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RADIATION RESEARCH 97, 237–245 (1984)

On the Determination of Microdosimetric Parameters in Time-Varying Radiation Fields: The Variance–Covariance Method

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KELLERER, A. M., AND ROSSI, H. H. On the Determination of Microdosimetric Parameters in Time-Varying Radiation Fields: The Variance–Covariance Method. *Radiat. Res.* **97**, 237–245 (1984).

Microdosimetric measurements in their usual form are difficult in fields of high intensity where events, i.e., passages of charged particles, cannot be individually resolved. The variance method can then be utilized to obtain the dose averages of the microdosimetric distributions from repeated measurements over finite time intervals. An essential condition for the method is that the dose per measurement interval is constant, but this condition cannot be fulfilled in accelerator-produced fields that consist of variable radiation pulses. A modified method is therefore proposed that does not require equal doses per measurement interval. It utilizes two detectors that operate in phase, and it derives the microdosimetric parameters from the difference between the variance of the signal from one detector and the covariance of the concomitant signals from the two detectors.

INTRODUCTION

Microdosimetric measurements are usually performed in radiation fields of sufficiently low intensity that individual events, i.e., charged-particle traversals through the detector, can be resolved. The determination of the probability distribution of energy imparted, ϵ, specific energy, z, or lineal energy, y, is then straightforward, and it is equally simple to obtain frequency averages or the weighted averages of these quantities.

The situation is more complicated when the fluence rates are so high that individual events cannot be resolved. This is especially pronounced when the radiation source is a pulsed accelerator. Although it may be possible to employ a detector of such small physical size that the mean number of events is substantially less than one, this would, especially with a small duty cycle, greatly prolong the time needed to obtain a representative distribution of y.

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If individual events cannot be resolved one may integrate the detector response over specified time intervals that correspond to constant doses. The distribution of the signals obtained for the individual intervals can then be utilized to derive the moments of the single-event distributions, and, in principle, it could even be used to reconstruct the distribution itself. The technique of measuring the detector response over specified time intervals was applied in early microdosimetric studies by Rossi *et al.* (*1*); it has more recently been made the basis of the *variance method* for the determination of the weighted averages $\bar{\epsilon}_D$, $\bar{z}_D$, or $\bar{y}_D$ of single events for a radiation field (*2, 3*). However, the variance method requires equal dose increments in successive measurement intervals. It is not applicable to fields from accelerators that produce high-intensity pulses of variable size. A modified technique will therefore be described that utilizes a pair of detectors and permits the determination of microdosimetric parameters in such cases.

GENERAL CONSIDERATIONS AND CONVENTIONS ON NOTATION

Before the two-detector method is presented in mathematical terms it may be useful to outline the underlying concepts.

When a dosimeter is exposed to a pulse of radiation which imparts a given absorbed dose, D, it will register a specific energy, z, which is variable. Consequently repeated exposures to D result in a distribution, $f(z)$. In conventional measurements of absorbed dose this distribution has negligible width, but at sufficiently low doses a suitable detector such as a proportional counter exhibits fluctuations that for equal D are greater for high-LET radiations and, as indicated above, the relative variance of the distribution $f(z)$ can in fact be employed to determine ζ, the weighted average of individual increments of z.

However, if D varies in successive exposures, the distribution $f(z)$ is broader and may be indistinguishable from a spectrum that would be obtained if the detector were exposed to radiation of higher LET (i.e., larger ζ) with constant increments of absorbed dose that are equal to the mean value, $\bar{D}$, of the absorbed dose when it is variable.

One method of overcoming the difficulty is to determine the variations of D between pulses. As will be shown, it is merely necessary for this purpose to measure a quantity that is proportional to D; the factor of proportionality need not be known. Thus one could, for instance, determine the variations of the output of an electrometer which registers the charge received by an accelerator target in each duty cycle. In this way a suitable correction can be derived that permits the derivation of ζ from the observed variance of z.

