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Exclusive vs Overlapping Viewers in Media Markets*

Attila Ambrus† and Markus Reisinger‡

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Abstract

This paper investigates competition for advertisers in media markets when viewers can subscribe to multiple channels. A central feature of the model is that channels are monopolists in selling advertising opportunities toward their exclusive viewers, but they can only obtain a competitive price for advertising opportunities to multi-homing viewers. Strategic incentives of firms in this setting are different than those in former models of media markets. If viewers can only watch one channel, then firms compete for marginal consumers by reducing the amount of advertising on their channels. In our model, channels have an incentive to increase levels of advertising, in order to reduce the overlap in viewership. We take an account of the differences between the predictions of the two types of models and find that our model is more consistent with recent developments in broadcasting markets. We also show that if channels can charge subscription fees on viewers, then symmetric firms can end up in an asymmetric equilibrium in which one collects all or most of its revenues from advertisers, while the other channel collects most of its revenues via viewer fees.

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1 Introduction

Advertising expenditures for television commercials have grown sharply in recent years in almost all industrialized countries. In Germany, expenditures for television advertising amounted to 7.74 bn. euros in 2004, which in real terms is three times higher than the expenditures in 1990.[1] Similar numbers can be found in almost all western countries. At the same time, there has been an ongoing discussion if regulation concerning the amount and the content of commercials is necessary.[2] These days in the European Union, there exists an advertising ceiling of 12 minutes per hour, although the media commission of the European Union is discussing whether to abolish this law. In contrast, an advertising cap in the United States was abolished already in 1981 by the Federal Communication Commission.[3] So the question arises if there is too much advertising in an unregulated broadcasting market, which would render a regulation necessary, or if such a regulation is harmful.

In a recent paper, Anderson and Coate [2005] provide a careful analysis of these issues. They model markets for media advertising as a two-sided market with both positive and negative externalities: advertisers care positively about the number of viewers on the same platform, but viewers dislike more advertising. Advertising is assumed to be informative. The paper provides useful insights in understanding strategic interaction in media markets and the welfare properties of equilibrium under different market configurations. It also inspired a rapidly growing literature (see a brief summary of this below) that builds on the model of Anderson and Coate.

In this paper, we revisit the analysis of markets for media advertising and point out that many of the results from the previous literature only apply to settings in which viewers are not allowed to multi-home, i.e. to connect to multiple platforms.[4] What makes this observation particularly

[1] This accounted for 44% of all advertising expenditures (including radio, newspapers, and magazines, but not including Internet advertising) in 2004, while it accounted for only 25% of advertising expenditures in 1990 (http://www.agf.de/daten/werbemarkt/werbespendings/).

[2] See Motta & Polo (1997) for a history of regulation of TV advertising. For a recent overview of the advantages and problems of advertising in the broadcasting industry, see Armstrong and Weeds (2005).

[3] There is a ceiling on the amount of advertising during children's programmes: 12 minutes per hour on weekdays and 10 minutes per hour during the weekend.

[4] Most of the time we will use the terminology "viewers" for ease of exposition, but the model we present applies to various forms of broadcasting, where the precise terminology

relevant is that in most media markets, restricting viewers to single-home is not a realistic assumption: viewers can subscribe to multiple TV channels, listeners can buy equipment to listen to many different radio stations, and readers can subscribe to multiple magazines with similar content.[5] We show that adding this extra element of realism alters the nature of competition in the market substantially, and it leads to different qualitative conclusions. We also confront our model's predictions with various stylized facts from broadcasting markets and find that our model explains these facts better than models with viewer single-homing.

In our model the sizes of exclusive and overlapping viewerships are endogenously determined. A central feature, which distinguishes our model from a standard Hotelling competition in which buyers can purchase multiple goods, is that overlapping viewers are less valuable for channels than exclusive ones.[6] There are two reasons for this. The first is a direct effect: viewers subscribing to a second channel might substitute time away from watching the first channel, which makes the first channel less attractive in a setting of informative advertising (there is less chance of "hitting the target"). The second effect comes from the fact that the platforms provide alternative ways of reaching a multi-homing viewer and therefore compete with each other in selling advertising opportunities to these viewers. Therefore, they can only obtain a competitive price for selling advertising opportunities to these viewers, which is equal to the incremental value of trying to reach a viewer through a second channel. On the other hand, platforms are monopolists with respect to selling advertising opportunities toward their exclusive viewers, and they can extract all the surplus for this transaction from advertisers. The reduced value of multi-homing viewers is the main driving force behind our results. It induces platforms to distort their advertising decisions, in order to reduce or eliminate the overlap either on the viewer side or on the advertiser side. Depending on which side it is less costly to reduce overlap, advertising levels can be distorted both upward or downward.

That multi-homing viewers are worth less to advertisers is consistent with the empirically well-documented fact that the per-viewer fee of an ad-

would be "listeners" or "readers."

[5] Of course, a viewer can only watch one programme at a time (and a listener only listens to one radio station), but over the course of a week, or even just an evening, viewers (listeners) typically switch among different channels.

[6] For a model that allows consumers to buy both products in a standard Hotelling framework, see Kim and Serfes (2005).

vertisement on programs with larger viewerships is larger. In the US, both Fisher et. al. (1980) and Chwe (1998) find this regularity. In the UK, television market ITV, the largest commercial network, enjoys a price premium on its commercials.[7] Our model is consistent with the regularity since reaching the same number of eyeball pairs through broadcasting a commercial to a large audience implies reaching more viewers than reaching the same number of eyeball pairs through a series of commercials to smaller audiences, because the latter audiences might have some viewers in common.[8]

We characterize equilibrium advertising levels both for the case when stations are owned by a monopolist and for the case of competing stations. We show that in case of a price-discriminating monopolist (one who can charge a different price to multi-homing advertisers than to single-homing ones), equilibrium always entails too much advertising relative to the social optimum. In all other cases, equilibrium can lead to either too much or too little advertising from the social point of view. We also show that advertising levels can be higher in duopoly competition than in case of a monopolist provider. This is a different conclusion than what can be obtained from a model with viewer single-homing, since in the latter the stations compete for viewers by decreasing the level of advertising. Therefore our model, as opposed to a model with viewer single-homing, can explain why competing firms might support a regulation that establishes advertising caps.

Our model has different implications on how a station reacts to the entrance of a competing station as well. When platforms are weakly differentiated in a single-homing model, entrance decreases the level of advertising on a platform. This is because the incumbent firm is forced to make its channel more appealing to viewers to avoid losing its marginal viewers. However, if viewers can multi-home and the value of overlapping viewers is low, then platforms increase advertising after the entry, to reduce the overlap in viewership. Moreover, in a model with viewer single-homing, it is never in the interest of a station to ban advertising on a competing station, since that

[7] In fact, this premium increased steadily in the last decade, despite the entry of several competitors and ITV's decreasing market share (see the 2003 Competition Commission Report). This is commonly referred to as the "ITV premium puzzle." We thank Helen Weeds for calling our attention to these facts.

[8] The fact that reaching the same potential buyer a second time is of less value than reaching him the first time is already recognized by Ozga (1960): "... as more and more of the potential buyers become informed of what is advertised, more and more of the advertising effort is wasted, because a greater and greater proportion of people who see the advertisements are already familiar with the object"(p. 40).

only makes the other channel more attractive for viewers. On the other hand, in our model this type of regulation is always in the interest of a station who can advertise, since this way it becomes a monopolist to provide advertising opportunities to all its viewers, even if there is overlap in viewerships. This is in line with the fact that private channels in Germany (where private and public channels have roughly equal viewerships) are putting up resistance against a proposal that would lift advertising regulations on public broadcasters.

We also point out that in a model with viewer multi-homing, platforms' profits are not always monotonically increasing in the attractiveness of viewers. The intuition is that, if there is overlapping viewership, an increase in the attractiveness also increases the overlap, which for a region of parameter values dominates the positive effect of channels becoming more desirable for viewers. This can explain why the price of commercials decreased significantly in the last five years in the German broadcasting market, despite the steady increase in the amount of time people spent watching TV, since an increase in the desirability of channels presumably increased the overlap in viewerships.

Finally, we show that if platforms are allowed to charge viewer fees, then a certain type of asymmetric equilibrium arises. In this equilibrium, one platform collects most or all of its revenues from advertising, while the other platform collects most of its revenues via viewer fees and provides little advertising. This type of differentiation can be observed in various settings, such as the cable television market ("standard" cable channels and HBO type channels).

There is an enormous and diverse literature on advertising. For a comprehensive summary, see Bagwell (2005). The informative view of advertising that we take in this paper is formulated by Ozga (1960). The traditional literature on broadcasting assumes advertising revenues to be fixed (e.g. Steiner (1952), Besen and Soligo (1973), Beebe (1977), and Spence and Owen (1977)). Moreover, they do not model the externalities that advertising is a nuisance to viewers and that more viewers are attractive for advertisers. There is also a large and growing literature on two-sided markets, starting with Rochet and Tirole (2003).[9] Below we only mention some recent papers which are at the intersection of the above lines of research, since they are the closest to our work. The recent literature on media markets

[9]For a survey of this literature, see Rochet and Tirole (2005).

started with the aforementioned paper by Anderson and Coate (2005). We use their model framework, with some modifications. Particularly related to our investigation is section 7 of their paper, where they briefly investigate a two-period model in which viewers can switch from one channel to the other after period 1. It is shown that in this case, the possibility of underadvertising is mitigated. Choi (2004) uses the Anderson and Coate framework to investigate how regulating the number of stations or the amount of advertising affects welfare. Crampes et al. (2005) compare price competition and quantity competition in media markets. Dukes and Gal-Or (2003) and Peitz and Valletti (2004) extend the analysis by examining location choices of stations. Finally, Armstrong (2005), using a two-sided market model with no negative externalities, investigates whether stations prefer charging advertisers on a per-consumer basis or on a lump-sum basis. For a recent empirical work on estimating advertising demand and advertising surplus in the US radio broadcasting industry, see Berry and Waldfogel (1999).

2 The Model

We use a simplified version of the model in Anderson and Coate (2005), in that we assume homogeneous advertisers, but extend the model with the possibility of multi-homing on the viewer side. The model features a two-stage game with three different types of players: platforms owner(s), viewers, and advertisers.

Platforms

There are two platforms (channels), indexed by $i \in \{0, 1\}$. We will investigate both the case in which the same owner operates both platforms, and the case in which the channels are competing. The owners of platforms set the advertising levels a_0 and a_1, with the aim of maximizing profits.[10] In the monopoly case, we also investigate the possibility that the monopolist can sell joint advertising slots for the two platforms. In this case, the monopolist sets three quantities: $\overline{a}_0$, $\overline{a}_1$, and $\overline{a}_{01}$, where $\overline{a}_0$ and $\overline{a}_1$ are the quantities of single advertising slots on the either of the platforms, while $\overline{a}_{01}$ is the quantity of joint advertising slots. The resulting total amount of advertising on

[10] Alternatively, we could consider a model in which platforms set prices. This leads to conclusions similar to those in the quantity competition we investigate, but the analysis (in particular, characterizing the range of parameter values resulting in different types of equilibria) is technically more difficult.

the channels is then $a_0 = \overline{a}_0 + \overline{a}_{01}$ and $a_1 = \overline{a}_1 + \overline{a}_{01}$. We refer to the latter case as a price-discriminating monopolist (since the equilibrium price of a joint advertising slot will typically be different from the sum of the prices of single advertising slots), while we refer to the case when a monopolist can only set two quantities as a nondiscriminating monopolist. The profit function of the nondiscriminating monopolist is $\Pi_{ND} = a_0 p_0 + a_1 p_1$, with p_i being the market clearing price of an advertising slot on platform i. The profit function of platform i in duopoly is $\Pi_i = a_i p_i$, and the profit of the discriminating monopolist is $\Pi_{dis} = \overline{a}_0 \overline{p}_0 + \overline{a}_1 \overline{p}_1 + \overline{a}_{01} \overline{p}_{01}$. This specification assumes that platforms collect all their revenues from advertisers. In Section 5, we consider the case when platforms can also charge subscription fees on viewers.

Viewers

There is a continuum of viewers with mass M, uniformly distributed on $[0, 1]$. Viewers can decide to watch both channels, meaning that they can subscribe to both channels and split their time between watching the two programs.[11] A viewer who is located at position x_j obtains a net viewing benefit of $\beta - \gamma a_0 - \tau x_j$ if only watching channel 0, and $\beta - \gamma a_1 - \tau(1 - x_j)$ if only watching channel 1. Viewers have heterogeneous tastes, with those located at point 0 liking channel 0 most and those located at point 1 liking channel 1 most. The marginal travel cost is τ. β represents the base level of utility from watching one's ideal channel. Finally, viewers dislike advertising and γ represents the nuisance cost parameter concerning commercials. As in Anderson and Coate (2005), viewers do not get any positive value from advertising, because advertisers are monopolist producers of differentiated products and therefore can extract all consumer surplus (see below in more detail).

