

OFFICIAL ORGAN OF THE RADIATION RESEARCH SOCIETY

Handwritten signature

RADIATION RESEARCH

EDITOR-IN-CHIEF: DANIEL BILLEN

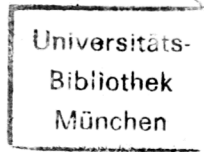
Volume 98, 1984



ACADEMIC PRESS, INC.

San Diego Orlando San Francisco New York London
Toronto Montreal Sydney Tokyo São Paulo

~~BIBLIOTHEK
Tierärztlichen Fakultät
Königinstraße 10
8000 München 22~~



6402/6 245

Copyright © 1984 by Academic Press, Inc.

All rights reserved

No part of this publication may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopy, recording, or any information storage and retrieval system, without permission in writing from the copyright owner.

The appearance of the code at the bottom of the first page of an article in this journal indicates the copyright owner's consent that copies of the article may be made for personal or internal use, or for the personal or internal use of specific clients. This consent is given on the condition, however, that the copier pay the stated per copy fee through the Copyright Clearance Center, Inc. (21 Congress Street, Salem, Massachusetts 01970), for copying beyond that permitted by Sections 107 or 108 of the U. S. Copyright Law. This consent does not extend to other kinds of copying, such as copying for general distribution, for advertising or promotional purposes, for creating new collective works, or for resale. Copy fees for pre-1984 articles are as shown on the article title pages; if no fee code appears on the title page, the copy fee is the same as for current articles.

0033-7587/84 \$3.00

MADE IN THE UNITED STATES OF AMERICA



RADIATION RESEARCH

OFFICIAL ORGAN OF THE RADIATION RESEARCH SOCIETY

Editor-in-Chief: DANIEL BILLEN, University of Tennessee–Oak Ridge Graduate School of Biomedical Sciences, Biology Division, Oak Ridge National Laboratory, P.O. Box Y, Oak Ridge, Tennessee 37830

Managing Technical Editor: MARTHA EDINGTON, University of Tennessee–Oak Ridge Graduate School of Biomedical Sciences, Biology Division, Oak Ridge National Laboratory, P.O. Box Y, Oak Ridge, Tennessee 37830

ASSOCIATE EDITORS

- | | |
|--|--|
| H. I. ADLER, Oak Ridge National Laboratory | L. R. PAINTER, University of Tennessee |
| W. D. BLOOMER, The Mount Sinai Medical Center | L. J. PETERS, University of Texas |
| J. M. CLARKSON, University of Texas | J. A. RALEIGH, Cross Cancer Institute, Edmonton, Alberta, Canada |
| A. COLE, University of Texas | J. L. ROTI ROTI, University of Utah |
| L. L. DEAVEN, Los Alamos National Laboratory | R. A. SCHLENKER, Argonne National Laboratory |
| S. S. DONALDSON, Stanford University | M. SODICOFF, Temple University |
| A. HAN, University of Southern California | J. R. STEWART, University of Utah |
| R. E. MEYN, JR., University of Texas | R. L. ULLRICH, Oak Ridge National Laboratory |
| S. M. MICHAELSON, University of Rochester | J. F. WARD, University of California, San Diego |
| J. H. MILLER, Battelle, Pacific Northwest Laboratory | D. W. WHILLANS, Ontario Hydro, Pickering, Canada |

OFFICERS OF THE SOCIETY

President: ERIC J. HALL, Radiological Research Laboratory, Columbia University College of Physicians and Surgeons, New York, New York 10032

Vice President and President-Elect: JOHN F. WARD, Department of Radiology, University of California, San Diego, M010, La Jolla, California 92093

Secretary-Treasurer: JANET S. RASEY, Department of Radiation Oncology, University of Washington, Seattle, Washington 98195

Editor-in-Chief: DANIEL BILLEN, University of Tennessee–Oak Ridge Graduate School of Biomedical Sciences, Biology Division, Oak Ridge National Laboratory, P.O. Box Y, Oak Ridge, Tennessee 37830

Administrative Director: JOHN J. CURRY, 925 Chestnut Street, Philadelphia, Pennsylvania 19107

ANNUAL MEETING

1985: May 26–31, Chicago, Illinois

Titus C. Evans, Editor-in-Chief Volumes 1–50
Oddvar F. Nygaard, Editor-in-Chief Volumes 51–79



