GOVERNANCE AND THE EFFICIENCY OF ECONOMIC SYSTEMS
GESY

Discussion Paper No. 286

Competitive Careers as a Way to Mediocracy

Matthias Kräkel*

*University of Bonn, Adenauerallee 24-42, D-53113 Bonn, Germany,
tel: +49228733914, fax:+49 228739210,
E-mail: m.kraekel@uni-bonn.de

November 2009

Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

Sonderforschungsbereich/Transregio 15 · www.sfbtr15.de
Universität Mannheim · Freie Universität Berlin · Humboldt-Universität zu Berlin · Ludwig-Maximilians-Universität München
Rheinische Friedrich-Wilhelms-Universität Bonn · Zentrum für Europäische Wirtschaftsforschung Mannheim

Speaker: Prof. Dr. Urs Schweizer. · Department of Economics · University of Bonn · D-53113 Bonn,
Phone: +49(0228)739220 · Fax: +49(0228)739221

Competitive Careers as a Way to Mediocracy*

Matthias Kräkel†

Abstract

We show that in competitive careers based on individual performance the least productive individuals may have the highest probabilities to be promoted to top positions. These individuals have the lowest fall-back positions and, hence, the highest incentives to succeed in career contests. This detrimental incentive effect exists irrespective of whether effort and talent are substitutes or complements in the underlying contest-success function. However, in case of complements the incentive effect may be be outweighed by a productivity effect that favors high effort choices by the more talented individuals.

Key Words: career competition; contest; mediocracy.

JEL Classification: D72; J44; J45; M51

*I would like to thank Oliver Gürtler, Daniel Müller, Petra Nieken, Anja Schöttner and the participants of the economics workshop of the Leibniz University of Hannover, in particular Hendrik Hakenes, Vilen Lipatov and Georgios Katsenos, for helpful comments. Financial support by the Deutsche Forschungsgemeinschaft (DFG), grant SFB/TR 15, is gratefully acknowledged.

†University of Bonn, Adenauerallee 24-42, D-53113 Bonn, Germany, tel: +49 228 733914, fax: +49 228 739210, e-mail: m.kraekel@uni-bonn.de.

"mediocracy = A society in which people with little (if any) talent and skill are dominant and highly influential." (dictionary)

1 Introduction

Career systems that are not based on individual performance but on criteria like seniority may end up in a situation where people with average or less than average talent are assigned to key positions. At the level of society, the direct consequence would be the emergence of a mediocracy. At first sight, one might suspect that competitive career systems with a performance-based job assignment should lead to a significantly better outcome. However, in our paper we show that competitive careers systems have an inherent tendency to promote the least productive individuals, thus leading to mediocracy. The intuition for this result comes from the fact that more productive people have better fall-back positions than less productive ones when failing in the competition for top positions. Hence, highly productive people have only moderate incentives to win the competition for top jobs, whereas individuals with low productivity have strong incentives to avoid their rather unattractive fall-back positions.

We use a contest model based on Skaperdas (1996) to analyze competitive careers of heterogeneous individuals. At the beginning, all individuals have the chance to reach the top position with the highest possible career income. However, during the career we have losers and winners, where winners still competing for the top position and losers competing for less important jobs in a so-called consolation match.[1] Winning such consolation match defines the fall-back position of an individual.

We consider two different set-ups. First, we analyze a career model with effort and talent (or productivity) being substitutes in the contest-success function of an individual. Our results show that near the top position – individuals only have to pass one hurdle for being assigned to the top job – the individuals with the worst fall-back positions will be most likely to win the contest for the top position, due to the *incentive effect* mentioned above. If individuals have to pass more than one hurdle to reach the top, it will be crucial whether productivity has a higher impact on the fall-back position or the future career and how dissipative the competition is on different career levels. For example, if consolation contests are sufficiently dissipative (i.e., expected utility from participating in these contests is rather low), fall-back positions will be quite unattractive, but most unattractive for the least productive individuals. In this situation, again the less productive individuals are most likely to reach the top.

In a second set-up, we consider a career model where talent and effort are complements in the contest-success function. Now there is an additional effect that works into the opposite direction of the incentive effect: The larger an individual's productivity the more effective will be the exertion of effort to win the contest. For this reason, a highly talented individual may prefer to spend high effort despite an attractive fall-back position. If this *productivity effect* is dominated by the incentive effect, society will still tend to mediocracy. Otherwise, the most talented people will assert themselves in the career competition and be assigned to the top position.

Besides the application to society in general, our model also offers insights for com-

[1] See, for example, Uehara (2009) on consolation contests in a Japanese corporation. For a theoretical analysis of consolation contests see Kiyotaki (2004).

petitive careers in a more concrete situation. For example, it can be applied to career competition between politicians, who first struggle for being elected as local leader of their political party. Thereafter, they may further compete for becoming national party leader, governor or president of a certain state. As another example, consider the case of internal careers within corporate hierarchies. These hierarchies are often organized as internal labor markets with clearly structured career paths.[2] Here, a manager first wants to be promoted to become division head. In further career steps, the manager may be elected to the board of directors or even compete for the position as chief executive officer.

In these applications, those policies would work against the problem of mediocracy that ensure highly talented individuals a significantly higher career income at the top position than less talented candidates. In other words, if wages are not attached to jobs but depend on the productivity of the respective individual, the top position should be quite attractive for the most talented candidates. For example, top positions could be combined with performance-based incentive schemes so that highly productive individuals have sufficiently large expected incomes when reaching the top. By this policy, the detrimental incentive effect mentioned above could be turned into a beneficial one.

However, for some career positions another solution may be appropriate. As an extreme kind of solution, the career designer might rely on the seniority rule for job-promotion decisions. Of course, this policy would completely eliminate career incentives and, therefore, also the detrimental incentive effect. Moreover, the seniority rule has the further advantage of tying promotion to the accumulation of human capital, because the most senior candidate is also the one with the highest amount of on-the-job training. The

[2] See, for example, Baker, Gibbs and Holmström (1994).

seniority rule also works against the problem of influence activities and favoritism.[3] If possible, verifiable performance signals or self-enforcing implicit contracts can be used to outweigh the missing career incentives.

Our paper is related to the literature on rank-order tournaments and contests. Lazear and Rosen (1981), Nalebuff and Stiglitz (1983) and Rosen (1986) belong to the first papers that analyze (job-promotion) tournaments in the context of labor markets. A wide range of papers discuss rent-seeking contests based on the seminal work by Tullock (1980). Our paper builds on the axiomatized contest-success function introduced by Skaperdas (1996), which has been used by a lot of subsequent papers (e.g., Wärneryd (2001)). Konrad (2009) offers a comprehensive overview of the most frequently used contest-success functions, applications of contest models and the most important results in contest theory.

