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**Expectation Damages,
Divisible Contracts, and
Bilateral Investment**
Susanne Ohlendorf*

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*Bonn Graduate School of Economics, University of Bonn, Adenauerallee 24-26, 53113 Bonn, Germany; email: susanne.ohlendorf@uni-bonn.de.

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Speaker: Prof. Dr. Urs Schweizer · Department of Economics · University of Bonn · D-53113 Bonn,
Phone: +49(228) 73 9220 · Fax: +49 (228) 73 9221

Expectation Damages, Divisible Contracts, and Bilateral Investment

Susanne Ohlendorf*

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Abstract

This paper examines the efficiency of expectation damages as a breach remedy in a bilateral trade setting with renegotiation and relationship-specific investment by the buyer and the seller. As demonstrated by Edlin and Reichelstein (1996), no contract that specifies only a fixed quantity and a fixed per-unit price can induce efficient investment if marginal cost is constant and deterministic. We show that this result does not extend to more general payoff functions. If both parties face the risk of breaching, the first best becomes attainable with a simple price-quantity contract.

(JEL: K12, D86, L14)

Real-world contracts sometimes look surprisingly simple given the complexity of the environment. A production contract between a buyer and a seller might specify only

*Bonn Graduate School of Economics, University of Bonn, Adenauerallee 24-26, 53113 Bonn, Germany; email: susanne.ohlendorf@uni-bonn.de. Many people have helped to improve the paper with their comments and suggestions. In particular, I would like to thank the two anonymous referees, as well as Urs Schweizer and Patrick Schmitz. I am also grateful to Bastian Goldlücke, Martin Hellwig, John Moore, Timofiy Mylovanov, Stefan Reichelstein, Andreas Roeder, Alexander Stremitzer, and seminar audiences in Bonn. Support by Deutsche Forschungsgemeinschaft through SFB TR 15 is gratefully acknowledged.

a fixed quantity and a price per unit, although between the signing of the contract and actual trade many contingencies may arise that affect the value of trade. As a consequence, after the uncertainty about valuation and production costs is resolved the parties might observe that they can make a larger profit by trading more or less than the stipulated quantity. In this case, they can resort to renegotiation to reach a trade decision that is optimal given the relationship-investments they made. The main purpose of their contract is then to set the right incentives to invest.

The contract affects the investment incentives by serving as a disagreement point in the renegotiations. Here, standard breach remedies play a role, since the payoff that one party can realize unilaterally also depends on the consequences of breach. In this paper, we focus on the standard expectation damages remedy. It is assumed that courts can verify all relevant information to award expectation damages, which compensate the victim of breach for the loss of profit. Our result is that contracts do not need to be contingent under this damage rule: price and quantity can be adjusted to induce both parties to invest efficiently.

Two effects of renegotiation and standard breach remedies on investment have been identified in the literature. If the parties leave the trade decision to ex post negotiations, they will underinvest relative to the first best, due to a hold-up problem (see e.g. Oliver S. Williamson (1985), Oliver Hart and John H. Moore (1988)). In contrast, the economic analysis of breach remedies by Steven Shavell (1980,1984), William P. Rogerson (1984) and others reveals that if contracts and breach remedies are available then there can also be an overinvestment effect. Remedies such as expectation damages act as an insurance against breach. The victim of breach invests more than if he or she internalized the lost investment in case of breach.

These two intuitions are integrated by Aaron S. Edlin and Stefan Reichelstein (1996) (henceforth ER), who show that it is possible to balance the hold-up effect against the overinvestment effect when contracts are enforced by the standard breach remedies of expectation damages or specific performance. They find that a continuous quantity in the contract is a powerful tool to adjust the investment incentives of one party.¹ In

¹The idea that a continuous variable can help to reach the first best by balancing one party's investment incentives also appears in related articles such as Tai-Yeong Chung (1991), Philippe Aghion,

addition, when the breach remedy is specific performance, the incentives of two parties can be aligned with a single quantity if the payoff functions have a particular form. In contrast, for the regime of expectation damages ER show that for a deterministic and linear cost function no fixed price-quantity contract exists that achieves first best investment decisions. They conclude that specific performance is better suited for two-sided investment problems than expectation damages.

The present paper extends the analysis to more general cost and valuation functions and finds that the scope of expectation damages to solve a bilateral hold-up problem is much larger than this counterexample suggests. It turns out that in ER's framework with divisible contracts², the per-unit price can be used as an additional instrument to fine-tune both parties' incentives to invest. ER establish their positive results for one-sided investment through the adjustment of quantity alone, while the price is set high or low such that always the same party breaches. With an intermediate price, any party may breach the contract, and the parties' probabilities of breach vary with price. Therefore, a contract that specifies an up-front transfer, a quantity, and a per-unit price often suffices to obtain efficient two-sided investment.

The reason why the first best can not be obtained under the expectation damages remedy in the case of constant and deterministic marginal cost is that the seller's investment decision completely determines who breaches. The breaching party never gains from having invested more than the efficient amount, hence the hold-up effect dominates this party's incentives. In order to balance both investment decisions, both parties must face the risk of breach. While no single per-unit price has this effect in ER's example, we demonstrate that the parties can use a lottery between an extremely high and extremely low per-unit price. Like the intermediate per-unit price, the probability of a high price can be used as an additional continuous variable to fine-tune investment incentives.

The paper is organized as follows: Section I introduces the model, while in Section II the ex post consequences of expectation damages with divisible contracts are

Mathias Dewatripont and Patrick Rey (1994), and Georg Nöldeke and Klaus M. Schmidt (1995). In these papers, the first best can be reached because renegotiation leaves one party with the full surplus.

²A divisible contract consists of several items and the price to be paid is apportioned to each item. It can be broken into its component parts, such that each unit together with the per-unit price can be treated as a separate contract, which is fulfilled or breached independently.

