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Bremsstrahlung in Heavy-Ion Reactions

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Abstract. We investigate bremsstrahlung processes induced by heavy ions: nuclear dipole and quadrupole radiation, radiation from bound target electrons into the continuum and secondary electron bremsstrahlung (SEB), which contribute to the background of the X-ray spectra. A comparison with experiment is presented.

Introduction

It is well known [1] that heavy-ion induced radiation is produced in form of characteristic X-rays and also as a continuous X-ray spectrum. The continuous radiation originates from radiative electron capture (REC) [2–4], from nuclear and electron bremsstrahlung and from electronic transitions during the temporary formation of molecular orbitals (MO) (see, for example, Ref. 5).

This paper deals with bremsstrahlung phenomena [6] in the keV region, induced by heavy ions of several MeV energy per nucleon.

Nuclear bremsstrahlung is produced when the projectile is accelerated in the Coulomb field of the target nucleus [7, 8]. As the charge to mass ratio of target and projectile nucleus is almost the same for light nuclei, the dipole radiation is reduced and higher multipolarities will be of comparable magnitude. This nuclear bremsstrahlung is investigated in Sect. 1.

Bremsstrahlung originates also from the electrons of the target and the projectile. Apart from being captured radiatively into bound states of the projectile (REC), the target electrons can be scattered into continuum states by the radiation field of the heavy ion. Electron bremsstrahlung resulting from this one-step process is calculated in Sect. 2.

Radiation from a two-step process is called secondary electron bremsstrahlung (SEB). In the first step target atoms are ionized [9, 10] via Coulomb excitation. The free electrons produced in this step radiate in the field of another target nucleus. The maximum energy which can be transferred from a projectile with velocity v to a free electron with mass m is $2mv^2$. For weakly bound electrons the ionization cross section shows a local maximum [11] at this energy and a rapid decrease of intensity at higher energies. Thus bremsstrahlung from secondary electrons decreases slowly up to $2mv^2$. We calculate the cross section of SEB in Sect. 3.

In Sect. 4 we give a comparison with experiment [3].

1. Nuclear Bremsstrahlung

In the region of interest the radiated energy is small compared to the energy of the projectile so that we can use the Born approximation.

For the dipole radiation with energy $\hbar\omega$ one obtains [6, 7]

$$\left.\frac{d\sigma}{d(\hbar\omega)}\right|_{\text{dip}} = \frac{16}{3}\frac{e^2}{\hbar c}(Z_P Z_T e^2)^2 \frac{1}{(M_P c^2)^2}(c/v)^2 \cdot (Z_P/A_P - Z_T/A_T)^2 \frac{1}{\hbar\omega} \ln\left((\sqrt{E}+\sqrt{E-\hbar\omega})^2/\hbar\omega\right) \quad (1.1)$$

where Z_P, A_P, Z_T, A_T is charge and mass number of projectile and target, M_P is the proton mass and E the c.m. energy.

If both nuclei have the same charge to mass ratio the dipole term vanishes. The leading term is then quadrupole bremsstrahlung. The corresponding cross section

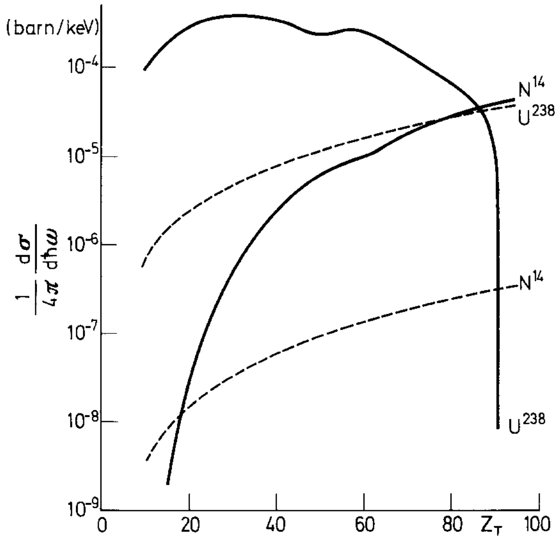

Fig. 1. Nuclear bremsstrahlung at 10 keV photon energy from N and U projectiles with relative velocity $v/c=0.1$ as a function of the target charge. The full lines are dipole and the dashed lines quadrupole radiation

in Born approximation turns out to be

$$\left.\frac{d\sigma}{d(\hbar\omega)}\right|_{\text{quad}}=\frac{32}{15}\frac{e^2}{\hbar c}(Z_P Z_T e^2)^2\frac{1}{(M_P c^2)^2}(\mu/M_P)^2$$

$$\cdot(Z_P/A_P^2+Z_T/A_T^2)^2\frac{1}{\hbar\omega}((E-\hbar\omega)/E)^{1/2}. \tag{1.2}$$

μ is the reduced mass of the nuclei. In case of identical nuclei there is an additional factor of 2.

