GOVERNANCE AND THE EFFICIENCY OF ECONOMIC SYSTEMS
GESY

Discussion Paper No. 61

A Note on Sabotage in Collective Tournaments

Oliver Gürtler*

October 2005

*Oliver Gürtler, Department of Economics, BWL II, University of Bonn, Adenauerallee 24-42, D-53113 Bonn, Germany. Tel.:+49-228-739214, Fax:+49-228-739210. oliver.guertler@uni-bonn.de

Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

Sonderforschungsbereich/Transregio 15 · www.gesy.uni-mannheim.de
Universität Mannheim · Freie Universität Berlin · Humboldt-Universität zu Berlin · Ludwig-Maximilians-Universität München
Rheinische Friedrich-Wilhelms-Universität Bonn · Zentrum für Europäische Wirtschaftsforschung Mannheim

Speaker: Prof. Konrad Stahl, Ph.D. · Department of Economics · University of Mannheim · D-68131 Mannheim,
Phone: +49(0621)1812786 · Fax: +49(0621)1812785

A Note on Sabotage in Collective Tournaments*

Oliver Gürtler**

Department of Economics, University of Bonn, Germany

Abstract

In this paper a tournament between teams (a collective tournament) is analyzed, where each contestant may spend productive effort in order to increase his team's performance or sabotage the members of the opponent team. It is shown that sabotaging the weaker members of a team always decreases their team's performance more significantly than sabotaging stronger members does. As a consequence, sabotage activities are only directed at a team's weaker members. This finding is quite interesting, as previous results on individual tournaments indicate that oftentimes only the stronger participants should be sabotaged.

Key words: Collective Tournament, Sabotage, Complementarities
JEL classification: C72, J33, M52

*I would like to thank Matthias Kräkel for helpful comments. Financial support by the Deutsche Forschungsgemeinschaft (DFG), SFB-TR 15 (Governance and the efficiency of economic systems), is gratefully acknowledged.

**Oliver Gürtler, Department of Economics, BWL II, University of Bonn, Adenauerallee 24-42, D-53113 Bonn, Germany. Tel.:+49-228-739214, Fax:+49-228-739210; E-mail: oliver.guertler@uni-bonn.de

1 Introduction

"A chain is only as strong as its weakest link."

The relevance of this saying can be confirmed in many real-world settings: Computer hackers, for instance, find new weaknesses in Microsoft's programs nearly every weak. In the past, these weaknesses allowed several computer worms to disperse and to do considerable harm: According to the Computer Crime Research Center, estimations of economic losses in January 2004 caused by malicious software range from $ 63.4 to $ 77.5 billion.

Similarly, important advantages in historic battles have been achieved by employing a battle strategy exploiting a weakness of the opponent army. Examples include the first Battle of Narva, where Charles XII of Sweden made use of a weakness in fortification to defeat Russian army that was clearly predominant in size, or the siege of Alesia, where, again a weakness in fortification, almost led to the end of Roman emperor Julius Caesar.

The observation that a chain is only as strong as its weakest link, should also affect strategies in tournaments between teams (so-called collective tournaments).[1] These tournaments are ubiquitous in practice: For the reconstruction of the World Trade Centre, e.g., eight international teams of architects presented suggestions, how to design the new building. Finally, the proposal of an architect team from Berlin was selected. The selection process had all the characteristics of a tournament: The architect teams were in competition for a given prize (here the fame and monetary gain from designing the new World Trade Centre) and spent resources, while developing their proposals. Other examples for collective tournaments include tournaments between research teams or, most obviously, sports tournaments.

As is known from the literature on tournaments between individuals (see e.g. Lazear (1989), Konrad (2000), Chen (2003), Münster (2004) or Kräkel (2005)), sabotage is an effective instrument to improve one's own position in a tournament and is empirically relevant (see Garicano & Palacios-Huerta (2000)). Yet, sabotage in collective tournaments has not been analyzed so far, although it raises interesting, new questions. Most importantly, if competing against a heterogeneous team, who should be sabotaged more, the weaker or the stronger members? The introductory examples indicate that the answer should be the weaker members, as these members are less robust to sabotage activities.