There are also parallels to the public economics literature on bad politicians. These models offer an explanation for why political leaders and presidents often have rather low qualifications or are of low quality. Such models typically follow either the political-agency framework or the citizen-candidate approach (see, among many others, Caselli and Morelli (2004), Messner and Polborn (2004), Mattozzi and Merlo (2007)).

Finally, the paper is related to the work on systematic decision failures in job promotion. For instance, Chakravarty (1993) uses a Bayesian approach to show that too ambitious definitions of elite leads to an inherent problem of selection failures at the top of a hierarchy.

The paper is organized as follows. We start with the analysis of talent and effort being substitutes in the contest-success function. Section 2 deals with the case where individuals are near the top and only have to pass one hurdle to reach the highest possible position.

[3] See Milgrom and Roberts (1988) and Prendergast and Topel (1996).

Section 3 considers a two-hurdle career model with productivity influencing both an individual's fall-back position and his future career. Section 4 offers a robustness check. Here, we analyze whether the main findings also hold under the assumption of talent and effort being complements. Section 5 concludes.

2 One-Hurdle Career

2.1 Basic Model

We consider a career game between $N \geq 3$ risk neutral individuals (e.g., workers of the same cohort entering a firm in a certain period). The structure of this game is sketched in Figure 1.

[Figure 1]

The N players start by simultaneously choosing productive activities (e.g., efforts) $a_i \geq 0$ ($i = 1, \ldots, N$) to become the winner of the major career contest. This winner receives a high career income Y_H. The $N-1$ other players enter a consolation match where they compete for less attractive positions. The winner of this consolation contest earns Y_M, whereas the $N-2$ losers get Y_L with $0 < Y_L < Y_M < Y_H$.[4] We assume that the three career incomes are sufficiently large so that players choose positive efforts in equilibrium.

In both, the major contest for Y_H and the consolation contest each player has to bear the costs of his activity (e.g., disutility of effort). For simplicity, when player i exerts activity level a_i let his costs be equal to a_i. Moreover, in both contests, players face the

[4]Note that in our model career incomes are exogenously given, but as we can see below the general effects still exist in a situation where a career designer endogenously chooses incomes.

same contest-success function based on the one suggested by Skaperdas (1996):[5] If player i chooses a_i and the other players $j \neq i$ choose activity levels a_j, then i's probability of winning is given by

$$p_i = \frac{f(a_i - \alpha\delta_i)}{f(a_i - \alpha\delta_i) + \sum_{j \neq i} f(a_j - \alpha\delta_j)} \tag{1}$$

with $f(\cdot)$ as monotonically increasing impact function that is concave and strictly positive.[6] Hence, each player increases his winning probability by exerting more effort, but the other contestants' efforts as well as luck or measurement error also influence the outcome of the contest. The parameter $\delta_i > 0$ indicates player i's productivity.[7] The lower the value of δ_i the more productive will be the player. If, for example, all players choose identical activity levels, the most productive player will have the highest winning probability, whereas the player with the lowest productivity is least likely to win the contest. The scaling parameter $\alpha > 0$ is identical for all contestants. As can be seen from the equilibrium outcomes below, α measures how dissipative the contest is (i.e., the larger α the smaller will be an individual player's expected utility from participating).

We assume that productivity parameters $\delta_1, \ldots, \delta_N$ are common knowledge. This assumption can be justified for at least two reasons. First, players typically observe each others' qualifications, which are positively correlated with productivity.[8] Second, note that the common-knowledge assumption is only introduced for the contestants.[9] These players often observe each other when working close together at the same organization

[5] This assumption can be motivated by the fact that both contests take place between the same individuals excepting the first-round winner, who is missing in the consolation round.

[6] As the player's activity is assumed to be productive, $f(a_i - \alpha\delta_i)$ may describe a worker's or a politician's output.

[7] Hence, effort and ability/productivity are substitutes in the players' impact function. We will comment on the case of complements in Section 4.

[8] On the one hand, following human capital theory based on Becker (1962), investment in a player's skills rises his productivity, which is certified by a formal qualification. On the other hand, according to Spence (1973), high qualifications can be a credible signal for corresponding high ability.

[9] We do not have a contest designer, who shares the same information as the contestants. Note that our results will not change when introducing an uninformed designer, who endogenously chooses optimal tournament prizes but cannot observe individual productivities.

every day. For example, the two arguments hold for a situation where workers compete for job promotion in internal labor markets and for a situation where politicians compete for becoming the leader of their political party.

Although we do not explicitly specify a welfare function, we assume that welfare will be highest if the most productive player (i.e., the one with parameter value $\min\{\delta_1, \ldots, \delta_N\}$) passes the decisive hurdle by winning the major contest and is assigned to the most important position, which is associated with career income Y_H.

2.2 Consolation Contest

We start the analysis by solving the contest game of those $N-1$ players that have failed in the major contest and, therefore, enter the consolation stage. Here, the winner earns career income Y_M but the $N-2$ losers end up with the lower income Y_L. Player i chooses activity a_i to maximize expected utility

$$\widetilde{EU}_i(a_i) = Y_L + (Y_M - Y_L)\frac{f(a_i - \alpha\delta_i)}{f(a_i - \alpha\delta_i) + \sum_{j\neq i} f(a_j - \alpha\delta_j)} - a_i.$$

Since this objective function is strictly concave and, by assumption, career incomes are sufficiently large to prevent a corner solution where neither player chooses positive effort, the equilibrium is described by the $N-1$ first-order conditions

$$\frac{(Y_M - Y_L) f'(a_i - \alpha\delta_i) \sum_{j\neq i} f(a_j - \alpha\delta_j)}{\left[f(a_i - \alpha\delta_i) + \sum_{j\neq i} f(a_j - \alpha\delta_j)\right]^2} = 1,$$

which can be rearranged to

$$\frac{Y_M - Y_L}{\left[\sum_{j=1}^{N-1} f\left(a_j - \alpha\delta_j\right)\right]^2} = \frac{1}{f'\left(a_i - \alpha\delta_i\right)\left[f\left(a_k - \alpha\delta_k\right) + \sum_{j\neq i,k} f\left(a_j - \alpha\delta_j\right)\right]}$$

with player k denoting an arbitrary opponent of player i. In analogy, the first-order condition of player $k \neq i$ can be written as

$$\frac{Y_M - Y_L}{\left[\sum_{j=1}^{N-1} f\left(a_j - \alpha\delta_j\right)\right]^2} = \frac{1}{f'\left(a_k - \alpha\delta_k\right)\left[f\left(a_i - \alpha\delta_i\right) + \sum_{j\neq i,k} f\left(a_j - \alpha\delta_j\right)\right]}.$$

