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The Choice of Prices vs. Quantities under Uncertainty*

Markus Reisinger† and Ludwig Ressner‡

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Abstract

This paper analyzes a duopoly model with stochastic demand in which firms first choose their strategy variable and compete afterwards. Contrary to the existing literature, we show that firms do not always choose a quantity which is the variable that induces a smaller degree of competition. The reason is that demand uncertainty and the degree of substitutability have countervailing effects on variable choice. Higher uncertainty favors prices, while closer substitutability favors quantities. Moreover, for intermediate values firms choose different strategy variables in equilibrium.

JEL classification: D43, L13

Keywords: Competition, Strategy Variables, Demand Uncertainty

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1 Introduction

The two classic papers in the theory of strategic interaction among firms are those by Cournot (1838) and Bertrand (1883). The first one proposes quantities as the strategy variable while the latter one suggests prices. Since then it is well known that quantities induce a lower degree of competition. So if firms are free to choose their strategy variable, they would prefer quantities rather than prices. This result was first confirmed by Singh and Vives (1984) and Cheng (1985) in a deterministic two-stage game in which duopolistic firms first choose their strategy variable and compete afterwards. They show that quantities are a dominant action for both firms.[1]

However, a deterministic model might not be fully appropriate because firms often face uncertainty at the time the strategy variable has to be chosen. For example, firms may be uncertain about the size of the market or about the distribution of consumers' reservation prices. Our analysis incorporates this aspect by introducing uncertainty via shocks that affect the slope and the intercept of the demand curve. In our set-up, we show that the dominance of quantities no longer holds since a higher amount of uncertainty lowers firms' profits under quantity competition and favors prices. So if uncertainty is high compared to the degree of substitutability, it is a dominant strategy for each firm to set a price. Moreover, we find that for an intermediate amount of uncertainty the unique equilibrium outcome is that one firm selects a price and the other one a quantity.

Our analysis employs the same game structure as Singh and Vives (1984) and Cheng (1985), namely firms first select their strategy variable independently of each other and then compete. Yet, while these authors analyze the deterministic case, we consider stochastic demand. We set out by developing the main insights in the simplest possible framework. Specifically, we consider a linear demand system where a shock affects the slope of the demand curves. We demonstrate that there is a relative advantage of price setting due to uncertainty. The reason is that a firm's choice of its strategy variable determines how the shock affects its expected profit. If a firm fixes its price, the quantity responds to the shock in a more favorable way than the price would respond if it fixed the quantity instead. This effect increases in the size of the shock. On the other hand, the relative advantage

[1] Tanaka (2001) analyzes a more general model with n firms and finds that all firms selecting quantities is a subgame perfect equilibrium. Recently, Tasnadi (2006) showed for the case with homogeneous goods and n firms that the only equilibrium is that all firms select quantities if they are not capacity constrained.

of quantities is that they induce a smaller degree of competition and this becomes more pronounced the larger the degree of substitutability. Thus, the degree of substitutability and the degree of uncertainty have countervailing effects on variable choice. We show that the subgame perfect equilibrium outcome under uncertainty is unique. Firms select prices as their strategic variable if uncertainty is high relative to the degree of substitutability and select quantities if the reverse holds true. Moreover, for every degree of substitutability there exists an intermediate range of uncertainty in which both effects balance each other and the unique equilibrium outcome involves one firm setting a price and the other one choosing a quantity.[2]

We extend our analysis by introducing a shock to the intercept that might be correlated with the shock to the slope. The same line of reasoning applies. However, now it is the covariance that in addition to the variance of the slope shock drives firms' choice of strategy variables. If the covariance is positive and high relative to the degree of substitutability, both firms select prices rather than quantities and vice versa. The "hybrid" outcome in which firms choose different strategy variables arises only for sufficiently high degrees of substitutability.

There are only few papers that deal with the choice of prices versus quantities in a stochastic environment. Weitzman (1974) analyzes the incentives of a social planner to regulate prices or quantities in the presence of demand uncertainty. Reis (2006) considers the choice of a monopolist under general demand conditions. The only paper that explicitly analyzes this choice in an oligopolistic setting is Klemperer and Meyer (1986). They consider a one-shot duopoly game in which a firm chooses the strategy variable and its magnitude at the same time. As the game has a simultaneous structure, firms do not choose the mode of competition. Instead, each firm acts as a monopolist given its expected residual demand curve. Therefore in their framework the relative advantage of quantities is not present. Our analysis shows that there is a trade-off between uncertainty and the degree of competition and derives conditions under which either strategy variable's relative advantage dominates.

The rest of the paper is organized as follows. Section 2 sets out the model. In Section 3 we solve for the subgame perfect equilibrium in case of a shock affecting the slope. In Section 4 we extend the model by incorporating a shock to the

[2]That the choice of strategy variables varies across firms seems to be in line with empirical research. For example, Aiginger (1999) asked managers of 930 manufacturing firm in Austria if they select prices or quantities as their decision variable. Roughly, 2/3 charge prices and 1/3 set quantities.

intercept. Section 5 concludes.

2 The Model

Consider a duopoly with differentiated products. Assume that firms face the linear inverse demand system

$$p_i = \alpha - \frac{\beta}{\theta} q_i - \frac{\gamma}{\theta} q_j, \tag{1}$$

$$p_j = \alpha - \frac{\beta}{\theta} q_j - \frac{\gamma}{\theta} q_i, \tag{2}$$

with $\alpha > 0$ and $\beta > \gamma \geq 0$.[3] When $\gamma \to \beta$, products become perfect substitutes, whereas with $\gamma = 0$ they are independent. θ is a random variable with $E[\theta] = 1$ and $\text{Var}(\theta) = \sigma_\theta^2 > 0$. We denote $E[\frac{1}{\theta}]$ by z. By Jensen's inequality, $z > 1$ and increases in σ_θ^2. To avoid unnecessary complications we require the support of θ to be sufficiently small such that no equilibria emerge in which a price setting firm sells a negative quantity or a quantity setting firm receives a negative price. We further assume that firms have zero marginal costs.[4]

Competition between firms takes the form of a two-stage game. In stage 1 firms simultaneously and irrevocably choose their strategy variables. Each firm observes the other firm's choice and competes in stage 2 contingent on the chosen strategy variables. Thereafter the shock realizes, markets clear, and profits accrue. We solve for the subgame perfect equilibrium by backward induction.

3 Solution to the Model

3.1 The Second Stage

First, suppose that both firms set prices as their strategy variable. Solving equations (1) and (2) for q_i and q_j gives firm i's demand curve, $q_i = \frac{\theta(\alpha(\beta-\gamma)-\beta p_i+\gamma p_j)}{\beta^2-\gamma^2}$. As $E[\theta] = 1$, firm i thus solves

$$\max_{p_i} \quad p_i \left(\frac{\alpha(\beta - \gamma) - \beta p_i + \gamma p_j}{\beta^2 - \gamma^2} \right).$$

[3]The demand system and the way in which the shock affects the slopes of the inverse demand curves is the same as in Klemperer and Meyer (1986).

[4]As Singh and Vives (1984) show, the analysis would not change if firms faced positive constant marginal costs c because this would only lower the effective intercept from α to $\alpha - c$.

Computing the solution to the maximization problems of firm i and j yields equilibrium prices of

$$p_i = p_j = \frac{(\beta - \gamma)\alpha}{2\beta - \gamma}.$$

Therefore each firm's expected profit is equal to

$$\Pi^{pp} = \frac{\alpha^2(\beta - \gamma)\beta}{(\beta + \gamma)(2\beta - \gamma)^2}.$$

Next, suppose that both firms set quantities as their strategy variable. Since $E[\frac{1}{\theta}] = z$, firm i solves

$$\max_{q_i} \quad q_i(\alpha - z(\beta q_i + \gamma q_j)).$$

Computing the solution to the maximization problem of both firms yields equilibrium quantities of

$$q_i = q_j = \frac{\alpha}{z(2\beta + \gamma)}$$

and an expected profit of

$$\Pi^{qq} = \frac{\alpha^2\beta}{z(2\beta + \gamma)^2}$$

for each firm.

