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Project scheduling under uncertainty using fuzzy modelling and solving techniques

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A B S T R A C T

In the real world, projects are subject to numerous uncertainties at different levels of planning. Fuzzy project scheduling is one of the approaches that deal with uncertainties in project scheduling problem. In this paper, we provide a new technique that keeps uncertainty at all steps of the modelling and solving procedure by considering a fuzzy modelling of the workload inspired from the fuzzy/possibilistic approach. Based on this modelling, two project scheduling techniques, Resource Constrained Scheduling and Resource Leveling, are considered and generalized to handle fuzzy parameters. We refer to these problems as the Fuzzy Resource Constrained Project Scheduling Problem (FRCPSP) and the Fuzzy Resource Leveling Problem (FRLP). A Greedy Algorithm and a Genetic Algorithm are provided to solve FRCPSP and FRLP respectively, and are applied to civil helicopter maintenance within the framework of a French industrial project called Helimaintenance.

1. Introduction

A project is informally defined as a unique undertaking, composed of a set of precedence related tasks that have to be executed using diverse and mostly limited company resources. Project scheduling consists of deciding when tasks should be started and finished, and how many resources should be allocated to them (Creemers et al., 2008). Project scheduling respecting precedence and resource constraints is a research problem which is generally known to be NP-Hard. Many uncertainties can affect the project scheduling problem and hence increase its complexity (Bidot, 2005). These uncertainties can be grouped into three subsets; uncertainties in tasks, uncertainties in resources and temporal uncertainties (Elkhayari, 2003).

Among the applications that are considered by this study, we cite the civil helicopter heavy maintenance activity. This activity is almost carried out by an external maintenance center called Maintenance Repair and Overhaul (MRO) center that maintains a multi-customers relation. Each customer's helicopter is viewed as a unique project with its release and due date that should be respected. The presence of uncertainties is the major issue of the maintenance activity. How to deal with these uncertainties at the operational level of planning is studied in this paper. A quite similar problem exists in heavy maintenance of (other) complex systems e.g. trains and boats (De-Boer, 1998).

To deal with uncertainties in project scheduling, Herroelen and Leus (2005) distinguish between five main approaches: reactive scheduling, stochastic project scheduling, stochastic project networks, fuzzy project scheduling and proactive/robust scheduling. Particularly, the fuzzy project scheduling, based on the assumption that task durations rely on human estimations, is used when theory of probabilities is not compatible with the decision-making situation because of the lack of historical data (Pierre et al., 2004; Herroelen and Leus, 2005), which is the case for helicopter maintenance activity (Masmoudi and Haït, 2010) (see Section 2.2).

Resource availability is one of the important constraints to take into account to obtain feasible scheduling. Thus, two major techniques; *resource constrained scheduling* (RCS) and *resource leveling* (RL) are employed (Kim et al., 2005a). As far as we know, in fuzzy scheduling literature, dates and durations are considered fuzzy, but deterministic workload plans are provided (Hapke and Slowinski, 1996; Leu et al., 1999). In this paper, we deal with fuzzy project scheduling problems and provide a fuzzy solution with a fuzzy workload. A new approach is provided based on the idea to keep uncertainty in all calculations at each step. Firstly, we exploit the fuzzy/possibilistic approach to model a new concept that we call fuzzy workload. Secondly, based on this modelling concept, two techniques RCS and RL are generalized to handle fuzzy parameters. Finally, these techniques are supported by adapted Genetic Algorithm and Greedy Algorithm, respectively.

This paper is organized as follows. Section 2 recalls the state of the art for project scheduling problem under uncertainty and defines the

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specific problem to address. In Section 3, we recall some basics of fuzzy sets modelling and possibilistic approach. Section 4 describes a new modelling approach to deal with resource management problem in fuzzy area. Sections 5 and 6 contain a generalization of two algorithms to uncertain data: Greedy and Genetic Algorithms. In Section 7, these two algorithms are applied to instances from civil helicopters maintenance activity. Section 8 is a conclusion of the work.

2. Project scheduling under uncertainty

2.1. State of the art

To deal with uncertainties in project scheduling issues, both fuzzy sets and probabilities are considered in the literature (Hillier, 2002; Herroelen and Leus, 2005). The literature on project stochastic scheduling is rather sparse (Subhash et al., 2010) and most of the efforts concentrate on the Stochastic Resource Constrained Project Scheduling Problem (Herroelen and Leus, 2005). Since the early 90s, fuzzy logic has become a very promising mathematical approach to model uncertainty and imprecision in manufacturing problems (Wong and Lai, 2011) and scheduling problems (Slowinski and Hapke, 2000). Below, how the precedence and resource issues within fuzzy project scheduling are treated in the literature.

To deal with precedence constraint, Program Evaluation and Review Technique (PERT) and Critical Path Method (CPM) are considered. They are based on two successive steps; a *forward propagation* to determine the earliest starting and finishing dates (and consequently the project duration and the free floats) and a *backward propagation* for the latest starting and finishing dates (and the total floats). The majority of the research on the fuzzy project scheduling topic has been devoted to fuzzy PERT and CPM techniques (Chanas et al., 2002; Guiffrida and Nagi, 1998; Zareei et al., 2011). In the fuzzy case, forward propagation is done using fuzzy arithmetic, leading to fuzzy earliest dates and a fuzzy end-of-project event. Unfortunately, backward propagation is no longer applicable because uncertainty would be taken into account twice. Dubois et al. (2003) show that the boundaries of uncertain parameters like the tasks' latest dates and floats are reached in extreme configurations. Fortin et al. (2005) justify the problem complexity and propose some algorithms to calculate the tasks' latest dates and floats while uncertainties are represented by intervals.

To deal with resources in operational level of planning, RCS and RL techniques are considered. The study of fuzzy scheduling has been initiated in Hapke et al. (1994) and Hapke and Slowinski (1996). Many techniques particularly the serial and the parallel scheduling schemes (Hapke and Slowinski, 1996), and the resource levelling technique (Leu et al., 1999) were generalized to handle fuzzy parameters. To decide the feasibility of a project schedule, the workload plan is established and compared to the available capacity. In the literature, the majority of authors who work with fuzzy sets in scheduling problems transform the fuzzy scheduling into crisp scheduling by applying either alpha-cuts (see Section 4) or a defuzzification technique. Thus, they generate deterministic workload plans (Hapke and Slowinski, 1996; Leu et al., 1999). On the other hand, Masmoudi and Haït (2011a,b) use the possibility theory to define a new concept of fuzzy workload plan in operational level of planning. This original idea is developed in detail in this paper and applied to a real multi-project environment such as the civil helicopter maintenance activity which is described below.

2.2. Helicopter maintenance scheduling problem

Almost the totality of research in helicopter maintenance field are carried out in the military domain. To the best of our knowledge, only few works have been published on civil helicopter maintenance (Glade, 2005; Djeridi, 2010), and none on scheduling heavy inspections. Addressing civil customers involves a great heterogeneity of helicopters. Indeed, the average number of helicopters by civil owner is between two and three, and the conditions of use can radically vary from one customer to another (sea, sand, mountain, etc.). On the contrary, in the military domain, there are important homogeneous fleets, and the missions for which the helicopters are assigned are quite similar. Moreover, the management process in Civil MROs is different from the process in military MROs. In fact, in the military domain, the helicopters maintenance is managed respecting planned and expected missions (Sgaslink, 1994). This is similar to the maintenance of machines in production industry that is managed respecting the orders due dates (Nakajima, 1989). On the contrary, in the civil domain, heavy maintenance is carried out by an external maintenance center that is not concerned by the exploitation, but maintains a highly multi-customers relation, and considers each customer's helicopter as a unique project with its release and due date that should be respected. The application of global optimization approaches, as can be found in the military domain for important homogeneous fleets and one single customer (Hahn and Newman, 2008), is not necessarily pertinent for civil helicopter maintenance.

The helicopter maintenance visits contain planned maintenance tasks and also corrective maintenance tasks since several failures are discovered during the helicopters inspections. Precedence constraints exist between the tasks, due to technical or accessibility considerations. Hence helicopter maintenance visits may be seen as projects involving various resources as operators, equipment and spare parts. Minimizing the helicopters immobilization gives a competitive advantage to the company. Consequently, the management of a maintenance center is viewed as multi-project management, where every project duration should be minimized while respecting capacity constraints.

Managing helicopter maintenance activity is a complex task as it is affected by high uncertainties (Masmoudi and Haït, 2010) that should be taken into account when dealing with scheduling optimization. We can identify three main sources of uncertainty:

- Tasks durations: differ according to skills level of assigned operator. It differs also from one helicopter to another according to the compactness, state, and mission use. Tasks starting dates are consequently uncertain.
- Maintenance program updates: manufacturers and authorities send regularly new documents (Service Bulletin, Airworthiness Directives, etc.) to add, eliminate or modify some tasks from the maintenance program document.
- Absence of operators: the unexpected lack of resources causes the delay of several tasks and hence some tasks' durations are increased.

According to our knowledge, dealing with uncertainties is a main issue of civil helicopter maintenance scheduling problem that has never been studied in the literature. Considering the non-repetitive aspect of the problem (each helicopter has its own history, the customers are numerous and the conditions of use are highly different), the difficulty to predict the exploitation or establish statistics on corrective tasks or tasks' durations and the very limited available data, we propose a fuzzy set modelling for tasks' dates and durations, and hence a possibilistic approach instead of stochastic approach.

3. Fuzzy/possibilistic approach

3.1. Fuzzy set modelling

An ensemblist representation can be either a simple interval or a more complex and complete form as triangular or trapezoidal

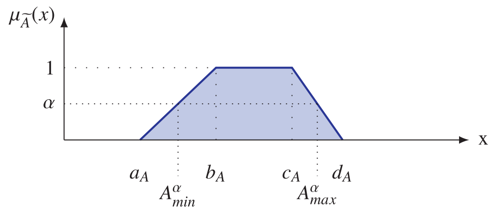

Fig. 1. Trapezoidal fuzzy set.

fuzzy profile (Fig. 1). A fuzzy model has the advantage to be supported by the possibility theory (see Section 3.2) for decision making.

