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Prizes and Lemons: Procurement of Innovation under Imperfect Commitment

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Abstract

The literature on R&D contests implicitly assumes that contestants submit their innovation regardless of its value. This ignores a potential adverse selection problem. The present paper analyzes the procurement of innovations when the procurer cannot commit to never bargain with innovators who bypass the contest. We compare fixed-prize tournaments with and without entry fees, and optimal scoring auctions with and without minimum score requirement. Our main result is that the optimal fixed-prize tournament is more profitable than the optimal auction since preventing bypass is more costly in the optimal auction.

JEL classification: C70, D44, D89, L12, O32
Keywords: innovation, contests, tournaments, auctions, bargaining, adverse selection

1 Introduction

Contests are a widely used and well documented method to procure innovations. In the past, fixed-prize tournaments were employed to procure major bottleneck innovations. For example, in 1795 Napoleon Bonaparte offered a prize of FF12.000 for a method of food preservation that was in high need to serve his military excursions across Europe. The winner of that tournament was Nicolas Appert, who invented the method of food canning, which is still widely used today.[1] In 1714 the British Parliament offered

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[1] In-container sterilization is known as "appertisation" in francophone regions, in memory of Nicolas Appert.

a prize of £20.000 for a method of determining longitude at sea, following a series of maritime disasters. That tournament was won by John Harrison who invented the first mechanical chronometer to provide reliable time measurement service at sea.[2] More recent examples range from the procurement of weapon systems, energy efficient refrigerators,[3] and pharmaceutical innovations, to the awarding of academic grants and fellowships, to name just a few.

Contests in the form of a scoring auction are also widely used in the procurement of goods and services that involve innovative activity. For example, when the World Bank procures the design or construction of a power plant or a national health care system, potential contractors compete not only with price, but also with technical proposals that lay out innovative solutions to problems of technical or institution design (see The World Bank, 2004a,b).

Elements of a research contest are also present in architecture competitions where designers and contractors are typically asked to present pilot proposals that are rewarded with fixed cash prizes and that play a crucial role in the final selection.

Inspired by these and other examples, R&D contests were analyzed extensively in the recent theoretical literature. In his seminal paper Taylor (1995) introduced a model of innovation activity that has been widely used in the subsequent literature. In that model, innovations are measured by their value added (the increment in wealth that their application would induce), and innovation activities are viewed as costly draws from a given i.i.d. probability distribution of innovations, similar to the independent private-values model in auction theory.

Fullerton and McAfee (1999) extended this analysis by introducing asymmetric innovators whose cost of innovation differ, and addressed the important issue of how to induce the best selection of contestants through auctioning participation rights. And Fullerton, Linster, McKee, and Slate (2002), as well as Schöttner (2008) compared the profitability of procuring innovations by fixed-prize tournaments and auctions, while Che and Gale (2003) analyzed private-value contests from an optimal mechanism design perspective, assuming a deterministic innovation technology and excluding entry fees.

The literature on R&D contests implicitly assumes that contestants submit their best innovation regardless of its value. This assumption ignores that innovators may withhold innovations that are worth considerably more than the prize, so that only the lemons, i.e., the inferior innovations are submitted. If there is only one potential user of the innovation, i.e., if the procurer is a monopsonist, the procurer can in principle prevent this adverse selection problem by committing himself to never bargain with innovators who bypass the contest. However, such a commitment is difficult to achieve.

There are many cases where innovations were inspired by a contest, but innovators ultimately decided to bypass the contest when they felt that their innovation had a substantially higher commercial value than the prize offered by the contest, and then successfully negotiated more profitable license agreements after bypassing the con-

[2]However, the prize committee was dominated by astronomers who pursued their own agenda, sabotaged the work of the clockmaker, and tried to withhold the prize from him. For a vivid account of these incidents see Sobel (1996).

[3]In 1991 a $10 million prize was sponsored. Whirlpool won the tournament but never collected the prize because it failed to sell the 250.000 units required within the first five years after the tournament (see Langreth, 1994).

test.[4]

For example, the inventor John Wesley Hyatt was encouraged to develop a new substance after he saw an advertisement by Phelan & Collander, offering $10,000 to the person who invented a usable substitute for ivory in billiard balls. Hyatt eventually succeeded by inventing celluloid, which seemed to be a perfect and cheap substitute for ivory in billiard balls, but finally decided to patent his innovation in 1869 instead of submitting it to the tournament and collecting the prize.[5] This bypass of the fixed-prize tournament allowed him to more profitably license his innovation not only for use in billiard balls, but also in a variety of other products, ranging from film and ping-pong balls to dental plates.[6]

Motivated by this and other examples, the present paper analyzes the procurement of innovations when the procurer is unable to commit to never bargain with innovators and innovators consider to bypass a contest in the event that they draw a high value innovation, when the value of innovation is not verifiable to third parties, and when the benefits of innovation accrue exclusively to the procurer. We compare two different methods to procure innovations: fixed-prize tournaments and incentive compatible (scoring) auctions, and determine which of these mechanisms is more profitable for the procurer if both mechanisms are potentially subject to a bypass and subsequent lemons problem.[7]

Our main finding is that this imperfect commitment generally affects the profitability of both mechanisms, but in substantially different ways, and depending on whether one employs a standard fixed-prize tournament or amends it by requiring advance registration and entry fees, just like in the optimal auction.

Altogether, we show that the optimal fixed-prize tournament outperforms the optimal auction. Specifically, we construct a simple fixed-prize tournament that prevents bypass (just like the optimal auction) and matches the profitability of the optimal auction. However, we also identify cases in which the optimal fixed-prize tournament is strictly more profitable than the optimal auction. Interestingly, a standard fixed-prize tournament that does not employ entry fees can be more profitable than the optimal auction that employs entry fees.

The intuition for this surprising result is as follows. When no registration/entry fee is required in a fixed-prize tournament, both innovators have the option to submit their innovation. Therefore, in order to prevent bypass it is sufficient to employ a simple fixed-prize tournament without entry fees and set the smallest prize that assures submission when *both* innovators have the option to submit. Whereas, once registration/entry fees are imposed, preventing bypass requires not only that innovators always exercise an option to submit, but also that they buy the option to submit. Altogether, assuring both advance registration and subsequent submission can be more costly than

[4]See the excellent survey by Cabral, Cozzi, Denicoló, Spagnolo, and Zanza (2006).

[5]As reported in *Wikipedia* (see the entry under "John Wesley Hyatt"), "the English inventor Daniel Spill developed the same product which he patented in England as 'Xylonite', and later pursued Hyatt in a number of costly court cases between 1877 and 1884. The eventual outcome found that the true inventor of celluloid was Alexander Parkes, and that all manufacturing of celluloid could continue, including Hyatt's."

[6]Despite its initial success, the popularity of celluloid billiard balls diminished rapidly after a number of incidents caused by the high flammability of celluloid.

[7]For a survey of alternative methods to procure innovations see Scotchmer (2005).

adopting a simple fixed-prize contest that requires no registration/entry fees.

In addition to proving the optimality of the optimal fixed-prize tournament, we also provide sufficient conditions for ranking the optimal standard fixed-prize tournament (without entry fees) with the optimal auction. And we also show that the auction cannot be improved by adding a minimum score requirement in addition to entry fees.

The plan of the paper is as follows: sections 2 and 3 introduce the model and show that the optimal auction and fixed-prize tournament are revenue equivalent under perfect commitment. In section 4 we compare the profitability of the two mechanisms under imperfect commitment, when innovators may bypass the mechanism. In section 5 we show that the appropriately generalized fixed-prize tournament is more profitable than the optimal auction. And in section 6 we also show that the auction cannot be improved by applying a minimum score requirement in addition to requestding payment of entry fees. The paper closes with a discussion in section 7.

2 The model

A risk neutral procurer wishes to buy an innovation from one of two short-listed innovators, using either a fixed-prize tournament or a (scoring) auction. The procurer can commit to employ one of these mechanisms, but is unable to commit to never trade with an innovator who bypassed it.

Innovation technology: Innovation is modeled as one i.i.d. random draw x from the c.d.f. $G : [\underline{x}, \bar{x}] \to [0, 1]$ with positive density g everywhere, at cost $c > 0$. The random variable X measures the increment in wealth that result if the procurer adopts it. The innovation has no value for anyone other than the procurer. G is such that $H(x) := \int_0^x G(y)dy$ is log-concave for all x.[8] For convenience, the support of G is normalized to $[0, 1]$. Order statistics of the sample of two random draws are denoted by $X_{(1)} \geq X_{(2)}$ (as a rule, random variables are denoted by capital and realizations by lowercase letters).

Information: Innovations are innovators' private information. That information becomes known to the procurer only after the innovation has been submitted either to the contest or to the bargaining table (and at the bargaining may be revealed to another innovator). Innovations are not verifiable to third parties, which restricts the set of feasible auction rules and rules out the use of bilateral contracts.

Contest mechanisms: The procurer adopts either a fixed-prize contest or a (scoring) auction. In the fixed-prize contest the procurer sets a prize p to be paid to the best submitted innovation and possibly an entry fee, f, to be paid by innovators who register for the contest. If an innovator registers, innovates, and submits his innovation, he earns $p - f - c$ if he wins and $-f - c$ if he loses. In the auction, the procurer selects a scoring rule, a pricing rule, and an entry fee f. The scoring rule maps an innovation, x, and a financial proposal, b (the smallest price requested for the innovation), into a score, S, and then selects the highest scoring bid as winner. The pricing rule maps bids into a price that the winner shall pay.

Since innovations are not verifiable by third parties, the only incentive-compatible scoring rule is the non-discriminating rule, $S(x, b) = x - b$, that scores bids by the net

[8]Log-concavity is frequently assumed in information economics (see the survey by Bagnoli and Bergstrom, 2005). The assumed log-concavity of H is obviously weaker than log-concavity of G.

surplus that they promise to deliver to the procurer; and the only incentive-compatible pricing rule is that of the first-score auction that requires the procurer to pay the winner the price he requested, b (see Che and Gale, 2003). This does, however, not rule out, that one may use a non-incentive-compatible auction, like the second-score auction, as a proxy of the first-score auction.

Since innovators may bypass the contest prescribed by the procurer, the contest rules do not fully describe the game played between the procurer and innovators. In addition, we need to consider the bargaining game between innovator and procurer that applies in the event when one or both innovators bypass the contest.

Bargaining in the event of bypass: Neither do innovators know each other nor does the procurer reveal their identity. Therefore, if a bypass has occurred, the subsequent bargaining game is one between two players: the procurer and the innovator who owns the best bypassed innovation, provided that the bypassed innovation is superior to the submitted innovation, if any innovation was submitted.

The outcome of the two-players bargaining game is described by the symmetric Nash bargaining solution in which the gains from trade are shared equally.[9]

Specifically, if one innovator submitted his innovation while the other bypassed, the procurer already owns the submitted innovation. He then bargains only if the innovator who proposed to bargain has the superior innovation. In that event, the procurer's threat point is equal to $X_{(2)}$; therefore, the Nash bargaining equilibrium price is equal to $P = 1/2\left(X_{(1)} - X_{(2)}\right)$.

