An Efficient Decomposition Approach for Surgical Planning

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1. Introduction and problem description

The operating theater (OT), consisting of several *operating rooms* (ORs), is one of the most critical resources in a hospital because it has a strong impact on the quality of health service and represents one of the main sources of costs (surgical teams, equipment etc.). A growing literature exists on methods, models and algorithms for carefully planning elective surgeries over time.
Given the patients' waiting list and various information on OT characteristics and status, OT planning problems consist in deciding the time schedule of surgeries in a given time horizon, with the aim of optimizing several performance measures such as operating room utilization, throughput, surgeons' overtime, lateness etc.

This study focuses on the problem of allocating elective surgeries to operating rooms over a given time horizon (e.g., one week) covering the problems denoted as MSSP and SCAP. The Master Surgical Scheduling Problem (MSSP), is the problem to decide the surgical discipline that will be performed in each OR and its output as the *Master Surgical Schedule* (MSS). While the and is therefore denoted as Surgical Case Assignment Problem (SCAP) and outputs the *Surgical Case Assignment* (SCA). Solving the SCAP consists in selecting elective surgeries to be performed in each OR session.

Literature on all these problems is wide and growing, and it has thoroughly been reviewed by several researchers e.g., (Cardoen et al 2010), (Guerriero et al. 2011), (Testi et al. 2008), (Sier et al. 1997) among others.

The objective is to determine MSS and SCA in order to maximize the score of the cases selected for the next time horizon. While finding the optimal solution to this integrated MSSP+SCAP problem may be time-consuming and may require significant computational resources, we will show that an efficient, though rather sophisticated, decomposition approach produces very good solutions in a fraction of computational time. This is especially true when considering medium-to-large operating theaters, since computational times of integrated models may grow very fast.

All elective surgeries are grouped into *surgical disciplines* (e.g., orthopedics, day surgery). The main input to the overall planning problem is the *waiting list* of each discipline, containing all the case surgeries that currently need to be performed. For each case surgery, the following information is specified in the waiting list:

- *Processing time*. Expected duration of the surgery (including setup times due to cleaning and OR preparation for the next surgery). We assume all these times to be deterministic; i.e., they are not affected by uncertainty.
- *Due date*. Nominal time within which the surgery should be performed. It is set on the basis of the entrance time and the priority class.

Cases are carried out in *OR sessions* each assigned to a single surgical discipline. These are of three *types*, lasting either half a day (morning and afternoon sessions) or the whole day (full-day session). Therefore during one day, an OR can be either assigned one morning session and one afternoon session (for two different disciplines), or a single full-day session. All sessions of the same type have the same duration, which must not be exceeded by the total processing time of the surgeries allocated to that session.

In general, a MSS may be subject to various types of restrictions, which must be accounted for when planning:

- *Discipline-to-OR restrictions*. Certain disciplines can only be performed in a restricted set of ORs, due to size and/or equipment constraints.
- *Limits on discipline parallelism*. Typically, there is a limit on the number of OR sessions of a certain discipline can take place at the same time, e.g. because only a limited number of equipes for that discipline are available.
- *OR sessions-per-discipline restrictions*. Lower and upper limits to the number of OR sessions assigned to each discipline throughout one week can be specified.
- *OR reservation*. The hospital management may reserve one or more OR sessions to certain disciplines every day.

The objectives of the overall problem concern several issues, which can be encompassed by the following two:

- *Resource perspective*. OR sessions capacity and actual demand should be matched as much as possible, since both under- and over-utilization of an operating room are wasteful;
- *Patient perspective*. In organizational terms, the quality of service is expressed by the due date performance of the service, which in turn is related to having cases done within the respective due dates as much as possible.

We define an objective function that allows to account for both these aspects. Namely, we define the *score* of each surgery as the product of the surgery processing time and a coefficient which depends on how close is the surgery due date. The problem is to decide the MSS and the SCA so that the total score of selected cases is maximized. We also allow the maximum flexibility when generating the MSS, i.e., no information on the previous MSS is used during its generation.