In a tournament model, this conclusion will be shown to be right. Sabotage activities are always directed at the team members that are most essential for team success, i.e., the team members whose marginal product is highest. We will see that these are always the weaker (or, in the terminology of the model, the less able) team members. This result is due to two effects, a "decreasing-returns effect" and a "complementarity effect". The former effect stems from the assumption that there are decreasing returns to effort so that team members exerting higher effort (that are the more able team members) are, on the margin,

[1] See, for a formal analysis of collective tournaments, Drago et al. (1996).

less productive. Complementarities between team members' activities yield the second effect. As able team members' efforts are higher, complementarities in team production lead to a relatively higher increase in productivity of less able members. Summarizing, both effects imply that less able team members are, on the margin, more productive and, as a consequence, are subject to all sabotage activities.

The note is organized as follows: The next section describes the model, while section 3 contains the model solution and the main results. Section 4 concludes.

2 Description of the model and notation

Consider a tournament between two teams, each consisting of one able and one less able player. All players are assumed to be risk-neutral. The teams compete for an exogenously given winner-prize w that is equally divided between the winner-team's members. Let players 1 and 2 be the members of the first team and let the second team consist of players 3 and 4. Player i (i=1, 2, 3, 4) exerts productive effort $e_i \geq 0$ and sabotage efforts $s_{ij} \geq 0$ (j= 1, 2, 3, 4, $j \neq i$) at cost $\lambda_i C(e_i) + K(\sum s_{ij})$. The parameter λ_i stands for player i's ability. A lower λ_i corresponds to a more able player, i.e., a player being able to perform at lower costs. Suppose that $\lambda_1 = \lambda_3 = 1$, $\lambda_2 = \lambda_4 =: \lambda_L > 1$ and that the players' abilities are common knowledge.[2] $C(\cdot)$ is an increasing and convex function satisfying $C(0) = 0$, $C_e(e_i) = 0$, for $e_i \in [0, \delta]$ with $\delta > 0$ and $\delta \to 0$ and $C_e(e_i) = \infty$, for $e_i \to \infty$, where the subscript denotes the first derivative with respect to effort. The second condition on $C(\cdot)$ says that, up to a strictly positive level of δ, marginal effort costs are zero. Sabotage costs are assumed to only depend on overall sabotage. The single sabotage activities are therefore perfect substitutes for each player. Further, $K(\cdot)$ is supposed to be increasing, convex and satisfying $K(0) = 0$, $K_s(0) = 0$ and $K_s(\sum s_{ij}) = \infty$, for $\sum s_{ij} \to \infty$.[3] Define $x_i := \max\left\{e_i - \sum_{j \neq i} s_{ji}, 0\right\}$ as the "effective effort" of a player, that is, his effort net off incurred sabotage.[4] Team outputs are then assumed to be given by

$$y_1 = f(x_1, x_2) + \epsilon_1 \quad \text{and} \quad y_2 = f(x_3, x_4) + \epsilon_2. \tag{1}$$

The function f is assumed to satisfy the following four properties: (i) f is continuously differentiable and increasing in both arguments, (ii) f exhibits decreasing returns to effective effort, given the other effective effort remains constant, (iii) there are complementarities between both effective efforts, (iv) f is symmetric. Formally, the second, third and fourth property translate into $f_{11} < 0$

[2] Note that the "L" stands for low-ability players and not for low-cost player.

[3] The subscript denotes the first derivative with respect to sabotage effort.

[4] The sabotage cost function is assumed to be sufficiently steeper than the productive effort cost function. Thus, effective efforts are non-negative on the equilibrium path so that, when solving the model, the maximum operator can be omitted.

(and $f_{22} < 0$), $f_{12} > 0$ and $f(a,b) = f(b,a)$.[5] ϵ_1 and ϵ_2 describe random components being continuously, identically and independently distributed with mean μ and variance σ^2. Using (1), the first team's winning-probability can be written as $P_1 := \Pr ob\{y_1 > y_2\} =: H(f(x_1,x_2) - f(x_3,x_4))$, where $H(\cdot)$ denotes the distribution function of the composed random variable $\epsilon_2 - \epsilon_1$, and $h(\cdot)$ the corresponding density function. Similarly, the winning-probability of team 2 is given by $P_2 = 1 - H(f(x_1,x_2) - f(x_3,x_4))$.

3 Solution to the model

We explicitly derive the solution for player 1. Other derivations are completely analogous. Player 1 chooses (productive and destructive) efforts to maximize his expected payoff, which equals his expected payment minus costs entailed by (both kinds of) effort.

$EU_1 = \frac{w}{2} H(f(x_1,x_2) - f(x_3,x_4)) - \lambda_1 C(e_1) - K(s_{12} + s_{13} + s_{14})$.

