To cite this document: Roussouly, Nicolas and Salaün, Michel and Petitjean, Frank and Buffe, Fabrice and Carpine, Anne *Reliability approach in spacecraft structures*. (2009) In: ECSSMMT 2009 - 11th European Conference on Spacecraft Structures, Materials and Mechanical Testing, 15-17 Sep 2009, Toulouse, France.

RELIABILITY APPROACH IN SPACECRAFT STRUCTURE

Nicolas ROUSSOULY(1)(2), **Michel SALAUN**(2), **Frank PETITJEAN**(1)
Fabrice BUFFE (3), **Anne CARPINE** (4)

(1) *Institut Catholique d'Arts et Métiers (ICAM), 75 avenue de Grande Bretagne, 31300 Toulouse, France*
nicolas.roussouly@icam.fr, frank.petitjean@icam.fr, 05.34.50.50.16
(2) *Université de Toulouse, ISAE, 10 avenue Edouard Belin 31055 Toulouse, France*
michel.salaun@isae.fr, 05.61.33.92.83
(3) *Centre National d'Etudes Spatiales (CNES), 18 Avenue Edouard Belin 31401 Toulouse, France*
fabrice.buffe@cnes.fr, 05.61.28.34.65
(4) *Thales Alenia Space, 100 boulevard du Midi 06156 Cannes la Bocca, France*
anne.carpine@thalesaleniaspace.fr, 04.92.92.65.07

ABSTRACT

This paper presents an application of the probabilistic approach with reliability assessment on a spacecraft structure. The adopted strategy uses meta-modeling with first and second order polynomial functions. This method aims at minimizing computational time while giving relevant results. The first part focuses on computational tools employed in the strategy development. The second part presents a spacecraft application. The purpose is to highlight benefits of the probabilistic approach compared with the current deterministic one. From examples of reliability assessment we show some advantages which could be found in industrial applications.

INTRODUCTION

Up to now, mechanical analysis in spacecraft structure has been deterministic. Numerical values of input variables are specified in order to guarantee structural performance in the worst cases. Structure validation is based on safety margin calculation which contains safety factors. This approach aims at simplifying the problem of uncertainties by makeing sure they are covered but is unable to bring under control neither risk of failure nor oversizing of the structure.

For several years, certain approaches to take into account uncertainties in modeling have arisen. The probabilistic approach is one of them and consists of considering uncertainties as probabilistic ones, i.e with random variables characterised by their probability density function (pdf). This procedure gives more information like output uncertainties, reliability with respect to a failure mode or sensitivity of input variables. These aspects are very helpful in industrial applications especially in risk analysis and in order to find an optimal design from an economical point of view that deals with technical and financial information.

To formalize the reliability concept we consider $\boldsymbol{X} = \{X_1, ..., X_n\}$ an input random vector with n random variables with a joint probability density function $f_{\boldsymbol{X}}(\boldsymbol{x})$ and a performance function $G(\boldsymbol{X})$ which defines three mechanical domains : (a) domain of success : $G(\boldsymbol{X}) > 0$, (b) limit state : $G(\boldsymbol{X}) = 0$ and (c) domain of failure : $G(\boldsymbol{X}) < 0$. Then probability of failure reads

$$P_f = \int_{G(\boldsymbol{X})<0} f_{\boldsymbol{X}}(\boldsymbol{x}) d\boldsymbol{x}$$

and reliability, called F, is equal to $F = 1 - P_f$. Therefore, the probability of failure calculation implies knowledge of the joint pdf and evaluation of the performance function. Joint pdf is generally assumed because of the lack of information on marginal laws and variable correlation. In large scale structures, performance function stems from a finite element code and the number of parameters to take into account grow significantly which is increasingly time consuming. Moreover, it is often necessary to determine reliability with respect to several performance functions because of the number of mechanical components and load cases.