Lastly, if firm i sets a price while firm j sets a quantity, the demand curve of firm i is $q_i = \frac{\alpha\theta - \theta p_i - \gamma q_j}{\beta}$ and the inverse demand curve of firm j is $p_j = \frac{(\alpha\theta - q_j(\beta+\gamma))(\beta-\gamma)}{\beta\theta}$ $+\frac{\gamma p_i}{\beta}$. Computing the profit functions, maximizing and solving for the equilibrium yields a price of

$$p_i = \frac{\alpha^2(\beta - \gamma)(2z(\beta + \gamma) - \gamma)}{4z(\beta^2 - \gamma^2) + \gamma^2}$$

and a quantity of

$$q_j = \frac{\alpha(2\beta - \gamma)}{4z(\beta^2 - \gamma^2) + \gamma^2}.$$

The expected profit of the price setting firm is

$$\Pi^{pq} = \frac{(\beta - \gamma)^2(2z\alpha^2(\beta + \gamma) - \gamma)^2}{(4z(\beta^2 - \gamma^2) + \gamma^2)^2\beta},$$

while the expected profit of the quantity setting firm is

$$\Pi^{qp} = \frac{z\alpha^2(\beta^2 - \gamma^2)(2\beta - \gamma)^2}{(4z(\beta^2 - \gamma^2) + \gamma^2)^2\beta}.$$

Before we continue with the analysis we introduce some notation.[5] Conditional on firm j setting a quantity the difference in profits of firm i between setting a price and setting a quantity is defined as

$$\Delta\Pi^q(\gamma, z) := \Pi^{pq} - \Pi^{qq}.$$

If firm j sets a price this difference is defined as

$$\Delta\Pi^p(\gamma, z) := \Pi^{pp} - \Pi^{qp}.$$

3.2 The First Stage

As spelled out before, if the game is deterministic ($\sigma_\theta^2 = 0$) it is the dominant strategy for firms to set quantities in the first stage since they induce a lower degree of competition.[6] The following Lemma shows that this is no longer true under uncertainty.

Lemma 1

For every $\gamma \in (0, \beta)$ there exists a unique z, labeled $z^q(\gamma)$, such that

$$\Delta\Pi^q(\gamma, z) \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \;\; \textit{if} \;\; z \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} z^q(\gamma),$$

and there exists a unique z, labeled $z^p(\gamma)$, such that

$$\Delta\Pi^p(\gamma, z) \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \;\; \textit{if} \;\; z \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} z^p(\gamma).$$

$z^q(\gamma)$ and $z^p(\gamma)$ are strictly increasing with $\lim_{\gamma\to 0} z^q(\gamma) = \lim_{\gamma\to 0} z^p(\gamma) = 1$ and $z^p(\gamma) > z^q(\gamma)$ for $\gamma \in (0, \beta)$.

Proof. See Appendix.

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[5] In the following analysis we hold α and β fixed and consider only changes in γ and z to point out the tension between the degree of substitutability and the amount of uncertainty.

[6] This is easy to check since for all $\gamma \in (0, \beta)$, $\Delta\Pi^q(\gamma, 1) < 0$ and $\Delta\Pi^p(\gamma, 1) < 0$.

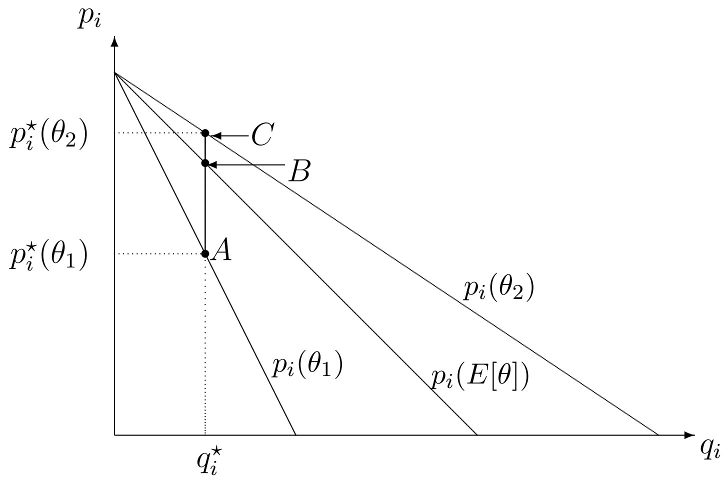

Figure 1: Example with quantity setting

The result that $z^q(\gamma)$ and $z^p(\gamma)$ are strictly increasing highlights the trade-off between uncertainty and the degree of substitutability. While a higher amount of risk (higher z) favors price setting, a higher degree of substitutability (higher γ) favors quantity setting. An increase in the degree of substitutability makes the market more competitive. Since quantities are the less aggressive strategy variable, such an increase makes quantity setting more attractive relative to price setting. The opposite holds true concerning uncertainty. Let us explain this in more detail. First look at the case in which both firms select quantities. As can be seen from the demand system, the shock enters the inverse demand function in a non-linear way. The bigger the variance σ_θ^2 of the shock, the larger is z. As a consequence the expected price is decreasing in z. This is illustrated in an example in Figure 1.

Here θ can take on two values, either $\theta_1 = \frac{1}{2}$ or $\theta_2 = \frac{3}{2}$ with equal probability. Thus, $E[\theta] = 1$ and $z = \frac{4}{3} > 1$. The consequence is that $\frac{1}{2}\Big(p_i^\star(\theta_1) + p_i^\star(\theta_2)\Big) < p_i^\star(E[\theta])$. As is obvious, the price decrease following a bad shock ($\overline{AB}$ in Figure 1) is larger than the price increase following a good shock of same size ($\overline{BC}$ in Figure 1). Thus, as is evident in Π^{qq}, firms' profits in the quantity setting case are decreasing in the size of the shock. Next, consider the case in which both firms select prices. The expression for Π^{pp} shows that firms' expected profits are not affected by the size of the shock. The reason is that the shock enters the demand equation in a linear way and therefore cancels out in expectation. Lastly, in the hybrid case the shock affects the demand of the price setting firm linearly but it enters the inverse demand for the quantity setting firm non-linearly. Thus, the larger is σ_θ^2, the higher is the expected price-decrease that the quantity setting firm experiences if it produces a higher amount. As a consequence, it produces less, thereby leaving a larger residual demand curve to the price setting firm which in turn reacts by setting a higher price. Thus, as can be seen in Π^{pq}, the profit of the price setting firm increases in the size of the shock. This highlights that a higher amount of uncertainty favors the choice of prices while a higher degree of substitutability favors the choice of quantities.

This line of reasoning does not depend on the way we introduced uncertainty in the demand system. If, for example, the demand system were $p_i = \alpha - \beta\theta q_i - \gamma\theta q_j$, $i, j \in \{1, 2\}$, $i \neq j$, the result would be the same. In that case the shock would have no influence on expected profits if both firms set quantities but would increase expected profits if both firms select prices.

Now we characterize the subgame perfect equilibrium outcome of the two stage game under uncertainty.

Proposition 1

The subgame perfect equilibrium outcome of the two-stage game under uncertainty is the following:

Both firms select a quantity in the first stage if

$$z < z^q(\gamma),$$

and at least one firm selects a quantity if $z = z^q(\gamma)$.

One firms selects a price and the other firm a quantity in the first stage if

$$z^p(\gamma) > z > z^q(\gamma),$$

and at most one firm selects a quantity if $z = z^p(\gamma)$.

Both firms select a price in the first stage if $z > z^p(\gamma)$.

Proof

From Lemma 1 we know that if $z > (<)\ z^p(\gamma)$ a firm prefers to set a price (quantity) conditional on the other firm selecting a price. If $z = z^p(\gamma)$ it is indifferent between setting a price or a quantity.

Furthermore, if $z > (<)\ z^q(\gamma)$ a firm prefers to set a price (quantity) conditional on the other firm selecting a quantity. If $z = z^q(\gamma)$ it is indifferent between setting a price or a quantity.

Since $z^p(\gamma) > z^q(\gamma)$ it follows from the above that for $z < z^q(\gamma)$ the unique equilibrium involves both firms setting a quantity and for $z = z^q(\gamma)$ at least one

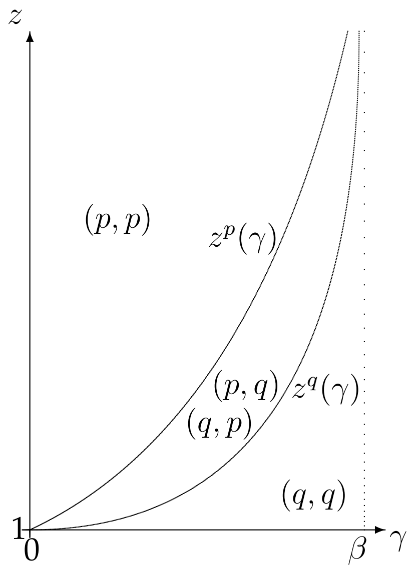

Figure 2: Equilibrium outcome with one-dimensional uncertainty

firm sets a quantity. Moreover, for $z > z^p(\gamma)$ the unique equilibrium involves both firms setting a price and for $z = z^p(\gamma)$ at least one firm sets a price. Now for $z^p(\gamma) > z > z^q(\gamma)$ firm i prefers to set a price conditional on firm j setting a quantity while firm j prefers to set a quantity conditional on firm i setting a price. ■

The equilibrium outcome of the game with one-dimensional uncertainty is displayed in Figure 2.

