

CONCEPTUAL BASIS FOR CALCULATIONS OF ABSORBED-DOSE DISTRIBUTIONS

Recommendations of the
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Appendix C

A Compilation of Geometric Reduction Factors for Standard Geometries

In the following, typical geometries are considered and the resulting geometric-reduction factors are given. The point-pair distance distributions are not listed but can readily be obtained from the geometric-reduction factor:

$$p_{AB}(x) = 4\pi x^2 U_{AB}(x) / V_B. \quad (\text{C.1})$$

C.1 The Autologous Case (A = B)

As source region A and receptor B coincide in the autologous case, the simplified notation $U(x)$ can be used instead of $U_{AB}(x)$. Outside the specified intervals for x , all functions $U(x)$ or $U_{AB}(x)$ are zero.

a) *Infinite slab of height h :*

$$U(x) = \begin{cases} 1 - \frac{x}{2h}, & 0 \leq x \leq h \\ \frac{h}{2x}, & x \geq h. \end{cases} \quad (\text{C.2})$$

b) *Sphere of radius r :*

$$U(x) = 1 - \frac{3x}{4r} + \frac{x^3}{16r^3}, \quad 0 \leq x \leq 2r. \quad (\text{C.3})$$

c) *Spherical shell with outer radius R and inner radius r:*

For $r \leq R/3$:

$$U(x) = \frac{1}{R^3 - r^3} \begin{cases} R^3 - r^3 - 3(R^2 + r^2)x/4 + x^3/8, & 0 \leq x \leq 2r \\ R^3 - 2r^3 - 3R^2x/4 + x^3/16, & 2r \leq x \leq R - r \\ -r^3 + 3(R^2 - r^2)^2/8x + 3r^2x/4 - x^3/16, & R - r < x \leq R + r \\ R^3 - 3R^2x/4 + x^3/16, & R + r < x \leq 2R. \end{cases} \quad (C.4)$$

For $r \geq R/3$:

$$U(x) = \frac{1}{R^3 - r^3} \begin{cases} R^3 - r^3 - 3(R^2 + r^2)x/4 + x^3/8, & 0 \leq x \leq R - r \\ 3(R^2 - r^2)^2/8x, & R - r \leq x \leq 2r \\ -r^3 + 3(R^2 - r^2)^2/8x + 3r^2x/4 - x^3/16, & 2r \leq x \leq R + r \\ R^3 - 3R^2x/4 + x^3/16, & R + r \leq x \leq 2R. \end{cases} \quad (C.5)$$

Limit of thin spherical shell with radius r and thickness δ ($\delta \ll r$):

$$U(x) = \begin{cases} 1 - \frac{x}{2\delta}, & 0 \leq x \leq \delta \\ \frac{\delta}{2x}, & \delta < x \leq 2r. \end{cases} \quad (C.6)$$

d) *Spheroid with two axes d and the third axis e·d (Kellerer, 1984)*

$$U(x) = 1 - \frac{3x}{2d} \frac{c_1}{e} + \frac{x^3}{2d^3} \frac{c_2}{e} \quad (C.7)$$

$$+ \frac{3}{8} \frac{\epsilon}{(e^{-1} - e)} \left[\sqrt{\left| \frac{d^2}{x^2} - 1 \right|} \left(\frac{x^2}{2d^2} + 1 \right) + \left(\frac{x^3}{2d^3} - \frac{2x}{d} \right) \text{ci} \left(\frac{d}{x} \right) \right],$$

for $0 < x \leq \text{Max}(d, e \cdot d)$.

The expression in the first line of Equation (C.7) applies for $x \leq \text{Min}(d, e \cdot d)$. In the case of $e < 1$, the expression in the second line applies for $x > e \cdot d$; in the case of $e > 1$, the sum of the expressions in the first and in the second lines is used for $x > d$.

