Balancing performance measures when agents behave competitively in an environment with technological interdependencies

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Overview

- Technological interdependencies are a major characteristic in many decentralized firms. They represent situations, where agents are supposed to work together or where production externalities impose consequences of own efforts on other division results.

- Previous studies in agency theory have examined incentive compatible wage compensation systems in the presence of technological interdependencies focusing on the conflict between cooperation and competition. They rely on the assumption of pure egoism.

- Recently, experimentalists, psychologists and others have shown, that this assumption does not hold under all circumstances. Instead, social preferences are often also of concern. This particularly applies to work environments where agents are expected to work together due to technological interdependencies.

- This study enriches a principal-multi-agent model with technological and stochastic interdependencies by a generalized utility conception. We focus on competitive preferences and examine how the weighting and the combination of performance measures are affected.

- We find that the principal reacts to his agents' rivalry through a reallocation of incentive intensity. Thus, various differences in the balancing of performance measures arise. We further show that the principal does not want both of his agents to behave equally competitively. Instead, he can only profit when the agents are asymmetrical. Then the principal wants the more productive agent to exhibit rivalry while the other ideally should behave completely egoistically.

1 Motivation

This paper addresses the question, what metrics should be used for performance evaluation and in particular how they should be weighted and combined in the presence of technological interdependencies when the agents exhibit variedly strong developed rivalry.[1] It is further examined, whether the principal can exploit his managers' striving for personal advancement to further his own merits and how these results depend on the underlying work environment.

Technological interdependencies are a constitutive element in many decentralized companies. Increasing complexity of technological processes, time pressure and rising customer demands on the one side as well as the need for specialized know-how on the other cause numerous mutual intra-firm dependencies, making cooperation between employees an indispensable success factor. Thus, teamwork has received increasing attention over the last few years. At the same time monetary incentives are considered to be of crucial importance as an instrument of employee motivation in order to guarantee for goal congruent behavior.[2] Therefore, many firms nowadays employ contracts using performance-related components as an effort enhancing device. In particular, piece rates, where agents participate in certain performance indicators have received growing attention. However, if only individual division results were used as performance measures, this would enhance departmental self-interests and therefore contradict the necessity of trans-divisional cooperation. Thus, a positive combination of individual metrics called *team based compensation* can prove to be superior. A major drawback is the agents' higher exposure to risk, mainly if their division results are affected by common factors outside of the firm like the overall business cycle. In this case a negative combination of performance measures called *relative performance evaluation*[3] may be advantageous.[4]

The principal's decision of whether to *induce cooperation* or *competition* does not depend on these external factors alone. Incentives have the purpose of affecting people's efforts. Therefore they need to take their receivers' diverse motivations into account in order to be effective. Teamwork due to technological interdependencies makes the social component of human interaction come to the fore since employees mutually affect their division results and usually work closely together for longer periods so that they are able to control and compare themselves with each other. Recently, it has been substantiated by experimental econometricians[5], psychologists and neuro scientists[6], that *social preferences* primarily determine behavior in such personal face-to-face interactions, that occur on a regular basis,[7] where no complete contracts can be written to govern people's actions.[8] Most notably, *rivalry*[9] seems to be of high practical relevance as can be seen from the current discussion on wage justice in Germany. Furthermore, the secrecy of remuneration and its composition in many industrial enterprises indicates, that the executive board fears counterproductive effects, if agents have the opportunity to compare their compensation.

Our paper shifts the discussion on the field of „Behavioral Contract Theory“ towards an accounting perspective of how to balance performance measures in a situation with heterogeneous competitive agents under various circumstances. As we will show below, the principal reacts to his agents' social preferences with a reallocation of incentive intensity through adjusting the relative weight of performance measures in their respective wage compensation systems. Depending on the established trade-off between the opposing external effects as well as the comparative strength of rivalry, changes in the decision between cooperation and competition inducing incentive systems can occur. This paper's main contribution is to provide a thorough understanding of the underlying interactions of the considered types of interdependencies with their motivational consequences. Building on these results, we establish, that the principal profits from his agents' competitive preferences, if only the higher productive agent exhibits rivalry while the other behaves preferably egoistically. This result holds independent of the underlying production technology, as long as separate performance measures for each of the agents are available.

The remainder of the paper is organized as follows. *Section 2* briefly reviews the related literature. *Section 3* describes the model with its basic assumptions. *Sections 4, 5 and 6* present the main results of our theoretical analysis. The impacts of rivalry on the weighting and the combination of performance measures are separately discussed for two forms of technological interdependencies. *Section 4* considers a situation with productive interaction, in *section 5* the case of a production externality is examined. Finally, the impacts of rivalry on firm profitability are considered in *section 6*. *Section 7* concludes.

2 Related literature

Previous papers in the field of „Behavioral Contract Theory“ have broadly studied the single-agent case where one agent compares himself to the principal.[10] In addition to that, horizontally directed social preferences have been incorporated in principal-multi-agent models. Rey Biel (2007) examines the impact of inequity aversion[11] under the assumption of complete certainty and finds, that in case of team production social comparisons between agents have an intrinsic motivation effect. Itoh (2004) confirms these results in a situation of uncertainty and generalizes them to the case of competitive preferences. Additionally, he compares team based compensation and relative performance evaluation in dependence of the degree of uncertainty as well as the agents' personnel characteristics. However, he does not allow for asymmetries between the agents and neglects technological dependencies. Both authors rely on the assumption of limited liability. In contrast to their results, Bartling/von Siemens (2005) show, that the principal cannot make use of his agents' inequity aversion, if the limited liability constraints are not imposed. They further prove that the „Sufficient Statistics Result“[12] loses validity when the agents exhibit social preferences, since the principal then tends to provide more equal forms of remuneration.[13] Similar results under the assumption of linear incentive contracts are achieved by Dierkes/Harreiter (2006) and Neilson/Stowe (2008).[14] In contrast to these studies we take an accounting perspective and focus on the balancing of performance measures in presence of social preferences. In particular we examine how the trade-off between team based payment and relative performance evaluation is influenced by two-sided rivalry in a situation of technological and stochastic interdependencies. This is distinguishable from the case of team production in the sense of Alchian/Demsetz (1972), which was previously considered by Rob/Zemsky (2002), Bartling/von Siemens (2004) and Huck/Kübler/Weibull (2006).

3 Model description

For our theoretical analysis we employ a principal-multi-agent LEN model[15] with technological interdependencies as introduced by Itoh (1992) as well as Choi (1993) and enrich their specifications by a generalized utility conception accounting for social preferences between agents on the same horizontal layer. We consider two agents denoted by A and B, each of whom is responsible for the profits of the decentralized unit he is presiding. Let x_1 be the performance metric of agent A and x_2 be that of agent B.[16] Each of the performance measures is linear contingent on the agents' actions a_i , b_i as well as an additively linked error term ϵ_1 and ϵ_2. The error terms are assumed to be normally distributed with zero mean and variance σ_i^2. Their correlation is denoted by ϱ. By means of the underlying production technology, two cases of technological interdependencies are differentiated in the subsequent analysis: (1) The first production technology of the form

$$x_1 = p_{A1}a_1 + p_{B1}b_1 + \epsilon_1 \qquad ; \qquad x_2 = p_{A2}a_2 + p_{B2}b_2 + \epsilon_2, \tag{1}$$

describes a situation where each of the agents cannot only influence his own performance measure but also, through specific actions, the one of the respective other agent. The effort parameters a_2 and b_1 denote those actions of agent A (B) that precipitate in agent B's (A's) division result. Positive values are interpreted as help, negative values as sabotage. The parameters p_{A1} and p_{A2} as well as p_{B1} and p_{B2} in equation 1 reflect differential productivities of the two agents with respect to their various activities. (2) The second production technology has the form

$$x_1 = a_1 + p_{B1}b_2 + \epsilon_1 \qquad ; \qquad x_2 = b_2 + p_{A2}a_1 + \epsilon_2, \tag{2}$$

and represents a situation, in which both agents' effort choices not only contribute to their own margins but are also automatically reflected in the division profits of the respective other. The parameters p_{B1} and p_{A2} in equation 2 are measures for the strength of the production externality between the two decentralized divisions and are restricted to lie in the interval $p_{B1}, p_{A2} \in [0,1]$. For the sake of semantical clearness, the subsequent analysis refers to the production technology in equation 2 as production technology with a production externality. The wording production technology with productive interaction however identifies a production technology of the type given in equation 1.

Further differentiation is possible with respect to the values of parameter ϱ. For $\varrho = 0$ the production technology is called stochastically independent while for $\varrho \neq 0$ the two divisions are also interrelated through stochastic interdependencies. In the latter case, the absolute value of the correlation coefficient ϱ is a measure for the strength of the stochastic interdependencies and restricted to lie in the interval $\varrho \in [0;1]$.[17] For the sake of generality, we assume $\varrho > 0$ throughout our analysis.

Effort costs are assumed to be quadratic in the agents' actions and modeled by the use of the general function

$$V_A(a_1,a_2) = \frac{1}{2}c_{A1}a_1^2 + \frac{1}{2}c_{A2}a_2^2 \quad ; \quad V_B(b_1,b_2) = \frac{1}{2}c_{B1}b_1^2 + \frac{1}{2}c_{B2}b_2^2, \tag{3}$$

which, in the case of the production technology in equation 2 reduces to:

$$V_A(a_1) = \frac{1}{2}c_{A1}a_1^2 \quad ; \quad V_B(b_2) = \frac{1}{2}c_{B2}b_2^2. \tag{4}$$

The parameters $c_{A1}, c_{A2}, c_{B1}, c_{B2}$ are again weighting factors, describing task specific differences in the strength of perceived disutility. Both agents are compensated by a linear incentive scheme[18] of the form

$$S_A = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 \quad ; \quad S_B = \beta_0 + \beta_1 x_1 + \beta_2 x_2, \tag{5}$$

where α_0, β_0 identify the fixed and $\alpha_1, \beta_1, \alpha_2, \beta_2$ the variable wage compensation components. Therefore, both agents' remuneration does not only depend on their own performance measure, but also on the one of the respective other agent. Through the values of parameters α_2 and β_1 various types of compensation are differentiated:

a) $\alpha_2 = 0$; $\beta_1 = 0$ (Individual Compensation): Solely the own performance measure determines wage compensation.

b) $\alpha_2 < 0$; $\beta_1 < 0$ (Relative Performance Evaluation): Own remuneration is negatively influenced by a better result of the other agent leading to a competitive relationship between the decentralized divisions.

c) $\alpha_2 > 0$; $\beta_1 > 0$: In this case two subcases have to be differentiated depending on the signs of α_1 and β_2:

   i) $\alpha_1 > 0$; $\beta_2 > 0$ (Team Based Compensation): Own remuneration is positively influenced not only by a better own result but also by a better result of the other agent. Therefore, there are additional incentives for cooperation between the agents. The resulting form of cooperation is called induced cooperation.

ii) $\alpha_1 < 0\,;\, \beta_2 < 0$ (Relative Compensation to the own Division Result): Own remuneration is raised by higher values of the other agent's performance measure while at the same time it is alleviated for better own results.

The agents' utility functions are exponential with $r_A > 0$ and $r_B > 0$ as coefficients of absolute risk aversion. Higher values of $r_A\,, r_B$ indicate higher degrees of risk aversion. The principal by contrast is assumed to be risk neutral and therefore maximizes the residuum, consisting of the two division profits lessened by the agents' wage compensation payments:

$$U_P = E[x_1 + x_2 - S_A(\cdot) - S_B(\cdot)]. \tag{6}$$

It is further assumed that the agents can differ in their (social) preferences. Apart from individual monetary interest they also care for the well-being of the respective other. In our model we restrict the analysis to competitive preferences (rivalry) meaning that an agent endowed with this sort of social preference strives to differentiate himself from the other in terms of remuneration, independent of how total wage compensation is distributed. Formally, we represent such behavior by an additional utility component influencing the agent's effort choices. This social preference function is denoted by $G_A\left(S_A(\cdot)\,, S_B(\cdot)\right)$ as well as $G_B\left(S_A(\cdot)\,, S_B(\cdot)\right)$ and compares ex post compensation. If it takes positive values, the considered agent feels disadvantaged and suffers an additional disutility. Therefore, the principal has to heighten remuneration in order to guarantee his participation in the cooperation. By contrast, if a negative value results, the agent receives higher utility and accepts smaller remuneration. As a consequence, he works more for unchanged wage payments. In the subsequent analysis, the social preference function enters the agents' utility calculus with a negative sign and is assumed to take the following form:

$$G_A\left(S_A(\cdot)\,, S_B(\cdot)\right) = k_A(l_A S_B(\cdot) - S_A(\cdot)) \tag{7}$$

$$G_B\left(S_A(\cdot)\,, S_B(\cdot)\right) = k_B(l_B S_A(\cdot) - S_B(\cdot)). \tag{8}$$

The social preference parameter k_i denotes the strength of rivalry and can differ between the agents accounting for their possible heterogeneity ($k_A\,, k_B \geq 0$). It measures the degree of disutility resulting from an unequal distribution of remuneration. For $k_A = 0$ ($k_B = 0$) the agent behaves completely egoistically. Rising values of k_A (k_B) cause higher utility changes when wage payments are unequally distributed. Consequently, k_A and k_B are assumed to lie in the interval $k_A\,, k_B \in [0; \infty]$. The parameter l_A is agent A's aspiration level. Formally, it defines for what ratio of remuneration the effect of the social preference changes signs from utility reducing to utility enhancing. For example, if $l_A = 1$, agent A is envious and suffers a disutility if he does not earn as much as agent B. However, as soon as his remuneration surpasses the one of agent B he is spiteful and gains additional utility. For values of $l_A > 1$ there exists a region where agent A is unhappy with his own remuneration relative to agent B's despite his higher earnings since he does not even grant agent B his smaller wage compensation. For $l_A = 2$ agent A suffers a disutility if he does not at least earn twice as much as agent B. In this case he first becomes spiteful, when the ratio of remuneration $S_A : S_B$ exceeds the value l_A. Values of $l_A < 1$ however would mean that there exist distributions of income among the agents, where agent A is spiteful although his remuneration is smaller than the one of agent B. This is not very intuitive and therefore l_A is restricted to the interval $l_A \in [1; \infty]$. The same analogously holds true for agent B's aspiration level l_B.[19]

Both agents' exponential utility functions are additively separable in the described monetary as well as non-monetary components and, under the LEN assumptions, can also be written using certainty equivalent notation:

$$CE_A = E[S_A(\cdot)(1+k_A) - k_A l_A S_B(\cdot)] - V_A(\cdot) - \frac{r_A}{2}\text{Var}[S_A(\cdot)(1+k_A) - k_A l_A S_B(\cdot)] \tag{9}$$

$$CE_B = E[S_B(\cdot)(1+k_B) - k_B l_B S_A(\cdot)] - V_B(\cdot) - \frac{r_B}{2}\text{Var}[S_B(\cdot)(1+k_B) - k_B l_B S_A(\cdot)]. \tag{10}$$

$E(\cdot)$ is the expectation and $Var(\cdot)$ the variance operator. The term $\frac{r_i}{2}Var(\cdot)\,,\, i = A, B$ constitutes the risk premium, that has to be paid to the agents for imposing risk on them. It ascends in the risk aversion coefficients r_i and, everything else being positive, also in the piece rates $\alpha_1\,, \beta_1\,, \alpha_2\,, \beta_2$, the variances of the respective performance metric $\sigma_1^2\,, \sigma_2^2$ as well as the correlation coefficient ϱ. For $r_i = 0$ the agents are risk neutral and fluctuations in wage compensation are of no relevance.

