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Risk Aversion, Intertemporal Substitution, and the Aggregate Investment-Uncertainty Relationship

Risk aversion, intertemporal substitution, and the aggregate investment-uncertainty relationship*

Enrico Saltari†, Davide Ticchi ‡§

Abstract

We analyze the role of risk aversion and intertemporal substitution in a simple dynamic general equilibrium model of investment and savings. Our main finding is that risk aversion cannot by itself explain a negative relationship between aggregate investment and aggregate uncertainty, as the effect of increased uncertainty on investment also depends on the intertemporal elasticity of substitution. In particular, the relationship between aggregate investment and aggregate uncertainty is positive even if agents are very risk averse, as long as the elasticity of intertemporal substitution is low. A negative investment-uncertainty relationship requires that the relative risk aversion and the elasticity of intertemporal substitution are both relatively high or both relatively low. We also show that the implications of our model are consistent with the available empirical evidence.

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†Department of Public Economics, University of Rome "La Sapienza", via del Castro Laurenziano 9, 00161, Roma, Italy.

‡Department of Economics, University of Urbino, via Saffi 42, 61029, Urbino, Italy.

§Corresponding author. Tel.: +39-0722305556; fax: +39-0722305550. *E-mail address*: ticchi@uniurb.it

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1 Introduction

Economic theory has been analyzing the effect of uncertainty on investment for more than forty years. One seminal strand of the literature starts with Oi (1961), followed by Hartman (1972) and Abel (1983). They show that, in a perfectly competitive environment, an increase in output-price uncertainty raises the investment of a risk-neutral firm with a constant returns to scale technology. Intuitively, this is because constant returns to scale imply that the marginal revenue product of capital rises more than proportionally with the output price when firms can adjust employment after uncertainty is resolved. Hence, the marginal revenue product of capital is convex in the output price and, by Jensen's inequality, greater price variability translates into a higher expected return to capital and higher investment.

This theoretical conclusion has been contradicted by empirical research as no study has found a positive investment-uncertainty correlation; estimates range from negative to zero. Most of the empirical evidence is about the relationship between investment and uncertainty at the aggregate level. Many studies are based either on country data (see Ramey and Ramey, 1995, Aizenman and Marion, 1999, Pindyck and Solimano, 1993, Calcagnini and Saltari, 2000, Alesina and Perotti, 1996) or on highly aggregated data (see Huizinga, 1993, Ferderer, 1993a, 1993b). Only Leahy and Whited (1996), Guiso and Parigi (1999) and Bloom, Bond and Van Reenen (2005) do empirical work at the micro level.

Investment irreversibility has been one of the first elements considered by economic theory to explain the negative effect of uncertainty on investment. Bernanke (1983), McDonald and Siegel (1986), Pindyck (1988) and Bertola (1988) show that, if the firm cannot resell its capital goods, then the optimal investment policy derived under reversibility, equalization of the marginal revenue product of capital and the Jorgensonian user cost of capital (Jorgenson, 1963), does not hold anymore. In particular, if investment is irreversible, the firm invests only when the marginal revenue product of capital is higher than a threshold that exceeds the Jorgensonian user cost of capital because the firm takes into account that the irreversibility constraint may be binding in the following periods. The difference between this threshold and the Jorgensonian user cost of capital represents the value of the option of investing in the future. A higher degree of uncertainty implies a higher threshold for investing since the value of the option is always increasing in the variance of the stochastic variable.

The higher threshold for investing under irreversibility does not necessarily translates into lower investment however. For this to happen, two additional conditions must be satisfied. The first condition, highlighted by Caballero (1991), Pindyck (1993) and Abel and Eberly (1997), is that the marginal revenue product of capital is a decreasing function of the capital stock, i.e. that the firm operates under imperfect competition and/or decreasing returns to scale.[1] Under perfect competition and constant returns to scale the marginal revenue product of capital is independent of the capital stock so that current investment does not affect the current and future marginal profitability of capital, which implies that investment irreversibility does not change optimal investment.

The second condition required for the higher threshold for investing under irreversibility to generate lower investment is that the current capital of the firm is zero, which would be the case for a firm just getting started. This condition has been noted by Abel and Eberly (1999) who analyze the effect of irreversibility and uncertainty on the long-run capital stock (so that capital must be positive). They show that irreversibility and uncertainty have two effects on investment. One is the increase in the user cost of capital described above that tends to reduce the capital stock compared to the case with reversibility. But there is also a hangover effect, which implies a higher capital stock under irreversibility than under reversibility because investment irreversibility prevents the firm from selling capital when the marginal revenue product of capital is low. Abel and Eberly demonstrate that neither of the two effects dominates globally, so that irreversibility may increase

[1] A decreasing marginal revenue product of capital was necessary in the initial models of irreversible investment under uncertainty to bound the size of the firm given the standard assumptions of complete irreversibility and absence of upward adjustment costs. Later contributions to this literature, as Abel and Eberly (1994, 1996, 1997), have provided solutions to the problem of optimal investment under uncertainty in more general frameworks allowing, for example, for fixed costs of investment, adjustment costs and partial irreversibility.

or decrease capital accumulation in the long-run. Higher uncertainty reinforces both the user cost effect and the hangover effect and, therefore, does not help in obtaining an unambiguous result. If the firm has zero capital stock, the hangover effect is inoperative and the user cost effect is the only effect at work, which implies that an increase in uncertainty with investment irreversibility always lower the level of capital stock compared to the case with reversibility. It is also worthwhile noticing that the works with adjustment costs and irreversibility use partial equilibrium models with an exogenous risk-free interest rate so that it is not clear whether the results of these papers are about sectoral investment or aggregate investment.

To obtain a robust negative relationship between investment and uncertainty, economic theory has taken into consideration the role of risk aversion in general equilibrium frameworks, so incorporating the role of savings into the model. Craine (1989) uses a model with many sectors and risk averse households to show that an increase in exogenous risk in one sector may lead, under some conditions, to capital being reallocated toward less risky sectors. Zeira (1990) makes a similar point in a model where sectors differ in the intensity of capital and labor used. He shows that, in some cases, higher labor cost uncertainty may shift capital from labor intensive sectors toward less risky, capital intensive, sectors. Even though Craine and Zeira use general equilibrium models, they both concentrate on the effect of uncertainty on the reallocation of savings and investment across sectors and in their work there is no effect of uncertainty on aggregate savings/investment.[2]

Our goal here is instead to analyze the effect of an increase in aggregate, and hence non-diversifiable, uncertainty on aggregate equilibrium investment when agents are risk averse. Therefore, we propose a dynamic general equilibrium model where households are risk averse and firms

[2] The increase in sectoral uncertainty in the models of Craine and Zeira also leads to an increase in aggregate uncertainty. The increase in aggregate risk does not affect aggregate investment in Craine's model, however, because the household's instantaneous utility function is logarithmic, which implies that aggregate savings and aggregate investment are a fixed fraction of total output. Therefore, aggregate risk makes the time path for investment more volatile but does not affect the aggregate savings/investment decision rule. This is also the case in the overlapping generations model of Zeira where he assumes that each individual of the young generation, independently on the realization of the (real wage) shock, always works one unit of time, gets the real wage and saves it all.

are subject to aggregate exogenous shocks. This setting allows us to focus on the effect of aggregate uncertainty on aggregate investment instead of the effect of uncertainty on the distribution of investment across sectors as analyzed in Craine (1989) or Zeira (1990). A key feature of our model is that we use Kreps-Porteus nonexpected utility preferences (recursive preferences) in order to separate the role of risk aversion from that of intertemporal substitution. As is well-known, the conventional expected utility set up with constant relative risk aversion ($CRRA$) preferences makes it impossible to separate the role of these two parameters.

We show that risk aversion cannot by itself explain a negative relationship between investment and uncertainty at the aggregate level as the effect of increased uncertainty on investment also depends on the intertemporal elasticity of substitution. For example, we show that if the elasticity of intertemporal substitution is low, then an increase in aggregate uncertainty has a positive effect on aggregate investment even if risk aversion is very high. If the elasticity of intertemporal substitution is high, however, then even small degrees of risk aversion imply a negative investment-uncertainty relationship. Intuitively, in a dynamic framework, a high degree of risk aversion reduces the certainty equivalent of the return to capital. This does not necessarily lower investment however. The reason is that a lower rate of return to capital generates a substitution effect and an income effect affecting aggregate savings and, therefore, aggregate investment in opposite directions. The substitution effect reduces aggregate savings and investment while the income effect increases aggregate savings/investment. The relative strength of these two effects is determined by the elasticity of intertemporal substitution. If the elasticity of substitution is lower than unity, the income effect dominates and the equilibrium investment increases as a result of increased uncertainty. The opposite happens if the elasticity of substitution is greater than unity.

We characterize the aggregate investment-uncertainty relationship for all possible parameter values of the Kreps-Porteus nonexpected utility preferences as well as for the standard $CRRA$ expected utility preferences (which are a special case of the recursive preferences). We show that the relationship is generally ambiguous and depends on the value of technological and preference parameters. A negative relationship between aggregate investment and aggregate uncertainty requires that the relative risk aversion and the elasticity of intertemporal substitution are both relatively high or both relatively low. If this is not the case, the relationship is positive. With *CRRA* preferences the region of the parameter values where the relationship is negative is generally small and the fact that the elasticity of intertemporal substitution is the inverse of the coefficient of relative risk aversion implies that high values of risk aversion always lead to a positive correlation between aggregate investment and aggregate uncertainty.[3] We also study the investment-uncertainty relationship implied by empirically plausible values of the relative risk aversion and the elasticity of intertemporal substitution and find that the wide range of estimates available in the literature implies that our model is compatible with a negative, positive, or no relationship between aggregate investment and aggregate uncertainty.

Our results therefore suggest that risk aversion, as well as irreversibility, is not enough to generate a theoretically robust negative investment-uncertainty relationship. Indeed, even though increased uncertainty in one sector may reduce investment in that sector, the same needs not to be true at the aggregate level as the effect of uncertainty on aggregate savings/investment is different than the effect of uncertainty on the allocation of savings and investment across sectors.

The paper is organized as follows. Section 2 presents the model with recursive preferences and analyzes the relationship between aggregate uncertainty and aggregate investment for all possible values of the coefficients of relative risk aversion and intertemporal substitution elasticity. Section 3 relates the implications of the model to the evidence on the investment-uncertainty relationship. Section 4 concludes. Detailed proofs of the main propositions and an extension of the baseline model can be found in the Appendix.

[3] It is immediate that this result can be understood only using recursive preferences that allows us to separate the role of risk aversion from the role of intertemporal substitution.

