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Kinematic analysis of spatial geared mechanisms

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ABSTRACT

In this paper, a general method for kinematic analysis of complex gear mechanisms, including bevel gear trains and non-collinear input and output axes, is presented. This new approach is based on the nullspace of the kinematic constraint matrix computed from the mechanism graph or its adjacency matrix. The novelty is that the elements of the adjacency matrix are weighted with complex coefficients allowing bevel gears to be taken into account and the angular velocity of each link to be directly expressed using polar coordinates. This approach is illustrated on a two-degree-of-freedom car differential and applied to an helicopter main gear box. A MATLAB *open source software was developed to implement this method.*

Keywords: Gear Mechanisms, Kinematic Analysis, Kinematic Graph, Car Differential.

1 Introduction

Gear trains are commonly used in power transmission mechanisms. In transportation (vehicles, helicopters,...), such devices are quite complex mechanisms involving several degrees-of-freedom (car differential) or several stages of epicyclic gear trains (helicopter main gear box). They involve several bevel gear trains and are qualified of *spatial-geared* mechanisms by opposition to *planar-geared* mechanims. Furthermore in such mechanisms, input and output axes are not collinear.

In the field of helicopter engineering, lots of studies are conducted in Health and Usage Monitoring Systems (HUMS) [1] and more particularly on the vibration analysis of the Main Gear Box (MGB) which is the most critical part in the transmission system. HUMS are used to detect or predict faults and prevent possible failures. Actually, each contact between two links in the MGB creates an harmonic disturbance at a precise angular frequency in the vibration signal of the whole MGB. Some promising vibration analsysis methods are based on signal processing on angularly sampled signals [2] and required a good knowledge of angular frequencies of all contacts between the various links inside the mechanism. With this goal in mind, efficient kinematic analysis, able to provide angular velocities (magnitude and direction) of all the links in spatial-geared devices, are required.

A lot of kinematic analysis methods have already been studied, for different kinds of mechanisms [3–10]. The tabular method is often used but requires a lot of calculation and does not apply to mechanisms with non collinear input and output axes [5]. The vector analysis method is an efficient method for the mechanisms with a bevel gear, but is very complicated to compute and to implement on a computer [4]. The graph theory based method is quite generic, easy to implement [6] and can be adapted to bevel gear. It has been used by NELSON and CIPRA in [8] so as to find the angular velocities of all links in bevel epicyclic gear trains to perform power-flow and efficiency analysis. But, as regards the tabular method, it is not applicable to systems with non collinear input and output axes. Furthermore, for mechanism with several degrees-of-freedom (d.o.f) i.e. various operating modes, such an analysis requires to identify which of the links are to be considered as inputs and so must be performed for each operating mode. More recent kinematic analyses of gear mechanisms are based on the block diagram representation [10] which implies that the mechanism is also input/output oriented. Other representations of kinematic chains are proposed such as the unified topological representation in [9] but are restricted to planar geared mechanisms.

The method proposed here is based on the former work of NELSON and CIPRA [8]. The main contribution of this method is an extension to mechanisms with non collinear input and and output axes. Furthermore, the method is consequently simplified using complex number in a new adjacency matrix (named *adjacency table*) to take into account bevel gear trains. The mechanism overall kinematic is characterized by the (complex) kernel or nullspace of the *kinematic constraint matrix* and does not require to specify the input links. The input links to analyse a particular operating mode are specified at the very last step to normalize this kernel (defined up to a multiplication by a factor). Then, the components of the kernel (named *velocity ratio matrix*) provide the ratios of angular velocities of all the links in polar coordinates (magnitude and angle) with respect to input link(s) angular velocity(ies). Thus, the analysis of multi d.o.f mechanisms is quite simplified.

The next section presents the kinematic analysis method and an illustration on a car differential, a quite complex two d.o.f mechanism with various operating modes. In the last section, the interest and the generality of this method is highlighted on an helicopter Main Gear Box (MGB) analysis, that is a mechanism with several stages of epicyclic trains and with non-collinear input and output axes. A general tool implemented in the MATLAB environment is available (go to: `http://personnel.isae.fr/daniel-alazard/matlab-packages`).

