To link to this article: DOI: 10.1049/el.2012.3101
URL: http://dx.doi.org/10.1049/el.2012.3101

To cite this version: Bandiera, Francesco and Besson, Olivier and Ricci, Giuseppe *Direction detector for distributed targets in unknown noise and interference.* (2013) IEEE Electronics Letters, vol. 49 (n° 1). pp. 68-69. ISSN 0013-5194

Direction detector for distributed targets in unknown noise and interference

F. Bandiera, O. Besson and G. Ricci

Adaptive detection of distributed radar targets in homogeneous Gaussian noise plus subspace interference is addressed. It is assumed that the actual steering vectors lie along a fixed and unknown direction of a preassigned and known subspace, while interfering signals are supposed to belong to an unknown subspace, with directions possibly varying from one resolution cell to another. The resulting detection problem is formulated in the framework of statistical hypothesis testing and solved using an ad hoc algorithm strongly related to the generalised likelihood ratio test. A performance analysis, carried out also in comparison to natural benchmarks, is presented.

Introduction: Adaptive radar detection of distributed targets requires proper strategies taking into account the nature of the targets as shown in [1,2]. In those papers returns from the target are modelled as signals known up to multiplicative factors, namely they are supposed to belong to a one-dimensional subspace of the observables. The case of returns modelled in terms of signals having the same direction which is not *a priori* known, but for the fact that it belongs to a given subspace of the observables, has also been considered [3,4]. Subsequently, in [5] it has been assumed that the target is also buried by interference belonging to a known subspace linearly independent of the signal subspace. In this Letter, we attack the detection problem addressed in [5], but assuming that the interference subspace is unknown (but for its rank). Since a closed form of the generalised likelihood ratio test (GLRT) is not available, we derive an ad hoc algorithm capable of effectively dealing with the considered scenario.

Problem formulation: Assume that an array of N (possibly space-time) sensors probes K_P range cells. Denote by r_k, $k \in \Omega_P \equiv \{1, \ldots, K_P\}$, the N-dimensional vector containing returns from the kth cell. We want to decide between the H_0 hypothesis that the r_ks contain disturbance only and the H_1 hypothesis that they also contain signals backscattered from target scattering centres. We also assume that the overall disturbance is the sum of coloured noise and deterministic interference. In symbols, the detection problem to be solved can be formulated in terms of the following binary hypothesis test:

$$\begin{cases} H_0 : \boldsymbol{r}_k = \boldsymbol{J}\boldsymbol{q}_k + \boldsymbol{n}_k, & k \in \Omega_P, \quad \boldsymbol{r}_k = \boldsymbol{n}_k, k \in \Omega_S \\ H_1 : \boldsymbol{r}_k = \alpha_k \boldsymbol{H}\boldsymbol{p} + \boldsymbol{J}\boldsymbol{q}_k + \boldsymbol{n}_k, & k \in \Omega_P, \quad \boldsymbol{r}_k = \boldsymbol{n}_k, k \in \Omega_S \end{cases} \tag{1}$$

where the useful signals $\alpha_k \boldsymbol{Hp}$ and the interference signals $\boldsymbol{Jq_k}$ are assumed to belong to the range spaces of the full-column-rank matrices $\boldsymbol{H} \in C^{N\times r}$ and $\boldsymbol{J} \in C^{N\times q}$, respectively, with $\boldsymbol{p} \in C^{r\times 1}$, $\boldsymbol{q_k} \in C^{q\times 1}$, and $q + r1 < N$. In the following we assume that the space spanned by $\boldsymbol{H}$ is known while that spanned by $\boldsymbol{J}$ is not (but for its rank q). The noise vectors $\boldsymbol{n}_k$s, $k \in \Omega_P$, are modelled as N-dimensional complex normal random vectors, i.e. $\boldsymbol{n_k} \sim CN_N(\boldsymbol{0}, \boldsymbol{M})$, $k \in \Omega_P$, with $\boldsymbol{M}$ being in turn a positive-definite matrix; we assume that M is unknown. We suppose that $K_S \geq N$ secondary data, $\boldsymbol{r}_k$, $k \in \Omega_S \equiv \{K_P + 1, \ldots, K_P + K_S\}$, containing noise only, are available and that these returns share the same statistical characterisation of the noise components in the primary data. Finally, we assume that the $\boldsymbol{n}_k$s, $k \in \Omega_P \cup \Omega_S$, are independent random vectors.

