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**Licensing Complementary
Patents: “Patent Trolls”, Market
Structure, and “Excessive”
Royalties**

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LICENSING COMPLEMENTARY PATENTS: “PATENT TROLLS”, MARKET STRUCTURE, AND “EXCESSIVE” ROYALTIES

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ABSTRACT: The infamous Blackberry case brought new attention to so-called “patent trolls” and began the general association of trolls with “non-practicing” patent holders. This has had important legal consequences: Namely, patent holders have been denied injunctive relief because they did not practice the patents themselves. In this paper we analyze how patent holders — both non-practicing and vertically integrated — choose their royalties depending on the structure of the upstream and downstream markets and the types of licensing agreements available. We show that a vertically integrated firm has an incentive to raise its rivals’ costs and to restrict entry on the downstream market; incentives that do not hold for non-integrated patent holders. An automatic presumption that a non-integrated patent holder will charge higher royalties than a vertically integrated company is therefore unfounded. Whether a company charges “excessive” royalties depends on whether there is scope for hold-up, either because of sunk investments on the part of potential licensees or because of “weak” patents held by the licensor. These factors are orthogonal to whether patent holders are practicing or not.

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1. Introduction

In early 2006 the email correspondence of millions of BlackBerry users nearly came to a halt when NTP, Inc. accused Research in Motion, Ltd. (“RIM”), the maker of the popular communication device, of infringing several of its patents.¹ The court found that BlackBerry’s email retrieval system was indeed infringing on some of NTP’s patents and awarded damages and a permanent injunction against RIM. Armed with the injunction, NTP threatened to shut down BlackBerry email services if RIM did not pay royalties for the future use of NTP’s patents. In March 2006 RIM agreed to pay \$612.5 million in a last minute settlement that averted the injunction, an amount significantly more than the past damages of \$33.5 million awarded by the court.²

This highly visible case is often quoted as an example of a so called “patent troll”.³ This term is used to describe a company that uses a patent to “hold-up” manufacturing companies and to extort “excessive” royalties that are higher than the “fair share” dictated by the contribution of its patent. Unfortunately, it is notoriously difficult to determine whether royalties are “excessive” and to distinguish between aggressive (but legitimate) bargaining and hold-up. Because a workable definition is lacking, patent trolls are often associated with “non-practicing” or “non-manufacturing” patent holders (often referred to as NPEs, for non-practicing entities).⁴ In contrast to a vertically integrated firm (that both holds a patent and uses it to produce a good on a downstream market), an NPE does not require cross licenses from competitors on the

¹ NTP, Inc. v. Research in Motion, Ltd., No. 3:01-cv-767 (E.D. Va. 2006).

² For a more detailed discussion of this case see Mark A. Lemley and Carl Shapiro, *Patent Hold-up and Royalty Stacking* 85 TEX. L. REV. 1991, 2009 (2007); James F. McDonough III, *The Myth of the Patent Troll: An Alternative View of the Function of Patent Dealers in an Idea Economy* 56 EMORY L. J. 189, 194-195 (2006).

³ The term “patent troll” was coined by Peter Detkin, former assistant general counsel for Intel, after Intel was sued for libel for its use of the term “patent extortionist”: “A patent troll is somebody who tries to make a lot of money off a patent that they are not practicing and have no intention of practicing and in most cases never practiced.” McDonough III, *supra* note 2, at 192. Ian Austen and Lisa Guernsey, *A Payday for Patents ‘R’ US*, N.Y. TIMES, May 2, 2005.

⁴ See e.g. McDonough III, *supra* note 2, at 189; Carl Shapiro, *Injunctions, Hold-Up and Patent Royalties* 24 (Working Paper, 2006), available at <http://faculty.haas.berkeley.edu/SHAPIRO/royalties.pdf>; John M. Golden, “Patent Trolls” and Patent Remedies 85 TEX. L. REV. 2111, 2112 (2007). As Lemley and Shapiro, *supra* note 2, 2009 put it: “Defining a patent troll has proven a tricky business, but that does not mean the problem does not exist. Nonpracticing entities file 30–40% of all patent suits in the computing and electronics industries, for example.”

downstream market. Therefore, so the argument goes, an NPE is not constrained in its behavior and may choose unjustifiably high royalty rates that are not in proper relation to the contribution of its patents. In the example of the Blackberry case, NTP, Inc. is a non-practicing entity. It does not produce cell phones or any other goods but owns a portfolio of patents that it licenses to manufacturing companies.

The identification of NPEs and patent trolls has important legal consequences. In the US Supreme Court's 2006 *eBay v. MercExchange* decision⁵, Justice Kennedy⁶ stated in his concurring opinion that there are firms that "use patents not as a basis for producing and selling goods but, instead, primarily for obtaining licensing fees. For these firms, an injunction ... can be employed as a bargaining tool to charge exorbitant fees". The implied message behind Kennedy's statement was that the lower courts should be careful in granting injunctive relief to non-practicing patent holders. In fact, since *eBay v. MercExchange* many district courts have been regularly denying injunctive relief to non-manufacturing and non-competing firms.⁷

In this paper we challenge this legal practice. We analyze how patent holders choose their royalties depending on their business model (vertically integrated or not), the structure of the upstream and downstream markets, and the type of licensing agreements feasible (linear and non-linear royalties, cross-licensing, patent pools). We show that an NPE does indeed have different incentives regarding royalty rates than a vertically integrated company. However, we argue that there is no reason to presume that a non-integrated patent holder will charge *higher* rates. To the

⁵ *eBay Inc. v. MercExchange, L.L.C.*, 126 S.Ct. 1837 (2006).

⁶ Joined by Justices Stevens, Souter, and Breyer.

⁷ See John L. Dauer Jr. and Sarah E. Cleffi, *Trends In Injunctive Relief In Patent Cases Post-eBay* THE METROPOLITAIN CORPORATE COUNSEL 16 (Feb. 2007): "... a survey of the cases decided since eBay proves useful in identifying one trend in the decisions. What has become apparent thus far is the district courts' attention to the considerations expressed in Justice Kennedy's concurrence and whether the parties are in direct competition. ... When the parties are not in direct competition, the courts ... would likely find monetary damages are adequate. ... To date, district courts appear to have thus far heeded Justice Kennedy's warnings in his eBay concurrence and not issued injunctions to such parties." See also, Bernard H. Chao, *eBay, Inc. V. Mercexchange: The Changing Landscape for Patent Remedies* 9 MINN. J. L. SCI. & TECH. 543, 553 (2008) stating "...the existence of direct competition generally results in a permanent injunction. The converse is also true. Lack of direct competition generally results in the denial of a permanent injunction."

contrary, we show that it is the vertically integrated firm that can have an incentive to raise its royalties in order to raise its rivals' costs and to restrict entry on the downstream market, neither of which motivations hold for non-integrated patent holders. Thus, an integrated firm may charge higher royalties than an NPE.

