A Robinsonian Approach to Discrimination

by Ekkehart Schlicht

Journal of Institutional and Theoretical Economics

(Zeitschrift für die Gesamte Staatswissenschaft)

1982, 138(1), pages 64-83

Zeitschrift für die gesamte Staatswissenschaft (ZgS) 138 (1982), 64 83
Journal of Institutional and Theoretical Economics

A Robinsonian Approach to Discrimination*

by

EKKEHART SCHLICHT

1. Introduction

Wage discrimination might simply come about when firms offer lower wages to applicants whom they expect to accept comparatively low wage offers. This is the issue upon which the present paper tries to shed light.

The approach adopted here might supplement other contributions, notably by Becker and Arrow[1]. Becker's pioneering work explains discrimination by a taste for discrimination: Employers or employees of group 1 dislike working together with members of group 2. This induces discrimination. The problem with this approach is, however, that there cannot be discrimination in the long run since firms which employ a larger fraction of the cheaper workers will earn higher profits and will drive out other firms[2]. Thus, discrimination will disappear eventually.

Another possibility is that discrimination arises because race or sex are used as indicators for productivity[3]. Hence these indicators will be coupled with different wages. In order to explain persistent discrimination one has to assume, however, a feedback by which higher wages induce higher productivity, thereby making the use of these indicators competitively dominant[4]. The additional problem with this approach seems to be, however, that there are often better indicators for productivity which are readily available or can be made available by employees in order to avoid discrimination: The particular certificate of a certain educational institution and previous employment records provide better indicators for productivity and qualification than race or sex; and the social

* An earlier draft of this paper has been presented at the 1981 Econometric Society European Meeting in Amsterdam under the title "A Search-Theoretic Approach to Discrimination". I thank P. Rice and an unnamed referee for some critical comments. The paper is dedicated to Emmi.

1 For a survey of the literature, see MARSHALL [1974].

2 See BECKER [1971], p. 56; ARROW [1972a], pp. 90–96; THUROW [1975], pp. 160.

3 See ARROW [1972a], pp. 96–98, [1972b], pp. 199–203, [1972c], section 4; DOERINGER and PIORE [1971], p. 139; PHELPS [1972].

4 See MARSHALL [1974], p. 855. Akerlof's analysis of the origins of social custom circumvents this problem by introducing the assumption that products cannot be trated anonymously (AKERLOF [1976], pp. 606–616). Hence his solution of the problem will not apply in the present context.

background and working attitudes can better be forecasted by information concerning the occupation, social status, and neighborhood of the parents than by race. For this reason, it seems to me, the "indicator" approach has its particular shortcomings, too, if indicators are thought to refer to productivity: The typical "social" indicators like race or sex will usually not be the best. Furthermore the approach presupposes productivity differentials in a statistical sense and will not be applicable in cases where the expected productivities of the two groups are identical.

It might however be that the "social" indicators are linked to different supply attitudes. If women are, for instance, more closely tied to their homes than men, they will respond differently to wage offers, and this might lead to discrimination. The approach has been introduced in the thirties by the monopolistic competition theorists, most notably by Joan Robinson[5]. In the more recent discrimination literature it has been somewhat neglected[6].

The argument runs as follows: Assume two groups of workers who are equally qualified and are hence perfect substitutes in production. Denote the wage rate paid to group 1 workers by v and denote the wage rate paid to group 2 workers by w; denote furthermore the supply elasticities of the two groups by $\varepsilon(v)$ and $\eta(w)$, respectively. The additional expenditure for one additional unit of type 1 workers is $v(1+1/\varepsilon(v))$ and the marginal expenditure for type 2 workers is $w(1+1/\eta(w))$. A necessary condition for a profit maximum is now that these marginal expenditures are equated to the marginal product m:

$$v(1+1/\varepsilon(v)) = m, \quad w(1+1/\eta(w)) = m .$$

This implies that the wage differential v/w is related to the supply elasticities

$$\frac{v}{w} = \frac{1+1/\eta(w)}{1+1/\varepsilon(v)} . \tag{1}$$

Loosely speaking, one might say that the group with a higher elasticity will receive a higher wage in equilibrium. It might be argued that this will also hold true in the long run when all pure profits are competed away and the "tangency solution" obtains[7]. Hence this type of theory will not run into the long-run difficulties of the other approaches. There is, however, a hitch in the argument since it is reasonable to assume that supply elasticities, although very small in the short run, will become very large in the long run. Hence discrimination will vanish.

The argument developed in this paper will yield formulas similar to those given above, but with a different interpretation which avoids this hitch. It rests

5 ROBINSON [1969], pp. 228, 302–304.

6 The survey article by MARSHALL [1974], e.g., does not mention it. See MADDEN [1977] for one of the few recent approaches in this tradition.

7 CHAMBERLIN [1962], pp. 81–100.

on the assumption that the probability of finding a new worker within one period is an increasing function of the wage offer. Hence the firms will choose a wage offer which is neither so high as to induce unnecessarily high labor costs nor so low that they have to wait very long in order to find a candidate at all. If the two groups respond differently in their supply probabilities to wage changes, this will induce discrimination[8].

The argument is developed in part 1 of the paper in a single-firm short-run framework, and will be generalized in part 2 to include some effects of competition and changes of aspiration levels.

Part 1. Short-Run Analysis

2. The Problem

Consider a firm which intends to fill a vacancy. There are two types of applicants who can perform the job equally well: They are perfect substitutes in production.

They differ, however, in the probability of applying for the job opening and of accepting it. The firm is able to distinguish the members of the two groups directly and to offer different wages according to group affiliation. The problem is to describe an optimal wage-setting behavior of the firm.

More specifically, let m denote the marginal product of the job under consideration. Let v denote the wage rate offered to applicants of group 1 and denote by w the wage offer to applicants of group 2. The probability of finding a suitable applicant of group 1 within one time period at the wage offer v is denoted by p and is called the *access probability* of group 1. Similarly, the access probability of group 2 is denoted by q and gives the probability of finding a suitable applicant of group 2 at the wage offer w within one time period.