Both types of radiation have roughly the same ω^{-1} dependence. While the dipole term decreases with projectile energy the quadrupole term does not depend on it (if $E \gg \hbar\omega$). Fig. 1 compares both terms as a function of the charge of natural target isotopes. We take ${}_7N^{14}$ and ${}_{92}U^{238}$ projectiles at $v/c=0.1$.

The quadrupole increases steadily with Z_T^2 whereas the dipole shows violent oscillations which are smoothed out in Fig. 1 to show the general dependence.

2. Electron Bremsstrahlung

In this section we calculate the direct radiative transition of the target electron to a continuum state of the projectile. In this case the energy of the ejected electron can be smaller than the photon energy so we have to use a Coulomb wave $\chi(\boldsymbol{\kappa}_f, \mathbf{r})$ for the outgoing electron. We start with evaluating the first order term of the perturbation series. Next we sum up all higher order graphs which contain the Coulomb interaction between electron and projectile in the entrance channel by taking a Coulomb wave instead of the plane wave part of the initial electron wave function. This will lead us to the "impulse approximation" known from REC calculations [4]. We then compare the plane wave limit of the impulse approximation with the Born approximation which consists of the two possible second order graphs. This will give a criterion of the validity of the impulse approximation.

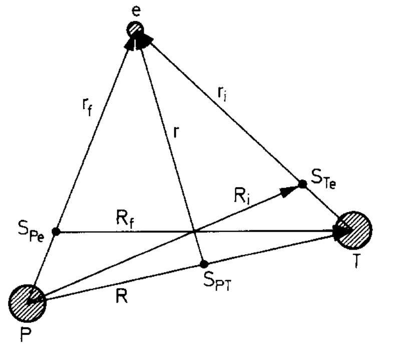

Fig. 2. Coordinate system for a three body problem consisting of projectile nucleus P, target nucleus T and electron e. S_{Pe}, S_{Te} and S_{PT} are the center of mass of projectile-electron, target-electron and projectile-target, respectively

a) Radiative Ionization in Perturbation Theory

The Hamiltonian of the three body problem – we consider only one target electron – is given by

$$H=-\frac{\hbar^2}{2m}\Delta_e+i\frac{e\hbar}{mc}\mathbf{A}\nabla-\frac{Z_P e^2}{r_f}-\frac{Z_T e^2}{r_i}-\frac{\hbar^2}{2M}\Delta_{\mathbf{R}}. \tag{2.1}$$

The last term is the kinetic energy of the relative motion of the two nuclei and M their reduced mass. The coordinates are shown in Fig. 2.

The wave function in the entrance channel consists of a plane wave for the motion of the nuclei and the bound state function of the target electron

$$|\psi_i\rangle=\exp(i\,\mathbf{K}_i\,\mathbf{R}_i)\,\varphi_i(\mathbf{r}_i). \tag{2.2}$$

It is important to take care of the recoil, so that

$$\mathbf{R}_i=\frac{m_T}{m_T+m}\mathbf{R}+\frac{m}{m_T+m}\mathbf{r}_f.$$

In the exit channel we have a free electron in the Coulomb field of the projectile, a photon created by the field $\mathbf{A}$, the dipole part of which is

$$\mathbf{A}=\left(\frac{\hbar c^2}{\omega(2\pi)^2}\right)^{1/2}\mathbf{u}_\lambda\, a_\lambda^+$$

where $\mathbf{u}_\lambda$ denotes the direction of photon polarization. The final-state wave function reads

$$|\psi_f\rangle=a_\lambda^+\exp(i\,\mathbf{K}_f\,\mathbf{R}_f)\,\chi(\boldsymbol{\kappa}_f,\mathbf{r}_f) \tag{2.3}$$