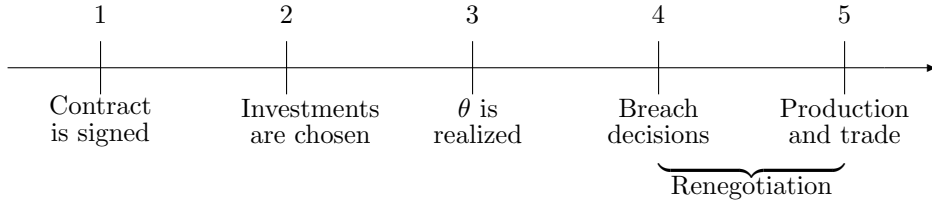


Figure 1: Timeline of the model.

discussed. The main result on the optimality of price-quantity contracts is presented in Section III, and Section IV deals with stochastic prices. Concluding remarks can be found in Section V. Proofs are relegated to the appendix.

I Model Description

The sequence of events is illustrated in Figure 1. A seller and a buyer, both of whom are risk-neutral, have to incur relationship-specific investments in preparation of future trade. To protect these investments, they sign a contract at date 1, specifying a per-unit price \bar{p} and a quantity $\bar{q} \in [0, q^{max}]$ of the good to be traded.³ The parties may also exchange an up-front transfer T to divide the gains from trade after price and quantity are chosen to maximize joint surplus.

At date 2, the seller invests to decrease his marginal cost, and the buyer invests to increase her marginal benefit of the good. The costs of their investments (or reliance expenditures, in legal terms) are denoted by $\sigma \in [0, \sigma^{max}]$ and $\beta \in [0, \beta^{max}]$, respectively. These investment decisions may be difficult to describe or to observe, and consequently are not contractible. The exact shape of the cost and valuation functions becomes commonly known at date 3, when the state of the world $\theta \in \Theta$ is realized. The contingency θ reflects exogenous uncertainty and is drawn from a compact state space $\Theta \subset \mathbb{R}^n$ according to a distribution function F .

At date 4, both seller and buyer decide whether they want to breach to a quantity lower than the one specified in the contract. The consequences of breach are determined

³The quantity of the traded good could also be interpreted as duration of the business relationship. With this interpretation the problem has a dynamic structure which is analyzed by Sergei Guriev and Dmitriy Kvasov (2005).

by expectation damages, either as the default breach remedy or because the contract explicitly specifies this breach remedy. A more detailed description of the consequences of breach follows in Section II. The payoff as determined by the legal consequences constitutes the disagreement point in subsequent renegotiations. The outcome of the negotiations is assumed to be the (generalized) Nash bargaining solution, where $\gamma \in [0, 1]$ denotes the seller's bargaining power. We use the following notation and assumptions:

- The seller's payoff of producing q is $-C(\sigma, \theta, q) - \sigma$. The cost function C is increasing and strictly convex in q , and $(\sigma, q) \mapsto C(\sigma, \theta, q)$ is twice continuously differentiable for all $\theta \in \Theta$, with $C_{\sigma q} \leq 0$.
- The buyer's payoff of obtaining q is $V(\beta, \theta, q) - \beta$. The valuation function V is increasing and strictly concave in q , and $(\beta, q) \mapsto V(\beta, \theta, q)$ is twice continuously differentiable for all $\theta \in \Theta$, with $V_{\beta q} \geq 0$.

Ex post, trade of a quantity q creates a joint surplus of

$$W(\beta, \sigma, \theta, q) := V(\beta, \theta, q) - C(\sigma, \theta, q).$$

This is maximized at the efficient quantity

$$Q^*(\beta, \sigma, \theta) := \arg \max_{q \in [0, q^{max}]} W(\beta, \sigma, \theta, q).$$

We denote by (β^*, σ^*) the efficient investment levels which maximize

$$\int W(\beta, \sigma, \theta, Q^*(\beta, \sigma, \theta)) dF - \beta - \sigma.$$

in $[0, \beta^{max}] \times [0, \sigma^{max}]$. We assume that these are the unique maximizers.

II Breach decisions

The contract and the legal consequences define a game between seller and buyer, which we will solve by backward induction. In this section we analyze the ex post subgame, when cost and valuation functions are realized and observable by both players and the court. Renegotiation will lead the parties to the ex post efficient trade decision. In order to determine their payoffs, we first explore the consequences of breach to see how

much a party can achieve unilaterally. We abstract from issues like litigation costs or difficulties to assess damages.

According to the expectation damage rule, the breaching party has to compensate the victim of breach for the loss caused by the breach. The goal of this remedy is to put the victim in as good a position as if the contract had been fulfilled. This rule is, however, not applied literally in cases where this party faces negative profits from completion of the contract. If one party's breach is advantageous for the other, it is not possible to sue for a reward, but damage payments are zero.

The buyer and the seller face symmetric decisions in this subgame. While the seller can breach by producing and delivering less than ordered, the buyer can breach by announcing her breach decision before the unwanted units are produced. In that case, the seller can only recover the profit margin of the canceled goods, but not their cost of production. Suppose that first the buyer announces her anticipatory breach decision $q_B \leq \bar{q}$, followed by the seller's announcement $q_S \leq \bar{q}$.⁴ Then the seller subsequently delivers the quantity $q = \min\{q_S, q_B\}$, as he will not be compensated for producing a larger quantity.

Since the contract explicitly specifies a price per unit, it is divisible, and the seller is entitled to a payment of $\bar{p}q$ for his partial performance. The payoffs from this part of the contract are

$$S(\sigma, \theta, q) := \bar{p}q - C(\sigma, \theta, q)$$

and

$$B(\beta, \theta, q) := V(\beta, \theta, q) - \bar{p}q.$$

Damages are confined to that part of the contract that was breached. If $q < q_B$, the seller is liable for the buyer's loss on the units between q and q_B . On all units above q_B the contract counts as consensually canceled. The seller has to pay

$$\max\{B(\beta, \theta, q_B) - B(\beta, \theta, q), 0\}^5$$

⁴The order of announcements does not matter for the outcome of the subgame.