Combining the right-hand sides of both equations yields

$$\frac{f\left(a_i - \alpha\delta_i\right) + \sum_{j\neq i,k} f\left(a_j - \alpha\delta_j\right)}{f'\left(a_i - \alpha\delta_i\right)} = \frac{f\left(a_k - \alpha\delta_k\right) + \sum_{j\neq i,k} f\left(a_j - \alpha\delta_j\right)}{f'\left(a_k - \alpha\delta_k\right)}.$$

As both sides describe the same monotonically increasing function of $a_i - \alpha\delta_i$ and $a_k - \alpha\delta_k$, respectively, we have

$$a_i - \alpha\delta_i = a_k - \alpha\delta_k, \forall\left(i,k\right).$$

Hence, the higher the productivity of a player the lower will be his equilibrium effort. Intuitively, a highly productive player prefers to save effort costs by choosing a low activity level since activities and productivity parameters are substitutes in the impact function $f\left(\cdot\right)$. By inserting $a_i - \alpha\delta_i = a_k - \alpha\delta_k, \forall\left(i,k\right)$, in player i's first-order condition, the equilibrium activity level a_i^* is described by

$$\begin{aligned}\frac{\left(Y_M - Y_L\right)}{\left(N-1\right)^2 f\left(a_i^* - \alpha\delta_i\right)} &= \frac{1}{f'\left(a_i^* - \alpha\delta_i\right)\left(N-2\right)} \Leftrightarrow \\ a_i^* &= A\left(\frac{\left(N-2\right)\left(Y_M - Y_L\right)}{\left(N-1\right)^2}\right) + \alpha\delta_i\end{aligned}$$

with $A(\cdot)$ denoting the monotonically increasing inverse function of f/f'. Equilibrium activity increases in the size of the career income difference $Y_M - Y_L$, because any contestant earns at least Y_L in the consolation round. Furthermore, a_i^* is non-increasing and for $N > 3$ strictly decreasing in the number of contestants. This effect can be labeled discouragement effect: Each player exerts less effort when the number of opponents increases since his relative impact on the outcome of the contest becomes smaller.

Finally, we insert equilibrium activities in the objective function $\widetilde{EU}_i(a_i)$. Thus, in equilibrium player i's expected utility is given by

$$\widetilde{EU}_i^*(a_i^*) = Y_L + \frac{Y_M - Y_L}{N-1} - A\left(\frac{(N-2)(Y_M - Y_L)}{(N-1)^2}\right) - \alpha\delta_i.$$

Now, we can see why α measures the contest's degree of dissipation. Note that, in equilibrium, a player's winning probability is always $1/(N-1)$, irrespective of the value of α. However, a player's equilibrium activity and, therefore, his effort costs rise in α. Consequently, the higher α, the lower will be each player's expected utility from participating in the consolation match. Nevertheless, a player strictly benefits from higher productivity (i.e., $\widetilde{EU}_i^*(a_i^*)$ decreases in δ_i).

2.3 Major Contest

A player will earn the highest career income Y_H, if he passes the hurdle of winning the major contest. In case of losing, he will be relegated to the consolation contest associated with expected utility $\widetilde{EU}_i^*(a_i^*)$. Hence, player i's objective function in the major contest can be written as

$$\begin{aligned}
\widehat{EU}_i(a_i) &= Y_H \frac{f(a_i - \alpha\delta_i)}{f(a_i - \alpha\delta_i) + \sum_{j\neq i} f(a_j - \alpha\delta_j)} \\
&\quad + \widetilde{EU}_i^*(a_i^*)\left(1 - \frac{f(a_i - \alpha\delta_i)}{f(a_i - \alpha\delta_i) + \sum_{j\neq i} f(a_j - \alpha\delta_j)}\right) - a_i \\
&= \widetilde{EU}_i^*(a_i^*) + \frac{\left(Y_H - \widetilde{EU}_i^*(a_i^*)\right) \cdot f(a_i - \alpha\delta_i)}{f(a_i - \alpha\delta_i) + \sum_{j\neq i} f(a_j - \alpha\delta_j)} - a_i.
\end{aligned}$$

In analogy to the consolation contest, the first-order conditions of two arbitrary players i and k are given by

$$\frac{Y_H - \widetilde{EU}_i^*(a_i^*)}{\left[\sum_{j=1}^N f(a_j - \alpha\delta_j)\right]^2} = \frac{1}{f'(a_i - \alpha\delta_i)\left[f(a_k - \alpha\delta_k) + \sum_{j\neq i,k} f(a_j - \alpha\delta_j)\right]}$$

and

$$\frac{Y_H - \widetilde{EU}_k^*(a_k^*)}{\left[\sum_{j=1}^N f(a_j - \alpha\delta_j)\right]^2} = \frac{1}{f'(a_k - \alpha\delta_k)\left[f(a_i - \alpha\delta_i) + \sum_{j\neq i,k} f(a_j - \alpha\delta_j)\right]}.$$

Hence, if $\delta_i < \delta_k$, then we have $\widetilde{EU}_i^*(a_i^*) > \widetilde{EU}_k^*(a_k^*)$ and

$$\begin{gathered}
\frac{1}{f'(a_i - \alpha\delta_i)\left[f(a_k - \alpha\delta_k) + \sum_{j\neq i,k} f(a_j - \alpha\delta_j)\right]} < \\
\frac{1}{f'(a_k - \alpha\delta_k)\left[f(a_i - \alpha\delta_i) + \sum_{j\neq i,k} f(a_j - \alpha\delta_j)\right]} \Leftrightarrow
\end{gathered}$$

$$\frac{f(a_i - \alpha\delta_i) + \sum_{j\neq i,k} f(a_j - \alpha\delta_j)}{f'(a_i - \alpha\delta_i)} < \frac{f(a_k - \alpha\delta_k) + \sum_{j\neq i,k} f(a_j - \alpha\delta_j)}{f'(a_k - \alpha\delta_k)},$$

which implies $a_i - \alpha\delta_i < a_k - \alpha\delta_k$ since $\left[f(x) + \sum_{j\neq i,k} f(a_j - \alpha\delta_j)\right]/f'(x)$ is a monotonically increasing function of x. Let $\delta_{(1)} < \delta_{(2)} \cdots < \delta_{(N)}$ denote the order of the players' productivity parameters (i.e., player (1) is the most productive one), $a^*_{(1)}, a^*_{(2)}, \ldots, a^*_{(N)}$ the respective equilibrium efforts and $p^*_{(1)}, p^*_{(2)}, \ldots, p^*_{(N)}$ the winning probabilities in the major contest. Then we obtain the following result:

Proposition 1 *The players' winning probabilities in the major contest satisfy* $p^*_{(1)} < p^*_{(2)} < \ldots < p^*_{(N)}$ *and the corresponding equilibrium efforts* $a^*_{(1)} < a^*_{(2)} < \ldots < a^*_{(N)}$.

