GOVERNANCE AND THE EFFICIENCY OF ECONOMIC SYSTEMS
GESY

Discussion Paper No. 184

Jostling for Advantage: Licensing and Entry into Patent Portfolio Races

Ralph Siebert*
Georg von Graevenitz**

September 2006

*Ralph Siebert, Department of Economics, Krannert School of Management, Purdue University, 403 West State Street, West Lafayette, IN 47907-2056, USA. rsiebert@purdue.edu
**Georg von Graevenitz, INNO-tec, Munich School of Management, University of Munich, Kaulbachstraße 45, 80539 Munich, Germany. graevenitz@bwl.uni-muenchen.de

Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

Sonderforschungsbereich/Transregio 15 · www.gesy.uni-mannheim.de
Universität Mannheim · Freie Universität Berlin · Humboldt-Universität zu Berlin · Ludwig-Maximilians-Universität München
Rheinische Friedrich-Wilhelms-Universität Bonn · Zentrum für Europäische Wirtschaftsforschung Mannheim

Jostling for Advantage: Licensing and Entry into Patent Portfolio Races

Ralph Siebert (Purdue University and CEPR)*

Georg von Graevenitz (University of Munich)†

September 12, 2006

Abstract

Licensing in a patent thicket allows firms to either avoid or resolve hold-up. Firms' R&D incentives depend on whether they license ex ante or ex post. We develop a model of a patent portfolio race, which allows for endogenous R&D efforts, to study firms' choice between ex ante and ex post licensing. The model shows that firms' relationships in product markets and technology space jointly determine the type of licensing contract chosen. In particular, product market competitors are more likely to avoid patent portfolio races, since the threat of hold-up increases. On the other hand, more valuable technologies are more likely to give rise to patent portfolio races. We also discuss the welfare implications of these results.

JEL: L13, L49, L63.

Keywords: Hold-Up Problem, Licensing, Innovation, Patent Race, Patent Thicket, Research Joint Ventures.

Acknowledgements: We would like to thank Tilman Börgers, Steffen Brenner, Walter Elberfeld, Bronwyn Hall, Dietmar Harhoff and his group at INNO-tec, Heidrun Hoppe, Jos Jansen and Klaus Schmidt for their helpful suggestions and comments. Georg von Graevenitz also gratefully acknowledges the support of the ESRC through its Postdoctoral Fellowship scheme and its support for ELSE, as well as the support of the SFB Transregio 15 during the writing of this paper. All remaining errors are our own.

*Ralph Siebert, Purdue University, Department of Economics, Krannert School of Management, 403 West State Street, West Lafayette, IN 47907-2056, rsiebert@purdue.edu.

†Georg von Graevenitz, graevenitz@bwl.uni-muenchen.de, INNO-tec, Munich School of Management, Kaulbachstraße 45, $D-80539$, Munich.

1 Introduction

Rival firms, in complex product industries, are often owners of complementary assets.[1] Therefore, firms in these industries are more frequently forced to license technologies from each other than rival firms in other industries. Increasingly, rival firms in complex product industries protect these component technologies with patents [Hall (2004)]. It is, therefore, likely that ownership of technologies underlying a complex product is highly dispersed and a "patent thicket" emerges [Heller and Eisenberg (1998); Hall and Ziedonis (2001); Shapiro (2001)]. Firms caught in a patent thicket must cooperate by licensing technologies, otherwise mutual blocking of technological improvements is likely and competition degenerates into litigation.

Licenses in complex product industries can cover large numbers of component technologies. The terms of such licensing agreements depend on the relative quality of firms' entire patent portfolios. Consequently, licensing induces rivalry: firms enter into patent portfolio races to guarantee favourable bargaining outcomes in the future. Ziedonis (2004) shows that this is particularly true if ownership of patent rights for a complex technology is very dispersed. Previous work on licensing in complex product industries [Grindley and Teece (1997); Shapiro (2001)] focuses on ex post licensing, which takes place once firms have acquired patents. However licensing may also take place ex ante: at the start of a research program. This type of licensing prevents a patent portfolio race for a technology. Siebert and von Graevenitz (2006) find that the majority of licensing contracts, signed in the semiconductor industry between 1989-1999, were ex ante contracts. This raises both the positive question paraphrased in the title: when do firms enter into patent portfolio races leading to ex post licensing, and the normative question: how are R&D efforts and welfare affected by licensing in patent thickets?

To address these questions we develop a model of R&D competition in a patent thicket. Our model endogenizes the choice of licensing contract and firms' R&D efforts. Licensing type is endogenized by modelling of the choice between ex ante and ex post licensing. Economic theory suggests that R&D efforts will depend on the type of licensing contract firms adopt. We define ex ante licensing as an agreement to share future research results prior to R&D investments. In contrast a patent portfolio race will lead to ex post licensing if blocking patents exist. Here ex post licensing resolves the threat of hold-up based on these blocking patents.

The existence of blocking patents is key to modelling the choice between ex ante and ex post licensing. Where blocking patents exist firms must license in order to realize the full surplus from a new technology, since competitors could potentially block its adoption. This additional incentive to license is characteristic of a patent thicket. Blocking patents are introduced into our model by allowing for firms' stocks of previous patents. We assume that firms anticipate the advent of a new technology which is a complement to technologies protected by their patent stocks. The extent of complementarity between a new technology and existing technologies determines the *blocking strength* of patent stocks.

[1] Complex products, such as semiconductors, are based on modular technologies, which means that a single product incorporates many component technologies.

To model a patent portfolio race we extend the patent race models presented by Beath et al. (1989) and Loury (1979). Both distinguish uncertainties about the eventual winner of the race, market uncertainty, and the length of the race, technological uncertainty. Furthermore they focus on races for individual patents. In contrast we model competition for a dominant patent portfolio covering a new technology. This implies that firms anticipate bargaining over the surplus created by the new technology after a patent portfolio race. The need for such bargaining arises if there are blocking patents. An ex post bargaining stage is absent from existing patent race models that focus on individual patents as blocking patents have no role in such models. Additionally, the existence of blocking patents implies that the expected value of ex post bargaining may be so low as to make ex ante licensing more attractive. Therefore we endogenize the choice between ex ante and ex post licensing.

We find that a firms' choice between an ex ante contract and entry into a patent portfolio race depends on the blocking strength of existing patents and the nature of competition in the product market. In particular, a higher blocking strength of rival firms' patent portfolios increases the probability of ex ante licensing, ceteris paribus, if firms are competitors in production and the R&D cost function is not too steep. We also show higher blocking strength of firms' patent portfolios can reduce the likelihood of ex ante cooperation if firms are complementors in production. When firms produce substitute (complementary) products, higher blocking strength of patent portfolios will lower (raise) expected payoffs to innovation. Finally we show ex post licensing becomes more attractive relative to ex ante licensing when the value of a new technology rises.

Our paper is related to current research on patent thickets, licensing and patent races. Clark and Konrad (2005) also model patent portfolio races, but assume blocking arises from contemporaneously granted patents in a patent portfolio race. Their model excludes technological uncertainty and they assume blocking is either absent or full. Furthermore they concentrate on ex post licensing. They find that the details of ex post licensing agreements have no effect on firms' R&D efforts. This finding complements our own work in which we do not model the detail of the ex post licensing agreement. The result suggests that our comparison of ex ante and ex post licensing will not depend on our modelling of ex post licensing. We assume that the solution to the ex post bargaining problem conforms to the Shapley value.

Recent work by Doraszelski (2003) and Hörner (2004) extends the patent race literature by incorporating knowledge accumulation effects and by allowing for multiple prizes respectively. Both papers may be viewed as movement away from earlier models which largely focused on memoryless races for individual patents . Both papers show that the dynamics of R&D races are more complex than suggested by the earlier literature on patent races. In this paper we maintain the assumption of memoryless racing, in order to extend patent race models to races for patent portfolios in complex product industries. We deviate from the earlier literature by considering a complex prize structure which captures the existence of complementarities between firms' patent stocks. Maintaining the memorylessness assumption allows us to build on the work of Nti (1997) to derive comparative statics results. Our main innovation is to consider effects of blocking patents on firms' R&D incentives. Additionally we endogenize entry into the race by allowing firms to choose between ex ante and ex post licensing. This allows us to provide comments on the welfare effects of patent portfolio races extending the canonical results for patent races derived by Loury (1979).

Though we model patent portfolio races, our results also apply to contests and rent seeking. Nti (1997) notes several similarities between the modelling frameworks employed in these literatures and Hoppe and Baye (2003) provide a formal proof of the equivalence of the basic models employed by both. Our model may be interpreted as a contest for a better bargaining position between parties who have a strong interest in cooperation. In particular our model captures situations in which the terms of a bargain are renegotiated from time to time and where parties' relative bargaining strength depends on a contest over bargaining chips.

The remainder of the paper is structured as follows: we next describe and solve the model. This is followed by a discussion of the welfare implications of our results. Section four concludes.

2 The model

In this section we describe and solve our model of licensing in complex product industries. The model captures how the expected value of ex ante and ex post licensing depend on the value of a technology and the strength of blocking patents held by rival firms. We employ the model to determine conditions under which firms are likely to prefer ex ante to ex post licensing contracts.

In our model N firms compete in the product and technology markets. These firms may produce substitute or complementary products and may be undertaking R&D that is more or less complementary to that of their rivals. As a result of previous R&D activity all firms are endowed with symmetrical stocks of patents.

