Schneeweiss:

Resolving the Ellsberg Paradox by Assuming that People Evaluate Repetitive Sampling

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Resolving the Ellsberg Paradox by Assuming that People Evaluate Repetitive Sampling

Hans Schneeweiss

Ellsberg (1961) designed a decision experiment where most people violated the axioms of rational choice. He asked people to bet on the outcome of certain random events with known and with unknown probabilities. They usually preferred to bet on events with known probabilities. It is shown that this behavior is reasonable and in accordance with the axioms of rational decision making if it is assumed that people consider bets on events that are repeatedly sampled instead of just sampled once.

Key words: Ellsberg's paradox, rational decision making, Sure Thing Principle, subjective probabilities.

1 Introduction

Ellsberg (1961) designed an experiment, where people had to decide between bets on risky lotteries with known probabilities or on uncertain events, where probabilities were not known. They usually preferred to bet on the lottery thereby blatantly violating the rationality axioms of Bayesian decision theory, sometimes also referred to as Subjective Expected Utility (SEU) theory. This behavior has therefore been called a "paradox". The Ellsberg Paradox has since become the paradigm for a new concept in decision theory: ambiguity. Probability statements can be more or less ambiguous depending on how strong an individual believes in the assertion of these probabilities.

Otherwise rational people seem to express a preference for unambiguous probabilities, i.e., for probabilities that are objective and completely known to them. They shrink from ambiguous probabilities, i.e., probabilities that are only of a subjective kind or that are objective but only vaguely known. A typical example would be an urn (I) with red and black balls of an unknown proportion, out of which one ball is to be drawn. If indeed nothing is known about the proportion of red and black balls in the urn, then one might be indifferent of whether to bet on Red or on Black, just as with an urn (II), where the red and the black balls are in equal number and where this proportion is known. So in both cases the indifference between betting on Red or on Black can be regarded as an expression of assigning equal probabilities to both colors. In the second case (urn II), however, the probability of drawing a red ball, say, is objectively given as $\frac{1}{2}$, whereas in the first case (urn I), the objective probability of the same event is unknown and it is only a subjective probability which can be asserted as being $\frac{1}{2}$. It turns out that, although in both cases most people are indifferent when confronted with a choice of betting on Red or on Black, they typically prefer to have the ball drawn from the urn II with known proportion ($\frac{1}{2}$), regardless of whether they bet on Red or on Black. This behavior is well documented by numerous experiments. It was first made public by Ellsberg (1961) and has since been known as the Ellsberg paradox. It is regarded to be paradoxical because in both urns the subjective probabilities for Black and Red are equal and therefore $\frac{1}{2}$ and yet Black I $\succ$ Black II and at the same time Red I $\succ$ Red II. (Here $\succ$ means "is strictly preferred to"and "Black I" denotes the bet on the event of a black ball being drawn if the ball is drawn from urn I).

Ellsberg also designed another experiment with only one urn but with balls of three different colors, where the nature of the paradox can be studied more closely. We shall analyze this situation in the next section.

The findings of Ellsberg have been verified in a large number of similar betting experiments and many suggestions have been proposed for understanding the apparent paradox. One can, of course, simply ignore the problem and discard the observed behavior as being irrational and not worth any further study. But as the observed behavior in the Ellsberg experiment is rather persistent and therefore can hardly be dismissed on the basis of being irrational, an explanation for it is called for, in particular as this kind of behavior is probably prevalent in many practical decisions, e.g., in economics or in business, see, e.g., Sarin and Weber (1993).

The central trait of the observed behavior seems to be that most people shrink from uncertain events the objective probabilities of which are unknown or only vaguely known. Betting on an event with known objective probability, like Red II, is preferred to betting on an event with the same subjective probability, which however is not substantiated by an objective probability and is therefore ambiguous, like Red I. Ambiguity is a quality attached to probability assertions. A person may assign a probability to some event, but may be more or less certain about the value of this probability. It is questionable whether a measure of ambiguity adequate for all kinds of uncertain circumstances can be found, but as a concept to describe situations as in Ellsberg's experiment it is worth studying.