Proof. The first part immediately follows from the fact that $\delta_i < \delta_k$ implies $a_i - \alpha\delta_i < a_k - \alpha\delta_k$ and that $p_i = f(a_i - \alpha\delta_i) / [f(a_i - \alpha\delta_i) + \sum_{j\neq i} f(a_j - \alpha\delta_j)]$ is strictly increasing in $a_i - \alpha\delta_i$. The second part follows from $a_i - \alpha\delta_i < a_k - \alpha\delta_k \Leftrightarrow a_i - a_k < -\alpha(\delta_k - \delta_i) < 0$.
■

Proposition 1 shows that the more productive a player, the less likely he will win the major contest and the less effort he will choose. The intuition for the first result comes from the players' different fall-back positions in the major contest. If a player has a large productivity (i.e., a small δ_i), then he will also be a strong player in the consolation match, which will guarantee him a large expected utility $\widetilde{EU}^*_i(a^*_i)$ as a kind of fall-back position. This fact reduces his incentives in the major contest so that we have a tendency to mediocracy where key positions are filled by less productive individuals, leading to welfare losses. In other words, participation in the consolation match is rather unattractive for the less productive players so that they have very strong incentives to win the major contest.

Note that although we assume career incomes to be exogenously given, our results of Proposition 1 as well as the following results will qualitatively hold under endogenous incomes or prizes that are optimally chosen by a contest designer before the competition starts. The career incomes or contest prizes of course influence the levels of all players' equilibrium efforts. However, they neither have an impact on the players' effort differences

$a_i - a_k$ in the consolation match nor an impact on the ranking between $a_i - \alpha\delta_i$ and $a_k - \alpha\delta_k$ in the major contest. As the same holds for the findings in the following sections our results are robust with respect of exogeneity/endogeneity of the contest prizes.

The result on effort ranking $a^*_{(1)} < a^*_{(2)} < \ldots < a^*_{(N)}$ stems from the intuition before together with the fact that effort and productivity are substitutes in the impact function. Hence, even if all players had identical fall-back positions, the order $a^*_{(1)} < a^*_{(2)} < \ldots < a^*_{(N)}$ would not change. Welfare was solely defined via the career decision based on the outcome of the major contest. However, if the activity levels a^*_i $(i = 1, \ldots, N)$, which are productive by assumption, were also important for welfare considerations, we might have a second source for welfare losses since the most productive individuals choose the lowest activity levels.

3 Two-Hurdle Career

In this section, we consider the case where a player has to pass two career hurdles to reach the top. This situation is described by Figure 2.

[Figure 2]

First, a player must assert himself in the organizational unit that he belongs to (e.g., a division or affiliate of a corporation, a certain political party). Second, when being successful in becoming head of the organizational unit (e.g., division head or party leader), he enters a higher-order contest to reach the top position in his career path (e.g., CEO of the corporation or president of a state). As crucial difference to the one-hurdle career, now a player has to succeed two times before reaching the top and a player's productivity has both an influence on his fall-back position and an influence on his future career.

3.1 Model Modifications

There are two organizational units, A and B, that consist of N_A and N_B members, respectively, with $N_O \geq 3$ ($O \in \{A, B\}$). In each unit O, the N_O members compete for becoming unit head (*level-I contests*). These two contests are modeled analogously to the one in Subsection 2.1. Each member $i \in \{1, \ldots, N_O\}$ chooses activity level $a_{Oi} \geq 0$ (at costs a_{Oi}), which influences his probability of winning the level-I contest for unit head O,

$$p_{Oi} = \frac{f\left(a_{Oi} - \alpha_I \delta_{Oi}\right)}{f\left(a_{Oi} - \alpha_I \delta_{Oi}\right) + \sum_{j \neq i} f\left(a_{Oj} - \alpha_I \delta_{Oj}\right)}. \tag{2}$$

Again, δ_{Oi} indicates the productivity of the respective player, with lower values corresponding to higher productivities. The parameter $\alpha_I > 0$ measures the grade of dissipation in the level-I contests.[10] If player i wins and becomes unit head he will enter the higher-order level-II contest where he competes against the other unit head to reach the top of his career. However, the $N_O - 1$ losers are relegated to a consolation contest in unit O where the winner earns career income $Y_L > 0$ and the $N_O - 2$ losers get 0.

As outcome of the level-I contests, two players – the unit heads A and B – enter the *level-II contest*. Here, they compete for the unique top position, which is associated with career income Y_H. Whereas the winner of the level-II contest is assigned to this top position, the loser gets a lower career income Y_M with $0 < Y_L < Y_M < Y_H$. Career incomes are assumed to be sufficiently large so that the players exert strictly positive efforts in each contest. The contest-success function for the level-II contest is again

[10] Our results will not change qualitatively, if we define different parameters α_{AI} and α_{BI} for the two units.

a Skaperdas-type function like (1) and (2). The scaling parameter for measuring the degree of dissipation at level II is denoted by $\alpha_{II} > 0$.

As in Section 2, we assume that from a welfare perspective the player with the highest productivity (i.e., the one with parameter $\min\{\delta_{O1}, \ldots, \delta_{ON_O} | O \in \{A, B\}\}$) should be assigned to the top position with career income Y_H.

3.2 Level-II Contest

Let δ_{Ai} and δ_{Bi} denote the productivity parameters of the two unit heads, who enter the level-II contest, and a_{Ai} and a_{Bi} the corresponding activity variables. In the contest, the head of unit O maximizes

$$\widehat{EU}_{Oi}(a_{Oi}) = Y_M + (Y_H - Y_M) \frac{f(a_{Oi} - \alpha_{II}\delta_{Oi})}{\sum_{\Omega \in \{A,B\}} f(a_{\Omega i} - \alpha_{II}\delta_{\Omega i})} - a_{Oi}.$$

Proceeding in the same way as in Section 2, we find that, in equilibrium, activity levels are described by

$$\begin{aligned} \frac{Y_H - Y_M}{\left[\sum_{\Omega \in \{A,B\}} f(a_{\Omega i} - \alpha_{II}\delta_{\Omega i})\right]^2} &= \frac{1}{f(a_{Bi} - \alpha_{II}\delta_{Bi}) f'(a_{Ai} - \alpha_{II}\delta_{Ai})} \\ &= \frac{1}{f(a_{Ai} - \alpha_{II}\delta_{Ai}) f'(a_{Bi} - \alpha_{II}\delta_{Bi})} \end{aligned}$$

yielding $f(a_{Ai} - \alpha_{II}\delta_{Ai})/f'(a_{Ai} - \alpha_{II}\delta_{Ai}) = f(a_{Bi} - \alpha_{II}\delta_{Bi})/f'(a_{Bi} - \alpha_{II}\delta_{Bi})$ and, hence, $a_{Ai} - \alpha_{II}\delta_{Ai} = a_{Bi} - \alpha_{II}\delta_{Bi}$. Altogether, in the level-II contest the head of organizational unit O optimally chooses

$$a_{Oi}^* = A\left(\frac{Y_H - Y_M}{4}\right) + \alpha_{II}\delta_{Oi}$$

and gets expected utility

$$\widehat{EU}^*_{Oi}(a_{Oi}) = Y_M + \frac{Y_H - Y_M}{2} - A\left(\frac{Y_H - Y_M}{4}\right) - \alpha_{II}\delta_{Oi}.$$

We can see that in equilibrium the more productive unit head exerts less effort than the other head, but has the same probability of being promoted to the top position since the higher productivity completely outweighs the effort deficit. Thus, it is pure luck whether the better head is assigned to the top job or not.