$\hbar\boldsymbol{\kappa}_f$ is the momentum of the outgoing electron and

$$\mathbf{R}_f=\mathbf{R}-\frac{m}{m_P+m}\mathbf{r}_f.$$

m_P and m_T refer to the masses of projectile and target nucleus respectively.
For the gradient operator in the radiation field

$$H' = \frac{i e \hbar}{m c} \mathbf{A} \nabla \tag{2.4}$$

we take [4]

$$\nabla = \nabla_{\mathbf{r}} + i \frac{m_T}{m_T + m_P} \mathbf{k}_i$$

because the electron is ionized in the field of the projectile. $\hbar \mathbf{k}_i$ is the initial momentum of a free electron in the rest system of the projectile

$$\hbar \mathbf{k}_i = m \mathbf{v} \tag{2.5}$$

and the coordinate on which $\nabla_{\mathbf{r}}$ acts is

$$\mathbf{r} = \mathbf{r}_f - \frac{m_T}{m_T + m_P} \mathbf{R}.$$

Thus we obtain for the transition amplitude

$$\begin{aligned} W_{fi} &= \langle \psi_f | H' | \psi_i \rangle \\ &= \frac{i e \hbar^2 \mathbf{u}_\lambda}{(2\pi)^7 m (\hbar \omega)^{1/2}} \int d\mathbf{r}_f \, d\mathbf{R} \, d\mathbf{k}_0 \, \chi^*(\boldsymbol{\kappa}_f, \mathbf{r}_f) \\ &\cdot \exp\left(-i \mathbf{K}_f \left(\mathbf{R} - \frac{m}{m_P + m} \mathbf{r}_f\right)\right) \left(\nabla_{\mathbf{r}} + i \frac{m_T}{m_T + m_P} \mathbf{k}_i\right) \\ &\cdot \exp\left(i \mathbf{K}_i \left(\frac{m_T}{m_T + m} \mathbf{R} + \frac{m}{m_T + m} \mathbf{r}_f\right)\right) \psi_i(\mathbf{k}_0) \\ &\cdot \exp(i \mathbf{k}_0 (\mathbf{r}_f - \mathbf{R})). \end{aligned} \tag{2.6}$$

$\psi_i(\mathbf{k}_0)$ is the Fourier transform of the bound electron wave function. The integration over $\mathbf{R}$ leads to the momentum conservation

$$\mathbf{K}_f = \frac{m_T}{m_T + m} \mathbf{K}_i - \mathbf{k}_0 \tag{2.7}$$

and we eliminate $\mathbf{K}_f$ by use of this relation. Expressing $\mathbf{K}_i$ by

$$\mathbf{K}_i = M_i \mathbf{v} / \hbar = \frac{m_P (m_T + m)}{m (m_P + m + m_T)} \mathbf{k}_i \tag{2.8}$$

and writing $\mathbf{r}$ instead of $\mathbf{r}_f$, W_{fi} can be written up to the order m/m_P

$$\begin{aligned} W_{fi} &= -\frac{e \hbar^2 \mathbf{u}_\lambda}{(2\pi)^4 m (\hbar \omega)^{1/2}} \int d\mathbf{r} \, \chi^*(\boldsymbol{\kappa}_f, \mathbf{r}) (\mathbf{k}_0 + \mathbf{k}_i) \\ &\cdot \exp(i (\mathbf{k}_0 + \mathbf{k}_i) \mathbf{r}) \psi_i(\mathbf{k}_0). \end{aligned} \tag{2.9}$$

The transition amplitude has been reduced to electronic properties alone. The initial electron is described by a plane wave and its momentum is a superposition of the free motion $\mathbf{k}_i$ and the momentum due to the binding to the target nucleus, weighted with the momentum distribution $\psi_i(\mathbf{k}_0)$. This means that W_{fi} has the form

$$W_{fi} = \langle \psi_{e,f} | H' | \psi_{e,i} \rangle$$

and the gradient operator in H' acts on the electron coordinate $\mathbf{r}$.

b) Radiative Ionization in Case of Strong Distortion

The initial function $|\psi_i\rangle$ is no eigenfunction to $H - H'$ but only to $H - H' + Z_P e^2/r_f$. Therefore we must treat the projectile Coulomb field as perturbation in the entrance channel. If the effective charge of the incident heavy ion is greater than or comparable with the charge of the target nucleus, the electron orbits will be strongly polarized by the approaching ion, and the corresponding Coulomb excitation cannot be treated by perturbation theory. If, however, at the same time the velocity of the projectile is greater than the orbital velocity of the target electron, such that the electron may be considered as quasi free in the field of the projectile, one can approximate the eigenfunction of $H - H'$ by taking a Coulomb wave instead of the plane wave which enters (2.9):

$$\begin{aligned} W_{fi} &= \frac{i e \hbar^2 \mathbf{u}_\lambda}{(2\pi)^4 m (\hbar \omega)^{1/2}} \psi_i(\mathbf{k}_0) \int d\mathbf{r} \, \chi^*(\boldsymbol{\kappa}_f, \mathbf{r}) \nabla_{\mathbf{r}} \chi(\mathbf{k}_0 + \mathbf{k}_i, \mathbf{r}) \\ &:= \psi_i(\mathbf{k}_0) \, W_{\text{free}}(\mathbf{k}_0 + \mathbf{k}_i, \boldsymbol{\kappa}_f). \end{aligned} \tag{2.10}$$