4 Incentive system in case of productive interaction

4.1 Calculation of the optimal incentive system with productive interaction

Division of labor and specialization in an increasingly complex work environment give rise to situations where agents cannot make their decisions independently of one another. Instead they are expected to cooperate in the fulfillment of their tasks in order to exploit possible efficiency gains. This particularly concerns the internal transfer of technical know-how as well as the support of other divisions, if they either lack capacity or necessary specific knowledge. Such situations, where agents are supposed to work together, can best be captured by a production technology as given in equation 1.

We proceed by solving the model described above with respect to the decision parameters in case of productive interaction. The agents' *reaction functions* are obtained by differentiation of their *incentive compatibility constraints* with respect to $a_1\,, a_2$ and $b_1\,, b_2$:

$$a_1 = \frac{(1+k_A)p_{A1}\alpha_1 - k_A l_A p_{A1}\beta_1}{c_{A1}} \quad ; \quad a_2 = \frac{(1+k_A)p_{A2}\alpha_2 - k_A l_A p_{A2}\beta_2}{c_{A2}} \tag{11}$$

$$b_1 = \frac{(1+k_B)p_{B1}\beta_1 - k_B l_B p_{B1}\alpha_1}{c_{B1}} \quad ; \quad b_2 = \frac{(1+k_B)p_{B2}\beta_2 - k_B l_B p_{B2}\alpha_2}{c_{B2}}. \tag{12}$$

The *reservation utilities* are normalized to zero and the two agents' *participation constraints* bind in the optimum. Performing the typical optimization steps leads to:

Proposition 1 (Wage compensation system with rivalry in the case of productive interaction):

$$\alpha_1 = \frac{1+k_B}{1+k_B+k_B l_B} \cdot \frac{P_{A1}}{P} + \frac{k_A l_A}{1+k_A+k_A l_A} \cdot \frac{Q_{B1}}{Q} \tag{13}$$

$$\alpha_2 = \frac{1+k_B}{1+k_B+k_B l_B} \cdot \frac{P_{A2}}{P} + \frac{k_A l_A}{1+k_A+k_A l_A} \cdot \frac{Q_{B2}}{Q} \tag{14}$$

$$\beta_1 = \frac{1+k_A}{1+k_A+k_A l_A} \cdot \frac{Q_{B1}}{Q} + \frac{k_B l_B}{1+k_B+k_B l_B} \cdot \frac{P_{A1}}{P} \tag{15}$$

$$\beta_2 = \frac{1+k_A}{1+k_A+k_A l_A} \cdot \frac{Q_{B2}}{Q} + \frac{k_B l_B}{1+k_B+k_B l_B} \cdot \frac{P_{A2}}{P} \tag{16}$$

with

$$P_{A1} = p_{A1}^2 p_{A2}^2 + r_A\sigma_2(p_{A1}^2 c_{A2}\sigma_2 - p_{A2}^2 c_{A1}\sigma_1\varrho) > 0 \tag{17}$$

$$P_{A2} = p_{A1}^2 p_{A2}^2 + r_A\sigma_1(p_{A2}^2 c_{A1}\sigma_1 - p_{A1}^2 c_{A2}\sigma_2\varrho) \gtrless 0 \tag{18}$$

$$Q_{B1} = p_{B1}^2 p_{B2}^2 + r_B\sigma_2(p_{B1}^2 c_{B2}\sigma_2 - p_{B2}^2 c_{B1}\sigma_1\varrho) \gtrless 0 \tag{19}$$

$$Q_{B2} = p_{B1}^2 p_{B2}^2 + r_B\sigma_1(p_{B2}^2 c_{B1}\sigma_1 - p_{B1}^2 c_{B2}\sigma_2\varrho) > 0 \tag{20}$$

as well as

$$P = p_{A1}^2 p_{A2}^2 + r_A(p_{A1}^2 c_{A2}\sigma_2^2 + p_{A2}^2 c_{A1}\sigma_1^2) + r_A^2 c_{A1}c_{A2}\sigma_1^2\sigma_2^2(1-\varrho^2) > 0 \tag{21}$$

$$Q = p_{B1}^2 p_{B2}^2 + r_B(p_{B1}^2 c_{B2}\sigma_2^2 + p_{B2}^2 c_{B1}\sigma_1^2) + r_B^2 c_{B1}c_{B2}\sigma_1^2\sigma_2^2(1-\varrho^2) > 0. \tag{22}$$

Agent A reacts through his effort choices

$$a_1^* = \frac{p_{A1}}{c_{A1}} \cdot \frac{(1+k_A)(1+k_B) - k_A l_A k_B l_B}{1+k_B+k_B l_B} \cdot \frac{p_{A1}^2 p_{A2}^2 + r_A\sigma_2(p_{A1}^2 c_{A2}\sigma_2 - p_{A2}^2 c_{A1}\sigma_1\varrho)}{P} \tag{23}$$

$$a_2^* = \frac{p_{A2}}{c_{A2}} \cdot \frac{(1+k_A)(1+k_B) - k_A l_A k_B l_B}{1+k_B+k_B l_B} \cdot \frac{p_{A1}^2 p_{A2}^2 + r_A\sigma_1(p_{A2}^2 c_{A1}\sigma_1 - p_{A1}^2 c_{A2}\sigma_2\varrho)}{P}. \tag{24}$$

For agent B one receives correspondingly

$$b_1^* = \frac{p_{B1}}{c_{B1}} \cdot \frac{(1+k_A)(1+k_B) - k_A l_A k_B l_B}{1+k_A+k_A l_A} \cdot \frac{p_{B1}^2 p_{B2}^2 + r_B\sigma_2(p_{B1}^2 c_{B2}\sigma_2 - p_{B2}^2 c_{B1}\sigma_1\varrho)}{Q} \tag{25}$$

$$b_2^* = \frac{p_{B2}}{c_{B2}} \cdot \frac{(1+k_A)(1+k_B) - k_A l_A k_B l_B}{1+k_A+k_A l_A} \cdot \frac{p_{B1}^2 p_{B2}^2 + r_B\sigma_1(p_{B2}^2 c_{B1}\sigma_1 - p_{B1}^2 c_{B2}\sigma_2\varrho)}{Q}. \tag{26}$$

The fixed wage compensation components $\alpha_0\,,\beta_0$ are defined, such that both agents' participation constraints are binding. The principal thus reacts to each of his agents' rivalry through a reallocation of incentive intensity, where piece rates α_1 and β_1 as well as α_2 and β_2 are intertwined. This measure has major consequences for the weighting and the combination of performance measures. These consequences depend on the relative strength of the two agents' respective social preferences as well as on the type of compensation they would receive in a situation of purely egoistical behavior. (Proof: See the appendix)

First, it has to be stated, that a separation of the decision problem in the presence of stochastic dependencies and social preferences is not possible,[20] since every piece rate simultaneously fulfills several functions. The subsequent analysis of the piece rates given in proposition 1, seeks to provide a thorough understanding of these interacting effects and draws conclusions of how to balance performance measures depending on the characteristics of the *three interdependencies* considered. From the previous literature it is well-known, that *stochastic dependencies* further the merits of relative performance evaluation. This form of remuneration enables the principal to optimally exploit all the information contained in the agents' performance measures, reduces their exposure to risk and thus leads to higher values of the objective function. In the presence of productive interaction (*technological interdependency*) however, the resulting competitive relation between the agents yields the danger of counterproductive actions (sabotage). Therefore, team based compensation can be beneficial, depending in particular on the value of the correlation coefficient, that must not exceed a certain critical value. Building on this work and introducing social preferences, our main goal is to show how the optimality conditions for relative performance evaluation and team based compensation as well as the weightings of performance measures are affected through the consideration of rivalry (*psychological interdependency*). We therefore suppose that a threshold value for the correlation coefficient exists, meaning that the formal relations $p_{A1}^2\sigma_2 c_{A2} - p_{A2}^2\sigma_1 c_{A1} > \frac{p_{A1}^2 p_{A2}^2}{r_A\sigma_1}$ as well as $p_{B2}^2\sigma_1 c_{B1} - p_{B1}^2\sigma_2 c_{B2} > \frac{p_{B1}^2 p_{B2}^2}{r_B\sigma_2}$ hold. They express, that it is easier for the principal to motivate an agent for his own task rather than the one of the other agent. This assumption generalizes our analysis since we allow for piece rates α_2 and β_1 to switch signs depending on the value of ϱ. For the sake of expositional clearness, we proceed by differentiating the cases of one- ($k_A > 0\,, k_B = 0$) and two-sided ($k_A\,, k_B > 0$) rivalry.

4.2 Functions and interrelations of the piece rates for one-sided rivalry

4.2.1 Examination of piece rates β_1 and α_1

When agent B behaves completely egoistically ($k_B = 0$), the expressions for the optimal values of piece rates α_1 and β_1 in equations 13 and 15 reduce to:

$$\alpha_1 = \frac{p_{A1}^2 p_{A2}^2 + r_A\sigma_2(p_{A1}^2 c_{A2}\sigma_2 - p_{A2}^2 c_{A1}\sigma_1\varrho)}{P} + \frac{k_A l_A}{1+k_A+k_A l_A}\cdot\frac{Q_{B1}}{Q} \tag{27}$$

$$\beta_1 = \frac{1+k_A}{1+k_A+k_A l_A}\cdot\frac{Q_{B1}}{Q}. \tag{28}$$

Parameter β_1 in equation 28 simultaneously serves three functions, each of which has its foundation in a different form of interdependency between the agents: Its *first function* is the impact as insurance parameter and is primarily due to the stochastic interdependency expressed through positive values of the correlation coefficient ϱ. From standard agency models[21] it is known, that this parameter can be used to reduce agent B's exposure to risk, if it is chosen by the principal with a negative sign. However, more generally spoken it can be stated, that a smaller β_1 leads to smaller risk and therefore this first effect implies a reduction of its value. The *second function* is the effect, that in a situation of productive interaction β_1 not only has an influence on agent B's exposure to risk but also determines the efforts he takes to affect agent A's performance measure. This additional incentive effect is caused by the structure of the production technology in equation 1, which contains a technological interdependency furthering the need to provide incentives for help rather than sabotage. This raises β_1's value, as can also be seen from agent B's reaction function in equation 12. Finally, in the *third function* agent A's rivalry, imposing a psychological interdependency between the agents, is incorporated. Its impact on the value of piece rate β_1 and thus on agent A's effort a_1, depends on which one of the above stated first two effects dominates, since this determines whether the principal links agent B's remuneration positively or negatively to agent A's performance measure.

As a consequence, the principal's decision of whether to pay agent B according to relative performance evaluation or a team based compensation scheme is independent of A's rivalry. The threshold of B's correlation coefficient takes the same value as in a situation of purely egoistical behavior, meaning that although A's rivalry impacts the absolute value of β_1, it does not affect its sign. Hence the influence of A's rivalry on piece rate β_1 and therefore on A's effort a_1 reverses exactly for the critical value of the correlation coefficient. If the insurance effect dominates the effect of providing B with incentives for cooperative behavior, piece rate β_1 is negative. According to the reaction function in equation 11, it then simultaneously provides agent A with additional incentives for efforts in his own division. To see why this is the case, bear in mind that higher values of a_1 raise the expected value of performance measure x_1, and α_1 , β_1 are the two agents' respective shares in this performance measure. When $\alpha_1 > 0$ and $\beta_1 < 0$, higher values of x_1 and thus higher efforts a_1 benefit the competitive agent A in two ways. Directly through higher own remuneration and indirectly through reduced remuneration of agent B. Therefore, agent A's rivalry furthers his ambition to differentiate himself as much as possible from agent B in terms of wage compensation, leading to a higher intrinsic motivation when β_1 is below zero and preferably small. Nevertheless, this additional incentive effect not only has positive consequences for the principal but at the same time also diminishes utility, since he has to compensate agent A for his higher effort costs and for his increased exposure to risk. The latter arises since uncertainty ascends in decreasing β_1 ($\beta_1 < 0$):

$$Risk_A = \frac{r_A}{2}\{[\alpha_1(1+k_A) - \boldsymbol{k_A l_A \beta_1}]^2\sigma_1^2 + [\alpha_2(1+k_A) - k_A l_A \beta_2]^2\sigma_2^2 + 2[\alpha_1(1+k_A) - \boldsymbol{k_A l_A \beta_1}][\alpha_2(1+k_A) - k_A l_A \beta_2]\sigma_1\sigma_2\varrho\}. \tag{29}$$

This causes the need for the principal to establish an additional trade-off between incentive provision and exposure to risk, that only indirectly comes into being through the existence of the psychological interdependency. Thus, the value of β_1 increases with a stronger development of A's rivalry $k_A \uparrow$.