3.3 Level-I Contests

Consider the contest in organizational unit O. The objective function of member i can be written as

$$EU_{Oi}(a_{Oi}) = \widetilde{EU}^*_{Oi} + \frac{\left(\widehat{EU}^*_{Oi} - \widetilde{EU}^*_{Oi}\right) \cdot f(a_{Oi} - \alpha_I\delta_{Oi})}{f(a_{Oi} - \alpha_I\delta_{Oi}) + \sum_{j\neq i} f(a_{Oj} - \alpha_I\delta_{Oj})} - a_{Oi}.$$

If player i wins, he will earn the expected utility from participating in the level-II contest, $\widehat{EU}^*_{Oi}$. In case of losing, he will enter the consolation contest of his unit O, where he receives expected utility $\widetilde{EU}^*_{Oi}$. Hence, his fall-back position is characterized by $\widetilde{EU}^*_{Oi}$ and the extra utility from being successful at level I by $\widehat{EU}^*_{Oi} - \widetilde{EU}^*_{Oi}$. From the first-order condition we obtain the following description of player i's equilibrium activity:

$$\frac{\widehat{EU}^*_{Oi} - \widetilde{EU}^*_{Oi}}{\left[\sum_{j=1}^{N_O} f(a_{Oj} - \alpha_I\delta_{Oj})\right]^2} = \frac{1}{f'(a_{Oi} - \alpha_I\delta_{Oi})\left[\sum_{j\neq i} f(a_{Oj} - \alpha_I\delta_{Oj})\right]}. \tag{3}$$

To further characterize the equilibrium activities, we have to calculate player i's fall-back position $\widetilde{EU}^*_{Oi}$. Applying the results for the consolation contest of Subsection 2.2 yields

$$\widetilde{EU}^*_{Oi}\left(a^*_{Oi}\right) = \frac{Y_L}{N_O - 1} - A\left(\frac{\left(N_O - 2\right) Y_L}{\left(N_O - 1\right)^2}\right) - \alpha_I \delta_{Oi},$$

and, therefore,[11]

$$\begin{aligned} \widehat{EU}^*_{Oi} - \widetilde{EU}^*_{Oi} = \; & \frac{Y_H + Y_M}{2} - \frac{Y_L}{N_O - 1} + \left(\alpha_I - \alpha_{II}\right) \delta_{Oi} \\ & + A\left(\frac{\left(N_O - 2\right) Y_L}{\left(N_O - 1\right)^2}\right) - A\left(\frac{Y_H - Y_M}{4}\right). \end{aligned}$$

Hence, if $\alpha_I > \alpha_{II}$ then for two arbitrary members i and k of unit O in the level-I contest we have $\widehat{EU}^*_{Oi} - \widetilde{EU}^*_{Oi} < \widehat{EU}^*_{Ok} - \widetilde{EU}^*_{Ok} \Leftrightarrow \delta_{Oi} < \delta_{Ok}$. The first-order conditions (3) of players i and k can be written as

$$\begin{aligned} & \frac{\widehat{EU}^*_{Oi} - \widetilde{EU}^*_{Oi}}{\left[\sum_{j=1}^{N_O} f\left(a_{Oj} - \alpha_I \delta_{Oj}\right)\right]^2} \\ = \; & \frac{1}{f'\left(a_{Oi} - \alpha_I \delta_{Oi}\right)\left[f\left(a_{Ok} - \alpha_I \delta_{Ok}\right) + \sum_{j \neq i,k} f\left(a_{Oj} - \alpha_I \delta_{Oj}\right)\right]} \end{aligned}$$

and

$$\begin{aligned} & \frac{\widehat{EU}^*_{Ok} - \widetilde{EU}^*_{Ok}}{\left[\sum_{j=1}^{N_O} f\left(a_{Oj} - \alpha_I \delta_{Oj}\right)\right]^2} \\ = \; & \frac{1}{f'\left(a_{Ok} - \alpha_I \delta_{Ok}\right)\left[f\left(a_{Oi} - \alpha_I \delta_{Oi}\right) + \sum_{j \neq i,k} f\left(a_{Oj} - \alpha_I \delta_{Oj}\right)\right]}. \end{aligned}$$

[11] Recall that, by assumption, career incomes guarantee interior solutions for the equilibrium activities. Hence, we must have $\widehat{EU}^*_{Oi} - \widetilde{EU}^*_{Oi} > 0$.

Thus, if $\widehat{EU}^*_{Oi} - \widetilde{EU}^*_{Oi} < \widehat{EU}^*_{Ok} - \widetilde{EU}^*_{Ok}$ then

$$\frac{f(a_{Oi} - \alpha_I \delta_{Oi}) + \sum_{j \neq i,k} f(a_{Oj} - \alpha_I \delta_{Oj})}{f'(a_{Oi} - \alpha_I \delta_{Oi})} < \frac{f(a_{Ok} - \alpha_I \delta_{Ok}) + \sum_{j \neq i,k} f(a_{Oj} - \alpha_I \delta_{Oj})}{f'(a_{Ok} - \alpha_I \delta_{Ok})},$$

which implies $a_{Oi} - \alpha_I \delta_{Oi} < a_{Ok} - \alpha_I \delta_{Ok}$, analogously to the findings in Subsection 2.3. Since this comparison holds for any two members of organizational unit $O \in \{A, B\}$ and since players have identical winning probabilities in the level-II contest, we have proven the following result:

Proposition 2 *If $\alpha_I > \alpha_{II}$, then the most (least) productive player of each unit $O \in \{A, B\}$, i.e., the player with $\delta_{Oi} = \min\{\delta_{O1}, \ldots, \delta_{ON_O}\}$ (with $\delta_{Oi} = \max\{\delta_{O1}, \ldots, \delta_{ON_O}\}$), has the lowest (highest) probability to reach the top position with career income Y_H.*

Proposition 2 shows that the mediocracy result of the one-hurdle career will prevail under the assumptions of the two-hurdle career if the level-I contests are more dissipative then the level-II contest. The intuition for this result is the following. The higher α_I relative to α_{II}, the less attractive will be participation in the consolation match for the players. Moreover, since expected utility increases in the productivity of a player (i.e., decreases in δ_{Oi}), the less productive a player the stronger will be his incentives to avoid a consolation match and, hence, to pass the hurdle that leads to participation in the level-II contest.