A viewer who watches both channels obtains utility $u(\beta - \gamma a_0 - \tau x_j, \beta - \gamma a_1 - \tau(1 - x_j))$. We assume that u is increasing in both variables, and that $u(\beta - \gamma a_0 - \tau x_j, 0) = \beta - \gamma a_0 - \tau x_j$ and $u(0, \beta - \gamma a_1 - \tau(1 - x_j)) = \beta - \gamma a_1 - \tau(1 - x_j)$. The above specification implies that a viewer watches channel 0 iff $\beta - \gamma a_0 - \tau x_j > 0$, and watches channel 1 iff $\beta - \gamma a_1 - \tau(1 - x_j) >$

[11] If the media platforms are radio or TV stations, then multi-homing implies that the listener/viewer subscribes to both channels (or has equipment to tune into both stations) and over a unit interval of time, which can be a day or a week, spends a positive amount of time listening to/watching each channel. If the media platforms are magazines with similar content, multi-homing implies that the reader reads both magazines.

0 (in particular, if both of the above terms are positive, then the viewer watches both channels).

Advertisers

There is a continuum of advertisers with mass N. To simplify the analysis, we restrict attention to the case when advertisers are homogeneous. In Section 6, we briefly discuss how our results extend to the case of heterogeneous advertisers.

Advertising is informative. Each advertiser is a monopolist producer of a differentiated product, with constant marginal cost of zero. Consumers have to get informed of the product through advertising in order to be able to buy it. For each product, a fraction q of the viewers (randomly selected) has a reservation value normalized to A, while the rest of the viewers have reservation value 0. Because the advertiser is a monopolist producer of its product, it charges a price of exactly A.

The gross value of an advertising slot on a platform for an advertiser is equal to the expected increase in sales revenues that the commercial generates, which depends on the number of viewers who watch this channel, the time they spend watching it, and whether the same viewers can get informed about the product by watching the other channel. The amount of time a viewer spends watching a channel over a unit interval of time is $t \in [0, 1]$. There is $\varepsilon \in [0, 1]$ probability that a viewer does not pay attention to a commercial when it appears on the screen. Commercials appear at random times on the channel, and so the expected value of advertising to a consumer is $t(1-\varepsilon)qA$. If the same viewer watches two channels, and spends t amount of time watching each channel, then advertising to this viewer through both channels yields an expected value $[t(1-\varepsilon)+t(1-\varepsilon)(t\varepsilon)]qA$. In what follows we simplify the model by assuming that each viewer who only watches one channel spends the same amount of time t' on watching the channel. Similarly, each viewer who watches both channel spends the same amount of time t'' on watching each channel.[12] Furthermore, we assume that $t'' \in [t'/2, t']$. In the limit case $t'' = t'$, a viewer does not substitute any time away from the first channel when subscribing to a second channel.

[12] More general formulations, in which the amount of time a viewer spends on watching a channel depends on the utility the viewer obtains from connecting to the channel, would imply the same qualitative conclusions.

This formulation leads to a reduced form in which the expected value of advertising to a viewer who watches exactly one channel is ω, the expected value of advertising through exactly one channel to a viewer who watches two channels is $\omega' \in [\omega/2, \omega]$, and the expected value of advertising through both channels to a viewer who watches two channels is $\omega' + \omega''$, where $\omega'' \in (0, \omega')$.[13] The incremental value of reaching the viewer with a commercial through a second channel (ω'') is smaller than the value of reaching her through the first channel (ω') because there is a chance that the viewer already got informed about the product through the first channel.[14]

Let n_0 denote viewers who watch channel 0 (including those who also watch channel 1), n_1 denote viewers who only watch channel 1 (including those who also watch channel 0), and n_{01} denote viewers who watch both channels. Then the profit of an advertiser is:

$$\pi = \begin{cases} 0 \text{ if he does not advertise} \\ (n_i - n_{01})\omega + n_{01}\omega' - p_i \text{ if he advertises only on platform } i \\ (n_0 + n_1 - 2n_{01})\omega + n_{01}(\omega' + \omega'') - p_0 - p_1 \text{ if he advertises on both.} \end{cases}$$

We note that the above payoff structure need not apply to types of advertising other than informative. For example, in case of persuasive advertising, it is not clear if the value of reaching a given viewer the second or third time is less valuable for an advertiser than the first time. However, even in this case, there is a reason why overlapping viewers can be less valuable than exclusive ones, namely that it is easier to reach viewers the ideal number of times if they only watch one station (in the case of overlapping viewerships, an advertiser can only reach all viewers the desired number of times if some people are reached inefficiently many times). Moreover, the empirical findings in media markets from the US and UK are consistent with the assumption that overlapping viewers are less valuable for advertisers. In the UK, the ITV network (commonly known as "Channel 3") is the biggest commercial television network. Because of entry of many new smaller channels (like "Channel 4" in 1983 or "Channel 5" in 1997), the audience share of ITV decreased steadily over the last 15 years.[15] Yet the percentage of ITV's

[13] In particular, $\omega = t'(1-\varepsilon)qA$, $\omega' = t''(1-\varepsilon)qA$, and $\omega'' = [t''(1-\varepsilon)(t''\varepsilon)]qA$.

[14] The same applies when the same advertiser buys a second advertising slot on the same platform. The latter phenomenon does not arise in equilibrium in our model if there are enough potential advertisers (which we assume below).

[15] The audience share in 1987 was 40 percent, while in 2003 it was only 22.2 percent. See Competition Commission (2003), p. 96.

net advertising revenue out of total TV advertising revenue has decreased by far less. Moreover, from 1992 on, the advertising price (per adult impact) of ITV has increased.[16] This increase was even higher from 1997 to 2002, a time during which many new competing channels entered the market. This phenomenon is known as the "ITV premium puzzle." A possible explanation for the puzzle, which is mentioned in a 2003 Competition Commission Report, is that if a commercial reaches one million pair of eyeballs on ITV, it is likely to reach a million different people, since ITV has many mass audience programmes.[17] On the other hand, reaching a million pairs of eyes on other channels means reaching much less than a million viewers, because there are many viewers whom the commercial reaches multiple times. The fact that the price per viewer of a commercial is increasing in audience size is a strong pattern in US data as well: see for example Fisher et al. (1980) and Chwe (1998). In fact, Fisher et al. (1980) provides a reasoning similar to our main argument: "a spot on a large audience provides a greater number of different viewers than two spots on a smaller audience, because the smaller audience may have some viewers in common," and so advertisers are willing to pay more for it than for the two spots together.

Timing of the game

First, platform owners simultaneously set advertising levels (number of advertisement slots) on their platforms, that is, $\overline{a}_0$, $\overline{a}_1$, and $\overline{a}_{01}$ in the case of a discriminating monopolist, and a_0 and a_1 in the other cases. The chosen advertising levels determine the prices that clear the market. In the next section, we show that these market clearing prices are unique. Finally, viewers observe the levels of advertising on the platforms and decide whether to watch both channels, only one, or neither of them.

3 Equilibrium Levels of Advertising

In this section, we solve for the subgame perfect Nash equilibria of the game introduced in the previous section, both for when the platforms are owned

[16] The advertising price is measured as the expenditure of an advertiser divided by the impact on its targeting group, where the latter is measured as percentage of viewers of the targeted group. This variable does not account for double viewing, so if a viewer watches the commercial twice, she is counted in the same way as two viewers who have seen the commercial once.

[17] See Competition Commission (2003), p. 123 and p. 136.

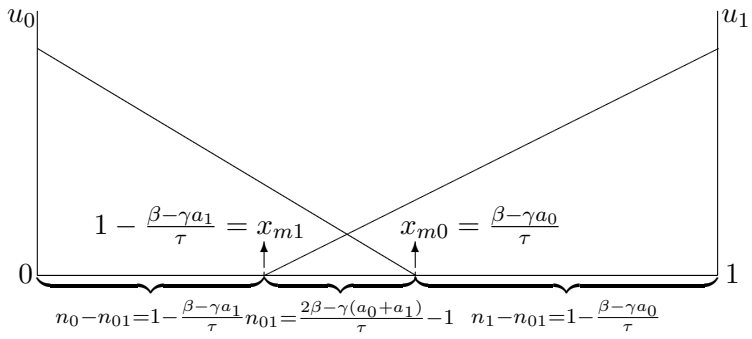

Figure 1: Viewerships (if viewers overlap)

by a monopolist provider and for the case of competing platforms. For the characterization of the equilibrium of the corresponding model when viewers are not allowed to multi-home (which is essentially the model in Anderson and Coate (2005) restricted to a setting with homogeneous advertisers), see Appendix A.

3.1 General considerations

First, note that the amount of viewers is determined uniquely by the advertising quantity decisions of channels. In the nondiscriminating monopoly case and the duopoly case, it is given by $n_0 = \max(0, \min(\frac{\beta-\gamma a_0}{\tau}, 1)M)$ for platform 0, and $n_1 = \max(0, \min(\frac{\beta-\gamma a_1}{\tau}, 1)M)$ for platform 1. The marginal viewers in an inner solution are $x_{m0} = \frac{\beta-\gamma a_0}{\tau}$ and $x_{m1} = 1 - \frac{\beta-\gamma a_1}{\tau}$. The overlapping viewership is $n_{01} = \max(0, x_{m0} - x_{m1})M$, which is equal to $\max(0, \frac{2\beta-\gamma(a_0+a_1)}{\tau} - 1)M$ for an inner solution. In the discriminating monopoly case, the viewerships are $n_0 = \max(0, \min(\frac{\beta-\gamma(\overline{a}_0+\overline{a}_{01})}{\tau}, 1)M)$ and $n_1 = \max(0, \min(\frac{\beta-\gamma(\overline{a}_0+\overline{a}_{01})}{\tau}, 1)M)$ for platforms 0 and 1, respectively. The marginal viewers and the viewerships are displayed in Figure 1.

Next, it can be shown (see Lemmas 1 and 2 in Appendix B) that for almost all advertising level choices the market clearing prices are uniquely determined, and given by:

$$p_i = \begin{cases} \omega n_i \text{ if } n_0 + n_1 \leq M \\ (n_i - n_{01})\omega + n_{01}\omega' \text{ if } n_0 + n_1 > M \text{ and } a_0 + a_1 \leq N \\ (n_i - n_{01})\omega + n_{01}\omega'' \text{ if } n_0 + n_1 > M \text{ and } a_0 + a_1 > N. \end{cases}$$

for advertising slots on platform i in the nondiscriminating monopoly and in the

duopoly case. In the discriminating monopoly case, they are given by $p_i = \begin{cases} \omega n_i \text{ if } n_0 + n_1 \leq M \\ (n_i - n_{01})\omega + n_{01}\omega' \text{ if } n_0 + n_1 > M \end{cases}$ for an advertising slot on platform i and by $p_{01} = \omega(n_0 + n_1 - 2n_{01}) + (\omega' + \omega'')n_{01}$ for the joint advertising slots. Moreover, market clearing prices can be assumed without loss of generality to be given by the above formulas for all pairs of advertising levels in equilibrium.[18]

Equilibrium prices depend on the number of overlapping and nonoverlapping viewers, and on whether advertisers overlap or not.

For ease of exposition, we impose the following two-parameter restriction for the subsequent analysis:

$$\max(\frac{\beta}{2\gamma}, \frac{2\beta - \tau}{2\gamma}) \leq N \tag{1}$$

$$\beta - \gamma\frac{N}{2} \leq \tau. \tag{2}$$

The first one rules out boundary cases in which all potential advertisers advertise on a platform. The second one implies that the nuisance parameter is high enough such that if the amount of advertising is at least $\frac{N}{2}$ (half of the potential advertisers are present), then not every viewer would watch both channels. This assumption rules out boundary cases in which even for relatively high levels of advertising, all viewers watch both channels in equilibrium. The above restrictions are only made for analytical convenience, in order to avoid checking whether the corresponding boundary conditions $a_0, a_1 \leq N$ and $x_0 \leq 1$, $x_1 \geq 0$ bind. Dropping these restrictions does not change the qualitative features of the analysis.

3.2 Monopoly

A provider who owns both stations can choose to set advertising levels such that in the resulting continuation equilibrium: (i) there is no overlap either on the viewer side or the advertiser side; (ii) viewers overlap but advertisers

[18] For certain advertising level choices there are multiple market clearing prices. For example if $a_i = N$ then any price below what is specified above clears the market, too. The reason this does not matter in equilibrium is that a channel can always choose an advertising level arbitrarily close to the original level, such that the market clearing prices are now uniquely determined and they are given by the formulas above.

do not; (iii) advertisers overlap but viewers do not; and (iv) there is overlap on both sides. Which of the above is optimal depends on the parameters of the model, in particular on the relative scarcity of advertisers, the nuisance parameter, and the travel cost. For a formal derivation of equilibrium advertising levels for different parameter values, see Appendix B. Here we only state the results and provide intuition for the optimal strategy of the monopolist.

Proposition 1: Assume that the platforms are owned by a nondiscriminating monopolist. Then advertising levels chosen in equilibrium are generically unique, and depending on the parameters they can take the following values:

(a) $a_0 = a_1 = \frac{\beta}{2\gamma}$ (viewerships do not overlap)

(b) $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$ (exactly half of the viewers watch each channel)

(c) $a_0 = a_1 = \max(\frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega'-\omega)}, \frac{\beta-\tau}{\gamma})$ (overlap of viewers, but not of advertisers)

(d) $a_0 = a_1 = N/2$ (exactly half of the potential advertisers advertise on each channel)

(e) $a_0 = a_1 = \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega''-\omega)}$ (overlap on both sides).

