To link to this article: DOI:10.1016/S0093-6413(02)00262-8
http://dx.doi.org/10.1016/S0093-6413(02)00262-8

To cite this version:
Welemane, Hélène and Cormery, Fabrice *A critical review of some damage models with unilateral effect.* (2002) Mechanics Research Communications, vol. 29 (n° 6). pp. 391-395. ISSN 0093-6413

A critical review of some damage models with unilateral effect

F. Cormery *, H. Welemane

Laboratoire de Mécanique de Lille, Ecole Universitaire D'Ingenieurs de Lille, Boulevard Paul Langevin, URA CNRS 1441, 59655 Villeneuve d'Ascq Cedex, France

Abstract

The concern here is the macroscopic modeling of the brittle damage unilateral effect (due to the opening-closure of microcracks). Several formulations have been proposed in recent years to solve the problems pointed out by Chaboche (Int. J. Damage Mech. 1 (1992) 148). In this paper, we examine precisely two of these new formulations (Int. J. Damage Mech. 2 (1993) 311; Int. J. Damage Mech. 5 (1996) 384) and show that they still exhibit some major inconsistencies.

1. Introduction

The particularities of the mechanical response of quasi-brittle materials such as some rocks, concrete, ceramics have been widely explained by the existence, nucleation and growth of microcracks. The oriented nature of these microdefects, coupled with the unilateral contact of their lips (i.e. microcracks can be either open or closed depending on loading), lead to a complex anisotropic behavior notably characterized by a recovery of some effective properties at the closure of microcracks.

In an extensive critical review paper, Chaboche (1992) has pointed out that no existing continuum damage model could account accurately for the damage activation–deactivation process (referred to as unilateral effect). Generally, the description of this phenomenon led to either a non-symmetric elastic stiffness tensor or the occurrence of discontinuities in the stress–strain response. To address this critical issue, several new damage formulations have been proposed in the literature. In this paper, we examine two of these new formulations (Chaboche, 1993; Halm and Dragon, 1996) and show that, although they offer a better overall description of damage, they still exhibit some internal inconsistencies.

Usual intrinsic notation is employed throughout. In particular, the tensor products of two second-order tensors **a** and **b** are defined by:

$$[\mathbf{a} \otimes \mathbf{b}] : \mathbf{x} = (\mathbf{b} : \mathbf{x})\mathbf{a}, \quad [\mathbf{a} \underline{\otimes} \mathbf{b}] : \mathbf{x} = \mathbf{a} \cdot \mathbf{x} \cdot \mathbf{b}^{\mathrm{T}}, \quad [\mathbf{a} \overline{\otimes} \mathbf{b}] : \mathbf{x} = \mathbf{a} \cdot \mathbf{x}^{\mathrm{T}} \cdot \mathbf{b}^{\mathrm{T}}, \quad \mathbf{a} \overline{\underline{\otimes}} \mathbf{b} = [\mathbf{a} \overline{\otimes} \mathbf{b} + \mathbf{a} \; \_\_$$

* Corresponding author.

E-mail address: fabrice.cormery@eudil.fr for any second-order tensor $\mathbf{x}$. Moreover, $\mathbf{n}^{\otimes p} = \mathbf{n} \otimes \mathbf{n} \otimes \cdots \otimes \mathbf{n}$ describes the pth tensor product power of any vector $\mathbf{n}$, $\mathbf{I}$ denotes the second-order identity tensor, H represents the Heaviside function and the set of unit vectors $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ forms an orthonormal basis.

2. Presentation of the models

The models are formulated within the framework of irreversible thermodynamics with internal variables, in which the single dissipative mechanism considered is nucleation and growth of microcracks. In the undamaged state, the material is assumed to be isotropic and linear elastic, the corresponding elastic stiffness tensor is denoted by $\mathbf{C}_0$ and the Lamé coefficients are λ_0 and μ_0. Since the discussion presented below is restricted to the investigation of the elastic response, we just present the thermodynamic potential postulated in these models. Let denote by $\boldsymbol{\varepsilon}$ the strain tensor and by $\mathbf{D}$ the damage internal variable(s).