In Equation (C.7), $\epsilon = \sqrt{|e^2 - 1|}$ is the eccentricity; then

$$c_1 = \frac{1}{2} + \frac{e^2}{2\epsilon} \text{ci} \left(\frac{1}{e} \right), \quad c_2 = \frac{1}{4e^2} + \frac{3}{4} c_1, \quad (C.8)$$

and

$$ci(x) = \begin{cases} \cos^{-1}(x), & \text{for } 0 \leq x \leq 1, \\ \cosh^{-1}(x) = \ell n(x + \sqrt{x^2 - 1}), & \text{for } x > 1. \end{cases} \quad (C.9)$$

The auxiliary function $ci(x)$ is introduced to permit one common formula for the oblate and the prolate spheroid (Kellerer, 1984). Throughout Appendix C, $f^{-1}(x)$ is defined as the inverse function of $f(x)$.

e) *Right cylinder with arbitrary cross section of width d and height h (Kellerer, 1981):*

$$U(x) = \frac{1}{x} \int_{z_1}^{z_2} \left(1 - \frac{z}{h}\right) U_c(\sqrt{x^2 - z^2}) dz, \quad x \leq \sqrt{h^2 + d^2}, \quad (C.10)$$

$$z_1 = \sqrt{\text{Max}(0, x^2 - d^2)}; \quad z_2 = \text{Min}(x, h). \quad (C.11)$$

$U_c(x)$ is the geometric-reduction factor of the cross section ($x \leq d$) as follows:

Circular cross section:

$$U_c(x) = \frac{2}{\pi} \left[\cos^{-1}\left(\frac{x}{d}\right) - \frac{x}{d} \sqrt{1 - \frac{x^2}{d^2}} \right], \quad x \leq d. \quad (C.12)$$

Square cross section (see Coleman, 1969):

$$U_c(x) = \frac{1}{\pi} \begin{cases} \frac{x^2}{d^2} - \frac{4x}{d} + \pi, & x \leq d \\ \pi - 2 - 4\cos^{-1}\left(\frac{d}{x}\right) + 4\sqrt{\frac{x^2}{d^2} - 1} - \frac{x^2}{d^2}, & d < x \leq d\sqrt{2}. \end{cases} \quad (C.13)$$

Integrating the expression, one obtains for the unit cube the relation derived by Piefke (1978):

$$U(x) = \begin{cases} 1 - \frac{3}{2}x + \frac{2}{\pi}x^2 - \frac{1}{4\pi}x^3, & 0 \leq x \leq 1 \\ \frac{6\pi - 1}{4\pi x} - 2 + \frac{3x}{2\pi} + \frac{x^3}{2\pi} + \frac{6}{\pi x} \cos^{-1}\left(\frac{1}{x}\right) - \frac{2}{\pi x} (2x^2 + 1) \sqrt{x^2 - 1}, & 1 < x \leq \sqrt{2} \\ \frac{6\pi - 5}{4\pi x} + 1 - \frac{3(1 + \pi)x}{2\pi} - \frac{x^3}{4\pi} + \frac{2}{\pi x} (x^2 + 1) \sqrt{x^2 - 2} - \frac{6}{\pi x} A(x), & \sqrt{2} < x \leq \sqrt{3}, \end{cases} \quad (C.14)$$

where

$$A(x) = \tan^{-1}(\sqrt{x^2-2}) + 2x \tan^{-1}(x^2-1-x\sqrt{x^2-2}) - x^2 \tan^{-1}\left(\frac{1}{\sqrt{x^2-2}}\right).$$

Figure C.1 shows geometric-reduction factors and point-pair distance distributions for the autologous case according to some of the preceding equations.

C.2 The Heterologous Case (A ≠ B)

a) *Two concentric spheres A and B with radii R and r, respectively:*

$$U_{AB}(x) = \begin{cases} \text{Min}(1, r^3/R^3), & x < |R-r| \\ \frac{3}{4R^3} \left[\frac{2}{3}(R^3+r^3) - \frac{1}{4}(R^2-r^2)^2 \frac{1}{x} - \frac{1}{2}(R^2+r^2)x + \frac{1}{12}x^3 \right], & |R-r| < x < R+r. \end{cases} \quad (C.15)$$

b) *A spherical surface, A, of radius R and a concentric sphere, B, of radius r:*

$$U_{AB}(x) = \begin{cases} \frac{1}{2} + \frac{r^2-R^2}{4Rx} - \frac{x}{4R}, & |R-r| < x < R+r \\ 1, & R < r \text{ and } x < r-R. \end{cases} \quad (C.16)$$

c) *A spherical surface, A, of radius R and a concentric shell, B, of inner radius r₁ and outer radius r₂:*

The geometric-reduction factor is equal to the preceding solution with $r=r_2$ minus the solution with $r=r_1$.

