# Flexible resources allocation techniques: characteristics and modelling


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**Abstract:** At the interface between engineering, economics, social sciences and humanities, industrial engineering aims to provide answers to various sectors of business problems. One of these problems is the adjustment between the workload needed by the work to be realised and the availability of the company resources. The objective of this work is to help to find a methodology for the allocation of flexible human resources in industrial activities planning and scheduling. This model takes into account two levers of flexibility, one related to the working time modulation, and the other to the varieties of tasks that can be performed by a given resource (multi-skilled actor). On the one hand, multi-skilled actors will help to guide the various choices of the allocation to appreciate the impact of these choices on the tasks durations. On the other hand, the working time modulation that allows actors to have a work planning varying according to the workload which the company has to face.



**Keywords:** multi-skills; working time modulation; flexible resources; allocation; modelling; optimisation, planning; scheduling; genetic algorithms.

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## 1 Introduction and background of this study

In industrial groups, the achievement of an activity is organised by involving resources (Baptiste et al., 2005) that lead to the creation of goods or services. These resources are generally one of the four major types: the operators (they are involved in the realisation process), equipment (machines, tools, facilities, etc.), the materials (which undergo transformations) and technical as well as procedural information (scales, guidelines, nomenclature, etc.). To find an adequate solution, engineers are required to formalise various problems showing an important combinatorial aspect. One of these problems is the adjustment between the workload represented by the work to be done and the availability of company resources. Among all these resources, the human resource is one of the most difficult to control and manage because of the multiple regulation aspects, their limited capacity and unexpected events – e.g. workload limited time, illness, holidays, etc. – so, there is an availability problem (Lin and Gen, 2008; Vidal, 2000). However, the realisation of a good or a service is ensured by a succession of tasks, consuming human resources (Bennour, 2004), which are by nature renewable from period to period, cumulative (in which an amount of their availability is consumed to carry out some tasks thus they are depleted and replenished overtime periods) (Esquirol and Lopez, 1999; Schwindt, 2005) and transforming the morphological or space matter characteristics (Giard, 2003). Therefore, to carry out an activity, companies are organised around a common concept, i.e. planning. However, this work of planning runs up against double problems: firstly, to respect scheduling constraints between the tasks and secondly, to check the availability of the actors to perform these tasks. If the former is generally imposed by the logic of carrying out the activities, then the availability problem of human resources is often considered as a part of their predetermined capacity to work

regularly while respecting the working time regulations. Thus the improvement of the allocation systems through the development and modelling of flexibility factors gives the ability to choose between several options, because the market requests become increasingly variable and turbulent. Thus, one of the best ways of facing such turbulent environment changes is the workforce flexibility (Davis et al., 2009).

Recently, many academic research works were conducted dealing with the investigation of workforce flexibility in different applications, and the importance of implementing cross training programmes (shifting operators to work for different workstations) to develop multi-skilled workforce; this results from a strategy to preserve and develop the firms' core competences. In addition, their ability to react and cover the non-predictable changes of the working environment will be developed. For example, the project scheduling problem with the flexibility of multi-skilled workforce was introduced by Bellenguez-Morineau (2006), and then the work of Bellenguez-Morineau and Néron (2007), both works optimising the project duration in presence of traditional finish–start precedence and resources availability constraints. In such problems, each task requires a number of skills for its realisation, and each mission, corresponding to a given skill, can be carried out by one or more resource(s) at a time; furthermore, each worker may master one or more skill(s) with consideration to the homogeneous nature of operators' efficiencies in different competences, i.e. each actor neither master nor the considered competence. This homogeneous nature of workers' competences has been introduced in many works and applications, e.g. production management (Franchini et al., 2001), software production technology (Li and Womer, 2009) or software development (Drezet and Billaut, 2008), and the power stations and energy production (Eitzen et al., 2004). In the homogeneous modelling of operators' efficiencies, the tasks are often approached with predetermined durations and release/due dates: this considerably reduces the flexibility of carrying out the activity and the complexity of the problem – all together as it reduces the model's accuracy.

However, Duquenne et al. (2005) introduced an industrial application methodology for workforce allocation based on their multi-competency with task execution times influenced by the actor's different efficiencies. We can find this heterogeneous nature of the multi-skilled actors' efficiencies in different applications, e.g. in the service centres (Valls et al., 2009), information technologies' projects (Heimerl and Kolisch, 2009), product design and development (Hlaoittinun et al., 2010) and the projects portfolio selection (Gutjahr et al., 2008). There are many forms of modelling the workforce efficiencies, aiming at quantifying each one, and from which one can calculate the project tasks' durations. Some research works transpose this parameter as a real value $\in [0, 1]$ (see e.g. Duquenne et al., 2005; Gutjahr et al., 2008). But, Hlaoittinun et al. (2010) described it as a multiplication factor within the interval [1, 2]: if the actor has a value of unity for a given competence, he will work with the standard duration to execute the corresponding task; but if he has a value of '2', the task execution time will be doubled. For Heimerl and Kolisch (2009), these efficiencies take values greater than zero. Others such as Valls et al. (2009) classified the actors into groups (senior, standard and junior), each one has a given productivity factor with respect to a standard one.

As well known, when human resources are involved in a problem, they always come with their working time regulations. So, Edi and Duquenne (2006) and Drezet and Billaut (2008) presented their problems of scheduling multi-skilled actors with respect to resources legislation constraints. On the other hand, the reduction of working hours and the use of annualised working time allowed workers to work according to a varying

timetable below a maximum number of working hours per year, to cover the seasonal variations. Many research works have been conducted to workforce scheduling with such new working time flexibility, with or without the overtime qualitative flexibility (see e.g. Azmat and Widmer, 2004; Hertz et al., 2010; Hung, 1999; Inman, 1996; Kane, 2001). These work do not take into account the flexibility of the resources allocation offered by actor's multiple competences. In this context, we wondered about the possibility of modular tasks realisation times, starting from the possibilities of allocating the actors according to their multi-skills, with taking into account the working time modulation (Edi and Duquenne, 2006). This work falls under a logic of search for a flexible methodology of actors allocation, to give reactivity to the companies during the scheduling of their business, while enabling them to adjust their availability with the workloads of the activities. This paper is dedicated to the characterisation and the modelling of the problem.

This paper is organised as follows: Section 2 details the problem characterisation. Section 3 discusses the problem modelling approach. Section 4 describes the problem resolution method and Section 5 illustrates its application to a light numerical example and describes the results obtained. In Section 6, we present our conclusions and perspectives.

## 2 Problem characterisations

As mentioned above, our approach is motivated by the importance of developing flexibility within firms and of finding a compromise between the project costs and the consideration of the resources' availabilities. Therefore, in this section, we present the characterisation of the different aspects of the project scheduling with multi-skilled human resources, in which each individual masters a static and heterogeneous set of skills. Here we do not consider the dynamic aspects of the workforce skills (i.e. the evolution of competences with growing experience – see e.g. the works of Gutjahr et al. (2008) and Hlaoittinun et al. (2010)): our only interest is to present a flexible model that can be used to reduce the project cost, by using multi-skilled workforce and flexible working time strategy. In the following sections, we will introduce the different model significations and its different dimensions.

### *2.1 Writing conventions*

Several terms have emerged to describe the various quantities that we handled – and they sometimes have in our text a specific meaning that may differ from the usual one. We define here some vocabulary in use in this work, and following are the way it should be understood:

*Activity*: an activity here is taken in the broad sense, and may refer to a part either of a project (seen a group of unique and original tasks) or of a manufacturing process (series of tasks better defined and calibrated, in the execution of a planning sheet). Thereafter, we will deal activity as a set of scheduled tasks without pre-conceived ideas about the application domain.

*Actor*: in this work, we will call 'actor' a human resource (taken individually).

*Efficiency*: refers to an actor and for a given competence or 'skill': it characterises the possibility for an actor to perform or not some tasks requiring this competence; it also quantifies the effort needed for the proper execution of this task. It is represented, for one actor and one competence, by a real number between zero (inability of the actor to do this competence) and one (full capacity with nominal efficiency).

*Equivalent workforce*: relative to one competence, it is the sum of the efficiencies of all actors allowed to practise this competence. This number may not be an integer.

*Real workforce*: the number of actors, in whole value.

*Workload*: refers to a task and denotes the working time required to perform it. This number of hours is task specific, assuming that all actors allocated to perform it have a nominal efficiency (equal to 1). In this study, the concept of workload will be often discussed in reference to a given competence: by extension, the execution of a task that requires several competences is then defined by a separate set of workloads – this is to avoid using the term 'sub-tasks', quite large: we will often refer to 'workloads' to describe these sub-divisions.

*Work*: concerns actors: number of hours actually needed by an actor to perform a task. Contrary to the workload, this value involves efficiency of the actor.

## 2.2 Problem typology

We consider an activity consisting of $I$ tasks, the precedence relationships between tasks are known and identified; this activity is performed within a company where there are $K$ competences or skills. For each task $i$ in the activity, we know the set of workloads $w_{i,k}$ (expressed in hours) that represent the required workload for the execution of the task's competence $k$ (note that any task may involve several competencies). For each task $i$ in the activity, we know the minimum duration $D_i^{\min}$, its standard duration $D_i$ and a maximum duration $D_i^{\max}$. The standard duration is used to calculate the initial scheduling of tasks to know the fixed contractual duration of the activity $L$. But the required duration ($d_{i,k}$) to complete task $i$ on competence $k$ is not known in advance, but it is a variable defined within a tolerance period $d_{i,k} \in [D_i^{\min}, D_i^{\max}]$. The company lays out multi-skilled actors and each actor $a$ is characterised by his efficiency $\theta_{a,k}$ (lower than or equal to 1) on each competence which can govern his allocation (as will be seen in Section 2.3.1). We assume that the company has, at any time, an efficiency evaluation of its actors for their respective competences. The choice of allocating an actor $a$ on a task $i$ with respect to a competence $k$, $\sigma_{a,k,i}$ is also a decision variable of the problem. For each competence $k$, we know the workforce $A_k$ (number of actors having competence $k$); this number of actors can provide an available quantity of equivalent working hours $Q_k$ over a given time period that can be calculated according to the regulatory durations of the working time and to the efficiency of each actor. In addition, for each actor, we associate a standard hourly cost rate, $U_a$, for calculating the activity execution cost. The problem consists in minimising the human labour cost, by respecting, firstly, the precedence constraints between tasks and, secondly, ensuring that at any given day $j$, the sum of the workloads of running tasks does not exceed the available amount of equivalent working time offered by the resources.

