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Modern Production Concepts

Theory and Applications

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HANS-ULRICH KÜPPER

1. Relations Between Production Planning and Managerial Accounting

Profits and costs are important objectives in production planning. Therefore many production models involve *cost parameters*. Their values must be calculated within the system of cost accounting.

Production models are frequently constructed without paying attention to the problems in calculating these cost parameters. Sometimes it is very difficult to find their correct values. For example lot size models include parameters for set up costs, storage costs and - rarely - shortage costs. The values of these parameters cannot be identified directly. Set up costs consist of personnel costs for preparing the machine and sometimes of energy costs. The calculation of storage and especially of shortage costs is still more difficult. Storage costs include personnel costs for the workers in the storage and interest costs on capital. Do they also include parts of the costs for the storage equipment? Shortage costs are opportunity costs. They cannot be derived from expenses as they reflect the loss of revenues.

The fundamental reason for these problems originates in the *separation of decisions*. The size and the equipment of a storage is determined by the stocks of several products and the expectations of the further needs of inventory space. A single lot only has minimal influence on these factors.

In general there are two ways how to solve these problems. The first way is to *estimate* those cost parameters by allocating costs. Often this can only be done arbitrarily. The other way is shown by *simultaneous* models. In some of these models, for example models of production and investment planning, the expenses for machines and workers can be related directly to variables of their utilization times [5, p. 259]. The relations between product quantities, lot sizes, sequencing and the utilization of machines as well as personnel are worked out within the model [10]. The calculation of the cost parameters becomes easier. There are fewer problems of allocating full costs. On the other hand the planning models get more complicated.

In consequence we perceive some sort of *symmetric* relation between planning and managerial accounting. If independent partial models are used for different problems of production planning as product quantities, lot sizes, staff assignment, sequencing etc., it is very difficult to determine the required cost parameters correctly. These parameters express the mutual dependencies of isolated decisions. In managerial accounting their determination becomes very important for the coordination between the decisions. Hence, the problems of coordination are partially shifted to managerial accounting.

Simultaneous models facilitate the calculation of those cost parameters. Such complex models however can frequently not be used for *practical* problems. Lot sizes and sequencing often lead to mixed integer models which cannot be solved and require extensive data. Furthermore central planning is not convenient for many organizations.

The symmetric relation between planning and managerial accounting displays a *dilemma*: simultaneous models only give a theoretical, but not a practical solution. The problem of separation and coordination between several decision problems has to be solved by other means of planning and managerial accounting.

One approach will be demonstrated in the following chapters. It illustrates the relations between production planning and managerial accounting by means of an example of multi-period programme planning. The critical cost parameter of this problem is depreciation. We will try to find a way to determine variable depreciation by managerial accounting in order to coordinate one-period programme planning with the long-term objective and planning.

2. Integration of One-Period and Multi-Period Production Planning

2.1. Central Elements of the Investment Approach

The calculation of *depreciation* is an important and difficult problem of cost accounting [3, 15]. In recent years a new approach for managerial accounting has been developed [2, 4, 6]. It is based on *investment theory*. The central idea of this approach is to derive the theory of managerial accounting from the models of investment theory. The investment approach proceeds on the assumption of a long-term plan. In the simplest case infinite identical investment repetitions are assumed, according to the principle of "going concern". The objective of managerial accounting is to determine the implications of short-term decisions on long-term profit objectives. The common objective of all profit-oriented calculations is the net present value. It is calculated in terms of *cash-flows*.

In order to get the relevant information it is necessary to know how the net present value varies with the decision variable. Given that, it is possible to identify the influence of short-term changes of the decision variable on profit. These changes are the relevant costs.

For example: The net present value G_t of a good at time t is a function of the cash-flows caused by input of material, labor, and equipment. If its value depends on lifetime and cumulative utilization Y_t, we get the *net present value* of equation 1:

$$G_t = G(t, Y_t) \tag{1}$$

The information which is relevant for a decision on utilization is determined by infinitely small *changes of utilization*. This is given within the total differential:

$$\frac{dG_t}{dt} = \frac{\partial G_t}{\partial t} + \frac{\partial G_t}{\partial Y_t} \cdot \frac{dY_t}{dt} \tag{2}$$

The degree of utilization is embodied in this approach by means of cumulative utilization. This might be the miles done by a truck, the time of utilization for an engine or the working time for labor. The differential ratio of the function of the net present value represents the costs, which are relevant to short-term decisions.

2.2. Multi-Period Planning of the Production Programme

As a consequence of this approach the relations between one-period and multi-period planning of the production programme can be demonstrated. Let us consider an example [7]. Three types of products have to be produced by three machines. In long-term planning the periodical product quantities will be fixed simultaneously with the life and replacement time of the machines. For the purpose of simplification we presume constant data and infinite identical investments. *Table 1* shows the data of the example. There are the acquisition and the residual values of the three machines. These machines cause *current expenses* C for energy, maintenance etc. Their values may be determined by the variables time t, periodical utilization y_t and cumulative utilization Y_t. The *capacity* of each machine in each period t is limited. The *contribution margins* CM of the products are calculated without machinery costs.