Whereas if both innovators have bypassed, the procurer bargains only with the owner of the superior innovation but threatens to buy the inferior innovation. Again, assuming a symmetric Nash bargaining solution, the procurer's threat point is $1/2 X_{(2)}$. Therefore, the procurer buys only the superior innovation at a price equal to $P = 1/2\left(X_{(1)} - 1/2 X_{(2)}\right) = 1/2 X_{(1)} - 1/4 X_{(2)}$.

Timeline: At date 0, the procurer announces the contest rule. At date 1, each innovator simultaneously registers for the contest and pays an entry fee or does not register (bypass may already occur at this point), if registration is required. At date 2, innovators simultaneously draw an innovation (or do not innovate), not knowing whether their rival has registered for the contest. At date 3, innovators privately observe their innovation and either submit it to the contest or bypass it. If an auction is adopted, submission requires choice of a financial bid. If at least one innovation was submitted, at date 4 the mechanism game is executed, the winner/loser is selected and the winner is paid. If an innovator has bypassed the mechanism, at date 5, this innovator proposes bargaining, and the procurer bargains with him if and only if he has drawn the superior innovation.

Finally, we assume that the procurer is subject to a sufficiently high loss in the event that no procurement takes place. This is the case when the support of the random innovation is sufficiently bounded away from zero, so that $\underline{x} > 0$.[10] Normalizing that support of X to $[0, 1]$ then means that a zero profit from no procurement is transformed into a negative profit of no procurement, equal to $-\underline{x}$.[11]

[9] This is supported by the solutions of standard bargaining games such as the Rubinstein (1982) model of sequential offer-counteroffer bargaining (provided players are equally impatient, and the time interval between offers is small) and the axiomatic bargaining theory by Nash (1950).

[10] This is typically the case when the procurement concerns some bottleneck innovation.

[11] This fact is often overlooked. For example, it is often claimed that the optimal standard single-unit

We also assume that the cost of drawing an innovation is sufficiently low so that procuring innovations does not require subsidizing innovators. This is assured by assuming that $c \leq 1/2 E\left[X_{(1)} - X_{(2)}\right]$.

3 Optimal mechanisms under perfect commitment

As a benchmark, we first assume that the procurer can commit to never negotiate with innovators who do not participate in the mechanism, and does so. We consider a standard fixed-prize tournament where the procurer offers to pay a fixed prize p for the best submitted innovation, and a scoring auction described by a scoring rule, a pricing rule, and entry fee f.

Optimal fixed-prize tournament The tournament game has a unique equilibrium outcome: the procurer sets the smallest prize that assures that both innovators innovate and submit their innovation (see Taylor, 1995). The equilibrium prize, p^*, and payoffs of the procurer, π_p^*, and innovators, π_i^*, are

$$p^* = 2c \tag{1}$$
$$\pi_p^* = E[X_{(1)}] - p^* = E[X_{(1)}] - 2c \tag{2}$$
$$\pi_i^* = \frac{1}{2} p^* - c = 0. \tag{3}$$

Note, if the procurer would set a smaller prize, $p \in [c, 2c)$, innovators would play mixed strategies. Given our assumption concerning the cost of "no procurement", the procurer will never set a price that induces mixed strategies since this would involve that procurement fails with positive probability.

Also, note that the tournament is optimal for the procurer and thus cannot be improved by another mechanism, because it is constrained efficient, (maximizes the gain from trade conditional on having two innovation draws) and gives the entire surplus to the procurer.

Scoring Auction In a scoring auction, bids are two-dimensional, (x_i, b_i); where x_i is the value of the innovation, and b_i the minimum price requested for this innovation. Bids are scored by a non-discriminatory scoring rule $S_i(x_i, b_i) := x_i - b_i$ that ranks innovations by their value-added for the procurer, and, for analytical simplicity, assume a second-score auction format. There, the highest score wins, and the winner receives the price that makes his score match the second highest score.

At the time of bidding, the cost of innovation, c, is already sunk. Therefore, in the second-score auction, it is an equilibrium in dominant strategies to bid a score equal to the value of the innovation, x_i.[12] The associated equilibrium price, P, then solves the equation $X_{(1)} - P = S_{(2)} = X_{(2)}$, which gives

$$P = X_{(1)} - X_{(2)}. \tag{4}$$

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auction involves a high minimum bid which gives rise to a positive probability of no sale. However, this is not true if the support of buyers' valuations is sufficiently bounded away from the seller's own valuation.

[12] To prove this, not that bidding a higher score can only change something if $x_1 < x_2$, in which case the price becomes negative. Similarly, one can show that it never pays to bid a lower score.

We stress that the second-score auction is not incentive compatible if the value of innovations is not verifiable to third parties. However, in the present framework, first- and second-score auctions are payoff equivalent since the value of innovation is the result of a pure random draw and at the time of bidding bidders only choose their financial bid b_i. Therefore, we can view the second-score auction as a proxy for the first-score auction, which allows us to highlight the performance of auctions without getting entangled in unnecessarily complex bidding strategies.[13]

In the optimal second-score auction, the procurer levies an entry fee, f, which bidders have to pay in advance to register for the auction, before they draw their innovation.

The equilibrium expected price, entry fee, and payoffs are

$$p := E[P] = E\left[X_{(1)} - X_{(2)}\right] \quad \text{(expected price)} \tag{5}$$

$$f = \frac{1}{2}E[X_{(1)} - X_{(2)}] - c \quad \text{(entry fee)} \tag{6}$$

$$\pi_p = E[X_{(1)}] - p + 2f = E[X_{(1)}] - 2c = \pi_p^* \tag{7}$$

$$\pi_i = \frac{1}{2}E[X_{(1)} - X_{(2)}] - c - f = 0. \tag{8}$$

Obviously, the optimal auction and the optimal tournament are payoff equivalent.

Note, the registration or entry fee is collected before innovators draw their innovation. Only registered innovators can participate in the auction.[14]

Comparing the procurer's expected profits, we conclude,

Proposition 1 (revenue equivalence). *Under perfect commitment the two mechanisms are equally profitable for the procurer and achieve full surplus extraction.*

Whereas the auction achieves full surplus extraction only by charging entry fees, the optimal fixed-prize tournament can do without entry fees.

4 Auction vs. fixed-prize tournament under imperfect commitment

Now assume the procurer cannot commit to never trade with an innovator who did not participate in the contest. In that case, innovators may bypass the contest and engage in bargaining after the mechanism game has been played. We analyze the optimal auction and the optimal standard fixed-prize tournament (without entry fees) and show, among other results, that the optimal fixed-prize tournament may outperform the optimal auction, even if the tournament does not employ entry fees. In the later section 5 we also allow the procurer to charge an entry fee in fixed-prize tournament.

4.1 Optimal fixed-prize tournament (without entry fees)

We now characterize the optimal standard fixed-prize tournament without entry fees, assuming the procurer cannot commit to never trade with an innovator who did not

[13] Note, this revenue-equivalence of first- and second-score auctions does not apply in models of scoring auctions in which bidders choose the value of their innovation by choosing effort *and* price (as in the models by Che and Gale, 2003, Schöttner, 2008).

[14] In procurement this corresponds to the commonly employed short-listing procedure.

participate in the mechanism. We show that for each given prize p the game played between innovators has a unique symmetric equilibrium in which innovators play a cutoff strategy, $\gamma_i \in [0,1]$, i.e., *submit* their innovation if $x_i \leq \gamma_i$, and *bypass* if $x_i > \gamma_i$. If p is sufficiently high, innovators play the equilibrium strategies (submit, submit), i.e., $\gamma = 1$, whereas if p is sufficiently low, they play the equilibrium strategy $0 < \gamma < 1$.

Lemma 1. $\gamma = 1$ *is innovators' equilibrium strategy if and only if* $p \geq \bar{p}$*, where*

$$\bar{p} := \frac{1}{2}\int_0^1 G(x)dx = \frac{1}{2}(1 - E[X]). \tag{9}$$

Proof. *(submit, submit)*, i.e., $\gamma = 1$, is a symmetric equilibrium if and only if

$$\begin{aligned}\pi(\text{submit, submit} \mid x, p) &\geq \pi(\text{bypass, submit} \mid x, p)\\ G(x)p &\geq G(x)E[1/2(x - Y) \mid Y < x]\\ p &\geq \frac{1}{2}\int_0^x \frac{G(y)}{G(x)}dy = \frac{1}{2}\frac{H(x)}{H'(x)}.\end{aligned}$$

Recall that $H(x) := \int_0^x G(y)dy$ is log-concave; hence, $H(x)/H'(x)$ is increasing, and thus,

$$\bar{p} := \inf\left\{p \mid p \geq \frac{1}{2}\frac{H(x)}{H'(x)}, \forall x\right\} = \frac{1}{2}\frac{H(1)}{G(1)} = \frac{1}{2}(1 - E[X]).$$

□

Now suppose $p < \bar{p}$. We show in two lemmas that for each $p < \bar{p}$ the game played between innovators has a unique symmetric equilibrium in which both innovators play the same cutoff strategy $0 < \gamma < 1$.

Lemma 2. *Suppose one innovator, say innovator 2, plays the cutoff strategy* γ_2*. Then, the best response of innovator 1 is also a cutoff strategy* γ_1*.*

This lemma implies that, as we analyze the best response to a cutoff strategy, we can restrict our attention to cutoff strategies. The proof of this lemma is in Appendix A.1.

In order to solve the symmetric equilibrium cutoff strategy γ, consider one player, say player 1, who contemplates the deviating strategy $\gamma_1 \geq \gamma$, while his rival, player 2, plays the equilibrium strategy γ. To compute the payoff function of player 1, $\pi_1(\gamma_1, \gamma)$, take a look at the state space representation of that innovator's payoffs in Figure 1. Using the joint density $g_{12}(x_1, x_2) = g(x_1)g(x_2)$, one can then compute the payoff function by integrating over the relevant subsets of the state space $[0,1] \times [0,1]$:

$$\begin{aligned}\pi_1(\gamma_1, \gamma) = p\Bigg(&\int_0^{\gamma}\int_0^{x_1} dG(x_2)dG(x_1) + \int_{\gamma}^{\gamma_1}\int_0^{\gamma} dG(x_2)dG(x_1)\\ &+ \int_0^{\gamma_1}\int_{\gamma}^{1} dG(x_2)dG(x_1)\Bigg) + \frac{1}{2}\int_{\gamma_1}^{1}\int_0^{\gamma}(x_1 - x_2)\,dG(x_2)dG(x_1)\\ &+ \frac{1}{2}\int_{\gamma_1}^{1}\int_{\gamma}^{x_1}\left(x_1 - \frac{1}{2}x_2\right)dG(x_2)dG(x_1) - c\end{aligned} \tag{10}$$

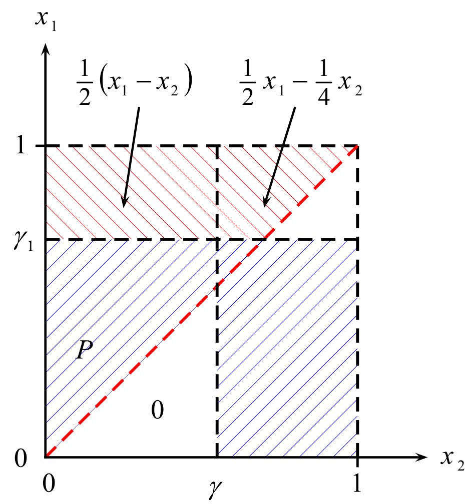


Figure 1: Payoffs of innovator 1 for $\gamma_1 \geq \gamma$ in the state space $[0, 1] \times [0, 1]$

Lemma 3. *Assume $p < \bar{p}$. The game played between innovators has a unique symmetric equilibrium strategy $\gamma \in (0, 1]$, which is implicitly defined as the solution of*

$$p = \frac{1}{2} \int_0^{\gamma} G(x)dx. \tag{11}$$

γ is strictly increasing in p for all $0 < p < \bar{p}$.