Detector design: Denote by $\boldsymbol{R} = [\boldsymbol{R}_P \boldsymbol{R}_S] \in C^{N\times K}$ the overall data matrix, with $\boldsymbol{R}_P = [\boldsymbol{r}_1 \ldots \boldsymbol{r}_{K_P}] \in C^{N\times K_P}$ being the primary data matrix, $R_S = [\boldsymbol{r}_{K_P+1} \ldots \boldsymbol{r}_{K_P+K_S}] \in C^{N\times K_S}$ the secondary data matrix, and $K = K_P + K_S$. Let us also introduce the following matrices $\boldsymbol{Q} = [\boldsymbol{q}_1 \cdots \boldsymbol{q}_{K_P}] \in C^{q\times K_P}$ and $\alpha = [\alpha_1 \cdots \alpha_{K_P}] \in C^{K_P\times 1}$. The GLRT for the above hypothesis testing problem can be written as

$$\frac{\max\limits_{\boldsymbol{p}} \max\limits_{\boldsymbol{\alpha}} \max\limits_{\boldsymbol{J}} \max\limits_{\boldsymbol{Q}} \max\limits_{\boldsymbol{M}} f_1(\boldsymbol{R}; \boldsymbol{p}, \boldsymbol{\alpha}, \boldsymbol{J}, \boldsymbol{Q}, \boldsymbol{M})}{\max\limits_{\boldsymbol{J}} \max\limits_{\boldsymbol{Q}} \max\limits_{\boldsymbol{M}} f_0(\boldsymbol{R}; \boldsymbol{J}, \boldsymbol{Q}, \boldsymbol{M})} \mathop{\gtrless}_{H_0}^{H_1} \gamma \tag{2}$$

where $f_j(\boldsymbol{R}; \cdot)$ is the probability density function of $\boldsymbol{R}$ under the H_j, $j = 0, 1$, hypothesis and γ the threshold value to be set in order to ensure the desired probability of false alarm (P_{fa}). The denominator of (2) has been computed in [6] and it is given by

$$\frac{[K/e\pi]^{NK}}{\| S \|^K} \prod_{i=q+1}^{N} [1 + \lambda_i(S^{-1/2} R_P R_P^\dagger S^{-1/2})]^{-K} \tag{3}$$

where $\boldsymbol{S} = \boldsymbol{R}_S \boldsymbol{R}_S^\dagger$, $\| \cdot \|$ is the determinant of the matrix argument, $\lambda_i(\cdot)$ are the eigenvalues of the matrix argument arranged in decreasing order, and † denotes conjugate transpose. As to the numerator of (2), it is possible to reach the following intermediate step [5]

$$\max_{\boldsymbol{p},\boldsymbol{J}} \frac{[K/e\pi]^{NK}}{\| \boldsymbol{S} \|^K} \left\| \boldsymbol{I}_{K_P} + \boldsymbol{R}_P^\dagger \boldsymbol{S}^{-1/2} (\boldsymbol{I}_N - \boldsymbol{P}_{\boldsymbol{W}_S}) \boldsymbol{S}^{-1/2} \boldsymbol{R}_P \right\|^{-K} \tag{4}$$

where $\boldsymbol{I}_n$ denotes the identity matrix of dimension $n \in \mathbb{N}$ and $\boldsymbol{P}_{W_S}$ is the projector onto the span of $\boldsymbol{W}_S = \boldsymbol{S}^{-1/2}[\boldsymbol{Hp} \;\; \boldsymbol{J}] = [\boldsymbol{H}_S\boldsymbol{p} \;\; \boldsymbol{J}_S]$ which is assumed to be full rank (i.e. $q + 1$). Combining (3) and (4) into (2) provides the following equivalent decision rule

$$\frac{\prod_{i=q+1}^{N} [1 + \lambda_i(\boldsymbol{S}^{-1/2} \boldsymbol{R}_P \boldsymbol{R}_P^\dagger \boldsymbol{S}^{-1/2})]}{\min\limits_{\boldsymbol{p}} \min\limits_{\boldsymbol{J}} \left\| \boldsymbol{I}_{K_P} + \boldsymbol{R}_P^\dagger \boldsymbol{S}^{-1/2} (\boldsymbol{I}_N - \boldsymbol{P}_{\boldsymbol{W}_S}) \boldsymbol{S}^{-1/2} \boldsymbol{R}_P \right\|} \mathop{\gtrless}_{H_0}^{H_1} \gamma \tag{5}$$

