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The Effects of Product Bundling in Duopoly*

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Abstract

This paper studies the incentives for multiproduct duopolists to sell their products as a bundle. It is shown that contrary to the monopoly case bundling may reduce profits and increase consumer rent. This is the case if consumers' reservation values are negatively correlated. The reason is that bundling reduces consumer heterogeneity and makes price competition more aggressive. This effect can dominate the sorting effect that is well known for the monopoly case. Firms are in a prisoner's dilemma situation because they would be better off without bundling. Despite the lower prices a welfare loss occurs because some consumers do not buy their prefered product which results in distributive inefficiency. If firms can influence the correlation by choosing their location in the product range they try to avoid negative correlation and choose minimal differentiation in one good.

1 Introduction

Product bundling refers to the practice of selling two or more goods together in a package at a price which is below the sum of the independent prices. This practice can be observed very often in the real world. For example in the USA internet access is sold by long distance telephone companies. If a consumer buys internet access and long distance service together from the same company this is cheaper than if he buys both services independently. Another well known example is the selling of stereo systems. Big electronic companies always supply a package consisting of CD-player, stereo deck and receiver which is sold at a low price. There are many other examples of bundling in big department stores or cultural organizations, e.g. theaters and concert halls always offer season tickets.

In the industrial organization literature bundling has been extensively studied for monopolists and it is shown that mixed bundling, that is selling the goods individually and bundled together in a package, will in general increase the monopolist's profit.[1] However, the industry structure in the examples above is clearly not monopolistic. This shows that there is a need to examine bundling in oligopolistic or competitive markets. The objective of this paper is to analyze, how the ability to bundle affects profits and consumer rents in a duopolistic market structure.

It is shown that duopolists generically have an incentive to use mixed bundling, but the consequences on profits are ambiguous. If consumers are homogeneous, i.e. correlation of their reservation prices is positive, firms are better off with bundling. If instead consumers are heterogeneous, i.e. their reservation values are negatively correlated, profits are lower than without bundling. This is in sharp contrast to the monopoly case, where bundling raises the monopolist's profit, especially if consumers are heterogeneous.

The intuition behind this result is the following. First look at the monopoly case. If correlation of reservation values is negative there exist many consumers with extreme preferences, that means with a high valuation for good A but a low valuation for good B and vice versa. The optimal pricing strategy for a monopolist

[1] See Varian (1989) for an overview.

is to charge a high individual price for each good and the consumers with these extreme preferences buy only the good for which they have a high valuation. But still there are some consumers with middle range valuations for both goods and they buy the bundle at some discount. Thus bundling has a sorting effect. It allows the monopolist to sort its consumers into three categories instead of two and it can therefore extract more consumer rent.

Now let us look at a situation with two firms. Each firm must compete for demand and will do this with the help of the bundle. So beside the sorting effect, bundling now causes a second effect, which is called 'business-stealing' effect. This effect goes in the opposite direction than the sorting effect, because it results in a higher degree of competition and thus in lower profits. Whether bundling is profitable for the firms depends on which effect is dominating the other one.

The first result is that there is always an incentive for the duopolists to engage in mixed bundling as long as the correlation of valuations is not perfectly positive. This result is in line with the monopoly case. Since the firms have an additional instrument to sort their consumers they will use it.

Now assume consumers are homogeneous. This means that many of them have a strong preference for both goods of one firm. Therefore firms can act in some sense as local monopolists and can extract more consumer rent with bundling. There are only few consumers who are undecided between both firms. So it does not pay for a firm to undercut its competitor's prices to get these consumers at the margin. Thus prices and profits are relatively high. The sorting effect dominates the business-stealing effect. The consequences of bundling are very similar to the monopoly case.

If instead consumer preferences are heterogeneous the situation is completely different. In this case many consumers prefer good A from firm 1 and good B from firm 2 and vice versa. For simplicity, assume first that both firms can sell their goods only in a bundle. These bundles are now almost perfect substitutes to each other. Each firm can gain many new customers by lowering the price of its own bundle. Thus harsh price competition arises. If the firms can sell their products independently as well, this business-stealing effect endures. The price of the bundle is driven down to nearly marginal costs and this influences the unbundled prices which are now very low. Thus profits are low and consumer rent is high. The initial idea of the bundle, namely to price discriminate in a more skilful manner, is dominated by the business-stealing effect. So the result is completely opposite to the monopoly case. In this second case firms are in a prisoner's dilemma situation. It would be better for both of them not to bundle.

There is also an interesting welfare effect. Since the bundle is cheaper than the sum of the two independent prices, consumers are encouraged to buy the bundle. If heterogeneity increases firms react in equilibrium with an increase of their independent prices. Thus more consumers buy the bundle. This results in distributive inefficiency because some consumers prefer the products from different firms. So if markets are covered bundling reduces social welfare as it can only cause consumers to purchase the wrong good.

It is also analyzed what will happen if firms can influence the correlation of valuations. This can be done with the introduction of an additional stage in which firms choose their location in the product range. It is shown that firms may choose minimal differentiation in one product and thus forego profits with that product. They do this to avoid competition on the bundle which is very fierce if correlation is negative. Such firm behavior can be observed in the US by telephone companies which sell long distance service and internet access in one package. The long distance service offer is very similar in each package while firms try to differentiate themselves a lot in the offer of internet access with each firm offering different rates and amounts of installation gifts.

In the literature economists' attention on bundling was first drawn by the seminal paper of Adams and Yellen (1976). They show in a series of examples with an atomistic distribution of consumers that selling goods through bundling will raise the profit of a monopolist. This result was generalized by Schmalensee (1984) to a joint normal distribution and by McAfee, McMillan and Whinston (1989) to general distribution functions. They all show that bundling will raise the monopolist's profit, because it is an additional instrument to sort its customers. This is especially the case if reservation values for different goods are negatively correlated.

There are some papers which study bundling in a more competitive environment. The focus of these papers is if and how a multiproduct firm, which has monopoly power in one market, can increase its profit through bundling. Such a strategy is called tying. Whinston (1990) analyzes whether a firm which has monopoly power in the first market can monopolize a second market with duopolistic market structure by committing to engage in pure bundling. He shows that this is possible. The reason is that the monopolist sets a low bundle price in order to keep the consumers in its monopoly market. The consequence is that many consumers will now buy the bundle and the profit of the rival is low, which induces him to exit. Whinston (1990) shows that this effect can also be present if a tying commitment is not possible. Carbajo, deMeza and Seidmann (1990) study a model with a similar market structure. They present another idea why a tying commitment can be profitable for a monopolist. This is that with pure bundling products in the second market are differentiated and thus competition is reduced. Profits of both firms are higher.[2] Nalebuff (2004) shows that under a variety of circumstances pure bundling is more profitable for an incumbent even if commitment is not possible. This is the case if the entrant can enter only in one market. The intuition is, as in Whinston (1990), that the entrant must compete for consumers in the first market as well since the incumbent only offers the bundle. This greatly reduces the profit of the entrant. Choi (1996) analyzes the effects of bundling on research & development. In his model there is originally duopoly in both markets but both firms can invest in R&D to lower their production costs before reaching the price competition stage. If the difference in production costs for one good is large after the R&D game the market for this good is monopolised by the low cost firm. Choi (1996) shows that in this case bundling serves as a channel to monopolize the second market. Finally, Mathewson and Winter (1997) study a model with monopoly in one market and perfect competition in the other. They show that requirements tying is profitable for the monopolist provided that demands

[2] Chen (1997) presents a model with the same intuition only the market structure is different. He assumes duopoly in one market and perfect competition in the second. The duopolists can differentiate themselves by one firm selling the bundle and the other firm selling the goods only independently.

are stochastically dependent. For a great parameter range the optimal prices are Ramsey prices.[3]

There are two papers which have the same market structure as in my model (duopoly in both markets), namely Matutes and Regibeau (1992) and Anderson and Leruth (1993). The result in both papers is that if firms cannot commit not to bundle in equilibrium they choose mixed bundling. But this results in increased competition and lowers profits. A prisoner's dilemma dilemma arises because profits would be higher without bundling. However, in these papers the driving force of the monopoly case, the correlation of consumers' reservation values, is not modelled. In my model it is shown that this is also the crucial variable for the oligopoly case, but can create opposite effects. Also these papers are not concerned with welfare and location choice.

This paper is also in the spirit of a relatively new literature which studies the effects of price discriminating methods in a competitive environment. An extensive overview of the different branches of these literature is given in a paper by Stole (2003) which is prepared for the forthcoming volume of the Handbook of Industrial Organization. In the section about bundling Stole (2003) summarizes many of the recent papers which are concerned with the question how bundling affects profits and market structure when commitment is possible or not.

The structure of the paper is as follows. Section 2 sets out the model. The particular structure of consumer heterogeneity and the correlation of reservation values is presented in Section 3. Equilibrium selling and price policy is determined in Section 4. Section 5 studies the welfare consequences of bundling. Section 6 analyzes the effects if firms have the possibility to choose their location and influence the reservation price correlation. An application of the model to the US telephone industry is considered in Section 7. Section 8 concludes the chapter. The proofs of all results are given in the Appendix of the paper.

[3] Seidmann (1991) and Denicolo (2000) analyze the consequences of bundling in other market structures.

2 The Model

The model is a variant of Salop's (1979) model of spatial competition on the circle but with two goods.

There are two firms $i = 1, 2$. Both firms produce two differentiated goods $j = A, B$ at the same constant marginal costs c_A and c_B.[4] The product space for each good is taken to be the unit-circumference of a circle. The product variants are then the locations of the firms on each circle. It is assumed that firm 1 is located at point 0 on both circles and firm 2 is located at point $\frac{1}{2}$ on both circles. So there is maximum product differentiation in both goods. The firms have the choice to sell their products not only independently but also together as a bundle. So each firm i can choose between two possible selling strategies. It can sell its goods separately at prices p^i_A and p^i_B (independent pricing) or it can sell the goods independently and as a bundle at prices p^i_A, p^i_B and p^i_{AB} (mixed bundling).[5] Firms have to decide simultaneously about their selling and price strategies. It is assumed that they cannot monitor the purchases of consumers. So the strategy space for each firm i is to quote three prices p^i_A, p^i_B and p^i_{AB}. If $p^i_{AB} < p^i_A + p^i_B$ firm i engages in mixed bundling while if $p^i_{AB} \geq p^i_A + p^i_B$ firm i practice independent pricing as no consumer would buy the bundle from firm i. Last, resale by consumers is impossible.

There is a continuum of consumers and without loss of generality we normalize its total mass to 1. Each consumer is described by her location on both circles, $\mathbf{x} = (x_A, x_B)^T$. Every consumer has a unit demand for both goods and purchases each good independently of the other. So there is no complementarity between the products. This allows me to focus on the pure strategic effect of bundling. The consumers are uniformly distributed on each circle j. This is mainly for tractability reasons and to compare the results with previous papers.[6] In the next section

[4]The assumption of the same cost function for both firms is made for simplicity and is not crucial to the results.

