To link to this article: DOI: 10.1007/s11630-012-0575-9
URL: http://dx.doi.org/10.1007/s11630-012-0575-9

To cite this version: Carbonneau, Xavier and Binder, Nicolas *Performance Back-deduction from a Loading to Flow Coefficient Map: Application to Radial Turbine.* (2012) Journal of Thermal Science, vol. 21 (n° 6). pp. 509-517. ISSN 1003-2169

Performance back-deduction from a loading to flow coefficient map: application to radial turbine

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Abstract

Radial turbine stages are often used for applications requiring off-design operation, as turbocharging for instance. The off-design ability of such stages is commonly analyzed through the traditional turbine map, plotting the reduced mass-flow against the pressure-ratio, for reduced-speed lines. However, some alternatives are possible, such as the flow-coefficient (ψ) to loading-coefficient (ϕ) diagram where the pressure-ratio lines are actually straight lines, very convenient property to perform prediction. A robust method re-creating this map from a predicted ψ–ϕ diagram is needed. Recent work has shown that this back-deduction quality, without the use of any loss models, depends on the knowledge of an intermediate pressure-ratio. A modelization of this parameter is then proposed. The comparison with both experimental and CFD results is presented, with quite good agreement for mass flow rate and rotational speed, for the intermediate pressure ratio.

The last part of the paper is dedicated to the application of the intermediate pressure-ratio knowledge to the improvement of the deduction of the pressure ratio lines in the ψ–ϕ diagram. Beside this improvement, the back-deduction method of the classical map is structured, applied and evaluated.

Keywords

Radial turbine, off-design, flow coefficient, load coefficient, performance

Nomenclature

Diameter (mm)	D
Enthalpy (J/kg)	H
Rotational speed (rpm)	N
Mass flow rate (kg/s)	Q
Specific gas constant (J/kg/K)	R
Stator section (mm^2)	S_3
Rotor section (mm^2)	S_5
Temperature (K)	T
Linear rotational speed (m/s)	U
Absolute flow velocity	V
Relative flow velocity	W
Exit pressure ratio	P_{t5}/P_5

Subscripts

Stage inlet	1	Stator inlet	2
Stator throat	3	Rotor inlet	4
Rotor outlet	5		

Absolute flow angle (degree)	α
Relative flow angle (degree)	β
Specific heats ratio	γ
Rotor static expansion ratio	ξ
Flow coefficient	ϕ
Loading coefficient	ψ
Total-total efficiency	η_{tt}
Total-static efficiency	η_{ts}
Total-total pressure ratio	π_{tt}
Total-total pressure ratio	π_{ts}
Total reference	t
Static reference	s
Tangential projection	θ
Corrected value	cor
Reduced value	*

Introduction

In turbomachinery, the mapping of stages allows not only the evaluation of the performance, but also the characterization of the off-design behavior. This characterization is challenging to predict since complex phenomenon appear in different location of the stage when the inlet conditions differ from the design point. However, for radial turbines, [1] states that a one-dimensional approach is accurate enough to predict the off-design behavior with acceptable accuracy. Actually, most of the prediction tools for radial turbines are built around a one-dimension treatment of the elementary thermodynamic equations. These equations are expressed at the different intermediate planes of the stage. Basically, it leads to the application of the fundamental conservative principles of the physics (mass conservation, momentum conservation, first and second principles of the thermodynamics...) at the different interfaces found between the elements of the stage (volute inlet, stator inlet, rotor inlet, rotor outlet...). Such processes are found in [1], [2] or [3], where a detailed description of the equations is proposed. The process in itself generally leads to physically sound results, but since the real situation in the machine generates complex flows, some complement of information is needed to reach the required accuracy. That's why the "1D-backbone" is improved with specific models. These models are essential because the actual complexity of the flow (tri-dimensional, turbulent, involving separation...) is obviously above a mere one-dimensional expression of the problem for inviscid flows. The viscosity influence producing this complexity is thus treated separately as divergence from the one-dimensional expectation, and for which the generic expression is "secondary flows". As a synthesis of all the contributions found in the literature, it can be said that the interference with the initial model is categorized in three families of corrections: the angular deviation, the blockage and the losses. The angular deviation is useful at the trailing edge of stator/rotor blades, since the flow does not follow strictly the direction imposed by the geometry. This has some importance for the angle of attack of the following element, or for the application of the Euler's theorem. The blockage expresses the boundary layer displacement thickness influence, which modifies the effective value of cross section areas in the flow path. The losses are generally total pressure losses, which accounts for the entropy rise due to irreversible phenomenon, such as frictions at the wall. A lot of corrections can be found in the literature, for example in [4] where numerous models are presented, or in [5] for which deviation and losses are proposed and implemented (for instance [6] and [7]). However, in each of these cases, the mapping envisaged is the usual plot of mass-flow versus pressure-ratio, and efficiency for speed-lines. Recent work has shown that another choice for the axis can be convenient in the scope of prediction. In [8] is stated the fact that pressure-ratio lines are straight lines in a map, thus very easy to interpolate or extrapolate. Two problems were then identified. First, a displacement of the intercept point of the expected lines was observed, which limits the prediction potential of the method. This aspect has been worked on and presented in [9], stating the necessity to take into account the variation of an intermediate pressure-ratio, for a global pressure-ratio line. Second, the back deduction of the usual map from the alternative map, to build the classical representation of the stage behavior, was difficult to obtain for the case of non-chocked functioning. This study focuses on this second point. A basic resolution of the equations allows the segregation of the mass-flow and of the rotational-speed, necessary to process the back deduction of the initial map. This process, which could be the first step of the development of a prediction code, is applied in the field of interpolation, in order to deduce the value of certain regions of the map difficult to measure, and to reduce the number of test necessary to have a good description in the mapping. The first part of the paper presents the theoretical background of the approach. The second part describes the iterative process bases on a 1-D description of the flow, which is the core of the back-deduction method. The last two parts of the paper evaluates the iterative process and the back-deduction method, through the comparison with CFD and experimental data.