Obviously, player 1 chooses $s_{12} = 0$. This is very intuitive, for sabotaging one's own teammate decreases the probability of winning and additionally leads to higher costs. The first-order conditions concerning his other effort choices are given by[6]

$$\begin{aligned}
\frac{\partial EU_1}{\partial e_1} &= \frac{w}{2} h(f(x_1,x_2) - f(x_3,x_4)) f_1(x_1^*, x_2^*) - \lambda_1 C_e(e_1) \leq 0 \qquad (2) \\
&\quad (= 0, \text{ for } e_1^* > 0), \\
\frac{\partial EU_1}{\partial s_{13}} &= \frac{w}{2} h(f(x_1,x_2) - f(x_3,x_4)) f_1(x_3^*, x_4^*) - K_s(s_{13} + s_{14}) \leq 0 \qquad (3) \\
&\quad (= 0, \text{ for } s_{13}^* > 0), \\
\frac{\partial EU_1}{\partial s_{14}} &= \frac{w}{2} h(f(x_1,x_2) - f(x_3,x_4)) f_2(x_3^*, x_4^*) - K_s(s_{13} + s_{14}) \leq 0 \qquad (4) \\
&\quad (= 0, \text{ for } s_{14}^* > 0).
\end{aligned}$$

Note that, as $\lambda_1 = \lambda_3 = 1$ and $\lambda_2 = \lambda_4 = \lambda_L$, we get a symmetric solution with $e_1 = e_3 =: e_H$, $e_2 = e_4 =: e_L$, $s_{12} = s_{21} = s_{34} = s_{43} = 0$, $s_{13} = s_{23} = s_{31} = s_{41} =: s_H$ and $s_{14} = s_{24} = s_{32} = s_{42} =: s_L$. The system of first-order

[5] Here and in all what follows, a subscript "j" accompanying the function $f(\cdot)$ denotes a first derivative with respect to component j, a subscript "jj" a second derivative with respect to component j, and a subscript "jk" with $j \neq k$ a mixed derivative with respect to components j and k.

[6] As pointed out by e.g. Lazear and Rosen (1981), the second-order conditions are not necessarily satisfied. To guarantee the existence of the pure-strategy equilibrium specified by the first-order conditions, the density function $h(\cdot)$ has to be sufficiently flat or the effort cost functions sufficiently convex. In what follows, it is assumed that these conditions are fulfilled so that the second-order conditions hold.

conditions then simplifies to

$$\begin{aligned}
\frac{w}{2}h(0)\,f_1(x_H^*,x_L^*)-C_e(e_H) &\leq 0\,(=0,\ \text{for } e_H^*>0), & (5)\\
\frac{w}{2}h(0)\,f_2(x_H^*,x_L^*)-\lambda_L C_e(e_L) &\leq 0\,(=0,\ \text{for } e_L^*>0), & (6)\\
\frac{w}{2}h(0)\,f_1(x_H^*,x_L^*)-K_s(s_H+s_L) &\leq 0\,(=0,\ \text{for } s_H^*>0), & (7)\\
\frac{w}{2}h(0)\,f_2(x_H^*,x_L^*)-K_s(s_H+s_L) &\leq 0\,(=0,\ \text{for } s_L^*>0), & (8)
\end{aligned}$$

where $x_H^* = e_H^* - 2s_H^*$ and $x_L^* = e_L^* - 2s_L^*$.

Before turning to a closer examination of these conditions, it is advisable to say something about the relation between $f_1(x_H^*, x_L^*)$ and $f_2(x_H^*, x_L^*)$. It is, in this context, true that $x_H^* > x_L^* \Longleftrightarrow f_1(x_H^*, x_L^*) < f_2(x_H^*, x_L^*)$. This ranking of marginal productivities is due to two reasons. First, if x_H^* exceeds x_L^*, increasing x_H will be (relatively) less valuable, as f exhibits decreasing returns to effective effort. Second, the complementarities between effective efforts in combination with $x_H^* > x_L^*$ rises $f_2(x_H^*, x_L^*)$ more strongly than $f_1(x_H^*, x_L^*)$. The relation between marginal productivities is extremely helpful in characterizing the tournament equilibrium. Deriving this equilibrium we proceed in three steps, i.e., we derive three lemmas that partly build on each other. Let us start with the first one:

Lemma 1 *There exists no equilibrium with $x_H^* = 0$ or $x_L^* = 0$ (or both).*

Proof: Lemma 1 is proved by contradiction. Suppose that, for some $k = H, L$, $x_k^* = 0$. Effective effort may equal zero for two reasons. Either productive effort is positive, but k is fully sabotaged, or productive effort is zero. The first case cannot constitute an equilibrium. If k is fully sabotaged, he will always have an incentive to choose productive effort slightly above δ, as this effort level is the highest one leading to no costs for the agent. From the first-order conditions, it follows that $\frac{w}{2}h(0)\,f_k(x_H^*, x_L^*) = 0$. Then, $K_s(s_H + s_L) = 0$. This, however, will only be possible, if $e_k - 2s_k > 0 \Longleftrightarrow x_k > 0$. It can easily be shown that the second case is also never part of equilibrium. As condition $C_e(0) = 0$ holds, it will always be in k's interest to deviate and choose positive effort, if no sabotage occurs. This completes the proof of Lemma 1. ■

Notice that $K(\cdot)$ being much steeper than $C(\cdot)$ (footnote 4) implies that productive efforts are substantially higher than destructive efforts. From the fact that there exists no equilibrium with $x_H^* = 0$ or $x_L^* = 0$, it then follows that $x_H^*, x_L^* > 0$. In words, equilibrium effective efforts are always strictly positive. With this assumption in mind, Lemma 2 can be derived.

Lemma 2 *Sabotage is only aimed at the player exerting lower effort. That is, $s_H^* = 0$, if $e_H^* > e_L^*$ and $s_L^* = 0$, if $e_L^* > e_H^*$.*

Proof: Again, the lemma is proved by contradiction. Suppose that $e_H > e_L$ and $s_H > 0$ together hold. We proceed in two steps. First, we show that, with

$e_H > e_L$, $x_H < x_L$ can never hold in equilibrium. Let both, $e_H > e_L$ and $x_H < x_L$, hold and denote the shift in sabotage activities from the high to the low-ability player that is needed to equate x_H and x_L by $\hat{s}$. Consider now a shift in sabotage activities from the high to the low-ability player in the amount of $2\hat{s}+\theta$, with $\theta > 0$ and $\theta \to 0$. This shift leaves sabotage costs unchanged, but changes the opponent team's performance by $\int_{-\hat{s}}^{0} [f_1 (x_L + t, x_L) - f_2 (x_L + t, x_L)] \, dt + \int_{0}^{\hat{s}+\theta} [f_1 (x_L + t, x_L) - f_2 (x_L + t, x_L)] \, dt$. This change in performance is negative, so the shift in sabotage activities is profitable. Further, it is always feasible, as the subsequent calculations will show: Initially, we have $(e_L - 2s_L) - (e_H - 2s_H) = 2\hat{s}$. Rearranging yields $(e_L - 2s_L) - 2\hat{s} = (e_H - 2s_H)$. As $e_H - 2s_H$ is strictly positive in equilibrium, the term $e_L - 2s_L - 2\hat{s} - \theta$ is non-negative for small enough θ. Hence, it is possible and beneficial to shift sabotage activities in the described way. It can therefore never be the case that, in equilibrium, $e_H > e_L$ and $x_H < x_L$ together hold. Hence, with $e_H > e_L$, we must have $x_H \geq x_L$. Let us turn to the case, where $x_H = x_L$. Consider now a shift of two marginal sabotage units from the high to the low-ability player. This shift again leaves sabotage costs unchanged. Further, it is profitable. The first shifted unit does not affect the opponent team's performance. The second, however, leads to a performance decrease. Note that the shift is also feasible, for $x_L^* > 0$. Hence, it cannot be that $e_H > e_L$ and $x_H = x_L$ together hold. Finally, consider the case, where $e_H > e_L$, $x_H > x_L$ and $s_H > 0$. Let sabotage activities be marginally shifted from the high to the low-ability player. While effort costs remain unchanged, the associated performance change is given by $f_1 (x_H, x_L) - f_2 (x_H, x_L)$, which is strictly negative. Hence, members of the opponent team always prefer to shift sabotage activities from high to low-ability players. Further, as $x_L^* > 0$, these shifts are always possible in equilibrium. As a result, in equilibrium we must have $s_H^* = 0$. The proof is completely analogous for the case $e_H < e_L$. This proves Lemma 2. ■