Nomenclature

$\sqrt{-1}$: imaginary unit,
N: number of links in the mechanism,
L: number of gear pairs in the mechanism,
$|x|$: modulus (magnitude) of complex number x,
$\mathrm{rank}(M)$: rank of the matrix M,
$\ker(M)$: kernel (or nullspace) of the matrix M,
M^T: transpose of matrix M
N_i: number of teeth of the gear i,
θ_i: pitch angle of gear i (rad),
ω_i: angular velocity of link i (rad/s),
N_{dof}: number of degrees of freedom (d.o.f).

2 Kinematic Analysis Method

The main steps of the method are as follows:

- Build the graph and the associated adjacency table T of the mechanism. This table T describes the interactions between all the links of the system.
- Determine the reference link (carrier) associated with each gear pair.
- Compute the null space (kernel) $\ker(M)$ of the kinematic constraint matrix M. M is the matricial expression of the WILLIS Formula applied to all gear pairs. $\ker(M)$ is the vector (single d.o.f case) or the set of vectors (several d.o.f case) of links angular velocities satisfying all kinematic constraints.

Each of these steps is detailled in the following sections and illustrated on a 2 d.o.f car differential. A picture of the car differential and the corresponding kinematic sketch are presented in Figures 1 and 2, respectively.

2.1 Graph representation of the mechanism

The graph representation (see HSU and LAM in [6]) is quite usefull to describe mechanism kinematics. Note that for computer implementation of the method, the associated adjacency table (presented in the next section) in sufficient. In this graph, links are represented by vertexes, gear pairs by dashed lines and turning pairs by solid lines. Each turning pair edge can be characterized by a level which describes the location of the rotation axis in space. When several turning pairs are on the same level, they form a multiple turning pair. The multiple turning pairs are represented by shaded polygons.

The Figure 3 shows the graph representation of the car differential: this mechanism has seven links ($N = 7$), five gear pairs ($L = 5$) and six turning pairs with various levels. Turning pairs 1-3, 1-7, and 1-4 are on the wheel axis level (level #1), turning pairs 5-7 and 6-7 are on the radial axis (level #2) and turning pair 1-2 is on the drive shaft level (level #3). Thus, this graph involves 2 shaded polygons: one for links # 1, 3, 4 and 7 and anoher one for links # 5, 6 and 7.

Remark: It is important to note that the car differential is a very interesting kinematic example since one of its links (# 7) is involved in two multi-turning pairs. It is quite rare and worth to be mentioned.

Fig. 1. Picture of the car differential.

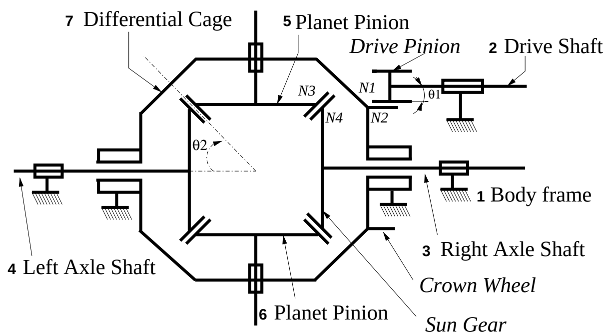

Fig. 2. Car differential kinematic sketch.

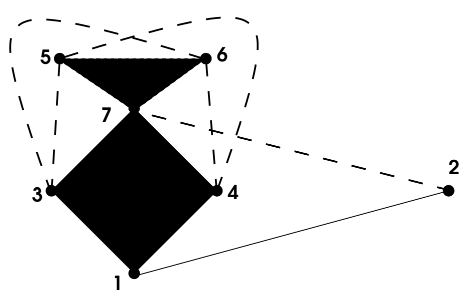

Fig. 3. Graph representation of the car differential (gear pair edges 4-5 and 3-6 are rounded for the legibility of the graph).