Theoretical background

Through the development of the usual Euler equation[1],

$$\Delta H = U_4 V_{\theta 4} - U_5 (U_5 - W_{\theta 5})$$

a specific expression for the pressure-ratio lines is found if expressed in a flow-coefficient to loading-coefficient map. All the details of the approach, the hypotheses and the calculations can be found in [8]. It leads to the following relation,

[1] The outlet radius considered in this expression is a critical choice. In the literature, the "rms" radius is usually chosen. From a theoretical point of view, the proper choice is the mass averaged expression of the tangential momentum of the flow. This is discussed in a following section of the paper.

$$\psi = -\left(\frac{D_5}{D_4}\right)^2 + \phi.\frac{\pi}{4}\left\{\left[\xi.\pi_{ts}\right]^{\frac{1}{\gamma}}\left(S_3^*\right)^{-1} + \left[\frac{P_{t5}}{P_5}\right]^{\frac{1-\gamma}{\gamma}}\frac{D_5}{D_4}\left(S_5^*\right)^{-1}\pi_{ts}\left[1-\eta_{ts}\left[1-\left(\frac{1}{\pi_{ts}}\right)^{\frac{\gamma-1}{\gamma}}\right]\right]\right\} \quad \textbf{(1)}$$

where

$$\psi = \frac{\Delta H}{U_4^2} \quad \textbf{(2)}$$

and

$$\phi = \frac{rT_{t1}Q}{U_4 P_{t1}(\pi/4)D_4^2}$$

It means that the ϕ-ψ map is a very convenient representation of the stage behavior since the pressure-ratio lines are straight lines. The different features of the two maps are presented in Figure 1 & Figure 2. The building of this alternative map from the usual map is very easy, but not very useful. At the contrary, the ϕ-ψ map is convenient for some prediction, some interpolation processes or for analyzing the off-design functioning. But the back-deduction from this map to the classical one is not obvious since the four parameters (reduced mass-flow, reduced rotational-speed, pressure-ratio and efficiency) of the usual map must be back-deduced from the only three parameters (flow-coefficient, loading-coefficient, and pressure ratio) of the alternative one. When the blockage is reached, the mass flow gets constant and a simple back-deduction process is accessible. But for "subsonic" situations the process is definitely more complex.

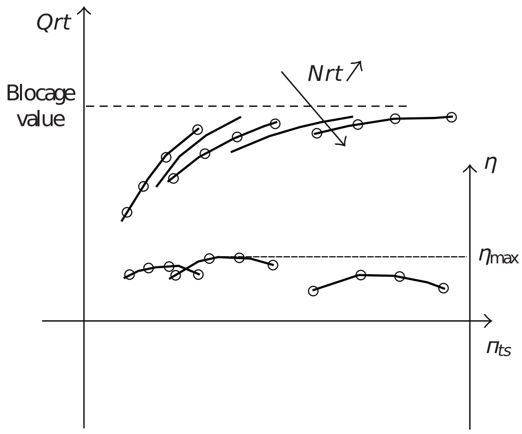

Figure 1: Usual turbine map (mass-flow and efficiency against pressure-ratio for speed-lines).