VOLUME 98, 1984

Councilors, Radiation Research Society 1984–1985

PHYSICS

G. L. Brownell, Massachusetts General Hospital
C. C. Ling, George Washington University

BIOLOGY

D. B. Leeper, Thomas Jefferson University
E. A. Blakely, University of California, Berkeley

MEDICINE

M. A. Bagshaw, Stanford University
L. J. Peters, University of Texas

CHEMISTRY

J. E. Biaglow, Case Western Reserve University
G. E. Adams, Institute of Cancer Research, Sutton, Surrey, England

AT-LARGE

J. D. Chapman, Cross Cancer Institute, Edmonton, Alberta, Canada
E. A. L. Travis, University of Texas

CONTENTS OF VOLUME 98

NUMBER 1, APRIL 1984

NOBUO ODA AND SHIGERU IWANAMI. Microdosimetric Distributions for Nanometer-Size Targets in Water Irradiated with ⁶⁰ Co Gamma Rays: Frequency Distributions of Effective Primary Events by Individual Tracks of Electrons	1
D. J. BRENNER AND M. ZAIDER. The Application of Track Calculations to Radiobiology. II. Calculations of Microdosimetric Quantities	14
HIE-JOON KIM, LORNA K. MEE, S. JAMES ADELSTEIN, AND IRWIN A. TAUB. Binding-Site Specificity of the Radiolytically Induced Crosslinking of Phenylalanine to Glucagon	26
JOHN O. ARCHAMBEAU, ALFRED INES, AND LUIS F. FAJARDO. Response of Swine Skin Microvasculature to Acute Single Exposures of X Rays: Quantification of Endothelial Changes	37
HOWARD H. VOGEL, JR., AND SARA ANTAL. Neutron Irradiation of Late Mouse Embryos (15-19 Days) <i>in Utero</i>	52
TAKASHI ITO, TSUNEO KADA, SHIGEFUMI OKADA, KOTARO HIEDA, KATSUMI KOBAYASHI, HIROSHI MAEZAWA, AND ATSUSHI ITO. Synchrotron System for Monochromatic uv Irradiation (>140 nm) of Biological Material	65
KOTARO HIEDA, KATSUMI KOBAYASHI, ATSUSHI ITO, AND TAKASHI ITO. Comparisons of the Effects of Vacuum-uv and Far-uv Synchrotron Radiation on Dry Yeast Cells of Different uv Sensitivities	74
DENNIS H. J. SCHAMHART, HENDRIKA S. VAN WALRAVEN, FREDERIK A. C. WIEGANT, WILBERT A. M. LINNEMANS, JOHANNES VAN RIJN, JAAP VAN DEN BERG, AND ROELAND VAN WIJK. Thermotolerance in Cultured Hepatoma Cells: Cell Viability, Cell Morphology, Protein Synthesis, and Heat-Shock Proteins	82
KOICHI ANDO, SACHIKO KOIKE, NOBUO FUKUDA, AND CHIHIRO KANEHIRA. Independent Effect of a Mixed-Beam Regimen of Fast Neutrons and Gamma Rays on a Murine Fibrosarcoma	96
J. L. ROTI ROTI, R. HIGASHIKUBO, AND M. MACE. Protein Cross-Migration during Isolation of Nuclei from Mixtures of Heated and Unheated HeLa Cells	107
OVE V. SOLESVIK, EINAR K. ROFSTAD, AND TOR BRUSTAD. Vascular Changes in a Human Malignant Melanoma Xenograft following Single-Dose Irradiation	115
GEORGE R. MERRIAM, JR., BASIL V. WORGUL, CECILY MEDVEDOVSKY, MARCO ZAIDER, AND HARALD H. ROSSI. Accelerated Heavy Particles and the Lens. I. Cataractogenic Potential	129
C. J. KOCH, C. C. STOBBE, AND E. A. BUMP. The Effect on the K_m for Radiosensitization at 0°C of Thiol Depletion by Diethylmaleate Pretreatment: Quantitative Differences Found Using the Radiation Sensitizing Agent Misonidazole or Oxygen	141
F. IANZINI, L. GUIDONI, P. L. INDOVINA, V. VITI, G. ERRIU, S. ONNIS, AND P. RANDACCIO. Gamma-Irradiation Effects on Phosphatidylcholine Multilayer Liposomes: Calorimetric, NMR, and Spectrofluorimetric Studies	154
G. P. RAAPHORST, J. A. VADASZ, AND E. I. AZZAM. Thermal Sensitivity and Radiosensitization in V79 Cells after BrdUrd or IdUrd Incorporation	167
DONALD G. BAKER, HOLLIDAY SAGER, WILLIAM CONSTABLE, AND NIGEL GOODCHILD. The Response of Previously Irradiated Skin to Combinations of Fractionated X Radiation, Hyperthermia, and <i>cis</i> -Diamminedichloroplatinum	176
BOBBY R. SCOTT. Methodologies for Predicting the Expected Combined Stochastic Radiobiological Effects of Different Ionizing Radiations and Some Applications	182
J. B. M. JORRITSMAN AND A. W. T. KONINGS. The Occurrence of DNA Strand Breaks after Hyperthermic Treatments of Mammalian Cells with and without Radiation	198

NUMBER 2, MAY 1984

R. SILBERBERG, C. H. TSAO, J. H. ADAMS, JR., AND J. R. LETAW. Radiation Doses and LET Distributions of Cosmic Rays	209
HIROSHI MAEZAWA, TAKASHI ITO, KOTARO HIEDA, KATSUMI KOBAYASHI, ATSUSHI ITO, TOMOYUKI MORI, AND KENSHI SUZUKI. Action Spectra for Inactivation of Dry Phage T1 after Monochromatic	

(150–254 nm) Synchrotron Irradiation in the Presence and Absence of Photoreactivation and Dark Repair	227
GABRIEL A. INFANTE, PABLO GUZMAN, RAMON ALVAREZ, ANTONIO FIGUEROA, JOSÉ N. CORREA, JOHN A. MYERS, LINDA J. LANIER, T. M. WILLIAMS, SONIA BURGOS, JORGE VERA, AND P. NETA. Radiosensitization by Derivatives of Isoindole-4,7-dione	234
JAMES V. WIEROWSKI, ROLLAND R. THOMAS, AND KENNETH T. WHEELER. DNA Repair Kinetics in Mammalian Cells following Split-Dose Irradiation	242
J. O. ARCHAMBEAU AND G. W. BENNETT. Quantification of Morphologic, Cytologic, and Kinetic Parameters of Unirradiated Swine Skin: A Histologic Model	254
MASAMI WATANABE, MASAKATSU HORIKAWA, AND OSAMU NIKAIIDO. Induction of Oncogenic Transformation by Low Doses of X Rays and Dose-Rate Effect	274
MICHAEL J. TILBY, PAMELA S. LOVEROCK, AND E. MARTIN FIELDEN. An Effect of Elevated Post-irradiation pH on the Yield of Double-Strand Breaks in DNA from Irradiated Bacterial Cells	284
U. SÜTTERLIN, W.-G. THIES, H. HAFFNER, AND A. SEIDEL. Comparative Studies on the Lysosomal Association of Monomeric ²³⁹ Pu and ²⁴¹ Am in Rat and Chinese Hamster Liver: Analysis with Sucrose, Metrizamide, and Percoll Density Gradients of Subcellular Binding as Dependent on Time	293
DETLEF GABEL, RALPH G. FAIRCHILD, BÖRJE LARSSON, AND HANS G. BÖRNER. The Relative Biological Effectiveness in V79 Chinese Hamster Cells of the Neutron Capture Reactions in Boron and Nitrogen	307
J. DENEKAMP, M. C. JOINER, AND R. L. MAUGHAN. Neutron RBEs for Mouse Skin at Low Doses per Fraction	317
WILHELM NOTHDURFT, KARL HEINZ STEINBACH, AND THEODOR M. FLIEDNER. Dose- and Time-Related Quantitative and Qualitative Alterations in the Granulocyte/Macrophage Progenitor Cell (GM-CFC) Compartment of Dogs after Total-Body Irradiation	332
T. S. HERMAN, K. J. HENLE, W. A. NAGLE, A. J. MOSS, AND T. P. MONSON. Exposure to Pretreatment Hypothermia as a Determinant of Heat Killing	345
RAYMOND L. WARTERS AND O. LEE STONE. The Sedimentation Coefficient and Buoyant Density of Nucleosomes from Replicating Chromatin in Heated Cells	354
SAORI FUJITA AND NOBUO EGAMI. Effect of Gamma Irradiation on the Reproductive System of the Pond Snail <i>Physa acuta</i>	362
EDWARD P. CLARK, EDWARD R. EPP, JOHN E. BIAGLOW, MICHELE MORSE-GAUDIO, AND EWE ZACHGO. Glutathione Depletion, Radiosensitization, and Misonidazole Potentiation in Hypoxic Chinese Hamster Ovary Cells by Buthionine Sulfoximine	370
KAZUTO SATO, JAMES F. FLOOD, AND TAKASHI MAKINODAN. Influence of Conditioned Psychological Stress on Immunological Recovery in Mice Exposed to Low-Dose X Irradiation	381
MARTIN H. SCHNEIDERMAN AND G. S. SCHNEIDERMAN. G ₂ Cells: Progression Delay and Survival	389
WILLIAM F. WARD, AGOSTINO MOLteni, CHUNG-HSIN TS'AO, AND NORMAN H. SOLLIDAY. Radiation Injury in Rat Lung. IV. Modification by D-Penicillamine	397
F. A. STEWART, J. A. SORANSON, E. L. ALPEN, M. V. WILLIAMS, AND J. DENEKAMP. Radiation-Induced Renal Damage: The Effects of Hyperfractionation	407
ACKNOWLEDGMENTS	421