[5]There can also be a third strategy, namely to sell the goods only as a bundle at price p^i_{AB}. Adams & Yellen (p. 483) and McAfee, McMillan & Whinston (p. 334) have shown that this cannot be the unique optimal strategy because mixed bundling with prices $p^i_A = p^i_{AB} - c_B$ and $p^i_B = p^i_{AB} - c_A$ always does weakly better. This also holds in my model.

[6]For analyzes without uniform distributions see Neven (1986), Tabuchi & Thisse (1995) and Anderson, Goeree & Ramer (1997).

we give some structure to the joint distribution and present the modelling of the correlation of reservation values.

A consumer who is located at $0 \leq x_A, x_B \leq \frac{1}{2}$ and buys good A from firm 1 and good B from firm 2 enjoys an indirect utility of

$$V(x_A, x_B) = K_A - p_A^1 - t_A(x_A)^2 + K_B - p_B^2 - t_B\left(\frac{1}{2} - x_B\right)^2. \tag{1}$$

A similar expression holds for consumers who are located somewhere else or buy different products. K_A and K_B are the surpluses from consumption (gross of price and transportation cost) of good A and B. p_j^i is the price of variant i of product j. The transportation cost function is the weighted squared distance between the location of the consumer and the variant produced by the firm where she buys. The weight is the salience coefficient for each product, t_j, and without loss of generality we assume that $t_A > t_B > 0$.[7] The reservation price of a consumer for variant i of good j, R_j^i, is thus $K_j - t_j(\mathrm{d}_i)^2$, where d_i is the shortest arc length between the consumer's location and firm i on circle j. It is also assumed that K_j is sufficiently large such that both markets are covered. This means that the reservation values are high enough such that in each price equilibrium all consumers buy both goods. When dealing with welfare considerations this means that there is no welfare loss due to exclusion of consumers who should buy the product from a social point of view. The form of utility in (1) looks special but it is the standard form in models with spatial competition if consumers can buy many products.[8]

The consumers thus have the choice between four alternative consumption combinations. They can buy the bundle from firm 1 ($AB1$), the bundle from firm 2 ($AB2$), good A from firm 1, good B from firm 2 ($A1B2$), and good B from firm 1, good A from firm 2 ($B1A2$).

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[7]The cases $t_B \to t_A$ and $t_B \to 0$ are analyzed in Section 4.

[8]In the literature the assumption of a quadratic transportation cost function is usually made to guarantee existence of an equilibrium if firms can choose their locations before setting prices (see e.g. D'Aspremont, Gabszewicz & Thisse (1979) and Irmen & Thisse (1998)). In my basic model this assumption is not necessary since firms are maximally differentiated and one could also work with a linear transportation cost function. However, in Section 6 the model is extended to allow for location choice of firms. To keep the analysis consistent quadratic transportation costs are assumed right from the beginning.

3 Dependence between Location and Correlation

In the monopoly case the correlation of reservation values is crucial for the incentive to bundle. It is a known result that especially in case of independence or negative correlation bundling dominates unbundled sales.

In our case it is possible to infer the joint distribution function of reservation values $G(R_A^i, R_B^i)$ for firm i and therefore the correlation between the reservation values from the joint distribution function of consumer location $F(x_A, x_B)$. If for example every consumer has the same location on both circles then the conditional density function of x_A given x_B is

$$f(x_A \mid x_B) = \begin{cases} 0 & \text{if } x_A \neq x_B \\ 1 & \text{if } x_A = x_B. \end{cases}$$

The conditional density function $g(R_A^i \mid R_B^i)$ of reservation values for firm i is then

$$g(R_A^i \mid R_B^i) = \begin{cases} 0 & \text{if } R_A^i - R_B^i \neq K_A - K_B - (t_A - t_B)(\mathrm{d}_i)^2 \\ 1 & \text{if } R_A^i - R_B^i = K_A - K_B - (t_A - t_B)(\mathrm{d}_i)^2. \end{cases}$$

This would imply a reservation price correlation of $\rho[R_A^i, R_B^i] = 1$. This is a simple example and there are possibly infinitely many ways how the consumers can be distributed on one circle given the location on the other circle. To keep the model tractable, we have to give some structure to this conditional distribution, which still captures the main point of expressing different correlations. This is done in a very simple way. It is assumed that if a consumer is located at x_A on circle A then she is located at

$$x_B = \begin{cases} x_A + \delta & \text{if } x_A + \delta \leq 1 \\ x_A + \delta - 1 & \text{if } x_A + \delta > 1 \end{cases}$$

on circle B, where $0 \leq \delta \leq \frac{1}{2}$.[9] This means a δ-shift of all consumers on circle B. So a δ of 0 corresponds to the former example. The advantage of doing this is that with this simple structure correlations of values can be obtained easily by altering

[9]It suffices to consider δ between 0 and $\frac{1}{2}$. A δ greater than $\frac{1}{2}$ expresses the same correlation as one between 0 and $\frac{1}{2}$. For example a δ of 0.8 expresses the same correlation as a δ of 0.2.

δ.

Remark 1

The function $\rho[R_A, R_B](\delta) = \frac{\mathrm{Cov}[R_A,R_B](\delta)}{\sigma(R_A)\sigma(R_B)}$ is given by $1 - 30\delta^2 + 60\delta^3 - 30\delta^4$.[10]

Thus correlation is strictly decreasing in δ.[11] If $\delta = 0$, $\rho(\delta) = 1$, i.e. perfect positive correlation while if $\delta = 0.5$, $\rho(\delta) = -0.875$.[12] Correlation here relates to the products of one firm. So negative correlation means that a consumer who values product A from firm i highly has a low valuation for product B of firm i.

Obviously this simple structure has important characteristics. First, there is a one-to-one mapping between positions on circles. This implies that there is no stochastic in the model.

Second given the location on circle A the location on circle B is exactly ordered by δ and can not be crisscross.

However, this structure captures the main point of correlation. With a low δ, there are many consumers having high reservation values for both goods of firm i. For a high δ, many people have extremely different reservation values for both goods of firm i. So this structure represents exactly what is meant with correlation. Its main advantage is that it keeps the model tractable and gives clear cut results.

4 Equilibrium Price and Selling Strategies

In this section the equilibrium price and selling strategies of a firm conditional on the correlation of values is analyzed.

Before doing this the equilibrium of the game without the bundling option is determined. The result will later be used as a benchmark.

[10] The proof of this and all other results can be found in the Appendix.

[11] The term correlation does not mean a stochastic correlation in this model, because there is no stochastic element. It describes the relation between known reservation values. So it is a term from descriptive statistics.

[12] We do not get the whole range of correlation coefficients because distance enters quadratically in the utility function. With a linear transportation cost function the whole range of coefficients could be reached but the results of the analysis would stay the same.

If bundling is not possible there is no connection between the two products. Each market is independent and we are in a standard situation of product differentiation on the circle. The Nash equilibrium can be determined in the usual way. In this equilibrium firms set prices

$$p_A^1 = p_A^2 = p_A^\star = c_A + \frac{1}{4}t_A,$$

$$p_B^1 = p_B^2 = p_B^\star = c_B + \frac{1}{4}t_B$$

and earn profits

$$\Pi_1^\star = \Pi_2^\star = \frac{1}{8}(t_A + t_B).$$

Now assume that bundling is possible. In the following the profit functions of the firms for different correlations are determined. First, the question arises if firms have an incentive to bundle.

Proposition 1

If $\delta > 0$, i.e. $\rho < 1$, then in equilibrium both firms choose mixed bundling.

This is in line with the monopoly case. The firms have an additional instrument to sort their customers and so they will use it. The exception is, if $\delta = 0$, i.e perfect positive correlation. In this case all consumers have the same position on each circle. Thus firms do not need a third instrument because consumers cannot be sorted better than with independent prices.

Now the demand structure on the circles in dependence of δ can be derived. The special form of locations allows us to work only with one circle because the location on the other circle is then uniquely determined.

First, assume δ is small and start at a consumer with location $x_A = 0$. She has a high reservation value for both variants 1 and will therefore buy bundle $(AB1)$.[13] If we move clockwise on circle A then the consumer who is indifferent between

[13]Product combination (AB1) is only bought if p_{AB}^1 is not too high compared with other prices. In the proof of Proposition 2 in the appendix it is shown that this is the case in equilibrium .

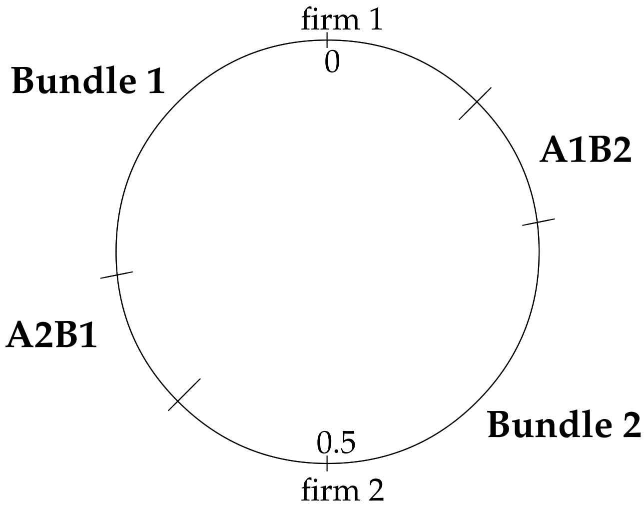

Figure 1: Demand structure if $\delta \leq \frac{1}{3} + \frac{t_B}{6t_A}$

$(AB1)$ and $(A1B2)$ is defined by

$$x_A = \frac{1}{4} + \frac{p_A^1 + p_B^2 - p_{AB}^1}{t_B} - \delta.$$

The product combination which is bought to the right of $(AB1)$ is $(A1B2)$. It is not bundle 2, because then no one would buy the independent products, which cannot be the case in equilibrium.[14] Moving further to the right the next combination which is bought is $(AB2)$ and the marginal consumer is located at

$$x_A = \frac{1}{4} + \frac{p_{AB}^2 - p_A^1 - p_B^2}{t_A}.$$

If we pass the point $\frac{1}{2}$ and move upward on the left side of the circle, we get the same product structure as on the right side, because of symmetry, only with firm 1 and 2 reversed. Consumers next to $\frac{1}{2}$ buy $(AB2)$, consumers in the middle buy $(A2B1)$ and consumers next to 1 buy $(AB1)$. Figure 1 illustrates the product combinations on circle A.

[14]Remember that firms always engage in mixed bundling.