Attempts have been made to model the behavior of the majority of people in Ellsberg's experiment and to study the conditions under which ambiguity is perceived by individuals in decisions under uncertainty. For a recent survey see Camerer and Weber (1992), see also Keppe and Weber (1995) and Eisenberger and Weber (1995). A famous axiomatic approach that results in a subjective expected utility theory with probabilities replaced by capacities has been proposed by Gilboa (1987), see also Schmeidler (1989). For a recent further development of this approach, where objective probabilities are incorporated in the theory, see Eichberger and Kelsey (1989). For an empirical test see Mangelsdorff and Weber (1994). Recently a different criterion for decision making in the face of uncertainty governed by interval probabilities was proposed by Weichselberger and Augustin (1998). Schneeweiß (1968, 1973) tried to explain Ellsberg's paradox by embedding Ellsberg's experiments in a game theoretic framework.

Here a different approach is chosen. I shall argue that the typical behavior of people in the Ellsberg experiment can be explained by assuming that they consider (subconsciously) the act of drawing a ball from an urn as a repetitive act, despite the fact that they are told the ball will be drawn only once. In evaluating the possible gains and losses from participating in a lottery, people imagine the lottery to be played several times and consider the average amount they might gain or lose. For a lottery (or urn) with known probabilities of gains or losses this average amount is rather certain due to the law of large numbers. But if the probabilities are unknown, the result of repeated lottery draws will also be unknown no matter how many repetitions are considered. When confronted with a choice between urn I with unknown probability and urn II with known probability $\frac{1}{2}$ of drawing a red ball, a person might assign the same subjective probability $\frac{1}{2}$ to Red for both urns as long as one draw is considered. But if that person (perhaps only subconsciously) imagines repeated draws from the urn she chooses and if she is risk averse, then she will choose urn II because it is with this urn only that the average gain of repeated draws will be rather certain, whereas the uncertainty of gains from repeated draws out of urn I will remain uncertain.

The paper will analyze the distribution of gains under repeated draws in Ellsberg's experiment and will show that risk averse people will always prefer the less ambiguous situation, in complete accordance with the axioms of rational choice and thus in accordance with Bayesian decision theory. In the next section the Ellsberg paradox is reviewed in a setting somewhat different from what was described above. Section 3 gives the main argument how to evaluate the result of repeated draws and why less ambiguous events are preferred to more ambiguous ones even if their (subjective) probabilities do not differ. Section 4 contains some concluding remarks.

2 The Ellsberg paradox

The Ellsberg experiment (or rather one of two suggested experiments) consists in bets on the outcome of a single draw from an urn which contains 30 red balls and 60 black or yellow balls in an unknown proportion. A person is given a choice to bet on the outcome of the draw to be Red or to be Black and another choice to bet on whether the outcome will be Red or Yellow or whether it will be Black or Yellow. In each case the person wins 1 Euro if the color he bets upon does indeed show up; otherwise nothing is gained or lost. This decision situation is depicted in the following diagram (Table 1), which shows the payoff function depending on the color of the ball and on the betting act chosen. When asked to choose between bets R or B, most people

Table 1: Payoffs in Ellberg's experiment

Number of balls		30	60	
Bet on	Symbol	Red	Black	Yellow
Red	R	1	0	0
Black	B	0	1	0
Red or Yellow	$R \vee Y$	1	0	1
Black or Yellow	$B \vee Y$	0	1	1

decide for R. When the same people are then asked to choose between $R \vee Y$ or $B \vee Y$, they typically decide for $B \vee Y$. Only few people would choose $R \vee Y$. Some people decide for B in the first decision problem and for $R \vee Y$ in the second problem.

Let us discuss the choice of the first group, the majority of people. (The arguments for the last group are completely analogous.) First one might think that, since nothing is known about the proportion of black and yellow balls, a person should assume, owing to the principle of insufficient reason, that Black and Yellow are equally likely to show up. Not that the person thinks both colors to be in equal proportion in the urn, he only bases his decision on the (implicit) assumption that the proportion is 30:30. Perhaps a better description of this attitude would be to say that the decision is made *as if* the proportion of black and yellow balls were equal, or more technically: the subjective probabilities of Black and Yellow are equal for this person. For, given that nothing is known about the way the balls were put into the urn, why should there be a higher subjective probability for a black ball to be drawn than for a yellow ball? If someone follows this argument, then he will be indifferent between R and B in the first problem and between $R \vee Y$ and $B \vee Y$ in the second problem. In symbols: $R \sim B$ and $R \vee Y \sim B \vee Y$, because $P(R) = P(B) = \frac{1}{3}$ and $P(R \vee Y) = P(B \vee Y) = \frac{2}{3}$, P being a subjective probability. This, however, is not what is observed as the actual behavior of most people.