Zadeh (1965) has defined a fuzzy set $\tilde{A}$, whose boundaries are gradual rather than abrupt, as a subset of a referential set X. The *membership function* $\mu_{\tilde{A}}$ of a fuzzy set assigns to each element $x \in X$ its degree of membership $\mu_{\tilde{A}}(x)$ taking values in (0,1].

To generalize some operations from classical logic to fuzzy sets, Zadeh has shown that it was possible to represent a fuzzy profile by an infinite family of intervals called α-cuts. Hence, the fuzzy profile $\tilde{A}$ can be defined as a set of intervals $A_\alpha = [A^{\alpha}_{\min}, A^{\alpha}_{\max}] = \{x \in X / \mu_{\tilde{A}}(x) \geq \alpha\}$ with $\alpha \in (0,1]$ (Fig. 1). It becomes consequently easy to use classical interval arithmetics and adapt it to fuzzy profiles. Dubois and Prade (1988) and Chen and Hwang (1992) have defined mathematical operations that can be performed on trapezoidal fuzzy sets. Let $\tilde{A}(a_A,b_A,c_A,d_A)$ and $\tilde{B}(a_B,b_B,c_B,d_B)$ be two trapezoidal fuzzy numbers, then:

$$\tilde{A} \oplus \tilde{B} = (a_A + a_B, b_A + b_B, c_A + c_B, d_A + d_B) \tag{1}$$

$$\tilde{A} \ominus \tilde{B} = (a_A - d_B, b_A - c_B, c_A - b_B, d_A - a_B) \tag{2}$$

$$\min(\tilde{A},\tilde{B}) = (\min(a_A,a_B), \min(b_A,b_B), \min(c_A,c_B), \min(d_A,d_B)) \tag{3}$$

$$\max(\tilde{A},\tilde{B}) = (\max(a_A,a_B), \max(b_A,b_B), \max(c_A,c_B), \max(d_A,d_B)) \tag{4}$$

$$\tilde{A} \cup \tilde{B} = \max_{x \in X}(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)) \tag{5}$$

$$\tilde{A} \cap \tilde{B} = \min_{x \in X}(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)) \tag{6}$$

$$\alpha\tilde{A} = \begin{cases} (\alpha a_A, \alpha b_A, \alpha c_A, \alpha d_A) & \text{if } \alpha > 0 \\ (\alpha d_A, \alpha c_A, \alpha b_A, \alpha a_A) & \text{if } \alpha < 0 \end{cases} \tag{7}$$

Other operations like multiplication and division have also been studied. For more details on fuzzy arithmetics, one can refer to Dubois and Prade (1988).

3.2. *Possibility theory*

To cope with decision making on fuzzy area, Zadeh (1978) has developed the concept of possibility, based on fuzzy subsets. Possibility theory introduces both *possibility* measure (denoted Π) and *necessity* measure (denoted N), in order to express plausibility and certainty of events.

Let τ be a variable in the fuzzy interval $\tilde{A}$ and t be a real value. To measure the truth of the event $\tau \leq t$, equivalent to $\tau \in (-\infty; t]$, we need the couple $\Pi(\tau \leq t)$ and $N(\tau \leq t)$ representing the fact that $\tau \leq t$ is respectively possibly true and necessarily true (Fig. 2).

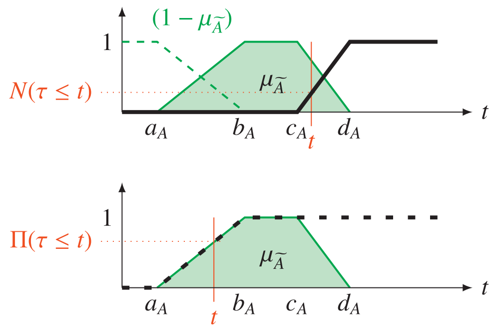

Fig. 2. Possibility and necessity of $\tau \leq t$ with $\tau \in \tilde{A}$.

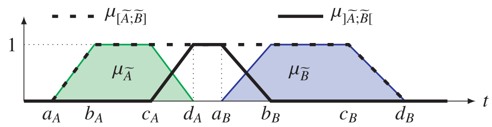

Fig. 3. Possibility and necessity of t being between $\tilde{A}$ and $\tilde{B}$.

Thus:

$$\Pi(\tau \leq t) = \sup_{u \leq t} \mu_{\tilde{A}}(u) = \mu_{[\tilde{A};+\infty)}(t) = \sup_{u} \min(\mu_{\tilde{A}}(u), \mu_{(-\infty;t]}(u)) \tag{8}$$

$$N(\tau \leq t) = 1 - \sup_{u > t} \mu_{\tilde{A}}(u) = \mu_{]\tilde{A};+\infty)}(t) = \inf_{u} \max(1 - \mu_{\tilde{A}}(u), \mu_{(-\infty;t]}(u)) \tag{9}$$

Consequently, let τ and σ be two variables in fuzzy intervals $\tilde{A}$ and $\tilde{B}$ respectively, and t a real value. To measure the truth of the event "t between τ and σ" we need both $\Pi(\tau \leq t \leq \sigma)$ and $N(\tau \leq t \leq \sigma)$ (Fig. 3). Thus:

$$\Pi(\tau \leq t \leq \sigma) = \mu_{[\tilde{A};\tilde{B}]}(t) = \mu_{[\tilde{A};+\infty)\cap(-\infty;\tilde{B}]}(t) = \min(\mu_{[\tilde{A};+\infty)}(t), \mu_{(-\infty;\tilde{B}]}(t)) \tag{10}$$

$$N(\tau \leq t \leq \sigma) = \mu_{]\tilde{A};\tilde{B}[}(t) = \mu_{]\tilde{A};+\infty)\cap(-\infty;\tilde{B}[}(t) = \min(\mu_{]\tilde{A};+\infty)}(t), \mu_{(-\infty;\tilde{B}[}(t)) \tag{11}$$

The expressions (10) and (11) will be considered in Section 4 to define the necessity and possibility of a task to be present between its starting and finishing times. This will permit to deduce the new concept of fuzzy workload.

4. Fuzzy task presence and fuzzy workload

The project dates and durations are represented by trapezoidal fuzzy numbers. Let $\tilde{S}(a_S,b_S,c_S,d_S)$ be the fuzzy starting time of a task T, and $\tilde{D}(w,x,y,z)$ its duration. Let $\tilde{F}(a_F,b_F,c_F,d_F)$ be its finishing time with $\tilde{F} = \tilde{S} + \tilde{D}$. Let C be the number of operators assigned to the task T. Once starting and finishing times of all tasks are defined, several deterministic resource workload plans are established by applying alpha-cuts (Fig. 4).

In this section, we provide a new technique to deal with fuzzy resource-constrained task scheduling. Instead of applying alpha-cuts on a fuzzy Gantt to get deterministic resource plans, both Gantt and workload plan are considered fuzzy.

Inspired from (10) and (11), we can define $]\tilde{S};\tilde{F}[$ (resp. $[\tilde{S};\tilde{F}]$), the domain where the task T presence is necessarily (resp. possibly) true. They represent the truth of the event "t between the starting and finishing times of T". Associated membership functions, $\mu_{]\tilde{S};\tilde{F}[}(t)$ and $\mu_{[\tilde{S};\tilde{F}]}(t)$, are respectively denoted $N(t)$ and $\Pi(t)$.

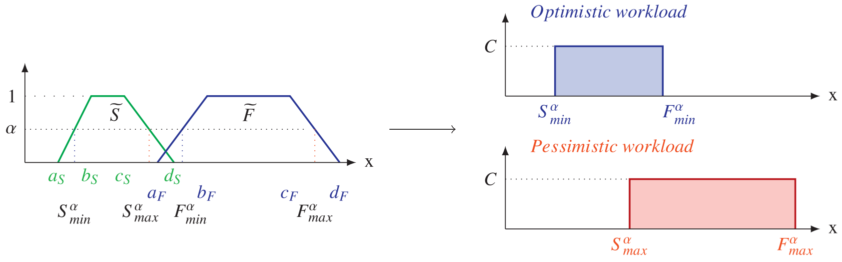

Fig. 4. Alpha-cuts and deterministic workloads.

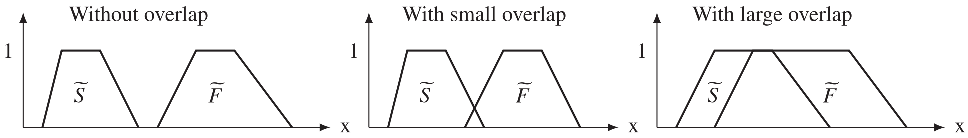

Fig. 5. Different configurations: with and without overlap.

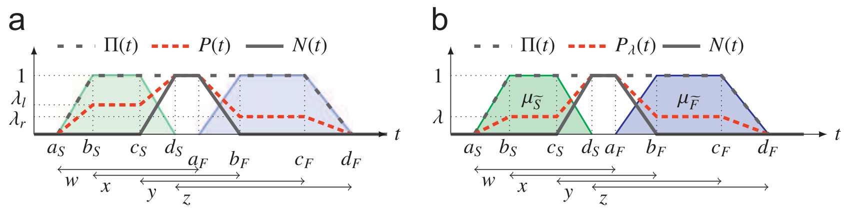

Fig. 6. Presence of a task: no overlap configuration. (a) General distribution: non-symmetric. (b) Particular distribution: symmetric.

We can distinguish three different configurations depending on the intersection degree between fuzzy starting and finishing times: a configuration without overlap ($d_S \leq a_F$), a configuration with small overlap ($d_S > a_F$ and $c_S \leq b_F$) and a configuration with large overlap ($c_S > b_F$).

