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INDEX TO VOLUME 15

* Starred items are "Shorter Notes"

- Abhyankar, Shreeram. *A remark on the nonnormal locus of an analytic space*, 505, 1000.
- Accola, R. D. M. *On a class of Riemann surfaces*, 607.
- Adler, R. L. and Rivlin, T. J. *Ergodic and mixing properties of Chebyshev polynomials*, 794.
- Almgren, F. J., Jr. *An isoperimetric inequality*, 284.
- Amir, D. *Projections onto continuous function spaces*, 396.
- Andrews, J. J. and Dristy, Forrest. *The Minkowski units of ribbon knots*, 856.
- Anselone, P. M. and Korevaar, J. *Translation invariant subspaces of finite dimension*, 747.
- Apostol, T. M. *A short proof of Shō Iseki's functional equation*, 618.
- Appel, K. I. and Djorup, F. M. *On the group generated by a free semigroup*, 838.
- Arkowitz, Martin. *An example for homotopy groups with coefficients*, 136.
- Austin, C. W. *Positive semicharacters of some commutative semigroups*, 382.
- Bauman, S. *Nonsolvable IC groups*, 823.
- Baxter, Glen. *On fixed points of the composite of commuting functions*, 851.
- Beck, Anatole. *A theorem on maximum modulus*, 345.
- *On rings on rings*, 350.
- Bender, E. A. *Numerical identities in lattices with an application to Dirichlet products*, 8.
- Berg, I. D. *A Banach algebra criterion for Tauberian theorems*, 648.
- Bergman, G. M. *A ring primitive on the right but not on the left*, 473, 1000.
- Berkson, A. J. *The u -algebra of a restricted Lie algebra is Frobenius*, 14.
- Bialynicki-Birula, A. *On the inverse problem of Galois theory of differential fields*, 960.
- Bojanic, R. and Musielak, J. *An inequality for functions with derivatives in an Orlicz space*, 902.
- Bouwsma, W. D. *Zeros of functions of regular growth on a line*, 938.
- Brannen, J. P. *Concerning Hausdorff matrices and absolutely convergent sequences*, 114.
- Brauer, Fred. *Nonlinear differential equations with forcing terms*, 758.
- Brauer, Richard. *A note on theorems of Burnside and Blichfeldt*, 31.
- Bridgland, T. F., Jr. *Asymptotic equilibria in homogeneous and nonhomogeneous systems*, 131.
- Browder, A. and Wermer, J. *A method for constructing Dirichlet algebras*, 546.
- Brown, A. *On the adjoint of a closed transformation*, 239.
- Brown, D. R. *On clans of non-negative matrices*, 671.
- Brown, Gordon. *A remark on semi-simple Lie algebras*, 518.
- Bruckner, A. M. *Some relationships between locally superadditive functions and convex functions*, 61.
- Bumby, R. T. *An elementary example in p -adic diophantine approximation*, 22.
- Butts, H. S. *Unique factorization of ideals into nonfactorable ideals*, 21.
- Calabi, Eugenio. *Linear systems of real quadratic forms*, 844.
- Callahan, F. P. *Density and uniform density*, 841.
- Cantor, D. G. *A simple construction of analytic functions without radial limits*, 335.
- Cantrell, J. C. *n -frames in euclidean k -space*, 574.
- Carlitz, L. *The coefficients of the reciprocal of a Bessel function*, 318.
- Carroll, F. W. *On bounded functions with almost-periodic differences*, 241.
- Carroll, Robert. *On the structure of the Green's operator*, 225.
- Cavior, S. R. *Exponential sums related to polynomials over the $GF(p)$* , 175.