A situation with level-I contests being more dissipative than the level-II contest seems quite realistic. For example, note that a single player is less visible in the large contest against $N_O - 1$ opponents on level I compared to the level-II contest with two players. Therefore, it is rather difficult for an individual to stand up to his opponents on level I compared to level II. In order to become winner on level I, each player has to spend a lot of effort, which makes participation in the contest costly and, hence, level-I contests quite dissipative.[12]

4 Activities and Productivity Parameters as Complements

So far we have assumed that activities (a_i) and productivity parameters (δ_i) are substitutes in the players' impact function $f(\cdot)$. This assumption drives part of the previous results, in particular the findings that players with higher productivities (i.e., lower values of δ_i) choose lower efforts in equilibrium. However, this paper does not focus on effort choice but on the probability that the most productive player is not assigned to the top career position. In this section, we will check the robustness of the finding that the correlation between productivity and promotion probability of a player may be strictly negative.

For this purpose, we reconsider the one-hurdle career model and assume that player i's contest-success function is given by

$$\check{p}_i = \frac{f\left(\frac{a_i}{\delta_i}\right)}{f\left(\frac{a_i}{\delta_i}\right) + \sum_{j \neq i} f\left(\frac{a_j}{\delta_j}\right)} \tag{4}$$

with $f(\cdot)$ denoting the same strictly positive, increasing and concave impact function as before. Again, the smaller δ_i the more productive will be the respective player, and for the case of identical activity levels by all players the most productive one has the highest

[12]Note that equilibrium efforts strictly increase in the dissipation parameters α, α_I and α_{II}, respectively.

winning probability. However, comparison of (1) and (4) shows that (besides skipping the dissipation parameter α) the contest-success functions p_i and $\check{p}_i$ differ significantly. Contrary to (1), now the activity variable and the productivity parameter are complements in the sense that lower values of δ_i make higher activity levels a_i more effective.[13]

As in Section 2, we first solve the consolation contest game and then turn to the major contest. In the consolation contest, player i maximizes $\widetilde{EU}_i(a_i) = Y_L + (Y_M - Y_L) \cdot \check{p}_i - a_i$. The first-order conditions of two arbitrary players i and k can be combined to

$$\begin{aligned} \frac{(Y_M - Y_L)}{\left[\sum_{j=1}^{N-1} f\left(\frac{a_j}{\delta_j}\right)\right]^2} &= \frac{\delta_i}{f'\left(\frac{a_i}{\delta_i}\right)\left[f\left(\frac{a_k}{\delta_k}\right) + \sum_{j\neq i,k} f\left(\frac{a_j}{\delta_j}\right)\right]} \\ &= \frac{\delta_k}{f'\left(\frac{a_k}{\delta_k}\right)\left[f\left(\frac{a_i}{\delta_i}\right) + \sum_{j\neq i,k} f\left(\frac{a_j}{\delta_j}\right)\right]}, \end{aligned} \tag{5}$$

which yields the following equilibrium outcomes:

Lemma 1 *In the consolation contest, if $\delta_i < \delta_k$ then (i) $\check{p}_i^* > \check{p}_k^*$ and (ii) $\widetilde{EU}_i^*(a_i^*) > \widetilde{EU}_k^*(a_k^*)$.*

Proof. Part (i) can be shown by contradiction. Suppose that

$$\check{p}_i^* \leq \check{p}_k^* \Leftrightarrow \frac{a_i^*}{\delta_i} < \frac{a_k^*}{\delta_k}. \tag{6}$$

From the first-order conditions we obtain

$$\frac{\delta_i\left[f\left(\frac{a_i^*}{\delta_i}\right) + \sum_{j\neq i,k} f\left(\frac{a_j^*}{\delta_j}\right)\right]}{f'\left(\frac{a_i^*}{\delta_i}\right)} = \frac{\delta_k\left[f\left(\frac{a_k^*}{\delta_k}\right) + \sum_{j\neq i,k} f\left(\frac{a_j^*}{\delta_j}\right)\right]}{f'\left(\frac{a_k^*}{\delta_k}\right)}. \tag{7}$$

Since $[f(x) + \sum_{j\neq i,k} f\left(\frac{a_j^*}{\delta_j}\right)]/f'(x)$ is a monotonically increasing function of x, (6) and (7) can only be satisfied at the same time if $\delta_i > \delta_k$, a contradiction. (ii) Since player i

[13] Since $f(\cdot)$ is concave, in (1) lower values of δ_i make activity choice even less effective.

can always choose the same effort level as k so that he has the same effort costs but a higher winning probability we must have $\widetilde{EU}_i^* (a_i^*) > \widetilde{EU}_k^* (a_k^*)$ in equilibrium. ■

Lemma 1 points out that, in the consolation match, more productive players have higher winning probabilities and larger expected utilities than less productive players. Result (ii) is also important for the major contest, where players compete for the top position with income Y_H. Due to the positive correlation between productivity and expected utility, more productive players have better fall-back positions $\widetilde{EU}_i^* (a_i^*)$ in the major contest, leading to less incentives. This effect also works in the models with substitutes (Sections 2 and 3) and will be called *incentive effect.*

In the major contest, player i maximizes $\widehat{EU}_i (a_i) = \widetilde{EU}_i^* (a_i^*) + (Y_H - \widetilde{EU}_i^* (a_i^*)) \cdot \check{p}_i - a_i$. The first-order conditions of two players i and k,

$$\begin{aligned} \frac{Y_H - \widetilde{EU}_i^* (a_i^*)}{\left[\sum_j f\left(\frac{a_j}{\delta_j}\right)\right]^2} &= \frac{\delta_i}{f'\left(\frac{a_i}{\delta_i}\right)\left[f\left(\frac{a_k}{\delta_k}\right) + \sum_{j\neq i,k} f\left(\frac{a_j}{\delta_j}\right)\right]} \\ \text{and } \frac{Y_H - \widetilde{EU}_k^* (a_k^*)}{\left[\sum_j f\left(\frac{a_j}{\delta_j}\right)\right]^2} &= \frac{\delta_k}{f'\left(\frac{a_k}{\delta_k}\right)\left[f\left(\frac{a_i}{\delta_i}\right) + \sum_{j\neq i,k} f\left(\frac{a_j}{\delta_j}\right)\right]}, \end{aligned} \tag{8}$$

together with $\widetilde{EU}_i^* (a_i^*) > \widetilde{EU}_k^* (a_k^*)$ imply

$$\frac{\delta_i \left[f\left(\frac{a_i}{\delta_i}\right) + \sum_{j\neq i,k} f\left(\frac{a_j}{\delta_j}\right)\right]}{f'\left(\frac{a_i}{\delta_i}\right)} < \frac{\delta_k \left[f\left(\frac{a_k}{\delta_k}\right) + \sum_{j\neq i,k} f\left(\frac{a_j}{\delta_j}\right)\right]}{f'\left(\frac{a_k}{\delta_k}\right)}. \tag{9}$$