In region (a), the travel cost parameter is so high ($\tau > \beta$) that viewerships in equilibrium do not overlap: $n_0 = n_1 < \frac{M}{2}$ and $n_{01} = 0$. The owner can set the optimal level of advertising on the platforms independently of each other, since advertising revenues are not interrelated.

In regions (b)-(e), the travel cost parameter is low enough so that the monopolist cannot set the levels of advertising that would be optimal for isolated platforms without establishing an overlap in the viewerships.

In region (b), ω' is low enough such that establishing an overlapping viewership is not profitable. In this case, the monopolist chooses advertising levels such that exactly half of the viewers watch each channel: $n_0 = n_1 = \frac{M}{2}$ and $n_{01} = 0$.

In region (c), there is a relative abundance of potential advertisers, and ω' is high. In this case, viewerships overlap, but advertisers do not: $n_{01} > 0$ and $a_0 = a_1 < \frac{N}{2}$.

In region (d), there is a relative scarcity of advertisers, and ω'' is low. In this case, the monopolist sets advertising levels such that exactly half of

the potential advertisers advertise on each platform, but there is overlap in viewerships: $a_0 = a_1 = \frac{N}{2}$ and $n_{01} > 0$.

In region (e), there is a relative scarcity of advertisers, and ω'' is high. In this case, the monopolist sets advertising levels such that both advertisers and viewers overlap: $a_0 = a_1 > \frac{N}{2}$ and $n_{01} > 0$.

We note that if the number of potential advertisers is high enough, such that $\frac{2\beta-\tau}{\gamma} \leq N$, then the equilibrium advertising levels cannot be the quantities in (d) or (e), since overlapping viewership in this region implies $a_0 + a_1 < N$. Therefore, if there is no scarcity of potential advertisers and the travel cost parameter is low enough, the monopolist's problem is simplified to choosing between establishing overlapping viewership or establishing viewerships that just do not overlap.

Next we consider the case when the monopolist can offer a price discount to advertisers who advertise on both platforms. The difference compared to the nondiscrimination case is that now the monopolist can extract the whole surplus $\omega' + \omega''$, as opposed to $2\omega''$, from advertising a product to a multi-homing viewer through both channels. This makes two-sided multi-homing a more desirable choice for the monopolist. Note that the value of advertising two different products on the two different channels to a multi-homing viewer is $2\omega'$, which still exceeds $\omega' + \omega''$.

Since the value of nonoverlapping viewers and the value of overlapping viewers in case of nonoverlapping advertisers are not affected by whether the monopolist can price discriminate or not, outside the region of two-sided overlapping potential equilibrium advertising levels are the same as in the nondiscriminating monopoly case.

Proposition 2: Assume that the platforms are owned by a nondiscriminating monopolist. Then advertising levels chosen in equilibrium are generically unique. If there is two-sided overlap in equilibrium, then they are given by $a_0 = a_1 = \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)-N\gamma(\omega'-\omega'')}{2(2\omega''-\omega)\gamma}$. Otherwise the possible equilibrium levels of advertising are as in Proposition 1.

Because of the increased value of overlapping viewers in the case of two-sided overlapping, the parameter region in which the monopolist chooses two-sided overlap is strictly larger than in case of a nondiscriminating monopolist. Conversely, the regions in which the monopolist chooses either

just nonoverlapping viewerships ($a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$) or just nonoverlapping advertisers ($a_0 = a_1 = \frac{N}{2}$) are smaller.

3.3 Duopoly

Here we analyze the case when the stations are controlled by different firms. If $\tau \geq \beta$, then advertising levels in equilibrium are the same as in the case of monopoly ownership, because viewerships do not overlap and the stations can act as local monopolists. From now on, we assume $\beta > \tau$.

The next proposition characterizes possible symmetric equilibrium advertising levels in the duopoly game, for different parameter values.

Proposition 3: Assume that the platforms are owned by competing providers and that $\beta > \tau$. Then the following levels of advertising can constitute symmetric equilibria:
(a) $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$
(b) $a_0 = a_1 = \max(\frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}, \frac{\beta-\tau}{\gamma})$
(c) $a_0 = a_1 = \frac{N}{2}$
(d) $a_0 = a_1 = \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$.

The duopoly game has exactly the same types of symmetric equilibria as the monopoly game. Advertising levels in equilibrium are such that viewers do not overlap (including the case when exactly half of them connect to each channel), or viewers overlap but advertisers do not (including the case when exactly half of them advertise on each channel), or there is two-sided overlap. Moreover, the conditions for the existence of these different types of equilibria are similar to the conditions that determine which type of equilibrium prevails in the monopoly case. However, both the boundaries of the regions and the equilibrium advertising levels within a region differ from the monopoly case. This is because the duopolist providers do not take into account the effects of their advertising decisions on the profits generated on the other platform. In particular, if the level of advertising decreases on a platform, then the number of multi-homing viewers can increase, which affects the amount of revenues the other platform can collect for providing advertising opportunities after their viewers.

In contrast with the monopoly case, it is not true that equilibrium advertising levels are generically unique, even if one restricts attention to symmetric equilibria. It can be shown that there is a nonempty range of parameter

values such that both $a_0 = a_1 = \frac{N}{2}$ and $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$ are equilibria. In this region, the value of overlapping viewers is low in case of two-sided overlapping, so firms set advertising levels to avoid two-sided multi-homing. They can achieve this either by setting advertising levels high and avoiding overlapping of viewers, or by setting advertising levels low and avoiding overlapping of advertisers. The reason for this multiplicity is that advertising levels are strategic complements: the optimal level of advertising of a station increases in the level of advertising chosen by the other station. In a similar vein, there is a range of parameter values such that both $a_0 = a_1 = \frac{N}{2}$ and $a_0 = a_1 = \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$ can be an equilibrium.

Besides the above symmetric equilibria, the duopoly game can have asymmetric equilibria as well, in two cases. One is when the stations set advertising levels such that viewers just do not overlap. Then besides the equilibrium in which exactly half of the viewers watch each channel, there are equilibria in which stations share the market asymmetrically. Similarly, when stations set advertising levels such that advertisers just do not overlap, there are equilibria in which stations divide advertisers unequally. We do not provide a formal characterization of these asymmetric equilibria here. In general, the above equilibria cannot be "too asymmetric"; otherwise the firm with the smaller market share would have an incentive to deviate and establish two-sided multi-homing. Furthermore, the lower the value of overlapping viewers for the stations is, the more asymmetric equilibria can exist. We note that in these asymmetric equilibria, the aggregate profit of the firms is strictly smaller than in the symmetric equilibrium.

4 Comparative investigation of advertising levels

In this section, we derive some qualitative conclusions of the model in which there can be multi-homing on both sides, and compare the predictions with those that can be obtained from a model in which the viewers are not allowed to multi-home. We also relate our findings to some stylized facts from television advertising markets in the US and Germany.

4.1 Ownership structure and advertising levels

We start out by comparing equilibrium advertising levels under monopoly ownership and duopoly competition and show that in our model, as opposed

to a model with viewer single-homing, competing stations might have an incentive to support a regulation establishing advertising ceilings.

In the benchmark model where viewers can only single-home, a monopolist platform provider always chooses levels of advertising (weakly) higher than the equilibrium levels in duopoly competition. In particular, if $\tau \geq 2/3\beta$, then monopoly and duopoly imply the same levels of advertising, while $\tau < 2/3\beta$ implies that the amount of advertising is strictly smaller in duopoly than in monopoly.[19]

This result no longer holds in the model with viewer multi-homing. Advertising level in a duopoly equilibrium can be both larger or smaller than the advertising levels set by either a nondiscriminating or a discriminating monopolist. This is despite the fact that competing platform owners, as opposed to a monopolist provider, do not take into account the positive effect of increasing their level of advertising on revenues obtained on the other platform, which is a force driving advertising levels downward. This effect can be offset for several different reasons. First, in the discriminating monopolist case, the platform owner can extract higher revenues of overlapping viewers in case of two-sided overlapping. Therefore, there is a region of parameter values such that the unique symmetric duopoly equilibrium involves high enough advertising levels such that viewerships do not overlap, while the discriminating monopolist chooses lower levels of advertising that induce two-sided multi-homing. Second, if there are multiple symmetric equilibria in duopoly competition, then one of these equilibria might imply higher levels of advertising than the levels chosen by either the discriminating or a nondiscriminating monopolist. For example, there is a nonempty intersection of region (d) in Proposition 1 and region (a) in Proposition 3. For these parameter values a nondiscriminating monopolist chooses advertising levels $\frac{N}{2}$ (advertisers just do not overlap), while there is a symmetric equilibrium in duopoly such that $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma} > \frac{N}{2}$ (viewers just do not overlap).

The above result is consistent with the empirical finding in Brown and Alexander (2004) that a decrease in concentration in the broadcasting industry leads to a lower amount of commercial time and to increased viewerships.

The result also implies that, as opposed to a model in which viewers can only single-home, competing firms in the multi-homing model might find it profitable to establish advertising ceilings (since profits are always higher in

[19] See Appendix A.

the monopoly cases). This can be an explanation of why stations in the US voluntarily established such a ceiling in the 1970s. The National Association of Broadcasters (NAB) had a code of conduct in place which limited the advertising time per hour to 9.5 minutes in primetime and to 16.5 minutes at all other times.[20] There are several other possible explanations, the most natural one being that the code of conduct was a standard cartel agreement, to keep prices of commercials high. Indeed, the Department of Justice alleged that the Code of Conduct and kept the price of advertising high and therefore violated antitrust laws. As a consequence, the NAB voluntarily quit the Code in 1983. However, this story is still difficult to reconcile with a model with viewer single-homing, unless either advertisers are credit constrained or their production technologies inhibit decreasing returns to scale. Otherwise, since viewerships are distinct in the latter model, and therefore the stations sell different "goods" to advertisers, a smaller supply of advertising spots on other stations does not increase demand for a station's advertising spots. Hence, stations would not have an incentive to form a cartel.

4.2 Social welfare

In Appendix A, we show that in a model in which viewers can only single-home and the channels collect their revenues from advertising fees, a monopolist provider always chooses a (weakly) higher level of advertising than what is socially optimal. The intuition is that the monopolist does not fully internalize the negative externality of advertising on viewers. On the other hand, whether duopoly competition induces too much or too little advertising relative to the socially optimal level depends on the magnitudes of the nuisance cost parameter and the traveling cost parameter. If both of these parameters are small enough, then competition implies too little advertising.

Below we show that in our model, the result that a monopolist always chooses too high levels of advertising only holds for the case of the discriminating monopolist.

Social welfare in our model is the sum of aggregate viewer surplus and the total surplus from advertising, which can be written as follows:

$$WF = \overline{a}_0 \left(\omega(n_0 - n_{01}) + \omega' n_{01}\right) +$$

[20] For further reference, see Campbell (1999).

$$\overline{a}_1\left(\omega(n_1-n_{01})+\omega' n_{01}\right)+$$
$$\overline{a}_{01}[\omega(n_0+n_1-2n_{01})+(\omega'+\omega'')n_{01}]+$$

$$M\int_0^{(n_0-n_{01})/M}(\beta-\gamma a_0-\tau x)dx+M\int_{1-(n_1-n_{01})/M}^{1}(\beta-\gamma a_1-\tau(1-x))dx)+$$

$$\int_{(n_0-n_{01})/M}^{n_0/M}u(\beta-\gamma a_0-\tau x,\beta-\gamma a_1-\tau(1-x))dx,$$

where $a_i=\overline{a}_i+\overline{a}_{01}$ for $i=\{0,1\}$.

Note that the sum of the first three terms is exactly equal to the profit function of a discriminating monopolist, and that the last three terms are decreasing in advertising levels. This can be used to establish the following result.

Proposition 4: A discriminating monopolist always chooses a higher advertising level than what is socially optimal.

A similar general result cannot be established for the nondiscriminating monopolist and the duopoly case. The intuition is that if stations cannot price discriminate, then they cannot extract the full surplus from advertising toward overlapping viewers. The result is that, for a range of parameter values, the welfare maximizing outcome implies two-sided overlapping (i.e., advertising levels higher than $\frac{N}{2}$), while the monopolist sets advertising levels equal to $\frac{N}{2}$ to avoid overlapping of advertisers.

One can derive further welfare implications by choosing a concrete model specification. Below we consider the case when $\omega'=\omega$, that is, when viewers do not substitute time away from viewing a channel if they start watching a second channel. In this case, there is a natural specification of the utility function of a multi-homing viewer located at x_i, given by:

$$U_j=s_0(\beta-\gamma a_0-\tau x_j)+s_1(\beta-\gamma a_1-\tau(1-x_j)),$$

where $s_i, i\in\{0,1\}$ denotes an indicator function that is 1 if $\beta-\gamma a_0-\tau x_j>0$ (resp. $\beta-\gamma a_1-\tau(1-x_j)>0$) and 0 if $\beta-\gamma a_0-\tau x_j\leq 0$ (resp.

$\beta - \gamma a_1 - \tau(1 - x_j) \leq 0$).That is, utility of a viewer is just the sum of utilities obtained from watching the channels.

Characterizing the welfare maximizing solution is a similar exercise to characterizing equilibrium levels for a discriminating monopolist; therefore here we only report the possible equilibrium levels of advertising.