- Chacon, R. V. *A class of linear transformations*, 560.
- Chacon, R. V. and Krengel, U. *Linear modulus of a linear operator*, 553.
- Chakerian, G. D. *On some geometric inequalities*, 886.
- Chao, C.-Y. *On a theorem of Sabidussi*, 291.
- Cohen, Eckford, *Arithmetical notes. X. A class of totients*, 534.
- Cohen, Haskell. *On fixed points of commuting functions*, 293.
- Cohen, H. B. *The k -norm extension property for Banach spaces*, 797.
- Coleman, D. B. *On the modular group ring of a p -group*, 511.
- Crowell, R. H. *The annihilator of a knot module*, 696.
- Dawson, D. F. *Some rate invariant sequence transformations*, 710.
- Deckard, Don and Percy, Carl. *On algebraic closure in function algebras*, 259.
- Dickson, D. G. *Infinite order differential equations*, 638.
- Distler, R. J. *The domain of univalence of certain classes of meromorphic functions*, 923.
- Djorup, F. M. See Appel, K. I.
- Dowker, Y. N. and Lederer, G. *On ergodic measures*, 65.
- Dristy, Forrest. See Andrews, J. J.
- Dudley, R. M. *Pathological topologies and random walks on abelian groups*, 231.
- Earle, C. J. *On the homotopy classes of self-mappings of bordered Riemann surfaces*, 622.
- Edelstein, Michael. *On nonexpansive mappings*, 689.
- Enochs, Edgar. *Homotopy groups of compact Abelian groups*, 878.
- Ferguson, J. D. *Entropy for noninvertible transformations*, 895
- Flanders, Harley, *Satellites of half exact functors, a correction*, 834.
- Foguel, S. R. *A counterexample to a problem of Sz.-Nagy*, 788.
- Foster, B. L. *Short proof of a theorem of Rado on graphs*, 865
- Fox, R. H. and Smythe, N. *An ideal class invariant of knots*, 707.
- Freedman, D. A. *Determining the time scale of a Markov chain*, 87.
- Gaifman, Haim and Specker, E. F. *Isomorphism types of trees*, 1.
- Gaughan, E. D. *The index problem for infinite symmetric groups*, 527.
- Gelbaum, B. R. *Von Neumann's theorem on Abelian families of operators*, 391.
- George, M. D. *Completely well-posed problems for nonlinear differential equations*, 96.
- Getoor, R. K. and Woll, J. W., Jr. *Multiplicative functionals of a Markov process*, 80.
- *Gil de Lamadrid, Jesús. *Some simple applications of the closed graph theorem*, 509.
- Gilmer, R. W., Jr. *Integral domains which are almost Dedekind*, 813.
- Ginsburg, Michael. *On the homology of fiber spaces*, 423.
- Goodman, R. W. *One-sided invariant subspaces and domains of uniqueness for hyperbolic equations*, 653.
- Goodner, D. B. *The closed convex hull of certain extreme points*, 256.
- Gottschalk, W. H. and Hedlund, G. A. *A characterization of the Morse minimal set*, 70.
- Grace, E. E. *A totally nonaposyndetic, compact, Hausdorff space with no cut point*, 281.
- Graham, Nancy. *Note on M -groupoids*, 525.
- Graham, R. L. *On quadruples of consecutive k th power residues*, 196.
- Guggenheimer, H. *Topology and elementary geometry. II. Symmetries*, 164.
- Halmos, P. R. *On Foguel's answer to Nagy's question*, 791.
- Hano, Jun-ichi. *On compact complex coset spaces of reductive Lie groups*, 159.
- Hasumi, Morisuke and Seever, G. L. *The extension and the lifting properties of Banach spaces*, 773.
- Head, T. J. *Note on the occurrence of direct factors in groups*, 193.
- Hedlund, G. A. See Gottschalk, W. H.
- Hempel, John. *A simply connected 3-manifold is S^3 if it is the sum of a solid torus and the complement of a torus knot*, 154.

- Herzog, J. O. *Phragmén-Lindelöf theorems for second order quasi-linear elliptic partial differential equations*, 721.
- Hilton, P. J. *Remark on loop spaces*, 596.
- Hinrichs, L. A. *Integer topologies*, 991.
- Hoyt, W. L. *Embeddings of Picard varieties*, 26.
- Hudson, A. L. *Note on point-wise periodic semigroups*, 700.
- Hudson, S. N. *Transformation groups in the theory of topological loops*, 872.
- Hummel, J. A. *The Grunsky coefficients of a schlicht function*, 142.
- *Hunt, R. A. and Weiss, Guido. *The Marcinkiewicz interpolation theorem*, 996.
- Insel, A. J. *A relationship between the complete topology and the order topology of a lattice*, 847.
- Jacobson, Bernard. *Sums of distinct divisors and sums of distinct units*, 179.
- Janowitz, M. F. *On the antitone mappings of a poset*, 529.
- Jensen, Chr. U. *A remark on arithmetical rings*, 951.
- Johnson, B. E. *Isometric isomorphisms of measure algebras*, 186.
- Johnson, H. H. *Bracket and exponential for a new type of vector field*, 432.
 ——— *A new type of vector field and invariant differential systems*, 675.
- Johnson, K. G. *B(S, Σ) algebras*, 247.
- Kahn, D. W. *A note on H-spaces and Postnikov systems of spheres*, 300.
- Kammerer, W. J. and Kasriel, R. H. *On contractive mappings in uniform spaces*, 288
- Kasriel, R. H. See Kammerer, W. J.
- Keisler, H. J. *Unions of relational systems*, 540.
- Kelly, J. B. *Partitions with equal products*, 987.
- Kennedy, P. B. *A problem on bounded analytic functions*, 325.
- Kim, Jehpill, *On (n-1)-dimensional factors of I^n* , 679.
- Kingman, J. F. C. and Orey, Steven. *Ratio limit theorems for Markov chains*, 907.
- Kirk, W. A. *On locally isometric mappings of a G-space on itself*, 584.
- Kishore, Nand. *The Rayleigh polynomial*, 911.
- Korevaar, J. See Anselone, P. M.
- Krause, E. F. *On the collection process*, 497.
 ——— *Groups of exponent 8 satisfy the 14th Engel congruence*, 491.
- Kreith, Kurt. *Oscillation theorems for elliptic equations*, 341.
- Krengel, U. See Chacon, R. V.
- Kwun, K. W. *Uniqueness of the open cone neighborhood*, 476.
- Langenhop, C. E. *On the stabilization of linear systems*, 735.
- Lederer, G. See Dowker, Y. N.
- Leger, G. *A note on free Lie algebras*, 517.
- Levinson, N. *The prime number theorem from $\log n!$* , 480.
 ——— *On an inequality of Opial and Beesack*, 565.
- Lima, E. L. *Commuting vector fields on S^2* , 138.
- Lindenstrauss, Joram. *On the extension of operators with range in a C(K) space*, 218.
 ——— *On projections with norm 1—an example*, 403.
- Lipschutz, Seymour. *An extension of Greendlinger's results on the word problem*, 37.
- Lodato, M. W. *On topologically induced generalized proximity relations*, 417.
- Lorch, Lee, and Szego, Peter. *Monotonicity of the differences of zeros of Bessel functions as a function of order*, 91.
- Lorentz, G. G. and Zeller, K. *Summation of sequences and summation of series*, 743.
- Loscy, Gerald. *A note on groups of prime power exponent satisfying an Engel congruence*, 209.
 ——— *On the structure of π -regular semigroups*, 955.
- McCarthy, C. A. *Commuting Boolean algebras of projections. II. Boundedness in L_p* , 781