If conversely the incentive effect dominates the insurance effect, the value of β_1 is positive leading to decreasing incentives for efforts of agent A in his own division, since higher levels of a_1 would not only cause him but also his rival B a higher remuneration. In the considered case of rivalry, A suffers a comparative utility loss when B earns more, and therefore he reduces all activities that on the one side may benefit him, but on the other at the same time benefit agent B, whom he looks at as an opponent. This indicates that ambition can sometimes also have harmful consequences and inhibit an employee's capability, if the principal does not appropriately take his agents' social preferences into consideration.[22] The value of β_1 shrinks with a stronger development of A's social preference k_A. Figure 1[23] three-dimensionally displays the values of piece rate β_1 (z-axis) in dependence of the strength of rivalry k_A (y-axis) and the strength of the stochastic dependency ϱ (x-axis).[24] The horizontally drawn supporting layer contains all ϱ-k_A-combinations, for which piece rate β_1 is zero. The graph illustrates, that the run of the $\beta_1(\varrho, k_A)$-layer gets flatter, as k_A takes greater values. This means that a change in the weighting of rivalry k_A has bigger influence on β_1 for smaller values of k_A. The stronger the development of agent A's rivalry, the lesser its marginal impact on β_1. The crossing curve of the layers drawn in the figure is a straight line separating the positive from the negative codomain of β_1 and thus reflecting the independence of the correlation's threshold coefficient from the strength of agent A's rivalry. The change of kurtosis contained in the $\beta_1(\varrho, k_A)$-layer represents the reversion of the impact of rivalry on the value of piece rate β_1 in either the positive or the negative direction.

The value of piece rate α_1 depends on β_1 as can be seen from the formal relation

$$\alpha_1 = \frac{p_{A1}^2 p_{A2}^2 + r_A\sigma_2(p_{A1}^2 c_{A2}\sigma_2 - p_{A2}^2 c_{A1}\sigma_1\varrho)}{P} + \frac{k_A l_A}{1+k_A}\beta_1. \tag{30}$$

It immediately becomes clear from equation 30, that the principal reduces the direct incentive intensity provided through parameter α_1, if β_1 takes negative values. As was

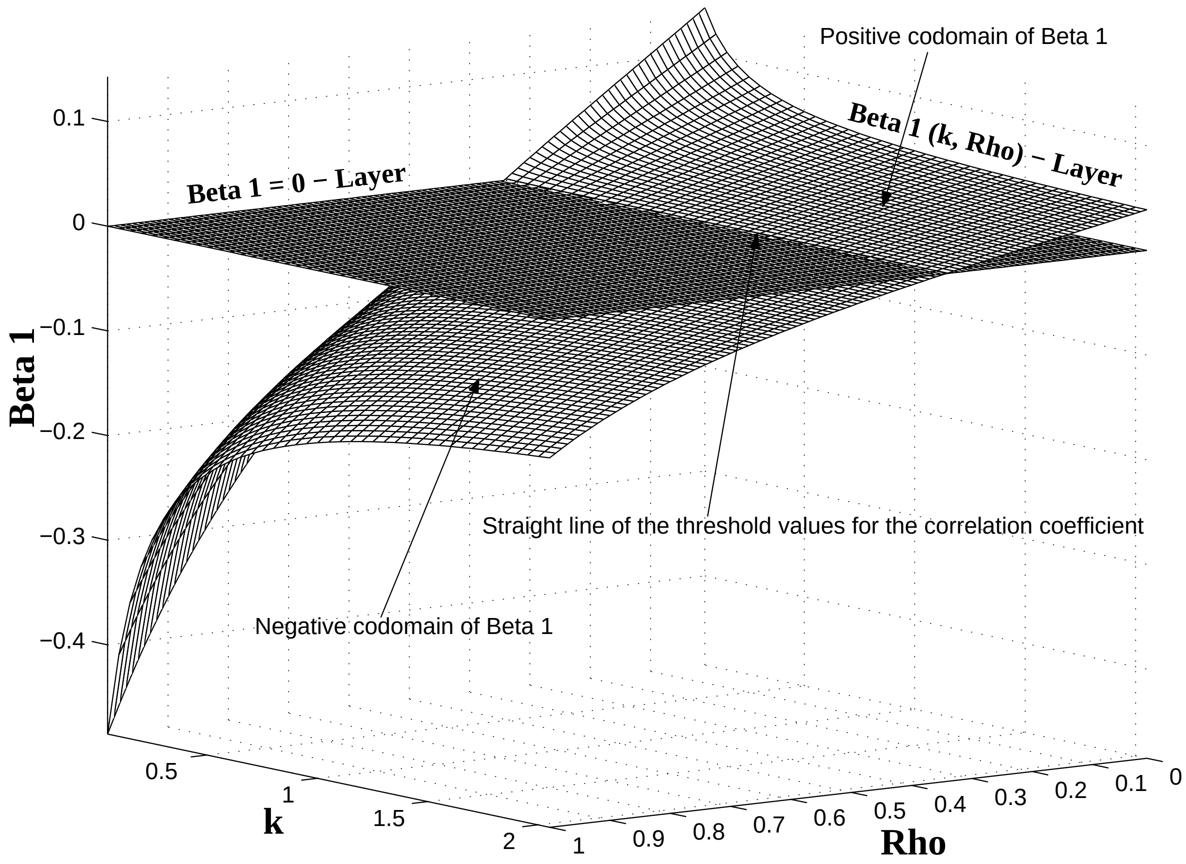

Figure 1: Codomain of piece rate β_1 in dependence of the correlation coefficient ϱ as well as the strength of rivalry k_A in a situation with productive interaction.

already mentioned above, in this situation agent A's effort a_1 increases with ascending absolute values of β_1. Thus, in order to avoid inefficiently high effort levels and to achieve a (pareto-)efficient risk allocation, the principal needs to reduce the direct incentive intensity α_1 by a corresponding amount. Conversely, if β_1 takes positive values, the principal needs to enhance this direct incentive intensity. Otherwise agent A's effort level falls short of his expectations.

4.2.2 Examination of piece rates α_2 and β_2

In case of $k_B = 0$ the expressions for the optimal values of piece rates α_2 and β_2 in equations 14 and 16 reduce to:

$$\alpha_2 = \frac{p_{A1}^2 p_{A2}^2 + r_A \sigma_1 (p_{A2}^2 c_{A1} \sigma_1 - p_{A1}^2 c_{A2} \sigma_2 \varrho)}{P} + \frac{k_A l_A}{1 + k_A + k_A l_A} \cdot \frac{Q_{B2}}{Q} \tag{31}$$

$$\beta_2 = \frac{1 + k_A}{1 + k_A + k_A l_A} \cdot \frac{Q_{B2}}{Q}. \tag{32}$$

When choosing piece rate α_2 for agent A in equation 31 the principal, according to his proceeding for the definition of β_1, has to establish a trade-off between efficient incentive provision and risk allocation taking rivalry into consideration. Differences arise since social preferences only impact the value of α_2 through the additive composition with piece rate β_2, as can be seen from the mathematical expression in equation 31, that can also be written as:

$$\alpha_2 = \frac{p_{A1}^2 p_{A2}^2 + r_A \sigma_1 (p_{A2}^2 c_{A1} \sigma_1 - p_{A1}^2 c_{A2} \sigma_2 \varrho)}{P} + \frac{k_A l_A}{1 + k_A} \beta_2. \tag{33}$$

Agent A, due to his rivalry, additionally participates in B's performance measure x_2 because of the principal's reallocation from β_2 to α_2. Since the codomain of β_2 is strictly above zero, this leads to higher values of α_2. The consequence for agent A is an increase in his incentive intensity for efforts to support B in the execution of his task meaning that the relative advantageousness of team based compensation compared to relative performance evaluation augments. The threshold value of the correlation coefficient, for which piece

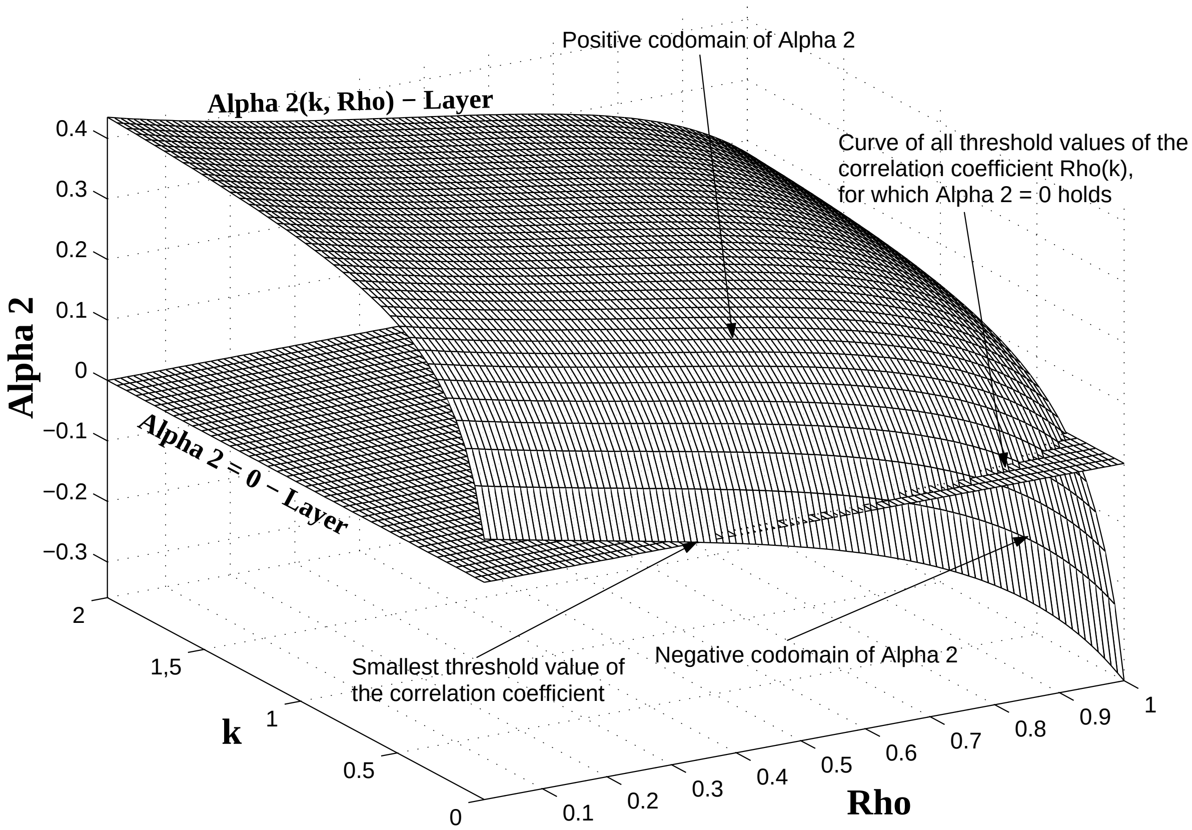

Figure 2: Codomain of piece rate α_2 in dependence of the correlation coefficient ϱ as well as the strength of rivalry k_A in a situation with productive interaction.

rate α_2 switches signs thus takes higher values when A's rivalry is stronger developed. Figure 2 exemplarily depicts the codomain of α_2 (z-axis) in dependence of the strength of rivalry k_A (y-axis) and the strength of the stochastic dependency ϱ (x-axis).[25] The horizontally running supporting layer is again the set of all ϱ-k_A-combinations, for which the value of piece rate α_2 is zero. In the diagram the kurtosis of the $\alpha_2(\varrho, k_A)$-layer becomes increasingly flat for ascending values of k_A on the one side and falling values of ϱ on the other meaning that the marginal impact of rivalry decreases for higher values of k_A and lower values of ϱ. The cutting curve of the two layers drawn in figure 2 again separates the positive codomain of α_2 from the negative and rises in k_A. Therefore, the threshold value of the correlation coefficient becomes larger when agent A's rivalry is stronger developed. The second derivative of the $\varrho^*(k_A)$-curve is negative reflecting the decreasing marginal impact of social preferences.

As a result of the principal's reallocation between the agents, piece rate β_2 in equation 32 is reduced because of agent A's rivalry. The principal hazards the negative consequences of a lower incentive intensity for agent B, since this measure is necessary for him to optimally account for A's rivalry.

4.3 Combined effects in the definition of the piece rates for two-sided rivalry

If both agents simultaneously exhibit rivalry, the reallocation measures the principal has to take are mutually overlapping. For the analysis of their respective relationship table 1 distinguishes four cases, in which the mathematical expressions $P_{A2} = p_{A1}^2 p_{A2}^2 + r_A\sigma_1(p_{A2}^2 c_{A1}\sigma_1 - p_{A1}^2 c_{A2}\sigma_2\varrho)$ and $Q_{B1} = p_{B1}^2 p_{B2}^2 + r_B\sigma_2(p_{B1}^2 c_{B2}\sigma_2 - p_{B2}^2 c_{B1}\sigma_1\varrho)$ are positive or negative, indicating whether relative performance evaluation or team based compensation would be optimal for purely egoistical behavior of A and B. In either case, the impact of a stronger developed rivalry on the weighting of performance measures x_1 and x_2 in each of the agents' wage compensation systems is displayed. Our purpose here is to focus on the qualitative effects of heterogeneously developed social preferences rather than going into explanatory detail. However, due to additive separability within the expressions for all four piece rates, the underlying economic mechanisms can be interpreted according to the exposition in the previous section.