The welfare maximizing levels of advertising in the above specification can take the following values:
(a) $a_i^{WF} = 0$
(b) $a_i^{WF} = \frac{\beta(\omega-\gamma)}{(2\omega-\gamma)\gamma}$
(c) $a_i^{WF} = \frac{N}{2}$
(d) $a_i^{WF} = \frac{\beta(\omega-\gamma)-N\gamma(\omega-\omega'')}{\gamma(4\omega''-2\omega-\gamma)}$
(e) $a_i^{WF} = \frac{2\beta-\tau}{2\gamma}$.

Region (a) corresponds to the case when the nuisance parameter is high ($\omega < \gamma$). In all other regions welfare maximizing advertising levels are similar to the types of equilibrium outcomes characterized in Section 3.

In this model specification, if we restrict attention to the region in which there is two-sided overlapping in equilibrium, then we can obtain a clear-cut ordering of equilibrium advertising levels relative to the welfare maximizing levels. Let a_{WF} denote the welfare maximizing level of advertising on a station, and let a_{dis}, a_{nd}, and a_{duo} denote the equilibrium levels of advertising in the discriminating monopoly, nondiscriminating monopoly, and duopoly cases, respectively.

Proposition 5: If $\omega' = \omega$ and there is overlap on both sides in equilibrium, then advertising levels are too high in all three regimes compared to the socially optimal level. The order of advertising levels is $a_{WF} < a_{dis} < a_{duo} < a_{nd}$.

Relationship $a_{WF} < a_{dis}$ is implied by Proposition 4. $a_{duo} < a_{nd}$ always holds within the region where equilibrium implies two-sided overlapping, because the duopolist stations do not take into account the negative externality of decreasing their levels of advertising on the profits of the other station. Finally, $a_{dis} < a_{duo}$ holds if $\omega' = \omega$ because in this specification, a discriminating monopolist values overlapping viewers in case of two-sided overlapping relatively high compared to duopolist firms, and therefore advertises less to increase the overlap.

4.3 Entrance of a competitor and advertising

In this subsection, we investigate how a channel adjusts the level of advertising after the entrance of a competing channel, in the two-sided multi-homing model introduced in Section 2, and in a corresponding model in which viewers can only single-home.

If only firm 0 is present in the market, it is straightforward to establish that in equilibrium it chooses $a_0 = \max(\frac{\beta}{2\gamma}, \frac{\beta-\tau}{\gamma})$.

Assume now that firm 1 enters the market. If viewers can only single-home, the equilibrium levels of advertising are given by:[21]

$$a_0 = a_1 = \begin{cases} \frac{\beta}{2\gamma} & \text{if } \beta \leq \tau \\ \frac{2\beta-\tau}{2\gamma} & \text{if } \beta > \tau \geq (2\beta)/3 \\ \frac{\tau}{\gamma} & \text{if } \tau < (2\beta)/3. \end{cases}$$

This implies that in the single-homing case, whether the entrance of a competitor increases the level of advertising depends on the traveling cost parameter. If $\tau > \frac{\beta}{2}$, then the amount of advertising on channel 0 increases (weakly). If differentiation between channels is high, then stealing marginal consumers from the competitor is costly. Then in equilibrium, channels advertise a lot, to a smaller audience. However, if $\tau < \frac{\beta}{2}$, then entrance decreases the amount of advertising on channel 0. If channels are less differentiated, then competition for viewers forces the channels to advertise less, to retain marginal consumers switching to the competitor.

If consumers can multi-home, then it can be shown that as long as the number of potential advertisers is high enough, entrance of a competitor increases the level of advertising on a station.

Proposition 6: If $N \geq \frac{2\beta-\tau}{\gamma}$, then entrance of a competitor always weakly increases the level of advertising on a station.

If $N < \frac{2\beta-\tau}{\gamma}$ then entrance of a competitor can either decrease or increase the level of advertising, depending on the type of equilibrium that prevails for the specific parameter values.

The above implies that for some parameter regions, the difference between predictions of the single-homing and multi-homing models is stark.

[21] See Appendix A.

For example, this is the case when the travel cost parameter is low (the stations are weakly differentiated) and the amount of potential advertisers is large. In these cases, competition for viewers in the single-homing model forces a channel to decrease the level of advertising, while in the multi-homing model, competition increases the level of advertising on the channel, because of the decreased value of overlapping viewers.

The entry of a competitor, as expected, decreases the equilibrium of profit of a firm both in the model with viewer single-homing and in the model with viewer multi-homing. However, there is a sharp difference between the predictions of the two models in how a station's profit is affected by a competing channel switching from no advertising to advertising. The investigation of this question is motivated by an ongoing policy debate in Germany concerning regulating the level of advertising on public stations.

The German broadcasting market consists of six major channels (ARD, Pro7, RTL, SAT1, ZDF, and the "third programmes," which are regional programmes in each state). Each of these channels has a market share between 10% and 14%, leading with ARD with 13.9% and ending with SAT1 with 10.3%.[22] Three of the channels are public channels (ARD, ZDF, and the "third programmes") and are financed partly by taxes and partly by advertising revenues. The other three channels (Pro7, RTL, SAT1) are private stations and collect revenues only through advertising. For the private stations, there is no law regulating their broadcasting of commercials besides the 12 minutes per hour limit set by the media commission of the European Union. The public stations, on the other hand, are not allowed to broadcast commercials after 8:00 p.m. The German government is currently discussing whether to abolish this law and to allow public stations to broadcast advertising after 8:00 p.m. It is well-publicized that the private stations strongly oppose this abolishment, arguing that the public stations have a social mission for their viewers and should therefore not be allowed to fill their viewing time with commercials.[23]

This stylized fact would be difficult to reconcile with models in which advertisers can multi-home but viewers can only single-home. If viewers can only single-home, then channels sell advertising opportunities toward distinct viewerships. Hence, as long as advertisers are not budget constrained,

[22] The source of these data is AGF/GfK Fernsehforschung. See also www.agf.de.

[23] For example, in July 2005, the president of the Association of Private Radio and Telecommunication demanded a complete advertising ban for public TV stations. See e.g. www.presseportal.de.

and their production technologies do not inhibit decreasing returns to scale, an increased supply of advertising slots by competing channels does not decrease the demand for advertising on a station. Therefore, the only effect of increased advertising on a competing station is through its impact on viewerships. And in the viewer single-homing model, more advertising on a rival station makes that station less attractive; therefore it increases the number of viewers on its own station and leads to higher profits. This implies that private stations should be supporting an initiative like that above. However, their behavior is in line with the results of our model. If viewers multi-home (and indeed there is a large overlapping viewership between public and private channels), then allowing a competing station to advertise decreases a station's profit, because it can only extract ω'' after selling advertising opportunities toward an overlapping viewer, while previously it could extract ω'.[24]

4.4 Comparative statics

In case of a monopolist provider, advertising levels and firms' profits are monotonic in the parameters of the model, with the expected signs. An increase in β or a decrease in either γ or τ increases advertising levels and profits, because the same viewership sizes are compatible with higher amounts of advertising. Similarly, increases in ω, ω', and ω'' increase advertising levels and profits.

In case of competing channels, these relationships can become nonmonotonic. In particular, firms' profits can be nonmonotonic in τ.

Proposition 7: There is a range of parameter values for which advertising levels are uniquely determined in duopoly competition, and equilibrium profits of the stations are strictly decreasing in τ.

An increase in τ on one hand has a direct negative effect on profits of the firms, because it reduces the total potential surplus (stations become less valuable for viewers). However, it decreases the amount of overlap in viewerships, which has a positive effect on firms' profits. For a range of

[24] Another, complementary explanation for this behavior is that public stations could obtain higher revenues, which if used to improve the quality of programs, could induce some viewers to substitute time away from watching private channels to watching public channels.

parameter values, the second effect dominates, implying that a decrease in the appeal of stations increases their profits.

This result can provide an explanation for a surprising recent development in the German broadcasting market: viewership in the last five years has grown steadily, but this was accompanied by decreasing advertising prices. The percentage of viewers in the population on a regular weekday has increased from 73.7% to 75.4% from 2001 to 2004, and the average minutes per day a viewer spends watching TV have increased from 192 to 210. But during this time, commercial prices have decreased by roughly 10%. The price per 1000 pairs of eyeballs of a 30-second commercial has dropped from 10.34 to 9.52 euros. Furthermore, television advertising expenditures increased from 7.636 bn to 7.744 bn euros, which together with the decreasing prices indicates that at the same time the level of advertising increased.[25] Our model provides an explanation to this seemingly puzzling observation. During this time period, it is notable that the content of public stations has moved closer to the one of private stations, especially in the afternoon and late-afternoon programme. This move was made to attract more viewers, in particular younger ones who are a particularly attractive consumer group for advertisers. In our model, this development can be associated with a decrease in τ, which makes the channels more attractive and induces more potential viewers to watch. But this also increases the overlapping viewership among channels and therefore lowers advertising prices.[26]

5 Introducing viewer fees

In this section, we analyze the case when stations, in addition to collecting revenues from advertisers, can charge fees on viewers for watching the station.[27] Because of new encryption techniques, viewer pricing is becoming increasingly important.[28] We denote the viewer fee on platform $i \in \{0, 1\}$ by f_i and restrict it to be nonnegative.

[25] See www.agf.de/daten/werbemarkt/werbespendings/.

[26] Another possible explanation, which is often given in newspapers, is the struggling German economy. This can explain part of the price drop. But it is hardly conceivable that prices have dropped in a growing market by such a large amount because of this reason alone.

[27] For an early discussion about the advantages and problems of viewer pricing, see Coase (1966).

[28] In the US, many special interest channels, especially sports or movie channels, can only be watched by paying additional fees. Similarly, in many European countries, recent movies or popular sport events can only be watched via pay TV.

To simplify the analysis in what follows, we assume that $\omega' = \omega$ and that a multi-homing viewer's utility is simply the sum of the net benefits obtained from watching each program:

$$u(x) = s_0(\beta - \gamma a_0 - \tau x - f_0) + s_1(\beta - \gamma a_1 - \tau(1-x) - f_1),$$

for a viewer located at x. These utilities imply that $n_0 = M \min(\frac{\beta - \gamma a_0 - f_0}{\tau}, 1)$ and $n_1 = M \min(\frac{\beta - \gamma a_1 - f_1}{\tau}, 1)$.

We focus on the case when the platforms are owned by competing firms. The analysis of the case when platforms are operated by a monopolist is analogous.[29]

In the above linear specification, a key determinant of whether stations try to collect revenues through advertising fees or through viewer fees is the relative magnitude of ω compared to γ. It is easy to show that if $\omega \leq \gamma$, then independently of the other parameters, it is optimal for each station to set $a_i^* = 0$ and $f_i^* = \frac{\beta}{2}$. If the nuisance parameter is too high relative to the value of advertising, then there is no advertising in equilibrium, and platforms collect all their revenues through viewer fees.

Assume from now on that $\omega > \gamma$. For the region $N \geq \frac{\beta}{\gamma}$, the platforms can act as local monopolists, and analysis remains the same as in Section 3: stations set $f_i^* = 0$ and $a_i^* = \frac{\beta}{2\gamma}$.

The most interesting case is when $\omega > \gamma$, $\beta \geq \tau$, and $N < \frac{\beta}{\gamma}$. For these parameter values, there can be two types of equilibria. One is when both stations only charge advertising fees. These equilibria are analogous to the ones derived in Subsection 3.3. In particular, equilibrium advertising levels can imply two-sided multi-homing, or overlapping viewerships and just not overlapping advertisers, or overlapping advertisers and just not overlapping viewerships. The second type of equilibrium is a new type of asymmetric equilibrium. It involves one station collecting most or all of its revenue through advertising, while the other station collects most of its revenue through viewer fees. This asymmetry is interesting, because our model is perfectly symmetric for the firms. This type of equilibrium can exist if the travel cost cost parameter is low, advertisers are relatively scarce, and ω'' is low. The intuition for this is the following. $\omega > \gamma$ implies that it is more

[29] This analysis is available from the authors upon request.

efficient to collect revenues through advertising fees than through viewer fees. However, for parameter values as above, if one station advertises a lot, the other station might be better off switching to positive viewer fees and only serving the remaining low fraction of advertisers, since otherwise it would either have to establish two-sided overlap (which is undesirable because of the low ω'') or nonoverlapping viewerships (which is undesirable because the station's market share would be low).

Proposition 8: There exists a range of parameter values for which there are asymmetric equilibria such that station i sets $a_i^* = \frac{N}{2} + \delta$ ($\delta \in (0, \frac{N}{2})$) and $f_j^* = \max(0, \frac{2\beta-(2\delta+N)(\omega+\gamma)}{4})$ and gets most or all of its revenues from advertising, while station $-i$ sets $a_{-i}^* = \frac{N}{2} - \delta$ and $f_{-i}^* = \frac{2\beta+(2\delta-N)(\omega+\gamma)}{4}$ and gets most of its revenue via viewer fees.

This type of differentiation among stations can be observed in various settings. For example, standard commercial TV channels in the US are cheap to subscribe to and broadcast relatively many commercials, while exclusive channels like HBO are more expensive to subscribe to but broadcast less or no commercials.