- MacGregor, T. H. *A class of univalent functions*, 311.
 ——— *A covering theorem for convex mappings*, 310.
- MacNerney, J. S. *Characterization of regular Hausdorff moment sequences*, 366.
- Mahowald, Mark. *On embedding manifolds which are bundles over spheres*, 579.
- Marcus, Marvin. *The Hadamard theorem for permanents*, 967.
- Meyers, N. G. *Mean oscillation over cubes and Hölder continuity*, 717.
- Michael, E. *Three mapping theorems*, 410.
 ——— *A short proof of the Arens-Eells embedding theorem*, 415.
 ——— *A linear mapping between function spaces*, 407.
- Milnor, J. *On the Betti numbers of real varieties*, 275.
- Mrówka, S. G. and Pervin, W. J. *On uniform connectedness*, 446.
- Munkres, James. *Obstructions to extending diffeomorphisms*, 297.
- Musielak, J. See Bojanic, R.
- Norman, E. See Wei, J.
- Nussbaum, A. E. *On the reduction of C^* -algebras*, 567.
- Olubummo, A. *A note on perturbation theory for semi-groups of operators*, 818.
- Orey, Steven. See Kingman, J. F. C.
- Orland, G. H. *On a class of operators*, 75.
- Pareigis, Bodo. *Cohomology of groups in arbitrary categories*, 803.
- Pasiencier, Samuel. *Homogeneous almost complex spaces of positive characteristic*, 601.
- Pearcy, Carl. *On certain von Neumann algebras which are generated by partial isometries*, 393.
 ——— See Deckard, Don.
- Pervin, W. J. See Mrówka, S. G.
 ——— See Sieber, J. L.
- Petro, J. W. *Some results on the asymptotic completion of an ideal*, 519.
- Pless, Vera. *On Witt's theorem for nonalternating symmetric bilinear forms over a field of characteristic 2*, 979.
- Poole, J. T. *A note on variational methods*, 929.
- Porter, G. J. *Homotopical nilpotence of S^3* , 681.
- Posner, E. C. See Rumsey, Howard, Jr.
- Pour-El, M. B. *Gödel numberings versus Friedberg numberings*, 252.
- Putnam, Hilary. *On hierarchies and systems of notations*, 44.
- Quintas, L. V. See Supnick, Fred.
- Rajeswara Rao, K. V. *Lindelöfian meromorphic functions*, 109.
 ——— *Remarks on the classification of Riemann surfaces*, 632.
- Rapaport, E. S. *Groups of order 1*, 828.
- Redheffer, R. M. *Differential and integral inequalities*, 715.
- Ree, Rimhak. *Commutators in semi-simple algebraic groups*, 457.
- Reiner, Irving. *On the number of irreducible modular representations of a finite group*, 810.
- Retherford, J. R. *A note on unconditional bases*, 899.
- Rhoades, B. E. *Some Hausdorff matrices not of type M*, 361.
- Rivlin, T. J. See Adler, R. L.
- Roberts, J. B. *Some orthogonal functions connected with polynomial identities*. II, 127.
- Robinson, C. E. *A note on Saalfrank's generalization of absolute retract*, 308.
- Rodin, Burton. *The sharpness of Sario's generalized Picard theorem*, 373.
 ——— *Extremal length of weak homology classes on Riemann surfaces*, 369.
- Rogers, Kenneth. *The Schnirelmann density of the squarefree integers*, 515.
- Rosenstein, G. M., Jr. *A further extension of Lebesgue's covering theorem*, 683.