2 The model

We assume that the technology of the competitive firm is described by the following constant returns to scale Cobb-Douglas production function

$$Y_t = B_t K_t^{1-\alpha} L_t^{\alpha} \tag{1}$$

where K_t is the stock of capital, L_t is the amount of labor employed and $B_t = Be^{\vartheta_t}$ is a multiplicative shock to the production function. We assume that B is a constant and ϑ_t is an identically and independently distributed normal random variable with variance σ^2 and mean $\overline{\vartheta} - \frac{1}{2}\sigma^2$.[4] This parametrization implies that the expected value of the multiplicative shock is a function of $\overline{\vartheta}$ only (does not depend on σ^2) and that the variance of the multiplicative shock is increasing in σ^2. Hence, an increase in σ^2 increases the variance of the multiplicative shock (and hence the degree of uncertainty) without affecting its expected value. The parameters $\overline{\vartheta}$ and σ^2 are assumed to be constant over time.[5] The *i.i.d.* assumption is crucial for the derivation of the investment function, and the log-normality assumption permits us to derive the effect of uncertainty on investment analytically.[6]

We assume that the firm can adjust the amount of labor employed in each period but that the capital stock is decided one period in advance. In each period, the firm first observes the realization of the shock and then adjust the amount of labor. Choosing output as the numeraire, the firm's operating profit (i.e. revenues minus the cost of variable inputs) is therefore equal to

[4] In this Section, we only discuss the effect of technological uncertainty on aggregate investment. In Appendix D, we extend this framework to analyze also the effect of preference shocks.

[5] As most models in this literature, we make a comparative static analysis and do not allow for time-varying uncertainty. See, for example, Guo, Miao and Morellec (2005) for a contribution that analyzes the dynamic of investment when the growth rate and volatility of the marginal revenue product of capital are subject to discrete regime shifts at random times.

[6] The *i.i.d.* assumption is also made in Craine (1989) and Zeira (1990). The investment function and the effect of uncertainty on investment cannot be derived analytically when the shock is subject to a trend or displays persistence. In this case it is necessary to use numerical solution methods.

$$\chi_t = \max_{\{L_t\}} \left\{ B_t K_t^{1-\alpha} L_t^{\alpha} - w L_t \right\} \tag{2}$$

where w is the real wage. We assume the real wage w to be constant, which will be true in equilibrium. The optimal amount of labor according to the maximization problem in (2) is

$$L_t^* = \left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}} B_t^{\frac{1}{1-\alpha}} K_t \tag{3}$$

and the operating profit is

$$\chi_t = \xi B_t^{\frac{1}{1-\alpha}} K_t \tag{4}$$

where $\xi \equiv (1-\alpha)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}$. If we define $A_t = Ae^{\vartheta_t}$ where $A = \xi^{\frac{1}{\eta}} B$ is a constant and $\eta = \frac{1}{1-\alpha} > 1$ (given that $\alpha < 1$), then the operating profit can be written as

$$\chi_t = A_t^{\eta} K_t. \tag{5}$$

We assume that one unit of capital is produced with one unit of output. Hence, the firm's cash flow at time t is

$$\pi_t = A_t^{\eta} K_t - I_t. \tag{6}$$

where A_t^{η} represents the marginal revenue product of capital at time t. Given that $\eta > 1$, the profit function is convex in the random variable as in Abel (1983) and Hartman (1972). This comes from the fact that the capital stock is chosen before the realization of the shock and the employment of labor.

The firm is owned by the representative household. The representative household supplies labor and chooses consumption for $t = 1, 2, ...$ to maximize the following Kreps-Porteus nonexpected utility

$$V_t(K_t, \vartheta_t) = \max_{\{C_t, L_t\}} U\left[C_t, L_t, E_t V_{t+1}\right]$$

$$\equiv \max_{\{C_t, L_t\}} \frac{\left\{(1-\beta)\left(C_t - \phi L_t\right)^{1-\rho} + \beta\left[1 + (1-\beta)(1-\gamma) E_t V_{t+1}\left(K_{t+1}, \vartheta_{t+1}\right)\right]^{\frac{1-\rho}{1-\gamma}}\right\}^{\frac{1-\gamma}{1-\rho}} - 1}{(1-\beta)(1-\gamma)}$$

where C_t is consumption and ϕ is the constant opportunity cost of supplying labor, which in equilibrium will be equal to the real wage w. The parameters that characterize this representation of preferences are $\beta \in (0,1)$, $\rho > 0$, and $\gamma > 0$. β is the subjective discount factor under certainty. Time preference under uncertainty is endogenous except $\gamma = \rho$, i.e. unless we have Von Neumann-Morgenstern (VNM) time- and state-separable isoelastic preferences. γ is the coefficient of relative risk aversion for timeless gambles and $\frac{1}{\rho} = \varepsilon$ is the elasticity of intertemporal substitution for deterministic consumption paths.[7] We start our analysis by assuming that both $\rho \neq 1$ and $\gamma \neq 1$. Then, we consider the case with unit intertemporal substitution elasticity ($\rho = 1$) and with unit relative risk aversion ($\gamma = 1$).

It can be shown easily that the competitive equilibrium allocation of this economy is equivalent to the allocation where the central planner chooses investment by solving the following maximization problem

$$V_t\left(K_t, \vartheta_t\right) = \max_{\{I_t\}} U\left[\pi_t, E_t V_{t+1}\right] \tag{7}$$

$$\equiv \max_{\{I_t\}} \frac{\left\{(1-\beta)\,\pi_t^{1-\rho} + \beta\left[1 + (1-\beta)(1-\gamma) E_t V_{t+1}\left(K_{t+1}, \vartheta_{t+1}\right)\right]^{\frac{1-\rho}{1-\gamma}}\right\}^{\frac{1-\gamma}{1-\rho}} - 1}{(1-\beta)(1-\gamma)}$$

subject to the capital accumulation equation

$$K_{t+1} = (1-\delta) K_t + I_t \tag{8}$$

[7] For a general discussion of the properties of these preferences and for a better understanding of the role played by the preference parameters see Kreps and Porteus (1978, 1979), Epstein and Zin (1989, 1991), Weil (1989, 1990) and Giovannini and Weil (1989). The last three papers clarify the importance of *i.i.d.* uncertainty for obtaining closed form solutions. For a two-period application of these preferences see Selden (1978).

where δ is the rate of capital depreciation. For notational simplicity, from now on we will use $V_t \equiv V_t(K_t, \vartheta_t)$ and $V_{t+1} \equiv V_{t+1}(K_{t+1}, \vartheta_{t+1})$.

Our guess of the value function for the maximization problem in (7) and (8) is the same as in the VNM isoelastic utility case:

$$V_t = \frac{\psi^{1-\gamma}\left[A_t^\eta + (1-\delta)\right]^{1-\gamma} K_t^{1-\gamma} - 1}{(1-\beta)(1-\gamma)} \tag{9}$$

with the investment function given by

$$I_t = A_t^\eta K_t - \mu\left[A_t^\eta + (1-\delta)\right] K_t \tag{10}$$

where ψ and μ are constants to be determined. Solving the maximization problem (details can be found in Appendix A) we obtain that the unknown constant ψ is

$$\psi = \left[(1-\beta)\,\mu^{-\rho}\right]^{\frac{1}{1-\rho}} \tag{11}$$

and that

$$\mu = 1 - \beta^{\frac{1}{\rho}} \left\{ \left[E_t \left(A_{t+1}^\eta + (1-\delta) \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \right\}^{\frac{1-\rho}{\rho}}. \tag{12}$$

2.1 The relationship between aggregate investment and aggregate uncertainty

The effect of uncertainty on investment can be obtained by differentiating equation (10) with respect to the volatility σ^2 of the shock

$$\frac{dI_t}{d\sigma^2} = -\frac{d\mu}{d\sigma^2}\left[A_t^\eta + (1-\delta)\right] K_t. \tag{13}$$

It is easy to see from equation (12) that it is not possible to derive a closed form solution for $d\mu/d\sigma^2$ except in the case where capital fully depreciates in production. Therefore, we assume $\delta = 1$ and obtain[8]

$$\mu = 1 - \beta^{\frac{1}{\rho}} \left\{ \left[E_t \left(A_{t+1}^{\eta(1-\gamma)} \right) \right]^{\frac{1}{1-\gamma}} \right\}^{\frac{1-\rho}{\rho}} = 1 - \beta^{\frac{1}{\rho}} A^{\eta \frac{1-\rho}{\rho}} \left\{ e^{\eta \overline{\vartheta} + \frac{1}{2}(\alpha-\gamma)\eta^2\sigma^2} \right\}^{\frac{1-\rho}{\rho}}. \tag{14}$$

This implies

$$\frac{dI_t}{d\sigma^2} = \frac{1}{2} \beta^{\frac{1}{\rho}} \eta^2 A^{\eta \frac{1-\rho}{\rho}} \frac{(1-\rho)(\alpha-\gamma)}{\rho} \left\{ e^{\eta \overline{\vartheta} + \frac{1}{2}(\alpha-\gamma)\eta^2\sigma^2} \right\}^{\frac{1-\rho}{\rho}} A_t^{\eta} K_t$$

and

$$sign\left(\frac{dI_t}{d\sigma^2}\right) = sign\left[(1-\rho)(\alpha-\gamma)\right] = sign\left[(\varepsilon-1)(\alpha-\gamma)\right]. \tag{15}$$

Table 1 summarizes the sign between investment and uncertainty for different values of preference and technology parameters.[9] It can be seen that the relationship between investment and uncertainty is negative when the coefficient of relative risk aversion γ and the intertemporal substitution elasticity ε are both relatively high or both relatively low. More precisely, there is a negative investment-uncertainty relationship in two situations: when the coefficient of relative risk aversion is greater than the elasticity of output to labor ($\gamma > \alpha$) and the coefficient of intertemporal substitution is greater than one ($\varepsilon > 1$); and when $\gamma < \alpha$ and $\varepsilon < 1$.

For a more intuitive understanding of our results imagine a consumer-producer facing the decision of allocating output between consumption and investment, where total output is equal to the sum of operating profits and labor income (see equation (2)).[10] The investment function in (10) (with $\delta = 1$) implies that this problem is solved by always investing a fraction $1 - \mu$ of operating profits and consuming a fraction μ. It is immediate to see that total consumption is $C_t = \left(\frac{\alpha}{1-\alpha} + \mu\right) A_t^{\eta} K_t$, where $\frac{\alpha}{1-\alpha} A_t^{\eta} K_t$ is equal to labor income. Hence, μ can be interpreted

[8] It is often the case that complete depreciation is necessary for analytical solutions in dynamic setting with capital. See for example Long and Plosser (1983) and (in the investment literature) Craine (1989).

[9] The particular case where $\rho = 1$ will be derived and discussed later.

