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Procurement with Costly Bidding, Optimal Shortlisting, and Rebates

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Abstract

We consider the procurement of a complex, indivisible good when bid preparation is costly, assuming a population of heterogeneous contractors. Shortlisting is introduced to implement the optimal number of bidders, and we explore whether the procurer should reimburse the nonrecoverable cost of preparing a bid in whole or in part. We find that a reimbursement policy is profitable for the procurer only if performance and bidding costs are negatively correlated. Moreover, negative rebates (entry fees) always dominate positive rebates.

1 INTRODUCTION

In many procurements the drafting of a bid is costly. For example, in many procurements funded by the *World Bank*, when contractors bid on the design or construction of a power plant or a national health care system, a bid consists of two envelopes, one containing the detailed technical proposal, and the other containing the financial proposal. The cost of drafting the technical proposal can easily amount to $100,000 and more. Yet, this cost cannot be recovered, except by winning the contract. Naturally, in these circumstances contractors are concerned not only about which proposals to make, but also whether to submit a bid at all.

The typical institutional response to maintain incentives for participation is to restrict the number of bidders. For example, the *World Bank* generally requires contractors to first submit an expression of interest (*EoI*), and then puts a certain number (typically up to six) of those who expressed interest on a "short list." Shortlisted contractors are invited to bid; no one else is allowed to bid.[1]

Another response is to reimburse the cost of bidding in whole or in part. For example, the U.S. Department of Defense has used multiple sourcing policies that subsidize the bid preparation for complex weapons systems (see McAfee and McMillan, 1987). However, if one counts in the cost of reimbursements, it is not obvious whether the procurer can actually benefit from such a policy.

The present paper examines this issue in the framework of a simple procurement game that captures the stylized features of complex procurements. The starting point is the "contractors' game" by Lang and Rosenthal (1991). We extend this game to allow for heterogeneous contractors, introduce shortlisting to implement the optimal number of bidders, and allow the procurer to reimburse the cost of bidding in whole or in part.

Our main finding is that a reimbursement policy is profitable only if contractors are heterogeneous and reimbursements can change their sorting. If these requirements are met, a profitable reimbursement policy exists only if performance and bidding costs are negatively correlated. Moreover, an optimal reimbursement always prescribes a negative rebate (entry fee); positive rebates are dominated in all circumstances.

There is a small literature on the role of reimbursement policies. Kaplan and Sela (2006) explore reimbursements in the framework of a second–price auction. More in line with the present paper, Gal, Nemirovski, and Landsberger (2003) assume a first–price auction; however, unlike in the present paper, they do not allow the procurer to restrict the number of bidders, ignore negative rebates, and assume a continuum of types. They claim that positive rebates are always profitable, which cannot be confirmed in the present model.

2 BASE MODEL

Our starting point is the "contractors' game" by Lang and Rosenthal (1991). There, $n \geq 2$ identical contractors bid for one indivisible contract in a first–price sealed–bid (reverse) auction. Contractors have two costs: the cost of performing the contract (performance cost), $c > 0$, and a nonrecoverable cost of preparing a bid, $d > 0$. The procurer's reservation price of the good (and

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[1] See The World Bank (2004a,b).

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highest accepted bid) is normalized to 1 and $c + d < 1$. Contractors have complete information, yet the procurer does not know contractors' costs.

The "contractors' game" has a unique and symmetric equilibrium in mixed strategies, (q, B). There, each contractor bids with probability $q \in (0, 1)$ according to the continuous mixed bidding strategy $B : [c + d, 1] \to [0, 1]$.

A key result is that the equilibrium price increases in n in the sense of first-order stochastic dominance.[2] Therefore, a profit maximizing procurer should restrict the number of bidders to two.