The profit function of firm 1 is therefore

$$\begin{aligned}\Pi_1 = \ & (p^1_{AB} - c_A - c_B)(\tfrac{1}{4} + \tfrac{p^1_A + p^2_B - p^1_{AB}}{t_B} - \delta + 1 - \tfrac{3}{4} - \tfrac{p^1_{AB} - p^1_B - p^2_A}{t_A}) \\ & +(p^1_A - c_A)(\tfrac{p^2_{AB} + p^1_A - p^2_B}{t_A} + \tfrac{p^1_A + p^2_B - p^1_{AB}}{t_B} + \delta) \\ & +(p^1_B - c_B)(\tfrac{p^1_{AB} + p^2_A - p^1_B}{t_A} + \tfrac{p^2_{AB} - p^2_A - p^1_B}{t_B} + \delta).\end{aligned} \tag{2}$$

Because of symmetry we get a similar function for firm 2. Calculating prices and profits we get

$$\begin{aligned} p^\star_A &= c_A + \tfrac{1}{4}t_A + \tfrac{1}{3}\delta\tfrac{t_A t_B}{t_A + t_B}, \\ p^\star_B &= c_B + \tfrac{1}{4}t_B + \tfrac{1}{3}\delta\tfrac{t_A t_B}{t_A + t_B}, \\ p^\star_{AB} &= c_A + c_B + \tfrac{1}{4}(t_A + t_B), \\ \Pi^\star &= \tfrac{1}{8}(t_A + t_B) + \tfrac{4}{9}\delta^2\tfrac{t_A t_B}{t_A + t_B}. \end{aligned} \tag{3}$$

for both firms.

Next assume that δ is large and start again at $x_A = 0$. The consumer located there has the highest reservation value for variant 1 of good A and a high reservation value for variant 2 of good B. If p^1_A and p^2_B are not much higher than other prices she will buy $(A1B2)$. Moving clockwise the next combination can only be bundle 1 or bundle 2, because it is shown in Claim 1 in the appendix, that $(A2B1)$ can never be in direct rivalry to $(A1B2)$. In equilibrium it will be bundle 1 because the position of the consumer on circle A is nearer to firm 1. Since $t_A > t_B$, the distance on circle A is more important than the one on circle B. The marginal consumer is given by

$$x_A = \frac{3}{4} + \frac{p^1_{AB} - p^1_A - p^2_B}{t_B} - \delta.$$

If we move further clockwise the distance to firm 2 becomes shorter than that to firm 1 and so consumers buy bundle 2. The marginal consumer between $(AB1)$ and $(AB2)$ is defined by

$$x_A = \frac{1}{(t_A - t_B)}\left(p^2_{AB} - p^1_{AB} + \frac{1}{4}t_A - \frac{3}{4}t_B + t_B\delta\right).$$

Next, consumers located near $\frac{1}{2}$ buy $(A2B1)$. The structure on the left side is the same only with firms reversed. The whole demand structure is illustrated in Fig-

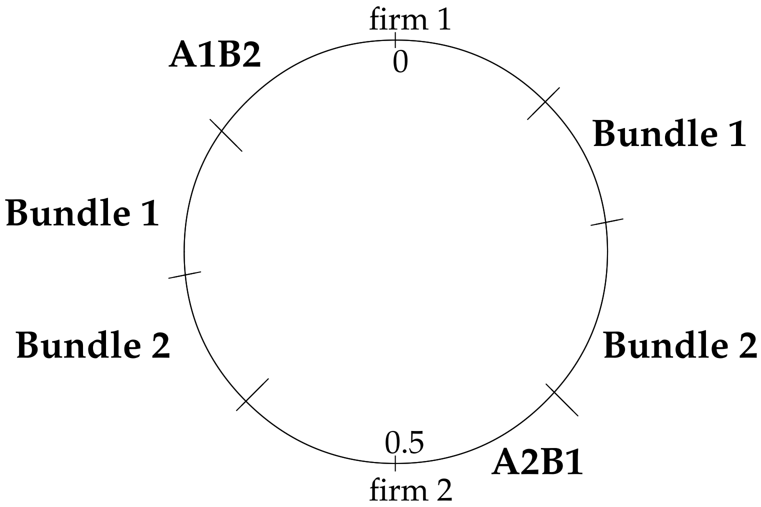

Figure 2: Demand structure if $\delta > \frac{1}{3} + \frac{t_B}{6t_A}$

ure 2.

The profit function of firm 1 is thus

$$
\begin{aligned}
\Pi_1 = \; & (p_A^1 - c_A)(\tfrac{3}{4} + \tfrac{p_{AB}^1 - p_A^1 - p_B^2}{t_B} - \delta - \tfrac{1}{4} + \delta - \tfrac{p_A^1 + p_B^2 - p_{AB}^1}{t_B}) \\
& + (p_{AB}^1 - c_A - c_B)\Big(\tfrac{p_{AB}^2 - p_{AB}^1 + \frac{1}{4}t_A - \frac{3}{4}t_B + t_B\delta}{(t_A - t_B)} - \tfrac{3}{4} - \tfrac{p_{AB}^1 - p_A^1 - p_B^2}{t_B} + \delta \\
& + \tfrac{5}{4} + \tfrac{p_A^1 + p_B^2 - p_{AB}^1}{t_B} - \delta - \tfrac{p_{AB}^1 - p_{AB}^2 + \frac{3}{4}t_A - \frac{5}{4}t_B + t_B\delta}{(t_A - t_B)}\Big) \\
& + (p_B^1 - c_B)(\tfrac{5}{4} + \tfrac{p_{AB}^2 - p_B^1 - p_A^2}{t_B} - \delta - \tfrac{3}{4} - \tfrac{p_A^2 + p_B^1 - p_{AB}^2}{t_B} + \delta).
\end{aligned} \tag{4}
$$

and equilibrium prices and profits are

$$
\begin{aligned}
p_A^\star &= c_A + \tfrac{1}{6}t_A - \tfrac{1}{6}t_B, \\
p_B^\star &= c_B + \tfrac{1}{12}t_B, \\
p_{AB}^\star &= c_A + c_B + \tfrac{1}{4}(t_A - t_B), \\
\Pi^\star &= \tfrac{1}{8}t_A - \tfrac{7}{72}t_B.
\end{aligned} \tag{5}
$$

for both firms. It remains to calculate at which value of δ the profit function is changing. The difference between the two profit functions is that on the right side of the circle the region (A1B2) is followed by (AB2) in profit function (2) while in profit function (4) (A1B2) is followed by (AB1). Likewise on the left side (A2B1) is followed by (AB1) in profit function (2) but by (AB2) in profit function (4). If profit function (2) is relevant there is some value of δ at which (A1B2) would no longer be followed by (AB2) but by (AB1) if firms charge equilibrium prices. Calculating this threshold yields $\delta = \frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B})$. At this value both firms begin to lower its prices in such a way that demand structure of Figure 1 is still valid. The prices and profits for $\delta > \frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B})$ are given by

$$\begin{aligned}
p_A^\star &= c_A + \tfrac{1}{4}t_A + \tfrac{t_A t_B}{2(t_A-t_B)^2}((5t_A+4t_B) - 2\delta(8t_A+t_B)), \\
p_B^\star &= c_B + \tfrac{1}{4}t_B + \tfrac{t_A t_B}{2(t_A-t_B)^2}((5t_A+4t_B) - 2\delta(8t_A+t_B)), \\
p_{AB}^\star &= c_A + c_B + \tfrac{1}{4}(t_A+t_B) + \tfrac{t_A t_B}{2(t_A-t_B)^2}(9(t_A+t_B) - 6\delta(5t_A+t_B)), \\
\Pi^\star &= \tfrac{1}{8}t_A + \tfrac{1}{8}t_B + \tfrac{t_A t_B}{2(t_A-t_B)^2}\left(4(t_A+t_B) - 2\delta(6t_A+t_B) - 4\delta^2 t_A\right).
\end{aligned} \tag{6}$$

But if δ increases further at some point it is profitable for both firms to deviate from the above strategy and keep their prices constant. At this value the demand structure changes and for all δ above this value profit function (4) is valid. Calculating this threshold yields $\delta = \frac{1}{3} + \frac{t_B}{6t_A}$.

The analysis above is summarized in the following proposition.

Proposition 2

If $\delta \leq \frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B})$, then in the unique Nash equilibrium firms set prices and earn profits according to (3).

If $\delta > \frac{1}{3} + \frac{t_B}{6t_A}$, then in the unique Nash equilibrium firm set prices and earn profits according to (5).

If $\frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B}) < \delta \leq \frac{1}{3} + \frac{t_B}{6t_A}$, then in the unique Nash equilibrium firm set prices and earn profits according to (6).

So the profit function is continuous but non-monotonic in δ. It is first increasing in δ then decreasing and for high values of δ it is constant. The profit function in dependence of δ is illustrated graphically in Figure 3.

What is the intuition behind this result? First look at the case where $\delta \leq \frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B})$. Because δ is small, the locations of consumers on both circles are similar. This means that there are a lot of consumers with high reservation values for both goods of one firm. From the perspective of these consumers, firms are very distinct. Thus firms have high market power and price competition is low. One can see this also in Figure 1. There are four product combination regions. But there are no bundle regions side by side. This means that if one firm lowers its bundle price, it will

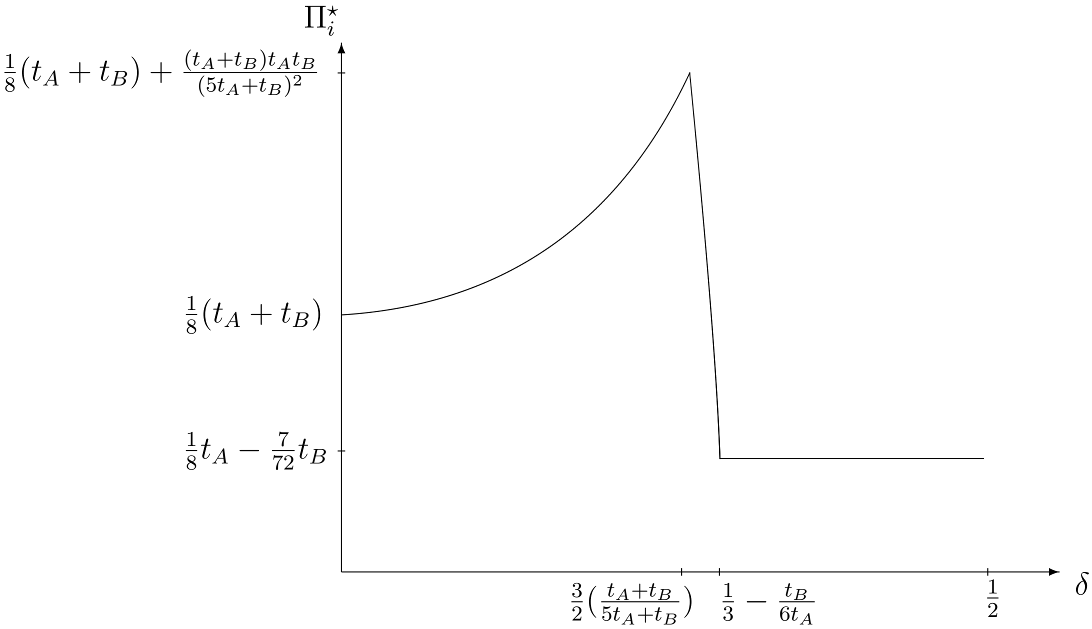

Figure 3: Equilibrium profits

get more bundle consumers but also lose demand on its own independent sales. So lowering a price has also a negative effect on a firm's own demand and thus there is only little incentive to lower prices. Note that for $\delta \to 0$ equation (3) implies that prices and profits are the same as without bundling. This is in line with Proposition 1 where it is shown that if $\delta = 0$, there is no incentive to bundle. From (3), $p^\star_{AB}$ is independent of δ. $p^\star_{AB}$ is the sum of the two prices that arise if bundling is not possible. So consumers buying the bundle have to pay the same amount of money if bundling is possible or not. Consumers located further away from the variants of the firms, thus buying $(A1B2)$ or $(A2B1)$, lose through bundling because $p^\star_A$ and $p^\star_B$ are increasing in δ. Calculating the breadth of the product combination ranges we get that demand for each bundle is $D_{AB1} = D_{AB2} = \frac{1}{2} - \frac{1}{3}\delta$ and demand for each two-variant-combination is $D_{A1B2} = D_{A2B1} = \frac{1}{3}\delta$. Despite the fact that $p^\star_A$ and $p^\star_B$ increase with δ, $D_{A1B2} = D_{A2B1}$ increase with δ as well. The reason is that preferences get more heterogeneous with higher δ and this effect is stronger than the price increase. Because of this increasing heterogeneity firms gain through product bundling. They charge higher independent prices and can better sort their consumers. Profits rise with δ and consumer rent decreases.