Starting with these notions, we can write down the probability of not finding an applicant of group 1 within one time period as $(1-p)$. Similarly, the probability of not finding an applicant of group 2 within one time period is $(1-q)$. Hence the probability of finding neither a candidate of type 1 nor a candidate of type 2 within one period of time is $(1-p)(1-q)$. The joint probability of not finding any worker within t periods of time can be written as $((1-p)(1-q))^t$. From this it follows that the probability of finding a suitable applicant within t periods of time is just $(1-(1-p)^t(1-q)^t)$.

It is agreed that the wage offered to an applicant initially is to be paid permanently and that search is stopped after the vacancy has been filled. Hence the expected remuneration to be paid to the job is $(p \cdot v+q \cdot w)/(p+q)$.

[8] The similarity between search theory and the theory of monopolistic competition has been noted repeatedly, e.g. by PHELPS and WINTER [1970], p. 323.

If the firm is risk neutral and has a discount rate of r, the expected discounted value of the job is given by the discounted sum of the expected returns $(m-(p \cdot v+q \cdot w)/(p+q))$ weighted with the probabilities of having filled the job at time t, which is $(1-(1-p)^t(1-q)^t)$:

$$E = \sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^t \left(m - \frac{p \cdot v + q \cdot w}{p+q}\right)(1-(1-p)^t(1-q)^t) \,. \tag{2}$$

An evaluation of the geometrical series leads to

$$E = \left(\frac{1+r}{r}\right) \cdot \left(m - \frac{pv+qw}{p+q}\right)\left(\frac{p+q-pq}{r+p+q-pq}\right) . \tag{3}$$

3. Switching to Continuous Time: Informal Argument

In order to simplify the analysis it seems appropriate to take the trouble of casting the analysis into continuous time instead of using formulae (2) and (3) as the starting point of the further analysis[9].

If the time intervals are taken to be very small, the access probabilities for these intervals can be made very small, too. Hence the terms involving the product of access probabilities $p \cdot q$ will become negligible in (3). Disregarding the constant factor $(1+r)/r$, the maximand turns into

$$M = \left(m - \frac{pv+qw}{p+q}\right)\left(\frac{p+q}{r+p+q}\right) . \tag{4}$$

For short time intervals – and hence small discount factors r – the factor $(1+r)/r$ can be approximated by $1/r$. Hence the value of an additional job will be just M/r.

The exact limiting process is described in the next section and can be omitted at the readers discretion.

4. Switching to Continuous Time: Formal Analysis

If a time unit is subdivided into n intervals of equal length $1/n$, the access probability for group one will be p/n, and that for group 2 will be q/n. The marginal product during an interval will be m/n and wages will be v/n and w/n, respectively. The discount factor applicable to an interval will be r/n. The expected discounted value of the job to the firm will be the discounted sum of interval returns weighted with the appropriate probabilities. This turns E from (2) into

[9] Alternatively – and equivalently – the accession process could have been envisaged as a Poisson process from the beginning – but this more elegant approach has not been used since it seems to be less intelligible than the more clumsy formulation adopted here.

$$(5)\qquad E^n = \sum_{\tau=1}^{\infty}\left(\frac{1}{1+r/n}\right)^{\tau n}\left(\frac{m}{n}-\frac{\frac{p}{n}\cdot\frac{v}{n}+\frac{q}{n}\cdot\frac{w}{n}}{\frac{p}{n}+\frac{q}{n}}\right)\left(1-\left(1-\frac{p}{n}\right)^{\tau n}\left(1-\frac{q}{n}\right)^{\tau n}\right)$$

Taking the limit $n\to\infty$ yields[10]

$$(6)\qquad E^{\infty} = \lim_{n\to\infty} E^n = \int_0^{\infty} e^{-r\tau}\left(m-\frac{pv+qw}{p+q}\right)\left(1-e^{-(p+q)\tau}\right)d\tau\ .$$

By performing the integration, the following expression is obtained[11]:

$$(7)\qquad E^{\infty} = \frac{1}{r}\left(m-\frac{pv+qw}{p+q}\right)\left(\frac{p+q}{r+p+q}\right).$$

If this expression is to be maximized for a given r, this is equivalent to the maximization of the expression M given in the last section, formula (4). In the following we will start from there[12].

5. *Assumptions About Access Probabilities*

Returning to the more economic aspects of the problem, it will be assumed that the access probability p of group 1 is an increasing function of the wage offer v made to members of this group. Likewise, the access probability q of group 2 is assumed to be an increasing function of the wage offer w. The general shape of these functions is depicted in Figure 1 for the access probability of group 1:

There is a certain minimum wage offer, $\underline{v}$, below which supply will be zero. If v increases beyond $\underline{v}$, access probability will increase first progressively up to $v=v_0$ and degressively for a further increase in v. For $v\to\infty, p(v)$ approaches an upper limit $\bar{p}\leq 1$ which captures the absolute scarcity of type 1 workers. The shape of the access probability function $q(w)$ for group 2 workers is assumed to be qualitatively similar although it will differ, in general, from $p(v)$.

[10] By using

$$\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = \lim_{n\to\infty}\frac{1}{\left(1-\frac{x}{n}\right)^n} = e^x$$

and the fact $\lim f(x)\cdot g(x) = \lim f(x)\cdot\lim g(x)$ together with the relation

$$\int_0^{\infty} f(s)\,ds = \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{\infty} f(i/n)$$

derived from the definition of the integral.

[11] By using $\int_0^{\infty} e^{-s\tau}\,d\tau = 1/s$.

[12] To be precise, the period rate of interest r in (2) is to be replaced by the associated continuous rate $\log(1+r)$. The rate of interest in (7) is to be understood in this sense.