Several method have been developed to handle these problems [8], [6]. Although simulation methods such as Monte Carlo or improved techniques are robust, they are computationally too demanding for assessing low probability of failure, i.e lower than 10^{-6}. Other methods, like first and second order reliability methods, commonly called FORM and SORM exist. They use an optimization algorithm to find the most probable failure point and then approximate the limit state (with a first or second order function) in order to assess probability of failure. Such methods are interesting with a regular limit state and a low probability of failure but require an optimization procedure per response. Another idea to achieve propagation of uncertainties is meta-modeling as an approximation of a finite element model. They are widely used in other domains of science and their efficiency in mechanical structure has already been proved. The most commonly used are first and second order response surface, polynomial chaos, kriging or radial basis function but there also exists more complicated kinds of meta-models such as neural networks or support vector machines [2]. In industrial cases, advantages can be found in meta-models. They can be treated with a relatively low computational cost by comparison with a finite element one. Moreover, as a result of lack of knowledge about uncertainty of variables, repeating a study with the lowest cost to assess importance of pdf is necessary. Finally, they may be useful in optimization and reliability based optimization.

In this paper we propose to use first or second order polynomial functions which are sufficiently well suited approximations in case of linear elastic behavior under static load with low variation on parameters. In the next section we present some computational tools used. The procedure is then applied to a spacecraft structure in order to obtain reliability results on components which can potentially fail at first. Since the finite element model is not updated after a first failure we are not able to take into account successive failures. Finally, the kind of results and benefits obtained will be discussed.

1 STRATEGY DEVELPOMENT

1.1 Sentivity analysis and variable selection

A finite element model of an industrial structure contains a lot of variables and it is often computationally too demanding to include all of them into the study.

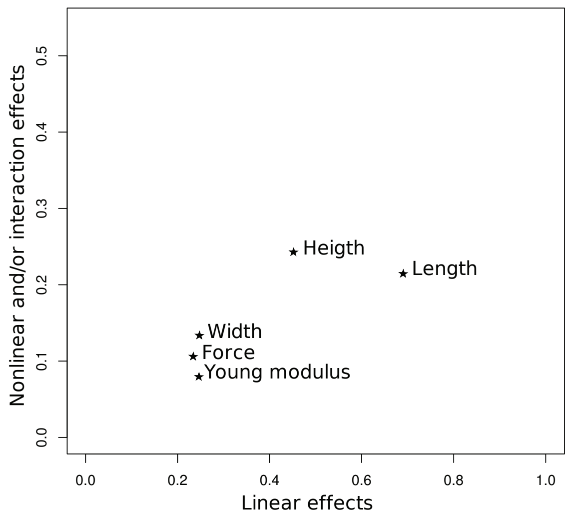

Figure 1: Graphical result of Morris OAT experiment

Generally, it appears that only a few influence studied responses. Therefore, detecting a couple of the most relevant is valuable and can be done through sensitivity analysis. Some methods exist in computer simulation whose purpose is to rank input variables in terms of their importance. These techniques are known as screening methods and are able to deal with hundreds variables (more information on screening methods can be found in [11]). One-at-a-time (OAT) design proposed by Morris [9] appeared to be a good compromise between computational cost and the relevance of results [7]. Morris OAT is a global sensitivity experiment because it covers the entire space over which variables vary. It is used to distinguish (a) negligible effects, (b) linear effects without interactions, and (c) non-linear effects and/or interactions. In practice, results are plotted on a graph which represents non-linear and/or interaction effects versus linear effects. Figure 1 shows an example of a Morris OAT experiment for the displacement at the end of a bending embedded beam in terms of force, young modulus, length, width and height. The number of computer runs is equal to $3(n+1)$ where n is the number of variables (it could be more but $3(n+1)$ is enough to obtain good information). Morris OAT doesn't really quantify the sensitivity of variables but rather gives a hierarchy. More than in probabilistic approach, this kind of method can help in model understanding, optimization and model updating. When it is used in variable selection, selected variables will be considered as random variables characterized by their distribution while others will be fixed to deterministic values. Therefore, meta-models will only link selected variables to studied responses. The meta-modeling process imply several steps which are presented now.