There are a few recent papers in the law and economics literature dealing with the problem of patent trolls and non-practicing patent holders. Several authors have pointed out that non-practicing patent holders can and do perform important and valuable functions in a market economy. Many NPEs, including universities, government sponsored research labs, and some high technology companies, specialize on their comparative advantage of conducting research and development while leaving the manufacturing of final products to other companies.⁸ NPEs can also foster the dissemination of new technologies and encourage entry on the downstream market because their primary, if not only, source of revenue relies on making their innovations tradable by patenting and licensing.⁹ Other NPEs can act as intermediaries, buying and selling patents to provide liquidity and increased efficiency to technology markets.¹⁰ This literature establishes that a general condemnation of non-practicing patent holders as “trolls” is clearly not warranted. However, none of these papers addresses the question of whether NPEs may have an incentive to charge higher royalties than vertically integrated firms.

In a seminal paper, Shapiro¹¹ identifies two sets of conditions under which a hold-up problem may give rise to patent troll behavior. First, the manufacturing firm is unaware of the patent when it invests in the production of its product. In this case, after the investment is sunk

⁸ Motivated by the example of universities, Lemley refines the definition of a patent troll. He argues that universities are not patent trolls because they are actively engaged in technology transfer, while patent trolls are non-manufacturing entities that do not engage in technology transfer but instead license only the right not to be sued. However, Lemley concedes that this definition is too abstract and could easily be gamed if applied by the courts. See Mark Lemley, *Are Universities Patent Trolls?* 19 (Stanford Public Law Working Paper No. 980776, 2007), available at <http://ssrn.com/abstract=980776>.

⁹ See Damien Gerardin, Anne Layne-Farrar, and Jorge Padilla, *Elves or Trolls? The Role of Non-Practicing Patent Owners in the Innovation Economy*, (TILEC Discussion Paper No. 2008-018, 2008), available at <http://ssrn.com/abstract=1136086>.

¹⁰ See McDonough III, *supra* note 2.

¹¹ Shapiro, *supra* note 4. See also Lemley and Shapiro, *supra* note 2, for a more detailed discussion of the implications of this model.

the patent holder can extort supra-normal royalties by threatening to obtain an injunction and shut down production of the entire product. Second, the manufacturing firm is aware of the patent before it invests in the production of its product, but the patent is “weak”, i.e. the probability that it will be declared valid by a court is considerably smaller than one. In this case Shapiro’s model assumes that the manufacturing firm can either start production without a license and wait for a decision of the court, facing the same sunk cost hold-up problem as described above, or it can threaten to invent around the patent, such that negotiations with the patent holder are based on the assumption that the patent is valid with certainty. In both cases the patent holder will get supra-normal royalties.

The cases considered by Shapiro are illustrations of the hold-up problem and seem to fit the stylized facts of the Blackberry case, but they have nothing to do with the distinction between non-practicing and vertically integrated patent holders. If a hold-up problem as described by Shapiro’s model arises, a non-practicing and a vertically integrated patent holder both have the exact same incentive to exploit it.

We consider a set-up that allows for different market structures and different types of licensing contracts. On the upstream market there are one or several patent holders each of whom has at least one patent that is essential for the production of the downstream good.¹² All parties are aware of all patents and their validity is not in dispute. Thus there is no hold-up problem stemming from weak patents,¹³ but each patent holder has considerable market power because he can threaten to interrupt downstream production if the downstream firms are unwilling to accept his royalties.

¹² We restrict our attention to the case in which the patents at issue are essential for downstream production, so there are no feasible substitutes. This is the most suitable case for our purposes since the presence of viable substitutes prevents hold-up. If there are imperfect but viable substitutes for the patent upstream, the analysis is more complicated. For an analysis of the welfare effects of patent pools when not all patents are essential, see Daniel Quint, *Economics of Patent Pools when Some (but not All) Patents Are Essential* (SIEPR Discussion Paper 06-028, 2008), available at, <http://www.ssc.wisc.edu/~dqint/papers/patent-pools-quint.pdf>.

¹³ We find this form of hold-up far less plausible since licensees can and do challenge the validity of a patent and need not negotiate as if the patent were valid and infringed.

Standard models of oligopolistic competition suggest that there are two effects that may induce firms to charge excessive royalties: First, when upstream patent holders choose their royalties they do not take into account that higher royalties reduce the profits of the downstream firms. This is the well known “double mark-up effect”. Second, an upstream patent holder does not take into account that if he raises the royalty rate for his patent he reduces the profits of the other patent holders, because the patents are perfect complements. This is called the “complements effect”.¹⁴ In the recent IP literature, the complements problem has gained the new moniker of “royalty stacking” because the many firms’ royalty rates stack up to form a large cumulative burden for manufacturers.¹⁵ Both effects may result in a cumulative royalty rate that is higher than the royalty rate chosen by a fully integrated monopolist (i.e. a monopolist who owns all essential patents and all downstream firms).

In Section 2 we analyze how patent holders choose their royalties depending on their business model (vertically integrated or not) and the structure of the upstream and downstream markets if all firms act non-cooperatively and set individual linear and non-discriminatory royalties. It is well known that vertical integration eliminates the vertical double mark-up problem within the vertically integrated firm (but not across firm boundaries), which leads to lower royalties. So, for example, if the firm that holds a patent on an essential element of a DVD player is separate from the firm that manufactures the DVD player, then the implied royalty rate for the patent may be higher than it would be if the manufacturer also held the patent (i.e., had integrated R&D and manufacturing) because the two separate firms both require a profit margin whereas for an integrated firm a single profit margin suffices. However, there is a second effect that goes in the opposite direction. If a vertically integrated firm raises its royalty rate, it does not

¹⁴ Both effects have been first described by Auguste A. Cournot, RESEARCHES INTO THE MATHEMATICAL PRINCIPLES OF THE THEORY OF WEALTH, New York: Kelly 1971 (English edition of RESEARCHES SUR LE PRINCIPES MATHÉMATIQUES DE LA THÉORIE DES RICHESSES, Paris 1838).