We conclude this section by pointing out that the welfare implications of introducing viewer fees are ambiguous, as in models with viewer single-homing like Anderson and Coate (2005). For many parameters, viewer fees improve welfare, because they help internalizing the negative externalities on the viewer side. For example, consider the case in which $\omega < \gamma$ and $N \geq \frac{\beta}{\gamma}$. In this case, the optimal policy in the duopoly and also in both monopoly regimes is $a_i^* = \frac{\beta}{2\gamma}$, which leads to a welfare of $\frac{\beta^2(2\omega+\gamma)}{4\tau\gamma}$. Instead, with the viewer fee, $a_i^* = 0$ and $f_i^* = \frac{\beta}{\gamma}$, giving a welfare of $\frac{3\beta^2}{4\,\gamma}$, which is higher than the one without viewer fee because $\omega < \gamma$. Here the possibility of a viewer fee lead both stations to reduce their advertising levels, which improves welfare. However, there exist also constellations such that viewer fees are welfare reducing. In all three regimes, there are cases in which the optimal policy without the viewer fee is $a_i^* = \frac{N}{2}$, while with the viewer fee, a station leaves its advertising level unchanged but sets $f_i^* > 0$, which is welfare reducing. Here the stations use the viewer fee only to extract more surplus from viewers.

6 Extensions

Our analysis can be extended in many directions. Here we only briefly discuss a few of them.

If advertisers are heterogeneous with respect to the surplus that their product can generate, as in Anderson and Coate (2005), then there is an extra incentive for firms to advertise less. This is the usual downward distortion effect resulting from imperfect competition, in both the monopoly and the duopoly cases (more profitable advertisers are willing to pay a higher advertising fee per viewer). However, most of the qualitative conclusions of our analysis still apply in this framework.

If viewers are heterogeneous with respect to the nuisance parameter for advertising and platforms can charge viewer fees, then there can be equilibria that are qualitatively different from the ones characterized in Section 5. One possibility is that the same platform offers different versions of the same content, with different levels of advertising. This phenomenon can be observed among Internet-based services: there are websites that offer different subscription fees to their members depending on whether they choose to access the same service with on-screen advertisements or without them.[30]

An interesting extension of the model would be explicitly modeling stations' decision on the timing of commercials.[31] Sweeting (2005, 2006) points out (and provides empirical evidence) that if channels can synchronize the timing of their commercial blocks, then they have an incentive to do so because it increases the value of overlapping viewers. The reason is that synchronizing prevents listeners from switching channels when commercials come on the air. Our model provides an additional reason for coordinating commercial block times. It increases the value of multi-homing listeners even if they do not flip channels to avoid commercials, by making it less likely that a listener will hear a commercial broadcasted on two different channels twice, bringing the value of ω'' closer to ω'.

[30] See for example Fileplanet at http://www.fileplanet.com/subscribe/subscribe.shtml, a web site that allows members to download PC games. Access with advertisements is \$6.95/month, while advertisement-free access is \$9.95/month.

[31] This issue is of course only relevant for the case of electronic media.

7 Conclusion

The main point of our paper is that in media markets, keeping a distinction between exclusive and overlapping viewership is important, because the value of a single-homing viewer and of a multi-homing viewer might be very different for a media platform. The possibility of overlapping viewership changes the nature of competition among stations, which needs to be taken into account in both theoretical and empirical investigations of these markets.

8 Appendix

8.1 Appendix A: Single-Homing Viewers

To provide a benchmark for comparison, in this appendix we characterize equilibrium in the same model as the one defined in Section 2 (in particular, assuming that advertisers are homogeneous), with the modification that viewers cannot multi-home. We show that the welfare implications of the resulting model are ambiguous, as in the more general setting of Anderson and Coate (2005). A monopolist platform owner either advertises too much in equilibrium or broadcasts the socially efficient level of advertising. The level of advertising in duopoly competition can be either too high, socially efficient, or too low. Finally, advertising is (weakly) higher in monopoly than in duopoly, and it is ambiguous whether monopoly or duopoly leads to a more efficient outcome. For the intuition behind these results, see Anderson and Coate (2005).

Just like in Section 3, we assume that $N \geq \max(\frac{2\beta-\tau}{2\gamma}, \frac{\beta}{2\gamma})$, to rule out cases when the total number of advertisers is a binding constraint on advertising levels in equilibrium. This is done to simplify the exposition; the propositions below can be easily extended to parameter values not satisfying the above restriction.

If $\tau \geq \beta$, then stations are local monopolists and do not compete for the same viewers, independently if there is a monopolist platform owner or competing platforms. If $\beta < \tau$, then the following observations can be made:

(i) if there is a set of viewers who would have positive net utility from watching both channels, then the marginal viewer is $x = \frac{1}{2} + \frac{\gamma(a_1-a_0)}{2\tau}$. Then

the profit of station 0 is $\Pi_0 = (\frac{1}{2} + \frac{\gamma(a_1 - a_0)}{2\tau})M\omega a_0$, and the profit of station 1 is $\Pi_1 = (\frac{1}{2} - \frac{\gamma(a_1 - a_0)}{2\tau})M\omega a_1$.

(ii) if there are no viewers who would have positive net utility from watching both channels, then simple considerations establish that the marginal viewer gets exactly 0 net utility when connecting to either channel; otherwise the channels could increase their profits.

This leads to the following characterization of equilibria:

Monopoly:

The equilibrium level of advertising for a monopolist platform provider if viewers cannot multi-home is given by:

$$a_i^{mon} = \begin{cases} \frac{\beta}{2\gamma} & \text{if } \beta \leq \tau \\ \frac{2\beta - \tau}{2\gamma} & \text{if } \tau < \beta. \end{cases}$$

The $\beta \leq \tau$ case is straightforward. If there is a set of viewers who would have positive net utility from watching both channels, then the monopolist could increase its profits by increasing advertising on both channels. Furthermore, the monopolist's profit is maximized if the marginal viewer is $x = \frac{1}{2}$. These imply the result for $\tau < \beta$.

Duopoly:

In this case, we only characterize symmetric equilibria. The asymmetric equilibria that arise for a range of parameter values are similar to the ones in the model in which viewers can multi-home.[32] It is straightforward to show that the asymmetric equilibria that can arise imply the same total amount of advertising and smaller total welfare than the (unique) symmetric equilibrium for the corresponding parameter values. This implies that the propositions below hold for asymmetric equilibria as well.

The equilibrium level of advertising in symmetric equilibrium for competing providers if viewers cannot multi-home is given by:

[32] Just like when viewers can multi-home, it can be shown that asymmetric equilibria can only arise when in equilibrium the marginal viewer gets 0 net utility from connecting to either channel. In this case, we focus on the case when the marginal viewer is $x = \frac{1}{2}$, although there is an interval around $\frac{1}{2}$ such that the marginal viewer can be any point of this interval.

$$a_i^{duo} = \begin{cases} \frac{\beta}{2\gamma} & \text{if } \beta \leq \tau \\ \frac{2\beta-\tau}{2\gamma} & \text{if } \beta > \tau \geq 2/3\beta \\ \frac{\tau}{\gamma} & \text{if } \tau < 2/3\beta. \end{cases}$$

The $\beta \leq \tau$ case is straightforward. For $\beta > \tau$, assume first that the marginal viewer gets positive utility from joining a channel. Then $\Pi_0 = (\frac{1}{2} + \frac{\gamma(a_1-a_0)}{2\tau})M\omega a_0$ and $\Pi_1 = (\frac{1}{2} - \frac{\gamma(a_1-a_0)}{2\tau})M\omega a_1$ imply that the best response functions of the platforms are $a_i = \frac{\tau+\gamma a_{-i}}{2\gamma a_{-i}}$ for $i = 0, 1$. This implies that the marginal viewer is $\frac{1}{2}$ and that $a_0 = a_1 = \frac{\tau}{\gamma}$. The condition for viewers located at $\frac{1}{2}$ getting positive utility when $a_0 = a_1 = \frac{\tau}{\gamma}$ is $\tau < 2/3\beta$. For $\beta > \tau \geq 2/3\beta$, the marginal consumer at $\frac{1}{2}$ gets exactly 0 net utility when connecting to a channel, which yields $a_0^{duo} = a_1^{duo} = \frac{2\beta-\tau}{2\gamma}$.

Comparing a_i^{mon} with a_i^{duo}, it is obvious that a_i^{mon} is always (weakly) higher than a_i^{duo}.

Social optimum:

The socially optimal level of advertising if viewers cannot multi-home is given by:

$$a_i^{WF} = \begin{cases} 0 & \text{if } \gamma \geq \omega \\ \frac{\beta(\omega-\gamma)}{\gamma(2\omega-\gamma)} & \text{if } \gamma < \omega \text{ and } \frac{2\beta\omega}{2\omega-\gamma} \leq \tau \\ \frac{2\beta-\tau}{2\gamma} & \text{if } \gamma < \omega \text{ and } \frac{2\beta\omega}{2\omega-\gamma} > \tau. \end{cases}$$

The case when $\gamma \geq \omega$ and the case when $\gamma < \omega$ and $\frac{2\beta\omega}{2\omega-\gamma} \leq \tau$ are the same as in the model in which viewers can multi-home. If $\frac{2\beta\omega}{(2\omega-\gamma)} > \tau$, then it is easy to establish that in the welfare maximizing outcome, all consumers connect to a channel. In this case, the welfare function is:

$$WF = (\frac{1}{2} + \frac{\gamma(a_1 - a_0)}{2\tau})M\omega a_0 + (\frac{1}{2} - \frac{\gamma(a_1 - a_0)}{2\tau})M\omega a_1 +$$

$$+M\int_0^{\frac{1}{2}+\frac{\gamma(a_1-a_0)}{2\tau}} (\beta - \gamma a_0 - \tau x)dx + M\int_{\frac{1}{2}+\frac{\gamma(a_1-a_0)}{2\tau}}^1 (\beta - \gamma a_0 - \tau(1-x))dx.$$

If $\gamma < \omega$, then the above expression is increasing in a_i $(i = 0, 1)$. Furthermore, $WF(a_0, a_1) \geq WF(\frac{a_0-a_1}{2}, \frac{a_0-a_1}{2})$. Then the welfare maximizing advertising levels are the maximum possible symmetric levels that are compatible with all consumers wanting to connect to a channel, which is $\frac{2\beta-\tau}{2\gamma}$.

Using the above results, we can compare the socially optimal level of advertising with the equilibrium levels of advertising in monopoly and in duopoly.

First consider a monopolist provider. If $\omega > \gamma$, then the socially efficient amount is given by $\frac{\beta(\omega-\gamma)}{\gamma(2\omega-\gamma)}$ if $\frac{2\beta\omega}{2\omega-\gamma} \leq \tau$ and by $\frac{2\beta-\tau}{2\gamma}$ if $\frac{2\beta\omega}{2\omega-\gamma} > \tau$. Instead, a monopolist advertises $\frac{\beta}{2\gamma}$ if $\beta \leq \tau$ and $\frac{2\beta-\tau}{2\gamma}$ if $\tau < \beta$. Comparing this shows that in the region $\frac{2\beta\omega}{2\omega-\gamma} \leq \tau$, a monopolist advertises too much because $\frac{\beta}{2\gamma} > \frac{\beta(\omega-\gamma)}{\gamma(2\omega-\gamma)}$ since $\gamma > 0$. In the region $\beta \leq \tau < \frac{2\beta\omega}{2\omega-\gamma}$, the monopolist also advertises too much because comparing $a_i^{mon} = \frac{\beta}{2\gamma}$ with $a_i^{WF} = \frac{2\beta-\tau}{2\gamma}$ shows that $a_i^{mon} > a_i^{WF}$ because $\beta \leq \tau$ in this region. Lastly, for $\beta > \tau$, both advertising amounts are the same. If $\omega \leq \gamma$, then the socially optimal level of advertising is zero while the monopolist chooses a positive level.

This concludes that if $\omega \leq \gamma$, or if $\omega > \gamma$ and $\tau \geq \beta$, then the monopolist advertises more on both platforms than the socially efficient amount. If $\omega > \gamma$ and $\tau < \beta$, then the monopolist in equilibrium chooses the socially efficient level of advertising.

Now consider the duopoly case. Since the advertising amounts here are the same as in monopoly for $\tau \geq 2/3\beta$, the result for this case is analogous to the result for a monopolist platform provider. For $\tau < 2/3\beta$, $a_i^{duo} = \frac{\tau}{\gamma}$, while the socially efficient amount is $\frac{2\beta-\tau}{2\gamma}$. Comparing these two equations shows that $\frac{\tau}{\gamma} < \frac{2\beta-\tau}{2\gamma}$ if $\tau < 2/3\beta$, which holds in this region. If $\omega \leq \gamma$, then the socially optimal level of advertising is zero, while market equilibrium always implies positive advertising levels.

This concludes that if $\omega \leq \gamma$, or if $\omega > \gamma$ and $\tau \geq \beta$, then there is too much advertising on both platforms in symmetric duopoly equilibrium compared to the social optimum. If $\omega > \gamma$ and $\beta > \tau \geq 2/3\beta$, then the level of advertising in symmetric duopoly equilibrium is socially efficient. If $\omega > \gamma$ and $\tau < 2/3\beta$, then the level of advertising is lower on both platforms in symmetric duopoly equilibrium than in social optimum.

The previous results imply that competition among platforms leads to a more efficient outcome than monopoly does if $\tau < 2/3\beta$ and $\gamma \geq \omega$, while monopoly leads to a more efficient outcome than competition if $\tau < 2/3\beta$ and $\gamma < \omega$. In all other cases, monopoly and platform competition are equally efficient from a social point of view.

