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Integrated Strategy for Generating Permutation

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Abstract

An integrated strategy for generating permutation is presented in this paper. This strategy involves exchanging two consecutive elements to generate the starter sets and then applying circular and reversing operations to list all permutations. Some theoretical works are also presented.

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1 Introduction

Permutation can be generated under several operations such as cycling, transposition, and exchanging [3]. Ibrahim *et. al*,(2010) introduced a new strategy to generate permutation generation based on starter sets by employing circular and reversing operations. This strategy, however produces an equivalence starter sets which will enumerate the similar permutations and need to be discarded. This process becomes tedious when the number of element increases as we need to search all $(n-1)!$ starter sets. To overcome this shortcoming, Karim *et. al*,(2010) proposed a new strategy for generating starter sets without eliminating the equivalence starter sets.

This paper, we integrate two strategies for permutation generation. We employ exchanging process for generating starter sets, and then the circular and reversing operations process will be employed on the starter sets to generate all permutations.

2 Preliminary Definition

The following definitions will be used throughout this paper.

Definition 2.1. *A starter set is a set that is used as a basis to enumerate other permutations.*

Definition 2.2. *The circular permutation process over k elements is the process where the k elements of permutation are rotated*

Definition 2.3. *The reverse set is a set that is produced by reversing the order of permutation set.*

Definition 2.4. *A latin square of order n is an $n \times n$ array in which n distinct symbols are arranged where each element occurs once in each row and column*

Definition 2.5. *The circular permutation of order n (CP) is a latin square of order n which obtained by employing the circular process over all elements*

Definition 2.6. *The reverse of circular permutation (RoCP) is also a latin square of order n which is obtained by reversing arrangement element in each row of circular permutation.*

3 Permutation Generation Algorithm

The general algorithm for permutation generation by employing CP and $RoCP$ on starter sets which generated by exchanging two consecutive elements as follows:

Step 1 : Let $\{1, 2, 3, 4, \ldots, k, k+1, \ldots, n-2, n-1, n\}$ as initial permutation and without loss of generality, the first elements is fixed.

Step 2 : Identify the elements in the $(n-2)$th position of the initial permutation in step 1. Exchange this element until it reaches the nth (last) position. Hereby three distinct starter sets are obtained

Step 3 : Identify the elements in the $(n-3)$th position of the initial permutation in step 2. Exchange this element until it reaches the n th (last) position. Hereby 12 distinct starter sets are obtained

$\vdots$

Step $n-2$: Identify the elements of in the 2nd position of each starter sets in step (n-3). Exchange this element until it reaches the n th (last) position At this step, the $\frac{(n-1)!}{2}$ distinct starter sets are obtained.

Step $n-1$: Perform CP and RoCP simultaneously to all n elements of $\frac{(n-1)!}{2}$ distinct starter sets and $n!$ distinct permutations are obtained.

Step n : Display all $n!$ permutations.

Step 1 until $n-2$, is a process for starter sets generation and the rest is to list all $n!$ permutations.

In order to illustrate the general algorithm, we present the starter sets generation and listing all permutation for case $n = 5$.

Example 3.1. *Let* $n = 5$ *and* $S = \{1, 2, 3, 4, 5\}$.

Step 1 : Suppose $\{1, 2, 3, 4, 5\}$ *as initial permutation and without loss of generality, the first elements is fixed.*

Step 2 : Identify the elements in the (n−2)th position of the initial permutation in Step 1 *i.e* 3. *Exchange this element until it reaches the* 5*th (last) position. Hereby three distinct starter sets are obtained.*

$$\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 4 & 3 & 5 \\ 1 & 2 & 4 & 5 & 3 \end{array}$$

Step 3 : Identify the elements in the (n−3)th position of the initial permutation in Step 2 i.e 2. *Exchange this element until it reaches the 5th (last) position. Hereby 12 distinct starter sets are obtained.*

$$\begin{array}{ccccccccccccccc} 1 & 2 & 3 & 4 & 5 & & 1 & 2 & 4 & 3 & 5 & & 1 & 2 & 4 & 5 & 3 \\ 1 & 3 & 2 & 4 & 5 & & 1 & 4 & 2 & 3 & 5 & & 1 & 4 & 2 & 5 & 3 \\ 1 & 3 & 4 & 2 & 5 & & 1 & 4 & 3 & 2 & 5 & & 1 & 4 & 5 & 2 & 3 \\ 1 & 3 & 4 & 5 & 2 & & 1 & 4 & 3 & 5 & 2 & & 1 & 4 & 5 & 3 & 2 \end{array}$$

After process of starter sets generation completed, they will be exploited to list down all n! permutation by employing CP and RoCP