Observe now that the optimisation problem at the denominator of (5) requires finding a projector onto the span of $\boldsymbol{W}_S$ which, in turn, is constrained to have one direction belonging to the span of $\boldsymbol{H}_S$ while the remaining ones are unconstrained (provided that $\boldsymbol{W}_S$ has rank $q + 1$). Since we do not know how to jointly solve such a problem, we herein propose a heuristic technique based upon the following rationale. Given that we know that one direction of $\boldsymbol{W}_S$ must lie in the span of $\boldsymbol{H}_S$, we start by finding a set of possible candidates as estimates of the vector $H_S p$. Subsequently, for each of these candidates, we solve problem (5) with respect to $\boldsymbol{J}$. In order to find the candidates, we observe that, in the absence of interference, the optimum (in the maximum-likelihood sense) choice for the vector $H_S p$ is given by the dominant eigenvector of the matrix

$$\boldsymbol{E} = \boldsymbol{P}_{H_S} \boldsymbol{S}^{-1/2} \boldsymbol{R}_P \boldsymbol{A}^{-1} \boldsymbol{R}_P^\dagger \boldsymbol{S}^{-1/2} \boldsymbol{P}_{\boldsymbol{H}_S} \tag{6}$$

where $\boldsymbol{A} = \boldsymbol{I}_{K_P} + \boldsymbol{R}_P^\dagger \boldsymbol{S}^{-1} \boldsymbol{R}_P$ and $\boldsymbol{P}_{\boldsymbol{H}_S}$ is the projector onto the span of $\boldsymbol{H}_S$ [5]. For this reason, we choose to construct the set of candidate estimates of the vector $\boldsymbol{H}_S\boldsymbol{p}$ as the r eigenvectors of $\boldsymbol{E}$ corresponding to the greatest eigenvalues. To be quantitative, let $v_1, \ldots, v_r$ be the eigenvectors corresponding to the greatest eigenvalues of $\boldsymbol{E}$; for each v_ℓ, $\ell = 1, \ldots, r$, we solve the problem

$$\min_{\boldsymbol{J}} \left\| \boldsymbol{I}_{K_P} + \boldsymbol{R}_P^\dagger \boldsymbol{S}^{-1/2} (\boldsymbol{I}_N - \boldsymbol{P}_{\boldsymbol{W}_S}) \boldsymbol{S}^{-1/2} \boldsymbol{R}_P \right\| \tag{7}$$

with the matrix $\boldsymbol{W}_S$ that is now given by $\boldsymbol{W}_S = [\boldsymbol{v}_\ell \;\; \boldsymbol{J}_S]$. The solution to such problem is known (see [6]) and it is given by

$$\prod_{i=q+1}^{N} [1 + \lambda_i(\boldsymbol{P}_{\boldsymbol{v}_\ell}^\perp \boldsymbol{S}^{-1/2} \boldsymbol{R}_P \boldsymbol{R}_P^\dagger \boldsymbol{S}^{-1/2} \boldsymbol{P}_{\boldsymbol{v}_\ell}^\perp)] \tag{8}$$

where $\boldsymbol{P}_{v_\ell}^\perp$ is the projector onto the orthogonal complement of the span of v_ℓ. Summarising, we propose to replace the statistic in (2) with the one computed as follows. First: extract the r eigenvectors corresponding to the greatest eigenvalues of $\boldsymbol{E}$ (given by (6)) and denote them by $v_1, \ldots, v_r$; secondly, for each v_ℓ, $\ell = 1, \ldots, r$, construct the statistic

$$\Lambda_\ell(R) = \frac{\prod_{i=q+1}^{N} [1 + \lambda_i(S^{-1/2} R_P R_P^\dagger S^{-1/2})]}{\prod_{i=q+1}^{N} [1 + \lambda_i(P_{v_\ell}^\perp S^{-1/2} R_P R_P^\dagger S^{-1/2} P_{v_\ell}^\perp)]} \tag{9}$$

thirdly, compute the maximum of $\Lambda_\ell(\boldsymbol{R})$ with respect to $\ell = 1, \ldots, r$. The resulting direction detector for unknown $\boldsymbol{J}$ will be denoted by DD-u$\boldsymbol{J}$.