⁵This is the formula used by ER and most of the literature. Another approach is to compare the utility resulting from breach with a hypothetical, "reasonably foreseeable" (Robert Cooter (1985)) valuation $B(\beta^*, \theta, q)$ instead of the actual valuation. As shown by Jim Leitzel (1989) and Urs Schweizer (2005) this solves the overreliance problem.

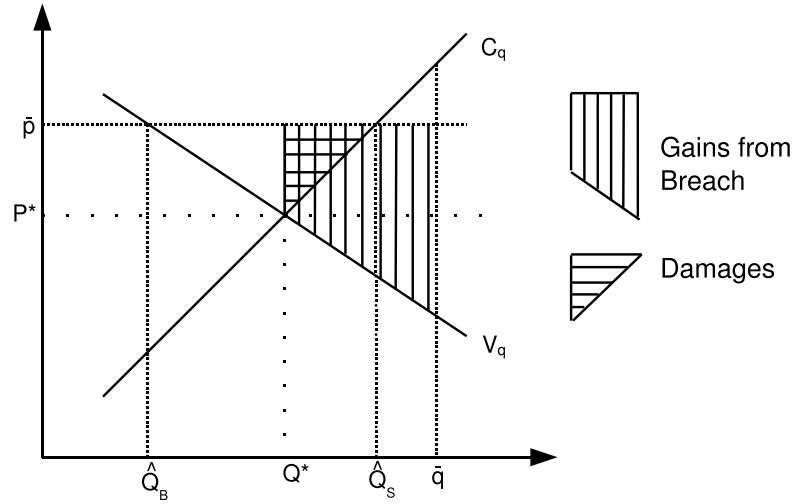


Figure 2: This figure shows that at Q^* , the buyer's marginal damage payments equal her marginal gain from breach.

in damages to the buyer. Similarly, if $q = q_B$, the seller can sue the buyer for the sum

$$\max\{S(\sigma, \theta, q_S) - S(\sigma, \theta, q), 0\}.$$

We will show that with these damage payments the *efficient breach* property of expectation damages continues to hold.⁶ An intuition for this result can be gained from Figure 2 which shows marginal cost and valuation. The two functions intersect at the efficient quantity Q^* , and in the case that is illustrated, this quantity is lower than the contracted quantity \bar{q} , and the “equilibrium price” P^* is lower than the contracted price \bar{p} . No damage is done by breach on the units above \hat{Q}_S , and no damages have to be paid. For breach on all other units, the buyer has to pay the difference between \bar{p} and the “supply curve” to the seller, while she gains the difference between \bar{p} and the “demand curve”. The buyer thus breaches on all units above the quantity Q^* , where these differences become equal.

To formalize this result we define two quantities that are related to supply and demand at the contract price:

$$\hat{Q}_S := \arg \max_{q \leq \bar{q}} S(\sigma, \theta, q) = \min\{\bar{q}, \arg \max_q S(\sigma, \theta, q)\}$$

⁶This would not be true if the court ignored the higher breach quantity and calculated damages with respect to the contracted quantity. Such a rule would aggregate gains and losses on breached units, such that in some contingencies breach leads to a less than efficient quantity.

and

$$\hat{Q}_B := \arg \max_{q \leq \bar{q}} B(\beta, \theta, q) = \min\{\bar{q}, \arg \max_q B(\beta, \theta, q)\},$$

where the second equality is due to concavity of the functions S and B . We also define

$$P^* := C_q(\sigma, \theta, Q^*).$$

Let us assume first that $Q^* \leq \hat{Q}_S$, which is equivalent to the case that $Q^* \leq \bar{q}$ and $P^* \leq \bar{p}$. We will show that equilibrium payoffs are equal to $S(\sigma, \theta, \hat{Q}_S)$ for the seller and $W(\beta, \sigma, \theta, Q^*) - S(\sigma, \theta, \hat{Q}_S)$ for the buyer. If the seller chooses $q_S = \hat{Q}_S$, the damage rule ensures him a payoff of $S(\sigma, \theta, \hat{Q}_S)$ plus a possible gain from renegotiation. This is true regardless of the buyer's breach decision, since even if the seller turned out to be the one to breach, he would pay no damages. On the other hand, if the buyer chooses the anticipatory breach decision $q_B = Q^*$, she gets at least $W(\beta, \sigma, \theta, Q^*) - S(\sigma, \theta, \hat{Q}_S)$. To see that, note that she gets $W(\beta, \sigma, \theta, Q^*) - S(\sigma, \theta, q_S)$ if she pays positive damages and at least $B(\beta, \theta, Q^*)$ if she has to pay no damages.

Therefore, in any Nash equilibrium the buyer gets at least $W(\beta, \sigma, \theta, Q^*) - S(\sigma, \theta, \hat{Q}_S)$, while the seller gets at least $S(\sigma, \theta, \hat{Q}_S)$. Since in sum the payoffs cannot be higher than the maximal joint surplus, these must be the parties' equilibrium payoffs, achieved by $q_B = Q^*$ and $q_S = \hat{Q}_S$.⁷

If $Q^* \leq \hat{Q}_B$, which is equivalent to the case that $Q^* \leq \bar{q}$ and $P^* \geq \bar{p}$, the buyer makes a positive profit on every unit that is efficient to trade. This time the seller breaches to $q_S = Q^*$ and pays damages to the buyer. Their payoffs are $B(\beta, \theta, \hat{Q}_B)$ for the buyer and $W(\beta, \sigma, \theta, Q^*) - B(\beta, \theta, \hat{Q}_B)$ for the seller.