For $R > r_2$, i.e., for A outside B:

$$U_{AB}(x) = \begin{cases} \frac{1}{2} + \frac{r_2^2-R^2}{4Rx} - \frac{x}{4R}, & \begin{cases} R-r_2 \leq x \leq R-r_1 \\ R+r_1 \leq x \leq R+r_2 \end{cases} \\ \frac{r_2^2-r_1^2}{4Rx}, & R-r_1 < x < R+r_1. \end{cases} \quad (C.17)$$

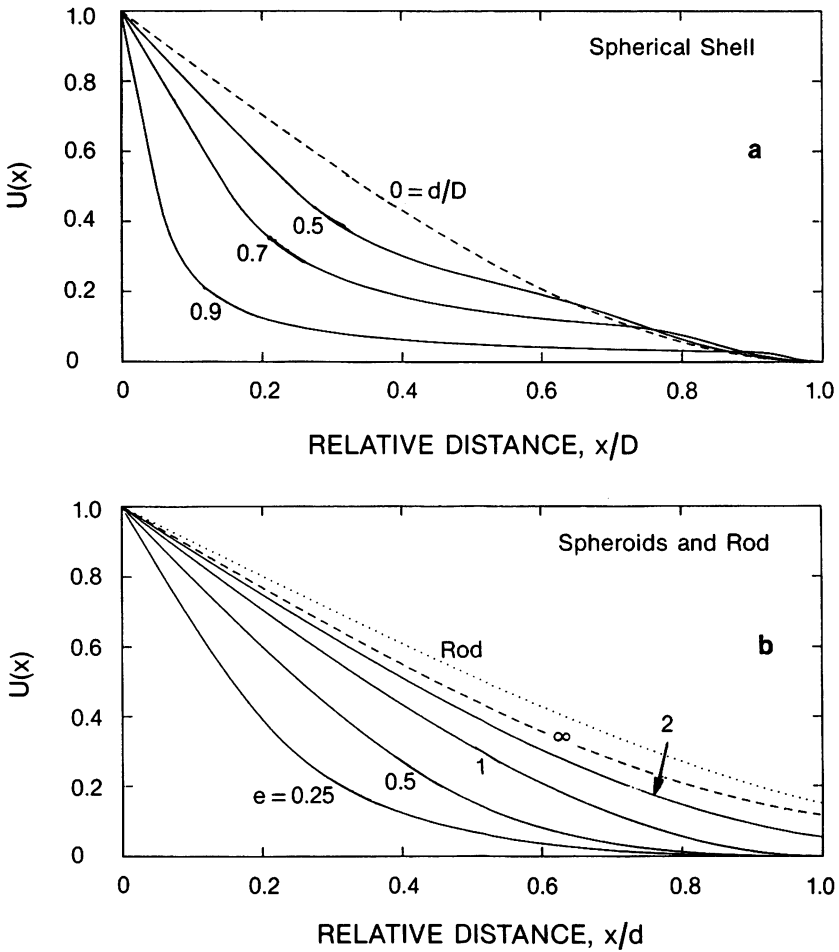


Fig. C.1. Geometric-reduction factors, $U(x)$, or the point-pair distance distributions, $p(x)$, for the autologous case.

a: Geometric-reduction factors for spherical shells according to Equations (C.4) and (C.5). The distance, x , is given relative to the outer diameter, D . The ratio of inner diameter, d , to outer diameter of the shell is indicated on the curves. The dashed curve corresponds to the sphere [Equation (C.3)].

b: Geometric-reduction factors for spheroids according to Equation (C.7). The distance, x , is given relative to the two equal diameters, d , of the spheroid. The ratio e of the third diameter to d is indicated on the curves. The dashed curve represents the limit of an infinitely extended prolate spheroid, while the dotted curve represents, for comparison, an infinitely long circular rod of diameter d [Equations (C.10) and (C.12)].