¹⁵ See Lemley and Shapiro, *supra* note 2.

affect its own cost of production — the higher royalty is a cost of manufacture but also a source of revenue for the upstream division — but that higher royalty does raise the costs of its competitors on the downstream market. By raising the costs of its downstream rivals the vertically integrated firm can increase its downstream market share and its profits. In the presence of these two countervailing forces, the overall effect on royalties is ambiguous.

In Section 3 we allow for coordinated royalty setting on the upstream market, i.e. cross-licensing agreements or patent pools. If firms set royalties jointly through patent pooling, an association of companies will internalize the effects of horizontal double mark-up/royalty stacking and therefore may reduce aggregate royalties as compared to a situation where royalties are set non-cooperatively. If such an association were comprised solely of vertically integrated firms, and thus included cross-licensing agreements, it could solve both the vertical double mark-up and the horizontal complements problems. However, if vertically integrated firms can charge discriminatory royalties, they have a strong incentive to charge prohibitively high royalties to non-integrated downstream producers in order to reduce downstream competition. Non-integrated patent holders, on the other hand, benefit from vigorous downstream competition because it tends to expand sales and lead to more royalty payments. We also show that if vertically integrated firms and non-integrated upstream firms agree to pool their patents and share profits equally, the royalties charged by the non-integrated upstream firm must be higher than the ones charged by the integrated company, because the upstream firm has to make all of its profits upstream.¹⁶

In Section 4 we extend the analysis to the case where patent holders can charge non-linear royalties. When firms are non-integrated, upstream firms can use non-linear royalty schemes to

¹⁶ Note that this finding is consistent with the findings reported in Aoki and Nagaoka, who observe that “there should be extra distribution to research firms [within the pool] to compensate for the lack of production profits.” Reiko Aoki and Sadao Nagaoka, *The Consortium Standard and Patent Pools* 17 (HISTAT Discussion Paper 32, 2004), available at, www.ier.hit-u.ac.jp/pie/Japanese/discussionpaper/dp2004/dp222/text.pdf.

eliminate any vertical and horizontal distortions, just as the vertically integrated firms do. Vertically integrated firms, however, still have an incentive to raise their rivals' costs and to use their royalties to force downstream competitors to exit the market.

These results show that non-integrated companies are constrained by the impact their behavior has on the downstream market and in some cases they may charge *lower* royalties than their vertically integrated counterparts. We conclude that whether a company charges excessive royalties depends on whether there is scope for hold-up, either because of sunk investments or because of weak patents (coupled with high litigation costs). These factors are orthogonal to whether patent holders are vertically integrated or not. Unfortunately, this result implies that the search goes on for readily identifiable characteristics that can objectively indicate a patent troll in advance of any hold-up behavior.

2. Complements, Double Mark-Ups, and Raising One's Rivals' Costs

Consider a high technology good such as a cell phone or a DVD player that is based on a technological standard requiring the use of a number of different patented technologies to be operational. Each of the patents is essential in the sense that no firm can legally produce the good without access to the patent. The essential patents are often owned by different companies. Thus, each firm that produces the good requires a license from each of the patent holders. We call the market for licenses the "technology" or "upstream market" and the market for the good (e.g., the cell phone or the DVD player) the "product" or "downstream market". Some firms may be vertically integrated in that they hold an essential patent and produce the final good. Other companies may be non-integrated: either they hold a patent but do not use it to produce the final good themselves, or they produce the final good but do not hold any essential patents. We call these non-integrated companies upstream and downstream firms, respectively. To fix ideas, we

assume that each company on the upstream market owns exactly one essential patent and that all firms on the downstream market have the same cost functions and produce end goods that are substitutes.

The focus of our analysis is on the royalties that will be charged by vertically integrated and non-integrated patent holders. In this section we assume that all patent holders are restricted to using linear, non-discriminatory royalties.¹⁷ The prevalence of linear royalty rates in real world licensing contracts can be explained by asymmetries of information between the licensee and the licensor and their risk-sharing properties.¹⁸ Non-discriminatory royalties are often explicitly noted in contracts and are a common commitment made in standard setting (known as RAND, or reasonable and non-discriminatory licensing). In Section 4 we extend the analysis to the case where firms can use two-part tariffs and where they can discriminate between different downstream producers.

What royalties will be charged by a vertically integrated vs. a non-integrated patent holder? The answer to this question depends on the market structure. It turns out that there are several effects that have opposing impacts on vertically integrated as compared to non-integrated patent holders. We will show that, in general, it is ambiguous whether a non-integrated patent holder charges higher or lower royalties than a vertically integrated firm. Nevertheless, it is highly instructive to understand the different effects in order to evaluate their relative importance.

Consider a non-integrated upstream firm owning one patent that is required for downstream production. This patent holder has a monopoly on its patent and will charge a royalty

¹⁷ In other words, for now we assume straight forward royalty rates comprise the license payment; non-linear royalty schedules and lump sum fees are not used.

¹⁸ If there are asymmetries of information, linear royalties can be used as optimal screening or signalling devices. For a discussion of optimal screening see A. W. Beggs, *The Licensing of Patents under Asymmetric Information* 10 INT'L J. INDUS. ORG. 171 (1992); for a discussion of signaling devices see Nancy T Gallini and Brian D Wright, *Technology Transfer under Asymmetric Information* 21 RAND J. ECON. 147 (1998). Choi shows that linear royalties are optimal when there is moral hazard. Jay P. Choi, *Technology Transfer with Moral Hazard* 19 INT'L J. INDUS. ORG. 249 (2001). Bouquet et al. characterize licensing agreements with optimal risk sharing properties. Alain Bousquet, Helmuth Cremer, Marc Ivaldi, and Michel Wolkowicz, *Risk Sharing in Licensing* 16 INT'L J. INDUS. ORG. 535 (1998).

that maximizes profits. If the patent holder increases its royalty rate, it will increase the marginal cost that each downstream firm has to incur. Downstream firms will (at least partially¹⁹) pass through this cost increase to their customers. Thus, downstream prices will increase and the quantity of the final good sold on the downstream market will decrease. When the monopolistic patent holder chooses its royalty rate, it takes into account that a higher royalty rate raises its profit on each unit but lowers the number of units sold. Because marginal costs are (essentially) zero for patent licensing, the patent holder will raise its royalties to the point that a one percent increase in the royalty rate yields a one percent decrease in the quantity sold. This is just the standard monopoly profit maximization problem as applied to patent licensing.