It will further be shown that the output of the second instrument may also be subject to statistical fluctuations. A special case is that of an identical counter receiving the same doses, D. While the spectrum generated by the two devices, A and B, is the same, correlation of concomitant pulse heights of the two counters is absent if D is fixed, but it is present if D varies. Thus when D is larger there is an enhanced probability that both counters register a larger z. From knowledge of the correlation one can correct the observed variance of z to obtain ζ.

In this method it is, however, essential that at a specified dose, D, the two devices

respond independently. If, for example, two proportional counters could both be traversed by the same energetic charged particles this kind of correlation would be mistaken for that which is associated with variability of D; the result would be an underestimate of ζ.

A few conventions on notation will be helpful to make the subsequent arguments and results more transparent. The first convention is to utilize the relative *variance*, $V(z)$, of a random variable z:

$$V(z) = \overline{(z - \bar{z})^2}/\bar{z}^2 = \overline{z^2}/\bar{z}^2 - 1. \tag{1}$$

The second convention is to utilize the relative *covariance*, $C(z_A, z_B)$, or for brevity, C_{AB}, of the two random variables z_A and z_B:

$$C_{AB} = \frac{\overline{(z_A - \bar{z}_A)(z_B - \bar{z}_B)}}{\bar{z}_A \cdot \bar{z}_B} = \frac{\overline{z_A z_B}}{\bar{z}_A \bar{z}_B} - 1. \tag{2}$$

It is apparent that the relative variance is merely the special form of the relative covariance in the case where the two random variables are identical:

$$V_A = C_{AA}. \tag{3}$$

The variance expresses the mean deviation of a random variable from its expectation value; the covariance expresses the mean *correlated* deviations of two random variables from their expectation values. If two random variables are statistically independent their covariance is zero.

It should be noted that the *relative*, i.e., normalized, quantities V_A, V_B, and C_{AB} are not dependent on detector calibrations and have the same value for any physical quantity that is proportional to that measured by the detector.

A gas-filled microdosimetric detector simulates a tissue region of linear dimensions that are smaller by the ratio, r, of the densities. Accordingly the mass, M, of the detector gas exceeds the mass, m, of the simulated region by the factor r^2, and the same applies to the number of events. It is usual in microdosimetry to relate the quantities z and D to the simulated mass, m, rather than to M. For example, if a detector of 1-cm diameter simulates a 1-μm tissue region, then the absorbed dose 0.1 mGy to the detector will correspond to an absorbed dose D = 10 kGy in the simulated site, and this latter quantity will be referred to in the subsequent equations.

THE VARIANCE METHOD AND ITS EXTENSION TO VARIABLE FIELDS

Consider a stable field and the measurement of the specific energy accumulated within successive time intervals of equal length. The mean, $\bar{z}$, and the mean square, $\overline{z^2}$, of these values are then, as has been shown earlier (*4*),

$$\bar{z} = D, \qquad \overline{z^2} = \zeta D + D^2. \tag{4}$$

D is the absorbed dose per interval, and ζ is the weighted average of specific energy in single events:[1]

[1] In agreement with common usage and for simplicity of notation the symbol ζ is used rather than the more explicit notation $\bar{z}_D$.

$$\zeta = \int z^2 f_1(z)dz \Big/ \int z f_1(z)dz. \tag{5}$$

$f_1(z)$ is the single-event distribution, i.e., $f_1(z)dz$ is the probability for an increment between z and $z + dz$ if an event occurs [see for example (*5, 6*)].

The microdosimetric parameter ζ can be utilized to characterize the quality of a radiation. It is connected to the weighted average, $\bar{y}_D$, of lineal energy that is the microdosimetric analogon of dose-average LET.