If participation has been restricted to $n = 2$ the equilibrium strategies (q, B) are:

$$q = 1 - \frac{d}{1 - c} \quad \text{(probability of bidding)} \tag{1}$$

$$B(b) = \frac{(1 - c)(b - c - d)}{(b - c)(1 - c - d)} \quad \text{(c.d.f. of bids).} \tag{2}$$

And the associated equilibrium expected profit of the procurer is equal to (random equilibrium bids are denoted by $\tilde{b}$):[3]

$$
\begin{aligned}
\pi : &= \left(1 - (1 - q)^2\right) - \left(2q(1 - q)E[\tilde{b}] + q^2 E[\min\{\tilde{b}_1, \tilde{b}_2\}]\right) \\
&= \left(1 - (1 - q)^2\right) - 2q(1 - q)\int_{c+d}^{1} b\, dB(b) - q^2 \int_{c+d}^{1} b\, 2(1 - B(b))\, dB(b) \\
&= \frac{(1 - c - d)^2}{1 - c}.
\end{aligned}
\tag{3}
$$

(An alternative method to determine π that avoids tedious computations is to compute the equilibrium expected surplus of the game, S:

$$S := \left(1 - (1 - q)^2\right)(1 - c) - q^2 2d - 2q(1 - q)d = \frac{(1 - c - d)^2}{1 - c}. \tag{4}$$

Contractors' equilibrium expected payoff is obviously equal to zero because they must be indifferent between bidding and not bidding; hence, $\pi = S$.)

We augment this base model successively in two directions: 1) We allow the procurer to reimburse d, in whole or in part, assuming bidding costs are *ex post* verifiable; 2) We introduce a population of heterogeneous contractors and incomplete information, and apply shortlisting to implement the optimal $n = 2$.

3   REBATES DO NOT PAY IF THEY CANNOT AFFECT "SORTING"

Here we maintain the assumption that contractors are identical, assume the optimal $n = 2$, and explore whether it pays to (partially) reimburse the bidding cost.

Suppose the procurer (partially) reimburses the bidding cost by paying a rebate $r = \alpha d$. This allows for full reimbursement ($\alpha = 1$), no reimbursement ($\alpha = 0$), as well as entry fees ($\alpha < 0$), and rewards for participation ($\alpha > 0$).

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[2]A similar adverse effect of increased competition was observed for Bertrand games in Elberfeld and Wolfstetter (1999).

[3]The first term on the RHS of (3) is the procurer's expected benefit; the second term his expected payment. The *c.d.f.* of $\min\{\tilde{b}_1, \tilde{b}_2\}$, conditional on receiving two bids, is $1 - (1 - B(b))^2$.

Introducing a rebate, reduces the bidding cost from d to $(1-\alpha)d$. This increases q and stochastically lowers bids. Altogether, the equilibrium price decreases in the sense of first-order stochastic dominance. Therefore, the procurer benefits, however, at a cost that, as we will show, exceeds the benefit.

To compute the procurer's equilibrium expected net profit:

$$\Pi := \pi - r \left(q^2 2 + 2q(1-q) \right), \tag{5}$$

one must distinguish between $\alpha < 1$ and $\alpha \geq 1$. For $\alpha < 1$, Π can be computed from (1), (2), (3), after replacing d by $d' := d - r$, $r = \alpha d$. Whereas, for $\alpha \geq 1$, the game has a unique pure strategy equilibrium where both contractors submit the bid $b = c$ with probability 1 and $\pi = 1 - c$. Combining these results, one finds, after a bit of rearranging,

$$\Pi = \begin{cases} \frac{(1-c-d)^2 - \alpha^2 d^2}{1-c}, & \text{if } \alpha \leq 1, \\ 1 - c - 2\alpha d, & \text{if } \alpha \geq 1. \end{cases} \tag{6}$$

Evidently, Π has a global maximum at $\alpha = 0$ (it has a stationary point at $\alpha = 0$ and is pseudoconcave), and we conclude:

PROPOSITION 1 *In the basic contractors' game with only one type of contractor the best refunding policy is no refunding, $r = 0$.*