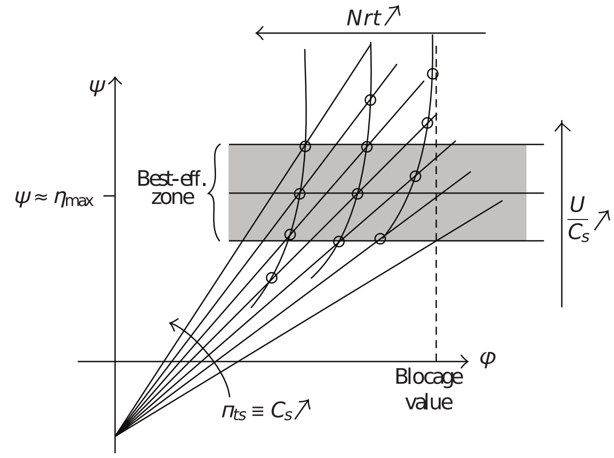

Figure 2: Alternative turbine map (loading-coefficient against flow coefficient for pressure-ratio lines).

Let's examine the relative variation of the different parameters involved in the stage functioning. The first step of this analysis is located in the usual map reference. It shows the well-known behavior of the stage: roughly like a convergent nozzle. But the effect of the rotational speed is so that the speed-lines do not collapse exactly on the same trend. For a given pressure ratio, there is a large number of possible pairs of [*Qrt* ; *Nrt*]. Before blockage, at fixed pressure-ratio, when *Nrt* increases, *Qrt* decreases. After the blockage, *Qrt* is constant whatever the variation of *Nrt*.

Let's now switch in the ϕ-ψ reference. The link between ϕ and the usual parameters is recalled in *Eq (3)*.

$$\phi = \frac{240}{\pi^2}\frac{Qrt}{Nrt} \quad \textbf{(3)}$$

This expression is at the very core of the approach proposed, because it means that whatever the situation (blockage or not), if the rotational speed of the machine (*Nrt*) increases at given pressure-ratio, then ϕ decreases. It can be observed on the Figure 2 where the speed-lines are plotted. As a consequence, for a given value of the pressure-ratio, and a given value of *Nrt*, only one value of ϕ is possible. Rephrasing this, we obtain the following assessment:

For a given map (i.e. a given stage geometric configuration), if π_{ts} *and* ϕ *are fixed, there is only one possible pair for [Qrt ; Nrt]. This pair depends only on the geometry of the stage.*

The knowledge of this pair closes the back-deduction process since π_{ts} , *Qrt* and *Nrt* are known. From the values of π_{ts} and ϕ, one can deduce very easily ψ in the ϕ-ψ map. The definition of ψ (eq 2) gives access to the efficiency value. Our problem is thus to obtain the pair [*Qrt ; Nrt*] , corresponding to a fixed pair of [π_{ts} ; ϕ]. Since this relation depends on the geometry of the stage, a deduction using basic information of the geometry should be possible. This approach is developed in this paper. An iterative process based on physical considerations has been developed in order to deduce [*Qrt ; Nrt*], from [π_{ts} ; ϕ] with the adjunction of geometric characteristics of the stage. This process is now presented.

Physical basics of the iterative process

The iterative process is based on a very simple 1D-description of the stage. In fact, both the stator and the rotor are treated as convergent nozzles, one in the absolute frame, and the other one on the relative frame. Only the inlet/outlet planes of those nozzles are involved in the process.
As usual when dealing with those kinds of model, some restrictive hypotheses are taken. The most important are recalled here:

- 1D, compressible and inviscid flows of perfect gas;
- Isentropic evolution inside the nozzles;
- Heat transfers neglected;
- Between rotor and stator, the angular momentum is considered constant, and the density variation is neglected;
- In the rotor, the flow can be divided in layers along the blade span in the meridian plane.

Obviously, it is not possible to predict an accurate performance of a given stage with those hypotheses without implementing some losses, deviation and blockage models. But the purpose of our iterative process is not to predict efficiency, but to find a correspondence between two pairs of variables.
Also, the process must be adapted to both on-design and off-design situations. Thus, our method needs a formulation for which none of the usual "on-design" simplification is required (such as neglecting the rotor-outlet gyration or choosing the value of the rotor inlet incidence; actually, those information will be provided by the iterative process). The fundamentals of such a process are now detailed.

Fundamentals

The position of an operating point in the flow characteristic of a given convergent nozzle is a function of only one parameter: the nozzle total-to-static pressure ratio. The initial pair of parameters of our process is [π_{ts} ; ϕ]. It means that the global pressure-ratio of the stage is known: the total pressure at inlet of the first nozzle (stator) is defined, and the static pressure at the outlet of the second nozzle (rotor) is set. But these two nozzles are not independent:

1. The outlet conditions of the first nozzle and the inlet condition of the second nozzle are linked through the passage from the absolute frame to the relative one;
2. The mass-flow rate is equal in the two nozzles.