## 2.3 Taking into account the characterisation of flexibility levers

### 2.3.1 The multi-skills

In our work, to characterise the concept of multi-skills in the companies, we associated a factor called 'efficiency' to each competence that can be hold by an actor (Duquenne et al., 2005). The efficiency of an actor $a$ on a competence $k$ will be noted as $\theta_{a,k}$ and it is a dimensionless quantity that is involved in determining the work required by this actor. In our model $\theta_{a,k} \in [\theta_{\min}, 1]$; the term $\theta_{\min}$ represents the lower limit below which the allocation is not considered as desirable, for reasons related to economic or quality aspects. Table 1 gives an example of efficiencies for actors' competences.

Further work gives values to efficiencies that could exceed 1 (e.g. Yoshimura et al., 2006). In their model, the actors' expertise is ranked with skill levels equal to {0, 0.5, 1, 2}, this four values corresponding to actors' skills of {novice, informed, experienced, expert}. Valls et al. (2009) sorted the workers according to their skills in three categories (senior, standard and junior) and give to each category a skill value (1, 2 and 3), respectively; in their model, the execution time of a given task varies according to the skill category with ±25% from the standard one. And, others such as Drezet and Billaut (2008) take the concept of actors' skills with the ability of performing a task or not: in their model, all actors having this skill were able to perform the given task with equal amount of time periods. For us, the value of 1 represents the nominal efficiency of the actor (e.g. in his principal competence), and non-zero values lower than or equal to 1 indicate his efficiencies in various additional competences which he would have acquired (Table 1). In this case, the planner may allocate some overloads on the available actors, even if their efficiencies are not optimal on the competences involved, in order to avoid or reduce overtime or the recruitment of external actors (Kane, 2001; Yang et al., 2007). A multi-skilled actor then has a range of competences allowing him to be assigned to various tasks involving different skills (Edi and Duquenne, 2006). This flexibility factor should enable the company to profit from a variable work by competence, departing from a constant real manpower.

In industrial practice, a multi-skilled actor does not spend the same time in the execution of a task whatever the competence it involves. So, for all his competences, the actor does not have the same level of efficiency. Pragmatically, a task requiring for its realisation only one resource of competence $k$, and defined by a workload $w_{i,k}$ (in hours), implicitly requires the same amount of work $\omega_{a,k} = w_{i,k}$, when it is assigned an actor of nominal efficiency ($\theta_{a,k} = 1$) in this competence; in cases where the actor has an efficiency lower than the nominal efficiency, the work $\omega_{a,k}$ needed to achieve this task will be:

$$\omega_{a,k} = \frac{w_{i,k}}{\theta_{a,k}}$$

Based on the previous equation, the execution of the global workload of the company should be minimised.

**Table 1** Example of the efficiencies of actors' competences

| | *Efficiency of actor's competence ($\theta_{a,k}$)* | | | |
|---|---|---|---|---|
| *No. of actors a* | *k = 1* | *k = 2* | *k = 3* | *k = 4* |
| 1 | 0.0 | 1 | 0.0 | 0.0 |
| 2 | 1 | 0.0 | 0.0 | 0.5 |
| 3 | 0.6 | 0.0 | 1 | 0.0 |
| 4 | 0.0 | 0.8 | 0.0 | 1 |
| 5 | 0.7 | 0.0 | 1 | 0.0 |

### *2.3.2 The timetables modulation*

The modulation consists in negotiating a collective agreement of annual smoothing of the working time to avoid a massive use of expensive overtime during high activity periods, as well as high unemployment in low activity seasons – this last solution being expensive too. The working time modulations offer an important flexibility; it makes it possible to vary the daily and weekly actors' timetables on individual as well as collective levels (workshops opening and closing hours). And it represents an economically good solution for adjusting productive capacity to cover the seasonal variation demand (Hertz et al., 2010), and so it enables companies to cope with fluctuating workloads without additional cost, by increasing the duration of the periodic work in the event of strong activity and by reducing it when the activity decreases.

In other words, an actor has an availability timetable that evolves throughout time. This actor then becomes a consumable resource, that is to say his overall (or cumulated) consumption throughout time is limited with a maximum capital of hours to perform over a given period. The maximum and minimum work per week for each actor is conducted on weekly average hours $C_{s0}$. In France, $C_{s0}$ = 35 hr.

## 3 Modelling approach

### *3.1 Representation of the planning unit*

Planning of an activity requires the choice of time unit. This unit can be: hour, day, week, month, etc. In the case of scheduling problems with allocation under resources constraints, the most frequent choice leads to a discrete and uniform time based on a division of the scheduling horizon in periods of equal durations. In our approach, we chose to work with the time unit 'day', and to express the workloads of the tasks and the work of the actors in 'hours'. To model the weekly constraints, we suppose that one week is equivalent to five consecutive days, which awards two consecutive days of rest to the actors. We will also suppose that the worked and non-worked days are the same ones for all the actors.

We can note that due to timetable modulation, all days do not have the same duration in terms of working hours, neither the same day from one actor to another nor even for a given actor from one day to the next.

### 3.2 Modelling of the problem constraints

The problem constraints are represented by a set of equations of type equality or inequality.

We distinguish four categories of constraints:

- *Temporal and precedence constraints*: this group of constraints make it possible to govern the realisation time of the activity and to provide the tasks execution order.
- *Timetable modulation constraints*: limits the exceedances of working time in accordance with the legislation in manpower force. To model the constraints related to the working time, we adopted a standard weekly schedule $C_{s0}$.
- *Allocation constraints*: make it possible to limit the allocations of the actors on the tasks by respecting the number of allocations of actors on the same task and the equivalent workforce of each competence.
- *Availability and competence constraints*: make it possible to limit the actor occupation and setting the minimum value of efficiency to be met before any allocation is accepted.

#### 3.2.1 Temporal and precedence constraints

These constraints will make it possible to limit the execution times of the tasks and, by deduction, that of the activity. Thus, the start date of a task is deduced, starting from these constraints, by taking into account the durations of its predecessors. We distinguished three fundamental constraints: constraints on the actual duration of tasks, the constraint on the global execution time of the activity and the precedence constraints between tasks.

*3.2.1.1 Tasks actual duration constraints* One of the characteristics of a task is the number of competences it involves. Thus, for each competence $k$ of the task $i$ an actual execution time $d_{i,k}$ is associated to it. Therefore, to define the actual execution time $d_i$ of a task $i$, it is necessary to identify the duration times $d_{i,k}$ for each of its competencies. Any of these durations must be comprised between the minimum and maximum durations of the considered task.

$$D_i^{\min} \le d_{i,k} \le D_i^{\max}, \quad \forall i, \forall k \tag{1}$$

$$d_i = \max\left(d_{i,k}\right)_{k=1,\ldots,K} \Rightarrow D_i^{\min} \le d_i \le D_i^{\max} \tag{2}$$

Note that $d_{i,k}$ is one of the variables of our problem. It will be determined by an algorithmic way.

*3.2.1.2 Activity total execution time constraints* We will suppose that the realisation of an activity comes from a contract between the company and a customer. Therefore, the completion date on which the result of the activity is delivered to the customer was pre-fixed. Within the framework of our problem, we suppose that the contractual fixed duration $L$ of the activity is determined from an initial sequence based on the standard durations of tasks. To this duration $L$, we add a flexible part $\beta$ positive or zero, allowing or not the company to have a time margin on the completion date of the activity. It will be considered that if the result is provided to the customer with a delay higher than $\beta$, the company will be charged with lateness penalties; in the same way, we avoid to complete the job sooner than $\beta$ to avoid storage costs on the finished products.

Thus, to avoid any penalty fees or storage costs, it is necessary that the actual duration LV of the activity is within the following interval:

$$L - \beta \leq \mathrm{LV} \leq L + \beta \tag{3}$$

The choice of a maximum delay from the contractual delivery date will be a function of the urgency of the activity and the negotiations between the company and the customer. On the other hand, the existence of a 'storage cost' means that finishing the products significantly before their contractual delivery date may be a nuisance – even if it does not actually induce a recordable cost; this 'storage cost' could even be negative, expressing schedule incentives. In the real-world, there is strictly no reason why cost variation should be symmetric in cases of advance or delay; but in the scope of our work, we chose to simplify this aspect which is not the core of our concern by introducing this unique variable $\beta$. In the case of our problem, we opted for $\beta = 5$ days (as an indication).

*3.2.1.3 Precedence constraints between tasks* These constraints will make it possible to respect the logic of realisation of the activity and to deduce the dates (start and finish) of the tasks from their predecessors, by admitting that the tasks without predecessors are likely to start at the date $\mathrm{dd}_i = 0$. We take into account four types of relations which can exist between the tasks: the relation Finish–Start, Finish–Finish, Start–Start and Start–Finish. These relations can be accompanied with a delay (positive or negative). In addition, these constraints must be respected at every stage of the problem solving.

For a simple presentation, we will briefly consider only two tasks $i$ and $n$, where $i$ is the predecessor of $n$. With the precedence constraints, we can calculate the start date of the task $n$ as the following:

- *Finish–Start*: the end of $i$ authorises the beginning of $n$:

$$\mathrm{dd}_n \geq \mathrm{dd}_i + d_i + \alpha_{i,n} \tag{4}$$

- *Finish–Finish*: the end of $i$ authorises the end of $n$:

$$\mathrm{dd}_n \geq \mathrm{dd}_i + d_i - d_n + \alpha_{i,n} \tag{5}$$

- *Start–Start*: the beginning of $i$ authorises the beginning of $n$:

$$\mathrm{dd}_n \geq \mathrm{dd}_i + \alpha_{i,n} \tag{6}$$

- *Start–Finish*: the beginning of $i$ authorises the end of $n$:

$$\mathrm{dd}_n \geq \mathrm{dd}_i - d_n + \alpha_{i,n} \tag{7}$$

In these equations, in addition to the relations between tasks, we can have a scheduling delay, $\alpha_{i,n}$, of positive, negative or zero value that represents the possibility or not of taking into account a time delay between tasks. In a scheduling calculation, a task can have more than one predecessor; if $\Delta_i$ represents a set of all predecessors of the task $i$, we calculate the start date of this task as many times as it has predecessors. In this case, the start date of the task $i$ will be the maximum of all the start dates calculated from all the constraints resulting from the existence of its predecessors:

$$\mathrm{dd}_i = \max\left(\mathrm{dd}_i\right)_{\mathrm{dd}_i \text{ calculated for each predecessor task} \in \Delta_i} \tag{8}$$

### *3.2.2 Timetables modulation constraints*

It is a group of constraints which comes from the regulatory data on the working time. There are five types of constraints concerning our problem, all dealing with the maximum availabilities of an actor: per day, per week and over one floating period of 12 consecutive weeks, as well as the constraints on the workloads of the tasks' competences and the constraints of overtime periods.