The transition probability is that for a free electron in the field of a nucleus and has been calculated by Sommerfeld [12]. It is weighted with the probability $|\psi_i(\mathbf{k}_0)|^2$ of finding this electron with momentum $\hbar(\mathbf{k}_0 + \mathbf{k}_i)$.
The differential cross section is given by

$$\begin{aligned} d\sigma &= 2\pi/\hbar \, |W_{fi}|^2 \, \delta(\varepsilon_f - \varepsilon_i) \frac{(\hbar \omega)^2}{(\hbar c)^3} \\ &\cdot d(\hbar \omega) \, d\Omega_q \, d\mathbf{k}_0 \, d\boldsymbol{\kappa}_f (2\pi)^3 / v. \end{aligned} \tag{2.11}$$

Taking the energy in entrance and exit channel

$$\varepsilon_i = \frac{\hbar^2 K_i^2}{2 M_i} + E_i$$

$$\varepsilon_f = \frac{\hbar^2 K_f^2}{2 M_f} + \frac{\hbar^2 \kappa_f^2}{2m} + \hbar \omega$$

where E_i is the (negative) binding energy of the target electron, we obtain with (2.7) and (2.8) the dispersion relation

$$\delta(\varepsilon_i - \varepsilon_f) = \delta\left(\frac{\hbar^2 k_i^2}{2m} + \frac{\hbar^2 \mathbf{k}_i \mathbf{k}_0}{m} + E_i - \frac{\hbar^2 \kappa_f^2}{2m} - \hbar \omega\right). \tag{2.12}$$

With W_{fi} from (2.10) and after performing the integration over the direction Ω_q of the emitted photon we find

$$\frac{d\sigma}{d(\hbar\omega)} = \int d\mathbf{k}_0\, |\psi_i(\mathbf{k}_0)|^2 \left.\frac{d\sigma}{d(\hbar\omega)}\right|_{\text{free}} (|\mathbf{k}_0+\mathbf{k}_i|, \kappa_f)\, B^2 \tag{2.13}$$

κ_f is given by (2.12). $\left.\frac{d\sigma}{d(\hbar\omega)}\right|_{\text{free}}$ is the well known bremsstrahlung cross section for free electrons [7, 12]. The factor B

$$B = \frac{\hbar\omega}{\hbar^2(\mathbf{k}_0+\mathbf{k}_i)^2/2m - \hbar^2\kappa_f^2/2m} \tag{2.14}$$

is 1 in the case of free electrons and <1 for bound electrons. (2.13) is called the "impulse approximation". In the limit of zero binding $\psi_i(\mathbf{k}_0)$ is nonvanishing only for $\mathbf{k}_0=0$ and we arrive at the free bremsstrahlung cross section.

c) Comparison with the Born Approximation

The identity of impulse approximation and perturbation theory in the limit of small perturbation is only true for first order processes such as REC. In this section we demonstrate that for electron bremsstrahlung, which is a second order process, the two methods show small deviations.

To compare both methods we calculate (2.10) in the limit of small Sommerfeld parameters $\eta(k) = Z_P e^2 m/(\hbar^2 k) \ll 1$. In this case one has

$$W_{fi}(\eta_i, \eta_f) \approx W_{fi}(0, \eta_f) + W_{fi}(\eta_i, 0) \tag{2.15}$$

with $\eta_i = \eta(|\mathbf{k}_0+\mathbf{k}_i|)$ and $\eta_f = \eta(\kappa_f)$.

Into (2.15), the first term of which is given by (2.9), enters the Fourier transform of a Coulomb wave

$$\chi^*(\mathbf{k}, \boldsymbol{\kappa}) = 1/(2\pi)^3 \int \chi^*(\boldsymbol{\kappa}, \mathbf{r}) \exp(i\,\mathbf{k}\,\mathbf{r})\, d\mathbf{r}. \tag{2.16}$$