Here we focus our study on the evaluation of a decomposition scheme for solving the MSSP+SCAP problem under stable conditions without considering the issues of long term evaluation and without considering limits to the flexibility in the MSS.

2. The proposed approach

The mathematical models introduced in (Agnetis et al. 2011) may in general require large computation times, even to find suboptimal solutions. Therefore, in order to quickly reach good solutions to MSSP and SCAP, we propose an efficient decomposition approach. The idea is to adopt a decomposition scheme addressing the MSSP and SCAP sequentially. The rationale is to first produce a MSS and next, given the MSS as input, determining the SCA.

2.1. Determining the MSS

The algorithm that produces the MSS works in three phases:

- In Phase 1, given the waiting list of each surgical discipline as input it quickly generate a set of *candidate OR* sessions. Each candidate OR session contains a set of surgeries such that the sum of their processing times does not exceed the maximum allowed duration of the session.

- Next, in Phase 2 a complete surgical case assignment is temporarily produced by solving a generalized assignment problem, i.e., assigning the candidate OR sessions to the available sessions during the week.
- Thereafter, in Phase 3 all surgical cases are discarded, and only the MSS is retained.

In Phase 1, for each surgical discipline we generate a number of candidate OR sessions, of various types. To this aim, we consider the problem of filling a number of bins (the OR sessions) of given capacity (the OR session capacity) with items (the surgical cases) each having a given size (the processing time of the surgical case) and a given value (the score associated to the surgery).
The half-day candidate OR sessions are filled as follows: we order all the surgical cases of discipline by non-increasing score, and sequentially fill the bins according to the first-fit-decreasing rule, i.e., assigning the current item to the first bin that fits. Bins corresponding to morning and afternoon sessions are considered alternately. In this way, we generate a set of candidate OR sessions. For each candidate OR session we compute the *value* as the sum of the individual contribution of the selected surgeries.
The same process is repeated from scratch but with full-day candidate OR sessions. This is done without considering if a given surgical case has been already inserted in a morning/afternoon session. Note that, half-day (morning or afternoon) candidate sessions are disjoint and the same holds true for candidate full-day sessions. However, a full-day and a half-day candidate sessions may have non-empty intersection. We say that two sessions are *incompatible* if they share some surgical case and we define a list of incompatible pairs of candidate OR sessions.

Phase 2 is solved by selecting a subset of the candidate OR sessions generated, to produce a feasible plan. Such problem is an assignment problem with complicating constraints that can be formulated and solved by as mathematical programming model. The output of the model is an assignment of sessions to ORs. The considered constraints allow to:
- set limits on the number of weekly OR sessions for each discipline
- enforce discipline-to-OR restrictions
- avoid the selection of incompatible candidate OR sessions.

Finally, in Phase 3, we only retain the MSS structure, discarding all surgical cases. The MSS is the input to the next phase, consisting in finding a heuristic solution to SCAP.

2.2. Solving the SCAP

It can be observed that solving the SCAP corresponds to solving a number of independent multiple-knapsack problems (Martello et al. 1990) one for each surgical discipline, where surgeries correspond to items and sessions to knapsacks. The problem are solved by means of a mathematical program and the detailed plan of the weekly surgeries is obtained.

3. Computational experiments

In our computational experiments, we generated 6 different sets of benchmark instances. This is done by considering three different OT sizes (5, 10 and 15 ORs) and for each OT size we generate 2 different waiting lists by considering 200 or 300 surgical cases for each OR. Each set is composed of 10 instances. More in details, we refer to these sets as X-β, where X is the number of rooms in the theater and β the multiplier used to obtain the waiting lists (i.e., 5-200 refers to instances with 5 ORs and having 1000 surgical cases in the waiting lists).

Based on a realistic case study, we consider a realistic operating theater composed of 5 operating rooms: they are all identically equipped, even though two of them are bigger than the others. Benchmark sets with 10 and 15 operating rooms are obtained by duplicating/triplicating ORs.