*3.2.2.1 Daily maximum availability constraints* For a given day $j$, the same actor can be assigned with several of his competences if his efficiency is considered to be sufficient on several different tasks, under the conditions that the sum of its daily work respects the regulation of daily maximum duration DMaxJ (e.g. in France DMaxJ = 10 hr). And the actor, when assigned to a workload of one task, will work there during the full execution of this workload, without any interruption. For example, if this workload will be carried out during three days, the affected actor will work for this period of time on the same task. In addition, all tasks are carried out without interruption. In other words, if a task starts, it runs without interruption until completion.

$$\sum_{i=1}^{I}\sum_{k=1}^{K}\sigma_{a,k,i}\times\omega_{a,k,i,j}\leq \text{DMaxJ},\qquad \forall a,\ \forall j \tag{9}$$

$$\omega_{a,k,i,j}\geq 0 \tag{10}$$

$\sigma_{a,k,i}$ is a binary variable of actor allocation on the tasks according to his competence.

*3.2.2.2 Weekly maximum availability constraints* The sum of work realised by an actor $a$ on the week $s$, $\omega_{a,s}$ must respect the weekly working time regulations DMaxS (e.g. in France the maximum availability is DMaxS = 48 hr):

$$\omega_{a,s}=\left(\sum_{j=(\text{NJS}\times(s-1))+1}^{\text{NJS}\times s}\left(\sum_{i=1}^{I}\sum_{k=1}^{K}\sigma_{a,k,i}\times\omega_{a,k,i,j}\right)\right)_{s=1,\,2,\,\ldots,\,\text{Int}[(\text{LV}-1)/\text{NJS}]+1},\ \forall a,\ \forall s \tag{11}$$

$$\omega_{a,s}\leq \text{DMaxS},\quad \forall a,\ \forall s \tag{12}$$

*3.2.2.3 Task workload by competence constraints* An actor $a$ assigned on a workload $w_{i,k}$ of a task provides a work $\omega_{a,i,k}$ that is function of his efficiency to cover the considered workload. This work provided by the actor will be equal to the workload if the actor has a nominal efficiency ($\theta_{a,k}=1$) on the corresponding competence. On the other hand, this work will be higher than the workload if his efficiency is lower than the nominal value, and can be calculated according to the following relation:

$$\omega_{a,i,k}=\frac{w_{i,k}}{\theta_{a,k}}$$

This increase of the working time compared to a standard execution (when $\omega_{a,i,k}>w_{i,k}$) will be the cost to be paid for the use of the multi-skill (additional competences acquired by an actor whose efficiency is lower than the nominal value). In all cases, the objective

is to cover the totality of the workload, with one or more actors over one or more days according to the planning horizon of the task.

$$\sum_{a \in \mathrm{ER}_{i,k}} \left( \sum_{j=\mathrm{dd}_{i,k}}^{\mathrm{dd}_{i,k}+d_{i,k}} \omega_{a,k,i,j} \times \sigma_{a,k,i} \times \theta_{a,k} \right) = w_{i,k}, \quad \forall i, \forall k \tag{13}$$

where $\mathrm{ER}_{i,k}$ represents the real workforce (integer number of the actors) assigned to carry out the workload $w_{i,k}$.

*3.2.2.4 Overtime constraints* For an actor, we will call the weekly overtime hours with $\mathrm{HS}_{a,s}$, the hours carried out by an actor $a$ in addition to the weekly limit of the modulation agreement DMaxMod. Indeed, the use of the modulation requires the establishment of a weekly limit beyond which we begin to implement the overtime. Thus, for one working week, the overtime hours that an actor will carry out will be lower than or equal to the difference of the weekly maximum duration of the working time DMaxS and the limit of fixed modulation DMaxMod.

$$0 \le \mathrm{HS}_{a,s} \le \mathrm{DMaxS} - \mathrm{DMaxMod}, \quad \forall a, \forall s \tag{14}$$

This constraint will be applied if the weekly work carried out by an actor exceeds DMaxMod:

$$\mathrm{HS}_{a,s} = \begin{cases} \omega_{a,s} - \mathrm{DMaxMod} & \text{if } \omega_{a,s} \ge \mathrm{DMaxMod} \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

The law of the working time modulation also limits the number of annual overtime that an actor can carry out; if this number is HSA, we have:

$$\sum_{s=1}^{\mathrm{Int}[(\mathrm{LV}-1)/\mathrm{NJS}]+1} \mathrm{HS}_{a,s} \le \mathrm{HSA} - \mathrm{HSR}_a, \quad \forall a \tag{16}$$

where $\mathrm{HSR}_a$ represents the number of overtime already carried out by an actor $a$ in the same year for other activities within this year.

*3.2.2.5 Constraints on a maximum capacity for a floating period of 12 consecutive weeks* The regulation on the working time modulation indicates that, for any actor, the weekly average work over 12 consecutive weeks should not exceed a certain threshold DMax12S (e.g. in France DMax12S is 44 hr). To put this constraint into practice, it is necessary that the number of weeks for activity execution is higher than or equal to 12 consecutive weeks, or that the history of actors work is well recorded in actors' data sheets.

$$\frac{1}{12} \times \left( \sum_{s=p}^{p+11} \omega_{a,s} \right)_{p=1 \ \ to \ \ (p+11) \le \mathrm{Int}\left[(\mathrm{LV}-1)/\mathrm{NJS}\right]} \le \mathrm{DMax12S}, \quad \forall a \tag{17}$$

### *3.2.3 Allocation constraints*

Facing the problem of allocation, there are choices to be made about the number of allocations of an actor on the same period. In the case of our problem, where we consider

the versatility of actors, and the fact that tasks can mobilise more than one competence, we need to define constraints to guide the allocation process. Thus, we have: the constraints of the number of possible assignments for an actor on a task, the constraint on the equivalent workforce which we can assign on a task and the constraint on the real workforce. Due to the disjunctive nature of the human resources (on a short-scale time horizon), we will consider that it is impossible for one actor to be allocated on many workloads in the same working time instance. Thus, the number of allocations and the number of resources that are allocated to a given workload are constrained.

*3.2.3.1 Constraint of the allocations number of an actor on a task* A multi-skilled actor holds sufficient efficiencies on more than one competence which can govern his allocation procedures. In the case where a task involves at least two competences of an actor, he can be assigned to anyone of the workloads $(w_{i,k})_{k=1,\ldots,K}$ and this, whatever the task. If $nk_a$ represents the number of competences which an actor $a$ holds, then we must have:

$$\sum_{k \in nk_a} \sigma_{a,k,i} \leq 1, \quad \forall a, \forall i \tag{18}$$

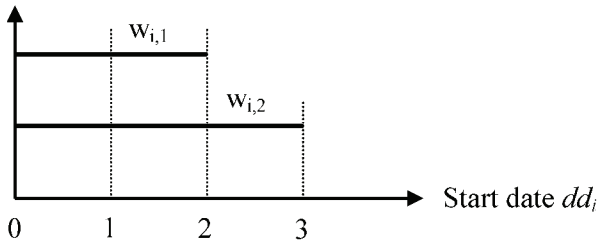
Equation (18) governs the allocation process of the actor on the task competences and makes sure that the actor is assigned to only one of the tasks' competences, i.e. because we assumed that all the workloads $(w_{i,k})_{k=1,\ldots,K}$ for a task have the same start date but not necessarily the same durations, because the duration is a variable depends on the allocated actors' efficiencies. That is to say, e.g. a task $i$ who mobilises two competences $k = 1$ and $k = 2$ of respective workloads $w_{i,1}$ and $w_{i,2}$, as shown in Figure 1, the duration of the workload $w_{i,1} = 2$ workdays and that of the workload $w_{i,2} = 3$ workdays, but the both competences begin with the same start date $\mathrm{dd}_i = 0$, and thus the finished date of task $i$ equal to the max (2; 3) = 3 days.

*3.2.3.2 Equivalent workforce assigned constraints* Taking into account the flexibility in activities' scheduling requires the identification of all actors' available competences, provided their efficiencies exceed the efficiency lower limit. At this step, we consider a total competence for every trade of the company. Facing a given workload $w_{i,k}$, the planner has to perform an evaluation of the total efficiency of the actors likely to provide this workload, to determine the equivalent working hours capacity $Q_k$ available and the equivalent workforce $\mathrm{EE}_{i,k}$ needed to achieve the job.

The equivalent workforce represents the real productivity of the actors assigned to a workload $w_{i,k}$. The actors being multi-skilled, they are not all equivalent for the realisation of a task even if they perform the same number of hours on this task. The actors, whose efficiency on competence is lower than the nominal value, will perform a less productive job than those whose efficiency is nominal. Thus, to know the equivalent workforce $\mathrm{EE}_{i,k}$ of the actors assigned to the task $i$ of competence $k$, we make a summation of their respective efficiencies.

$$\mathrm{EE}_{i,k} = \left( \sum_{a \in \mathrm{ER}_{i,k}} \theta_{a,k} \right)_{\theta_{a,k} \geq \theta_{\min}}, \quad \forall i, \forall k \tag{19}$$

**Figure 1** Example of start date of a task's competences workloads




However, for a task $i$ and a competence $k$, if the corresponding workload $w_{i,k}$ is non-zero, the equivalent workforce must cover this workload for the realisation process of the considered task. When the workload $w_{i,k}$ needed by a task $i$ can be scheduled on a maximum period $D_i^{\max}$, and if we call DMaxJ, the maximum number of working hours in one day for an actor, the respect of the schedule constraint for the task $i$ can be presented by:

$$\mathrm{EE}_{i,k} \geq \frac{w_{i,k}}{\mathrm{DMaxJ} \times D_i^{\max}}, \quad \forall i, \forall k \tag{20}$$

For a given competence, this constraint ensures that the equivalent workforce must be greater than or equal to the manpower capacity required by the workload: this is to indicate the feasibility of achieving the corresponding workload. Many papers introduced this concept of equivalent workforce to check the feasibility of performing or not an activity (e.g. Yoshimura et al., 2006). In their model, the activity or project is feasible to be conducted if the available skills of the workforce are greater than or equal to the activity requirements skills.