- **Case 1** ($P_{A2} < 0$ and $Q_{B1} < 0$): In the absence of social preferences both agents are rewarded according to relative performance evaluation. Their remuneration shrinks when the respective other performs well. In the case of two-sided rivalry it shows, that the agents' social preferences operate in the *same direction* for each of the piece rates $\alpha_1, \alpha_2, \beta_1, \beta_2$. This leads to a decrease of the agents' incentive intensities α_1 and β_2, which are used to enhance their efforts with respect to own division results. At the same time the incentive intensities α_2 and β_1 for the provision of efforts in the respective other agent's division result are increasing with stronger developed rivalry. Therefore the advantages of relative performance evaluation compared to team based compensation further diminish in the case of two-sided rivalry. This can even lead to an extreme form of relative performance evaluation, where both agents are paid on the basis of the respective other's performance measure and relative to their own.[26]
- **Case 2** ($P_{A2} > 0$ and $Q_{B1} > 0$): In the situation of purely egoistical behavior, a team based compensation scheme is optimal for both agents. If they exhibit rivalry,

Case		Agent A		Agent B	
		α_1	α_2	β_1	β_2
1	**FOC**	$\frac{\partial \alpha_1}{\partial k_A} < 0\,; \frac{\partial \alpha_1}{\partial k_B} < 0$	$\frac{\partial \alpha_2}{\partial k_A} > 0\,; \frac{\partial \alpha_2}{\partial k_B} > 0$	$\frac{\partial \beta_1}{\partial k_A} > 0\,; \frac{\partial \beta_1}{\partial k_B} > 0$	$\frac{\partial \beta_2}{\partial k_A} < 0\,; \frac{\partial \beta_2}{\partial k_B} < 0$
	$k_A \uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$
	$k_B \uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$
2	**FOC**	$\frac{\partial \alpha_1}{\partial k_A} > 0\,; \frac{\partial \alpha_1}{\partial k_B} < 0$	$\frac{\partial \alpha_2}{\partial k_A} > 0\,; \frac{\partial \alpha_2}{\partial k_B} < 0$	$\frac{\partial \beta_1}{\partial k_A} < 0\,; \frac{\partial \beta_1}{\partial k_B} > 0$	$\frac{\partial \beta_2}{\partial k_A} < 0\,; \frac{\partial \beta_2}{\partial k_B} > 0$
	$k_A \uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$
	$k_B \uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$
3	**FOC**	$\frac{\partial \alpha_1}{\partial k_A} < 0\,; \frac{\partial \alpha_1}{\partial k_B} < 0$	$\frac{\partial \alpha_2}{\partial k_A} > 0\,; \frac{\partial \alpha_2}{\partial k_B} < 0$	$\frac{\partial \beta_1}{\partial k_A} > 0\,; \frac{\partial \beta_1}{\partial k_B} > 0$	$\frac{\partial \beta_2}{\partial k_A} < 0\,; \frac{\partial \beta_2}{\partial k_B} > 0$
	$k_A \uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$
	$k_B \uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$
4	**FOC**	$\frac{\partial \alpha_1}{\partial k_A} > 0\,; \frac{\partial \alpha_1}{\partial k_B} < 0$	$\frac{\partial \alpha_2}{\partial k_A} > 0\,; \frac{\partial \alpha_2}{\partial k_B} > 0$	$\frac{\partial \beta_1}{\partial k_A} < 0\,; \frac{\partial \beta_1}{\partial k_B} > 0$	$\frac{\partial \beta_2}{\partial k_A} < 0\,; \frac{\partial \beta_2}{\partial k_B} < 0$
	$k_A \uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$
	$k_B \uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$

Table 1: Impacts of varying developments of both agents' rivalry on the four piece rates in a situation with productive interaction.

the principal's necessary reallocations in order to account for each of the agents' social preferences work in exactly the *opposite directions* for each of the four pieces. Resulting from this is, that the effects described in the previous section cancel each other more and more, if the two agents' rivalry is equally strong developed. A trade-off between team based compensation and relative performance evaluation is not necessary in this situation.

- **Case 3** ($P_{A2} > 0$ and $Q_{B1} < 0$): When social preferences are absent, Agent A is paid according to team based compensation while agent B receives his remuneration based on the relative performance compared to A. In a situation of two-sided rivalry, the necessary reallocations for piece rates α_1 and β_1 correspond to the ones in *case 1*, while for piece rates α_2 and β_2 they correspond to those in *case 2*. Accordingly, the impacts of the two agents' social preferences work in the same direction for piece rates α_1 and β_1 but in opposing directions for α_2 and β_2. The relative advantageousness of team based compensation for agent B augments. Agent A's basic type of compensation generically remains unchanged. Also in this case, only for extreme parameter constellations an evaluation relative to the own division result for him can turn out to be optimal.

- **Case 4** ($P_{A2} < 0$ and $Q_{B1} > 0$): In the treatment of purely egoistical behavior, for agent A relative performance evaluation, for agent B team based compensation proves to be optimal. According to the analysis of *case 3*, in a situation of two sided-rivalry their social preferences work in the same directions for α_2 and β_2, but in opposing directions for α_1 and β_1. The consequences with respect to the wage compensation system are analogous to *case 3*.

5 Incentive system in case of a production externality

5.1 Calculation of the optimal incentive system with a production externality

In the previous section we have analyzed the situation of a principal-agent relationship, where the agents could distribute their efforts on actions to manipulate their own performance measure or on actions to affect the respective other's division result. In reality however there often occur situations, in which other organizational units automatically profit from the efforts an agent primarily provides to affect his own performance measure. For example, one can think of a value chain with vertical integration[27], where one division produces a good, that is directly sold to the market and simultaneously serves as an input for internal usage in a different department. In this setting all divisions involved gain utility from higher efforts in the production of the good. Marketing actions to strengthen an overall brand name serve as a different example, since all product managers profit from these activities at the same time. More generally spoken, many synergy effects go back to the existence of production externalities. If the notation of production is, as is usual done in agency models, broadly interpreted as efforts to influence certain performance indicators, the findings with the model development below are of empirical relevance for a broad variety of practical applications.

The calculations are analogous to the case of productive interaction. This time the production technology has the form of equation 2 and the corresponding cost functions are given through equation 4. The agents' *reaction functions* thus are:

$$a_1 = \frac{(1+k_A)(\alpha_1 + p_{A2}\alpha_2) - k_A l_A(\beta_1 + p_{A2}\beta_2)}{c_{A1}} \tag{34}$$

$$b_2 = \frac{(1+k_B)(p_{B1}\beta_1 + \beta_2) - k_B l_B(p_{B1}\alpha_1 + \alpha_2)}{c_{B2}}. \tag{35}$$

The agents' *reservation utilities* are again scaled to zero. Conducting the same typical optimization steps as in the case of productive interaction leads to:

Proposition 2 (Wage compensation system with rivalry in the case of a production externality):

$$\alpha_1 = \frac{1+k_B}{1+k_B+k_B l_B} \cdot \frac{X_{A1}}{X} + \frac{k_A l_A}{1+k_A+k_A l_A} \cdot \frac{Y_{B1}}{Y} \tag{36}$$

$$\alpha_2 = \frac{1+k_B}{1+k_B+k_B l_B} \cdot \frac{X_{A2}}{X} + \frac{k_A l_A}{1+k_A+k_A l_A} \cdot \frac{Y_{B2}}{Y} \tag{37}$$

$$\beta_1 = \frac{1+k_A}{1+k_A+k_A l_A} \cdot \frac{Y_{B1}}{Y} + \frac{k_B l_B}{1+k_B+k_B l_B} \cdot \frac{X_{A1}}{X} \tag{38}$$

$$\beta_2 = \frac{1+k_A}{1+k_A+k_A l_A} \cdot \frac{Y_{B2}}{Y} + \frac{k_B l_B}{1+k_B+k_B l_B} \cdot \frac{X_{A2}}{X} \tag{39}$$

where

$$X_{A1} = r_A c_{A1} \sigma_2 (1+p_{A2}) \cdot (\sigma_2 - p_{A2}\sigma_1 \varrho) \gtreqless 0 \tag{40}$$

$$X_{A2} = r_A c_{A1} \sigma_1 (1+p_{A2}) \cdot (p_{A2}\sigma_1 - \sigma_2 \varrho) \gtreqless 0 \tag{41}$$

$$Y_{B1} = r_B c_{B2} \sigma_2 (1+p_{B1}) \cdot (p_{B1}\sigma_2 - \sigma_1 \varrho) \gtreqless 0 \tag{42}$$

$$Y_{B2} = r_B c_{B2} \sigma_1 (1+p_{B1}) \cdot (\sigma_1 - p_{B1}\sigma_2 \varrho) \gtreqless 0 \tag{43}$$

as well as

$$X = r_A c_{A1} \sigma_2^2 + p_{A2}^2 r_A c_{A1} \sigma_1^2 + r_A^2 c_{A1}^2 \sigma_1^2 \sigma_2^2 (1-\varrho^2) - 2 p_{A2} r_A c_{A1} \sigma_1 \sigma_2 \varrho > 0 \tag{44}$$

$$Y = p_{B1}^2 r_B c_{B2} \sigma_2^2 + r_B c_{B2} \sigma_1^2 + r_B^2 c_{B2}^2 \sigma_1^2 \sigma_2^2 (1-\varrho^2) - 2 p_{B1} r_B c_{B2} \sigma_1 \sigma_2 \varrho > 0. \tag{45}$$

Agent A's reaction can be expressed by

$$\begin{aligned} a_1^{**} = {} & \frac{1}{c_{A1}} \cdot \frac{(1+k_A)(1+k_B) - k_A l_A k_B l_B}{1+k_B+k_B l_B} \cdot \frac{r_A c_{A1} \sigma_2 (1+p_{A2}) \cdot (\sigma_2 - p_{A2}\sigma_1\varrho)}{X} + \\ & \frac{p_{A2}}{c_{A1}} \cdot \frac{(1+k_A)(1+k_B) - k_A l_A k_B l_B}{1+k_B+k_B l_B} \cdot \frac{r_A c_{A1} \sigma_1 (1+p_{A2}) \cdot (p_{A2}\sigma_1 - \sigma_2\varrho)}{X}. \end{aligned} \tag{46}$$

For agent B we accordingly obtain:

$$\begin{aligned} b_2^{**} = {} & \frac{p_{B1}}{c_{B2}} \cdot \frac{(1+k_A)(1+k_B) - k_A l_A k_B l_B}{1+k_A+k_A l_A} \cdot \frac{r_B c_{B2} \sigma_2 (1+p_{B1}) \cdot (p_{B1}\sigma_2 - \sigma_1\varrho)}{Y} + \\ & \frac{1}{c_{B2}} \cdot \frac{(1+k_A)(1+k_B) - k_A l_A k_B l_B}{1+k_A+k_A l_A} \cdot \frac{r_B c_{B2} \sigma_1 (1+p_{B1}) \cdot (\sigma_1 - p_{B1}\sigma_2\varrho)}{Y}. \end{aligned} \tag{47}$$

The fixed wage compensation components α_0, β_0 are defined such that the agents' participation constraints are binding. The principal again reacts to his agents' rivalry by reallocating incentive intensity through shifting between α_1 and β_1 as well as α_2 and β_2. The qualitative effects on the the weighting and the combination of performance measures depend on the sign, each of the four piece rates would receive for purely egoistical behavior. (Proof: See the appendix)

A comparison of the results given in propositions 1 and 2 reveals, that the formal structure is the same. In either case, the mathematical expressions for the piece rates are composed of two summands, each consisting of a weighting factor that is determined by the social preference parameters k_i and l_i as well as an additional multiplicatively linked term, known from the situation of purely egoistical behavior. Differences arise, since the latter have other properties than in the case of productive interaction. As is known from standard agency models[28], the principal in their definition again has to establish a trade-off between two opposing effects. He can either internalize the production externality by

	Agent A		Agent B	
	$\boldsymbol{\alpha_1}$	$\boldsymbol{\alpha_2}$	$\boldsymbol{\beta_1}$	$\boldsymbol{\beta_2}$
Numerator > 0, if	$p_{A2} < \frac{\sigma_2}{\sigma_1 \varrho}$	$p_{A2} > \frac{\sigma_2}{\sigma_1}\varrho$	$p_{B1} > \frac{\sigma_1}{\sigma_2}\varrho$	$p_{B1} < \frac{\sigma_1}{\sigma_2 \varrho}$
Team based compensation, if	$\frac{\sigma_2}{\sigma_1}\varrho < p_{A2} < \frac{\sigma_2}{\sigma_1 \varrho}$		$\frac{\sigma_1}{\sigma_2}\varrho < p_{B1} < \frac{\sigma_1}{\sigma_2 \varrho}$	
Relative evaluation with respect to the other's result, if	$p_{A2} < \frac{\sigma_2}{\sigma_1}\varrho$		$p_{B1} < \frac{\sigma_1}{\sigma_2}\varrho$	
Relative evaluation with respect to the own result, if	$p_{A2} > \frac{\sigma_2}{\sigma_1 \varrho}$		$p_{B1} > \frac{\sigma_1}{\sigma_2 \varrho}$	

Table 2: Optimality conditions for team based compensation and relative performance evaluation with a production externality in a situation of purely egoistical behavior.

applying team based compensation or choose to reduce the agents' exposure to risk when ϱ is large. Table 2 summarizes the optimality conditions. It shows, that in contrast to our previous analysis, three types of wage compensation can be optimal. Additionally, their advantageousness in the absence of social preferences does not only depend on the *correlation coefficient* ϱ but also on the *relative risk* $\frac{\sigma_2}{\sigma_1}$ $(\frac{\sigma_1}{\sigma_2})$ and the *strength of the production externality* p_{A2} (p_{B1}).[29] Building on these results, our purpose again is to examine how the optimality conditions for the different types of compensation change due to the principal's reallocations when the agents behave competitively. We therefore again differentiate the cases of one-sided and two-sided rivalry.

5.2 Balancing performance measures in case of one-sided rivalry

In case of one-sided rivalry ($k_B = 0$) the optimal values for the piece rates in equations 36–39 simplify to:

$$\alpha_1 = \frac{r_A c_{A1} \sigma_2 (1 + p_{A2}) \cdot (\sigma_2 - p_{A2}\sigma_1\varrho)}{X} + \frac{k_A l_A}{1 + k_A + k_A l_A} \cdot \frac{Y_{B1}}{Y} \tag{48}$$

$$\alpha_2 = \frac{r_A c_{A1} \sigma_1 (1 + p_{A2}) \cdot (p_{A2}\sigma_1 - \sigma_2\varrho)}{X} + \frac{k_A l_A}{1 + k_A + k_A l_A} \cdot \frac{Y_{B2}}{Y} \tag{49}$$

$$\beta_1 = \frac{1 + k_A}{1 + k_A + k_A l_A} \cdot \frac{Y_{B1}}{Y} \tag{50}$$

$$\beta_2 = \frac{1 + k_A}{1 + k_A + k_A l_A} \cdot \frac{Y_{B2}}{Y}. \tag{51}$$

Differences to the case of productive interaction arise since the mathematical expressions $Y_{B1} = r_B c_{B2} \sigma_2 (1 + p_{B1}) \cdot (p_{B1}\sigma_2 - \sigma_1\varrho)$ and $Y_{B2} = r_B c_{B2} \sigma_1 (1 + p_{B1}) \cdot (\sigma_1 - p_{B1}\sigma_2\varrho)$, that are affected by the principal's reallocations, can both either take positive or negative values. Therefore, the analysis proceeds by differentiating *three cases* according to agent

B's type of compensation, that are summarized in table 3. In addition to that we focus on the qualitative insights, since the underlying mechanisms of action can analogously be explained as was done in the case of productive interaction considered above.