We derive this expression from the transfer matrix element

$$T(\mathbf{k}, \boldsymbol{\kappa}, \gamma) = \int d\mathbf{r}\, \chi^*(\boldsymbol{\kappa}, \mathbf{r}) \exp(-\gamma r)/r\, \chi(\mathbf{k}, \mathbf{r})$$

which can be evaluated analytically [13]. We take $\eta(k)=0$ which means to replace the Coulomb wave in the entrance channel $\chi(\mathbf{k}, \mathbf{r})$ by a plane wave $\exp(i\,\mathbf{k}\,\mathbf{r})$, and obtain (2.16) by means of

$$\chi^*(\mathbf{k}, \boldsymbol{\kappa}) = -1/(2\pi)^3 \frac{d}{d\gamma} \left.\left\{\lim_{\eta(k)\to 0} T(\mathbf{k}, \boldsymbol{\kappa}, \gamma)\right\}\right|_{\gamma=0}$$

which leads (with Coulomb waves for an attractive potential) to

$$\chi^*(\mathbf{k}, \boldsymbol{\kappa}) = \kappa\,\eta_f/\pi^2 \exp(\pi\,\eta_f/2)\, \Gamma(1-i\,\eta_f) \frac{(k^2-\kappa^2)^{-i\eta_f-1}}{[(\mathbf{k}-\boldsymbol{\kappa})^2]^{1-i\eta_f}}. \tag{2.17}$$

Thus we obtain for the transition amplitude

$$\begin{aligned} W_{fi}(\eta_i, \eta_f) &= -\frac{Z_P e^3 \mathbf{u}_\lambda}{2\pi^3 (\hbar\omega)^{1/2}} \psi_i(\mathbf{k}_0) \frac{1}{(\mathbf{k}_0+\mathbf{k}_i-\boldsymbol{\kappa}_f)^2 \left((\mathbf{k}_0+\mathbf{k}_i)^2-\kappa_f^2\right)} \\ &\cdot \left[(\mathbf{k}_0+\mathbf{k}_i)\left(\frac{2\pi\,\eta_f}{1-\exp(-2\pi\,\eta_f)}\right)^{1/2} F(\eta_f) \right. \\ &\left. -\boldsymbol{\kappa}_f \left(\frac{2\pi\,\eta_i}{\exp(2\pi\,\eta_i)-1}\right)^{1/2} F(\eta_i)\right] \end{aligned} \tag{2.18}$$

where

$$F\ \eta) = \exp(i \arg \Gamma(1-i\,\eta)) \left[\frac{(\mathbf{k}_0+\mathbf{k}_i-\boldsymbol{\kappa}_f)^2}{(\mathbf{k}_0+\mathbf{k}_i)^2-\kappa_f^2}\right]^{i\eta}.$$

When $\eta_i = \eta_f = 0$, $F(\eta)$ reduces to 1.

We can simplify the integrations which are necessary for evaluating the cross section (2.11) by integrating over k_{0z} with the aid of the energy conservation (2.12); k_{0z} is the projection of $\mathbf{k}_0$ on the direction $\mathbf{k}_i$ of the incident electron. Since $|\psi_i(\mathbf{k}_0)|^2$ varies quickly with $\mathbf{k}_0$ compared to the remaining integrand we can take the latter outside the integral at $k_{0x} = k_{0y} = 0$ where $\psi_i(\mathbf{k}_0)$ is peaked.

Introducing the Compton profile [4]

$$J_i(k_{0z}) = \int dk_{0x}\, dk_{0y}\, |\psi_i(\mathbf{k}_0)|^2 \tag{2.19}$$

we obtain in the plane wave limit of (2.13) after summing over the two polarization directions of the emitted photon

$$\begin{aligned} \frac{d\sigma}{d(\hbar\omega)} &= 32/3\, Z_P^2 \left(\frac{e^2}{\hbar c}\right)^3 \frac{1}{k_i^2} \frac{1}{\hbar\omega} \int_0^\infty \kappa_f^2\, d\kappa_f\, J_i(k_{0z}) \\ &\cdot \left\{ BG \frac{1}{2\kappa_f (k_{0z}+k_i)} \ln \frac{k_{0z}+k_i+\kappa_f}{k_{0z}+k_i-\kappa_f} + \frac{1}{[(k_{0z}+k_i)^2-\kappa_f^2]^2} \right. \\ &\left. \cdot \left[\kappa_f^2 G^2 + (k_{0z}+k_i)^2 B^2 - BG\left((k_{0z}+k_i)^2+\kappa_f^2\right)\right] \right\}. \end{aligned} \tag{2.20}$$

The factor G was introduced to combine the plane wave limit of (2.13) with the second order Born approximation. In the case of the impulse approximation $G=B$ and only the logarithmic term survives. If on the other hand the electron bremsstrahlung is calculated in Born approximation we obtain (2.20) with $G=1$. This means that only the first term of the transition amplitude (2.15) agrees for $\eta_i = \eta_f = 0$ with the corresponding term in the Born approximation. The deviations resulting from the additional factor B in the second term in (2.15) are the smaller, the higher the electron momentum $\hbar\, k_i$ and the smaller the binding of the electron. We compared the impulse approximation and the Born theory in the case of N as target and found for $v/c \approx 0.1$ deviations up to 10% and for $v/c \approx 0.2$ up to 1% for K-shell electrons in the whole energy region of interest $4 \leqq \hbar\omega \leqq 16$ keV. For finite η_i and η_f the first term in (2.18) strongly dominates the second one, so that the agreement is even better. Therefore it is justified to use (2.13) for the electron bremsstrahlung calculations.