*3.2.3.3 Real workforce assigned constraints* When performing an activity, several tasks can be initiated on the same day. So these tasks can mobilise the same competence $k$. It is necessary that the model takes care that for a given day $j$, the number of actors of the same competence assigned to the tasks in the course of execution does not exceed the total number of actors having this competence. If $A_k$ is the number of actors having a competence $k$, and if $\rho_{j,k}$ represents the set of all tasks running on the day $j$ and mobilising competence $k$, we will have:

$$\sum_{i \in \rho_{j,k}} \mathrm{ER}_{i,k,j} \leq A_k, \quad \forall j, \forall k \tag{21}$$

### *3.2.4 The constraints of availability and competence*

This group of constraints will ensure a rational use of actors and the verification of the efficiency of each actor before his assignment, to make sure that it respects the authorised minimal efficiency allowed. We have: the constraint on the efficiency of an actor and the constraint on his availability.

*3.2.4.1 Actor efficiency constraints* The efficiencies of actors make it possible to appreciate the total behaviour of an actor in a competence. Thus, for the allocation of an

actor, a company may demand a minimal value $\theta_{\min}$ below which any assignment is impossible. The choice of this lower limit depends on the risk of the activity for the company. A lower limit which tends towards the nominal value makes it possible to optimise the use of the most efficient actors, but sacrifices the idea of multi-skills.

$$\theta_{\min} \leq \theta_{a,k} \leq 1, \quad \forall a, \forall k \tag{22}$$

(the above equation is not true, $\forall a$, $\forall k$: only if the actor $a$ is assigned to the competence $k$). In the case of our model, we arbitrarily set up $\theta_{\min}$ at 0.5 and the nominal value $\theta_{a,k}$ at 1.

*3.2.4.2 Actors' availability constraints* The implementation of the timetable modulation can be varied by the weekly work of actors within an interval admitted in the agreements. For that, an actor can be mobilised above the standard duration during one week (or more), and let at rest, or less requested, for the following one(s). Thus, the occupation of the actors is done according to the workload of the company, based on the sequence of tasks execution. Therefore, we will call occupancy rate $O_{a,s}$ of an actor $a$ on the week $s$, the ratio between the carried out weekly work $\omega_{a,s}$ and the standard weekly work $C_{s0}$,

$$O_{a,s} = \frac{\omega_{a,s}}{C_{s0}}, \quad \forall a, \forall s \tag{23}$$

Note that this rate may be higher than 1, meaning that this actor works below the company modulation hours DMaxMod, or with the overtime strategy, that can be calculated as shown in Equation (15). In all cases, the occupation will not exceed a threshold calculated according to the regulations of the maximum authorised work DMaxS.

$$0 \leq O_{a,s} \leq \frac{\text{DMaxS}}{C_{s0}} \tag{24}$$

$O_{a,s}$ will allow to appreciate the residual flexibility of an actor at the end of the activity. Here, we call 'residual flexibility' the future work time modulation that was preserved by his allocations during the current job. Pragmatically, if a task takes one week and that occupation of the actor $a$ is $O_{a,s}$, then the residual flexibility will be: $(1 - O_{a,s})$. If this value is positive, it will be concluded that this actor's working capacity has been preserved for the following week(s); thus he has gained some flexibility, which might be useful for future allocations. If we consider another activity that would follow this one, the value of $O_{a,s}$ will also indicate how much of the potential flexibility of the resources involved has been consumed, and therefore what part remains available for future activities. This consideration will be part of the evaluation of solutions via the objective function. But when the value of $(1 - O_{a,s})$ is negative, it indicates an over-consumption of the working hours: thus the corresponding actor has been working within the frame of quantitative flexibility by either working time modulation (company agreement) or overtime strategy. In this case, the consumption of flexibility must be optimised, so we add a term of virtual cost to the objective function to avoid abusive flexibility consumption. This term takes place only when the average working hours of an actor exceeds the standard working hours, i.e. he works within the flexibility zone between $C_{s0}$ and DMaxS.

### 3.3 The objective function

Considering the versatility of actors leads us to identify a criterion for evaluating the solutions obtained, which should be an economic criteria: the various solutions of allocations will involve the sum of variable work hours according to the efficiencies of the various actors on each of the selected tasks. The research for an assignment of the maximum efficiency $\theta_{a,k}$, thus contributes not only to the minimisation of the real duration of the task $d_{i,k}$, but also to an economic minimum. It is seen that this criterion also allow to evaluate the cost of using the multi-skills. Logically, our objective function is thus an evaluation of a cost of conducting an activity, this cost should be minimised. The overall costs to be minimised are then the sum of four different components: the cost of the workforce who carried out the work during normal working hours ($F_1$), the cost of overtime working hours ($F_2$), the cost of the variations compared to a desired completion date ($F_3$: lateness penalties or storage cost) and finally a fictive cost associated to an excessive erosion of actors' flexibility ($F_4$). Among these costs, $F_1$, $F_2$ and $F_3$ are functions that determine an actual cost in monetary units. On the other hand, the function $F_4$ measures residual flexibility at the end of the activity. $F_4$ is brought back to a cost appreciation to allow us to solve the allocation problem with a single criterion.

#### 3.3.1 $F_1$: normal working cost

The determination of the salary costs of the activity passes by the work identification of each assigned actor. For a workload $w_{i,k}$, we will suppose that all the assigned actors $\mathrm{ER}_{i,k}$ carry out the same number of working hours whatever the efficiency of each one of them. As a result, the total work of each actor $a$ assigned to a workload $w_{i,k}$ will be:

$$\omega_{a,k,i} = \left( \frac{w_{i,k}}{\mathrm{EE}_{i,k}} \right)_{a \in \mathrm{ER}_{i,k}} \quad \text{with} \quad \mathrm{EE}_{i,k} = \left( \sum_{a \in \mathrm{ER}_{i,k}} \theta_{a,k} \right)_{\theta_{a,k} \geq \theta_{\min}, \sigma_{a,k,i} \neq 0} \tag{25}$$

This results in a daily working time of $\omega_{a,k,i,j} = \omega_{a,k,i} / d_{i,k}$ by working day on duration of $d_{i,k}$ for the workload $w_{i,k}$. According to the start dates of tasks $\mathrm{dd}_i$, the daily total work of each actor $\omega_{a,j}$ will be calculated to be used as a basis for overtime costs calculations and residual flexibility, etc.

$$\omega_{a,j} = \sum_{k=1}^{K} \sum_{i=1}^{I} \omega_{a,k,i,j}, \quad \forall a, \forall j \tag{26}$$

This function of the normal working cost makes it possible to calculate the salary cost of the work completed within the normal hours, without taking into account overtime hours. If we name $U_a$ as the hourly cost rate of the actor $a$, then we have:

$$F_1 = \sum_{a=1}^{A} \sum_{s=1}^{\mathrm{Int}[(\mathrm{LV}-1)/\mathrm{NJS}]+1} \left( \omega_{a,s} - \mathrm{HS}_{a,s} \right) U_a \tag{27}$$

### 3.3.2 $F_2$: the overtime cost

The overtime hours are raised of a factor $(1 + u)$ compared to the normal hours with $u$ the rate of increase:

$$F_2 = \sum_{a=1}^{A} \left( \left( \sum_{s=1}^{\mathrm{Int}[(\mathrm{LV}-1)/\mathrm{NJS}]+1} \mathrm{HS}_{a,s} \right) \times U_a \times (1+u) \right) \tag{28}$$

### 3.3.3 $F_3$: cost resulting from immobilisation or lateness penalties

The contractual duration of an activity is an interval defined around a fixed duration $L$, called 'zone of flexible contractual duration'. This zone is defined by: $L - \beta \leq \mathrm{LV} \leq L + \beta$.

To take into account the calculation of the function $F_3$, it is necessary that the real duration LV of the activity is a part from this zone. When the real end of the activity is earlier than the zone of flexible contractual duration, the result of the activity products must be stored, while waiting for its delivery: for that reason, we calculate a storage cost that may figure the cost of the resulting financial immobilisation. Regarding the activity production cost, that consists of normal wages plus overtime costs, this economic cost is stated at the end of actual duration of the activity (represents the present value); but the enterprise cannot receive any financial resources within the period between the activity real achievement and the contractual milestone $(L - \beta)$. It is well known that the time value of money decreased, thus the difference between the future value and the present value of this money can be considered as a penalty cost of carrying out the activity sooner than needed. Thus this penalty cost can be formulated as a function of the activity realisation cost and a daily discount rate $\tau_j$:

$$\mathrm{LV} < (L - \beta) \Rightarrow F_3 = (F_1 + F_2) \times \left( \left(1 + \tau_j\right)^{(L - \mathrm{LV} - B)} - 1 \right) \tag{29a}$$

Note that in Equation (29a), we only took into account the cost related to human resources, and we neglected the cost of raw materials and purchased equipments. We assumed that normally they were ordered before the start date of the activity and were ready to be used around its start date, regardless to an earlier or later activity completion; some other costs such as amortising (depreciation) of tools, equipments or machines were neglected too, because they are fixed in all cases.

Anyway, this consideration about a storage cost, if it is not essential, can be omitted by choosing a value of zero for $\tau_j$. The choice of a negative value for $\tau_j$ can also reveal the existence of incentives paid by the customer in the event of an anticipated delivery compared to contractual duties.

On the other hand, if the real completion date of the activity exceeds the zone of flexible contractual duration, the time of going beyond is identified. The resulting penalty is calculated with a daily rate, which we suppose to be constant, US. This penalty cost will be negotiable and pre-noted in the contract between the buyer and the customer:

$$\mathrm{LV} > L + \beta \Rightarrow F_3 = \mathrm{US}(\mathrm{LV} - (L + \beta)) \tag{29b}$$

If the real duration of the activity is within the flexible zone of contractual duration, there will be neither 'storage cost' nor lateness penalties.

$$L - \beta \leq \mathrm{LV} \leq L + \beta \Rightarrow F_3 = 0 \tag{29c}$$

### *3.3.4 $F_4$: cost associated to the residual flexibility of the actors*

In contrary to the previous ones, this function represents a fictive cost, relative to a non-optimal use of the actors' flexibility; it will make it possible to appreciate the interest of preserving a minimum amount of flexibility, to ensure a maximum reactivity of the company in the future. This function aims at penalising any solution that would be efficient for the execution of a short- or middle-term activity but would not preserve the long-term flexibility margin of the company. The higher the value of this function will be, the more it will preserve flexibility for the company towards the future capacity of actors work. The goal of this sub-function is to favour solutions which offer an interesting future flexibility. If we call $\mathrm{UF}_a$ the cost associated for the flexibility of an actor $a$, and $O_{a,s}$ its occupation at the week $s$, we have:

$$F_4 = \sum_{a=1}^{A} \mathrm{UF}_a \left( 1 - \frac{\sum_{s=1}^{\mathrm{Int}[(\mathrm{LV}-1)/\mathrm{NJS}]+1} O_{a,s}}{\mathrm{Int}[(\mathrm{LV}-1)/\mathrm{NJS}]+1} \right) \tag{30}$$

$\mathrm{UF}_a$ is the unit costs of actor flexibility, it can be defined according to the importance of competence held by the actor $a$. We can give greater values for some actors than for others, to preserve their future flexibility according to the importance and the scarcity of the competences they master.