2.2 The adjacency table T

There are several definitions of the adjacency matrix associated with the mechanism graph. The definition used in the paper is quite different from the one used in [8] or [6]: for a mechanism with N links, the adjacency table T is a $N \times N$ matrix. The element (i, j) of table T describes the interaction between link i and link j. It is important to emphasize that $T(i, j)$ can be a complex number to describe bevel gears. This enables to deal with mechanisms the input and output axes of which are not collinear.

The table T is filled in based on these rules:

- For gear pairs: $T(i, j) = N_i e^{\theta_i \sqrt{-1}}$ and $T(j, i) = N_j e^{\theta_j \sqrt{-1}}$, where N_i (resp. N_j) is the number of teeth on the gear i (resp. j). θ_i (resp. θ_j) is the pitch angle of gear i (resp. j) from the gear axis to the tooth axis in contact with link j (resp. i). θ_i is positive clockwise so that $|\theta_i| < \pi/2$ for an external gear and $\pi > |\theta_i| > \pi/2$ for an internal gear (see example on Figure 4).
- For turning pairs: $T(i, j) = T(j, i)$ and is composed of a character string *'level ='* and an integer specifing the level of the turning pair (i, j),
- The other elements of T remain blank.

Thus, the elements of this table can be empty, a complex number or a cell with a character string and an integer. That is why this table is called *adjacency table* instead of adjacency matrix to do the distinction with other well-known definitions (for instance in [6] and [8]). Note also that the adjacency table is no more symmetric.

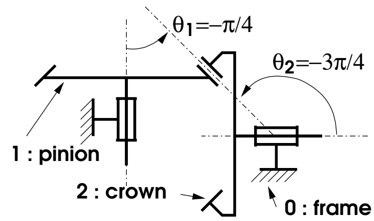

Fig. 4. Angle definition example for a pinion-crown bevel gear.

The Table 1 gives the complete adjacency table of the car-differential. The definition of the 7 links is detailled in Figure 2 and the numerical values for gear pairs are: $N_1 = 13$, $N_2 = 65$, $N_3 = 10$, $N_4 = 14$, $\theta_1 = 0$ and $\theta_2 = \pi/4$.

Table 1. Adjacency table for the car differential with: $N_1 = 13$, $N_2 = 65$, $N_3 = 10$, $N_4 = 14$, $\theta_1 = 0$ and $\theta_2 = \pi/4$.

	1	2	3	4	5	6	7
1		*level* = 3	*level* = 1	*level* = 1			*level* = 1
2	*level* = 3						$N_1 e^{\theta_1\sqrt{-1}}$
3	*level* = 1				$N_4 e^{-\theta_2\sqrt{-1}}$	$N_4 e^{\theta_2\sqrt{-1}}$	
4	*level* = 1				$N_4 e^{\theta_2\sqrt{-1}}$	$N_4 e^{-\theta_2\sqrt{-1}}$	
5			$N_3 e^{\theta_2\sqrt{-1}}$	$N_3 e^{-\theta_2\sqrt{-1}}$			*level* = 2
6			$N_3 e^{-\theta_2\sqrt{-1}}$	$N_3 e^{\theta_2\sqrt{-1}}$			*level* = 2
7	*level* = 1	$N_2 e^{-\theta_1\sqrt{-1}}$			*level* = 2	*level* = 2	

2.3 The reference link

For each gear pair, the reference link (carrier) is the link in which the contact point of the gear pair is motionless. It can be determined using the graph representation of the mechanism or equivalently, the associated adjacency table. From the graph representation of the mechanism, the reference link k associated with the gear pair (i, j) can be determined in the following way:

Let S_i (resp. S_j) be the union set of the links connected to link i (resp. j) through a turning pair or a multi-turning pair. Then:

$$k = S_i \cap S_j \tag{1}$$

where $\cap$ stands for the intersection of sets S_i and S_j.

Note that this intersection is:

- not empty since it is not possible to find a closed walk in the graph involving one gear pair and turning pairs with three different levels,
- reduced to a singleton, otherwise this would mean that there is a multi-turning pair between S_i and S_j.

In other words, considering the graph representation of the mechanism and the path from i to j involving only turning pairs, the reference link k is the link where the level changes [6].