The *objective* of long-term planning is to maximize the present value of the cash-flows G at t=0. They are the sum of the present values of the contributions for the continuously sold products and the present values of the expenses for the machines. To achieve an optimal solution we should maximize this non-linear objective function with linear capacity constraints.

In order to find an easier way to get a solution we only consider the *corners* of the decision space in a first step (see *Figure 1*). The present values of the two best alternatives are shown in *Table 2*. For both alternatives the periodical operating times of the machines A and B are equal. There is only one difference concerning machine C. Consequently both alternatives include the same present value of the cash-flows and the same replacement times for the machines A and B.

A better solution might be achieved if a *switching* between the alternatives is allowed. To examine this point we only regard the utilization of machine C and a switching between the two best alternatives. The switching between the alternatives and therefore the change of the periodical utilization of machine C may occur at τ. If y_I denotes the *periodical utilization* of the first selected alternative and y_{II} that of the second alternative the *cumulative utilization* Y_t before and after the switching time τ can be calculated as follows:

$$t < \tau \;\rightarrow\; Y_t = y_I \cdot t \tag{3.1}$$

$$t > \tau \;\rightarrow\; Y_t = y_{II} \cdot t + (y_I - y_{II}) \cdot \tau = y_{II} \cdot t + \Delta y \cdot \tau \tag{3.2}$$

In consequence the *current costs* of this machine before (C_I) and after (C_{II}) the switching in τ are equal to

Machine	A	B	C
Acquisition Value	$A_A = 600$	$A_B = 400$	$A_C = 500$
Residual Value	$L_A = 600 \cdot e^{-0,1T}$	$L_B = \frac{400}{T + 2} + \frac{1200}{Y_T + 6}$	$L_C = \frac{20000}{Y_T + 40}$
Current Expenses For Machine Per Time Unit; $t<\tau$: / $t\geq\tau$:	$C_A = 3t + y_t + 0,1Y_t$ $C_A = 3t + y_t + 0,1Y_t + 0,1\Delta Y$	$C_B = 2t + 3y_t + 0,2Y_t$ $C_B = 2t + 3y_t + 0,2Y_t + 0,2\Delta Y$	$C_C = 4t + 1,8y_t + 0,3Y_t$ $C_C = 4t + 1,8y_t + 0,3Y_t + 0,3\Delta Y$
Machine Operating Time	$2 \cdot x_1 + 2 \cdot x_2 + 1,6 \cdot x_3 \leq 32$	$1,5 \cdot x_1 + 1 \cdot x_2 + 1,5 \cdot x_3 \leq 21$	$1,5 \cdot x_1 + 2 \cdot x_2 + 1,5 \cdot x_3 \leq 30$
Contribution Before Variable Machinery Costs	$CM_1 = 39$; $CM_2 = 35$; $CM_3 = 35$		

Table 1: Data of an example of production programme planning

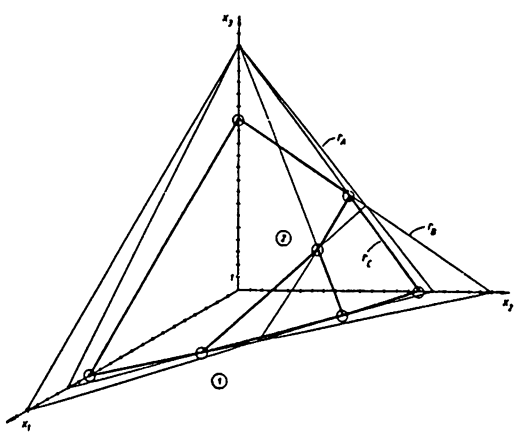

Figure 1: Solution space of the example of production programme planning

Alter-native	Product Quantities		Machines A	Machines B	Machines C	Net Present Value of Current Costs	Net Present Value of Contribution Margin	Net Present Value of Profits
1	$x_1 = 10$	y	32	21	27	4629,46	6000	1370,54
	$x_2 = 6$	T	10,908	12,970	9,300			
	$x_3 = 0$	K	1399,43	1486,30	1743,73			
2	$x_1 = 3$	y	32	21	30	4722,49	6070	1347,51
	$x_2 = 9$	T	10,908	12,970	8,998			
	$x_3 = 5$	K	1399,43	1486,30	1836,77			

Table 2: Best long-term programme alternatives without change of capacity utilization (i = 0,10)

$$C_I = \alpha \cdot t + \beta \cdot y_I + \varepsilon \cdot Y_t \quad (4.1)$$

$$C_{II} = \alpha \cdot t + \beta \cdot y_{II} + \varepsilon \cdot Y_t = \alpha \cdot t + \beta \cdot (y_I - \Delta y) + \varepsilon \cdot Y_t \quad (4.2)$$

Until τ we get the contribution CM_I of the first selected alternative, after τ the contribution CM_{II}. At the end of the service time of machine C we return to CM_I and so on. The *objective function* consists of the present values of the following terms:

- the *contribution* CM_I and *current costs* C_I of the first alternative from zero until the switching in τ,
- the *contribution* CM_{II} and *current costs* C_{II} of the second alternative from the switching in τ until the replacement in T,
- the *acquisition value* A for machine C and
- the *residual value* of machine C at replacement in T.