Proof. Consider one contestant, say contestant 1. We need to show that for each given p, the γ implicitly defined in Lemma 3 satisfies the equilibrium requirement

$$\gamma = \arg\max_{0 \leq \gamma_1 \leq 1} \pi_1(\gamma_1, \gamma). \tag{12}$$

For this purpose, first consider "upward" deviations from the equilibrium, $\gamma_1 \geq \gamma$, as in (10). Computing the partial derivative of π_1 w.r.t. γ_1 gives

$$\begin{aligned} \frac{\partial \pi_1}{\partial \gamma_1} &= p\left(G(\gamma)g(\gamma_1) + (1 - G(\gamma))g(\gamma_1)\right) \\ &\quad - \frac{1}{2} g(\gamma_1) \int_0^{\gamma} (\gamma_1 - x_2) dG(x_2) \\ &\quad - g(\gamma_1) \int_{\gamma}^{\gamma_1} \left(\frac{1}{2}\gamma_1 - \frac{1}{4}x_2\right) dG(x_2) \end{aligned} \tag{13}$$

$$\left.\frac{\partial \pi_1}{\partial \gamma_1}\right|_{\gamma_1 = \gamma} = \left(p - \frac{1}{2} \int_0^{\gamma} G(x)dx\right) g(\gamma) =: \xi(p, \gamma) g(\gamma). \tag{14}$$

Using the Lagrange function $\mathcal{L} := \pi_1 + \lambda(1 - \gamma_1)$, with the Lagrangian λ, and invoking the equilibrium requirement that γ must be such that the best response of innovator 1 to γ is $\gamma_1 = \gamma$ (see (12)), the equilibrium strategy γ must solve the Kuhn-Tucker (KT) conditions

$$\begin{aligned}\left.\frac{\partial \mathcal{L}}{\partial \gamma_1}\right|_{\gamma_1=\gamma} &= \left.\frac{\partial \pi_1}{\partial \gamma_1}\right|_{\gamma_1=\gamma} - \lambda = 0 \\ \left.\frac{\partial \mathcal{L}}{\partial \lambda}\right|_{\gamma_1=\gamma} &= 1-\gamma \geq 0 \quad \text{and} \quad \left.\frac{\partial \mathcal{L}}{\partial \lambda}\right|_{\gamma_1=\gamma} \lambda = 0.\end{aligned} \tag{15}$$

For $p \geq \bar{p}$ one finds (omitting the subscript 1) $\partial\pi/\partial\gamma|_{\gamma=1} \geq 0$; hence, the KT conditions are solved by $(\gamma = 1, \lambda = \partial\pi/\partial\gamma|_{\gamma=1})$. This confirms Lemma 1.

For $0 < p < \bar{p}$ one finds (omitting the subscript 1) $\partial\pi/\partial\gamma|_{\gamma=1} < 0$; hence, the KT conditions are solved by $(0 < \gamma < 1, \lambda = 0)$, where γ is implicitly defined as the unique solution of equation (11).

A similar argument deals with "downward" deviations, $\gamma_1 \leq \gamma$; it yields the same results.

Uniqueness of the solution for $p < \bar{p}$ follows from the fact that $\xi(p,\gamma)$ is strictly decreasing in γ and that $\gamma = 0 \Rightarrow \xi(p,\gamma) = p > 0$ and $\gamma = 1 \Rightarrow \xi(p,\gamma) = p - 1/2(1 - E[X]) < 0$. Monotonicity of $\gamma(p)$ follows easily.

Finally, we show that the unique solution of the condition

$$\left.\frac{\partial \pi_1}{\partial \gamma_1}\right|_{\gamma_1=\gamma} = \xi(p,\gamma)g(\gamma) = 0$$

is indeed a maximizer of the payoff of innovator 1 (assuming innovator 2 also plays the strategy γ). We prove this by showing that the function $\pi_1(\gamma_1,\gamma)$ is pseudoconcave in γ_1.[15] For this purpose, compute the cross derivative, using (13):

$$\frac{\partial^2}{\partial \gamma_1 \partial \gamma}\pi_1 = \frac{1}{4}\gamma g(\gamma_1) g(\gamma) \geq 0.$$

Together with the monotonicity of $\xi(p,\gamma)$ in γ, it follows that

$$\begin{aligned}\gamma_1 < \gamma \Rightarrow \frac{\partial}{\partial \gamma_1}\pi_1(\gamma_1,\gamma) \geq \frac{\partial}{\partial \gamma_1}\pi_1(\gamma_1,\gamma_1) = \xi(p,\gamma_1)g(\gamma_1) > \xi(p,\gamma)g(\gamma_1) = 0 \\ \gamma_1 > \gamma \Rightarrow \frac{\partial}{\partial \gamma_1}\pi_1(\gamma_1,\gamma) \leq \frac{\partial}{\partial \gamma_1}\pi_1(\gamma_1,\gamma_1) = \xi(p,\gamma_1)g(\gamma_1) < \xi(p,\gamma)g(\gamma_1) = 0.\end{aligned}$$

Therefore, $\pi_1(\gamma_1,\gamma)$ is increasing to the left of its stationary point (for $\gamma_1 < \gamma$) and decreasing to the right of its stationary point (for $\gamma_1 > \gamma$). Hence, the stationary point is a global maximum. □

Using this result, we now compute the procurer's payoff as a function of γ, eliminating the variable p. For this task, take a look at Figure 2, where the procurer's profits are represented in the order statistics space. The joint p.d.f. of $X_{(1)}, X_{(2)}$ is $g_{(1,2)}(x,y) = 2g(x)g(y)$. Therefore, one obtains, after a bit of rearranging, for all $\gamma \leq 1$ (resp. $p \leq \bar{p}$):

$$\begin{aligned}\pi_p(\gamma) = 2\int_0^\gamma \int_0^x (x-p)g(x)g(y)dydx + 2\int_\gamma^1 \int_0^\gamma \left(\frac{x+y}{2} - p\right) g(y)g(x)dydx \\ + 2\int_\gamma^1 \int_\gamma^x \left(\frac{1}{2}x + \frac{1}{4}y\right) g(y)g(x)dydx.\end{aligned} \tag{16}$$

[15] On pseudoconcavity see Avriel, Diewert, Schaible, and Zang (1988, p. 93 ff.).

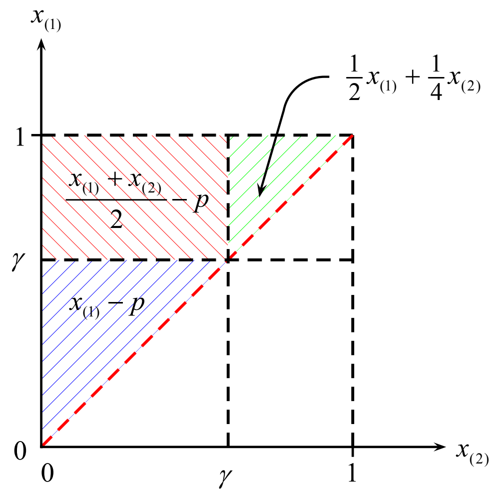


Figure 2: Payoffs of the procurer in the order statistics space

Whereas for $p \geq \bar{p}$ (resp. $\gamma = 1$) one has

$$\pi_p = E[X_{(1)}] - p. \tag{17}$$

The optimal fixed-prize tournament maximizes the procurer's expected profit over γ, resp. p, subject to the constraint that innovators' equilibrium expected payoff is nonnegative.

Example 1. *Suppose $G(x) \equiv x$ (uniform distribution) and $c \leq {}^{15}/_{128}$. Then, $\bar{p} = {}^{1}/_{2}\int_0^1 y dy = {}^{1}/_{4}$, innovators' equilibrium strategy is*

$$\gamma(p) = \begin{cases} 2\sqrt{p} & \text{if} \quad p < {}^{1}/_{4} \\ 1 & \text{if} \quad p \geq {}^{1}/_{4} \end{cases}$$

the procurer's payoff function, as a function of p, is $\pi_p(\gamma) = {}^{5}/_{12} + {}^{\gamma^2}/_{4} - {}^{\gamma^3}/_{2} + {}^{\gamma^4}/_{4}$, the optimal fixed-prize tournament is

$$\gamma^* = \arg\max_{\gamma} \pi_p(\gamma) = {}^{1}/_{2}, \quad \textit{resp.} \quad p^* = {}^{1}/_{16},$$

and innovators' equilibrium payoff is $\pi_i^* = {}^{15}/_{128} - c \geq 0$ (see Figure 3).

We mention:

Corollary 1. *The introduction of imperfect commitment*

1. *reduces the procurer's payoff if and only if $c < {}^{\bar{p}}/_{2}$,*

2. *does not affect the procurer's payoff if and only if $c \geq {}^{\bar{p}}/_{2}$.*

If the cost of innovation is sufficiently high, specifically if $c \geq {}^{\bar{p}}/_{2}$, the optimal prize under perfect commitment, $p^* = 2c$, is equal or greater than $\bar{p}$, so that the optimal fixed-prize tournament under perfect commitment implies that $\gamma = 1$ even under imperfect commitment. Therefore, in that case bypass is not an issue, and the optimal mechanism under imperfect commitment is the same as that under perfect commitment.

However, for all $c < \bar{p}/2$, the lack of perfect commitment entails either $\gamma < 1$ or $p > p^*$, and thus reduces the procurer's expected profit.

4.2 Optimal scoring auction

A bypass can occur in two ways: either the innovator does not register for the auction, yet innovates and engages in bargaining after the auction, or he registers for the auction, draws an innovation but then abstains from bidding and engages in bargaining after the auction. Of course, an innovator can always not register and not innovate, which we refer to as *quit*.

Innovators' equilibrium play is determined by the procurer's choice of entry fee. We will show that the optimal entry fee induces both innovators to register, innovate, and bid, so that in equilibrium no bypass ever occurs. The only effect of imperfect commitment is that the procurer lowers the entry fee below the rate that is optimal under full commitment,[16] and in this (and only in this) way suffers from the lack of commitment.[17]

The procurer's choice of participation fee, f, induces either an equilibrium in which both innovators play *register* or, as f is increased, an equilibrium in which only one innovator *registers* or, as f is further increased, an equilibrium in which no innovator plays *register*. We now show, in a sequence of lemmas, that the optimal participation fee induces both innovators to register, and determine the maximum payoff of the procurer.