4 EXTENSION: HETEROGENOUS CONTRACTORS, REBATES, AND "SORTING"

Now suppose there are two types of contractors that have different costs (c, d). Types are named in such a way that the higher type index indicates the higher total cost, $t_2 := c_2 + d_2 > c_1 + d_1 =: t_1$. The population of potential contractors is sufficiently large to include at least two contractors of each type. The strategies played if only type i submits an *EoI* are derived from (1) and (2) by setting $(c, d) = (c_i, d_i)$ and are denoted by (q_i, B_i), $i \in \{1, 2\}$.[4]

In order to qualify for bidding, contractors must first submit an expression of interest (*EoI*). The procurer selects two contractors who have submitted an *EoI*, if possible, on the short list. Only shortlisted contractors are allowed to bid.

Cost components are either positively or negatively correlated. Therefore, either one of the cases summarized in Table 1 applies, where $\Delta c := c_2 - c_1$, $\Delta d := d_2 - d_1$.

	$\Delta c > 0$	$\Delta c < 0$
$\Delta d > 0$	case a (+)	case b (-)
$\Delta d < 0$	case c (-)	$\varnothing$

Table 1: Positive (+) and negative (-) correlation

As a first result we show that, under certain conditions, shortlisting induces sorting of types that minimizes the total cost $c + d$:

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[4]Heterogeneity is also assumed in Samuelson (1985), Levin and Smith (1994), McAfee and McMillan (1987), and others. The main difference is that they assume a continuous distribution of types, which implies that contractors' costs differ with probability one.

PROPOSITION 2 *Suppose no reimbursement policy is employed. In equilibrium, in cases a) and b) only (the low cost) type 1-contractors submit an EoI and play the strategies (q_1, B_1). This applies also in case c) if and only if*[5]

$$\Delta d > -\frac{d_1}{1 - c_1} \Delta c. \tag{7}$$

PROOF: Suppose all type 1-contractors submit an *EoI* and, if shortlisted, play the strategy (q_1, B_1) while all but one type 2-contractors do not submit an *EoI*. The deviating type 2-contractor submits an *EoI*, and, if shortlisted, makes a bid $b \in (c_2 + d_2, 1]$. We show that if that deviator happens to be on the short list, his expected payoff, U_2', is negative. By definition of U_2' and the equilibrium strategy (q_1, B_1) one obtains

$$U_2' = -d_2 + q_1 (1 - B_1(b)) (b - c_2) + (1 - q_1)(b - c_2) \tag{8}$$
$$0 = -d_1 + q_1 (1 - B_1(b)) (b - c_1) + (1 - q_1)(b - c_1). \tag{9}$$

Deduct (9) from (8) and one has

$$U_2' = (c_1 + d_1) - (c_2 + d_2) + \Delta c q_1 B_1(b) \tag{10}$$
$$= -(1 - q_1 B_1(b)) \Delta c - \Delta d. \tag{11}$$

Combining case a) with (11) resp. case b) with (10) implies $U_2' < 0$. Finally, combining case c) with condition (7) one finds

$$0 > -\frac{d_1}{1 - c_1} \Delta c - \Delta d$$
$$= -(1 - q_1) \Delta c - \Delta d \quad \text{(by definition of } q_1)$$
$$\geq -(1 - q_1 B_1(b)) \Delta c - \Delta d \quad \text{(for all } b \in (c_1 + d_1, 1])$$

which implies $U_2' < 0$. □

However, this sorting does not always maximize the procurer's expected profit. A glance at (3) indicates that the procurer may wish to replace type 1- by type 2-contractors if the performance cost c_2 is higher than c_1 and the resulting increase in total cost is not too high. This suggests that rebates may serve a useful role to induce the right sorting of types.

A reimbursement policy cannot affect the sorting of types if rebates are paid uniformly. Therefore, rebates must be type dependent, $r_1 \neq r_2$.