Task presence distributions are used to build task resource usage profiles in a way that keeps track of uncertainty on starting and finishing times. Hence, the profile reflects the whole possible time interval while giving a plausible repartition of workload according to the duration parameter value. To this aim, the resource usage profiles are defined as projections onto the workload space of the task presence distributions. Each configuration of starting and finishing times is studied separately within two (symmetric and non-symmetric) distributions of the workload, used in the scheduling optimization algorithms (Fig. 5).

4.1. Configuration without overlap

In the configuration without overlap between the starting time $\tilde{S}$ and the finishing time $\tilde{F}$ (Fig. 6), we can identify the following intervals of possibility and necessity:

$$\begin{array}{lll} [d_S; a_F]: & \Pi = 1 & N = 1 \\ [c_S; d_S] \text{ and } [a_F; b_F]: & \Pi = 1 & N \geq 0 \\ [b_S; c_S] \text{ and } [b_F; c_F]: & \Pi = 1 & N = 0 \\ [a_S; b_S] \text{ and } [c_F; d_F]: & \Pi \geq 0 & N = 0 \\ [0; a_S] \text{ and } [d_F; +\infty[: & \Pi = 0 & N = 0 \end{array}$$

Then we characterize the probability of task T presence as a distribution $P(t)$ situated between the possibility and the necessity profiles: $N(t) \leq P(t) \leq \Pi(t)$. We propose a parametric piecewise linear distribution to represent the probability of the presence of task (dashed lines in Fig. 6).

Both symmetric and non-symmetric distributions are considered and will be used to establish resource requirement. The symmetric distribution is a particular case, and thus the non-symmetric distribution which is the general one is represented by a compound function depending on different intervals of possibility and necessity:

$$P(t) = \begin{cases} \dfrac{\lambda_l}{b_S - a_S}(t - a_S) & \text{if } t \in [a_S; b_S] \\ \lambda_l & \text{if } t \in [b_S; c_S] \\ \dfrac{1}{d_S - c_S}((1-\lambda_l)t + \lambda_l d_S - c_S) & \text{if } t \in [c_S; d_S] \\ 1 & \text{if } t \in [d_S; a_F] \\ \dfrac{1}{b_F - a_F}((\lambda_r - 1)t + b_F - \lambda_r a_F) & \text{if } t \in [a_F; b_F] \\ \lambda_r & \text{if } t \in [b_F; c_F] \\ \dfrac{-\lambda_r}{d_F - c_F}(t - d_F) & \text{if } t \in [c_F; d_F] \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

where parameters λ_l and λ_r, varying from 0 to 1, make profile $P(t)$ evolve from $N(t)$ ($\lambda_l = \lambda_r = 0$) to $\Pi(t)$ ($\lambda_l = \lambda_r = 1$).

Suppose that the resource requirement of the task is r. Resource workload then lies in $[r \cdot w, r \cdot z]$, according to the task duration. Fig. 7b presents the resource profiles $L_N(t)$ and $L_\Pi(t)$, projections of the necessity and possibility presence distributions.

We define the "equivalent durations" D_N and D_Π of the areas covered by resource profiles $L_N(t)$ and $L_\Pi(t)$:

$$r \cdot D_N = \int_0^{+\infty} L_N(t)\, dt = r(b_F - c_S + a_F - d_S)/2 \tag{13}$$

$$r \cdot D_\Pi = \int_0^{+\infty} L_\Pi(t)\, dt = r(d_F - a_S + c_F - b_S)/2 \tag{14}$$

In case of symmetric distribution, the link between task duration D and profile parameter λ is given by the following formula that expresses the equivalence of resource requirement:

$$\begin{aligned} r \cdot D &= \int_0^{+\infty} r \cdot P^*(t)\, dt = \int_0^{+\infty} (\lambda \cdot L_\Pi(t) + (1-\lambda) L_N(t))\, dt \\ &= \lambda \cdot r \cdot D_\Pi + (1-\lambda) r \cdot D_N \end{aligned}$$

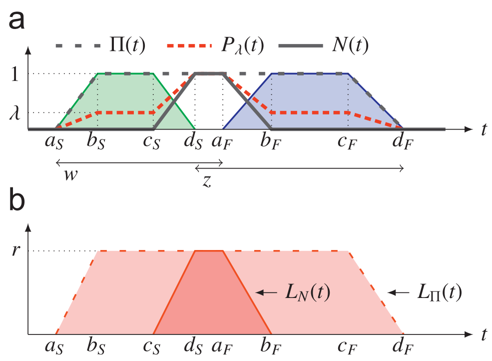

Fig. 7. Configuration without overlap: presence distributions (a) and resource profiles (b).

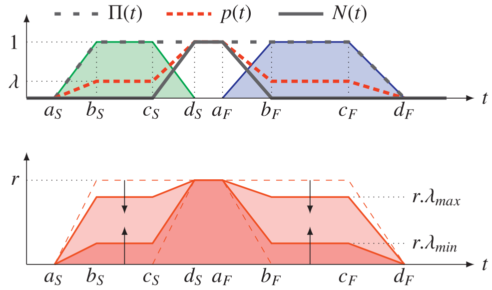

Fig. 8. Resource profiles: restriction to $\lambda_{\min}$ and $\lambda_{\max}$ in order to match with extreme workloads.

$$= \lambda \cdot r(d_F - a_S + c_F - b_S)/2 + (1-\lambda) r(b_F - c_S + a_F - d_S)/2 \tag{15}$$

In general case where distribution is non-symmetric, the link between the task duration and the profile is as follows:

$$\begin{aligned} r\cdot\ &= r\cdot\lambda_l\left(\frac{d_S-b_S}{2}+\frac{c_S-a_S}{2}\right)+r\cdot\lambda_r\left(\frac{c_F-a_F}{2}+\frac{d_F-b_F}{2}\right) \\ &+ r\cdot\left(\frac{a_F-d_S}{2}+\frac{b_F-c_S}{2}\right) \end{aligned} \tag{16}$$

In case of symmetric distribution, if D_N and D_Π do not match with task extreme durations w and z, i.e. $D_N < w$ or $z < D_\Pi$, then the profiles must be modified so that resource workload belongs to $[r\cdot w, r\cdot z]$. The extreme workloads are defined within a minimal or maximal values of λ, denoted respectively $\lambda_{\min}$ and $\lambda_{\max}$. Hence, the range of λ is reduced from $[0,1]$ to $[\lambda_{\min}, \lambda_{\max}]$ such as

If $D_N < w$, $\lambda_{\min} = (w - D_N)/(D_\Pi - D_N)$

If $z < D_\Pi$, $\lambda_{\max} = (z - D_N)/(D_\Pi - D_N)$

Fig. 8 shows an example of restricted extreme profiles.

Let us consider the particular case of task with a fuzzy duration, and deterministic starting time ($a_S = b_S = c_S = d_S = s$, Fig. 9a). If we choose $D = z$, then there is only one possible position for the task, between s and d_F. So the resource chart is fixed, rectangular shaped. One can remark that in this case, the projection $L_\Pi(t)$ of the probability distribution is not able to represent the resource consumption: even with $\lambda = 1$, the resource workload would be underestimated (Fig. 9b). Indeed, the surface of profile $L_\Pi(t)$ is $r\cdot(c_F - s + d_F - s)/2 = r\cdot(y+z)/2$, lower than $r\cdot z$.

For any duration D so that $(y+z)/2 < D \le z$, the area of resource profile $L_\Pi(t)$ is too small to represent the resource workload. To cope this problem, we modify the resource profile: in place of points (s, s, c_F, d_F), the new profile is defined by the points (s, s, c'_F, d_F), where $c'_F = c_F + \max(0, 2D - z - y)$. Hence, while $D \le (y+z)/2$, the initial profile is used and $\lambda \le 1$, then the new profile is used. When $D = z$, the rectangular profile is attained. A similar modification can be done for the minimal duration, when the area of the projected necessity distribution is greater than $r\cdot w$.

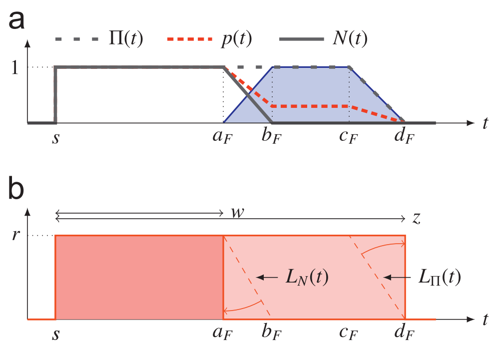

Fig. 9. Case of a deterministic starting time: presence distributions and maximal resource profile.

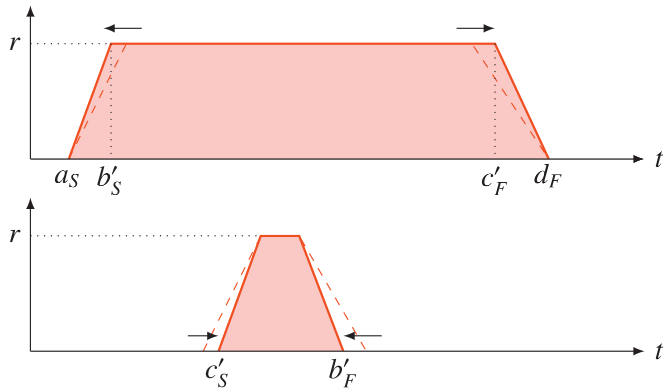

Fig. 10. Resource profiles: extension of maximal profile and reduction of minimal profile in order to match extreme workloads $r\cdot w$ and $r\cdot z$.

These modifications can be generalized to the case with fuzzy times and duration. Then the profiles, if needed, are modified on both sides. The extended maximal profile, defined by (a_S, b'_S, c'_F, d_F), is used when $D_\Pi < D \le z$. Values b'_S and c'_F are

$$b'_S = b_S - 2(D - D_\Pi)\frac{b_S - a_S}{b_S - a_S + d_F - c_F} \tag{17}$$

$$c'_F = c_F + 2(D - D_\Pi)\frac{d_F - c_F}{b_S - a_S + d_F - c_F} \tag{18}$$

The reduced minimal profile, defined by (c'_S, d_S, a_F, b'_F), is used when $w \le D < D_N$. Values c'_S and b'_F are

$$c'_S = c_S + 2(D_N - D)\frac{d_S - c_S}{d_S - c_S + b_F - a_F} \tag{19}$$

$$b'_F = b_F - 2(D_N - D)\frac{b_F - a_F}{d_S - c_S + b_F - a_F} \tag{20}$$

Fig. 10 shows an example of modified extreme profiles.