Each operating point of the map is in fact the result of a matching between the two nozzles, in order to meet the two requirements mentioned above. The importance of the intermediate pressure-ratio in that process is fundamental. It immediately sets the position in the flow characteristic of the first nozzle. Together with the rotational speed, and the stator outlet conditions, it defines the total inlet conditions of the second nozzle. On Figure 3 some experimental results are displayed (see [10]): the evolution of the static pressure at the shroud of the stage in the meridian plane is plotted. The pressure-ratio is fixed, and different values of the rotational speed are considered. The relation between the rotational speed evolution, and the value of the pressure between rotor and stator is highlighted. One value of the rotational speed is associated with one value of the intermediate pressure. When the rotational speed is very small, and the pressure-ratio fixed, a very small amount of the expansion is provided by the rotor: the stator provide most of the expansion and the intermediate pressure is very low (let's imagine the extreme case for which there is no rotation of the rotor: the intermediate pressure is almost equal to the outlet pressure of the stage). At the contrary, when the rotational speed is high, most of the expansion occurs in the rotor: the intermediate pressure is also high.

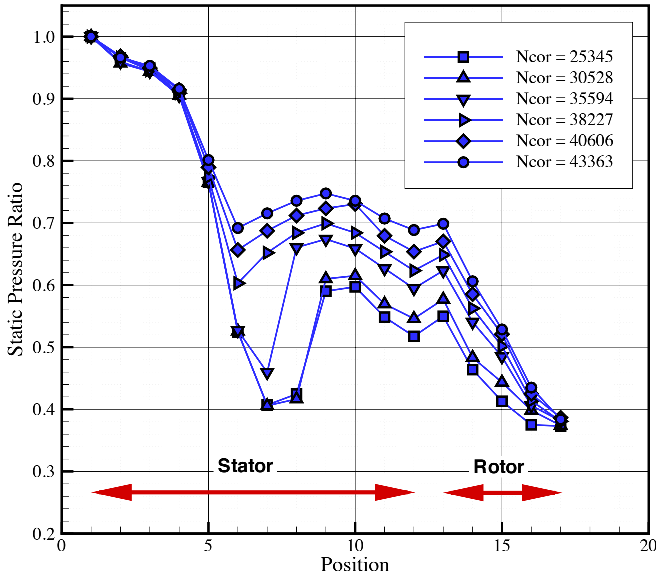

Figure 3: Stage static to static pressure ratio variation - Total to total pressure ratio fixed and equal to 3 – Experiments.

That's why the intermediate pressure, or intermediate pressure-ratio (ξ) in its non-dimensional form, is an excellent candidate to be used as the main iterative parameter. A second iterative parameter is necessary because there are two matching conditions to meet (the rotational speed is not known *a priori*). The rotational speed itself is used in a second loop, thus defining the final process, which is here summarized and illustrated in Figure 4.

For a fixed value of the pressure ratio, and a target value of ϕ :

- Choose an initial value of the rotational speed;
- Iterate on the intermediate pressure-ratio until convergence of the mass-flow between the two nozzles;
- Use the mass-flow and the rotational-speed to calculate ϕ ;
- Compare to the target value, and iterate on the rotational speed until convergence.

The intermediate calculation will not be presented in totality. But since the process implies a specific treatment for each nozzle, and for the interface, some information is now detailed.

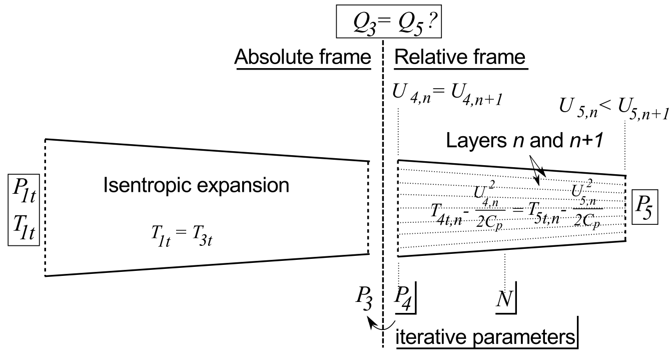

Figure 4: illustration of the iterative process. For a fixed value of pressure-ratio an iteration on the inlet conditions of the rotor allow reaching a specified value of the flow-coefficient.

Specific treatment in the equations

The information needed at the beginning of the process is split into two families:

- *Flow conditions:* at the inlet of the stage, the total conditions (pressure and temperature) are needed. The outlet static pressure is needed to define the pressure ratio. Finally, a target value of ϕ is necessary;
- *Geometry:* the outlet cross-sections of the rotor and stator are needed, together with mechanical angle of the channels. The outlet radius of the stator, and the inlet radius of the rotor must be specified. Finally, at the outlet of the rotor, the radius at tip and at hub is needed for the discretization in layers.