1.2 Specification of experimental design

Experimental design specifies a set of input values for which finite element computation is performed in order to get corresponding outputs. Well known techniques called Design of Experiment (DOE) are generally used in physical experiments. They focus on planning the experiment in order to satisfy some optimality criteria based on physical random error. In our study, the experiment is a computer deterministic simulation and some distinctions such as random error or number of factors and their range of variation can be noticed. In this way, many authors advise the use of "space filling" design in order to best cover the design space. Methods like latin hypercube sampling (LHS) or low discrepancy sequences which imply uniform distribution of points in space are often used in literature [14, 12, 11, 10]. We will afterward perform LHS technique.

1.3 Determination of meta-models

In this paper we use first and second order polynomial functions as an approximation of the finite element model in linear regression. The linear regression equation is given by

$$\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \tag{1}$$

where $\boldsymbol{Y}$ is the vector of output values Y_p with $p \in [1, P]$ at each sample, $\boldsymbol{X}$ is a $k \times p$ matrix where $k \in [1, N]$ which represents a matrix of regressors, $\boldsymbol{\beta}$ is the vector of parameters β_k which must be determined and $\boldsymbol{\varepsilon}$ is the vector of error ε_p at each sample point due to the approximation. The best way to determine parameter estimation, noted $\hat{\boldsymbol{\beta}}$, of this model is the least square method. It appears that $\hat{\beta}_k$ depends on chosen regressors and input sample and is therefore a random variable whose quality must be guaranteed. This quality is measured by the accuracy characterised by the bias $(\mathbb{E}[\hat{\beta}_k] - \beta_k)$ and the stability characterized by the variance $(Var[\hat{\beta}_k])$. Bias and variance are both to be minimized but are antagonists : fitting is improved when model dimension increases but the variability is worse. Quadratic risk (QR) takes into account both aspects and is given by

$$QR(\boldsymbol{X}\hat{\boldsymbol{\beta}}) = Var[\boldsymbol{X}\hat{\boldsymbol{\beta}}] - Bias[\boldsymbol{X}\hat{\boldsymbol{\beta}}]^2 \tag{2}$$

If we suppose that for all p, ε_p is an independently, normally distributed random variable of zero mean and with a variance noted σ^2, then QR can be estimated by the well known Mallows Cp criteria. Other criteria exist based on other formulations than QR to check model quality. The most well-known are Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) (see more details in [1, 5]). These criteria can be used to compute the best model from several potential regressors. This is known as stepwise regression. Another interesting measure used in stepwise regression is the R-square coefficient. It represents the fraction of variation of output explained by the model and therefore indicates how well the model reproduces output. R-square coefficient varies between 0 and 1, and is equal to 1 if the model perfectly fits the sample. Actually this coefficient increases in terms of the number of regressors and is equal to 1 when the number of regressors is equal to the number of samples. We will prefer to use the adjusted R-square, noted adjR-square, which is the R-squared penalized by the number of terms [5].

Mallows Cp, AIC and BIC are based on the normal distribution of error ε_p in linear regression which is a classical assumption in statistics to approximate physical experiments. Although in our case the experiment is deterministic and terms of error ε_p only specify the meta-model lack of fit, these tools can be used as a guideline but is unable to conclude about model validity. The validation step is done without assumptions.

1.4 Selection and validation of meta-models

Previous criteria used in stepwise regression are more or less restrictive and result in different models. Selection of the best model must be done with respect to its ability to predict responses from a new input sample. The best way would be to have two samples, one for learning step and another for selection but this process imply a larger computational time. A possible approach is the use of cross-validation techniques. This method consists of the following steps : (a) split the global sample into D samples, (b) leave one sample out (called the validation sample) and build the model with the others, (c) use the validation sample to assess the approximation error, (d) repeat previous steps with all samples. Then, prediction error is estimated as the mean of errors computed on all validation samples. The measure of error used is the classical root mean square error (RMSE). Therefore among all models proposed by stepwise regression, the one with the lowest prediction RMSE is chosen.

To assess model accuracy in the region of interest we also compute the error MAX on validation samples, given by $\max_{p\in[1,P]}(\left|Y_p - \hat{Y}_p\right|)$ which estimates a more local error than RMSE. The validation of the model is guaranteed if predicted RMSE and predicted MAX are lower than a threshold. In our case, we use as the threshold 5% of maximum variation of the response. This ensures that error due to the meta-model is low with respect to response variation. From a graphical point of view, a good way to assess model accuracy is to plot response of the finite element model versus response of the meta-model with the domain of admissible error (cf. Figure 2).