When we substitute Equation (23) in Equation (30), we get: $F_4 = \sum_{a=1}^{A} \mathrm{UF}_a \left(1 - (\text{average of } \omega_{a,s} / C_{s0})\right)$, thus this virtual cost $F_4$ is a function of the actors' weekly average work. The ratio between this average weekly work and the standard working hours probably could take any value between 0 and (DMaxS/$C_{s0}$). When this ratio takes a value of zero, it means that the corresponding actor is not assigned to any of the activity's workloads, thus he has a 100% availability for the standard working hours and a 100% preservation of the quantitative flexibility. But when this ratio takes a value within the interval ]0, 1], it means that the corresponding actor works under the limits of standard working hours; in addition, he preserves all of his qualitative flexibility. Lastly, when it takes a value within the interval ]1, DMaxS/$C_s$0], this means that the corresponding actor is working with a quantitative lever of flexibility. In this case, the virtual cost $F_4$ can be added to the other real costs in the objective function to penalise excessive consumption of flexibility; $F_4$ can also be negative, and then favour solutions that would not only totally preserve flexibility, but also minimise actors' work.

Overall, the objective function to be minimised is calculated in the following way:

$$F = \left(F_1 + F_2 + F_3 - F_4\right) \tag{31}$$

Thus, any solution which respects the whole of the constraints and which preserves a minimum of flexibility will be privileged.

### 3.4 Performance indicators

To evaluate the performance of the solution methodology, we defined two indicators:

- the allocation optimisation ratio to know the impact of the multi-skills
- the average ratio of flexibility per actor to appreciate the preservation of his availability.

#### 3.4.1 Allocation optimisation ratio

To measure the impact of the multi-skills on the result of the allocation, we define an indicator of evaluation 'allocation optimisation ratio: $\tau$opt'. This ratio will be the overall productivity of the actors on the realisation of the activity. It is the ratio between equivalent workforce assigned and real workforce:

$$\tau\text{opt} = \frac{\sum_{i=1}^{I}\sum_{k=1}^{K}\text{EE}_{i,k}}{\sum_{i=1}^{I}\sum_{k=1}^{K}\text{ER}_{i,k}} \tag{32}$$

$\tau$opt will be comprised between $0 \leq \tau\text{opt} \leq 1$. If we suppose that all the actors cost the same hourly rate $U$, then we can draw the following conclusions: the more $\tau$opt will be close to 1, the lower will be the overall work needed to carry out the activity. It will tend towards the sum of the workloads of the tasks which compose it, and it will be closer to an optimal solution, from the point of view of work efficiency. A high value of $\tau$opt (close to 1) also means that little use has been done of flexibility, if we assume that any actor has non-ideal efficiencies ($\theta_{a,k} < 1$) on his additional skills. On the other hand, if the best solutions we can find to our problem show little values of $\tau$opt, it indicates that multi-skills play a great part in these solutions – and we can infer that our planning problem would have had no solution without this flexibility lever. This indicator may thus be useful to measure the consistency of multi-skills management in a company, and help to determine if it is worth developing it or not.

#### 3.4.2 The average residual flexibility ratio $\tau FlexR_a$

This ratio measures the average availability preserved by an actor in the planning horizon, referring to weekly standard working time:

$$\tau\text{FlexR}_a = 1 - \frac{\sum_{s=1}^{\text{Int}[(\text{LV}-1)/\text{NJS}]+1} O_{a,s}}{\text{Int}[(\text{LV}-1)/\text{NJS}]+1} \tag{33}$$

Indeed, the average residual flexibility ratio $\tau\text{FlexR}_a$ is nothing else than the average for the whole activity of the weekly occupancy rates $O_{a,s}$ of Equation (23). Thus, according to the domain of occupancy rate based on Equation (24), we can conclude that $\tau\text{FlexR}_a$ of the actor '$a$' can take any value in the range $[-(\text{DMaxS}/C_{s0} - 1), 1]$. When it takes a value within $[-(\text{DMaxS}/C_{s0} - 1), 0[$, this means that this actor consumed some of his working time flexibility. But a value within $[0, 1[$, can be interpreted as an actor working under the limits of standard working hours, thus he preserved his working time flexibility.

Finally, if it takes a value of one, we can conclude that this actor has no work to do on this activity.

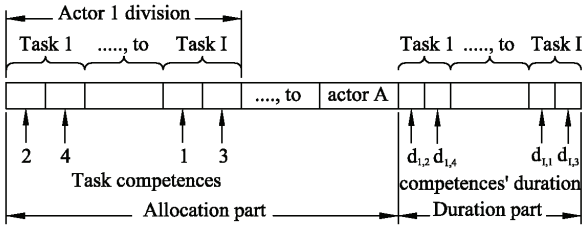
## 4 Problem solving by single criterion genetic algorithms

The single criterion genetic algorithms (GAs) are used to solve the current problem taking into account only one objective function. The implementation of GAs requires the definition of the procedures and the associated parameters (Goldberg, 1989): a suitable encoding of the problem variables that will provide a genotype, in which each decision variable can be found and expressed by a given gene. A complete set of these genes, representing an exhaustive set of decision variables, is called individual (ind), or chromosome. One can generate randomly a group of individuals to form a 'population', so that the entire space of solutions can be explored. These individuals are then evaluated referring to the objective function to calculate for each chromosome its fitness, force or adaptation; the fittest individuals in a population thus represent the best solutions to the problem amongst the ones that were explored. In the following, we will discuss the different genetic operators from the point of view of our allocation problem: first, chromosome encoding, constitution of an initial population with a number of individuals known as population size (IP) and fitness calculations; then we will describe the chromosomes' evolutions through selection, reproduction, crossover and mutation.

### *4.1 Chromosome encoding*

Here the chromosome is composed of two main parts: the first one represents the allocation variables $\sigma_{a,k,i}$. As shown in Figure 2, this first sub-chromosome is divided into as many divisions as actors considered by the model. Each of these divisions represents the allocation decisions for one actor: thus, it accounts for all the tasks with all of their required competences. Each task-related segment has at maximum only one gene allele (the gene value) with value of '1' to represent the actor allocation decision for performing the workload corresponding to skill $k$ of this task. Since we adopted the assumption that one given actor could be allocated on one task for only one unique competence, all the other genes within this segment should have an allele of '0'. If all the genes in a segment have a value of '0', it means that the corresponding actor is not assigned on this task. But we must take into account that if the actor shows an efficiency lower than the minimum required $\theta_{\min}$, the corresponding gene must be removed from the chromosome, because it will be considered as an aberrant decision variable: it will have a value of zero for all the ongoing procedures.

**Figure 2** Representation of the chromosome



The second sub-chromosome represents the durations of the tasks' workloads $d_{i,k}$. As shown in Figure 2, it contains only the tasks and their required competences. The value of each gene in this part represents the integer duration variable $d_{i,k} \in [D_i^{\min}, D_i^{\max}]$ of the mission.

### *4.2 Generation of the initial population*

The implementation of the GAs begins with the creation of the initial population of size (IP). Thus, our initial population is generated randomly while respecting some constraints – such as allocation constraints and temporal constraints – for any task $i$, $D_i^{\min} \le d_{i,k} \le D_i^{\max}$. To respect the temporal constraints of tasks and the data resulting from initial scheduling (based on the tasks standard durations and on project precedence constraints), for every task $i$, we define $\mathrm{DR}_i^{\max}$ which represents its real maximum duration that it can take, based on its total float $\mathrm{MT}_i$:

$$\mathrm{DR}_i^{\max} = \max\left(D_i^{\max}, D_i + \mathrm{MT}_i\right) \tag{34}$$

The initial population will be treated in a separate way for each of the sub-chromosomes. The allocation part will be generated by random selection of the actors showing sufficient efficiencies to perform each competence for all the tasks; then the durations sub-chromosomes will be created considering the precedence relations between tasks as well as tasks' durations that result from the previous allocation choices. This procedure is repeated until the entire population size (IP) is generated.

### *4.3 Fitness function calculation*

With GAs, we can afford the latitude of neglecting some soft constraints, provided the evaluation result will be penalised in case of constraints violation. The concept of using 'penalty functions' (Davis, 1996; Morz and Musliu, 2004); it makes it possible to consider an adaptation function made up from the objective function, to which are added the quantified penalties linked to constraints violations. When all these penalties are equal to zero, it means that all constraints of the problem are satisfied. In addition to the working time constraints, some other constraints must be respected for each individual. Thus, to take into account the violation of the working time constraints (e.g. the daily working time for an actor exceeds the ten authorised hours), we introduced a penalty function $F_5$ in the objective function to inhibit chromosomes that do not respect these types of constraints. Analysing the different cost components of the objective function indicates the penalties weight in the event of constraints violation for calculating the component $F_5$ whenever these violations are authorised in the exploration process. In Equation (35), we added the term $F_5$ to Equation (31) to represent the sum of penalties related to the violations of working time modulation constraints. These penalties are expressed in monetary units – thus, $F$ has to be minimised.

$$F = F_1 + F_2 + F_3 - F_4 + F_5 \tag{35}$$

The GA evaluation phase consists in calculating the adaptation for each individual in the population. But, the GAs are implemented to maximise the objective function

(Goldberg, 1989); however, the step of solution of our problem consists in minimising the cost-based objective function. Therefore, it is necessary to transform the problem so that the minimum value of the objective function will be corresponding to the strongest individuals. Thus the work of Goldberg makes it possible to associate to the objective function $F(ind)$ of each individual ($ind$) a constant CMAX as large as possible to give a new non-negative function $F_{ab}(ind)$:

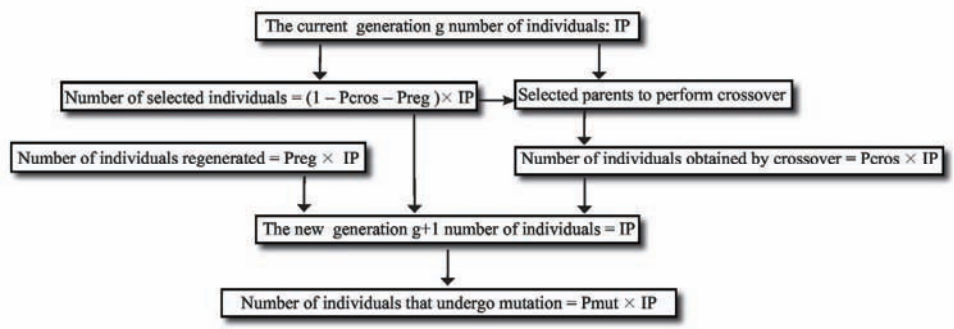
$$F_{ab}(\text{ind}) = \text{CMAX} - F(\text{ind}) \tag{36}$$

We call it the 'absolute force' of the individuals. This constant CMAX can be estimated from three cost terms: the first one corresponds to a project minimum cost assuming that we have a sufficient workforce to complete it without overtime, with fully efficient operators and right in time (thus encountering neither delay penalties nor storage fees). The second cost will be estimated from the maximum cost value of the constraints that may be violated. Finally, the third cost term will be related to the project duration when penalties costs exist. Taking into account all the probable maximum costs, one can estimate the value of CMAX that ensure non-negative results of the function $F_{ab}$ that has now to be maximised.