The stator

The case of the stator is treated as an ideal nozzle. The isentropic relations apply between the inlet and the outlet. It means that the total pressure and temperature are conserved, and if the nozzle is convergent, the flow is accelerated. With the iteration on the intermediate pressure ratio, it is very easy to reach, for example, the outlet Mach number of the nozzle using the relation:

$$M_3 = \sqrt{\frac{1}{\gamma - 1}\left[\left(\frac{\tilde{P}_{1t}}{P_3}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} \quad \textbf{(4)}$$

and the mas-flow of the first nozzle is given by:

$$Q_3 = \sqrt{\frac{\gamma}{r}}\frac{P_3}{\sqrt{T_3}}M_3\tilde{S}_3 \quad \textbf{(5)}$$

Once again, no loss, deviation or blockage model has been used to evaluate the process in its most simple expression. But such models could be easily implemented in the calculations. The quantities requiring such a correction are identified with a tilde ~ (P_{1t} and S_3 for instance).

The interface stator/rotor

At the interface, the relation between the two nozzles is created. In a real machine, the flow goes from the stator to the rotor trough the free space. In this region, the flow is also accelerated due to radial effects. It is also a location where the rotor/stator interactions have a lot of importance, but it is beyond the scope of this analysis. The process will thus be considered isentropic. Here the hypothesis of conservation of the angular momentum, together with the conservation of the mass-flow inside the free space is expressed in the two following expressions:

$$D_3V_3\cos(\alpha_3) = D_4V_4\cos(\alpha_4) \quad \textbf{(6)}$$

$$\rho_3V_3\sin(\alpha_3)\pi D_3H_3 = \rho_4V_4\sin(\alpha_4)\pi D_4H_4 \quad \textbf{(7)}$$

Assuming that $H_3 = H_4$, and assuming the variations of density are small (as stated in the general hypotheses), it is then possible to express the acceleration in the free space, and the conservation of the absolute angle of the flow:

$$V_4 = D_3V_3/D_4 \quad \textbf{(8)}$$

$$\alpha_3 = \alpha_4 \quad \textbf{(9)}$$

As a consequence, the static conditions are linked as follow:

$$T_4 = T_3 - \frac{V_3^2}{2C_p}\left(\frac{D_3}{D_4}^2 - 1\right) \text{ and } P_4 = P_3\frac{T_4}{T_3}$$

(10)

Since the iteration is operated through the value of P_4, it is possible to establish a test for blockage at the stator if $P_3 < 0.528\ P_{1t}$.

The rotor

The nozzle in the relative frame presents, as expected, the most complex situation. The inlet conditions of this nozzle are built through the value of the relative velocity, which is obtained by a velocity composition with the rotational speed:

$$W_4 = \sqrt{(V_4\cos\alpha_4)^2 + (\pi D_4N/60 - V_4\sin\alpha_4)^2} \quad \textbf{(11)}$$

With the knowledge of the static conditions established above, the inlet total-pressure and total-temperature are identified.

Anyway, the second nozzle does not behave purely as a perfect convergent nozzle: since the rotational-speed is different at the inlet and at the outlet, its variation must be taken into account in the first principle of thermodynamics. The total temperature in the relative frame (defined through the relative velocity) is no more constant, even if the transformation is still considered isentropic. The proper relation is:

$$T_{4t} - \frac{U_4^2}{2C_p} = T_{5t,n} - \frac{U_{5,n}^2}{2C_p} \quad \textbf{(12)}$$

The problem comes from the choice of a reference outlet diameter: the rotational speed is not constant along the blade span, and some reference position should be defined to express the different equations. No consensus is found in the literature. The assumption of no-swirl at outlet insures the validity of the relations whatever the radius on which it is expressed. Generally, the “rms” radius is chosen. A strict analysis based on the applicability of the Euler theorem leads to the definition of a mean radius with the mass average of the quantity ($r_5V_{\theta5}$), but it is not the most convenient formulation, since it is proper to each operating point. To avoid such approximations, it has been decided to discretize the outlet, and solve the equation in “layers”. A summation of the contribution of the different layers will give access to the different quantities of the flow. In each layer, the total pressure at outlet is given by:

$$P_{5t,n} = P_{4t}\left(\frac{T_{5t,n}}{T_{4t}}\right)^{\frac{\gamma}{\gamma-1}}$$

where *n* is the number of the layer.

The static pressure is fixed at the outlet; it is very easy to end the resolution through the establishment of the outlet Mach number, its projection using the mechanical angle at the outlet, and then the mass flow for the layer considered. The summation of the different layers gives access to the mass flow through the second nozzle, for comparison with that of the first one.

The process has been coded in a FORTRAN program, in its double loop configuration. Its ability to deduce [*Qrt* ; *Nrt*] from [π_{ts} ; ϕ] is now evaluated.

Evaluation of the iterative process

The iterative process outputs are confronted to both experimental and simulation results. The machine chosen for the evaluation is a radial turbine of intermediate size (diam. ≈ *150 mm*; nominal operating point: N ≈ *45 000 rpm* and π_{ts} ≈ 3.0).

