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Damping models for PDEs: a port-Hamiltonian formulation.

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Abstract

Some methodologies developped by practitioners to model damping phenomena for distributed parameter systems will be listed and revisited. We will try to put them in the framework of port-Hamiltonian systems, either linear or nonlinear, either finite or infinite-dimensional. The most significant distinction occurs between *static* and *dynamic* damping models: for the latter, a particular attention will be paid to the notion of memory variables, such as those defined in mechanical engineering.

The first part of the presentation is devoted to *static* damping models: for conservative mechanical systems, the so-called Caughey series are known to define the class of damping matrices that preserve eigenspaces. In particular, for finite-dimensional systems, these matrices prove to be a polynomial of one reduced matrix, which depends on the mass and stiffness matrices. Damping is ensured whatever the eigenvalues of the conservative problem if and only if the polynomial is positive for positive scalar values.
This work first recasts this result in the port-Hamiltonian framework by introducing a port variable corresponding to internal energy dissipation (resistive element). Moreover, this formalism naturally allows to cope with systems including gyroscopic effects (gyrators).
Second, generalizations to the infinite-dimensional case are considered. They consists of extending the previous polynomial class to rational functions and more general functions of operators (instead of matrices), once the appropriate functional framework has been defined. In this case, the resistive element is modelled by a given static operator, such as an elliptic PDE. These results are illustrated on several PDE examples: the Webster horn equation, the Bernoulli beam equation; the damping models under consideration are fluid, structural, rational and generalized fractional Laplacian or bi- Laplacian.

The second part of the presentation is devoted to *dynamic* damping models: the aim is to study a conservative wave equation coupled to a diffusion equation : this coupled system naturally arises in musical acoustics when viscous and

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thermal effects at the wall of the duct of a wind instrument are taken into account. The resulting equation, known as Webster-Lokshin model, has variable coefficients in space, and a fractional derivative in time. The port-Hamiltonian formalism proves adequate to reformulate this coupled system, and could enable another well-posedness analysis, using classical results from port-Hamiltonian systems theory.
First, an equivalent formulation of fractional derivatives is obtained thanks to so-called diffusive representations: this is the reason why we first concentrate on rewriting these diffusive representations into the port-Hamiltonian formalism; two cases must be studied separately, the fractional integral operator as a low-pass filter, and the fractional derivative operator as a high-pass filter.
Second, a standard finite-dimensional mechanical oscillator coupled to both types of dampings, either low-pass or high-pass, is studied as a coupled pHs. The more general PDE system of a wave equation coupled with the diffusion equation is then found to have the same structure as before, but in an appropriate infinite-dimensional setting, which is fully detailled.

Keywords: energy storage, port-Hamiltonian systems, damping, Caughey series, partial differential equations, memory variables, diffusive representations, fractional calculus, fractional Laplacian.

Collaborators: The first part of the talk is joint work with Thomas Hélie (IRCAM & CNRS, Paris, France), see [1]. The second part of the talk is joint work with Yann Le Gorrec (FEMTO-ST institute, Besançon, France), see [2].

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