Case	**Value of p_{B1}**		**Impact of $k_A \uparrow$**			
	β_1	β_2	α_1	α_2	β_1	β_2
1	$p_{B1} > \frac{\sigma_1}{\sigma_2}\varrho$	$p_{B1} < \frac{\sigma_1}{\sigma_2 \varrho}$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$
2	$p_{B1} < \frac{\sigma_1}{\sigma_2}\varrho$		$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$
3	$p_{B1} > \frac{\sigma_1}{\sigma_2 \varrho}$		$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$

Table 3: Impact of a change in agent A's strength of rivalry k_A on the four piece rates in dependence of the strength of agent B's production externality p_{B1}.

- **Case 1** ($Y_{B1} > 0\,, Y_{B2} > 0$): Agent B is paid according to team based compensation. The weighting of both performance measures x_1 and x_2 in his wage compensation system is reduced due to agent A's rivalry. His incentive intensity diminishes. In turn the performance measures are correspondingly stronger weighted in agent A's wage compensation system. For him the relative advantageousness of team based compensation compared to relative performance evaluation therefore augments.

- **Case 2** ($Y_{B1} < 0\,, Y_{B2} > 0$): Agent B is evaluated relative to A's division result. Due to the reduction of piece rate α_1 with simultaneous enhancement of α_2, changes in the type of compensation are possible for agent A. If, in the situation of purely egoistical behavior A would be rewarded relative to B, the possibility exists, that now because of his rivalry team based compensation becomes optimal from firm perspective.[30] Accordingly, the initial advantageousness of team based compensation can switch in a way, that after the principal's reallocations a compensation relative to the own division result is superior. This possibility only exists, if piece rate α_1 takes a very small value in the situation without social preferences. Under these circumstances the principal concentrates on piece rate α_2 in his provision of incentives. It is thus only a matter of a marginal effect.

- **Case 3** ($Y_{B1} > 0\,, Y_{B2} < 0$): Agent B is rewarded on basis of agent A's performance measure x_1 and relative to his own division result x_2. The reduced intrinsic motivation of agent A as the result of $\beta_1 > 0$ is counterbalanced by the principal with an increase of his incentive intensity through enhancing α_1. By contrast $\beta_2 < 0$ causes a gain in A's intrinsic motivation and is used for risk reduction purposes through decreasing α_2. The examination of these measures' effects on agent A's wage compensation type again takes his type of compensation in a situation of purely egoistical behavior as a starting point. If he was remunerated according to team aspects, in the case of rivalry a relative evaluation compared to B can prove to be optimal depending on the strength of his social preference. By contrast the possibility exists, that the optimality of a relative evaluation with respect to A's own division result reverses to the advantageousness of team based compensation.

5.3 Balancing performance measures in case of two-sided rivalry

In a situation with two-sided social preferences, each agent's rivalry affects all four piece rates. For this reason it again comes to an overlap of the previously described effects. We focus our analysis on whether an agent's stronger developed rivalry has an increasing or a decreasing effect on the definition of piece rates α_1 , α_2 , β_1 , β_2, whereupon the question of how the various influences interact has to be answered. Do they operate in the same or in opposing directions? The answer to this question primarily depends on the principal's type of compensation in the case of purely egoistical behavior. It determines whether the agents, due to the principal's reallocations, participate in positive or negative shares of the respective other. Given that each of the terms X_{A1}, X_{A2}, Y_{B1} and Y_{B2} in equations 40–43 can switch signs, for every agent three types of compensation can prove to be optimal leading to a total of *nine possible cases*, that are contrasted in table 4. In order to put

Case	**Value of p_{A2}**	**Value of p_{B1}**	$k_i \uparrow$	**Result**			
				α_1	α_2	β_1	β_2
1	$\frac{\sigma_2}{\sigma_1}\varrho < p_{A2} < \frac{\sigma_2}{\sigma_1 \varrho}$	$\frac{\sigma_1}{\sigma_2}\varrho < p_{B1} < \frac{\sigma_1}{\sigma_2 \varrho}$	k_A	↑	↑	↓	↓
			k_B	↓	↓	↑	↑
2	$p_{A2} < \frac{\sigma_2}{\sigma_1}\varrho$	$p_{B1} < \frac{\sigma_1}{\sigma_2}\varrho$	k_A	↓	↑	↑	↓
			k_B	↓	↑	↑	↓
3	$p_{A2} > \frac{\sigma_2}{\sigma_1 \varrho}$	$p_{B1} > \frac{\sigma_1}{\sigma_2 \varrho}$	k_A	↑	↓	↓	↑
			k_B	↑	↓	↓	↑
4	$\frac{\sigma_2}{\sigma_1}\varrho < p_{A2} < \frac{\sigma_2}{\sigma_1 \varrho}$	$p_{B1} < \frac{\sigma_1}{\sigma_2}\varrho$	k_A	↓	↑	↑	↓
			k_B	↓	↓	↑	↑
5	$\frac{\sigma_2}{\sigma_1}\varrho < p_{A2} < \frac{\sigma_2}{\sigma_1 \varrho}$	$p_{B1} > \frac{\sigma_1}{\sigma_2 \varrho}$	k_A	↑	↓	↓	↑
			k_B	↓	↓	↑	↑
6	$p_{A2} < \frac{\sigma_2}{\sigma_1}\varrho$	$\frac{\sigma_1}{\sigma_2}\varrho < p_{B1} < \frac{\sigma_1}{\sigma_2 \varrho}$	k_A	↑	↑	↓	↓
			k_B	↓	↑	↑	↓
7	$p_{A2} > \frac{\sigma_2}{\sigma_1 \varrho}$	$\frac{\sigma_1}{\sigma_2}\varrho < p_{B1} < \frac{\sigma_1}{\sigma_2 \varrho}$	k_A	↑	↑	↓	↓
			k_B	↑	↓	↓	↑
8	$p_{A2} < \frac{\sigma_2}{\sigma_1}\varrho$	$p_{B1} > \frac{\sigma_1}{\sigma_2 \varrho}$	k_A	↑	↓	↓	↑
			k_B	↓	↑	↑	↓
9	$p_{A2} > \frac{\sigma_2}{\sigma_1 \varrho}$	$p_{B1} < \frac{\sigma_1}{\sigma_2}\varrho$	k_A	↓	↑	↑	↓
			k_B	↑	↓	↓	↑

Table 4: Impacts of a change in the weighting of both agents' rivalry k_A and k_B on the four piece rates in dependence of the values for the production externalities p_{A2} and p_{B1}.

emphasis on the empirical relevance, we forgo discussing each of them in detail and deduce statements of a more general nature, which emerge from the overall-view.

The two agents' rivalry has no impact on the type of compensation employed by the principal, if their wage compensation systems in the absence of social preferences would equally depend (that is positive or negative connection) on the two performance measures. This applies to *cases one, eight* and *nine*. In these examinations the impacts of the two agents' rivalry tend to cancel each other, such that the principal only needs to apprehend minor corrections in his incentive system. In all other cases, changes in the principal's compensation type compared to the situation without social preferences can occur for both agents, where it has to be differentiated, whether these variations in the agents' wage compensation systems either refer to both performance measures (*cases two* and *three*) or only to one performance measure (*cases four, five, six* and *seven*). If for $k_A = k_B = 0$ both agents are remunerated relatively to one another, as is done in the *cases two* and *three*, the principal's measures point in the same direction for all four piece rates $\alpha_1, \alpha_2, \beta_1, \beta_2$. The initially negative parameter values augment, the positive ones decline meaning that in this consideration, the absolute values of all piece rates assimilate at a smaller level than in the case without social preferences. The originally highly asymmetric contract offers, where only one agent profits, if one of the measures x_1 and x_2 takes high values, converge for two-sided rivalry. The relative advantageousness of team based compensation therefore augments.[31] A corresponding harmonization for the absolute values of $\alpha_1, \alpha_2, \beta_1, \beta_2$ is asserted in *cases four, five, six* and *seven* with respect to the performance measure, in which the agents would be opposingly involved, if social preferences were not an issue. For the respective other metric the effects tend to cancel each other, since the agents' participations point in the same direction. These conclusions hold independently of what type of compensation in detail proves to be optimal.

Summarizing, the principal reacts to equally strong developed social preferences of his agents with an assimilation of their wage compensation systems. This becomes clear from the fact, that both agents' wage compensation does increasingly similar depend on each of the performance measures available.[32] Growing asymmetries in the strength of development of both agents' rivalry as well as the other parameters, characterizing their personality or the exogenous environmental influences, lead to more asymmetrical incentive contracts. These observations are also the driving forces for the results obtained in the case of productive interaction. They even generalize to a situation of technological independence[33] and consequently hold irrespective of the underlying production technology.

6 Impacts of rivalry on firm profitability

Rivalry leads to the ambition of the agents to differentiate as much as possible from one another in terms or remuneration. When technological interdependencies are present, this competitive spirit does not necessarily need to have only positive consequences, since the agents are supposed to take the effects of their efforts on the respective other's division result into account. It even has to be assumed that their willingness to take actions, that reduce the relevant reference agent's profits, increases in such an environment.[34] Thus, value for the principal would be destroyed causing negative consequences on firm profitability. We first consider the case of productive interaction.

Proposition 3 (Consequences of Rivalry on Firm Profitability in Case of Productive Interaction): *The principal can usually exploit an agent's rivalry in case of one-sided social preferences, if he defines the piece rates according to the terms given in equations 13–16 and if the egoistical agent's contribution to firm profits does not by far exceed the one from the competitive agent. The principal's utility gains are then the larger, the stronger developed rivalry is for the respective agent. By contrast, an agent's rivalry generally leads to a reduction of firm profits, if the extractable utility from the egoistical agent by far exceeds the one of the competitive agent. In case of two-sided social preferences it's optimal from firm perspective, if only one agent exhibits preferably strong developed rivalry, while the other behaves completely egoistically. If the principal could choose, he would want the agent to behave competitively, who contributed more to his utility, if social preferences were not an issue.*[35] *(Proof: See the appendix)*

In order to give an intuition for this result, we first consider the case of one-sided rivalry ($k_B = 0$). Agent A's contributions to the principal's goal function take the same values as in the situation of purely egoistical behavior, weighted by a social preference term $(1+k_A) > 1$, as can be seen from equations 23 and 24. This term indicates what we call the *intrinsic motivation effect*. Due to rivalry the agent wants to differentiate himself as much as possible from the other. Therefore, he values own remuneration higher, leading to higher sensitivities with respect to the own incentive intensities α_1 and α_2, as can be seen from the reaction functions in equation 11. Thus, for given incentives, the competitive agent invests higher efforts. This also holds true for counterproductive actions to lessen the respective other agent's division result (sabotage) in the case where the principal applies relative performance evaluation.[36] Then the *intrinsic motivation effect* not only has a positive but also a negative effect on firm profitability. However, since the principal's incentives are in absolute terms always higher for a_1 than they are for a_2, the positive consequences of the intrinsic motivation effect always dominate, leading to higher contributions from agent A to firm profitability. If, by contrast, team based compensation proves to be optimal, agent A's incentives for providing positive efforts are enlarged with respect to both divisions due to his competitive preferences. Thus, the *intrinsic motivation effect* has *positive impacts* on firm profitability.

When examining agent B's efforts in equations 25 and 26, we again observe, that they have the same form as in the case of egoistical behavior, but, contrary to A, are weighted by the terms $\frac{1+k_A}{1+k_A+k_A l_A} < 1$. We call this the *extrinsic motivations effect*, since it is caused by the principal's reallocations as a result of agent A's rivalry, leading to reduced extrinsic incentive provision for agent B. This entails an additional positive element, if the principal employs relative performance evaluation. The reduced extrinsic incentives have the consequence, that B not only reduces the efforts for his own division result but also decreases his counterproductive manipulations of A's measure. However, for analogous reasons as before, the negative consequences on firm profits dominate. In case of team based compensation the *extrinsic motivation effect* brings about reduced efforts in both divisions. Therefore, the *extrinsic motivation effect* in total has a *negative sign*.

When analyzing the total effect of agent A's rivalry, usually the positive intrinsic motivation effect dominates over the negative extrinsic motivation effect, since A's increased intrinsic motivation has a much higher impact on firm profitability compared to B's reduced extrinsic incentives. Only for extreme cases, where B contributes a lot more to firm profits with his actions than A, it can happen that A's competitiveness is actually harmful.

Next we consider two-sided rivalry. In this case the weighting factors take the form $\frac{(1+k_A)(1+k_B)-k_A l_A k_B l_B}{1+k_B+k_B l_B}$ for agent A and $\frac{(1+k_A)(1+k_B)-k_A l_A k_B l_B}{1+k_A+k_A l_A}$ for agent B (see equations 23–26). Thus, examining agent A, we observe that his positive intrinsic motivation effect, expressed by the factor $(1 + k_A)$ is overlaid, by the negative extrinsic motivation effect $\frac{1+k_B}{1+k_B+k_B l_B}$, which is due to agent B's rivalry. However, in the case of two-sided social preferences, an additional influence has to be included. Because of B's rivalry, his piece rate β_1 increases by $\frac{k_B l_B}{1+k_B+k_B l_B} \cdot \frac{p_{A1}^2 p_{A2}^2 + r_A \sigma_2 (p_{A1}^2 c_{A2} \sigma_2 - p_{A2}^2 c_{A1} \sigma_1 \varrho)}{P}$, which, according to the reaction function in equation 11, leads to a reduced intrinsic motivation of agent A, who, due to his own rivalry, dislikes falling behind. As a consequence, the intrinsic motivation effect has an additional negative component when both agents behave competitively at the same time. This is reflected in the term $-\frac{k_A l_A k_B l_B}{1+k_B+k_B l_B}$ as further part of A's weighting factor $\frac{(1+k_A)(1+k_B)-k_A l_A k_B l_B}{1+k_B+k_B l_B}$. The same interpretations analogously hold true for the weighting factor in agent B's effort choices.

From this it can be seen, why the principal only intends one of his agents to behave competitively.[37] If he wants to make use of an agent's rivalry, the principal has to optimally react to this agent's social preferences in the design of his wage compensation system. As we have seen in the analysis above, this implies, that the principal increases the competitive agent's variable wage compensation components at the expense of the other agent's remuneration. This reallocation accumulates with stronger developed social preferences. If, in such a situation, the other agent also exhibits rivalry, the principal's measure to account for the first agent's rivalry simultaneously has a negative, that is effort reducing effect on the second agent. This means, that the principal is no longer able to establish a beneficial trade-off through an adequately designed incentive system. Every reallocation leads to an improvement on the one side, but at the same time yields an aggravation on the other. The measures, the principal would have to take, if he simultaneously wanted to account for both agents' rivalry are inconsistent with one another. Therefore, he can no longer benefit from their social preferences, if they are equally strong developed. The value of his goal function remains unchanged at the same level as in the case of purely egoistical behavior. However, as soon as the agents differ in the strength of their social preferences, the principal's profits increase. They reach their maximum, if one agent behaves as competitively as possible, while the other is fully egoistical. This result somehow contrasts the one in Itoh (2004) (p. 42) who derives his conclusion, that the principal can benefit from the competitive spirit of agents who exhibit rivalry from a model, in which the agents are completely symmetrical.