If uncertainty is high relative to the degree of substitution both firms select prices while they both select quantities if the opposite holds true.[7] But for any γ there exists a range of z such that the unique equilibrium outcome is hybrid, i.e. one firm charges a price and the other one sets a quantity. The intuition behind this result is the following: For a given degree of substitutability, competition is the fiercest if both firms charge prices while it is the softest when both set quantities. As a consequence, the size of the shock that induces both firms to play the price game must be strictly higher than the one that makes a firm indifferent between setting a price and setting a quantity conditional on the other firm setting a quantity. Thus, there always exists a range of z such that firms set different strategy variables in equilibrium.

We have restricted our attention to the case in which products are substitutes or independent, i.e. $\gamma \geq 0$. Here we note briefly that if products are complements,

[7] If products are nearly perfect substitutes ($\gamma \to \beta$), then $z^q(\gamma) \to \infty$ and quantities are the preferred choice for every z.

i.e. $\gamma < 0$, then it is a dominant strategy for both firms to set a price irrespective of the degree of uncertainty. The reason for this result is that both the competition and the uncertainty effect favor prices. Firstly, as Singh and Vives (1984) have shown, if products are complements, setting prices is the dominant strategy in a deterministic environment. Secondly, the relative advantage of setting a price with respect to uncertainty is still present since this effect is independent of whether the products are substitutes or complements.[8]

A possible limitation of our analysis is that we merely consider a linear demand system. We do that for tractability reasons and to make our point as clear as possible. Yet, our results are more general. The reason is the following: We have demonstrated that whether uncertainty favors prices or quantities depends on how the shock affects the residual demand curve of each firm. The central question is how this residual demand curve would change in a more general demand system. To this end, consider the case of a monopolist who faces the inverse demand system $p = \frac{f(q)}{\theta}$ with $f'(q) < 0$. As pointed out by Reis (2006), the same line of reasoning as in the case of a linear demand implies that the optimal choice of a monopolist is to set a price. Now when looking at a duopoly model, the difference is that the competitor additionally affects the intercept of the residual demand curve but leaves the advantage of prices unchanged. Thus, even with a general demand system, as uncertainty increases, duopolists prefer to set prices rather than quantities.

4 Two-Dimensional Uncertainty

So far we considered a shock affecting the slope of the demand curve. This implies that a firm knows the range of consumers' reservation prices but does not know the distribution. In reality however, even the range of reservation prices might be uncertain. In order to incorporate this aspect, we additionally consider a shock to the intercept.

[8]The analysis with $\gamma < 0$ is straightforward since, as first pointed out by Sonnenschein (1968), quantity (price) competition with substitutes is the dual to price (quantity) competition with complements.

Now the inverse demand system is given by

$$p_i = \alpha + \epsilon - \frac{\beta}{\theta} q_i - \frac{\gamma}{\theta} q_j, \tag{3}$$

$$p_j = \alpha + \epsilon - \frac{\beta}{\theta} q_j - \frac{\gamma}{\theta} q_i, \tag{4}$$

where ϵ is a random variable.[9] Without loss of generality we set $E[\epsilon] = 0$. We denote the covariance between the shocks by $\sigma_{\theta\epsilon}$.

Proceeding in the same way as before now yields equilibrium prices of

$$p_i = p_j = \frac{(\beta - \gamma)(\alpha + \sigma_{\theta\epsilon})}{2\beta - \gamma}$$

and expected equilibrium profits of

$$\Pi^{pp} = \frac{(\alpha + \sigma_{\theta\epsilon})^2(\beta - \gamma)\beta}{(\beta + \gamma)(2\beta - \gamma)^2}$$

in the case when both firms select a price as the strategy variable in the first stage. If both firms choose a quantity, the equilibrium quantities are the same as in the case of one-dimensional uncertainty and so the expected equilibrium profits are also the same, namely

$$\Pi^{qq} = \frac{\alpha^2\beta}{z(2\beta + \gamma)^2}.$$

If firms select different strategy variables, the price setting firm sets

$$p_i = \frac{(\beta - \gamma)(2z(\beta + \gamma)(\alpha + \sigma_{\theta\epsilon}) - \alpha\gamma)}{4z(\beta^2 - \gamma^2) + \gamma^2},$$

while the quantity setting firm chooses

$$q_j = \frac{\alpha(2\beta - \gamma) + \gamma\sigma_{\theta\epsilon}}{4z(\beta^2 - \gamma^2) + \gamma^2}.$$

As a consequence, the price setting firm receives an expected equilibrium profit of

$$\Pi^{pq} = \frac{(\beta - \gamma)^2(2z(\beta + \gamma)(\alpha + \sigma_{\theta\epsilon}) - \alpha\gamma)^2}{(4z(\beta^2 - \gamma^2) + \gamma^2)^2\beta},$$

[9] We do not consider the case of a shock on the intercept alone. The reason is that ϵ enters both the inverse demand and the direct demand in a linear way. So this shock alone would cancel out, and it would be a dominant action for both firms to set a quantity.

while the quantity setting firm receives an expected equilibrium profit of

$$\Pi^{qp} = \frac{(\beta^2 - \gamma^2) z (\alpha(2\beta - \gamma) + \gamma \sigma_{\theta\epsilon})^2}{(4z(\beta^2 - \gamma^2) + \gamma^2)^2 \beta}.$$

As before we restrict our attention to cases in which the support of the shocks is such that realized prices and quantities are non-negative. As is evident from the equilibrium prices and quantities, if the covariance is 'too' negative, even the optimally chosen price or quantity becomes negative. Therefore we impose a lower bound on $\sigma_{\theta\epsilon}$. This restriction stems from the equilibrium price of the price setting firm in the hybrid equilibrium and is given by

$$\sigma_{\theta\epsilon} > -\alpha\Big(1 - \frac{\gamma}{2z(\beta+\gamma)}\Big) := \hat{\sigma}_{\theta\epsilon}(\gamma).$$

This ensures that all equilibrium prices and quantities are positive. Since we are only concerned with cases in which the support is such that realized prices or quantities are positive, $\hat{\sigma}_{\theta\epsilon}(\gamma)$ constitutes the lowest bound above which this is still possible for a small enough support of the noise.[10]

As in the case of one-dimensional uncertainty, before proceeding to the first stage we define $\Delta\Pi^q(\gamma, \sigma_{\theta\epsilon}) := \Pi^{pq} - \Pi^{qq}$ and $\Delta\Pi^p(\gamma, \sigma_{\theta\epsilon}) := \Pi^{pp} - \Pi^{qp}$, where $\Delta\Pi^q(\gamma, \sigma_{\theta\epsilon})$ denotes the difference in expected profits of firm i if firm j chooses a quantity in the first stage, while $\Delta\Pi^p(\gamma, \sigma_{\theta\epsilon})$ denotes the difference conditional on firm j selecting a price. The following Lemma summarizes the technicalities we need in order to derive the equilibrium.