If on the opposite $\delta > \frac{1}{3} + \frac{t_B}{6t_A}$, then profits are low. It is apparent from (3) that profits are lower than without bundling. This can be explained in the following way. Assume that firms can only offer the bundle. In this case the reservation value of a consumer for both bundles is nearly the same. An extreme case would be $\delta = \frac{1}{2}$ and $t_A = t_B$. Then each consumer has the same valuation for both bundles. Firms can gain many new consumers by lowering the bundle price. So competition in the bundle is very harsh and this affects also the unbundled prices. This business-stealing effect of bundling drives profits down. In terms of strategic substitutes and complements defined by Bulow, Geanakoplos and Klemperer (1985), the two bundles are direct strategic complements, $\frac{\partial^2 \Pi^i}{\partial p^1_{AB} \partial p^2_{AB}} > 0$. So if one firm lowers its bundle price, the other will do the same. This can also be seen in Figure 2. On the right as well as on the left side of the circle there is a region, where bundle 1 is side by side with bundle 2. If a firm lowers its bundle price then it gets new consumers, who formerly did not buy either good of that firm. Such a region does not exist in Figure 1. In case of profit function (2) there is no direct strategic complementarity. This result is in sharp contrast to the monopoly case. In monopoly the bundle helps the firm to reduce the dispersion of reservation values to get more consumer rent. This is especially profitable if correlation is negative. In duopoly there is the same effect, but with completely different consequences. The bundle also reduces dispersion, but competition gets harsher and profits lower.

In this region prices and profits are low and do not change with δ. The reason is that there is no incentive to decrease prices because they are already low and thus the gains from decreasing prices are low compared with the losses. There is also no incentive to increase prices because a firm would lose some consumers who have formerly bought the bundle and would buy both goods from the rival after the price increase.

In the remaining region $\frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B}) < \delta \leq \frac{1}{3} + \frac{t_B}{6t_A}$ prices are decreasing with δ. As δ is already high consumers are more homogeneous. Each firm has an incentive to exploit this and reduce its price to induce more consumers to buy the bundle. So both firms lower their prices. But since δ is not very high and consumers' bundle valuations are still heterogeneous the demand structure does not change. This effect of lowering prices becomes stronger the higher δ is. Thus prices and profits decrease with δ.

It is interesting to compare profits in case of bundling with profits if bundling is not possible. If bundling is not possible profits are $\Pi^\star = \frac{1}{8}t_A + \frac{1}{8}t_B$. Thus if $\delta \leq \frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B})$ bundling raises profits while if $\delta > \frac{1}{3} + \frac{t_B}{6t_A}$ profits are lower with bundling. Since the profit function is strictly and continuously decreasing in δ in the region $\frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B}) < \delta \leq \frac{1}{3} + \frac{t_B}{6t_A}$ there is one value of δ for which profits are the same. Calculating this value by comparing profits yields the following lemma.

Lemma 1

If $\delta > \frac{\sqrt{52t_A^2+28t_At_B+t_B^2}}{4t_A} - \frac{3}{2} - \frac{t_B}{4t_A}$ profits are lower than without bundling and firms are in a prisoner's dilemma.

Firms are in a prisoner's dilemma situation because as is shown in Proposition 1 they both choose to bundle. But this results in lower profits than if they did not bundle. Thus firms would be better off without the possibility to bundle.

It is also possible to analyze the thresholds where the profit function has kinks. The first threshold is given by $\delta_1^{TS} = \frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B})$. Since $t_A > t_B > 0$ the threshold lies in the range $\delta_1^{TS} \in]\frac{3}{10}, \frac{1}{2}[$. The maximal profit of the firms is reached at this threshold and is given by $\Pi^\star = \frac{1}{8}t_A + \frac{1}{8}t_B + \frac{(t_At_B)(t_A+t_B)}{(5t_A+t_B)^2}$. The second threshold is given by $\delta_2^{TS} = \frac{1}{3} + \frac{t_B}{6t_A}$. At this threshold the demand structure changes. Since $t_A > t_B > 0$ this threshold lies in the range $\delta_2^{TS} \in]\frac{1}{3}, \frac{1}{2}[$. Thus the intermediate region where the profit decreases is very small. Its maximal breadth is approximately 0.03. This is the case when $t_B \to 0$ which implies $\delta_1^{TS} = \frac{3}{10}$ and $\delta_2^{TS} = \frac{1}{3}$. Thus the profit decreases sharply from a high level to a level that is even lower than without bundling.

It is also interesting to look at two extreme cases of the transportation costs. First let us see what will happen if $t_B \to 0$. In this case $\lim_{t_B \to 0} \delta_2^{TS} = \frac{1}{3}$. In this case no consumer has a special preference for product B of one firm. The standard Bertrand argument leads to $p_B^* = c_B$. But also if firms bundle they can only make profits on good A. A look at the profit functions shows that $\Pi_i^* = \frac{1}{8}t_A$ independent of which profit function arises. The bundle has neither a sorting nor an additional competition effect since good B is offered in perfect competition. Another extreme is if $t_B \rightarrow t_A$ which results in $\lim_{t_B \rightarrow t_A} \delta_2^{TS} = \frac{1}{2}$. This shows that in this case only profit function (2.2) is relevant. Thus only the price discrimination effect of bundling is valid and profits are always increasing the more negative the correlation is. But for all values of t_B between 0 and t_A whenever $\delta > \frac{\sqrt{52t_A^2+28t_At_B+t_B^2}}{4t_A} - \frac{3}{2} - \frac{t_B}{4t_A}$ the ability to bundle reduces profits.

5 Welfare Consequences

The model has also interesting welfare implications. It is assumed that the reservation price of every consumer is high enough, so that in each price equilibrium all consumers are served. Thus there is no inefficiency that results from consumers whose valuations are higher than marginal costs and who do not buy the goods. But there is a distributive inefficiency. It arises because some consumers do not buy their preferred product.[15]

As a benchmark we can first calculate maximal welfare. Welfare is maximized if transportation costs are minimized. This is the case if on both circles consumers at $0 \leq x_j \leq \frac{1}{4}$ and $\frac{3}{4} \leq x_j \leq 1$ buy from firm 1 and consumers at $\frac{1}{4} \leq x_j \leq \frac{3}{4}$ buy from firm 2. The resulting welfare is

$$WF^{max} = K_A + K_B - c_A - c_B - \frac{1}{48}[t_A + t_B].$$

Maximal welfare is reached if the firms do not bundle.

If bundling is possible welfare depends on δ.

Proposition 3

If $\delta \leq \frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B})$ then

$$WF = K_A + K_B - c_A - c_B - \frac{1}{48}(t_A + t_B) - \frac{4}{9}\delta^2 \frac{t_At_B}{t_A + t_B}. \quad (7)$$

[15]Distributive inefficiency is also present in the monopoly case. Here some consumers who value a good higher than others do not buy it while the latter individuals do. See Adams & Yellen (1976).

If $\frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B}) < \delta \leq \frac{1}{3} + \frac{t_B}{6t_A}$ then

$$WF = K_A + K_B - c_A - c_B - \frac{1}{48}(t_A + t_B) - (\frac{1}{4} - \delta^2 + \delta)\frac{(t_A + t_B)t_A t_B}{(t_A - t_B)^2}. \quad (8)$$

If $\delta > \frac{1}{3} + \frac{t_B}{6t_A}$ then

$$WF = K_A + K_B - c_A - c_B - \frac{1}{48}(t_A + t_B) - \frac{1}{36}(t_A + t_B)\frac{t_B}{t_A}. \quad (9)$$

Thus welfare in case of bundling is always lower than without bundling. The reason is that the price of the bundle is lower than the sum of the independent prices. This induces some consumers to buy the bundle and therefore both goods from one firm although they prefer the goods from different firms. Bundling always causes a welfare loss if markets are covered.

In case of $\delta \leq \frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B})$ welfare decreases with δ. With an increase in δ consumers get more heterogeneous. This means that they wish to buy the goods from different firms. But in equilibrium independent prices are increasing in δ while the bundle price is constant. The difference between the independent prices and the bundle price is therefore increasing in δ. This tempts consumers to buy the bundle. Thus distributive inefficiency increases with δ.

In the region $\frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B}) < \delta \leq \frac{1}{3} + \frac{t_B}{6t_A}$ welfare slightly increases with δ because $\delta < \frac{1}{2}$. All three prices are decreasing in δ because competition rises. This reduces the distributive inefficiency slightly.

If $\delta > \frac{1}{3} + \frac{t_B}{6t_A}$ welfare is independent of δ because all prices are independent of δ as well.

As the profit functions the welfare function is also continuous but non-monotonic in δ. Figure 4 illustrates the shape of the welfare function, where $WF_{\mathrm{gr}} = K_A + K_B - c_A - c_B$.