[10] The consumer-producer interpretation is natural given that we use a representative consumer and a representative firm in a framework where there are no imperfections.

as the "marginal propensity to consume" out of operating profits: when μ increases consumption increases at the expense of investment. The key distribution coefficient μ can be rewritten as[11]

$$\mu = 1 - \beta^{\varepsilon} \check{Z}^{\varepsilon - 1} \tag{16}$$

where

$$\check{Z} \equiv \left[E_t \left(A_{t+1}^{\eta(1-\gamma)} \right) \right]^{\frac{1}{1-\gamma}} \tag{17}$$

is the certainty equivalent of the marginal revenue product of capital (or, given the assumption of full capital depreciation, the certainty equivalent of the return to capital). To see this, notice that the return to capital is A_{t+1}^{η}, which is what the consumer-producer will receive at time $t+1$ if she consumes one unit less at time t and invests it in capital. As the individual is risk averse, she will take her decision by considering the certainty equivalent of the return to capital, i.e. $\check{Z}$. Under the assumption that shocks are *i.i.d.* and lognormally distributed the certainty equivalent of the return to capital is

$$\check{Z} \equiv \left[E_t \left(A_{t+1}^{\eta(1-\gamma)} \right) \right]^{\frac{1}{1-\gamma}} = A^{\eta} e^{\eta \overline{\vartheta} + \frac{1}{2}(\alpha - \gamma)\eta^2 \sigma^2} \tag{18}$$

which is increasing in the variance σ^2 of the shock if $\gamma < \alpha$ and decreasing if $\gamma > \alpha$.[12] The intuition for this result is straightforward. When uncertainty increases there are two effects at work. The first, which might be called *flexibility effect*, comes from the fact that the consumer-producer can substitute labor for capital after observing the realization of the shock. This implies that the return to capital is convex with respect to the shock. Therefore, by Jensen's inequality, the expected return to capital is increasing in the volatility of the shock and the size of this relationship is positively related to the elasticity of output with respect to labor α. The second effect, which we call *risk aversion effect*, is generated by the agent's risk aversion. Indeed, given that the agent is risk averse,

[11] We are using the fact that $\frac{1}{\rho} \equiv \varepsilon$ and therefore $\frac{1-\rho}{\rho} \equiv \varepsilon - 1$.

[12] It is immediate to verify from (18) that $\frac{\partial \check{Z}}{\partial \sigma^2} \gtreqless 0$ if $\alpha \gtreqless \gamma$.

she does not take her decisions considering the expected return to capital but the correspondent certainty equivalent, which is negatively related to the riskiness of the return, here represented by the variance of the shock. It is clear that the magnitude of this effect increases with the degree of risk aversion γ of the consumer-producer. The final effect of uncertainty on the certainty equivalent of the return to capital $\check{Z}$ depends on which of the two effects is bigger. If risk aversion is sufficiently small ($\gamma < \alpha < 1$) for the *flexibility effect* to prevail over the *risk aversion effect*, then $\check{Z}$ will be increasing in the variance σ^2 of the shock. If risk aversion is big enough ($\gamma > \alpha$), then the *risk aversion effect* is stronger than the *flexibility effect* and $\check{Z}$ will be decreasing in the shock's volatility.

Hence, an increase in uncertainty changes the certainty equivalent of the return to capital. This gives rise to an income and a substitution effect affecting aggregate investment in opposite directions. The final effect of aggregate uncertainty on aggregate investment will depend on the magnitude of the elasticity of intertemporal substitution because ε determines the relative strength of income and substitution effects. To see how things work, let us assume that the coefficient of relative risk aversion is lower than the elasticity of output with respect to labor ($\gamma < \alpha < 1$) so that an increase in uncertainty produces an increase in the certainty equivalent of the return to capital $\check{Z}$. The substitution effect induces the consumer-producer to save and invest more (and consequently consume less) because capital is more productive. The income effect (due to the fact that a higher productivity of capital makes the consumer-producer richer) implies higher consumption and lower savings and investment. If the elasticity of intertemporal substitution is greater than one ($\varepsilon > 1$), then the substitution effect prevails over the income effect and investment will increase. This is immediate from equation (16): an increase in $\check{Z}$ leads to a decrease in μ whenever $\varepsilon > 1$. If the elasticity of intertemporal substitution is less than one ($\varepsilon < 1$), then the income effect more than balances the substitution effect. This leads the agent to consume more and invest less. Equation (16) shows how an increase in $\check{Z}$ implies an increase in μ whenever $\varepsilon < 1$.

A similar argument applies to the situation where the coefficient of relative risk aversion is higher than the elasticity of output to labor ($\gamma > \alpha$).[13] In this case an increase in uncertainty reduces the

[13] Even though the discussion on the empirically plausible values of the parameters is presented in the next Section,

certainty equivalent of the return to capital. The substitution effect induces the consumer-producer to invest less because the return to capital (in certainty equivalent terms) is lower. On the other hand, the income effect increases investment by pushing down the individual's consumption because the lower productivity of capital makes the consumer-producer poorer. Again, if the elasticity of intertemporal substitution is greater than one ($\varepsilon > 1$), then the substitution effect more than balances the income effect and investment decreases, while the opposite happens when $\varepsilon < 1$.[14]

2.2 Two special cases: unit elasticity of intertemporal substitution and CRRA

Let us now consider the case with unit elasticity of intertemporal substitution ($\rho = 1$). The maximization problem is now given by equation[15]

$$V_t = \max_{\{I_t\}} \left\{ \pi_t^{(1-\beta)(1-\gamma)} \left(E_t V_{t+1}\right)^{\beta} \right\}.$$

By making the same guess and performing the same steps of the previous maximization problem, we obtain that investment is still given by equation (10) but that

it is worthy to notice since now that $\gamma > \alpha$ is the relevant region of the parameter space given that $\alpha < 1$.

[14] At this point it may be useful to clarify the difference between our model and the standard model of intertemporal consumption choice. An increase in the volatility of the future income flows always increases savings in the standard model as it generates a precautionary savings effect (i.e. a negative income effect) given the widespread assumptions of convex marginal utility (see Leland, 1968) and existence of a risk-free asset. So one may think that in our model an increase in uncertainty would increase savings and investment for the same reason. But this is not the case because in our model uncertainty is in the return of the asset used to transfer wealth over time. In the terminology of Sandmo (1970), in our model there is a "capital risk" instead of the "income risk" of the standard model. An increase in uncertainty of the return to capital (assuming that $\gamma > \alpha$ so that the *flexibility effect* is dominated by the *risk aversion effect*) generates an income effect (precautionary savings effect) also in our model because it raises the probability of low levels of consumption in the following period. This leads the agents to insure themselves by consuming less today so increasing the current level of savings. However, in our model there is also a substitution effect as the agents try to reduce their exposure to risk by increasing current consumption and reducing savings (given that uncertainty is on the savings vehicle). The magnitude of the intertemporal elasticity of substitution defines the relative strength of the income and substitution effects.

[15] See Appendix *B* for the mathematical details.

$$\mu = 1 - \beta.$$

It is immediate that in this case investment is not affected by uncertainty for any given value of the coefficient of relative risk aversion: this result holds even with partial depreciation of capital. This is because, independently from what happens to the certainty equivalent of the return to capital (i.e. $\gamma \lesseqgtr \alpha$), income and substitution effects exactly offset each other (as in the logarithmic preference case).[16]

Another particular case is the one corresponding to the *CRRA* preferences: these preferences are obtained when the coefficient of relative risk aversion is equal to the inverse of the intertemporal substitution elasticity ($\gamma = \rho \equiv \frac{1}{\varepsilon}$). Clearly, the investment function is still given by equation (10) with μ (see equation (12)) equal to

$$\mu = 1 - \beta^{\frac{1}{\gamma}} \left\{ \left[E_t \left(A_{t+1}^{\eta} + (1-\delta) \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \right\}^{\frac{1-\gamma}{\gamma}}.$$

As before, to get a closed form solution for the effect of uncertainty on investment it is necessary to assume complete depreciation of capital ($\delta = 1$). In this case

$$\mu = 1 - \beta^{\frac{1}{\gamma}} \left\{ \left[E_t \left(A_{t+1}^{\eta(1-\gamma)} \right) \right]^{\frac{1}{1-\gamma}} \right\}^{\frac{1-\gamma}{\gamma}} = 1 - \beta^{\frac{1}{\gamma}} A^{\eta \frac{1-\gamma}{\gamma}} \left\{ e^{\eta\overline{\vartheta} + \frac{1}{2}(\alpha-\gamma)\eta^2\sigma^2} \right\}^{\frac{1-\gamma}{\gamma}}$$

and

$$\frac{dI_t}{d\sigma^2} = \frac{1}{2} \beta^{\frac{1}{\gamma}} \eta^2 \frac{(1-\gamma)(\alpha-\gamma)}{\gamma} \left\{ e^{\eta\overline{\vartheta} + \frac{1}{2}(\alpha-\gamma)\eta^2\sigma^2} \right\}^{\frac{1-\gamma}{\gamma}} A_t^{\eta} K_t$$

which means that

$$sign\left(\frac{dI_t}{d\sigma^2}\right) = sign\left[(1-\gamma)(\alpha-\gamma)\right].$$

[16] As we already said, logarithmic preferences correspond to the case where $\gamma = \rho = 1$. The maximization problem when the coefficient of relative risk aversion is equal to one ($\gamma = 1$) can be found in Appendix C. The results and the interpretation correspond to the case discussed above where $\gamma > \alpha$, given that α is always lower than one.

Table 2 summarizes the relationship between aggregate investment and aggregate uncertainty for different values of the relative risk aversion coefficient. It is easy to see that the effect of uncertainty on investment is generally positive except when $\alpha < \gamma < 1$.[17] This is because with *CRRA* preferences the coefficient of relative risk aversion is the inverse of the elasticity of intertemporal substitution ($\gamma = 1/\varepsilon$), which implies that only two main situations are possible. First, the coefficient of relative risk aversion is less than one ($\gamma < 1$): this implies that the elasticity of intertemporal substitution is greater than one ($\varepsilon > 1$). If $\alpha < \gamma < 1$, then greater uncertainty reduces the certainty equivalent of the return to capital $\check{Z}$. As $\varepsilon > 1$, the substitution effect prevails over the income effect, leading to lower investment. When risk aversion is small enough ($0 < \gamma < \alpha < 1$), then more uncertainty increases the certainty equivalent of the return to capital $\check{Z}$ and this raises investment.[18] The second situation is when the coefficient of relative risk aversion is greater than one ($\gamma > 1$). Hence, it is greater than the elasticity of output with respect to labor α and the elasticity of intertemporal substitution is less than one ($\varepsilon < 1$). In this case greater uncertainty decreases the certainty equivalent of the expected return to capital $\check{Z}$ (as $\gamma > \alpha$). The fact that the elasticity of intertemporal substitution is less than one implies that the income effect more than balances the substitution effect, and this implies higher investment.[19]

2.3 The investment-uncertainty relationship under partial capital depreciation

In the analysis developed above we have assumed that capital fully depreciates in production ($\delta = 1$) in order to obtain a closed form solution for the relationship between aggregate investment and aggregate uncertainty. We now relax this assumption and analyze what happens when the

[17] Moreover, what seems to be surprising is that the investment-uncertainty relationship is positive when agents are very risk averse ($\gamma > 1$).