8.2 Appendix B: Proofs

Lemma 1: In the nondiscriminating monopoly and the duopoly cases market clearing prices are uniquely determined for any $a_0, a_1 \in (0, N)$ such that $a_0 + a_1 \neq N$, and they are given by:

$$p_i = \begin{cases} \omega n_i \text{ if } n_0 + n_1 \leq M \\ (n_i - n_{01})\omega + n_{01}\omega' \text{ if } n_0 + n_1 > M \text{ and } a_0 + a_1 \leq N \\ (n_i - n_{01})\omega + n_{01}\omega'' \text{ if } n_0 + n_1 > M \text{ and } a_0 + a_1 > N. \end{cases}$$

In the discriminating monopoly case market clearing prices on platforms 0 and 1 are uniquely determined for any $\overline{a}_0, \overline{a}_1 > 0$ such that $\overline{a}_0 + \overline{a}_{01} < N$ and $\overline{a}_1 + \overline{a}_{01} < N$ and $\overline{a}_0 + \overline{a}_1 \neq N$, and they are given by:

$$p_i = \begin{cases} \omega n_i \text{ if } n_0 + n_1 \leq M \\ \omega(n_i - n_{01}) + \omega' n_{01} \text{ if } n_0 + n_1 > M \end{cases}$$

If moreover $\overline{a}_{01} \in (0, N)$ then the market clearing price of a joint advertising slot is uniqely determined as well, and it is given by:

$p_{01} = \omega(n_0 + n_1 - 2n_{01}) + (\omega' + \omega'')n_{01}$.

Proof of Lemma 1: Consider first the nondiscriminating monopoly and duopoly cases.

If $n_0 + n_1 \leq M$ then $p_i < \omega n_i$ would imply that the demand for advertising spots on channel i is N, therefore these prices do not clear the market. Similarly, $p_i > \omega n_i$ would imply that the demand for advertising spots on channel i is 0, therefore these prices do not clear the market. At $p_i = \omega n_i$ advertisers are indifferent between advertising on channel i or not.

If $n_0 + n_1 > M$ and $a_0 + a_1 < N$ then $p_i > (n_i - n_{01})\omega + n_{01}\omega'$ for some $i \in \{0, 1\}$ implies that the demand for advertising spots on channel i is 0, therefore these prices do not clear the market. And $p_i < (n_i - n_{01})\omega + n_{01}\omega'$ for some $i \in \{0, 1\}$ implies that the sum of the demands for advertising spots for the two channels is at least N, so prices like this cannot clear the market.

If $n_0 + n_1 > M$ and $a_0 + a_1 > N$ then $p_i > (n_i - n_{01})\omega + n_{01}\omega''$ for some $i \in \{0, 1\}$ implies that no advertiser buying an advertising spot on the other channel would buy an advertising spot on channel i as well. Therefore the sum of demands for advertising spots on the two channels is at most N, which means that these prices cannot clear the market. And $p_i < (n_i - n_{01})\omega + n_{01}\omega''$ for some $i \in \{0, 1\}$ implies that the demand for advertising spots on channel i is N, therefore these prices do not clear the market.

Consider next the discriminating monopoly case.

If either (i) $n_0 + n_1 \leq M$, or (ii) $n_0 + n_1 > M$, $\overline{a}_0 + \overline{a}_{01} < N$, $\overline{a}_1 + \overline{a}_{01} < N$ and $\overline{a}_{01} = 0$, then establishing the claim is analogous to the nondiscriminating monopoly case. Assume now that $n_0 + n_1 > M$, $\overline{a}_0 + \overline{a}_{01} < N$, $\overline{a}_1 + \overline{a}_{01} < N$ and $\overline{a}_{01} > 0$. Then if either $p_i < \omega(n_i - n_{01}) + \omega' n_{01}$ or

$p_{01} < \omega(n_0 + n_1 - 2n_{01}) + (\omega' + \omega'')n_{01}$ then the sum of demands for advertising slots on channel i and for the joint advertising slots is at least N, therefore these prices cannot clear the market. If $p_i > \omega(n_i - n_{01}) + \omega' n_{01}$ then demand for advertising slots on channel i is 0, therefore these prices cannot clear the market. Finally, if $p_{01} > \omega(n_0 + n_1 - 2n_{01}) + (\omega' + \omega'')n_{01}$ then demand for the joint advertising slots is 0, therefore these prices cannot clear the market. ■

Lemma 2: In the nondiscriminating monopoly and the duopoly cases, if in equilibrium $a_i = N$ for some $i \in \{0, 1\}$, or $a_i > 0$ and $a_0 + a_1 = N$, then the equilibrium price for advertising slots on channel i is given by:

$$p_i = \begin{cases} \omega n_i \text{ if } n_0 + n_1 \leq M \\ (n_i - n_{01})\omega + n_{01}\omega' \text{ if } n_0 + n_1 > M \text{ and } a_0 + a_1 \leq N \\ (n_i - n_{01})\omega + n_{01}\omega'' \text{ if } n_0 + n_1 > M \text{ and } a_0 + a_1 > N. \end{cases}$$

In the discriminating monopoly case, if in equilibrium $\overline{a}_i = N$ for some $i \in \{0, 1\}$, or $\overline{a}_i > 0$ and either $\overline{a}_i + \overline{a}_{01} = N$ or $\overline{a}_0 + \overline{a}_1 = N$, then the equilibrium price for advertising slots on channel i is given by:

$$p_i = \begin{cases} \omega n_i \text{ if } n_0 + n_1 \leq M \\ (n_i - n_{01})\omega + n_{01}\omega' \text{ if } n_0 + n_1 > M \end{cases}$$

If also $\overline{a}_{01} > 0$, then the equilibrium price for a joint advertising spot is given by:

$p_{01} = \omega(n_0 + n_1 - 2n_{01}) + (\omega' + \omega'')n_{01}$.

Proof of Lemma 2: Consider the nondiscriminating monopoly and duopoly cases. If $a_i = N$ and $n_0 + n_1 \leq M$ then any price $p_i \leq \omega n_i$ clears the market.

Suppose that in equilibrium $a_i = N$ and $p_i < \omega n_i$. Then the platform owner, by changing advertising level on channel i to $N - \varepsilon$ for small enough $\varepsilon > 0$ can increase its profit, since for such advertising levels the market clearing price is unique and given by $p_i = \omega n_i$. This concludes that the above cannot be an equilibrium. Similar erguments establish that if in equilibrium $a_i = N$, $n_0 + n_1 > M$ and $a_0 + a_1 \leq N$ then $p_i = (n_i - n_{01})\omega + n_{01}\omega'$, and that if in equilibrium $a_i = N$, $n_0 + n_1 > M$ and $a_0 + a_1 > N$ then $p_i = (n_i - n_{01})\omega + n_{01}\omega''$.

Suppose now that $a_i > 0$, $a_0 + a_1 = N$ and $n_0 + n_1 \leq M$. Then $p_i > \omega n_i$ implies that the demand for advertising spots on channel i is 0, therefore such prices cannot clear the market. Note that $p_i = \omega n_i$ is a market clearing price, since all advertisers are indifferent between advertising or not on channel i at that price. Assume that in equilibrium $p_i < \omega n_i$. Then channel i could increase its profit by decreasing the advertising level by ε for small enough $\varepsilon > 0$, since for such prices the market clearing price is unique, and given

by $p_i = \omega n_i$. Similar erguments establish that if in equilibrium $a_i > 0$, $a_0 + a_1 = N$ and $n_0 + n_1 > M$ then $p_i = (n_i - n_{01})\omega + n_{01}\omega'$.

The arguments that establish the claims for the discriminating monopoly case are analogous to the above. ■

The proofs of all subsequent propositions use the results in Lemma 1 and Lemma 2 on equilibrium prices.

Proof of Proposition 1: Let $\Pi_M^* = \frac{MN(2\tau\omega - 2\beta\omega + 4\beta\omega' - 2\tau\omega' + N\gamma\omega - 2N\gamma\omega')}{2\tau}$ and $\Pi_M^{**} = \frac{M[\omega''(2\beta-\tau)-\omega(\beta-\tau)]^2}{\gamma\tau(2\omega''-\omega)}$.

We need to consider three cases: when there is no overlap of viewers (either with or without advertiser overlap), when viewers overlap but advertisers do not, and finally when both viewers and advertisers overlap.

First assume that $n_{01} = 0$, which means no viewers overlap. The profit function of station i is then given by

$$\Pi_i = p_i a_i = \omega M(\frac{\beta - \gamma a_i}{\tau})a_i.$$

Solving this for a_0 and a_1 yields $a_i = \frac{\beta}{2\gamma}$. The condition for which the above values are compatible with the starting assumption $n_{01} = 0$ is $\beta \leq \tau$. This concludes region (a).

If $\beta > \tau$, then the best advertising level choices that lead to nonoverlapping viewerships are $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$ (resulting in $n_0 = n_1 = M/2$). The profit in the latter case is $\frac{\omega M(2\beta-\tau)}{2\gamma}$.

¿From now on, assume $\beta > \tau$. To analyze the monopolist's decision in this parameter region, we distinguish between the case when $\frac{2\beta-\tau}{\gamma} \leq N$ and the case when $\frac{2\beta-\tau}{\gamma} > N$.

Consider first $\frac{2\beta-\tau}{\gamma} \leq N$. In this case, it cannot be that in equilibrium both advertisers and viewers overlap, since overlapping viewership implies $a_0 + a_1 \leq N$. Suppose first that $n_{01} > 0$. Then the profit function in this region is given by:

$$\Pi_M = Ma_0\left(\omega \max(1 - A_1, 0) + \omega' \min(A_0 + A_1 - 1, 1)\right) + \\ + Ma_1\left(\omega \max(1 - A_0, 0) + \omega' \min(A_0 + A_1 - 1, 1)\right),$$

with $A_i = \frac{\beta-\gamma a_i}{\tau}$.

Solving this for a_0 and a_1 yields:

$$a_0 = a_1 = \max(\frac{\omega'(2\beta - \tau) - \omega(\beta - \tau)}{2\gamma(2\omega' - \omega)}, \frac{\beta - \tau}{\gamma}). \tag{3}$$

The monopolist's profit is then $\Pi_M = \min\left(\frac{M\omega'(\omega'(2\beta-\tau)-\omega(\beta-\tau))^2}{\gamma\tau(2\omega'-\omega)^2}, \frac{(\beta-\tau)2M\omega'}{\gamma}\right)$.

The above levels of advertising are consistent with the starting assumption $n_{01} > 0$ iff $\omega' > \frac{\omega\beta}{2\beta-\tau}$. If $\omega' \leq \frac{\omega\beta}{2\beta-\tau}$, then establishing an overlapping viewership is not optimal for the monopolist because ω' is too small and so $n_{01} = 0$. The optimal choice is $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$. This is one case in which region (b) occurs.

Consider next $\frac{2\beta-\tau}{\gamma} > N$. Assume first that $n_{01} > 0$, but $a_0 + a_1 < N$. Then equilibrium advertising levels are given by (3). This is only consistent with the starting assumption $a_0 + a_1 < N$ if $\frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)} < N$. In the latter case, (3) is a global optimum for the monopolist (note that $n_{01} > 0$ holds because $\frac{2\beta-\tau}{\gamma} > N$). In this case, only viewers multi-home, while there are some potential advertisers who do not advertise on either of the channels. This concludes region (c).

Now consider $\frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)} \geq N$. Then the optimal choice for the monopolist under which $n_{01} > 0$ and $a_0 + a_1 \leq N$ is setting $a_0 = a_1 = \frac{N}{2}$. This gives a profit of:

$$\Pi_M^* = \frac{MN(2\tau\omega - 2\beta\omega + 4\beta\omega' - 2\tau\omega' + N\gamma\omega - 2N\gamma\omega')}{2\tau}.$$

Here again viewers multi-home, but the overall number of potential advertisers is low enough such that they all advertise, but none of them are multi-homing.

If $\frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega'-\omega)} \geq \frac{2\beta-\tau}{2\gamma}$, which holds when $\omega' \leq \frac{\beta\omega}{2\beta-\tau}$, the optimal decision for the monopolist is $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$ (establishing as before viewerships that just do not overlap).

If $\omega' > \frac{\beta\omega}{2\beta-\tau}$ and $N \leq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)}$, then we have to consider three scenarios: nonoverlapping viewership (i.e. $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$), overlapping viewership and nonoverlapping advertisers (i.e. $a_0 = a_1 = \frac{N}{2}$), and overlap on both sides. Consider the latter scenario first: $n_{01} > 0$ and $a_0 + a_1 > N$. The profit function in this case is given by:

$$\Pi_M = Ma_0\left(\omega(1 - \frac{\beta - \gamma a_1}{\tau}) + \omega''(\frac{2\beta - \gamma(a_0 + a_1)}{\tau} - 1)\right) +$$

$$+Ma_1\left(\omega(1 - \frac{\beta - \gamma a_0}{\tau}) + \omega''(\frac{2\beta - \gamma(a_0 + a_1)}{\tau} - 1)\right).$$

Given the constraints $x_0 \leq 1$ and $x_1 \geq 0$, the optimal solution is:

$$a_0 = a_1 = \frac{\omega''(2\beta - \tau) - \omega(\beta - \tau)}{2\gamma(2\omega'' - \omega)}.$$

This gives a profit of

$$\Pi_M^{**} = \frac{M\omega''[\omega''(2\beta - \tau) - \omega(\beta - \tau)]^2}{\gamma\tau(2\omega'' - \omega)^2}.$$

This solution is only consistent with the assumptions $n_{01} > 0$ and $a_0 + a_1 > N$ if $\omega'' > \frac{\beta\omega}{2\beta-\tau}$ and $N \leq \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega''-\omega)}$. It can be shown that in the relevant parameter range, $\frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega''-\omega)} > \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)}$; therefore $N \leq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)}$ implies $N \leq \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega''-\omega)}$. So the above advertising levels indeed imply two-sided multi-homing. It still remains to check that this is more profitable than reducing advertisers to $a_0 = a_1 = \frac{N}{2}$, which holds iff $\Pi_M^{**} \geq \Pi_M^{*}$. This is one case of region (d) and also region (e).