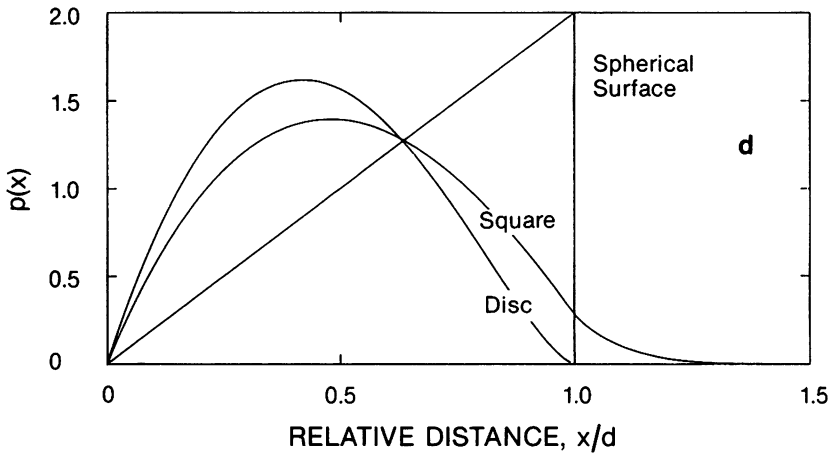
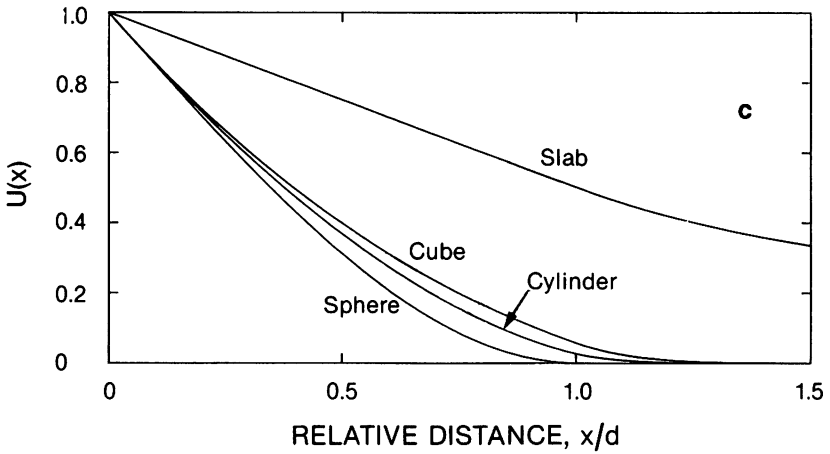


Fig. C.1. continued

c: Geometric-reduction factors for the sphere of diameter d [Equation (C.3)], the circular right cylinder of diameter and height d [Equations (C.10) and (C.12)], the cube of side length d [Equation (C.14)], and the infinite slab of height d [Equation (C.2)]. The distance, x , is given relative to d .

d: Point-pair distance densities for a disc of diameter d [Equation (C.12)], a square of side length d [Equation (C.13)], and a spherical surface of diameter d [Equation (C.6)]. The distance, x , is given relative to d .

For $R < r_1$, i.e., for A in the interior void of the shell for $R \geq (r_2 - r_1)/2$:

$$U_{AB}(x) = \begin{cases} \frac{1}{2} - \frac{r_1^2 - R^2}{4Rx} + \frac{x}{4R}, & r_1 - R \leq x \leq r_2 - R \\ \frac{r_2^2 - r_1^2}{4Rx}, & r_2 - R \leq x \leq R + r_1 \\ \frac{1}{2} + \frac{r_2^2 - R^2}{4Rx} - \frac{x}{4R}, & R + r_1 < x \leq R + r_2. \end{cases} \quad (C.18)$$

For $R \leq (r_2 - r_1)/2$

$$U_{AB}(x) = \begin{cases} \frac{1}{2} - \frac{r_1^2 - R^2}{4Rx} + \frac{x}{4R}, & r_1 - R \leq x \leq r_1 + R \\ 1, & r_1 + R < x \leq r_2 - R \\ \frac{1}{2} + \frac{r_2^2 - R^2}{4Rx} - \frac{x}{4R}, & r_2 - R < x \leq r_2 + R. \end{cases} \quad (C.19)$$