[18] It is immediate that if $\gamma = \alpha$ then volatility has no effect on $\check{Z}$ (because the *flexibility effect* and the *risk aversion effect* exactly compensate each other) and therefore it does not affect investment.

[19] If the coefficient of relative risk aversion is equal to one (and to the elasticity of intertemporal substitution) then the utility function is logarithmic. In this case uncertainty has no effect on investment because the income and substitution effect exactly offset each other (given that $\varepsilon = 1$).

depreciation of capital is only partial ($\delta < 1$) using numerical simulation methods. Similarly to the case of $\delta = 1$, the determination of the aggregate investment-uncertainty relationship requires the analysis of the behavior of the distribution parameter μ with respect to σ^2.[20] This parameter is still given by (16) but the certainty equivalent of the return to capital is now equal to

$$\check{Z} \equiv \left[E_t \left(A_{t+1}^{\eta} + (1-\delta) \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} . \tag{19}$$

This implies that removing the assumption of full capital depreciation may have an effect only on the certainty equivalent of the return to capital and therefore on the relative strength of the *flexibility effect* and the *risk aversion effect*. Indeed, it is immediate to verify from (16) that once we have determined the effect of uncertainty (represented by σ^2) on $\check{Z}$, the income and substitution effects work as usual. If the elasticity of intertemporal substitution is greater than one ($\varepsilon > 1$), the substitution effect prevails over the income effect and vice versa. This means that we can concentrate our analysis on the effect of uncertainty on the certainty equivalent of the return to capital $\check{Z}$.

In the previous section we have seen that $\check{Z}$ is increasing in σ^2 if $\gamma < \alpha$ and decreasing if $\gamma > \alpha$ when capital fully depreciates in production ($\delta = 1$). Given that it is not possible to obtain a closed form solution for the derivative of $\check{Z}$ with respect to σ^2 when δ is lower than one, we have made a numerical analysis. We have found that for each value of the capital depreciation δ, there exists a threshold value of the coefficient of relative risk aversion γ^* such that the certainty equivalent of the return to capital $\check{Z}$ is increasing in σ^2 if $\gamma < \gamma^*$ and decreasing if $\gamma > \gamma^*$.[21] Figure 1 presents three examples on the relationship between the derivative of $\check{Z}$ with respect to σ^2 and the coefficient of relative risk aversion γ for the following parameterization: $\sigma = 0.3$, $\overline{\vartheta} = 0.2$, $A = 1$, $\alpha = 0.67$, and for $\delta = 0$, 0.5 and 1. The results confirm that $\frac{\partial \check{Z}}{\partial \sigma^2}$ is positive if $\gamma < \gamma^*$, it is zero at γ^* and then becomes negative for $\gamma > \gamma^*$. The figure shows the derivative of $\check{Z}$ with respect to σ^2 for only three

[20] This is apparent from an inspection of the derivative of the investment function with respect to σ^2 in (13).

[21] In words, the threshold value of the coefficient of relative risk aversion that defines the behavior of $\check{Z}$ with respect to uncertainty is α when $\delta = 1$ and γ^* when $\delta < 1$. The properties of γ^* are discussed below.

values of δ, but its behavior is the same for all $\delta \in [0,1]$ as well as for other parameter values.[22]

Additional numerical simulations are presented in Table 3 and in Figures 2 and 3. Each column of Table 3 contains the threshold values of γ^* for different rates of capital depreciation δ. The value of uncertainty σ is indicated at the top of the column, the values of $\overline{\vartheta}$ can be found at the top of each sub-table and the other parameter values are $A = 1$ and $\alpha = 0.67$. From Table 3 (as well as from Figures 2 and 3 that we discuss below) we can notice two things. First, the threshold γ^* can be higher or lower than one depending on the value of the parameters. Second, the threshold γ^* is monotonically decreasing in δ and it is equal to α (as we already know) at $\delta = 1$. This can be explained as follows. A lower δ means that capital lasts longer. This allows the *flexibility effect* to operate for more periods, which in turn implies that the agent's risk aversion has to be relatively higher for the *risk aversion effect* to prevail over the *flexibility effect*.

Figure 2 provides a graphical representation of the negative relationship between γ^* and δ and shows how changes in the technological parameter $\overline{\vartheta}$ affect the threshold γ^*. We observe that an increase in $\overline{\vartheta}$ leads to a counterclockwise rotation of the schedule $\gamma^*(\delta)$ around the point where $\delta = 1$ and $\gamma^* = \alpha$, namely it leads to a reduction in the threshold γ^* for all $\delta < 1$. This result is generated by the fact that a higher $\overline{\vartheta}$ increases the marginal productivity of capital and this in turn reduces the relative importance of uncertainty so leading the threshold γ^* closer to α. Another parameter that affects positively the return to capital is A. The numerical simulations show that the qualitative effect of an increase in A on $\gamma^*(\delta)$ is the same as the increase in $\overline{\vartheta}$. For this reason we omit the presentation of an example.

Finally, Figure 3 presents the results of a variation of α on γ^*. An increase in the elasticity of output with respect to labor α strengthen the *flexibility effect* and leaves unaffected the *risk aversion effect*. This implies an upward shift of the schedule $\gamma^*(\delta)$ because for each value of δ the degree of risk aversion has now to be higher in order to allow the *risk aversion effect* to balance the *flexibility effect*.

Table 4 summarizes the relationship between aggregate investment and aggregate uncertainty

[22] More numerical simulations are available from the authors on request.

with recursive preferences when $\delta < 1$ and shows clearly that removing the assumption of full capital depreciation does not change the results from a qualitatively point of view. The only variation when $\delta < 1$ is on the value of the threshold of the coefficient of relative risk aversion where the *flexibility effect* and the *risk aversion effect* exactly offset each other. This does not correspond to the elasticity of output with respect to labor α anymore but will be $\gamma^* > \alpha$. However, a negative aggregate investment-uncertainty relationship still requires that the relative risk aversion and the elasticity of intertemporal substitution are both relatively high or both relatively low.

Removing the assumption of full capital depreciation when preferences display *CRRA* may lead to a variation in the results slightly greater than under recursive preferences because in this case the coefficient of the elasticity of intertemporal substitution and the coefficient of relative risk aversion are constrained to be one the reciprocal of the other. Indeed, it is now important to distinguish between two possible situations depending on the threshold γ^* being smaller or greater than one.[23] Assume first that $\gamma^* < 1$. In this case there is a restriction of the region of the values of γ where the relationship between aggregate investment and aggregate uncertainty is negative (see Table 5).[24] Then, let us consider the situation where $\gamma^* > 1$. The aggregate investment-uncertainty relationship is now negative when $1 < \gamma < \gamma^*$. Indeed, if $\gamma < \gamma^*$ the *flexibility effect* prevails over the *risk aversion effect* and an increase in uncertainty leads to an increase in the certainty equivalent of the return to capital $\check{Z}$. If γ is also lower than one, the elasticity of intertemporal substitution is greater than one, and the substitution effect more than compensate the income effect leading to more investment. Instead, if $1 < \gamma < \gamma^*$, then $\varepsilon < 1$ and the income effect more than balances the substitution effect, which implies lower investment. Finally, consider the region where $\gamma > \gamma^* > 1$. The *risk aversion effect* is now stronger than the *flexibility effect* and an increase in uncertainty lowers $\check{Z}$. Given that $\varepsilon < 1$, the income effect prevails over the substitution effect and

[23] The situation where γ^* is exactly equal to one cannot be excluded *a priori*. However, it is immediate that in this case the investment-uncertainty relationship is always positive and absent at $\gamma = 1$.

[24] We remind that under *CRRA* preferences and $\delta = 1$ the investment-uncertainty relationship is negative only when $\alpha < \gamma < 1$. Instead, in this case ($\delta < 1$) the relationship is negative when $\alpha < \gamma^* < \gamma < 1$.

investment increases. These results are presented in Table 6.[25]

From this analysis we conclude that under *CRRA* preferences relaxing the assumption of full capital depreciation does not change the main features of the aggregate investment-uncertainty relationship, namely that the region of the values of γ where this relationship is negative is close to one, and that a sufficiently high level of risk aversion always leads aggregate investment and aggregate uncertainty to be a positively related.

3 Discussion and empirical evidence

The aim of our work is to analyze the relationship between aggregate uncertainty and aggregate investment. To this purpose, in the previous Section we have proposed a closed economy model with only one asset (the representative firm) and we have analyzed the effects of an increase in the volatility of the returns of this asset on savings/investment. In our model we have assumed that there is no alternative asset where the agents can invest their savings, like for example an external asset, for the following reasons. First, we are interested in analyzing the response of aggregate savings/investment to a systemic increase in risk, namely to a risk that cannot be eliminated by households with a portfolio reallocation. In other words, we want to study the variation of aggregate savings/investment when all activities in the economy become riskier and not what happens to the investment in one sector when uncertainty in this sector (or other sectors) increases as the analysis of these reallocation effects is already well understood in Craine (1989) and Zeira (1990). Second, there are many situations where the access to external capital markets is available only to sophisticated investors while ordinary savers do not have this opportunity as a practical matter.

We now turn to the empirical evidence on the investment-uncertainty relationship. Even though the theory of investment under uncertainty has been developed with reference to the single firm, most of the evidence about the investment-uncertainty relationship is based on aggregate data.

[25] It is clear that uncertainty does not affect investment if $\gamma = 1$ or $\gamma = \gamma^*$ because in the first case $\varepsilon = 1$ and the income and substitution effects exactly compensate each other and in the second one the *flexibility effect* and the *risk aversion effect* have the same strength.

We know of only three papers, Leahy and Whited (1996), Guiso and Parigi (1999) and Bloom, Bond and Van Reenen (2005), where the investment-uncertainty relationship is investigated using micro data. Cross-country and time series studies using aggregate data are instead quite abundant. Ramey and Ramey (1995) in a sample of 92 countries (from 1960-1985) and 24 OECD countries (from 1950-1988) find that countries with higher volatility (of per capita annual growth rates or of the innovations to growth) have lower growth but their evidence suggests that investment is not an empirically important conduit between volatility and growth. Indeed, volatility appears to have a negative relationship with investment and is significant at the 10-percent level in the 92 country sample, but not in the case of the OECD sample. However, once the other standard control variables are included in the investment equation (see Levine and Renelt, 1992) the effect of uncertainty on investment is positive for the 92-country sample and negative in the OECD sample, but it is no longer significant in both cases. Thus, these authors find little evidence that the investment share of GDP is linked to volatility.