This shows that the shape of the welfare function in the first two regions is exactly opposite to the shape of the profit function. The intuition is the following. If δ is small an increase in consumer heterogeneity helps firms to extract more consumer rent through bundling. But this is done by increasing the independent prices thereby inducing consumers to buy the bundle which reduces welfare. If δ is

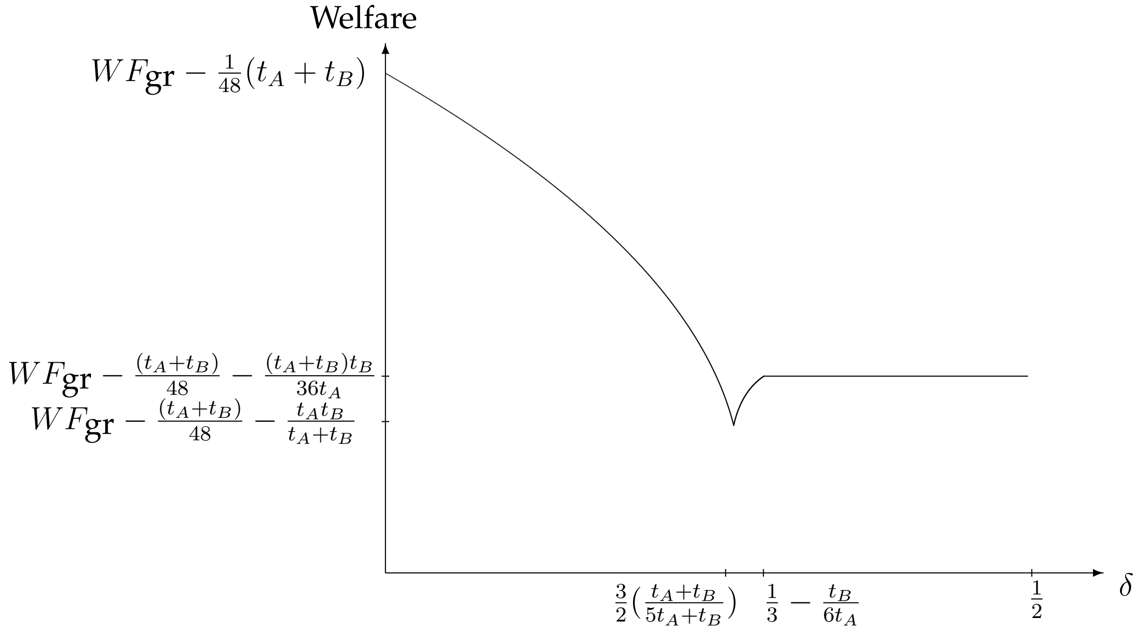

Figure 4: Welfare function

high consumers are heterogeneous and their valuation for both bundles is almost the same. Price competition is fierce and profits are low. But the difference between the sum of the independent prices and the bundle price is almost the same as with a δ in the middle range. Thus welfare stays unchanged.

6 Location Choice

In this section the model is extended by endogenizing the level of product differentiation. In choosing the locations the firms not only change the differentiation and with that the degree of competition but also the correlation of values. This effect of correlation change has interesting implications on firms' location choice. Before analyzing this let us look at a generalization of the basic model where products are no longer maximally differentiated. This is also a first step towards the later analysis of location choice.

The location of firm 1 is point 0 on both circles as before. Firm 2 is now located at α on both circles with $0 \leq \alpha \leq \frac{1}{2}$.[16] In calculating marginal consumers the same

[16] Assuming α between $\frac{1}{2}$ and 1 would give the same results since e.g. $\alpha = 0.8$ represents the same game as $\alpha = 0.2$.

analysis as in Section 4 can be conducted. This yields the following proposition.

Proposition 4

If $\delta \leq \frac{3}{2}(\frac{t_A+t_B}{3t_A-t_B+4\alpha(t_A+t_B)})$ prices of the firms are

$$\begin{aligned} p^\star_A &= c_A + \alpha(1-\alpha)\left(t_A + \tfrac{4}{3}\delta\tfrac{t_At_B}{t_A+t_B}\right), \\ p^\star_B &= c_B + \alpha(1-\alpha)\left(t_B + \tfrac{4}{3}\delta\tfrac{t_At_B}{t_A+t_B}\right), \\ p^\star_{AB} &= c_A + c_B + \alpha(1-\alpha)(t_A+t_B), \end{aligned}$$

and profits of the firms are given by

$$\Pi^\star = \frac{1}{2}\alpha(1-\alpha)((t_A+t_B) + \frac{8}{9}\delta^2\frac{t_At_B}{t_A+t_B}).$$

If $\delta > \frac{1}{2} - \frac{\alpha}{3} + \frac{t_B(1-\alpha)}{3t_A}$ prices of the firms are

$$\begin{aligned} p^\star_A &= c_A + \alpha(1-\alpha)\left(t_A - \tfrac{2}{3}t_B\right), \\ p^\star_B &= c_B + \alpha(1-\alpha)\tfrac{1}{3}t_B, \\ p^\star_{AB} &= c_A + c_B + \alpha(1-\alpha)(t_A-t_B) \end{aligned}$$

and profits of the firms are given by

$$\Pi^\star = \frac{1}{2}\alpha(1-\alpha)(t_A - \frac{7}{9}t_B).$$

If $\frac{3}{2}(\frac{t_A+t_B}{3t_A-t_B+4\alpha(t_A+t_B)}) < \delta \leq \frac{1}{2} - \frac{\alpha}{3} + \frac{t_B(1-\alpha)}{3t_A}$ prices of the firms are

$$\begin{aligned} p^\star_A &= c_A + \alpha(1-\alpha)\left(t_A + \tfrac{t_At_B}{(t_A-t_B)^2}(2(5t_A+4t_B) - 4\delta(8t_A+t_B)\right), \\ p^\star_B &= c_B + \alpha(1-\alpha)\left(t_B + \tfrac{t_At_B}{(t_A-t_B)^2}(2(5t_A+4t_B) - 4\delta(8t_A+t_B)\right), \\ p^\star_{AB} &= c_A + c_B + \alpha(1-\alpha)\left((t_A+t_B) + \tfrac{t_At_B}{(t_A-t_B)^2}(18(t_A+t_B) - 12\delta(5t_A+t_B))\right) \end{aligned}$$

and profits of the firms are given by

$$\Pi^\star = \frac{1}{2}\alpha(1-\alpha)((t_A+t_B) + \frac{t_At_B}{(t_A-t_B)^2}\left(16(t_A+t_B) - 8\delta(6t_A+t_B) - 16\delta^2 t_A\right)).$$

The method of proof is the same as in Section 4 and the proof is therefore omitted.

This shows that the results are qualitatively similar to the results with maximal product differentiation. The profit function is non-monotonic in δ and negative correlation hurts firms. The only difference is that profits are lower if $\alpha < \frac{1}{2}$. This is a result one would expect. Since product differentiation is no longer maximal the degree of competition is higher and thus prices are lower.

Now let us turn to the location choice of firms. As is standard in the literature this is modelled in a two-stage-game. In the first stage location is chosen, in the second stage firms set prices after observing the location choices. To keep the model tractable we have to make two additional assumptions which are not very restrictive. The first is that in the first stage only firm 2 chooses its location α_A, α_B on both circles while firm 1′s location is fixed. This assumption is not crucial although it sounds asymmetric. The reason is that in a model on the circle there is no possibility for one firm to have a better position than the other one.[17] Even with the connection between the circles through the bundle there is no advantage for firm 2 and in equilibrium both firms earn the same profits. The second assumption is that firm 1 is still located at $(0,0)^T$. This assumption is a bit more restrictive because the equilibrium values would be different if the exogenous positions of firm 1 were different from each other.[18] Yet, the qualitative results would be the same; only the values of the equilibrium prices and profits would be different but the location choice of firm 2 in the first stage would be the same. To compare the results with the former analysis a location of firm 1 at $(0,0)^T$ is assumed.
The game is solved by backward induction. In the second stage optimal prices can be calculated given α_A, α_B and in the first stage firm 2 chooses α_A and α_B. This is done in the appendix.

Proposition 5
If $\delta \leq \frac{\sqrt{53t_A^2+26t_At_B+2t_B^2}}{4t_A} - \frac{3}{2} - \frac{t_B}{4t_A} = \delta'$ firm 2 chooses maximal product differentiation for both goods ($\alpha_A = \alpha_B = \frac{1}{2}$).

[17]This stands in contrast to competition on the line where such a modelling would give firm 2 a huge advantage.

[18]The exception is if the distance is $\frac{1}{2}$. This would yield the same results as an equal location on the circles.

If $\delta > \frac{\sqrt{53t_A^2+26t_At_B+2t_B^2}}{4t_A} - \frac{3}{2} - \frac{t_B}{4t_A} = \delta'$ firm 2 chooses maximal product differentiation on circle A ($\alpha_A = \frac{1}{2}$) and minimal product differentiation on circle B ($\alpha_B = 0$).

Thus if $\delta \leq \delta'$ there is maximal product differentiation on both circles. But if $\delta > \delta'$ we have a sudden shift to minimal differentiation on the circle with lower transportation costs. What is the intuition behind this result?
If δ is small the result is not surprising. With maximal product differentiation firms have high market power and competition is best reduced with a location which is most distant. If δ is high we know from Proposition 2 that competition is fierce. This is the case because from the point of view of the bundle consumers are nearly homogeneous if firms are maximally differentiated. With the same location on circle B firms avoid the additional competition resulting from this homogeneity. They make no longer profits with good B because $p_B^* = c_B$. But consumer homogeneity is reduced because on circle A each consumer has a strict preference for one firm. Thus the business stealing effect of bundling is reduced and each firm earns profits of $\Pi_i^* = \frac{1}{8}t_A$.
The threshold value $\delta' = \frac{\sqrt{53t_A^2+26t_At_B+2t_B^2}}{4t_A} - \frac{3}{2} - \frac{t_B}{4t_A}$ can be compared with the value of δ at which the profit with mixed bundling is lower than the profit without bundling. From Lemma 1 this value of δ is given by $\frac{\sqrt{52t_A^2+28t_At_B+t_B^2}}{4t_A} - \frac{3}{2} - \frac{t_B}{4t_A}$. Thus δ' is slightly above this value. The reason is that in choosing minimal differentiation on circle B firms forego all profits with good B. Firm 2 therefore chooses $\alpha_B = 0$ only when profits with maximal differentiation are lower than $\frac{1}{8}t_A$. But the profits without bundling are given by $\frac{1}{8}t_A + \frac{1}{8}t_B$. Thus δ' is higher.
With the location choice the firms change the correlation of values. They have to balance the effect of increasing competition because of smaller differentiation with the effect of increasing competition because of homogeneity of the bundle. If the latter effect is dominating firms choose minimal differentiation in one product.
The result can also compared with the result of Irmen & Thisse (1998). They analyze a model with one product where firms have to compete in multidimensional characteristics. Each characteristic is independent from each other. Irmen & Thisse (1998) find that firms choose maximal differentiation in the characteristic with the

highest salience coefficient and minimal differentiation in all others. The intuition is that price competition is relaxed with differentiation in one characteristic but firms enjoy the advantage of a central location in all others. The argument for minimal differentiation is quite different in my model where firms want to avoid additional competition on the bundle that would arise with differentiation.

7 Application

In this section an application of the model to US telephone companies is presented. In the US many of these companies sell internet access and long distance service together in one package. The price of this package is by far lower than if both services are bought independently.

Here I look at three companies, AT&T, birch telecom, and Verizon. Each of them offers such a package. The long distant service in each package is almost the same, so there are no essential differences in offers. But internet access is supplied quite differently in each bundle. AT&T offers only 20 hours per month but gives a free installation kit and free live support. By contrast, birch telecom offers unlimited access but gives only standard support and no gifts. Verizon offers also unlimited access and free live support but no installation kit. In addition, consumers can choose at Verizon if they want to buy DSL or wireless where wireless is a bit more expensive.

This fits the results of the model in the last section, maximal differentiation in one good and minimal in the other, quite well. It is empirically hard to estimate in which good firms are more differentiated, which is represented by higher transportation costs, but the example points to the fact that it is more important for consumers from which firm they get internet access than which one offers them long distance service.

