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Frequency of α-Particles from $^{239}PuO_2$ in Lung Cells

A. M. Kellerer

Institut für Medizinische Strahlenkunde der Universität Würzburg,
Versbacher Landstraße 5, D-8700 Würzburg, Federal Republic of Germany

Summary. NCRP-Report 49 deals with the effect of $^{239}PuO_2$-particles in the lung. A basic aspect in the considerations is the frequency of cells traversed by one or by more α-particles. The present article contains the explicit derivation of the formulae underlying the data in the NCRP-report.

Introduction

The US NCRP-report 46 "Alpha-Emitting Particles in Lungs" [4] deals with the effect of plutonium α-particles in the lung and its dependence on the size of the $^{239}PuO_2$ particles.

If a given activity is uniformly dispersed, then a maximum number of cells is traversed by α-particles. The probability for multiple traversals of one cell is small. If, on the other hand, the activity occurs in $^{239}PuO_2$-particles, then cells close to these particles will be subject to multiple traversals; the total number of cells at risk will be accordingly reduced. In addition malignant transformations produced by the α-particles are less likely to express themselves. The reason is that multiply traversed cells will be very unlikely to retain their proliferative capacity.

The report deals with these matters in detail and arrives at the conclusion that a given activity will be less effective when it occurs in $^{239}PuO_2$ particles. Computed numbers of cells traversed by one or more α-particles are presented; these data are essential to the argumentation. Since the basis of the computations is given only in brief form in the appendix of the report, it is felt that a detailed derivation will be useful.

Computation of Traversal Frequencies

Total Number of Traversals

The lung cells will be considered as spheres of radius s. Then the mean chord length in the cell resulting from the traversal by an α-particle will be $\frac{4}{3}\,s$. This follows from

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a relation frequently called the Cauchy theorem. According to this theorem the mean traversal length through a convex domain of volume V and surface area S is equal to 4 V/S (see for example [3] or [2]).

The average density of the lung tissue is taken to be 0.2 g/cm^3. Therefore, if R is the average range of an α-particle in the lung, the actual tissue range is 0.2 R. Accordingly the total number of traversals due to the activity A during the time t is:

$$T(A, t) = \frac{3 \cdot 0.2\, R}{4\, s} A\, t = 0.15\, \frac{R}{s}\, A\, t\,. \tag{1}$$

With the commonly used units curie and day:

$$T(A, t) = 4.80 \cdot 10^{14}\, \frac{R}{s}\, \frac{A}{[\mathrm{Ci}]}\, \frac{t}{[\mathrm{d}]}\,. \tag{2}$$

Traversal Frequencies at Specified Distance from a PuO_2-Particle

If the activity is dispersed uniformly in the lung all cells have the same probability to be traversed by an α-particle. The probability for multiple traversals results then from the Poisson distribution. The mean of this Poisson distribution is equal to the ratio of $T(A, t)$ and the total number of cells in the lung. At dose levels relevant to radiation protection this ratio is much smaller than 1; multiple traversals can then be neglected.

The activity is, however, concentrated in $^{239}PuO_2$ particles, and cells close to these particles have a considerable probability to be traversed by many α-particles. In order to obtain the number of cells traversed once or several times, one must first deduce the probabilities for single and multiple traversals at a specified distance from a PuO_2-particle. Secondly one has to integrate over the cells at all distances up to R from the particle. These two steps will be considered in the following. The possibility can be disregarded that a cell is traversed by α-particles from two different PuO_2-particles.