However, in contrast to the standard monopoly problem, there are several additional externalities that the monopolistic patent holder imposes on other producers who are active in these markets. First, there is an external effect on the other patent holders in the upstream market. Recall that all patents are perfect complements: each of them is essential for producing the final good, so for each unit sold on the downstream market exactly one license of each of the patents is required. If one patent holder raises its royalty and thereby reduces the total quantity of the final good sold downstream, it reduces the revenues of all the other patent holders. This is the so called “complements problem” that was first described by Cournot.²⁰

Second, there is an external effect on the firms producing on the downstream market. If the downstream market is not perfectly competitive, the pass through rate is less than one. Thus, the portion of the increase in the royalty rate that is not passed through to end prices reduces the

¹⁹ The pass-through rate may be larger than 1, for example in the case of a constant elasticity demand function.

²⁰ In a famous example, Cournot, *supra* note 14, considered two monopolists, one controlling copper, the other zinc, both selling to the competitive brass industry. The production of brass requires copper and zinc in fixed proportions. Thus, if the copper monopolist increases his price, the production of brass is reduced, which imposes a negative external effect on the zinc monopolist who now also sells less. See also Carl Shapiro, *Theories of Oligopoly Behavior*, in 1 HANDBOOK OF INDUSTRIAL ORGANIZATION 329, 339, (Richard Schmalensee and Robert Willig eds., 1989) for a more detailed discussion. In another paper Shapiro extends this analysis to the case of complementary patents. See Carl Shapiro, *Navigating the Patent Thicket: Cross Licenses, Patent Pools, and Standard Setting*, in 1 INNOVATION POLICY AND THE ECONOMY 1,4-5 (Adam Jaffe et. al. eds, 2001).

profits of the downstream firms by reducing their margins, and the portion of the royalty rate increase that is passed through to end prices reduces the quantities that they can sell downstream. This is the well known “double mark-up problem” that arises between two vertically related firms that both have market power.

Both the complements effect and the double mark-up effect tend to raise royalties above their optimal level. In fact, if all patent holders set their royalty rates independently and non-cooperatively, then the sum of all royalties would be higher than the total royalties charged by a fully integrated monopolist, i.e. a monopolist who owns all the essential patents and all downstream firms. In this case, not only consumers, but also firms on the upstream and downstream markets would benefit if total royalties were lower.

A well known remedy to mitigate the double mark-up problem is vertical integration. In a simple chain of monopolies, vertical integration eliminates the double mark-up problem. However, in a more complicated world with several upstream and downstream firms, vertical integration need not improve social welfare. To see this, suppose that one upstream and one downstream firm vertically integrate. When the upstream division of this integrated firm increases its royalty rate, it fully internalizes the effect on the profits of its downstream division. This mitigates the double mark-up problem within the integrated firm. However, with vertical integration there is a new strategic effect pointing in the opposite direction. By raising its royalty rate the vertically integrated firm raises the costs of its downstream competitors without raising its own cost. Note that the royalty rate of its own downstream division is just an internal transfer payment that shifts profits from the downstream to the upstream division but leaves total firm profits unaffected. Thus, by raising the marginal costs of its rivals the integrated firm gains a

competitive advantage, pushes its rivals' prices higher, and as a result gains a higher market share downstream for the sale of its own good.²¹

The source of this "raising one's rival's costs effect" is vertical integration. A non-integrated patent holder does not participate in downstream profits. Thus an upstream patent holder benefits if the downstream market becomes more competitive, if there is market entry, and if more units of the final good are sold as a result. In contrast, a vertically integrated firm makes part (or most) of its profits on the downstream market. Therefore it wants this market to be less competitive, it will oppose market entry, and it will use the royalty rate for the patent of its upstream division to achieve an advantage over rival downstream firms that do not hold patents for cross licensing.

To illustrate this point suppose that there is just one upstream patent holder and a large number N of potential downstream firms with identical cost functions that compete in quantities on the downstream market. Suppose that a Cournot equilibrium exists on the downstream market for any royalty rate $r > 0$ charged by the upstream monopolist.²² We do not have to impose any other assumptions on the cost and demand functions. Suppose first that the patent holder is non-integrated and not active on the downstream market. In this case it will charge the monopolistic

²¹ The raising one's rivals' costs effect has first been described by Salop and Scheffman. See Steven C. Salop and David T. Scheffman, *Raising Rivals' Costs* 37 AM. ECON. REV. 267 (1983); Steven C. Salop and David T. Scheffman, *Cost-Raising Strategies* 36 J. INDUS. ECON. 19 (1987). However, they restrict attention to a dominant firm that can affect marginal and average costs of a competitive fringe. They show that the dominant firm will raise its rivals' cost in order to either foreclose the market or to induce competitors to raise their prices and to relax competition. The situation we are interested in is closer to Ordover, Saloner and Salop, and Kim. Ordover et al. consider a two-stage duopoly model with price competition and differentiated products. Their model looks at the more conventional case where the goods produced upstream are perfect substitutes while we look at the opposite case of perfect complements. In their model a vertically integrated firm must be able to commit ex ante to a price for the input good, even though there is an incentive to reduce this price ex post. No such commitment is necessary in our analysis. Janusz Ordover, Garth Saloner, and Steven C. Salop, *Equilibrium Vertical Foreclosure* 80 AM. ECON. REV. 127 (1990). Kim analyses a model similar to ours, but he restricts attention to the case of a linear demand curve. Sung H. Kim, *Vertical Structure and Patent Pools* 25 REV. INDUS. ORG. 231 (2004).

²² That is, an equilibrium where firms compete on quantities supplied and the equilibrium price is determined by the sum of these quantities. Novshek offers a set of fairly weak sufficient conditions that guarantee existence and uniqueness of an equilibrium in the Cournot model. He requires that there exists a \bar{Q} such that $P(Q) > 0$ for all $Q < \bar{Q}$ and that $P(Q) = 0$ for all $Q > \bar{Q}$, and that $P(Q)$ is twice continuously differentiable with $P'(Q) < 0$ for all $Q < \bar{Q}$. Furthermore, $P'(Q) + q \cdot P''(Q) < 0$ for all $0 \leq q \leq Q \leq \bar{Q}$. See William Novshek, *On the Existence of Cournot Equilibrium* 52 REV. ECON. STUD. 85 (1985).

royalty rate $r^M > 0$ that maximizes profits given the Cournot equilibrium on the downstream market. The larger N , the more competitive the downstream market, the smaller is the mark-up charged by the downstream firms, the greater is the volume of goods sold, and the higher are the profits of the patent holder. Suppose now that the upstream patent holder vertically integrates with one of the downstream firms. One possible strategy of the integrated firm is to charge a royalty rate that is so high that no other downstream firm can make a profit and are thus forced to exit the market. Note that the downstream division of the integrated firm is not affected by the rate increase, because within the integrated firm the royalty rate is a mere transfer price that has no effect on overall firm profits: each dollar spent on higher royalties by the downstream division is a dollar of revenues for the upstream division. Thus, by raising its royalty rate to a prohibitively high level, the integrated firm can foreclose all other downstream firms and monopolize the downstream market. Because profits are maximized by a fully integrated monopolist (who controls both the upstream and the downstream market) this strategy is indeed optimal.²³