NUMBER 3, JUNE 1984

A. M. KELLERER. Chord-Length Distributions and Related Quantities for Spheroids	425
VENKATARAMAN SRINIVASAN AND JOSEPH F. WEISS. Suppression of Delayed-Type Hypersensitivity to Oxazolone in Whole-Body-Irradiated Mice and Protection by WR-2721	438
JAMES R. OLESON, AYAAD ASSAAD, MARK W. DEWHIRST, DONALD W. DEYOUNG, KATHRYN J. GROCHOWSKI, AND DALICE A. SIM. Regional Hyperthermia by Magnetic Induction in a Beagle Dog Model: Analysis of Thermal Dosimetry	445
MASANORI OTAKE AND ROSS L. PRENTICE. The Analysis of Chromosomally Aberrant Cells Based on Beta-Binomial Distribution	456
J. VAN RIJN, J. VAN DEN BERG, D. H. J. SCHAMHART, AND R. VAN WIJK. Morphological Response and Survival of Hepatoma Cells during Fractionated Hyperthermia: Effect of Glycerol	471

KAZUO SAKAI AND SHIGEFUMI OKADA. Radiation-Induced DNA Damage and Cellular Lethality in Cultured Mammalian Cells	479
R. A. READ, M. H. FOX, AND J. S. BEDFORD. The Cell Cycle Dependence of Thermotolerance. II. CHO Cells Heated at 45.0°C	491
LAURIE ROIZIN-TOWLE, JOHN E. BIAGLOW, HERBERT L. MELTZER, AND MARIE E. VARNES. Factors Associated with the Preincubation Effect of Hypoxic Cell Sensitizers <i>in Vitro</i> and Their Possible Implications in Chemosensitization	506
D. CHMELEVSKY, A. M. KELLERER, J. LAFUMA, M. MORIN, AND R. MASSE. Comparison of the Induction of Pulmonary Neoplasms in Sprague-Dawley Rats by Fission Neutrons and Radon Daughters	519
JOHN E. MOULDER AND DOUGLAS F. MARTIN. Hypoxic Fraction Determinations with the BA1112 Rat Sarcoma: Variation within and among Assay Techniques	536
T. P. LIN AND JEAN Y. CHAN. Effect of Laser Microbeam Irradiation of the Nucleus on the Cleavage of Mouse Eggs in Culture	549
D. C. LLOYD, A. A. EDWARDS, J. S. PROSSER, D. BOLTON, AND A. G. SHERWIN. Chromosome Aberrations Induced in Human Lymphocytes by D-T Neutrons	561
AKIKO M. UENO, OSAMU TANAKA, AND HIROMICHI MATSUDAIRA. Inhibition of Gamma-Ray Dose-Rate Effects by D ₂ O and Inhibitors of Poly(ADP-ribose) Synthetase in Cultured Mammalian L5178Y Cells	574
TOKIKO NAKAYAMA AND WATARU NAKAMURA. Platelet Aggregation Induced in Mice by Whole-Body Hyperthermia	583
ETHEL S. GILBERT. Some Effects of Random Dose Measurement Errors on Analyses of Atomic Bomb Survivor Data	591
G. M. WOODS AND R. M. LOWENTHAL. Effects of Irradiation on PHA-Induced T-Lymphocyte Colonies: Differential Effects According to the Timing of Irradiation	606
R. D. LLOYD, C. W. JONES, C. W. MAYS, D. R. ATHERTON, F. W. BRUENGER, AND G. N. TAYLOR. ²²⁸ Th Retention and Dosimetry in Beagles	614
NANCY L. OLEINICK, SONG-MAO CHIU, AND LIBBY R. FRIEDMAN. Gamma Radiation as a Probe of Chromatin Structure: Damage to and Repair of Active Chromatin in the Metaphase Chromosome	629
THOMAS M. KOVAL. Multiphasic Survival Response of a Radioresistant Lepidopteran Insect Cell Line	642
NORIO SUZUKI. Radiation Response of "Clonogenic" Tumor Cell Release from NFSA2ALM1 Tumors	649
ANNOUNCEMENTS	656
AUTHOR INDEX FOR VOLUME 98	658

The Subject Index for Volume 98 will appear in the December 1984 issue as part of a cumulative index for the year 1984.

Chord-Length Distributions and Related Quantities for Spheroids

A. M. KELLERER

*Institut für Medizinische Strahlenkunde der Universität Würzburg,
Versbacher Strasse 5, D-8700 Würzburg*

KELLERER, A. M. Chord-Length Distributions and Related Quantities for Spheroids. *Radiat. Res.* 98, 425-437 (1984).

The chord-length distributions are derived that result when spheroids are randomly traversed by straight lines. The first part of the article applies generally to convex domains in three-dimensional or two-dimensional space; the relationships between the chord-length distributions and their moments for different types of randomness are summarized. Subsequently the chord-length distributions, the point-pair distance distributions, and the geometric reduction factors are derived by a suitable transformation from the distributions for the sphere. All integrals can be resolved and the resulting formulae are valid for both prolate and oblate spheroids. The moments of the chord-length distributions are obtained by the same transformation from those for the sphere. The solutions for ellipses are given in the Appendix and contain Legendre integrals.

INTRODUCTION

Chord-length distributions result when convex bodies are randomly intercepted by straight lines. These distributions and related concepts have applications in acoustics, microscopy, texture analysis, shielding or dose calculations, and microdosimetry. Early results have been obtained by Crofton (1, 2). Kingman (3, 4), Coleman (5, 6), and Enns and Ehlers (7) have made important recent contributions. Surveys are given by Kendall and Moran (8) and by Coleman (9). Weil (10) offers an excellent general overview with a comprehensive list of references. Simple analytical expressions of the chord-length distributions exist for the sphere and for the infinite slab, and, in the two-dimensional case, for the disc and for the rectangle (4). A formula containing one integration has been obtained for cylinders with convex cross sections (11). For circular cylinders the integral requires numerical integration (12). For parallelepipeds Coleman (13) has derived the explicit solutions.

The distance distributions of pairs of random points in convex bodies are linked to the chord-length distributions. They, too, have simple analytical expressions in the case of the sphere and the infinite slab; Borel (14) has considered the point-pair distance distributions for two-dimensional figures. A solution for general cylinders, including the case of circular cylinders and parallelepipeds (15), contains a somewhat simpler integral than the solution for the chord-length distributions.