One can argue against this reasoning that it assumes at the outset the existence of subjective probabilities. This assumption is indeed what Bayesian decision analysis is based upon. There are strong arguments for making this assumption. For a recent information based argument see Ferschl (1998). Moreover one can prove the existence of subjective probabilities for any "rational" decision maker from certain axioms of consistent (rational) choices between acts with uncertain outcome, Savage (1954). These axioms do not necessarily imply that $P(B) = P(Y)$. This follows only if in addition to the axioms of rational choice the principle of insufficient reason is taken as an extra assumption. But even without this additional principle, the observed behavior of individuals in the Ellsberg experiment stands in contrast to the Bayesian decision rule.

Suppose the decision maker, as a Bayesian, attaches subjective, not necessarily equal, probabilities to the events of drawing a ball with a specific color and suppose he has a Neumann-Morgenstern utility function $u(.)$, where we can take, without loss of generality, $u(1) = 1$ and $u(0) = 0$. Then the expected subjective utilities of the four acts of the Ellsberg experiment are just the subjective probabilities of the events betted upon. Thus if R is preferred to B ($R \succ B$), then $P(R) > P(B)$, which implies $P(R)+P(Y) > P(B)+P(Y)$, and this again implies $R \vee Y \succ B \vee Y$. These preferences, which follow naturally from a Bayesian decision framework are, however, contradicted by the observed behavior of people that instead prefer $B \vee Y$ to $R \vee Y$. (The same contradiction arises for those people — the minority — for which $B \succ R$ and $R \vee Y \succ B \vee Y$; only the very few with $R \succ B$ and $R \vee Y \succ B \vee Y$ have preferences in accordance with Bayesian decision theory.)

The conclusion of the Ellsberg experiment then is that most people, contrary to what Bayesians say, do not base their decision on subjective probabilities. This is seen as a paradox. Against this one may argue that it is a paradox only to Bayesians, that is, only to those that believe in subjective probabilities. But the paradox goes deeper.

The observed conjunction of preferences $R \succ B$ and $B \vee Y \succ R \vee Y$ is paradoxical in that it violates the Sure Thing Principle, Savage (1954). The preference of R to B in the first problem should not change if one gets 1 Euro not only for the color betted upon but also in case yellow turns up regardless of whether one betted on Red or on Black. Note that this 1 Euro is not an additional amount that you might get in addition to the prize of 1 Euro for betting on the right color. You get the extra prize only if you would have lost both of the two bets R and B, whatever your actual bet was; consult Table 1. It is a kind of consolation prize that you get if neither a red nor a black ball is drawn, and this consolation prize should not interfere with your decision whether to bet on Red or on Black. Thus $R \succ B$ should imply $R \vee Y \succ B \vee Y$. The fact that most people, in their behavior, seem to contradict this (almost logical) implication is seen to be paradoxical, at least by those that see the Sure Thing Principle as a self-evident principle of decision making. Note that this argument does not make use of any probability assertions; it does not even use the concept of probability.

3 Repetitive draws

The situation described in Section 2 changes dramatically if instead of having just one draw from the urn several independent draws are considered. After a bet has been made, a ball is drawn from the urn n times with replacements. Each time the color betted upon shows up, the amount of 1 Euro is payed to the decision maker. Let X be the average gain , i.e., the amount gained after n repeated draws divided by n. Then $nX \sim Bin(n, P(C))$, where Bin stands for the binomial distribution and $P(C)$ is the probability of drawing a ball with color C in a single draw. When C is Black or Yellow, this probability depends, in an obvious way, on the proportion p of black balls within the set of black and yellow balls, i.e., the number of black balls divided by 60.