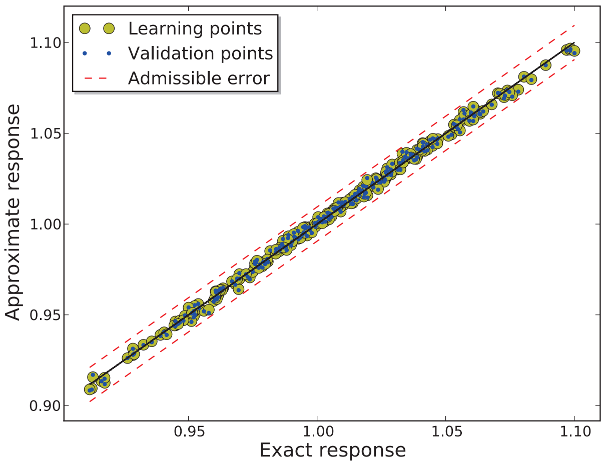

Figure 2: Meta-model accuracy

Once the meta-model is built and validated, it is used as a surrogate of the finite element model. It is then employed in probabilistic procedures in order to analyse output uncertainty and to assess reliability information.

1.5 Methodology in application

We present here the manner in which the previously described tools are used in application.

The morris OAT method is performed on the finite element model but no variable selection is done. Therefore, we decided to build linear meta-models with all variables from a Latin Hypercube sampling with $3(n + 1)$ points which are necessary for the validation step (n is the number of variables). For each response, three linear models are established through stepwise regression with Mallows Cp, BIC and adjR-square criteria (Mallows Cp and AIC are equivalent in linear regression [13]). The best one is chosen from predicted RMSE computed with the cross-validation technique. If predicted MAX error and predicted RMSE are lower than admissible error (5% of maximum variation of response), the linear model is validated. If not, we add quadratic and cross-product regressors of the most significant variables selected with Morris OAT from the value indicating non-linear and/or interaction effects. Since stepwise regression has removed useless linear regressors, added quadratic and cross-product terms do not involve more sampling points. This procedure avoids performing more finite element computation than necessary. It allows a sufficiently accurate model to be obtained in the following application. In other cases, if more regressors are useful, we must add sampling points and thus perform more finite element computation. Finally, if second order polynomial functions are not good enough, more sophisticated meta-models have to be used.

2 APPLICATION ON A SPACECRAFT STRUCTURE

The following example involves the TARANIS spacecraft whose platform belongs to the Myriade family developped by CNES. The analysis has been treated under 8 quasi-static load cases which are those of the qualification step.

2.1 Finite element model description

The mechanical model is deterministic and has been done with the finite element code MSC NASTRAN. Figure 3 presents the finite element model. It contains about 380 000 degrees of freedom. Loads consist of acceleration of $-9.5g$ in longitudinal direction (X direction) and $5.2g$ in lateral direction (Y and Z direction). Eight cases are different projection of lateral load on Y and Z axis, they are presented in Table 1. We have to notice that the model has been optimized and resultant values are not those provided by CNES.

Cases	Acceleration on axis (g)		
	X	Y	Z
100	-9.75	5.20	0.00
200	-9.75	3.68	3.68
300	-9.75	0.00	5.20
400	-9.75	-3.68	3.68
500	-9.75	-5.20	0.00
600	-9.75	-3.68	-3.68
700	-9.75	0.00	-5.20
800	-9.75	3.68	-3.68