The attention to parallelepipeds and circular cylinders stems from various applications. The case of spheroids has comparatively less pragmatic importance and has therefore rarely been treated. However, it is not without interest in applications of microdosimetry to radiobiology where one deals with cells, cell nuclei, or various cell

organelles that have rounded but frequently nonspherical shape. Such structures are often adequately approximated as spheroids, i.e., as ellipsoids with two axes of equal length. Accordingly the objective of the present article is the derivation of the chord-length distributions, the point-pair distance distributions, and the moments of these distributions for spheroids. Allisy and Boutillon (16) have, within the context of microdosimetric computations for neutrons, utilized a transformation that links the chord-length distribution of the spheroid to that of the sphere. The present approach is somewhat different, but the idea is also to apply a suitable transformation to the solutions for a sphere. The results are not entirely new. Enns and Ehlers (7) have earlier given the chord-length distribution of prolate spheroids and their moments, although they have not reported the details of their derivation.

INTERRELATIONS AMONG THE VARIOUS QUANTITIES

The Distributions

The earlier article with solutions for cylinders (15) deals also with general properties of the chord-length distributions and point-pair distributions, and with their interrelations. This treatment applies equally to the present article. A brief but somewhat more complete restatement of essential relations and their synopsis in tabular form may nevertheless be useful.

There are different types of randomness for the intercept of a convex body, K , by straight lines. Following the recent terminology of Coleman (13) one can distinguish three main types: *isotropic uniform randomness* results when the body is exposed to a uniform isotropic fluence of infinite straight lines; *weighted randomness* results when a uniformly distributed random point is chosen in K and is traversed by a straight line with uniform random direction; *two-point randomness* is obtained when a straight line traverses two random points that are independently and uniformly distributed in K . The subscripts μ , ν , λ , respectively, are used for these three cases.¹ For example, $f_\mu(x)$ designates the chord-length density under isotropic uniform randomness, and $F_\mu(x)$ designates the sum distribution. For convenience all sum distributions are summed from the left (see Eq. (14)). The letters μ , ν , and λ are also used for the moments; the order is given by an index. For example, μ_1 is the mean value and μ_k is the moment of order k of $f_\mu(x)$:

$$\mu_k = \int_0^\infty x^k f_\mu(x) dx. \quad (1)$$

The probability densities of the chord lengths for the three different types of randomness are related (see (4, 12)):

$$f_\mu(x) = ax^{-1}f_\nu(x) = bx^{-m}f_\lambda(x). \quad (2)$$

In three-dimensional space, R_3 ,

$$a = 4V/S, \quad b = 12V^2/\pi S \quad \text{and} \quad m = 4 \quad (3)$$

V : volume; S : surface of K . In two-dimensional space, R_2 ,

¹ $f_\mu(x)$ was formerly designated by $f_1(x)$, and the term *interior* randomness was used instead of *weighted* randomness.

TABLE I

Relations between Chord-Length Distributions for Different Types of Randomness^a

	μ	ν	i	p	λ
μ	$f_\mu(x)$	$= ax^{-1}f_\nu(x)$	$= -af'_i(x)$	$= aU''(x)$	$= bx^{-n-1}f_\lambda(x)$
ν	$xf_\mu(x)/a$	$= f_\nu(x)$	$= xf'_i(x)$	$= xU''(x)$	$= x^{-n}f_\lambda(x)/c$
i	$F_\mu(x)/a$		$= f_i(x)$	$= -U'(x)$	
p	$\int_s^x F_\mu(x)/a$		$= F_i(x)$	$= U(x)$	
λ	$x^{n+1}f_\mu(x)/b$	$= cx^n f_\nu(x)$	$= cx f'_i(x)$	$= cx^{n+1}U''(x)$	$= f_\lambda(x)$

^a For simplicity a number of relations are omitted; they can be obtained from the inverse relations. The geometric reduction factor, $U(x)$, is used instead of $f_p(x)$; $s(x)$ is the proximity function.

In R_3 : $n = 3$, $a = 4V/S$, $b = 12 V^2/\pi S$, $f_p(x) = 4\pi x^2 U(x)/V = s(x)/V$, $c = a/b$.

In R_2 : $n = 2$, $a = \pi A/C$, $b = 3A^2/C$, $f_p(x) = 2\pi x U(x)/A = s(x)/A$, $c = a/b$.

$$a = \pi A/C \quad \text{and} \quad b = 3A^2/C \quad \text{and} \quad m = 3 \tag{4}$$

A : area; C : perimeter of K . The constant a is equal to the mean chord length μ_1 (see Eqs. (12), (13) and Table II).

Other types of randomness involve the selection of random points on the surface of the body (17). They will not be considered here, since they stand in no known relation to μ -, ν -, or λ -randomness.

A further concept of importance in radiation dosimetry has been termed i -randomness (internal source randomness) (11); it refers to the distribution of the length, x , of randomly oriented rays from random points in K . The length, x , of the ray is the distance from the random point to the intersection of the ray with the surface of S . The probability density is designated by $f_i(x)$. It is related to the chord-length distribution (11)

$$f_i(x) = F_\mu(x)/a. \tag{5}$$

$F_i(x)$ is equal to the *geometric reduction factor*, $U(x)$, that is important in dose calculations with internal emitters (see (18, 19)) and that is closely linked to the point-pair distance distribution.² This latter distribution results if pairs of points are chosen independently and uniformly in the body, and if their distances are considered. The probability density of these distances is designated by $f_p(x)$; it is related to $U(x)$ or $F_i(x)$:

$$f_p(x) = 4\pi x^2 U(x)/V = 4\pi x^2 F_i(x)/V \text{ in } R_3 \tag{6}$$

$$f_p(x) = 2\pi x U(x)/A = 2\pi x F_i(x)/A \text{ in } R_2. \tag{7}$$

Table I gives a synopsis of the various interrelations that result from Eqs. (2)–(7). For simplicity the quantity $U(x)$ is utilized instead of the density $f_p(x)$. The proximity function, $s(x)$, (see (15)) equals $f_p(x) \cdot V$ in R_3 and $f_p(x) \cdot A$ in R_2 .

² In spite of their numerical identity for convex bodies, the concepts of $F(x)$ and $U(x)$ are distinguished. $U(x)$ has earlier been designated by $\phi(x)$ and $\Omega(x)$ (7, 18); it is defined as the probability that a shift by x in random direction from a random point in K leads again to a point in K . This definition, as the definition of $f_p(x)$, applies equally to convex and nonconvex bodies. The chord-length distributions, on the other hand, pertain only to convex bodies; they admit different extensions to nonconvex bodies.