Special cases:

An outer surface of the shell ($R = r_2$):

$$U_{AB}(x) = \begin{cases} \frac{1}{2} - \frac{x}{4r_2}, & \begin{cases} 0 < x < r_2 - r_1 \\ r_1 + r_2 \leq x \leq 2r_2 \end{cases} \\ \frac{r_2^2 - r_1^2}{4r_2x}, & r_2 - r_1 < x < r_1 + r_2; \end{cases} \quad (C.20)$$

An inner surface of the shell ($R = r_1$) and $R \geq r_2/3$:

$$U_{AB}(x) = \begin{cases} \frac{1}{2} + \frac{x}{4R}, & 0 \leq x \leq r_2 - r_1 \\ \frac{r_2^2 - r_1^2}{4r_1x}, & r_2 - r_1 < x \leq 2r_1 \\ \frac{1}{2} + \frac{r_2^2 - r_1^2}{4r_1x} - \frac{x}{4r_1}, & 2r_1 < x \leq r_1 + r_2. \end{cases} \quad (C.21)$$

d) *Two concentric shells A and B of infinitesimal thickness and with radii R and r:*

$$U_{AB}(x) = \frac{r\delta}{2Rx}, \quad |R - r| < x < R + r, \quad (C.22)$$

where δ is the (infinitesimal) thickness of the shell B.

Note: Solutions b) to d) apply also if A is part of the spherical surface, e.g., a sector, a spherical cap, a ring within the shell or a point.

e) *A plane A (or part of a plane) and a parallel plate B of thickness h at distance a:*

$$U_{AB}(x) = \begin{cases} \frac{1}{2} - \frac{a}{2x}, & a \leq x \leq a+h \\ \frac{h}{2x}, & a+h < x. \end{cases} \quad (C.23)$$

f) *Two parallel infinite plates A and B of thickness H and h, respectively, and separated by distance a (H < h)*

$$U_{AB}(x) = \begin{cases} \frac{1}{4} - \frac{a}{4x}, & a \leq x \leq a+H \\ \frac{x-a-H/2}{2x}, & a+H < x \leq a+h \\ \frac{h}{2x} - \frac{(H+h+a-x)^2}{4Hx}, & a+h < x < a+H+h \\ \frac{h}{2x}, & x > a+H+h \end{cases} \quad (C.24)$$

$$U_{BA}(x) = U_{AB}(x) \cdot \frac{H}{h} \quad (C.25)$$

The same solution applies if A is a section of the infinite plate, i.e., a right cylinder (finite plate).

g) *A point A and a right cylinder B:*

$$U_{AB}(x) = \frac{1}{2x} \int_{z_1}^{z_2} U_{AC}(\sqrt{x^2 - z^2}) dz, \quad \sqrt{y_1^2 + b^2} < x < \sqrt{(b+h)^2 + y_2^2} \quad (C.26)$$

$$z_1 = \text{Max}(b, \sqrt{\text{Max}(0, x^2 - y_2^2)}), \quad z_2 = \text{Min}(b+h, \sqrt{x^2 - y_1^2}).$$

$U_{AC}(x)$ is the two-dimensional geometric-reduction factor for the cross section C of the cylinder relative to the projection of point A,

P(A), into the plane which corresponds to C; y_1 and y_2 are the minimum and maximum distances in this plane between P(A) and C; h is the height of the cylinder; b is the projected distance on the cylinder axis between A and P(A). Point A is assumed to lie below the cylinder bottom. If A lies between the two planes through the faces of the cylinder, one obtains the solution as a sum by suitable subdivision of B into two cylinders.