4.2. Configuration with small overlap

For the small overlap configuration (as in the previous configuration), the general distribution is also represented by a compound function (dashed line in Fig. 11):

$$p(t)=\begin{cases}\dfrac{\lambda_l}{b_S-a_S}(t-a_S) & \text{if } t\in[a_S;b_S]\\ \lambda_l & \text{if } t\in[b_S;c_S]\\ \dfrac{1}{d_S-c_S}((1-\lambda_l)t+\lambda_l d_S-c_S) & \text{if } t\in[c_S;\alpha]\\ \dfrac{1}{b_F-a_F}((\lambda_r-1)t+b_F-\lambda_r a_F) & \text{if } t\in[\alpha;b_F]\\ \lambda_r & \text{if } t\in[b_F;c_F]\\ \dfrac{-\lambda_r}{d_F-c_F}(t-d_F) & \text{if } t\in[c_F;d_F]\\ 0 & \text{otherwise.}\end{cases} \tag{21}$$

where the higher point (α,β) is calculated as follows:

$$\alpha=\frac{(b_F-a_F)(\lambda_l d_S-c_S)+(d_S-c_S)(\lambda_r a_F-b_F)}{(b_F-a_F)(\lambda_l-1)+(d_S-c_S)(\lambda_r-1)} \tag{22}$$

$$\beta=\frac{(b_F-\lambda_r a_F)(\lambda_l-1)+(\lambda_l d_S-c_S)(\lambda_r-1)}{(b_F-a_F)(\lambda_l-1)+(d_S-c_S)(\lambda_r-1)} \tag{23}$$

And particularly while $\lambda_l=\lambda_r=\lambda$:

$$\alpha=\alpha_0=\frac{d_s\cdot b_f-a_f\cdot c_s}{(b_f-c_s)+(d_s-a_f)} \tag{24}$$

$$\beta=\frac{(b_f-c_s)+\lambda(d_s-a_f)}{(b_f-c_s)+(d_s-a_f)} \tag{25}$$

The β value varies in a range $[\beta_0,1]$ and the α value varies in a range $[a_F,d_S]$ along with parameters λ or λ_l and λ_r.

The areas of the projected necessity and possibility distributions are

$$r\cdot D_N=\int_0^{+\infty} r\cdot N(t)\,dt=r\cdot\beta_0\frac{b_F-c_S}{2}=r\frac{(b_F-c_S)^2}{2(d_S-a_F+b_F-c_S)} \tag{26}$$

$$r\cdot D_\Pi=\int_0^{+\infty} r\cdot\Pi(t)\,dt=r\cdot(d_F-a_S+c_F-b_S)/2 \tag{27}$$

If $r\cdot D_N$ is lower than the minimal workload $r\cdot w$ (respectively, $r\cdot D_\Pi$ greater than the maximal workload $r\cdot z$) we use the projection of the presence probability distribution and determine $\lambda_{\min}$ (respectively, $\lambda_{\max}$). Given D so that $D_N<D<D_\Pi$,

$$r\cdot D=\int_0^{+\infty} r\cdot P_\lambda(t)\,dt=\lambda\cdot r\cdot D_\Pi+(1-\lambda)r\cdot D_N$$

In general case where distribution is non-symmetric, the link between the task duration and the profile is given by the following formula:

$$\begin{aligned}r\cdot D&=\int_0^{+\infty} r\cdot p(t)\,dt\\&=r\cdot\lambda_l\left(\frac{c_S+\alpha}{2}-\frac{a_S+b_S}{2}\right)+r\cdot\lambda_r\left(\frac{d_F+c_F}{2}-\frac{\alpha+b_F}{2}\right)+r\cdot\beta\left(\frac{b_F-c_S}{2}\right)\end{aligned} \tag{28}$$

In case of symmetric distribution, when $D_N<w$, $\lambda_{\min}=(w-D_N)/(D_\Pi-D_N)$ and when $D_\Pi>z$, $\lambda_{\max}=(z-D_N)/(D_\Pi-D_N)$.

The extended maximal profile, defined by (a_S,b'_S,c'_F,d_F), is used when $D_\Pi<D\le z$. It is the same extended profile as the one of no overlap configurations.

The reduced minimal profile, defined by (c'_S,d_S,a_F,b'_F), is used when $w\le D<D_N$. Values c'_S and b'_F are

$$c'_S=\theta\cdot c_S+(1-\theta)d_S \tag{29}$$

$$b'_F=\theta\cdot b_F+(1-\theta)a_F \tag{30}$$

where $\theta=(1-\beta_0)/(1-\beta')$ and

$$\beta'=\frac{\sqrt{r^2D^2+2(d_S-a_F)r\cdot D}-r\cdot D}{d_S-a_F} \tag{31}$$

4.3. Configuration with large overlap

For the large overlap configuration (as in the previous configuration), the general distribution is also represented by a compound function (Fig. 12):

$$p(t)=\begin{cases}\dfrac{\lambda_l}{b_S-a_S}(t-a_S) & \text{if } t\in[a_S;b_S]\\ \lambda_l & \text{if } t\in[b_S;b_F]\\ \dfrac{1}{b_F-c_S}((\lambda_l-\lambda_r)t+\lambda_r b_F-\lambda_l c_S) & \text{if } t\in[b_F;c_S]\\ \lambda_r & \text{if } t\in[c_S;c_F]\\ \dfrac{-\lambda_r}{d_F-c_F}(t-d_F) & \text{if } t\in[c_F;d_F]\\ 0 & \text{otherwise}\end{cases} \tag{32}$$

The necessity presence distribution is $N(t)=0\ \forall t$. The areas of the projected necessity and possibility distributions are

$$r\cdot D_N=\int_0^{+\infty} r\cdot N(t)\,dt=0 \tag{33}$$

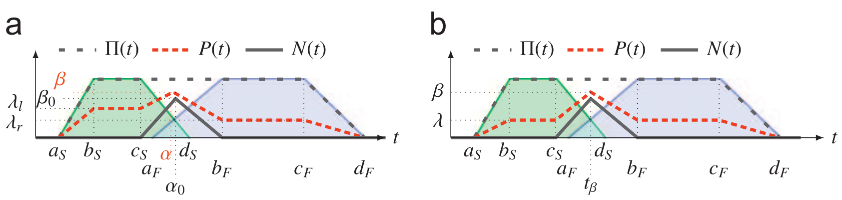

Fig. 11. Presence of a task: small overlap configuration. (a) Non-symmetric and (b) symmetric

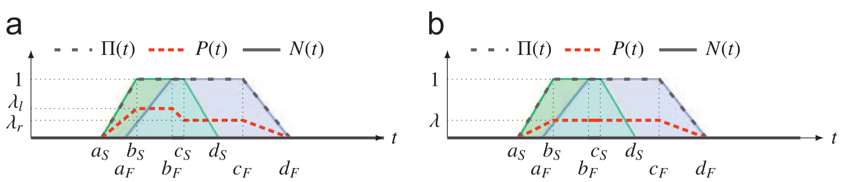

Fig. 12. Presence of a task: large overlap configuration. (a) Non-symmetric and (b) symmetric.

$$r \cdot D_\Pi = \int_0^{+\infty} r \cdot \Pi(t)\, dt = r \cdot (d_F - a_S + c_F - b_S)/2 \tag{34}$$

If the minimal workload $r \cdot w$ is greater than zero (respectively, $r \cdot D_\Pi$ greater than the maximal workload $r \cdot z$) we use the projection of the presence probability distribution and determine $\lambda_{\min}$ (respectively, $\lambda_{\max}$). Given D so that $0 < D < D_\Pi$,

$$r \cdot D = \int_0^{+\infty} r \cdot P_\lambda(t)\, dt = \lambda \cdot r \cdot D_\Pi$$

In general case where distribution is non-symmetric, the link between the task duration and the profile is given by the following formula:

$$r \cdot D = \int_0^{+\infty} r \cdot p(t)\, dt = r \cdot \lambda_l \left(\frac{c_S + b_F}{2} - \frac{a_S + b_S}{2} \right) + r \cdot \lambda_r \left(\frac{d_F + c_F}{2} - \frac{c_S + b_F}{2} \right) \tag{35}$$

In case of symmetric distribution, when $w > 0$, $\lambda_{\min} = w/D_\Pi$ and when $D_\Pi > z$, $\lambda_{\max} = z/D_\Pi$.

The extended maximal profile, defined by (a_S, b'_S, c'_F, d_F), is used when $D_\Pi < D \le z$. It is the same extended profile as the one of no overlap configurations. The minimal profile is never reduced.

In this section we studied the resource workload for a fuzzy task and provided symmetric and non-symmetric fuzzy distribution for the three possible configurations depending on the degree of intersection between the starting and finishing times. These modelling approaches will be used later to solve fuzzy scheduling problems.

5. Greedy algorithm for fuzzy resource constrained scheduling

The Schedule Generation Schemes (SGS) are the core of many heuristics for the RCPSP. The so-called Serial SGS performs activity incrementation and the Parallel SGS performs time incrementation (Kolish and Hartmann, 1999). In both procedures, tasks are ranked in some order and scheduled according to resource availabilities. Hapke and Slowinski (1996) have proposed a parallel scheduling procedure for fuzzy projects based on fuzzy priority rules and fuzzy time incrementation. However, resources are considered scarce and deterministic workload plans are deduced within alpha-cuts application on a fuzzy Gantt chart as shown in Fig. 4. The Parallel SGS that we propose in this section mainly differs from the latter on the resource availability test.