Table 1: Load cases

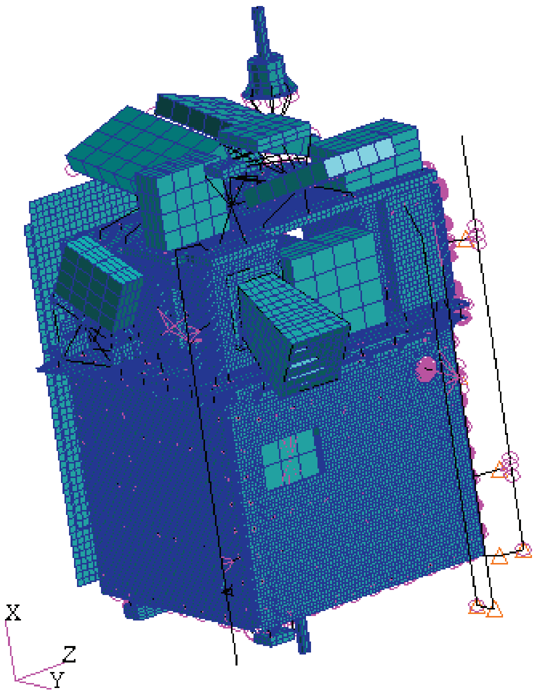

Figure 3: TARANIS finite element model

2.2 Study description

Studied responses are maximum Von Mises stress in lateral and top panels (honeycomb with aluminium skins), in bottom panel (bulk machining aluminium) and forces in interface screws between the bottom panel and lateral panels. As the bottom panel (cf. Figure 4) is more complicated than a simple plate (contrary to other panels), maximum stress in each part such as stiffener, coupling ring, bases, etc is considered as one response. In this way, taking into account all load cases, the number of the most critical responses to study is 60 (the most critical responses are these

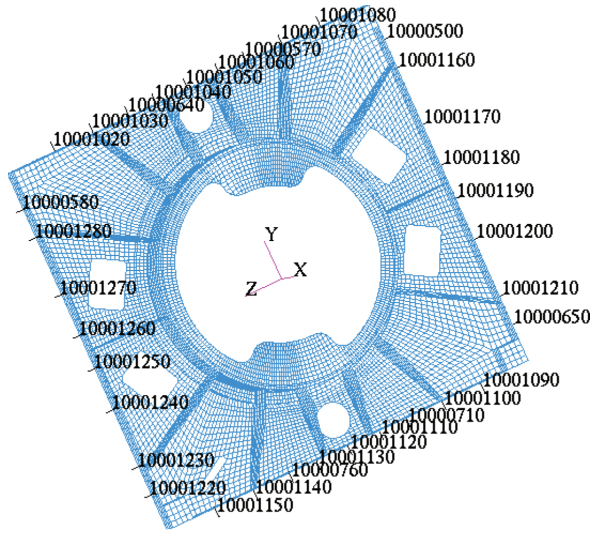

Figure 4: Bottom panel of TARANIS

whose deterministic safety margin is low). All input parameters of the previous components are taken as random variables (but remain constant on the substructure), i.e thicknesses and all material properties (Young modulus, Poisson coefficient, shear modulus, ...) which involves $n = 83$ variables. Random variables are considered uncorrelated and distributions are assumed uniform for thicknesses (with an uncertainty of ± 0.1 mm) and normal for material properties (with a coefficient of variation of 5%).

The methodology described in section 1.5 is applied on the spacecraft. Latin Hypercube sampling and Morris OAT are performed on $\pm 20\%$ of variation on each variable. These two methods each involve 252 finite element computations (see time computation in Table 3).

2.3 Uncertainty and sensitivity analysis

Uncertainty of responses can be analysed from a Monte Carlo sampling. It gives statistical measures of response such as its expectation and standard deviation (and higher order moment) but a probability density function can also be determined thanks to a statistical hypothesis test.

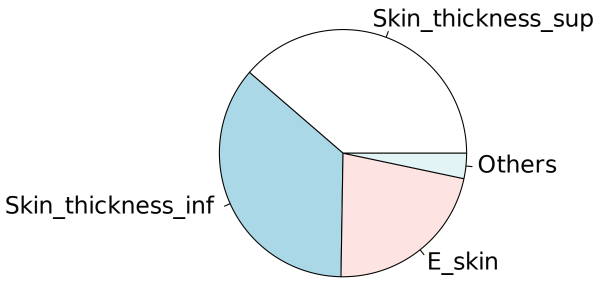

Figure 5: Variance decomposition on the +Z panel stress response

Another interesting analysis consists of variance decomposition of responses (see more details in [11]). This procedure aims at identifying the fraction of the total variance of a response which is due to any individual input variables. Figure 5 shows the variance decomposition of Von Mises stress in +Z panel (i.e panel whose normal direction is +Z).