If $\omega'' \leq \frac{\beta\omega}{2\beta-\tau}$, then the optimal choice for the monopolist is either $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$ or $a_0 = a_1 = \frac{N}{2}$, depending on the relative magnitudes of $\frac{\omega M(2\beta-\tau)}{2\gamma}$ and Π_M^{*}. It can be shown that $\frac{\omega M(2\beta-\tau)}{2\gamma} \geq \Pi_M^{*}$ iff $N \leq \frac{\omega\tau}{\gamma(2\omega'-\omega)}$. This is a second case in which either region (b) or (d) occurs. ■

Proof of Proposition 2: Let Π_M^{+} be the profit if each station contains $a_i = \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)-N\gamma(\omega'-\omega'')}{2(2\omega''-\omega)\gamma}$ advertising slots.

If advertisers overlap, then $\overline{a}_{01} = N - \overline{a}_0 - \overline{a}_1$. From this, the monopolist's profit function can be written as follows:

$$\Pi_m = \overline{a}_0 M\left(\omega(1 - \frac{\beta - \gamma(N - \overline{a}_0)}{\tau}) + \omega'(\frac{2\beta - \gamma(2N - \overline{a}_0 - \overline{a}_1)}{\tau} - 1)\right) +$$

$$\overline{a}_1 M\left(\omega(1 - \frac{\beta - \gamma(N - \overline{a}_1)}{\tau}) + \omega'(\frac{2\beta - \gamma(2N - \overline{a}_0 - \overline{a}_1)}{\tau} - 1)\right) +$$

$$(N - \overline{a}_0 - \overline{a}_1)M[\omega(2 - \frac{2\beta - \gamma(2N - \overline{a}_0 - \overline{a}_1)}{\tau}) + (\omega' + \omega'')(\frac{2\beta - \gamma(2N - \overline{a}_0 - \overline{a}_1)}{\tau} - 1)].$$

The first term is the revenue from advertisers only advertising on platform 0; the second term is the revenue from advertisers who only advertise on platform 1; and the third term is the revenue from multi-homing advertisers. Writing out the first-order conditions and solving for the optimal values of a_0 and a_1 yields:

$$\overline{a}_0 = \overline{a}_1 = \frac{\omega(\beta-\tau) - \omega''(2\beta-\tau) - N\gamma(\omega' + 3\omega'' - 2\omega)}{2(2\omega''-\omega)\gamma}. \tag{4}$$

This implies that total advertising on a platform is:

$$a_i = \overline{a}_i + \overline{a}_{01} = \frac{\omega''(2\beta-\tau) - \omega(\beta-\tau) - N\gamma(\omega' - \omega'')}{2(2\omega''-\omega)\gamma}.$$

Denote the resulting profit of the monopolist by Π^+. The above levels of advertising are only consistent with the assumptions $n_{01} \geq 0$ and $a_0 + a_1 \geq N$ if $\omega'' \geq \frac{\beta\omega}{2\beta-\tau}$ and $N \leq \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)-N\gamma(\omega'-\omega'')}{(2\omega''-\omega)\gamma}$. It can be shown that $N \leq \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)-N\gamma(\omega'-\omega'')}{(2\omega''-\omega)\gamma}$ is implied by $N \leq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)}$. Then $\frac{2\beta-\tau}{2\gamma} \geq \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)-N\gamma(\omega'-\omega'')}{2(2\omega''-\omega)\gamma}$ implies that the optimal choice for the monopolist is either $\frac{N}{2}$ or the levels specified by (4), depending on the relative magnitudes of Π_M^* and Π^+. Similarly, $\frac{2\beta-\tau}{2\gamma} \leq \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)-N\gamma(\omega'-\omega'')}{2(2\omega''-\omega)\gamma}$ implies that the optimal advertising level on each platform is either $\frac{2\beta-\tau}{2\gamma}$ or $\frac{N}{2}$, depending on N bigger or smaller than $\frac{\omega\tau}{\gamma(2\omega'-\omega)}$. ■

Proof of Proposition 3: Let $\Pi_i^{dd} = \frac{(2\tau(\omega-\omega'')+2\beta(2\omega''-\omega)-\gamma N(2\omega''-\omega))^2}{16\tau\omega''\gamma}$ and

$$\Pi_j^{dev} = \left(M(N - \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}\right)\left(\omega\left(1 - \frac{\beta - \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{(3\omega''-\omega)}}{\tau}\right) + \omega'\left(\frac{2\beta-\gamma N}{\tau} - 1\right)\right).$$

Assume first that viewers overlap, but advertisers do not. The profit function of firm i is then given by

$$\Pi_i = Ma_i\left(\max(\omega(1 - \frac{\beta-\gamma a_j}{\tau}), 0) + \min(\omega'(\frac{2\beta-\gamma(a_i+a_j)}{\tau} - 1), \omega')\right).$$

Taking the first order condition and solving for a_i yields:[33]
$a_i = \max[\frac{1}{2}\frac{\tau}{\gamma\omega'}\left(\omega\left(-\frac{1}{\tau}(\beta-\gamma a_j)+1\right) + \omega'\left(\frac{1}{\tau}(2\beta-\gamma a_j)-1\right)\right), \frac{\beta-\tau}{\gamma}]$,
which is increasing in a_j (levels of advertising are strategic complements). The equilibrium levels of advertising are:

$$a_0 = a_1 = \max(\frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}, \frac{\beta-\tau}{\gamma}). \tag{5}$$

This yields equilibrium profits of

$$\Pi_i = \min(\frac{M\omega'[(2\beta-\tau)\omega' - \omega(\beta-\tau)]^2}{\gamma\tau(3\omega'-\omega)^2}, \frac{M\omega'(\beta-\tau)}{\gamma}).$$

[33] The second order condition for maximum, $-2M\omega'\frac{\gamma}{\tau} < 0$, is satisfied.

The above equilibrium is only valid as long as at advertising levels given by (5) viewers overlap, but advertisers do not. The condition for no viewer overlap is $\omega' \geq \frac{\tau\omega}{2\beta-\tau}$. If $\omega' < \frac{\tau\omega}{2\beta-\tau}$, then the only possibility for an equilibrium is when the two platforms set their advertising levels such that viewers just do not overlap. This means that in equilibrium, $a_j + a_i = \frac{2\beta-\tau}{\gamma}$ and $n_0 + n_1 = M$.[34] That is, the sum of advertising levels and the sum of viewerships are uniquely pinned down. However, how the share of advertising (and total viewership) is divided between channels is not uniquely determined. For any parameter constellation satisfying $\omega' < \frac{\tau\omega}{2\beta-\tau}$, there is a $\overline{\delta} \in (0, \frac{1}{2})$ such that $a_0 = \frac{2\beta-\tau}{\gamma} + \delta$, $a_1 = (1-x)\frac{2\beta-\tau}{\gamma} - \delta$ constitute an equilibrium of the duopoly game for any $\delta \in [-\overline{\delta}, \overline{\delta}]$. Among these, we focus on the symmetric equilibrium $a_0 + a_1 = \frac{2\beta-\tau}{2\gamma}$, which both is the most efficient from the social point of view and maximizes the total profit of the platforms. The qualitative conclusions of our model would not change if instead we selected an asymmetric sharing rule of the market for the above parameter values. This concludes one case of region (a).

The condition for advertisers not to overlap at advertising levels given by (5) is $N \geq \frac{2\omega'(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$. Just like in the case of a monopolist provider, this automatically holds if $\frac{2\beta-\tau}{\gamma} \leq N$. This concludes region (b).

Assume from now on that $\omega' > \frac{\tau\omega}{2\beta-\tau}$ and $N < \frac{2\omega'(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$. Then there can be three types of equilibria: (i) $a_0 + a_1 > N$ and $n_{01} > 0$ (two-sided overlap); (ii) $a_0 + a_1 > N$ and $n_0 + n_1 = M$ (viewerships just do not overlap); (iii) $a_0 + a_1 = N$ and $n_{01} > 0$ (advertisers just do not overlap).

If $a_0 + a_1 > N$ and $n_{01} > 0$, then profit functions are given by

$$\Pi_i = Ma_i \left(\omega(1 - \frac{\beta - \gamma a_j}{\tau}) + \omega''(\frac{2\beta - \gamma(a_i + a_j)}{\tau} - 1) \right).$$

Solving for the equilibrium levels yields:

$$a_0 = a_1 = \frac{\omega''(2\beta - \tau) - \omega(\beta - \tau)}{\gamma(3\omega'' - \omega)}. \tag{6}$$

This gives a profit of $\frac{M\omega''[(2\beta-\tau)\omega''-\omega(\beta-\tau)]^2}{\gamma\tau(3\omega''-\omega)^2}$.

Next we characterize the range of parameter values for which each of the above symmetric profiles constitutes equilibrium.

[34] Note that the maintained assumption $\frac{\beta}{2\gamma} \leq N$ implies $\frac{2\beta-\tau}{2\gamma} \leq N$; therefore there are $a_0, a_1 \in [\frac{2\beta-\tau}{\gamma} - N, N]$ such that $a_0 + a_1 = \frac{2\beta-\tau}{\gamma}$.

First we consider the region $N > \frac{2\omega''(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$.[35] Note that $\omega'' < \frac{\tau\omega}{2\beta-\tau}$ implies the previous condition, since $\omega'' < \frac{\tau\omega}{2\beta-\tau}$ implies $\frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega''-\omega)} < \frac{2\beta-\tau}{2\gamma}$, and $N > \frac{2\omega'(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$ implies $\frac{2\beta-\tau}{\gamma} < N$. Then advertising levels in (6) imply $a_0+a_1 < N$. Therefore in this region, overlapping on both sides cannot be in equilibrium.

Out of the remaining two possibilities, $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$ constitutes an equilibrium if no firm wants to decrease advertising to $N - \frac{2\beta-\tau}{2\gamma}$, the point at which advertisers do not overlap anymore. The profit implied by this deviation is $M(N-\frac{2\beta-\tau}{2\gamma})\left(\omega(1-\frac{\beta-\frac{2\beta-\tau}{2}}{\tau})+\omega'(\frac{2\beta-\gamma N}{\tau}-1)\right)$. Therefore the condition for $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$ to constitute an equilibrium is

$$\frac{\omega}{2}\frac{2\beta-\tau}{2\gamma} \geq (N-\frac{2\beta-\tau}{2\gamma})\left(\omega(1-\frac{\beta-\frac{2\beta-\tau}{2}}{\tau})+\omega'(\frac{2\beta-\gamma N}{\tau}-1)\right). \quad (7)$$

This inequality holds whenever $\frac{2\beta-\tau+\tau\frac{\omega}{\omega'}}{2\gamma} \leq N$. By (1), this is always true; therefore for parameter range $\omega' \geq \frac{\tau\omega}{2\beta-\tau}$, $N < \frac{2\omega'(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$, and $\omega'' \leq \frac{\tau\omega}{2\beta-\tau}$, advertising levels $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$ always constitute an equilibrium. This concludes a second case of region (a).

Finally, $a_0 = a_1 = \frac{N}{2}$ constitutes an equilibrium if no firm wants to deviate and set $a_i = \frac{4\beta-2\tau-\gamma N}{2\gamma}$ such that viewers just do not overlap (but advertisers do). Advertising levels $a_0 = a_1 = \frac{N}{2}$ imply a profit of

$$\Pi_i = \frac{MN(2\tau(\omega-\omega')+2\beta(2\omega'-\omega)-\gamma N(2\omega'-\omega))}{4\tau}.$$

The above deviation gives a profit of $\Pi_i^{dev} = \frac{\omega M(4\beta-2\tau-\gamma N)(2\tau-2\beta+\gamma N)}{4\tau\gamma}$. Comparing these two profit levels shows that Π_i^{dev} is lower iff $\omega' \geq \frac{\gamma N-2(\beta-\tau)}{\gamma N}$. This concludes the first case of region (c).