A further insight of our model is, that rivalry can be best exploited by the principal, if it is endowed by an agent with high capability. This agent's already high productivity is further enhanced via the ambition to differentiate from his peer. The hereunto necessary reduction of the other agent's incentive intensity has comparatively negligible effects on the overall firm result. By contrast, it harms the principal, if somebody behaves competitively, who does not contribute a lot to firm profitability. This agent then rather causes trouble through the need to disadvantage an agent endowed with higher productivity in order to appropriately account for his rivalry, although he is of comparatively minor importance with respect to firm value.

Corollary 1 (Consequences of Rivalry on Firm Profitability in Case of a Production Function with a Production Externality): *In case of a production function with a production externality the same qualitative insights hold true as in the case of productive interaction. The principal's maximal goal function value again is achieved for preferably differential developments of the two agents' rivalry, where the agent ideally should behave competitively, who would make a larger contribution when social preferences were not an issue.*

Corollary 1 is due to the fact, that the weighting factors in the principal's optimal goal function value again take the same form as in the case of productive interaction. In addition to that, for each of the agents the total incentive intensity is always positive, independent of what type of compensation in detail proves to be optimal. Therefore, the above argumentation analogously holds, leading to:

Corollary 2 (Consequences of Rivalry on Firm Profitability in Case of Different Production Functions): *The qualitative impacts of (two-sided) rivalry on firm profits* *are independent of the underlying production technology, as long as separate cardinally scaled performance measures for each of the agents are available.*

Itoh (2004)'s (p. 42) conjecture, that the principal's possibility to make use of his agents' rivalry depends on the absence of technological interdependencies, is not confirmed in our model. However, the principal may not be able to exploit the agents' social preferences when they are engaged in team production in the sense of Alchian/Demsetz (1972), where their interaction takes the form of a public goods game.[38]

7 Conclusion

Our study contributes to both the literature on the topic of performance evaluation as well as in the field of „Behavioral Contract Theory" by examining the impact of two-sided rivalry on the weighting and the combination of performance measures in case of different technological interdependencies. Figure 3 summarizes our main results. Due to *one agent's*

	Purely Egoistical Behavior	**One-Sided Rivalry from Agent A**	**Two-Sided Rivalry from both Agents**
Productive Interaction	Threshold for correlation coefficient ρ^{**} $\rho < \rho^{**}$: Team based compensation $\rho > \rho^{**}$: Relative performance evaluation	**Agent A:** • Increase of the threshold value for the correlation • Increased incentive intensity for own task in case of team based comp. for B **Agent B:** • Unchanged treade-off between team based compensation and RPE • Decreased incentive intensity for both tasks	• Impact depends on both agents´ type of compensation in a situation of purely egoistical behavior: • **Team based comp.:** Both agents´ social preferences have opposing effects on the weighting of performance measures • **RPE:** Both agents´ social preferences work in the same direction
Production Externality	Decision between team based compensation and relative performance evaluation in dependence of • the strength of the production externality, • the relative risk and • the strength of the stochastic dependency	**Team based comp. of B:** • Reduction (increase) of B´s (A´s) shares • Advatages of team based compensation rise for A **RPE of B with respect to A:** • Share of A in his own division result (in B´s division result) decreases (increases) **Evaluation of B relative to his own result:** • A´s share in his own division result (in B´s division result) increases (decreases)	• Impact depends on both agents´ type of compensation in a situation of purely egoistical behavior: • Respective other agent´s rivalry causes reduction (enhancement) of positive (negative) own shares • Own rivalry leads to taking up the respective offsetting items concerning the reallocations

Figure 3: Impacts of rivalry on the optimal wage compensation system in a situation with stochastic and technological interdependencies.

rivalry, the respective other agent's shares in the two performance measures x_1 and x_2 are partitioned between the two of them. Therefore, the qualitative effects on agent A's piece rates depend on B's type of compensation. In this context, it is of particular importance whether B's shares in the performance measures are positive or negative. Positive values lead to an increase in the competitive agent's shares, whereupon the respective weightings of performance measures in B's wage compensation system simultaneously have to be reduced. Conversely, negative piece rates of the egoistical agent are augmented by the respective other agent's rivalry, while his shares in the corresponding performance measures take lower values. However, the type of compensation for the egoistical agent remains unchanged, since his shares in the performance measures are only proportionately affected by the principal's measures. In case of *two-sided rivalry*, the principal separately performs the above described reallocations due to each of the agent's social preferences at the same time. Because of the consequential overlapping effects, various changes in the types of compensation can emerge for each of the agents. When assuming a production technology with productive interaction, we have shown that, departing from relative performance evaluation, an agent's share in the respective other's division result increases with rising strength of both agents' rivalry, making team based compensation more appealing. By contrast the impacts of the social preferences on this performance measure neutralize, if team-based compensation was optimal in case of purely egoistical behavior. Therefore, the relative advantageousness of team based compensation compared to relative performance evaluation increases, yielding a possible explanation for why relative performance evaluation is not so commonly observed in practice despite its theoretical attractiveness. This general insight however does not unrestrictedly carry over to the case of a production externality, since in this context also a relative evaluation with respect to the own division result can prove to be optimal, giving rise to several further constellations that are worth of mention.

Building on these results concerning performance measurement, we have shown that the principal's actions in order to account for each of the agents' rivalry contradict one another leading to the fact that he only wants one of his agents to behave competitively. This ideally should be the agent who would also contribute more in a situation of egoistical behavior, because otherwise rivalry does not necessarily lead to an increase in firm profitability. However, these findings are constrained by the assumption, that the agents' rivalry only refers to a comparison of wages. Different results might emerge, if effort costs are also included in the specifications of the social preference terms. In addition to that we have taken an extreme point of view through only considering rivalry as a particularly negative type of social preference. Examinations of different, more positive forms like altruism, inequity aversion and reciprocity are important topics for further research.[39] In this context the extension to team production in the sense of Alchian/Demsetz (1972) seems important, since its public goods character furthers their occurrence. Examining these open cases will provide a thorough understanding of the interaction of different types of social preferences as well as the principal's ability to make use of them. Furthermore additional empirical and experimental studies have to be conducted. Only through the usage of diverse scientific methodological approaches, the area „Behavioral Contract Theory“ can in the long run come up to the claim of more realistic results *and* assumptions compared to classical contract theory.

Appendix (Only for Peer Review)

Proof of Proposition 1

Solving the participation constraints with respect to α_0, β_0 and plugging the resulting expressions together with the reaction functions in equations 11–12 into the principal's utility function 6 yields his optimization problem for the case of productive interaction:

$$\begin{aligned}
\max_{\alpha_1\,,\alpha_2\,,\beta_1\,,\beta_2} U_P = {} & (1+k_A)\frac{p_{A1}^2}{c_{A1}}\alpha_1 - k_Bl_B\frac{p_{B1}^2}{c_{B1}}\alpha_1 + (1+k_A)\frac{p_{A2}^2}{c_{A2}}\alpha_2 - k_Bl_B\frac{p_{B2}^2}{c_{B2}}\alpha_2 + \\
& (1+k_B)\frac{p_{B1}^2}{c_{B1}}\beta_1 - k_Al_A\frac{p_{A1}^2}{c_{A1}}\beta_1 + (1+k_B)\frac{p_{B2}^2}{c_{B2}}\beta_2 - k_Al_A\frac{p_{A2}^2}{c_{A2}}\beta_2 - \\
& \frac{1}{2}\cdot\frac{(1+k_A)^2(1+k_B+k_Bl_B)}{(1+k_A)(1+k_B)-k_Al_Ak_Bl_B}\cdot\left(\frac{p_{A1}^2}{c_{A1}}\alpha_1^2+\frac{p_{A2}^2}{c_{A2}}\alpha_2^2\right) - \\
& \frac{1}{2}\cdot\frac{k_B^2l_B^2(1+k_A+k_Al_A)}{(1+k_A)(1+k_B)-k_Al_Ak_Bl_B}\cdot\left(\frac{p_{B1}^2}{c_{B1}}\alpha_1^2+\frac{p_{B2}^2}{c_{B2}}\alpha_2^2\right) - \\
& \frac{1}{2}\cdot\frac{(1+k_B)^2(1+k_A+k_Al_A)}{(1+k_A)(1+k_B)-k_Al_Ak_Bl_B}\cdot\left(\frac{p_{B1}^2}{c_{B1}}\beta_1^2+\frac{p_{B2}^2}{c_{B2}}\beta_2^2\right) - \\
& \frac{1}{2}\cdot\frac{k_A^2l_A^2(1+k_B+k_Bl_B)}{(1+k_A)(1+k_B)-k_Al_Ak_Bl_B}\cdot\left(\frac{p_{A1}^2}{c_{A1}}\beta_1^2+\frac{p_{A2}^2}{c_{A2}}\beta_2^2\right) + \\
& \frac{k_Al_A(1+k_A)(1+k_B+k_Bl_B)}{(1+k_A)(1+k_B)-k_Al_Ak_Bl_B}\cdot\left(\frac{p_{A1}^2}{c_{A1}}\alpha_1\beta_1+\frac{p_{A2}^2}{c_{A2}}\alpha_2\beta_2\right) + \\
& \frac{k_Bl_B(1+k_B)(1+k_A+k_Al_A)}{(1+k_A)(1+k_B)-k_Al_Ak_Bl_B}\cdot\left(\frac{p_{B1}^2}{c_{B1}}\alpha_1\beta_1+\frac{p_{B2}^2}{c_{B2}}\alpha_2\beta_2\right) - \\
& \frac{1+k_A+k_Al_A}{(1+k_A)(1+k_B)-k_Al_Ak_Bl_B}\cdot Risk_B - \frac{1+k_B+k_Bl_B}{(1+k_A)(1+k_B)-k_Al_Ak_Bl_B}\cdot Risk_A. \qquad (52)
\end{aligned}$$

Differentiating with respect to the four piece rates $\alpha_1\,,\alpha_2\,,\beta_1\,,\beta_2$ leads after some rearrangements to the first order conditions:

$$\textbf{Condition 1: } \frac{\partial U_P}{\partial \alpha_1} = 0$$

$$\begin{aligned}
\Leftrightarrow {} & [(1+k_A)(1+k_B)-k_Al_Ak_Bl_B]\cdot[(1+k_A)p_{A1}^2c_{B1}-k_Bl_Bp_{B1}^2c_{A1}] - \\
& [(1+k_A)^2(1+k_B+k_Bl_B)(p_{A1}^2+r_Ac_{A1}\sigma_1^2)c_{B1} + \\
& k_B^2l_B^2(1+k_A+k_Al_A)(p_{B1}^2+r_Bc_{B1}\sigma_1^2)c_{A1}]\alpha_1 - \\
& [(1+k_A)^2(1+k_B+k_Bl_B)r_Ac_{A1}c_{B1}\sigma_1\sigma_2\varrho + k_B^2l_B^2(1+k_A+k_Al_A)r_Bc_{A1}c_{B1}\sigma_1\sigma_2\varrho]\alpha_2 + \\
& [k_Al_A(1+k_A)(1+k_B+k_Bl_B)(p_{A1}^2+r_Ac_{A1}\sigma_1^2)c_{B1} + \\
& k_Bl_B(1+k_B)(1+k_A+k_Al_A)(p_{B1}^2+r_Bc_{B1}\sigma_1^2)c_{A1}]\beta_1 + \\
& [k_Al_A(1+k_A)(1+k_B+k_Bl_B)r_Ac_{A1}c_{B1}\sigma_1\sigma_2\varrho + \\
& k_Bl_B(1+k_B)(1+k_A+k_Al_A)r_Bc_{A1}c_{B1}\sigma_1\sigma_2\varrho]\beta_2 = 0 \qquad (53)
\end{aligned}$$

$$\textbf{Condition 2: } \frac{\partial U_P}{\partial \alpha_2} = 0$$

$$\Leftrightarrow [(1+k_A)(1+k_B) - k_A l_A k_B l_B] \cdot [(1+k_A) p_{A2}^2 c_{B2} - k_B l_B p_{B2}^2 c_{A2}] -$$
$$[(1+k_A)^2 (1+k_B+k_B l_B) r_A c_{A2} c_{B2} \sigma_1 \sigma_2 \varrho + k_B^2 l_B^2 (1+k_A+k_A l_A) r_B c_{A2} c_{B2} \sigma_1 \sigma_2 \varrho] \alpha_1 -$$
$$[k_B^2 l_B^2 (1+k_A+k_A l_A)(p_{B2}^2 + r_B c_{B2} \sigma_2^2) c_{A2} +$$
$$(1+k_A)^2 (1+k_B+k_B l_B)(p_{A2}^2 + r_A c_{A2} \sigma_2^2) c_{B2}] \alpha_2 +$$
$$[k_A l_A (1+k_A)(1+k_B+k_B l_B) r_A c_{A2} c_{B2} \sigma_1 \sigma_2 \varrho +$$
$$k_B l_B (1+k_B)(1+k_A+k_A l_A) r_B c_{A2} c_{B2} \sigma_1 \sigma_2 \varrho] \beta_1 +$$
$$[k_B l_B (1+k_B)(1+k_A+k_A l_A)(p_{B2}^2 + r_B c_{B2} \sigma_2^2) c_{A2} +$$
$$k_A l_A (1+k_A)(1+k_B+k_B l_B)(p_{A2}^2 + r_A c_{A2} \sigma_2^2) c_{B2}] \beta_2 = 0 \quad (54)$$