We assume that the betting acts are decided upon by a "rational" person. This means that the decisions of this person are governed by a subjective probability distribution. But now the sample space consists not of the three colors, but rather of the various outcomes of n repeated draws. A convenient way to model probabilities for these outcomes is to assign a subjective probability to the proportion p of black balls and then to compute, in the usual way, conditional objective probabilities for the outcomes of n draws conditional on p. As we are only interested in the average gain X, we may use the conditional probability distribution of X given p: $f(x|p)$. This procedure is based on the obvious assumption that the subjective probability of an event coincides with its objective probability if the latter exists and is known to the decision maker. So the only truly subjective probability distribution is the one for p. Let its distribution function be denoted by $F(p)$. Any "rational" decision maker has a distribution $F(p)$, which is independent of the betting act chosen. The unconditional distribution of X is then given by

$$f(x) = \int f(x|p) dF(p).$$

In particular the distribution of X for the four betting acts is given by the following expressions for $P(X=x)$, $nx = 0, \ldots, n$:

$$\begin{aligned}
R &: \binom{n}{nx}\left(\frac{1}{3}\right)^{nx}\left(\frac{2}{3}\right)^{n(1-x)} \\
B &: \binom{n}{nx}\int_0^1\left(\frac{2}{3}p\right)^{nx}\left(1-\frac{2}{3}p\right)^{n(1-x)} dF(p) \\
R \vee Y &: \binom{n}{nx}\int_0^1\left(1-\frac{2}{3}p\right)^{nx}\left(\frac{2}{3}p\right)^{n(1-x)} dF(p) \\
B \vee Y &: \binom{n}{nx}\left(\frac{2}{3}\right)^{nx}\left(\frac{1}{3}\right)^{n(1-x)}
\end{aligned}$$

A "rational" person following SEU theory bases his decisions on a Neumann-Morgenstern utility function $u(X)$, which in our case should be increasing. That act is preferred for which the expected utility $E\{u(X)\}$ is largest. For a risk averse person the utility function $u(.)$ is concave. We assume, for simplicity, that $u(.)$ is quadratic. The results should, however, hold true also for more general concave utility functions. For a quadratic utility function, $E\{u(X)\}$ is a function of $\mu = E(X)$ and $\sigma^2 = V(X)$. If $u(.)$ is chosen so that $u(0) = 0$ and $u(1) = 1$ and is taken to be increasing for $x \leq 1$, then $u(x) = (a+1)x - ax^2$, $0 < a < 1$, and for any random variable X with range $[0,1]$, $E\{u(X)\}$ is increasing in μ and decreasing in σ^2. The parameters μ and σ^2 can be easily computed for the four betting acts, see Table 2. We

Table 2: Mean and variance of average gain X

betting act	μ	σ^2	$\lim_{n\to\infty}\sigma^2$
R	$\frac{1}{3}$	$\frac{2}{9n}$	0
B	$\frac{2}{3}E(p)$	$\frac{2}{3n}M(p) + \frac{4}{9}V(p)$	$\frac{4}{9}V(p)$
$R \vee Y$	$1 - \frac{2}{3}E(p)$	$\frac{2}{3n}M(p) + \frac{4}{9}V(p)$	$\frac{4}{9}V(p)$
$B \vee Y$	$\frac{2}{3}$	$\frac{2}{9n}$	0

$M(p) = E(p) - \frac{2}{3}E(p^2)$

indicate their derivation for the betting act B. We have

$$\begin{aligned}
\mu &= E\{E(X|p)\} = E\left(\frac{2}{3}p\right) = \frac{2}{3}E(p), \\
\sigma^2 &= E\{V(X|p)\} + V\{E(X|p)\} \\
&= E\left\{\frac{2}{3}p\left(1-\frac{2}{3}p\right)\frac{1}{n}\right\} + V\left(\frac{2}{3}p\right) \\
&= \frac{2}{3n}\left\{E(p) - \frac{2}{3}E(p^2)\right\} + \frac{4}{9}V(p).
\end{aligned}$$

The preferences between bets are determined by the value of

$$E\{u(X)\} = (a+1)\mu - a(\mu^2 + \sigma^2),$$

where $0 < a < 1$ and $0 \leq \mu \leq 1$.

For $n = 1$, μ is the probability of the bet to come true, and $E\{u(X)\} = \mu$. This means that of two bets that one is preferred which has the higher probability of coming true. Therefore $R \succ B$ implies $R \vee Y \succ B \vee Y$ as shown in Section 2. This is the betting behavior of a "rational" person if only one draw is considered.