Before going in detail through the new fuzzy Parallel SGS, the following subsection is dedicated to explain how to deal with complex configuration while fuzzy resource and fuzzy precedence constraints are both considered, and the consequent subsection will show a list of adapted priority rules to the context of fuzzy multi-project scheduling problem.

5.1. Precedence and resource constrained tasks

When considering a precedence constraint between two tasks, their workload profiles may not overlap because the constraint expresses the fact that the two tasks cannot be performed simultaneously.

Let us consider two tasks A and B so that A precedes B. Their resource consumptions are denoted r_A and r_B. We assume that the starting time of B is equal to the finishing time of A (e.g. in case of forward earliest times calculation). This means that between the starting time of A and the finishing time of B, an activity will occur successively induced by A then B. So between the necessity peaks of A and B, we can affirm that an activity will necessarily occur, induced by A or B. This necessary presence of A or B is projected onto the resource load space using the minimal resource requirement $\min(r_A, r_B)$, associated to pseudo-task $A \vee B$ starting at $\tilde{S}_A$ and finishing at $\tilde{F}_B$ (Fig. 14). The projected necessity and possibility load profiles of the sequence $A \to B$ are defined as follows:

if $r_B\lambda_B > max(r_B, r_A) - r_A\lambda_A^r$ **then**
  $\lambda_B^l = (max(r_B, r_A) - r_A\lambda_A^r)/r_B$
else if $r_B\lambda_B < min(r_B, r_A).N(A \vee B) - r_A\lambda_A^r$ **then**
  $\lambda_B^l = (min(r_B, r_A).N(A \vee B) - r_A\lambda_A^r)/r_B$
else
  $\lambda_B^l = \lambda_B$
end if
$\lambda_B^r = f(\lambda_B^l, D_B)$

Fig. 13. Workload modelling for two directly successive tasks.

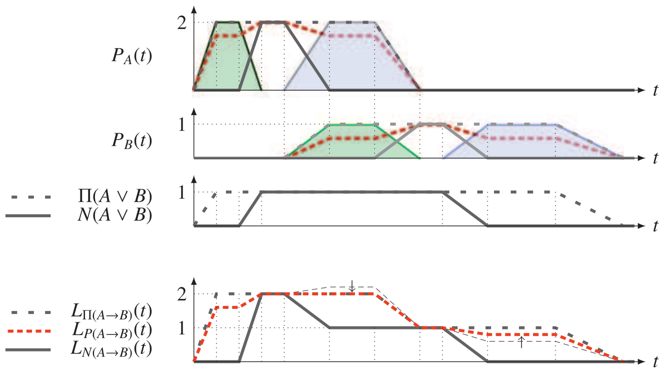

Fig. 14. Fuzzy continuous workload plan.

$$L_{N(A\to B)}(t) = \max(r_A \cdot N_A(t), r_B \cdot N_B(t), \min(r_A, r_B) \cdot N_{A\vee B}(t)) \tag{36}$$

$$L_{\Pi(A\to B)}(t) = \max(r_A \cdot \Pi_A(t), r_B \cdot \Pi_B(t)) \tag{37}$$

The probability workload profile is more complex to define. A constructive way can be provided; firstly the distribution of A is defined and then the distribution of B is deduced respecting resources and precedence constraints. Let us consider A without predecessors. Hence, we can assign to A its symmetric distribution while $\lambda_A^l = \lambda_A^r = \lambda_A$. For B we apply the following checks in Fig. 13: where D_B is the duration of B, f is a function deduced from (16), (28), and (35), and λ_B is the parameter value of task B distribution while considering $\lambda_B^l = \lambda_B^r$.

Once probabilistic distributions of A and B are defined respecting resource and precedence constraints, the sum of the two distributions corresponds to the total probabilistic workload:

$$L_{P(A\to B)}(t) = r_A \cdot P_A(t) + r_B \cdot P_B(t) \tag{38}$$

Fig. 14 shows the workload while $r_A = 2$, and $r_B = 1$. The integration of these profiles considering updates made by the aforementioned formula gives the total workload.

5.2. Fuzzy priority rules

Priority heuristics using crisp or fuzzy time parameters are found efficient by many researchers either for one project or multi-project scheduling (Kolish and Hartmann, 1999; Browning and Yassine, 2010; Hapke and Slowinski, 1996). It is generally useful to perform parallel scheduling with a set of rules instead of one as the computational complexity is low (Hapke and Slowinski, 1993). Some rules that appear to be good in minimizing Makespan are presented in Table 1.

The list is not exhaustive and many other interesting rules could be used, like the Minimum Worst Case Slack (*MINWCS*), the

Minimum Total Work Content (*MINTWC*) and some dynamic and combined rules presented in Browning and Yassine (2010).

5.3. Fuzzy parallel SGS

Let S (index $j = 1..S$) be the set of tasks to be scheduled. Within a loop, we calculate the distribution parameters of each task j (H_j^l then H_j^r) task by task within a new Parallel SGS technique based on the new fuzzy workload modelling provided in this paper. The structure of the new fuzzy Parallel SGS in shown in Fig. 15, where:

Table 1
Priority rules giving good results in Makespan minimization.

Rule	Name	Formula
EST	Early start time[a]	$\min(\tilde{E}_j^s)$
EFT	Early finish time[a]	$\min(\tilde{E}_j^f)$
LST	Late start time[a,b,c]	$\min(\tilde{L}_j^s)$
LFT	Late finish time[a,b,c]	$\min(\tilde{L}_j^f)$
MINSLK	Minimum slack[a,b,c]	$\min(\tilde{f}_j)$
MAXSLK	Maximum slack[c]	$\max(\tilde{f}_j)$
SPT	Shortest processing time[a,b,c]	$\min(\tilde{p}_j)$
LPT	Longest processing time[a,c]	$\min(\tilde{p}_j)$
LIS	Least immediate successors[a]	$\min(\lvert S_j\rvert)$
MIS	Most immediate successors[a]	$\max(\lvert S_j\rvert)$
MTS	Most total successors[b,c]	$\max(\lvert\overline{S_j}\rvert)$
GRD	Greatest resource demand[a]	$\tilde{p}_j \sum_{k=1}^{K} r_{jk}$
SASP	Shortest activity from shortest project[c]	$\min(\tilde{p}_{jl})$
LALP	Longest activity from longest project[c]	$\max(\tilde{p}_{jl})$
GRPW	Greatest rank positional weight[a,c]	$\max(\tilde{p}_j + \sum_{i \in S_j} \tilde{p}_i)$
LRPW	Least rank positional weight[a]	$\min(\tilde{p}_j + \sum_{i \in S_j} \tilde{p}_i)$

Where $\tilde{p}_j$: duration, $\tilde{f}_j$: margin, r_{jk} is the requirement for resource R_k.
$\tilde{L}_j^f$: last finishing, $\tilde{E}_j^f$: earliest finishing.
$\tilde{L}_j^s$: last starting, $\tilde{E}_j^s$: earliest starting.
S_j: direct successors, $\overline{S_j}$: total successors.
[a] Used by Hapke and Slowinski (1996) for a fuzzy RCPSP.
[b] Used by Kolish and Hartmann (1999) for deterministic RCPSP.
[c] Used by Browning and Yassine (2010) for multi-projects RCPSP (RCMPSP).

- $Av(\tilde{t})$ is the set of tasks whose defuzzification values of earliest starting times (set of Es_j) are less than or equal to the defuzzification value of $\tilde{t}$ ($Es_j \le t, \forall j \in Av(\tilde{t})$).
- $\tilde{l}(\tilde{t})$ is the least value among the earliest starting times of tasks from $A(\tilde{t})$ and the finishing times of tasks from $S(\tilde{t})$.
- $A(\tilde{t})$ is the set of tasks that are not yet scheduled and whose immediate predecessors have been completed by $\tilde{t}$.
- $S(\tilde{t})$ is the set of tasks present in $\tilde{t}$; a task j is considered as present in $\tilde{t}$ when $S_j \le t \le F_j$ (S_j and F_j are the defuzzifications of starting and finishing times of j, respectively.

The considered defuzzification technique is the mean value provided by Dubois and Prade (1987). Let t be the mean value of $\tilde{t}$, $t = (a_t + b_t + c_t + d_t)/4$.

This fuzzy Parallel SGS structure is similar to the one provided by Hapke and Slowinski (1996). However, there are two major differences. First, the possibility to schedule a task is checked according to the resource requirement and resource availability which are deterministic in Hapke and Slowinski's algorithm and fuzzy in ours. Second, to generalize the Parallel SGS dynamic time progression (Kolish and Hartmann, 1999) to fuzzy consideration, Hapke and Slowinski (1996) consider weak and strong inequalities to compare fuzzy times and make the adequate incrementation. In our approach, the same progression technique is considered but, according to our fuzzy workload consideration, an additional specific time progression technique is proposed when at least one task is available for scheduling but not yet scheduled because of resource availability issue.

We mention that the Parallel SGS algorithm is to be run as much time as priority rules we have. Hence, we talk about multi-priority rule method (Boctor, 1990). Other procedures based on Parallel and Serial SGS and called multi-pass methods (Kolish and

```
Choose a priority rule;
Initialize Ẽs_j (∀j), earliest starting time of task j, using the CPM
technique;
Initialize t̃ = t̃_0; the begin of the scheduling horizon
Initialize the total resources availabilities at all scheduling periods;
repeat
  Compose the set Av(t̃) of available tasks at t̃
  for each j from Av(t̃) according to the priority rule do
    Calculate the corresponding symmetric probabilistic distribution P_j
    if the symmetric probabilistic distribution P_j does not fit period by
    period the resources availabilities then
      Calculate a new P_j with a asymmetric shape considering the minimum
      possible value of the left parameter (H_j^l)
      if the configuration P_j fits the resource availabilities then
        Schedule j with corresponding starting and finishing times,
        Integrate the distribution P_j into the workload plan and update
        the total resources availabilities,
        Update the earliest starting time of all successors of j,
      end if
    end if
  end for
  if all tasks from Av(t̃) are scheduled  then
    t̃ = max(t̃, l̃(t̃))
  else
    t̃ = max(t̃, a_t + 1)
  end if
until all tasks are scheduled
```

Fig. 15. Fuzzy parallel SGS technique for resource leveling problem.