TABLE II
Relations between the Moments of the Chord-Length Distributions

μ	ν	i	$In R_3^a$		$In R_2^b$	
			p	λ	p	λ
$\mu_{-1} =$	av_{-2}			$= b\lambda_{-5}$		$= b\lambda_{-4}$
$1 =$	av_{-1}			$= b\lambda_{-4}$		$= b\lambda_{-3}$
$\mu_1 =$	a			$= b\lambda_{-3}$		$= b\lambda_{-2}$
$\mu_2 =$	$av_1 = 2ai_1$	$= b/6p_{-2}$		$= b\lambda_{-2} = b/3p_{-1}$		$= b\lambda_{-1}$
$\mu_3 =$	$av_2 = 3ai_2$	$= b/2p_{-1}$		$= b\lambda_{-1} = b$		
$\mu_4 =$	$av_3 = 4ai_3$	$= b$		$= 2bp_1$		$= b\lambda_1$
$\mu_5 =$	$av_4 = 5ai_4$	$= 5b/3p_1$		$= b\lambda_1 = 10b/3p_2$		$= b\lambda_2$
$\mu_k =$	$av_{k-1} = kai_{k-1}$	$= b \frac{k(k-1)}{12} p_{k-4}$		$= b\lambda_{k-4} = b \frac{k(k-1)}{6} p_{k-3}$		$= b\lambda_{k-3}$

^a In R_3 : $a = 4V/S$, $b = 12 V^2/\pi S$.

^b In R_2 : $a = \pi A/C$, $b = 3A^2/C$.

The Moments

Equations (2)–(4) yield the relations between the moments of the chord-length distributions:

$$\mu_k = av_{k-1} = b\lambda_{k-m}, \quad (k \geq -1).^3 \tag{8}$$

From Eqs. (5)–(7) one obtains by partial integration

$$\mu_k = kai_{k-1}, \quad (k \geq 1) \tag{9}$$

and

$$p_k = \frac{4\pi}{V(k+3)} i_{k+3} = \frac{12}{(k+3)(k+4)} \lambda_k, \quad (k \geq -2) \text{ in } R_3 \tag{10}$$

$$p_k = \frac{2\pi}{A(k+2)} i_{k+2} = \frac{6}{(k+2)(k+3)} \lambda_k, \quad (k \geq -1) \text{ in } R_2. \tag{11}$$

Table II gives a synopsis of these relations and contains several notable identities for moments that depend only on integral parameters of K . In R_3 one has for any convex body

$$\begin{aligned} \mu_1 &= a = 4V/S & \mu_4 &= b = 12V^2/\pi S \\ \nu_{-1} &= 1/a = S/4V & \nu_3 &= b/a = 3V/\pi \\ & & i_3 &= b/4a = 3V/4\pi \\ \lambda_{-4} &= 1/b = \pi S/12V^2 & \lambda_{-3} &= a/b = \pi/3V. \end{aligned} \tag{12}$$

³ The case $k = -1$ does not apply to figures with corners or to bodies with edges; μ_{-1} , ν_{-2} , and λ_{-4} (in R_2) or λ_{-5} (in R_3) are then infinite.

in R_2 one has

$$\begin{aligned} \mu_1 &= a = \pi A/C & \mu_3 &= b = 3A^2/C \\ \nu_{-1} &= 1/a = C/\pi A & \nu_2 &= b/a = 3A/\pi \\ & & i_2 &= b/3a = A/\pi \\ \lambda_{-3} &= 1/b = C/3A^2 & \lambda_{-2} &= a/b = \pi/3A. \end{aligned} \tag{13}$$

No such general relation is known for moments of the point-pair distance distributions.

The Distributions for the Sphere

The solutions for the sphere will be required subsequently; they can also serve to illustrate the various distributions. One obtains, with the diameter d and with $0 \leq x \leq d$:

$$\begin{aligned} f_\mu(x) &= 2x/d^2; & F_\mu(x) &= 1 - (x/d)^2 \\ f_\nu(x) &= 3x^2/d^3; & F_\nu(x) &= 1 - (x/d)^3 \\ f_\lambda(x) &= 6x^5/d^6; & F_\lambda(x) &= 1 - (x/d)^6 \\ f_i(x) &= \frac{3}{2d} - \frac{3x^2}{2d^3}; & F_i(x) &= U(x) = 1 - \frac{3}{2} \frac{x}{d} + \frac{1}{2} \left(\frac{x}{d}\right)^3 \\ f_p(x) &= \frac{24x^2}{d^3} \left(1 - \frac{3x}{2d} + \frac{x^3}{2d^3}\right); & F_p(x) &= 1 - 8\left(\frac{x}{d}\right)^3 + 9\left(\frac{x}{d}\right)^4 - 2\left(\frac{x}{d}\right)^6. \end{aligned} \tag{14}$$

The distributions are represented in Fig. 1. The moments will be listed subsequently in Eqs. (32)–(36) for spheroids; the solutions for the sphere result when all parameters e_k are set equal to unity.

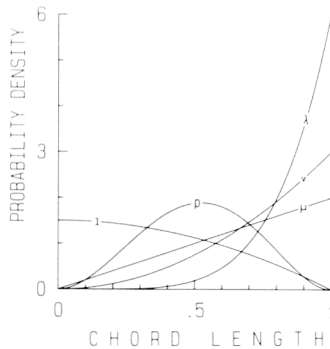


FIG. 1. Chord-length distributions for the sphere (see Eq. (14)): μ -randomness (uniform isotropic distribution of straight lines); ν -randomness (straight lines through random point in sphere); λ -randomness (straight lines through two random points in sphere); i -randomness (ray originating in random point); p : point-pair distance distribution.

SOLUTION FOR SPHEROIDS

Principle of the Solution and Derivation of the Transformation Kernel

The solutions will first be formulated in terms of the two distributions $f_\lambda(x)$ and $f_p(x)$ that result from the random choice of pairs of points in K . The related functions $s(x)$, $U(x)$, $f_u(x)$, $f_v(x)$, or $f_i(x)$ can then readily be obtained.

A unidirectional compression or expansion by the factor e transforms the sphere into an oblate ($e < 1$) or prolate ($e > 1$) spheroid, i.e., this transformation, T , establishes a one-to-one relation between the points of the sphere and their image points in the spheroid. Two independently, uniformly distributed random points in the sphere have images that are also independently, uniformly distributed in the spheroid. This will be utilized in the solution, i.e., the distributions $f_\lambda(x)$ and $f_p(x)$ in the spheroid will be obtained by applying a suitable transformation to the corresponding distributions in the sphere.

Consider two points in the sphere that are separated by the distance u . The distance, x , between their image points can then, depending on orientation, have any value between u and ue . The point pairs in the sphere are randomly oriented and the distribution, $h_u(x)$, of resulting distances is therefore obtained by considering a spherical surface of radius u , and by asking for the distribution in distance from the center that results after the points of the surface are subjected to the transformation T . Figure 2 indicates this schematically for an oblate spheroid; it will be noted that the result applies equally to prolate spheroids.