Let us return to repeated draws. If n becomes large then σ^2 will be negligible for the unambiguous bets R and $B \vee Y$, but will be large and will not go to zero for the ambiguous bets B and $R \vee Y$, see the last column of Table 2. Thus one may expect a tendency in each of the two comparisons to prefer the unambiguous bet to the ambiguous one, i.e., $R \succ B$ and $B \vee Y \succ R \vee Y$. Of course, whether this will, in fact, be true depends on the probability distribution of p, in particular, on $E(p)$ and $V(p)$.

To illustrate, let us suppose that the decision maker has a symmetric distribution $F(p)$, so that $E(p) = \frac{1}{2}$. For $n \to \infty$ the expected utilities of the four bets become as in Table 3.

Table 3: Expected utilities

R :	$\frac{1}{3} + \frac{2}{9}a$	B :	$\frac{1}{3} + \frac{2}{9}a - \frac{4}{9}aV(p)$
$B \vee Y$:	$\frac{2}{3} + \frac{2}{9}a$	$R \vee Y$:	$\frac{2}{3} + \frac{2}{9}a - \frac{4}{9}aV(p)$

From Table 3 it is obvious that, for any values of a and $V(p)$, $R \succ B$ and at the same time $B \vee Y \succ R \vee Y$. These are exactly the preferences observed for the majority of participants in Ellsberg's experiment. They follow from Bayesian arguments and are therefore the preferences of a "rational" person. It is however a person that determines his preferences from viewing the drawing of balls as being repeated an indefinite number of times.

One can generalize this result by allowing the number of red balls to vary. Let r be the ratio of red balls in the urn, which is supposed to be known to the decision maker. In Ellsberg's original experiment $r = \frac{1}{3}$. A simple computation, analogous to the previous one, leads to the expected utilities (for $n \to \infty$) of Table 4, where we assumed a symmetric distribution for p, as before. Noting that $0 \leq V(p) \leq \frac{1}{4}$, it is easy to see, by letting r

Table 4: Expected utilities with general r

$R:$	$r + ar(1-r)$	$B:$	$\frac{1}{2}(1-r)(1+\frac{a}{2}+\frac{a}{2}r)$
			$-a(1-r)^2V(p)$
$B \vee Y:$	$(1-r)(1+ar)$	$R \vee Y:$	$\frac{1}{2}(1+r)(1+\frac{a}{2}-\frac{a}{2}r)$
			$-a(1-r)^2V(p)$

go to zero, that $B \succ R$ and $B \vee Y \succ R \vee Y$ for sufficiently small r, where the point of switching from $R \succ B$ to $B \succ R$ depends on the subjectively perceived amount of risk, $V(p)$, and on the measure of aversion of risk, a. Such a switching strategy has actually been postulated by Ellsberg himself. Note that under a minimax criterion no switching of the kind described is allowed to take place.

Similar arguments lead to analogous conclusions for Ellsberg's first experiment described in the Introduction. Let p be the proportion of black balls in urn I and let, as before, X be the average gain of n repeated independent draws from the urn chosen and for the color betted upon. Then mean and variance of X are determined as in Table 5, corresponding to Table 2. For $n = 1$, $E\{u(X)\} = \mu$, which is just the probability of winning the bet. It follows , for a "rational" person, that $R\ II \succ R\ I$ implies $B\ I \succ B\ II$, which, however, typically is not observed. On the other hand, for n large ($n \to \infty$) and assuming $E(p) = \frac{1}{2}$, the expected utilities for $R\ II$ and $B\ II$ are both $\frac{1}{2} - \frac{a}{4}$ and for $R\ I$ and $B\ I$ they are $\frac{1}{2} - \frac{a}{4} - aV(p)$. It follows that a "rational" person will have preferences $R\ II \succ R\ I$ and at the same time $B\ II \succ B\ I$, which corresponds to the behavior of the majority of people in Ellsberg's experiment.