Lemma 2

For any $\gamma \in [0, \beta)$ there exists a unique $\sigma_{\theta\epsilon} > \hat{\sigma}_{\theta\epsilon}(\gamma)$, labeled $\sigma^q_{\theta\epsilon}(\gamma)$, such that

$$\Delta\Pi^q(\gamma, \sigma_{\theta\epsilon}) \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \;\; \textit{if} \;\; \sigma_{\theta\epsilon} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \sigma^q_{\theta\epsilon}(\gamma).$$

Moreover, there exists a unique $\gamma^+ \in (0, \beta)$, such that for any $\gamma \in [0, \gamma^+)$ there

[10] Moreover, a positive or not too negative correlation is the more realistic case. If some consumers are willing to pay a high price for the good (positive shock on the intercept), it is likely that the market becomes larger as well (positive shock on the slope). It is hard to imagine a market in which a positive shock on the intercept is coupled with the expectation of a decreasing market size.

exists a unique $\sigma_{\theta\epsilon} > \hat{\sigma}_{\theta\epsilon}(\gamma)$*, labeled* $\sigma^p_{\theta\epsilon}(\gamma)$*, such that*

$$\Delta\Pi^p(\gamma, \sigma_{\theta\epsilon}) \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \text{ if } \sigma_{\theta\epsilon} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \sigma^p_{\theta\epsilon}(\gamma).$$

For any $\gamma \in (\gamma^+, \beta)$*,* $\Delta\Pi^p(\gamma, \sigma_{\theta\epsilon}) < 0$ *for all* $\sigma_{\theta\epsilon} > \hat{\sigma}_{\theta\epsilon}(\gamma)$*.*

Finally $\sigma^q_{\theta\epsilon}(\gamma) > \sigma^p_{\theta\epsilon}(\gamma)$ *for* $\gamma \in (0, \beta(1 - \frac{1}{\sqrt{z}}))$*,* $\sigma^q_{\theta\epsilon}(\gamma) < \sigma^p_{\theta\epsilon}(\gamma)$ *for* $\gamma \in (\beta(1 - \frac{1}{\sqrt{z}}), \gamma^+)$*, and* $\sigma^q_{\theta\epsilon}(\gamma) = \sigma^p_{\theta\epsilon}(\gamma)$ *for* $\gamma \in \{0, \beta(1 - \frac{1}{\sqrt{z}})\}$*.*

Proof. See Appendix.

Now we state the subgame perfect equilibrium outcome of the game with two-dimensional uncertainty.

Proposition 2

If $0 < \gamma < \beta\Big(1 - \frac{1}{\sqrt{z}}\Big)$ *the subgame perfect equilibrium outcome of the game with two-dimensional uncertainty is the following:*

Both firms select a quantity in the first stage if

$$\sigma_{\theta\epsilon} < \sigma^p_{\theta\epsilon}(\gamma).$$

Either both firms select a price or both firms select a quantity in first stage if

$$\sigma^q_{\theta\epsilon}(\gamma) \geq \sigma_{\theta\epsilon} \geq \sigma^p_{\theta\epsilon}(\gamma).$$

Both firms select a price in the first stage if

$$\sigma_{\theta\epsilon} > \sigma^q_{\theta\epsilon}(\gamma).$$

If $\gamma > \beta\Big(1 - \frac{1}{\sqrt{z}}\Big)$ *the unique subgame perfect equilibrium outcome of the two-stage-game with two-dimensional uncertainty is the following:*

Both firms select a quantity in the first stage if

$$\sigma_{\theta\epsilon} < \sigma^q_{\theta\epsilon}(\gamma)$$

and at least one firm selects a quantity if $\sigma_{\theta\epsilon} = \sigma^q_{\theta\epsilon}(\gamma)$*.*

One firm selects a price and the other firm selects a quantity if

$$\sigma^p_{\theta\epsilon}(\gamma) > \sigma_{\theta\epsilon} > \sigma^q_{\theta\epsilon}(\gamma)$$

and at most one firm selects a quantity if $\sigma_{\theta\epsilon} = \sigma^p_{\theta\epsilon}(\gamma)$*.*

Both firms select a price in the first stage if

$$\sigma_{\theta\epsilon} > \sigma^p_{\theta\epsilon}(\gamma).$$

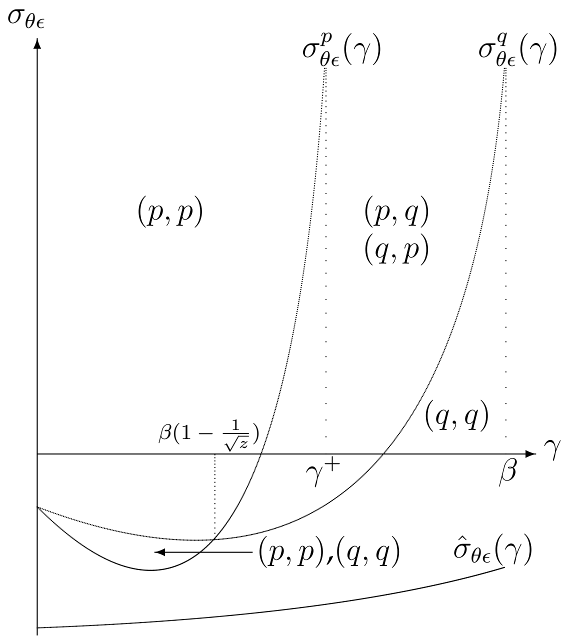

Figure 3: Equilibrium outcome with two-dimensional uncertainty

Proof

From Lemma 2 we know that if $\gamma \in [0, \gamma^+)$ a firm prefers to set a price (quantity) conditional on the other firm selecting a price if $\sigma_{\theta\epsilon} > (<)\ \sigma^p_{\theta\epsilon}(\gamma)$. If $\sigma_{\theta\epsilon} = \sigma^p_{\theta\epsilon}(\gamma)$ it is indifferent between setting a price or a quantity.

Furthermore, if $\sigma_{\theta\epsilon} > (<)\ \sigma^q_{\theta\epsilon}(\gamma)$ a firm prefers to set a price (quantity) conditional on the other firm selecting a quantity. If $\sigma_{\theta\epsilon} = \sigma^p_{\theta\epsilon}(\gamma)$ it is indifferent between setting a price or a quantity.

Since $\sigma^q_{\theta\epsilon}(\gamma) > \sigma^p_{\theta\epsilon}(\gamma)$ for $0 < \gamma < \beta\Big(1-\frac{1}{\sqrt{z}}\Big)$, and $\sigma^q_{\theta\epsilon}(\gamma) < \sigma^p_{\theta\epsilon}(\gamma)$ for $\beta\Big(1-\frac{1}{\sqrt{z}}\Big) < \gamma < \gamma^+$ the result follows from the above statements. ■

The equilibrium outcome of the game with two-dimensional uncertainty is depicted in Figure 3.

This shows that for the case of a positive covariance the equilibrium outcome is similar to the one with one-dimensional uncertainty. For the same reason as before, the degree of competition and the amount of uncertainty have offsetting effects on variable choice. In the following, we briefly discuss the main differences that arise compared to the analysis without a shock on the intercept.

The first difference is that the parameters determining the choice of the strategy variable are now the covariance of the shocks and the variance of the slope shock instead of the latter alone. The reason is that the interplay of the shocks is crucial for the position of the residual demand curve. For example, if a firm selects a price as its decision variable in the first stage, the interplay between the shocks determines the quantity that it receives. Thus, the covariance enters its expected profit and determines the strategy variable chosen in equilibrium.

The second difference is that now even for a low degree of competition both firms select quantities if the shocks are sufficiently negatively correlated. With a negative covariance, the shocks affect the slope and the intercept of the expected demand function differently, i.e. if there is a positive shock on the intercept, a steeper slope becomes more likely and vice versa. Thereby the interplay of the shocks reduces the relative advantage of prices. Since the equilibrium profits in the quantity setting game are not affected by the covariance, setting a price might no longer be the preferred action even if uncertainty is high.

The third difference is that we get an equilibrium region in which either both firms choose a quantity or both firms set a price if the degree of substitutability is relatively small and the covariance is sufficiently negative. The reason for this result is the following: It is easy to see that the equilibrium profits of a price setting firm, i.e. Π^{pq} and Π^{pp}, increase in the covariance irrespective of the degree of substitutability. In contrast, the equilibrium profit of a quantity setting firm depends positively on the covariance if and only if its competitor sets a price and if the degree of substitutability is positive.

In order to see that this generates multiple equilibria, we first consider the case in which firm j sets a price and firm i is indifferent between setting a price or a quantity. If γ increases, starting from $\gamma = 0$, Π^{qp} and Π^{pp} fall due to competition. But there is an additional effect only on Π^{qp} which stems from the fact that with a positive degree of substitutability the covariance comes into play. Since $\sigma^p_{\theta\epsilon}(0)$ is negative, this decreases Π^{qp} further. Thus, in order to restore equality between Π^{qp} and Π^{pp}, $\sigma^p_{\theta\epsilon}(\gamma)$ must decrease since the covariance has a larger impact on the profit of the price setting firm.