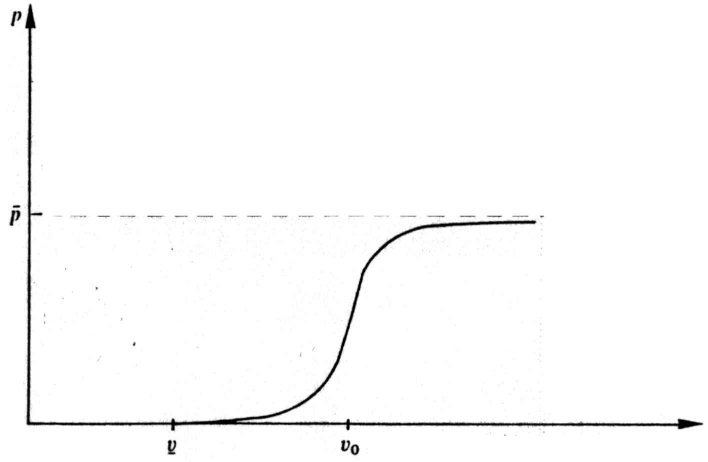

Fig. 1 Access probability p of group 1 as a function of the wage offer v

These assumptions are formalized algebraically as follows[13]:

$$(8)\qquad p = p(v)$$

with $p(v)=0$ for $0\leq v\leq \underline{v}$, for some $\underline{v}>0$; $p(v)>0$ for $v>\underline{v}$; $\operatorname{sign} p'(v)=\operatorname{sign} p(v)$; $p''(v)\geq 0$ for $0\leq v\leq v_0$, for some $v_0\in[\underline{v},\infty)$; $p''(v)<0$ for $v>v_0$; $\lim_{v\uparrow\infty} p(v)=\bar{p}$, for some $\bar{p}\in(0,1]$;

$$(9)\qquad q = q(w)$$

with $q(w)=0$ for $0\leq w\leq \underline{w}$, for some $\underline{w}>0$; $q(w)>0$ for $w>\underline{w}$; $\operatorname{sign} q'(w)=\operatorname{sign} q(w)$; $q''(w)\geq 0$ for $0\leq w\leq w_0$, for some $w_0\in[\underline{w},\infty)$; $q''(w)<0$ for $w>w_0$; $\lim_{w\uparrow\infty} q(w)=\bar{q}$, for some $\bar{q}\in(0,1]$.

From these access probability functions, the associated *access elasticities* can be defined. These elasticities, denoted by ε and η, respectively, measure the relative increase in access probability in response to a one percent increase in the wage offer. Hence they are independent of the choice of units. (They remain unaffected, for instance, by the slicing of time intervals discussed in section 3 above.)

[13] A convenient and rather flexible family of functions is given by

$$(*)\qquad p(v) = \bar{p}\,\frac{(a(v-\underline{v}))^b}{(a(v-\underline{v}))^b+c}$$

for $a,c>0$, $b>1$. Another family is described by

$$(**)\qquad p(v) = \bar{p}\cdot\exp\{-(a(v-\underline{v}))^{-b}\}\ .$$

Figure 1 has been selected from (*) by choosing $a=0.5$, $b=10$, $c=10$, $\underline{v}=2$, $\bar{p}=0.75$.

(10) $$\varepsilon(v) := \frac{p'(v) \cdot v}{p(v)},$$

(11) $$\eta(w) := \frac{q'(w) \cdot w}{q(w)}.$$

These elasticities can be read off from the probability functions as indicated n Figure 2.

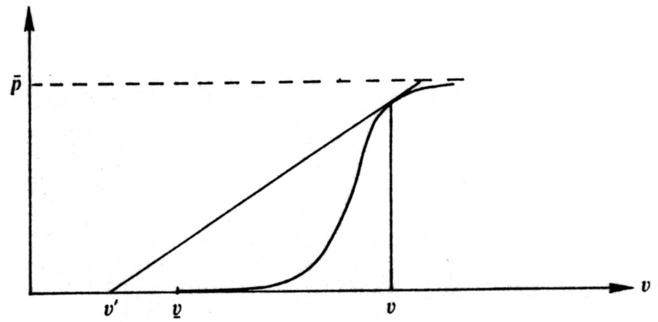

Fig. 2 The access elasticity at v equals $v/(v-v')$

Note that $\varepsilon(\underline{v}) = \infty$ and $\varepsilon(\infty) = 0$[14]. Similarly, $\eta(\underline{w}) = \infty$ and $\eta(\infty) = 0$ hold true.

If the access probability function is sufficiently smooth, the associated elasticity will be strictly decreasing.

14 $$\varepsilon(\underline{v}) := \lim_{v \downarrow \underline{v}} \varepsilon(v) = \lim_{v \downarrow \underline{v}} \frac{p'(v) \cdot v}{p(v)} = \lim_{v \downarrow \underline{v}} \left(\frac{p'}{p'} + v \cdot \frac{p''}{p'} \right)$$
$$= 1 + \underline{v} \lim_{v \downarrow \underline{v}} \frac{p''}{p'} = 1 + \underline{v} \lim_{v \downarrow \underline{v}} \frac{p^{n}}{p^{n-1}}$$

where p^n denotes the lowest non-vanishing derivative which is positive in order to guarantee $p(v) > 0$ for $v > \underline{v}$. Hence the limit is ∞.

On the other hand, define $P(v) := p(v) - p'(v) \cdot v$.

The tangency equation at v is now given by $y = P(v) + p'(v) \cdot x$.

For $v \to \infty$, the tangency approaches the asymptote $y = \bar{p}$. Hence $\lim_{v \uparrow \infty} P(v) = \bar{p}$. From this it follows that

$$\varepsilon(\infty) := \lim_{v \uparrow \infty} \frac{p'(v) \cdot v}{p(v)} = \lim_{v \uparrow \infty} \frac{p(v) - P(v)}{p(v)} = \frac{1}{\bar{p}} \lim_{v \uparrow \infty} \left(p(v) - P(v) \right) = 0$$

This will be assumed in the following[15]:

(12) $$\varepsilon'(v) < 0 \text{ for all } v > \underline{v}; \quad \varepsilon(\underline{v}) = \infty, \ \varepsilon(\infty) = 0$$

(13) $$\eta'(w) < 0 \text{ for all } w > \underline{w}; \ \eta(\underline{w}) = \infty, \ \eta(\infty) = 0 .$$

Assumptions (12) and (13) can be viewed as imposing a law of decreasing returns on expected pay: Expected pay in one period for a group 1 worker is $p \cdot v$. The percentage response of expected pay to a one percent increase in the wage offer is $\varepsilon + 1$. Assumption (12) states that this response is decreasing with increasing wage offers. Similarly, assumption (13) states that the response of expected pay to a group 2 worker is decreasing in the same sense.

