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CURVED CHANNEL FLOW WITH STENOSES AND ANEURYSMS

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1 Introduction

Curvature is everywhere, in man or nature made devices. Its implication on physiological flows may be as important as other inertial or viscous effects. Relatively to an otherwise straight vessel, the so called centrifugal forces induce secondary flow that modify the whole flow structure and give rise to Dean vortices. Since the pioneering work of Dean many fundamental and applied investigations were performed. More specifically, in the blood dynamics studies, experimental and numerical simulations were also achieved. On the other hand many studies dealt with stenoses or aneurysms mainly situated in otherwise straight vessels. Thus the coupling between the global curvature effects and local section variations due to stenoses or aneurysms is not enough investigated.

In the present work we will quantify how the impact of a stenosis or an aneurysm is modified when it occurs in a curved vessel compared to when it is located in a straight environment.

2 Methods

The configuration considered is of a U shaped 2D channel. Through an asymptotic method and a process similar to that used in [1], a uniformly valid reduced mathematical model was asymptotically established and is, given in local curvilinear coordinates (ξ,η), as follows:

$$\frac{\partial u}{\partial \xi}+\frac{\partial}{\partial \eta}\left(v-\frac{\alpha'}{\alpha}\eta u\right)=0\,. \qquad (1)$$

$$u\frac{\partial u}{\partial \xi}+\left(v-\frac{\alpha'}{\alpha}\eta u\right)\frac{\partial u}{\partial \eta}-\frac{\alpha'}{\alpha}u^2=-\alpha^2\frac{\partial p_1}{\partial \xi}+$$

$$.\frac{1}{\alpha R_e}\frac{\partial}{\partial \eta}\left(1+\alpha K\eta\right)\frac{\partial u}{\partial \eta} \qquad (2)$$

$$\frac{\partial p_1}{\partial \xi}=\left(A'''+\frac{K'}{\alpha}+\frac{K\alpha'}{\alpha^2}\right)\int_0^{\eta}u_0^2\,d\eta'+B'(\xi) \qquad (3)$$

Associated with the boundary conditions,

$$u=v=0 \text{ for } \eta=\pm 0.5.$$

All variables are non dimensional. u and v are longitudinal and radial velocities, $p_1(\xi,\eta)$ is the core pressure $.\alpha(\xi)$ is the local section's width, $K(\xi)=k(\xi)\delta$ where $\delta=H/R_c$ is the curvature , R_c being the radius of curvature and H the channel's width. R_e is the Reynolds number. Primes stand for derivatives relative to the longitudinal coordinate ξ.

Starting from Navier Stokes (NS) equations written in curvilinear coordinates, an $O(\delta)$ reduced version of NS was first obtained. Then equations (1-3) were established on the assumption that the Reynolds number is high and that the curvature δ is small. However, as was shown in [1], the validity of the model is relatively accurate even for moderate and small Reynolds number when the curvature δ is kept small, say 0.1.

The pressure gradient equation (3) is one of the main novelties of the model. This expression captures the salient aspects of the pressure field as induced by the curvature and section variations. It does show the induced contributions due to the different geometric variations and their couplings. A''' is the (inertial) contribution of any discontinuity such as that of curvature at the straight-curved junctions. The K'/α showcases the curvature variation effect weighted by the local section, while the $K\alpha'/\alpha^2$ represents the curvature effect as mediated by the local section reduction or dilatation.

The pressure gradient equation is the mediator between the core flow and boundary layers regions. It tunes the adjustment of the parameters of the flow field involved in the interaction between the two zones. The two unknowns $A(\xi)$ and $B(\xi)$ are the only unknowns to be provided to compute the pressure field at each section. The case of a uniform section is retrieved when $\alpha=1$ and corresponds to the cases treated in [1]. The uniformly valid asymptotic reduced model (UV-ARM) (1-3) was solved numerically as in [1].

3 Results and Discussion

Fig.1(a) shows the case of a stenosis located at the middle of the curved part so as to avoid any interactions with the entry or exit effects. The stenosis length is 4 times the channel (straight) width H, while the stenosis maximum is $0.3H$, hence equivalent to a 60% diameter reduction for this symmetrical case. The Reynolds number is equal to 1000.