$$\textbf{Condition 3: } \frac{\partial U_P}{\partial \beta_1} = 0$$

$$\Leftrightarrow [(1+k_A)(1+k_B) - k_A l_A k_B l_B] \cdot [(1+k_B) p_{B1}^2 c_{A1} - k_A l_A p_{A1}^2 c_{B1}] +$$
$$[k_A l_A (1+k_A)(1+k_B+k_B l_B)(p_{A1}^2 + r_A c_{A1} \sigma_1^2) c_{B1} +$$
$$k_B l_B (1+k_B)(1+k_A+k_A l_A)(p_{B1}^2 + r_B c_{B1} \sigma_1^2) c_{A1}] \alpha_1 +$$
$$[k_A l_A (1+k_A)(1+k_B+k_B l_B) r_A c_{A1} c_{B1} \sigma_1 \sigma_2 \varrho +$$
$$k_B l_B (1+k_B)(1+k_A+k_A l_A) r_B c_{A1} c_{B1} \sigma_1 \sigma_2 \varrho] \alpha_2 -$$
$$[k_A^2 l_A^2 (1+k_B+k_B l_B)(p_{A1}^2 + r_A c_{A1} \sigma_1^2) c_{B1} +$$
$$(1+k_B)^2 (1+k_A+k_A l_A)(p_{B1}^2 + r_B c_{B1} \sigma_1^2) c_{A1}] \beta_1 -$$
$$[k_A^2 l_A^2 (1+k_B+k_B l_B) r_A c_{A1} c_{B1} \sigma_1 \sigma_2 \varrho + (1+k_B)^2 (1+k_A+k_A l_A) r_B c_{A1} c_{B1} \sigma_1 \sigma_2 \varrho] \beta_2 = 0 \quad (55)$$

$$\textbf{Condition 4: } \frac{\partial U_P}{\partial \beta_2} = 0$$

$$\Leftrightarrow [(1+k_A)(1+k_B) - k_A l_A k_B l_B] \cdot [(1+k_B) p_{B2}^2 c_{A2} - k_A l_A p_{A2}^2 c_{B2}] +$$
$$[k_A l_A (1+k_A)(1+k_B+k_B l_B) r_A c_{A2} c_{B2} \sigma_1 \sigma_2 \varrho +$$
$$k_B l_B (1+k_B)(1+k_A+k_A l_A) r_B c_{A2} c_{B2} \sigma_1 \sigma_2 \varrho] \alpha_1 +$$
$$[k_B l_B (1+k_B)(1+k_A+k_A l_A)(p_{B2}^2 + r_B c_{B2} \sigma_2^2) c_{A2} +$$
$$k_A l_A (1+k_A)(1+k_B+k_B l_B)(p_{A2}^2 + r_A c_{A2} \sigma_2^2) c_{B2}] \alpha_2 -$$
$$[k_A^2 l_A^2 (1+k_B+k_B l_B) r_A c_{A2} c_{B2} \sigma_1 \sigma_2 \varrho + (1+k_B)^2 (1+k_A+k_A l_A) r_B c_{A2} c_{B2} \sigma_1 \sigma_2 \varrho] \beta_1 -$$
$$[(1+k_B)^2 (1+k_A+k_A l_A)(p_{B2}^2 + r_B c_{B2} \sigma_2^2) c_{A2} +$$
$$k_A^2 l_A^2 (1+k_B+k_B l_B)(p_{A2}^2 + r_A c_{A2} \sigma_2^2) c_{B2}] \beta_2 = 0 \quad (56)$$

Solving this system of four equations with four unknown parameters finally results in the optimal values for the piece rates given in equations 13–16 in the main text. The second order conditions are fulfilled.

Proof of Proposition 2

The procedure of calculation is the same as in the case of productive interaction above. This time the principal's optimization problem takes the form:

$$\max_{\alpha_1\,,\,\alpha_2\,,\,\beta_1\,,\,\beta_2} U_P = (1+k_A)\frac{1}{c_{A1}}(1+p_{A2})\alpha_1 - k_B l_B \frac{p_{B1}}{c_{B2}}(1+p_{B1})\alpha_1 +$$
$$(1+k_A)\frac{p_{A2}}{c_{A1}}(1+p_{A2})\alpha_2 - k_B l_B \frac{1}{c_{B2}}(1+p_{B1})\alpha_2 + (1+k_B)\frac{p_{B1}}{c_{B2}}(1+p_{B1})\beta_1 -$$
$$k_A l_A \frac{1}{c_{A1}}(1+p_{A2})\beta_1 + (1+k_B)\frac{1}{c_{B2}}(1+p_{B1})\beta_2 - k_A l_A \frac{p_{A2}}{c_{A1}}(1+p_{A2})\beta_2 -$$
$$\frac{1}{2}\cdot\frac{1}{c_{A1}}\cdot\frac{(1+k_A)^2(1+k_B+k_B l_B)}{(1+k_A)(1+k_B)-k_A l_A k_B l_B}\cdot(\alpha_1+p_{A2}\alpha_2)^2 -$$
$$\frac{1}{2}\cdot\frac{1}{c_{B2}}\cdot\frac{k_B^2 l_B^2(1+k_A+k_A l_A)}{(1+k_A)(1+k_B)-k_A l_A k_B l_B}\cdot(p_{B1}\alpha_1+\alpha_2)^2 -$$
$$\frac{1}{2}\cdot\frac{1}{c_{B2}}\cdot\frac{(1+k_B)^2(1+k_A+k_A l_A)}{(1+k_A)(1+k_B)-k_A l_A k_B l_B}\cdot(p_{B1}\beta_1+\beta_2)^2 -$$
$$\frac{1}{2}\cdot\frac{1}{c_{A1}}\cdot\frac{k_A^2 l_A^2(1+k_B+k_B l_B)}{(1+k_A)(1+k_B)-k_A l_A k_B l_B}\cdot(\beta_1+p_{A2}\beta_2)^2 +$$
$$\frac{k_A l_A(1+k_A)(1+k_B+k_B l_B)}{(1+k_A)(1+k_B)-k_A l_A k_B l_B}\cdot\left[\frac{1}{c_{A1}}(\alpha_1\beta_1+p_{A2}\alpha_1\beta_2)+\frac{p_{A2}}{c_{A1}}(p_{A2}\alpha_2\beta_2+\alpha_2\beta_1)\right] +$$
$$\frac{k_B l_B(1+k_B)(1+k_A+k_A l_A)}{(1+k_A)(1+k_B)-k_A l_A k_B l_B}\cdot\left[\frac{p_{B1}}{c_{B2}}(p_{B1}\alpha_1\beta_1+\alpha_1\beta_2)+\frac{1}{c_{B2}}(\alpha_2\beta_2+p_{B1}\alpha_2\beta_1)\right] -$$
$$\frac{1+k_A+k_A l_A}{(1+k_A)(1+k_B)-k_A l_A k_B l_B}\cdot Risk_B - \frac{1+k_B+k_B l_B}{(1+k_A)(1+k_B)-k_A l_A k_B l_B}\cdot Risk_A. \quad (57)$$

Differentiating with respect to the four piece rates yields:

$$\textbf{Condition 1: } \frac{\partial U_P}{\partial \alpha_1} = 0$$

$$\Leftrightarrow [(1+k_A)(1+k_B)-k_A l_A k_B l_B]\cdot[(1+k_A)(1+p_{A2})c_{B2} - k_B l_B(1+p_{B1})p_{B1}c_{A1}] -$$
$$[(1+k_A)^2(1+k_B+k_B l_B)(1+r_A c_{A1}\sigma_1^2)c_{B2} + k_B^2 l_B^2(1+k_A+k_A l_A)(p_{B1}^2+r_B c_{B2}\sigma_1^2)c_{A1}]\alpha_1 -$$
$$[(1+k_A)^2(1+k_B+k_B l_B)(p_{A2}+r_A c_{A1}\sigma_1\sigma_2\varrho)c_{B2} +$$
$$k_B^2 l_B^2(1+k_A+k_A l_A)(p_{B1}+r_B c_{B2}\sigma_1\sigma_2\varrho)c_{A1}]\alpha_2 +$$
$$[k_A l_A(1+k_A)(1+k_B+k_B l_B)(1+r_A c_{A1}\sigma_1^2)c_{B2} +$$
$$k_B l_B(1+k_B)(1+k_A+k_A l_A)(p_{B1}^2+r_B c_{B2}\sigma_1^2)c_{A1}]\beta_1 +$$
$$[k_A l_A(1+k_A)(1+k_B+k_B l_B)(p_{A2}+r_A c_{A1}\sigma_1\sigma_2\varrho)c_{B2} +$$
$$k_B l_B(1+k_B)(1+k_A+k_A l_A)(p_{B1}+r_B c_{B2}\sigma_1\sigma_2\varrho)c_{A1}]\beta_2 = 0 \quad (58)$$

$$\textbf{Condition 2: } \frac{\partial U_P}{\partial \alpha_2} = 0$$

$$\begin{aligned}
\Leftrightarrow [(1+k_A)(1+k_B) - k_A l_A k_B l_B] \cdot [(1+k_A)(1+p_{A2})p_{A2}c_{B2} - k_B l_B(1+p_{B1})c_{A1}] - \\
[(1+k_A)^2(1+k_B+k_B l_B)(p_{A2}+r_A c_{A1}\sigma_1\sigma_2\varrho)c_{B2}+ \\
k_B^2 l_B^2(1+k_A+k_A l_A)(p_{B1}+r_B c_{B2}\sigma_1\sigma_2\varrho)c_{A1}]\alpha_1 - \\
[(1+k_A)^2(1+k_B+k_B l_B)(p_{A2}^2+r_A c_{A1}\sigma_2^2)c_{B2} + k_B^2 l_B^2(1+k_A+k_A l_A)(1+r_B c_{B2}\sigma_2^2)c_{A1}]\alpha_2 + \\
[k_A l_A(1+k_A)(1+k_B+k_B l_B)(p_{A2}+r_A c_{A1}\sigma_1\sigma_2\varrho)c_{B2}+ \\
k_B l_B(1+k_B)(1+k_A+k_A l_A)(p_{B1}+r_B c_{B2}\sigma_1\sigma_2\varrho)c_{A1}]\beta_1 + \\
[k_A l_A(1+k_A)(1+k_B+k_B l_B)(p_{A2}^2+r_A c_{A1}\sigma_2^2)c_{B2}+ \\
k_B l_B(1+k_B)(1+k_A+k_A l_A)(1+r_B c_{B2}\sigma_2^2)c_{A1}]\beta_2 = 0
\end{aligned} \quad (59)$$

$$\textbf{Condition 3: } \frac{\partial U_P}{\partial \beta_1} = 0$$

$$\begin{aligned}
\Leftrightarrow [(1+k_A)(1+k_B) - k_A l_A k_B l_B] \cdot [(1+k_B)(1+p_{B1})p_{B1}c_{A1} - k_A l_A(1+p_{A2})c_{B2}] + \\
[k_A l_A(1+k_A)(1+k_B+k_B l_B)(1+r_A c_{A1}\sigma_1^2)c_{B2}+ \\
k_B l_B(1+k_B)(1+k_A+k_A l_A)(p_{B1}^2+r_B c_{B2}\sigma_1^2)c_{A1}]\alpha_1 + \\
[k_A l_A(1+k_A)(1+k_B+k_B l_B)(p_{A2}+r_A c_{A1}\sigma_1\sigma_2\varrho)c_{B2}+ \\
k_B l_B(1+k_B)(1+k_A+k_A l_A)(p_{B1}+r_B c_{B2}\sigma_1\sigma_2\varrho)c_{A1}]\alpha_2 - \\
[(1+k_B)^2(1+k_A+k_A l_A)(p_{B1}^2+r_B c_{B2}\sigma_1^2)c_{A1} + k_A^2 l_A^2(1+k_B+k_B l_B)(1+r_A c_{A1}\sigma_1^2)c_{B2}]\beta_1 - \\
[(1+k_B)^2(1+k_A+k_A l_A)(p_{B1}+r_B c_{B2}\sigma_1\sigma_2\varrho)c_{A1}+ \\
k_A^2 l_A^2(1+k_B+k_B l_B)(p_{A2}+r_A c_{A1}\sigma_1\sigma_2\varrho)c_{B2}]\beta_2 = 0
\end{aligned} \quad (60)$$

$$\textbf{Condition 4: } \frac{\partial U_P}{\partial \beta_2} = 0$$

$$\begin{aligned}
\Leftrightarrow [(1+k_A)(1+k_B) - k_A l_A k_B l_B] \cdot [(1+k_B)(1+p_{B1})c_{A1} - k_A l_A(1+p_{A2})p_{A2}c_{B2}] + \\
[k_A l_A(1+k_A)(1+k_B+k_B l_B)(p_{A2}+r_A c_{A1}\sigma_1\sigma_2\varrho)c_{B2}+ \\
k_B l_B(1+k_B)(1+k_A+k_A l_A)(p_{B1}+r_B c_{B2}\sigma_1\sigma_2\varrho)c_{A1}]\alpha_1 + \\
[k_A l_A(1+k_A)(1+k_B+k_B l_B)(p_{A2}^2+r_A c_{A1}\sigma_2^2)c_{B2}+ \\
k_B l_B(1+k_B)(1+k_A+k_A l_A)(1+r_B c_{B2}\sigma_2^2)c_{A1}]\alpha_2 - \\
[(1+k_B)^2(1+k_A+k_A l_A)(p_{B1}+r_B c_{B2}\sigma_1\sigma_2\varrho)c_{A1}+ \\
k_A^2 l_A^2(1+k_B+k_B l_B)(p_{A2}+r_A c_{A1}\sigma_1\sigma_2\varrho)c_{B2}]\beta_1 - \\
[(1+k_B)^2(1+k_A+k_A l_A)(1+r_B c_{B2}\sigma_2^2)c_{A1}+ \\
k_A^2 l_A^2(1+k_B+k_B l_B)(p_{A2}^2+r_A c_{A1}\sigma_2^2)c_{B2}]\beta_2 = 0
\end{aligned} \quad (61)$$

Solving this system of equations results in the optimal values for $\alpha_1, \alpha_2, \beta_1, \beta_2$, given in equations 36–39 in the main text. The second order conditions again are fulfilled.