Inserting Eq. (4) into Eq. (1) one obtains the relation for ζ:

$$\zeta = V(z) \cdot \bar{z} = \overline{z^2}/\bar{z} - \bar{z}. \tag{6}$$

$\overline{z^2}$ and $\bar{z}$ can be estimated from the values, z_i, obtained from measurements for a sufficiently large number, I, of intervals:

$$\overline{z^2} = \frac{1}{I} \sum_{i=1}^{I} z_i^2, \qquad \bar{z} = \frac{1}{I} \sum_{i=1}^{I} z_i. \tag{7}$$

This procedure for the determination of ζ—the variance method—is dependent on the condition that the absorbed dose per interval is constant. If, on the other hand, the dose, D, per interval is a random variable with probability density $p(D)$ and with mean $\bar{D}$ and mean square $\overline{D^2}$,

$$\bar{D} = \int Dp(D)dD \qquad \text{and} \qquad \overline{D^2} = \int D^2 p(D)dD \tag{8}$$

one obtains by integration of the terms in Eq. (4) over the distribution $p(D)$:

$$\bar{z} = \bar{D} \qquad \text{and} \qquad \overline{z^2} = \int (\zeta D + D^2)p(D)dD = \zeta\bar{D} + \overline{D^2}. \tag{9}$$

Inserting this into Eq. (1) one has

$$V(z) = \zeta/\bar{D} + V(D). \tag{10}$$

The relation for ζ is

$$\zeta = (V(z) - V(D)) \cdot \bar{z}. \tag{11}$$

This result is readily understood. Dose fluctuations cause the additional contribution, $V(D)$, to the observed relative variance of z that is irrelevant to the microdosimetric evaluation and that must therefore be subtracted in the formula for ζ. The correction term vanishes if the dose, D, per interval is constant; one obtains then the familiar formula of Eq. (6).

For actual numerical evaluations one can use the equation

$$\zeta = (\overline{z^2}/\bar{z}^2 - \overline{D^2}/\bar{D}^2) \cdot \bar{z} \tag{12}$$

with the estimates from Eq. (7) and with

$$\bar{D} = \frac{1}{I} \sum_{i=1}^{I} D_i, \qquad \overline{D^2} = \frac{1}{I} \sum_{i=1}^{I} D_i^2 \tag{13}$$

where D_i are the doses determined with a dose monitor that operates in phase with the microdosimetric detector.

Equation (12) has the evident advantage that it is independent of the dose-monitor calibration, and, more importantly, that it remains valid if the dose rates at the location of the monitor differ by a constant factor from those at the location of the microdosimetric detector. Nevertheless it can be impractical to use a dose monitor. If an instrument is to measure absorbed dose, rather than specific energy, it needs to have a sufficiently large size to overcome the random fluctuations of energy deposition. In variable nonuniform fields, the absorbed doses measured by the monitor may then not be proportional to those at the location of the microdosimetric detector. An additional problem can be the uncertain influence of the limited resolution of any dose monitor on measurements that depend critically on fluctuations of the observed values. A better method is therefore required.

THE VARIANCE-COVARIANCE METHOD

Consider a pair of microdosimetric detectors, A and B, that operate in phase and determine concomitant values $z_{A,i}$ and $z_{B,i}$ in a series of measurement intervals, i. The detectors are assumed to be in sufficient proximity that the doses, $D_{A,i}$ and $D_{B,i}$, at their respective locations differ at most by a constant factor:

$$D_{A,i} = D_i, \qquad D_{B,i} = f \cdot D_i. \tag{14}$$

On the other hand, the two detectors must be sufficiently separated to be *uncoupled* in the sense that correlated events, i.e., energy depositions due to the same particle or its secondaries, can be disregarded. The two detectors may, but need not, be identical. The weighted mean event sizes, ζ_A and ζ_B, in the two detectors depend on their effective diameters and can, accordingly, be different. There is, furthermore, no need to postulate that the radiation qualities are identical at the locations of the two detectors.