### *4.4 Construction of the next generation*

After the generation of the initial population and once the methodology of calculating the fitness value for each individual is defined, we need a construction strategy to build the generations $g + 1$ from the previous one $g$. As for all GAs, this construction is based on selection, crossover and mutation, and is repeated until a stopping criterion is reached. As shown in Figure 3, displaying the reconstitution of the individuals from a generation $g$ to a generation $g + 1$, the new population contains three groups of individuals; this proposed construction is similar to that of Mendes et al. (2009). The first group represents the individuals that were selected to survive with a given percentage and inserted directly to the generation $g + 1$. The second set is the individuals that were produced from the crossover operation; the third group results from the generation of some individuals, exactly as for the initial population.

**Figure 3** Reconstitution of individuals from generation $g$ to generation $g + 1$



First, the selection procedure consists in choosing, according to the fitness values, the individuals who will constitute a part of the next generation via the reproduction approach. The individuals' selection is based on the cost function (Equation (36)): each individual has a probability of contributing to the next generation according to its genetic inheritance, and this probability will be all the higher as its adaptation $F$(ind) is important. This operator is a conceptual data processing version of the natural selection and corresponds to Darwinian survival of the most adapted individuals.

The procedure of selection that we used is that operated with the Goldberg's roulette (Goldberg, 1989). It makes it possible to select randomly the individuals having the higher adaptations, still not prohibiting the selection of less adapted individuals: indeed, it is important to authorise the selection of weak individuals, to preserve a certain genetic variety within the future generations, ensuring a good exploration of the whole solutions domain. The selection is carried out by drawing only one individual each time of rotating a casino roulette, in which each individual of the population is assigned a sector of the wheel proportional to its relative force $F_{\text{relative}}$(ind). This relative force corresponds to its absolute force $F_{ab}$(ind) in relation to the total sum of the absolute forces of all the individuals in the generation:

$$F_{\text{relative}}(\text{ind}) = \frac{F_{ab}(\text{ind})}{\sum_{\text{ind}=1}^{\text{IP}} F_{ab}(\text{ind})} \quad \text{with} \quad \sum_{\text{ind}=1}^{\text{IP}} F_{\text{relative}}(\text{ind}) = 1 \tag{37}$$

The cumulation of the individuals' relative forces can be calculated to determine the identification zones of individuals with a generated random number between 0 and 1. The selection is done with a probability of survival $P_{\text{surv}}$ that is complementary to the added probabilities of reproduction $P_{\text{cros}}$ and regeneration $P_{\text{reg}}$. So, we have: $P_{\text{surv}} = 1 - (P_{\text{cros}} + P_{\text{reg}})$. The selection process is based on spinning the roulette wheel IP × $P_{\text{surv}}$ times each time a chromosome would be selected.

After that, the set of individuals that will apply the process of crossover to be a part of the next population will be selected with a number of IP × $P_{\text{cros}}$, each time two parents should be selected to produce only one individual, the later will be inserted to the new generation, as shown in Figure 3. Due to the heterogeneous nature of the chromosome encoding (containing two parts), the crossover process was done for each part separately. For the 'allocation' part of the chromosomes, the two-point crossover was used, taking into account that the exchange process takes place at the level of actors division (described in Section 4.1) not at the level of the genes themselves, since they represent the different competences of the task. But for the second part of the chromosome, the one-point crossover was used (Davis, 1996).

However, to enforce some renewal in the next generation, we introduce a probability of regeneration $P_{\text{reg}}$. This aims at reproducing a set of individuals, as in the case of the initial population, but with a rate lower than that of the crossover to preserve the spirit of GAs.

As for the crossover, the mutation is performed in the actor allocation part (the first part) through permuting two actors' work allocations. For the tasks' duration part, it is simply done by regenerating a value of the task skill's duration in $[D_i^{\min}, D_i^{\max}]$. After reproduction and mutation, the produced population perhaps shows some distortions, so a preparation procedure was presented to fix this distortion.

### *4.5 Stopping criteria of the algorithm*

As in any iterative algorithm, the implementation of a GA requires the definition of criteria that make it possible to stop the exploration procedure. For this model, we define two of them:

- The first criterion relates to the evolution, from generation to generation, of the average of the objective function. Indeed, the average of the objective function is not calculated from all the individuals of the population: we record in a file a fixed number of the best individuals ever found from all the generations already explored. If we call $g$ the number of generations, we consider the average for these best individuals at each generation, called Moy($g$). If we state no evolution of Moy($g$) during a given number IdMoy of successive generations (fixed in advance), then the exploration will be stopped.
- The second criterion relates to a maximum number of generations noted '*generation*' also fixed in advance. This second criterion acts if the first one is not satisfied after a long processing time: it just makes it possible to stop a research which does not seem to succeed.

## 5 Numerical example and results

This application example will be used to illustrate our methodology. For presenting it in an easier way, it is deliberately simple, too much simple to claim to be representative of a real industrial application. However, an application of important size is described in the thesis manuscript of Edi (2007), in conformity with what can be encountered in 'the real world'. In this application, we consider a project where there are $K = 4$ required competences and $A = 10$ actors; they are all multi-skilled, each one having one main competence (with an efficiency of $\theta_{a,k} = 1$) and additional competences for which $\theta_{a,k} \in [\theta_{\min}, 1]$ (as shown in Table A1); the minimal efficiency was set at $\theta_{\min} = 0.5$. In addition, operators are working under the working time regulations displayed in Table A2; the company has to carry out an activity consisting of $I = 10$ tasks for which the data are represented in Table A3. We assume that all the actors cost the same hourly rate $U$ (in monetary unit: mu per hour), and that the overtime hours are raised of $u = 25\%$ compared to the normal hours. Without the loss of generality in this example, the relations between tasks are of the *Finish–Start* type, and the delays $\alpha_{i,n}$ on these relations are all zero. According to the initial scheduling established with the standards durations $D_i$ and considering only the tasks precedence relations, the project duration is $L = 25$ days, which will be considered as the contractual duration. We will allow a tolerance of $\beta = 5$ days related to this contractual duration: to avoid paying lateness penalties as well as supporting storage costs, it is necessary that the real duration LV lies between:

$$L - \beta \leq \mathrm{LV} \leq L + \beta \Rightarrow 20 \leq \mathrm{LV} \leq 30 \text{ days}$$

This application was tested on a computer machine of the type: 'Intel dual core Xeon 2.4 GHz with a 2.5 GB RAM. The coding was carried out on the programming software 'Visual C++'.

The methodological approach consists of three inter-dependent parts. Initially, we read the data of the model. Then we start a feasibility pre-study procedure to investigate the adequacy between the workload and the availability of the company resources; if we find insufficient evidence of the availability to cover the workload, then we validate the study of non-feasibility of the activity – in other words, we stop looking for a solution, knowing that there cannot be any. Otherwise, we begin the third step that represents the exploration process with GAs.

For this application, the following simulation data were used: the study considers a population size of IP = 200 individuals, for which the probabilities of crossover, regeneration and mutation were fixed at: $P_{\text{cros}} = 0.65$, $P_{\text{reg}} = 0.10$, $P_{\text{mut}} = 0.20$, respectively. The algorithm is stopped for non-evolution of the average after IdMoy = 100 generations – this average concerning the 15 best individuals of the whole of the already explored generations (stopping condition). The maximum number of generations is set at generation = 8,000 (stopping condition for non-convergence). The penalties in case of modulation constraints violation are presented in Table A4. To appreciate the application of GAs to the problem, we are treating, ten simulations have been conducted. Table A5 gives the results concerning computation times, number of generations, total real durations of the activity as well as the values of the objective function for the best individual by generation according to the evaluation criteria. We can check that the limit of 8,000 generations was never reached: for each run, convergence caused the process stop. In this set, the simulation no. 4 presents the best value of the objective function, thus, scheduling data of this solution (the durations and the start dates of tasks) are listed in Table A6, and the components of the objective function are detailed in Table A7. To check the duration constraints, for instance: we take the example of the task 3 of which the global duration is $d_3 = \max(d_{3,k})_{k=1,\ldots,4} = 7$ days. From the activity data, we know that its standard duration is $D_3 = 4$ days, its maximum duration $D_3^{\max} = 7$ days and its minimum duration $D_3^{\min} = 3$ days. So, using the initial scheduling, we can obtain the total float $\text{MT}_3 = 0$ day: thus the maximum duration of this task is $\text{DR}_3^{\max} = \max(D_3^{\max}, D_3 + \text{MT}_3) = 7$ days. Taking into account this information, we can check that task 3 respects its duration constraints $3 \leq d_3 \leq 7$. However, the durations on competences are different. Thus, actors affected on the workload $w_{3,2}$ can be released at the end of four working days, whereas other workloads of the same task continue to be carried out ($w_{3,3}$ lasts five days and $w_{3,4}$ seven days). The analysis is the same for the other tasks; however, they all respect the constraints of definite durations. With regard to the precedence constraints, e.g. in the activities data file task 3 has tasks 5 and 8 as successors, and as predecessor task 2. The result provided by Table A5 indicates that the start date of task 3 is $\text{dd}_3 = 8$ with one duration of $d_3 = 7$ days, which provides an end date of $\text{dd}_3 + d_3 = 15$. However, the start dates of its successors are: for task 5 we have $\text{dd}_5 = 15 \geq \text{dd}_3 + d_3$ and for task 8 we have $\text{dd}_8 = 20 \geq \text{dd}_3 + d_3$. For its predecessor task we have $\text{dd}_3 \geq (\text{dd}_2 + d_2 = 4 + 4 = 8)$. We notice that the precedence constraints are fully respected. From the data of Table A5, we can deduce the real duration of execution of the activity $\text{LV} = \max(\text{dd}_i + d_i)_{i=1,\ldots,10} = 30$ days.