Table 5: Mean and variance of X in Ellsberg's first experiment

betting act	μ	σ^2	$\lim_{n\to\infty}\sigma^2$
$R\ II$	$\frac{1}{2}$	$\frac{1}{4n}$	0
$R\ I$	$1-E(p)$	$\frac{1}{n}E\{p(1-p)\}+V(p)$	$V(p)$
$B\ I$	$E(p)$	$\frac{1}{n}E\{p(1-p)\}+V(p)$	$V(p)$
$B\ II$	$\frac{1}{2}$	$\frac{1}{4n}$	0

4 Conclusion

We can explain Ellsberg's paradox by assuming that people, subconsciously and erroneously, evaluate the possible outcomes of the experiment as if the draws from the urn were repeated an indifferent number of times. They are assumed to have a concave (possibly quadratic) utility function and a subjective probability distribution over the unknown "states of the world", which in this case are the unknown values of the proportion of balls in the urn. They then "compute" the subjective expected utility of the average gain from a bet on a particular color and choose the bet with the biggest expected utility. In doing so, they are in accordance with Bayesian SEU as well as with the actual behavior observed in Ellsberg's experiment. It turns out that ambiguity aversion is nothing but risk aversion in a SEU framework.

I must admit, though, that this explanation replaces one contradiction of rational decision theory with another one. In the original interpretation, where people are supposed to fully understand the experimental set up, which is that indeed just one ball is to be drawn, their behavior is in conflict with the Sure Thing Principle and so they are seen to behave irrationally. In the interpretation proposed in this paper, their behavior is irrational in that they misconceive the important, albeit a bit artificial, trait of the experiment that only one draw is to be executed. In the first interpretation people show ambiguity aversion in the second they show risk aversion.

Is there a "rational" explanation why people in a situation like Ellsberg's experiment intuitively think of repeated draws even though they are told that only one draw will be executed? I should like to argue that in practical decision situations under uncertainty the repetitive element is prevalent.

Consider an entrepreneur who wants to invest in an enterprise with uncertain profits. Suppose the entrepreneur can buy shares of a firm that for a long time in the past showed varying yearly profits with constant mean and variance and that will supposedly continue with that performance in the future. To be more specific, yearly profits of that firm are *i.i.d.* $N(\mu_1, \sigma_1^2)$ with known μ_1 and σ_1^2. Now suppose there is another possibility for investment. The entrepreneur can invest in a newly founded firm, which will have a rather constant, hardly varying stream of yearly profits π, the amount of which however is unknown. Let us, for the sake of simplicity, assume that π is constant (but unknown). Depending on the future development of the market the firm has business in, π can turn out to be high or low, but whatever value it will have in the future, this value will stay constant.

If the entrepreneur is a “rational” decision maker in the Bayesian sense , he will assign a probability distribution to π. Suppose $\pi \sim N(\mu_2, \sigma_2^2)$, where it so happens that the mean of this subjective distribution is the same as the mean of the objective distribution in the first case: $\mu_2 = \mu_1$, but where $\sigma_2^2 < \sigma_1^2$. If variances were also equal the two investments would only differ in their ambiguity, the first one being unambiguous, the second one being very ambiguous. Now if the entrepreneur invested his money just to receive a profit after one year and then went out of business, he would prefer the uncertain second alternative to the first one because the second alternative has the lower variance and both have equal means for the profit of this one year. However such a situation is rather unrealistic. An entrepreneur cannot so easily quit his engagement, especially not in the second case. If profits π turn out to be low, he will, of course, come to know this very soon, in fact after one year, but so will everybody else. So he will be able to disengage from his investment only with a loss of money. Therefore the entrepreneur has to consider not just the profit of one year but rather the whole stream of profits over the years. He might wish to evaluate an average profit (or rather an average of discounted profits, a case, where we could argue in a similar, but more complicated, way and that we shall not follow up here). In doing so he will most probably prefer the first investment even though it has the higher variance (the means being equal). The reason is that for the average profit the variance in the first investment goes to zero if the number of years increases but remains positive and constant for the second investment no matter how many years the investor takes into account. What looks as a case of ambiguity aversion actually is risk aversion.

As this kind of situation seems to arise quite often in practice, we should not be surprised to discover that people behave in Ellsberg’s experiment as if they considered not just one draw but several draws and thus come to a conclusion which seems to contradict the axioms of rationality but is in fact in accordance with these axioms if seen from the perspective of a long term investment.

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After the paper was finished I learned about an unpublished paper by Halevy and Feltkamp (1999) that, in part, deals with the same problem using a similar approach as in the present paper.

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