3. Bremsstrahlung from Secondary Electrons (SEB)

This is a two step process and therefore density dependent. We first give the cross section for ionization of the target atom in the Coulomb field of the projectile and then derive the cross section for bremsstrahlung of these electrons in the field of the target nuclei.

a) Ionization

The ionization cross section can be calculated in the classical binary-encounter approximation [8]. For projectile energies higher than 1 MeV/nucleon as well as for strongly bound electrons, however, the Born approximation should be preferred [14].
Choosing the target nucleus as origin which means that in Fig. 2 $\mathbf{R}$ has to be replaced by $-\mathbf{R}$, the transition amplitude in first order Born approximation is given by

$$W_{fi} = -Z_P e^2/(2\pi)^6 \int d\mathbf{r}_f\, d\mathbf{R}\, d\mathbf{k}_0 \exp(-i\,\mathbf{K}_f\,\mathbf{R}) \cdot \exp(-i\,\boldsymbol{\kappa}_f(\mathbf{r}_f+\mathbf{R}))\, 1/r_f \exp(i\,\mathbf{K}_i\,\mathbf{R}) \cdot \exp(i\,\mathbf{k}_0(\mathbf{r}_f+\mathbf{R}))\, \psi_i(\mathbf{k}_0) \quad (3.1)$$

where $\hbar\,\boldsymbol{\kappa}_f$ is the momentum of the outgoing electron. Following the same way as in Section 2 we arrive at the differential cross section per electron energy $E_e = \hbar^2\,\kappa_f^2/2m$

$$\frac{d\sigma}{d\Omega_e\, dE_e} = 4(Z_P e^2)^2 (m/\hbar^2)^2\, 1/k_i \int d\mathbf{k}_0\, |\psi_i(\mathbf{k}_0)|^2 \cdot \kappa_f/(\mathbf{k}_0-\boldsymbol{\kappa}_f)^4\, \delta(\varepsilon_f-\varepsilon_i) \quad (3.2)$$

and the energy conservation reads

$$\hbar^2/m\, \mathbf{k}_i(\boldsymbol{\kappa}_f-\mathbf{k}_0)+E_i = \hbar^2\,\kappa_f^2/2m. \quad (3.3)$$

We are interested in the high energy tail of the photon distribution with $\hbar\omega > 1/2\,\mathrm{mv}^2$. To produce these photons, E_e has to be larger than $1/2\,\mathrm{mv}^2$. In this region the dominating part of the integrand in (3.2) is $|\psi_i(\mathbf{k}_0)|^2$. Thus we take the slowly varying $\kappa_f/(\mathbf{k}_0-\boldsymbol{\kappa}_f)^4$ before the integral and introduce the Compton profile (2.19) as we did in Sect. 2. Thus we get the simple form

$$\frac{d\sigma}{d\Omega_e\, dE_e} = 4(Z_P e^2)^2 (m/\hbar^2)^3\, 1/k_i^2\, J_i(k_{0z})\, \kappa_f/(\mathbf{k}_{0z}-\boldsymbol{\kappa}_f)^4. \quad (3.4)$$

If the binding energy E_i is small or k_i is large so that E_i can be neglected in (3.3), k_{0z} is zero at $\kappa_f = 2k_i\cos\vartheta$. ϑ is the angle between $\mathbf{k}_i$ and $\boldsymbol{\kappa}_f$. This means that $J_i(k_{0z})$ can take on its peak value and the cross section increases for energies $E_e \lesssim 2\,\mathrm{mv}^2$ (in the forward direction). This is not true for higher binding energies. For $E_e > 2\,\mathrm{mv}^2$ (3.4) rapidly goes to zero which means that the maximum photon energy emitted by these electrons is $\hbar\omega \approx 2\,\mathrm{mv}^2$, whereas in the direct bremsstrahlung (Sect. 2) the maximum photon energy is $\approx 1/2\,\mathrm{mv}^2$ and one has only small tails from the strongly bound electrons.