Aizenman and Marion (1999) investigate the investment-uncertainty relationship using a sample of 46 developing countries over the period 1970-1992. They find a statistically significant negative correlation between various volatility measures (two internal and one external) and private investment even when standard control variables are added. They also find that there is no aggregate investment-uncertainty correlation due to a positive relationship between uncertainty and public investment spending.

Pindyck and Solimano (1993) use cross section and time series data for a set of developing and industrialized countries and find that the volatility of the marginal profitability of capital (which they use as summary measure of uncertainty) affects aggregate investment negatively but that the size of this effect is moderate (overall larger for developing countries). A detailed analysis of the shortcomings of their measure of uncertainty and their results can be found in Eberly (1993).

Calcagnini and Saltari (2000), using data on the Italian economy for the period 1971-1995, find that changes in the level of volatility of expected demand have a negative impact on aggregate investment.[26]

Alesina and Perotti (1996) analyze a sample of 71 countries for the period 1960-1985 and find a negative correlation between indices of political and social instability, taken as proxies of uncertainty, and aggregate investment. A similar result is obtained by Barro (1991) who finds that measures of political instability and aggregate investment are negatively related for 98 countries in the period 1960-1985. These results are consistent with the finding of several other papers like, for example, Aizenman and Marion (1993).

All the studies cited so far consider investment at the country level. There are also empirical studies that use important parts of aggregate investment and that can therefore provide insightful information on the aggregate investment-uncertainty relationship. For example, Huizinga (1993) provides evidence that inflation uncertainty reduces aggregate investment using U.S. manufacturing data over the period 1954-1989. Ferderer (1993a) explores the empirical relationship between uncertainty and real gross expenditures on producers' durable equipment and the real value of contracts and orders for new plant and equipment using U.S. data from 1969 to 1989. He measures uncertainty about interest rates and other macroeconomic variables using the risk premium embedded in the term structure and finds a significant negative impact of uncertainty on investment. Ferderer (1993b) obtains the same result with a different sample and methodology.

The aggregate investment-uncertainty relationship predicted by our model depends crucially on the coefficient of relative risk aversion and the elasticity of intertemporal substitution. Two studies that estimate these two parameters separately are Attanasio and Weber (1989) and Epstein and Zin (1991). Attanasio and Weber find an elasticity of intertemporal substitution between 1.946 and 2.247 and a coefficient of relative risk aversion ranging from 5.1 to 29.9. For these values, our model predicts a negative relationship between aggregate investment and aggregate uncertainty even when we consider the case of partial capital depreciation given that the values of the coefficient of relative

[26] In the paper the investment rate refers to the whole economy while demand variables refer to industrial firms. More precisely, the authors used monthly survey data about Italian industrial firms' expectations regarding the growth in orders over the 3-4 months to come.

risk aversion obtained by Attanasio and Weber are pretty high. Epstein and Zin find a coefficient of relative risk aversion close to one and a coefficient of intertemporal substitution between 0.2 and 0.87. These parameter values would lead to a positive correlation between aggregate investment and aggregate uncertainty in the baseline version of our model (i.e. with complete capital depreciation). If we consider the model with partial capital depreciation (which is clearly more realistic), the threshold γ^* is higher than the elasticity of output with respect to labor ($\alpha = 0.67$) and it can also be higher than one. This means that with the Epstein and Zin's estimates of ε and γ our model is compatible with both a positive or a negative (or even absent) relationship between aggregate investment and aggregate uncertainty.[27]

Most empirical work estimates the key parameters of our model using an expected utility framework with a *CRRA* utility function (which implies that the coefficient of relative risk aversion and the elasticity of intertemporal substitution are linked). Hansen and Singleton (1983), Eichennbaum, Hansen, and Singleton (1988) and Gourinchas and Parker (2002) estimate a range of values for the constant relative risk aversion parameter that is consistent with both a negative and a positive aggregate investment-uncertainty relationship in our model (more or less independently on the degree of capital depreciation that we may consider).[28] Hansen and Singleton (1982) obtain an estimate of the coefficient of relative risk aversion in the range of 0.52 to 0.97, which in our model implies that the relationship between aggregate investment and aggregate uncertainty would be mostly negative if we consider the case of complete depreciation of capital and mostly positive under partial capital

[27] It is worthy to notice that the values of the coefficient of relative risk aversion and the elasticity of intertemporal substitution are clearly important to determine the sign of the investment-uncertainty relationship. However, from a quantitative point of view also the difference between γ and the threshold γ^* and ε and one are key. Indeed, if γ is very close to γ^*, as well as if ε is very close to one, then the effect of uncertainty on investment is likely to be low or statistically not significantly different from zero. On the other hand, we have just seen in the review of the empirical literature that the absence of a statistically significant relationship between investment and uncertainty is obtained in some empirical works.

[28] Hansen and Singleton (1983) find a coefficient of relative risk aversion γ in the range of zero to two. Eichennbaum, Hansen, and Singleton (1988) estimate a γ that varies between 0.5 and three while Gourinchas and Parker (2002) find a coefficient of relative risk aversion from 0.5 to 1.4.

depreciation.

The estimates of the parameters of Hansen and Singleton (1982) as well of Epstein and Zin (1991) are such that considering the results of our model with complete or partial capital depreciation may change the sign of the aggregate investment-uncertainty relationship. However, the range of the estimates of the preference parameters is generally so wide that considering our model with complete or partial capital depreciation does not change the result, namely that both a positive or a negative relationship is possible.

4 Conclusions

This paper has analyzed the relationship between aggregate investment and aggregate uncertainty when agents are risk averse. We have demonstrated that risk aversion does not necessarily imply a negative aggregate investment-uncertainty relationship. This is somewhat surprising as the existing literature appears to take for granted that the effect of increased uncertainty on investment is negative if agents are risk averse. We have clarified that the difference in the results is explained by the fact that the existing literature has analyzed the role of risk aversion in the investment-uncertainty relationship at the sectoral level while our work do it at the aggregate level.

Using recursive preferences, we show that understanding the effect of uncertainty on aggregate investment requires to separate the role of risk aversion from the role of intertemporal substitution. This allows us also to explain why the effect of uncertainty on aggregate investment can be positive even when agents are very risk averse. In particular, we find that a low elasticity of intertemporal substitution leads to a positive association between investment and uncertainty even if (or, better, especially if) agents are very risk averse. A negative relationship between aggregate investment and aggregate uncertainty requires that the relative risk aversion and the elasticity of intertemporal substitution are both relatively high or both relatively low. This result also clarifies why high levels of risk aversion always give rise to a positive aggregate investment-uncertainty relationship when preferences display $CRRA$.

Solving the dynamic investment problem analytically has required making simplifying assumptions, but it is not evident (as we have shown for the case of partial capital depreciation) that these assumptions drive our results. Still it would be interesting in future research to apply numerical solution methods to a more general version of the framework proposed in this paper.

Appendix A

This Appendix shows the mathematical details of maximization problem (7). The first order condition of this problem is

$$\begin{aligned} & (1-\beta)(1-\rho)\pi_t^{-\rho} \\ = \; & \beta(1-\rho)\left[1+(1-\beta)(1-\gamma)E_t V_{t+1}\right]^{\frac{1-\rho}{1-\gamma}-1}(1-\beta)E_t\left(\frac{dV_{t+1}}{dI_t}\right). \end{aligned} \tag{A.1}$$

Given (9), the value function at time $t+1$ is

$$V_{t+1} = \frac{\psi^{1-\gamma}\left[A_{t+1}^{\eta}+(1-\delta)\right]^{1-\gamma}K_{t+1}^{1-\gamma}-1}{(1-\beta)(1-\gamma)}. \tag{A.2}$$

From (A.2) and (8) we can determine

$$\frac{dV_{t+1}}{dI_t} = \frac{\psi^{1-\gamma}\left[A_{t+1}^{\eta}+(1-\delta)\right]^{1-\gamma}K_{t+1}^{-\gamma}}{(1-\beta)}. \tag{A.3}$$

The capital accumulation equation (8) and the investment function (10) imply that

$$K_{t+1} = (1-\mu)\left[A_t^{\eta}+(1-\delta)\right]K_t. \tag{A.4}$$

Substituting equation (A.4) into equations (A.3) and (A.2) leads respectively to

$$\frac{dV_{t+1}}{dI_t} = \frac{\psi^{1-\gamma}\left[A_{t+1}^{\eta}+(1-\delta)\right]^{1-\gamma}(1-\mu)^{-\gamma}\left[A_t^{\eta}+(1-\delta)\right]^{-\gamma}K_t^{-\gamma}}{(1-\beta)} \tag{A.5}$$

and

$$V_{t+1} = \frac{\psi^{1-\gamma}\left[A_{t+1}^{\eta} + (1-\delta)\right]^{1-\gamma}(1-\mu)^{1-\gamma}\left[A_t^{\eta} + (1-\delta)\right]^{1-\gamma} K_t^{1-\gamma} - 1}{(1-\beta)(1-\gamma)}. \tag{A.6}$$

The investment function in (10) implies that the cash flow at time t is

$$\pi_t = \mu\left[A_t^{\eta} + (1-\delta)\right] K_t. \tag{A.7}$$

Substituting equations (A.7), (A.6) and (A.5) into the first order condition (A.1) and rearranging terms we get

$$\left\{E_t\left[A_{t+1}^{\eta} + (1-\delta)\right]^{1-\gamma}\right\}^{\frac{1-\rho}{1-\gamma}} = \frac{1-\beta}{\beta}\mu^{-\rho}\psi^{-(1-\rho)}(1-\mu)^{\rho}. \tag{A.8}$$

Substituting (9), (A.6) and (A.7) into (7), after some manipulations we obtain

$$\psi^{1-\rho} = (1-\beta)\mu^{1-\rho} + \beta\psi^{1-\rho}(1-\mu)^{1-\rho}\left\{E_t\left[A_{t+1}^{\eta} + (1-\delta)\right]^{1-\gamma}\right\}^{\frac{1-\rho}{1-\gamma}}. \tag{A.9}$$

Using the first order condition (A.8) into equation (A.9) leads to the following equation

$$\psi^{1-\rho} = (1-\beta)\mu^{1-\rho} + \beta\psi^{1-\rho}(1-\mu)^{1-\rho}\frac{1-\beta}{\beta}\mu^{-\rho}\psi^{-(1-\rho)}(1-\mu)^{\rho}. \tag{A.10}$$