- Roth, R. *Flag-transitive planes of even order*, 485.
- Rothenberg, Melvin, *The J functor and the nonstable homotopy groups of the unitary groups*, 264.
- Royster, W. C. *A Poisson integral formula for the ellipse and some applications*, 661.
- *Rubel, L. A. *A complex-variables proof of Hölder's inequality*, 999.
- Rudin, Walter. *An arithmetic property of Riemann sums*, 321.
- Rumsey, Howard, Jr. and Posner, E. C. *On a class of exponential equations*, 974
- Ryan, Robert. *Representative sets and direct sums*, 387.
- Sacks, G. E. *A simple set which is not effectively simple*, 51.
- Samelson, Hans. *A note on the Bockstein operator*, 450.
- Sandler, Reuben. *Some theorems on the automorphism groups of planar ternary rings*, 984.
- Saworotnow, P. P. *On a realization of a complemented algebra*, 964.
- Seall, Robert and Wetzal, Marion. *Quadratic forms and chain sequences*, 729.
- Seeley, R. T. *Extension of C^∞ functions defined in a half space*, 625.
- Seever, G. L. See Hasumi, Morisuke.
- Selden, John. *A note on compact semirings*, 882.
- Schaefer, H. H. *On the point spectrum of positive operators*, 56.
- Schue, J. R. *The structure of hyperreducible triangular algebras*, 766.
- Sharma, P. L. *On the sequence of Fourier coefficients*, 337.
- Shields, Allen L. *On fixed points of commuting analytic functions*, 703.
- Shisha, O. See Walsh, J. L.
- Shuka, U. *A relative cohomology for associative algebras*, 461.
- Sieber, J. L. and Pervin, W. J. *Connectedness in syntopogeneous spaces*, 590.
- Silverman, Edward. *Geodesics and Lebesgue area*, 775.
- Smullyan, R. M. *Effectively simple sets*, 893.
- Smythe, N. See Fox, R. H.
- Sobczyk, Andrew. *Convex polygons*, 438.
- Specker, E. F. See Gaifman, Haim.
- Srivastava, K. N. *Inversion integrals involving Jacobi's polynomials*, 635.
- *Errata to A class of integral equations involving ultraspherical polynomials as Kernel*, 1000.
- Stearns, R. E. *On the axioms for a cooperative game without side payments*, 82.
- Stoneham, R. G. *The reciprocals of integral powers of primes and normal numbers*, 200.
- Stong, R. E. *Relations among Stiefel Whitney classes*, 151.
- Stuth, C. J. *A generalization of the Cartan-Brauer-Hua Theorem*, 211
- Sudler, Culbreth, Jr. *Two algebraic identities and the unboundedness of a restricted partition function*, 16, 1000.
- Supnick, Fred and Quintas, L. V. *Extreme Hamiltonian circuits. Resolution of the convex-odd case*, 454.
- Szego, Peter. See Lorch, Lee.
- Thaler, A. I. *On the Newton polytope*, 944.
- Tits, J. *A theorem on generic norms of strictly power associative algebras*, 35.
- Tomić, M. *A convergence criterion for Fourier series*, 612.
- Tomonaga, Yasurō. *A genus and indecomposability of differentiable manifolds*, 586.
- Treybig, L. B. *Concerning continuous images of compact ordered spaces*, 866.
- Tung, Shih-Hsiung. *Harnack's inequality and theorems on matrix spaces*, 375.
- Vermes, Robert. *On Wronskians whose elements are orthogonal polynomials*, 124
- Wachman, Murray. *Generalized Laurent series for singular solutions of elliptic partial differential equations*, 101.

- Walsh, J. L. *A theorem of Grace on the zeros of polynomials, revisited*, 354.
- Walsh, J. L. and Shisha, O. *Extremal polynomials and the zeros of the derivative of a rational function*, 753.
- Waltman, Paul. *A bifurcation theorem*, 627.
- *On the asymptotic behavior of solutions of a nonlinear equation*, 918
- Wang, J. S. P. *On completely decomposable groups*, 184.
- Watari, Chinami. *Contraction of Walsh Fourier series*, 189.
- Wei, J. and Norman, E. *On global representations of the solutions of linear differential equations as a product of exponentials*, 327.
- Weiss, Guido. See Hunt, R. A.
- Wells, James. *Restrictions of Fourier-Stieltjes transforms*, 243.
- Wermer, J. See Browder, A.
- Wetzel, Marion. See Seall, Robert.
- Wilf, H. S. *The stability of smoothing by least squares*, 933.
- Wolf, J. A. *Curvature in nilpotent Lie groups*, 271.
- Wolk, E. S. *On decompositions of partially ordered sets*, 197.
- Woll, J. W., Jr. See Getoor, R. K.
- Wong, Yuen-Fat. *Similarity transformations of hypersurfaces*, 286.
- Wright, E. M. *Direct proof of the basic theorem on multipartite partitions*, 469.
- Young, P. R. *On reducibility by recursive functions*, 889.
- Zeitlin, David. *On the sums $\sum_{k=0}^n k^p$ and $\sum_{k=0}^n (-1)^k k^p$* , 642.
- Zeller, K. See Lorentz, G. G.