The experimental results were published in [10], (including the results presented in Figure 3). For the self-viability of the present paper, we will recall the essential information. The turbine is driven by a supply of pressurized air. The mass flow is measured with a normalized diaphragm (2,5% of precision). A local measurement of the static pressure is performed all along the meridian plane (sensor precision +/- 5 mBars). Total temperature is measured at inlet and outlet by using A-Class platinum probes (+/- 0.3°). The rotational-speed of the turbine is piloted by acting on the inlet/outlet conditions of the compressor connected to it. Six pressure-ratio lines were explored to describe the complete map of the machine.

Some numerical simulations were performed on the same stage, with the Navier-Stokes solver Euranus (FINETurbo package of NUMECA Int.). The results are published in [11], together with an exhaustive presentation of the methodology. Again, the main elements are recalled here. The RANS calculations were performed with the Spalart-Allmaras turbulence model, and the mixing-plane interface treatment. The mesh (1 million cells in the stator and more than 2 million in the rotor) satisfies $y^+<5$ everywhere. The data are extracted in both the rotor and stator in a cut plane located at 1 mm upstream the leading edges, or 1mm downstream the trailing edges. Integral values are obtained by a mass-weighted averaging.

For three values of the pressure ratio (π_{ts} = 2; π_{ts} = 3 and π_{ts} = 3.5), the mass-flow and the rotational speed were back deduced from different values of ϕ (ϕ = *[0.025; 0.03; 0.035…0.065]*) with the iterative process. The results are presented in both Figure 5 and Figure 6 where a comparison with the experimental results is proposed.

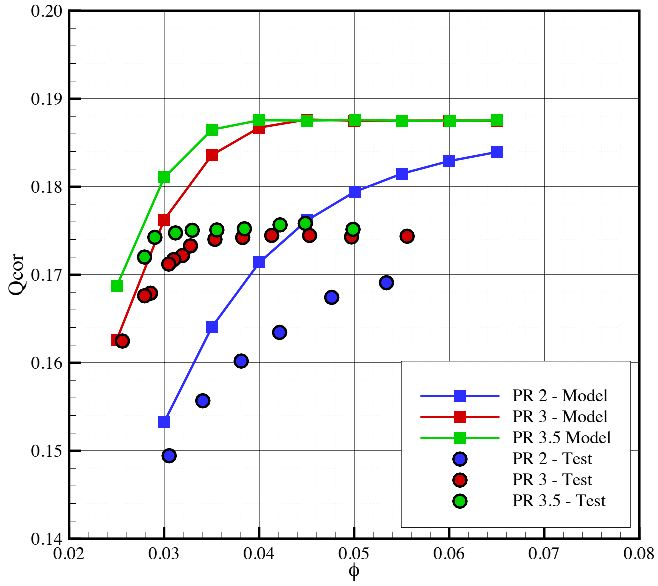

Figure 5: Corrected mass-flow rate variation for pressure-ratios from 2 to 3.5. Comparison between the experimental results and the iterative-process outputs.

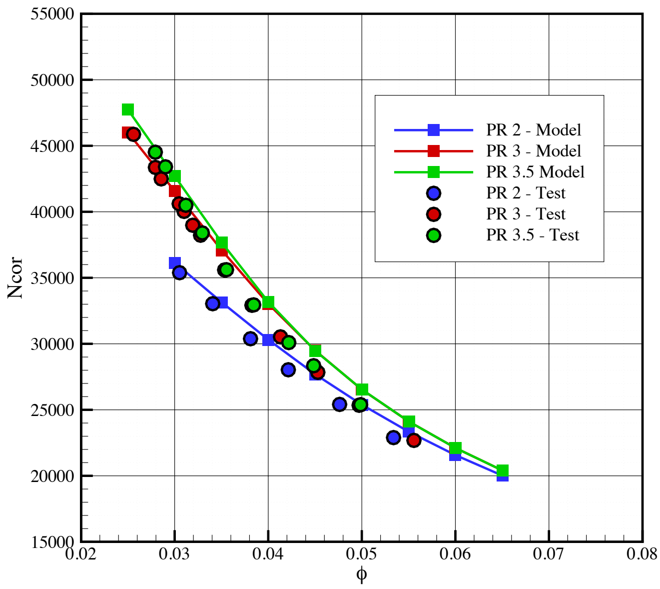

Figure 6: Corrected rotational-speed variation for pressure-ratios from 2 to 3.5. Comparison between the experimental results and the iterative-process outputs.

For the mass-flow evolution, the qualitative behavior of the process is quite satisfying. The general trend is quite similar to that of the experimental data. The blockage is well predicted. From a quantitative point of view, the matching is not perfect. An overestimation of the mass flow reaching 7% of the experimental value is observed. The blockage is predicted at lower flow-coefficient than expected. It can be observed that the discrepancy between experiment and modelization increases with the flow-coefficient value, and more generally once the blockage is reached. To soften this severe statement, the fact that no correction model has been used must be recalled. The overestimation of the mass-flow is not surprising since the geometric information were used without rectification due to blockage. In those conditions, the ability of the process to deduce the value of the mass-flow from the flow coefficient and the pressure ratio is judged surprisingly good.