Table 1: List of 5! distinct permutations

1	**2**	**3**	**4**	**5**	5	4	3	2	1
2	3	4	5	1	1	5	4	3	2
3	4	5	1	2	2	1	5	4	3
4	5	1	2	3	3	2	1	5	4
5	1	2	3	4	4	3	2	1	5
1	**3**	**2**	**4**	**5**	5	4	2	3	1
3	2	4	5	1	1	5	4	2	3
2	4	5	1	3	3	1	5	4	2
4	5	1	3	2	2	3	1	5	4
5	1	3	2	4	4	2	3	1	5
1	**3**	**4**	**2**	**5**	5	2	4	3	1
3	4	2	5	1	1	5	2	4	3
4	2	5	1	3	3	1	5	2	4
2	5	1	3	4	4	3	1	5	2
5	1	3	4	2	2	4	3	1	5
1	**3**	**4**	**5**	**2**	2	5	4	3	1
3	4	5	2	1	1	2	5	4	3
4	5	2	1	3	3	1	2	5	4
5	2	1	3	4	4	3	1	2	5
2	1	3	4	5	5	4	3	1	2
1	**2**	**4**	**3**	**5**	5	3	4	2	1
2	4	3	5	1	1	5	3	4	2
4	3	5	1	2	2	1	5	3	4
3	5	1	2	4	4	2	1	5	3
5	1	2	4	3	3	4	2	1	5
1	**4**	**2**	**3**	**5**	5	3	2	4	1
4	2	3	5	1	1	5	3	2	4
2	3	5	1	4	4	1	5	3	2
3	5	1	4	2	2	4	1	5	3
5	1	4	2	3	3	2	4	1	5
1	**4**	**3**	**2**	**5**	5	2	3	4	1
4	3	2	5	1	1	5	2	3	4
3	2	5	1	4	4	1	5	2	3
2	5	1	4	3	3	4	1	5	2
5	1	4	3	2	2	3	4	1	5
1	**4**	**3**	**5**	**2**	2	5	3	4	1
4	3	5	2	1	1	2	5	3	4
3	5	2	1	4	4	1	2	5	3
5	2	1	4	3	3	4	1	2	5
2	1	4	3	5	5	3	4	1	2
column A					**column B**				

1	**2**	**4**	**5**	**3**	3	5	4	2	1
2	4	5	3	1	1	3	5	4	2
4	5	3	1	2	2	1	3	5	4
5	3	1	2	4	4	2	1	3	5
3	1	2	4	5	5	4	2	1	3
1	**4**	**2**	**5**	**3**	3	5	2	4	1
4	2	5	3	1	1	3	5	2	4
2	5	3	1	4	4	1	3	5	2
5	3	1	4	2	2	4	1	3	5
3	1	4	2	5	5	2	4	1	3
1	**4**	**5**	**2**	**3**	3	2	5	4	1
4	5	2	3	1	1	3	2	5	4
5	2	3	1	4	4	1	3	2	5
2	3	1	4	5	5	4	1	3	2
3	1	4	5	2	2	5	4	1	3
1	**4**	**5**	**3**	**2**	2	3	5	4	1
4	5	3	2	1	1	2	3	5	4
5	3	2	1	4	4	1	2	3	5
3	2	1	4	5	5	4	1	2	3
2	1	4	5	3	3	5	4	1	2
column A					**column B**				

As shown in Table 1, all $5!$ *permutation are listed down after performing* CP *and* $RoCP$ *over 12 starter sets.*

Remark 3.2. *The* ***bold*** *of the permutation in* ***column A*** *represents the starter sets.*

4 Some Theoretical Results

The following lemmas and theorem are produced from the recursive permutation generation.

Lemma 4.1. *There are* $\frac{(n-1)!}{2}$ *distinct starter sets which generated recursively for* $n \geq 3$ *under exchange two elements.*

Proof. Suppose we have $\{1, 2, 3, ..., n-3, n-2, n-1, n\}$ as initial starter for any $n \geq 3$. The first element will be selected from $(n-2)$th position ie. element $n-2$. Then by moving that element to the right until it reaches nth position, three distinct starters are produced, as shown below:

1	2	3	...	$n-3$	**n-2**	$n-1$	n	(starter 1)
1	2	3	...	$n-3$	$n-1$	**n-2**	n	(starter 2)
1	2	3	...	$n-3$	$n-1$	n	**n-2**	(starter 3)

Then for each previous starter sets, element in $(n-3)$th will be selected i.e element $n-3$. Then by moving that element to the right until it reaches nth position from each previous starter sets, four distinct starters are produced, as shown below:

From starter 1:	**1**	**2**	**3**	...	**n-3**	**n-2**	**n-1**	**n**
	1	2	3	...	$n-2$	**n-3**	$n-1$	n
	1	2	3	...	$n-2$	$n-1$	**n-3**	n
	1	2	3	...	$n-2$	$n-1$	n	**n-3**
From starter 2:	**1**	**2**	**3**	...	**n-3**	**n-1**	**n-2**	**n**
	1	2	3	...	$n-1$	**n-3**	$n-2$	n
	1	2	3	...	$n-1$	$n-2$	**n-3**	n
	1	2	3	...	$n-1$	$n-2$	n	**n-3**
From starter 3:	**1**	**2**	**3**	...	**n-3**	**n-1**	**n**	**n-2**
	1	2	3	...	$n-1$	**n-3**	n	$n-2$
	1	2	3	...	$n-1$	n	**n-3**	$n-2$
	1	2	3	...	$n-1$	n	$n-2$	**n-3**

Thus at this stage, the total starter sets are $3 \times 4 = 12$. We continue doing the processes recursively until $2nd$ position is reached.

$$\begin{array}{ccc}
(n-2)th \text{ position} & \Rightarrow & 3 \text{ starter sets} \\
(n-3)th \text{ position} & \Rightarrow & 4 \text{ starter sets} \\
(n-4)th \text{ position} & \Rightarrow & 5 \text{ starter sets} \\
(n-5)th \text{ position} & \Rightarrow & 6 \text{ starter sets} \\
\vdots & & \vdots \\
(n-i+1)th \text{ position} & \Rightarrow & i \text{ starter sets} \\
(n-i)th \text{ position} & \Rightarrow & i+1 \text{ starter sets} \\
(n-i-1)th \text{ position} & \Rightarrow & i+2 \text{ starter sets} \\
\vdots & & \vdots \\
3rd \text{ position} & \Rightarrow & n-2 \text{ starter sets} \\
2nd \text{ position} & \Rightarrow & n-1 \text{ starter sets}
\end{array}$$

By product rule, we produce

$$(3 \times 4 \times ... \times n-1) = \frac{1 \times 2}{2} \times (3 \times 4 \times ... \times n-1)) = \frac{(n-1)!}{2} \text{distinct starter sets}$$

$\square$

Remark 4.2. *For case $n = 2$ is impossible since it has only one distinct starter set while $\frac{(2-1)!}{2} = \frac{1}{2}$.*

Lemma 4.3. *$2n$ distinct permutations are produced by each distinct starters set by performing CP and RoCP.*

Proof. Suppose we have starter set of A,$\{1, 2, ..., n-1, n\}$ with n distinct elements. By using circular permutation (CP) process where the all element is rotated to the left.

$$\begin{array}{cccccc}
1 & 2 & 3 & ... & n-1 & n \\
2 & 3 & 4 & ... & n & 1 \\
3 & 4 & 5 & ... & 1 & 2 \\
4 & 5\dots & \dots & \dots & 3 & \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
n & 1 & \dots & \dots & n-2 & n-1
\end{array}$$

Thus n distinct permutations are produced. Reversing each row of CP will also produced other n distinct permutations as follows:

$$\begin{array}{cccccc}
n & n-1 & ... & 3 & 2 & 1 \\
1 & n & ... & 4 & 3 & 2 \\
2 & 1 & ... & 5 & 4 & 3 \\
3 & \dots & \dots & \dots & 5 & 4 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
n-1 & n-2 & \dots & \dots & 1 & n
\end{array}$$

We obtained other n distinct permutations. From CP and RoCP process, $2n$ distinct permutations are produced. □

Theorem 4.4. *The generation of all $n!$ distinct permutation can be obtained by $\frac{(n-1)!}{2}$ distinct starter sets.*

Proof. From Lemma 4.1, there are $\frac{(n-1)!}{2}$ distinct starter sets are produced by employing exchanging two consecutive elements . Then from Lemma 4.3, $2n$ distinct permutation are obtained by employ circular and reversing process on the starter sets.
Thus $\frac{(n-1)!}{2} \times 2n = n!$ permutations are generated. □

5 Conclusion

Enhancing exchanging two consecutive elements strategy for starter sets generation is highlighted and supported by some theoretical results. Exploiting distinct starter sets avoiding redundancy. Since starter sets were used for listing down all permutations, decomposition of starter sets generation task for parallel implementation is one of our future works.

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References

[1] H. Ibrahim, Z. Omar, and A. M. Rohni, New algorithm for listing all permutations. *Modern Applied Science,*(2010) 89-94.

[2] S. Karim, Z. Omar, and H. Ibrahim, K.I. Othman, and M. Suleiman, New recursive circular algorithm for listing all permutations,(2010)(paper submitted for Pertanika Journal).

[3] R. Sedgewick, Permutation generation methods, *Computing Surveys*, **9(2)** (1977), 137-164.

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