In the case $Q^* > \bar{q}$ the optimal quantity can be reached by renegotiation only. In these contingencies, one of the parties makes a profit on each unit traded under the contract. The analysis here is the same as in ER. In this case, there will be no breach, and renegotiation can take place before or after delivery of \bar{q} . The parties share the additional gain from efficient trade

$$\Delta(\beta, \sigma, \theta, \bar{q}) := W(\beta, \sigma, \theta, Q^*) - W(\beta, \sigma, \theta, \bar{q})$$

⁷It can be shown that these strategies form the unique equilibrium if $\gamma \in (0, 1)$. The key intuition here is that the breaching party receives all gains from the breach decision.

according to bargaining power, leaving the seller with a share of γ , the buyer with a share of $1 - \gamma$ of the renegotiation surplus. Hence, their payoffs are $S(\sigma, \theta, \bar{q}) + \gamma\Delta(\beta, \sigma, \theta, \bar{q})$ for the buyer and $B(\beta, \theta, \bar{q}) + (1 - \gamma)\Delta(\beta, \sigma, \theta, \bar{q})$ for the seller.

III Optimal Contracts

In this section, we turn to the ex ante perspective of the game and analyze the investment choices that are induced by the contract. Taking into account the possible ex post equilibrium payoffs as described in the last section we obtain the following expression for the seller's expected payoff:

$$(1) \quad s(\beta, \sigma) = \int_{[Q^* > \bar{q}]} S(\sigma, \theta, \bar{q}) + \gamma\Delta(\beta, \sigma, \theta, \bar{q}) dF + \int_{[\hat{Q}_B \geq Q^*]} W(\beta, \sigma, \theta, Q^*) - B(\beta, \theta, \hat{Q}_B) dF + \int_{[\hat{Q}_S \geq Q^*]} S(\sigma, \theta, \hat{Q}_S) dF - \sigma.$$

The buyer's expected payoff, denoted by $b(\beta, \sigma)$, is derived analogously to the seller's expected payoff. The payoff functions are easiest to analyze for extreme contracts, for which at efficient investment at least one of the events “renegotiation”, “buyer breaches” and “seller breaches” never occurs. We define

$$q_H := \max_{\theta} Q^*(\beta^*, \sigma^*, \theta), \quad q_L := \min_{\theta} Q^*(\beta^*, \sigma^*, \theta)$$

and

$$p_L := \min_{\theta, \beta, \sigma} P^*(\beta, \sigma, \theta), \quad p_H := \max_{\theta, \beta, \sigma} P^*(\beta, \sigma, \theta).$$

Moreover, let

$$\sigma_S(q, p) := \arg \max_{\sigma} s(\beta^*, \sigma)$$

denote the seller's best response to β^* and

$$\beta_B(q, p) := \arg \max_{\beta} b(\beta, \sigma^*)$$

the buyer's best response to σ^* if the contract specifies a quantity q and a price p .

LEMMA 1. *For all prices p , it holds that $\sigma_S(q_L, p) \leq \sigma^* \leq \sigma_S(q_H, p)$ and $\beta_B(q_L, p) \leq \beta^* \leq \beta_B(q_H, p)$. Moreover, $\sigma_S(q_H, p_L) = \sigma^*$ and $\beta_B(q_H, p_H) = \beta^*$.*

PROOF: See the Appendix.

The intuition is that, given efficient investment, a contracted quantity as low as q_L means that the contract will always be renegotiated to a higher quantity. In the renegotiations, a party receives only a fraction of the surplus generated by the investment, therefore both parties underinvest (hold-up effect). A high contracted quantity q_H means renegotiation never occurs, and being sometimes the non-breaching party induces the parties to prepare for trade of a high quantity. Both parties overinvest, except for the case of a very high or very low price: In this case one party always breaches and invests efficiently, in anticipation of the efficient breach decision. This last result has been studied in detail in Edlin (1996), who uses the term “Cadillac contract” for a contract that is always breached.

This extreme kind of contracts will only in exceptional cases be able to induce efficient investment in equilibrium.⁸ To infer from the behavior of best responses for extreme contracts to the behavior for contracts with an intermediate price and quantity, we need a continuity assumption. In line with the analysis of the one-sided investment case in ER, we make the following assumption.

ASSUMPTION 1. *The best response correspondences $(q, p) \mapsto \sigma_S(q, p)$ and $(q, p) \mapsto \beta_B(q, p)$ have a continuous selection.*

It would be desirable to have a characterization of the cost and valuation functions for which this condition holds. Sufficient conditions can be found, as for example the following assumption.

ASSUMPTION 2. *Let $(\sigma, q) \mapsto C(\sigma, \theta, q)$ be strictly convex and $(\beta, q) \mapsto V(\beta, \theta, q)$ be strictly concave for all $\theta \in \Theta$.*

LEMMA 2. *Assumption 2 implies Assumption 1.*

⁸One of these special cases is $\gamma \in \{0, 1\}$. It has been known since Chung (1991) that specific performance can lead to two-sided efficient investment if one party has all the bargaining power. The same is true for expectation damages if the price is set such that this party always breaches. The quantity can then be used to generate investment incentives for the other party.

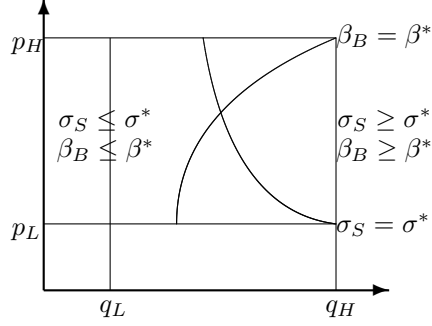


Figure 3: This figure shows the space of price-quantity contracts. For low and high quantities it is indicated whether the best response to efficient investment of the other party is underinvestment, overinvestment or efficient investment. The contracts on the paths have the property that the best response to efficient investment is equal to efficient investment.

PROOF: See the Appendix.