For the circular area of radius r relative to the point at distance a from the center, one has:

$$U_{AC}(x) = \frac{1}{\pi} \cos^{-1} \left[\text{Max} \left(-1, \frac{x^2 + a^2 - r^2}{2ax} \right) \right], \quad \text{Max} (a - r, 0) \leq x \leq a + r. \quad (\text{C.27})$$

h) *A point A and a planar domain B of infinitesimal thickness δ :*

Let h be the distance of A from the plane C of B, and let $U_2(y)$ be the 2-dimensional geometric-reduction factor ($y_1 < y < y_2$) of B relative to the projection of A in C. Then:

$$U_{AB}(x) = \frac{\delta}{2x} U_2(\sqrt{x^2 - h^2}), \quad \sqrt{h^2 + y_1^2} < x < \sqrt{h^2 + y_2^2}. \quad (\text{C.28})$$

For a disc of radius r and a projected distance a of A from its center, one obtains

$$U_{AB}(x) = \frac{\delta}{2\pi x} \cos^{-1} \left[\text{Max} \left(-1, \frac{x^2 + a^2 - r^2 - h^2}{2a \sqrt{x^2 - h^2}} \right) \right]. \quad (\text{C.29})$$

For a rectangle with side lengths a and b , we restrict to a special case for A: The straight lines which come out from the sides divide the plane into 8 rectangles with infinite content; the projection of A, P(A), is assumed to lie in the left lowest rectangle with infinite content; s is the distance of P(A) to the next rectangle line parallel to a , t is analogously related to b . (Different positions of A can be reduced to the described special case.) For the two dimensional geometrical factor, one has:

$$U_2(y) = \frac{1}{2\pi} \left[\sin^{-1} \left(\frac{\text{Min} (a + s, \sqrt{y^2 - t^2})}{y} \right) - \sin^{-1} \left(\frac{\text{Max} (s, \sqrt{\text{Max} (0, y^2 - (b + t)^2})}}{y} \right) \right]; \quad (\text{C.30})$$

therefore,

$$U_{AB}(x) = \frac{\delta}{4\pi x} \left[\sin^{-1} \left(\frac{\text{Min}(a+s, \sqrt{x^2 - h^2 - t^2})}{\sqrt{x^2 - h^2}} \right) \right. \quad (\text{C.31})$$

$$\left. - \sin^{-1} \left(\frac{\text{Max}(s, \sqrt{\text{Max}(0, x^2 - h^2 - (b+t)^2)})}{\sqrt{x^2 - h^2}} \right) \right]$$

The diagrams in Figure (C.2) give geometric-reduction factors and point-pair distance distributions for the heterologous case according to some of the preceding equations.