2.1. Formulation of Chaboche (1993)

This formulation constitutes a general framework that can be applied to any macroscopic damage model. In the case of a strain formulation, the thermodynamic potential w takes the form:

$$w(\boldsymbol{\varepsilon}, \mathbf{D}) = -$$

where d_i and $\mathbf{v}_i$ are the eigenvalues and eigenvectors of $\mathbf{D}$. According to the previous spectral decomposition (4), any damage configuration is thus equivalent to three mutually orthogonal sets of parallel microcracks.

The thermodynamic potential proposed in (Halm and Dragon, 1996) has the following expression:

$$w(\boldsymbol{\varepsilon}, \mathbf{D}) = g\,\mathrm{tr}(\boldsymbol{\varepsilon}\cdot\mathbf{D}) + \frac{\lambda_0}{2}\mathrm{tr}^2\boldsymbol{\varepsilon} + \mu_0\,\mathrm{tr}(\boldsymbol{\varepsilon}\cdot\boldsymbol{\varepsilon}) + \alpha\,\mathrm{tr}\,\boldsymbol{\varepsilon}\,\mathrm{tr}(\boldsymbol{\varepsilon}\cdot\mathbf{D}) + 2\beta\,\mathrm{tr}(\boldsymbol{\varepsilon}\cdot\boldsymbol{\varepsilon}\cdot\mathbf{D}) - (\alpha+2\beta)\boldsymbol{\varepsilon} : \left[\sum_{i=1}^{3} H(-\mathbf{v}_i\cdot\boldsymbol{\varepsilon}\cdot\mathbf{v}_i) d_i \mathbf{v}_i^{\otimes 4}\right] : \boldsymbol{\varepsilon} \tag{5}$$

where the constant g characterizes residual effects due to damage, whereas α and β are two coefficients related to the degradation of elastic properties.

In the formulation (5), the fourth-order tensorial operator $d_i\mathbf{v}_i^{\otimes 4}$ ensures the cancellation of the contribution of the equivalent set of microcracks with normal $\mathbf{v}_i$ to the degradation of the normal stiffness in this direction when $\mathbf{v}_i\cdot\boldsymbol{\varepsilon}\cdot\mathbf{v}_i \leqslant 0$. The model postulated in (Halm and Dragon, 1996) is thus based on the spectral decomposition of $\mathbf{D}$ to account for damage unilateral effect.

3. Critical analysis

As indicated in the introduction, the above formulations show some inconsistencies. Let point them out through two simple examples.

Let us examine first the formulation (1) proposed in (Chaboche, 1993) when the set of unit vectors $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ corresponds to a principal basis of the second-order damage variable $\mathbf{D}$. Consider a state $(\boldsymbol{\varepsilon}, \mathbf{D})$ for which strain is uniaxial $\boldsymbol{\varepsilon} = \varepsilon_0\mathbf{e}_1^{\otimes 2}$ with $\varepsilon_0 < 0$ and tensor $\mathbf{D}$ is isotropic, i.e. $\mathbf{D} = d_0\mathbf{I}$. Since the material is assumed to be isotropic in the case of active damage, the stiffness tensor $\widetilde{\mathbf{C}}$ is isotropic and has the form:

$$\widetilde{\mathbf{C}} = (\lambda_0 + a)\mathbf{I}^{\otimes 2} + 2(\mu_0 + b)\mathbf{I}\,\underline{\overline{\otimes}}\,\mathbf{I} \tag{6}$$

where a and b are scalar functions of $\mathbf{D}$. Besides, as damage is described by a spherical tensor, tensor $\mathbf{D}$ has an infinite number of principal bases and the set of vectors $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ can be identified with any of these bases. In particular, if we choose the basis $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ for the set $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$, then Eqs. (1) and (6) yield:

$$w(\boldsymbol{\varepsilon}, \mathbf{D}) = \tfrac{1}{2}[\lambda_0 + 2\mu_0 - \eta(a + 2b)]\varepsilon_0^2 \tag{7}$$