In a first lemma we show that bypass in the form of not submitting a bid after having registered for the auction can be ruled out by elimination of dominated strategies:

Lemma 4. *Suppose an innovator has registered for the auction and paid the entry fee. Then, participation in the auction dominates bypass for all values of the innovation.*

Proof. Consider an innovator who has registered for the auction and then has drawn innovation x. Suppose the other innovator has also registered for the auction. Then participation in the auction yields a strictly higher payoff than bypass, since

$$\begin{aligned}\pi(x \mid \text{participate}) &= \int_0^x (x-y)g(y)dy \\ &> \int_0^x \frac{1}{2}(x-y)\,g(y)dy \\ &= \pi(x \mid \text{bypass}).\end{aligned}$$

Next, suppose the other innovator did not register. Then, participation is even more profitable than bypass, since being the only bidder yields a price for the innovation equal to the full value of the innovation x. Therefore, conditional upon registration, participation in the auction is the dominant strategy. □

[16]The restriction imposed on entry fees by the possibility of bypass is similar to the restriction based on listing fees due to search costs in auction hosting site pricing (see Deltas and Jeitschko, 2007).

[17]As we show later, in section 6, profitability of the auction cannot be increased by adopting a minimum score requirement.

We can thus reduce innovators' strategies to: *register*' (short for register and innovate and submit), *not register* (short for not register and innovate and bargain), and *quit* (short for not register and not innovate). Table 1 summarizes the payoffs of innovator 1 for all combinations of innovators' strategies.

		Innovator 2		
		register	not register	quit
Innovator 1	register	$\frac{1}{2}E[X_{(1)}-X_{(2)}]-c-f$	$E[X]-c-f$	$E[X]-c-f$
	not register	$\frac{1}{2}\frac{1}{2}E[X_{(1)}-X_{(2)}]-c$	$\frac{1}{2}\frac{1}{2}E[X_{(1)}-\frac{1}{2}X_{(2)}]-c$	$\frac{1}{2}E[X]-c$
	quit	0	0	0

Table 1: (Reduced form) entry game in the auction

Let

$$f^* := \frac{1}{4}E\left[X_{(1)}-X_{(2)}\right] \tag{18}$$

$$f^{**} := \frac{1}{2}E\left[X_{(1)}-X_{(2)}\right]-c. \tag{19}$$

Lemma 5. *(*Register, register*) is innovators' unique equilibrium strategy profile if and only if* $f \le \min\{f^*, f^{**}\}$. *Moreover, if the procurer sets* $f = \min\{f^*, f^{**}\}$, *his expected profit is equal to*

$$\pi_p^* = \begin{cases} E[X] & \text{if} \quad c \le \bar{c} \\ E[X_{(1)}]-2c & \text{if} \quad c \ge \bar{c} \end{cases} \tag{20}$$

$$\text{where} \quad \bar{c} := \frac{1}{4}E\left[X_{(1)}-X_{(2)}\right]. \tag{21}$$

Proof. Suppose innovator 2 plays *register*. Then, *register*' is the best reply of innovator 1 if and only if $1/2E\left[X_{(1)}-X_{(2)}\right]-f-c \ge \max\{1/4E\left[X_{(1)}-X_{(2)}\right]-c, 0\}$ (see the payoff matrix, Table 1). Solving these inequalities for f proves that *register, register* is innovators' equilibrium strategy if and only if $f \le \min\{f^*, f^{**}\}$. In Appendix A.2 we show that if $f \le \min\{f^*, f^{**}\}$, *register*' is actually the dominant strategy of innovator 1. Therefore, *register, register* is the unique equilibrium in this case.

Suppose the procurer sets $f = \min\{f^*, f^{**}\}$. If $c \le \bar{c}$, one has $\min\{f^*, f^{**}\} = f^*$, and the procurer's payoff is equal to

$$\begin{aligned} \pi_p &= E[X_{(1)}]-E[X_{(1)}-X_{(2)}]+2f \\ &= E[X_{(2)}]+\frac{1}{2}E[X_{(1)}-X_{(2)}] \\ &= \frac{1}{2}E[X_{(1)}+X_{(2)}] = E[X]. \end{aligned} \tag{22}$$

If $c \ge \bar{c}$, one has $\min\{f^*, f^{**}\} = f^{**}$, and the procurer's payoff is equal to

$$\pi_p = E[X_{(1)}]-E[X_{(1)}-X_{(2)}]+2f = E[X_{(1)}]-2c. \tag{23}$$

□

Lemma 6. *If the procurer sets an entry fee, f, that induces an asymmetric equilibrium in which one innovator* registers *while the other plays* not register*, his payoff is equal to*

$$\pi_p = \frac{1}{4}E[X_{(1)}] - \frac{3}{8}E[X_{(2)}] + f < \pi_p^*. \tag{24}$$

Proof. The proof is in Appendix A.3. □

Lemma 7. *If the procurer sets an entry fee, f, that induces an equilibrium in which both innovators play* not register*, his payoff is*

$$\pi_p = \frac{1}{2}E[X_{(1)}] + \frac{1}{4}E[X_{(2)}] < \pi_p^*. \tag{25}$$

Proof. The proof is in Appendix A.4. □

We mention that this result implies that pure bargaining (which is obtained by inducing both innovators to play *not register*) is strictly less profitable for the procurer than the optimal auction.

Combining these lemmas implies:

Proposition 2 (optimal auction). *The optimal auction involves the registration fee, $f = \min\{f^*, f^{**}\}$, which induces all innovators to register, innovate, and bid (no bypass). The procurer's expected profit is equal to*

$$\pi_p^a = \begin{cases} E[X] & \text{if} \quad c \le \bar{c} \\ E[X_{(1)}] - 2c & \text{if} \quad c \ge \bar{c}. \end{cases} \tag{26}$$

Corollary 2. *The introduction of imperfect commitment*

1. *reduces the procurer's payoff from $E[X_{(1)}] - 2c$ to $E[X]$ if $c < \bar{c}$*
2. *does not affect the procurer's payoff if $c \ge \bar{c}$.*

The intuition for this result is as follows: a sufficiently high cost of innovation entails a low entry fee in the optimal auction under perfect commitment; at sufficiently low entry fees, bypass is not an issue; therefore, when the cost of innovation is sufficiently high, the lack of commitment power does not affect the optimal auction. This is similar to how the lack of commitment affects the optimal fixed-prize tournament, depending on the cost of innovation. Note however, that the smallest cost that neutralizes the lack of commitment is not the same in the two mechanisms. Moreover, when the lack of commitment has bite, it has a different impact on the procurer's payoff across the two mechanisms, as we show now.

4.3 Which mechanism is more profitable?

We now compare the profitability of the optimal auction and the optimal standard fixed-prize tournament (without entry fees). The main purpose will be to identify cases in which the tournament is more profitable, even without entry fees. This result will be used to prove the optimality of the fixed-prize tournament in section 5, where we allow the procurer to employ entry fees also in the fixed-prize tournament.

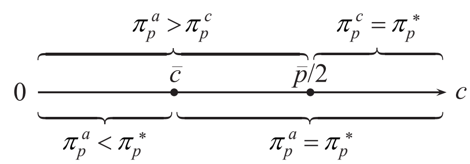
In the following it is useful to distinguish probability distributions that either satisfy the following condition:

$$\eta(\xi) := E\left[\frac{3}{2}X_{(1)} - \frac{1}{2}X_{(2)} \mid X \leq \xi\right] < \xi, \quad \forall \xi \in (0,1]. \tag{27}$$

or violate it for $\xi = 1$. Condition (27) is satisfied for a variety of standard probability distributions, ranging from truncated normal to uniform distribution, and it is violated if probability mass is concentrated on high values (as explained in more detail below).

We first state a sufficient condition for the superiority of the optimal auction:

Proposition 3. *The optimal scoring auction is more profitable than the optimal fixed-prize tournament (without entry fees),* $\pi_p^a \geq \pi_p^c$*, if* $\eta(\xi) < \xi$ *for all* ξ*, as illustrated below.*



The proof is in Appendix A.5.

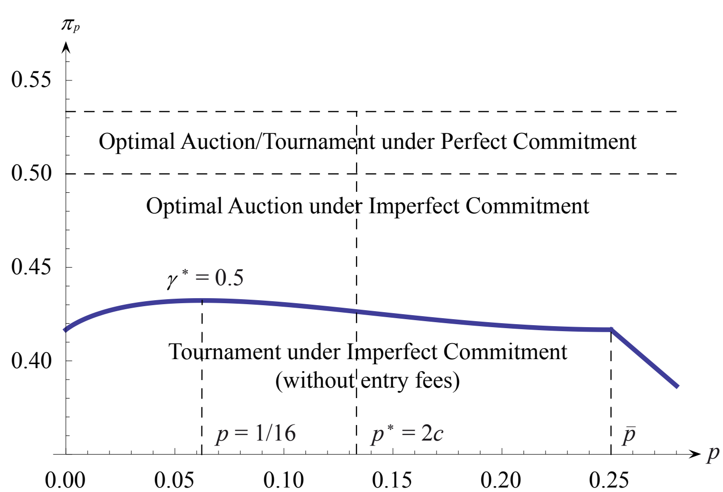
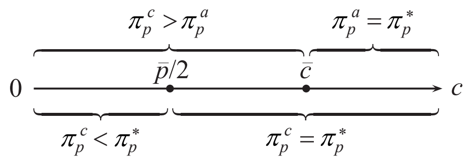


Figure 3: Comparing the optimal auction with the optimal fixed-prize tournament (without entry fees), assuming a uniform distribution

This result is illustrated for the example of the uniform distribution in Figure 3, assuming $c = 1/15$. There, the solid curve plots the procurer's expected profit in the fixed-prize tournament, as a function of the prize p. It has a kink at $p = \bar{p} = 1/4$ (which is the smallest prize that prevents bypass). The optimal prize is equal to $p = 1/16$, which is substantially lower than the optimal prize under perfect commitment ($p^* = 2/15$), and the optimal γ is equal to $\gamma^* = 1/2$. Therefore, in the optimal fixed-prize tournament, all innovations $X > 1/2$ bypass the tournament. Evidently, the lack of perfect commitment hurts the procurer in the fixed-prize tournament as well as in the auction. However, in the optimal fixed-prize tournament, the procurer tolerates bypass of the best innovations by setting $p < \bar{p}$, resp. $\gamma < 1$, because preventing bypass by setting $p = \bar{p}$ would be too costly. This induces a distortion, unlike in the optimal auction, where bypass is never a problem, because the entry fee is set sufficiently low to avoid bypass. Altogether, the lack of commitment is considerably more costly in the optimal fixed-prize tournament than in the optimal auction.