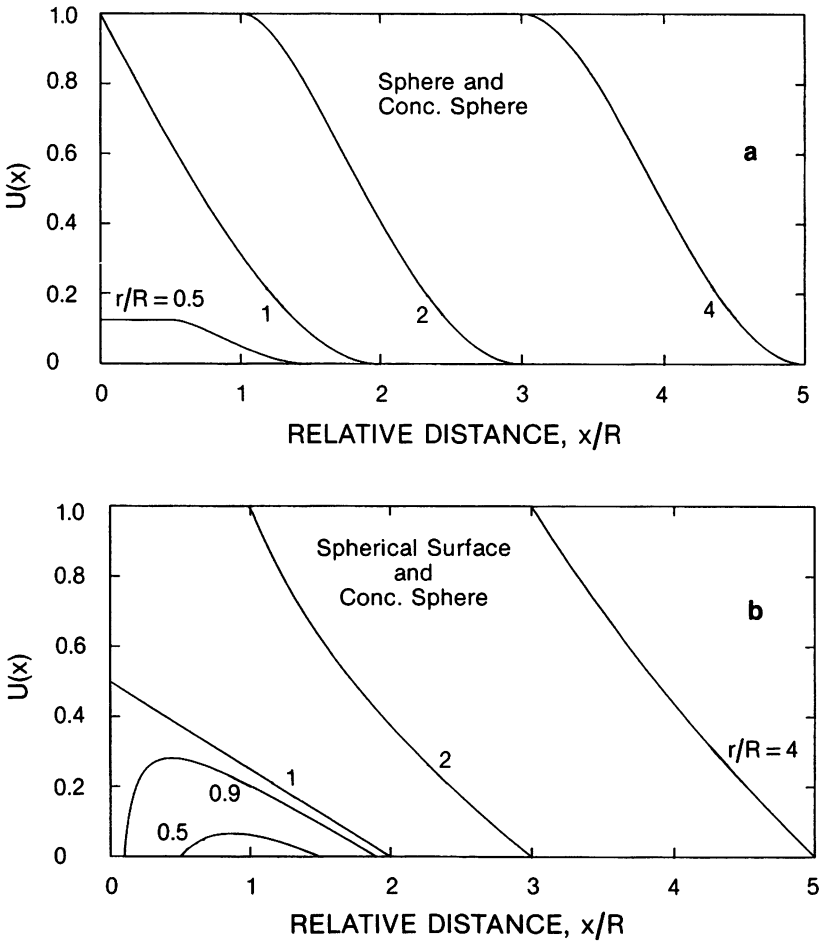


Fig. C.2. Geometric reduction factors, $U(x)$, or the point-pair distance distributions, $p(x)$, for the heterologous case.

a: Geometric-reduction factors for a sphere, A, of radius R , and a concentric sphere, B, of radius r according to Equation (C.15). The ratio r/R is indicated on the curves. The distance, x , is given relative to R .

b: Geometric-reduction factors for a spherical surface, A, of radius R and a concentric sphere, B, of radius r according to Equation (C.16). The ratio r/R is indicated on the curves. The distance, x , is given relative to R .

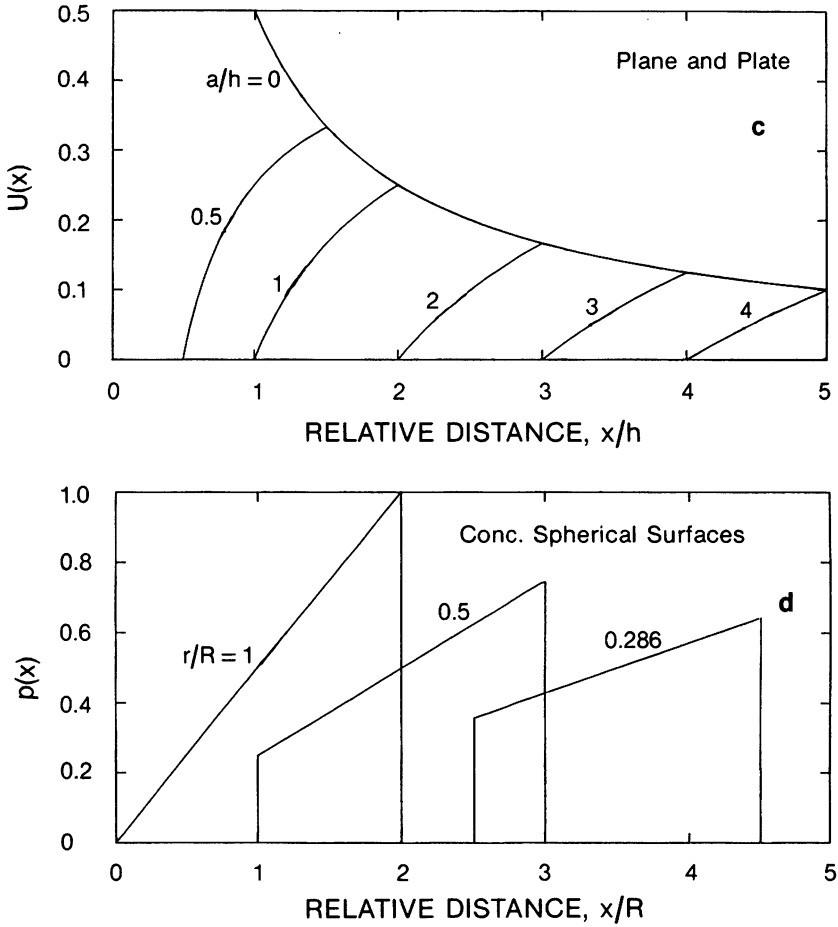


Fig. C.2. continued

c: Geometric-reduction factors for a plane, A, separated by distance a from an infinite plate, B, of height h according to Equation (C.23). The ratio a/h is indicated on the curves. The distance, x , is given relative to h .

d: Point-pair distance distributions between a spherical surface of radius R and a concentric spherical surface of radius r according to Equation (C.22). The ratio r/R is indicated on the curves. The distance, x , is given relative to r .