A desired change in sorting can be induced equally by a positive rebate ("carrot") and by a negative rebate or entry fee ("stick"):

LEMMA 1 *One can always find a negative rebate (entry fee) $(r_1 < 0, r_2 = 0)$ that crowds out type 1 and crowds in type 2. The smallest positive subsidy that also achieves this is $(r_1 = 0, r_2 = \Delta c + \Delta d + \epsilon$, with $\epsilon = 0$ in cases a) and c) and $\epsilon \geq -\frac{\Delta c (1 - c_1 - d_1)}{1 - c_1}$ in case b).*

PROOF: Obviously, $r_1 = -(1 - c_1 - d_1)$ (entry fee) achieves the assumed sorting.

Consider a positive rebate $(r_1 = 0, r_2 > 0)$. The assumed sorting is induced if only type 2-contractors participate and no type 1-contractor has an incentive

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[5]If (7) does not hold, perfect sorting is not an equilibrium.

to deviate and submit an *EoI*, and type 2 plays the strategies (q_2, B_2) which are derived from (1), (2) by setting $(c, d) = (c_2, d_2 - r_2)$. For convenience we continuously extend the domain of B_2 to $[0, 1]$ by setting $B_2(b) = 0$ for all $b < c_2 + d_2 - r_2$.

By an argument similar to the proof of Proposition 2 this is assured if and only if for all $b \in (c_1 + d_1, 1]$ the payoff of the deviating type 1-contractor is negative. Setting $r_2 = \Delta c + \Delta d + \epsilon$, one obtains

$$
\begin{aligned}
U_1' &= \Delta d - r_2 + q_2(1 - B_2(b))\Delta c + (1 - q_2)\Delta c \\
&= -q_2 B_2(b)\Delta c - \epsilon < 0.
\end{aligned}
\tag{12}
$$

In cases a) and c) this inequality is obviously satisfied for $\epsilon = 0$, but not for $\epsilon < 0$, in which case $c_2 + d_2 - r_2 > c_1 + d_1$ and hence $B(b) = 0$ for $b \in (c_1 + d_1, c_2 + d_2 - r_2]$. Therefore, in these cases the stipulated positive rebate is the smallest one that induces the desired sorting.

In case b) the inequality (12) can only be satisfied for

$$
\epsilon \geq -\frac{\Delta c (1 - c_1 - d_1)}{1 - c_1}.
\tag{13}
$$

Therefore, the stipulated $r_2 = \Delta c + \Delta d + \epsilon$ is also the smallest positive rebate that achieves the assumed sorting in this case. □

However, negative rebates (entry fees) always dominate positive rebates:

PROPOSITION 3 (CARROT VS. STICK) *Consider reimbursement policies that crowd out type 1 and crowd in type 2 either by subsidizing d_2 (positive rebate) or by taxing d_1 (entry fee). The policy that employs a negative rebate (entry fee) is in all cases more profitable for the procurer than that which uses a positive rebate.*

PROOF: Consider the negative rebate (tax) and alternatively the smallest positive rebate (subsidy) that induce the assumed sorting, stated in Lemma 1. The procurer's expected net profits under the tax, Π_t, resp. the subsidy, Π_s, are

$$
\Pi_t = \frac{(1 - c_2 - d_2)^2}{(1 - c_2)}
\tag{14}
$$

$$
\Pi_s = \frac{(1 - c_2 - d_2 + r_2)^2}{1 - c_2} - \left(q_2^2 2 r_2 + 2 q_2(1 - q_2) r_2\right).
\tag{15}
$$

Assume cases a) and c). Insert the subsidy rate $r_2 = \Delta c + \Delta d$ into (15) and one finds $\Pi_t - \Pi_s = (\Delta c + \Delta d)^2 / (1 - c_2) > 0$.