Let $H_u(x)$ be the sum distribution that belongs to $h_u(x)$, i.e., the probability that the transformation changes the distance u into a distance larger than x . It is apparent that $H_u(x)$ is equal to that fraction of the surface of the hemisphere in Fig. 2, that lies to the right of the broken line for $e > 1$ and to the left for $e < 1$. These fractions are $1 - z/u$ and z/u :

$$H_u(x) = \begin{cases} z/u = \sqrt{(1 - x^2/u^2)/(1 - e^2)}, & ue \leq x \leq u \text{ for } e < 1 \\ 1 - z/u = 1 - \sqrt{(x^2/u^2 - 1)/(e^2 - 1)}, & u \leq x \leq ue \text{ for } e > 1 \end{cases} \quad (15)$$

$$h_u(x) = -\frac{dH_u(x)}{dx} = \frac{x}{u\sqrt{(1 - e^2)(u^2 - x^2)}}. \quad (16)$$

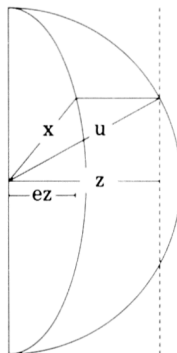


FIG. 2. Diagram for the derivation of the transformation kernel $H_u(x)$.

The transformation will be utilized to obtain the distributions and the moments for spheroids from the solutions for the sphere.

Evaluation of the Transformation Formula

If the distributions for the sphere are marked by a star, one has the following formulae for the spheroid:

$$f_\lambda(x) = \int_{u_1}^{u_2} h_u(x) f_\lambda^*(u) du \tag{17}$$

and

$$f_p(x) = \int_{u_1}^{u_2} h_u(x) f_p^*(u) du \tag{18}$$

with:

$$\begin{aligned} 0 \leq x \leq d, & \quad u_1 = x & \text{and} & \quad u_2 = \text{Min}(d, x/e), & \text{for} & \quad e < 1 \\ 0 < x < ed, & \quad u_1 = x/e & \text{and} & \quad u_2 = \text{Min}(d, x), & \text{for} & \quad e > 1. \end{aligned} \tag{19}$$

Derivation of the chord-length distribution. Equation (17) for the chord-length distribution will be evaluated first. The distribution, $f_\mu(x)$, for uniform isotropic randomness has more pragmatic importance than $f_\lambda(x)$; utilizing Eq. (2) one can rewrite Eq. (17) in terms of $f_\mu(x)$ and $f_\mu^*(x)$:

$$\begin{aligned} f_\mu(x) &= \frac{e^2 \pi d^2}{Sx^4} \int_{u_1}^{u_2} h_u(x) u^4 f_\mu^*(u) du \\ &= \frac{2e^2 \pi}{Sx^3 \epsilon} \int_{u_1}^{u_2} \frac{u^4}{|u^2 - x^2|^{.5}} du \end{aligned} \tag{20}$$

$\epsilon = |1 - e^2|^{.5}$ is the linear excentricity; S is the surface area of the spheroid.

To obtain a common formulation, in terms of real functions, for the oblate and the prolate spheroid, it is practical to use a function that merges the inverse cosine and the inverse hyperbolic cosine:

$$ci(x) = \begin{cases} \cos^{-1}(x), & \text{for } 0 \leq x \leq 1 \\ \text{Cos}^{-1}(x) = \ln(x + (x^2 - 1)^{.5}), & \text{for } x > 1. \end{cases} \tag{21}$$

Furthermore one can use the two constants

$$c_1 = \frac{1}{2} + \frac{e^2}{2\epsilon} ci\left(\frac{1}{e}\right) \quad \text{and} \quad c_2 = \frac{1}{4e^2} + \frac{3}{4} c_1. \tag{22}$$

The surface of the spheroid is $S = c_1 \cdot \pi \cdot d^2$, and the integration of Eq. (20) gives the chord-length distribution for the spheroid⁴

$$f_\mu(x) = \frac{2x}{c_1 d^2} \left[c_2 + \frac{\epsilon}{4(e^{-2} - 1)} \left\{ \left| \frac{d^2}{x^2} - 1 \right|^{.5} \left(\frac{d^3}{x^3} + \frac{3d}{2x} \right) + \frac{3}{2} ci\left(\frac{d}{x}\right) \right\} \right]. \tag{23}$$

⁴ The corresponding distribution, $f_\mu(x) = (3c_1 x / 2ed) f_\mu(x)$, for weighted randomness agrees with the expression given by Enns and Ehlers (7, Eq. (31)) for the prolate spheroid, although the identity of the two expressions is not readily evident.

The first term, c_2 , in the square bracket applies only for $0 < x < ed$, the second term for $ed < x < d$ or $d < x < ed$. The distributions are illustrated in Fig. 3.

Derivation of the point-pair distance distributions. Inserting the expression from Eq. (14) into Eq. (18) one obtains

$$f_p(x) = \frac{24x}{d^3 \epsilon} \int_{u_1}^{u_2} \frac{\left(u - \frac{3}{2} \frac{u^2}{d} + \frac{u^4}{2d^3}\right)}{|u^2 - x^2|^{5/2}} du. \quad (24)$$

The integration yields

$$f_p(x) = \frac{24x^2}{d^3 e} \left[1 - \frac{3x}{2d} \frac{c_1}{e} + \frac{x^3}{2d^3} \frac{c_2}{e} + \frac{3}{8} \frac{\epsilon}{(e^{-1} - e)} \times \left\{ \left| \frac{d^2}{x^2} - 1 \right|^{5/2} \left(\frac{x^2}{2d^2} + 1 \right) + \left(\frac{x^3}{2d^3} - \frac{2x}{d} \right) ci\left(\frac{d}{x}\right) \right\} \right]. \quad (25)$$

The expression in the first line of the equation applies for $0 < x < ed$, the expression in the second line for $ed < x < d$ or $d < x < ed$.

The distributions are represented in Fig. 4. The related quantity $U(x) = F_i(x) = f_p(x)d^3e/24x^2$ is equal to the expression in the square brackets in Eq. (25).

The Moments

A direct derivation of the moments will make it unnecessary to integrate the complicated distributions.

If u is the distance of two random points in the sphere, or the length of the chord defined by these random points, then the distribution of the corresponding distances, x , in the spheroid is $h_u(x)$ and the expectation of x^k for this fixed value u is

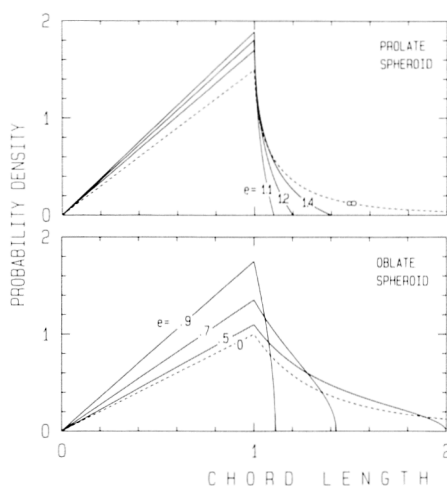


FIG. 3. Chord-length distributions (μ -randomness) for spheroids of elongation e and of unit length smaller axis ($d = 1$ for $e > 1$; $d = 1/e$ for $e < 1$).