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I certify that the statements made by me above are correct and complete.—Gordon L. Walker

COHOMOLOGY OF GROUPS IN ARBITRARY CATEGORIES

BODO PAREIGIS¹

1. **Introduction.** In this paper we give a short sketch of a method of doing cohomology theory of group-like objects in arbitrary categories. The way of approach is closely connected with the usual theory of cohomology of groups and has also been used by D. K. Harrison in [6]. Specifically the equivalence of the homogeneous, inhomogeneous, and normalized theories will be shown. Since we consider arbitrary categories we must give all definitions by properties of maps and cannot apply explicit computations with elements. But instead of using the diagrams which contain the maps of those definitions, we consider these maps as elements of the morphism sets and use the algebraic structures of the morphism sets which are induced by the abstractly defined structures of the objects.

In [6] two examples of this theory, Harrison's complex and Amitsur's complex, have already been mentioned. Since the homogeneous and inhomogeneous definitions of Harrison's complex are equivalent we can prove that Harrison's complex is a subcomplex of Amitsur's complex.

This theory may be developed in greater generality using certain functorial properties of the cohomology theory of groups as I. Bernstein pointed out to me. I hope that the possibilities of explicit computation as described in this paper, might also be of some interest.

2. **Notation.** Let \mathcal{C} be a category with finite direct products. This means that for a finite collection of objects B_1, \dots, B_n in \mathcal{C} there exists an object $\prod B_i = B_1 \times \dots \times B_n$ in \mathcal{C} and morphisms $p_i: \prod B_i \rightarrow B_i$, so called projections, such that for any object A in \mathcal{C} and any system of morphisms $b_i: A \rightarrow B_i$, $i = 1, \dots, n$ there exists a unique morphism $b: A \rightarrow \prod B_i$ with $p_i b = b_i$. We write (b_1, \dots, b_n) instead of b . It is easy to see that $(p_1, \dots, p_n) = \text{id}$, the identity on $\prod B_i$. The direct product of n copies of B will be written as B^n . Furthermore we denote by $\text{Mor}(A, B)$ the set of morphisms from A to B . For any finite set of morphisms $b_i \in \text{Mor}(A_i, B_i)$, $i = 1, \dots, n$ we denote by $(b_1 \times \dots \times b_n)$ the morphism $(b_1 p_1, \dots, b_n p_n) \in \text{Mor}(\prod A_i, \prod B_i)$, where $p_j \in \text{Mor}(\prod A_i, A_j)$. For the composi-

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tion of these morphisms we have the following rules [3, Propositions 3.5, 3.6, 3.7]: let $b \in \text{Mor}(A, B)$, $c_i \in \text{Mor}(B, C_i)$, $d_i \in \text{Mor}(C_i, D_i)$, and $e_i \in \text{Mor}(D_i, E_i)$, then

$$(2.1) \quad \begin{aligned} (a) \quad & (c_i, \dots, c_n)b = (c_1b, \dots, c_nb), \\ (b) \quad & (d_1 \times \dots \times d_n)(c_1, \dots, c_n) = (d_1c_1, \dots, d_nc_n), \\ (c) \quad & (e_1 \times \dots \times e_n)(d_1 \times \dots \times d_n) = (e_1d_1 \times \dots \times e_nd_n). \end{aligned}$$

Since we do not assume that the category \mathcal{C} is a category with zeros in the sense of [3] we have to use another axiom for the neutral element for group-like structures.

The complete set of axioms which we shall use is:

(I) There exists a morphism $\mu: X \times X \rightarrow X$. μ is called a multiplication on X .

(II) There exists an element $0 \in \text{Mor}(X, X)$ such that

(a) $0f = 0g$ for all objects A and all $f, g \in \text{Mor}(A, X)$,

(b) $\mu(0, \text{id}) = \text{id} \in \text{Mor}(X, X)$,

(c) $\mu(\text{id}, 0) = \text{id} \in \text{Mor}(X, X)$.

(III) $\mu(\mu \times \text{id}) = \mu(\text{id} \times \mu) \in \text{Mor}(X^3, X)$.

(IV) There exists a morphism $s \in \text{Mor}(X, X)$ such that $\mu(\text{id}, s) = \mu(s, \text{id}) = 0 \in \text{Mor}(X, X)$.

(V) Let $\text{id} = (p_1, p_2)$ and $\tau = (p_2, p_1) \in \text{Mor}(X^2, X^2)$, then $\mu = \mu\tau$.

If (I) and (III) hold, then X is called a semigroup, if (I), (II), (III), and (IV) hold, then X is called a group and if (I), (II), (III), (IV) and (V) hold, then X is called a commutative group. We write (X, μ) to indicate that μ is the multiplication on X under consideration.

One easily proves that $0 \in \text{Mor}(A, A)$ is unique. We call 0 the neutral element of the multiplication μ . If the necessary axioms hold for the multiplication μ in X , we shall use the following notation with $x_i \in \text{Mor}(A, X)$

$$(2.2) \quad \begin{aligned} (a) \quad & \mu(x_1, x_2) = x_1 \cdot x_2 \quad (= x_1 + x_2), \\ (b) \quad & 0x_i = 1 \quad (= 0), \\ (c) \quad & sx_i = x_i^{-1} \quad (= -x_i). \end{aligned}$$

The notation in parentheses will be used, if the multiplication is commutative, i.e., if axiom (V) holds.