The PuO_2 will be considered as a point source and self-absorption will be disregarded. It will accordingly be assumed that each decay results in an α-particle of range R. The fluence [1] at distance r from the PuO_2-particle is then:

$$\phi(r) = A\, t/4\pi r^2 \tag{3}$$

where A is the activity of the particle and t is its time of residence at the specified position. The expected number of traversals through a sphere is equal to the fluence times the cross-section of the sphere. Accordingly the expected number of traversals through a cell of radius s at distance r from the PuO_2 particle is:

$$\mu = k/r^2 \tag{4}$$

where the constant is:

$$k = \frac{A\, t}{4}\, s^2 \tag{5}$$

or with the convenient choice of the units curie and day:

$$k = 8.0 \cdot 10^{14} \frac{A}{[\mathrm{Ci}]} \frac{t}{[\mathrm{d}]} s^2 . \tag{6}$$

Strictly, these relations apply only if the radius s of the cell is small compared to the distance r. The resulting inaccuracy can, however, be accepted in view of the approximate nature of the calculations which account neither for the nonspherical shape of the cells nor for their non-uniform arrangement and the resulting fluctuations of the range of α-particles in the lung.

While μ is the expected number of traversals at distance r from a PuO_2-particle the actual number of traversals is a random variable which follows the Poisson distribution. The probability for ν events is:

$$p(\nu) = e^{-\mu} \frac{\mu^\nu}{\nu!} = e^{-k/r^2} \, (k/r^2)^\nu / \nu! \tag{7}$$

Integration over all Distances

Let N_R be the number of cells contained in the sphere of radius R around a PuO_2 particle. These are the cells within the range of the α-particles from the PuO_2-particle.

The number of cells within a distance r from the PuO_2-particle is then:

$$N_r = \left(\frac{r}{R}\right)^3 N_R . \tag{8}$$

Accordingly one has the following number of cells per unit interval of distance at distance r:

$$n(r) = \frac{dN_r}{dr} = \frac{3\, r^2}{R^3} N_R . \tag{9}$$

This equation, too, is written, as if the dimension of the cell were small compared to r.

Integrating the product of $p(\nu)$ and $n(r)$ over all distances from 0 to R one obtains the number of cells in the vicinity of a PuO_2-particle which are traversed by ν α-particles:

$$N(\nu) = \frac{3\, k^\nu N_R}{R^3 \nu!} \int_0^R e^{-k/r^2} \, r^{-2\nu+2} \, dr . \tag{10}$$

This equation comprises the relations for $\bar{p}_0 = N(0)/N_R$ and $\bar{p}_1 = N(1)/N_R$ which are given in the appendix of NCRP-report 46.

Equation (10) is simplified by introduction of the new variable $x = r/R$:

$$N(\nu) = \frac{3\, c^\nu N_R}{\nu!} \int_0^1 e^{-c/x^2} \, x^{2(1-\nu)} \, dx \tag{11}$$

with:

$$c = \frac{A\,t}{4}\left(\frac{s}{R}\right)^2 = 8.0 \cdot 10^{14}\,\frac{A}{[\mathrm{Ci}]}\,\frac{t}{[\mathrm{d}]}\left(\frac{s}{R}\right)^2 . \tag{12}$$

c is the expected number of traversals for those cells which are at distance R from the PuO_2-particle, i.e., just within the range of the α-particle.

Equation (11) could be expressed in terms of the incomplete gamma-function. This need, however, not be considered, as the equation is readily evaluated numerically.

Numerical Evaluation

NCRP-Report 46 presents as a function of the activity of the PuO_2-particle the number, $N(1)$, of cells within the surrounding of the PuO_2-particle which are traversed by exactly one α-particle. In addition it gives the quantity $[N_R - N(0) - N(1)]$, i.e. the number of cells traversed by several α-particles. The data result from the following values of the range R of the α-particle in the lung, the radius s of the cell (represented by type 2 pneumocytes), the time of residence t of the PuO_2-particle at a location in the lung, and the number N_R of cells within a sphere of radius R[1]:

$$\begin{aligned} R &= 200\ \mu\mathrm{m}, \\ s &= 6\ \mu\mathrm{m}, \\ t &= 1400\ \mathrm{d}, \\ N_R &= 6600 . \end{aligned} \tag{13}$$