b) Bremsstrahlung (SEB)

We obtain the photon number per energy and solid angle by multiplying the bremsstrahlung cross section for free electrons with the probability of finding such a free electron. The electron production, as well as the consecutive bremsstrahlung, depends on the density of target atoms ρ_T and on the linear dimension l of the sample. Therefore this quantity enters into the radiation cross section. We find

$$\frac{d\sigma_{\mathrm{rad}}}{d(\hbar\omega)\, d\Omega_q} = \int d\Omega_e\, dE_e \left.\frac{d\sigma_{\mathrm{brems}}}{d(\hbar\omega)\, d\Omega_q}\right|_{\mathrm{free}} (\boldsymbol{\kappa}_f, \mathbf{k}_f) \cdot \frac{d\sigma_{\mathrm{ion}}}{d\Omega_e\, dE_e}(\mathbf{k}_i, \boldsymbol{\kappa}_f)\, 1/2\,\rho_T\, l. \quad (3.5)$$

One has to integrate over all intermediate electron states $\boldsymbol{\kappa}_f$. Since we are only interested in high energy electrons we describe these states by plane waves. So $d\sigma_{\mathrm{ion}}$ is given by (3.2). The bremsstrahlung cross section $d\sigma_{\mathrm{brems}}$ for free electrons is readily calculated if we have a plane wave in the entrance channel and a Coulomb wave in the exit channel. One has only to multiply the plane wave cross section by a factor C [12]

$$C = \frac{2\pi\,\eta_k}{1-\exp(-2\pi\,\eta_k)} \quad (3.6)$$

which also appears in (2.18). $\hbar\,\mathbf{k}_f$ is the momentum of the outgoing electron and $\eta_k = \eta(k_f)$.
After performing the integration over all photon directions we obtain

$$\frac{d\sigma_{\mathrm{rad}}}{d(\hbar\omega)} = 64/3\,(Z_P Z_T e^2)^2 \left(\frac{e^2}{\hbar c}\right)^3 (m/\hbar^2)^2 \frac{1}{k_i}\frac{1}{\hbar\omega} \cdot \int dE_e\, d\Omega_e\, d\mathbf{k}_0\, |\psi_i(\mathbf{k}_0)|^2 \frac{1}{\kappa_f(\mathbf{k}_0-\boldsymbol{\kappa}_f)^4} \ln\frac{\kappa_f+k_f}{\kappa_f-k_f} \cdot \frac{2\pi\,\eta_k}{1-\exp(-2\pi\,\eta_k)}\, \delta(\hbar^2/m\, \mathbf{k}_i(\boldsymbol{\kappa}_f-\mathbf{k}_0)+E_i-\hbar^2\,\kappa_f^2/2m) \cdot 1/2\,\rho_T\, l. \quad (3.7)$$

We again make use of the peaking approximation and introduce the Compton profile (2.19). Because of energy conservation

$$\hbar\omega = \hbar^2/2m(\kappa_f^2 - k_f^2) = E_e - \hbar^2 k_f^2/2m \tag{3.8}$$

the minimum value of E_e is $\hbar\omega$ for a given photon energy. Therefore

$$\frac{d\sigma_{\rm rad}}{d(\hbar\omega)} = 32/3\,\rho_T\, l(Z_P Z_T e^2)^2 \left(\frac{e^2}{\hbar c}\right)^3 \frac{1}{v^2}\frac{1}{\hbar\omega}\int_{\kappa_{\min}}^{\infty} d\kappa_f\, d\Omega_e$$

$$\cdot J_i(k_{0z})/(\mathbf{k}_{0z} - \boldsymbol{\kappa}_f)^4 \ln\frac{\kappa_f + k_f}{\kappa_f - k_f}\,\frac{2\pi\eta_k}{1-\exp(-2\pi\eta_k)} \tag{3.9}$$

where k_{0z} and k_f are given by (3.3) and (3.8) respectively. We evaluated (3.9) for N and Ne as target nuclei and found an energy dependence of ω^{-3} in the region of $\hbar\omega = 1/2\,\mathrm{mv}^2$, ω^{-4} at twice this energy and ω^{-6} at $\hbar\omega \approx 3/2\,\mathrm{mv}^2$.

4. Comparison with Experiment and Discussion

In recent experiments [3] at Berkeley, heavy-ion induced radiation was studied at high energies, ranging from 7 MeV per nucleon to 18 MeV per nucleon. The projectiles were highly stripped: N(7$^+$), Ne(10$^+$) and Ar(17$^+$). These ions passed through various gas targets at normal pressure. The target cell was 6 mm thick.