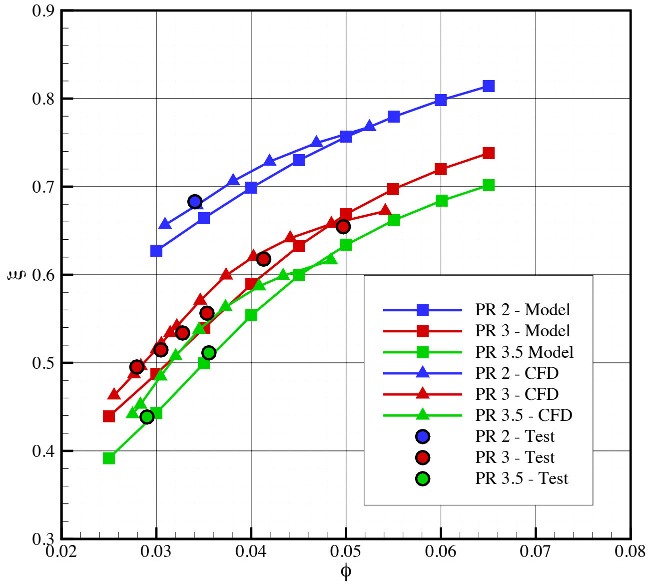

Figure 7: Rotor static expansion ratio evolution vs. flow coefficient for pressure ratios from 2 to 3.5.

For the rotational-speed, the analysis of the comparison between the output of the iterative process and the experimental results lead to the same conclusion. The trend is very well reproduced. Here the discrepancy is even smaller: 3 to 4% of over-estimation of the experimental value.

The Figure 7 presents the evaluation of the iterative process to converge on the good value of the intermediate pressure. Here ξ ($\xi = P_5/P_4$, were P_5 is fixed) is plotted against the value of the flow-coefficient. The iterative process output is confronted to the CFD results, and to the experiment results. The local matching is very satisfying. The gap within all data doesn't exceed 4% and the global evolution is well predicted.

CFD results highlight a curvature of the pressure ratio-line. This tendency is less prominent on the tests results for PR3. The model in a less proportion reproduces this curvature.

Both the global and local results are satisfying, even if the accuracy is far from being sufficient. Anyway, the fact that the physical behavior of the stage (even in off-design conditions) is reliably reproduced by the iterative process, without the implementation of a single correction model, gives full confidence in the initial formulation. This process could be improved very easily, to the extend of becoming a full prediction process. Until then, let's discuss the initial point of this paper: the interpolation of the performance through the back-deduction from a ϕ-ψ map.

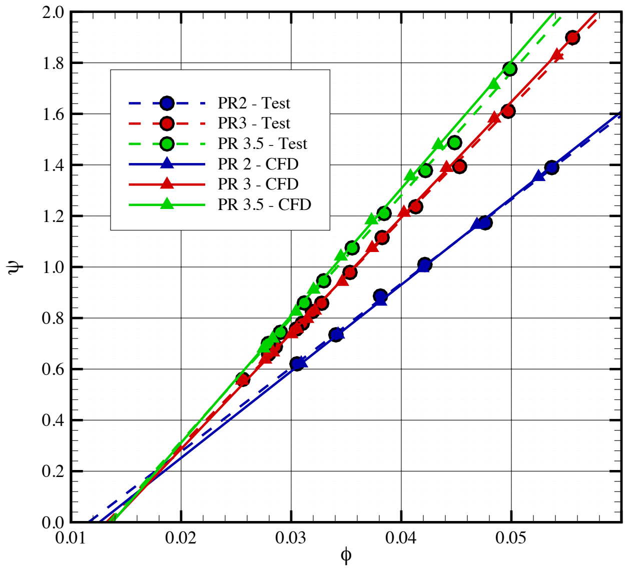

Figure 8: Presentation of the φ-ψ map of the stage, for pressure ratios from 2 to 3.5 (experimental and simulation data).

Interpolation of the performances

The ϕ-ψ map of the stage is presented in the Figure 8 for three pressure-ratio lines ($\pi_{ts} = 2$; $\pi_{ts} = 3$ and $\pi_{ts} = 3.5$). Whatever the origin of the results is (numerical simulation and experiments), the expected linearity of the pressure-ratio lines is demonstrated. The equations of the lines are extracted from this map. The back-deduction process is run to obtain [Qrt ; Nrt], from [π_{ts} ; ϕ] information. Then, using the different equations of the lines and the definition of ψ, the evolution of the efficiency is established. The different results are presented in the Figure 9 and Figure 10.