Let us check the uniqueness of the representation (7). If we identify $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ with an other principal basis of $\mathbf{D}$, say $(\mathbf{t}_1, \mathbf{t}_2, \mathbf{t}_3)$ such that:

$$\mathbf{t}_1 = \tfrac{1}{\sqrt{2}}(\mathbf{e}_1 + \mathbf{e}_2), \qquad \mathbf{t}_2 = \tfrac{1}{\sqrt{2}}(-\mathbf{e}_1 + \mathbf{e}_2), \qquad \mathbf{t}_3 = \mathbf{e}_3 \tag{8}$$

then we obtain:

$$w(\boldsymbol{\varepsilon}, \mathbf{D}) = \frac{1}{2}\left[\lambda_0 + 2\mu_0 - \frac{\eta}{2}(a + 2b)\right]\varepsilon_0^2 \tag{9}$$

Comparison between (7) and (9) clearly shows that $w(\boldsymbol{\varepsilon}, \mathbf{D})$ is not unique. Thus, a state $(\boldsymbol{\varepsilon}, \mathbf{D})$ can be associated with several different values of the free energy (an infinite number in the present case); this shows that w is not a thermodynamic potential.

The formulation proposed in (Halm and Dragon, 1996) leads to the same mathematical anomaly. Indeed, Eq. (5) can be written in the form (residual effects due to damage being neglected, thus $g = 0$):

$$w(\boldsymbol{\varepsilon}, \mathbf{D}) = \frac{1}{2}\boldsymbol{\varepsilon} : \widetilde{\mathbf{C}} : \boldsymbol{\varepsilon} + -$$

with

$$\widetilde{\mathbf{C}} = \mathbf{C}_0 + \alpha(\mathbf{I} \otimes \mathbf{D} + \mathbf{D} \otimes \mathbf{I}) + 2\beta(\mathbf{I} \overline{\underline{\otimes}} \mathbf{D} + \mathbf{D} \overline{\underline{\otimes}} \mathbf{I}) \tag{11}$$

which shows that the formulation postulated in (Halm and Dragon, 1996) enters the general framework proposed in (Chaboche, 1993) that we have just investigated above. From this remark, we can conclude that the introduction of the damage unilateral condition in the basic model proposed in (Dragon et al., 1994) makes w lose its status of thermodynamic potential. Note that we arrive to the same conclusion when residual effects are taken into account.

When the set of unit vectors $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ corresponds to a principal basis of the strain tensor $\boldsymbol{\varepsilon}$, the formulation (1) postulated in (Chaboche, 1993) associates each state $(\boldsymbol{\varepsilon}, \mathbf{D})$ with a single value of the free energy $w(\boldsymbol{\varepsilon}, \mathbf{D})$. It can be shown however that this choice of unit vectors $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ does not ensure the uniqueness of the representation of the elastic stiffness tensor $\mathbf{C} = \mathbf{C}(\boldsymbol{\varepsilon}, \mathbf{D})$. Consider a state $(\boldsymbol{\varepsilon}, \mathbf{D})$ characterized by uniform strain $\boldsymbol{\varepsilon} = \varepsilon_0 \mathbf{I}$ with $\varepsilon_0 < 0$ and damage variable(s) $\mathbf{D}$ such that tensor $\widetilde{\mathbf{C}}$ is isotropic (thus of the form (6)). In this case, the set of unit vectors $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ can be identified with any of the principal bases of the strain tensor (as tensor $\boldsymbol{\varepsilon}$ is spherical, it has an infinite number of principal bases). Let us determine the expression for tensor $\mathbf{C}$ when the set $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ corresponds to either the basis $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ or the basis $(\mathbf{t}_1, \mathbf{t}_2, \mathbf{t}_3)$ defined in (8). In view of (2) and (6), we obtain:

$$\mathbf{C} = \widetilde{\mathbf{C}} - \eta(a + 2b)(\mathbf{e}_1^{\otimes 4} + \mathbf{e}_2^{\otimes 4} + \mathbf{e}_3^{\otimes 4}) \quad \text{when } \mathbf{v}_i = \mathbf{e}_i \quad (i = 1, 2, 3) \tag{12}$$

and

$$\mathbf{C} = \widetilde{\mathbf{C}} - \eta(a + 2b)(\mathbf{t}_1^{\otimes 4} + \mathbf{t}_2^{\otimes 4} + \mathbf{t}_3^{\otimes 4}) \quad \text{when } \mathbf{v}_i = \mathbf{t}_i \quad (i = 1, 2, 3) \tag{13}$$