Focusing on elective surgeries only, the weekly surgery plan spans five days, from Monday to Friday. A morning session lasts 6.5 hours, an afternoon session 5 hours, and a full-day session 11.5 hours. However, in order to account for possible delays and/or uncertainties affecting surgery duration we introduce a planned slack time of 30 minutes (60 minutes) for half-day (full-day) sessions, respectively. All session types are divided into time slots of 15 minutes each.

We considered 6 different specialties. Namely, general surgery, otolaryngology, gynaecology, orthopaedic surgery, urology and day surgery. Note that, when considering instances with 10 and 15 Operating Rooms the number of specialties is unchanged.

To represent some additional operational constraint we consider that some specialties have a set of non-available ORs. That is, gynaecology surgeries should always be performed in the same OR, and orthopaedic surgeries have to be performed in a bigger OR. Nevertheless, these ORs are not exclusively assigned to these disciplines.

With X = 10 (X = 15) the number of unavailable rooms is duplicated (triplicated). General surgery and orthopaedic surgery allow 2 parallel OR sessions, whereas all the other surgical disciplines do not admit parallel sessions. However, when considering larger instances with 10 rooms the limit is set to 4 for general surgery and orthopaedic and to 2 for all other specialities. Analogously when X =15 the limits are set to 6 and 3, respectively.

We were provided by the San Giuseppe hospital of Empoli with the current waiting lists. The waiting list of all the specialities in our benchmark instances are obtained through nonparametric bootstrapping, which consists in sampling surgeries with replacement from the original waiting list. Therefore the sizes of the waiting list of each specialty is variable even though the total number of surgical cases is given.

Tests have been performed on a 3.2 GHz Intel Core i3 processor with 4 GB of RAM, using OPL Studio 6.1 and the CPLEX 11.2 MILP solver for the mathematical programming models and standard C++ for the heuristic algorithms. The maximum computation time has been set to 60 minutes for each optimization run of the exact model. Whereas, the decomposition approach are allowed to run for 300 and at most 60 seconds to solve the SCAP.

The main results are presented in Table 1. The first column shows the name of the benchmark set X-β). The next 5 columns report the performance indices of the mathematical model, while in the last 5 columns we report the same performance indices for the decomposition approach. More specifically Column 2 and 6 report the value of the objective function respectively. The next column shows the gap for the lower bound/optimal solution of the two approaches. Then Columns 4 and 8 detail the number of time unit left empty. Note that the total number of available time units are 1050, 2100 and 3150 for instances having 5, 10 and 15 operating rooms, respectively. Finally the last column reports the CPU time needed by the two approaches. Each row of the table reports the averaged values over ten instances belonging to the same benchmark set.

The results in Table 1 show that the proposed decomposition approach is able to obtain good quality solution requiring only a fraction of the CPU time. When comparing the solutions, we observe that the two models are able to produce comparable quality solution and also having comparable number of empty slots. The decomposition approach turns out to be slightly better in both of these indicators. When considering the larger instances (set 15-300) we observe that the allowed CPU time for the decomposition approach is not sufficient to find a good quality solution however there are only few empty slots.

Table 1:**Error! No text of specified style in document.** Comparison between exact and decomposition approaches

X-β	*Gap*	*Empty slots*	*CPU time*	*Gap*	*Empty slots*	*CPU time*
5-200	0.50%	2.3	603.00	0.45%	1.6	16.54
10-200	0.36%	10.4	3428.85	0.22%	6.2	173.00
15-200	0.41%	28.9	3173.17	0.20%	20.1	340.02
5-300	0.60%	2.3	603.80	0.47%	2.1	17.95
10-300	0.37%	10.8	3346.77	0.28%	5.1	182.14
15-300	0.45%	43.1	3178.03	9.26%	17.2	332.96

3. Conclusions and future research

In this paper we introduced a decomposition approach to tackle both the Master Surgical Schedule Problem and the Surgical Case Assignment Problem. We compared this decomposition approach against an exact model to solve the same MSSP+SCAP.
Preliminary results on several realistic instances of different sizes suggest that the proposed decomposition scheme represents a good trade-off between solution quality and computational effort. This makes it a suitable tool also to what-if analysis, or to quickly recompute feasible plans in the face of unpredicted events.

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