The objective function for the net present value of the profit G at the beginning of the whole planning period t=0

$$G = \frac{1}{1-e^{-iT}} \cdot [\int_0^{\tau} (CM_I - C_I) \cdot e^{-it} dt + \int_{\tau}^{T} (CM_{II} - C_{II}) \cdot e^{-it} dt - A + L(Y_T) \cdot e^{-iT}] \quad (5)$$

depends on two variables, the switching time τ and the replacement time T of machine C. To get the optimum of this function we must differentiate by these variables. With respect to *replacement time* T we get

$$\frac{\partial G}{\partial T} = \frac{1}{1-e^{-iT}} \cdot [(CM_{II} - C_{II}) \cdot e^{-iT} + \frac{dL}{dY_T} \cdot \frac{dY_T}{dT} \cdot e^{-iT} - i \cdot e^{-iT} \cdot L(Y_T) - i \cdot e^{-iT} \cdot G] = 0 , \quad \frac{\partial^2 G}{\partial T^2} < 0 , \quad (6.1)$$

or $$CM_{II} - C_{II}(T) + \frac{dL}{dY_T} \cdot \frac{dY_T}{dT} - i \cdot L(Y_T) - i \cdot G = 0 \quad (6.2)$$

The equation 6.2 corresponds to the well-known condition of optimal replacement in investment theory.

The differentiation by τ gives the optimal *switching time*:

$$\frac{\partial G}{\partial \tau} = \frac{1}{1-e^{-iT}} \cdot [(CM_I - C_I(\tau)) \cdot e^{-i\tau} - (CM_{II} - C_{II}(\tau)) \cdot e^{-i\tau} - \int_{\tau}^{T} \frac{\partial C_{II}}{\partial \tau} \cdot e^{-it} dt + \frac{dL}{dY_T} \cdot \frac{dY_T}{d\tau} \cdot e^{-iT}] = 0 , \quad \frac{\partial^2 G}{\partial \tau^2} < 0 , \text{ or} \quad (7.1)$$

$$(CM_I - C_I(\tau)) - (CM_{II} - C_{II}(\tau)) = e^{i\tau}\cdot\left[\int_\tau^T \frac{\partial C_{II}}{\partial \tau}\cdot e^{-it}dt - \frac{dL}{dY_T}\cdot\frac{dY_T}{d\tau}\cdot e^{-iT}\right] \quad (7.2)$$

The equations 4.1 and 4.2 lead to

$$C_I - C_{II} = \beta\cdot(y_I - y_{II}) = \beta\cdot\Delta y, \quad (8)$$

$$\frac{\partial C_{II}}{\partial \tau} = \varepsilon\cdot\Delta y = \frac{\partial C}{\partial Y_t}\cdot\Delta y \quad (9) \quad \text{and} \quad \frac{dY_t}{d\tau} = \Delta y \quad (10)$$

Now we can transform the equation 7

$$CM_I - CM_{II} - \Delta y\cdot\beta = e^{i\tau}\cdot\Delta y\cdot\left[\int_\tau^T \frac{\partial C}{\partial Y_t}\cdot e^{-it}dt - \frac{dL}{dY_T}\cdot e^{-iT}\right] \quad (7.3)$$

Setting

$$e^{i\tau}\cdot\left[\int_\tau^T \frac{\partial C}{\partial Y_t}\cdot e^{-it}dt - \frac{dL}{dY_T}\cdot e^{-iT}\right] = d_N \quad (11)$$

we finally get:

$$CM_I - y_I\cdot(\beta + d_N) = CM_{II} - y_{II}\cdot(\beta + d_N) \quad (12)$$

Each side of this equation stands for the difference between the periodical contributions and costs of machine C. The optimal switching time is reached as soon as the *periodical profits* become equal. Both sides represent a one-period objective function. As β denotes the costs proportional to utilization time y_t the parameter d_N may be interpreted as variable depreciation.

If we are able to determine this cost parameter in managerial accounting, we can solve the problem in one-period planning. It has to be shown that this way is given by the investment approach.