However, the more important finding is that the optimal fixed-prize tournament can be more profitable than the optimal auction, even if the tournament does not include entry fees:

Proposition 4. *The optimal fixed-prize tournament (without entry fees) is more profitable than the optimal auction, $\pi_p^c \geq \pi_p^a$, if $\eta(1) > 1$, as illustrated below.*



Proof. The assumption $\eta(1) > 1$ implies $\bar{c} > \bar{p}/2$ (the proof is similar to part 1) of the proof of Proposition 3 (see Appendix A.5). It follows immediately that for all $c \geq \bar{c}$ one has $\pi_p^a = \pi_p^c = \pi_p^*$. Therefore, it only remains to be shown that $c < \bar{c} \Rightarrow \pi_p^c > \pi_p^a$.

Suppose $c < \bar{c}$. Then, $\pi_p^a = E[X]$, and since $\pi_p^c \geq \pi_p^c\big|_{p=\bar{p}} = E[X_{(1)}] - \bar{p}$ (i.e. the procurer's maximum expected profit cannot be lower than that from setting the prize $p = \bar{p}$ that induces $\gamma = 1$), one has:

$$\begin{aligned} \pi_p^c - \pi_p^a &\geq \pi_p^c\big|_{p=\bar{p}} - \pi_p^a \\ &= E[X_{(1)}] - \bar{p} - E[X] \\ &= \frac{1}{2}\left(\frac{3}{2}E[X_{(1)}] - \frac{1}{2}E[X_{(2)}] - 1\right) \\ &= \frac{1}{2}\Big(\eta(1) - 1\Big) > 0, \end{aligned} \tag{28}$$

as asserted. □

Two conditions must be met to find a probability distribution that satisfies the sufficient condition for the superiority of the fixed-prize tournament, $\eta(1) > 1$: the distribution must exhibit a concentration of probability mass on high values *and* a relatively high spread between the order statistics $X_{(1)}, X_{(2)}$. In order to understand these requirements, it is useful to compute the "cost of imperfect commitment" (defined as the loss in profit due to imperfect commitment) in the auction resp. the tournament, defined as $\Delta_a := \pi_p^* - \pi_p^a$, resp. $\Delta_c := \pi_p^* - \pi_p^c$. Using (28), one finds

$$\Delta_a = \pi_p^* - E[X] = \frac{1}{2}E[X_{(1)} - X_{(2)}] - 2c \tag{29}$$

$$\Delta_c \leq \pi_p^* - \pi_p\big|_{p=\bar{p}} = \frac{1}{2}(1 - E[X]) - 2c =: \bar{\Delta}_c. \tag{30}$$

Evidently, the loss of imperfect commitment in the optimal tournament, Δ_c, can be made small by choosing a probability distribution that exhibits a concentration on

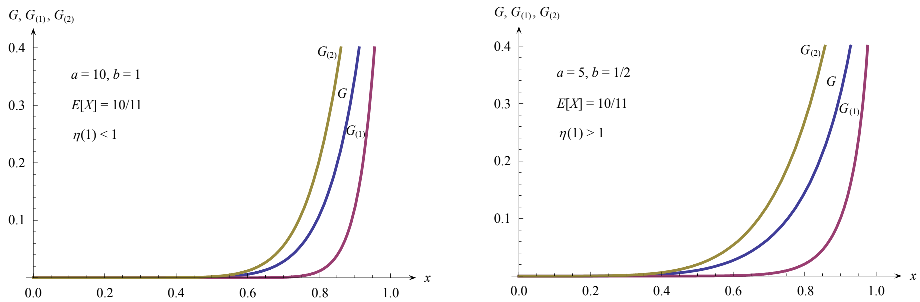


Figure 4: c.d.f.'s of beta-distributions with a concentration on high values

high values, so that $E[X]$ is relatively large. However, in order to make the fixed-prize tournament superior one also needs a relatively high cost of imperfect commitment in the auction, Δ_a, in addition to a relatively low Δ_c. And that requires that the spread between $X_{(1)}$ and $X_{(2)}$ be relatively large, as one can see immediately from (29)), (30).

The construction of the example plotted in Figure 4 is guided by this simple intuition.[18] There, we start with a beta-distribution that exhibits a concentration on high values, leading to a high $E[X]$ and thus a low Δ_c (see the figure on the left). However, for this initial example, Δ_a is still lower than Δ_c, i.e., the auction is still superior to the fixed-prize tournament. Thus we change the parameters of the beta-distribution in such a way that $E[X]$ and thus Δ_c remain unchanged, while the spread between $X_{(1)}$ and $X_{(2)}$ goes up, as one can see clearly by comparing the probability distributions of the two order statistics on the left and right of Figure 4. That change in the spread of the two order statistics is sufficiently strong to reverse the ranking of mechanisms, making the optimal fixed-prize tournament superior to the optimal auction.[19]

We also mention that the superiority of the fixed-prize tournament for certain probability distributions resembles a result by Schöttner (2008), who compared auctions and tournaments under perfect commitment, when cost functions are subject to random shocks, and entry fees are used neither in the auction nor in the fixed-prize tournament. Schöttner showed that fixed-prize tournaments may outperform auctions if the values of innovations tend to differ a great deal.

5 Optimality of the fixed-prize tournament (with/without entry fees)

So far we have assumed a standard fixed-prize tournament in which the procurer offers a prize, but does not require that contestants register and pay an entry fee. We now examine how fixed-prize tournaments perform if they may include registration and entry fees, just like in auctions. The fixed-prize tournament is then characterized by two parameters: the prize, p, and the entry fee, f. The innovator who registers and pays

[18] Incidentally, both distribution functions satisfy the assumed log-concavity of $H(x) = \int_0^x G(y)dy$, for all x.

[19] Similar examples can be found for the class of Kumaraswamy (1980) distributions, $G(x) = 1 - \left(1-(x/\gamma)^{\alpha}\right)^{\beta}$ that are often used in lieu of the beta-distribution. For $\alpha = 1, \beta = 1/5$, one finds $\eta(1) - 1 = 1/14 > 0$, and for all $c \leq \bar{c}$ one finds $\pi_p^c \geq 73/84 > 5/6 = \pi_p^a$, which confirms the superiority of the optimal fixed-prize tournament in this case.

the entry fee acquires the option to submit his innovation; innovators who do not register cannot submit. Altogether, we will show that the optimal fixed-prize tournament, (p, f), is more profitable for the procurer than the optimal auction. Thereby, we use the convention to write the simple fixed-prize contest that does not require registration (and does not collect entry fees) as $(p, 0)$.

We show that a fixed-prize tournament (p, f) exists that always matches the expected profit of the optimal auction:

Proposition 5. *The fixed-prize tournament* $(\hat{p}, \hat{f})$*,*

$$\hat{p} = \frac{1}{2} - \frac{1}{4}E[X] \tag{31}$$

$$\hat{f} = \begin{cases} \frac{1}{4} - \frac{1}{16}\left(5E[X_{(1)}] - 3E[X_{(2)}]\right) & \text{if } c \leq \bar{c} \\ \frac{1}{4} - \frac{1}{8}E[X] - c & \text{if } c \geq \bar{c}. \end{cases} \tag{32}$$

achieves the same expected profit of the procurer as the optimal auction.

The following result gives an interpretation of $\hat{p}$ and prepares the proof of this proposition:

Lemma 8. *The smallest prize that assures that an option to submit an innovation will be exercised, regardless of the value of the innovation and regardless of whether the other innovator also has the option to submit, is equal to* $\hat{p}$*, which exceeds* $\bar{p}$*,*

$$\hat{p} = \frac{1}{2} - \frac{1}{4}E[X] > \bar{p}. \tag{33}$$

Proof. Suppose the innovator has the option to submit while his rival did not register. Then, that innovator submits if and only if $\Delta(p, x) \geq 0$, for all x:

$$\begin{aligned} \Delta(p, x) :&= \pi_i(\text{submit}) - \pi_i(\text{bypass}) \\ &= p - G(x)E\left[\frac{1}{2}(x - \frac{1}{2}Y) \mid Y < x\right] \\ &= p - \frac{1}{2}\int_0^x (x - \frac{1}{2}y)g(y)dy \\ &= p - \frac{1}{4}xG(x) - \frac{1}{4}\int_0^x G(y)dy. \end{aligned} \tag{34}$$

Evidently, $\Delta(p, x)$ is strictly increasing in p and decreasing in x, and one has

$$\Delta(p, x)|_{p=\hat{p}, x=1} = \hat{p} - \frac{1}{4} - \frac{1}{4}\int_0^1 G(y)dy = \hat{p} - \frac{1}{2} + \frac{1}{4}E[X] = 0.$$

$$\Delta(p, x)|_{p=\bar{p}, x=1} = \frac{1}{4}\left(\int_0^1 G(y)dy - 1\right) < \frac{1}{4}\left(\int_0^1 dy - 1\right) = 0$$

Therefore, $\hat{p}$ is the smallest prize that assures $\Delta(p, x) \geq 0$, for all x, and $\bar{p} < \hat{p}$, since $\Delta(\bar{p}, x) < 0$ for high values of x.

Recall that $p = \bar{p}$ is the smallest prize that assures $\Delta(p, x) \geq 0$ for all x, if both innovators have the option to submit. Since $\hat{p} > \bar{p}$ it follows that $p = \hat{p}$ also assures that the innovator submits if both innovators have the option to submit. □

We are now ready to prove Proposition 5.

Proof. By construction of $\hat{p}$, an innovator who registers will later submit his innovation to the tournament. Therefore, the simultaneous entry game can be described by the following payoff matrix (since the game is symmetric we list only the payoffs of innovator 1):[20]

		Innovator 2		
		register	not register	quit
Innovator 1	register	$\frac{1}{2}\hat{p}-f-c$	$\hat{p}-c-f$	$\hat{p}-f-c$
	not register	$\frac{1}{2}\left(\frac{1}{2}E\left[X_{(1)}-X_{(2)}\right]\right)-c$	$\frac{1}{2}\left(\frac{1}{2}E\left[X_{(1)}-\frac{1}{2}X_{(2)}\right]\right)-c$	$\frac{1}{2}E[X]-c$
	quit	0	0	0

Table 2: (Reduced form) entry game in the fixed-prize tournament with entry fees

Suppose $c \leq \bar{c} = {}^1\!/_4 E[X_{(1)} - X_{(2)}]$. Then, for $f = \hat{f}$ one has

$$\begin{aligned}\frac{1}{2}\hat{p}-\hat{f}-c &= \frac{1}{2}\left(\frac{1}{2}E\left[X_{(1)}-X_{(2)}\right]\right)-c \geq 0\\ \hat{p}-\hat{f}-c &> \frac{1}{2}\left(\frac{1}{2}E\left[X_{(1)}-\frac{1}{2}X_{(2)}\right]\right)-c > \frac{1}{2}\left(\frac{1}{2}E\left[X_{(1)}-X_{(2)}\right]\right)-c \geq 0.\end{aligned}$$

Therefore, *register* dominates '*quit*, and after eliminating *quit*, *register* dominates *not register*. Hence, "register/register" is the unique equilibrium of the entry game, by iterated elimination of dominated strategies, and the procurer's equilibrium expected profit is $\pi_p^c = E[X_{(1)}] - \hat{p} + 2\hat{f} = E[X] = \pi_p^a$.