Hartmann, 1999) can be studied, but this is not the objective of this paper.

6. GA for fuzzy resource leveling

Resource leveling, also called smoothing technique, aims at completing projects respecting their due dates within a resource usage that is levelled as possible throughout the total project durations. Based on the result of the PERT/CPM technique, the result of the resource leveling is a schedule respecting precedence constraints. In this paper, a schedule will be defined by the tasks starting times that are between the earliest and latest starting times.

Many exact and heuristic techniques were developed to solve resource leveling problems (Zhao et al., 2006; Easa, 1989). Since 1975, the Genetic Algorithm has proven its effectiveness for complex problems like particularly the multi-projects and multi-objectives scheduling problems (Kim et al., 2005b). A GA is a search heuristic that follows the natural evolutionary process. The technique of GA is quite known, thus to get more complete information about it we refer readers to Goldberg (1989).

6.1. Genetic algorithm description

In multi-projects context, the Resource Leveling Problem can be defined as a set of tasks with precedence constraints and predetermined durations. A schedule is defined by a set of tasks starting times. Let n be the total number of tasks, P be the number of projects to schedule and n_j be the number of tasks in project j ($n=\sum_{j=1}^{P} n_j$). A schedule is defined by the set $S=(S_{11},S_{21},\ldots,S_{n_1 1},\ldots,S_{ij},\ldots,S_{1P},\ldots,S_{n_P P})$ where S_{ij} is the starting time of the task i from the project j.

The CPM technique is applied to a scheduling problem without considering resources in order to define the lower and upper bounds of each value S_{ij} which are respectively the earliest starting time (ES_{ij}) and the latest starting time (LS_{ij}) of the task i from the project j.

The objective L is to smooth resource utilization which can be mathematically expressed as follows:

$$L: \min \sum_{k=1}^{K}\sum_{t=1}^{T}\left[\sum_{j=1}^{P}\sum_{i=1}^{n_j} r_{kijt} - r_k^*\right]^2 \tag{39}$$

where:

L	the resource leveling index that indicates the sum of squared differences between period resource usage and average resource usage
r_{kijt}	the partial resource k demand of the activity i from the project j at the period of time t
D	the projects duration
K	the number of resource types
P	the number projects
n_j	the number of tasks in project j
r_k^*	average of resource k per period ($r_k^*=[\sum_{t=1}^{T}\sum_{j=1}^{P}\sum_{i=1}^{n_j} r_{kit}/D$)

The issue of applying Genetic Algorithm is to select an appropriate form of the chromosome representation. In resource leveling problem, the well-appropriate form is the one considering the task starting times as decision variables being coded as genes values. Thus, the sequence of the tasks in the chromosome corresponds to the sequence of tasks project by project sorted by their Id number. Each gene value is equal to a possible starting time of corresponding task (Fig. 16). The starting time of each task T_{ij} is chosen randomly in its domain rate respecting precedence constraints.

The fitness function needed to evaluate chromosomes is the resource leveling index L defined in (39). The adopted selection technique is the roulette wheel method that we combine with Elitist method (Goldberg, 1989) in order to improve selection efficiency. Thus, the selection probability for a chromosome k is proportional to the ratio $f_k/\sum_{j=1}^{n_{pop}} f_j$, where f_k is the fitness value of the chromosome k and n_{pop} is the population size. According to the Elitist method, the best chromosomes of the current generation are kept and preserved into the next generation.

The GA operators are uniform 1-point crossover and uniform mutation. Table 2 presents an example of multi-projects that will be considered afterward to show the different GA operators.

The crossover starts with randomly selecting a cut point and parent's chromosomes. The right parts of the chromosomes are swapped and hence children are generated (Fig. 17).

Some children generated in this way do not satisfy precedence constraints. To deal with this situation, a reparation technique is applied (Fig. 18).

Let k be the one-cut-point value and task T_{ij} the corresponding task of gene k. All the gene values of the successors of k must be checked to deal with precedence constraints. Hence, task $k+1$ is the first task to be checked if it is part of project j, otherwise no repair is needed. The repairing formula is as follows:

$$S_{lj} = \max\left(S_{lj}, \max_{p\in pred(T_{lj})}(S_{pj}+D_{pj})\right) \quad \forall l\in[i+1,n] \tag{40}$$

where:

$pred(T_{lj})$	the set of predecessors of task T_{lj}
D_{pj}	the duration of the task T_{pj}

Table 2
Small multi-projects example.

Task	Duration	Predecessors
T_{11}	3	–
T_{12}	1	–
T_{13}	3	–
T_{21}	2	T_{11}
T_{22}	6	T_{12}
T_{23}	3	T_{13}
T_{31}	3	T_{11}
T_{32}	2	–
T_{33}	2	T_{13}
T_{42}	1	T_{22}

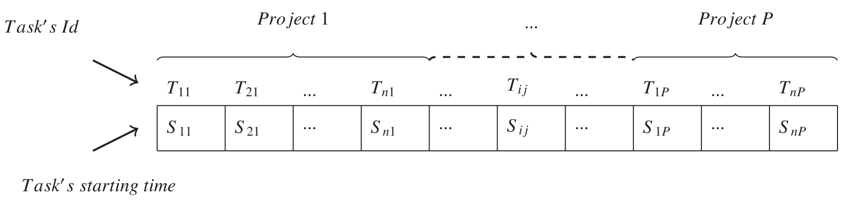

Fig. 16. Chromosome representation in multi-project resource leveling

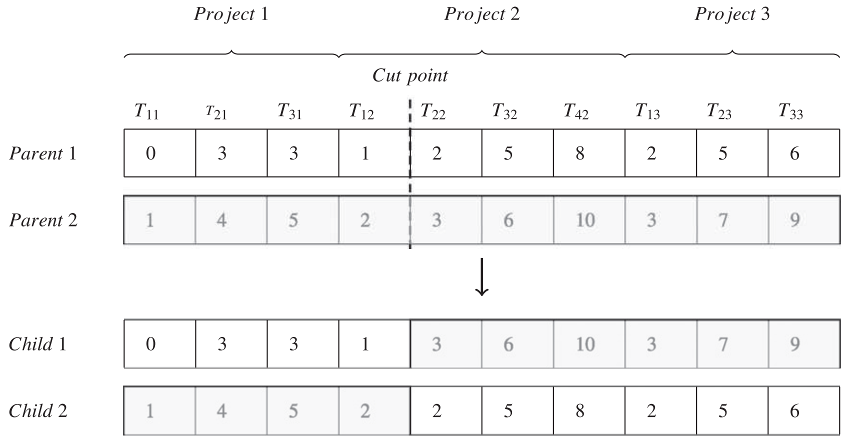

Fig. 17. Uniform 1-point crossover.

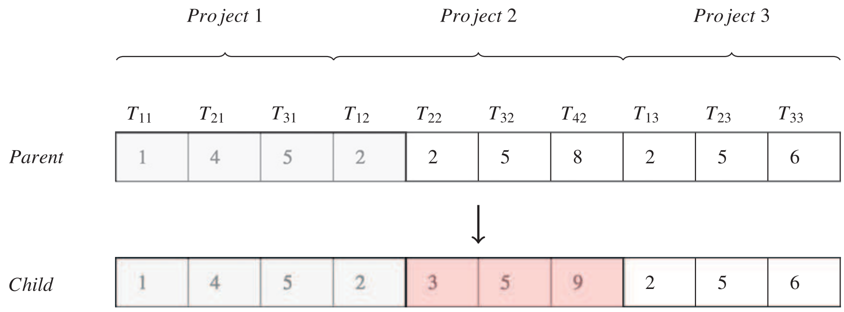

Fig. 18. Reparation after crossover.

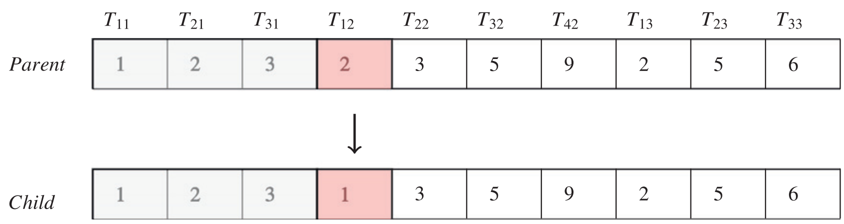

Fig. 19. Uniform mutation.

The mutation consists of randomly replacing at least one gene with a random value within the range of the corresponding task's starting time (Fig. 19).

Let k be a selected gene to mutate and task T_{ij} its associated task. The new value of the gene is chosen randomly between the maximum finishing time of predecessor tasks ($\max_{p \in pred(T_{ij})}(S_{pj}+D_{pj})$) and the minimum starting time of successor tasks ($\min_{p \in succ(T_{ij})}(S_{pj})$) minus D_{ij}, duration of T_{ij}.

6.2. GA generalization for fuzzy resource leveling

Resource Leveling technique for Fuzzy Scheduling Problem is studied in some recent papers (Zhao et al., 2006; Leu et al., 1999) where genetic algorithm is adapted to projects with fuzzy time parameters. The idea in these papers is to make different α-cuts on tasks' durations to obtain pessimistic and optimistic scenarios for each α-cut, and then apply deterministic Genetic Algorithm to each scenario to find the corresponding best plan.

In this section, a new vision of fuzzy resource leveling is provided. The Genetic Algorithm developed in Section 6.1 copes well with deterministic multi-projects and multi-resources scheduling problems. To be generalized to handle fuzzy parameters, some useful hypothesis and extensions are suggested, where the main idea is to make just one couple of fuzzy Genetic Algorithm instead of numerous deterministic ones.

A trapezoidal fuzzy number is numerically represented by four deterministic values. Genetic algorithm becomes very heavy in computation when considering four numbers for each fuzzy decision variable. To deal with this problem only one value is considered and then the encoding and decoding of each solution (chromosome) is done according to the principle of linearity that is explained below.