The point of this extreme example is to show that a vertically integrated firm has a natural inclination to use its royalty rate to raise the costs of its rivals in order to increase the profits it makes on the downstream market. It can be shown that if the downstream firms are not all identical, but instead sell sufficiently differentiated products or have lower marginal costs than the downstream division of the integrated firm, then the integrated firm will not want to completely shut out all of its downstream rivals from the market.²⁴ The reason is that the presence of other (differentiated) downstream firms extends the market and thus increases the royalty income of the upstream division. However, it is still the case that the integrated firm

²³ And has thus been a real world problem. See, e.g., Case T-5/97, *Industrie des Poudres Sphériques SA v Commission*, E.R.C. II-3755; *United States v. Aluminum Co. of America*, 148 F.2d 416 (2d Cir. 1945).

²⁴ See also X.H. Wang and B.Z. Yang, *On Licensing under Bertrand Competition*, AUSTRALIAN ECONOMIC PAPERS 38 (1999) for a similar result with Bertrand competition.

charges a royalty rate that discriminates against its downstream rivals and that may be higher than the royalty rate that a non-integrated upstream firm would have chosen for otherwise identical circumstances and patented technology.

If there are several vertically integrated upstream firms, each of them has an incentive to raise the costs of the other vertically integrated firms by raising its royalties. This gives rise to a prisoners' dilemma. All vertically integrated upstream firms would be better off if they would all charge lower royalties, but for each firm it is optimal to charge high royalties. In the next section we will see that cross-licensing agreements or patent pools can be used to solve this prisoners' dilemma, although they do not necessarily solve the problem of raising rivals' costs.

We conclude that vertical integration has two effects on the market outcome. On the one hand, vertically integrated firms internalize the double mark-up effect between upstream and downstream divisions, which tends to improve efficiency. On the other hand, the "raising one's rivals' cost effect" tends to raise royalties and to reduce efficiency. Which of the two effects dominates depends on the specific structure of the cost and demand functions. Schmidt²⁵ shows in a much more general model, with arbitrarily many firms on the upstream and downstream markets and a general model of downstream competition, that a market in which all firms are vertically integrated may give rise to higher *or* lower total royalties than a market in which all firms are non-integrated. For the special but natural example of Cournot competition with linear demand and cost functions and identical firms, Kim²⁶ shows that if the number of vertically integrated firms is not too large, then vertical integration induces an equilibrium price on the downstream market that is strictly higher than the equilibrium price that obtains under non-

²⁵ Klaus M. Schmidt, *Complementary Patents and Market Structure* (CEPR Discussion Paper Series, No. 7005), available at www.cepr.org/pubs/new-dps/dplist.asp?dpno=7005.

²⁶ Kim, *supra* note 21, Theorem 3.

integration. Thus, vertical integration may well reduce total output, total industry profit, and social welfare.

3. Pooling Agreements

So far we have assumed that all firms active on the upstream market choose their royalty rates independently and non-cooperatively. However, it is quite common that firms coordinate their behavior through bilateral cross-licensing agreements. Moreover, within standard setting contexts it is becoming more common for at least some patent holders among a group of firms with complementary patents to coordinate their licensing through patent pools. For instance, many of the video and audio MPEG standards already involve patent pools, a pool is under development for RFID (radio frequency identification) standards, and a number of pools are under development for mobile telecom standards.²⁷ Such cross-licensing and pooling agreements are often reciprocal. Each firm agrees to a low royalty rate for its own patents as long as the other firms involved in the agreement also charge a low royalty rate.

In this section we show that if all firms are vertically integrated — as was largely the case within standard setting in past decades — it is indeed possible to solve the complements and the double mark-up problem and to sustain the fully integrated monopoly outcome with a set of cross-licensing agreements. If the firms are symmetric, they will charge symmetric royalty rates. Because everything is symmetric, the royalty payments just cancel out and there are no net payments in equilibrium for other vertically integrated firms. However, if downstream specialists

²⁷ See Press Release, Via Licensing Corporation, Via Licensing Announces Progress Toward Establishing AVC License Terms (Oct. 15, 2003), available at http://www.vialicensing.com/news/details.cfm?VIANEWS_ID=309; Sabra Chartrand, *The Federal Government Will Allow a Group of Companies to Unify Administration of 27 Patent*, N.Y. TIMES, June 30, 1997; Olga Kharif, *The RFID Patent Pool: Will It Make Waves?*, BUSINESSWEEK, Sept. 11, 2006. **ADD CITE FOR CDMA-2000 and LTE.**

are present in the market (such as, for example, semiconductor chip fabricators that are not active in chip design), the problem of raising rivals' costs returns.

It is sometimes argued that because a non-integrated patent holder's behavior is not restricted by the need to get cross licenses from fellow patent holders, it will turn into a patent troll charging unjustifiably high royalties. Without the constraint that reciprocal cross-licensing imposes, so the argument goes, non-integrated patent holders will hold up vertically integrated firms and charge excessively high royalties. However, this argument misses the mark. We show that it is also possible to solve the complements and the double mark-up problem with a pooling agreement even if some upstream firms are non-integrated (keeping with the chip example, these firms would be the design-only shops, with no manufacturing plants). However, in this case firms are no longer symmetric. The non-integrated patent holders make their profits on the upstream market only and must insist on charging relatively higher royalties, while the integrated firms make their profits both upstream and downstream and tend to prefer lower royalties that shift profits downstream. If all (integrated and non-integrated) patent holders want to split profits equally, then the non-integrated patent holders must charge higher royalties than the integrated firms.

To make these arguments more precise, let us start with the case where there are $N \geq 2$ symmetric and vertically integrated companies. Suppose that these firms negotiate a set of cross-licensing agreements according to which each firm charges each other firm the royalty $r \geq 0$ for using its patent. Thus, the total royalty that each firm has to pay for each unit of the final good it sells on the downstream market is equal to sum of rival upstream firm royalty rates, which (given the symmetry assumed) amounts to multiplying the royalty rate r by the number of rivals: $R = (N-1)r$. Let R^* denote the total royalty payment that induces each firm to produce $1/N$ of the

monopoly output on the downstream market. If the firms choose $r = r^* = R^* / (N - 1)$, then, given the set of cross-licensing agreements, each firm will produce $1/N$ of the monopoly quantity downstream and each firm will earn $1/N$ of the fully integrated monopoly profit. Thus, if all vertically integrated firms agree to charge the same royalty r^* the complements and the double mark-up problems disappear, just as if the firms had literally merged into a single monopoly firm. Furthermore, the cross licensing agreements prevent the firms from raising their (vertically integrated) rivals' costs.