Continuity of best responses indeed ensures existence of an optimal contract, as is illustrated in Figure 3, which provides the intuition for our main result:

PROPOSITION 1. *Given Assumption 1, there exists a non-contingent contract (\bar{q}, \bar{p}) such that the first best investment levels (β^*, σ^*) constitute a Nash equilibrium of the induced game.*

PROOF: See the Appendix.

In order to find the optimal contract given a particular problem we can use the first order conditions of the parties' maximization problem. For example, the derivatives of the expected payoff functions take a particularly simple form if cost and valuation functions belong to the following class of functions:

ASSUMPTION 3.

$$\begin{aligned}
 C(\sigma, \theta, q) &= C_1(\sigma)q + C_2(\theta, q) + C_3(\sigma, \theta) \\
 V(\beta, \theta, q) &= V_1(\beta)q + V_2(\theta, q) + V_3(\beta, \theta).
 \end{aligned}$$

ER show that with this functional form, the incentives for the two parties can be aligned with a single quantity if the breach remedy is specific performance.

COROLLARY 1. *If Assumption 3 holds, and (β^*, σ^*) is an interior solution, the first*

best contract (\bar{q}, \bar{p}) has to fulfill the conditions

$$(2) \quad \int_{[\hat{Q}_S \geq Q^*]} (\hat{Q}_S - Q^*) dF = (1 - \gamma) \int_{[Q^* > \bar{q}]} (Q^* - \bar{q}) dF$$

$$(3) \quad \int_{[\hat{Q}_B \geq Q^*]} (\hat{Q}_B - Q^*) dF = \gamma \int_{[Q^* > \bar{q}]} (Q^* - \bar{q}) dF,$$

where all quantities are evaluated at $\sigma = \sigma^*$ and $\beta = \beta^*$.

PROOF: See the Appendix.

As an example, consider the case that cost and valuation are correlated in a way that P^* is independent of θ . Balancing incentives in such a case is potentially hard to do, because the investment decisions completely determine the identity of the breaching party. From the necessary conditions (2) and (3) it follows that if $\gamma \in (0, 1)$, the only candidate for a first best contract is $\bar{q} = q_H$ and $\bar{p} = P^*(\beta^*, \sigma^*, \theta)$. If the payoff functions are sufficiently concave, the parties make a larger profit with efficient investment than by preparing for a higher quantity, because overinvestment leads to renegotiation in some contingencies. To make sure that this is indeed the case, we have to appeal to Assumption 2. Two explicit examples, exploring the importance of this assumption, can be found in Appendix B.

IV Price adjustment clauses

In this section, we explore what kind of contracts the trading partners can write if Assumption 1 and Proposition 1 do not hold. One of these cases is identified by ER, who show that for $\gamma \in (0, 1)$ and the cost function $C(\sigma, \theta, q) = C_1(\sigma)q$ there exists no contract (\bar{q}, \bar{p}) that can achieve the first best if the valuation function has positive variance. With this kind of cost function the seller faces a choice between two conflicting roles: Either he invests low and breaches the contract, or he invests high and seeks damages if necessary. Whenever such a conflict leads to a discontinuity in one party's best response, the parties may have to write more complicated contracts in order to attain the first best. We will show that if the parties can stipulate a stochastic price, they can obtain first best outcomes for a larger class of payoff functions, including linear ones.

Using a lottery between a very low and a very high price instead of an intermediate price \bar{p} is a more direct way to achieve breach of both parties. Let the contract specify a lottery between p_L with probability λ and p_H with probability $1 - \lambda$. Equivalently, the parties can choose an event that occurs with probability λ independently of cost and valuation functions, and condition the low price p_L on the event and p_H on its complement. This resembles so-called *price escalator clauses* or *price adjustment clauses*, which parties can use to share the risk of breach.⁹

PROPOSITION 2. *Assume that the best responses are continuous in q and λ . Then there is a quantity $\bar{q} \in [q_L, q_H]$ and a $\lambda \in [0, 1]$, such that a contract specifying \bar{q} and a lottery over p_L with probability λ and p_H with probability $1 - \lambda$ induces the first best.*

PROOF: See the Appendix.

Since this result does not require continuity of best responses in price, it holds for a larger class of payoff functions than Proposition 1.¹⁰ The optimal contract illustrates how the performance of expectation damages depends on who will breach the contract. This is especially true for the case of the payoff functions defined in Assumption 3, for which the contract takes a very intuitive form.

PROPOSITION 3. *When Assumption 3 holds, a contract for $\bar{q} = \int Q^*(\beta^*, \sigma^*, \theta) dF$ at a price p_L with probability γ (the seller's bargaining power) and p_H with probability $1 - \gamma$ induces the first best.*

PROOF: See the Appendix.

As ER show, the same contracted quantity leads to efficient investment with specific performance if Assumption 3 holds. One could ask whether stochastic prices are also able to improve the performance of this breach remedy. The answer depends on the bargaining game. While ER keep the bargaining process very general in the one-sided investment analysis, for two-sided investment they also assume the Nash bargaining solution with a constant sharing rule. When the price is so high or so low that it is

⁹Usually, one would think of price adjustment clauses as insurance against events that are correlated with either cost or valuation. Such a clause can also help to balance incentives, but the point is much simpler to make for the independent case.

¹⁰A sufficient condition corresponding to Assumption 2 is that $W(\beta, \sigma, \theta, q)$ is strictly concave in (σ, q) and (β, q) .

always in one party's interest to sue for performance, the seller's expected profit as derived in ER is:

$$\int S(\sigma, \theta, \bar{q}) + \gamma \Delta(\beta, \sigma, \theta, \bar{q}) dF - \sigma.$$

The derivative of this expression with respect to σ does not depend on the price. Hence, investment incentives can only be generated through the contracted quantity, and there is no analog to Proposition 2.