Firms compete for ownership of a new, complex technology which will be protected by patents. The outcome of competition for the technology will be an asymmetric distribution of patents covering the technology among rival firms. There will be a winner and one or more losers. Due to complexity of the underlying technology it is very likely that the losers own blocking patents, allowing them to hold-up part or all of the gains from the new technology. Therefore firms bargain over access to each others' patent portfolios and agree to license. They may write a licensing contract before (ex ante licensing) or after (ex post licensing) the research into the new technology has begun.

In a departure from the patent race literature we assumes that firms race for a patent portfolio covering a new technology. Thus firms set their R&D expenditure for the entire portfolio, rather than individual patents. The patent portfolio race is over when the technology is ready for development. At this point, if blocking is not, and has not been, resolved through licensing the technology will probably never come to market.

We analyse the following game to determine when firms prefer ex ante to ex post licensing:

Stage 1 Each firm simultaneously chooses whether or not to contract ex ante with each one of their R&D competitors. A contract comes about only if both sides agree to it.

Stage 2 Firms enter a patent portfolio race for patents on a new technology and each chooses a hazard rate h_i of winning this race. Thereafter the firm owning the majority of patents on the new technology emerges the winner.

Stage 3 If firms have not previously contracted, they bargain over surplus created by the combination of new patents covering the new technology and existing patent stocks of losers. We characterise the value of bargaining using the Shapley value.

Using subgame perfect equilibrium we solve the game by backwards induction.

2.1 General assumptions

In this section we introduce assumptions that allow us to capture the effects of blocking on firms' profits. We also discuss several other assumptions we rely upon in our analysis.

We model ex post asymmetry between patent stocks of the winner and losers through the quality of firms' respective patent portfolios. The quality of the winner's patent portfolio ex post ($\bar{q}$) will be:

$$\bar{q}(B, C) = q_e \left(q_n + 1\right)^{(C-B)}, \tag{1}$$

where $C \in [0, 1]$ is the strength of the *forward complementarity* between new and existing patents in each firm's patent portfolio. $B \in [0, C]$ denotes the anticipated *blocking strength of rivals' patent stocks* - this captures the extent to which the winner's technology is blocked by losers' patents in the absence of a license. q_e and q_n are the quality of *e*xisting patent stocks and of the *n*ew patents respectively.

In absence of a licensing contract the winner will have a patent stock quality of $\bar{q}(B, C)$ while the quality of losers' patent stocks is q_e. Equation (1) shows that the ex post quality of the leader's patent stock increases in the forward complementarity and decreases in the blocking strength of rival firms' patent stocks. When firms license patents, removing the threat of blocking, patent stocks of all firms will be of quality $\bar{q}(0, C)$.

This representation of the quality of firms' patent stocks captures the idea that blocking patents reduce the value of new technologies to the holder of patents covering that technology. It also incorporates the complementarity between new patents covering the technology and existing patents that is necessary for blocking to arise.

To capture the effects of patent stock quality on a firm's profits we employ a simple reduced form representation of profits. We assume each firm's profits π depend on the quality (q) of its own patent portfolio and on the average quality of its rivals' patent portfolios (Q):

$$\pi(q, Q) \qquad \text{where} \qquad \frac{\partial \pi(q, Q)}{\partial q} > 0 \qquad . \tag{P}$$

We assume own profits increase in the quality of a firm's own patent portfolio. The direct effect of the average quality of others' portfolios on own profits is nil where the firms are only competitors in the market for technology. It is negative where they are competitors in the product market and positive where they are complementors in the product market. As we make use of this general property of the profit function extensively below we show in Appendix A.3 that it applies to linear demand.

Our assumption that firms can profitably share patents, either ex ante or ex post is innocuous as long as firms are not competitors in the product market. If, however, they are competitors, not licensing may be more profitable than licensing as Katsoulacos and Ulph (1998) show. We rule these cases out by assumption allowing us to focus on the determinants of licensing type. This implies:

$$2\pi(\bar{q}, \bar{q}) > \pi_w(\bar{q}, q) + \pi_l(q, Q) \quad \text{where} \quad Q(B, C) = \frac{(N-2)q + \bar{q}(B, C)}{(N-1)} \qquad . \tag{S}$$

When a group of firms raises the quality of patent stocks jointly, we assume the direct effect of such an improvement outweighs any negative indirect effects:

$$\frac{d\pi}{dq} = \frac{\partial \pi}{\partial q} + \frac{\partial \pi}{\partial Q}\frac{\partial Q}{\partial q} > 0 \, . \tag{D}$$

Our model of R&D competition is taken from the patent race literature. [2] The date of innovation, τ_i, by firm i is randomly distributed with the exponential distribution:

$$Pr(\tau \leq t) = 1 - e^{-ht} \, ,$$

where h represents the hazard rate with which firm i innovates.

The level of each firm's R&D investment is determined by an R&D cost function $\gamma(h)$. We impose the following conditions on the R&D cost function:

$$\text{(i) } \gamma(0) = \gamma'(0) = 0, \ \gamma''(0) > 0 \qquad \text{(ii) } \forall h > 0, \ \ \gamma(h) > 0, \ \gamma'(h) > 0, \ \gamma''(h) > 0$$
$$\text{(iii) } \lim_{h \to \infty} \gamma'(h) = \infty \tag{G}$$

This R&D cost function satisfies the following assumptions: (i) firms always find it optimal to do some R&D, (ii) the costs of R&D are strictly increasing in the probability of successful innovation, (iii) no firm can ever innovate with certainty in the following instant.

[2] For a survey of this literature refer to Reinganum (1989) or Beath et al. (1994) The model we adopt here is restrictive due to the memorylessness property which makes it tractable.

2.2 Stage 3: Ex post bargaining

By assumption (S) firms not already cooperating on R&D will find it profitable to sign an ex post licensing contract. Since we model patent portfolio races between two or more firms we rely on the Shapley value to determine the expected value of bargaining to leaders and losers. The Shapley value is a widely used method of allocating gains from a coalition amongst its members. In this section we describe bargaining between two firms for which the Shapley value corresponds to Nash bargaining. In Appendix A.2 we also discuss cases with three and four firms.

We denote the expected value of winning the patent portfolio race as $v_w(i)$ and of being a loser as $v_l(i)$, where i denotes the number of firms in the race. These are defined as the Shapley values of the bargaining game between the winner and one or several losers of the technology race. The Shapley value must be calculated independently for each number of competitors (N) in technology space. In this paper we investigate the cases of two, three and four firms.[3] The Shapley value of player i is defined as follows:

$$\phi_i \equiv \sum_{K \leq N} \frac{(N-K)!(K-1)!}{N!} \left(v(K) - v(K_{-i})\right) \tag{2}$$

where $K \leq N$ is a coalition of size K whose members license patents to one another and $v(K)$ is the payoff to each member of such a coalition. The Shapley value of firm i is the average marginal gain over each possible coalition out of N firms from having firm i join that coalition.

For convenience we refer to the ex post quality level in the absence of blocking, $\bar{q}(0, C)$, as $\bar{q}$ and where all rival firms share the same quality level we replace $Q(B, C)$ by that value, e.g. with $\bar{q}$ if all firms share the new technology.

The following table describes the characteristic function for two firms. It sets out the payoffs for each coalition between these firms:

Coalition	Payoff
W	$\pi(\bar{q}(B, C), q)$
L	$\pi(q, \bar{q}(B, C))$
WL	$2\pi(\bar{q}, \bar{q})$

For convenience we sometimes denote the value of a coalition XY as $v(XY)$. This makes it easy to simplify the long expressions that arise in the appendix where coalitions between up to four firms are considered. Using this notation the Shapley values of the winner (v_w) and the loser (v_l) are:

$$v_w(2) = \frac{1}{2}v(W) + \frac{1}{2}\Big[v(WL) - v(L)\Big] \qquad v_l(2) = \frac{1}{2}v(L) + \frac{1}{2}\Big[v(WL) - v(W)\Big] \quad , \tag{3}$$

[3] Empirical research (Siebert and von Graevenitz (2006); Anand and Khanna (2000)) on licensing has shown the vast majority of technology licensing contracts are written by two firms. Contracts between more than two firms arise in 11% of cases.

and it follows that:

$$v_w(2) + v_l(2) = v(WL) \qquad \text{and } v_w(2) - v_l(2) = v(W) - v(L) \tag{4}$$

These composite values are used further below. [4]

2.3 Stage 2: R&D investment

At this stage there is either a group of N firms with ex ante contracts to share future patents or there are N R&D competitors. We derive the expected value of both alternatives.

The expected value of the patent portfolio race

Here all N firms are symmetrical, hence we solve their R&D investment problem using a single value function:

$$V_p(h_p, H_p) = \frac{\frac{v_w}{r} h_p + \frac{v_l}{r} H_p + \pi(q,q) - \gamma(h_p)}{h_p + H_p + r}, \tag{5}$$

where h_p denotes the hazard rate chosen by each firm, $H_p \equiv \sum_{j=1, j\neq i}^{N-1} h_j$ is the sum of hazard rates of all remaining $(N-1)$ R&D competitors. The first order condition determining the optimal hazard rate $\hat{h}_p$ of each firm is:

$$\frac{\partial V_p}{\partial h_p} = \frac{1}{(h_p + H_p + r)^2} \Bigg[\frac{(v_w - v_l)}{r} H_p + [v_w - \pi(q,q)] + \gamma(h_p) - \gamma'(h_p)[h_p + H_p + r] \Bigg] = 0. \tag{6}$$

In equilibrium symmetry of all N firms implies $\hat{h}_p = h_p = \frac{H_p}{N-1}$. Assumption (G) on the R&D cost function implies $\hat{h}_p$ is a local maximum[5].