Rearranging terms allows us to get the unknown constant ψ of the value function

$$\psi = \left[(1-\beta)\mu^{-\rho}\right]^{\frac{1}{1-\rho}}. \tag{A.11}$$

Substituting this expression into equation (A.9) gives

$$(1-\beta)\mu^{-\rho} = (1-\beta)\mu^{1-\rho} + \beta(1-\beta)\mu^{-\rho}(1-\mu)^{1-\rho}\left\{E_t\left[A_{t+1}^{\eta} + (1-\delta)\right]^{1-\gamma}\right\}^{\frac{1-\rho}{1-\gamma}} \tag{A.12}$$

so that after some algebra we find

$$\mu = 1 - \beta^{\frac{1}{\rho}} \left\{ \left[E_t \left(A_{t+1}^{\eta} + (1-\delta) \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \right\}^{\frac{1-\rho}{\rho}} \tag{A.13}$$
$$= 1 - \beta^{\frac{1}{\rho}} \left\{ E_t \left[\left(A^{\eta} e^{\eta \vartheta_{t+1}} + 1 - \delta \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \right\}^{\frac{1-\rho}{\rho}} .$$

Given that the shock ϑ is *i.i.d.*, equations (A.11) and (A.13) imply that ψ and μ are constants and therefore that our guess for the value function (9) was correct.[29]

Appendix B

This Appendix provide the mathematical details for the derivation of the investment function when the elasticity of intertemporal substitution is equal to one ($\rho = 1$). We start by computing the limit of the aggregator function in (7) using l'Hopital's rule:

$$V_t = \lim_{\rho \to 1} \frac{\left\{ (1-\beta) \pi_t^{1-\rho} + \beta \left[1 + (1-\beta)(1-\gamma) E_t V_{t+1} \right]^{\frac{1-\rho}{1-\gamma}} \right\}^{\frac{1-\gamma}{1-\rho}} - 1}{(1-\beta)(1-\gamma)} \tag{B.1}$$
$$= \pi_t^{(1-\beta)(1-\gamma)} \left(E_t V_{t+1} \right)^{\beta} .$$

Therefore, the maximization problem is

$$V_t = \max_{\{I_t\}} \pi_t^{(1-\beta)(1-\gamma)} \left(E_t V_{t+1} \right)^{\beta} \tag{B.2}$$

where π_t is given by (6). The guess of the functional form for the value function is the same of the VNM isoelastic utility

$$V_t = \frac{\psi^{1-\gamma} \left[A_t^{\eta} + (1-\delta) \right]^{1-\gamma} K_t^{1-\gamma}}{(1-\beta)(1-\gamma)} \tag{B.3}$$

with the investment function still given by equation (10).

[29] We restrict the values of the parameters ρ, γ, β, δ, η, $\overline{\vartheta}$, σ^2 and A to the case where the solution is such that $\mu > 0$.

The first order condition of maximization problem (B.2) is

$$-\left(1-\beta\right)\left(1-\gamma\right)\pi_t^{-1} = \beta\left(E_t V_{t+1}\right)^{-1} E_t\left(\frac{dV_{t+1}}{dI_t}\right). \tag{B.4}$$

From (B.3) we know that

$$V_{t+1} = \frac{\psi^{1-\gamma}\left[A_{t+1}^{\eta}+(1-\delta)\right]^{1-\gamma} K_{t+1}^{1-\gamma}}{\left(1-\beta\right)\left(1-\gamma\right)} \tag{B.5}$$

and by using this equation with the capital accumulation equation (8) we obtain

$$\frac{dV_{t+1}}{dI_t} = \frac{\psi^{1-\gamma}\left[A_{t+1}^{\eta}+(1-\delta)\right]^{1-\gamma} K_{t+1}^{-\gamma}}{\left(1-\beta\right)}. \tag{B.6}$$

Remember that the investment function in (10) implies that the capital accumulation equation and the cash flow at time t can be expressed by equations (A.4) and (A.7) respectively. Therefore, substituting equation (A.4) into (B.6) yields

$$\frac{dV_{t+1}}{dI_t} = \frac{\psi^{1-\gamma}\left[A_{t+1}^{\eta}+(1-\delta)\right]^{1-\gamma}\left(1-\mu\right)^{-\gamma}\left[A_t^{\eta}+(1-\delta)\right]^{-\gamma} K_t^{-\gamma}}{\left(1-\beta\right)}. \tag{B.7}$$

Using the capital accumulation equation (8), the value function (B.5) can be rewritten as

$$V_{t+1} = \frac{\psi^{1-\gamma}\left[A_{t+1}^{\eta}+(1-\delta)\right]^{1-\gamma}\left(1-\mu\right)^{1-\gamma}\left[A_t^{\eta}+(1-\delta)\right]^{1-\gamma} K_t^{1-\gamma}}{\left(1-\beta\right)\left(1-\gamma\right)}. \tag{B.8}$$

Substituting equations (B.7), (B.8) and (A.7) into the first order condition (B.4) implies that

$$\begin{aligned} & \left(1-\beta\right)\left(1-\gamma\right)\mu^{-1}\left[A_t^{\eta}+(1-\delta)\right]^{-1} K_t^{-1} \qquad \text{(B.9)} \\ = & \left\{\beta\left(1-\beta\right)\left(1-\gamma\right)\psi^{1-\gamma} E_t\left[A_{t+1}^{\eta}+(1-\delta)\right]^{1-\gamma}\left(1-\mu\right)^{1-\gamma}\left[A_t^{\eta}+(1-\delta)\right]^{1-\gamma} K_t^{1-\gamma}\right\}^{-1} \\ & \frac{\psi^{1-\gamma} E_t\left[A_{t+1}^{\eta}+(1-\delta)\right]^{1-\gamma}\left(1-\mu\right)^{-\gamma}\left[A_t^{\eta}+(1-\delta)\right]^{-\gamma} K_t^{-\gamma}}{\left(1-\beta\right)}. \end{aligned}$$

Simplifying we get

$$\mu = 1-\beta. \tag{B.10}$$

Substituting this result in the functional equation (B.1) and using the equations (B.3), (B.8) and (A.7) we get

$$\begin{aligned} &\frac{\psi^{1-\gamma}\left[A_t^{\eta}+(1-\delta)\right]^{1-\gamma}K_t^{1-\gamma}}{(1-\beta)(1-\gamma)} \\ = \; &(1-\beta)^{(1-\beta)(1-\gamma)}\left[A_t^{\eta}+(1-\delta)\right]^{(1-\beta)(1-\gamma)}K_t^{(1-\beta)(1-\gamma)}(1-\beta)^{-\beta}(1-\gamma)^{-\beta} \\ &\psi^{\beta(1-\gamma)}\left\{E_t\left[A_{t+1}^{\eta}+(1-\delta)\right]^{1-\gamma}\right\}^{\beta}\beta^{\beta(1-\gamma)}\left[A_t^{\eta}+(1-\delta)\right]^{\beta(1-\gamma)}K_t^{\beta(1-\gamma)}. \end{aligned} \tag{B.11}$$

Rearranging terms

$$\psi=(1-\beta)^{\frac{1}{1-\gamma}+1}(1-\gamma)^{\frac{1}{1-\gamma}}\beta^{\frac{\beta}{1-\beta}}\left\{E_t\left[A_{t+1}^{\eta}+(1-\delta)\right]^{1-\gamma}\right\}^{\frac{\beta}{(1-\beta)(1-\gamma)}} \tag{B.12}$$

which is a constant given that A_{t+1} is *i.i.d.*. Hence, our guess was correct.

Appendix C

In this Appendix we derive the investment function for the case of unit coefficient of relative risk aversion ($\gamma=1$). The limit of the aggregator function in (7) is

$$\begin{aligned} V_t &= \lim_{\gamma\to 1}\frac{\left\{(1-\beta)\pi_t^{1-\rho}+\beta\left[1+(1-\beta)(1-\gamma)E_tV_{t+1}\right]^{\frac{1-\rho}{1-\gamma}}\right\}^{\frac{1-\gamma}{1-\rho}}-1}{(1-\beta)(1-\gamma)} \\ &= \frac{\ln\left\{(1-\beta)\pi_t^{1-\rho}+\beta e^{(1-\rho)(1-\beta)E_tV_{t+1}}\right\}}{(1-\rho)(1-\beta)} \end{aligned} \tag{C.1}$$

and the corresponding maximization problem

$$V_t=\max_{\{I_t\}}\frac{\ln\left\{(1-\beta)\pi_t^{1-\rho}+\beta e^{(1-\rho)(1-\beta)E_tV_{t+1}}\right\}}{(1-\rho)(1-\beta)}. \tag{C.2}$$

We guess the following functional form for the value function

$$V_t=(1-\beta)^{-1}\ln\left[A_t^{\eta}+(1-\delta)\right]+(1-\beta)^{-1}\ln K_t+\psi \tag{C.3}$$

which again implies that the investment function is given by equation (10). This also means that equations (A.4) and (A.7) hold.

The first order condition of (C.2) is

$$\pi_t^{-\rho} = \beta e^{(1-\rho)(1-\beta)E_t V_{t+1}} E_t \left(\frac{dV_{t+1}}{dI_t} \right). \tag{C.4}$$

From the value function (C.3) it follows that

$$V_{t+1} = (1-\beta)^{-1} \ln \left[A_{t+1}^{\eta} + (1-\delta) \right] + (1-\beta)^{-1} \ln K_{t+1} + \psi. \tag{C.5}$$

Using equation (C.5) and the capital accumulation equation (8) we obtain

$$\frac{dV_{t+1}}{dI_t} = \frac{1}{(1-\beta)\, K_{t+1}} \tag{C.6}$$

and with the capital accumulation equation (A.4)

$$\frac{dV_{t+1}}{dI_t} = \frac{1}{(1-\beta)\,(1-\mu)\,[A_t^{\eta} + (1-\delta)]\, K_t}. \tag{C.7}$$

Taking into account the equations (A.7), (C.5) and (C.7), the first order condition (C.4) becomes

$$\begin{aligned} & \mu^{-\rho}\, [A_t^{\eta} + (1-\delta)]^{-\rho}\, K_t^{-\rho} \qquad \text{(C.8)} \\ = \; & \beta \cdot e^{\left\{(1-\rho)(1-\beta)E_t\left[(1-\beta)^{-1}\ln\left[A_{t+1}^{\eta}+(1-\delta)\right]+(1-\beta)^{-1}\ln K_{t+1}+\psi\right]\right\}} \cdot \\ & \cdot \left[(1-\beta)\,(1-\mu)\,(A_t^{\eta} + (1-\delta))\, K_t\right]^{-1}. \end{aligned}$$

After some algebra this first order condition can be rewritten as

$$\frac{(1-\mu)^{\rho}}{\mu^{\rho}} \frac{1-\beta}{\beta} = e^{\left\{E_t \ln\left[A_{t+1}^{\eta}+(1-\delta)\right]^{(1-\rho)}+(1-\rho)(1-\beta)\psi\right\}}. \tag{C.9}$$