8 Conclusion

This paper has shown that commodity bundling in duopoly has inherently different consequences than in the monopoly case. In duopoly there is a high incentive to bundle. But if the correlation of reservation values is negative, profits of the firms decrease through bundling. This is contrary to the monopoly case where bundling is particularly profitable if correlation is negative. The decrease in consumer heterogeneity which renders bundling profitable in monopoly creates a higher degree of competition in duopoly and lowers profits. Thus firms are in a prisoner's dilemma situation. It has also been shown that welfare decreases with bundling because of distributive inefficiency. If firms can choose their location and thus influence the correlation they want to avoid high negative correlation of reservation values and choose minimal product differentiation in one good.

An interesting way in which the model could be extended is to introduce uncertainty. I assumed a one-to-one mapping of consumer locations on both circles to get clear cut results. A possible way to introduce uncertainty might be to assume that a consumer's location on circle B conditional on her location on circle A is uniformly distributed between $x_A+\delta-\epsilon$ and $x_A+\delta+\epsilon$, with $\epsilon \in [0, 1/2]$. So an ϵ of zero is the model analyzed in this paper while $\epsilon = 1/2$ means that x_B is independent of x_A. My intuition is that if ϵ is small the qualitative results would not change because uncertainty is small. If instead ϵ is high one may get different results. So the model also offers a framework to deal with questions of uncertainty.

9 Appendix

9.1 Proof of Remark 1

The goal is to calculate the function $\rho[R_A, R_B](\delta) = \frac{\text{Cov}[R_A,R_B](\delta)}{\sigma(R_A)\sigma(R_B)}$. The proof is done from the perspective of firm 1 but we get the same result for firm 2 because of symmetry.

The gross utility from buying the good, $K_j, j = 1, 2$, is constant and the same for all consumers. It can thus be ignored in the calculation of $\sigma(R_A), \sigma(R_B)$ and $\text{Cov}(R_A, R_B)$.

First we calculate of $\sigma(R_A) = \int_0^1 t_A^2(\text{d}(x_A))^2 dx_A - \bar{\text{d}}_A^2$, where $\bar{\text{d}}_A$ is the expected value of the transportation costs. We start with calculating $\bar{\text{d}}_A$,

$$\bar{\text{d}}_A = t_A \int_0^{\frac{1}{2}} (x_A)^2 dx_A + t_A \int_{\frac{1}{2}}^1 (1 - x_A)^2 dx_A = \frac{1}{12} t_A.$$

Next, calculating $\int_0^1 t_A^2(\text{d}(x_A))^2 dx_A$ yields

$$\int_0^1 t_A^2(\text{d}(x_A))^2 dx_A = t_A^2 \int_0^{\frac{1}{2}} x_A^4 dx_A + t_A^2 \int_{\frac{1}{2}}^1 (1 - x_A)^4 dx_A = \frac{1}{80} t_A^2.$$

Thus

$$\sigma(R_A) = \frac{1}{80} t_A^2 - \frac{1}{144} t_A^2 = \frac{1}{180} t_A^2.$$

Turning to circle B, $\bar{\text{d}}_B$ is given by

$$\bar{\text{d}}_B = t_B \int_0^{\frac{1}{2}-\delta} (x_A+\delta)^2 dx_A + t_B \int_{\frac{1}{2}-\delta}^{1-\delta} (1-x_A-\delta)^2 dx_A + t_B \int_{1-\delta}^1 (x_A+\delta-1)^2 dx_A = \frac{1}{12} t_B.$$

Calculating $\sigma(R_B)$ gives

$$\begin{gathered}\sigma(R_B) = t_B^2 \int_0^{\frac{1}{2}-\delta} (x_A + \delta)^4 dx_A + t_B^2 \int_{\frac{1}{2}-\delta}^{1-\delta} (1 - x_A - \delta)^4 dx_A \\ + t_B^2 \int_{1-\delta}^1 (x_A + \delta - 1)^4 dx_A - (\tfrac{1}{12})^2 t_B^2 = \tfrac{1}{180} t_B^2.\end{gathered}$$

The covariance $\text{Cov}(R_A, R_B)$ is thus given by

$$\begin{aligned}\text{Cov}(R_A, R_B)(\delta) = &\textstyle\int_0^{\frac{1}{2}-\delta}(t_A x_A^2 - \frac{1}{12}t_A)(t_B(x_A+\delta)^2 - \frac{1}{12}t_B)dx_A \\ &+ \textstyle\int_{\frac{1}{2}-\delta}^{\frac{1}{2}}(t_A x_A^2 - \frac{1}{12}t_A)(t_B(1-x_A-\delta)^2 - \frac{1}{12}t_B)dx_A \\ &+ \textstyle\int_{\frac{1}{2}}^{1-\delta}(t_A(1-x_A)^2 - \frac{1}{12}t_A)(t_B(1-x_A-\delta)^2 - \frac{1}{12}t_B)dx_A \\ &+ \textstyle\int_{1-\delta}^{1}(t_A(1-x_A)^2 - \frac{1}{12}t_A)(t_B(x_A+\delta-1)^2 - \frac{1}{12}t_B)dx_A\end{aligned}$$

which after some manipulations yields

$$\text{Cov}(R_A, R_B)(\delta) = t_A t_B[\frac{1}{180} - \frac{1}{6}\delta^2 + \frac{1}{3}\delta^3 - \frac{1}{6}\delta^4].$$

Thus

$$\rho(R_A, R_B)(\delta) = 1 - 30\delta^2 + 60\delta^3 - 30\delta^4.$$

q.e.d.

9.2 Proof of Proposition 1

Consider the case where both firms do not bundle. Since the equilibrium is symmetric both firms charge the same independent prices, p_A^{ind} and p_B^{ind}, and earn profits of $\Pi_i^* = \frac{1}{2}(p_A^{ind} - c_A + p_B^{ind} - c_B)$.

Now let us look if there is an incentive for firm 1 to introduce a bundle, that means selling both goods together at a price $p_{AB}^1 < p_A^1 + p_B^1$. We analyze the case where $p_{AB}^1 = p_A^{ind} + p_B^{ind}$ and $p_j^1 = p_j^{ind} + \epsilon_1$, with $\epsilon_1 > 0$, but small. So firm 1 increases its independent prices by ϵ_1 and sets the bundle price equal to the sum of the prices if firms do not bundle.

We have to distinguish between two cases, either if δ is "near" $\frac{1}{2}$ or not, because this changes the demand structure on the circles. First look at the case where δ is not near $\frac{1}{2}$. If firms do not bundle there are four demand regions on the circles, namely $(AB1), (A1B2), (AB2)$ and $(A2B1)$. The frontiers between this regions (or the marginal consumers) are the following,

1. frontier between $(AB1)$ and $(A1B2)$: $\frac{1}{4} - \delta$,

2. frontier between $(A1B2)$ and $(AB2)$: $\frac{1}{4}$,

3. frontier between $(AB2)$ and $(A2B1)$: $\frac{3}{4} - \delta$,

4. frontier between $(A2B1)$ and $(AB1)$: $\frac{3}{4}$.

If firm 1 introduces the bundle the frontiers are changed to

1. frontier between $(AB1)$ and $(A1B2)$: $\frac{1}{4} - \delta + \frac{\epsilon_1}{t_B}$,

2. frontier between $(A1B2)$ and $(AB2)$: $\frac{1}{4} - \frac{\epsilon_1}{t_A}$,

3. frontier between $(AB2)$ and $(A2B1)$: $\frac{3}{4} - \delta + \frac{\epsilon_1}{t_B}$,

4. frontier between $(A2B1)$ and $(AB1)$: $\frac{3}{4} - \frac{\epsilon_1}{t_A}$.

The new profit function of firm 1 is

$$\begin{gathered}\Pi_1^{**} = (p_A^1 + p_B^1 - c_A - c_B)(\tfrac{1}{2} - \delta + \epsilon_1(\tfrac{1}{t_A} + \tfrac{1}{t_B})) \\ +(p_A^1 - c_A + \epsilon_1)(\delta - \epsilon_1(\tfrac{1}{t_A} + \tfrac{1}{t_B})) + (p_B^1 - c_B + \epsilon_1)(\delta - \epsilon_1(\tfrac{1}{t_A} + \tfrac{1}{t_B}))\end{gathered}$$

or

$$\begin{aligned}\Pi_1^{**} &= (p_A^1 - c_A + p_B^1 - c_B)\tfrac{1}{2} + 2\delta\epsilon_1 - 2(\epsilon_1)^2(\tfrac{1}{t_A} + \tfrac{1}{t_B}) \\ &= \Pi_1^* + 2\delta\epsilon_1 - 2(\epsilon_1)^2(\tfrac{1}{t_A} + \tfrac{1}{t_B}).\end{aligned}$$

This is always higher than the old profit Π_1^* as long as $\delta > 0$, because ϵ_1 can made arbitrary small and so $(\epsilon_1)^2$ tends faster to 0 then ϵ_1.
Up to now we have shown that firm 1 has an incentive to introduce a bundle. The question is now if firm 2 has an incentive to bundle if firm 1 is already bundling. The profit of firm 2 if firm 1 bundles while firm 2 not is given by

$$\begin{gathered}\Pi_2^* = (p_A^2 + p_B^2 - c_A - c_B)(\tfrac{1}{2} - \delta + \epsilon_1(\tfrac{1}{t_A} + \tfrac{1}{t_B})) \\ +(p_A^2 - c_A)(\delta - \epsilon_1(\tfrac{1}{t_A} + \tfrac{1}{t_B})) + (p_B^2 - c_B)(\delta - \epsilon_1(\tfrac{1}{t_A} + \tfrac{1}{t_B})) \\ = (p_A^2 + p_B^2 - c_A - c_B)\tfrac{1}{2}.\end{gathered}$$

If firm 2 chooses to bundle and set $p_{AB}^2 = p_A^{ind} + p_B^{ind}$ and $p_j^2 = p_j^{ind} + \epsilon_2$, with $\epsilon_2 > 0$, but small, the frontiers are given by

1. frontier between $(AB1)$ and $(A1B2)$: $\frac{1}{4} - \delta + \frac{\epsilon_1 + \epsilon_2}{t_B}$,

2. frontier between $(A1B2)$ and $(AB2)$: $\frac{1}{4} - \frac{\epsilon_1 + \epsilon_2}{t_A}$,

3. frontier between $(AB2)$ and $(A2B1)$: $\frac{3}{4} - \delta + \frac{\epsilon_1+\epsilon_2}{t_B}$,

4. frontier between $(A2B1)$ and $(AB1)$: $\frac{3}{4} - \frac{\epsilon_1+\epsilon_2}{t_A}$.