Similarly, in case $c \geq \bar{c}$ one has

$$\begin{aligned}\frac{1}{2}\hat{p}-\hat{f}-c = 0 &\geq \frac{1}{2}\left(\frac{1}{2}E\left[X_{(1)}-X_{(2)}\right]\right)-c\\ \hat{p}-\hat{f}-c &> \frac{1}{2}\left(\frac{1}{2}E\left[X_{(1)}-\frac{1}{2}X_{(2)}\right]\right)-c \geq 0\\ \hat{p}-\hat{f}-c &> 0.\end{aligned}$$

Again, "register/register" is also the unique equilibrium in this case and the procurer's equilibrium expected profit is $\pi_p^c = E[X_{(1)}] - \hat{p} + 2\hat{f} = E[X_{(1)}] - 2c = \pi_p^a$. □

Having established that the optimal tournament is at least as profitable for the procurer as the optimal auction, now recall that the optimal fixed-price tournament can be *more* profitable than the optimal auction. Indeed,

[20] Suppose innovator 1 registers; if innovator 2 does not register or quits, 1 wins the prize for sure, whereas if 2 registers as well, 1 wins the prize with probability 1/2. Suppose innovator 1 does not register and thus speculates on bargaining; he wins only if he has the better innovation, which occurs with probability 1/2, and in that event the procurer's threat point depends on whether innovator 2 has registered, as explained before.

Lemma 9. *Suppose $c < \bar{c}$. The simple contest, $(\bar{p}, 0)$, that does not require registration/entry fees is more profitable than the optimal auction if and only if $\eta(1) > 1$.*

Proof. 1) In the proof of proposition 4 we have already shown that the simple fixed-prize contest $(\bar{p}, 0)$ is more profitable than the optimal auction if $\eta(1) > 1$.

2) Assume $\pi_p^c(\bar{p}, 0) > \pi_p^a$. By proposition 5, $\pi_p^a = \pi_p^c(\hat{p}, \hat{f})$, and therefore $\hat{p} - \bar{p} > 2\hat{f}$. Using the definitions of $\hat{p}, \hat{f}, \bar{p}$, it follows, after a bit or rearranging, that $\eta(1) = E[{}^3/_2 X_{(1)} - {}^1/_2 X_{(2)}] > 1$, as asserted. □

Corollary 3 (**Optimality of the fixed-prize tournament**). *The optimal fixed-prize tournament is never less and sometimes more profitable than the optimal auction.*

The intuition for the surprising fact that the simple contest $(\bar{p}, 0)$ can be superior to the fixed-prize contest $(\hat{p}, \hat{f})$ that replicates the optimal auction is as follows. When no registration/entry fee is required, both innovators have the option to submit; therefore, in order to prevent bypass it is sufficient to employ the simple fixed-prize contest $(\bar{p}, 0)$, where $\bar{p}$ is the smallest prize that assures submission when both innovators have the option to submit. However, once registration/entry fees are imposed, preventing bypass requires not only that innovators always exercise an option to submit, but also buy the option to submit, which may be more costly for the procurer.

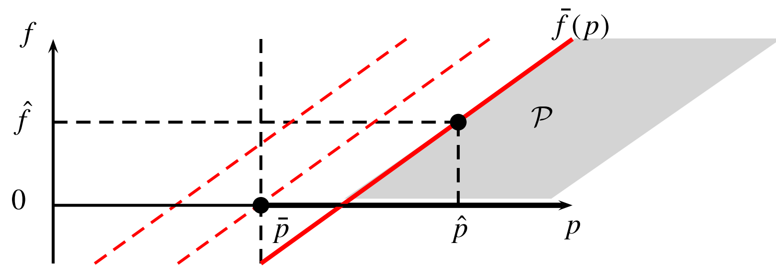


Figure 5: Why entry fees may reduce the procurer's expected profit

Figure 5 displays the set of all fixed-prize contests (p, f) that prevent bypass and their profitability for the procurer if $\eta(1) > 1$. The shaded area is the set of all (p, f) that prevent bypass if registration/entry fees are required, $\mathcal{P} := \{(p, f) \mid p \geq \bar{p}, 0 < f \leq \bar{f}(p)\}$, where $\bar{f}(p) := {}^p/_2 - {}^1/_4 E[X_{(1)} - X_{(2)}]$.[21] The solid line and the parallel dashed lines are indifference curves of the procurer (assuming both innovators play "register/always submit").

If no registration/entry fee is required, bypass is prevented by all fixed-prize contests from the set $\{(p, f) \mid p \geq \bar{p}, f = 0\}$, by definition of $\bar{p}$. Since $\eta(1) > 1 \Rightarrow (\hat{p} - \bar{p}) > 2f$,[22] one can see that the optimal simple fixed-prize contest $(\bar{p}, 0)$ reaches a higher level indifference curve than all fixed-prize contests $(p, f) \in \mathcal{P}$. In other

[21] Sketch of proof: Consider one innovator whose rival plays "register and always submit". Then "register and always submit" is a best reply if this strategy is more profitable than 1) "not submit (and bargain), and than 2) "register and submit iff $x \leq \gamma$". Requirement 1) is equivalent to $f \leq \bar{f}(p)$, and requirement 2) is equivalent to $p \geq \bar{p}$.

[22] Continuously extend $\bar{f}(p)$ to allow for $\bar{f}(p) \leq 0$. Then, $\eta(1) > 1 \Rightarrow \bar{f}(\bar{p}) = {}^1/_4(1 - E[{}^3/_2 X_{(1)} - {}^1/_2 X_{(2)}]) = {}^1/_4(1 - \eta(1)) < 0$.

words, preventing bypass becomes more costly if one replaces a simple fixed-prize contest by one that requires registration/entry fees.

6 The auction cannot be improved by a minimum score

One may think that the profitability of the auction can be improved by adding a minimum score requirement, similar to the standard optimal auction problem where the seller benefits from strategically setting a binding reserve price above his own valuation.[23]

In order to examine this conjecture, suppose the procurer accepts only bids that match or exceed a stated minimum score, which is denoted by R. This changes the auction as follows: if exactly one bidder, say bidder 1, submits a score $S_1 = x_1 - b_1 \geq R$, that bidder wins the auction and is paid a price equal to $x_1 - R$ (instead of a price equal to $x_1 - x_2$); if no bidder submits a score equal to R or more, no trade occurs in the auction; and if both bidders submit a score $S \geq R$, the minimum score does not bind, and the auction proceeds as before.

In the presence of a minimum score requirement, bidders play cutoff strategies and bid if and only if the value of their innovation is equal or greater than a threshold value, which is denoted by r. We look for a symmetric equilibrium. In such an equilibrium, a bidder with value $x = r$ must be indifferent between submitting a score $S = R$ and not bidding, and bidding must be profitable for all $x > r$, and unprofitable for all $x < r$.

Using the indifference condition for $x = r$ yields the following unique and strictly increasing relationship between the minimum score R and the threshold value r

$$R = \frac{1}{2}r + \frac{1}{4}E[X \mid X \leq r], \tag{35}$$

which in turn allows us to eliminate the variable R, compute the procurer's expected profit as a function of r, and then maximize that payoff over r.

An analysis of the optimal auction with minimum score shows that the auction cannot be improved by adding a minimum score. Indeed,

Proposition 6. *The optimal minimum score is equal to zero; hence, the procurer cannot raise his expected profit by employing a minimum score requirement in addition to charging entry fees.*

The proof is in Appendix A.6.

7 Discussion

In the present paper we analyze the procurement of innovations, assuming that the procurer cannot commit to never bargain with innovators who did not participate in the mechanism of his choice. We compare two methods of procurement from one of two innovators: auctions vs. fixed-prize tournaments. We show that the optimal fixed-prize tournament is more profitable than the optimal auction. While auctions are never

[23] Myerson (1981) showed that a binding reserve price that excludes participation of bidders with low values is optimal for the seller except if buyers' valuations are considerably larger than the seller's own valuation.

attractive without positive entry fees, surprisingly a standard fixed-prize contest that does not include entry fees can be more profitable than the optimal auction. Moreover, we show that one cannot improve the profitability of the auction by employing minimum score requirement in addition to entry fees.

Our analysis assumes that the value of an innovation is not verifiable to third parties. This excludes the use of contracts to procure innovations. It also precludes the use of a discriminatory scoring rule as well as a second-score auction in which the price paid by the procurer depends on the difference between the values of the two best innovations. Nevertheless, since in the present framework first- and second-score auctions are revenue equivalent, we are able to highlight the performance of auctions without getting entangled in unnecessarily complex bidding strategies, by using the second-score auction as a proxy for the first-score auction,

Our analysis has been carried out in a simple framework, with only two contestants and a stylized innovation technology. In further research one may wish to extend the analysis to cases in which it is optimal to short-list more than two contestants, introduce asymmetries between innovators, either with respect to their cost functions or with respect to the probability distributions from which they draw their innovations, and extend the analysis to a common (or affiliated) value framework.

A Appendix

A.1 Proof of Lemma 2

Suppose innovator 2 plays cutoff strategy γ_2, and innovator 1 has drawn innovation x_1. In order to show that the best response of innovator 1 is also a cutoff strategy, it is sufficient to prove the following statements:

1. if *not submit* is the best reply for x_1, then *not submit* is also a best reply for all innovation values greater than x_1;
2. if *submit* is the best reply for x_1, then *submit* is also a best reply for all innovation values smaller than x_1.

To prove the first statement, we distinguish between $x_1 < \gamma_2$ and $x_1 > \gamma_2$.

If $x_1 < \gamma_2$, the assumption that *not submit* is the best reply to γ_2 is equivalent to

$$\frac{1}{2}\int_0^{x_1}(x_1-y)dG(y) \geq p\,(G(x_1)+(1-G(\gamma_2)))$$

$$\frac{1}{2}\int_0^{x_1}G(y)dy \geq p\,(G(x_1)+(1-G(\gamma_2)))$$

$$\frac{1}{2}\frac{H(x_1)}{H'(x_1)} \geq p\left(1+\frac{1-G(\gamma_2)}{G(x_1)}\right), \quad \text{by} \quad H(x)=\int_0^x G(y)dy.$$

Since H is log-concave, the LHS of the last inequality is increasing in x_1 and the RHS is decreasing in x_1. Therefore, this inequality holds also for all innovations valued higher than x_1.

If $x_1 > \gamma_2$, the assumption that *not submit* is the best reply to γ_2 is equivalent to

$$\frac{1}{2}\int_0^{\gamma_2}(x_1-y)dG(y)+\frac{1}{2}\int_{\gamma_2}^{x_1}\left(x_1-\frac{1}{2}y\right)dG(y) \geq p.$$

Evidently, the LHS of the last inequality is increasing in x_1 and the RHS is constant. Therefore, this inequality holds also for all innovations valued higher than x_1.

The proof of the second statement is similar and hence omitted.

A.2 Supplement to the proof of Lemma 5

We have already shown in the proof of Lemma 5 that "register, register" is an equilibrium if $f \leq \min\{f^*, f^{**}\}$. Therefore, to prove the uniqueness of this equilibrium, we only need to show that *register* is also a best reply to *not register* and to *quit*.