2.3. One-Period Planning with Variable Depreciation

In the investment approach we calculate the depreciation corresponding to equation 2. In order to get the variable depreciation we have to differentiate the objective function $G_t(t, y_t, Y_t)$ for the net present value *at each time* t by the *cumulative utilization* variable Y_t:

$$\frac{\partial G_t}{\partial Y_t} = e^{it}\cdot[e^{-i\tau}\cdot\frac{d\tau}{dY_t}\cdot\{CM_I - CM_{II} - C_I(\tau, Y_\tau) + C_{II}(\tau, Y_\tau)\}$$

$$+ e^{-iT}\cdot\frac{dT}{dY_t}\cdot\{CM_{II} - C_{II}(T, Y_T) + \frac{dL}{dY_T}\cdot\frac{dY_t}{dT} - i\cdot L(Y_T) - i\cdot G\} \qquad (13.1)$$

$$- \int_t^\tau \frac{\partial C_I}{\partial Y_s}\cdot e^{-is}ds - \int_\tau^T \frac{\partial C_{II}}{\partial Y_s}\cdot e^{-is}ds - \int_t^\tau \frac{\partial C_I}{\partial \tau}\cdot\frac{\partial \tau}{\partial Y_s}\cdot e^{-is}ds - \int_\tau^T \frac{\partial C_{II}}{\partial \tau}\cdot\frac{\partial \tau}{\partial Y_s}\cdot e^{-is}ds]$$

Because of

$$Y_T = Y_t + (T-t)\cdot y \quad (14) \qquad \text{and} \qquad \frac{dY_T}{dY_t} = 1 \qquad (15)$$

we get:

$$\frac{dL}{dY_T}\cdot\frac{dY_t}{dT} = \frac{dL}{dY_T}\cdot\frac{dY_T}{dT} \qquad (16)$$

In the optimum we can add the equations 7.2 and 6.2 and come to:

$$\frac{\partial G_t}{\partial Y_t} = e^{it}\cdot[\frac{d\tau}{dY_t}\cdot\{\int_\tau^T \frac{\partial C_{II}}{\partial \tau}\cdot e^{-it}dt - \frac{dL}{dY_T}\cdot\frac{dY_T}{d\tau}\cdot e^{-iT}\}$$

$$- \int_t^T \frac{\partial C}{\partial Y_s}\cdot e^{-is}ds - \int_t^\tau \frac{\partial C_I}{\partial \tau}\cdot\frac{\partial \tau}{\partial Y_s}\cdot e^{-is}ds - \int_\tau^T \frac{\partial C_{II}}{\partial \tau}\cdot\frac{\partial \tau}{\partial Y_t}\cdot e^{-it}dt] \qquad (13.2)$$

Because of equations 4 and 3 it follows:

$$\frac{\partial C_I}{\partial \tau} = 0 \quad (17) \qquad \text{and for } t < \tau \qquad \frac{d\tau}{dY_t}\cdot\frac{dY_T}{d\tau} = -1 \qquad (18)$$

and we finally get:

$$\frac{\partial G_t}{\partial Y_t} = e^{it}\cdot[\int_\tau^T \frac{\partial C_{II}}{\partial \tau}\cdot\frac{d\tau}{dY_t}\cdot e^{-it}dt + \frac{dL}{dY_T}\cdot e^{-iT}$$

$$- \int_t^T \frac{\partial C}{\partial Y_s}\cdot e^{-is}ds - \int_\tau^T \frac{\partial C_{II}}{\partial \tau}\cdot\frac{\partial \tau}{\partial Y_t}\cdot e^{-it}dt]$$

$$= - e^{it}\cdot[\int_t^T \frac{\partial C}{\partial Y_s}\cdot e^{-is}ds - \frac{dL}{dY_T}\cdot e^{-iT}] = -d_N \qquad (13.3)$$

This equation gives us the negative *variable depreciation*. In consideration of equation 12 this depreciation leads to the optimal switching time τ. If we calculate the variable depreciation in the investment approach, the maximization of the objective function of the *periodical profits*

$$G_t^* = CM - y \cdot (\beta + d_N) \qquad (19)$$

for one-period planning leads to the optimum of the multi-period planning. *Figure 2* shows that the periodical profits of the alternatives change over time. Their point of intersection delivers the optimal switching time τ. *Table 3* demonstrates this result for the established example.

But there is one *problem*. To calculate the depreciation by means of the investment approach of the equation 13.3 we must know the optimal replacement time T of machine C. Its value must be fixed corresponding to equation 6.2. The solution of this optimization condition depends on the switching time τ. Therefore the exact solution of the one-period optimization problem presumes the solution of the multi-period problem. This dilemma is well-known from dual variables in linear programming.

The example of *Table 3* shows that approximate values of the replacement time T also lead to a good solution. Therefore this dilemma can reasonably be evaded by working with realistic *approximate* values of T. The investment approach seems to be a suitable concept to determine cost parameters which may coordinate short-term and long-term planning.