The above example can be seen as a patent pool. Note that such a pool formed by the N vertically integrated firms is an agreement that fixes input prices. This could be regarded as an illegal cartel by antitrust authorities. In fact, the vertically integrated firms charge each other royalties that are passed through to consumers and induce downstream divisions to charge the monopoly price. But, as we have just seen, this price is lower than the price that would obtain if firms did not coordinate their behavior. The reason is of course the complements problem. There are N monopolists upstream. If they do not coordinate their behavior each will try to exploit his monopoly power and total royalties will be even higher than the royalty charged by a fully integrated monopolist.

To be sure, the outcome established by the patent pool is a monopoly outcome. But, the monopoly is based on the IP rights of the patent holders. These rights have been granted to reward them for their innovations. The patent pool makes sure that none of the patent holders monopolizes his invention on his own, thereby imposing negative externalities on everybody else, but patent holders instead coordinate their behavior so as to make everyone better off.

Nevertheless, antitrust authorities have tended to view patent pools that fix input prices with great suspicion, while they are generally much less skeptical about bilateral, reciprocal cross

licensing agreements.²⁸ However, in terms of economic effects there is not much difference between a patent pool and bilateral cross licensing agreements. In the appendix we show, for the example of a linear Cournot game, that the same outcome can be sustained in equilibrium if only bilateral cross-licensing agreements are feasible, rather than a full fledged pool. Here, two firms i and j agree to a reciprocal royalty rate among themselves. This royalty rate has to be optimal given the royalty rates that all other pairs of firms have already agreed to. We show that if all firms bilaterally agree to charge each other r^* , then no pair of firms has an incentive to deviate and to agree to a different royalty rate among themselves.²⁹ Thus, the patent pool outcome can also obtain as an equilibrium outcome if firms are constrained to write bilateral cross-licensing agreements.

If, in addition to the N vertically integrated firms, there are some non-integrated downstream firms, the double mark-up and the raising one's rivals' cost effects reappear. This scenario, where the more traditional large vertically integrated firms compete with both up- and downstream specialists, is the best description of the composition of firms participating in most cooperative standard setting bodies today. In this common case, under our model the vertically integrated firms will charge the low reciprocal royalty rate r^* only among themselves but a higher royalty rate $r > r^*$ to their non-integrated downstream competitors. Part of the increased royalty rate is due to the downstream specialists' lack of patents to offer in cross license,³⁰ which

²⁸U.S. DEP'T JUSTICE & FED. TRADE COMM'N, ANTITRUST ENFORCEMENT AND INTELLECTUAL PROPERTY RIGHTS: PROMOTING INNOVATION AND COMPETITION (2007), available at <http://www.ftc.gov/reports/innovation/P040101PromotingInnovationandCompetitionrpt0704.pdf>; Commission Notice, Guidelines on the Application of Article 81 of the EC Treaty to Technology Transfer Agreements, 2004 O.J. (C 1001/2). See also Richard Schmalensee, STANDARD SETTING, INNOVATION SPECIALISTS AND COMPETITION POLICY, mimeo 2009, available at <http://ssrn.com/abstract=1219784>, who argues that antitrust policy should not allow or encourage collective negotiation of patent royalty rates.

²⁹ This equilibrium need not be unique. However, even if there are multiple equilibria, the equilibrium that implements the monopoly outcome maximizes total surplus of all firms and is a natural focal point.

³⁰ For a general introduction to the interaction between contract terms see Eric Brousseau and Jean-Michel Glachant, THE ECONOMICS OF CONTRACTS: THEORIES AND APPLICATIONS (2002).

eliminates a payment in kind. Some portion of the increase, however, is set in order to discriminate against the non-integrated downstream rivals.

What happens if in addition to the N vertically integrated firms there are also some number $M > 0$ of non-integrated patent holders? Is it still possible to solve the complements and the double mark-up problem with a multi-lateral cross-licensing agreement? If the total royalty rate that each downstream division has to pay equals the sum set before, that is R^* , then each firm will again produce $1/N$ of the monopoly output, just as before. However, the non-integrated patent holders have no interest in cross-licensing agreements because they do not require a license for the other patents. Furthermore, note that the integrated firms make part of their profits upstream and part of them downstream, while the non-integrated patent holders rely entirely on their royalty income. Finally, vertically integrated firms can also accept a portion of their upstream payment in the form of a cross license, which tends to reduce the explicit royalty rate sought. For these reasons non-integrated patent holders tend to ask for higher royalties than vertically integrated firms. In particular, if all firms have the same bargaining power and agree to share profits equally, the non-integrated patent holders must get a higher royalty rate than the vertically integrated firms.

If firms were constrained to charge equal royalties independent of whether they are active on the downstream market or not, a conflict of interest would arise. Vertically integrated firms prefer a royalty rate r that is somewhat smaller than the royalty rate $r^* = R^*/(M+N-1)$ that implements the monopoly outcome. This lowers total industry profits, but it does so at the expense of the non-integrated upstream firms, while the vertically integrated firms benefit from the lower downstream costs. On the other hand, the non-integrated upstream firms prefer a royalty rate that is somewhat higher than the monopoly rate $r^* = R^*/(M+N-1)$, because they want to shift profits from the downstream to the upstream market.

4. Non-linear and Discriminatory Royalties

Firms may also use non-linear royalty schemes, such as two-part tariffs. A two-part tariff is a license with an upfront lump sum fee paid in conjunction with a linear running royalty rate based on sales spread over a specified time period. Such licenses are common in practice, where the upfront fee reflects the agreed upon value of the patented technology while the running royalty shares the risk over any remaining uncertainty regarding the commercial success of the product employing the patented technology. If cross-licensing agreements can be written, the ability to employ non-linear payment schedules of this sort does not change the analysis. The parties can set the linear portion of the royalties such that the monopoly outcome is implemented and then use the fixed fees to redistribute profits.