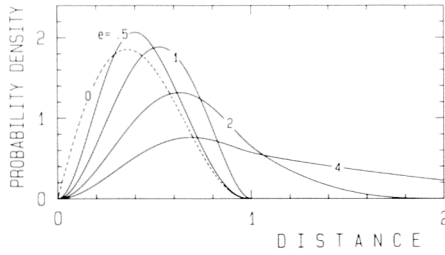


FIG. 4. Point-pair-distance distributions for the spheroid of elongation e and with two equal axes of unit length ($d = 1$).

$$\begin{aligned} \langle x_u^k \rangle &= \int x^k h_u(x) dx \\ &= \frac{1}{\epsilon} \int \frac{x^{k+1}}{u|u^2 - x^2|^{.5}} dx \\ &= e_k u^k \end{aligned} \tag{26}$$

with

$$e_k = \frac{\epsilon}{1 - e^2} \int_e^1 \frac{z^{k+1}}{|1 - z^2|^{.5}} dz, \quad (e_k = 1 \text{ for } e = 1) \tag{27}$$

Evaluation of these integrals gives

$$\begin{aligned} e_{-5} &= \frac{2}{3e} + \frac{1}{3e^3} & e_{-1} &= \frac{1}{\epsilon} ci(e) \\ e_{-4} &= \frac{1}{2e^2} + \frac{1}{2\epsilon} ci\left(\frac{1}{e}\right) & e_0 &= 1 \\ e_{-3} &= \frac{1}{e} & e_1 &= \frac{e}{2} + \frac{1}{2\epsilon} ci(e) \\ e_{-2} &= \frac{1}{\epsilon} ci\left(\frac{1}{e}\right) & e_2 &= \frac{2}{3} + \frac{e^2}{3}. \end{aligned} \tag{28}$$

Accordingly one has the relation between the moments for the spheroid and the moments, λ_k^* , of the sphere

$$\lambda_k = \int_0^d \langle x_u^k \rangle f_\lambda^*(u) du = e_k \lambda_k^*, \quad k \geq -5. \tag{29}$$

One has (see Eqs. (13), (14)):

$$\lambda_k^* = 6/(k + 6)d^k, \quad k \geq -5 \tag{30}$$

$$b = 1/\lambda_{-4} = d^4/3e_{-4} \quad \text{and} \quad b/a = 1/\lambda_{-3} = d^3/2e_{-3}. \tag{31}$$

Therefore one obtains, according to the relations in Table II,

$$\mu_k = \frac{e_{k-4}}{e_{-4}} \frac{2}{(k + 2)} d^k, \quad k \geq -1 \tag{32}$$

$$\nu_k = \frac{e_{k-3}}{e_{-3}} \frac{3}{(k+3)} d^k, \quad k \geq -2 \tag{33}$$

$$i_k = \frac{e_{k-3}}{e_{-3}} \frac{3}{(k+1)(k+3)} d^k, \quad k \geq 0 \tag{34}$$

$$\lambda_k = e_k \frac{6}{(k+6)} d^k, \quad k \geq -5 \tag{35}$$

$$p_k = e_k \frac{3 \cdot 4 \cdot 6}{(k+3)(k+4)(k+6)} d^k, \quad k \geq -2 \tag{36}$$

For the sphere all e_k 's are equal to 1.

Figure 5 gives, as numerical example, the mean values $\mu_1, \nu_1 (=2i_1)$, and $x_1 (=5/3p_1)$ for spheroids of unit length smaller axis.

APPENDIX: SOLUTION FOR ELLIPSES

The solution for ellipses is analogous to that for spheroids; however, the integrals cannot be solved analytically. It will be sufficient to cite the results; all notations correspond to those for the three-dimensional case.

Distributions for the Circle

For the circle of diameter d and with the abbreviation $X = x/d$ one has

$$f_\mu(x) = \frac{1}{d} X \sqrt{1 - X^2}; \quad F_\mu(x) = \sqrt{1 - X^2} \tag{A.1}$$

$$f_\nu(x) = \frac{4}{\pi d} X^2 \sqrt{1 - X^2}; \quad F_\nu(x) = \frac{2}{\pi} [\cos^{-1}(X) + X \sqrt{1 - X^2}] \tag{A.2}$$

$$f_i(x) = \frac{4}{\pi d} \sqrt{1 - X^2}; \quad F_i(x) = \frac{2}{\pi} [\cos^{-1}(X) - X \sqrt{1 - X^2}] \tag{A.3}$$

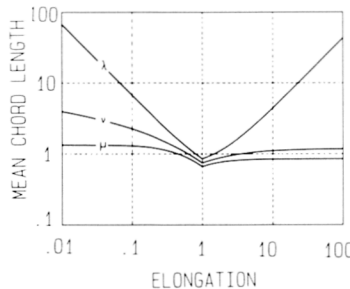


FIG. 5. The mean values $\mu_1, \nu_1 (=2i_1)$, and $\lambda_1 (=5/3p_1)$ for the different types of randomness in the spheroid. The smaller axis is taken to be of unit length ($d = 1$ for $e > 1$; $d = 1/e$ for $e < 1$).

$$f_\lambda(x) = \frac{16}{3\pi d} X^4/\sqrt{1 - X^2}; \quad F_\lambda(x) = \frac{2}{\pi} \left[\left(X + \frac{2}{3} X^3 \right) \times \sqrt{1 - X^2} - \sin^{-1}(X) \right] \quad (\text{A.4})$$

$$f_\rho(x) = \frac{16}{\pi d} X[\cos^{-1}(X) - X\sqrt{1 - X^2}]; \quad F_\rho(x) = \frac{2}{\pi} \left[\left(X + \frac{1}{2} X^3 \right) \times \sqrt{1 - X^2} + (1 - 4X^2) \cos^{-1}(X) \right]. \quad (\text{A.5})$$

Transformation Kernel

It is sufficient to consider the case $e > 1$. One obtains

$$H_u(x) = \frac{2}{\pi} \cos^{-1} \left[\left(\frac{x^2/u^2 - 1}{e^2 - 1} \right)^{.5} \right], \quad u \leq x \leq eu \quad (\text{A.6})$$

and

$$h_u(x) = \frac{2x}{\pi[(e^2u^2 - x^2)(x^2 - u^2)]^{.5}}, \quad u \leq x \leq eu. \quad (\text{A.7})$$

The Distribution for Ellipses

Using Eqs. (A.1), (A.4), and (A.7) one obtains

$$f_u(x) = \frac{2e}{C x^2} \int_{u_1}^{u_2} \frac{u^4 du}{[(u^2 - x^2/e^2)(x^2 - u^2)(d^2 - u^2)]^{.5}}, \quad 0 < x < ed \quad (\text{A.8})$$

with $u_1 = x/e$ and $u_2 = \text{Min}(x, d)$. C is the perimeter of the ellipse:

$$C = 2ed \int_0^{\pi/2} [1 - (e^2 - 1) \sin^2 z]^{.5} dz \approx \frac{(1 + e)\pi}{2} \frac{64 - 3\lambda^4}{64 - 16\lambda^2} d, \quad (\text{A.9})$$

with $\lambda = \frac{e - 1}{e + 1}$.