Now suppose that firm j sets a quantity and that the degree of substitutability increases starting from $\gamma = 0$. This decreases both Π^{pq} and Π^{qq} due to competition, with the latter decreasing relatively more than the former if the covariance is sufficiently negative. In contrast to the first case, there is no additional effect stemming from the interplay between the degree of substitutability and the covariance. Thus, the difference between the profits from price and quantity setting is larger for firm i if firm j selects a price. As a consequence, the decrease in the covariance that is needed in order to restore firm i's indifference is larger in this case. Thus, for relatively small degrees of substitutability, there exists a range of negative covariances such that the hybrid equilibrium does not exist because it is dominated by the price-price and the quantity-quantity equilibrium.

As in the case of one-dimensional uncertainty, the intuition of our results carries over to the case of a more general demand system, but with the qualification that demand is not too concave. Consider again the monopoly case where demand is given by $p = \frac{f(q)}{\theta} + \theta$, with $f'(q) < 0$. Combining the results of Klemperer and Meyer (1986) and Reis (2006) one can easily check that a monopolist prefers to set a price if $f(q)$ is convex or not too concave. By continuity reasons, this also holds if the shocks to the slope and the intercept are not perfectly but still positively correlated. Since this result also applies to the residual demand curve of a duopolist, uncertainty favors prices if demand is not too concave.

5 Conclusion

We show that the superiority of quantity competition for firms might no longer hold if there is a substantial amount of uncertainty concerning demand conditions. In the setting with a shock affecting the slope, we find that if uncertainty is high relative to the degree of substitutability, firms prefer to set prices rather than quantities. Moreover, for an intermediate range of uncertainty the unique equilibrium outcome is that firms choose different strategy variables. We also demonstrate that if there is a shock to the intercept and a shock to the slope, the important variable determining the mode of competition is the covariance between the shocks, but the basic intuition from the analysis with one-dimensional uncertainty carries over to this case. The paper provides the testable implication that if firms have some degree of freedom to choose their strategy variable, they should tend to choose quantities in industries with relatively stable and certain demand, but choose prices if demand is fluctuating and uncertain.

6 Appendix

6.1 Proof of Lemma 1

Existence and Uniqueness of $z^q(\gamma)$ and $z^p(\gamma)$

First, we show existence and uniqueness of $z^q(\gamma)$ and $z^p(\gamma)$ for any $\gamma \in (0, \beta)$.

The profit functions are rational functions defined on the domain $\gamma \in [0, \beta), z > 1$. Thus, $\Delta\Pi^q(\gamma, z)$ and $\Delta\Pi^p(\gamma, z)$ are continuous in γ and z and at least once continuously differentiable.

For arbitrary $\gamma \in (0, \beta)$,

$$\lim_{z\to 1} \Delta\Pi^q(\gamma, z) = \frac{\alpha^2\gamma^3(6\gamma^2\beta - 8\beta^3 + \gamma^3)}{(4\beta^2 - 3\gamma^2)^2\beta(2\beta + \gamma)^2} < 0,$$

and

$$\lim_{z\to\infty} \Delta\Pi^q(\gamma, z) = \frac{\alpha^2}{4\beta} > 0.$$

Since

$$\frac{\partial\Delta\Pi^q(\gamma, z)}{\partial z} = \frac{4\alpha^2(\beta-\gamma)^2(2z(\beta+\gamma)-\gamma)(\gamma+\beta)\gamma(2\beta-\gamma)}{(4z(\beta^2-\gamma^2)+\gamma^2)^3\beta} + \frac{\alpha^2\beta}{(2\beta+\gamma)^2 z^2} > 0, \tag{5}$$

we have shown that $z^q(\gamma)$ exists and that it is unique. An immediate consequence of this and the facts that $\lim_{z\to 1} \Delta\Pi^q(\gamma, z) < 0$ and $\lim_{z\to\infty} \Delta\Pi^q(\gamma, z) > 0$ is that $\Delta\Pi^q(\gamma, z) \gtreqless 0$ if $z \gtreqless z^q(\gamma)$.

Now we turn to the existence and uniqueness of $z^p(\gamma)$. For an arbitrary $\gamma \in (0, \beta)$

$$\lim_{z\to 1} \Delta\Pi^p(\gamma, z) = -\frac{(\beta-\gamma)\alpha^2\gamma^3(8\beta^3 - 6\gamma^2\beta + \gamma^3)}{(\beta+\gamma)(2\beta-\gamma)^2(4\beta^2-3\gamma^2)^2\beta} < 0,$$

and

$$\lim_{z\to\infty} \Delta\Pi^p(\gamma, z) = \frac{\beta(\beta-\gamma)\alpha^2}{(\beta+\gamma)(2\beta-\gamma)^2} > 0.$$

Since

$$\frac{\partial\Delta\Pi^p(\gamma, z)}{\partial z} = \frac{\alpha^2(2\beta-\gamma)^2(\beta^2-\gamma^2)(4z(\beta^2-\gamma^2)-\gamma^2)}{(4z(\beta^2-\gamma^2)+\gamma^2)^3\beta}, \tag{6}$$

which is negative for $z < \tilde{z}(\gamma) := \frac{\gamma^2}{4(\beta^2-\gamma^2)}$ and positive for $z > \tilde{z}(\gamma)$, we have shown that for every γ there exists a unique $z^p(\gamma) > \tilde{z}(\gamma)$. Combining the uniqueness of $z^p(\gamma)$ with the facts that $\lim_{z\to 1} \Delta\Pi^p(\gamma, z) < 0$ and $\lim_{z\to\infty} \Delta\Pi^p(\gamma, z) > 0$ yields $\Delta\Pi^p(\gamma, z) \gtreqless 0$ if $z \gtreqless z^p(\gamma)$.

If $\gamma \to 0$, then

$$\lim_{\gamma\to 0} \Delta\Pi^q(\gamma, z) = \frac{\alpha^2(z-1)}{4\beta z}$$

and

$$\lim_{\gamma\to 0} \Delta\Pi^p(\gamma, z) = \frac{\alpha^2(z-1)}{4\beta z}.$$

Thus, for $\gamma \to 0$, $z^q(\gamma) \to 1$ and $z^p(\gamma) \to 1$.

Characterization of $z^q(\gamma)$ and $z^p(\gamma)$

In the following we show that $\frac{\partial z^q(\gamma)}{\partial\gamma} > 0$ and $\frac{\partial z^p(\gamma)}{\partial\gamma} > 0$. This is done via the Implicit Function

Theorem.[11]

We already know that (5) is globally strictly positive. So it is also strictly positive when evaluated at $z^q(\gamma)$. Thus, $\left.\frac{\partial \Delta\Pi^q(\gamma,z)}{\partial z}\right|_{z=z^q(\gamma)} > 0$.

In the following, we show that the derivative of $\Delta\Pi^q(\gamma, z)$ with respect to γ is negative if it is evaluated at $z^q(\gamma)$.

Differentiating $\Delta\Pi^q(\gamma, z)$ with respect to γ yields

$$-\frac{2\alpha^2(\beta-\gamma)(2z(\beta+\gamma)-\gamma)(4z(\beta^2-\gamma\beta+\gamma^2)-\gamma^2)}{(4z(\beta^2-\gamma^2)+\gamma^2)^3}+\frac{\alpha^2\beta}{(2\beta+\gamma)^3 z}.$$

Evaluating $\frac{\partial \Delta\Pi^q(\gamma,z)}{\partial \gamma}$ at $z^q(\gamma)$ yields

$$\frac{2\alpha^2(\beta-\gamma)(2z'(\beta+\gamma)-\gamma)}{(4z'(\beta^2-\gamma^2)+\gamma^2)^3\beta(2\beta+\gamma)}\,\phi(\gamma, z^q(\gamma)) \tag{7}$$

with

$$\phi(\gamma, z) = 8(\beta^2-\gamma^2)^2 z^2 + 2(\beta^2\gamma^2 - 4\beta^4 - 3\gamma^4)z + \gamma^2(\gamma^2+2\beta^2).$$

Since the first factor of (7) is strictly bigger than zero, the sign of the derivative is determined by the sign of $\phi(\gamma, z)$ at $z^q(\gamma)$. Since $\phi(\gamma, z)$ is a quadratic function in z with a positive leading term, it is convex and has two real roots. The one that involves values of $z > 1$ is denoted by $\hat{z}(\gamma)$, where

$$\hat{z}(\gamma) = \frac{4\beta^4 - \beta^2\gamma^2 + 3\gamma^4 + \sqrt{\chi(\gamma)}}{8(\beta^2-\gamma^2)^2},$$

with

$$\chi(\gamma) = 49\beta^4\gamma^4 - 24\beta^6\gamma^2 - 6\beta^2\gamma^6 + 16\beta^8 + \gamma^8.$$

It can be shown that $\chi(0)$ and $\chi(\beta)$ are strictly positive. Furthermore, for $\gamma \in (0, \beta)$, $\chi(\gamma)$ attains its minimum value, $13\beta^8$, at $\gamma \approx \frac{\beta}{2}$. Thus $\hat{z}(\gamma)$ is well defined.