Substituting this result in the functional equation (C.2) and using the equations (C.3), (C.5) and (A.7), after some manipulation we get

$$(1-\rho)\left[\ln A_t^{\eta}+\ln K_t+(1-\beta)\,\psi\right] \tag{C.10}$$
$$= \ln\left\{(1-\beta)\,\mu^{1-\rho}A_t^{\eta(1-\rho)}K_t^{1-\rho}+\beta\cdot e^{(1-\rho)\ln K_{t+1}}\cdot e^{E_t\ln A_{t+1}^{\eta}+(1-\rho)(1-\beta)\psi}\right\}.$$

Using the fact that $e^{(1-\rho)\ln K_{t+1}}=K_{t+1}^{1-\rho}$ and the first order condition (C.9) we obtain

$$(1-\rho)\ln\left[A_t^{\eta}+(1-\delta)\right]+(1-\rho)\ln K_t+(1-\rho)\,(1-\beta)\,\psi \tag{C.11}$$
$$= \ln\{(1-\beta)\,\mu^{1-\rho}\left[A_t^{\eta}+(1-\delta)\right]^{1-\rho}K_t^{1-\rho}+$$
$$+\beta\,(1-\mu)^{1-\rho}\left[A_t^{\eta}+(1-\delta)\right]^{1-\rho}K_t^{1-\rho}\frac{(1-\mu)^{\rho}}{\mu^{\rho}}\frac{1-\beta}{\beta}\}$$

and after some algebra

$$\psi=\frac{\ln(1-\beta)-\rho\ln\mu}{(1-\rho)\,(1-\beta)}. \tag{C.12}$$

Substituting this expression in the maximization problem (C.2) we can recover the value of μ

$$\mu=1-\beta^{\frac{1}{\rho}}e^{\frac{1-\rho}{\rho}\left\{E_t\ln\left[A_{t+1}^{\eta}+(1-\delta)\right]\right\}} \tag{C.13}$$

which is a constant. Therefore, also ψ is constant and this implies that our guess was correct.

Similarly to the general case, a closed-form solution for the derivative of the investment with respect to the volatility of the shock can be obtained only if we assume $\delta=1$. Then,

$$\begin{aligned}\mu &= 1-\beta^{\frac{1}{\rho}}e^{\frac{1-\rho}{\rho}\left\{E_t\ln\left(A_{t+1}^{\eta}\right)\right\}} \\ &= 1-\beta^{\frac{1}{\rho}}A^{\eta\frac{1-\rho}{\rho}}e^{\frac{1-\rho}{\rho}\{E_t(\eta\vartheta_{t+1})\}} \\ &= 1-\beta^{\frac{1}{\rho}}A^{\eta\frac{1-\rho}{\rho}}e^{\eta\frac{1-\rho}{\rho}\left(\overline{\vartheta}-\frac{1}{2}\sigma^2\right)} \\ &= 1-\beta^{\varepsilon}A^{\eta(\varepsilon-1)}e^{\eta(\varepsilon-1)\left(\overline{\vartheta}-\frac{1}{2}\sigma^2\right)}.\end{aligned} \tag{C.14}$$

In this case an increase in aggregate uncertainty has a negative effect on aggregate investment if the intertemporal substitution elasticity is greater than one ($\varepsilon>1$, i.e. $\rho<1$) and a positive effect when $\varepsilon < 1$. This result and the interpretation correspond to the case discussed above where $\gamma > \alpha$. Although the procedure followed has involved a transformation of the utility function, the final result could be obtained directly form equations (16) and (18) substituting $\gamma = 1$.

Appendix D. Preference shocks and real wage uncertainty

In this Appendix we propose an extension to the baseline version of our model to discuss the effects of real wage uncertainty on aggregate investment. To this purpose, we assume that the opportunity cost of supplying labor ϕ can change over time due to a preference shock and that ϕ_t is an identically and independently distributed log-normal random variable. In particular, $\phi_t = \Phi e^{\varphi_t}$, where Φ is a positive constant and φ_t is *i.i.d.* and normally distributed with variance v^2 and mean $\varpi - \frac{1}{2}v^2$ so that an increase in v^2 increases the variance of ϕ_t without affecting its expected value. We also assume that the parameters ϖ and v^2 are constant over time and that the shocks ϑ_t and φ_t are independent.[30]

The representative agent knows the value of ϕ_t before taking the labor and consumption decisions at time t but she only knows the distribution of ϕ for the following periods. In equilibrium, $w_t = \phi_t$ and therefore an increase in the variance v^2 of the preference shock implies a higher real wage uncertainty.

From the optimization problem of the firm we obtain that the cash flow at time t is still given by (6) but that

$$A_t = De^{\vartheta_t} w_t^{-\alpha} \tag{D.1}$$

where $D \equiv (1-\alpha)^{1-\alpha} \alpha^{\alpha} B$. It is straightforward to verify that the solution of the model derived in Section 2 is still valid and that the aggregate investment function is given by (10) and (12).

[30] The *i.i.d.* assumption of ϕ_t is essential to have an analytical solution for the investment function, while the assumptions of log-normality of ϕ_t and of independence between the preference and the technological shocks are key to obtain an analytical solution for the relationship between each form of uncertainty and aggregate investment.

What changes is the marginal revenue product of capital A_t^η that now contains two sources of uncertainty (see (D.1)). Again, to derive analytically the relationship between aggregate investment and aggregate uncertainty we need to assume that capital fully depreciates in production ($\delta = 1$). The investment function is still $I_t = (1-\mu)A_t^\eta K_t$ and the effects of technological and preference uncertainty on investment are given by $\frac{\partial I_t}{\partial \sigma^2} = -\frac{\partial \mu}{\partial \sigma^2} A_t^\eta K_t$ and $\frac{\partial I_t}{\partial v^2} = -\frac{\partial \mu}{\partial v^2} A_t^\eta K_t$ respectively. Therefore, as before, to determine the effect of uncertainty on investment we need to analyze the effect of uncertainty on the parameter μ. This parameter is still given by (16), namely $\mu = 1 - \beta^\varepsilon \check{Z}^{\varepsilon - 1}$, but the certainty equivalent of the return to capital is now equal to[31]

$$\check{Z} \equiv \left[E_t \left(A_{t+1}^{\eta(1-\gamma)} \right) \right]^{\frac{1}{1-\gamma}} = D^\eta \Phi^{-\alpha\eta} e^{\eta\overline{\vartheta} - \alpha\eta\varpi + \frac{1}{2}(\alpha-\gamma)\eta^2\sigma^2 + \frac{1}{2}\alpha\eta^2(1-\alpha\gamma)v^2}. \tag{D.2}$$

It is immediate that the results obtained in Section 2 on the relationship between aggregate technological uncertainty and aggregate investment are also valid in this framework. Hence, we now concentrate our analysis on the effects of real wage uncertainty (generated by the preference shocks) on aggregate investment. From (D.2) we observe that the certainty equivalent of the return to capital $\check{Z}$ is increasing in the variance v^2 of the preference shock if $\gamma < 1/\alpha$ and decreasing if $\gamma > 1/\alpha$. Again, we have the usual two effects at work. One is the *flexibility effect* that originates from the fact that the consumer-producer chooses the optimal amount of labor after observing the realization of the preference shock. This effect implies that higher uncertainty increases the expected return to capital. The other is the *risk aversion effect* that comes from the individual's risk aversion and that reduces $\check{Z}$ when uncertainty increases. If the coefficient of relative risk aversion is higher than $1/\alpha \approx 1.5$ (assuming that $\alpha = 0.67$), then the *risk aversion effect* prevails over the *flexibility effect* and the certainty equivalent of the return to capital goes down as uncertainty increases. The opposite holds for $\gamma < 1/\alpha$.

The investment-wage uncertainty relationship is straightforward to obtain. If we define $\gamma^* \equiv 1/\alpha$, then Table 4 and Table 6, that we have derived for the case of partial capital depreciation, also

[31] Equation (D.2) is obtained using (D.1) with the equilibrium condition $w_{t+1} = \phi_{t+1}$ and taking into account that the shocks ϑ_t and φ_t are independent.

summarize the relationship between real wage uncertainty and aggregate investment under recursive and *CRRA* preferences respectively. A negative relationship between real wage uncertainty and aggregate investment under recursive preferences requires that the relative risk aversion and the elasticity of intertemporal substitution are both relatively high or both relatively low. This is the same result obtained with technological uncertainty. The only difference is on the threshold of the coefficient of relative risk aversion γ^* which is now $(1/\alpha) > 1$ instead of $\alpha < 1$. This also implies that the region of γ where there is a negative correlation between investment and uncertainty under *CRRA* preferences is between one and $1/\alpha$.

This analysis allows us to conclude that the existence of uncertainty on the real wage does not change the main results of our model.

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Table 1. The aggregate investment-uncertainty relationship with recursive preferences and $\delta = 1$.

$\frac{dI}{d\sigma^2}$	$\gamma > \alpha$	$\gamma = \alpha$	$\gamma < \alpha$
$\varepsilon < 1$; $\rho > 1$	> 0	$= 0$	< 0
$\varepsilon = 1$; $\rho = 1$	$= 0$	$= 0$	$= 0$
$\varepsilon > 1$; $\rho < 1$	< 0	$= 0$	> 0

Table 2. The aggregate investment-uncertainty relationship with *CRRA* preferences and $\delta = 1$.

	$0 < \gamma < \alpha$	$\gamma = \alpha$	$\alpha < \gamma < 1$	$\gamma = 1$	$\gamma > 1$
$\frac{dI}{d\sigma^2}$	> 0	$= 0$	< 0	$= 0$	> 0

Table 3. Threshold values of γ^*. Parameter values: $A = 1$, $\alpha = 0.67$.