V Conclusion

We have shown that in the framework of expectation damages with bilateral investment in Edlin and Reichelstein (1996), the first best can be restored if both parties face the risk of breaching. With divisible contracts, this can always be achieved with a lottery between a high and a low per-unit price, or with a fixed price if the payoff functions are sufficiently concave. In both cases, each party's probability of breaching varies with price, such that price and quantity are sufficient to fine-tune both sides' incentives to invest. Consequently, also in this framework the trading parties can write non-contingent contracts and obtain efficient outcomes, relying on renegotiation and the standard breach remedy of expectation damages.

Nevertheless, there is a general truth behind ER's inefficiency example: the expectation damage rule treats the breaching party and the party suffering from breach asymmetrically. The only contract that overcomes the hold-up problem of the breaching party specifies such a high quantity that the non-breaching party invests too much. This intuition is likely to carry over to more general settings, as long as only quantity has an effect on investment. The contribution of the current paper is to recognize that investment incentives need not be generated by quantity alone, price matters as well. It remains to be explored how much price and quantity can achieve if contracts are not divisible, or for other legal rules and contractual environments.

Appendix

A Proofs

Proof of Lemma 1. The steps of the proof are exercised in detail only for the seller's payoff function, the result for the buyer can then be derived in a similar way. In a first step, we calculate the derivative of the seller's expected profit. For this, note that as a direct application of the envelope theorem (for constrained maximization) we get for all $\theta \in \Theta$

$$(A1) \quad \frac{\partial}{\partial \sigma} W(\beta, \sigma, \theta, Q^*(\beta, \sigma, \theta)) = -C_\sigma(\sigma, \theta, Q^*(\beta, \sigma, \theta)),$$

and

$$(A2) \quad \frac{\partial}{\partial \sigma} S(\sigma, \theta, \hat{Q}_S(\sigma, \theta)) = -C_\sigma(\sigma, \theta, \hat{Q}_S(\sigma, \theta)).$$

Next, to calculate the derivative $\frac{\partial}{\partial \sigma} s(\beta, \sigma)$, note that for each θ the integrand in s is the piecewise defined function

$$\sigma \mapsto \begin{cases} (1 - \gamma)S(\sigma, \theta, \bar{q}) + \gamma W(\beta, \sigma, \theta, Q^*) - \gamma B(\beta, \theta, \bar{q}) & \text{if } Q^* > \bar{q} \\ S(\sigma, \theta, \hat{Q}_S) & \text{if } Q^* \leq \hat{Q}_S \\ W(\beta, \sigma, \theta, Q^*) - B(\beta, \theta, \hat{Q}_B) & \text{if } Q^* \leq \hat{Q}_B \end{cases}$$

It turns out that the piecewise defined derivative of this function is continuous, i.e. the pieces of this function are joined smoothly. We assume integrability of C_σ , so that we can interchange integration and differentiation, and get:

$$(A3) \quad \begin{aligned} \frac{\partial}{\partial \sigma} s(\beta, \sigma) &= - \int_{[Q^* > \bar{q}]} ((1 - \gamma)C_\sigma(\sigma, \theta, \bar{q}) + \gamma C_\sigma(\sigma, \theta, Q^*)) dF - 1 \\ &\quad - \int_{[\hat{Q}_S \geq Q^*]} C_\sigma(\sigma, \theta, \hat{Q}_S) dF - \int_{[\hat{Q}_B \geq Q^*]} C_\sigma(\sigma, \theta, Q^*) dF \\ &= -(1 - \gamma) \int_{[Q^* > \bar{q}]} \Delta_\sigma(\beta, \sigma, \theta, \bar{q}) dF - \int_{[\hat{Q}_S \geq Q^*]} \Delta_\sigma(\beta, \sigma, \theta, \hat{Q}_S) dF \\ &\quad - \int C_\sigma(\sigma, \theta, Q^*) dF - 1 \end{aligned}$$

Because we already know that for $\beta = \beta^*$ the expected joint surplus is uniquely maximized at σ^* , we will study the function

$$\tilde{s}(\sigma) := s(\beta^*, \sigma) - \left(\int W(\beta^*, \sigma, \theta, Q^*(\beta^*, \sigma, \theta)) dF - \sigma \right).$$

which has derivative

$$(A4) \quad \tilde{s}'(\sigma) = -(1 - \gamma) \int_{[Q^* > \bar{q}]} \Delta_\sigma(\beta^*, \sigma, \theta, \bar{q}) dF - \int_{[\hat{Q}_S \geq Q^*]} \Delta_\sigma(\beta^*, \sigma, \theta, \hat{Q}_S) dF.$$

By exploiting $C_{\sigma q} \leq 0$, it is straightforward to see that $\Delta_\sigma(\beta^*, \sigma, \theta, q)$ is weakly decreasing in q , and that the first term in $\tilde{s}'(\sigma)$ is negative and the second is positive (if they do not vanish).

Now, in order to prove the lemma, consider first $\bar{q} = q_L$. In this case, for all $\sigma \geq \sigma^*$

$$\tilde{s}'(\sigma) = -(1 - \gamma) \int \Delta_\sigma(\beta^*, \sigma, \theta, q_L) dF \leq 0,$$

i.e. \tilde{s} is a monotonically decreasing function in this range. Therefore, all $\sigma > \sigma^*$ lead to a lower payoff than σ^* , hence $\sigma_S(q_L, p) \leq \sigma^*$. For a contract over q_H , since Q^* is nondecreasing in σ , the first term in \tilde{s}' vanishes for $\sigma \leq \sigma^*$, i.e. \tilde{s} is a weakly increasing function. Therefore, at q_H all $\sigma < \sigma^*$ are dominated by σ^* , and $\sigma_S(q_H, p) \geq \sigma^*$. Finally, consider q_H and a low price p_L . By definition of p_L it holds that $\hat{Q}_S(\sigma, \theta) \leq Q^*(\beta^*, \sigma^*, \theta)$ for all $\theta \in \Theta$ and σ . Therefore, the function \tilde{s} is weakly decreasing for $\sigma \geq \sigma^*$, hence $\sigma_S(q_H, p_L) = \sigma^*$. For the buyer, the corresponding claims follow from the assumption that $V_{\beta q} \geq 0$.