6. *Short-Run Equilibrium Conditions*

The typical firm will try to maximize the present value of the job under consideration by choosing a suitable wage offer (v, w), thereby taking into account that the access probability of each group will be affected by the offer made to that group. In other words: (8) and (9) are inserted into (4), and this expression is maximized with respect to v and w:

(14) $$M(v, w; m) = \frac{m\left(p(v) + q(w)\right) - p(v) \cdot v - q(w) \cdot w}{r + p(v) + q(w)} = \max_{v, w \geq 0}! \ .$$

15 This is not a restriction on its steepness but simply a restriction on its general shape as can be seen from the fact that the functions (*) and (**) mentioned in note 13 above fulfill this requirement for all admissible parameter values and in particular for arbitrarily high values of the steepness parameter b.

Assumptions (12), (13) are made mainly in order to enhance the ease of exposition. For all results of the paper, the following far less restrictive, but less intuitive, assumptions on the elasticities of the partial derivatives would be sufficient:

(12′) $$\frac{p''(v) \cdot v}{p'(v)} < 2 \cdot \varepsilon(v)$$

(13′) $$\frac{q''(w) \cdot w}{q'(w)} < 2 \cdot \eta(w) .$$

Note that conditions (12′), (13′) are always fulfilled for $p''(v) < 0$, $q''(w) < 0$. Since $\varepsilon(\underline{v}) = \infty$ and $\eta(\underline{w}) = \infty$, $\varepsilon' < 0$ and $\eta' < 0$ must hold true close to $\underline{v}$ and $\underline{w}$, respectively. On the other hand,

$$\varepsilon' < 0 \text{ implies } \frac{p''v}{p'} < \varepsilon - 1 \text{ and hence (12′)}$$
$$\eta' < 0 \text{ implies } \frac{q''w}{q'} < \eta - 1 \text{ and hence (13′) .}$$

Therefore, (12′) and (13′) will always be satisfied close to $\underline{v}$ and $\underline{w}$, respectively: Conditions (12′) and (13′) put restrictions only on the convex part of the probability functions.

Since $M \to -\infty$ for $v \to \infty$ or $w \to \infty$, there exists a maximum of M[16]. Denote this maximum by M^*. Since $M(0,0;m)=0$, it is nonnegative:

$$(15) \qquad M^* := \max_{v,w} M(v,w;m) \geq 0 \ .$$

We will be interested only in solutions which yield positive access probabilities for each group. (If both access probabilities were zero, there would be no unemployment, and hence no discrimination; if one access probability were positive and the other zero, there would be segregation but not discrimination.) Hence we are interested only in cases where the necessary conditions for an inner maximum are fulfilled:

$$(16) \qquad v\left(1+\frac{1}{\varepsilon(v)}\right) = m - M^* \ ,$$

$$(17) \qquad w\left(1+\frac{1}{\eta(w)}\right) = m - M^* \ .$$

Since the elasticities are, according to (12), (13), decreasing functions, the left-hand sides are strictly increasing in v and w, respectively. Hence the maximum will be unique[17].

Conditions (16) and (17) imply

$$(18) \qquad \frac{v}{w} = \frac{1+1/\eta(w)}{1+1/\varepsilon(v)} \ .$$

This expression is algebraically identical to the monopolistic competition formula (1), although the interpretation of the elasticities is different. Guided by this analogy, one might say that the group with a more elastic supply will get a higher wage rate. This is not quite correct since the elasticities themselves are determined by the wage offers. The conjecture, however, leads to the following proposition.

Proposition 1: Assume v, w to be an equilibrium. If $\varepsilon(v) > \eta(v)$ or $\varepsilon(w) > \eta(w)$ or both, this implies $v > w$. In other words, if both groups were made the same wage offer v or w and one group had a higher elasticity at this offer, it would get the higher wage in equilibrium.

Proof: 1. Assume $\varepsilon(v) > \eta(v)$. This implies

$$v(1+1/\varepsilon(v)) < v(1+1/\eta(v)) \ .$$

According to (16) and (17), equilibrium requires

$$v(1+1/\varepsilon(v)) = w(1+1/\eta(w)) \ .$$

[16] Since $M < (m-(pv+qw)/(p+q))$ and since $p \to \bar{p}$ for $v \to \infty$ and $q \to \bar{q}$ for $w \to \infty$, the bracket tends to minus infinity if either $v \to \infty$ or $w \to \infty$ or both. Note that M is continuous and well defined for all nonnegative (v, w).

[17] I.e. if there exists an inner maximum, it will be a unique maximum. Since the existence of a maximum has been established, it is not necessary to look at the second-order conditions which happen to be (12'), (13') and are implied by (12), (13).

Since (13) says that η is strictly decreasing, the right-hand side is strictly increasing in w. Hence $v > w$ follows.

2. Assume $\varepsilon(w) > \eta(w)$. This implies

$$w(1+1/\varepsilon(w)) < w(1+1/\eta(w)) \ .$$

Since (12) says that ε is strictly decreasing, the equilibrium condition $v(1+1/\varepsilon(v)) = w(1+1/\eta(w))$ can only hold true for $v > w$. This proves the proposition.

Proposition 1 implies that $\varepsilon(v) > \eta(w)$ for all identical wage offers $v = w$ will lead to discrimination of group 2, i.e. to $v > w$.

7. *Some Applications of the Short-Run Equilibrium Conditions*

Proposition 1 might shed light on some issues:

1. Social determinants of discrimination. Take for instance the case that the two groups have different work attitudes for exogenous social reasons. It might be that women respond less to changes in wage offers since they place more weight on work attributes like proximity and comfortable working hours in order to meet their customary duties as housewives, whereas men tie their self-esteem more to remuneration and are more mobile. Under these conditions the access elasticity of males would be higher than that of women if both groups were offered identical wages. According to proposition 1 this would lead to a lower pay for women. Firms would pay less to women than to males since they knew that they would get them for lower pay, and their willingness to accept an offer would not increase sufficiently if the wage offer were increased. Hence it would not be worth while for the firm to offer higher wages.

2. Restricted opportunities and discrimination. The access elasticities, as faced by the typical firm, will be affected by the scope of employment and wage opportunities open to the two groups: If there is a lack of alternatives for members of one group, this will restrict their access elasticity. If the alternatives are very unattractive with regard to remuneration, i.e. if discrimination prevails in the rest of the economy, this will have the same effect of reducing access elasticities, but only in the short run. (We shall see later that this kind of discrimination will be eliminated in the long run.)