At the same time,

$$\mathbf{t}_1^{\otimes 4} + \mathbf{t}_2^{\otimes 4} + \mathbf{t}_3^{\otimes 4} = \tfrac{1}{2}[\mathbf{e}_1^{\otimes 4} + \mathbf{e}_2^{\otimes 4} + \mathbf{e}_1^{\otimes 2} \otimes \mathbf{e}_2^{\otimes 2} + \mathbf{e}_2^{\otimes 2} \otimes \mathbf{e}_1^{\otimes 2}] + \mathbf{e}_1^{\otimes 2} \overline{\underline{\otimes}} \mathbf{e}_2^{\otimes 2} + \mathbf{e}_2^{\otimes 2} \overline{\underline{}}$$

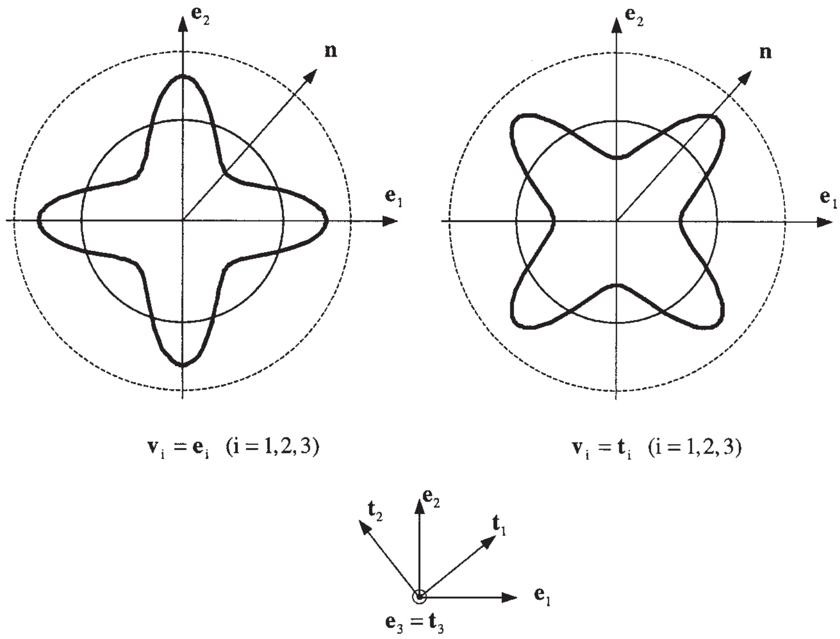

Fig. 1. Youngs modulus related to direction **n** normalized by its initial value E_0 for a Vosges sandstone ($\lambda_0 = 3250$ MPa, $\mu_0 = 4875$ MPa, $\alpha = 9925$ MPa, $\beta = -11\,180$ MPa, $\mathbf{D} = 0.1\mathbf{I}$, $\boldsymbol{\varepsilon} = \varepsilon_0\mathbf{I}$): (—) $E_{(\mathbf{n})}/E_0$ when $\varepsilon_0 > 0$, (

4. Conclusion

The macroscopic modeling of the damage unilateral effect remains a difficult and open research field. Indeed, besides the evidence of the inconsistencies of the formulations proposed by Chaboche (1993) and Halm and Dragon (1996), this study illustrates more generally the shortcomings induced by the use of spectral decompositions to represent the damage activation–deactivation process. Note that such a conclusion has also been pointed out by Carol and William (1996). The proper description of the unilateral effect in constitutive formulations would then require to abandon such decompositions and to consider the problem through a new approach. This research is currently under work.

References

Chaboche, J.L., 1992. Int. J. Damage Mech. 1, 148.
Chaboche, J.L., 1993. Int. J. Damage Mech. 2, 311.
Carol, I., Willam, K., 1996. Int. J. Solids Struct. 33 (20–22), 2939.
Dragon, A., Cormery, F., Desoyer, T., Halm, D., 1994. In: R. Chambon et al. (Ed.), Localization and Bifurcation Theory for Soils and Rocks. Rotterdam, p. 127.
Halm, D., Dragon, A., 1996. Int. J. Damage Mech. 5, 384.