Non-linear royalties are more interesting if firms have to set their rates independently and non-cooperatively, as is often the case in standard setting bodies. In the following analysis we show that if all firms are non-integrated, the monopoly outcome will still obtain. To see this, suppose that each non-integrated upstream patent holder u , $u = 1, \dots, N$, makes a take-it-or-leave-it offer of a two-part tariff consisting of a fixed fee R_u and a linear royalty rate r_u .³¹ The first step of the argument is to show that all of the non-integrated downstream firms must make zero economic profits in equilibrium.³² Suppose to the contrary that downstream firms make an economic profit that is strictly positive. Then an upstream patent holder could raise its fixed fee

³¹ The combination of take-it-or-leave-it offers and two-part tariffs implies that upstream patent holders have all the bargaining power and that downstream firms do not get any rents. If we allow for a more complex bargaining procedure that gives some bargaining power to downstream firms, this extreme result disappears. However, all the qualitative results described below still hold.

³² Recall that accounting profits are very different from economic profits, which account for opportunity costs in addition to costs of production. Zero economic profits simply indicate a competitive market, where downstream firms do not earn anything above the competitive return for their production investments.

R_u in order to capture this profit for itself, without affecting the downstream firm's decision to continue participating in the market. As long as economic profits are non-negative, the downstream firm will stay in the market. In fact, if the sum of the fixed fees of the other upstream firms leaves any profit for the downstream firms, then each upstream firm has an incentive to further raise its fixed fee until all downstream firms make zero economic profits. Thus, in equilibrium upstream firms will extract the entire surplus from downstream firms. Note that this extreme outcome is due to the fact that the upstream firms provide all of the IP while the downstream firms are solely responsible for assembly. If a downstream firm controls an input factor that increases the value of the downstream good (such as a brand name, for example) its bargaining power would be stronger and the allocation of profits would surely differ.

The second step of the argument is to show that linear royalties will be set efficiently, i.e. so as to maximize the downstream profits that can then be captured by the upstream firms. To see this suppose that the sum of the linear royalties is higher than the royalty rate that implements the monopoly price. This circumstance has been described as “royalty stacking” in the literature.³³ If rates were to stack to a level higher than the monopolistic rate, the upstream patent holder u could reduce its royalty rate r_u , which would increase total profits, because rates higher than a monopolist's yield strictly smaller profits for the patent holder. The patent holder can simultaneously increase its fixed fee R_u as a means of capturing the entire increase in total profits for itself, leaving the downstream firms with zero economic profits, as just described above. Thus, if the sum of the linear royalties does not lead to the monopoly outcome, each upstream firm has an incentive to change its royalty structure in order to obtain the resulting increase in profits through an increase of the fixed fee. There is a symmetric pure strategy equilibrium in which all patent holders charge the same linear royalties and fixed fees such that the sum of the

³³ See Lemley and Shapiro, *supra* note 2.

linear royalties implements the monopoly outcome and the sum of the fixed fees captures all the downstream profits. Thus, with the flexibility of two-part tariffs both the double mark-up and the complements problem (royalty stacking) disappear. To summarize, if all firms are non-integrated and two-part tariffs can be used, we get the same outcome that obtains with a patent pool even if no cross-licenses can be written.

However, this result holds only if all patent holders are non-integrated – an unlikely scenario. Today, most complex industries, where the complements and double mark-up problems are likely to emerge, are characterized by a mixture of vertically integrated and non-integrated firms. If there are also some vertically integrated firms, the equilibrium described above no longer exists. In fact, if there are at least two vertically integrated firms along with the other non-integrated firms, then there is no symmetric pure strategy equilibrium where all patent holder royalty rates are the same – some asymmetry must persist. To see why this is so, suppose that there is an equilibrium in which all firms charge symmetric fees and royalties (r, R) . First of all, it must be the case that the fixed fees extract all the profits from the downstream market. Otherwise, each firm would have an incentive to further raise its fixed fee. But if fees and royalties extract all of the downstream economic profits, then all vertically integrated firms are indifferent whether or not to produce downstream. Suppose that vertically integrated firm i further increases its linear royalty rate and/or its fixed fee. The increase does not affect firm i 's own costs, but it does raise the costs of its downstream rivals. As a result, some other firms will now decide not to buy the licenses at all and to produce a quantity of zero – all to the benefit of firm i . Therefore, firm i has an incentive to deviate from the proposed equilibrium candidate. Thus, we conclude that the “raising rivals’ costs” effect implies that a symmetric, pure strategy equilibrium does not exist when both integrated and non-integrated firms are present.

As we saw above, when all downstream firms are vertically integrated, then firms can deal with the “raising rivals’ costs” effect by writing a cross-licensing agreement. However, they nonetheless have an incentive to discriminate against non-integrated downstream firms in order to jointly monopolize the downstream market. Non-integrated upstream firms do not have this incentive. To the contrary, they benefit from more downstream competition because it increases downstream quantities sold and therefore increases their royalty income.

5. Conclusions

Our analysis has shown that the presumption that non-practicing patent holders always charge higher royalties than vertically integrated companies is not warranted. It is true that if firms set linear royalties non-cooperatively, a vertically integrated firm will internalize the vertical double mark-up problem which tends to reduce royalties. On the other hand, a vertically integrated firm also has an incentive to raise its royalties in order to raise its rivals’ costs and to restrict entry on the downstream market, especially among non-integrated downstream firms. The overall effect on rates is ambiguous. In fact, under some circumstances a vertically integrated firm may charge *higher* royalties than its non-integrated counterpart.

If firms can coordinate their royalties through patent pools or cross-licensing agreements they can solve the complements and the double-mark-up problem independently of whether they are vertically integrated or not. However, if there are non-integrated downstream firms, vertically integrated firms will nonetheless want to increase royalties in order to discriminate against them, while non-integrated patent holders benefit from more competition and more entry downstream.

When both non-integrated and integrated firms hold patents and are able to coordinate their royalties, a conflict of interests arises. Non-integrated firms have to make their profits upstream, while the vertically integrated firms make some part or even the bulk of their profits on

the downstream market. Furthermore, vertically integrated firms may reduce their royalty rates in exchange for a cross license payment in kind — an option not available to upstream firms. Therefore, if firms want to split *profits* equally, or if they want to achieve the same aggregate licensing payments, then non-integrated firms must get higher explicit *royalties* than the non-integrated firms.

Our analysis suggests that there is no justification for the presumption that non-integrated patent holders always charge higher royalties than vertically integrated companies. Moreover, even when non-integrated patent holders do charge “higher” royalties than their vertically integrated counterparts do, it does not imply that the rates are “excessive” or that the firm is exhibiting troll-like behavior. Rather, non-integrated patent holders naturally require higher royalty earnings because they earn no profits downstream and receive no payments in kind in the form of cross licenses.