3. Minority status and discrimination. If we consider two groups of identical behavior, where group 2 is a minority comprising only a (positive) fraction $\mu < 1$ of the number of group 1 workers, this will not lead to discrimination: Since both groups behave identically, we have $q(w) = \mu \cdot p(v)$ for all identical wage offers $w = v$. This implies that $\varepsilon(v) = \eta(w)$ for all $v = w$ and hence that the equilibrium condition $v(1+1/\varepsilon(v)) = w(1+1/\eta(w))$ can only be achieved without discrimination, i.e. at $v = w$. (Remember that $w(1+1/\eta(w))$ is strictly increasing due to (13).)

Part 2: Long-Run Analysis

8. Determinants of Access Probabilities

The access probability functions as introduced in section 5 above are faced by the representative firm *ceteris paribus*. The *ceteris paribus* clause entails, in particular, that other firms do not change their wage offers and their labor demand. In order to study the properties of market equilibrium, these determinants have to be given explicit consideration now.

We write the access probability p of group 1, then, as

(19) $$p = \lambda \cdot f(v/\alpha) \qquad \alpha > 0, \quad \lambda \in (0,1) \ .$$

The function f is assumed to have the same qualitative properties as the access probability function $p(v)$ described in (8)[18]. The function $p(v)$ can be viewed as being generated by (19) for fixed values of the parameters α and λ. The parameter α represents the *aspiration level of group 1 workers*: The higher it is, the higher has to be the wage offer to group 1 workers in order to induce a given access probability. The aspiration level will be viewed as being determined by the general wage level for group 1 workers prevailing in the economy. The parameter λ represents the scarcity of group 1 workers: It is the *scarcity parameter for group 1*. The larger the demand for group 1 workers, the smaller will be the share of type 1 applicants to the representative firm. This can be described by a low value of λ[19].

Similarly, we consider the access probability q of group 2 as being determined not only by the wage offer w but also by the *aspiration level β for group 2 workers* and of the *scarcity parameter μ of group 2*:

(20) $$q = \mu \cdot g(w/\beta) \qquad \beta > 0, \quad \mu \in (0,1) \ .$$

Again, the *ceteris paribus* access probability function $q(w)$ can be viewed as being generated by (20) for given parameter values β and μ. Hence $g(\cdot)$ satisfies the qualitative properties postulated for $q(\cdot)$[20].

The access elasticities associated with (19) and (20) turn out to be independent of the scarcity parameters λ and μ. Call them $\tilde{\varepsilon}$ and $\tilde{\eta}$. Because of (12) and (13), they are strictly decreasing in the relevant region:

(21) $$\tilde{\varepsilon}(v/\alpha) := \frac{\partial p}{\partial v} \cdot \frac{v}{p} = \frac{f'}{f} \cdot \frac{v}{\alpha}, \qquad \tilde{\varepsilon}' < 0$$

(22) $$\tilde{\eta}(w/\beta) := \frac{\partial q}{\partial w} \cdot \frac{w}{q} = \frac{g'}{g} \cdot \frac{w}{\beta}, \qquad \tilde{\eta}' < 0 \ .$$

[18] I.e. take one function $p(\cdot)$ satisfying (8), (12), and $\underline{v} < 1$ and call it $f(\cdot)$.

[19] The more general formulation of (19) as $p = f(v; \alpha, \lambda)$ would complicate the analysis unduly. More specific alternatives like $p = \lambda \cdot f(v - \alpha)$ would not affect the qualitative results. There are good psychological reasons, however, to assume that the ratio v/α is relevant since there exists no absolute standard of the wage level.

[20] I.e. take one function $q(\cdot)$ satisfying (9), (13), $\underline{w} < 1$ and call it $g(\cdot)$.

Of paricular interest are the elasticities at wage offers which are equal to the aspiration level, i.e. at $v/\alpha = w/\beta = 1$. Denote these by $\bar{\varepsilon}$ and $\bar{\eta}$ and call them the *long-run access elasticities*:

(23) $$\bar{\varepsilon} := \tilde{\varepsilon}(1) ,$$

(24) $$\bar{\eta} := \tilde{\eta}(1) \ .$$

Equipped with these notions and definitions we can now proceed to discuss some longer-run issues.

9. Long-Run Equilibrium Conditions

We start from short-run equilibrium. The firm realizes the maximum present value M^*/r of the job by choosing suitable wage offers. Assume now that the capital needed to create an additional job opening is K. If the difference $M^*/r - K$ is positive, the firm will create additional jobs, and other firms will, typically, do the same. Hence the scarcity parameters λ and μ in (19) and (20) will decrease, *shifting the short-run access probability functions* $p(v)$ and $q(w)$ downward. This will decrease M for all offers (v, w), as can be seen immediately from (14). Hence M^* will decrease[21].

Conversely, if the value of a job is less than the capital outlays required to establish it, i.e. if $M^*/r - K$ is negative, this will lead to depreciation and to a reduction in the number of jobs. In this way, an equilibrium will be achieved which satisfies[22]

(25) $$M^* = r \cdot K \ .$$

This turns conditions (16) and (17) into the analogues of the conventional monopolistic competition conditions

(26) $$v(1 + 1/\bar{\varepsilon}) = m - r \cdot K$$

(27) $$w(1 + /\bar{\eta}) = m - r \cdot K \ .$$

From these conditions, the impact of a change in aspiration levels can be deduced:

Proposition 2: A one percent increase in the aspiration level will increase the corresponding wage by less than one percent. Algebraically

[21] In addition, the marginal product m in (14) might decrease since the expanded production might run into decreasing returns and decreasing product prices. This would reduce M^* as well.

[22] Note that condition (25) will be satisfied whether or not pure profits have been competed away in the long-run, i.e. whether or not the "tangency solution" obtains. The argument applies also if the marginal product happens to change during the process of adaption.

$$0 < \frac{dv}{d\alpha} \cdot \frac{\alpha}{v} < 1 \tag{28}$$

$$0 < \frac{dw}{d\beta} \cdot \frac{\beta}{w} < 1 \; . \tag{29}$$

Proof: According to (21), $\tilde{\varepsilon}$ increases with α. Hence $v \cdot (1+1/\tilde{\varepsilon})$ is decreasing in α. On the other hand, this term is decreasing in v. If (26) is to be maintained, this requires $dv/d\alpha > 0$. On the other hand, $v(1+1/\tilde{\varepsilon})$ is homogenous of degree one in v and α. This implies that a proportionate increase in v and α would increase the expression. In order to keep it constant, v should increase less than proportionally. The second part of the proposition is proved analogously.