In the following, we compare $\hat{z}(\gamma)$ with $z^q(\gamma)$ and use the fact that $\Delta\Pi^q(\gamma, z^q(\gamma)) = 0$. Evaluating $\Delta\Pi^q(\gamma, z)$ at an arbitrary $\gamma \in (0, \beta)$ and the corresponding $\hat{z}(\gamma)$ yields

$$\Delta\Pi^q(\gamma, \hat{z}(\gamma)) = \frac{(4\gamma^3\beta - \gamma^4 + 3\gamma^2\beta^2 - 4\gamma\beta^3 + 4\beta^4 + \sqrt{\chi(\gamma)})^2\alpha^2}{4\beta(4\beta^4+\gamma^2\beta^2+\gamma^4+\sqrt{\chi(\gamma)})^2}$$
$$-\frac{8\alpha^2\beta(\beta^2-\gamma^2)^2}{(2\beta+\gamma)^2(4\beta^4-\gamma^2\beta^2+3\gamma^4+\sqrt{\chi(\gamma)})}.$$

To see that $\Delta\Pi^q(\gamma, \hat{z}(\gamma)) > 0$ for all $\gamma \in (0, \beta)$ we rewrite the right hand side of the previous equation as

$$\gamma^2\varphi_1(\gamma)\Big(\sqrt{\chi(\gamma)}(36\beta^6\gamma^2 + 40\gamma^3\beta^5 + 47\beta^4\gamma^4 + 96\beta^8 + 64\gamma\beta^7 + 40\gamma^5\beta^3 + 2\beta^2\gamma^6 - \gamma^8)$$
$$+960\gamma^4\beta^8 + 123\gamma^8\beta^4 + 24\gamma^9\beta^3 + \gamma^{12} + \varphi_2(\gamma) - \varphi_3(\gamma)\Big)$$

[11] In principle, we could solve for $z^q(\gamma)$ and $z^p(\gamma)$ explicitly. However, the expressions involved in determining the sign of the derivatives are hardly accessible. Thus, to prove the result we use the Implicit Function Theorem for the sake of exposition.

with

$$
\begin{aligned}
\varphi_1(\gamma) &= \frac{4\alpha^2}{\beta(\gamma^4+\beta^2\gamma^2+4\beta^4+\sqrt{\chi(\gamma)})^2(2\beta+\gamma)^2(4\beta^4-\beta^2\gamma^2+3\gamma^4+\sqrt{\chi(\gamma)})} > 0,\\
\varphi_2(\gamma) &= 600\beta^7\gamma^5+80\beta^5\gamma^7+256\beta^{11}\gamma+384\beta^{12} > 0,\\
\varphi_3(\gamma) &= 127\beta^6\gamma^6+21\beta^2\gamma^{10}+240\beta^{10}\gamma^2+96\beta^9\gamma^3 \geq 0.
\end{aligned}
$$

Obviously $\gamma = 0$ is one of the roots of $\Delta\Pi^q(\gamma, \hat{z}(\gamma))$. Now we need to show that it has none for $\gamma \in (0, \beta)$. Since

$$\varphi_2(\gamma) > 127\beta^7\gamma^5 + 21\beta^5\gamma^7 + 240\beta^{11}\gamma + 96\beta^{12} > \varphi_3(\gamma),$$

for $\gamma \in (0, \beta)$, we have shown that $\Delta\Pi^q(\gamma, \hat{z}(\gamma))$ has no real root for $\gamma \in (0, \beta)$. Thus, $\Delta\Pi^q(\gamma, \hat{z}(\gamma)) > 0$ for all $\gamma \in (0, \beta)$. As $\Delta\Pi^q(\gamma, z)$ is increasing in z, $z^q(\gamma) < \hat{z}(\gamma)$ for every $\gamma \in (0, \beta)$. Thus, $\phi(\gamma, z^q(\gamma)) < 0$ and thereby the derivative of $\Delta\Pi^q(\gamma, z^q(\gamma))$ with respect to γ is negative.

Since $\Delta\Pi^q_\gamma(\gamma, z^q(\gamma)) < 0$ and $\Delta\Pi^q_z(\gamma, z^q(\gamma)) > 0$ for $\gamma \in (0, \beta)$, the Implicit Function Theorem implies that

$$\frac{dz^q(\gamma)}{d\gamma} = -\frac{\Delta\Pi^q_\gamma(\gamma, z^q(\gamma))}{\Delta\Pi^q_z(\gamma, z^q(\gamma))} > 0.$$

Now we turn to the function $\Delta\Pi^p(\gamma, z) = 0$. If (6) is evaluated at $z^p(\gamma)$ it is strictly positive, since $z^p(\gamma) > \tilde{z}(\gamma)$. Thus, $\frac{\partial\Delta\Pi^p(\gamma,z)}{\partial z}\Big|_{z=z^p(\gamma)} > 0$.

In the following, we show that $\frac{\partial\Delta\Pi^p(\gamma,z)}{\partial\gamma}\Big|_{z=z^p(\gamma)} < 0$. Differentiating $\Delta\Pi^p(\gamma, z)$ with respect to γ yields:

$$2\alpha^2\left(\frac{(2\beta-\gamma)z(4z(\beta^3-2\beta^2\gamma-\beta\gamma^2+2\gamma^3)-\gamma(2\gamma^2+\beta\gamma+4\beta^2))}{(4z(\beta^2-\gamma^2)+\gamma^2)^3} - \frac{\beta(\beta^2-\gamma\beta+\gamma^2)}{(\beta+\gamma)^2(2\beta-\gamma)^3}\right).$$

Evaluating $\frac{\partial\Delta\Pi^p(\gamma,z)}{\partial\gamma}$ at $z^p(\gamma)$ yields:

$$\frac{2\alpha^2(2\beta-\gamma)(\beta+\gamma)\gamma}{(4z''(\gamma)(\beta^2-\gamma^2)+\gamma^2)^3\beta}\,\psi(\gamma, z^p(\gamma)), \tag{8}$$

with

$$\psi(\gamma, z) = (4z-1)(6\beta-\gamma)\gamma - (5z-1)4\beta^2.$$

Since the first factor in (8) is bigger than zero for all $\gamma \in (0, \beta)$, the sign of this derivative is negative if $\psi(\gamma, z^p(\gamma))$ is negative.

In order to check the sign of $\psi(\gamma, z^p(\gamma))$, we solve $\Delta\Pi^p(\gamma, z^p(\gamma)) = 0$ to obtain

$$z^p(\gamma) = \frac{\kappa(\gamma) + 8\beta^2\gamma^2(\beta-\gamma) + (2\beta-\gamma)^2\sqrt{\kappa(\gamma)}\sqrt{\beta+\gamma}}{32\beta^2(\beta-\gamma)^2(\beta+\gamma)} \tag{9}$$

with

$$\kappa(\gamma) = 16\beta^5 - 8\beta^2\gamma(2\beta^2+3\beta\gamma-4\gamma^2) - \gamma^4(7\beta-\gamma).$$

Obviously, $\kappa(0)$ and $\kappa(\beta)$ are strictly positive. Moreover, for $\gamma \in (0, \beta)$ this expression attains its minimum at $\gamma \approx 0.88\beta$. Evaluating $\kappa(\gamma)$ at this value yields $1.47\beta^5$. Thus, $\kappa(\gamma) > 0$.

Inserting $z^p(\gamma)$ into $\psi(\gamma, z)$ yields:

$$
\begin{aligned}
&-\frac{(2\beta-\gamma)}{8(\beta^2-\gamma^2)\beta^2}\Big(24\beta^5+31\beta^2\gamma^3+\gamma^5-2\gamma\beta(6\beta^3+13\beta^2\gamma+5\gamma^3)\\
&+(5\beta-\gamma)(2\beta-\gamma)\sqrt{\kappa(\gamma)}\sqrt{\beta+\gamma}\Big).
\end{aligned}
$$

Since

$$24\beta^5 + 31\beta^2\gamma^3 + \gamma^5 - 2\gamma\beta(6\beta^3 + 13\beta^2\gamma + 5\gamma^3) > \kappa(\gamma) > 0,$$

$\psi(\gamma, z^p(\gamma))$ and $\frac{\partial \Delta\Pi^p(\gamma,z)}{\partial\gamma}\Big|_{z=z^p(\gamma)}$ are negative.