Following Beath et al. (1989) we identify two innovation incentives in the above expression: the *competitive threat* and the *profit incentive*. These determine R&D investments of the N R&D complementors. The competitive threat is defined as the limit of the first order condition where the R&D investment of the competing firms is infinite:

$$\lim_{h_j \to \infty} \frac{\partial V_p}{\partial h_i} = \frac{(v_w - v_l)}{r} - \gamma'(\bar{h}_i) = 0. \tag{CT}$$

It captures the marginal benefit of just pipping the other contestants in the tournament at the post, given that they are almost entirely certain of innovating in the following instant. The competitive threat is the dominant R&D incentive where R&D investments are high and

[4] For exposition of cases involving more firms refer to Appendix A.2.

[5] Notice that $\frac{\partial^2 V_p}{\partial {h_p}^2} = -\left(\frac{\partial^2 \gamma}{\partial {h_p}^2}\right) \frac{1}{h_p + H_p + r} < 0$.

innovations occur more frequently.

The profit incentive is defined as the limit of the first order condition where the R&D investment of all rival firms is zero:

$$\lim_{h_j \to 0} \frac{\partial V_p}{\partial h_i} = [v_w - \pi(q,q)] + \gamma(\underline{h}_i) - \gamma'(\underline{h}_i)\,[\underline{h}_i + r] = 0 \tag{PI}$$

Here $(\bar{h}_i, \underline{h}_i)$ represent the limits of the interval from which a firm will choose its equilibrium hazard rate[6].

Below we derive comparative statics results from the first order condition (6). As Nti (1997) shows derivation of comparative statics results here is complicated by the fact that we are interested in comparative statics results with respect to *all* contestants in the R&D tournament. These are derived from the first order condition in which the symmetry of all contestants is explicitly recognised [7]:

$$R_p(B, C, \hat{h}_p) = \frac{1}{(N\hat{h}_p + r)^2}\left[\frac{\triangle v}{r}(N-1)\hat{h}_p + [v_w - \pi(q,q)] + \gamma(\hat{h}_p) - \gamma'(\hat{h}_p)\left[N\hat{h}_p + r\right]\right] = 0 \;, \tag{7}$$

where $\triangle v = v_w - v_l$.

Similarly the expected value of ex post contracting becomes:

$$V_p(\hat{h}_p) = \frac{\left(\frac{v_w}{r} + \frac{v_l}{r}(N-1)\right)\hat{h}_p + \pi(q,q) - \gamma(\hat{h}_p)}{N\hat{h}_p + r}. \tag{8}$$

The expected value of ex ante licensing

We assume that, irrespective of the type of ex ante contract (e.g. RJV or licensing), the new technology will be shared amongst contracting parties. As we show below this has the effect of eliminating the competitive threat as an innovation incentive. This drives our main results in the comparison of ex ante and ex post licensing.

Nonetheless the strength of the profit incentive *will* depend on the precise nature of the ex ante contract. In the following analysis we adopt the special case of independent R&D investments to derive our results on ex ante contracting[8]

$$V_a(h_a) = \frac{\frac{\pi(\bar{q},\bar{q})}{r}(h_a + H_a) + \pi(q,q) - \gamma(h_a)}{h_a + H_a + r} \tag{9}$$

where h_a is the hazard rate chosen by firms under ex ante contracting and $\pi(q,q)$ are profits

[6] Given assumption (P) above $\bar{h}_i > \underline{h}_i$.

[7] The corresponding second order condition is: $S = -\frac{\partial^2 \gamma}{\partial \hat{h}_p^2}\frac{1}{N\hat{h}_p + r} < 0$

[8] In case of an RJV in which the firms jointly maximize profits and centralise research in one facility the value function would be: $V_a(h_a) = (h_a + r)^{-1}\left[\frac{N\pi(\bar{q},\bar{q})}{r}h_a + \pi(q,q) - \gamma(h_a)\right]$.

of a member firm from its current products.

The symmetric first order condition in this case is[9]:

$$R_a(C, h_a) = \frac{1}{(N\hat{h}_a + r)^2}\left[(\pi(\bar{q}, \bar{q}) - \pi(q, q)) + \gamma(\hat{h}_a) - \gamma'(\hat{h}_a)(N\hat{h}_a + r)\right] = 0. \tag{10}$$

Here our assumption on the R&D cost function implies $\hat{h}_a$ marks an interior optimum[10]. Note the level of investment by partners under an ex ante contract is determined only by the profit incentive. Each firm's expected value of the contract in equilibrium is:

$$V_a(\hat{h}_a) = \frac{\frac{\pi(\bar{q}(C), \bar{q}(C))}{r} N\hat{h}_a + \pi(q, q) - \gamma(\hat{h}_a)}{N\hat{h}_a + r} \tag{11}$$

2.4 Stage 1: The decision to contract ex ante

In this section the choice between ex ante and ex post licensing is studied. The model we have thus far developed is too general for us to calculate when the expected value of ex ante licensing is greater than that of ex post licensing. Furthermore the value of such a result would be limited as other factors, such as transactions costs, will influence the choice between ex ante and ex post licensing. In fact in Siebert and von Graevenitz (2006) we find transactions costs have large effects on the firms' choice of licensing contract.

Nonetheless our model allows us to derive comparative statics results on the expected value of the difference between ex ante and ex post licensing:

$$\frac{\partial(V^p - V^a)}{\partial \nu} \quad ,$$

where ν represents any exogenous variable such as the blocking strength of firms' patent portfolios, the forward complementarity between new patents and existing patents and the quality of the new patents. In Siebert and von Graevenitz (2006) we test the resulting comparative statics results of this model on a sample of licensing contracts between semiconductor firms.

Next we briefly comment on the derivation of comparative statics results in this section. Thereafter we derive propositions about the effects of changes in blocking strength, forward complementarity and quality of new patents on the choice between ex ante and ex post licensing contracts.

General discussion As noted above we derive comparative statics effects on the difference between the expected values of ex ante and ex post licensing. To keep these derivations simple

[9] In case of RJV formation the symmetric first order condition is:

$$\frac{\partial V_a}{\partial h_a} = \frac{1}{(\hat{h}_a + r)^2}\left[N\left(\pi(\bar{q}, \bar{q}) - \pi(q, q)\right) + \gamma(\hat{h}_a) - \gamma'(\hat{h}_a)(\hat{h}_a + r)\right] = 0\ .$$

Notice that there is no competitive threat in this expression.

[10] $\frac{\partial^2 V_r}{\partial {h_r}^2} = -\frac{\partial^2 \gamma}{\partial {h_r}^2}\frac{1}{h_r + r} < 0$

we adopt slightly different approaches when considering the effects of the blocking strength of patent portfolios (B) and of the forward complementarity (C).

Consider the expected value of ex post contracting first. Equation (8) shows this depends on the forward complementarity and the blocking strength of firms' patent portfolios *directly* through $v_w + (N-1)v_l$. As we demonstrate in Appendix A.2 this sum is always the expected value of the coalition in which all parties agree to share the technology: $N\pi(\bar{q}, \bar{q})$. By definition the value of this coalition is independent of the blocking strength of patent portfolios (B) but is increasing in the forward complementarity (C). The expected value of ex post contracting also depends on these parameters *indirectly* through the endogenous hazard rate of innovation ($\hat{h}_p$).

Turning to the value of ex ante contracting (equation (11)) it clearly depends solely on the expected profit from innovating jointly: $\pi(\bar{q}, \bar{q})$. It is not a function of the blocking strength of patent portfolios. This implies that the *forward* complementarity affects both the expected value of ex post and ex ante contracting, while the *blocking strength of patent portfolios* affects only the value of ex post contracting.

Variation in blocking strength (B)

The blocking strength of firms' patent portfolios allows us to measure the proportion of expected profits from a new technology that a rival firm may be able to hold up, given that firm's patent stocks. Variation in the blocking strength will only affect the expected value of ex post licensing. Therefore it is an important determinant of the choice between ex ante and ex post licensing.

The effects of an increase in the blocking strength on the propensity to license ex ante may be summarised as follows:

Proposition 1
Let (B) denote the blocking strength of a loser's patent stock towards a new technology. Then a greater blocking strength has the effect of making ex ante licensing between two product market competitors less (more) likely if the R&D cost function is (not) steep.

We define a "steep" R&D cost function below. Where firms produce complementary products or there are more than two firms there are no general results on the effects of increased blocking. For these cases we investigate a particular demand function below and derive several corollaries to Proposition 1.