Considering Figure 3, the reference link of the gear pair 3-5 is the link # 7 because:

- the link 3 belongs to the multi-turning pair $\{1,3,4,7\} = S_3$,
- the link 5 belongs to the multi-turnung pair $\{5,6,7\} = S_5$,
- the intersection between $\{1,3,4,7\}$ and $\{5,6,7\}$ is 7.

Table 2. Gear pairs and reference links for the car differential.

Gear pair	Reference link
2-7	1
3-5	7
3-6	7
4-5	7
4-6	7

In the same way, the reference link of the gear pair 2-7 is the link # 1 as $S_2 = \{1\}$ and $S_7 = \{1,3,4,5,6,7\}$. The Table 2 sums up the list of reference links for all the gear pairs. Another example is proposed in section 3 to illustrate such a systematic search for the reference link.

Such a rule (equation (1)) is usefull to automatically identify the reference link and can be easily implemented on a computer. The only required data is the adjacency table of the mechanism. This rule is an alternative of the method used in [6] and [8], based on the adjacency matrix.

2.4 The kinematic constraint matrix M

Let us consider $\Omega = [\omega_1, \omega_2, ..., \omega_N]^T$ the vector of angular velocities of the N links in the mechanism. The kinematic constraint matrix M is the matrix such that: $M\Omega = 0$.

The first row of M must be equal to:

$$C = \underset{1 \;\cdots\; f-1 \;\; f \;\; f+1 \;\cdots\; N}{[\; 0 \cdots \;\; 0 \;\; 1 \;\; 0 \;\; \cdots 0 \;]},$$

where link f is the body frame (i.e.: ω_f=0).

Once the reference link k of each gear pair (i, j) is known, it is possible to fill the kinematic constraint matrix M row by row, using the WILLIS Formula and the adjacency table T:

$$T(i, j)(\omega_i - \omega_k) + T(j,i)(\omega_j - \omega_k) = 0 \tag{2}$$

From the upper triangular part of T and for each gear pair (i, j) with a reference link k, a new line l is added to M to take into account the WILLIS constraint (2):

- $M(l,i) = T(i, j)$,
- $M(l, j) = T(j,i)$,
- $M(l,k) = -T(i, j) - T(j,i)$,
- else 0.

Thus, M is a $(L+1) \times N$ complex matrix where L is the number of gear pairs in the mechanism.

For example, the kinematic constraint matrix of the car differential depicted in Figure 2 (considering link # 1 as the body frame) is given in equation (3).

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -78 & 13 & 0 & 0 & 0 & 0 & 65 \\ 0 & 0 & 14e^{-\pi/4\sqrt{-1}} & 0 & 10e^{\pi/4\sqrt{-1}} & 0 & -14e^{-\pi/4\sqrt{-1}} - 10e^{\pi/4\sqrt{-1}} \\ 0 & 0 & 14e^{\pi/4\sqrt{-1}} & 0 & 0 & 10e^{-\pi/4\sqrt{-1}} & -14e^{\pi/4\sqrt{-1}} - 10e^{-\pi/4\sqrt{-1}} \\ 0 & 0 & 0 & 14e^{\pi/4\sqrt{-1}} & 10e^{-\pi/4\sqrt{-1}} & 0 & -14e^{\pi/4\sqrt{-1}} - 10e^{-\pi/4\sqrt{-1}} \\ 0 & 0 & 0 & 14e^{-\pi/4\sqrt{-1}} & 0 & 10e^{\pi/4\sqrt{-1}} & -14e^{-\pi/4\sqrt{-1}} - 10e^{\pi/4\sqrt{-1}} \end{bmatrix} \tag{3}$$

2.5 Velocity Ratio Matrix

To meet all WILLIS kinematic constraints, Ω must be a solution of $M\Omega = 0$. It means that $\Omega \in \ker(M)$. Such a nullspace of a complex matrix can be easily computed using linear algebra tools (in MATLAB for instance).