Let now (X, μ) be a semigroup and (Y, ν) a commutative group and $\lambda \in \text{Mor}(X \times Y, Y)$. Then we shall write for all A and all morphisms $x \in \text{Mor}(A, X)$, $y \in \text{Mor}(A, Y)$

$$(2.3) \quad \lambda(x, y) = x \cdot y.$$

This notation should not interfere with the notation (2.2.a).

We call Y an X -module, if Y is a commutative group and X operates on Y as in (2.3) and if for all $x, x_1, x_2 \in \text{Mor}(A, X)$; $y, y_1, y_2 \in \text{Mor}(A, Y)$

$$(2.4) \quad \begin{aligned} (a) \quad & x \cdot (y_1 + y_2) = x \cdot y_1 + x \cdot y_2, \\ (b) \quad & (x_1 \cdot x_2) \cdot y = x_1 \cdot (x_2 \cdot y) \end{aligned}$$

and if $\text{Mor}(X, Y) \neq \emptyset$.

We shall call the X -module structure on Y trivial if $\lambda(x, y) = y$ for all A and all $x \in \text{Mor}(A, X)$, ultratrivial if $\lambda(x, y) = 0$ for all A and all $x \in \text{Mor}(A, X)$. If (X, μ) has a neutral element 1 and if $\lambda(1, y) = y$, then we call Y a unitary X -module. Certainly a trivial X -module structure on Y implies that Y is a unitary X -module. We remark furthermore that with the above definitions $\text{Mor}(A, Y)$ is a group if Y is a group [3, Theorem 4.10].

All these definitions are already well known [3] but by using the notation (2.2) and (2.3) computation will become easier and we can easily refer to computations made in the classical case. Thus axioms (2.4) are already given in this notation and stand for certain commutative diagrams.

3. Cohomology of groups. In the notation given in the preceding paragraph it is now easy to generalize the definition of the cohomology of a semigroup (X, μ) with coefficients in an X -module (Y, ν) . We define the differentiation $\partial^n: \text{Mor}(X^n, Y) \rightarrow \text{Mor}(X^{n+1}, Y)$ by

$$\begin{aligned} \partial^n(f) &= \partial^n(f)(p_1, \dots, p_{n+1}) \\ &= p_1 \cdot (f(p_2, \dots, p_{n+1})) \\ &\quad + \sum_{i=1}^n (-1)^i f(p_1, \dots, p_i \cdot p_{i+1}, \dots, p_{n+1}) \\ &\quad + (-1)^{n+1} f(p_1, \dots, p_n), \end{aligned}$$

where $p_i \in \text{Mor}(X^{n+1}, X)$, $f \in \text{Mor}(X^n, Y)$ and $(-1)^i f = s^i f$. As in ordinary cohomology theory [4] one checks $\partial^{n+1} \partial^n(f) = 0$ and that ∂^n is a homomorphism. Furthermore we define a set which we denote by $\text{Mor}(X^0, Y) = \text{Mor}(Y, Y)0$, where $0 \in \text{Mor}(X, Y)$. It is easy to see that $\text{Mor}(X^0, Y)$ is a commutative group under the induced multiplication of Y . We define $\partial^0: \text{Mor}(X^0, Y) \rightarrow \text{Mor}(X, Y)$ by

$$\partial^0(f_0)(p) = p \cdot (f_0) - (f_0) \in \text{Mor}(X, Y),$$

where $(p) = \text{id} \in \text{Mor}(X, X)$. ∂^0 is a homomorphism too and we get $\partial^1 \partial^0(f_0) = 0$.

If the category \mathfrak{C} has a final object F [5, p. 332], i.e., if there exists an object F such that $\text{Mor}(A, F)$ contains exactly one element for all A in \mathfrak{C} , and if $\text{Mor}(F, Y)$ is nonempty, then $0 \in \text{Mor}(Y, Y)$ admits the factorization

$$Y \rightarrow F \xrightarrow{0} Y$$

and $\text{Mor}(X^0, Y) \cong \text{Mor}(F, Y)$ by a natural isomorphism, which explains the definition of $\text{Mor}(X^0, Y)$. In the examples in §4 we always shall have categories with a final object.

We thus have constructed a complex of abelian groups:

$$(3.1) \quad 0 \xrightarrow{\partial^{-1}} \text{Mor}(X^0, Y) \xrightarrow{\partial^0} \text{Mor}(X, Y) \xrightarrow{\partial^1} \text{Mor}(X^2, Y) \cdots$$

We define the inhomogeneous cohomology groups of X with coefficients in Y by

$$H^n(X, Y) = \text{Ker} \partial^n / \text{Im} \partial^{n-1}, \quad n \geq 0.$$