To prove Proposition 1 we derive the effect of variation in the blocking strength (B) on the expected value of ex post licensing:

$$\frac{\partial V_p}{\partial B} = \frac{\partial}{\partial \hat{h}_p}\left(\frac{\frac{\pi(\bar{q},\bar{q})}{r} N\hat{h}_p + \pi(q,q) - \gamma(\hat{h}_p)}{N\hat{h}_p + r}\right)\frac{\partial \hat{h}_p}{\partial B}$$
$$= \frac{1}{(N\hat{h}_p + r)^2}\left(N\left(\pi(\bar{q},\bar{q}) - \pi(q,q)\right) + N\gamma(\hat{h}_p) - \gamma'(\hat{h}_p)(N\hat{h}_p + r)\right)\frac{\partial \hat{h}_p}{\partial B}$$

$$= -\frac{1}{N\hat{h}_p + r}\underbrace{\left[\frac{\triangle v}{r} - \gamma'(\hat{h}_p)\right]}_{\xi}(N-1)\frac{\partial \hat{h}_p}{\partial B} \quad . \tag{12}$$

where we make use of the first order condition in equation (7) to substitute out terms. By definition of the competitive threat ξ is positive and approaches zero in the limit as the competitive threat becomes infinitely large. Therefore the sign of this derivative depends only on the sign of the effect of the blocking strength (B) on the equilibrium hazard rate of innovation ($\hat{h}_p$).

We derive the sign of this effect with the implicit function theorem:

$$\frac{\partial \hat{h}_p}{\partial B} = -\frac{\partial R_p}{\partial B}\left(\frac{\partial R_p}{\partial \hat{h}_p}\right)^{-1} \quad . \tag{13}$$

First we derive the sign of the denominator. As the following expression shows the sign of the denominator depends on the convexity of the R&D cost function:

$$\frac{\partial R_p}{\partial \hat{h}_p} = \frac{1}{(N\hat{h}_p + r)^2}\left[(N-1)\underbrace{\left[\frac{\triangle v}{r} - \gamma'(\hat{h}_p)\right]}_{\xi>0} - \gamma''(\hat{h}_p)\left(N\hat{h}_p + r\right)\right] \quad . \tag{14}$$

As discussed above ξ will be positive. The convexity of the R&D cost function (G) implies that the second term in square brackets is increasing in the equilibrium hazard rate $\hat{h}_p$. Thus the difference of these two terms may be positive or negative at the equilibrium hazard rate. Therefore we may define two regimes: in the first the R&D cost function is not too convex such that $\xi(N-1) > \gamma''(\hat{h}_p)\left(N\hat{h}_p + r\right)$ while in the second the R&D cost function is sufficiently convex such that: $\xi(N-1) < \gamma''(\hat{h}_p)\left(N\hat{h}_p + r\right)$.

Given this distinction we now consider the numerator of the expression in equation (13). To sign the numerator we must consider the effect of greater blocking strength on the difference between the values of winning and losing the patent portfolio race ($\triangle v$) and on the expected value of winning by itself (v_w). As equation (7) above shows these two terms determine the strength of the competitive threat and the profit incentive, respectively.

The derivation of the Shapley values in Appendix A.2 demonstrates that the definition of the competitive threat ($\triangle v$) and of the profit incentive (v_w) depend on the size of the group of firms in the patent portfolio race. As we show next it is sufficient to focus on the cases of two and three firms to prove Proposition 1.

N=2 - Here we can show that:

$$\frac{\partial R_p}{\partial B}_{|N=2} = \frac{1}{2r(2\hat{h}_p + r)}\left[\overset{+}{\frac{\partial \pi}{\partial \bar{q}}}\overset{-}{\frac{\partial \bar{q}}{\partial B}} - \frac{\partial \pi}{\partial Q}\overset{-}{\frac{\partial Q}{\partial B}}\right] \tag{15}$$

This shows that when two firms are producers of substitute products ($\frac{\partial \pi}{\partial Q} < 0$), the derivative $\frac{\partial R_p}{\partial B}$ is negative. If firms produce complementary products ($\frac{\partial \pi}{\partial Q} > 0$) we cannot sign $\frac{\partial R_p}{\partial B}$.

N=3 - With three firms we can show that:

$$\frac{\partial R_p}{\partial B}_{|N=3} = \frac{1}{3r(3\hat{h}_p + r)} \left[\frac{\partial \pi(\bar{q}(B), q)}{\partial B} + \frac{\partial 2\pi(\bar{q}(B), Q(B))}{\partial B} - \frac{\partial \pi(q, \bar{q}(B))}{\partial B} - \frac{\partial 2\pi(q, Q(B))}{\partial B} \right] \quad (16)$$

If three firms are producers of substitute products then this term is negative whenever the second term in brackets is negative. In general, however, this will not be the case:

$$\frac{\partial 2\pi(\bar{q}(B), Q(B))}{\partial B} = \frac{\partial \pi(\bar{q}(\overset{+}{B}), Q(B))}{\partial \bar{q}} \frac{\overset{-}{\partial \bar{q}}}{\partial B} + \frac{\partial \pi(\bar{q}(B), Q(B))}{\partial Q} \frac{\overset{-}{\partial Q}}{\partial B}. \quad (17)$$

If firms produce substitute products the second component of this derivative is positive; therefore the sign of the entire term is ambiguous.

This demonstrates that *in general* we can only sign the effect of the blocking strength on the expected value of ex post licensing when there are two product market rivals. In all other cases the sign of $\frac{\partial R_p}{\partial B}$ is indeterminate. This fact is useful in empirical applications.

Specialising to linear demand Proposition 1 shows general comparative statics results for the effect of stronger blocking are unavailable where firms produce complementary products or there are more than two competitors in the patent portfolio race.

Here we briefly investigate both of these cases using the linear demand example developed in Appendix A.3. In this example holding a higher quality patent stock reduces firms' marginal costs $\frac{\partial \tilde{c}}{\partial \bar{q}} < 0$. Expected profits of N firms with access to a new technology are derived in the appendix. We assume the $n - N$ firms without the new technology have costs $\bar{c}$. Inverse demand is assumed to take the form:

$$p_i = a - x_i - \sigma \sum_{j \neq i}^{n} x_j \qquad \text{where } \sigma \in \left] -\frac{2}{n-1}, 1 \right]. \quad (18)$$

Here σ captures the degree of substitution between firms' products. We employ the profit functions derived from this example to derive the sign of $\frac{\partial R_p}{\partial B}$.

The case of two firms: $\frac{\partial R_p}{\partial B}$ may be rewritten as:

$$\frac{1}{2r(2\hat{h}_p + r)} \left[\frac{\partial \pi}{\partial \bar{q}(B)} - \frac{\partial \pi}{\partial Q} \right] \frac{\partial \bar{q}(B)}{\partial B} = -\frac{1}{r(2\hat{h}_p + r)} \frac{(1+2\sigma)\left[(a-\bar{c})(2-\sigma) + (\bar{c} - \tilde{c})\right]}{(2+\sigma(n-1))^2 (2-\sigma)^2} \frac{\partial \tilde{c}}{\partial \bar{q}} \frac{\partial \bar{q}(B)}{\partial B}. \quad (19)$$

This expression is negative (as shown above) where $\sigma > 0$. It will be positive if $1 + 2\sigma < 0$, i.e. when $\sigma < -\frac{1}{2}$. Thus two firms facing linear demand and producing very complementary products would gravitate towards ex post licensing if the blocking strength of their patent portfolios rises.

The case of three firms: $\frac{\partial R_p}{\partial B}$ may be rewritten as:

$$\frac{1}{3r(3\hat{h}_p + r)}\left[\frac{\partial\pi(\bar{q}(B), q)}{\partial B} + \frac{\partial 2\pi(\bar{q}(B), Q(B))}{\partial B} - \frac{\partial\pi(q, \bar{q}(B))}{\partial B} - \frac{\partial 2\pi(q, Q(B))}{\partial B}\right]$$
$$= -\frac{2}{3r(3\hat{h}_p + r)}\left[\frac{(1+3\sigma)\left[3(a-\bar{c})(2-\sigma) + (\bar{c}-\tilde{c})(3-\sigma)\right]}{(2+\sigma(n-1))^2\,(2-\sigma)^2}\right]\frac{\partial\tilde{c}}{\partial\bar{q}}\frac{\partial\bar{q}(B)}{\partial B} \quad . \tag{20}$$

This expression is positive when $\sigma > 0$, i.e. if firms produce substitute products. The term is negative for $\sigma < -\frac{1}{3}$. Thus three firms facing linear demand and producing fairly complementary products are driven towards ex post licensing as the blocking strength of their patent portfolios rises.

The case of four firms: $\frac{\partial R_p}{\partial B}$ may be rewritten as:

$$\frac{1}{4r(4\hat{h}_p + r)}\left[\frac{\partial\pi(\bar{q}(B), q)}{\partial B} + \frac{\partial 2\pi(\bar{q}(B), Q(B))}{\partial B} + \frac{\partial 3\pi(\bar{q}(B), Q(2B))}{\partial B}\right.$$
$$\left. - \frac{\partial\pi(q, \bar{q}(B))}{\partial B} - \frac{\partial 2\pi(q, Q(2B))}{\partial B} - \frac{\partial 2\pi(q, Q(B))}{\partial B}\right]$$
$$= -\frac{1}{2r(4\hat{h}_p + r)}\left[\frac{(1+4\sigma)\left[6(a-\bar{c})(2-\sigma) + (\bar{c}-\tilde{c})\,(6-4\sigma)\right]}{(2+\sigma(n-1))^2\,(2-\sigma)}\right]\frac{\partial\tilde{c}}{\partial\bar{q}}\frac{\partial\bar{q}(B)}{\partial B} \quad . \tag{21}$$

This expression is positive if $\sigma > 0$, i.e. if firms produce substitute products. The term is negative for $\sigma < -\frac{1}{4}$. Thus four firms facing linear demand and producing fairly complementary products tend towards ex post licensing if the blocking strength of their patent portfolios rises.