The number of degrees of freedom N_{dof} of the mechanism is:

$$N_{dof} = N - \text{rank}(M) \;, \tag{4}$$

that is the number of links N minus the number of independent kinematic constraints. In other words $\Omega_0 = \ker(M)$ is the $N \times N_{dof}$ matrix composed of the N_{dof} vectors describing the relationships between link angular velocities for each d.o.f. In most of mechanism, $N_{dof} = 1$, then Ω_0 is defined up to multiplication by a scalar and can be normalized in such a way that $\Omega_0(r) = 1$ where r is the index of the input link. This way, $\Omega_0(i)$ corresponds to the velocity ratio of the link i w.r.t to link r. Note that in the general case $\Omega_0(i)$ is a complex number involving polar coordinates of the angular velocity vector ($\Omega_0(i) = |\omega_i| e^{\varphi_i\sqrt{-1}}$). φ_i represents the relative attitude of the angular velocity vector of link i w.r.t. the angular velocity vector of the input link (φ_i is positive clockwise). The use of complex numbers is the main novelty of this approach which enables to extend to any spatial geared mechanisms the classical kinematic analysis restricted to planar geared mechanisms.

In the multi-d.o.f. case, Ω_0 can be normalized w.r.t. angular velocities of the N_{dof} input links. Later in the document, Ω_0 is called the *velocity ratio matrix* (or *vector* in the single-d.o.f. case) and the vector of angular velocities Ω can be determined from Ω_0 by:

$$\Omega = \Omega_0 \Lambda \tag{5}$$

where Λ is the vector of the N_{dof} input link angular velocities.

Illustration: Considering the car differential ($N = 7$), the rank of the kinematic constraint matrix M (equation (3)) is 5, that is a 2 d.o.f. mechanism. Therefore $\Omega_0 = \ker(M)$ is a 2-dimension subspace. The velocity ratio matrix Ω_0 can be normalized with respect to the angular velocities of the two wheels (links 3 and 4) and reads:

$$\Omega_0 = \begin{bmatrix} 0 & 0 \\ -2.5 & -2.5 \\ 1 & 0 \\ 0 & 1 \\ 0.86e^{0.95\sqrt{-1}} & 0.86e^{-0.95\sqrt{-1}} \\ 0.86e^{-0.95\sqrt{-1}} & 0.86e^{0.95\sqrt{-1}} \\ 0.5 & 0.5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -2.5 & -2.5 \\ 1 & 0 \\ 0 & 1 \\ 0.5+0.7\sqrt{-1} & 0.5-0.7\sqrt{-1} \\ 0.5-0.7\sqrt{-1} & 0.5+0.7\sqrt{-1} \\ 0.5 & 0.5 \end{bmatrix} . \tag{6}$$

It is well known that the car differential is a 2-degree-of-freedom mechanism, as long as it is made so that the two wheels of the car can spin at different velocities. The two columns of Ω_0 give the velocity ratios when one wheel is locked and the other one is free.

Note that the complex notation is quite useful and allows the direction of absolute and relative angular velocity to be determined in the plane of the mechanism description (see Figure 2). Indeed, considering the case where the left wheel is locked (first column of Ω_0), then the orientation of the absolute angular velocity of planet pinion (# 5) is $0.95\,rad$ with respect to wheel axis. Note also that $\omega_5 - \omega_7 = 0.7\sqrt{-1}$, that is the angular velocity of planet pinion (# 5) w.r.t differential cage (# 7) is along the wheel normal axis (the axis of the turning pair 5-7). The angular velocity vectors of the links in this operating mode (i.e. the left wheel is locked) are depicted in Figure 5. In the following, the velocity ratio matrix Ω_0 and equation (5) are used to analyze various operation modes.