Let $\tilde{ES}_{ij}=[es_1,es_2,es_3,es_4]$ be the earliest starting time and $\tilde{LS}_{ij}=[ls_1,ls_2,ls_3,ls_4]$ be the latest starting time of task T_{ij}. To generate a possible starting time $\tilde{S}_{ij}=[s_1,s_2,s_3,s_4]$, we choose randomly a value of s_4 between es_4 and ls_4. Let $\beta=(s_4-ls_4)/(es_4-ls_4)$. Thus, $\tilde{S}_{ij}$ is simply calculated according to the principle of linearity within $s_i=\beta es_i+(1-\beta)ls_i\ \forall i\in\{1,2,3,4\}$. In Fig. 20, four examples of possible starting times are shown; $\tilde{ES}$ with $\beta=1$, $\tilde{S1}$ with $\beta=2/3$, $\tilde{S2}$ with $\beta=1/3$ and $\tilde{LS}$ with $\beta=0$.

Some algorithms in Fortin et al. (2005) are provided to calculate fuzzy latest starting times and fuzzy total floats. However, no algorithms are provided in the same framework to calculate fuzzy latest finishing times. As these parameters are necessary for our

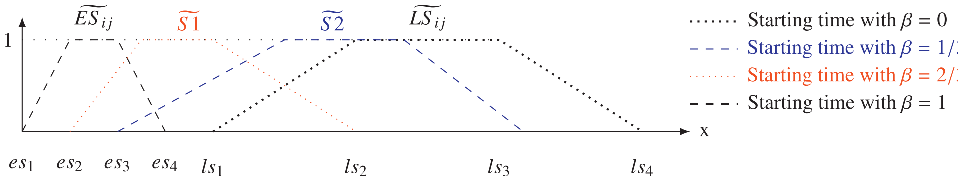

Fig. 20. Linearity.

study, the following formula is provided to calculate them:

$$\tilde{LF}_{ij} = \min(\tilde{LS}_{ij} + \tilde{D}_{ij}, \min(\tilde{LS}_{succ(ij)}), \tilde{Dd}(j)) \tag{41}$$

where:

$\tilde{LF}_{ij}$ the fuzzy latest finishing time of task T_{ij}
$\tilde{Dd}_j$ the fuzzy due-date of the project j

As latest starting times are calculated within the consideration of extreme configuration as explained in Dubois et al. (2003), the value of $\tilde{LS}_{ij} + \tilde{D}_{ij}$ can exceed the range domain of $\tilde{LF}_{ij}$. In fact, the duration $\tilde{D}_{ij}$ of task T_{ij} is not necessarily totally in the range of the extreme configurations provided by the forward propagation. Thus, Eq. (41) provides meaningful computable results respecting precedence constraints. Considering the same explanation, the finishing time is calculated as follows:

$$\tilde{F}_{ij} = \min(\tilde{S}_{ij} + \tilde{D}_{ij}, \tilde{LF}_{ij}) \tag{42}$$

Once starting and finishing times are calculated for each task, fuzzy workload is established as explained in Section 4. Symmetric distributions are considered because tasks are not necessarily critical i.e. task *B* is a successor of task *A*, but *B* does not start exactly at the end of *A*. The concept of *possible and necessary criticality* is explained in Chanas et al. (2002).

For each solution (chromosome), the corresponding fuzzy fitness $\tilde{L}$ is calculated as follows:

$$\tilde{L} = \min \sum_{k=1}^{K} \sum_{t=1}^{T} \left[\sum_{j=1}^{P} \sum_{i=1}^{n_j} \tilde{r}_{kijt} - \tilde{r}_k^* \right]^2 \tag{43}$$

with $\tilde{r}_k^* = [\sum_{t=1}^{T} \sum_{j=1}^{P} \sum_{i=1}^{n_j} \tilde{r}_{kit}]/\tilde{D}$

Many defuzzification techniques are provided in literature (Fortemps, 1997; Dubois and Prade, 1987) to cope with fuzzy rules particularly while using Genetic Algorithm (Sánchez et al., 2009). We can consider the extreme durations *w* or *z* to get the corresponding optimistic and pessimistic workload plans. Moreover, we can convert the continuous workload plan into a periodic workload plan, and apply the robustness functions defined in Masmoudi et al. (2011c). In this paper, we solve the problem after applying the defuzzification technique of Dubois and Prade (1987). $\tilde{D}$ is always projected to the maximum value of the projects duration.

Leu et al. (1999) consider a fuzzy profile to represent the uncertain activity duration and employ also Genetic Algorithm and fuzzy set theory to develop a resource leveling model under uncertainty. However, they apply different alpha-cuts (called acceptable risk levels) on all activity durations and keep for each alpha-level the two deterministic problems corresponding to all lower (optimistic) and all upper (pessimistic) bounds. Then, for each deterministic problem, they apply deterministic CPM techniques to get the margin of each activity and apply a deterministic GA-based approach to solve the problem. Finally, for each alpha in (0,1] they get a solution for the two corresponding deterministic (pessimistic and optimistic) problems. On the contrary, we apply a generalization of the Pert technique per interval provided by Boctor (1990) to fuzzy activities durations to get the fuzzy times. Then based on the fuzzy modelling of resource usage provided in Section 4, we proposed a complete fuzzy Genetic Algorithm procedure to generate only one fuzzy solution instead of multiple deterministic solutions.

The two algorithms described in Sections 5 and 6 are basically a generalization to fuzzy area of existing deterministic algorithms such as the Parallel SGS of Kolish and Hartmann (1999) and the Genetic Algorithm for RLP of Leu et al. (2000). In this paper, we have added a layer of specific treatments to these algorithms to support the new fuzzy modelling of resource workload provided in Section 4. An application to helicopter maintenance projects is presented in the next section.

7. Application to helicopter maintenance

Uncertainty affecting the scheduling problem in MROs can be managed by a fuzzy set modelling of tasks' dates and durations based on expert knowledge. For the following, we adopt 4-point trapezoidal number for each uncertain duration and consider several checks to carry out on components from PUMA helicopter:

- Main rotor: The work is carried out by 1 expert during 35–70 h.
- Propeller: The work is carried out by 1 expert during 70–105 h.
- Hydraulic system: The work is carried out by 1–2 experts during 18–35 h.

Each Component Check can be considered as a small project containing several tasks subject to precedence constraints. The MRO's resources (technicians and equipments) are limited, and thus will be shared by all projects. We consider that the technicians have the necessary qualifications to inspect the different components. Hence, the problem is to schedule small projects respecting both precedence constraints and workshop resource constraints.

For each task *j*, we need to transform the work content p_j into a duration D_j based on 35-h working week and the number of operators n_j assigned to *j*: $D_j = p_j/(35*n_j)$.

Table 3 contains the instance data on which we will apply our algorithms. Fig. 21 shows the earliest workload plan without consideration of resource constraints. Dealing with resources consideration, additional decisions on MRO's capacities limit and Projects due dates will be specified before the application of the Parallel SGS and the Genetic Algorithm, respectively. As notified before, the defuzzification formula that we have considered is the mean average provided in Dubois and Prade (1987). By applying other defuzzification functions, we get different results. Finding the best defuzzification technique for our application would be interesting, but, this is out of scope of our study.

For the resource scheduling, we consider the case where three operators are available at one time, only one test bench, one non-destructive testing equipment, and one cleaning machine exist in the workshop. We apply the Parallel SGS with the consideration of the aforementioned priority rules and the best result is provided by the *LPRW* rule. Fig. 22 shows the result.

For the Genetic Algorithm, we considered the due date of the three projects equal to 10 days. The values of the GA are chosen as follows:

- n_{pop}: population size ($n_{pop} = 60$).

Table 3
Real mechanical tasks from a PUMA HMV.

Part name	Tasks Id	Id	Pred.	Experts	Equipments	Duration (days)
Main rotor	Put off muff	1	–	1	–	[0.5, 0.7, 1, 1.5]
	Put off bearings	2	1	1	–	[1, 1.2, 1.4, 1.6]
	Put off flexible components	3	–	1	–	[0.1, 0.13, 0.17, 0.2]
	Clean	4	2-3	1	Cleaning machine	[1, 1.2, 1.4, 1.5]
	Non-destructive test	5	4	1	Testing equipment	[0.2, 0.3, 0.5, 0.6]
	Assemble components	6	5	1	–	[1, 1.2, 1.4, 1.5]
	Check water-tightness	7	6	1	–	[0.2, 0.3, 0.4, 0.5]
	Touch up paint	8	7	1	–	[0.1, 0.13, 0.17, 0.2]
	Tight screws	9	8	1	–	[0.3, 0.5, 0.6, 0.7]
Propeller	Put off axial compressor	10	–	1	–	[1.2, 1.5, 1.8, 2]
	Put off centrifugal compressor	11	10	1	–	[1.5, 1.6, 1.8, 2]
	Purchase	12	10	0	–	[0, 1, 2, 4]
	Put off turbine	13	–	1	–	[0.5, 0.7, 0.8, 1]
	Clean	14	11–13	1	Cleaning machine	[0.2, 0.4, 0.5, 0.6]
	Non-destructive test	15	14	1	Testing equipment	[0.2, 0.3, 0.4, 0.5]
	Assemble components	16	12–15	1	–	[2, 2.2, 2.8, 3.2]
	Touch up paint	17	16	1	–	[0.1, 0.13, 0.16, 0.2]
	Tight screws	18	17	1	–	[0.12, 0.17, 0.2, 0.3]
	Test	19	18	1	Test Bench	[0.12,0.17,0.2,0.23]
Hydraulic system	Evacuate oil	20	–	2	–	[0.1, 0.13, 0.16, 0.2]
	Put off servos	21	20	2	–	[0.6, 0.7, 0.8, 1]
	Clean	22	21	1	Cleaning machine	[0.2, 0.3, 0.4, 0.6]
	Non-destructive test	23	22	1	Testing equipment	[0.2, 0.3, 0.4, 0.6]
	Assemble then remove joints	24	23	2	–	[0.8, 1, 1.2, 1.4]
	Test	25	24	1	Test Bench	[0.1, 0.13, 0.16, 0.2]
	Tight screws	26	25	2	–	[0.1, 0.13, 0.16, 0.2]