Performance assessment: We carry out a Monte Carlo simulation to evaluate the performance of the proposed algorithm, also in comparison to the direction detector that assumes perfect knowledge of the interference subspace $\boldsymbol{J}$ [5] (denoted by DD-k$\boldsymbol{J}$) and the subspace detector for unknown $\boldsymbol{J}$ [6] (denoted by SD-u$\boldsymbol{J}$). In order to evaluate the thresholds necessary to ensure a preassigned value of P_{fa} and the probabilities of detection (P_ds) we resort to $100/P_{fa}$ and 10^4 independent trials, respectively. We assume $N = 16$, $K_S = 32$, $r = 6$, $q = 2$, $P_{fa} = 10^{-4}$. Matrices $\boldsymbol{H}$ and $\boldsymbol{J}$ are randomly generated at each Monte Carlo run as matrices whose entries are independent and identically distributed

(I.I.D) random variables taking on values $\pm 1/\sqrt{N}$ with equal probability. Vector $\boldsymbol{p}$ is generated as $\boldsymbol{p} \sim CN_r(\boldsymbol{0}, \boldsymbol{I_r})$. Moreover, at each run of the Monte Carlo simulation, we check the condition that the matrix $[\boldsymbol{Hp}\ \boldsymbol{J}]$ is full rank (i.e. $q+1$). We also assume $|\alpha_k| = |\alpha|$, $k \in \Omega_P$, ($|\cdot|$ being the modulus of a complex number). The signal-to-noise ratio (SNR) is defined as $\mathrm{SNR} = K_P|\alpha|^2 \frac{r}{N} \mathrm{tr}(\boldsymbol{M}^{-1})$ where tr$(\cdot)$ is the trace of the matrix argument. In addition, the interference coefficients $\boldsymbol{q}_k$, $k = 1, \ldots, K_P$, are I.I.D and $\boldsymbol{q}_k \sim CN_q(0, \sigma_J^2 \boldsymbol{I}_q)$. The noise vectors $\boldsymbol{n}_k$, $k = 1, \ldots, K$, are I.I.D and $\boldsymbol{n}_k \sim CN_N(\boldsymbol{0}, \boldsymbol{M})$ with the (i, j)th element of $\boldsymbol{M}$ given by $\sigma_n^2 0.95^{|i-j|}$, $\sigma_n^2 = 1$. Finally, the interference-to-noise ratio (INR), defined as σ_J^2/σ_n^2, is set to INR $= 20$ dB. Figs. 1 and 2 report P_d against SNR for DD-u$\boldsymbol{J}$, DD-k$\boldsymbol{J}$, and SD-u$\boldsymbol{J}$ for $K_P = 9$ and $K_P = 32$, respectively. From the Figures it is seen that, on one hand, the proposed DD-u$\boldsymbol{J}$ can provide a performance better than that of the SD-u$\boldsymbol{J}$ and, on the other hand, that its loss with respect to its non-adaptive counterpart (i.e. the DD-k$\boldsymbol{J}$) is limited. Such relations are emphasised as K_P increases; in fact, a higher value of K_P means that more data are available to estimate $\boldsymbol{Hp}$ and $\boldsymbol{J}$.

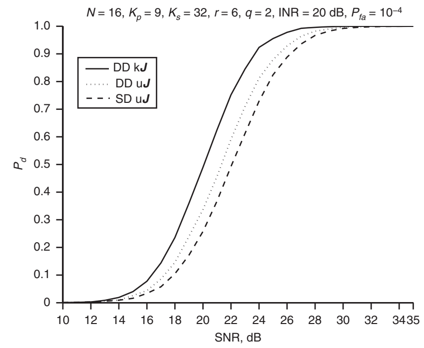

Fig. 1 *P_d against SNR for considered detectors, $K_p = 9$*

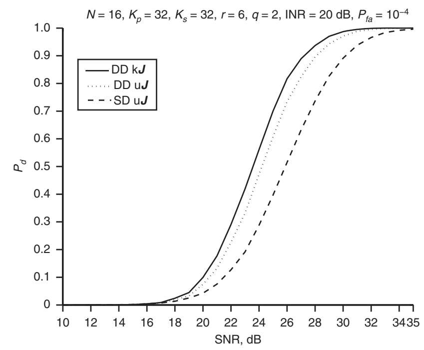

Fig. 2 *P_d against SNR for considered detectors, $K_p = 32$*

Conclusion: We have addressed adaptive detection of distributed targets in Gaussian noise plus interference, proposing a heuristic procedure to approximate the GLRT for the case that the interference subspace is unknown. The analysis has shown that the DD-u$\boldsymbol{J}$ can guarantee a better performance than the more conventional SD-u$\boldsymbol{J}$ and that it 'converges' to its non-adaptive counterpart, the DD-k$\boldsymbol{J}$, as the number of data increases.

31 August 2012
doi: 10.1049/el.2012.3101

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