Consider now $N \leq \frac{2\omega''(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$. As shown above, this implies $\omega'' \geq \frac{\tau\omega}{2\beta-\tau}$. In this case, advertising levels in (6) are consistent with two-sided multi-homing. The condition for this to constitute an equilibrium is that no firm wants to deviate. Set $a_i = N - \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$ to avoid overlap

[35] It can be shown that this cannot be the case if $\tau \geq \frac{\beta}{2}$, because in the latter case, $\frac{2\omega'(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega'-\omega)} > N$ implies $\frac{2\omega''(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega''-\omega)} > N$.

in advertisers. This deviation yields a profit of

$$\Pi_j^{dev} = \left(M(N - \frac{\omega''(2\beta - \tau) - \omega(\beta - \tau)}{\gamma(3\omega'' - \omega)} \right) \times$$

$$\left(\omega \left(1 - \frac{\beta - \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{(3\omega''-\omega)}}{\tau} \right) + \omega' \left(\frac{2\beta - \gamma N}{\tau} - 1 \right) \right).$$

The deviation is therefore profitable if $\Pi_j^{dev} > \frac{M\omega''[(2\beta-\tau)\omega''-\omega(\beta-\tau)]^2}{\gamma\tau(3\omega''-\omega)^2}$. Thus, $a_0 = a_1 = \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$ constitutes an equilibrium if $N \leq \frac{2\omega''(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$ and $\Pi_j^{dev} > \frac{M\omega''[(2\beta-\tau)\omega''-\omega(\beta-\tau)]^2}{\gamma\tau(3\omega''-\omega)^2}$.

It is straightforward to show that $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$ cannot be in equilibrium if $N \leq \frac{2\omega''(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$. This concludes region (d).

Finally, the condition for $a_0 = a_1 = \frac{N}{2}$ to constitute an equilibrium is that no firm has an incentive to deviate to increase advertising (resulting in two-sided overlap). It can be shown that the optimal deviation of this type is $a_i^{dd} = \frac{2\tau(\omega-\omega'')+2\beta(2\omega''-\omega)-\gamma N(2\omega''-\omega)}{4\omega''\gamma}$, giving a profit of

$$\Pi_i^{dd} = \frac{(2\tau(\omega - \omega'') + 2\beta(2\omega'' - \omega) - \gamma N(2\omega'' - \omega))^2}{16\tau\omega''\gamma}.$$

Therefore the condition for the existence of this type of equilibrium is

$$\Pi_i^{dd} \leq \frac{MN(2\tau(\omega - \omega') + 2\beta(2\omega' - \omega) - \gamma N(2\omega' - \omega))}{4\tau}.$$

This concludes the second case of region (c). ■

Proof of Proposition 4: First note that a welfare maximizing vector of advertising levels $\overline{a}_0, \overline{a}_1, \overline{a}_{01}$ cannot imply $n_i = 0$ for some $i \in \{0, 1\}$; otherwise, setting $\overline{a}'_0, \overline{a}'_1, \overline{a}'_{01}$ such that $\overline{a}'_{-i} = \overline{a}_{-i} + \overline{a}_{01}$ and $\overline{a}_i = \overline{a}_{01} = 0$ would lead to strict welfare improvement: it would generate positive as opposed to zero social welfare on platform i while leaving social welfare generated on the other platform unchanged. As shown in Subsection 3.1, $n_0 = \max(0, \min(\frac{\beta-\gamma a_0}{\tau}, 1)M)$ and $n_1 = \max(0, \min(\frac{\beta-\gamma a_1}{\tau}, 1)M)$. This can be used to show that if $\overline{a}_0 \neq \overline{a}_1$ and if advertising levels $\overline{a}_0, \overline{a}_1, \overline{a}_{01}$ imply $n_i > 0$ for $i \in \{0, 1\}$, then setting $\overline{a}'_0, \overline{a}'_1, \overline{a}'_{01}$ such that $\overline{a}'_0 = \overline{a}'_1 = \frac{\overline{a}_0+\overline{a}_1}{2}$ and $\overline{a}'_{01} = \overline{a}_{01}$ leads to strict welfare improvement. This complements the result in Subsection 3.2 that if $\overline{b}_0, \overline{b}_1, \overline{b}_{01}$ are equilibrium advertising levels for a discriminating monopolist, then $\overline{b}_0 = \overline{b}_1$. Note that if $\overline{a}_0, \overline{a}_1, \overline{a}_{01}$ is

either a welfare maximizing or a discriminating monopoly equilibrium advertising vector, then the following statements hold: (i) if $\overline{a}_0, \overline{a}_1, \overline{a}_{01}$ implies $n_{01} > 0$, then $\overline{a}_{01} = \max(0, a_0 + a_1 - N)$; otherwise, social welfare or the monopolist profit could be increased by increasing $\overline{a}_0, \overline{a}_1$ and decreasing $\overline{a}_{01}$; (ii) if $\overline{a}_0, \overline{a}_1, \overline{a}_{01}$ implies $n_{01} > 0$, then any $\overline{a}'_0, \overline{a}'_1, \overline{a}'_{01}$ such that $a'_i = a_i$ for $i \in \{0, 1\}$ would imply the same level of social welfare and monopolist profit. The above establish that if $\overline{a}_0, \overline{a}_1, \overline{a}_{01}$ is either a welfare maximizing or a discriminating monopoly equilibrium advertising vector, then it can be described simply by the scalar $a_0 = a_1$.

Let now $a_0 = a_1 = a^*$ in an equilibrium with a discriminating monopolist. This implies that the profit function

$$\Pi(a) = 2(a - \max(0, 2a - N)) \left(\omega(n_0 - n_{01}) + \omega' n_{01}\right) +$$

$$\max(0, 2a - N)[\omega(n_0 + n_1 - 2n_{01}) + (\omega' + \omega'')n_{01}]$$

is such that $\Pi(a^*) \geq \Pi(a)$ for any $a \geq a^*$. But note that the welfare function is

$$WF(a) = \Pi(a) +$$

$$M \int_0^{(n_0 - n_{01})/M} (\beta - \gamma a - \tau x) dx + M \int_{1-(n_1 - n_{01})/M}^{1} (\beta - \gamma a - \tau(1 - x)) dx) +$$

$$\int_{(n_0 - n_{01})/M}^{n_0/M} u(\beta - \gamma a - \tau x, \beta - \gamma a - \tau(1 - x)) dx,$$

where the last three terms are strictly decreasing in a. Therefore $WF(a^*) > WF(a)$ for any $a > a^*$, implying that any welfare maximizing advertising vector has to be such that $a_i < a^*$. ■

Proof of Proposition 5: Applying the results of Proposition 1-3 in the specification of the current proposition implies that if equilibrium implies two-sided overlapping, then $a_{dis} = \frac{\omega''(2\beta - \tau) - \omega(\beta - \tau) - \gamma N(\omega - \omega'')}{2\gamma(2\omega'' - \omega)}$, $a_{nd} = \frac{\omega''(2\beta - \tau) - \omega(\beta - \tau)}{2\gamma(2\omega'' - \omega)}$, and $a_{duo} = \frac{\omega''(2\beta - \tau) - \omega(\beta - \tau)}{\gamma(3\omega'' - \omega)}$. The relationship $a_{duo} < a_{nd}$ is immediate from $a_{nd} = \frac{\omega''(2\beta - \tau) - \omega(\beta - \tau)}{2\gamma(2\omega'' - \omega)}$ and $a_{duo} = \frac{\omega''(2\beta - \tau) - \omega(\beta - \tau)}{\gamma(3\omega'' - \omega)}$. Comparing a_{duo} with $a_{dis} = \frac{\omega''(2\beta - \tau) - \omega(\beta - \tau) - \gamma N(\omega - \omega'')}{2\gamma(2\omega'' - \omega)}$ reveals that $a_{duo} > a_{dis}$

if $\omega''(2\beta-\tau)-\omega(\beta-\tau)<\gamma N(3\omega''-\omega)$. From assumption (1), the right hand side is at least $\frac{2\beta-\tau}{2}(3\omega''-\omega)$; therefore it is always higher than the left-hand side if $\omega''\geq\frac{\omega\tau}{2\beta-\tau}$. But this inequality always holds in case of two-sided multi-homing, and so $a_{duo}>a_{dis}$. Finally, $a_{WF}<a_{dis}$ by Proposition 4. ■

Proof of Proposition 6: $N\geq\frac{2\beta-\tau}{\gamma}$ implies that there cannot be overlapping on both sides in equilibrium. Entry of a competitor implies the following for channel 0:

(i) If $\beta<\tau$, then there is no change in the level of advertising.

(ii) If $\beta\geq\tau$ and $\omega'<\frac{\tau\omega}{2\beta-\tau}$, then advertising increases from $\frac{\beta}{2\gamma}$ to $\frac{2\beta-\tau}{2\gamma}$.

(iii) If $\beta\geq\tau$ and $\omega'\geq\frac{\tau\omega}{2\beta-\tau}$, then advertising increases from $\frac{\beta}{2\gamma}$ to $\frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega'-\omega)}=\frac{\beta}{2\gamma}+\frac{(\omega-\omega')\tau}{2\gamma(2\omega'-\omega)}$. ■

Proof of Proposition 7: Consider a neighborhood of parameter values such that $\beta>\tau$, $\omega'>\frac{\tau\omega}{2\beta-\tau}$, and $\frac{2\omega'(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}<N$. For these parameter values, the unique equilibrium in duopoly competition implies $a_0=a_1=\frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$. The equilibrium profit of a station is given by $\frac{M\omega'[(2\beta-\tau)\omega'-\omega(\beta-\tau)]^2}{\gamma\tau(3\omega'-\omega)^2}$. Differentiating this $\frac{M\omega'[(2\beta-\tau)\omega'-\omega(\beta-\tau)]^2}{\gamma\tau(3\omega'-\omega)^2}$ with respect to τ yields

$$\frac{M\omega'}{\gamma(3\omega'-\omega)^2}[(2(2\beta-\tau)\omega'-2\omega(\beta-\tau))(\omega-\omega')\tau-[(2\beta-\tau)\omega'-\omega(\beta-\tau)]^2]. \quad (8)$$

After simplifying, the sign of this expression is given by:

$$\text{sign}\left(\frac{\partial\Pi_i}{\partial\tau}\right)=\text{sign}\left((\tau^2-4\beta^2)(\omega')^2+(4\beta^2\omega-2\tau^2\omega)\omega'+\omega^2\tau^2-\omega^2\beta^2\right).$$

This sign is positive if $\left(\frac{\omega(4\beta^2-2\tau^2-2\beta\tau)}{8\beta^2-2\tau^2}\right)\leq\omega'\leq\left(\frac{\omega(4\beta^2-2\tau^2+2\beta\tau)}{8\beta^2-2\tau^2}\right)$. Since in this region, $\omega'>\frac{\omega\tau}{2\beta-\tau}$ and $\beta>\tau$, there exists such ω'. ■

Proof of Proposition 8: Consider advertising levels $a_i^*=\frac{N}{2}+\delta$ and $a_{-i}^*=\frac{N}{2}-\delta$ for some $\delta\in(0,\frac{N}{2})$. For these advertising levels, station $-i$'s profit is $\Pi_{-i}=M(N/2-\delta+f_{-i})(\frac{\beta-\gamma(\frac{N}{2}-\delta)-f_{-i}}{\tau})$. Maximizing this with respect to f_{-i} gives

$$f_{-i}^*=\frac{2\beta+(2\delta-N)(\omega+\gamma)}{4}.$$

f^*_{-i} is here always positive, because $N < \frac{\beta}{\gamma}$ and $\omega > \gamma$. This gives station $-i$ a profit of $\Pi^*_{-i} = \frac{M(2\beta+(N-2\delta)(\omega-\gamma))^2}{16\tau}$.

Calculating the optimal fee of station i yields $f^*_i = \frac{2\beta-(2\delta+N)(\omega+\gamma)}{4}$. But this fee is only positive as long as $\delta < \frac{2\beta-N(\omega+\gamma)}{2(\omega+\gamma)}$, and station i then gets a profit of $\Pi^*_i = \frac{M(2\beta+(N+2\delta)(\omega-\gamma))^2}{16\tau}$. Thus if $\delta \geq \frac{2\beta-N(\omega+\gamma)}{2(\omega+\gamma)}$, then $f^*_i = 0$ and station i only gets revenue from advertising.

To check if this profile is indeed an equilibrium, we need to check that neither of the firms has a profitable deviation. The above implies that any profitable deviation by i involves choosing a level of advertising different from a^*_i, and any profitable deviation by $-i$ involves choosing a level of advertising different from a^*_{-i}. It can be shown that i (which obtains a higher profit in this equilibrium) has a profitable deviation only if $-i$ also has one. Therefore, below we only consider deviations by $-i$.

Consider first the deviation such that $-i$ advertises less than a^*_{-i}. It can be shown that this deviation is not profitable if $\frac{M(2\beta+(N-2\delta)(\omega-\gamma))^2}{16\tau} \geq \frac{M\beta^2}{4\tau}$, or $N \geq 2\delta$. But the last inequality always holds since the maximal possible value of δ is $\frac{N}{2}$.

If $-i$ chooses an advertising level higher than a^*_{-i}, then either viewerships do not overlap, or there is two-sided overlap. Establishing two-sided overlap is not profitable if ω'' is low enough. To establish nonoverlapping viewerships, $-i$ has to set an advertising level that is at least $a^{dev}_{-i} = \frac{4\beta-2\gamma-\gamma N-2\delta}{2\gamma}$. This deviation is not profitable, though, for example if $\tau \leq \frac{\beta}{2}$ and $\omega'' \geq \frac{\omega\tau}{2\beta-\tau}$. Note that the latter can hold for arbitrarily small positive ω'' if β is large enough. ■

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