The most common behavior (*driving straight ahead*) corresponds when both wheels spin at the same velocity $\omega_3 = \omega_4 = \omega$ that is: $\Lambda = [\omega\ \omega]^T$ in equation (5). Hence the angular velocity vector reads:

$$\Omega = \Omega_0 \begin{bmatrix} \omega \\ \omega \end{bmatrix} = [0\ -5\ 1\ 1\ 1\ 1\ 1]^T \omega \,.$$

Another well-known behavior appears when there is no transmission ($\omega_2 = 0$) and the car is jacked up. Then the two wheels

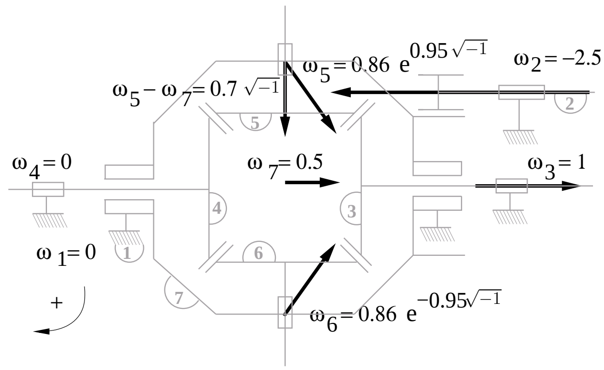

Fig. 5. Angular velocity vectors of car differential links when left wheel is locked.

spin in opposite directions at the same velocity $\omega_3 = -\omega_4 = \omega$. Indeed:

$$\Omega = \Omega_0 \begin{bmatrix} \omega \\ -\omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \\ 1.4\sqrt{-1} \\ -1.4\sqrt{-1} \\ 0 \end{bmatrix} \omega \, . \tag{7}$$

Numerical application: let us consider the example of a car turning a right corner at a velocity of $30km/h$. It can be shown that the angular velocities of the two wheels are $\omega_3 = 26\,rad/s$ and $\omega_4 = 30\,rad/s$. Hence the angular velocity vector reads:

$$\Omega = \Omega_0 \begin{bmatrix} 26 \\ 30 \end{bmatrix} = \begin{bmatrix} 0 \\ -140 \\ 26 \\ 30 \\ 28.14e^{-0.10\sqrt{-1}} \\ 28.14e^{0.10\sqrt{-1}} \\ 28 \end{bmatrix} (rad/s) \, . \tag{8}$$

3 Example: helicopter Main Gear Box

In this section the Main Gear Box (MGB) of an helicopter (ALOUETTE III) is considered. This mechanism is characterized by several stages of epicyclic trains and non-collinear input and output axes. The kinematic sketch is depicted in Figure 6 where $N_1 = 20$, $N_2 = 17$, $N_3 = 20$, $N_4 = 41$, $N_5 = 51$, $N_6 = 21$, $N_7 = 93$, $N_8 = 51$, $N_9 = 21$, $N_{10} = 93$ and the angle a each bevel gear is $(\pm)\pi/4\,(rad)$. The corresponding adjacency table T is given in Table 3.

The MGB is a mechanism with 8 links ($N = 8$), 6 gear pairs ($L = 6$) and 7 turning pairs. The turning pairs 1-3, 1-5 and 1-7 are on the same level. They are represented by a shaded polygon on the graph (Figure 7). Reference links of each gear pair can be found using the procedure proposed in section 2.3 (see Table 4).

The velocity ratio vector normalized w.r.t. the link # 2 (the input link) reads:

$$\Omega_0 = \begin{pmatrix} 0 \\ 1 \\ 0.488e^{\pi/2\sqrt{-1}} \\ 0.415 \\ 0.173e^{\pi/2\sqrt{-1}} \\ -0.592e^{\pi/2\sqrt{-1}} \\ 0.0612e^{\pi/2\sqrt{-1}} \\ -0.210e^{\pi/2\sqrt{-1}} \end{pmatrix} . \tag{9}$$