Proof of Lemma 2. Again, we prove the claim only for the seller. First, let us state the required conditions more precisely. For each θ , whenever $Q^*(\beta^*, \sigma, \theta) \leq \hat{Q}_S(\sigma, \theta)$ we need that $S(\sigma, \theta, \hat{Q}_S)$ is concave in σ , i.e.

$$C_{\sigma\sigma}(\sigma, \theta, \hat{Q}_S) - \frac{C_{q\sigma}(\sigma, \theta, \hat{Q}_S)^2}{C_{qq}(\sigma, \theta, \hat{Q}_S)} \geq 0.$$

This condition follows from Assumption 2, because the determinant of the Hessian matrix of $(\sigma, q) \mapsto C(\sigma, \theta, q)$ is positive at $q = \hat{Q}_S$. One can see here why a linear cost function might be a problem: as C_{qq} becomes small, this condition becomes harder to fulfill. Furthermore we need the condition that $W(\beta^*, \sigma, \theta, Q^*)$ is concave, meaning that

$$C_{\sigma\sigma}(\sigma, \theta, Q^*) + \frac{C_{\sigma q}(\sigma, \theta, Q^*)^2}{W_{qq}(\beta^*, \sigma, \theta, Q^*)} \geq 0,$$

which also follows from Assumption 2. Last, we need the condition $C_{\sigma\sigma}(\sigma, \theta, \bar{q}) \geq 0$, which is also implied by convexity of C in both variables.

Since s is continuous in q , p and σ (which is straightforward to check), according to Berge's theorem, the argmax correspondence $\sigma_S(q, p)$ is upper hemicontinuous. Since upper hemicontinuity coincides with continuity if the correspondences are functions, for Assumption 1 to hold it suffices that the function $\sigma \mapsto s(\sigma, \beta^*)$ has a unique maximizer for all q and p . We therefore show that s is strictly concave, given that Assumption 2 holds. For this we need that the derivative (see equation A3) is decreasing in σ . It suffices to show that the continuous integrand is piecewise decreasing, which can be done by calculating the piecewise derivatives and using the above conditions.

Proof of Proposition 1. Since because of Assumption 1 the best responses have a continuous selection, we may assume that $\sigma_S(q, p)$ and $\beta_B(q, p)$ are continuous functions. For all $p \in [p_L, p_H]$, define

$$\bar{q}_S(p) := \{q \in [0, q_H] : \sigma_S(q, p) = \sigma^*\}$$

and

$$\bar{q}_B(p) := \{q \in [0, q_H] : \beta_B(q, p) = \beta^*\}.$$

From Lemma 1 and the intermediate value theorem it follows that these sets are nonempty for each p . Note from equation (A3) that $C_{\sigma q} \leq 0$ implies that s' is weakly increasing in q . Together with Assumption 1 this implies that \bar{q}_S and \bar{q}_B are compact and convex valued upperhemicontinuous correspondences. Consider first the case that they are functions.¹¹ Lemma 1 tells us that $\bar{q}_S(p_L) = q_H \geq \bar{q}_B(p_L)$ and $\bar{q}_B(p_H) = q_H \geq \bar{q}_S(p_H)$. Applying the intermediate value theorem again yields existence of a \bar{p} such that $q_S(\bar{p}) = q_B(\bar{p}) =: \bar{q}$. This contract (\bar{q}, \bar{p}) thus leads to β^* as a best response to σ^* and σ^* as a best response to β^* . If the correspondences $\bar{q}_S(p)$ and $\bar{q}_B(p)$ are not single-valued, their graphs are still pathwise connected and we can apply the same reasoning as before.

Proof of Corollary 1. The derivative of $\tilde{s}(\sigma)$, as calculated in the proof of Lemma 1 (equation A4), evaluated at σ^* , must vanish at the optimal contract. The corollary follows since for the kind of functions defined in Assumption 3 it holds that

$$(A5) \quad \Delta_\sigma(q) = -C'_1(\sigma)(Q^* - q) \text{ and } \Delta_\beta(q) = V'_1(\beta)(Q^* - q).$$

¹¹This holds for example if the inequalities $C_{\sigma q} \leq 0$ and $V_{\beta q} \geq 0$ hold strictly everywhere, Q^* is continuous in θ , $\gamma \in (0, 1)$, and σ^* and β^* are interior solutions.

Proof of Proposition 2. When the price is p_L , the buyer makes a profit on each unit, i.e. $\hat{Q}_B = \bar{q}$ for all θ . When price is p_H , it holds that $\hat{Q}_S = \bar{q}$ for all θ . Expected payoff is analogous to the case with an intermediate price and can be rearranged to look as follows (again only for the seller):

$$(A6) \quad s(\sigma, \beta) = \int W(\beta, \sigma, \theta, Q^*)dF - \sigma - \int B(\beta, \theta, \bar{q})dF \\ - (1 - \gamma) \int_{[Q^* > \bar{q}]} \Delta(\beta, \sigma, \theta, \bar{q})dF - (1 - \lambda) \int_{[Q^* \leq \bar{q}]} \Delta(\beta, \sigma, \theta, \bar{q})dF$$

with $p = \lambda p_L + (1 - \lambda)p_H$. The claim can now be proved following the same steps as in the proof of Proposition 1, the role of the price being played by λ .