The expectation value, $\overline{z_A z_B}$, of the product of the concomitant detector signals involves a double average. Consider first a fixed dose, $D_A = D$ and $D_B = f \cdot D$, per measurement interval. The expectation value $\overline{z_A z_B}$ is then equal to the product of the expectation values $\bar{z}_A = D$ and $\bar{z}_B = f \cdot D$. The reason is that the two random variables z_A and z_B are, for a fixed D, uncorrelated. This follows from the assumption that the two detectors are uncoupled, i.e., that they are not traversed by the same particles. As a second step one considers the variations of the dose, D, per interval. These variations cause correlation between z_A and z_B; by integration over the distribution, $p(D)$, of D one obtains

$$\overline{z_A z_B} = f \int D^2 p(D) dD = f\overline{D^2}. \tag{15}$$

The relative covariance of z_A and z_B is therefore equal to the relative variance of the dose

$$C_{AB} = \overline{z_A z_B}/(\bar{z}_A \cdot \bar{z}_B) - 1 = \overline{D^2}/\bar{D}^2 - 1 = V(D). \tag{16}$$

Applied to Eq. (11) this permits a determination, without dose monitor, of the parameters ζ_A or ζ_B.

It is a simple but notable result that the variance of the dose per interval can be

obtained from a pair of uncoupled but simultaneously operating detectors, and that no dose monitor is required for this purpose.

Eq. (11) can now be replaced by

$$\zeta_A = (V_A - C_{AB}) \cdot \bar{z}_A \tag{17}$$

with the analogous relation for ζ_B being

$$\zeta_B = (V_B - C_{AB}) \cdot \bar{z}_B. \tag{18}$$

For the actual numerical evaluation the result can be expressed in terms of the quantities $\overline{z_A}$, $\overline{z_A^2}$, z_B, $\overline{z_B^2}$, $\overline{z_A z_B}$ determined with the twin detectors:

$$V_A = \overline{z_A^2}/\bar{z}_A^2 - 1, \qquad V_B = \overline{z_B^2}/\bar{z}_B^2 - 1$$
$$C_{AB} = \overline{z_A z_B}/(\bar{z}_A \bar{z}_B) - 1. \tag{19}$$

The estimates are based on the values observed in the series of I measurement intervals:

$$\bar{z}_A = \frac{1}{I}\sum_{i=1}^{I} z_{A,i} \qquad \overline{z_A^2} = \frac{1}{I}\sum_{i=1}^{I} z_{A,i}^2$$
$$\bar{z}_B = \frac{1}{I}\sum_{i=1}^{I} z_{B,i} \qquad \overline{z_B^2} = \frac{1}{I}\sum_{i=1}^{I} z_{B,i}^2$$
$$\overline{z_A z_B} = \frac{1}{I}\sum_{i=1}^{I} z_{A,i} \cdot z_{B,i}. \tag{20}$$

ILLUSTRATION OF THE METHOD BY A MONTE CARLO SIMULATION

The clouds of points in the two panels of Fig. 1 represent distributions of concomitant values z_A and z_B obtained in Monte Carlo simulations of a series of exposures of two

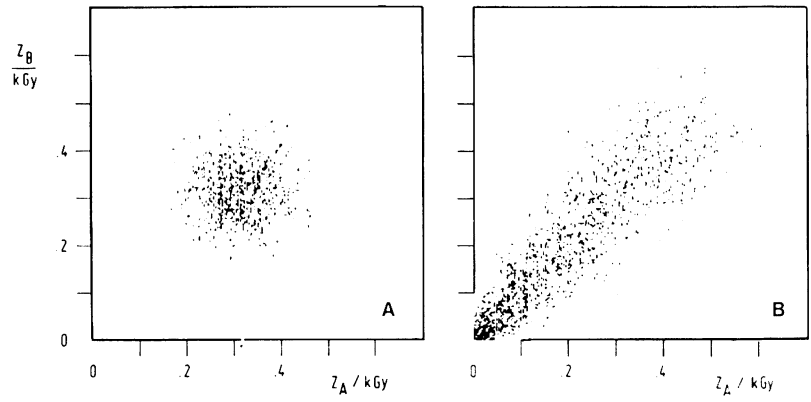

FIG. 1. Monte Carlo simulation of the repeated exposure of a pair of microdosimetric detectors (diameter of simulated spheres: 1 μm) to 340-keV neutrons. Each point represents the pair of values, z_A and z_B, obtained in an exposure. For panel A (2000 exposures) the dose per exposure is equidistributed between 300 and 330 Gy, for panel B (4000 exposures) it is equidistributed between 0 and 500 Gy.