In the longer run, the aspiration levels will move in the direction of actual experience, i.e. actual pay. The rationale for this is that aspiration levels tend to correspond to actual achievements of the relevant reference group[23]. In addition, the individual firm's wage offer will be judged in the light of the prevailing market remuneration, i.e. actual pay. We can stipulate, therefore, an adaptive process which explains the relative change of aspiration levels by the relative difference between actual pay and the relevant aspiration level,

$$\hat{\alpha} = \sigma_1 \cdot \frac{v-\alpha}{\alpha} \, , \qquad \hat{\alpha} := \frac{d\alpha}{dt} \cdot \frac{1}{\alpha} \, , \qquad \sigma_1 > 0 \tag{30}$$

$$\hat{\beta} = \sigma_2 \cdot \frac{w-\beta}{\beta} \, , \qquad \hat{\beta} := \frac{d\beta}{dt} \cdot \frac{1}{\beta} \, , \qquad \sigma_2 > 0 \; . \tag{31}$$

Together with (26), (30) describes an adjustment process in the aspiration level of group 1 which tends to a long-run equilibrium value equal to actual pay for group 1. This can be seen as follows: If $v > \alpha$, this will increase α according to (30). From (28) we know that this increase in α increases v less than proportionally. Hence v/α will decline. This will continue until $v/\alpha = 1$ and $\hat{\alpha} = 0$. Conversely, $v < \alpha$ will reduce α and will increase v/α until $v/\alpha = 1$ is reached. So we can write down as a long-run equilibrium condition that the wage equals the aspiration level of group 1,

$$\alpha = v \; . \tag{32}$$

The same argument holds true for group 2[24]:

$$\beta = w \; . \tag{33}$$

These long-run equilibrium conditions can be inserted into (26) and (27). This amounts to substituting the access elasticities $\tilde{\varepsilon}$ and $\tilde{\eta}$ by the long run access elasticities $\bar{\varepsilon}$ and $\bar{\eta}$ as defined in (23) and (24). Hence we get explicit formulae for the determination of long-run remuneration:

[23] See the references given in SCHLICHT [1981].

[24] Note that the process remains stable, and leads to conditions (32) and (33) if the marginal product is – due to an optimal firm policy – an increasing function of the wage rates: This will render only the left-hand sides of the inequalities (28) and (29) invalid.

$$v = \frac{\bar{\varepsilon}}{1+\bar{\varepsilon}} \cdot (m - rK) \tag{34}$$

$$w = \frac{\bar{\eta}}{1+\bar{\eta}} \cdot (m - rK) \; . \tag{35}$$

The wage differential will simply be determined by the long-run access elasticities (which are behavioral constants):

$$\frac{v}{w} = \frac{1+1/\bar{\eta}}{1+1/\bar{\varepsilon}} \; . \tag{36}$$

This enables us to state directly the long-run analogue to proposition 1:

Proposition 3: In the long run, the group with the higher long-run access elasticity will receive the higher wage.

10. Some Applications of the Long-Run Equilibrium Conditions

1. Social determinants of discrimination. The argument put forward in the context of the short-run analysis (section 7.1) still applies in the long-run: If monetary incentives are more effective for one group than for the other, the favored group will get higher pay. In addition, we can evaluate numerically the resulting wage differential v/w: For instance, if males have a long-run access elasticity of $\bar{\varepsilon} = 1$ (i.e. if their access probability increased by one percent if remuneration were increased by one percent above the aspiration level) and if women have a lower long-run elasticity of, say, $\bar{\eta} = 0.5$ for reasons mentioned before, this would induce a remuneration of women which is one-third below that of males, i.e. $v/w = 3/2$. Competition will not erode this differential.

2. Consequences of equal-pay enforcement. If equal pay is enforced legally, expression (14) is to be maximized under the assumption that a common wage rate u is paid to both groups, i.e. $v = w = u$. This yields the necessary condition

$$u\left(1 + \frac{1}{e}\right) = m - M^* \, , \tag{37}$$

where the access elasticity e referring to both groups is defined as

$$e = \frac{p}{p+q} \cdot \varepsilon + \frac{q}{p+q} \cdot \eta \; . \tag{38}$$

Using the long-run equilibrium conditions (25), (32), (33), condition (37) turns into

$$u = \frac{\bar{e}}{1+\bar{e}} (m - rK) \tag{39}$$

where the long-run access elasticity $\bar{e}$ is a weighted average of the long-run access elasticities $\bar{\varepsilon}$ and $\bar{\eta}$

(40) $$\bar{e} = \sigma\bar{\varepsilon} + (1-\sigma)\,\bar{\eta} \qquad 0 < \sigma < 1 \ .$$

The weight σ denotes the fraction of type 1 workers in the work force of the typical firm. It is determined by the scarcity parameters λ and μ which emerge in the long run. If (39) is compared with (34) and (35), it can be seen that the nondiscriminating wage rate will lie in between the wage rates emerging under discrimination, i.e.

(41) $$v > u > w \quad \text{for} \quad v > w \ .$$

Since the expression $1/(1+1/x)$ is strictly concave in x, we get furthermore

(42) $$u > \sigma v + (1-\sigma)\,w \quad \text{for} \quad v \neq w$$

in long run equilibrium. This means that the average wage will increase from $\sigma v + (1-\sigma)\,w$ to u if an anti-discrimination law is introduced[25].

Furthermore, it can be seen from (4) that M will decrease if either p or q is put to zero and $v = w$ holds. Hence it will not be profitable for the firm to hire only one type of workers: An equal-pay law will not lead to job discrimination or segregation.

3. Restricted opportunity and discrimination. Again, the group with restricted opportunities will supply with a lower long run elasticity and will suffer discrimination. Thus the short run argument put forward in section 7.2 above will remain valid.

However, the argument that there will be discrimination within one firm because there is discrimination in the rest of the economy will not hold true in the long run: Assume that in the short run there has been discrimination, and aspiration levels have adapted to that, i.e. assume $\alpha > \beta$. Assume in addition that both groups are identical in the sense that the functions f and g as introduced in (19) and (20) are identical. Because of proposition 2, there will be discrimination in the short run, but the wage differential v/w will be smaller than the ratio of aspiration levels α/β. Aspiration levels will adapt to the new situation. In the long run, discrimination will disappear according to (36), since $\bar{\varepsilon}$ and $\bar{\eta}$, as defined by (21)–(24), are identical.

