Notices

OF THE AMERICAN MATHEMATICAL SOCIETY

CONTENTS

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QUERIES

Edited by Hans Samelson

QUESTIONS WELCOMED from AMS members regarding mathematical matters such as details of, or references to, vaguely remembered theorems, sources of exposition of folk theorems, or the state of current knowledge concerning published or unpublished conjectures.

REPLIES from readers will be edited, when appropriate, into a composite answer and published in a subsequent column. All answers received will ultimately be forwarded to the questioner.

QUERIES AND RESPONSES should be typewritten if at all possible and sent to Professor Hans Samelson, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940.

◉ QUERIES

120. G. F. Kohlmayr (Mathmodel Consulting Bureau, Glastonbury, Connecticut 06033). In numerous textbooks one finds without credit to, or mentioning of, a particular author the following theorem (or proposition): A complete ordered field is Archimedean. Does anyone know by whom and where this theorem was originally published and please supply history and/or reference?

121. G. F. Kohmayr (Mathmodel Consulting Bureau, Glastonbury, Connecticut 06033). A. Robinson writes in *Model theory as a framework for algebra,* MAA Studies in Mathematics, Vol. 8, Studies in Model Theory (1973), p. 151): "... the class of differentially closed fields which are extensions of a given differential field of characteristic 0 includes prime models, and recent work by S. Shelah shows these are all isomorphic. However, according to the latest news they may possess endomorphisms over the groundfield, in which case they are not minimal." Could someone please provide me the reference(s)?

122. G. F. Kohlmayr (Mathmodel Consulting Bureau, Glastonbury, Connecticut 06033). In *Nonstandard arithmetic,* Bull. Amer. Math. Soc. 73 (1967), 818–843 (MR **36** #1319), A. Robinson states without proof (p. 842): "In particular, we may thus obtain the real numbers R by taking a countable direct power of the rational numbers Q^N and a free ultrafilter D on N," and "The quotient ring Q_0/Q_1 is isomorphic to the field of real numbers." A similar statement occurs in *What is nonstandard analysis,* Amer. Math. Monthly **80** (1973), no. 6, part II, 38–67 (MR **48** #10802) by W. A. J. Luxemburg. Has anyone seen a proof of these statements, and if so, could he please provide me the references?

123. William Sit (Department of Mathematics, City College, New York, New York 10031). I am interested in finding out how successful (or unsuccessful) computer assisted instruction (CAI) has been in the area of remedial mathematics and elementary Calculus. I would like to obtain copies of reports (or references to reports) from individuals (or institutions) who have actually implemented CAI. I am aware of articles published in The Monthly.

◉ RESPONSES

The replies below have been received to queries published in recent issues of these *Notices*. The editor would like to thank all who have replied.

To Zorn's Lemma. (vol. 23, p. 214, June 1976, Minty). Professor George J. Minty is wondering why in referring to Zorn's Lemma one writes quite often "Of course it wasn't invented by Zorn ...".

This is true: it wasn't invented by Zorn.

One can find the needed reference in the well-known Kelley's *General Topology,* (Van Nostrand, New York, 1955; MR **16**, 1136), p. 33, where Kelley cites "Zorn" Lemma among other similar or almost identical statements (one of them – as Kelley mentions – was found by me as early as in 1922, i.e. 13 years prior to Zorn). (Contributed by K. Kuratowski)

111. (vol. 24, p. 82, Jan. 1977, Parker). Although it is not quite clear how to measure the difficulty of a proof, we present an example of an equivalence relation where according to our feeling transitivity is easier achieved than reflexivity and symmetry.

Let P and L be sets. Assume that P operates on L, i.e. there is given a map $t\colon P \times L \longrightarrow L$, $(p, l) \longmapsto pl$. Further assume the following conditions to be satisfied

(i) $p(ql) = pl$ for all $p, q \in P$ and $l \in L$

(ii) t is surjective.

Then define a relation $\sim$ on L by taking $k \sim l$ (for $k, l \in L$), if there is a $p \in P$ such that $pk = l$. The transitivity of this relation is an immediate consequence of (i), while reflexivity and symmetry depend on (i) and (ii) – more precisely: under the assumption of (i) the condition (ii) is equivalent to the reflexivity as well as to the symmetry of this relation. (The situation described here is very familiar in affine geometry: There P and L denote the point set and the line set respectively of an affine space. pl means the line parallel to l through p; thus the considered relation is the parallelism. Incidence comes out from the equation $pl = l$; thus the surjectivity of t means that any line contains at least one point.) (Contributed by Rudolf Fritsch)

113. (vol. 24, p. 136, Feb. 1977, Gunter). The Gossip problem: $f(n)$ = minimum number of calls between n people to exchange all information; $f(2) = 1$, $f(3) = 3$; $f(n) = 2n - 4$ for $n \geqslant 4$. This can be generalized to (connected) graphs; conjecture (Harary-Schwenk): for $n \geqslant 2$, if the graph contains no 4-cycle, then $f(n) = 2n - 3$. References: B. Baker and R. Shostak, Discrete Math. **2**(1972), 191–193 (MR **46** #48); R.K. Guy, Amer. Math. Monthly **82**(1975), 995–1004; A. Hajnal, E.C. Milner and E. Szmerédi, Canad. Math. Bull. **15**(1972), 447–450 (MR **47** #3184); F. Harary and A. J. Schwenk, J. Franklin Inst. **297**(1974), 491–495 (MR **50** #1980); W. Knödel, Discrete Math. **13**(1975), 95 (MR **51** #15169); K. Lebensold, Studies in Appl. Math. **52**(1973), 345–358 (MR **49** #4797); R. Tijdeman, Nieuw Arch. Wisk. (3) **19**(1971), 188–192 (MR **49** #7151). The provenance of the problem is uncertain; Boyd or Chesters and Silverman may be the originators. First (unpublished) proof probably by Bumby and Spencer (1970). (Contributed by W.W. Adams, R.T. Bumby, J.R. Griggs, N.L. Johnson, D.J. Kleitman, Samuel Kotz, K.R. Rebman, Rochelle Ring, Eric Rosenthal, A.J. Schwenk, Daniel Zwillinger)

115. (vol. 24, p. 136, Feb. 1977, Demys). Write the class number of $Q(\sqrt[16]{-1})$ as $h_1 \cdot h_2$, with h_2 corresponding to the maximal real subfield. Then $h_1 = h_2 = 1$ (and so $h = 1$); this is essen-