). & (21)\end{aligned}$$

Since $v'' < 0$, the second order conditions are satisfied for $p < 1/(1+\alpha)$ such that the equations (19), (20), and (21) define implicitly the solutions $a_h^*(p)$, $a_{hl}^*(p)$, and $a_{hh}^*(p)$ of the relaxed problem.

More specifically, (19) shows that the optimal value for a_h coincides with the second best, i.e. $a_h^*(p) = a_h^{sb}$, and is independent of p. For $p = 1/2$ the first order conditions (20) and (21) also coincide with (3) such that $a_{hl}^*(1/2) = a_{hh}^*(1/2) = a_h^{sb}$. Differentiating w.r.t. p and rearranging terms yields

$$\frac{\partial a_{hl}^*}{\partial p} = \frac{(v'(a_{hl}^*)-\theta_h)+\alpha(v'(a_{hl}^*)-\theta_l)}{(1-p-\alpha p)v''(a_{hl}^*)}. \tag{22}$$

and

$$\frac{\partial a_{hh}^*}{\partial p} = \frac{\alpha(v'(a_{hh}^*)-\theta_l)+(v'(a_{hh}^*)-\theta_h)}{-(p-\alpha(1-p))v''(a_{hh}^*)} \tag{23}$$

Due to (20) the numerator in (22) is positive and, for $p < 1/(1+\alpha)$, the denominator is negative. Hence, $a_{hl}^*(p)$ is decreasing in p for $p < 1/(1+\alpha)$. The numerator of (23) is positive due to $v'(a_{hh}) > \theta_h > \theta_l$. The denominator is positive, because $v''(.) < 0$. Hence, $a_{hh}^*(p)$ increases with p. Since $\partial a_{hh}/\partial p > 0$ and $\partial a_{hl}/\partial p < 0$ it follows for $p \in (1/2, 1/(1+\alpha))$ that $a_{hl}^*(p) < a_h(p) = a_h^{sb} < a_{hh}^*(p)$.

We now check whether the solution to the relaxed problem satisfies the individual rationality constraint (7) of agent l. Substitution of (16), (17), and (18), yields

$$\begin{aligned}U_l(p) &= (\delta a_h + (1-\delta)((1-p)a_{hh} + pa_{hl}))(\theta_h-\theta_l)\\ &\quad +(1-\delta)(2p-1)(S_h(a_{hl})-S_h(a_{hh})), \qquad (24)\end{aligned}$$

where $S_i(a) \equiv v(a) - \theta_i a$ represents the joint surplus of the action a. Since the solution for $p = 1/2$ coincides with the second best solution, the individual rationality constraint is slack, i.e., $U_l(1/2) > 0$.

Differentiating w.r.t. p and using (20)-(22) yields

$$U_l'(p) = -(1-\delta)\left\{[\underbrace{S_h(a_{hh}) - S_h(a_{hl})}_{>0} + \underbrace{S_l(a_{hh}) - S_l(a_{hl})}_{>0}] - (1-\alpha)\alpha(1-p)p\times \right.$$
$$\left. (\theta_h - \theta_l)^2 \underbrace{\left[\frac{1}{(1-(1+\alpha)p)^3 v''(a_{hl})} - \frac{1}{(p-(1-p)\alpha)^3 v''(a_{hh})}\right]}_{\leq 0}\right\} < 0.$$

The first term in the square brackets is negative since $a_{hl} < a_{hh} < a_h^{fb} < a_l^{fb}$ and $S_i(a)$ is increasing for $a < a_i^{fb}$. The second term in the square brackets is non–positive since $1-(1+\alpha)p \leq p-(1-p)\alpha$ and $v''(a_{hh}) \leq v''(a_{hl}) < 0$, due to $v''' \leq 0$. Hence, starting from $p = 1/2$ the utility of type θ_l is decreasing in p. As p approaches $1/(1+\alpha)$ the first part of the second term in the square brackets approaches negative infinity. Hence, there exists some $\bar{p}(\delta) \in (1/2, 1/(1+\alpha))$ such that

$$U_l(\bar{p}(\delta)) = 0.$$

Solving for $\bar{p}(\delta)$ and differentiating with respect to δ yields

$$\bar{p}'(\delta) = \frac{(\theta_h - \theta_l)a_h}{(1-\delta)^2((a_{hh} - a_{hl})(\theta_h - \theta_l) + 2(S_h(a_{hh}) - S_h(a_{hl})))} > 0.$$

Hence, the critical level $\bar{p}(\delta)$ is increasing in δ.

For $p > \bar{p}(\delta)$ the individual rationality constraint of type θ_l is binding for $p > \bar{p}$. By identical arguments the derivative of the right hand side of (6) with respect to p equals $U_l'(p)$. As the left hand is zero the constraint remains satisfied as p rises. This shows that (6) is indeed slack.

To see that, as of $\bar{p}(\delta)$, the principal reduces the distortion on the first period allocation a_h, consider the Lagrangian of a maximization of $V(p)$ subject to $U_l(p) \geq 0$,

$$V(p) + \lambda U_l(p).$$

The first order condition with respect to a_h yields

$$(1-\alpha)(v'(a_h(p)) - t_h) = (\alpha - \lambda(p))(\theta_h - \theta_l).$$

Per definition the constraint becomes binding at $\bar{p}(\delta)$ such that for $p \leq \bar{p}$ the Kuhn-Tucker condition specify $\lambda(p) = 0$ so that $a_h(p) = a_h^{sb}$. For $p > \bar{p}(\delta)$ the constraint is binding so that $\lambda(p) > 0$ and a_h is closer to the first best.

It remains to be shown that for $p = 1$ the principal can achieve the first best. Note that if $p = 1$ a contract γ with $a_l = a_{ll} = a_l^{fb}$ and $a_h = a_{hh} = a_h^{fb}$, $w_l = w_{ll} = \theta_l a_l^{fb}$ and $w_h = w_{hh} = \theta_h a_h^{fb}$ yields, if incentive compatible, the principal the first best outcome. To ensure incentive compatibility we may set a_{hl} and a_{lh} large enough and $w_{hl} = v(a_{hl}) - v(a_{hh}) + w_{hh}$ and $w_{lh} = v(a_{lh}) + v(a_{ll}) - w_{ll}$ to satisfy condition (14) and (15) in equality. Q.E.D.

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