The workforce assigned on each competence are also presented in Table A6; we notice that for task 3, the assigned real workforce on the workload $w_{3,2}$ is: $\text{ER}_{3,2} = 5$ actors, all five representing an equivalent workforce of $\text{EE}_{3,2} = 4$: this means that some of

these actors have efficiencies lower than the nominal value. The global productivity of realisation is then:

$$\tau\text{opt} = \frac{\sum_{i=1}^{10}\sum_{k=1}^{4}\text{EE}_{i,k}}{\sum_{i=1}^{10}\sum_{k=1}^{4}\text{ER}_{i,k}} = \frac{79.8}{93} = 0.8580$$

corresponding to a real work of: $\omega = \sum_{a=1}^{10}\sum_{j=1}^{30}\omega_{a,j} = 1{,}307.43\text{ hr}$, to achieve a total standard workload of $w = \sum_{i=1}^{10}\sum_{k=1}^{4} w_{i,k} = 1{,}128\text{ hr}$.

The analysis of the daily work indicates that no daily constraint was violated whatever the actors and the days: $\omega_{a,j} \leq (\text{DMaxJ} = 10\text{ hr})$, $\forall a$, $\forall j$ as shown in Figures A1–A3. Summation of work performed by each actor does not show any violation of the weekly constraints (the verification was not performed over a floating period of 12 consecutive weeks because the real total duration is 30 days = 6 weeks).

The residual flexibility of each actor that has been introduced by Equation (33) after the assignment is shown in Figure A4. It raises the rate of availability preservation of each actor at the end of the realisation of the activity. Based on the flexible working time strategy, this reservation of working hours can be stored to be consumed for further peaks of work requirements. For example, the operator $a_1$ shows a residual flexibility of about 0.4 at the end of the activity: it means that, according to the annualised working time, an average of about 40% of the standard weekly working hours of this actor remain available for other activities either in parallel or to come. In other words, the actors' average occupation rate during the project period is of about 60% of the standard weekly working hours.

GAs for solving the problem of resource allocation taking into account the actors' multi-skills give an acceptable solution. First of all, the solution does not violate any constraint of the working time regulations, then it provides a final acceptable tasks scheduling, respecting the contractual duration, neither immobilisation cost nor lateness penalty. Moreover, the computing time for this example makes it possible to carry out several simulations, to get a good solution – or at least an acceptable compromise by using the properties of stochastic convergence of the GAs. This study allows validating the use of GAs for obtaining a human resources allocation with flexible working time and multi-skilled actors.

The use of actors' multi-skills has a great influence on managerial implications by providing the opportunity to face situations that would be impossible or at least very difficult to handle without such flexibility. For example, in this simple application and during the feasibility study that investigates the ratio between the required daily workloads and the actors availabilities, one can check easily that without multi-skills, there is a shortage of resources concerning skills 2 and 4 during part of the schedule horizon: this makes it impossible to perform the programme within the time limits. Thanks to GAs, our model benefits from quick evaluations of what could be the best acceptable solutions to a given problem of industrial activities scheduling – even with more realistic problem sizes – as explored by Edi (2007): using it as a simulator may provide the decision-maker with fast evaluations of feasibility for different scenario (involving for instance more or less flexibility via monitoring either the operators' additional competencies or the value of $\theta_{\min}$, spreading or restricting the combinatorial

exploration); and each feasible solution is supplied with its consequences in terms of programme duration and execution costs.

This concerns the manager in charge of the execution of a given programme; from the company management's point of view, such a model can be used to evaluate how they will face a provisional projects portfolio. This will help to highlight upcoming gaps between available competences and the workloads to undertake. Thus, a human resources manager may have a reliable forecast of what are the competencies that will have to be developed in the future, to what extent and with what reprieve – simultaneously, he will also know which skills will lose their importance in the future. This projection facility will help in defining future policy in terms of skills management: which competences are to develop via training or to enforce via recruitment, and who are the operators requiring some conversion. So, from a managerial point of view, results of this work may be considered as a decision-making tool for skills as well as human resources management, and more generally, for company activities steering.

## 6 Conclusions

The model we developed benefits from an accurate description of the many data that companies have to handle when they face the problem of planning and scheduling future activities. Taking into account flexible working time and opportunity to switch operators from a skill to another at the same time, it also considers the impact of allocation decisions on activities' durations and costs. Few works in the literature may claim for such a level of accuracy. Moreover, according to the tests performed so far, dealing with issues of growing size and complexity, it shows to provide reliable and fast answers to resource-constrained allocation problems.

At present, we are considering two future short-term developments for this model. The first one concerns resolution method and aims at reducing the computing time required by the search for solutions. The computing times we encountered during our tests were all acceptable – but we considered case studies counting at most a few hundred tasks, and we logically experienced increasing computing times with case complexities. The stochastic nature of GAs results in a non-ideal use of calculation means. This led us to imagine ways to enhance a random quest through progressive restrictions of the research domain.

Another issue we will face within a near future concerns the consistency of the skills model. Until now, we have considered that an operator's efficiency is a fixed data; but for decades, many academic works as well as industrial practise have well established that human's efficiency is strongly influenced by experience (Wright, 1936). So, the next step in making the model more accurate will be to take into account past allocations and corresponding working time for every actor in each of his competences. We will also have to describe rates of learning for operators – as well as the rhythm at which they loose competence if they do not implement it. This will allow us to determine for each actor whether his efficiency growths or regress along the project – and to what extent.

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## Appendix

### *A1 Nomenclature*

*Subscripts*

| | |
|---|---|
| $a$ | Actor index |
| $i$ or $n$ | Task index |
| $j$ | To indicate the day number |
| $k$ | Competence index |
| Max | To indicate the largest value of whole |
| Min | To indicate the smallest value of whole |
| $s$, $p$ | To indicate the week number |

*Data*

| | |
|---|---|
| $A$ | Set of actors – also appoints the total number of actors, integer number |
| $C_{s0}$ | Standard duration of weekly working time, expressed in hours, integer number. In France: $C_{s0}$ = 35 hr per week |
| $D$ | Standard duration of the tasks to be performed, expressed in days, integer number |
| DMax12S | The maximum average of weekly working hours for a period of 12 consecutive weeks, in hours, integer number |
| DMaxJ | Maximum daily working time, expressed in hours, integer number |
| DMaxMod | Duration of the maximum weekly working time according to the agreements of company modulation, expressed in hours, integer number |
| DMaxS | Maximum weekly working time, expressed in hours, integer number |
| DSA | Annual working time of any actor, expressed in hours, integer number |
| HAS | Maximum annual overtime, integer number |
| $I$ | Indicate the total number of activity tasks, integer number |
| $IP$ | number of individuals in the Initial Population |
| $K$ | Set of competences; also indicates the total number of competences, integer number |
| $L$ | Fixed contractual period of the activity, expressed in days, integer number |
| NJS | The number of working days per week, integer number; we suppose that this number of days is identical for all the actors. By default, we suppose that NJS = 5 days |
| $nk$ | Indicate the number of competences held by an actor, integer number |
| $U_a$ | Actors' standard hourly cost, in monetary units, real number |
| $u$ | Overtime allowance rate, multiplicative factor applied to standard hourly cost, real number, dimensionless |
| UF | Fixed cost associated to residual flexibility, in monetary units, real number |
| US | Daily cost of lateness penalties, in monetary units, real number |
| $w$ | Indicates the workload in hours, real number |
| $\alpha$ | Time difference added to a temporal relation between tasks, expressed in days, integer number |
| $\beta$ | Flexible part of the fixed contract period of the activity, expressed in days, integer number (overrun or delay permitted without risk of penalty) |
| $\Delta$ | Set of all predecessor of a given task; also indicates their total number, integer number |
| $\theta$ | Efficiency of an actor for a given competence, real number between 0 and 1, dimensionless |
| $\tau$ | Daily penalty rate for storage of finished product, real number, dimensionless |

*Decision variables*

$\sigma_{a,k,i}$ Allocation of the actor $a$ on the task $i$ for the competence $k$, binary variable.

$$\sigma_{a,k,i} = \begin{cases} 1 & \text{if the actor } a \text{ is assigned to the competence } k \text{ of the task } i \\ 0 & \text{otherwise} \end{cases}$$

$d_{i,k}$ Actual execution time period of a competence $k$ from task $i$, in days, integer number.

*Other variables*

| | |
|---|---|
| $\omega$ | Work, in hours, real number |
| $d$ | Global execution period of a task ($d_i = \max(d_{i,k})_{k=1,\ldots,K}$), integer number |
| dd | Start date, integer number |
| EE | Equivalent workforce, real number, dimensionless |
| ER | Real workforce, integer number |
| HS | Overtime hours, in hours, real number |
| LV | Total real duration of activity execution, in days, integer number |
| $O$ | Actor rate of occupation, real number, dimensionless |
| $\rho$ | Set of running tasks; also indicates the total number of them, integer number |
| $\tau$FlexR | Average residual flexibility of the actors, real number, dimensionless |
| $\tau$opt | Allocation optimisation rate, real number, dimensionless |

## *A2 Example data*

**Table A1** The company data

| | $\theta_{a,k}$ | | | |
|---|---|---|---|---|
| *Actor* | *k = 1* | *k = 2* | *k = 3* | *k = 4* |
| 1 | 0.8 | 1 | 0 | 0.5 |
| 2 | 1 | 0.0 | 0.8 | 0.0 |
| 3 | 0 | 0.6 | 0.0 | 1 |
| 4 | 0.7 | 0.0 | 1 | 0.6 |
| 5 | 0.0 | 1 | 0.7 | 0.0 |
| 6 | 0.9 | 0.0 | 0.0 | 1 |
| 7 | 1 | 0.8 | 0.0 | 0.6 |
| 8 | 0.0 | 0.7 | 1 | 0.0 |
| 9 | 1 | 0.8 | 0.0 | 0.5 |
| 10 | 0 | 0.9 | 1 | 0.0 |