4. Regional discrimination. As a slight modification of the model under consideration, one might consider the case of two identical firms, one situated in an industrial region and the other in an underdeveloped district. The workers of firm 1 – call them group 1 workers – have many employment alternatives. The workers of firm 2 live in a scarcely populated region with only a few job opportunities within a reasonable distance. Hence group 2 workers will have a

[25] This proposition is derived under the assumption that the fraction of type 1 workers σ is not changed by such a move in the long run, i.e. that the ratio λ/μ is not affected. If σ actually decreases, this might cancel the effect or even overcompensate it.

lower access elasticity than group 1 workers. Assume that $q(w) \equiv 0$ for firm 1 and $p(v) \equiv 0$ for firm 2, i.e. that each firm can draw applicants only from the relevant pool. The maximand (4) for firm 1 now becomes

(43) $$M_1 = \frac{p(v)}{r + p(v)} \cdot (m - v)$$

The maximand for firm 2 is

(44) $$M_2 = \frac{q(w)}{r + q(w)} \cdot (m - v) \ .$$

The necessary conditions for a maximum are[26]

(45) $$v\,(1 + 1/\varepsilon(v)) = m - M_1^*$$

(46) $$w\,(1 + 1/\eta(w)) = m - M_2^* \ .$$

In the long run, we have to assume $M_1^* = r \cdot K$ and $M_2^* = r \cdot K$ as the analogues to (25), and have to replace the elasticities ε and η by their long run values $\bar{\varepsilon}$ and $\bar{\eta}$ to capture the adaption of aspiration levels. Hence formulae (34)–(36) remain valid and the workers in the industrialized region will receive a higher income than the workers in the less-developed region. Note that these differentials are not eroded by competition since $M_1^* = M_2^* = r \cdot K$ is fulfilled: The lower wage prevailing in the less-developed region is assumed to have expanded the number of jobs and of firms, thereby increasing vacancies up to the point where the value of an additional job and of an additional firm is equal to the capital outlays necessary to create it[27].

The argument would require, therefore, a labor market which is, in a sense, tighter in the less-industrialized region than in the industrialized one. As it stands, it cannot explain the often-observed presence of both unemployment and low wages in the less-industrialized region. Note, however, that marginal products might be affected by the scarcity in kinds of labor outside of our consideration, where the scarcity argument might apply. These scarce factors of production might induce differentials in the marginal products of the workers under consideration between the two regions. Thus we might be led to consider the impact of productivity differentials on remuneration in the context of regional discrimination.

The issue transcends, however, the purely regional context and will be discussed in the following section.

[26] The second order conditions are satisfied. M_1^* denotes the maximum of M_1 and M_2^* the maximum of M_1, analogous to (15).
[27] See the argument leading to (25) above.

11. *Levelling and Boosting of Productivity Differentials*

Leaving the context of discrimination proper it might be asked: What are the effects of productivity differentials between the workers of the two groups on relative remuneration?

The question, although often unconsidered in the theory of discrimination (which deals by definition with differential treatment of essentially identical subjects), might be a very important one if it turns out, for instance, that minor productivity differentials cause very large wage differentials. We will refer to this as *boosting*. Take the case that women are slightly less productive than men because of pregnancy, child rearing, and induced slightly higher adjustment and turnover costs on the side of the firms. Will this slight productivity differential be boosted into a very large wage differential between males and females? Or, vice versa, will *levelling* take place in that resulting wage differentials are smaller than productivity differentials?

Assume, then, that the marginal product of a group 1 worker m_1 is larger than that of a group 2 worker m_2. The expected return of a job is now

$$\frac{p \cdot (m_1 - v) + q\,(m_2 - w)}{p+q} \quad (47)$$

This can replace the expression $(m-(pv+qw)/(p+q))$ in formula (2), and this expression can be placed in front of the summation. Hence the geometrical series remains unaffected and we get the maximand

$$M = \frac{p\,(m_1 - v) + q\,(m_2 - w)}{r+p+q} \quad (48)$$

which replaces (4). The short run equilibrium conditions now become

$$v \cdot (1 + 1/\varepsilon(v)) = m_1 - M^* , \quad (49)$$

$$w \cdot (1 + 1/\eta(w)) = m_2 - M^* . \quad (50)$$

If both groups behave alike, i.e. if $\varepsilon(v) = \eta(w)$ for all identical wage offers $v = w$, a higher marginal product of group 1 will cause a higher wage rate for that group. It turns out, however, that the resulting wage differential v/w will be less than the ratio of $(m_1 - M^*)/(m_2 - M^*)$[28]. If the number of jobs is adjusted optimally such that $M^* = rK$, there can be levelling or boosting, depending on the shape of the access elasticity function.

The long-run equilibrium conditions (25), (34), and (35) turn into

$$M^* = r \cdot K \quad (51)$$

$$v = \frac{\bar{\varepsilon}}{1+\bar{\varepsilon}} \cdot (m_1 - rK) \quad (52)$$

[28] This is so because the left-hand sides of (49), (50), which are identical functions in the case under discussion, are increasing more than proportionally in the wage offer.

$$w = \frac{\bar{\eta}}{1+\bar{\eta}} \cdot (m_2 - rK) . \quad (53)$$

The underlying argument remains the same. Hence, if both groups behave alike, we have $\bar{\varepsilon} = \bar{\eta}$ and there will be boosting in the long run: The long run wage differential will exceed the ratio of the marginal products:

$$\frac{v}{w} = \frac{m_1 - rK}{m_2 - rK} > \frac{m_1}{m_2} \quad \text{for} \quad m_1 > m_2 . \quad (54)$$

There might, additionally, be a strong "social" boosting effect if there is social interdependence between the groups and the long run elasticities are affected by wages paid to the members of the other group: If women are slighly less productive and if a couple has to decide about who to earn money and who to rear the children, this will favor men looking for employment, and thus induce the expectation in males and females that men will take a job with a higher probability. This expectation might crystallize in social rôles which tie women more closely to the home than men, inducing a decrease in their access elasticity. This will increase the wage differential and reinforce the custom, increasing the wage differential even more, etc. We might call this a *social* boosting effect since it works through social channels, even if the family, as we know it, might have its ultimate cause in economic pressures. This question need not be decided here.