2.4 Reliability analysis

Reliability assessment implies the definition of a performance function for each response. By comparison with a deterministic approach, failure modes considered are the elastic limit for panels and the sliding limit for interface screws. These two limits involve new random variables (elastic limit, friction coefficient, preload) which are assumed independent (with a normal distribution and a coefficient of variation equal to 10%) for each component and with respect to those already defined in the finite element model. This can be a strong assumption especially between the elastic limit and the young modulus of a material which could be correlated.

As the mechanical model is a system, components depend on each others and only potential first failures can be determined. If one component fails, the finite element model does not enable to know what happen afterward and it is not able to take into account successive failures. The probabilities of failure are assessed through a Monte Carlo analysis with 10^7 samples (see time computation in Table 3).

2.4.1 Probability of failure results

Results are given in Table 2. It only contains the worst load case, i.e the case which involves the largest probability of failure. For this case, components which are able to fail are presented with their probability of failure and their percentage of first failure.

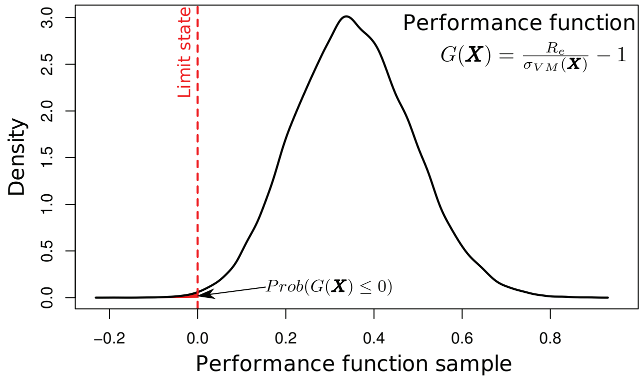

Figure 6: Representation of performance function distribution with reliability result on +Z panel

Case	Component	Probability of failure	Percentage of first failure
400	+Z panel	$1.9\,10^{-3}$	98.94
	Screw 125	$1.87\,10^{-5}$	0.96
	Screw 112	$< 10^{-6}$	0.05
	Screw 126	$< 10^{-6}$	0.03
	Screw 113	$< 10^{-6}$	0.02

Table 2: Reliabilty results

Results show that +Z panel under case 400 is the most critical element. The probability that Von Mises stress exceeds the elastic limit of material is $1.9\,10^{-3}$. Figure 6 shows the probability of failure of +Z panel with the distribution of performance function. For this kind of component and failure mode, such a high value could be problematic. This kind of problem would be missed in a deterministic approach since calculated safety margin is equal to 0.23. Another interesting result is the percentage of first failure which indicates how many times the component fails in a first instance. This information could be helpful in envisaging construction of system failure scenario.

2.4.2 Some examples of the benefits

We present here two examples which emphasize the benefits of a reliability approach compared to a deterministic one.

The first one is an illustration of safety margin calculation versus reliability assessment. We consider here the sliding limit of two interface screws between -Z panel and -X panel under load case 700. We assume that the friction coefficient and pre-load follow a normal distribution with 0.23 and $17260N$ respectively for the mean and 10% for both coefficient of variation [4]. In a deterministic approach, the safety margin is calculated with a B-value for friction coefficient and with a value lower than a B-value for pre-load (all other variables are considered at their mean). Moreover a safety coefficient of 1.25 is considered. From this configuration the sliding margin is negative and is equal to -0.022 for one screw and -0.017 for the other. This can create some problems in a qualification step. With a reliability approach random variables are considered, characterized by the previous distributions. Moreover, the safety coefficient is removed. In this case, probabilities that screws slide are $7.2\,10^{-4}$ and $6.5\,10^{-4}$ respectively. This kind of probability of failure could be considered as acceptable in this context.