The new profit of firm 2 is then

$$\Pi_2^{**} = (p_A^2 + p_B^2 - c_A - c_B)(\tfrac{1}{2}\delta + (\epsilon_1+\epsilon_2)(\tfrac{1}{t_A} + \tfrac{1}{t_B}))$$
$$+(p_A^1 - c_A + \epsilon_2)(\delta - (\epsilon_1+\epsilon_2)(\tfrac{1}{t_A} + \tfrac{1}{t_B})) + (p_B^1 - c_B + \epsilon_2)(\delta - (\epsilon_1+\epsilon_2)(\tfrac{1}{t_A} + \tfrac{1}{t_B}))$$
$$= \Pi_2^* + 2\epsilon_2\delta - 2[(\epsilon_2)^2 + \epsilon_1\epsilon_2](\tfrac{1}{t_A} + \tfrac{1}{t_B}).$$

Thus for ϵ_1 and ϵ_2 small, bundling is profitable if $\delta > 0$ since $(\epsilon_2)^2$ and $\epsilon_1\epsilon_2$ tends faster to 0 then ϵ_2.

Now let us turn the case where δ is near $\frac{1}{2}$ and look if firm 1 has an incentive to introduce a bundle. The difference to the former analysis is that in the surrounding of $x_A = \frac{1}{4}$ there are now some consumers who buy $(AB1)$ because they have almost the same preferences for all combinations but the bundle has a lower price than all other combinations. Thus moving clockwise on circle A starting at point zero the product combination $(A1B2)$ is followed by $(AB1)$ and no one buys $(AB2)$. The frontiers are given by

1. frontier between $(A1B2)$ and $(AB1)$: $\frac{3}{4} - \delta - \frac{\epsilon_1}{t_B}$,

2. frontier between $(AB1)$ and $(A2B1)$: $\frac{1}{4} + \frac{\epsilon_1}{t_A}$,

3. frontier between $(A2B1)$ and $(AB1)$: $\frac{3}{4} - \frac{\epsilon_1}{t_A}$,

4. frontier between $(AB1)$ and $(A1B2)$: $\frac{5}{4} - \delta + \frac{\epsilon_1}{t_B}$.

The profit of firm 1 if it bundles is

$$\Pi_1^{**} = (p_A^1 + p_B^1 - c_A - c_B)(2\epsilon_1(\tfrac{1}{t_A} + \tfrac{1}{t_B}))$$
$$+(p_A^1 - c_A + \epsilon_1)(\tfrac{1}{2} - 2\epsilon_1(\tfrac{1}{t_B}) + (p_B^1 - c_B + \epsilon_1)(\tfrac{1}{2} - 2\epsilon_1(\tfrac{1}{t_A})$$

or

$$\Pi_1^{**} = \Pi_1^* + 2(p_A^1 - c_A)\frac{\epsilon_1}{t_A} + 2(p_B^1 - c_B)\frac{\epsilon_1}{t_B} + \epsilon_1 - 2\frac{(\epsilon_1)^2}{t_A} - 2\frac{(\epsilon_1)^2}{t_B}.$$

Thus Π_1^{**} is independent of δ and always greater than Π_1^* if ϵ_1 is small.

Let us now look at firm 2 if firm 1 is already bundling. If firm 2 chooses not to bundle its profit is

$$\Pi_2^* = (p_A^1 - c_A)(\frac{1}{2} - 2\epsilon_1(\frac{1}{t_A})) + (p_B^1 - c_B)(\frac{1}{2} - 2\epsilon_1(\frac{1}{t_B})).$$

If firm 2 introduces a bundle itself the region where consumers buy that bundle returns and frontiers are given by

1. frontier between $(A1B2)$ and $(AB1)$: $\frac{3}{4} - \delta - \frac{\epsilon_1+\epsilon_2}{t_B}$,
2. frontier between $(AB1)$ and $(AB2)$: $\frac{1}{t_A-t_B}(\frac{1}{4}t_A - \frac{3}{4}t_B + \delta t_B)$,
3. frontier between $(AB2)$ and $(A2B1)$: $\frac{3}{4} - \delta + \frac{\epsilon_1+\epsilon_2}{t_B}$,
4. frontier between $(A2B1)$ and $(AB2)$: $\frac{5}{4} - \delta - \frac{\epsilon_1+\epsilon_2}{t_B}$,
5. frontier between $(AB2)$ and $(AB1)$: $\frac{1}{t_A-t_B}(\frac{3}{4}t_A - \frac{5}{4}t_B + \delta t_B)$,
6. frontier between $(AB1)$ and $(A1B2)$: $\frac{5}{4}\delta + \frac{\epsilon_1+\epsilon_2}{t_B}$.

Profit of firm 2 if both firms bundle is then

$$\begin{aligned}\Pi_2^{**} = (p_A^2 + p_B^2 - c_A - c_B)(2(\tfrac{\epsilon_1+\epsilon_2}{t_B}) - \tfrac{1}{2} + \tfrac{1}{t_A-t_B}(\tfrac{3}{4}t_A - \tfrac{5}{4}t_B - \tfrac{1}{4}t_A + \tfrac{3}{4}t_B)\\ +(p_A^2 - c_A + \epsilon_2)(\tfrac{1}{2} - 2(\epsilon_1+\epsilon_2)(\tfrac{1}{t_B}) + (p_B^2 - c_B + \epsilon_2)(\tfrac{1}{2} - 2(\epsilon_1+\epsilon_2)(\tfrac{1}{t_B})\\ = \Pi_2^* + \epsilon_2 - 4[\tfrac{(\epsilon_2)^2+\epsilon_1\epsilon_2}{t_B}] + 2(p_A^2 + p_B^2 - c_A - c_B)\tfrac{\epsilon_1}{t_B}.\end{aligned}$$

If ϵ_1 and ϵ_2 are small $\Pi_2^{**} > \Pi_2^*$, so firm 2 also has an incentive to bundle.

q.e.d.

9.3 Proof of Proposition 2

Before proving Proposition 2 we have to establish several claims:

Claim 1

There cannot exist direct rivalry between product combination $(A1B2)$ and $(A2B1)$.

Proof:

Assume that the consumer on x_A with x_A between 0 and $\frac{1}{2} - \delta$ is the marginal consumer between product combination $(A1B2)$ and $(A2B1)$ and she buys either of these alternatives. Thus $(A2B1)$ must be better for her then $(AB2)$. This is only the case if

$$p_A^2 + p_B^1 + t_B(x_A + \delta)^2 \leq p_{AB}^2 + t_B\left(\frac{1}{2} - x_A - \delta\right)^2 \tag{10}$$

or

$$p_A^2 + p_B^1 \leq p_{AB}^2 + \frac{1}{4}t_B x_A t_B - \delta t_B. \tag{11}$$

Since in equilibrium both firms bundle we know that $p_{AB}^2 < p_A^2 + p_B^2$. Thus we can write $p_A^2 + p_B^2 - \kappa$ with $\kappa > 0$ instead of p_{AB}^2. Then from (11) we get

$$p_B^1 \leq p_B^2 - \kappa + \frac{1}{4}t_B - x_A t_B - \delta t_B. \tag{12}$$

For the consumer indifferent between (A1B2) and (A2B1) it must also be optimal to buy $(A1B2)$ instead of $(AB2)$. This is only the case if (knowing that $p_{AB}^1 = p_A^1 + p_B^1 - \lambda$ with $\lambda > 0$)

$$p_A^1 + p_B^1 - \lambda + t_B\left(x_A + \delta\right)^2 \geq p_A^1 + p_B^2 + t_B\left(\frac{1}{2} - x_A - \delta\right)^2.$$

or

$$p_B^1 - \lambda \geq p_B^2 + \frac{1}{4}t_B - x_A t_B - \delta t_B.$$

But this is a contradiction to (12) because $\kappa, \lambda > 0$. Therefore it cannot be optimal for a consumer at x_A to buy $(A1B2)$.

One can show that the same holds for x_A between $\frac{1}{2} - \delta$ and $\frac{1}{2}$. Because of symmetry a similar condition holds on the second half of the circle.

q.e.d.

Claim 2

(i) Take x_A and x'_A with $0 \leq x_A, x'_A \leq \frac{1}{2}$ and $x'_A < x_A$.
If $(AB1)$ is optimal at x_A then at x'_A $(AB2)$ can never be optimal.

(ii) Take x_A and x'_A with $\frac{1}{2} \leq x_A, x'_A \leq 1$ and $x'_A < x_A$.
If $(AB2)$ is optimal at x_A then at x'_A $(AB1)$ can never be optimal.

Proof:

Assume that x_A lies between 0 and $\frac{1}{2} - \delta$. At x_A we have

$$p^1_{AB} + t_A(x_A)^2 + t_B(x_A + \delta)^2 \leq p^2_{AB} + t_A\left(\frac{1}{2} - x_A\right)^2 + t_B\left(\frac{1}{2} - x_A - \delta\right)^2$$

and therefore

$$(t_A + t_B)x_A \leq p^2_{AB} - p^1_{AB} - t_B\delta + \frac{1}{4}(t_A + t_B).$$

If $(AB2)$ were optimal at x'_A then

$$(t_A + t_B)x'_A \geq p^2_{AB} - p^1_{AB} - t_B\delta + \frac{1}{4}(t_A + t_B).$$

But since $x'_A < x_A$ this cannot be the case.
One gets a similar condition for $\frac{1}{2} - \delta \leq x_A \leq \frac{1}{2}$. If $\frac{1}{2} \leq x_A, x'_A \leq 1$ the method of proof is exactly similar only with $(AB1)$ and $(AB2)$ reversed.

q.e.d.

Claim 3

(i) Take x_A and x'_A with $0 \leq x_A, x'_A \leq \frac{1}{2}$ and $x'_A < x_A$.
If $(A1B2)$ is optimal at x_A then at x'_A $(A2B1)$ can never be optimal.

(ii) Take x_A and x'_A with $\frac{1}{2} \leq x_A, x'_A \leq 1$ and $x'_A < x_A$.
If $(A2B1)$ is optimal at x_A then at x'_A $(A1B2)$ can never be optimal.

Proof:

Assume that x_A lies between 0 and $\frac{1}{2} - \delta$. At x_A we have

$$p^1_A + p^2_B + t_A(x_A)^2 + t_B\left(\frac{1}{2} - x_A - \delta\right)^2 \leq p^1_B + p^2_A + t_A\left(\frac{1}{2} - x_A\right)^2 + t_B(x_A + \delta)^2$$

and therefore

$$(t_A - t_B)x_A \leq p_B^1 + p_A^2 - p_A^1 - p_B^2 + t_B\delta + \frac{1}{4}(t_A - t_B).$$

If $(A2B1)$ were optimal at x'_A then

$$(t_A - t_B)x'_A \geq p_B^1 + p_A^2 - p_A^1 - p_B^2 + t_B\delta + \frac{1}{4}(t_A - t_B).$$

But since $x'_A < x_A$ this cannot be the case.

One gets a similar condition for $\frac{1}{2} - \delta \leq x_A \leq \frac{1}{2}$.

If $\frac{1}{2} \leq x_A, x'_A \leq 1$ the method of proof is exactly similar only with $(A1B2)$ and $(A2B1)$ reversed.

q.e.d.