Inserting these values one obtains from Equations (11) and (12):

$$N(0) = 1.98 \cdot 10^4 \int_0^1 e^{-c/x^2}\, x^2\, dx . \tag{14}$$

$$N(1) = 1.98 \cdot 10^4\, c \int_0^1 e^{-c/x^2}\, dx \tag{15}$$

and generally:

$$N(v) = 1.98 \cdot 10^4\, \frac{c^v}{v!} \int_0^1 e^{-c/x^2}\, x^{2-2v}\, dx \tag{16}$$

with:

$$c = 1.01 \cdot 10^{15}\,\frac{A}{[\mathrm{Ci}]} . \tag{17}$$

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[1] The values $R = 200\ \mu\mathrm{m}$ and $N_R = 6600$ correspond to the average density 0.2 g/cm³ of the lung. In those cases where the PuO_2 particle is surrounded by connective tissue with a compact spacing of cells, the values $R = 40\ \mu\mathrm{m}$ and $N_R = 300$ will be more appropriate. The mean number of traversals per irradiated cell is then even higher, and the number of irradiated cells is even further reduced

NCRP-Report 46 gives, in addition to $N(0)$ and $[1 - N(0) - N(1)]$, the quantity $T(A,t)$ [see Eq. (2)] for the activity A of the PuO_2-particle. This is the total number of traversals during time t due to this activity; it is equal to the number of singly traversed cells in the case of uniformly dispersed activity.

Figure 1 represents the essential results in a form somewhat different from that chosen in NCRP-report 46. The curve on the left gives the number of cells in the vicinity of a PuO_2-particle which are traversed by one α-particle; these are the cells which are presumed to contribute most significantly to the risk of lung cancer. The second curve gives the number of cells traversed by more than one but less than 10

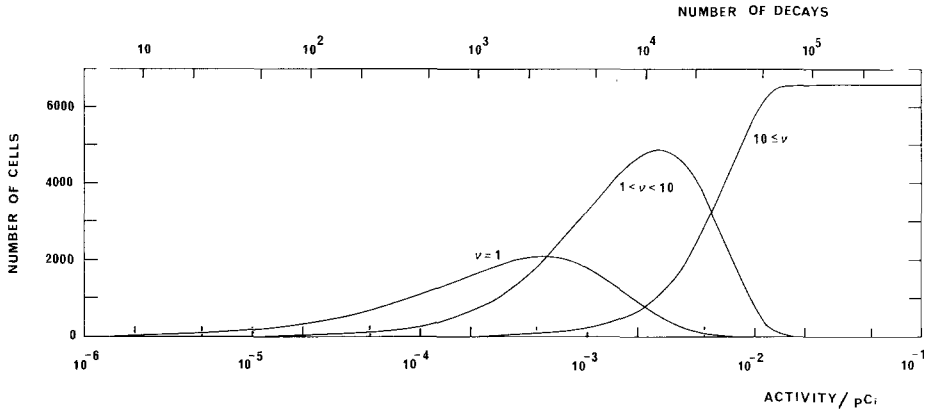

Fig. 1. Number of lung cells traversed by single α-particles, by 2–9 α-particles, and by 10 and more α-particles from a $^{239}PuO_2$-particle of specified activity A. The lower abscissa refers to an assumed exposure time $t = 1400$ days. The upper abscissa pertains to the general case. It gives the number, $A\ t$, of decays per $^{239}PuO_2$-particle