The second example illustrates the importance of reliability assessment in order to compare two manufacturing processes of aluminium panel skins. Skin thicknesses are assumed to follow a uniform distribution with a mean of 0.6 mm. With such a low thickness, two manufacturing processes are considered, one expensive with good accuracy which implies a thickness variation of $\pm5\%$; one cheaper with a lower accuracy which implies a thickness variation of $\pm20\%$. Probabilities that Von Mises stresses exceed elastic resistances (under case 400) are $1.9\,10^{-3}$ and $3.47\,10^{-4}$ respectively. Considering manufacturing costs of both processes, a well-founded decision can be taken which deals with the technical risk and financial aspect [3].

CONCLUSION

This paper presents two main issues. On one hand, we attempted to show the relevance of results which arise from a probabistic approach and more precisely in reliability assessment. Although current deterministic approach is firmly fixed in structural analysis, it doesn't enable to improve designs with respect to robustness or reliability linked with economical constraints whihc are always more and more restrictive. A probabilistic approach is one of possible means and is starting to be applied in several industrial domains. Of course, current practices will not be replaced but it is a way to enrich them with more accurate information especially in order to help in decision. Making this kind of approach can bring benefits to the definition of less conservative sizing rules. On the other hand, we presented a resolution strategy based on meta-modeling. It can be employed on quite large structures and is not too computationally time demanding. Table 3 resumes the computational time of each step of methodology. Even if it could appear large relatively to a deterministic approach, it is very fast when compared with a direct Monte Carlo on finite element model which would last 43 years. Finally advantages of meta-models are numerous. Structural behaviour can be represented well even with simple models : linear polynomial functions are often sufficient. Although in our case the structure is linear elastic under quasi-static load with low variations, more sophisticated meta-models could deal with more complicated mechanical behaviour. Meta-models are also re-usable in order to measure the influence of stochastic chosen input parameters. This seems to be an essential point as distribution of random variables are usually assumed because of the lack of databases. Another interest of meta-models can be found in optimization or more precisely reliability based optimization.

Step	Time
Morris OAT	9h30
Latin Hypercube Sampling	9h30
Meta-models construction	20 min
10^7 samples on meta-models	1h

Table 3: Time computation of each step in methodology. Desktop computer used is a dual core 2.8 GHz, 1.97 Go RAM

References

[1] J. Azaïs and J. Bardet. Le modèle linéaire par l'exemple. Tech. rep., Université Paul Sabatier.

[2] R. R. Barton. Metamodeling : a state of the art review. In *Proceedings of the 1994 Winter Simulation Conference*. 1994.

[3] J. Buffe. Aide à la prise de décision, gestion des risques. Tech. rep., Thales Alenia Space, 2006.

[4] A. Carpine. Faisabilité d'un dimensionnement basé sur une approche stochastique. Tech. rep., Thales Alenia Space, 2009.

[5] P. Cornillon and E. Matzner-Lober. *Régression, Théorie et applications*. Springer, 2007.

[6] O. Ditlevsen and H. Madsen. Structural reliability methods. Tech. rep., Section of Coastal, Maritime and Structural Engineering, Department of Mechanical Engineering, 2007.

[7] B. Garnier. *Etude des méthodes d'analyse de sensibilité, application aux structures spatiales*. Master's thesis, ENSMA, 2009.

[8] M. Lemaire. *Fiabilité des structures*. Hermes, 2005.

[9] M. D. Morris. Factorial sampling plans for preliminary computational experiments. *Technometrics*, 33:161–174, 1991.

[10] J. Sacks, W. Welch, et al. Design and analysis of computer experiments. *Statistical science*, pp. 409–423, 1989.

[11] A. Saltelli, K. Chan, et al. *Sensitivity analysis. Series in Probability and Statistics*. Wiley, 2000.

[12] T. Simpson, J. Poplinski, et al. Metamodels for Computer-based Engineering Design: Survey and recommendations. *Engineering with Computers*, 17(2):129–150, 2001.

[13] M. Stone. An asymptotic equivalence of choice of model by cross-validation and Akaike's criterion. *Journal of the Royal Statistical Society. Series B (Methodological)*, pp. 44–47, 1977.

[14] G. Wang and S. Shan. Review of Metamodeling Techniques in Support of Engineering Design Optimization. *Journal of Mechanical Design*, 129:370, 2007.