First, assume $c \leq \bar{c}$, i.e., $f \leq \min\{f^*, f^{**}\} = f^*$. Then *register* is a best reply to *not register* since being the only bidder ensures that one earns the price equal to the value of the own innovation, x, and thus a payoff equal to

$$\begin{aligned}
\pi &= E[X] - f - c \\
&\geq \frac{1}{2}E\left[X_{(1)} + X_{(2)}\right] - \frac{1}{4}E\left[X_{(1)} - X_{(2)}\right] - c \\
&= \frac{1}{4}E[X_{(1)}] + \frac{3}{4}E[X_{(2)}] - c > 0.
\end{aligned} \tag{36}$$

Whereas if one also plays *not register*, the payoff is

$$\pi' = \frac{1}{4}E[X_{(1)}] - \frac{1}{8}E[X_{(2)}] - c.$$

Evidently, $\pi > \pi'$, $\pi > 0$, and therefore *register* is the best reply to *not register*.

Similarly, if $c \geq \bar{c}$, one obtains $\min\{f^*, f^{**}\} = f^{**}$, and hence

$$\begin{aligned}
\pi &\geq E[X] - c - \left(\frac{1}{2}E[X_{(1)} - X_{(2)}] - c\right) \\
&= \frac{1}{2}E[X_{(1)} + X_{(2)}] = E[X] > 0,
\end{aligned} \tag{37}$$

$$\begin{aligned}
\pi' &= \frac{1}{2}\left(\frac{1}{2}E[X_{(1)} - \frac{1}{2}X_{(2)}]\right) - c \\
&= \frac{1}{4}E[X_{(1)}] - \frac{1}{8}E[X_{(2)}] - c \\
&\leq \frac{1}{4}E[X_{(1)}] - \frac{1}{8}E[X_{(2)}] - \frac{1}{4}E[X_{(1)}] + \frac{1}{4}E[X_{(2)}] \\
&= \frac{1}{8}E[X_{(2)}] < \pi.
\end{aligned} \tag{38}$$

Therefore, *register* is a best reply to *not register*, in all cases. Similarly, one can show that *register* is a best reply to *quit*.

A.3 Proof of Lemma 6

Suppose one innovator, say innovator 1, registers, while innovator 2 plays *not register*. Then innovator 1 earns the expected payoff

$$\pi_1 = E[X] - f - c. \tag{39}$$

If innovator 1 deviates to *not register*, his payoff is

$$\pi_1' = \frac{1}{2}\left(\frac{1}{2}E[X_{(1)} - \frac{1}{2}X_{(2)}]\right) - c. \tag{40}$$

And if he deviates to *quit*, his payoff is equal to zero. Therefore, in an asymmetric equilibrium one must have: $\pi_1 \geq \pi_1'$ and $\pi_1 \geq 0$. These conditions are equivalent to

$$f \leq \frac{1}{4}E[X_{(1)}] + \frac{5}{8}E[X_{(2)}], \quad \text{and } f \leq E[X] - c. \tag{41}$$

Similarly, the payoff of innovator 2 (who plays *not register* while innovator 1 plays *register*) is

$$\pi_2 = \frac{1}{2}\left(\frac{1}{2}E[X_{(1)} - \frac{1}{2}X_{(2)}]\right) - c. \tag{42}$$

If he deviates to *quit*, his payoff is equal to zero. Therefore, one must have $\pi_2 \geq 0$, which implies

$$\frac{1}{4}E[X_{(1)}] - \frac{1}{8}E[X_{(2)}] \geq c. \tag{43}$$

And if he deviates to *register*, his payoff is

$$\pi_2' = \frac{1}{2}E[X_{(1)} - X_{(2)}] - f - c. \tag{44}$$

Therefore, one must have: $\pi_2 \geq \pi_2'$. This condition is equivalent to

$$f \geq \frac{1}{4}E[X_{(1)}] - \frac{3}{8}E[X_{(2)}]. \tag{45}$$

We conclude that such an asymmetric equilibrium is induced by all

$$f \in \left[\frac{1}{4}E[X_{(1)}] - \frac{3}{8}E[X_{(2)}, \frac{1}{4}E[X_{(1)}] + \frac{5}{8}E[X_{(2)}]\right]. \tag{46}$$

It follows that in such an equilibrium the procurer's expected profit is

$$\begin{aligned}\pi_p &= E[X_{(1)}] - E[X] + f - \frac{1}{2}\left(\frac{1}{2}E[X_{(1)} - \frac{1}{2}X_{(2)}]\right) \\ &= \frac{1}{4}E[X_{(1)}] - \frac{3}{8}E[X_{(2)}] + f.\end{aligned} \tag{47}$$

If $c \leq \bar{c}$, it follows by (41) that

$$\begin{aligned}\pi_p &\leq \frac{1}{4}E[X_{(1)}] - \frac{3}{8}E[X_{(2)}] + \frac{1}{4}E[X_{(1)}] + \frac{5}{8}E[X_{(2)}] \\ &= \frac{1}{2}E[X_{(1)}] + \frac{1}{4}E[X_{(2)}] < E[X] = \pi_p^*.\end{aligned} \tag{48}$$

And if $\bar{c} \leq c \leq \frac{1}{4}E[X_{(1)}] - \frac{1}{8}E[X_{(2)}]$, one finds,

$$\begin{aligned}\pi_p^* &= E[X_{(1)}] - 2c \\ &\geq E[X_{(1)}] - 2\left(\frac{1}{4}E[X_{(1)}] - \frac{1}{8}E[X_{(2)}]\right) \\ &= \frac{1}{2}E[X_{(1)}] + \frac{1}{4}E[X_{(2)}] \geq \pi_p.\end{aligned} \tag{49}$$

A.4 Proof of Lemma 7

If innovators innovate but do not register, innovators' and the procurer's payoffs are

$$\begin{aligned}
\pi &= \frac{1}{2}\left(\frac{1}{2}E[X_{(1)} - \frac{1}{2}X_{(2)}]\right) - c \\
&= \frac{1}{4}E[X_{(1)}] - \frac{1}{8}E[X_{(2)}] - c \geq 0 \\
\pi_p &= E[X_{(1)}] - \frac{1}{2}E[X_{(1)} - \frac{1}{2}X_{(2)}] \\
&= \frac{1}{2}E[X_{(1)}] + \frac{1}{4}E[X_{(2)}].
\end{aligned} \tag{50}$$

Hence, one must have $c \leq 1/4E[X_{(1)}] - 1/8E[X_{(2)}]$.

If $c \leq \bar{c}$, one finds

$$\pi_p^* = E[X] = \frac{1}{2}E[X_{(1)} + X_{(2)}] > \pi_p. \tag{51}$$

And if $\bar{c} \leq c \leq 1/4E[X_{(1)}] - 1/8E[X_{(2)}]$, one has

$$\begin{aligned}
\pi_p^* &= E[X_{(1)}] - 2c \\
&\geq E[X_{(1)}] - \frac{1}{2}E[X_{(1)}] + \frac{1}{4}E[X_{(2)}] \\
&= \frac{1}{2}E[X_{(1)}] + \frac{1}{4}E[X_{(2)}] = \pi_p.
\end{aligned} \tag{52}$$

A.5 Proof of Proposition 3

We distinguish between $c \geq \bar{c}$ and $c \leq \bar{c}$.

1) Suppose $c \geq \bar{c}$. Since $\eta(\xi) < \xi, \forall \xi \in (0,1]$, one has in particular $\eta(1) < 1$. In that case the procurer's expected profit in the optimal auction is equal to $E[X_{(1)}] - 2c$, by Proposition 2, which is exactly the same as in the optimal auction and in the optimal fixed-prize tournament under perfect commitment. Since the optimal fixed-prize tournament under imperfect commitment cannot be more profitable than under perfect commitment, it follows that $\pi_p^c \leq \pi_p^a$.

If in addition $\eta(1) < 1$, one finds

$$\begin{aligned}
\frac{3}{2}E[X_{(1)}] - \frac{1}{2}E[X_{(2)}] &< 1 \\
\frac{1}{2}E[X_{(1)} - X_{(2)}] + \frac{1}{4}E[X_{(1)} + X_{(2)}] &< \frac{1}{2} \\
\frac{1}{2}E[X_{(1)} - X_{(2)}] + \frac{1}{2}E[X] &< \frac{1}{2} \\
\frac{1}{2}E[X_{(1)} - X_{(2)}] &< \frac{1}{2}(1 - E[X]) \\
\frac{1}{2}E[X_{(1)} - X_{(2)}] &< \frac{1}{2}\int_0^1 G(x)dx \\
2\bar{c} &< \bar{p}.
\end{aligned}$$

Hence, $\bar{p}/2 > \bar{c}$. By Corollary 2 the auction is not affected by the lack of commitment for all $c \geq \bar{c}$, whereas, by Corollary 1, the fixed-prize tournament is not affected only for all $c \geq \bar{p}/2$. Hence, $\pi_p^a = \pi_p^c$ for all $c \geq \bar{c}$ and $\pi_p^a > \pi_p^c$ for all $c \in [\bar{c}, \bar{p}/2]$. (Recall, under perfect commitment, the optimal fixed-prize tournament and the optimal auction yield the same expected payoffs, by Proposition 1.)

2) Suppose $c \leq \bar{c}$. A direct payoff comparison yields $\Delta := \pi_p^c - \pi_p^a < 0$, by the following reasoning:[24]

$$
\begin{aligned}
\Delta &= \int_0^\gamma \int_0^x (2x-2p)\, g(x)\, g(y)\, dydx + \int_\gamma^1 \int_0^\gamma (x+y-2p)\, g(y)\, g(x)\, dydx \\
&\quad + \int_\gamma^1 \int_\gamma^x \left(x + \frac{1}{2}y\right) g(y)\, g(x)\, dydx - \int_0^1 \int_0^x (x+y)\, g(x)\, g(y)\, dydx \\
&= \int_0^\gamma \int_0^x (2x-2p)\, g(x)\, g(y)\, dydx + \int_\gamma^1 \int_0^\gamma (x+y-2p)\, g(y)\, g(x)\, dydx \\
&\quad + \int_\gamma^1 \int_\gamma^x \left(x + \frac{1}{2}y\right) g(y)\, g(x)\, dydx \\
&\quad - \int_0^\gamma \int_0^x (x+y)\, g(x)\, g(y)\, dydx - \int_\gamma^1 \int_0^x (x+y)\, g(x)\, g(y)\, dydx \\
&= \int_0^\gamma \int_0^x (x-y-2p)\, g(x)\, g(y)\, dydx + \int_\gamma^1 \int_0^\gamma (x+y-2p)\, g(y)\, g(x)\, dydx \\
&\quad + \int_\gamma^1 \int_\gamma^x \left(x + \frac{1}{2}y\right) g(y)\, g(x)\, dydx \\
&\quad - \int_\gamma^1 \int_\gamma^x (x+y)\, g(x)\, g(y)\, dydx - \int_\gamma^1 \int_0^\gamma (x+y)\, g(x)\, g(y)\, dydx \\
&= \int_0^\gamma \int_0^x (x-y-2p)\, g(x)\, g(y)\, dydx - \int_\gamma^1 \int_0^\gamma 2pg(y)\, g(x)\, dydx \\
&\quad - \int_\gamma^1 \int_\gamma^x \frac{1}{2} yg(y)\, g(x)\, dydx \\
&= \int_0^\gamma \int_0^x (x-y)\, g(x)\, g(y)\, dydx - \int_\gamma^1 \int_\gamma^x \frac{1}{2} yg(y)\, g(x)\, dydx \\
&\quad - \int_0^\gamma \int_0^x 2pg(x)\, g(y)\, dydx - \int_\gamma^1 \int_0^\gamma 2pg(y)\, g(x)\, dydx \\
&= \int_0^\gamma \int_0^x (x-y)\, g(x)\, g(y)\, dydx - \int_\gamma^1 \int_\gamma^x \frac{1}{2} yg(y)\, g(x)\, dydx + pG(\gamma)^2 - 2pG(\gamma) \\
&= \frac{1}{2} G(\gamma)^2 E\left[X_{(1)} | X_{(1)} \leq \gamma\right] - \frac{1}{2} E\left[X_{(2)} | X_{(1)} \leq \gamma\right] - \frac{1}{4} (1 - G(\gamma))^2 E\left[X_{(2)} | X_{(2)} \geq \gamma\right] \cdot \\
&\quad + pG(\gamma)^2 - 2pG(\gamma) \\
&= \frac{1}{2} G(\gamma)^2 E\left[X_{(1)} | X_{(1)} \leq \gamma\right] - \frac{1}{2} G(\gamma)^2 E\left[X_{(2)} | X_{(1)} \leq \gamma\right] - \frac{1}{2} (1 - G(\gamma)^2) E\left[X_{(2)} | X_{(1)} \leq \gamma\right]
\end{aligned}
$$