We summarise our findings from these examples in the following corollary:

Corollary 1
If demand is linear then an increase in the blocking strength of firms' patent portfolios makes ex ante licensing more (less) likely if firms are product market competitors (complementors) and the R&D cost function is not too steep. These predictions are reversed if the R&D cost function is steep.

Convexity of R&D costs Above we demonstrated that the sign of the effect of blocking on the propensity to engage in ex ante licensing depends on the slope of the R&D cost function at the equilibrium hazard rate. Here, to provide some intuition, we further investigate this condition. We begin by using the first order condition in equation (7) to re-express equation

(14):

$$\frac{\triangle v}{r}(N-1)-\gamma'(\hat{h}_p)(N-1)>\gamma''(\hat{h}_p)\left(N\hat{h}_p+r\right)$$
$$\Leftrightarrow \gamma'(\hat{h}_p)(\hat{h}_p+r)-\gamma(\hat{h}_p)>\gamma''(\hat{h}_p)\left(N\hat{h}_p+r\right)+(v_w-\pi) \quad (22)$$

If we parameterise the R&D cost function as $\gamma = e^{\alpha h} - \alpha h$, where $0 < \alpha < 1$, then we find that this inequality is fulfilled when:

$$e^{\alpha\hat{h}_p}\left[\alpha\hat{h}_p(1-\alpha N)+\alpha r(1-\alpha)\right]>(v_w-\pi)+r\,. \quad (23)$$

Clearly this condition can be fulfilled if the profit incentive $(v_w - \pi)$ is small, the competitive threat is large and $\alpha < N^{-1}$.

Variation in the forward complementarity (C)

The forward complementarity C measures the strength of complementarity between existing patent stocks of a firm and new patents which that firm is competing for. A stronger forward complementarity, indicates the combination of these new patents with the firm's existing patents is more valuable. Using our model we derive the following proposition:

Proposition 2
If there are two product market competitors an increase in the forward complementarity will make ex post licensing less (more) likely if the R&D cost function is (not) steep.

To prove this result the difference between the expected values of ex post and ex ante licensing is first derived:

$$\begin{aligned} V^p - V^a =& \frac{\frac{\pi(\bar{q}(C),\bar{q}(C))}{r}N\hat{h}_p+\pi(q,q)-\gamma(\hat{h}_p)}{N\hat{h}_p+r}-\frac{\frac{\pi(\bar{q}(C),\bar{q}(C))}{r}N\hat{h}_a+\pi(q,q)-\gamma(\hat{h}_a)}{N\hat{h}_a+r}\\ =&\gamma'(\hat{h}_a)-\frac{(\pi(\bar{q},\bar{q})-\pi(q,q))+\gamma(\hat{h}_p)}{N\hat{h}_p+r}\qquad, \end{aligned} \quad (24)$$

using the first order condition (10) for ex ante licensing to substitute out terms. We are now in a position to do comparative statics:

$$\begin{aligned} &\frac{\partial\left(V^p-V^a\right)}{\partial C}\\ &=\gamma''(\hat{h}_a)\frac{\partial\hat{h}_a}{\partial C}-\frac{\partial\pi(\bar{q},\bar{q})}{\partial C}\frac{1}{N\hat{h}_p+r}-\frac{\partial\hat{h}_p}{\partial C}\frac{1}{(N\hat{h}_p+r)}\left[\frac{\triangle v}{r}-\gamma'(\hat{h}_p)\right](N-1)\\ &=\frac{1}{\frac{\partial R_p}{\partial\hat{h}}}\left[\frac{\partial R_p}{\partial\hat{h}}\left(\gamma''(\hat{h}_a)\frac{\partial\hat{h}_a}{\partial C}-\frac{\partial\pi(\bar{q},\bar{q})}{\partial C}\frac{1}{N\hat{h}_p+r}\right)+\frac{\partial R_p}{\partial C}\frac{1}{(N\hat{h}_p+r)}\left[\frac{\triangle v}{r}-\gamma'(\hat{h}_p)\right](N-1)\right]\quad. \end{aligned} \quad (25)$$

The last line of this expression shows how the implicit function theorem and the method developed by Nti (1997) may be employed in order to clarify the expression. Using this technique, we are able to derive a sign for it in some cases.

To do this we must determine the sign of the effect of the forward complementarity on firms' equilibrium hazard rates $\frac{\partial R_p}{\partial C}$. As before we must separately examine this sign for each number of competing firms.

N=2 - We demonstrate in Appendix A.1 that for two firms the derivative in equation (25) may be re-expressed as follows:

$$\frac{\partial\left(V^p - V^a\right)}{\partial C}_{|N=2} = \frac{1}{\frac{\partial R_p}{\partial \hat{h}_p}}\Bigg[\frac{\partial \pi(\bar{q},\bar{q})}{\partial C}\frac{\gamma''(\hat{h}_p)}{(2\hat{h}_p + r)^2}\left(\frac{\xi}{\gamma'(\hat{h}_a)+\gamma''(\hat{h}_a)(2\hat{h}_a+r)} + \left(1 - \frac{\gamma''(\hat{h}_a)(2\hat{h}_a+r)}{\gamma'(\hat{h}_a)+\gamma''(\hat{h}_a)(2\hat{h}_a+r)}\right)\right) + \frac{\xi}{2r(2\hat{h}_p+r)^2}\left[\frac{\partial \pi}{\partial \bar{q}}\frac{\partial \bar{q}}{\partial C} - \frac{\partial \pi}{\partial Q}\frac{\partial Q}{\partial C}\right]\Bigg] . \tag{26}$$

Note that $\frac{\partial \pi(\bar{q},\bar{q})}{\partial C}$ is always positive by assumption (D). Therefore the entire expression has a positive sign if $\frac{\partial R_p}{\partial \hat{h}_p} > 0$ and $\left[\frac{\partial \pi}{\partial \bar{q}}\frac{\partial \bar{q}}{\partial C} - \frac{\partial \pi}{\partial Q}\frac{\partial Q}{\partial C}\right] > 0$. We discuss these two conditions in turn:

- In the previous section we found $\frac{\partial R_p}{\partial \hat{h}_p} > 0$ if the R&D cost function faced by firms is not too steep.
- The expression $\left[\frac{\partial \pi}{\partial \bar{q}}\frac{\partial \bar{q}}{\partial C} - \frac{\partial \pi}{\partial Q}\frac{\partial Q}{\partial C}\right]$ is the effect of the forward complementarity on $\triangle v$ with two firms. This term is positive if the firms compete in the product market, since this implies $\frac{\partial \pi}{\partial Q} < 0$, and all other derivatives in the term are positive.

Therefore Proposition 2 holds in the case of two firms.

N=3 - As shown in Appendix A.1 when $N = 3$ we may express equation (25) as:

$$\frac{\partial\left(V^p - V^a\right)}{\partial C}_{|N=3} = \frac{1}{\frac{\partial R_p}{\partial \hat{h}_p}}\Bigg[\frac{\partial \pi(\bar{q},\bar{q})}{\partial C}\frac{\gamma''(\hat{h}_p)}{(3\hat{h}_p + r)^2}\left(\frac{2\xi}{2\gamma'(\hat{h}_a)+\gamma''(\hat{h}_a)(3\hat{h}_a+r)} + \left(1 - \frac{\gamma''(\hat{h}_a)(3\hat{h}_a+r)}{\gamma'(\hat{h}_a)+\gamma''(\hat{h}_a)(3\hat{h}_a+r)}\right)\right) + \frac{2\xi}{3r(3\hat{h}_p+r)^2}\left[\frac{\partial \pi(\bar{q}(C),q)}{\partial C} + \frac{\partial 2\pi(\bar{q}(C),Q(C))}{\partial C} - \frac{\partial \pi(q,\bar{q}(C))}{\partial C} - \frac{\partial 2\pi(q,Q(C))}{\partial C}\right]\Bigg] . \tag{27}$$

The only difference between this and the previous case arises in the last term in square brackets. This term represents the effects of an increase in the forward complementarity on the competitive threat ($\triangle v$). This term cannot be unambiguously signed in the case of three firms, as shown below.

If the three firms are product market competitors the last two terms in this expression have negative signs, since an increase in the forward complementarity raises the quality of rivals' patent portfolios. The first term in the above expression is always positive since an increase in the forward complementarity that only increases the quality of a firm's own patent portfolio always has a positive effect. Therefore the entire expression will be positive if the second term can be signed. However the sign of the second term is ambiguous in general when firms are competitors in the product market:

$$2\frac{\partial \pi(\bar{q}(C),Q(C))}{\partial C} = 2\frac{\partial \pi(\bar{q}(\overset{+}{C}),Q(C))}{\partial \bar{q}(C)}\frac{\partial \bar{q}(\overset{+}{C})}{\partial C} + 2\frac{\partial \pi(\bar{q}(\overset{-}{C}),Q(C))}{\partial Q(C)}\frac{\partial Q(\overset{+}{C})}{\partial C} \tag{28}$$

A general comparative statics result for the case of $N = 3$ is not available for the same reason as above when considering the effects of blocking on the expected value of ex post licensing.

Specialising to linear demand We proceed as before, to provide some results that go beyond Proposition 2 by specialising demand to the linear case. We derive results for the two, three and four firm cases. Our aim is to sign the second component of expression (25).