	Threshold values of γ^* with $\overline{\vartheta}$=0.1					
δ	σ=.05	σ=.1	σ=.2	σ=.3	σ=.4	σ=.5
0	1.164	1.161	1.148	1.127	1.1	1.069
0.1	1.115	1.113	1.104	1.088	1.065	1.039
0.2	1.066	1.064	1.059	1.047	1.03	1.007
0.3	1.016	1.016	1.013	1.005	0.992	0.976
0.4	0.967	0.967	0.967	0.964	0.955	0.942
0.5	0.918	0.919	0.921	0.921	0.916	0.907
0.6	0.868	0.87	0.873	0.876	0.875	0.871
0.7	0.819	0.82	0.825	0.83	0.832	0.832
0.8	0.769	0.771	0.775	0.781	0.786	0.788
0.9	0.72	0.721	0.724	0.729	0.734	0.739
1	0.67	0.67	0.67	0.67	0.67	0.67

	Threshold values of γ^* with $\overline{\vartheta}$=0.2					
δ	σ=.05	σ=.1	σ=.2	σ=.3	σ=.4	σ=.5
0	1.035	1.035	1.031	1.022	1.006	0.988
0.1	0.999	0.999	0.997	0.991	0.98	0.964
0.2	0.963	0.963	0.963	0.96	0.952	0.939
0.3	0.926	0.927	0.929	0.928	0.923	0.913
0.4	0.89	0.891	0.894	0.895	0.893	0.887
0.5	0.853	0.855	0.859	0.862	0.862	0.859
0.6	0.817	0.818	0.823	0.828	0.83	0.83
0.7	0.78	0.782	0.786	0.792	0.796	0.798
0.8	0.744	0.745	0.749	0.754	0.76	0.764
0.9	0.707	0.707	0.71	0.714	0.719	0.724
1	0.67	0.67	0.67	0.67	0.67	0.67

	Threshold values of γ^* with $\overline{\vartheta}$=0.3					
δ	σ=.05	σ=.1	σ=.2	σ=.3	σ=.4	σ=.5
0	0.94	0.941	0.942	0.94	0.934	0.923
0.1	0.913	0.914	0.917	0.917	0.912	0.904
0.2	0.886	0.888	0.891	0.892	0.89	0.884
0.3	0.859	0.861	0.865	0.868	0.868	0.862
0.4	0.832	0.834	0.838	0.843	0.844	0.843
0.5	0.805	0.807	0.812	0.817	0.82	0.82
0.6	0.778	0.78	0.785	0.79	0.795	0.797
0.7	0.751	0.753	0.757	0.763	0.768	0.771
0.8	0.724	0.725	0.729	0.734	0.739	0.744
0.9	0.697	0.698	0.7	0.703	0.708	0.712
1	0.67	0.67	0.67	0.67	0.67	0.67

	Threshold values of γ^* with $\overline{\vartheta}$=0.4					
δ	σ=.05	σ=.1	σ=.2	σ=.3	σ=.4	σ=.5
0	0.87	0.871	0.875	0.877	0.876	0.872
0.1	0.85	0.851	0.855	0.859	0.859	0.857
0.2	0.83	0.831	0.836	0.84	0.842	0.841
0.3	0.81	0.812	0.816	0.821	0.824	0.824
0.4	0.79	0.792	0.796	0.802	0.806	0.807
0.5	0.77	0.772	0.776	0.782	0.787	0.789
0.6	0.75	0.751	0.756	0.761	0.767	0.77
0.7	0.73	0.731	0.735	0.74	0.746	0.75
0.8	0.71	0.711	0.714	0.718	0.723	0.728
0.9	0.69	0.691	0.692	0.695	0.699	0.702
1	0.67	0.67	0.67	0.67	0.67	0.67

Table 4. The aggregate investment-uncertainty relationship with recursive preferences and $\delta < 1$.

$\frac{dI}{d\sigma^2}$	$\gamma > \gamma^*$	$\gamma = \gamma^*$	$\gamma < \gamma^*$
$\varepsilon < 1$; $\rho > 1$	> 0	$= 0$	< 0
$\varepsilon = 1$; $\rho = 1$	$= 0$	$= 0$	$= 0$
$\varepsilon > 1$; $\rho < 1$	< 0	$= 0$	> 0

Table 5. The aggregate investment-uncertainty relationship with *CRRA* preferences, $\delta < 1$ and $\gamma^* < 1$.

	$0 < \gamma < \gamma^*$	$\gamma = \gamma^*$	$\gamma^* < \gamma < 1$	$\gamma = 1$	$\gamma > 1$
$\frac{dI}{d\sigma^2}$	> 0	$= 0$	< 0	$= 0$	> 0

Table 6. The aggregate investment-uncertainty relationship with *CRRA* preferences, $\delta < 1$ and $\gamma^* > 1$.

	$0 < \gamma < 1$	$\gamma = 1$	$1 < \gamma < \gamma^*$	$\gamma = \gamma^*$	$\gamma > \gamma^*$
$\frac{dI}{d\sigma^2}$	> 0	$= 0$	< 0	$= 0$	> 0

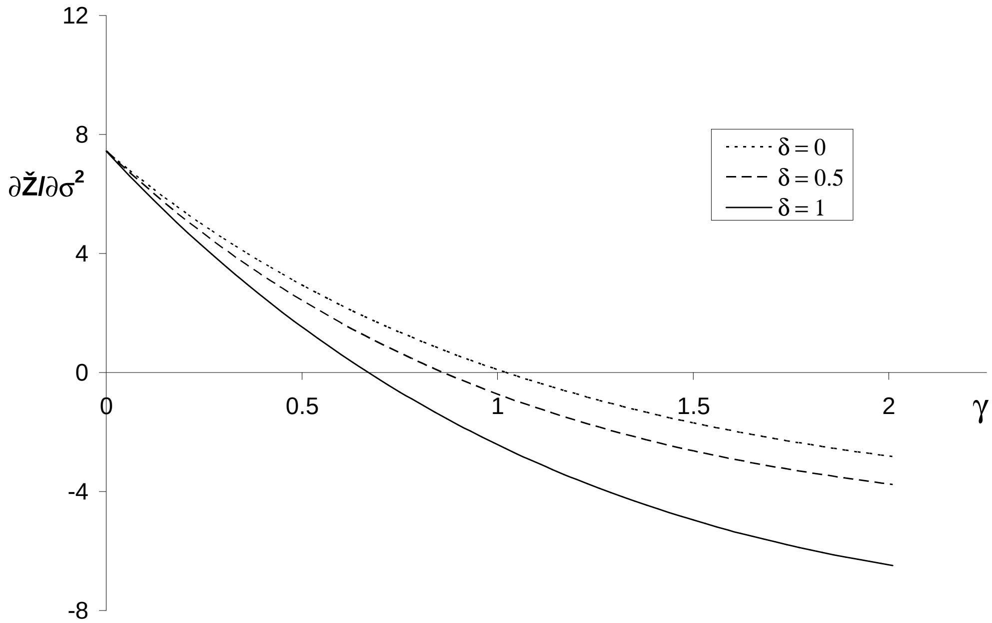

Figure 1. The derivative of $\check{Z}$ with respect to σ^2. Parameter values: $\sigma = 0.3$, $\overline{\vartheta} = 0.2$, $A = 1$, $\alpha = 0.67$.

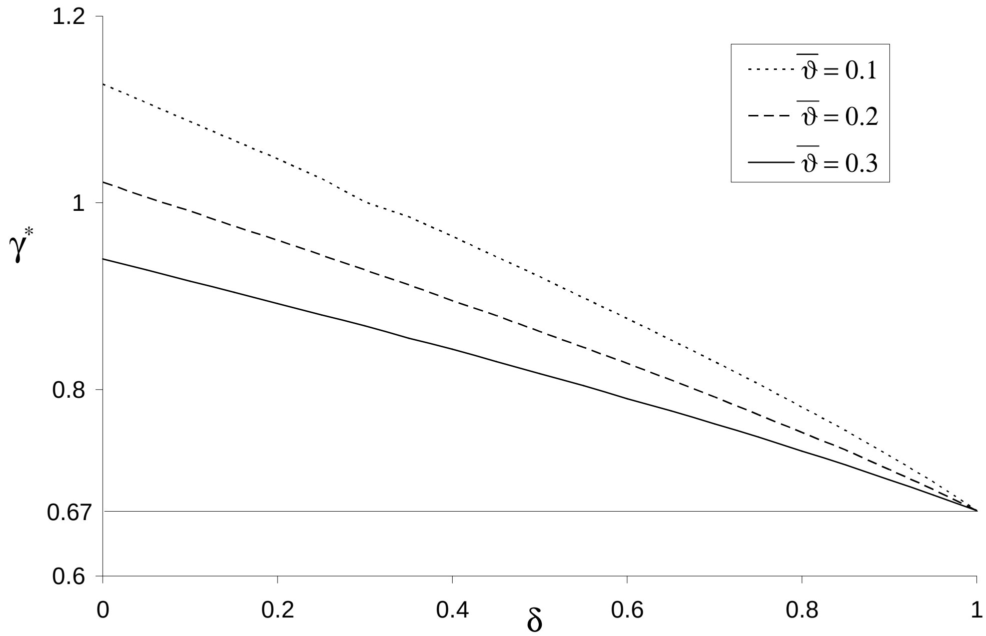

Figure 2. The effect of a variation of $\overline{\vartheta}$ on $\gamma^*(\delta)$. Parameter values: $\sigma = 0.3$, $A = 1$, $\alpha = 0.67$.

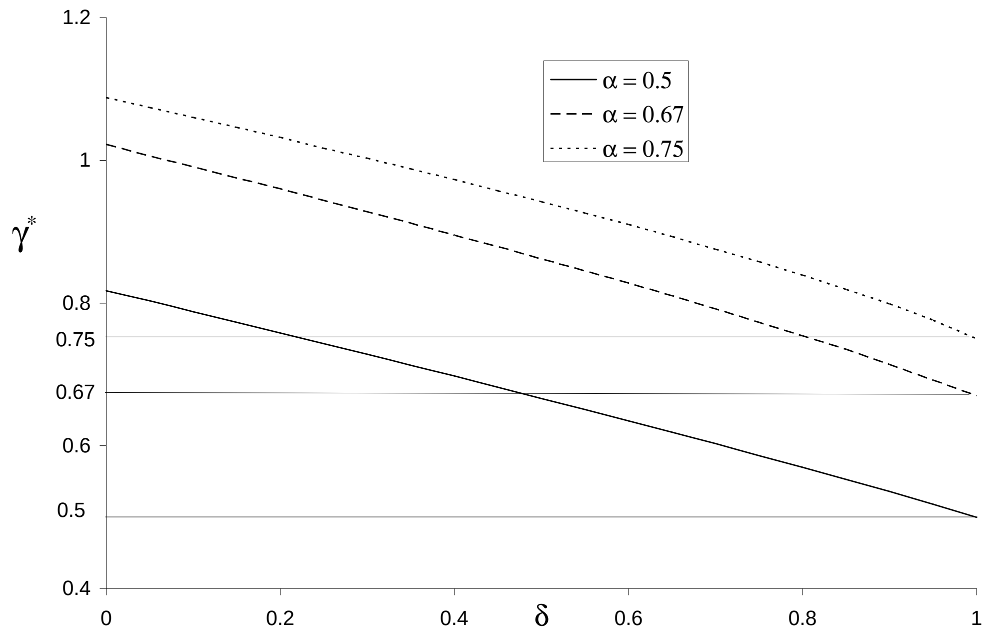

Figure 3. The effect of a variation of α on $\gamma^{*}(\delta)$. Parameter values: $\sigma = 0.3$, $\overline{\vartheta} = 0.2$, $A = 1$.