Assume case b). Insert the subsidy rate $r_2 = \Delta c + \Delta d - \frac{\Delta c (1 - c_1 - d_1)}{1 - c_1}$ into (15) and one finds $\Pi_t - \Pi_s = \frac{(c_1 d_2 - c_2 d_1 - \Delta d)^2}{(1 - c_1)^2 (1 - c_2)} > 0$. □

PROPOSITION 4 *In cases a) and b) one cannot find a reimbursement policy that increases the procurer's expected profit.*

PROOF: We show that in both cases there is no profitable reimbursement policy that "taxes" d_1, i.e., with $r_1 < 0, r_2 = 0$. By Proposition 3 taxes dominate subsidies. Therefore, there is no profitable subsidy policy either.

1) Without reimbursement, by Proposition 2 only type 1–contractors participate and play the strategy (q_1, B_1). Therefore, the procurer's equilibrium expected profit without reimbursement is $\Pi_1 = (1 - c_1 - d_1)^2 / (1 - c_1) = \pi_1$. A reimbursement policy can only change the procurer's expected profit if it induces a different sorting, i.e., if it crowds out type 1 and crowds in type 2 and gives rise to the expected gross profit $\pi_2 := (1 - c_2 - d_2)^2 / (1 - c_2)$. However, this is not profitable, since

$$\Delta\Pi := \Pi_2 - \Pi_1 \leq \pi_2 - \pi_1 \equiv \pi_1 \left(\frac{\Delta c}{1 - c_1} - \frac{2(\Delta c + \Delta d)}{1 - c_1 - d_1} \right) \qquad (16)$$

is negative in cases a) and b). In equilibrium, the "tax" is not collected. Therefore, the procurer earns no additional income due to the negative rebate. □

PROPOSITION 5 *In case c) the procurer can raise his expected profit by "taxing" the bidding cost d_1 (entry fee) if and only if Δd is bounded by*

$$-\Delta c \frac{d_1}{1 - c_1} < \Delta d < -\Delta c \frac{1 - c_1 + d_1}{2(1 - c_1)}. \qquad (17)$$

PROOF: By Lemma 1 we know that there is a negative rebate ("tax") that induces the required sorting. It gives rise to the expected profit $\Pi_2 = \pi_2$. By Proposition 2, in the absence of rebates only type 1 participates if and only if $\Delta d > -\Delta c \frac{d_1}{1-c_1}$ in which case the procurer's expected profit is equal to $\Pi_1 = \pi_1$.

By (16) it follows that $\pi_2 > \pi_1$ if in addition $\Delta d < -\Delta c \frac{1-c_1+d_1}{2(1-c_1)}$ holds.

This parameter set is not empty since $\Delta c > 0$ and

$$\frac{1 - c_1 + d_1}{2(1 - c_1)} < \frac{1 - c_1 + d_1}{1 - c_1} < \frac{d_1}{1 - c_1}.$$

Finally, one must make sure that Δc is such that the total cost is increasing, i.e., $\Delta d > -\Delta c$. However, this is automatically assured by $\Delta d > -\Delta c \frac{d_1}{1-c_1}$. □

5 DISCUSSION

In the present paper we generalized the well-known contractors' game to include heterogeneous contractors. We used that model to explore whether the procurer should reimburse contractors' bidding costs in whole or in part.

Altogether, we found that a reimbursement policy is profitable only if it can induce a more desirable sorting of types. Interestingly, this can occur only if it crowds out contractors with the lower performance cost, which requires that performance and bidding costs are negatively correlated. Moreover, the "stick approach" to sorting that employs negative rebates as a deterrent always dominates the "carrot approach" that employs positive rebates.

The three critical assumptions of our analysis are: a "large" population of potential contractors that contains at least two bidders of the same type, *ex post*

verifiability of the cost of bidding, and the restriction to populations with only two types.[6] In future extensions one may wish to relax these assumptions.

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[6]Moreover, we only considered equilibria in which there is perfect sorting. The cases in which both participate are interesting; in these cases rebates serve the additional purpose to eliminate undesirable equilibria.