Proof of Proposition 3. We prove this result independently of previous results in this paper, because it holds without Assumption 1, and would hold also for arbitrary investment decisions and linear functions. For $\lambda = \gamma$, the seller's expected payoff functions as stated in equation (A6) equals

$$s(\beta, \sigma) = \bar{p}\bar{q} + (1 - \gamma) \left(\int -C(\sigma, \theta, \bar{q})dF - \sigma \right) \\ + \gamma \left(\int W(\beta, \sigma, \theta, Q^*)dF - \sigma \right) - \gamma \int V(\beta, \theta, \bar{q})dF$$

with $\bar{p} = \gamma p_L + (1 - \gamma)p_H$. In this case, the payoff functions are identical to the ones that result from specific performance in ER. Next, consider the defining equation of σ^* , which is that for all other σ

$$(A7) \quad \int W(\beta^*, \sigma^*, \theta, Q^*(\sigma^*, \beta^*, \theta))dF - \sigma^* \geq \int W(\beta^*, \sigma, \theta, Q^*(\sigma, \beta^*, \theta))dF - \sigma.$$

Furthermore, from the definition of Q^* we know that

$$(A8) \quad W(\beta^*, \sigma, \theta, Q^*(\sigma, \beta^*, \theta)) \geq W(\beta^*, \sigma, \theta, Q^*(\sigma^*, \beta^*, \theta)) \quad \text{for all } \sigma, \theta.$$

From these two equations, it follows that

$$(A9) \quad \sigma^* \in \arg \max_{\sigma} \int -C(\sigma, \theta, Q^*(\sigma^*, \beta^*, \theta))dF - \sigma$$

Since we assumed the special payoff functions defined in Assumption 3 it follows that with $\bar{q} = \int Q^*(\beta^*, \sigma^*, \theta)dF$

$$(A10) \quad \sigma^* \in \arg \max_{\sigma} \int -C(\sigma, \theta, \bar{q})dF - \sigma.$$

Hence, when $\beta = \beta^*$, all terms in the seller's payoff function are maximized at σ^* , and it is straightforward to show that the same holds symmetrically for the buyer.

B Examples

In this appendix we compute two examples. Let $\gamma = \frac{1}{2}$ and

$$\begin{aligned} C(\sigma, \theta, q) &= \frac{1}{2\sigma}q + c\frac{q^2}{2\theta}, \\ V(\beta, \theta, q) &= \left(\frac{4}{3}c + \frac{7}{3} - \frac{1}{2\beta}\right)q - \frac{q^2}{2\theta}. \end{aligned}$$

In the specification of the model the investment cost was normalized to be linear, but it can as well be any convex function. For this example, we take $\sigma^2/2$ to be the cost of investment σ . The uncertainty parameter θ is assumed to be uniformly distributed on the interval $[1, 2]$. The efficient quantity is

$$Q^*(\beta, \sigma, \theta) = \left(\frac{4}{3}c + \frac{7}{3} - \frac{1}{2\beta} - \frac{1}{2\sigma}\right) \frac{\theta}{1+c}.$$

Calculations reveal that $\sigma^* = \beta^* = 1$. Since the equilibrium price

$$P^*(\beta, \sigma, \theta) = \frac{c}{1+c} \left(\frac{4}{3}c + \frac{7}{3} - \frac{1}{2\beta} - \frac{1}{2\sigma}\right) + \frac{1}{2\sigma}.$$

does not depend on θ , the candidate for an efficient contract is $\bar{q} = \frac{8}{3}$ and $\bar{p} = \frac{4}{3}c + \frac{1}{2}$. The sufficient condition in Assumption 2 is fulfilled if $c > 3/16$.¹² For very low c , this contract leads to a saddle point instead of a maximum of the seller's payoff function at σ^* . This example is one in which, once investment is sunk, only one party breaches the contract. Nevertheless, since the overinvesting party faces hold-up and non-breach contingencies, the equilibrium is efficient if the payoff functions are sufficiently concave. As the cost function approaches a linear and deterministic one, the first best ceases to be attainable.

This does not necessarily hold if there is a random element in the linear term, such that always both parties face the risk of breaching. Consider the following variant of the preceding example:

$$\begin{aligned} C(\sigma, \theta, q) &= \left(\frac{1}{2\sigma} + \theta_1\right)q \\ V(\beta, \theta, q) &= \left(\frac{7}{3} - \frac{1}{2\beta} + \theta_1\right)q - \frac{q^2}{2\theta_2}. \end{aligned}$$

That is, we set $c = 0$ and to the contingency we add a new component which makes marginal cost volatile. The part θ_1 is a market shock which affects both the buyer's

¹²This bound is even lower if the convex investment cost is taken into account.

valuation and the seller's cost (which could be opportunity cost). The part θ_2 only affects the buyer, and is again uniformly distributed on $[1, 2]$. With regard to θ_1 , we assume that it is uniformly distributed on $[0, 1]$. The efficient quantity is now

$$Q^*(\beta, \sigma, \theta) = \left(\frac{7}{3} - \frac{1}{2\beta} - \frac{1}{2\sigma} \right) \theta_2.$$

Looking for the optimal contract, we get the following equation from the seller's maximization problem:

$$\int_{[Q^* \leq \bar{q}]} \left(\bar{p} - \frac{1}{2} \right) (\bar{q} - Q^*) d\theta_2 = \int_{[Q^* > \bar{q}]} \frac{1}{2} (Q^* - \bar{q}) d\theta_2$$

One obvious solution is $\bar{q} = 2$ and $\bar{p} = 1$. All solutions are characterized by

$$\bar{p}_S(q) = \frac{1}{2} + \frac{(\frac{3}{4}\bar{q} - 2)^2}{2(\frac{3}{4}\bar{q} - 1)^2}$$

for all $q_H = \frac{8}{3} > \bar{q} > q_L = \frac{4}{3}$. The buyer's payoff fulfills all assumptions. Unfortunately, the condition that characterizes the optimal contract for the buyer becomes quite complex. As numerical solutions of the two equations we get $\bar{q} = 2.039$ and $\bar{p} = 0.8956$.

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