We also can define the homogeneous cohomology groups. For this purpose we consider the set of morphisms $f \in \text{Mor}(X^{n+1}, Y)$ with the property

$$p \cdot (f(p_0, \dots, p_n)) = f(p \cdot p_0, \dots, p \cdot p_n)$$

and denote this set by $\text{Mor}_X(X^{n+1}, Y)$. One can easily prove that this definition implies

$$x \cdot (f(x_0, \dots, x_n)) = f(x \cdot x_0, \dots, x \cdot x_n)$$

for $x, x_i \in \text{Mor}(A, X)$. Obviously $\text{Mor}_X(X^{n+1}, Y)$ is still a commutative group. We define homomorphisms

$$\delta^n: \text{Mor}_X(X^n, Y) \rightarrow \text{Mor}_X(X^{n+1}, Y), \quad n \geq 1$$

by

$$\delta^n(f)(p_0, \dots, p_n) = \sum_{i=0}^n (-1)^i f(p_0, \dots, \hat{p}_i, \dots, p_n),$$

where $\hat{}$ means that the projection under this sign is to be omitted. Here again one easily checks that $\delta^{n+1}\delta^n(f) = 0$. Thus one obtains the complex

$$(3.2) \quad 0 \xrightarrow{\delta^0} \text{Mor}_X(X, Y) \xrightarrow{\delta^1} \text{Mor}_X(X^2, Y) \xrightarrow{\delta^2} \cdots$$

and defines homogeneous cohomology groups by

$$\bar{H}^n(X, Y) = \text{Ker} \delta^{n+1} / \text{Im} \delta^n, \quad n \geq 0.$$

THEOREM 3.1. *Let (X, μ) be a group and (Y, ν) a unitary X -module*

and let $\text{Mor}(Y, X) \neq \emptyset$. Then

$$\bar{H}^n(X, Y) \cong H^n(X, Y), \quad n \geq 0.$$

PROOF. This proof is exactly the same as in the ordinary cohomology of groups [4]; one proves that $\text{Mor}_X(X^{n+1}, Y) \cong \text{Mor}(X^n, Y)$ and that the differentiation operators δ^n and ∂^n commute with these isomorphisms.

If now (X, μ) is a semigroup with neutral element and (Y, ν) is an X -module then we consider those elements $f \in \text{Mor}(X^n, Y)$ with $f(x_1, \dots, x_n) = 0$ if one of the $x_i \in \text{Mor}(A, X)$ is the neutral element. This subset forms a group $\text{Mor}^N(X^n, Y)$ of normalized cochains for $n \geq 1$. For $n = 0$ we define $\text{Mor}^N(X^0, Y) = \text{Mor}(X^0, Y)$. If we denote the differentiation induced by ∂^n on these subgroups also by ∂^n we get the complex

$$(3.3) \quad 0 \xrightarrow{\partial^{-1}} \text{Mor}^N(X^0, Y) \xrightarrow{\partial^0} \text{Mor}^N(X, Y) \xrightarrow{\partial^1} \text{Mor}^N(X^2, Y) \cdots$$

and the cohomology groups

$$\hat{H}^n(X, Y) = \text{Ker} \partial^n / \text{Im} \partial^{n-1}, \quad n \geq 0.$$

As in the classical case [4] one proves

PROPOSITION 3.2. *Let (X, μ) be a semigroup with neutral element and (Y, ν) a unitary X -module. Then*

$$\hat{H}^n(X, Y) \cong H^n(X, Y), \quad n \geq 0.$$

4. **Examples.** Let us now consider two examples of this theory. In the first example let \mathcal{C} be the category of sets and set maps; then we get the ordinary cohomology of groups.

Another example is the following, due to Harrison [6]: For a commutative ring K with identity consider the category \mathcal{A} of commutative K -algebras with identity and K -algebra homomorphisms which preserve the identity. Let $\mathcal{C} = \mathcal{A}^0$ be the dual of this category. Since \mathcal{A} has finite inverse products, namely the tensor products of K -algebras, \mathcal{C} has finite direct products. Since \mathcal{A} has an initial object K (in the sense of [5]), i.e., every set of K -algebra homomorphisms from K to any K -algebra A consists of exactly one element, \mathcal{C} has a final object K^0 .

Let now A be any arbitrary K -algebra in \mathcal{A} , $\langle z \rangle = Z$ the infinite cyclic group with generator z , and G any commutative multiplicative group. We denote by $K(Z)$ and $K(G)$ the group rings of Z and G over K . Then the following definitions make A^0 into a semigroup, $K(Z)^0$ and $K(G)^0$ into groups, and induce an ultratrivial A^0 -module structure and a trivial $K(G)^0$ -module structure on $K(Z)^0$:

$$\begin{aligned}
 \mu_1: A &\rightarrow A \otimes A & \mu_1(a) &= 1 \otimes a \\
 \mu_2: K(G) &\rightarrow K(G) \otimes K(G) & \mu_2(g) &= g \otimes g \\
 \nu: K(Z) &\rightarrow K(Z) \otimes K(Z) & \nu(z) &= z \otimes z \\
 \lambda_1: K(Z) &\rightarrow A \otimes K(Z) & \lambda_1(z) &= 1 \otimes 1 \\
 \lambda_2: K(Z) &\rightarrow K(G) \otimes K(Z) & \lambda_2(z) &= 1 \otimes z.
 \end{aligned}$$

We notice that all K -algebra homomorphisms from $K(Z)$ to A are uniquely determined by the image of the generator z of Z and that the range for the images of z is just the group of units A^* of A . So we get $\text{Mor}((A^0)^n, K(Z)^0) \cong (A^n)^*$.