The inequality shows that solutions of type $\frac{a_i}{\delta_i} < \frac{a_k}{\delta_k}$ are always possible. Such outcomes coincide with the findings above where more productive players are less likely to obtain the top career position. However, the parameters δ_i and δ_k in the numerators of the two sides in (9) indicate that we cannot rule out solutions with $\frac{a_i}{\delta_i} > \frac{a_k}{\delta_k}$ if δ_i is sufficiently small and δ_k sufficiently large. This effect can be labeled *productivity effect.* Hence, if activities and productivity parameters are complements in the impact function, we will have two effects that work into opposite directions. Coming back to our question regarding efficient assignment at the top we obtain the following result:

Proposition 3 *Consider the major contest for the top position with income Y_H and let $\delta_i < \delta_k$. If*

$$\frac{\delta_k}{\delta_i} < \frac{Y_H - \widetilde{EU}_k^* (a_k^*)}{Y_H - \widetilde{EU}_i^* (a_i^*)}, \tag{10}$$

then $\check{p}_i^* < \check{p}_k^*$.

Proof. Combining the first-order conditions (8) yields

$$\frac{f\left(\frac{a_i}{\delta_i}\right) + \sum_{j \neq i,k} f\left(\frac{a_j}{\delta_j}\right)}{f'\left(\frac{a_i}{\delta_i}\right)} = \frac{\delta_k \left[Y_H - \widetilde{EU}_i^* (a_i^*)\right]}{\delta_i \left[Y_H - \widetilde{EU}_k^* (a_k^*)\right]} \frac{f\left(\frac{a_k}{\delta_k}\right) + \sum_{j \neq i,k} f\left(\frac{a_j}{\delta_j}\right)}{f'\left(\frac{a_k}{\delta_k}\right)}.$$

If (10) is satisfied, we will have

$$\frac{f\left(\frac{a_i}{\delta_i}\right) + \sum_{j \neq i,k} f\left(\frac{a_j}{\delta_j}\right)}{f'\left(\frac{a_i}{\delta_i}\right)} < \frac{f\left(\frac{a_k}{\delta_k}\right) + \sum_{j \neq i,k} f\left(\frac{a_j}{\delta_j}\right)}{f'\left(\frac{a_k}{\delta_k}\right)}$$

and, thus, $\frac{a_i}{\delta_i} < \frac{a_k}{\delta_k}$, which implies $\check{p}_i^* < \check{p}_k^*$ for the winning probabilities in equilibrium according to (4). From Lemma 1(ii) we know that both sides of inequality (10) are larger than one so that there exist parameter constellations for which (10) holds and others for which (10) does not hold. ■

Condition (10) points out the two opposing effects. While the right-hand side describes the incentive effect, the left-hand side characterizes the productivity effect. If the incentive effect dominates the productivity effect, more productive players will have lower winning probabilities in the major contest than less productive ones.

5 Conclusions

At first sight, one might expect that career contests perfectly correspond to the well-known phrase "survival of the fittest". According to Darwin, there should be a natural selection among heterogeneous individuals so that the best suited ones will win the competition for reproduction. Relating to career competition, the most talented or most productive players should win and be promoted to the top positions within structured career paths. However, in our paper we show that, contrary to the Darwinian view, the least productive players may have the highest probability of winning career competition. The intuition comes from the fact that the phrase "survival of the fittest" implicitly assumes that all individuals choose the same activity level. Maybe, in biology this crucial assumption holds. In an economic context, however, equilibrium behavior of heterogeneous players usually differs. Since the least productive players are characterized by the lowest fall-back positions, these individuals have the highest incentives to win career competition and, thereby, to avoid their unattractive fall-back options. In this sense, we have a natural tendency that the least productive players succeed. It is important to emphasize that under identical activity levels our model would replicate the Darwinian outcome: the individuals with the highest productivity would most likely win the career contest.

In our paper, we used a game-theoretic perspective to show how the detrimental effect of fall-back positions may lead to adverse career outcomes for society. Switching to a contract-theoretic view would not qualitatively alter our results. Allowing endogenously chosen, optimal career incomes would probably lead to a change in the levels of equilibrium activities. However, as mentioned in Section 2 the ranking between the players' activities and winning probabilities would not change. Introducing reservation utilities for the players may even reinforce our findings. If the most productive players have also the highest reservation utilities, these players may prefer their outside options and decide not to participate in the consolation contest. If the fall-back positions are now determined by the players' different reservation utilities, we will have the same natural tendency that the most productive players have the lowest incentives to succeed in a given career contest. All these considerations are based on the assumption that the contest designer cannot use a mechanism to reveal the players' types and then choose type-dependent contest prizes to adjust individual incentives. As has been emphasized by Malcomson (1984, 1986), identical prizes for all contestants are important if individuals only have unverifiable but observable performance signals. In that case, under different prizes the principal would ex-post always claim that the player with the lowest winner prize has performed best, thereby saving labor costs. Since the players can anticipate such opportunistic behavior, contest incentives would break down if prizes differ.

According to the Peter Principle, individuals are promoted as long as they reach their level of incompetence. This observation was made by Peter and Hull (1969). The outcome of the Peter Principle is based on two rules – currently good performance is rewarded by job promotion and demotions are not possible. In the subsequent economic work, the Peter Principle has been used as a synonym for the misallocation of managers at (high) hierarchy levels. For example, Prendergast (1992) explains such misallocation by the personnel policy of hiding good talents, whereas the explanation of Lazear (2004) is based on temporary luck. In the light of our model, misallocation at higher hierarchy levels can be explained by a detrimental incentive effect that gives less talented individuals strong incentives to assert themselves in competitive careers.

References

Baker, G.P., Gibbs, M. and B. Holmström (1994): The Internal Economics of the Firm: Evidence from Personnel Data, *Quarterly Journal of Economics* **109**, 881–919.

Becker, G.S. (1962): Investment in Human Capital: A Theoretical Analysis, *Journal of Political Economy* **70**, Supplement, 9–49.