**Table A2** The regulatory data

| | | |
|---|---|---|
| Annual number of working hours for any actor | DSA | 1,600 hr |
| Maximum annual overtime working hours | HAS | 180 hr |
| Maximum weekly working hours | DMaxS | 48 hr |

**Table A2** The regulatory data (continued)

| | | |
|---|---|---|
| Maximum average of weekly working hours for a period of 12 consecutive weeks | DMax12S | 44 hr |
| Maximum weekly standard working time according to the agreements of company modulation | DMaxMod | 39 hr |
| Weekly standard working hours | $C_{s0}$ | 35 hr |
| Maximum daily working hours | DMaxJ | 10 hr |
| Number of working days per week | NJS | 5 days |
| Actors' standard hourly cost | $U$ | 11 money unit $hr^{-1}$ |
| Overtime allowance rate | $u$ | 0.25 |

**Table A3** Activity data

| *Task i no.* | $D_i$ | $D_i^{min}$ | $D_i^{max}$ | $w_{i,k}$ *(hr)* | | | | *Successors* | *Relation* | $\alpha_{i,n}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | *k = 1* | *k = 2* | *k = 3* | *k = 4* | | | |
| 1 | 4 | 2 | 6 | 0 | 60 | 0 | 50 | 2–3–4 | F-S | +0 |
| 2 | 5 | 3 | 7 | 45 | 68 | 0 | 0 | 3–5–7 | F-S | +0 |
| 3 | 4 | 3 | 7 | 0 | 63 | 45 | 35 | 5–6 | F-S | +0 |
| 4 | 7 | 5 | 10 | 53 | 0 | 60 | 0 | 6–9 | F-S | +0 |
| 5 | 4 | 2 | 6 | 0 | 65 | 0 | 60 | 7–8 | F-S | +0 |
| 6 | 3 | 1 | 5 | 60 | 0 | 35 | 0 | 8–9 | F-S | +0 |
| 7 | 5 | 3 | 7 | 35 | 56 | 0 | 40 | 10 | F-S | +0 |
| 8 | 5 | 3 | 8 | 0 | 0 | 47 | 50 | 10 | F-S | +0 |
| 9 | 4 | 2 | 5 | 0 | 45 | 26 | 0 | 10 | F-S | +0 |
| 10 | 3 | 2 | 4 | 35 | 30 | 35 | 30 | – | – | – |

**Table A4** Example of penalties in case of modulation constraints violation (in monetary units)

| | |
|---|---|
| Daily violation constraint | 6,000.00 |
| Weekly violation constraint | 3,500.00 |
| Floating period of 12 consecutive weeks violation constraints | 2,000.00 |

## *A3 Example results*

**Table A5** Results of simulations with GAs

| *Simulation no.* | *Computation time (sec)* | *No. of generations* | *Objective function* | $F_5$ | *LV (days)* |
|---|---|---|---|---|---|
| 1 | 76 | 613 | 7,833.151 | 0 | 29 |
| 2 | 144 | 1,055 | 9,325.489 | 0 | 29 |
| 3 | 102 | 573 | 9,897.428 | 0 | 31 |
| 4 | 155 | 709 | 6,834.938 | 0 | 30 |
| 5 | 84 | 686 | 7,674.276 | 0 | 30 |

**Table A5** Results of simulations with Gas (continued)

| *Simulation no.* | *Computation time (sec)* | *No. of generations* | *Objective function* | $F_5$ | *LV (days)* |
|---|---|---|---|---|---|
| 6 | 114 | 849 | 7,646.877 | 0 | 30 |
| 7 | 150 | 1,196 | 7,275.291 | 0 | 30 |
| 8 | 59 | 467 | 7,715.610 | 0 | 29 |
| 9 | 74 | 377 | 7,748.657 | 0 | 29 |
| 10 | 122 | 1,023 | 7,220.690 | 0 | 30 |

**Table A6** GAs results, tasks' durations, start dates and real/equivalent workforce for each skill

| *Task no.* | $d_i$ *(days)* | $dd_i$ | $d_{i,k}$ *(days)* | | | | $ER_{i,k}$ | | | | $EE_{i,k}$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $k = 1$ | $k = 2$ | $k = 3$ | $k = 4$ | $k = 1$ | $k = 2$ | $k = 3$ | $k = 4$ | $k = 1$ | $k = 2$ | $k = 3$ | $k = 4$ |
| 1 | 4 | 0 | 0 | 4 | 0 | 4 | 0 | 4 | 0 | 4 | 0 | 3.7 | 0 | 3.1 |
| 2 | 4 | 4 | 3 | 4 | 0 | 0 | 5 | 5 | 0 | 0 | 4.7 | 4.5 | 0 | 0 |
| 3 | 7 | 8 | 0 | 4 | 5 | 7 | 0 | 5 | 2 | 3 | 0 | 4 | 1.8 | 2.5 |
| 4 | 10 | 4 | 10 | 0 | 5 | 0 | 4 | 0 | 4 | 0 | 3.4 | 0 | 3.5 | 0 |
| 5 | 5 | 15 | 0 | 4 | 0 | 5 | 0 | 7 | 0 | 2 | 0 | 5.8 | 0 | 2 |
| 6 | 4 | 15 | 4 | 0 | 3 | 0 | 4 | 0 | 3 | 0 | 3.7 | 0 | 2.8 | 0 |
| 7 | 5 | 20 | 5 | 3 | 0 | 5 | 2 | 5 | 0 | 2 | 1.9 | 4.5 | 0 | 1.6 |
| 8 | 6 | 20 | 0 | 0 | 3 | 6 | 0 | 0 | 3 | 5 | 0 | 0 | 2.8 | 3.6 |
| 9 | 5 | 19 | 0 | 3 | 5 | 0 | 0 | 6 | 3 | 0 | 0 | 4.8 | 2.7 | 0 |
| 10 | 4 | 26 | 3 | 2 | 4 | 4 | 3 | 5 | 3 | 4 | 2.6 | 4.4 | 2.8 | 2.6 |

**Table A7** Results data of the exploration method with objective: $F = F_1 + F_2 + F_3 - F_4 + F_5$

| *Comparison criteria* | *Results*[a] |
|---|---|
| $\tau$ opt | 0.858 |
| Objective function ($F$) in monetary unit | 6,834.94 |
| Workforce costs without overtime ($F_1$) | 14,316.54 |
| Overtime cost ($F_2$) in monetary unit | 66.68 |
| $F_3$ (storage or lateness penalty cost) | 0.00 |
| $F_4$ (cost of residual flexibility) | 7,548.28 |
| $F_5$ (cost of violation of modulation constraints) | 0.00 |
| Computing time in second | 155 |
| Total duration of the activity LV in days | 30 |

[a]These results can be compared to the minimum cost for achieving the project, assuming that all the operators allocated are fully efficient and that no extra hours needed, the minimum cost will be = 12,408.00 money unit.

**Figure A1** Daily works by actors ($a_1$, $a_2$ and $a_3$)

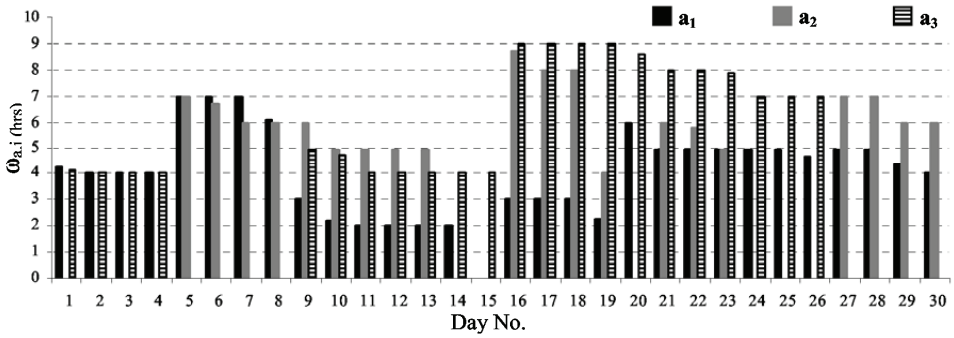


**Figure A2** Daily works by actors ($a_4$, $a_5$ and $a_6$)

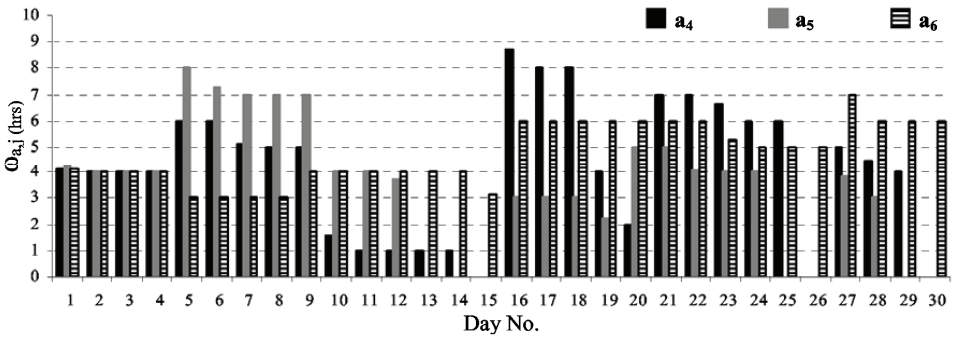


**Figure A3** Daily works by actors ($a_7$, $a_8$, $a_9$ and $a_{10}$)

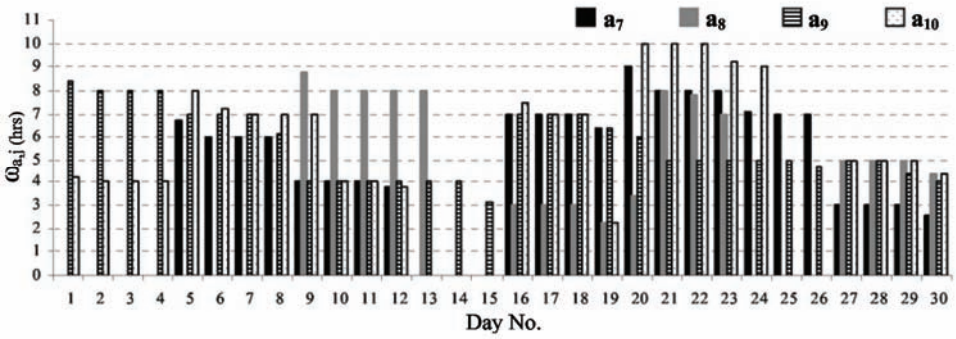

**Figure A4** Residual flexibility by actor