If we evaluate the inhomogeneous differentiation operators ∂^n , we get Amitsur's complex ([1] and [7])

$$(4.1) \quad \mathfrak{A}(A/K): 1 \rightarrow K^* \xrightarrow{\Delta^0} A^* \xrightarrow{\Delta^1} (A \otimes A)^* \xrightarrow{\Delta^2} \dots$$

from the A^0 -module $K(Z)^0$. For the $K(G)^0$ -module $K(Z)^0$ we get

$$(4.2) \quad \mathfrak{S}(G): 1 \rightarrow K^* \xrightarrow{\nabla^0} K(G)^* \xrightarrow{\nabla^1} (K(G) \otimes K(G))^* \xrightarrow{\nabla^2} \dots$$

which is Harrison's complex [6] in case $G = \mathcal{Q}/\mathbf{Z}$ and K is a field, where \mathcal{Q} are the rational numbers and \mathbf{Z} are the integers.

This example has already been given by Harrison in [6], but the definitions are repeated for the convenience of the reader.

By Theorem 3.1 we can also apply the homogeneous cohomology theory for the complex (4.2). By evaluating the maps δ^n we easily verify that this complex may be regarded as a subcomplex of Amitsur's complex $\mathfrak{A}(K(G)/K)$. Indeed the subgroups which form the complex (4.2) in the homogeneous representation consist of all elements of $(K(G)^n)^*$ of the form

$$\sum_{i=1}^m k_i g_{1,i} \otimes \dots \otimes g_{n,i}$$

such that $\prod_{j=1}^n g_{j,i} = 1$ for all i . In case $G = \mathcal{Q}/\mathbf{Z}$ and $q_{j,i} \in \mathcal{Q}/\mathbf{Z}$, we get that $\mathfrak{S}(\mathcal{Q}/\mathbf{Z})$ is a subcomplex of $\mathfrak{A}(K(\mathcal{Q}/\mathbf{Z})/K)$ and that the last conditions read:

$$\sum_{i=1}^m k_i q_{1,i} \otimes \dots \otimes q_{n,i} \in (K(\mathcal{Q}/\mathbf{Z})^n)^*$$

such that $\sum_{j=1}^n q_{j,i} = 0$ for all i .

Thus we get a factor complex \mathfrak{U} of $\mathfrak{A} = \mathfrak{A}(K(\mathcal{Q}/\mathbf{Z})/K)$ over $\mathfrak{S} = \mathfrak{S}(\mathcal{Q}/\mathbf{Z})$ and an exact sequence

$$(4.3) \quad 1 \rightarrow \mathfrak{S} \rightarrow \mathfrak{A} \rightarrow \mathfrak{U} \rightarrow 1.$$

THEOREM 4.1. $H^n(\mathfrak{S}) \cong H^{n-1}(\mathfrak{U})$, $n \geq 1$.

PROOF. The exact sequence (4.3) gives rise to an exact cohomology sequence

$$\cdots \rightarrow H^n(\mathfrak{A}) \rightarrow H^n(\mathfrak{U}) \rightarrow H^{n-1}(\mathfrak{S}) \rightarrow H^{n-1}(\mathfrak{A}) \rightarrow \cdots$$

We want to prove $H^n(\mathfrak{A}) = 0$ for all n .

For $H^n(\mathfrak{A})$ we have $H^n(\mathfrak{A}) = H^n(K(\mathfrak{Q}/\mathfrak{Z})/K)$, and $K(\mathfrak{Q}/\mathfrak{Z}) \cong \varinjlim K(G)$ for finite cyclic groups G and by [7, p. 345] $H^n(\mathfrak{A}) \cong \varinjlim H^n(K(G)/K)$.

Now let G be a cyclic group and 1 be the unit element of G . We define $\alpha: K \rightarrow K(G)$ by $\alpha(k) = k \cdot 1$ where $k \in K$, and $\beta: K(G) \rightarrow K$ by $\beta(\sum k_i g^i) = \sum k_i$, where $k_i \in K$ and g the generator of G . Then as in the proof of [9, Lemma 3.1] we have that the chain maps defined by α and β induce maps $\alpha^*: H^n(K/K) \rightarrow H^n(K(G)/K)$ and $\beta^*: H^n(K(G)/K) \rightarrow H^n(K/K)$ which are isomorphisms. So $H^n(K(G)/K) = 0$ for all finite cyclic groups G and thus $H^n(K(\mathfrak{Q}/\mathfrak{Z})/K) = 0$. By the exact cohomology sequence (4.4) we get the desired result of Theorem 4.1.

We have studied the homogeneous form of Harrison's complex. If one tries to construct homogeneous cohomology groups for Amitsur's cohomology one will find that the cohomology groups vanish, due to the more general fact that for ultratrivial module structures the complex (3.2) vanishes.

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