3. The Investment Approach as a General Concept for Managerial Accounting

3.1. Fundamental Characteristics of the Investment Approach

The investment approach forms a theoretical foundation of managerial accounting. It points out the way to generate relevant information for decision problems. There are four important characteristics [6]:

(1) The investment approach integrates *short- and long-term* profit calculations into one accounting system. Its basis is the long-term investment theory.

(2) The investment approach takes into account the aspect of time. Therefore it has to be developed as a *dynamic theory* [9].

(3) All calculations are based on payments or expenses and revenues, i.e. on *observable* figures. These are cash-flows with respect to profit. The other variables, such as periodical costs, should be derived from those by clear rules.

(4) All profit-oriented calculations are directed towards a *common objective*. The same profit objective should exist for all decisions. Hence, the point of departure is the long-term profit objective.

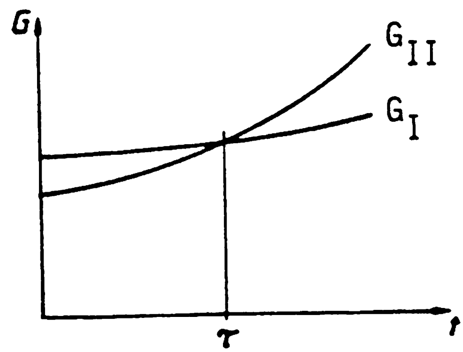

Figure 2: Development of short-term periodical profits in consideration of depreciation

Case	T_C	t	Short-term Objective Functions	Periodical Profits Alt. 1	Alt. 2	Chosen Alternative	Net Present Value of Profit
Without Depreciation	8,998	∀ t	$29,8x_1+26,4x_2+26,2x_3$	Alt. 2: 458		2	1.347,51
With	9,306 (Optimal)	0,5 8,146 9,0	$23,6x_1+20,0x_2+20,3x_3$ $27,4x_1+24,1x_2+23,9x_3$ $28,0x_1+24,8x_2+24,5x_3$	356,041 418,108 428,415	352,126 418,107 429,062	1 switching 2	1.370,88
Depre-	9,300 (Alt. 1)	0,5 8,146 9,0	$23,7x_1+20,1x_2+20,3x_3$ $27,4x_1+24,1x_2+23,9x_3$ $28,0x_1+24,8x_2+24,5x_3$	357,158 418,141 428,445	353,245 418,144 429,097	1 switching 2	1.370,88
ciation	8,998 (Alt. 2)	0,5 8,146 8,998	$23,7x_1+20,1x_2+20,4x_3$ $27,5x_1+24,2x_2+24,0x_3$ $28,2x_1+25,0x_2+24,6x_3$	357,984 419,816 431,358	354,162 420,005 432,334	1 2 2	1.370,55

Table 3: Short-term objective functions with and without regard to variable depreciation (i = 0,10)

3.2. The Determination of Several Cost Types in the Investment Approach

One-period and multi-period production planning are linked by depreciation. As a general concept for managerial accounting the investment approach has to prove its performance in determining the different types of costs and by solving decision problems of managerial accounting.

It can be shown for several types of costs how relevant costs are derived. The basic approach is the same for all types of costs: Firstly, the *cash-flows* have to be determined. Secondly, it has to be found out how the net present value depends on the *decision variable* and other variables. Thirdly, the *function* of the present value of cost has to be *differentiated* by the decision variable.

In order to derive *material costs*, for example, an infinite repetition of purchases can be assumed in the simplest case. The present value K_t of material input is determined by taking material quantities β, price q and manufacturing time π per product unit as well as interest rate i into account. The planned purchasing interval may be T^* and the production quantity x. Then we get the net present value of material input at time t as follows:

$$K_t = \beta \cdot q \cdot \frac{T^*}{\pi} \cdot \frac{e^{-i[T(x)-t]}}{1 - e^{-iT^*}} \tag{20}$$

If a further product unit, which has not been included in long-term planning, shall be produced, the next purchasing point T(x) will be reached earlier and the residual chain of cash-flows will be realized sooner. That causes a variation of net present value, which can be interpreted as material costs:

$$\frac{dK_t}{dx} = \frac{\partial K_t}{\partial T} \cdot \frac{dT}{dx} = \frac{\beta \cdot q \cdot T^* \cdot i \cdot e^{-i(T-t)}}{1 - e^{-iT^*}} \tag{21}$$

These costs seem to differ significantly from the *traditional* material costs. This is only caused by interest. Converging interest rate i or planned purchasing interval T^* towards zero the traditional material costs show up as marginal value of the present value:

$$\lim_{i \to 0} \frac{dK_t}{dx} = \lim_{i \to 0} \beta \cdot q \cdot T^* \cdot \frac{e^{-i(T-t)} - i \cdot (T-t) \cdot e^{-i(T-t)}}{T^* \cdot e^{-iT^*}} = \beta \cdot q \tag{22}$$

$$\lim_{T^* \to 0} \frac{dK_t}{dx} = \lim_{T^* \to 0} \beta \cdot q \cdot i \cdot \frac{e^{-i(T-t)}}{i \cdot e^{-iT^*}} = \beta \cdot q \qquad t \leq T \leq T^* \tag{23}$$

Hence, direct material costs are *marginal values* of the investment approach when interest is treated as a separate type of cost.