Proof of Proposition 3

To show the proposition, we calculate the principal's optimal goal function value in case of productive interaction through plugging the optimal values for the piece rates 13–16 together with the agents reactions 23–26 in his goal function 6. This leads us to the formal expression:

$$\begin{aligned}
&U_P^*(k_A\,,k_B) = \\
&\frac{p_{A1}^2}{2c_{A1}} \cdot \frac{[(1+k_A)(1+k_B)-k_Al_Ak_Bl_B]}{1+k_B+k_Bl_B} \cdot \frac{p_{A1}^2p_{A2}^2 + r_A\sigma_2(p_{A1}^2c_{A2}\sigma_2 - p_{A2}^2c_{A1}\sigma_1\varrho)}{P} + \\
&\frac{p_{A2}^2}{2c_{A2}} \cdot \frac{[(1+k_A)(1+k_B)-k_Al_Ak_Bl_B]}{1+k_B+k_Bl_B} \cdot \frac{p_{A1}^2p_{A2}^2 + r_A\sigma_1(p_{A2}^2c_{A1}\sigma_1 - p_{A1}^2c_{A2}\sigma_2\varrho)}{P} + \\
&\frac{p_{B1}^2}{2c_{B1}} \cdot \frac{[(1+k_A)(1+k_B)-k_Al_Ak_Bl_B]}{1+k_A+k_Al_A} \cdot \frac{p_{B1}^2p_{B2}^2 + r_B\sigma_2(p_{B1}^2c_{B2}\sigma_2 - p_{B2}^2c_{B1}\sigma_1\varrho)}{Q} + \\
&\frac{p_{B2}^2}{2c_{B2}} \cdot \frac{[(1+k_A)(1+k_B)-k_Al_Ak_Bl_B]}{1+k_A+k_Al_A} \cdot \frac{p_{B1}^2p_{B2}^2 + r_B\sigma_1(p_{B2}^2c_{B1}\sigma_1 - p_{B1}^2c_{B2}\sigma_2\varrho)}{Q}.
\end{aligned} \tag{62}$$

Each of the summands in 62 consists of two terms: One that is constituted by social preferences and one that is known form the situation of purely egoistical behavior. Therefore, we proceed by first considering the case of one-sided social preferences ($k_B = 0$). In this case $\frac{[(1+k_A)(1+k_B)-k_Al_Ak_Bl_B]}{1+k_B+k_Bl_B}$ becomes $(1+k_A)$ and $\frac{[(1+k_A)(1+k_B)-k_Al_Ak_Bl_B]}{1+k_A+k_Al_A}$ simplifies to $\frac{1+k_A}{1+k_A+k_Al_A}$. Since $(1+k_A)$ increases at a much higher rate than $\frac{1+k_A}{1+k_A+k_Al_A}$ decreases, the principal always profits from A's social preferences, when

$$\begin{aligned}
&\frac{p_{A1}^2p_{A2}^2 + r_A\sigma_2(p_{A1}^2c_{A2}\sigma_2 - p_{A2}^2c_{A1}\sigma_1\varrho)}{P} + \frac{p_{A1}^2p_{A2}^2 + r_A\sigma_1(p_{A2}^2c_{A1}\sigma_1 - p_{A1}^2c_{A2}\sigma_2\varrho)}{P} \geq \\
&\frac{p_{B1}^2p_{B2}^2 + r_B\sigma_2(p_{B1}^2c_{B2}\sigma_2 - p_{B2}^2c_{B1}\sigma_1\varrho)}{Q} + \frac{p_{B1}^2p_{B2}^2 + r_B\sigma_1(p_{B2}^2c_{B1}\sigma_1 - p_{B1}^2c_{B2}\sigma_2\varrho)}{Q}.
\end{aligned} \tag{63}$$

If this condition is not fulfilled he still normally profits from A's rivalry, if k_A is sufficiently large. Only if

$$\begin{aligned}
&\frac{p_{B1}^2p_{B2}^2 + r_B\sigma_2(p_{B1}^2c_{B2}\sigma_2 - p_{B2}^2c_{B1}\sigma_1\varrho)}{Q} + \frac{p_{B1}^2p_{B2}^2 + r_B\sigma_1(p_{B2}^2c_{B1}\sigma_1 - p_{B1}^2c_{B2}\sigma_2\varrho)}{Q} \gg \\
&\frac{p_{A1}^2p_{A2}^2 + r_A\sigma_2(p_{A1}^2c_{A2}\sigma_2 - p_{A2}^2c_{A1}\sigma_1\varrho)}{P} + \frac{p_{A1}^2p_{A2}^2 + r_A\sigma_1(p_{A2}^2c_{A1}\sigma_1 - p_{A1}^2c_{A2}\sigma_2\varrho)}{P},
\end{aligned} \tag{64}$$

A's stronger developed rivalry usually leads to a reduction of the principal's goal function value. This establishes part one of the proposition. To see the interplay of the agents' social preferences in case of two-sided rivalry, suppose that the agents are equal apart from their social preferences. The weighting factor $\frac{[(1+k_A)(1+k_B)-k_Al_Ak_Bl_B]}{1+k_B+k_Bl_B}$ takes its highest value when k_A is preferably large while k_B at the same time is small. Analogously $\frac{[(1+k_A)(1+k_B)-k_Al_Ak_Bl_B]}{1+k_A+k_Al_A}$ is the highest when, when k_B is large and k_A is small. If k_A and k_B increase at the same rate for $l_A = l_B = 1$, the values of $\frac{[(1+k_A)(1+k_B)-k_Al_Ak_Bl_B]}{1+k_B+k_Bl_B}$ and $\frac{[(1+k_A)(1+k_B)-k_Al_Ak_Bl_B]}{1+k_A+k_Al_A}$ remain unchanged. Therefore firm profits take their maximum if the difference between k_A and k_B is possibly large. If additionally

$$\frac{p_{A1}^2 p_{A2}^2 + r_A\sigma_2(p_{A1}^2 c_{A2}\sigma_2 - p_{A2}^2 c_{A1}\sigma_1\varrho)}{P} + \frac{p_{A1}^2 p_{A2}^2 + r_A\sigma_1(p_{A2}^2 c_{A1}\sigma_1 - p_{A1}^2 c_{A2}\sigma_2\varrho)}{P} \tag{65}$$

and

$$\frac{p_{B1}^2 p_{B2}^2 + r_B\sigma_2(p_{B1}^2 c_{B2}\sigma_2 - p_{B2}^2 c_{B1}\sigma_1\varrho)}{Q} + \frac{p_{B1}^2 p_{B2}^2 + r_B\sigma_1(p_{B2}^2 c_{B1}\sigma_1 - p_{B1}^2 c_{B2}\sigma_2\varrho)}{Q} \tag{66}$$

differ, the principal wants the agent to exhibit rivalry, whose contribution to his goal function value would be larger in the situation of purely egoistical behavior.

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Notes

[1] For contributions regarding the weighting and the combination of performance measures in case of purely egoistical behavior see Spremann (1987), pp. 22–26, Banker/Datar (1989), Feltham/Xie (1994), Datar/Kulp/Lambert (2001) as well as Christensen/Feltham (2005), especially p. 573 et sqq.

[2] The problem of effort reduction because of asymmetrically distributed information and conflicting objectives is referred to as moral hazard in agency theory. See Mirrlees (1976), Mirrlees (1999), Jensen/Meckling (1976), Holmström (1979) and Grossman/Hart (1983).

[3] See Holmström (1982), pp. 334–338. For the underlying information criterion see Holmström (1979).

[4] For previous agency studies addressing the conflict between cooperation and competition see Itoh (1992), Ramakrishnan/Thakor (1991), Choi (1993) and Macho-Stadler/Pérez-Castrillo (1993).

[5] See Fehr/Schmidt (2003), Fehr/Falk (2002), Fehr/Fischbacher (2002) and Camerer (2003), pp. 43–113.

[6] See Fehr/Fischbacher/Kosfeld (2005).

[7] The non-anonymity of players as well as the opportunity to reward and punish among themselves is of crucial importance in this context. See Gintis (2000), p. 240 et seq.

[8] By contrast, social preferences are only of minor importance on anonymous markets, where all relevant information can be contractually fixed (e.g. auction and oligopoly markets). See Gintis (2000), pp. 239–241 and Fehr/Schmidt (2003), pp. 242–244.

[9] Rivalry comprises the combination of envy and spite.

[10] See Englmaier/Wambach (2005), Itoh (2004), Dur/Glazer (2008), Mayer/Pfeiffer (2004) and Mayer (2006).

[11] See Fehr/Schmidt (1999).

[12] See Holmström (1979) and Holmström (1982).

[13] Bartling/von Siemens (2006) similarly reveal a tendency towards flat-wage contracts when agents are envious. They further provide formal arguments for this form of remuneration to be more widespread in firms than in markets.

[14] Further studies, that are more distantly related to our subject are Demougin/Fluet (2003), Grund/Sliwka (2005) and Demougin/Fluet/Helm (2006).

[15] See Spremann (1987) and Holmström/Milgrom (1987).

[16] For the importance of monetary performance measures see Vancil (1979), pp. 82–85.

[17] Positive values of ϱ seem to be of higher importance in practical applications, since several decentralized units within the same firm are usually exposed to exogenous shocks following similar patterns like the general business cycle, for example.

[18] The optimality of linear incentive contracts is controversial in the literature. For theoretical foundations see Holmström/Milgrom (1987), Ewert/Wagenhofer (1993) and Pfingsten (1995). A critical assessment of the LEN-assumptions is given in Hemmer (2004).

[19] An additional characteristic of the employed specification is, that higher values of l_A and l_B enhance the impact of the social preference through the multiplicative combination with k_A and k_B. Thereby the model represents, that particularly ambitious persons with a high aspiration level tend to behave more competitively than people with a comparatively low aspiration level.

[20] For purely egoistical behavior, the decision problem can be separated in one for agent A and one for agent B. When $\varrho = 0$ an analogous separation in the two performance measures x_1 and x_2 is feasible.

[21] See for example Holmström (1982) as well as Itoh (1992).

[22] We return to the question under which circumstances the principal benefits from his agents' social preferences in section 6.

[23] Compared to the other three-dimensional graphics, this figure is turned by 180° for perspective reasons. The scaling of the x- and y-axis are consequently decreasing.

[24] The index of k_A is suppressed in the diagram. The underlying parameter constellation for the graphic is $p_{A1} = p_{A2} = p_{B1} = p_{B2} = c_{A1} = c_{B2} = r_A = r_B = \sigma_1^2 = \sigma_2^2 = 1\,, c_{B1} = c_{A2} = 6\,, l_A = 4$.

[25] The parameter constellation like in figure 1 is $p_{A1} = p_{A2} = p_{B1} = p_{B2} = c_{A1} = c_{B2} = r_A = r_B = \sigma_1^2 = \sigma_2^2 = 1\,, c_{B1} = c_{A2} = 6\,, l_A = 4$. The index of k_A again is suppressed.

[26] See Sandner (2008), pp. 60–62.

[27] See Pedell (2000), p. 107 et sqq. for the depiction of a firm's value creation network with the various vertical as well as horizontal interdependencies.

[28] See Choi (1993).

[29] For the interaction of the different effects with interpretations see Choi (1993).

[30] For a more detailed analysis of this situation see Sandner (2008), pp. 55–59.

[31] For extreme parameter constellations it is theoretically even possible that both of an agents' piece rates switch signs.

[32] A similar result was already achieved earlier for the case of inequity aversion. See Itoh (2004), pp. 36–37 and Bartling/von Siemens (2005), pp. 10–11.

[33] See Sandner (2008), pp. 45–66.

[34] For a corresponding conjecture see Itoh (2004), p. 42.

[35] This is the case for the agent, who is endowed with higher productivity, lower effort costs as well as smaller risk aversion and has the responsibility for the less volatile division result.

[36] It has to be stated that agent A can even have an incentive to lessen agent B's division result, although he positively takes part in it through his wage compensation system. The reason is, that A also accounts for the consequences of his actions on B's remuneration when choosing efforts. Positive effects not only enhance own remuneration but also heighten B's reward at the same time. Thus, A may be willing to forgo own remuneration in order to avoid an increase in B's wage compensation or in order to achieve a decline in his rewards. The trade-off established through the principal's incentive system causes, that in case of rivalry sabotage exactly occurs under circumstances where it would have also occurred under purely egoistical behavior.

[37] In the subsequent explanations we imply, that the agents only differ in the strength of their social preferences. If not explicitly stated otherwise, we further assume $l_A = l_B = 1$.

[38] This case is not part of our study. For examinations see Kandori (2003) as well as Bartling/von Siemens (2004). As is well-known, in the presence of social preferences multiple equilibria can arise in a public goods game. While Kandori (2003) explicitly includes the possibility of multiple equilibria in his model and focuses on the analysis of their stability, Bartling/von Siemens (2004) ex ante exclude such constellations by the construction of their model.

[39] Sandner (2008), p. 135 et sqq. studies the impacts of altruism on the weighting and the combination of performance measures. However, in doing so he only considers the case of technological independence.

Summary

This paper addresses the question, what metrics should be used for performance evaluation and in particular how they should be weighted and combined in the presence of technological interdependencies when the agents exhibit variedly strong developed rivalry. We find that the principal reacts to his agents' competitive preferences through a reallocation of incentive intensity. As a consequence, depending on the underlying sort of technological interdependency, various differences in the balancing of performance measures compared to the case of purely egoistical behavior arise and changes in the agents' basic types of compensation can occur. We further show that the principal does not want both of his agents to behave equally competitively. Instead, he can only profit when the agents are asymmetrical. Then the principal wants the more productive agent to exhibit rivalry while the other ideally should behave completely egoistically.

Zusammenfassung

Der Beitrag analysiert die Fragestellung, wie Performancemaße bei Bestehen technologischer Abhängigkeiten zu gewichten und verknüpfen sind, wenn sich hierarchisch gleichgestellte Agenten untereinander rivalistisch verhalten. Das Ergebnis ist, dass der Prinzipal auf das Wettbewerbsdenken seiner Agenten durch eine Umverteilung in ihren Anreizintensitäten reagiert. Verglichen mit der Situation rein egoistischen Verhaltens, ergibt sich daraus ein veränderter Ausgleich in der Gewichtung von Performancemaßen. Als Folge sind Wechsel in den jeweils anzuwendenden Entlohnungsarten möglich, wobei die Auswirkungen von der Art der zugrunde gelegten Produktionstechnologie abhängig sind. Wir zeigen weiterhin, dass der Prinzipal keinen Nutzen daraus zieht, wenn sich beide Agenten gleichermaßen rivalistisch verhalten. Er kann nur dann profitieren, wenn sie sich in ihren (sozialen) Präferenzen unterscheiden. In diesem Fall ist es aus seiner Sicht wünschenswert, dass sich der produktivere Agent möglichst rivalistisch, der andere hingegen vollständig eigennützig verhält.