It is clearly seen that the model is quiet accurate and reproduces, qualitatively and quantitatively, the entire main features such as upstream phenomenon, the accident local and vicinity effects, the established flows as well as their up and downstream transitions.

One of the main features of Fig.1(a) is the existence of a separate zone at the inner wall while at the outer wall the flow remains attached although the stenosis is geometrically symmetric.

It is also worth noting the absence of any upstream anticipation of the presence of this symmetrical stenosis in the curved part while there is one at the straight-curved junctions, due to their non symmetrical character.

In the symmetrical stenoses with a straight environment there is a symmetric separation zone and the shear stress remains symmetric. Thus the location of the same stenosis at the curved part with all other things alike prevent the outside wall flow from separating.

4 Conclusions

An asymptotic reduced model (ARM) was established and compared favourably to complete Navier Stokes numerical resolution. It does incorporate global and local curvature's effects, section variations as well as curvature discontinuities. It was then applied to simulate the case of stenoses in a straight or in a curved environment. It was shown that curvature may suppress the separation zone at the outside wall in the case of a symmetric stenosis, while in the straight case the flow separates at both walls symmetrically. Asymmetrical constrictions and dilatations were also investigated and will be shown in the presentation.

In the 3D situation the centrifugal effects create an inside to outside oriented secondary core flow concomitant with the azimuthal secondary flow from the outer wall to the inner wall..

A future work will be to check how far the median plane of a 3D case, due to the secondary flow, differs qualitatively from this 2D model and its separation zones.

Reference

[1] Zagzoule M., Cathalifaud P, Cousteix J. and Mauss J., Uniformly valid asymptotic flow analysis in curved channels. *Physics of fluids, 24(1), 2012*

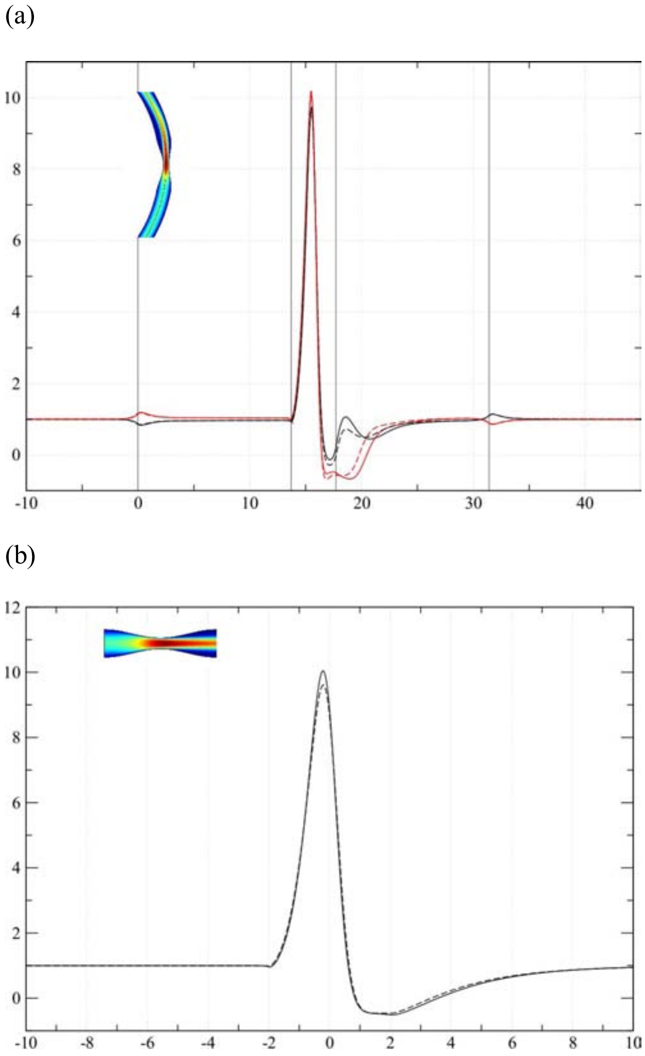

Fig 1. Shear stress at both walls (a): the curved case and (b) the straight case. Red: inner wall and black: for outer wall. Continuous curves are the results of complete Navier Stokes resolution while the discontinuous ones are the results of the present model (1-3).