When using targets with strongly bound electrons, the X-ray spectrum showed a high-energy tail which was ascribed to electron bremsstrahlung because neither radiative electron capture (REC) nor radiative transitions between intermediate molecular orbits could account for it.

We calculated the X-ray spectrum from the three types of radiation, REC [3, 4], radiative ionization (2.13) and SEB (3.9). For the bound electrons we used nonrelativistic hydrogen functions with screened nuclear charges [15], and we summed the contributions of all target electrons.

We note that, classically, radiation of frequency ω originates mainly from impact parameters $b \approx v/\omega$. For the frequencies under consideration the corresponding value of b is comparable with the K-shell radius of the target atom. This means that for energies larger than the REC maximum the full nuclear target charge enters into Eq. (3.9).

The result is shown in Figs. 3 and 4. In the case of N^{7+} (Fig. 3) the absolute cross section is not known, and we normalized the experimental differential cross section to the REC peak. This is compatible with REC calculations for experiments where the absolute value is given (see, for instance, Fig. 4).

To summarize, nuclear bremsstrahlung is negligible in the considered energy region. Radiative ionization becomes important for high collision energies, but its intensity decreases strongly for $\hbar\omega \gtrsim 1/2\,\mathrm{mv}^2$. Radiation from secondary electrons depends on density and

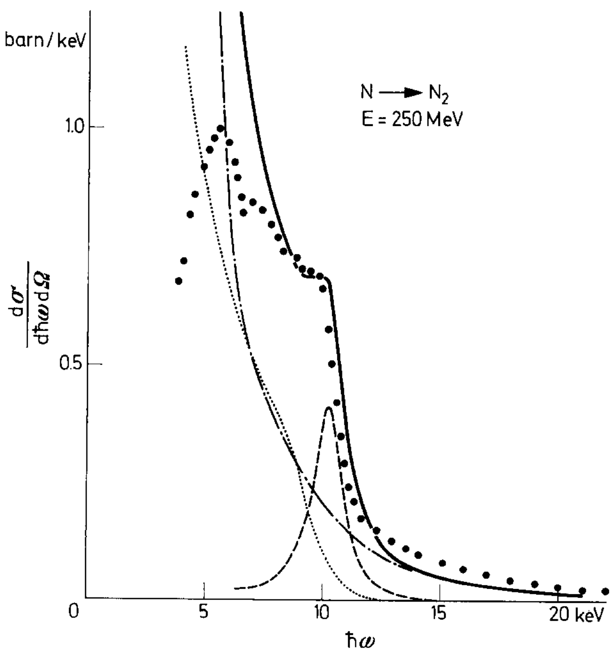

Fig. 3. Energy dependence of photons emitted during the scattering of N^{7+} with lab energy 250 MeV on N_2. The points are experimental data [16] at $\vartheta = 90°$. Below 7 keV the intensity is reduced because of absorption. The broken line is REC, the dotted line is radiative electron ionization and the dash-dotted line is SEB. The full line is a superposition of the three radiation processes, the cross sections of which are averaged over the scattering angle

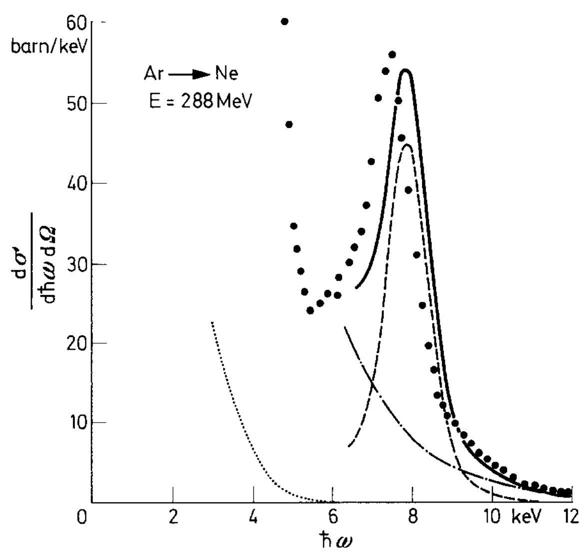

Fig. 4. Energy dependence of photons emitted during the scattering of Ar^{17+} with lab energy 288 MeV on Ne. The points are experimental data ($\vartheta = 90°$) taken from Ref. 3. The theoretical curves have the same meaning as in Fig. 3

thickness of the target and its cross section shows a slow decrease up to $2\,mv^2$. In our case, SEB explains the high energy tail of the radiation spectra.

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