The case of two firms The second term in equation (26) may be rewritten as:

$$\frac{\xi}{2r(2\hat{h}_p+r)^2}\left[\frac{\partial \pi}{\partial \bar{q}} - \frac{\partial \pi}{\partial Q}\right]\frac{\partial \bar{q}}{\partial C} = - \frac{\xi}{2r(2\hat{h}_p+r)^2}\left[\frac{(1+2\sigma)\left[(a-\bar{c})(2-\sigma)+(\bar{c}-\tilde{c})\right]}{(2+\sigma(n-1))^2\,(2-\sigma)^2}\right]\frac{\partial \tilde{c}}{\partial \bar{q}}\frac{\partial \bar{q}}{\partial C} \quad , \tag{29}$$

which is positive for $\sigma > 0$ and negative for $\sigma < -\frac{1}{2}$. Thus for two firms producing strongly complementary products the second component of expression (25) will be negative; hence the entire expression is positive if $\frac{\partial R_p}{\partial \hat{h}_p} < 0$.

The case of three firms The second term in equation (27) may be rewritten as:

$$\frac{2\xi}{3r(3\hat{h}_p+r)^2}\left[\frac{\partial \pi(\bar{q}(C),q)}{\partial C} + \frac{\partial 2\pi(\bar{q}(C),Q(C))}{\partial C} - \frac{\partial \pi(q,\bar{q}(C))}{\partial C} - \frac{\partial 2\pi(q,Q(C))}{\partial C}\right] = - \frac{2\xi}{3r(3\hat{h}_p+r)^2}\left[\frac{(1+3\sigma)\left[3(a-\bar{c})(2-\sigma)+(\bar{c}-\tilde{c})(3-\sigma)\right]}{(2+\sigma(n-1))^2\,(2-\sigma)^2}\right]\frac{\partial \tilde{c}}{\partial \bar{q}}\frac{\partial \bar{q}}{\partial C} \quad , \tag{30}$$

which is positive for $\sigma > 0$ and negative for $\sigma < -\frac{1}{3}$. Thus for three firms producing strongly complementary products the second component of expression (25) will be negative; hence the entire expression is positive if $\frac{\partial R_p}{\partial \hat{h}_p} < 0$.

The case of four firms We derive the last term in equation (25):

$$\frac{\xi}{2r(4\hat{h}_p+r)^2}\Bigg[\frac{\partial\pi(\bar{q}(C),q)}{\partial C}+\frac{\partial 2\pi(\bar{q}(C),Q(C))}{\partial C}+\frac{\partial 3\pi(\bar{q}(C),Q(2C))}{\partial C}$$
$$-\frac{\partial\pi(q,\bar{q}(C))}{\partial C}-\frac{\partial 2\pi(q,Q(2C))}{\partial C}-\frac{\partial 2\pi(q,Q(C))}{\partial C}\Bigg]$$
$$=-\frac{1}{2r(4\hat{h}_p+r)}\left[\frac{(1+4\sigma)\left[6(a-\bar{c})(2-\sigma)+(\bar{c}-\tilde{c})\,(6-4\sigma)\right]}{(2+\sigma(n-1))^2\,(2-\sigma)}\right]\frac{\partial\tilde{c}}{\partial\bar{q}}\frac{\partial\bar{q}(C)}{\partial C} \quad (31)$$

This expression is positive if $\sigma > 0$ and negative for $\sigma < -\frac{1}{4}$. Thus four firms producing fairly complementary products the second component of expression (25) will be negative; hence the entire expression is positive if $\frac{\partial R_p}{\partial \hat{h}_p} < 0$.

Our results from the previous examples are summarised in the following corollary:

Corollary 2
If demand is linear and firms produce substitute products then an increase in the forward complementarity has the effect of making ex post licensing
less (more) likely if the R&D cost function is (not) sufficiently steep;

as demonstrated the prediction of Proposition 2 may be extended to more than two firms if demand is linear. However, we cannot sign the effect of an increase in the forward complementarity when firms produce complementary products. This is clear if we note that the first component of equations (26) and (27) is always positive and that the second component may be negative if firms' products are sufficiently complementary.

Propositions 1 and 2 imply that, given the slope of the R&D cost function at equilibrium, we can make quite general statements about firms' choice between ex ante and ex post licensing when there are two firms producing substitute products.

3 Welfare

Here we consider whether firms' privately optimal choices regarding the form of R&D cooperation are also optimal for society. We adopt a second best welfare standard in which the social planner takes bargaining by the firms and the number of R&D competitors as given but may seek to influence the choice between ex post and ex ante licensing.

At the second stage of the model set out above the socially optimal level of R&D investment is derived from the following objective function:

$$V_S(h_S)=\frac{\frac{N\pi(\bar{q},\bar{q})+CS(\bar{q})}{r}Nh_S+(N\pi(q,q)+CS(q))-N\gamma(h_S)}{Nh_S+r}\quad, \quad (32)$$

where $CS(q)$ is the level of consumer surplus as a function of the average quality of all firms' patent stocks. We assume all firms undertake their research independently.

This objective function captures the fact that the social planner will not be concerned with the identity of the winner, nor with the distribution of surplus which the innovation generates amongst the competing firms. Also, the innovation will become available to all firms whether they cooperate ex ante or ex post. The social planner is only concerned with the intensity of effort which the firms expend on innovating.

The second-best level of R&D investment $\hat{h}_S$ corresponding to this objective function is implicitly defined by the following first order condition:

$$R_S(C, h_S) = \frac{1}{(N\hat{h}_S + r)^2} \left[\underbrace{N\left(\pi(\bar{q}, \bar{q}) - \pi(q, q)\right) + \left(CS(\bar{q}) - CS(q)\right)}_{\text{Profit Incentive}} + N\gamma(\hat{h}_S) - \gamma'(\hat{h}_S)(N\hat{h}_S + r) \right] = 0. \quad (33)$$

This first order condition shows several sources of welfare loss from private decisions on R&D:

i The increase in *consumer surplus* arising from innovation contributes to the "profit incentive" of the social planner but is absent from firms' R&D incentives under ex ante and ex post licensing. This indicates a tendency for firms to underinvest.

ii The increase in total *producers' surplus* arising from innovation also contributes to the "profit incentive" of the social planner. In contrast each firm only values the gains in its own profits provided by R&D investment. This effect will be much stronger if firms produce complements than if they produce substitutes. Where firms produce complements under-investment by complementors directly detracts from own profits. In contrast when firms are competitors in the product market under-investment by rivals will have some positive effects on own profits. This also suggests firms will under-invest in R&D, particularly when rivals in technology space produce complementary products.

iii The absence of a *competitive threat* in the social planner's first order condition suggests that cooperating on R&D ex post leads to overinvest in R&D. A comparison of first order conditions of the social planner above (equation 33) and firms under ex post licensing (6) shows that there may be overinvestment in R&D.

Therefore firms which license ex ante are likely to underinvest relative to the second-best welfare criterion we adopt above. Propositions 1 and 2 show ex ante licensing is more likely when (a) firms produce substitute products and (b) the strength of blocking patents is high if (c) the R&D cost function is not too steep. In this case there will be underinvestment in both consumer and producer surplus as described at i and ii above.

Corollary 1 indicates that where (a) firms produce complementary products and (b) blocking patents are strong the propensity for firms to choose ex post licensing rises if (c) the R&D cost function is not too steep. In these cases the underinvestment effects described above will also be at work. In particular underinvestment in producer surplus will be high, since firms are producing complementary products. Here the tendency to underinvest in these equilibria is counteracted by the effects of the competitive threat. As noted above this may encourage over-investment. In summary we observe particularly weak R&D investment incentives relative to the social optimum on the side of the profit incentive are paired with the excessive investment incentives of the competitive threat as a natural result of firms private decisions on licensing.

Our theoretical results also suggest all of these implications reverse, if the R&D cost function firm's face is steep enough in the sense defined above. Our empirical results in Siebert and von Graevenitz (2006) are commensurate with a flat R&D function for the semi-conductor industry.

It is unlikely that firms face socially correct R&D incentives in complex product industries if patent portfolio races arise. While it is unclear whether firms over- or underinvest our model suggests that underinvestment is more likely to be counteracted by overinvestment exactly where underinvestment is greatest.

4 Conclusion

In this paper we model patent portfolio races in complex product industries. To model competition in complex product industries we study patent portfolio races between firms endowed with patent portfolios. In this context a patent portfolio race is a contest to extend each firm's patent stock to a new technology. The winning firm holds the largest proportion of these new patents. However the complementarity of technologies in a complex product industry implies the losers of a patent portfolio race will be able to hold-up a proportion of the gains accruing to the winner.

Therefore firms anticipate the winner and the losers of the patent portfolio race must license technologies ex post. We assume firms anticipate the extent of ex post blocking. This determines the expected value of ex post licensing. Then we study how variation in the value of the new technology and in the blocking strength of firms' patent portfolios affects the relative expected values of ex ante and ex post licensing. We derive a number of predictions about the comparative statics of the premium to ex ante licensing. These predictions are tested and confirmed in Siebert and von Graevenitz (2006).

We find the predictions of our model depend on whether firms are product market rivals and on the extent of technological opportunity. We show firms are driven towards ex ante licensing by an increase in the blocking strength of patent portfolios if technological opportunity is high and firms are product market rivals. This result may be reversed if firms produce complementary products. Finally we show increasing the value of a new technology raises the likelihood that firms choose ex post licensing as long as technological opportunity is high and firms are product market rivals.