[24] The first inequality follows from $pG(\gamma)^2 \leq pG(\gamma)$, $E[X_{(1)} \mid X_{(1)} < \gamma] \leq E[X_{(1)} \mid X < \gamma]$, and $E[X_{(2)} \mid X_{(1)} < \gamma] \geq E[X_{(2)} \mid X < \gamma]$.

$$
\begin{aligned}
&-\frac{1}{4}(1-G(\gamma))^2 E\left[X_{(2)}|X_{(2)} \geq \gamma\right] + pG(\gamma)^2 - 2pG(\gamma) \\
&\leq \frac{1}{2}G(\gamma)^2 E\left[X_{(1)} - X_{(2)}|X_{(1)} \leq \gamma\right] - \frac{1}{2}(1-G(\gamma)^2)E\left[X_{(2)}|X_{(1)} \leq \gamma\right] \\
&\quad -\frac{1}{4}E\left[X_{(2)}|X_{(2)} \geq \gamma\right](1-G(\gamma))^2 - G(\gamma)\frac{1}{2}G(\gamma)(\gamma - E[X|X \leq \gamma]) \\
&= \frac{1}{2}G(\gamma)^2\left(E\left[X_{(1)} - X_{(2)}|X \leq \gamma\right] + E[X|X \leq \gamma] - \gamma\right) \\
&\quad -\frac{1}{4}(1-G(\gamma))^2 E\left[X_{(2)}|X_{(2)} \geq \gamma\right] - \frac{1}{2}(1-G(\gamma)^2)E\left[X_{(2)}|X_{(1)} \leq \gamma\right] \\
&< \frac{1}{2}G(\gamma)^2\left(E\left[X_{(1)} - X_{(2)}|X \leq \gamma\right] + \frac{1}{2}E[X_{(1)} + X_{(2)}|X \leq \gamma] - \gamma\right) \\
&= \frac{1}{2}G(\gamma)^2\left(E\left[\frac{3}{2}X_{(1)} - \frac{1}{2}X_{(2)}\,\middle|\, X \leq \gamma\right] - \gamma\right) = \frac{1}{2}G(\gamma)^2(\eta(\gamma) - \gamma) < 0.
\end{aligned}
$$

A.6 Proof of Proposition 6

To ensure that the auction has a symmetric equilibrium in cutoff strategies, we assume that $K := -G$ is starshaped, which is weaker than concavity but stronger than subadditivity of G. The function K is starshaped if for each $\alpha \in [0,1]$, and all x: $K(\alpha x) \leq \alpha K(x)$ (see Bruckner and Ostrow, 1962). Starshapedness implies that $K(x)/x$ is increasing, resp. $G(x)/x$ is decreasing. This property is used in the proof below.

In a symmetric equilibrium, a bidder with value $x = r$ is indifferent between submitting a score $S = R$ and not bidding, and bidding is profitable for all $x > r$ and unprofitable for all $x < r$.

Indifference between bidding and not bidding for $x = r$ means that

$$
G(r)(r-R) = \frac{1}{2}\int_0^r (r - \frac{1}{2}y)g(y)dy.
$$

This implies the relationship between R and r stated in (35).

Denote the difference between bidders' payoff when bidding and not bidding by Δ. Assume $x > r$. Then, using (35)

$$
\begin{aligned}
\Delta &= G(r)(x-R) + \int_r^x (x-y)g(y)dy - \frac{1}{2}\int_0^r (x - \frac{1}{2}y)g(y)dy - \frac{1}{2}\int_r^x (x-y)g(y)dy \\
&= G(r)(x-R) + \frac{1}{2}\int_r^x (x-y)g(y)dy - \frac{1}{2}\int_0^r (x-\frac{1}{2}y)g(y)dy \\
&= G(r)x - G(r)r + \frac{1}{2}\int_0^r (r - \frac{1}{2}y)g(y)dy + \frac{1}{2}\int_r^x (x-y)g(y)dy - \frac{1}{2}\int_0^r (x - \frac{1}{2}y)g(y)dy \\
&= G(r)(x-r) + \frac{1}{2}\int_0^r (r-x)g(y)dy + \frac{1}{2}\int_r^x (x-y)g(y)dy \\
&= G(r)(x-r) + \frac{1}{2}(r-x)G(r) + \frac{1}{2}\int_r^x (x-y)g(y)dy \\
&= \frac{1}{2}G(r)(x-r) + \frac{1}{2}\int_r^x (x-y)g(y)dy \\
&> 0.
\end{aligned}
$$

Similarly, one obtains for $x \leq r$:[25]

$$
\begin{aligned}
\Delta &= G(r)(x-R) - \frac{1}{2}\int_0^x (x-\frac{1}{2}y)g(y)dy \\
&= G(r)x - G(r)r + \frac{1}{2}\int_0^r (r-\frac{1}{2}y)g(y)dy - \frac{1}{2}\int_0^x (x-\frac{1}{2}y)g(y)dy \\
&= G(r)(x-r) + \frac{1}{2}rG(r) - \frac{1}{2}xG(x) - \frac{1}{4}\int_x^r yg(y)dy \\
&\leq G(r)(x-r) + \frac{1}{2}rG(r) - \frac{1}{2}xG(x) - \frac{1}{4}\int_x^r xg(y)dy \quad \text{(since } y \leq x\text{)} \\
&= G(r)(x-r) + \frac{1}{2}rG(r) - \frac{1}{2}xG(x) - \frac{1}{4}x(G(r)-G(x)) \\
&= \frac{3}{4}xG(r) - \frac{1}{4}xG(x) - \frac{1}{2}rG(r) \\
&\leq \frac{1}{2}xG(r) - \frac{1}{4}xG(x) - \frac{1}{2}rG(r) + \frac{1}{4}rG(x) \quad \text{(since } -G \text{ is starshaped)} \\
&= \left(G(r) - \frac{1}{2}G(x)\right)\left(\frac{1}{2}x - \frac{1}{2}r\right) < 0
\end{aligned}
$$

The addition of a minimum score implies restrictions on the entry fee. For a given r the procurer sets the highest entry fee that ensures that both innovators register. Consider an innovator whose rival registers. Denote his payoff if he also registers by π^r and if he does not register by π^n. Then, the procurer sets the highest fee that ensures $\pi^r \geq \pi^n$ and $\pi^r \geq 0$.

After some rearranging and changing the order of integration one finds

$$
\begin{aligned}
\pi^r &= \frac{1}{2}\int_0^r \int_0^x (x-\frac{1}{2}y)g(x)g(y)dydx + \int_r^1 \int_r^x (x-y)g(x)g(y)dydx \\
&\quad + \int_r^1 \int_0^r (x-R)g(x)g(y)dydx - f - c \\
\pi^n &= \frac{1}{2}\int_0^r \int_y^1 (x-\frac{1}{2}y)g(x)g(y)dxdy + \frac{1}{2}\int_r^1 \int_y^1 (x-y)g(x)g(y)dxdy - c.
\end{aligned}
$$

The highest entry fee, f^*, that ensures $\pi^r \geq \pi^n$ is

$$
f^* := \int_r^1 \int_r^x (\frac{1}{2}x - \frac{1}{2}y)g(x)g(y)dydx + \int_r^1 \int_0^r (\frac{1}{2}x - \frac{1}{4}y - R)g(x)g(y)dydx.
$$

And the highest entry fee, f^{**}, that ensures $\pi^r \geq 0$ is

$$
\begin{aligned}
f^{**} :&= \frac{1}{2}\int_0^r \int_0^x (x-\frac{1}{2}y)g(x)g(y)dydx \\
&\quad + \int_r^1 \int_r^x (x-y)g(x)g(y)dydx + \int_r^1 \int_0^r (x-R)g(x)g(y)dydx - c.
\end{aligned}
$$

Therefore, the optimal entry fee is $f = \min\{f^*, f^{**}\}$.

[25] Note, for some interval of x values below r one has nevertheless $x > R$.

Finally, compute the procurer's expected profit, using the optimal registration fee and the relationship between R and r, (35), writing π_p as a function of r. If $f^* \leq f^{**}$, one finds

$$\begin{aligned}\pi_p &= 2\int_0^r \int_0^x (\frac{1}{2}x + \frac{1}{4}y)g(x)g(y)dydx + 2\int_r^1 \int_r^x yg(x)g(y)dydx \\ &\quad + 2\int_r^1 \int_0^r Rg(x)g(y)dydx + 2f^* \\ &= \int_0^1 \int_0^x (x + \frac{1}{2}y)g(x)g(y)dydx + \int_r^1 \int_r^x \frac{1}{2}yg(x)g(y)dydx,\end{aligned}$$

which is decreasing in r and thus reaches the maximum at $r = 0$, associated with $R = 0$. Thus, in this case, the procurer cannot benefit from including a minimum score requirement.

Similarly, if $f^{**} \leq f^*$,

$$\begin{aligned}\pi_p &= 2\int_0^r \int_0^x (\frac{1}{2}x + \frac{1}{4}y)g(x)g(y)dydx + 2\int_r^1 \int_r^x yg(x)g(y)dydx \\ &\quad + 2\int_r^1 \int_0^r Rg(x)g(y)dydx + 2f^{**} \\ &= \int_0^1 \int_0^x 2xg(x)g(y)dydx - 2c \\ &= E[X_{(1)}] - 2c.\end{aligned}$$

Since $E[X_{(1)}] - 2c$ is the procurer's expected profit in the auction without minimum score, it follows also in this case that the procurer cannot benefit from a minimum score requirement.

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