The Implicit Function Theorem implies that

$$\frac{dz^p(\gamma)}{d\gamma} = -\frac{\Delta\Pi^p_\gamma(\gamma, z^p(\gamma))}{\Delta\Pi^p_z(\gamma, z^p(\gamma))} > 0.$$

Relation of $z^q(\gamma)$ and $z^p(\gamma)$

Consider an arbitrary $\gamma \in (0, \beta)$ and the associated $z^p(\gamma)$. Evaluating $\Delta\Pi^q(\gamma, z)$ at that $z^p(\gamma)$ yields:

$$\frac{4\alpha^2\gamma^2(2\beta-\gamma)^2}{\beta(2\beta+\gamma)^2}\lambda_1(\gamma)\Big(\sqrt{\kappa(\gamma)}\sqrt{\beta+\gamma}(2\beta-\gamma)\lambda_2(\gamma) + \lambda_3(\gamma)\Big) \tag{10}$$

with

$$\begin{aligned}\lambda_1(\gamma) = \Big(&(16\beta^3(\beta^2 - \beta\gamma - \gamma^2) + \gamma^3(24\beta^2 - 7\gamma\beta + \gamma^2) + (2\beta-\gamma)^2\sqrt{\kappa(\gamma)}\sqrt{\beta+\gamma}) \\ &(8\beta^3(2\beta^2 - 2\beta\gamma - \gamma^2) + \gamma^3(16\beta^2 - 7\gamma\beta + \gamma^2) + (2\beta-\gamma)^2\sqrt{\kappa(\gamma)}\sqrt{\beta+\gamma})^2\Big)^{-1},\end{aligned}$$

$$\lambda_2(\gamma) = (288\beta^7 - 304\beta^6\gamma - 176\beta^5\gamma^2 + 304\beta^4\gamma^3 - 70\beta^3\gamma^4 - 13\beta^2\gamma^5 + 8\beta\gamma^6 - \gamma^7),$$

and

$$\begin{aligned}\lambda_3(\gamma) = &\gamma^{11} - 13\beta\gamma^{10} + 67\beta^2\gamma^9 - 95\beta^3\gamma^8 - 592\beta^4\gamma^7 + 2832\beta^5\gamma^6 - 3536\beta^6\gamma^5 \\ &- 1920\beta^7\gamma^4 + 7168\beta^8\gamma^3 - 3072\beta^9\gamma^2 - 2560\beta^{10}\gamma + 1792\beta^{11}.\end{aligned}$$

Now we need to determine the sign of $\lambda_1(\gamma)$, $\lambda_2(\gamma)$, and $\lambda_3(\gamma)$. Since

$$\begin{aligned}16\beta^3(\beta^2 - \beta\gamma - \gamma^2) + \gamma^3(24\beta^2 - 7\gamma\beta + \gamma^2) &\geq \kappa(\gamma) > 0 \text{ and} \\ (8\beta^3(2\beta^2 - 2\beta\gamma - \gamma^2) + \gamma^3(16\beta^2 - 7\gamma\beta + \gamma^2) &\geq \kappa(\gamma) > 0,\end{aligned}$$

$\lambda_1(\gamma)$ is strictly bigger than zero.

It can be shown that $\lambda_2(\gamma)$ and $\lambda_3(\gamma)$ have no root in $(0, \beta)$. Since $\lambda_2\Big(\frac{\beta}{2}\Big) \approx 125\beta^7 > 0$ and $\lambda_3\Big(\frac{\beta}{2}\Big) \approx 449\beta^{11} > 0$, both expressions are strictly bigger than zero. This implies that (10) is bigger than zero. Since $\Delta\Pi^q(\gamma, z)$ is increasing in its second argument, $z^q(\gamma)$ has to be smaller than $z^p(\gamma)$ in order for this condition to hold. ■

6.2 Proof of Lemma 2

Existence and Uniqueness of $\sigma^q_{\theta\epsilon}(\gamma)$ and $\sigma^p_{\theta\epsilon}(\gamma)$

First, consider the case in which firm j sets a quantity. Firm i is indifferent between setting a price or a quantity if

$$\frac{(\beta-\gamma)^2(2z(\beta+\gamma)(\alpha+\sigma_{\theta\epsilon}) - \alpha\gamma)^2}{(4z(\beta^2-\gamma^2)+\gamma^2)^2\beta} = \frac{\alpha^2\beta}{z(2\beta+\gamma)^2}. \tag{11}$$

The solutions of (11) are

$$\begin{aligned}\sigma^{(1)}_{\theta\epsilon}(\gamma) &= \frac{\alpha(\sqrt{z}\beta(4z(\beta^2-\gamma^2)+\gamma^2)-z(2\beta+\gamma)(\beta-\gamma)(2z(\beta+\gamma)-\gamma))}{2z^2(\beta^2-\gamma^2)(2\beta+\gamma)},\\ \sigma^{(2)}_{\theta\epsilon}(\gamma) &= -\frac{\alpha(\sqrt{z}\beta(4z(\beta^2-\gamma^2)+\gamma^2)-z(2\beta+\gamma)(\beta-\gamma)(2z(\beta+\gamma)-\gamma))}{2z^2(\beta^2-\gamma^2)(2\beta+\gamma)}.\end{aligned}$$

We have to check how $\sigma^{(1)}_{\theta\epsilon}(\gamma)$ and $\sigma^{(2)}_{\theta\epsilon}(\gamma)$ relate to $\hat{\sigma}_{\theta\epsilon}(\gamma)$. In order to do that we simply subtract $\hat{\sigma}_{\theta\epsilon}(\gamma)$ from both threshold covariances. It turns out that $\sigma^{(1)}_{\theta\epsilon}(\gamma)-\hat{\sigma}_{\theta\epsilon}(\gamma)=\Delta$, and $\sigma^{(2)}_{\theta\epsilon}(\gamma)-\hat{\sigma}_{\theta\epsilon}(\gamma)=-\Delta$, where Δ is given by

$$\frac{\alpha(\sqrt{z}\beta(4z(\beta^2-\gamma^2)+\gamma^2)}{2z^2(\beta^2-\gamma^2)(2\beta+\gamma)}>0.$$

Thus, $\sigma^{(2)}_{\theta\epsilon}(\gamma)$ is never in the admissible range of $\sigma_{\theta\epsilon}$ while $\sigma^{(1)}_{\theta\epsilon}(\gamma)$ is. We denote $\sigma^{(1)}_{\theta\epsilon}(\gamma)$ by $\sigma^{q}_{\theta\epsilon}(\gamma)$.

Now we have to determine under which conditions firm i prefers to set a price or a quantity conditional on firm j choosing a quantity. Comparing Π^{pq} with Π^{qq} it is easy to see that firm i sets a price if $\sigma_{\theta\epsilon}(\gamma)>\sigma^{q}_{\theta\epsilon}(\gamma)$ since Π^{pq} is increasing in $\sigma_{\theta\epsilon}(\gamma)$ while Π^{qq} is independent of $\sigma_{\theta\epsilon}(\gamma)$. Thus, $\Delta\Pi^{q}(\gamma,\sigma_{\theta\epsilon})\gtreqless 0$ if $\sigma_{\theta\epsilon}\gtreqless\sigma^{q}_{\theta\epsilon}(\gamma)$.