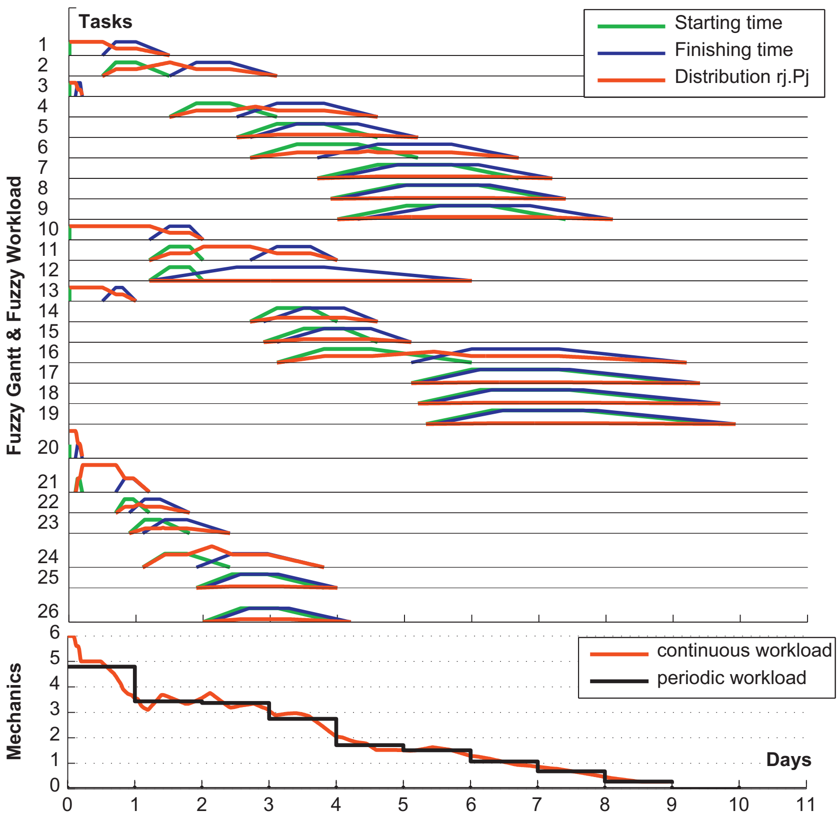

Fig. 21. Earliest workload plan.

- m_n: the best candidates to keep ($m_n = \max(2, n_{pop}/20)$).
- m_k: number of candidate to crossover ($m_k = 2* round(2*(n_{pop} - m_n)/5)$).
- m_d: number of candidate to mutate ($d = round(3*(n_{pop} - m_n)/5)$).
- $gmut$: number of genes to mutate by candidate ($gmut = \min(2, round(n/10))$).
- n_{iter}: number of iterations ($n_{iter} = 14$).
- n_{stop}: stop algorithm condition (with $n_{stop} = 5$, if the result is the same for five successive iterations then stop algorithm).

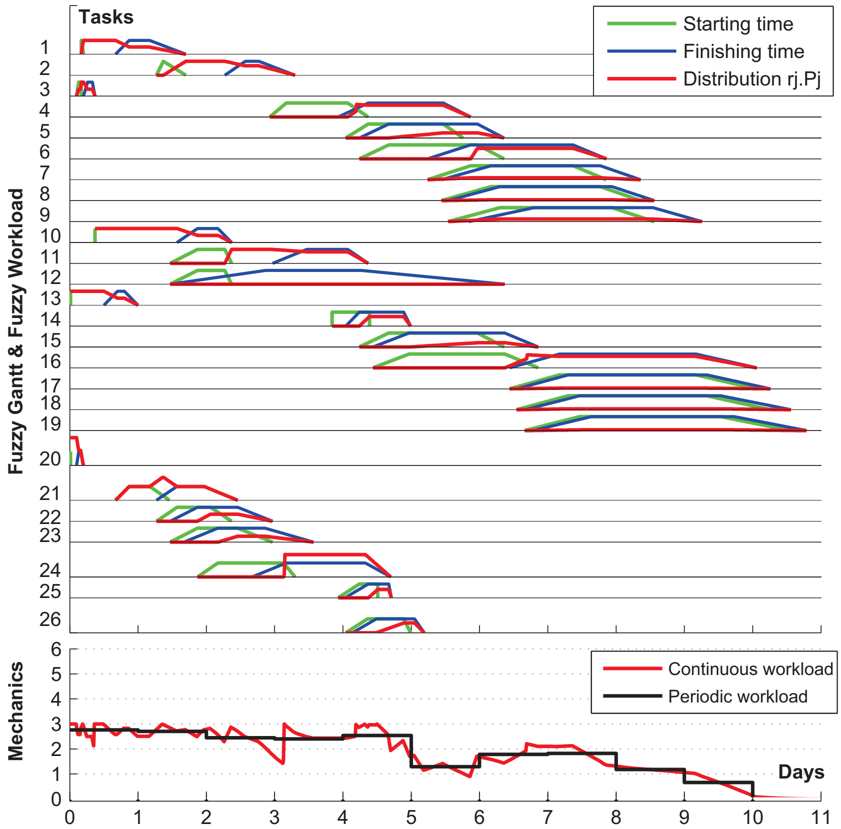

Fig. 22. The Gantt and the workload plan: result of the parallel SGS schedule (LRPW rule).

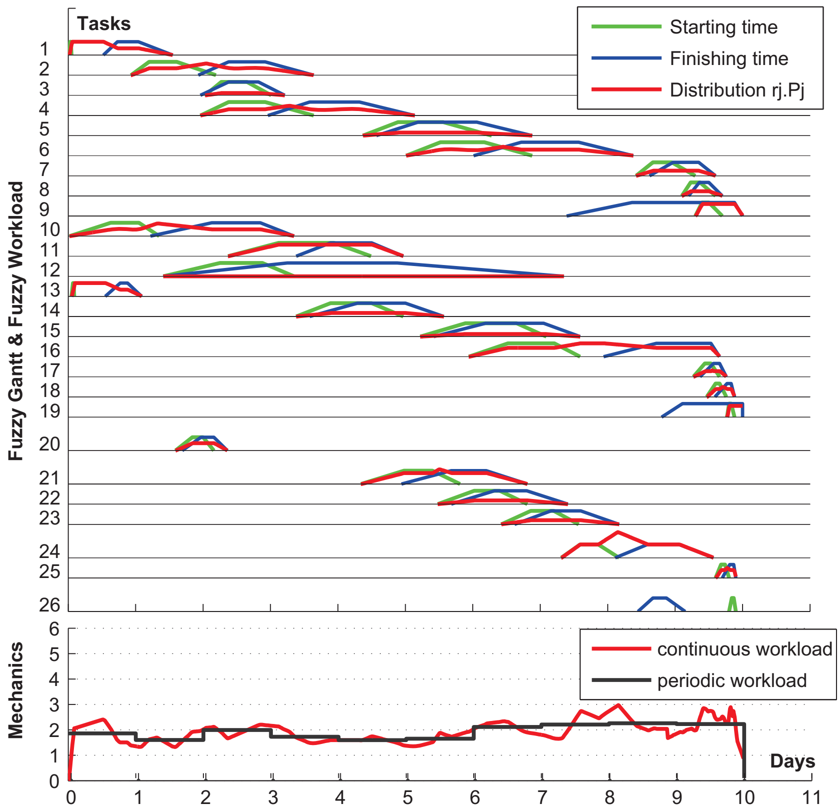

Fig. 23. The Gantt and the workload plan: result of the GA.

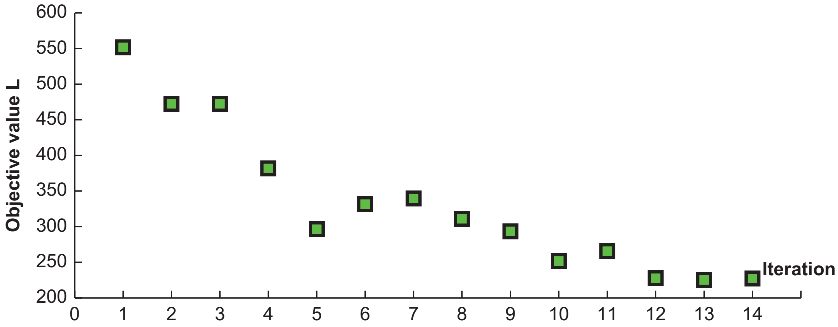

Fig. 24. Convergence of the GA.

Figs. 23 and 24 show the result and the convergence of the GA, respectively.

8. Conclusion

In this paper, we have presented a fuzzy model for project scheduling problems. A method to establish a resource workload is proposed for both tactical and operational levels of planning. Provided models are applied to the helicopter maintenance domain. Based on these modelling approaches, some recent papers provide a generalization of several scheduling heuristics to handle fuzzy parameters; a Genetic Algorithm is generalized to solve Fuzzy Resource Levelling problem (Masmoudi and Haït, 2011b) and a Parallel SGS is generalized to solve Fuzzy RCSPS problem (Masmoudi and Haït, 2011a). These two techniques can be applied simultaneously within a decisional loop handling projects due dates and production capacity simultaneously i.e. we can increase/decrease a project due date and apply resource leveling technique or increase/decrease the production capacity and apply Resource scheduling technique (Kim et al., 2005a). Future work will focus on applying such technique and dealing with the complexity of different possible fuzzy profiles (rectangular, triangular, exponential, etc.). The comparison of our fuzzy approaches (models and solving techniques) to existing stochastic ones is under study. The afore developed fuzzy techniques will be included into a Decisional Support System to manage a Maintenance Repair and Overhaul center.

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