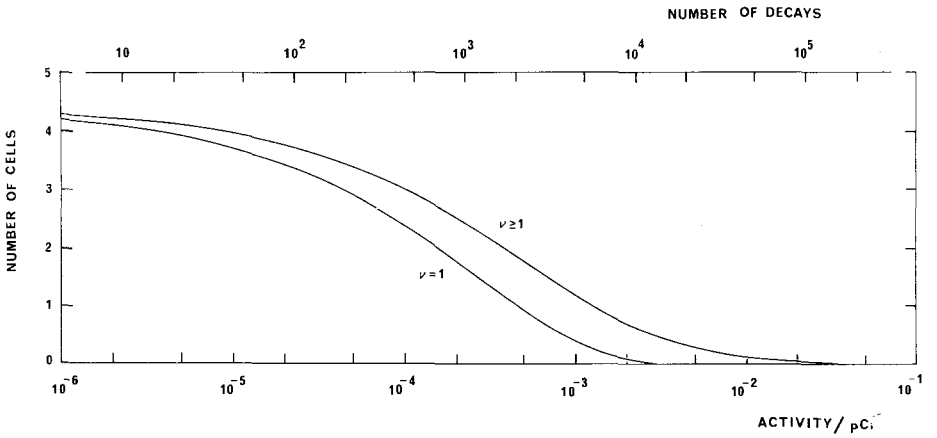

Fig. 2. Number of lung cells traversed by exactly one α-particle and number of pneumocytes traversed by at least one α-particle per decay for $^{239}PuO_2$-particles of specified activity A. The lower abscissa refers to an assumed exposure time $t = 1400$ days. The upper abscissa pertains to the general case. It gives the number, $A\ t$, of decays per $^{239}PuO_2$-particle

Table 1. Reduction, due to the non-uniform distribution of activity, of the number of cells traversed once ($\nu = 1$) and of cells traversed at least once ($\nu \geq 1$). The numbers apply to an exposure time of 1440 days

Activity per $^{239}PuO_2$-particle (pCi)	Diameter of the particle (μm)	Reduction factor	
		$\nu = 1$	$\nu \geq 1$
0.0003	0.1	0.3	0.4
0.0026	0.2	0.01	0.1
0.04	0.5	0	$7 \cdot 10^{-3}$
0.32	1	0	$9 \cdot 10^{-4}$
2.6	2	0	10^{-5}

α-particles. The third curve gives the number of cells traversed by at least 10 α-particles.

Figure 2 shows in still another way how the number of singly traversed cells and even the total number of cells subject either to single or to multiple traversals decreases if the activity is concentrated in PuO_2-particles of increasing size. The number of cells traversed by one α-particle and the number of cells with one or more traversals are in this figure given per α-particle emitted.

The decrease of the number of affected cells is also shown in Table 1. The reduction factors which are here quoted are defined as the number of cells traversed by α-particles from a $^{239}PuO_2$-particle of activity A, divided by the number of cells traversed by an α-particle if the activity A is uniformly dispersed in the lung. The numbers refer to an exposure time of 1440 days. Column 3 gives the reduction factor for the cells which are traversed once, column 4 for all cells traversed either by one or by several α-particles.

Comment

The relations which are here presented, are the explicit form of the equations underlying the numerical data in NCRP-report 46.

The calculations are based on several approximations. The self-absorption in the PuO_2-particles is disregarded; also disregarded is the non-uniform arrangement of lung cells which leads to varying ranges of the α-particles in lung tissue. These factors as well as the variations of size of the PuO_2-particles lead to deviations from the theoretical curves. An even more important factor could be the partial traversal of cells at the end of the range of the α-particles.

It is therefore to be assumed that somewhat more singly traversed or, generally speaking, more non-lethally damaged cells will occur at a specified activity per PuO_2-particle than indicated by Equations (15) and (16) and by Figures 1 and 2.

A similar shift from multiple traversals towards single traversals occurs if only traversals through the nucleus of the cell are considered. Equations (11) and (12) apply to this case if s is taken to be the radius of the cell nucleus, not the radius of the cell; the curves in Figure 1 remain valid if they are shifted by the square of the ratio of radii towards higher abscissa values.

These additional factors do not alter the essential conclusion that the number of cells at risk in the lung will be greatly reduced if plutonium occurs not uniformly dispersed but in individual PuO_2-particles.

References

1. ICRU, Report 19, Radiation Quantities and Units. Internat. Commission on Radiation Units and Measurements, Washington, D.C. 1971
2. Kellerer, A. M.: Considerations on the random traversal of convex bodies and solutions for general cylinders. Radiat. Res. **47**, 359–376 (1971)
3. Kendall, M. G., Moran, P. A. P.: Geometrical probability. London: Griffin 1963
4. NCRP, Report 46, Alpha-Emitting Particles in Lungs. National Council on Radiation Protection and Measurements, Washington, D.C. 1975

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