Claim 4

(i) Take x_A and x'_A with $0 \leq x_A, x'_A \leq \frac{1}{2}$ and $x'_A < x_A$.
If $(AB1)$ is optimal at x_A then at x'_A $(A2B1)$ can never be optimal.

(ii) Take x_A and x'_A with $\frac{1}{2} \leq x_A, x'_A \leq 1$ and $x'_A < x_A$.
If $(A2B1)$ is optimal at x_A then at x'_A $(AB1)$ can never be optimal.

Proof:

Assume that x_A lies between 0 and $\frac{1}{2} - \delta$. At x_A we have

$$p_A^1 + p_B^1 - \lambda + t_A(x_A)^2 + t_B(x_A + \delta)^2 \leq p_B^1 + p_A^2 + t_A\left(\frac{1}{2} - x_A\right)^2 + t_B(x_A + \delta)^2$$

and therefore

$$t_A x_A \leq p_A^2 - p_A^1 + \lambda + \frac{1}{4}t_A. \tag{13}$$

If $(A2B1)$ were better than $(AB1)$ at x'_A then we would have

$$t_A x'_A \geq p_A^2 - p_A^1 + \lambda + \frac{1}{4}t_A.$$

But since $x'_A < x_A$ this is a contradiction to (9.3).

One gets a similar condition for $\frac{1}{2} - \delta \leq x_A \leq \frac{1}{2}$.

If $\frac{1}{2} \leq x_A, x'_A \leq 1$ the method of proof is exactly similar only with $(AB1)$ and $(A2B1)$ reversed.

q.e.d.

Claim 5

(i) Take x_A and x'_A with $0 \leq x_A, x'_A \leq \frac{1}{2}$ and $x'_A < x_A$.
If $(A1B2)$ is optimal at x_A then at x'_A $(AB2)$ can never be optimal.

(ii) Take x_A and x'_A with $\frac{1}{2} \leq x_A, x'_A \leq 1$ and $x'_A < x_A$.
If $(AB2)$ is optimal at x_A then at x'_A $(A1B2)$ can never be optimal.

Proof:
Assume that x_A lies between 0 and $\frac{1}{2} - \delta$. At x_A we have

$$p_A^1 + p_B^2 + t_A(x_A)^2 + t_B\left(\frac{1}{2} - x_A - \delta\right)^2 \leq p_A^2 + p_B^2 - \kappa + t_A\left(\frac{1}{2} - x_A\right)^2 + t_B\left(\frac{1}{2} - x_A\delta\right)^2$$

and therefore

$$t_A x_A \leq p_B^2 - p_A^1 - \kappa + \frac{1}{4}t_A. \tag{14}$$

If $(AB2)$ were optimal at x'_A then we would have

$$t_A x'_A \geq p_B^2 - p_A^1 + \kappa + \frac{1}{4}t_A.$$

But since $x'_A < x_A$ this is not possible.
One gets a similar condition for $\frac{1}{2} - \delta \leq x_A \leq \frac{1}{2}$.
If $\frac{1}{2} \leq x_A, x'_A \leq 1$ the method of proof is exactly similar only with $(A1B2)$ and $(AB2)$ reversed.

q.e.d.

As a result in equilibrium there can only be three possible demand structures on the circle A.[19]

[19] This means that e.g. at demand structure (i) at point zero we have product combination $(AB1)$ followed clockwise by product combination $(A1B2)$ which in turn is follwed by $(AB2)$. $(AB2)$ is followed by $(A2B1)$ and arriving at point 1 we again have $(AB1)$.

(i) $(AB1), (A1B2), (AB2), (A2B1), (AB1)$

(ii) $(A1B2), (AB2), (A2B1), (AB1), (A1B2)$

(iii) $(A1B2), (AB1), (AB2), (A2B1), (AB2), (AB1), (A1B2)$

Calculating the profit function for each demand structure we get profit function (2.2) for demand structures (i) and (ii) and profit function (4) for demand structure (iii). Maximizing each profit function with respect to p_{AB}^1, p_A^1 and p_B^1 yields equation (3) for profit function (2) and equation (5) for profit function (4).

It remains to calculate for which values of δ the profit functions are valid.
For profit function (2) to arise (A1B2) must be followed by (AB2) and not by (AB1). The frontier between (A1B2) and (AB2) at the equilibrium prices is given by

$$x_A = \frac{1}{4} - \frac{2}{3}\delta\frac{t_B}{t_A + t_B}. \tag{15}$$

The frontier between (A1B2) and (AB1) at the equilibrium prices is given by

$$x_A = \frac{3}{4} - \delta\frac{5t_A + 3t_B}{3(t_A + t_B)}. \tag{16}$$

For demand structure (i) or (ii) to arise (15) must be smaller than (16). This gives the first threshold

$$\delta_1^{TS} = \frac{3}{2}(\frac{t_A + t_B}{5t_A + t_B}).$$

For profit function (4) to arise (A1B2) must be followed by (AB1) and not by (AB2). Calculating in the same way as before by inserting the equilibrium prices of profit function (4) gives that demand structure (iii) arises only if

$$\delta_2^{TS} > \frac{1}{3} + \frac{t_B}{6t_A}.$$

This gives the second threshold.
In the region in between $\frac{3}{2}\frac{t_A+t_B}{5t_A+t_B} < \delta \leq \frac{1}{3} + \frac{t_B}{6t_A}$ firms set their prices in such a way that demand structure (ii) arises. Routine manipulations show that equilibrium

prices and profits are given by (6). They exactly satisfy the constraint that

$$\frac{1}{4} + \frac{p_{AB}^2 - p_A^1 - p_B^2}{t_A} \geq \frac{3}{4} - \delta + \frac{p_{AB}^1 - p_A^1 - p_B^2}{t_B},$$

which says that (A1B2) is followed by (AB1) and not (AB2).
This completes the proof.

q.e.d.

9.4 Proof of Proposition 3

Welfare is calculated by inserting the equilibrium prices in the formulas for the frontiers of each product combination and calculating the resulting transportation costs on each circle. If $\delta < \frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B})$ welfare is given by

$$\begin{aligned} WF = \;& K_A + K_B - c_A - c_B \\ & -t_A\{ \int_0^{\frac{1}{4}-\frac{2}{3}\delta\frac{t_B}{t_A+t_B}} (x)^2 dx + \int_{\frac{1}{4}-\frac{2}{3}\delta\frac{t_B}{t_A+t_B}}^{\frac{1}{2}} (\tfrac{1}{2}-x)^2 dx \\ & \int_{\frac{1}{2}}^{\frac{3}{4}-\frac{2}{3}\delta\frac{t_B}{t_A+t_B}} (x-\tfrac{1}{2})^2 dx + \int_{\frac{3}{4}-\frac{2}{3}\delta\frac{t_B}{t_A+t_B}}^{1} (1-x)^2 dx\} \\ & -t_B\{ \int_0^{\frac{1}{4}+\frac{2}{3}\delta\frac{t_A}{t_A+t_B}} (x)^2 dx + \int_{\frac{1}{4}+\frac{2}{3}\delta\frac{t_A}{t_A+t_B}}^{\frac{1}{2}} (\tfrac{1}{2}-x)^2 dx \\ & \int_{\frac{1}{2}}^{\frac{3}{4}+\frac{2}{3}\delta\frac{t_A}{t_A+t_B}} (x-\tfrac{1}{2})^2 dx + \int_{\frac{3}{4}+\frac{2}{3}\delta\frac{t_A}{t_A+t_B}}^{1} (1-x) dx\}, \end{aligned}$$

which after some manipulations yields

$$WF = K_A + K_B - c_A - c_B - \frac{1}{48}(t_A + t_B) - \frac{4}{9}\delta^2 \frac{t_A t_B}{t_A + t_B}$$

which is equation (7).
Welfare is calculated in the same way if $\frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B}) < \delta \leq \frac{1}{3} + \frac{t_B}{6t_A}$ and if $\delta > \frac{1}{3} + \frac{t_B}{6t_A}$ which gives equations (8) in the first case and (9) in the second.

q.e.d.

9.5 Proof of Proposition 5

Calculating prices and profits for arbitrary values of α_A and α_B is done in the standard way. This yields profits of

$$\Pi^\star = \frac{1}{2}\alpha_A(1-\alpha_A)t_A + \frac{1}{2}\alpha_B(1-\alpha_B)t_B + \frac{16}{9}\delta^2\frac{\alpha_A(1-\alpha_A)t_A\alpha_B(1-\alpha_B)t_B}{\alpha_A(1-\alpha_A)t_A + \alpha_B(1-\alpha_B)t_B},$$

if $\delta \leq \frac{3(t_A\alpha_A(1-\alpha_A)+t_B\alpha_B(1-\alpha_B))}{2\alpha_B(1-\alpha_A)[6t_A\alpha_A-2t_B(1-\alpha_B+8(t_A+t_B)\alpha_A(1-\alpha_B))]}$
Differentiating $\Pi^\star$ with respect to α_A and α_B yields that profit is maximal if $\alpha_A = \alpha_B = \frac{1}{2}$.

If $\delta > \frac{1}{2}(1-\alpha_A) + \alpha_B\left(\frac{1}{6}+\frac{t_B}{3t_A}\right)$ profits are given by

$$\Pi^\star = \frac{1}{2}t_A\alpha_A(1-\alpha_A) - \frac{7}{18}t_B\alpha_B(1-\alpha_B).$$

Differentiating this profit with respect to α_A and α_B yields that profit is maximal if $\alpha_A = \frac{1}{2}$ and $\alpha_B = 0$ since α_B can only be between 0 and $\frac{1}{2}$.

If $\delta \leq \frac{3(t_A\alpha_A(1-\alpha_A)+t_B\alpha_B(1-\alpha_B))}{2\alpha_B(1-\alpha_A)[6t_A\alpha_A-2t_B(1-\alpha_B+8(t_A+t_B)\alpha_A(1-\alpha_B))]} < \delta \leq \frac{1}{2}(1-\alpha_A) + \alpha_B\left(\frac{1}{6}+\frac{t_B}{3t_A}\right)$ profits are given by

$$\Pi^\star = \tfrac{1}{2}((\alpha_A(1-\alpha_A)t_A + (\alpha_B(1-\alpha_B)t_B)) +$$
$$\tfrac{t_At_B}{(t_A-t_B)^2}\left(64\alpha_A(1-\alpha_B)(t_A+t_B) - 4\alpha_B(1-\alpha_A)\delta(6t_A+t_B) - 8\alpha_A(1-\alpha_A)\delta^2 t_A\right).$$

Differentiating this profit with respect to α_A yields that profit is always maximal if $\alpha_A = \frac{1}{2}$. Differentiating with respect to α_B yields that $\alpha_B = \frac{1}{2}$ if $\delta \leq \frac{\sqrt{53t_A^2+26t_At_B+2t_B^2}}{2t_A} - \frac{3}{2} - \frac{t_B}{4t_A}$ and $\alpha_B = 0$ if $\delta > \frac{\sqrt{53t_A^2+26t_At_B+2t_B^2}}{2t_A} - \frac{3}{2} - \frac{t_B}{4t_A}$.

q.e.d.

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