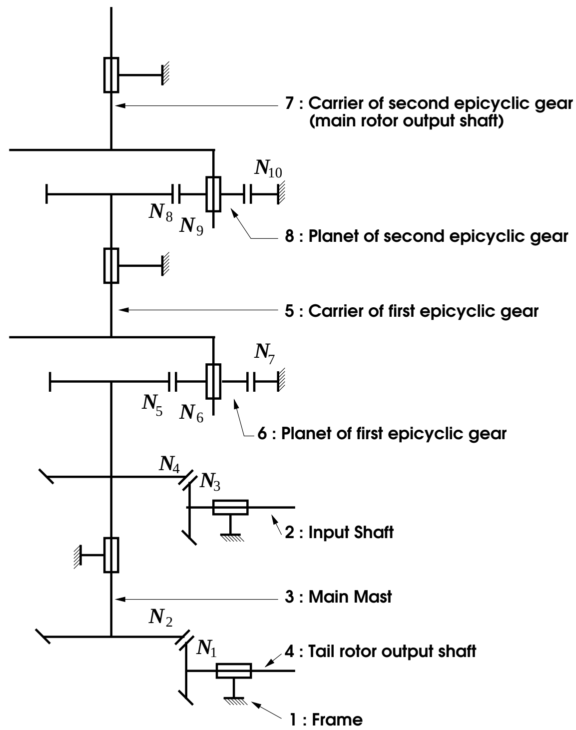

Fig. 6. Kinematic sketch of the Main Gear Box of Alouette III.

Table 3. Adjacency table of the MGB

	1	2	3	4	5	6	7	8
1		*level* = 2	*level* = 1	*level* = 3	*level* = 1	$-N_7$	*level* = 1	$-N_{10}$
2	*level* = 2		$N_3e^{-\pi/4\sqrt{-1}}$					
3	*level* = 1	$N_4e^{\pi/4\sqrt{-1}}$		$N_2e^{\pi/4\sqrt{-1}}$		N_5		
4	*level* = 3		$N_1e^{-\pi/4\sqrt{-1}}$					
5	*level* = 1					*level* = 4		N_8
6	N_6		N_6		*level* = 4			
7	*level* = 1							*level* = 5
8	N_9				N_9		*level* = 5	

Obviously, the reduction ratio between input shaft and:

- tail rotor shaft is $1/0.415$,
- main rotor shaft is $1/0.0612$ (and along the axis normal to the input axis).

4 Available software

This method has been implemented using MATLAB. The software is available at: `http://personnel.isae.fr/daniel-alazard/matlab-packages` and computes the following:

- the reference link of all the gear pairs of the mechanism,
- the kinematic constraint matrix M,
- the velocity ratio matrix Ω_0 (or vector if the system is a one d.o.f. mechanism).

The input data is a stuctured variable describing the adjacency table of the mechanism. This software can also compute contact frequencies in all gear pairs and also in all roll (or ball) bearings involved in turning pairs [11]. Indeed, the input structured variable can include a description of roll (or ball) bearings (that is: the mean diameter, the number and the diameter of balls (or rolls) and the angle of contact between the balls (or rolls) and the bearing rings).

The detailed instructions on how to use this software are available in the *read me* file contained in the software folder.

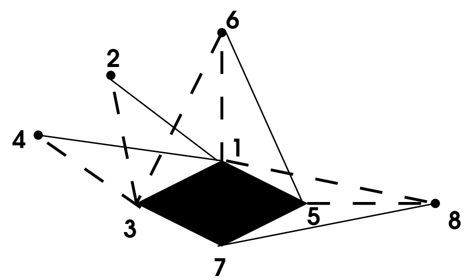

Fig. 7. Graph representation of the MGB.

Table 4. Gear pairs and reference links for the MGB.

Gear pair	Reference link
1-6	5
1-8	7
2-3	1
3-4	1
3-6	5
5-8	7

5 Conclusion

A general method for kinematic analysis of complex mechanical systems was presented in this paper. This approach based on the null space of the kinematic constraint matrix can be applied to multi-degree-of-freedom mechanisms with non collinear input and output axes. The use of complex coefficients in the adjacency table of the mechanism and the kinematic constraint matrix enables to find directly the angular velocity vector expressed in polar coordinates and constitutes the main novelty and contribution of this work. The proposed method is very simple to analyse multi-degrees-of-freedom with various operating modes. This approach was illustrated on a two-degree-of-freedom car differential and the main gear box of an helicopter. Finally, an open-source MATLAB code was developed to implement this approach.

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