12. Consequences of equal-pay enforcement. Denote as in section 10.2, the ratio of type 1 workers in the work force by σ. The necessary condition for maximum of (48) under the restriction $v = w = u$ (where u denotes the non-discriminating wage rate) turns into

$$u = \frac{\bar{e}}{1+\bar{e}}\,(\bar{m} - rK) \quad (55)$$

instead of (52) and (53), where $\bar{e}$ denotes the common access elasticity and $\bar{m}$ is average productivity:

$$\bar{e} = \bar{\varepsilon} = \bar{\eta}, \quad \bar{m} = \sigma m_1 + (1-\sigma)\,m_2 . \quad (56)$$

Hence, it turns out that the nondiscriminating wage is in the long run exactly equal to the average wage paid under discrimination[29].

The question arises, however, whether equal-pay enforcement might lead to job discrimination or segregation in the sense that it will become profitable for the firm to employ only one type of workers.

In order to decide this, the firm has to compare – for given aspiration levels and scarcity parameters – the maximum value of the job attainable by employing both groups at a uniform wage with the maximum value which can

[29] *The qualification made in note 25 above applies here, too.*

be obtained by employing only the more productive type 1 workers. Hence the anti-discrimination law will work, if the following expression is positive, and it will lead to segregation (job discrimination) if it is negative

$$\max_u (\bar{m} - u) \frac{p(u) + q(u)}{r + p(u) + q(u)} - \max_v (m_1 - v) \frac{p(v)}{r + p(v)} . \tag{57}$$

From this it can be seen that both cases are possible: If the discount factor r is sufficienty small, the expression will become negative for a productivity m_1 of group 1 which exceeds the average productivity $\bar{m}$. Hence the firm will employ only type 1 workers in this case. On the other hand, if productivity differentials are small, m_1 will be close to $\bar{m}$. For a sufficiently high discount rate, the firm will employ members of both groups on equal terms[30].

12. Concluding Remarks

The reformulation of Joan Robinson's theory of discrimination in terms of search theory might supplement the existing approaches. It has less difficulties in explaining persistent discrimination, thereby reinforcing the argument of the other approaches: Discrimination might lead, through habit formation, to a taste for discrimination as well as to productivity differentials in the "indicator" sense.

Zusammenfassung

Eine Theorie der Lohndiskriminierung à la Joan Robinson

Bei der Festsetzung des Lohnsatzes für eine bestimmte Tätigkeit wird die Unternehmung den Lohnsatz weder unnötig hoch wählen noch so gering, daß es unwahrscheinlich wird, eine geeignete Arbeitskraft zu finden. So werden die Lohnsätze unter Berücksichtigung ihres Einflusses auf die Wahrscheinlichkeit einer erfolgreichen Stellenbesetzung optimal fixiert.

Wenn Indikatoren wie Geschlecht oder Rasse typischerweise mit unterschiedlichem Angebotsverhalten verknüpft sind, wird die Lohnsetzung der Unternehmung zu Diskriminierung gemäß diesen Indikatoren führen.

Der Ansatz kann als eine moderne Neuformulierung der Diskriminierungstheorie von Joan Robinson verstanden werden. Er führt zu mathematisch identischen Gleichgewichtsbedingungen, allerdings mit abweichender Interpretation und unter Vermeidung einiger Schwierigkeiten des ursprünglichen

[30] A more exact statement of the underlying mathematical conditions is beyond the scope of the present paper.

Ansatzes. Innerhalb des entwickelten Rahmens läßt sich die Möglichkeit langfristig anhaltender Diskriminierung aufweisen und so ein Beitrag zur Erklärung eines Phänomens leisten, das für andere Ansätze (Joan Robinson's eingeschlossen) gewisse Schwierigkeiten bietet.

References

AKERLOF, G. [1976], "The Economics of Caste and of the Rat Race and Other Woeful Tales", *Quarterly Journal of Economics*, 90, 599–617.

ARROW, K.J. [1972a], "Models of Job Discrimination"; pp. 83–102, in: A.H. Pascal (ed.), *Racial Discrimination in Economic Life*, Lexington.

-- [1972b], "Some Mathematical Models of Race Discrimination in the Labor Market"; pp. 187–203, in: A.H. Pascal (ed.), *Racial Discrimination in Economic Life*, Lexington.

-- [1972c], "The Theory of Discrimination"; pp. 3–33, in: O. Ashenfelter and A. Rees (eds.), *Discrimination in Labor Markets*, Princeton.

BECKER, G.S. [1971], *The Economics of Discrimination*, 2nd ed., Chicago.

CHAMBERLIN, E.H. [1962], *The Theory of Monopolistic Competition*, Cambridge, Mass.

DOERINGER, P.S. and PIORE, M.J. [1971], *Internal Labor Markets and Manpower Analysis*, Lexington, Mass.

MADDEN, J.F. [1977], "A Spatial Theory of Sex Discrimination", *Journal of Regional Science*, 17, 369–380.

MARSHALL, R. [1974], "The Economics of Racial Discrimination: A Survey", *Journal of Economic Literature*, 12, 849–871.

PHELPS, E.S. [1972], "The Statistical Theory of Racism and Sexism", *American Economic Review*, 62, 659–661.

-- and WINTER, S. [1970], "Optimal Price Policy under Atomistic Competition"; pp. 309–337, in: E.S. Phelps (ed.), *Microeconomic Foundations of Employment and Inflation Theory*, New York.

ROBINSON, J. [1969], *The Economics of Imperfect Competition*, 2nd ed., London.

SCHLICHT, E. [1981], "Reference Group Behaviour and Economic Incentives", *Zeitschrift für die gesamte Staatswissenschaft/Journal of Institutional and Theoretical Economics*, 137, 125–127.

STIGLITZ, J.E. [1973], "Approaches to the Economics of Discrimination", *American Economic Review*, 63, 287–295.

THUROW, L. [1975], *Generating Inequality*, New York.

Professor Dr. Ekkehart Schlicht
Technische Hochschule Darmstadt
Fachgebiet Wirtschaftstheorie
Residenzschloß
D-6100 Darmstadt
Bundesrepublik Deutschland