In a recent empirical paper Allison, Lemley and Walker³⁴ analyse a data set on litigated patents. Almost by definition litigated patents are patents that are highly valuable. A large fraction of these patents is owned by non-practicing patent holders, held mainly by invention specialists and by patent holding companies. This shows that non-practicing entities play a major role in the modern patent system. Allison et al. compare the characteristics of the most litigated patents (patents that have been litigated at least eight times) to the patents that have been litigated only once. They find that the most litigated patents are much more likely to be owned by non-practicing patent holders. This is consistent with our analysis that suggests that there is more potential for conflict if non-practicing patent holders are involved. Furthermore, in the overwhelming majority of cases studied by Allison et al. the patent holders were invention specialists. Only 7 percent of all lawsuits involved patent-holders that acquired the patent and did

³⁴ John R. Allison, Mark A. Lemley and Joshua Walker, EXTREME VALUE OR TROLLS ON TOP? THE CHARACTERISTICS OF THE MOST-LITIGATED PATENTS, June 2009, available at <http://ssrn.com/abstract=1407796>.

not invent it themselves. This suggests that court-based discrimination against all non-practicing patent holders could have severe adverse effects on the investment incentives of innovation specialist who play an important role in advanced market economies.

We conclude that remedy rules, such as entitlement to injunctive relief, should not depend on the plaintiff's business model status. Whether a company charges "excessive" royalties depends on whether there is scope for hold-up, either because of sunk investments or weak patents. These factors are orthogonal to whether patent holders are vertically integrated or not.

Appendix: Bilateral Cross Licensing

Consider N vertically integrated firms producing with identical constant marginal production cost k . Each firm owns one essential patent. Firms compete in quantities on the downstream market and face a linear inverse demand function $P(Q) = a - bQ$. Each pair (i, j) , $i, j \in \{1, \dots, N\}$, of firms agrees to symmetric cross-licensing at rate $r_{ij} = r_{ji}$, where r_{ij} is the linear royalty charged by firm i to firm j . Thus, the total licensing costs of firm i are given by $R_i = \sum_{j \neq i} r_{ji}$.

We look for a subgame perfect equilibrium with the two following properties:

- Given the licensing cost R_i the firms play a Cournot-Nash equilibrium at the second stage of the linear Cournot game.
- There does not exist a pair (i, j) of firms that could increase its joint profits by agreeing to a different cross licensing rate.

Claim: *There is a symmetric subgame perfect equilibrium in the cross-licensing game in which all firms agree to charge the same cross-licensing royalty rate*

$r_{ij} = r = \frac{a-k}{2N}$. *This licensing rate implements the monopoly outcome.*

Proof: Suppose that each pair of firms agreed to the cross licensing rate $r = \frac{a-k}{2N}$. Then each

firm has marginal cost $c = k + \frac{(N-1)(a-k)}{2N}$. It is straightforward to compute the symmetric

Cournot-Nash equilibrium at the second stage of the game. In equilibrium each firm produces

$q = \frac{a-k}{2bN}$, so the total quantity supplied is $Q = \frac{a-k}{2b}$ which is equal to the monopoly quantity.

The resulting price is the monopoly price $P = \frac{a+k}{2}$, and each firm makes $\Pi_i = \frac{(a-k)^2}{4bN}$ which is

$1/N$ of the monopoly profit .

Suppose now that two firms 1 and 2 consider a deviation and charge each other $r_{12} = r_{21} = r - d$, (where d may be positive or negative). This deviation changes the marginal costs of these two firms to $\underline{c} = k + (N-2)r + r - d = (N-1)r - d$, while the marginal costs of all other firms remain unchanged. We now have to solve for the Cournot Nash equilibrium on the downstream market with asymmetric cost functions. Note that downstream profit functions are given by

$$\begin{aligned}\Pi_i &= (a - b \sum_{j=1}^N q_j - k - (N-1)r + d) \cdot q_i \quad \text{if } i \in \{1, 2\} \\ \Pi_i &= (a - b \sum_{j=1}^N q_j - k - (N-1)r) \cdot q_i \quad \text{if } i \in \{3, \dots, N\}\end{aligned}$$

The first order conditions for profit maximization are given by

$$\begin{aligned}\frac{\partial \Pi_i}{\partial q_i} &= a - 2bq_i - b \sum_{j \neq i} q_j - k - (N-1)r + d = 0 \quad \text{if } i \in \{1, 2\} \\ \frac{\partial \Pi_i}{\partial q_i} &= a - 2bq_i - b \sum_{j \neq i} q_j - k - (N-1)r = 0 \quad \text{if } i \in \{3, \dots, N\}\end{aligned}$$

Symmetry of firms 1 and 2 and firms 3, ..., N requires $q_1 = q_2 = \bar{q}$ and $q_3 = \dots = q_N = \underline{q}$. Solving

for \bar{q} , \underline{q} and P and plugging in $r = \frac{a-k}{2N}$ yields:

$$\begin{aligned}\bar{q} &= \frac{a - k - (N-1)(r-d)}{b(N+1)} = \frac{a-k}{2Nb} + \frac{N-1}{N+1}d \\ \underline{q} &= \frac{a - k - (N-1)r - 2d}{b(N+1)} = \frac{a-k}{2Nb} - \frac{2d}{N+1} \\ P &= \frac{a + Nk + rN(N-1) - 2d}{N+1} = \frac{a+k}{2} - \frac{2d}{N+1}\end{aligned}$$

Thus, the profit of firm $i \in \{1, 2\}$ is given by

$$\Pi_i = \underbrace{(P - k - (N - 1)r + d) \cdot \bar{q}}_{\text{downstream profit}} + \underbrace{(r - d)\bar{q} + (N - 2)r\underline{q}}_{\text{upstream profit}}$$

Substituting P , r , \bar{q} and \underline{q} yields

$$\Pi_i = \frac{-4akN + a^2N^2 + k^2N^2 + 2a^2N + a^2 + 2k^2N + k^2 - 2ak - 2aN^2k - 8d^2N(N - 1)}{4bN(N + 1)^2}$$

Differentiating with respect to d we get:

$$\frac{\partial \Pi_i}{\partial d} = \frac{-16dN(N - 1)}{4bN(N - 1)} = -\frac{4d}{b}$$

Note that this profit function is globally concave and maximized at $d=0$. Thus, no pair of firms has an incentive to deviate and to change its bilateral royalty rate. *Q.E.D.*