Caselli, F. and M. Morelli (2004): Bad Politicians, *Journal of Public Economics* **88**, 759–782.

Chakravarty, S.P. (1993): Why are Bosses Incompetent? *European Journal of Political Economy* **9**, 293–302.

Kiyotaki, F. (2004): The Effects of a Consolation Match on the Promotion Tournament, *Journal of the Japanese and International Economies* **18**, 264–281.

Konrad, K.A. (2009): *Strategy and Dynamics in Contests*, Oxford University Press: Oxford, New York.

Lazear, E.P. (2004): The Peter Principle: A Theory of Decline, *Journal of Political Economy* **112**, S141–S163.

Lazear, E.P. and S. Rosen (1981): Rank-Order Tournaments as Optimum Labor Contracts, *Journal of Political Economy* **89**, 841–864.

Malcomson, J.M. (1984): Work Incentives, Hierarchy, and Internal Labor Markets, *Journal of Political Economy* **92**, 486–507.

Malcomson, J.M. (1986): Rank-Order Contracts for a Principal with Many Agents, *Review of Economic Studies* **53**, 807–817.

Mattozzi, A. and A. Merlo (2007): Mediocracy, *NBER Working Paper No. W12920.*

Messner, M. and M. Polborn (2004): Paying Politicians, *Journal of Public Economics* **88**, 2423–2445.

Milgrom, P.R. and J. Roberts (1988): An Economic Approach to Influence Acrivities in Organizations, *American Journal of Sociology* **94**, Supplement, S-154–S-179.

Nalebuff, B.J. and J.E. Stiglitz (1983): Prizes and Incentives: Towards a General Theory of Compensation and Competition, *Bell Journal of Economics* **14**, 21–43.

Peter, L.J. and R. Hull (1969): *The Peter Principle.* New York.

Prendergast, C. (1992): The Insurance Effect of Groups, *International Economic Review* **33**, 567–581.

Prendergast, C. and R.H. Topel (1996): Favoritism in Organizations, *Journal of Political Economy* **104**, 958–978.

Rosen, S. (1986): Prizes and Incentives in Elimination Tournaments, *American Economic Review* **76**, 701–715.

Skaperdas, S. (1996): Contest Success Functions, *Economic Theory* **7**, 283–290.

Spence, A.M. (1973): Job Market Signaling, *Quarterly Journal of Economics* **87**, 355–374.

Tullock, G. (1980): Efficient Rent Seeking, in Buchanan, J.M., Tollison, R.D. and G. Tullock (Eds.): *Toward a Theory of the Rent-Seeking Society*, A&M University Press: College Station, 97–112.

Uehara, K. (2009): Early or Late Promotion/Screening? Empirical Analysis of Career Ladders for Japanese White-Collar Workers Using Employees' List, *Japan Labor Review* **6**, 25–58.

Wärneryd, K. (2001): Replicating Contests, *Economics Letters* **71**, 323–327.

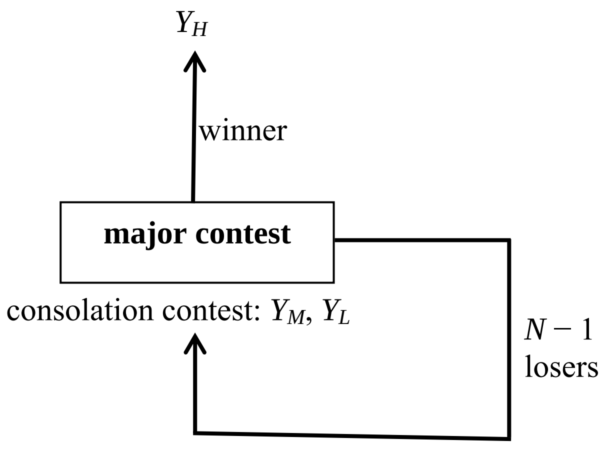

Figure 1: One-Hurdle Career

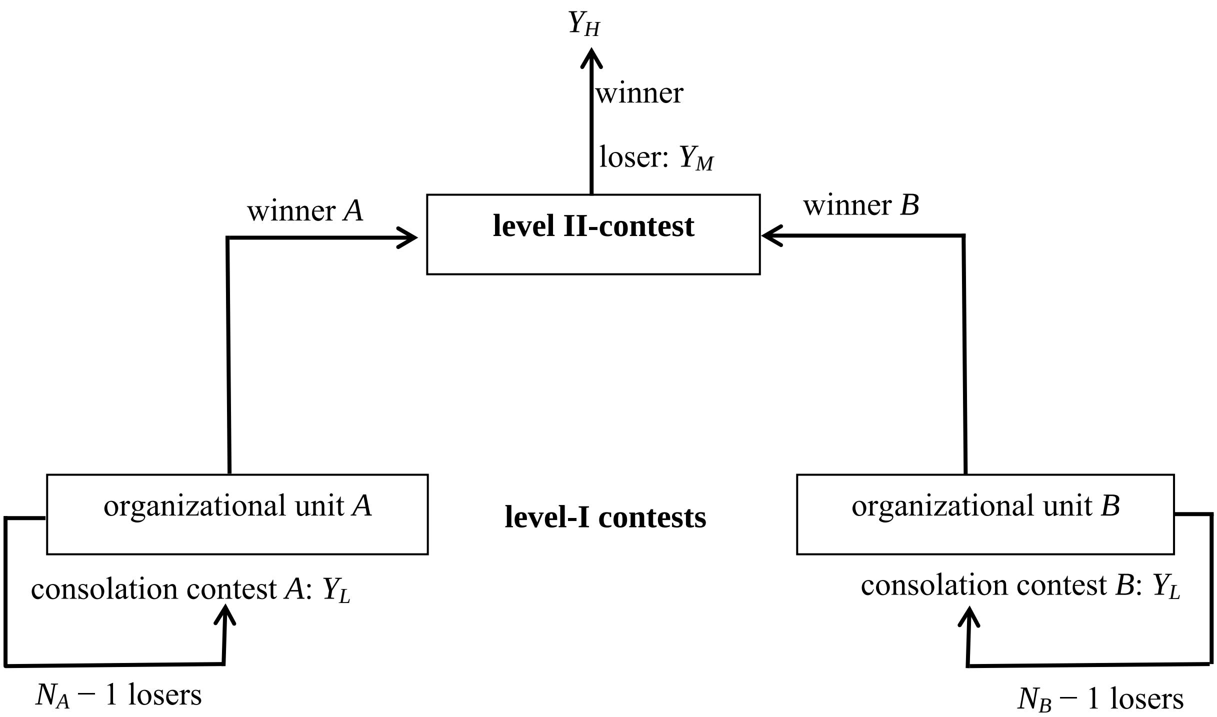

Figure 2: Two-Hurdle Career