Lemma 2 has interesting implications. It says that the players exerting lowest effort are the only ones subject to sabotage. This result is due to two effects, a "decreasing-returns effect" and a "complementarity effect". The decision of whom to sabotage depends on the answer to the question of whose effective effort is most essential for team performance. With the above team performance function, this question yields an unambiguous answer, namely the players exerting lowest effort. Sabotaging these players is most beneficial as, for given efforts of all other players, they have higher marginal productivities and are more relevant for complementarities. These effects may best be highlighted by considering an example. Suppose that $f(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2} + kx_1x_2$, with $k > 0$. The parameter k measures the strength of complementarities between x_1 and x_2. Moreover, let e_1 equal 5 and e_2 equal 2. Assume further that initially there is no sabotage, but that a member of the opponent team wants to spend 1 unit of sabotage. Sabotage will then be completely directed at player 2. The decreasing returns effect implies that it is more beneficial to sabotage player 2, as $\sqrt{5} - \sqrt{4} < \sqrt{2} - \sqrt{1}$. The complementarity effect also makes sabotaging player 2 more worthwhile, as $k \cdot 5 \cdot 1 < k \cdot 4 \cdot 2$. Hence, both effects work into the same direction so that it is always player 2 who is sabotaged.

Up to this point, we only said that the player exerting lower effort is subject to all sabotage activities. What we did not mention was the identity of that player. In other words, the following question is still unanswered: Does the high or the low-ability player exert higher effort? Lemma 3 gives an answer.

Lemma 3 *The high-ability players exert higher effort than the low-ability ones.*

Proof: Suppose that $e_L > e_H$. From Lemma 2, it follows that $x_L > x_H$. This implies that $f_1(x_H, x_L) > f_2(x_H, x_L)$. As a direct consequence, $C_e(e_H) > \lambda_L C_e(e_L)$ must also hold. However, this can only be the case, if $e_H > e_L$, which contradicts our initial assumption. Hence, $e_H > e_L$ must hold so that Lemma 3 is proved. ■

As high-ability players have a cost advantage, they always exert higher efforts than their low-ability counterparts. As a result, they are not sabotaged so that the effective effort difference is even higher than the difference in productive efforts.

Finally, the following proposition summarizes the derived results:

Proposition 4 *In equilibrium, high-ability players exert higher productive effort than low-ability players, i.e.* $e_H^* > e_L^* > 0$. *Sabotage activities are only aimed at the low-ability players.*

The model therefore supports the intuition based on the saying from the introduction. Under quite general conditions, it was shown to be preferable to aim all sabotage activities at the team members of relatively low ability, for this reduces the team's performance most significantly.

4 Conclusion

This note introduced sabotage into a collective tournament. An interesting result was derived, namely that relatively able players are subject to no sabotage. This result extremely differs from findings in individual tournaments (see e.g. the papers of Chen and Münster), where oftentimes very able players suffer most sabotage activities.

Hence, the analysis of collective tournaments may lead to important insights that are not captured by the extensive analysis of individual tournaments. Economic analysis should therefore focus more strongly on the analysis of collective tournaments. Interesting aspects such as optimal team composition or the optimal division of tournament prizes within teams are, until now, hardly explored.[7]

References:

[7] An exception is Gürtler (forthcoming) who deals with optimal composition of teams in the context of centralized and decentralized marketing in team sports.

Chen, K.-P., 2003, Sabotage in Promotion Tournaments. Journal of Law, Economics and Organization, 19, 119-140.

Drago, R., Garvey, G.T., Turnbull, G.K., 1996, A collective tournament. Economics Letters 50, 223-227.

Garicano, L., Palacios-Huerta, I., 2000, An Empirical Examination of Multidimensional Effort in Tournaments, Working Paper, Brown University.

Gürtler, O., A rationale for the coexistence of central and decentral marketing in team sports. German Economic Review, forthcoming.

Konrad, K.A., 2000, Sabotage in Rent-Seeking Contests. Journal of Law, Economics and Organization, 16, 155-165.

Kräkel, M., Helping and Sabotaging in Tournaments. International Game Theory Review, forthcoming.

Lazear, E.P., 1989, Pay Equality and Industrial Politics. Journal of Political Economy, 97, 561-580.

Lazear, E.P., Rosen, S., 1981, Rank-Order Tournaments as Optimal Labor Contracts, Journal of Political Economy, 89, 841-864.

Münster, J., 2004, Selection Tournaments, Sabotage, and Participation. Mimeo, Free University of Berlin.