We argue these results can provide us with an indication of the welfare implications of licensing in complex product industries. We do not, however, consider the possibility of collusion which is frequently cited as a concern in the literature (Shapiro (2003)). Rather we focus on R&D incentive effects of ex ante and ex post licensing. We show ex post licensing is most likely when underinvestment would be particularly severe under ex ante licensing. This suggests that firms have private incentives which can counteract underinvestment in the presence of licensing. Furthermore our model suggests that any intervention to mitigate under- or overinvestment is unlikely to work very well in the presence of licensing, due to the complexity of the factors that determine choice of licensing contract. From this we conclude that any regulation of licensing in complex product industries should seek to avoid distorting the choice firms make between ex ante and ex post licensing.

References

ANAND, B. AND T. KHANNA (2000): "The Structure of Licensing Contracts," *The Journal of Industrial Economics*, XLVIII, 103–135.

BEATH, J., Y. KATSOULACOS, AND D. ULPH (1989): "Strategic R&D Policy," *The Economic Journal*, 99, 74–83, conference Papers.

——— (1994): "Strategic R&D and Innovation," in *Current Issues in Industrial Economics*, ed. by J. Cable, Macmillan, chap. 8, 160–191.

CLARK, D. J. AND K. A. KONRAD (2005): "Fragmented Property Rights, R&D and Market Structure," Working Paper 4, University of Tromso, Department of Economics and Management.

DORASZELSKI, U. (2003): "An R&D Race with Knowledge Accumulation," *RAND Journal of Economics*, 34, 20–42.

GRINDLEY, P. C. AND D. J. TEECE (1997): "Managing Intellectual Capital: Licensing and Cross-Licensing in Semiconductors and Electronics," *California Management Review*, 39, 8–41.

HALL, B. H. (2004): "Exploring the Patent Explosion," Working Paper 10605, NBER.

HALL, B. H. AND R. H. ZIEDONIS (2001): "The Patent Paradox Revisited: An Empirical Study of Patenting in the U.S. Semiconductor Industry, 1979-1995," *RAND Journal of Economics*, 32, 101–128.

HELLER, M. AND R. EISENBERG (1998): "Can Patents Deter Innovation ? The Anticommons in Biomedical Research," *Science*, 698–701.

HOPPE, H. AND N. R. BAYE (2003): "The Strategic Equivalence of Rent-seeking, Innovation and Patent Race Games," *Games and Economic Behaviour*, 44, 217–226.

HÖRNER, J. (2004): "A Perpetual Race to Stay Ahead," *Review of Economic Studies*, 71, 1065–1088.

KATSOULACOS, Y. AND D. ULPH (1998): "Endogenous Spillovers and the Performance of Research Joint Ventures," *Journal of Industrial Economics*, 46, 333–357.

LOURY, G. C. (1979): "Market Structure and Innovation," *The Quarterly Journal of Economics*, 93, 395–410.

NTI, K. O. (1997): "Comparative Statics of Contests and Rent-seeking Games," *International Economic Review*, 38, 43–59.

REINGANUM, J. F. (1989): "The Timing of Innovation: Research, Development, and Diffusion." in *Handbook of Industrial Organization*, ed. by R. Schmalensee and R. D. Willig, North-Holland, 850–908.

SHAPIRO, C. (2001): "Navigating the Patent Thicket: Cross-Licenses, Patent Pools, and Standard-Setting," in *Innovation Policy and the Economy*, ed. by A. Jaffe, J. Lerner, and S. Stern, NBER.

——— (2003): "Antitrust limits to patent settlements," *RAND Journal of Economics*, 34.

SIEBERT, R. AND G. VON GRAEVENITZ (2006): "How Licensing Resolves Hold-Up: Evidence from a Dynamic Panel Data Model with Unobserved Heterogeneity," Discussion Paper 5436, CEPR.

ZIEDONIS, R. H. (2004): "Don't Fence Me In: Fragmented Markets for Technology and the Patent Acquisition Strategies of Firms," *Management Science*, 50, 804–820.

APPENDIX A

Proofs

Here we derive the comparative statics results that underpin our propositions above.

A.1 Proposition 2

Here we derive the comparative statics of the premium to ex post licensing $(V^p - V^a)$ with respect to the competitive threat. We demonstrate in equation (25) that in order to sign this expression we must derive the signs of $\frac{\partial \hat{h}_a}{\partial C}$ and $\frac{\partial \hat{h}_p}{\partial C}$. The implicit function theorem implies that:

$$\frac{\partial \hat{h}_a}{\partial C} = -\frac{\partial R_a}{\partial C}\left(\frac{\partial R_a}{\partial \hat{h}_a}\right)^{-1} \qquad \frac{\partial \hat{h}_p}{\partial C} = -\frac{\partial R_p}{\partial C}\left(\frac{\partial R_p}{\partial \hat{h}_p}\right)^{-1} \tag{A1}$$

We have already derived $\frac{\partial R_p}{\partial \hat{h}_p}$ above. It is easily shown that:

$$\frac{\partial R_a}{\partial C} = \frac{\partial \pi(\bar{q},\bar{q})}{\partial C}\frac{1}{(N\hat{h}_a + r)^2} \tag{A2}$$

$$\frac{\partial R_a}{\partial \hat{h}_a} = -\frac{1}{(N\hat{h}_a + r)^2}\Big(\gamma'(\hat{h}_a)(N-1) + \gamma''(N\hat{h}_a + r)\Big) \tag{A3}$$

$$\frac{\partial R_p}{\partial C}_{|N=2} = \frac{1}{(N\hat{h}_p + r)^2}\left[\left(\frac{1}{r}\hat{h}_p + \frac{1}{2}\right)\left[\overset{+}{\frac{\partial \pi}{\partial \bar{q}}}\overset{+}{\frac{\partial \bar{q}}{\partial C}} - \frac{\partial \pi}{\partial Q}\overset{+}{\frac{\partial Q}{\partial C}}\right] + \frac{\partial \pi(\bar{q},\bar{q})}{\partial C}\right] \tag{A4}$$

This implies that the hazard rate of ex ante innovation is increasing in the forward complementarity:

$$\frac{\partial \hat{h}_a}{\partial C} = \frac{\partial \pi(\bar{q},\bar{q})}{\partial C}\frac{1}{\Big(\gamma'(\hat{h}_a)(N-1) + \gamma''(N\hat{h}_a + r)\Big)} > 0 \tag{A5}$$

Reinserting this result into equation (25) we find that the derivative has two components:

$$\frac{\partial\left(V^p - V^a\right)}{\partial C} = \frac{\partial \pi(\bar{q},\bar{q})}{\partial C}\left[\frac{\gamma''(\hat{h}_a)}{\gamma'(\hat{h}_a)(N-1) + \gamma''(\hat{h}_a)(N\hat{h}_a + r)} - \frac{1}{N\hat{h}_p + r}\right] - \frac{\partial \hat{h}_p}{\partial C}\frac{1}{N\hat{h}_p + r}\underbrace{\left[\frac{\triangle v}{r} - \gamma'(\hat{h}_p)\right]}_{\xi}(N-1)\ . \tag{A6}$$

In spite of the apparent complexity of this expression we can show that its sign is positive if firms are competitors in the product market and the R&D cost function is not very convex. To do this we employ the method used by Nti (1997). Thus we may rewrite the above expression as:

$$\frac{\partial\left(V^{p}-V^{a}\right)}{\partial C}=\frac{1}{\frac{\partial R_p}{\partial \hat{h}_p}}\left[\frac{\partial R_p}{\partial \hat{h}_p}\frac{\partial \pi(\bar{q},\bar{q})}{\partial C}\frac{\gamma''(\hat{h}_a)}{\gamma'(\hat{h}_a)(N-1)+\gamma''(\hat{h}_a)(N\hat{h}_a+r)}-\frac{\partial R_p}{\partial \hat{h}_p}\frac{\partial \pi(\bar{q},\bar{q})}{\partial C}\frac{1}{N\hat{h}_p+r}\right.$$
$$\left.+\frac{\partial R_p}{\partial C}\frac{1}{N\hat{h}_p+r}\underbrace{\left[\frac{\triangle v}{r}-\gamma'(\hat{h}_p)\right]}_{\xi}(N-1)\right]. \quad \text{(A7)}$$

Now we simplify the above expression by inserting for $\frac{\partial R_p}{\partial C}$ and $\frac{\partial R_p}{\partial \hat{h}_p}$ in two cases. In order to insert for $\frac{\partial R_p}{\partial C}$ we assume that $N=2$ here:

$$\frac{\partial\left(V^{p}-V^{a}\right)}{\partial C}_{|N=2}=$$
$$\frac{1}{\frac{\partial R_p}{\partial \hat{h}_p}}\left[\frac{\partial R_p}{\partial \hat{h}_p}\frac{\partial \pi(\bar{q},\bar{q})}{\partial C}\frac{\gamma''(\hat{h}_a)}{\gamma'(\hat{h}_a)+\gamma''(\hat{h}_a)(2\hat{h}_a+r)}-\frac{1}{(2\hat{h}_p+r)^3}\left[\xi-\gamma''(\hat{h}_p)\left(2\hat{h}_p+r\right)\right]\frac{\partial \pi(\bar{q},\bar{q})}{\partial C}\right.$$
$$\left.+\frac{\xi}{(2\hat{h}_p+r)^3}\left[\left(\frac{1}{r}\hat{h}_p+\frac{1}{2}\right)\left[\frac{\partial \pi}{\partial \bar{q}}\frac{\partial \bar{q}}{\partial C}-\frac{\partial \pi}{\partial Q}\frac{\partial Q}{\partial C}\right]+\frac{\partial \pi(\bar{q},\bar{q})}{\partial C}\right]\right]. \quad \text{(A8)}$$