In the previous section, the evaluation of the iterative process has shown an overestimation of both the mass-flow and the rotational-speed. The overestimation is quite logically reported in the back-deduction process. In a rotational-speed-to-mass-flow map the discrepancy between the predicted and the experimental pressure-ratio lines is still in the range of 7%. The fact that the iterative process does not converge exactly on the good value of the mass-flow and of the rotational-speed

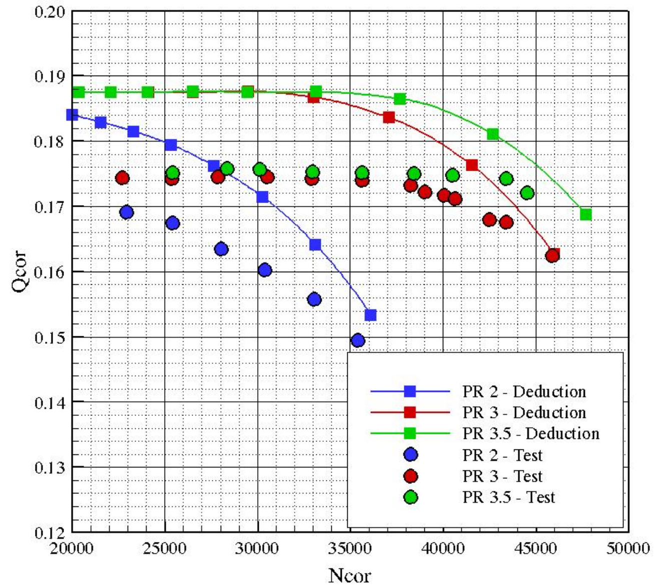

Figure 9: Comparison between the back-deduced and the experimental evolutions of the usual functioning parameters (mass-flow versus rotational speed for pressure-ratio lines).

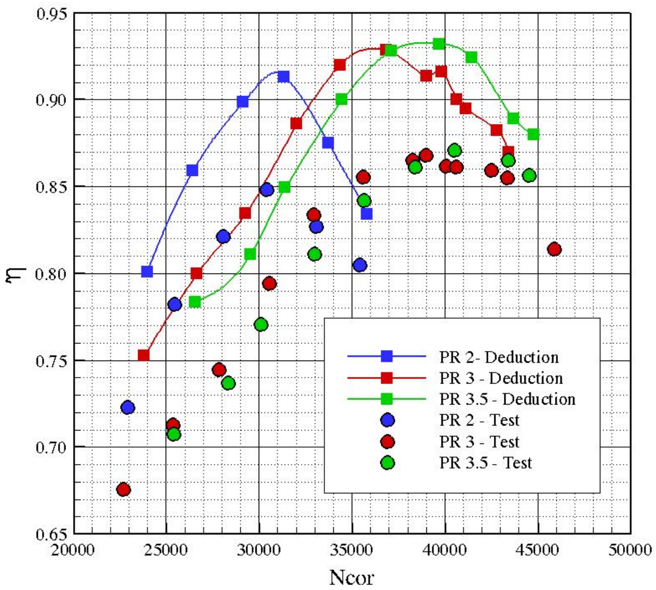

Figure 10: Comparison between the back-deduced and the experimental evolutions of efficiency along three pressure-ratio lines.

wraps the evaluation of the efficiency. An overestimation of the overall performance in the range of 7 points of efficiency is shown. Anyway, the inlet of the interpolation lines in just two elements: the equation of the pressure-ratio line in the ϕ-ψ map, and the iterative process (which means basic geometric information). With just those two elements one can observe that the global trend of efficiency is quite well anticipated. For example, the position of the best-efficiency points is well predicted, even if the value of the peak is clearly no satisfying.

Some more work must be conducted to increase the accuracy of the process, but as it is, we can say that a back-deduction is possible, even in "subsonic" functioning conditions. The final objective, of deducing the totality of the map, from very few experimental or numerical points cannot be achieved precisely. The implementation of some correction model should help us in that objective.

Conclusions

In the field of prediction, interpolation, or modeling of turbine stages, the relevance of the ϕ-ψ map presentation has been recalled. A back-deduction process of the usual map from this alternative diagram has been presented. The construction of this process gave the opportunity to build an iterative process, based on a simplified 1D-approach, able to model the behavior of a stator/rotor stage. The fact that no "on-design" simplification is applied makes the qualitative behavior of the process very similar to that of our reference stage, even in severe off-design conditions. Since no correction to model viscous effects (blockage, losses, deviation…) has been implemented, the quantitative results are not satisfying, even if the range of the discrepancy is judged surprisingly small in regard of the simpleness of the model.

Finally, the complete back-deduction process was applied. The results show a very good agreement in the trends, but still lack some accuracy for real application.
The implementation of correction model should improve the accuracy of the process. Moreover the good prediction of the intermediate pressure-ratio opens some perspectives to make this process a complete prediction tool, for which the only information needed is based on basic geometric description.

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