The determination of *depreciation* has been illustrated above. It is possible to show that total depreciation under certain conditions converges to a linear depreciation. Once again a well known application proves to be the marginal value of the presented theory.

Other types of costs like *personnel costs* [6], *replacement costs* [16], *interest costs* etc. can be determined in the same manner. It enables us to test how far traditional approaches of cost accounting provide the relevant costs for decision problems.

3.3. Application of the Theory of Managerial Accounting to Typical Decision Problems

The theory of managerial accounting has to show which costs are relevant to a decision problem in order to arrive at an optimal solution. In this respect the performance of the theory of managerial accounting can only be demonstrated by solving typical decision problems. In order to illustrate the performance of the investment approach *two* other typical decision problems for managerial accounting may be sketched, the determination of optimal order quantities and the determination of minimum prices.

In applying the investment approach to the determination of the optimal *order quantity* one has to focus on those cash-flows that are caused by the purchasing decision [12, 14]. The present value of purchases can be calculated under the usual assumptions. In order to derive the optimal order quantity, one has to determine the minimum of the present value function. We assume that the quantity x will be ordered for a cycle of w periods. In each period r units are required, each purchase causes fixed ordering expenses F and variable expenses $q \cdot r \cdot w$ for the delivered units, with q representing the price per unit. The storage costs without interest are c per unit of quantity and per unit of time. These parameters lead to the net value of the expenses for *one order cycle* K_C:

$$K_C = F + q \cdot r \cdot w + \int_0^w c \cdot r \cdot (w-t) \cdot e^{-it} dt \qquad (24)$$

In the simplest case we assume an infinite identical repetition of order cycles. Therefore we get the net present value K for *all cycles*:

$$K = \frac{K_C}{1-e^{-iw}} = \frac{1}{1-e^{-iw}} \cdot (F+qrw+crw) \cdot \int_0^w e^{-it} dt - c \cdot r \cdot \int_0^w t \cdot e^{-it} dt \qquad (25)$$

The order quantity x depends on the requirements r per period and the duration w of one order cycle:

$$x = r \cdot w \qquad (26)$$

To derive the optimal order quantity x we have to minimize the function of the net present value depending on the order cycle w. We get the optimum condition:

$$\frac{dK}{dw} = 0 \tag{27a}$$

$$r \cdot (q \cdot i + c) \cdot \frac{e^{iw} - 1}{i} = i \cdot F + q \cdot r \cdot i \cdot w + c \cdot r \cdot w \tag{27b}$$

Because of the compound interest this equation is difficult to solve analytically. Therefore we need a close-up solution. It is given by the first two parts of the *Taylor approximation*:

$$\frac{e^{iw} - 1}{i} = \int_0^w e^{it}\, dt \approx w + \frac{i \cdot w^2}{2} \tag{28}$$

As equation 29 demonstrates, this approximation neglects the compound interest:

$$w + i \cdot [(0,5+w-1) + (0,5+w-2) + \ldots + (0,5+w-w)] = w + \frac{i \cdot w^2}{2} \tag{29}$$

A revision shows that this approximation is satisfying for relatively low costs of interest and/or small order cycles (see *Table 4*).

Interest Rate i	Purchasing Interval w	Accurate Value $\frac{e^{iw}-1}{i}$	Approximate Value $w + \frac{i \cdot w^2}{2}$
0,002	3	3,009	3,009
0,002	5	5,025	5,025
0,002	10	10,101	10,100
0,1	1	1,052	1,050
0,1	10	17,183	15,000
0,5	10	294,826	35,000

Table 4: Revision of approximation

When applying the approximation to the optimizing equation we get:

$$r \cdot (q \cdot i + c) \cdot (w + \frac{i \cdot w^2}{2}) = i \cdot F + q \cdot r \cdot i \cdot w + c \cdot r \cdot w \tag{30}$$

Now the *optimal order cycle* w* can be determined by:

$$w^* = \sqrt{\frac{2 \cdot F}{r \cdot (q \cdot i + c)}} \tag{31}$$

One arrives at the well-known traditional formula for *the optimal order quantity* x*:

$$x^* = r \cdot w^* = \sqrt{\frac{2 \cdot F \cdot r}{q \cdot i + c}} \tag{32}$$

The way of approximation and the neglection of compound interest determine the deviation from the long-term profit objective.

The determination of *minimum prices* is another link to integrate the investment approach and other accounting approaches [6]. In this determination the whole lifecycle of a product including research, design, equipment purchase and manufacturing has to be considered. The minimum price is the premium on top of the profit contribution, which sets the present value to zero.