Cancelling excess terms we have:

$$\frac{\partial\left(V^{p}-V^{a}\right)}{\partial C}_{|N=2}=$$
$$\frac{1}{\frac{\partial R_p}{\partial \hat{h}_p}}\left[\frac{\partial \pi(\bar{q},\bar{q})}{\partial C}\left(\frac{\partial R_p}{\partial \hat{h}_p}\frac{\gamma''(\hat{h}_a)}{\gamma'(\hat{h}_a)+\gamma''(\hat{h}_a)(2\hat{h}_a+r)}+\frac{\gamma''(\hat{h}_p)}{(2\hat{h}_p+r)^2}\right)+\frac{\xi}{2r(2\hat{h}_p+r)^2}\left[\frac{\partial \pi}{\partial \bar{q}}\frac{\partial \bar{q}}{\partial C}-\frac{\partial \pi}{\partial Q}\frac{\partial Q}{\partial C}\right]\right]. \quad \text{(A9)}$$

Here we investigate the case in which $N=3$:

$$\frac{\partial\left(V^{p}-V^{a}\right)}{\partial C}_{|N=3}=\frac{1}{\frac{\partial R_p}{\partial \hat{h}_p}}\left[\frac{\partial R_p}{\partial \hat{h}_p}\frac{\partial \pi(\bar{q},\bar{q})}{\partial C}\frac{\gamma''(\hat{h}_a)}{2\gamma'(\hat{h}_a)+\gamma''(\hat{h}_a)(3\hat{h}_a+r)}\right.$$
$$-\frac{1}{(3\hat{h}_p+r)^3}\left[2\xi-\gamma''(\hat{h}_p)\left(3\hat{h}_p+r\right)\right]\frac{\partial \pi(\bar{q},\bar{q})}{\partial C}$$
$$+\frac{2\xi}{(3\hat{h}_p+r)^3}\left(\left(\frac{1}{r}\hat{h}_p+\frac{1}{3}\right)\left[\frac{\partial \pi(\bar{q}(C),q)}{\partial C}+\frac{\partial 2\pi(\bar{q}(C),Q(C))}{\partial C}\right.\right.$$
$$\left.\left.\left.-\frac{\partial \pi(q,\bar{q}(C))}{\partial C}-\frac{\partial 2\pi(q,Q(C))}{\partial C}\right]+\frac{\partial \pi(\bar{q},\bar{q})}{\partial C}\right)\right]. \quad \text{(A10)}$$

Cancelling excess terms we have:

$$\frac{\partial\left(V^{p}-V^{a}\right)}{\partial C}=\frac{1}{\frac{\partial R_p}{\partial \hat{h}_p}}\left[\frac{\partial \pi(\bar{q},\bar{q})}{\partial C}\left(\frac{\partial R_p}{\partial \hat{h}_p}\frac{\gamma''(\hat{h}_a)}{2\gamma'(\hat{h}_a)+\gamma''(\hat{h}_a)(3\hat{h}_a+r)}+\frac{\gamma''(\hat{h}_p)}{(3\hat{h}_p+r)^2}\right)\right.$$

$$+\frac{2\xi}{3r(3\hat{h}_p+r)^2}\left(\frac{\partial\pi(\bar{q}(C),q)}{\partial C}+\frac{\partial 2\pi(\bar{q}(C),Q(C))}{\partial C}-\frac{\partial\pi(q,\bar{q}(C))}{\partial C}-\frac{\partial 2\pi(q,Q(C))}{\partial C}\right)\Bigg]. \quad (A11)$$

A.2 Shapley value calculations

$\boldsymbol{N=3}$

Coalition	Payoff
W	$\pi(\bar{q}(B,C),q)$
L	$\pi(q,\bar{q}(B,C))$
WL	$2\pi(\bar{q}(B,C),Q(B,C))$
LL	$2\pi(q,Q(B,C))$
WLL	$3\pi(\bar{q},\bar{q})$

The Shapley values of the winner ($v_w(3)$) and the loser ($v_l(3)$) here are:

$$v_w(3)=\frac{1}{3}v(W)+\frac{1}{3}\Big[v(WL)-v(L)\Big]+\frac{1}{3}\Big[v(WLL)-v(LL)\Big] \quad (A12)$$

$$v_l(3)=\frac{1}{3}v(L)+\frac{1}{6}\Big[v(WL)-v(W)\Big]+\frac{1}{6}\Big[v(LL)-v(L)\Big]+\frac{1}{3}\Big[v(WLL)-v(WL)\Big] \quad (A13)$$

from this it follows that:

$$v_w(3)+2v_l(3)=v(WLL) \quad \text{and } v_w(3)-v_l(3)=\frac{1}{2}v(W)+\frac{1}{2}\Big[v(WL)-v(L)\Big]-\frac{1}{2}v(LL) \quad (A14)$$

$\boldsymbol{N=4}$

Coalition	Payoff
W	$\pi(\bar{q}(B,C),q)$
L	$\pi(q,\bar{q}(B,C))$
WL	$2\pi(\bar{q}(B,C),Q(B,C))$
LL	$2\pi(q,Q(2B,C))$
WLL	$3\pi(\bar{q}(B,C),Q(2B,C))$
LLL	$3\pi(q,Q(B,C))$
WLLL	$4\pi(\bar{q},\bar{q})$

$$v_w(4)=\frac{1}{4}v(W)+\frac{1}{4}\Big[v(WL)-v(L)\Big]+\frac{1}{4}\Big[v(WLL)-v(LL)\Big]+\frac{1}{4}\Big[v(WLLL)-v(LLL)\Big] \quad (A15)$$

$$\begin{aligned}v_l(4)=&\frac{1}{4}v(L)+\frac{1}{6}\Big[v(LL)-v(L)\Big]+\frac{1}{12}\Big[v(WL)-v(W)\Big]+\frac{1}{6}\Big[v(WLL)-v(WL)\Big]\\&+\frac{1}{12}\Big[v(LLL)-v(LL)\Big]+\frac{1}{4}\Big[v(WLLL)-v(WLL)\Big]\end{aligned} \quad (A16)$$

which implies:

$$v_w(4) + 3v_l(4) = v(WLLL) \qquad \text{and} \tag{A17}$$
$$v_w(4) - v_l(4) = \frac{1}{3}v(W) + \frac{1}{3}\Big[v(WL) - v(L)\Big] + \frac{1}{3}\Big[v(WLL) - v(LL)\Big] - \frac{1}{3}v(LLL)$$

A.3 Product market examples

Linear demand

Assume that the inverse demand function is linear:

$$p_i = a - x_i - \sigma \sum_{j \neq i}^{n} x_j \qquad \text{where } \sigma \in \left]-\frac{2}{n-1}, 1\right] \tag{A18}$$

There are $1 \leq N \leq n$ efficient firms with costs $\tilde{c}(B)$ and $n - N$ inefficient firms with costs $\bar{c}$ in the market. We denote all variables related to the efficient firms with $\tilde{}$ and all those related to the inefficient firms with $\bar{}$. Assuming that the firms compete in quantities the first order conditions for profit maximization imply that:

$$(\bar{p} - \bar{c}) = \bar{x} \qquad \text{and} \qquad (\tilde{p} - \tilde{c}) = \tilde{x} \tag{A19}$$

Combining the first order conditions with the inverse demand function we can show that:

$$\tilde{p} = a - (\tilde{p} - \tilde{c})\,(1 + \sigma(N-1)) - (\bar{p} - \bar{c})\,\sigma(n - N) \tag{A20}$$
$$\bar{p} = a - (\tilde{p} - \tilde{c})\,\sigma N - (\bar{p} - \bar{c})\,(1 + \sigma(n - N - 1)) \tag{A21}$$

which implies that:

$$\tilde{p} - \bar{p} = -\,(\bar{c} - \tilde{c})\,\frac{(1-\sigma)}{(2-\sigma)}, \quad \tilde{p} = \frac{A + (\bar{c} - \tilde{c})\left(\frac{1+\sigma(n-N)}{2-\sigma}\right)}{(2 + \sigma(n-1))} + \tilde{c}, \quad \bar{p} = \frac{A - (\bar{c} - \tilde{c})\,\frac{\sigma N}{(2-\sigma)}}{(2 + \sigma(n-1))} + \bar{c} \tag{A22}$$

where we define $A = a - \bar{c}$. Then firms' profits in equilibrium are:

$$\bar{\pi} = \left(\frac{A - (\bar{c} - \tilde{c})\,\frac{\sigma N}{(2-\sigma)}}{2 + \sigma(n-1)}\right)^2 \qquad \tilde{\pi} = \left(\frac{A + (\bar{c} - \tilde{c})\left(\frac{1+\sigma(n-N)}{2-\sigma}\right)}{(2 + \sigma(n-1))}\right)^2 \tag{A23}$$

Below we will often need to take the derivative of the difference of these profits:

$$\frac{\partial\,(\tilde{\pi} - \bar{\pi})}{\partial \tilde{c}} = \frac{(1 + \sigma n)}{(2 + \sigma(n-1))^2(2-\sigma)^2}\Big(2A(2-\sigma) + (\bar{c} - \tilde{c})\,(1 + \sigma(n - 2N))\Big) \tag{A24}$$