TABLE 1

Numerical Values from the Monte Carlo Simulations in Fig. 1

	Panel	
	Left	*Right*
$\bar{z}_A$/Gy	315.9	248.4
$\bar{z}_B$/Gy	315.8	249.4
$\overline{z_A^2}$/Gy^2	104854	85962
$\overline{z_B^2}$/Gy^2	104904	86530
$\overline{z_A z_B}$/Gy^2	100063	82452
V_A	0.05072	0.3932
V_B	0.05088	0.3911
C_{AB}	0.00303	0.3309
ζ_A/Gy	15.07	15.47
ζ_B/Gy	15.43	15.03

detectors to 340-keV neutrons. The detectors were taken to simulate spherical regions of 1-μm diameter, and the computations were based on a measured single-event spectrum (*7*). The left panel represents the outcome of 2000 exposures with the dose per measurement interval equidistributed between 300 and 330 Gy; the right panel gives the results for 4000 measurement intervals with the dose per interval equidistributed between 0 and 500 Gy. As stated earlier, these are nominal doses that refer to the simulated system. The actual doses to be applied to a detector of 1-cm diameter would range from 3 to 3.3 μGy and from 0 to 5 μGy.

It is evident that there is no appreciable correlation of the detector responses in panel A, where the dose per measurement interval has been nearly constant. In panel B where there have been large variations of dose per measurement interval the correlation of concomitant detector signals is substantial. The resultant numerical values are given in Table 1; the inferred values, ζ_A and ζ_B, of the dose-average specific energy are in good agreement with the true value, $\zeta = 15.21$ Gy, of the single-event spectrum that has been utilized for the Monte Carlo simulations.

CONCLUSION

Two microdosimetric detectors operating in phase during a series of equal or variable time intervals are sufficient to derive the parameter ζ for a radiation field, or the related quantity $\bar{y}_D$ that is the microdosimetric analogon of dose-average LET. The technique does not require a constant dose rate and is applicable to the fluctuating fields of accelerators.

The method is based on the determination of the variance and the covariance of the detector signals. It utilizes the fact that the observed variance of specific energy contains two separate contributions, one from the fluctuations of energy deposition—

the process relevant to microdosimetry—the other from the fluctuations of absorbed dose per interval. The covariance of the signals of two uncoupled detectors contains only the second term, and is unaffected by the fluctuations of energy deposition. The difference between variance and covariance is therefore the quantity relevant to the microdosimetric analysis. The present communication deals only with ζ, the parameter of main pragmatic interest for the characterization of radiation quality. However, other parameters linked to higher moments of the microdosimetric distributions can be obtained by analogous methods.

In practical applications it may be advantageous to use pairs of identical detectors. However, other configurations are equally possible. In particular it may be noted that the second detector need not be calibrated absolutely, and that it need not be a microdosimetric detector in the usual sense. It is sufficient that the expectation value of its signal is proportional to absorbed dose.

The determination of the various quantities required by the two-detector method poses technical complications that can be resolved by the utilization of microprocessors or suitable circuitry. The requirements will be discussed for the likely situation in which both instruments are proportional counters. Similar considerations apply for other devices.

The utilization of a pair of instruments and the determination of the variables z_A and z_B merely requires duplication of standard equipment. The derivation of the parameter ζ necessitates determination of the quantities $\bar{z}_A$, $\bar{z}_B$, $\overline{z_A^2}$, $\overline{z_B^2}$, and $\overline{z_A z_B}$. They are obtained by summation of the detector signals and their squares and of the products of concomitant values of z_A and z_B. Appropriate, sufficiently fast circuitry needs to be provided for this purpose.

If D is received in pulses that are short compared to the time over which the charge produced in the counter is collected, no further complications arise, i.e., fast pulses are registered individually (*8*). If, however, the pulses cannot be resolved in time or if some pulses are below the bias necessary to reject noise, one may integrate over fixed times by summing charges before they are applied to the input of the preamplifier (*2, 3*).

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