For example, one can assume that the expenses for *research* E_R have to be paid at the beginning of the first period, the expenses for *design* E_D in t=1 and the expenses for *equipment* E_M in t=2. Production and sale will start after two periods and will amount to x product units per period. Considering only *one product cycle*, production will last until t=T=8. During this time fixed costs F have to be paid at the beginning of each production period, whereas variable costs k per unit have to be incurred continuously.

The minimum prices will be calculated as a proportional charge α % to variable costs k. The net present value G of one product cycle is given by equation 33:

$$G = - E_R - E_D \cdot e^{-i} - E_M \cdot e^{-2i} - F \cdot \sum_{t=2}^{T-1} e^{-it} + k \cdot \frac{\alpha}{100} \cdot x \cdot \int_{t=2}^{T} e^{-it} dt \qquad (33)$$

In order to get the minimum price $\alpha_0{}^*$ at t=0 we have to solve this equation for G=0:

$$\alpha_0{}^* = \frac{E_R + E_D \cdot e^{-i} + E_M \cdot e^{-2i} + F \cdot \sum_{t=2}^{T-1} e^{-it}}{k \cdot x \cdot \int_{t=2}^{T} e^{-it} \, dt} \cdot 100 \qquad (34)$$

The minimum prices $\alpha_t{}^*$ vary over time. They are calculated for several periods in *Table 5a*. Apparently they go down to the variable costs. In this case, the minimum price of the last period obtained by the investment approach is equal to the well known absolute minimum price of variable accounting.

If, in contrast, one assumes *succeeding products* with identical cash-flows, the lower price limit varies around an average value (see *Table 5b*). As *Figure 3* illustrates this value is equal to the value of full costing. When converging the interest costs towards zero or making the payments continuous during the whole product lifecycle this value results as a satisfying approximation. The solutions of full and direct costing prove to be special cases of the investment approach.

3.4. The Investment Approach as a Special Case of the Control Theory

If the investment approach is applied to the model of control theory under identical assumptions, the same costs per good and period are obtained. This is illustrated by an example in *Table 6* [9]. The investment approach is therefore a special case of the control theory [8].

Thus it is linked with an useful instrument to analyze dynamic problems [11, 13]. However, there are already difficulties with simple models. The application of control theory requires the full understanding of

Table 5: Determination of minimum prices (i = 0,10)

t	0	0	1	1	2	2	3	3	4	4
α^*_t	65,94	55,35	55,35	43,31	43,31	8,30	10,48	7,97	10,48	7,48
t	5	5	6	6	6,5	7	7	8		
α^*_t	10,48	6,65	10,48	4,99	6,82	10,48	0	0		

Table 5a: Minimum prices of one product cycle

t	0	0	1	1	2	2	3	3	3,5			
α^*_t	66,78	62,17	62,17	56,93	56,93	41,68	45,84	44,89	47,08			
t	4	4	5	5	6	6	7	7	8	8	9	9
α^*_t	49,38	48,43	53,27	52,32	57,55	51,83	57,47	42,22	56,93	41,68	45,84	44,89
t	10	10	11	11	12	12	13	13	14	14	15	15
α^*_t	49,38	48,43	53,27	52,32	57,55	51,83	57,47	42,22	56,93	41,68	45,84	44,89