For the point-pair distance distribution one finds:

$$f_p(x) = \frac{32x}{d^2\pi^2} \int_{u_1}^{u_2} \frac{u \left[\cos^{-1} \left(\frac{u}{d} \right) - \frac{u}{d} \left(1 - \frac{u^2}{d^2} \right)^{.5} \right] du}{[(e^2u^2 - x^2)(x^2 - u^2)]^{.5}}, \quad 0 \leq x \leq ed. \quad (\text{A.10})$$

Eqs. (A.8) and (A.10) can not be solved in closed form, but can be expressed in terms of standard Legendre functions.

The Moments

From Eq. (A.1) one obtains the moments for the circle

$$\mu_k^* = j_k d^k, \quad k \geq -1$$

with

$$j_k = \int_0^1 \frac{z^{k+1}}{(1-z^2)^5} dz = \frac{\sqrt{\pi}}{2} \cdot \Gamma\left(\frac{k}{2} + 1\right) / \Gamma\left(\frac{k}{2} + \frac{3}{2}\right)$$

$$= \frac{\pi}{2}, 1, \frac{\pi}{4}, \frac{2}{3}, \frac{3\pi}{16}, \frac{8}{15}, \frac{5\pi}{32}, \frac{16}{35}, \dots \quad \text{for } k = -1, 0, 1, 2 \dots \quad (\text{A.11})$$

and from Eq. (8) with $b = 3\pi d^3/16$

$$\lambda_k^* = (16 j_{k+3}/3\pi) d^k. \quad (\text{A.12})$$

In analogy to Eq. (26) one obtains

$$\langle x_u^k \rangle = e_k u^k \quad \text{with} \quad e_k = \frac{2}{\pi} \int_1^e \frac{z^{k+1}}{[(e^2 - z^2)(z^2 - 1)]^{.5}} dz. \quad (\text{A.13})$$

Therefore the moments for the ellipse are

$$\lambda_k = e_k (16 j_{k+3}/3\pi) d^k, \quad k \geq -4 \quad (\text{A.14})$$

and with $b = 1/\lambda_{-3}$ and $a/b = \lambda_2$ (see Eq. (11)) and the relations in Table II:

$$\mu_k = \frac{\lambda_{k-3}}{\lambda_{-3}} = \frac{e_{k-3}}{e_{-3}} j_k d^k, \quad k \geq -1$$

$$\nu_k = \frac{\lambda_{k-2}}{\lambda_{-2}} = \frac{e_{k-2}}{e_{-2}} \frac{4 j_{k+1}}{\pi} d^k, \quad k \geq -2$$

$$i_k = \frac{\lambda_{k-2}}{(k+1)\lambda_{-2}} = \frac{e_{k-2}}{e_{-2}} \frac{4 j_{k+1}}{(k+1)\pi} d^k, \quad k \geq 0$$

$$p_k = \frac{6\lambda_k}{(k+2)(k+3)} = e_k \frac{32 j_{k+3}}{(k+2)(k+3)\pi} d^k, \quad k \geq -1 \quad (\text{A.15})$$

ACKNOWLEDGMENT

This work was supported by EURATOM Contract BIO-286-81-D.

RECEIVED: August 30, 1983; REVISED: November 4, 1983

REFERENCES

1. M. W. CROFTON, Geometrical theorems relating to mean values. *Proc. London Math. Soc.* **8**, 304–309 (1877).
2. M. W. CROFTON, Probability. In *Encyclopaedia Britannica*, 9th ed., 1885. [Partly contained also in Probability, *Encyclopaedia Britannica*, 11th ed., 1913.]
3. J. F. C. KINGMAN, Mean free paths in a convex reflecting region. *J. Appl. Prob.* **2**, 162–168 (1965).
4. J. F. C. KINGMAN, Random secants of a convex body. *J. Appl. Prob.* **6**, 660–672 (1969).
5. R. COLEMAN, Random paths through convex bodies. *J. Appl. Prob.* **6**, 430–441 (1969).
6. R. COLEMAN, Random paths through rectangles and cubes. *Metallography* **6**, 103–114 (1973).
7. E. G. ENNS and P. F. EHLERS, Random paths through a convex region. *J. Appl. Prob.* **15**, 144–152 (1978).
8. M. G. KENDALL and P. A. P. MORAN, *Geometrical Probability*. Griffin, London, 1963.

9. R. COLEMAN, An introduction to mathematical stereology. Memoir Series, Department of Theoretical Statistics, Institute of Mathematics, Aarhus University, Denmark, 1979.
10. W. WEIL, *Stereology. A Survey for Geometers*, in press.
11. A. M. KELLERER, Considerations on the random traversal of convex bodies and solutions for general cylinders. *Radiat. Res.* **47**, 359–376 (1971).
12. U. MÄDER, Chord length distributions for circular cylinders. *Radiat. Res.* **82**, 454–466 (1980).
13. R. COLEMAN, Intercept lengths of random probes through boxes. *J. Appl. Prob.* **18**, 276–282 (1981).
14. E. BOREL, *Principes et formules classiques du calcul des probabilités. Traité du calcul des probabilités et de ses applications*. Gauthier-Villars, Paris, 1925.
15. A. M. KELLERER, Proximity functions for general right cylinders. *Radiat. Res.* **86**, 264–276 (1981).
16. A. ALLISY and M. BOUTILLON, Distribution de l'énergie déposée par des neutrons à l'intérieur d'ellipsoïdes de révolution. In *Proceedings, Second Symposium on Microdosimetry* (H. G. Ebert, Ed.) pp. 183–192. Commission of the European Communities, Brussels, 1970.
17. P. F. EHLERS and E. G. ENNS, Random secants of a convex body generated by surface randomness. *J. Appl. Prob.* **18**, 157–166 (1981).
18. M. J. BERGER, Beta-ray dosimetry calculations with the use of point kernels. In *Medical Radionuclides: Radiation Dose and Effects* (R. J. Cloutier, C. L. Edwards, and W. S. Snyder, Eds.) pp. 63–86. USAEC Division of Technical Information Extension, Oak Ridge, TN, [Available as Report CONF-691212 from National Technical Information Service, Springfield, VA 22161.]
19. M. J. BERGER, Distribution of absorbed dose around point sources of electrons and β -particles in water and other media. *J. Nucl. Med.*, Suppl. 5, 5–23 (1971).