Now suppose firm j sets a price. Then, firm i is indifferent between choosing a price or a quantity if

$$\frac{(\alpha+\sigma_{\theta\epsilon})^2(\beta-\gamma)\beta}{(\beta+\gamma)(2\beta-\gamma)^2}=\frac{(\beta^2-\gamma^2)z(\alpha(2\beta-\gamma)+\gamma\sigma_{\theta\epsilon})^2}{(4z(\beta^2-\gamma^2)+\gamma^2)^2\beta}. \tag{12}$$

The solutions of (12) are

$$\begin{aligned}\tilde{\sigma}^{(1)}_{\theta\epsilon}(\gamma) &= \frac{\alpha\sqrt{z}(\beta^2-\gamma^2)(2\beta-\gamma)}{h(\gamma)}\Big(2\sqrt{z}\gamma(\beta+\gamma)+\beta(4z(\beta^2-\gamma^2)+\gamma^2)\Big)-\alpha,\\ \tilde{\sigma}^{(2)}_{\theta\epsilon}(\gamma) &= \frac{\alpha\sqrt{z}(\beta^2-\gamma^2)(2\beta-\gamma)}{h(\gamma)}\Big(2\sqrt{z}\gamma(\beta+\gamma)-\beta(4z(\beta^2-\gamma^2)+\gamma^2)\Big)-\alpha\end{aligned}$$

with

$$h(\gamma):=16\beta^2(\beta^2-\gamma^2)^2z^2+\gamma^2(\beta+\gamma)(4\beta^3-8\beta^2\gamma+3\beta\gamma^2-\gamma^3)z+\beta^2\gamma^4.$$

It is easy to show that $h(0)=16z^2\beta^6>0$, $h(\beta)=\beta^6(1-4z)<0$, and $\frac{\partial h(\gamma)}{\partial\gamma}<0$. This implies that there exists a unique $\gamma\in(0,\beta)$, labeled γ^+, that solves $h(\gamma)=0$. Thus, $\tilde{\sigma}^{(1)}_{\theta\epsilon}(\gamma)$ and $\tilde{\sigma}^{(2)}_{\theta\epsilon}(\gamma)$ are defined on $\gamma\in[0,\beta)\setminus\{\gamma^+\}$.

Now we analyze how $\tilde{\sigma}^{(1)}_{\theta\epsilon}(\gamma)$ relates to $\hat{\sigma}_{\theta\epsilon}(\gamma)$. Subtracting the latter from the former yields

$$\frac{\alpha(4z(\beta^2-\gamma^2)+\gamma^2)}{2z(\beta+\gamma)h(\gamma)}\Big((4z^{\frac{3}{2}}(\beta+\gamma)\beta(2\beta-\gamma)(\beta^2-\gamma^2)+\gamma^2(2z\beta(2\beta^2-\gamma^2)+\beta^2\gamma(z-1)+z\gamma^3)\Big). \tag{13}$$

The numerator of (13) is nonnegative for all $\gamma\in[0,\beta)$ while the sign of the denominator depends on the sign of $h(\gamma)$. The argument above implies that $h(\gamma)>0$ for $\gamma\in[0,\gamma^+)$ and $h(\gamma)<0$ for $\gamma\in(\gamma^+,\beta)$. As a consequence, we have $\tilde{\sigma}^{(1)}_{\theta\epsilon}(\gamma)>\hat{\sigma}_{\theta\epsilon}(\gamma)$ if and only if $\gamma<\gamma^+$.

Now we investigate the relation between $\tilde{\sigma}^{(2)}_{\theta\epsilon}(\gamma)$ and $\hat{\sigma}_{\theta\epsilon}(\gamma)$. Subtracting the minimum of the latter ($-\alpha$) from the former yields

$$\frac{\alpha\sqrt{z}(\beta^2-\gamma^2)(2\beta-\gamma)}{h(\gamma)}\Big(2\sqrt{z}\gamma(\beta+\gamma)-\beta(4z(\beta^2-\gamma^2)+\gamma^2)\Big).$$

This expression is equal to zero for $\gamma = \beta$ and smaller than zero for all $\gamma \in [0, \beta)$.[12] Thus, $\tilde{\sigma}^{(2)}_{\theta\epsilon}(\gamma)$ is never in the admissible range of $\sigma_{\theta\epsilon}$. For every $\gamma \in [0, \gamma^+)$ we denote $\tilde{\sigma}^{(1)}_{\theta\epsilon}(\gamma)$ by $\sigma^p_{\theta\epsilon}(\gamma)$.

Now we show that firm i prefers to set a price contingent on firm j choosing a price if $\sigma_{\theta\epsilon} > \sigma^p_{\theta\epsilon}(\gamma)$. Firm i sets a price if Π^{pp} is bigger than Π^{qp}. Differentiating $\Delta\Pi^p(\gamma, \sigma_{\theta\epsilon})$ with respect to $\sigma_{\theta\epsilon}$, and evaluating this difference at $\Delta\Pi^p(\gamma, \sigma_{\theta\epsilon}) = 0$ yields

$$\frac{\partial(\Pi^{pp} - \Pi^{qp})}{\partial \sigma_{\theta\epsilon}}\Big|_{\Delta\Pi^p(\gamma,\sigma_{\theta\epsilon})=0} = \frac{4\sqrt{z}\alpha(\beta-\gamma)^2}{(2\beta-\gamma)(4z(\beta^2-\gamma^2)+\gamma^2)} > 0.$$

This implies that at $\sigma^p_{\theta_\epsilon}(\gamma)$ firm i's marginal benefit from an increase in the covariance is bigger if it selects a price than if it chooses a quantity. Thus, $\Delta\Pi^p(\gamma, \sigma_{\theta\epsilon}) \gtreqless 0$ if $\sigma_{\theta\epsilon} \gtreqless \sigma^p_{\theta\epsilon}(\gamma)$.

Finally, for $\gamma \in (\gamma^+, \beta]$ there exists no $\sigma_{\theta\epsilon} > \hat{\sigma}_{\theta\epsilon}$ such that (12) holds. As a consequence, the right hand side of (12) is bigger than the left hand side and so $\Delta\Pi^p(\gamma, \sigma_{\theta\epsilon}) < 0$ for any $\gamma \in (\gamma^+, \beta]$.

Relation of $\sigma^q_{\theta\epsilon}(\gamma)$ and $\sigma^p_{\theta\epsilon}(\gamma)$

In the following we determine how $\sigma^q_{\theta\epsilon}(\gamma)$ and $\sigma^p_{\theta\epsilon}(\gamma)$ relate to each other. It is easy to check that at $\gamma = 0$, $\sigma^q_{\theta\epsilon}(0) = \sigma^p_{\theta\epsilon}(0) = -\alpha\Big(1 - \frac{1}{\sqrt{z}}\Big)$. Subtracting $\sigma^p_{\theta\epsilon}(\gamma)$ from $\sigma^q_{\theta\epsilon}(\gamma)$ reveals that there exists a unique $\gamma \in (0, \beta)$, namely $\gamma = \beta\Big(1 - \frac{1}{\sqrt{z}}\Big)$, for which both threshold covariances are equal. It remains to show that $\sigma^q_{\theta\epsilon}(\gamma)$ and $\sigma^p_{\theta\epsilon}(\gamma)$ cross at $\gamma = \beta\Big(1 - \frac{1}{\sqrt{z}}\Big)$. Differentiating $\sigma^q_{\theta\epsilon}(\gamma)$ with respect to γ and evaluating the derivative at the intersection point yields

$$\frac{\alpha\Big(1 - 3\sqrt{z} - 3z + 15z^{\frac{3}{2}} - 10z^2\Big)}{2\beta\sqrt{z}\Big(2\sqrt{z}-1\Big)^2\Big(3\sqrt{z}-1\Big)^2}, \tag{14}$$

while differentiating $\sigma^p_{\theta\epsilon}(\gamma)$ with respect to γ and evaluating the derivative at the intersection point yields

$$\frac{\alpha\Big(1 - 7\sqrt{z} + 13z + 25z^{\frac{3}{2}} - 128z^2 + 106z^{\frac{5}{2}} + 251z^3 - 523z^{\frac{7}{2}} + 101z^4 + 491z^{\frac{9}{2}} - 430z^5 + 100z^{\frac{11}{2}}\Big)}{2\beta z^{\frac{3}{2}}\Big(15z^2 - 11z^{\frac{3}{2}} - 4z + 5\sqrt{z} - 1\Big)^2}. \tag{15}$$

Since $z > 1$ it is easy to check that (15) is strictly bigger than (14). This shows that at the intersection point $\sigma^p_{\theta\epsilon}(\gamma)$ crosses $\sigma^q_{\theta\epsilon}(\gamma)$ from below and so $\sigma^p_{\theta\epsilon}(\gamma) < \sigma^q_{\theta\epsilon}(\gamma)$ for $0 < \gamma < \beta\Big(1 - \frac{1}{\sqrt{z}}\Big)$ and $\sigma^p_{\theta\epsilon}(\gamma) > \sigma^q_{\theta\epsilon}(\gamma)$ for $\beta\Big(1 - \frac{1}{\sqrt{z}}\Big) < \gamma < \gamma^+$. ■

[12] Here both the numerator and the denominator are zero at $\gamma = \gamma^+$, and one can show by using the Rule of L'Hospital that the expression is negative at $\gamma = \gamma^+$.

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