Table 5b: Minimum prices of infinitely repeated, identical product cycles

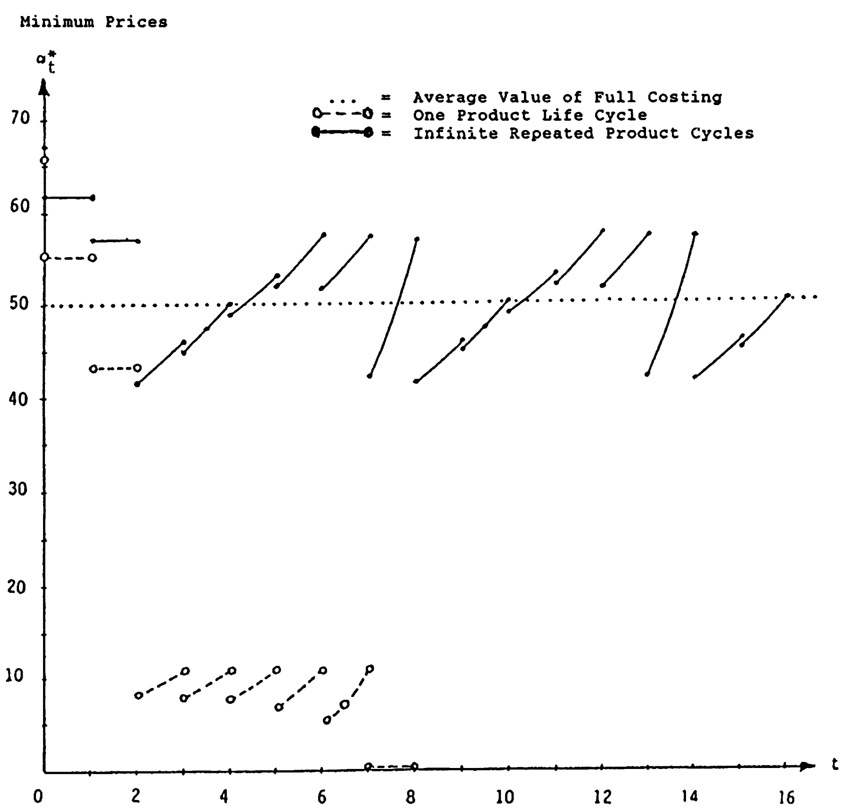

Figure 3: Example of the determination of minimum prices

	Control Theory					Investment Approach	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Condition of Equipment	Change of Condition	Dual Price		Costs per Period	Future Value	Change of Future Value
t	Z(t)	$\frac{dZ}{dt}$	p(t)	$p(t) \cdot \frac{dZ}{dt}$	$\int_a^b p \cdot \frac{dZ}{dt} dt$	G_t	$\frac{dG_t}{dt}$
	$=60e^{-0,2t}+140$	$=-12e^{-0,2t}$	$=50 - 5,09 \cdot e^{0,3t}$	$=61,10e^{0,1t} -600e^{-0,2t}$	$=610,95 \cdot (e^{0,1a}-e^{0,1b}) + 3000 \cdot (e^{-0,2a}-e^{-0,2b})$	$=610,95e^{0,1t} +3000e^{-0,2t} +20900$	$=61,10e^{0,1t} -600e^{-0,2t}$
0	200,00					24510,95	
0,5		-10,86	44,08	-478,68	479,55		-478,68
1	189,12					24031,40	
1,5		- 8,89	42,01	-373,51	374,22		-373,51
2	180,22					23657,18	
2,5		- 7,28	39,22	-285,47	286,05		-285,47
3	172,93					23371,13	
3,5		- 5,96	35,45	-211,25	211,71		-211,25
4	166,96					23159,42	
4,5		- 4,88	30,36	-148,13	148,49		-148,13
5	162,07					23010,93	
5,5		- 4,00	23,49	- 93,83	94,12		- 93,83
6	158,07					22916,81	
6,5		- 3,27	14,21	- 46,49	37,34		- 46,49
6,70	155,71	- 3,14	12,00	- 37,70		22879,47	- 37,70
			L(T) = 1868,52		= 1631,48	24510,95 - 22879,74 = 1631,48	A - L(T) = 1631,48

Table 6: Example of comparing the control theory to the investment approach

rather sophisticated instruments. "Analytical solutions to practical dynamic decision problems are rare ... " [1, p. 3]. A direct application of control theory to practical problems seems not to be possible.

Therefore the investment approach is important as a *simpler* concept to coordinate long-term and short-term planning. It avoids difficulties in solving differential equations by identifying the past utilization of the equipment with the cumulative utilization. The marginal-principle behind it is easier to understand than the Hamilton-Function and the maximum-principle.

4. Implications of the Dynamic Theory for Managerial Accounting

4.1. Performance of the Presented Theory

The presented theory accomplishes the combination of managerial accounting and investment theory. It provides the *theoretical basis* for integrated managerial planning. That seems to be very important for the relations between managerial accounting and production planning. Long-, medium- and short-term planning calculations can be derived from the investment approach. For this purpose separation theorems have to be formulated, which indicate the conditions under which simplified approaches can be used. At the same time the investment approach accomplishes the link to control theory. By this a connection to an overall dynamic theory is found.

The investment approach enables to determine the relevant information for short-term decision situations. This becomes particularly obvious with depreciations. The most important contrast to the traditional approaches is the different *way of thinking*. Within the investment approach you need not ask how to split up costs. The only important thing is the impact of decisions on *future* cash-flows. Therefore we search for the functions of the relationship between decision variables and the future cash-flows. Even if these relationships are not exactly known, the information obtained is more relevant to the decision than the information obtained by traditional systems.

Under certain assumptions the traditional approaches can be derived from the presented theory. It therefore enables the assessment of the *traditional systems* of managerial accounting. One can show under which conditions the systems of marginal costing and full costing have to be used. It becomes obvious that the systems of full costing can be useful for medium- and long-term decisions. Under special conditions they approximate the precise investment theory to a satisfying extent.

4.2. Perspectives for Further Development of Managerial Accounting

Besides the continuing work on existing elements, two directions seem to be important for further development:

(1) From a long-term perspective, *uncertainty* becomes more and more important. Therefore one has to leave the premise of perfect information. Under this assumption it might be possible to gain new knowledge about the derivation of relevant information and systems for decision making.

(2) Other important aspects of managerial accounting, besides the planning aspect, are *behavioral influence and control*. These have to be included in order to arrive at an overall theory of managerial accounting.

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